MULTI MODES COUPLING GALLOPING OF SLENDER STRUCTURES

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(Received: September 09, 2016; Revised: November 03, 2016; Accepted: December 06, 2016)

ABSTRACT

The paper presents a general theoretical framework for galloping analysis of slender structures, taking into account the coupling among modes, the modes shapes, and the variation along the structure of mass per unit length and mean wind velocity. The theory is then applied to a real structure. In the galloping analysis, aerodynamic coefficients are obtained from wind tunnel tests. The results of the analysis show crucial points different from conventional analysis.

Keywords: Aeroelasticity; Aerodynamics; wind-induced instability; wind tunnel tests.

1. Introduction

Aeroelasticity is the study of the interaction among aerodynamic forces and structural motion. Phenomenon which is characterized by rapid increase of the structural response due to the aeroelastic forces is referred to as aeroelastic instability phenomenon.

Aeroelastic stability in the quasi-steady range has been analysed since over 90 years ago but not all aeroelastic phenomena are entirely understood. The very first study the aeroelastic instability in quasi-steady range, referred to galloping, was introduced by Glauert (1919) and Den Hartog (1932) with a criterion for the critical condition for one degree of freedom (1DOF) transversal galloping. According to that, the galloping occurs in the crosswind direction when the aerodynamic damping is negative. This criterion is referred to as Glauert-Den Hartog's criterion, which has become a crucial method in engineering application. By including the effect of Reynolds number and relative angle between wind and structure, a generalised across-wind galloping condition was developed and applied to cables (Macdonald and Larose, 2006).

It is important to note that, in those approaches, only one mode is employed, and the correlations among modes are neglected. This, however, may be not correct when natural frequencies are close to each other leading to the correlations among modes. In addition, most of analyses have been carried out for simple structures or single elements extracted from the whole structural system with a hypothesis that the aerodynamic coefficients, mean wind velocity, mass per unit length, size of cross sections are constant along the structure. This assumption is actually not accurate since those parameters, in reality, usually vary along the structure. In other words, the traditional analyses can cause to significant error.

This paper aims to present a theoretical framework of galloping analysis. The coupling among modes, the general direction of wind with respect to structures, the variation of mean wind, aerodynamic coefficients, and mass per unit length along the structure are taken into account. Besides, essentially particular cases in which the analytical solutions can be derived are discussed.

The theory is applied to a real frame, in which aerodynamic coefficients are obtained from wind tunnel tests. Critical results are finally discussed.

2. Galloping analysis

Let us assume that the structure vibrates only in translational directions while the torsion is ignored. The structural motion at position z of the structure is related to two translational directions x and y (Fig. 1). The wind direction makes an angle γ with axis x.

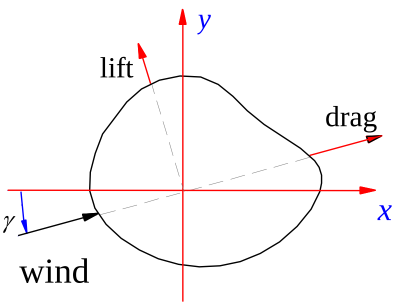

Figure 1. A bluff body section subjected wind action, indicating angle of attack between wind and principle axes x

The structural displacement $\mathbf{q}(z,t)=\left[x(z,t)\quad y(z,t)\right]^{\mathrm{T}}$, being T the transposition operator, can be expressed through the following principal transformation:

$$\mathbf{q}(z,t)=\boldsymbol{\Psi}(z)\mathbf{P}(t)=\sum_{i=1}^{N}\boldsymbol{\Psi}_i(z)P_i(t) \quad (1)$$

where $x(z,t)$ and $y(z,t)$ are the displacements along the x and y axes, t is the time, N is the number of modes, $\boldsymbol{\Psi}(z)=\left[\boldsymbol{\Psi}_1(z)\quad \boldsymbol{\Psi}_2(z)\ \ldots\ \boldsymbol{\Psi}_N(z)\right]$ is the modal matrix, $\mathbf{P}(t)=\left\{P_1(t)\quad P_2(t)\ \ldots\ P_N(t)\right\}^{\mathrm{T}}$ is the vector of the principal coordinates, $\boldsymbol{\Psi}_i(z)$ = $[\psi_{ix}(z)\quad \psi_{iy}(z)]^{\mathrm{T}}$ and $P_i(t)$ are the i-th mode and principal coordinate, respectively.

For the aeroelastic instability analysis, the equation of motion can be written as follows (Nguyen 2014):

$$\tilde{\mathbf{M}}\ddot{\mathbf{P}}(t)+\left(\tilde{\mathbf{C}}+\tilde{\mathbf{C}}_a\right)\dot{\mathbf{P}}(t)+\tilde{\mathbf{K}}\mathbf{P}(t)=\mathbf{0} \quad (2)$$

where $\tilde{\mathbf{M}}$, $\tilde{\mathbf{C}}_a$ and $\tilde{\mathbf{K}}$ are the principal mass, aerodynamic damping and stiffness matrices.

$$\tilde{\mathbf{C}}_a=\frac{1}{2}\rho\int_0^H \bar{u}(z)b(z)\boldsymbol{\Psi}^{\mathrm{T}}(z)\mathbf{R}^{\mathrm{T}}(z)\mathbf{c}_a(z)\mathbf{R}\boldsymbol{\Psi}(z)dz \quad (3)$$

$$\mathbf{c}_a(z)=\begin{bmatrix} 2c_d(z) & c_d'(z)-c_l(z) \\ 2c_l(z) & c_d(z)+c_l'(z)\end{bmatrix} \quad (4)$$

$$\mathbf{R}=\begin{bmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma\end{bmatrix} \quad (5)$$

$c_d(z)$ and $c_l(z)$ are the mean drag and lift coefficients, respectively, depending on the angle of attack γ, $c_d'(z)$ and $c_l'(z)$ are the prime derivatives of the mean drag and lift coefficients, respectively.

Rearranging Eq. (2) in state space form results in the following:

$$\dot{\mathbf{p}}(t)=\tilde{\mathbf{A}}\mathbf{p}(t) \quad (6)$$

where:

$$\mathbf{p}(t)=\begin{Bmatrix}\mathbf{P}(t)\\ \dot{\mathbf{P}}(t)\end{Bmatrix};$$

$$\tilde{\mathbf{A}}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\tilde{\mathbf{M}}^{-1}\tilde{\mathbf{K}} & -\tilde{\mathbf{M}}^{-1}\left(\tilde{\mathbf{C}}+\tilde{\mathbf{C}}_a\right)\end{bmatrix} \quad (7)$$

in which $\mathbf{p}(t)$ is the state vector and $\tilde{\mathbf{A}}$ is the dynamic matrix. The dynamical system represented by Eq.(15) is asymptotically stable if (and only if) all the real parts of the eigenvalues of the matrix $\tilde{\mathbf{A}}$ are negative (Meirovitch, 1986).

The eigenvalue problem given in Eq. (7) is normally solved numerically. A particular case which is the most common case dealing with in the literature refers to the crosswind galloping of a 1DOF system. This problem has been usually formulated by defining the system Oxy such that the axis x is aligned with the mean wind direction and by focusing on the sole oscillation corresponding to the crosswind modes. Consequently, the necessary condition for galloping occurrence in the *k*-th crosswind mode results (Nguyen et al. 2015):

$$(c_d + c_l')_{k,eq} < 0 \quad (8)$$

where $(c_d+c'_l)_{k,eq}$ is the equivalent galloping coefficient in the *k*-th crosswind mode:

$$(c_d + c_l')_{k,eq} = \int_0^H \left[c_d(z) + c_l'(z)\right]\Pi_k(z)\,dz \quad (9)$$

in which:

$$\Pi_k(z) = \frac{\bar{\mu}(z)b(z)\psi_k^2(z)}{\bar{\mu}(z_e)b(z_e)\int_0^H \psi_i^2(z)\,dz} \quad (10)$$

The critical velocity corresponding to the *k*-th crosswind mode at the reference height z_e is provided by:

$$\bar{u}_{cr,k}(z_e) = -\frac{4\omega_k \xi_k \tilde{m}_{k,eq}}{\rho b(z_e)(c_d + c_l')_{k,eq}} \quad (11)$$

3. Application

The proposed aeroelastic instability theory given in the previous section is applied to analyse the stability of a real frame, called “megaframe”, which is belong to a building located in Varesine - Milan (Fig. 2). The frame is composed by very slender columns and transversal bars, whose cross sections are square sections. Therefore, it is complex and dynamically sensitive to wind actions, leading to difficulty in the aeroelastic analysis.

Figure 2. The view of the "megaframe"

3.1. Modal analysis

The full frame model is shown in Fig. 3. It is composed by three frame groups, denoted as POD1, POD2 and POD3. Due to the complexity of the full frame with 3 dimensions, one plane frame of a frame group POD1, which is tallest, is studied at the preliminary stage. The structural properties of the vertical and horizontal bars are maintained as original structure. Such a system is modeled by using software SAP2000 as shown in Fig.4. For the convenience of illustrating the stability analysis presented in section 2, in this figure, four columns of the frame are marked with the square border, while seven elements of the column No.1 are marked with the ellipse one. The marked element is defined as a beam between two consecutive horizontal bars. From that it is apparent that the structure is very slender. Concerning to the system axes, it is defined that the plane YZ contains the plane of the frame, where the global axes Y and Z are horizontal and vertical, respectively. The global axis X is perpendicular to the frame plane. The local system of each column is xyz, in which axes x, y, and z are correspondingly parallel to the global X, Y, and Z.

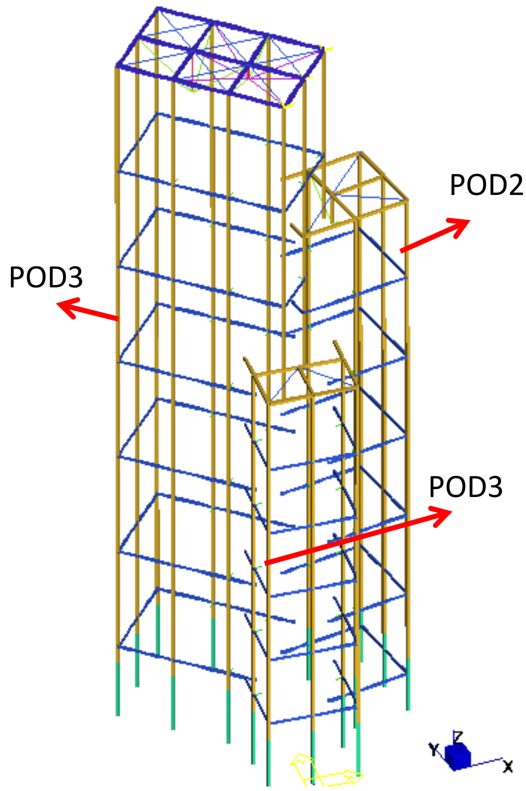

Figure 3. Model of the "megaframe"

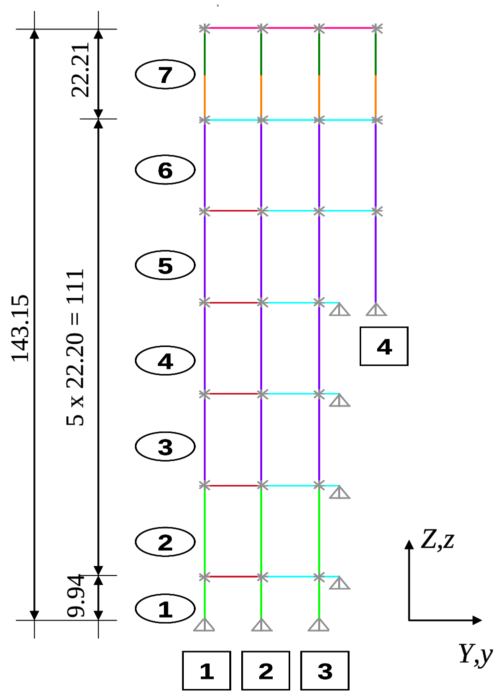

Figure 4. Finite element model of the plane frame (the length unit is meter)

Table 1 reports the first ten natural frequencies of the plane frame. It can be realized that they are close to each other. For more details, first four modes almost have the same frequencies. The frequency of the fifth mode is separated from the previous one and almost coincident with the one of the sixth and seventh modes. It is similar with the eighth, ninth and tenth modes.

Table 1

Natural frequencies for the first 10 modes

Mode	Natural frequency (Hz)
1	0.6655
2	0.6657
3	0.6676
4	0.6683
5	0.7859
6	0.7910
7	0.7936
8	0.9462
9	0.9529
10	0.9586

3.2. Aerodynamic parameters

In order to obtain the static aerodynamic coefficients, static wind tunnel tests have been carried out at the Wind Tunnel of Genoa University (DICAT 2010). Fig.5. shows the installation of the sectional test and definition of drag and lift directions. The test is conducted for 16 angles of attack, varying from 0° to 15° with angle increment 1°. The aerodynamic coefficients are shown in Fig. 6.

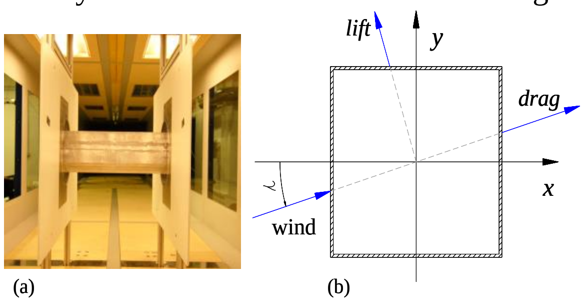

Figure 5. Wind tunnel test on the section model (a) and definition of aerodynamic force directions (b)

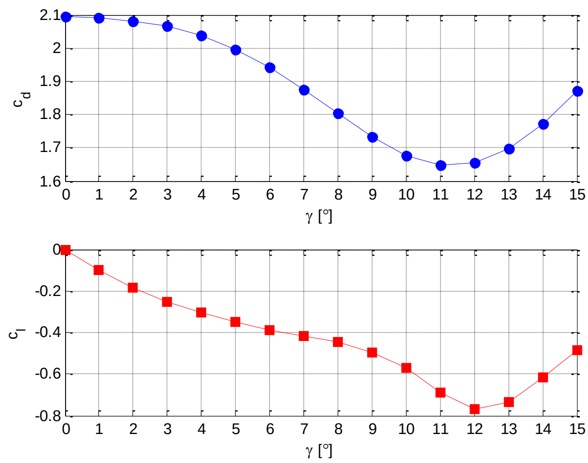

Figure 6. Drag (C_d) and lift (C_l) coefficients

Fig. 7 shows the critical velocity at the reference height $Z_r = 10\,m$ for the coupling of *k* modes (*k*=1,2,...5) related to the top element of the frame (element 7 in Fig.4), namely $\bar{u}_{cr,ck,el}(Z_r)$. This value is evaluated by numerically solving the eigenvalue problem given in Eq. (7), in which $\bar{u}_{cr,ck,el}(Z_r)$ is the lowest mean wind velocity $\bar{u}(Z)$ such that there exists a positive value in real part of the solutions.

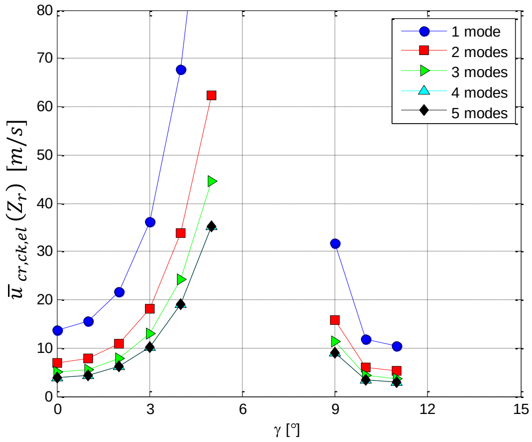

Figure 7. Critical velocity for coupling modes at reference height Z_r

It can be seen that the critical velocity at each attack angle where the instability can potentially occur is decreasing when more modes in coupling are considered and becomes constant when more than four modes are used. Especially, at the attack angle 11°, when the coupling among first four modes is taken into account, the critical velocity decreases more than three times compared with the case of considering only the first mode of vibration. This may due to the reason that first four modes are very close to each other, and their mode shapes have similar form. This result stresses the significance of considering the coupling among modes. Using only one mode in the stability analysis as in tradition may change completely the situation in the unsafe side.

4. Conclusion

The paper generalised the aeroelastic stability analysis of slender structures, taking into account the coupling among modes, the modes shapes, and the variation along the structure of mass per unit length and mean wind velocity. The well-known Den Hartog criterion is obtained as a particular case.

The theory is applied to analyse the aeroelastic stability of a real frame. The aerodynamic coefficients obtained from wind tunnel tests result in the negative galloping coefficients, giving rise to potential instability to the structure.

In addition, the analysis highlights the importance of considering the coupling among modes, which is usually underestimated in common analyses. Using only one mode in the stability analysis as in tradition may change completely the situation in the unsafe side for structures■

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