

# ADAPTIVE FILTER AND THRESHOLD FOR IMAGE DENOISING IN NEW GENERATION WAVELET

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## ABSTRACT

In reality, the nature images have the noise values because of many reasons. These values make the quality of images to decrease. Wavelet transform is proposed for denoising and it gives the better results. But with curvelet transform, one of the new generations of wavelet, the quality of images continues to be improved. In this paper, my proposed method is to combine filter and threshold to calculate the denoising coefficients in curvelet domain. The result of proposed method is compared with other previous methods and shows an improvement.

**Keywords:** image denoising; median filter; bayesian thresholding; curvelet transform.

## 1. Introduction

Nowadays, images are one of the popular tools for the saving information. However, their quality is reduced because of many reasons. These reasons include: environment, capture devices, technician's skills, transmission process, etc. The improved quality of images is a matter of interest in recent times. Denoising process not only helps to remove noise out of the corrupted images but also to maintain the edge features.

In recent years, the authors proposed transforms for denoising, and the first proposed object is wavelet transform (G. Strang, 1989). In wavelet transform, the input image is mutated into space instead of frequency as in other previous methods. The wavelet has the dyadic subbands to be  $[2^s, 2^{s+1}]$ . In each subband, the filter or threshold is applied to calculate the coefficients for reconstructing. The discrete wavelet transform (DWT) (Tim Edwards, 1992; Marcin Kociolk. et al., 2001) applies the directional in each subband and gives a positive result, but it has three disadvantages such as (N.T.Binh, Ashish Khare, 2013): lack of information, shift-sensitivity, and poor directionality. The new generation of wavelet is proposed and has overcome these disadvantages. The new generation of wavelet

transform is contourlet transform (Minh N. Do & Martin Vetterli, 2005) with single and not multi-directional in filter bank. With non-subsampled contourlet transform (Arthur L. da Cunha, J. Zhou and Minh N. Do, 2006), the images have multi-directional in filter bank. Contourlet or non-subsampled contourlet transform uses two processes: decomposition and reconstruction. Because the corrupted images must adapt to many filters and thresholds in each direction, the image denoising also lacks a lot of information and does not show well in the representation of edges. The ridgelet transform (J. Candes, 1998) is proposed to solve this problem. This is the first generation of curvelet transform (D.L. Donoho and M. R. Duncan, 2000; Starck J L, Candès E J, Donoho D L, 2002). Curvelet is outstanding representative of the presented curves. Besides, the filter or threshold is proposed to remove noise, such as (N.T.Binh, Ashish Khare, 2013): bayesian thresholding, cycle spinning, steerable wavelet, etc.

The combination between filters and thresholds in (Arthur L. da Cunha, J. Zhou and Minh N. Do, 2006; N.T.Binh, V.T.H.Tuyet and P.C.Vinh, 2015) or the combination between thresholds and transforms in (Abramovich, T.

Sapatinas and B. W. Silverman, 1998; Wei Zhang, Fei Yu and Hong-mi Guo, 2009; N.T.Binh, Khare A., 2010) will give spectacular results. It proves that the combination can give better results, but we must look at the keeping information, multi-directional and surmount the shift-sensitivity.

This paper proposes a method for image denoising. My algorithm is to adapt bayesian threshold to median filter in curvelet domain. For demonstrating the superiority of the proposed method, I compare the result of the proposed method with the other recent methods available in literature, such as: curvelet transform (Starck J L, Candès E J, Donoho D L, 2002), curvelet combining with cycle spinning (N.T.Binh, Khare A., 2010) by two values of Peak Signal to Noise Ratio (PSNR) and Mean Square Error (MSE). The results showed that the present method is better than the other methods. The outline of this paper is as follows - the basic of image denoising, curvelet transform and its application are shown in section 2; the proposed method is depicted clearly in section 3; the experimental and results are presented in section 4; section 5 is the conclusion.

## 2. Background

In this section, the basis of denoising process and curvelet transform are presented.

### 2.1. Image denoising

Noisy images are the images that have been added to an agent of the signal (the noise values) in the original image. Common types of natural interference current are Gaussian noise and speckle noise. The form of each type of noise will be presented in equation (1) and (2). Gaussian noise is the kind of interference and noise values are distributed evenly over the signals of pixels. The model of Gaussian noise is added to form interference patterns (additive), and given by the following equation:

$$w(x, y) = s(x, y) + n(x, y) \quad (1)$$

Speckle noise is a multiplicative form. It appears in most of image systems and speckle's form is:

$$w(x, y) = s(x, y) \times n(x, y) \quad (2)$$

In (1) and (2),  $(x, y)$ : coordinates of the image,  $s(x, y)$ : original image,  $n(x, y)$ : the noise values and  $w(x, y)$ : noisy images. Image denoising is a process which removes  $n(x, y)$  out of  $w(x, y)$ . This value only has two cases: show or not show depending on the values of coefficients. The process for denoising in the above methods is similar to (G. Strang, 1989) which includes 4 steps:

- Decomposition.
- Calculating the detail coefficients.
- Removing the impact of the existence of images.
- Reconstruct (the inverse transform).

In G. Strang's algorithm (1989), the author used two concepts which are the hard threshold ( $T_{\text{hard}}$ ) and the soft threshold ( $T_{\text{soft}}$ ). These concepts are calculated by equation (3) and (4):

$$T_{\text{hard}}(\hat{d}_{jk}, \lambda) = \hat{d}_{jk} I\left(\left|\hat{d}_{jk}\right| > \lambda\right) \quad (3)$$

and

$$T_{\text{soft}}(\hat{d}_{jk}, \lambda) = \text{sign}(\hat{d}_{jk}) \max\left(0, \left|\hat{d}_{jk}\right| - \lambda\right) \quad (4)$$

where  $\lambda \geq 0$  is parameter wavelet,  $I$  is normal parameter value.

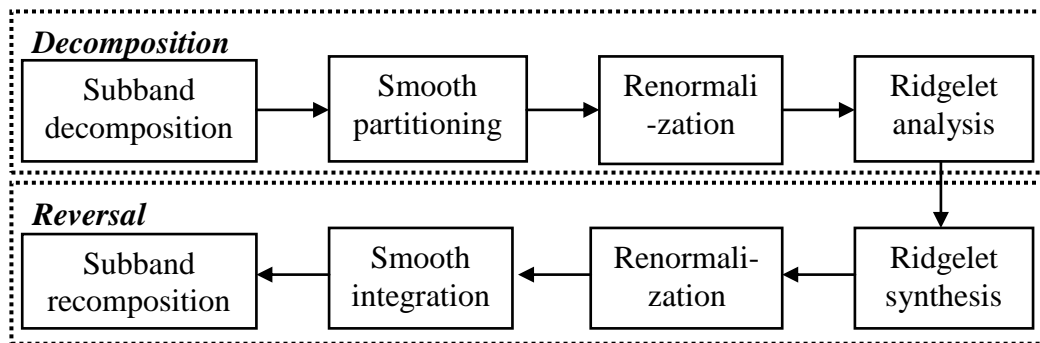
However, the calculation of the thresholds based on the sigma-hat values which include the estimate noise variance  $\sigma$  and signal variance  $\sigma_s$  is extended in new generation wavelet. In order to, the transform must have the values to recreate from space to normal domain and these values depend on threshold or filter to give. So, the combination between them is the primary key for denoising process.

### 2.2. Curvelet transform

Wavelet transform is used popularly in denoising. But curvelet transform is useful for representing edges by smoothing curves. Like wavelet, curvelet transform can be translated and dilated. But in the first decomposing, the curves of each subband are displayed with width  $\approx \text{length}^2$ . Then, a local ridgelet transform will be applied in each scale. Although ridgelets have global length and

variable widths, curvelets in addition to a variable width have a variable length and so does a variable anisotropy (Zhang, J. M. Fadili, and J. L. Starck, 2008). The basic process of the digital realization for curvelet

transform is described clearly in (D.L. Donoho and M. R. Duncan, 2000). In this paper, this process is abridged by the author as in figure 1:



**Figure 1.** The curvelet transform process

In period 1, the decomposition includes four steps. *Firstly*, the filter decomposes noise images into subbands. The subbands are:

$$f \mapsto (P_0 f, \Delta_1 f, \Delta_2 f, \dots) \quad (5)$$

where  $P_0$  is the lowpass filter, and  $\Delta_1, \Delta_2, \dots$  is the high-pass filters.

After subband decomposition, input images are put into space domain of curvelet transform. *Secondly*, each subband will be applied smoothly windowed into “squares” of an appropriate scale with sidelength  $\sim 2^{-s}$  by equation:

$$\Delta_s f \mapsto (w_Q \Delta_s f)_{Q \in Q_s} \quad (6)$$

where  $w_Q$  is a collection of smooth window localized around dyadic squares:

$$Q = [k_1/2^s, (k_1+1)/2^s] \times [k_2/2^s, (k_2+1)/2^s] \quad (7)$$

*Thirdly*, each square from the applying smoothly windowed of the previous step will be renormalized to unit scale by:

$$g_Q = (T_Q)^{-1} (w_Q \Delta_s f), Q \in Q_s \quad (8)$$

*Finally* in period 1 is the ridgelet analysis. In this step, each square is analyzed via the discrete ridgelet transform by merging two dyadic subbands  $[2^{2s}, 2^{2s+1}]$  and  $[2^{2s+1}, 2^{2s+2}]$ .

In period 2, the curvelet transform which reversed from domain to frequency domain image is inverted with the period 1. The period 1 is concluded with ridgelet analysis.

So, period 2 is begun with *ridgelet synthesis*. The equation (8) is reconstructed by (9) to have orthogonal ridgelet system:

$$g_Q = \sum_{\lambda} \alpha_{(Q,\lambda)} \times p_{\lambda} \quad (9)$$

The *renormalization* is calculated by the formula:

$$h_Q = T_Q g_Q, Q \in Q_s \quad (10)$$

This is the input smoothly windowed for *smooth integration* step. In the next step, smooth integration, each square is obtained from the previous step to restructure the algorithm:

$$\Delta_s f = \sum_{Q \in Q_s} w_Q \times h_Q \quad (11)$$

The final step of period 2 is the *subband recomposition*. Pairing the squares together using the equation:

$$f = P_0(P_0 f) + \sum_s \Delta_s(\Delta_s f) \quad (12)$$

We know that curvelet transform also includes two processes similar to non-subsampled contourlet transform. But curvelet has smoothing step and uses ridgelet in each subband. This is the local transform and improves the representing edges.

### 3. Image denoising by combining filter and threshold in curvelet domain

Because the positive results are curvelet transform, I propose a algorithm for denoising

image in curvelet domain by combining median filter and bayesian thresholding. In proposed method, I also apply decomposition similar to period 1 of figure 1. Then, I use median filter to remove noise in each square and calculate coefficient to depend on

bayesian thresholding. Based on this coefficient, I continue my method with period 2 of curvelet transform. Of course, this entire process will take place in curvelet domain. The proposed method can be summarized as in figure 2:

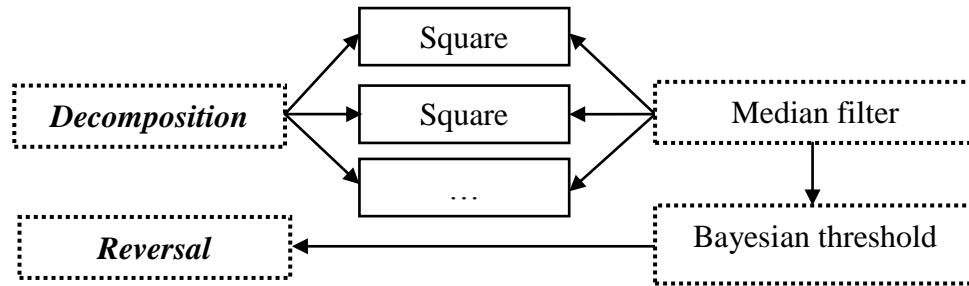


Figure 2. The process of proposed method

The decomposition of proposed method began with the kind of space domain. Here, the proposed method chooses the db2 for the decomposition step and is divided into 5 subbands:  $P_0(f)$ ,  $\Delta_1(f)$ ,  $\Delta_2(f)$ ,  $\Delta_3(f)$ ,  $\Delta_4(f)$ .  $P_0(f)$  is low-pass and other subbands are high-pass. The stage of curvelet transform is as follows (N.T.Binh, Khare A., 2010):

- apply the à trous algorithm to scales and set  $b_1 = b_{min}$
- for  $j=1, \dots, j$  do
  - partition the subband  $w_j$  with a block size  $b_j$  and apply the digital ridgelet transform to each block;
  - if  $j \text{ modulo } 2 = 1$  then  $b_{j+1} = 2b_j$ ;
  - else  $b_{j+1} = b_j$

The sidelength of the localizing windows is doubled at every other dyadic subband. In each high-pass, my algorithm applies median filter. Median filter is a nonlinear method which preserves edges.

The reason of using median filter is that it works by moving through the image pixel by pixel, replacing each value with the median value of neighbouring pixels. The median value is calculated by first sorting all the pixel values from the pattern of neighbours into numerical order, and replacing the pixel under consideration with the middle pixel value. The results of this processing overcome the

sharpening of pixels.

Then, the Bayesian threshold continues with calculating the estimate noise variance  $\sigma$  and signal variance  $\sigma_s$  by equations (13) and (14):

$$\sigma = \left( \frac{\text{median}(|w_{i,j}|)}{0.6745} \right)^2 \tag{13}$$

$$\sigma_s = \sqrt{\max(\sigma_w^2 - \sigma^2, 0)} \tag{14}$$

$$\text{with } \sigma_w^2 = \frac{1}{n^2} \sum_{i,j=1}^n w^2(i,j) \tag{15}$$

where  $w_{i,j}$  is the lowest frequency coefficient after performing transformations. Continued this process, the threshold is reconstructed by equation:

$$\text{Threshold}_{\text{Bayes}} = \begin{cases} \frac{\sigma^2}{\sigma_s}, \sigma^2 < \sigma_s^2 \\ \max\{|A_m|\}, \sigma^2 \geq \sigma_s^2 \end{cases} \tag{16}$$

When reconstructing the image based on the bayesian thresholded coefficients, if the value of pixel detail coefficients is less than the

thresholding, then the result is 0. Else the result is array Y, where each element of Y is 1 if the corresponding element of pixel is greater than zero, 0 if the corresponding element of pixel equals to zero, -1 if the corresponding element of pixel is less than zero.

After all calculated threshold steps, this algorithm continues with the curvelet transform reverse and gives the result image.

#### 4. Experimental results

In this section, we present about our denoising experiments and compare the results with other methods. For performance evaluation, the author compares the results of the proposed method based on the curvelet transform to combine median filter and Bayesian thresholding (CT-MF-BT) with the methods: curvelets transform (CT) (Starck J L, Candès E J, Donoho D L, 2002), and curvelet transform with cycle spinning (CT-CC) (N.T.Binh, Khare A., 2010). The quality of image is increasing by comparison with the value of Mean Square Error (MSE) and Peak Signal-to-Noise Ratio (PSNR).

$$MSE = \sqrt{\frac{1}{NxN} \sum_{i=1}^N \sum_{j=1}^N (x_{i,j} - y_{i,j})^2} \quad (17)$$

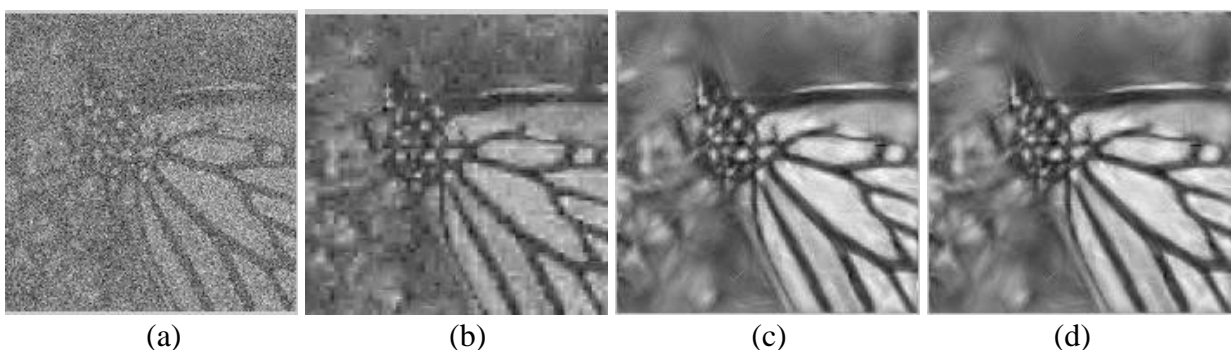
where x is image noisy, y is image denoising and NxN is size of image. PSNR is used as a measurement of quality of reconstruction of image denoising, defined as:

$$PSNR = 20 \log_{10} \left( \frac{MAX_1}{MSE} \right) \quad (18)$$

where,  $MAX_1$  is the maximum pixel value of the image. The smaller the value of MSE is the better. In the contrary, the higher value of PSNR is the better.

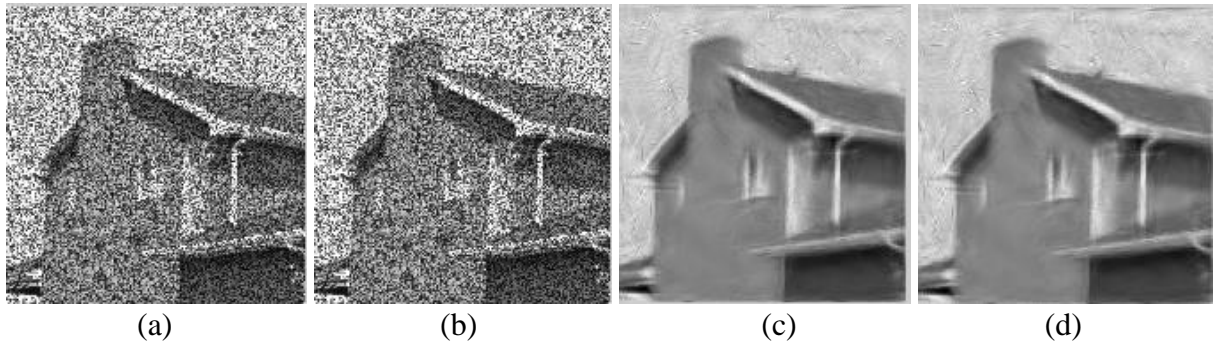
The experiments were tested on different noise levels of additive and multiplicative noise. Various types of noise, such as Gaussian, Speckle, Salt & Pepper, were added to these images. I test in a standard image dataset for image processing. It is a set of images which frequently found in literature such as: Lena, peppers, cameraman, lake, etc... This dataset is free and available at [http://www.imageprocessingplace.com/root\\_files\\_V3/image\\_databases.htm](http://www.imageprocessingplace.com/root_files_V3/image_databases.htm).

Figure 3 shows the denoising result by Gaussian, figure 4 shows the denoising result by speckle, figure 5 shows the denoising result by salt & sepper. In each figure, the comparison of results between the proposed method and other methods is presented.



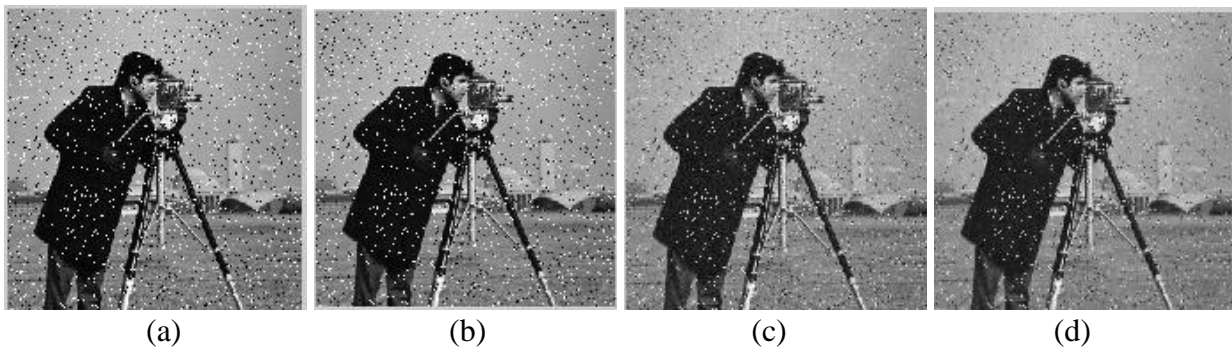
**Figure 3.** Noisy image with Gaussian noise and denoised images by different methods

- (a). Noisy image (PSNR = 9.0460 db).
- (b). Denoised image by CT (Starck J L, Candès E J, Donoho D L, 2002) (PSNR = 20.3996 db).
- (c). Denoised image by CT-CC (N.T.Binh, Khare A., 2010) (PSNR = 21.3686 db).
- (d). Denoised image by CT-MF-BT (PSNR = 21.8951 db).



**Figure 4.** Noisy image with Speckle noise and denoised images by different methods

- (a). Noisy image (PSNR = 12.7963db).
- (b). Denoised image by CT (Starck J L, Candès E J, Donoho D L, 2002) (PSNR = 23.3027 db).
- (c). Denoised image by CT-CC (N.T.Binh, Khare A., 2010) (PSNR = 23.9953 db).
- (d). Denoised image by CT-MF-BT (PSNR = 24.5136 db).

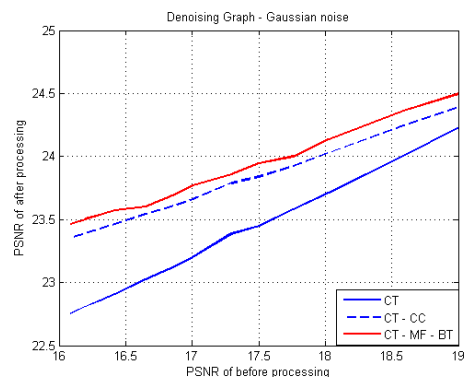


**Figure 5.** Noisy image with Salt & pepper noise and denoised images by different methods.

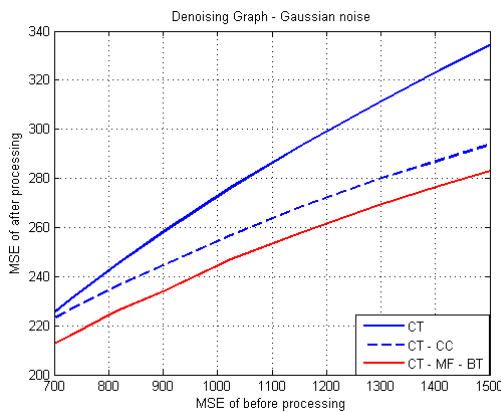
- (a). Noisy image (PSNR = 17.3262 db).
- (b). Denoised image by CT (Starck J L, Candès E J, Donoho D L, 2002) (PSNR = 17.7439 db).
- (c). Denoised image by CT-CC (N.T.Binh, Khare A., 2010) (PSNR = 19.6608 db).
- (d). Denoised image by CT-MF-BT (PSNR = 20.7492 db).

In figure 3, figure 4 and figure 5, (a) is the PSNR value of noisy image; and (d) is the PSNR value of the proposed method. This value, the result of my algorithm, is higher than the PSNR value of CT method to be (b) and CT-CC method to be (c).

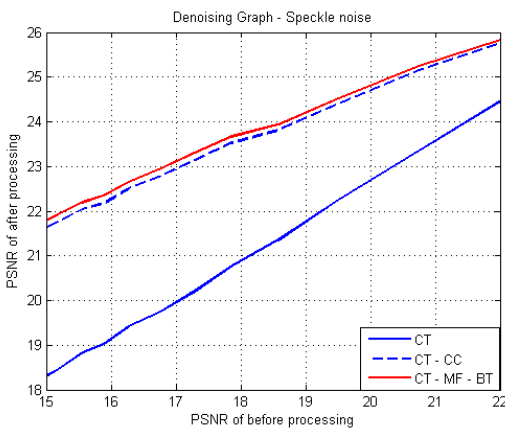
Figure 6 and 7 show the plot of PSNR and MSE values of different denoising methods with Gaussian noise. In these figures, we can see that the proposed method performs better than the other methods.



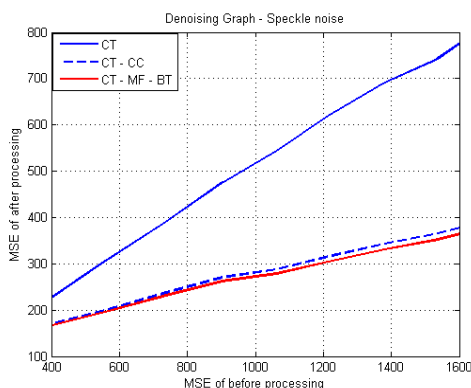
**Figure 6.** Plot of PSNR values of denoised images with Gaussian noise using different methods



**Figure 7.** Plot of MSE values of denoised images with Gaussian noise using different methods



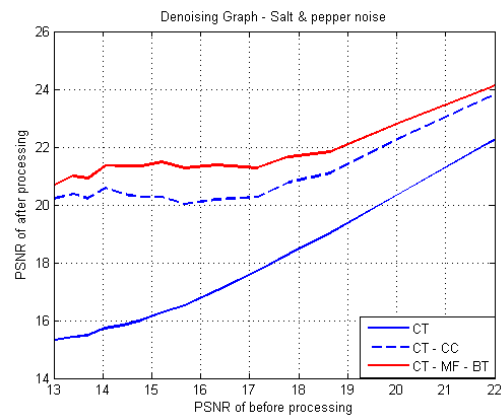
**Figure 8.** Plot of PSNR values of denoised images with speckle noise using different methods



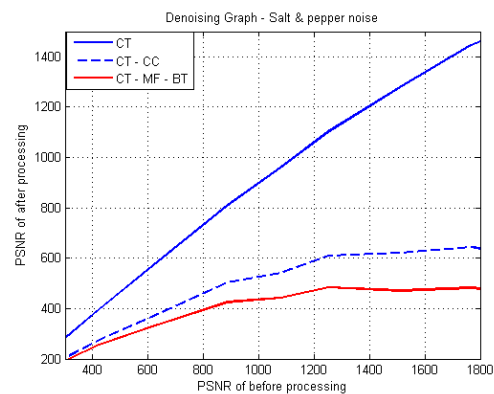
**Figure 9.** Plot of MSE values of denoised images with speckle noise using different methods

Figure 8 and 9 show the plot of PSNR and MSE values of different image denoising methods corrupted with speckle noise. In

these figures, the proposed method also performs better than the other methods. Figure 10 and 11 show the plot of PSNR and MSE values of different image denoising methods corrupted with salt & pepper noise. In these figures, the proposed method also performs better than the other methods.



**Figure 10.** Plot of PSNR values of denoised images with salt & pepper noise using different methods



**Figure 11.** Plot of MSE values of denoised images with salt & pepper noise using different methods

The above results show that the proposed method performs better than curvelet transform (Starck J L, Candès E J, Donoho D L, 2002) and curvelet transform combined with cycle spinning (N.T.Binh, Khare A., 2010).

### 5. Conclusion

In this paper, the proposed method bases on the combination between median filter and Bayesian thresholding in curvelet domain. The proposed technique allows denoising for

various types of noise such as Gaussian, speckle and salt & pepper in medical images. In medical image field, the thresholding must be chosen carefully to keep the information of images. From the results of the above section, I conclude that my algorithm works well and

better than other recent methods available in literature. With this idea, I think that a combination of filter or thresholding can upgrade the quality of image noising. Yet the execution time is still issues of further concern■

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