APPLICATION OF MONTE CARLO SIMULATION METHOD TO ESTIMATE THE RELIABILITY OF DESIGN PROBLEM OF THE BORED PILE ACCORDING TO LIMITED STATE

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ABSTRACT

This paper applies Monte Carlo simulation method to estimate the reliability of bored pile for designing problem at "Tax Office of Phu Nhuan District". Surveyed random variables are physico-mechanical properties of soil and loads are assumed that they follow the normal distribution. Limit state functions developed from design requirements of Ultimate limit state (ULS) and Serviceability limit state (SLS). Results show that the probability of failure is 0 and the reliability index of ULS is 9.493 and of SLS is 37.076 when examining coefficient of variation of soil and loads of 10%. The paper also considers the safety level when evaluating different coefficients of variation in the range of 10 ~30%. The authors suggest applying the reliability method to design calculation for other construction to help the engineers have a visual perspective, increase safety and avoid wastage.

Keywords: reliability; Monte Carlo; probability of failure; bored pile; limit state; settlement.

1. Introduction

In Vietnam, engineers often believe that the input parameters such as physico-mechanical properties obtained from soils investigation and loads applied to the structural system are constant values. This viewpoint has not reflected the reality because the objective factors from the environment (rain, wind, changes of groundwater table, etc.), construction conditions, and experimental processes can affect to the survey results. Therefore, the input parameters vary randomly and are usually assumed that they follow the normal distribution. Although they are rejected by factors of safety (FS), the selection of them still depends on designer's experiences. Thus, it leads to lack of safety design or wastage.

Meanwhile, application of probability theories, reliability is becoming popular all over the world in designing and evaluating safety level of the structural calculation, since it overcomes the disadvantages of FS. Reliability research results are also updated and added in the standards and designing software in many developed countries such as the European Union, Canada, USA, etc. Some typical examples are EN 1990: Eurocode - Basis of structural design, FHWA-NHI-10-016, Geostudio software, etc.

According to Gomesa and Awruch (2004), methods of estimating the reliability are being applied widely in many researches. They are Monte Carlo simulation (MCS), First Order Reliability Method (FORM), Second Order Reliability Method (SORM), etc. Among that, MCS is one of the most outstanding methods to analyze the reliability with specific mathematical analysis. Some studies of using MCS to estimate the reliability of bored pile design are being applied in the world. Wang et al. (2011) applied MCS method using MatLab software to analyze the reliability of bored pile for designing problems. Two geometrical parameters of pile which are diameter (B) and length (D) are considered to be random variable values follow normal distribution rule. The number of samples needed to ensure the expected accuracy is n_{min}= 10000000. The probability of failure (P_f) is based on the conditions of Ultimate limit state and Serviceability limit state load applied on pile head (F) is greater than ultimate bearing capacity (Q_{uls}) or the serviceability bearing capacity (Q_{sls}) corresponds to the vertical displacement y_a = 25mm at pile tip when B and D change. Fan and Liang (2012) also applied MCS; however, surveyed random variables are geological input parameters. Limit state function requests that pile tip displacement must not be greater than 25mm. The number of samples needed is n_{min} = 10000 to 20000. Studies of Nguyen et al. (2014) and Nguyen (2015) are typical studies in Vietnam that also confirm the importance of reliability. These results suggested that effects of the random input parameters should be considered in term of the effect of design and economy.

Based on the published studies, authors suggest applying MCS method to estimate the reliability (β) of bored pile for designing problems at "Tax Office of Phu Nhuan District" according to ULS and SLS. Surveyed random variables are physico-mechanical properties of soil (c',φ',e) and applied loads at column base (N,M,Q) which are assumed to follow the normal distribution have the same coefficient of variation (COV) of 10% by suggested design. In addition, this paper also evaluates the effects of different coefficients of variation of soil and loads in the range of 10~30% to reliability and failure probability results.

2. Application of reliability theories to problem and Monte Carlo simulation method

2.1. Application of reliability theories to problem

According to Nowak and Collins (2010), limit state function or safety margin can be determined as follow:

$$g = R - Q \quad (1)$$

where R is resistance capacity of structure, Q is load effect or behavior of structure under loading.

The limit state, corresponding to the boundary between desired and undesired performance, would be when g=0. If g ≥ 0, the structure is safe if g < 0, the structure is not safe.

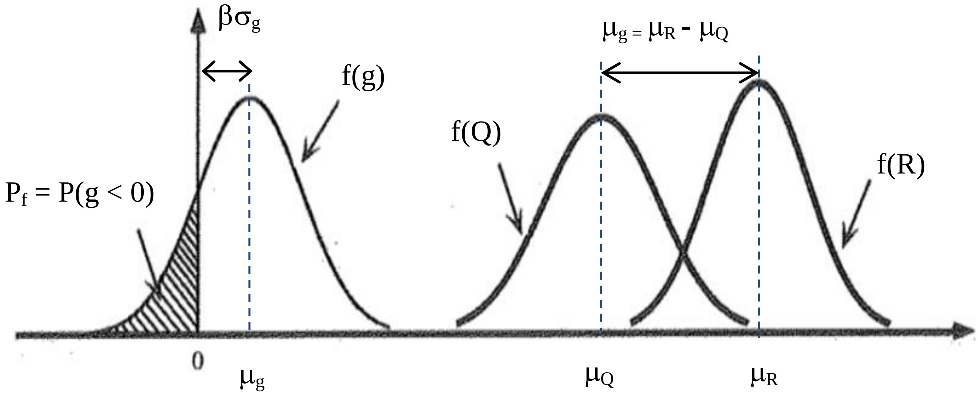

Figure 1. PDFs of load, resistance, and safety margin

Probability of failure :

$$P_f = P(R - Q < 0) = P(g < 0) \quad (2)$$

Reliability index of a random distribution rule which is determined by the formula: $\beta = \frac{\mu_g}{\sigma_g}$ (3) indicated how many standard deviations (σ_g) from which the average value of safety margin (μ_g) is far away the border of safety/failure. The larger the value of β, the higher the safety level, the lower the P_f and vice versa (Phan, 2001).

Limit state function is established by the request of ULS. The ULS requests that the force applied on the pile head must not exceed the ultimate bearing capacity of pile:

$$g_{(ULS)} = Q_u - P_{max} \quad (4)$$

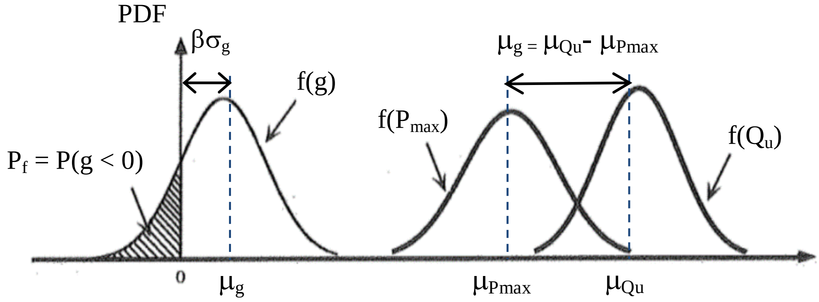

Figure 2. Safety margin and reliability index based on request of ULS

Ultimate bearing capacity of pile (Q_u) is conventionally taken as consisting of frictional resistance (Q_s) and point bearing capacity (Q_p):

$$Q_u = Q_s + Q_p = A_s f_s + A_p q_p = u\sum f_{si} l_i + A_p q_p \quad (5)$$

Unit friction resistance (f_{si}) is determined by:

$$f_{si} = \sigma_h' \tan\varphi'_{ai} + c'_{ai} \quad (6)$$

The ultimate load-bearing capacity (q_p) using semi-empirical formula of Terzaghi & Peck and Vesic's method is calculated as follows:

$$q_{p,Terzaghi\&Peck} = 1.3c'N_c + \sigma'_{vp}N_q + 0.3\gamma D_c N_\gamma \quad (7)$$

$$q_{p,Vesic} = c'N_c^* + \overline{\sigma'_o}N_\sigma^* \quad (8)$$

The bearing capacity factors N_c, N_q, N_γ and N_c^*, N_σ^* are found from document of Das (2004).

The maximum load applied at the pile head is determined by the formula:

$$P_{max} = \frac{N_t}{n_p} + \frac{M_y \times x_{max}}{\sum (x_i)^2} + \frac{M_x \times y_{max}}{\sum (y_i)^2} \quad (9)$$

where n_p: Number of piles in pile cap.

N_t, M_x, M_y: Total of vertical force and moment applied on the bottom of pile cap.

x_i, y_i: Distance from the center of pile number i to the axis passing through the center of pile cap plan.

x_{max}, y_{max}: Distance from the farthest center of pile to the axis passing through the center of pile cap plan.

Limit state function is established from the request of SLS. Limit state requests that the settlement of group piles (S) must not exceed the allowable settlement value (S_a):

$$g_{(SLS)} = S_a - S \quad (10)$$

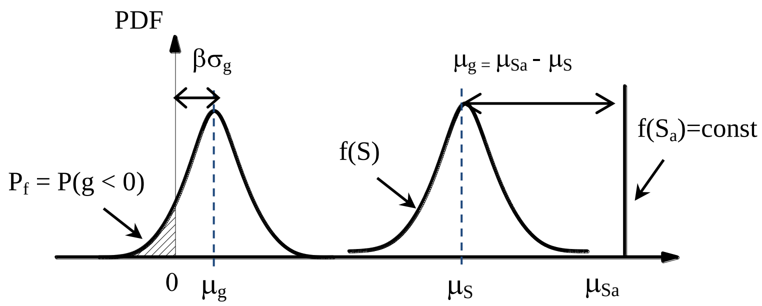

Figure 3. Safety margin and reliability index based on request of SLS

Settlement of group piles (S) is the sum of divided layers settlement under pile tip as follows:

$$S = \sum_{1}^{n} S_i = \sum_{1}^{n} \frac{e_{1i} - e_{2i}}{1 + e_{1i}} \times h_i \quad (11)$$

Where e_{1i} ,e_{2i} are void ratios of soil which are between layer i^{th} before and after applying load, respectively. These void ratios are determined from compression curve of the consolidation test; h_i is thickness of layer i^{th} under group piles.

2.2. Monte Carlo simulation method

According to Nguyen et al. (2014), MCS is one of the most typical methods to estimate reliability. It can be briefly described as follow: assuming we have N randomly evaluated samples of limit state based on random variables. Then the failure probability of structure using MCS method will be determined as follows:

$$P_f = P(g < 0) = \frac{n}{N} \quad (12)$$

where N: Total samples of evaluating limit state based on assumed random variables.

n: Number of evaluated samples in N samples which has limit state $g < 0$.

The number of N samples needed can be calculated based on below formula (Nowak & Collins, 2010):

$$N = \frac{1 - P_T}{(COV_P)^2 \times P_T} \quad (13)$$

with Targeted probability of failure, P_T = 1/1000 and maximum coefficient of variation of result, COV_P=10%, the number of samples needed 99900 or more.

The coefficient of variation of random variables X (COV_X) is defined as standard deviation (σ_X) divided by the mean (μ_X):

$$COV_X = \frac{\sigma_X}{\mu_X} \quad (14)$$

3. Characteristics of studied site and the sequences of determining reliability problem

3.1. Characteristics of studied site

"Tax Office of Phu Nhuan District" site is located at 145 alleys, Nguyen Van Troi Street, Phu Nhuan district, Ho Chi Minh City. This building includes 8 stories, 3 basements. Total area is 1060 m^2.

3.1.1. Soil profile of the site

There are 4 main soil layers and the groundwater table is located at a depth of 4.5 m below ground surface. The soil parameters of each layer are presented in Table 1 and Table 2.

Table 1

Physico-mechanical properties of soil layers at site

Number	Layer name	Thickness h_i (m)	G_s	γ_n (kN/m^3)	γ_{sat} (kN/m^3)	e_o	W (%)	c' (kN/m^2)	φ' (degree)
1	Soft clay	9.4	2.690	19.45	19.814	0.722	24.4	26.1	12.5
2	Fine sand	32.1	2.668	19.21	19.911	0.683	24.1	3.4	26.5
3	Semi-stiff clay	2.1	2.700	19.60	19.965	0.706	23.8	32	17.5
4	Stiff clay	26.4	2.699	19.57	20.347	0.642	19.4	37.5	16.5

Table 2

Void ratio at different consolidation pressures

Number	Layer name	e_o	$e_{12.5}$	e_{25}	e_{50}	e_{100}	e_{200}	e_{400}	e_{800}
1	Soft clay	0.722	0.699	0.690	0.676	0.655	0.626	0.582	0.541
2	Fine sand	0.683	0.667	0.656	0.639	0.616	0.594	0.573	0.542
3	Semi-stiff clay	0.706	0.688	0.681	0.674	0.662	0.643	0.615	0.582
4	Stiff clay	0.642	0.629	0.624	0.617	0.610	0.601	0.588	0.572

3.1.2. The surveyed pile cap and applied loads

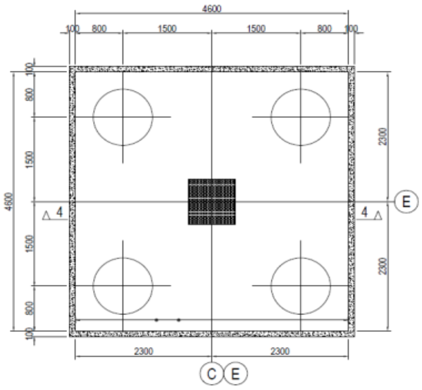

Figure 4. Geometry in plan of pile cap F4

F4 is pile cap of pile group that consists of 4 bored piles (each bored pile is 1m in diameter and 44m in length). This pile cap has size B_c x L_c x H_c = 4.6x4.6x1.6 (m), and depth of foundation D_f = 9 (m).

Table 3

Applied loads at column base

N^{tt} (kN)	M^{tt}_x (kNm)	M^{tt}_y (kNm)	Q^{tt}_x (kN)
9642	177.6	81.5	91.1

3.2. Sequences of determining reliability of problem

Authors use software programming in MatLab. This software has block diagram to determine reliability index based on requests of ULS, SLS and the input parameters that are assumed above.

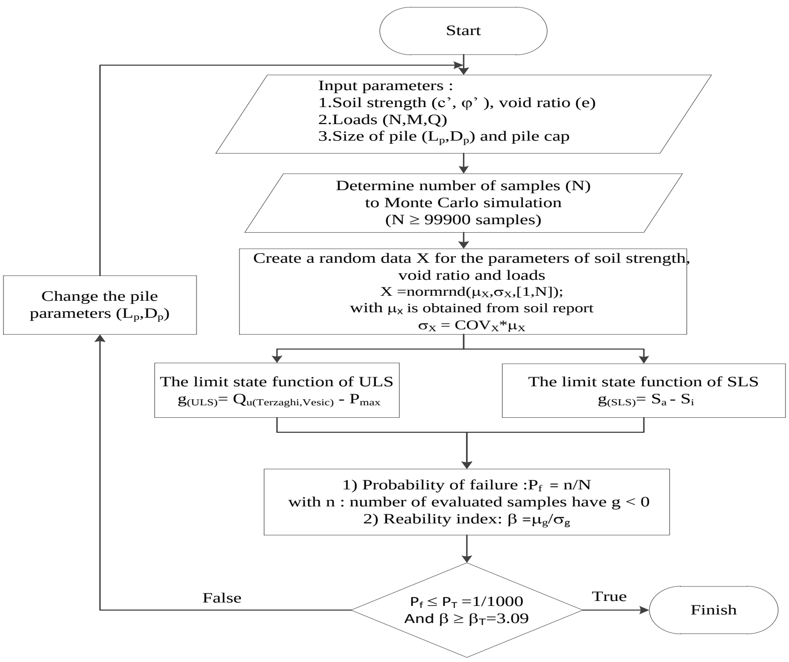

Figure 5. Block diagram to determine the reliability index and failure probability of bored pile approach to limit state

4. Results

4.1. Reliability index and failure probability based on request of ULS with $COV_{soil} = COV_{loads}$ =10%

Figure 6 shows the relationship between the ultimate bearing capacity of pile (Q_u) and depth. Based on the basic of normal distribution verification method (Nowak & Collins, 2010), (Phan, 2001), more than 95% Q_u values are not on the verification line of the normal probability paper as in Figure 7. Since the ultimate bearing capacity of piles in two cases, $Q_{u(Terzaghi\&Peck)}$ and $Q_{u(Vesic)}$, do not follow the normal distribution rule, the probability distribution density of ultimate bearing capacity and limit state function $g_{(ULS)} = Q_u - P_{max}$ as on Figures 8 and 9 do not follow normal distribution like P_{max}. Particularly, $Q_{u(Terzag\&Peck)}$ is a positively skewed distribution while $Q_{u(Vesic)}$ is a negatively skewed distribution. The reliability index and probability of failure by using MCS method are described in Table 4.

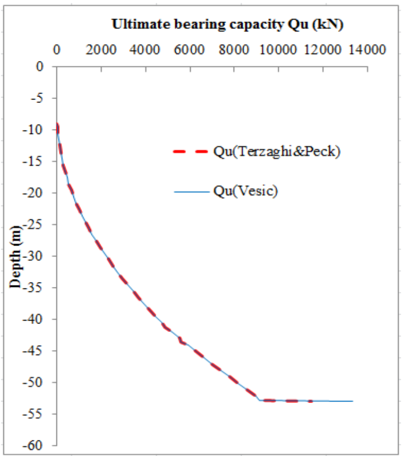

Figure 6. Relationship between ultimate bearing capacity (Q_u) and depth

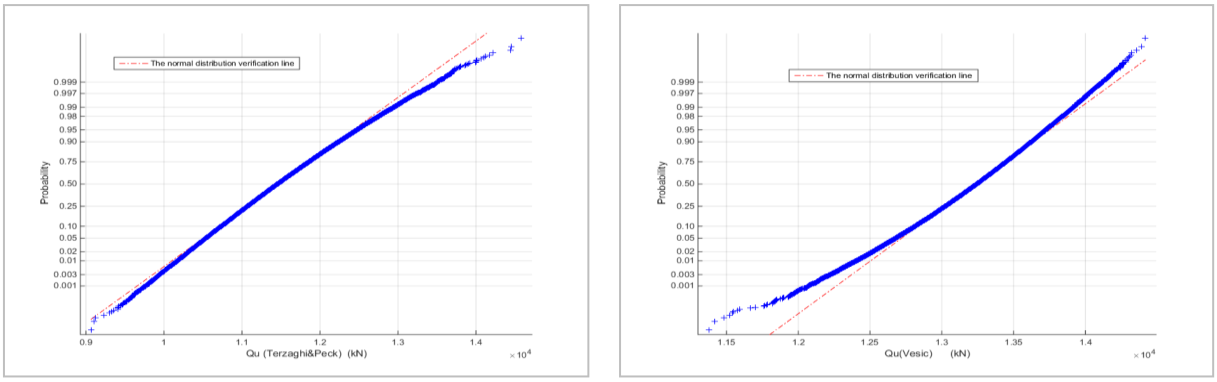

Figure 7. Normal distribution verification of the ultimate bearing capacity $Q_{u(Terzaghi\&Peck)}$ and $Q_{u(Vesic)}$.

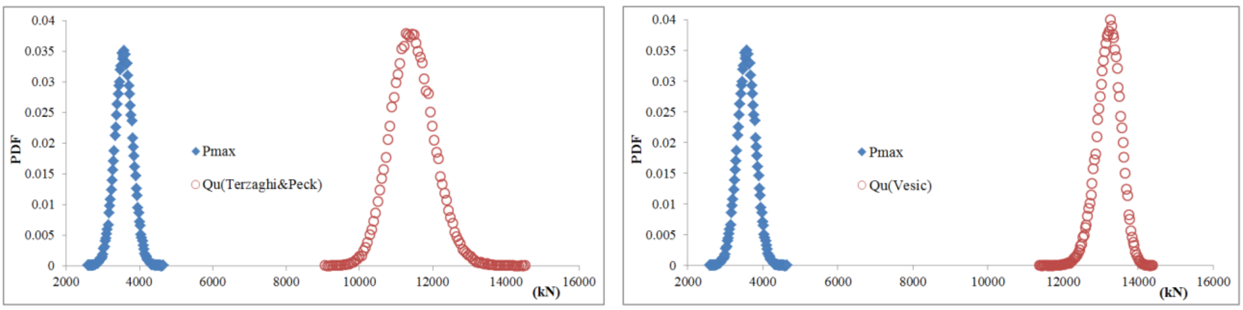

Figure 8. Probability distribution density of P_{max} and Q_u

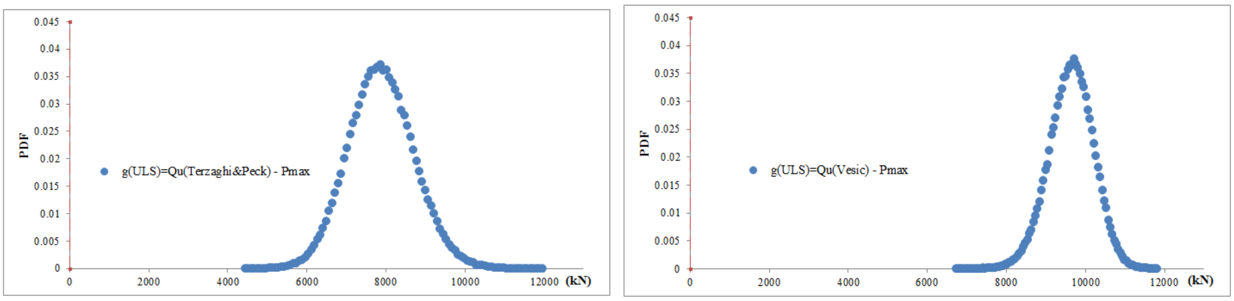

Figure 9. Probability distribution density of limit state function $g_{(USL)}$

Table 4

Reliability index and failure probability results of limit state function $g_{(ULS)}$

Method	μ_g (kN)	σ_g (kN)	β	P_f	$\beta_{(ULS)}$	$P_{f(ULS)}$	Evaluation
Terzaghi & Peck	7895.083	831.644	9.493	0	9.493	0	Safe
Vesic	9648.500	561.122	17.195	0			Safe

4.2. Reliability index and failure probability based on request of SLS with COV_{soil} = COV_{loads} =10%

In figure 10, normal distribution verification results show that the settlement of group piles follows normal distribution rule. Figure 11, the relationship between the increase in effective stress (caused by the construction of the foundation) and consolidation settlement of group piles indicates that the settlement data scatters very little. As in figure 12, both probability distribution density of settlement S and limit state function $g_{(SLS)}$ = S_a –S are normal distribution function. According to Table 5, the reliability index of SLS is $\beta_{(SLS)}$ = 37.076 which is higher than the reliability index of ULS, $\beta_{(ULS)}$ =9.493. The failure probability is determined to have value of 0.

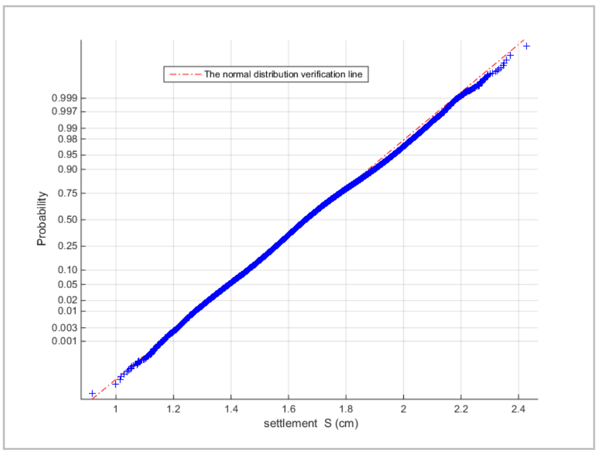

Figure 10. Normal distribution verification of settlement of group piles

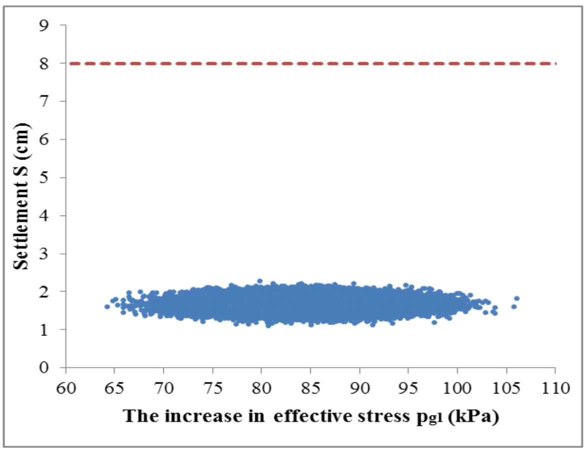

Figure 11. The increase in effective stress (p_{gl}) and consolidation settlement (S) of group piles relationship

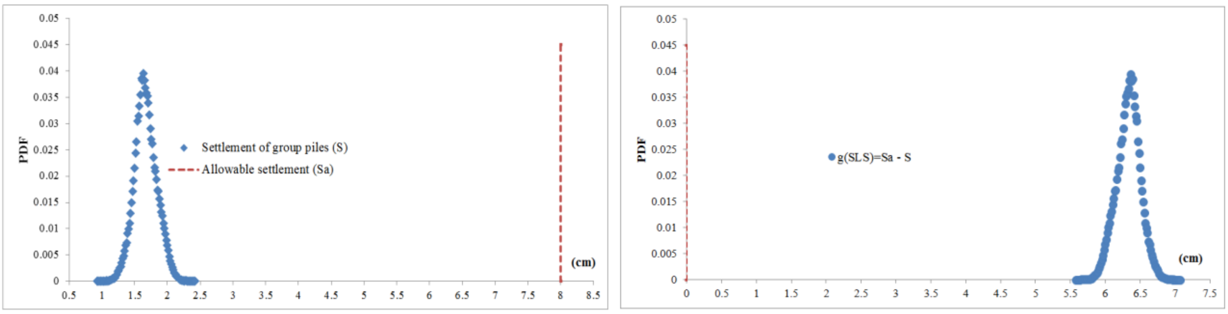

Figure 12. Probability distribution density of settlement S and limit state function $g_{(SLS)}$

Table 5

Reliability and failure probability result of limit state function $g_{(SLS)}$

μ_g (cm)	σ_g (cm)	$\beta_{(SLS)}$	$P_{f\,(SLS)}$	Evaluation
6.332	0.171	37.076	0	Safe

4.3. The effect of soil and loads coefficients of variation to the reliability index and failure probability results

To estimate the effect of soil and loads coefficient of variation to the reliability results as well as probability of failure of bored pile problems, authors conducted the change of coefficient of variation by two ways:

1) Coefficient of variation of soil is constant, coefficient of variation of loads is changed.

2) Coefficient of variation of loads is constant, coefficient of variation of is soil changed.

The calculated results are presented in Table 6.

Table 6

Reliability index with different coefficients of variation

Coefficients of variation COV (%)		ULS $g_{(ULS)} = Q_u - P_{max}$						SLS $g_{(SLS)} = S_a - S$		Evaluation
		Terzaghi & Peck		Vesic		$\beta_{(ULS)}$	$P_{f\,(ULS)}$			
Soil	Loads	β	P_f	β	P_f			$\beta_{(SLS)}$	$P_{f\,(SLS)}$	
10	10	9.493	0	17.195	0	9.493	0	37.076	0	Safe
15	10	6.934	0	12.720	0	6.934	0	30.477	0	Safe
20	10	5.361	0	9.612	0	5.361	0	24.925	0	Safe
25	10	4.317	0	7.218	0.00031	7.218	0.00031	20.822	0	Safe
30	10	3.528	0.00012	**5.391**	**0.00214**	**5.391**	**0.00214**	17.687	0	**Not safe**
10	15	8.286	0	14.138	0	8.286	0	28.506	0	Safe
10	20	7.355	0	12.016	0	7.355	0	22.605	0	Safe
10	25	6.603	0	10.458	0	6.603	0	18.527	0	Safe
10	30	5.991	0	9.213	0	5.991	0	15.409	0	Safe

5. Conclusions

Monte Carlo method is applied to estimate the reliability of bored pile problem according to limit states at "Tax Office of Phu Nhuan District". The main conclusions are summarized as follows.

(1) Calculation results proved that for problems involving complicated analysis equations such as bored pile design, the probability distribution density of ultimate bearing capacity cannot be a normal distribution even though random variables surveyed are physico-mechanical properties (c',φ',e) of soil and loads at column base. When $COV_{soil} = COV_{loads} = 10\%$ as in design, $\beta_{(ULS)} = 9.493$, $\beta_{(SLS)} = 37.076$ and probability of failure $P_f = 0$. Therefore, the design is evaluated to be safe.

(2) Although both ultimate bearing capacity values and the reliability index calculated by Vesic method are greater than that of Terzaghi & Peck method, the probability distribution density of $Q_{u(Vesic)}$ does not follow the normal distribution rule but is a negatively skewed distribution. Therefore, probability of failure is very high in case of large coefficient of variations. In Table 6, when $COV_{soil} = 30\%$ and $COV_{loads} = 10\%$ then $P_f = 0.00214 > P_T = 0.001$. As a result, the estimation of safety level of bored pile cannot be only based on factors of safety and allowable load-carrying capacity. It is important to pay attention to the soil coefficient of variation, loads and types of statistical distribution by estimating the reliability and probability of failure.

In overall, analysis of the reliability by Monte Carlo method is a straightforward method, based on strong scientific foundation, and can be widely applicable in reality. This method helps engineers to gain more knowledge in calculations, select appropriate strategies, increase safety and avoid wastage■

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