

# HIGH PERFORMANCE CONCRETE MIXTURE PROPORTIONING: MULTI-OBJECTIVE OPTIMIZATION APPROACH

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## ABSTRACT

This paper presents the application of multi-objective optimization approach to high performance concrete mixture proportioning. An integrated mathematical model was developed in order to optimize six criteria, which are the chlorine ion diffusion coefficient, per cubic meter cost, the amount of cement, fly ash, slag, chemical admixture. This model needs to satisfy with ten functional constraints and seven design variables. The Visual Interactive Analysis Method (VIAM) was used to solve the multicriteria task. Eventually, twelve solutions have been found for the different cases in terms of criteria during the process of proportioning high performance concrete mixture. They are all Pareto solutions, which allow experts to choose in the proposed cases.

**Keywords:** High performance concrete; mix proportion; multi-objective optimization; Pareto solution; Visual Interactive Analysis Method; VIAM.

## 1. Introduction

The parts of the world in which large-scale concrete construction takes place have extended enormously. Due to the recent trends in construction industries (i.e., increased number of heavily reinforced concrete structures), construction of large and taller structures, and developments of construction techniques (i.e., efficient concrete pumping techniques), the industries and companies in general strive to cast massive volume of concrete. When this large volume of concrete is used for construction, the safety and durability of cast concrete become fundamental issues. To ensure these issues, much effort has been focused on the developments of high-performance concrete (Neville and Aitcin, 1998).

High-performance concrete is designed to give optimized performance characteristics for a given set of materials, usage, and exposure conditions, consistent with strength, workability, service life, and durability. Engineers and constructors all over the world are finding that using high performance

concrete allows them to build more serviceable structures at comparable cost. High-performance concrete is being used for structures in aggressive environments: marine structures, highway bridges and pavements, nuclear structures, tunnels, precast units, etc. (Aitcin, 2000).

Meanwhile, in Vietnam in recent years, high-performance concrete has played an important role in the engineering structures like bridges, roads, high-rise buildings in the big cities (Hanoi, Ho. Ho Chi Minh City, Da Nang). Especially, in the construction of reinforced concrete bridge and tunnel by new technology high-performance concrete was used properly, such as intersections at Chuong Duong Bridge in Hanoi, Hai Van tunnel in Da Nang or Thu Thiem tunnel in Ho Chi Minh (Pham, 2008).

The major difference between conventional concrete and high-performance concrete is essentially the use of chemical and mineral admixtures. The use of chemical admixtures reduces the water content, thereby at the same time reduces the porosity within

the hydrated cement paste. The reduction in the water content to a very low value with high dosage of chemical admixtures is undesirable, and the effectiveness of chemical admixtures such as superplasticizer principally depends on the ambient temperature, cement chemistry, and fineness. Mineral admixtures, also called as cement replacement materials, act as pozzolanic materials as well as fine fillers; thereby, the microstructure of hardened cement matrix becomes denser and stronger. At ambient temperature, their chemical reaction with calcium hydroxide is generally slow. However, the finer and more vitreous the pozzolan is, the faster will be this reaction. If durability is of primary interest, then the slow rate of setting and hardening associated with the incorporation of fly ash or slag in concrete is advantageous. Also, the mineral admixtures are generally industrial by-products and their use can provide a major economic benefit. Therefore, the combined use of superplasticizer and cement replacement materials can lead to economical high-performance concrete with enhanced strength, workability, and durability. It is also reported

that the concrete containing cement replacement materials typically provides lower permeability, reduced heat of hydration, reduced alkali–aggregate reaction, higher strength at later ages, and increased resistance to attack from sulfates. However, the effect of cement replacement materials on the performance of concrete varies markedly with their properties (Hassan et al. 2000). To obtain the special combinations of performance and uniformity requirements, a near-optimum mix proportion of high-performance concrete is very important.

In this paper, high-performance concrete of class 60 MPa is a selected object used for the multi-objective optimization. The constituent materials of this concrete are Portland cement, water, fly ash, fine slag, sand, stone and chemical admixture, as illustrated in Figure 1. The costly materials such as cement, slag, fly ash and admixture, cost of 1m<sup>3</sup> concrete, and diffusion factor, which represents concrete durability are the objective functions. The optimal solution for mix proportion should be a concrete with low costly materials content, low diffusivity and low total cost of 1m<sup>3</sup> concrete.



**Figure 1.** Concrete constituent materials for high-performance concrete

## 2. Problem statement

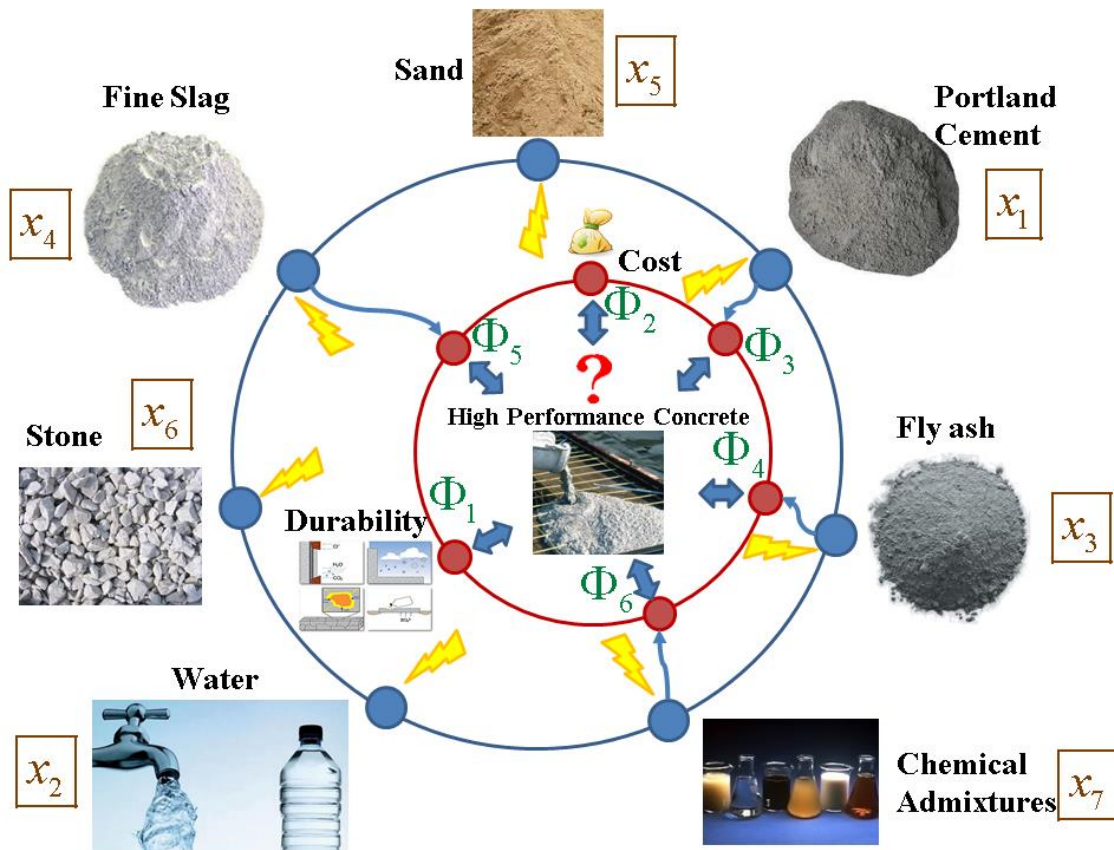
The literature review has revealed that in Xie's work (Xie et al., 2011), a mathematical model for multi-objective optimization of concrete mix has been established. However,

these authors only have considered two criteria such as the chlorine ion diffusion coefficient and cost of 1m<sup>3</sup>. In fact, the amounts of costly components like Portland cement, fly ash, slag and, chemical

admixtures, which are also criteria in objective function, need to be minimized when designing a concrete mix. Therefore, in this paper, an integrated mathematical model was developed for multicriteria design of high performance concrete, which is better adapted to the production process in real conditions in

Vietnam. Therefore, the cost of constituent materials, which is considered in this paper, was taken at the current circumstance at the area of Ho Chi Minh City.

Mathematical model of the problem in this paper are presented in the diagram below (Figure 2).



**Figure 2.** Model for multicriteria design of high performance concrete mix

In this model, three factors are variables, constraints and criteria, which are stated as follows:

**Design variable**

The control variables and their corresponding constraints in the mathematical model are included in Table 1.

**Table 1**

Design variables and their constraint

<i>Design variable</i>	<i>Meaning: Amount of materials</i>	<i>Units</i>	<i>Initial lower admissible value</i>	<i>Initial upper admissible value</i>
$x_1$	Portland cement	kg/m <sup>3</sup>	300	500
$x_2$	Water	kg/m <sup>3</sup>	130	210
$x_3$	Fly ash	kg/m <sup>3</sup>	45	155
$x_4$	Fine slag	kg/m <sup>3</sup>	60	200

<i>Design variable</i>	<i>Meaning: Amount of materials</i>	<i>Units</i>	<i>Initial lower admissible value</i>	<i>Initial upper admissible value</i>
$x_5$	Sand	kg/m <sup>3</sup>	500	1000
$x_6$	Stone	kg/m <sup>3</sup>	900	1400
$x_7$	Chemical Admixtures	kg/m <sup>3</sup>	2.5	12

### Functional constraints

The functional constraints are given by the following equality and inequalities (see Table 2).

**Table 2**

Functional constraints

<i>Function</i>	<i>Expression</i>	<i>Type of constraint</i>	<i>Meaning</i>
$f_1$	$-\frac{x_2}{x_1 + x_3 + x_4} + 0.2$	$\leq 0$	The range of water to binder ratio
$f_2$	$\frac{x_2}{x_1 + x_3 + x_4} - 0.4$	$\leq 0$	
$f_3$	$-\frac{x_5}{x_5 + x_6} + 0.35$	$\leq 0$	The range of sand ratio, which is the ratio of the amount of sand to the amount of overall aggregates
$f_4$	$\frac{x_5}{x_5 + x_6} - 0.4$	$\leq 0$	
$f_5$	$450 - (x_1 + x_3 + x_4)$	$\leq 0$	The range of the amount of cementitious material including cement, fly ash and slag.
$f_6$	$x_1 + x_3 + x_4 - 600$	$\leq 0$	
$f_7$	$-\frac{x_7}{x_1 + x_3 + x_4} + 0.01$	$\leq 0$	The High-Range Water-Reducing Admixture (HRWRA) is used to improve the workability and micro-structure of concrete. These are its ratio to cement
$f_8$	$\frac{x_7}{x_1 + x_3 + x_4} - 0.02$	$\leq 0$	
$f_9$	$\sum_{i=1}^7 \frac{x_i}{\rho_i} - 990$	$= 0$	The volume of concrete mixture is made up of the absolute volume of each content and the volume of the air captured in the mixture. The following expression should be met for the amount of materials for each cubic meter of concrete mixture
$f_{10}$	$-0.304\lambda_c f_{ce,k} \left( \frac{x_1 + x_3 + x_4}{x_2} + 0.62 \right) + f_{cu,k} - t\sigma$	$\leq 0$	The strength of concrete, which is affected by various factors, is the most important parameter in concrete design

where  $\rho_i$  ( $i = 1..7$ ) represents the density of each ingredient ( $\text{ton/m}^3$ ):  $\rho_1 = 3.11$ ;  $\rho_2 = 1$ ;  $\rho_3 = 2.11$ ;  $\rho_4 = 2.45$ ;  $\rho_5 = 2.61$ ;  $\rho_6 = 2.76$ ;  $\rho_7 = 1.08$ .  $\lambda_c$  is the affluence coefficient of the strength class of concrete. It should be determined according to statistics and in general cases it can be 1.13;  $f_{ce,k}$  represents the grading strength of cement and  $f_{ce,k} = 50.5$ ;  $f_{cu,k}$  is the standard value of compressive

strength of concrete and  $f_{cu,k} = 68$ ;  $t$  is the degree of probability and  $t = -1.64$ ;  $\sigma$  is the standard deviation of concrete strength. It is determined according to the national standard code for acceptance of constructional quality of concrete structure and  $\sigma = 5$  (Pham, 2008).

**Performance criteria**

The performance criteria are shown in Table 3:

**Table 3**  
Performance criteria

Criteria	Expression	Meaning
$\Phi_1 \rightarrow$ MIN	$\left\{ \begin{aligned} & 5.760 + 5.81 \cdot \left( \frac{x_2}{x_1 + x_3 + x_4} - 0.45 \right) / 0.2 \\ & - 0.567 \cdot (x_1 + x_3 + x_4 - 425) / 175 + 1.323 \\ & + 0.74 \cdot \left( \frac{x_3}{x_1 + x_3 + x_4} \cdot 100 - 22.5 \right) / 22.5 \\ & - 2.117 \cdot \left( \frac{x_4}{x_1 + x_3 + x_4} \cdot 100 - 35 \right) / 35 \\ & - (2.78 \cdot 0.472) - (0.254 \cdot 0.286) - (0.368 \cdot 1) \\ & + 1.171 \cdot \frac{\frac{x_2}{x_1 + x_3 + x_4} - 0.45}{0.2} \cdot \frac{\frac{x_3}{x_1 + x_3 + x_4} \cdot 100 - 22.5}{22.5} \\ & - 2.891 \cdot \frac{\frac{x_2}{x_1 + x_3 + x_4} - 0.45}{0.2} \cdot 0.472 \\ & - 1.053 \cdot \frac{\frac{x_3}{x_1 + x_3 + x_4} \cdot 100 - 22.5}{22.5} \cdot 0.472 \end{aligned} \right\} \cdot 10^{-6}$	The chlorine ion diffusion coefficient on the 28 <sup>th</sup> day for concrete without microsilica under a molding temperature of 21 Celsius degree. ( $\text{m}^2/\text{s}$ )
$\Phi_2 \rightarrow$ MIN	$\sum_{i=1}^7 (y_i \cdot x_i)$	Per cubic meter cost (VND/ $\text{m}^3$ )
$\Phi_3 \rightarrow$ MIN	$x_1$	Amount of Portland cement per cubic meter ( $\text{kg}/\text{m}^3$ )
$\Phi_4 \rightarrow$ MIN	$x_3$	Amount of Fly ash per cubic meter ( $\text{kg}/\text{m}^3$ )
$\Phi_5 \rightarrow$ MIN	$x_4$	Amount of Fine slag per cubic meter ( $\text{kg}/\text{m}^3$ )
$\Phi_6 \rightarrow$ MIN	$x_7$	Amount of Chemical Admixtures per cubic meter ( $\text{kg}/\text{m}^3$ )

where  $y_i$  ( $i = 1..7$ ) the unit price of each ingredient (VND/kg):  $y_1 = 1500$ ;  $y_2 = 12$ ;  $y_3 = 550$ ;  $y_4 = 5050$ ;  $y_5 = 118$ ;  $y_6 = 135$ ;  $y_7 = 21000$ .

In this mathematical model, we need to optimize 6 standard criteria  $\Phi_i$  ( $i = 1..6$ ), which are necessary to satisfy with 10 functional constraints and 7 design variables  $x_k$  ( $k = 1..7$ ).

### 3. Method of solution and calculation

In recent years, the single-objective and multi-objective optimization methods have been used commonly. However, most of the preceding studies have focused on the development of optimization algorithms for a single-objective function. The problem of a multicriteria task most of the time was converted into a representative single criteria by means of the methods, for instance, Weighted Maximin (Maximin), Compromise Programming, Weighted Sum, Bounded Objective Function, Modified Tchebycheff, Weighted Product, Exponential Weighted Sum, etc.

Xie and colleagues (Xie et al., 2011) have also chosen that option. After proposing an equivalent objective function, those authors used the method of Sequential Quadratic Programming to find out the minimum. It is important to note that there are many methods to find the minimum of an equivalent function, such as algorithms Cooko, Fireflies, Hybrid, Genetic, Swarm, ect. Every algorithm gives the minimum with a small discrepancy. However, the problem is that the solution of the equivalent function does not represent the solution of the individual function. This means that one criteria reaches the optimum by using a certain algorithm, but another criteria does not reach the optimum by using another algorithm.

There are two questions that have not been reviewed in detail in the abovementioned work applied to a single-objective function:

- Will the equivalent criteria be able to actually substitute for the individual analysis of single criteria, when importance grade of every single criteria at certain moment and

production circumstance is different from one expert to another?

- In the course of preparation and real production process, how will the experts be able to analyze directly, and opt for the priority consideration of criteria flexibly, which in turn make an appropriate decision?

The significance of the optimization algorithm is enormous, however in practice when a flexible compromise needs to be made to find out the most feasible production option, the criteria should be analyzed individually and repeatedly in comparative process. Then the “give and take” process should be done in order to achieve an agreement among the criteria. Therefore, it is necessary to have a tool or an approach to solve a multicriteria task with high applicability. In this paper, an application of Visual Interactive Analysis Method (VIAM) is proposed to tackle with the issue of high performance concrete mixture proportioning.

The VIAM was described in details, elsewhere (Gavriushin and Dang, 2016). The main idea of this method includes: i) set up an interactive table, containing the range value of criteria, which satisfies with all constraints; ii) based on the current circumstance and determined production demand, the experts would give the threshold values of the criteria (the threshold is within the range value); iii) the final step is to find the variable vector, which satisfy with the threshold values.

There are many ways to find a valid variable vector. VIAM uses two main approaches; such as filling and spatial parameter survey, and space conversion variables - functional constraints - criteria. In this paper, the authors will take into account the second approach. The process to solve the mathematical task is presented below.

Determination of the range value of criteria and set it up in the interactive table.

Using an available single-objective optimization method, we can find the minimum of the objective function and the interactive table is presented as follows:

**Table 4**

The Interactive Table

$\min\Phi_1 =$ <b>0</b>	$\min\Phi_2 =$ <b><math>1.1 \times 10^6</math></b>	$\min\Phi_3 =$ <b>300</b>	$\min\Phi_4 =$ <b>45</b>	$\min\Phi_5 =$ <b>60</b>	$\min\Phi_6 =$ <b>4.5</b>
...	...	...	...	...	...
<b><math>[\Phi_1]</math></b>	<b><math>[\Phi_2]</math></b>	<b><math>[\Phi_3]</math></b>	<b><math>[\Phi_4]</math></b>	<b><math>[\Phi_5]</math></b>	<b><math>[\Phi_6]</math></b>
...	...	...	...	...	...
$\max\Phi_1 =$ <b><math>5.78 \times 10^{-13}</math></b>	$\max\Phi_2 =$ <b><math>2.04 \times 10^6</math></b>	$\max\Phi_3 =$ <b>495</b>	$\max\Phi_4 =$ <b>155</b>	$\max\Phi_5 =$ <b>200</b>	$\max\Phi_6 =$ <b>12</b>
The chlorine ion diffusion coefficient ( $m^2/s$ )	Per cubic meter cost (VND/ $m^3$ )	Amount of Portland cement ( $kg/m^3$ )	Amount of Fly ash ( $kg/m^3$ )	Amount of Fine slag ( $kg/m^3$ )	Amount of Chemical Admixtures ( $kg/m^3$ )

When using the interactive table in the production process, there are many different cases and the corresponding production methods. In this paper, three production cases are solved by using VIAM.

Case 1: there is a hypothesis that the experts have discussed and indicated the required threshold value of criteria, as included in Table 5:

**Table 5**

Case 1

$\Phi_2 \rightarrow$	$\Phi_3 \rightarrow$	$\Phi_1 \rightarrow$	$\Phi_5 \rightarrow$	$\Phi_4 \rightarrow$	$\Phi_6 \blacksquare$
$1.3 \times 10^6$	400	$4.5 \times 10^{-13}$	100	100	8

• First of all, we have  $\min\Phi_2$ , and it has been set before that  $\Phi_2^{\oplus} = \min\Phi_2 = 1.3 \times 10^6$ . Since this threshold is within the range value of  $\Phi_2$ , there exist definitely satisfied variable vectors. Three of those vectors are represented

in the matrix form in Figure 3. In the first row, there are 7 variables, in the second row there are functional constraints and in the last row they are criteria values.

$$\begin{bmatrix}
 363.416639110167 & 170.217943644146 & 77.0888438524496 & 64.4699760167256 & 649.887293902405 & 1055.44355292330 & 7.88993346107516 & NULL & NULL & NULL \\
 -0.13708 & -0.06292 & -0.03110 & -0.01890 & -54.979 & -95.02 & -0.005624 & -0.004376 & 0.02 & -0.230 \\
 5.1300 \cdot 10^{-13} & \boxed{1.3000 \cdot 10^6} & 363.416639110167 & 77.0888438524496 & 64.4699760167256 & 7.88993346107516 & NULL & NULL & NULL & NULL
 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix}
 374.816976356103 & 174.317185191679 & 154.992402183237 & 61.8924516786018 & 559.002420565760 & 1035.20793240516 & 6.29355574116044 & NULL & NULL & NULL \\
 -0.09461 & -0.10539 & -0.00065 & -0.04935 & -141.70 & -8.30 & -0.000636 & -0.009364 & 0.01 & -11.254 \\
 2.3526 \cdot 10^{-13} & \boxed{1.3000 \cdot 10^6} & 374.816976356103 & 154.992402183237 & 61.8924516786018 & 6.29355574116044 & NULL & NULL & NULL & NULL
 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix}
 398.241887940642 & 176.410623360612 & 78.2600293856075 & 64.7658412622898 & 605.294308698476 & 1058.69614362576 & 5.52671515505546 & NULL & NULL & NULL \\
 -0.12592 & -0.07408 & -0.01376 & -0.03624 & -91.266 & -58.73 & -0.000211 & -0.009789 & -0.01 & -2.849 \\
 4.4340 \cdot 10^{-13} & \boxed{1.3000 \cdot 10^6} & 398.241887940642 & 78.2600293856075 & 64.7658412622898 & 5.52671515505546 & NULL & NULL & NULL & NULL
 \end{bmatrix} \tag{3}$$

**Figure 3.** Obtained solution  $\Phi_2^{\oplus} = \min\Phi_2 = 1.3 \times 10^6$

The solutions (1) – (3) satisfy the criteria 2, 3, 5, and 6. However, only the solution (2) satisfies the criteria 1, but does not for the criteria 4 from the expert’s point of view. Although the solutions (1) and (3) do not satisfy the criteria 1, they excel for the criteria 4. Therefore, only the solution (3) satisfies all of criteria from the expert’s standpoint. Nevertheless, the value of criteria 1 is  $4.43 \times 10^{-13}$ , which is very close to  $4.5 \times 10^{-13}$  or

it is not really optimized. Additionally, it is still unknown what the optimum value of criteria 2 can be reached, when compromising that the criteria 2 is the most important one. Thus, let’s move to the next step.

• Adding to the constraints the condition  $|\Phi_2 - \Phi_2^\oplus| \leq \varepsilon_2 = 10^{-6}$  to find  $\min \Phi_3$ . We obtain the following three results, as shown in Figure 4:

$$\begin{bmatrix} 300. & 146.74 & 103.34 & 79.515 & 632.95 & 1143.4 & 7.6580 & NULL & NULL & NULL \\ -0.10390 & -0.09610 & -0.00631 & -0.04369 & -32.855 & -117.14 & -0.005860 & -0.004140 & 0.02 & -8.576 \\ 2.9355 \cdot 10^{-13} & 1.3000 \cdot 10^6 & 300. & 103.34 & 79.515 & 7.6580 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} 300. & 144.51 & 104.29 & 79.268 & 633.67 & 1147.8 & 7.6611 & NULL & NULL & NULL \\ -0.09885 & -0.10115 & -0.00569 & -0.04431 & -33.558 & -116.44 & -0.005843 & -0.004157 & 0.01 & -10.009 \\ 2.7219 \cdot 10^{-13} & 1.3000 \cdot 10^6 & 300. & 104.29 & 79.268 & 7.6611 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} 300. & 136.67 & 152.83 & 82.754 & 690.28 & 1047.0 & 5.8846 & NULL & NULL & NULL \\ -0.05518 & -0.14482 & -0.04733 & -0.00267 & -85.584 & -64.42 & -0.000987 & -0.009013 & 0. & -24.77 \\ 8.2135 \cdot 10^{-14} & 1.3000 \cdot 10^6 & 300. & 152.83 & 82.754 & 5.8846 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (6)$$

**Figure 4.** Obtained solutions  $\Phi_2^\oplus = \min \Phi_2 = 1.3 \times 10^6$  và  $\Phi_3^\oplus \leq 400$

Three solutions (4) – (6) satisfy the criteria 1, 2, 3, 5, and 6. Particularly, the criteria 1, 3, 5, and 6 excel the purposes of the experts. However, these solutions do not satisfy the criteria 4, because all of them are out of allowable limits according to the experts. Besides, for the criteria 3 the minimum value

$\Phi_3^\oplus = 305$  can be obtained. Nevertheless, there is still no solution satisfying all of requirements from the experts at this step.

• Adding to the constraints the condition  $|\Phi_3 - \Phi_3^\oplus| \leq \varepsilon_3 = 10^{-6}$  to find  $\min \Phi_1$ . We obtain the following three results, as shown in Figure 5:

$$\begin{bmatrix} 305.00 & 158.08 & 155.00 & 73.444 & 668.71 & 1008.5 & 8.0663 & NULL & NULL & NULL \\ -0.09634 & -0.10366 & -0.04871 & -0.00129 & -83.444 & -66.56 & -0.005121 & -0.004879 & 0. & -10.743 \\ 2.2755 \cdot 10^{-13} & 1.3000 \cdot 10^6 & 305.00 & 155.00 & 73.444 & 8.0663 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} 305.00 & 170.38 & 144.49 & 86.341 & 634.96 & 1016.3 & 5.3725 & NULL & NULL & NULL \\ -0.11797 & -0.08203 & -0.03452 & -0.01548 & -85.831 & -64.17 & -0.000026 & -0.009974 & -0.02 & -4.825 \\ 3.0212 \cdot 10^{-13} & 1.3000 \cdot 10^6 & 305.00 & 144.49 & 86.341 & 5.3725 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} 305.00 & 158.90 & 95.878 & 91.394 & 691.73 & 1046.9 & 4.9227 & NULL & NULL & NULL \\ -0.12279 & -0.07721 & -0.04787 & -0.00213 & -42.272 & -107.73 & 0. & -0.010000 & -0.01 & -3.617 \\ 3.5325 \cdot 10^{-13} & 1.3000 \cdot 10^6 & 305.00 & 95.878 & 91.394 & 4.9227 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (9)$$

**Figure 5.** Obtained solution  $\Phi_2^\oplus = \min \Phi_2 = 1.3 \times 10^6$ ,  $\Phi_3^\oplus \leq 400$ ,  $\Phi_1^\oplus \leq 4.5 \times 10^{-13}$



Three solutions (7) – (9) satisfy the criteria 1, 2, 3, and 5. Looking at the criteria 5 and 6 for the solutions (7) – (9), they are opposite. At this moment, the solution (9) seems to be satisfied all of requirements from the experts. In principle, we can stop the work at this step. However, if more severely  $\Phi_1^\oplus = 3.022 \times 10^{-13}$  is set for the

criteria 1, we do not have any satisfied solution, because the solutions (7) and (8) do not satisfy the criteria 4. Thus, let's carry on the next step.

• Adding to the constraints the condition  $|\Phi_1 - \Phi_1^\oplus| \leq \varepsilon_1 = 10^{-6}$  to find  $\min \Phi_5$ . We obtain the following four results, as shown in Figure 5:

$$\begin{bmatrix} 305.01 & 161.36 & 155. & 63.232 & 648.60 & 1025.9 & 10.521 & NULL & NULL & NULL \\ -0.10839 & -0.09161 & -0.03734 & -0.01266 & -73.242 & -76.76 & -0.010107 & 0.000107 & 0. & -7.346 \\ 3.0220 \cdot 10^{-13} & 1.3000 \cdot 10^6 & 305.01 & 155. & 63.232 & 10.521 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} 305.01 & 167.99 & 154.90 & 74.802 & 586.76 & 1066.8 & 7.8212 & NULL & NULL & NULL \\ -0.11417 & -0.08583 & -0.00484 & -0.04516 & -84.712 & -65.29 & -0.004627 & -0.005373 & -0.02 & -5.805 \\ 3.0224 \cdot 10^{-13} & 1.3000 \cdot 10^6 & 305.01 & 154.90 & 74.802 & 7.8212 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} 304.99 & 170.98 & 154.96 & 79.922 & 575.88 & 1067.2 & 6.6479 & NULL & NULL & NULL \\ -0.11671 & -0.08329 & -0.00048 & -0.04952 & -89.872 & -60.13 & -0.002314 & -0.007686 & -0.02 & -5.149 \\ 3.0222 \cdot 10^{-13} & 1.3000 \cdot 10^6 & 304.99 & 154.96 & 79.922 & 6.6479 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} 304.98 & 175.35 & 154.99 & 87.397 & 600.66 & 1025.0 & 4.9639 & NULL & NULL & NULL \\ -0.12035 & -0.07965 & -0.01948 & -0.03052 & -97.367 & -52.63 & 0.0009314 & -0.010931 & 0. & -4.222 \\ 3.0210 \cdot 10^{-13} & 1.2996 \cdot 10^6 & 304.98 & 154.99 & 87.397 & 4.9639 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (13)$$

**Figure 6.** Obtained solutions  $\Phi_2^\oplus = \min \Phi_2 = 1.3 \times 10^6$ ,  $\Phi_3^\oplus \leq 400$ ,  $\Phi_1^\oplus \leq 4.5 \times 10^{-13}$ ,  $\Phi_5^\oplus \leq 100$

The minimum value of criteria 5, which can be reached after passing the system of 10 functional constraints, is 64 (at solution (10)). However, these solutions do not satisfy the criteria 4, thus we need to look into the criteria 4 at this step. At the moment, there is still no satisfied solution. Nevertheless, if select the

threshold value of the criteria 4 according to the solutions (10) and (11), the criteria will be rarely satisfied. Thus, we opt for  $\Phi_5^\oplus = 80$ .

• Adding to the constraints the condition  $|\Phi_5 - \Phi_5^\oplus| \leq \varepsilon_5 = 10^{-6}$  to find  $\min \Phi_4$ . We obtain the following three results, as shown in Figure 7:

$$\begin{bmatrix} 305.00 & 148.14 & 99.948 & 79.999 & 695.50 & 1073.8 & 7.3666 & NULL & NULL & NULL \\ -0.10547 & -0.09453 & -0.04309 & -0.00691 & -34.947 & -115.05 & -0.005190 & -0.004810 & 0. & -8.139 \\ 3.0222 \cdot 10^{-13} & 1.3000 \cdot 10^6 & 305.00 & 99.948 & 79.999 & 7.3666 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} 304.98 & 139.76 & 79.973 & 79.998 & 712.16 & 1104.7 & 7.6042 & NULL & NULL & NULL \\ -0.10059 & -0.09941 & -0.04196 & -0.00804 & -14.951 & -135.05 & -0.006355 & -0.003645 & -0.02 & -9.511 \\ 3.0226 \cdot 10^{-13} & 1.3000 \cdot 10^6 & 304.98 & 79.973 & 79.998 & 7.6042 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (15)$$

$$\begin{bmatrix} 305.03 & 138.04 & 75.864 & 80.002 & 676.56 & 1152.3 & 7.6026 & NULL & NULL & NULL \\ -0.09951 & -0.10049 & -0.01993 & -0.03007 & -10.896 & -139.11 & -0.006495 & -0.003505 & 0.01 & -9.819 \\ 3.0218 \cdot 10^{-13} & 1.3000 \cdot 10^6 & 305.03 & 75.864 & 80.002 & 7.6026 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (16)$$

**Figure 7.** Obtained solutions  $\Phi_2^\oplus = \min \Phi_2 = 1.3 \times 10^6$ ,  $\Phi_3^\oplus \leq 400$ ,  $\Phi_1^\oplus \leq 4.5 \times 10^{-13}$ ,  $\Phi_5^\oplus \leq 100$ ,  $\Phi_4^\oplus \leq 100$

All of solutions (14), (15), (16) satisfy all of the criteria requirements, therefore they are satisfied solutions. However, we need to analyze whether the criteria 6 can be optimized more. Looking into the criteria (4), (5), (6) of the solutions (15) and (16), the minimum value of the criteria 4 does not worsen the value of criteria 6, and only

influences on the value of criteria 5, besides it is within the allowable limits. Thus, we opt for  $\Phi_4^\oplus = 76$ .

• Adding to the constraints the condition  $|\Phi_4 - \Phi_4^\oplus| \leq \varepsilon_4 = 10^{-6}$  to find  $\min\Phi_6$ . We obtain the following two results, as shown in Figure 8:

305.00	138.09	76.000	80.000	656.31	1173.4	7.5799	NULL	NULL	NULL
-0.09954	-0.10046	-0.00870	-0.04130	-11.000	-139.00	-0.006442	-0.003558	0.	-9.809
$3.0220 \cdot 10^{-13}$	$1.3000 \cdot 10^6$	305.00	76.000	80.000	7.5799	NULL	NULL	NULL	NULL

305.00	138.09	76.000	80.002	640.70	1189.9	7.5615	NULL	NULL	NULL
-0.09954	-0.10046	0.00001	-0.05001	-11.002	-139.00	-0.006402	-0.003598	0.	-9.809
$3.0220 \cdot 10^{-13}$	$1.3000 \cdot 10^6$	305.00	76.000	80.002	7.5615	NULL	NULL	NULL	NULL

**Figure 8.** Obtained solutions  $\Phi_2^\oplus = \min\Phi_2 = 1.3 \times 10^6$ ,  $\Phi_3^\oplus \leq 400$ ,  $\Phi_1^\oplus \leq 4.5 \times 10^{-13}$ ,  $\Phi_5^\oplus \leq 100$ ,  $\Phi_4^\oplus \leq 100$ ,  $\Phi_6^\oplus \leq 8$

For the criteria 6, the solutions (17) and (18) do not turn out the significant optimization in comparison with the solution (14)-(16). However, they all satisfy the requirements from the experts included in Table 5. Therefore, for the case 1 we have 7 satisfied solution, those are solutions (3), (9), (14) – (18), all of them are Pareto solutions, which are not able to be optimized simultaneously at all of criteria.

Case 2: the experts focus on the three criteria, which have a similar importance. The experts do not allow lowering the limit value

of the criteria, as included in Table 6.

**Table 6**  
Case 2

$[\Phi_1]$	$[\Phi_2]$	$[\Phi_3]$
$1.8 \times 10^{-13}$	$1.3 \times 10^6$	390

We add to the constraints three conditions  $\min\Phi_1 \leq \Phi X_1 \leq [\Phi_1]$ ,  $\min\Phi_2 \leq \Phi X_2 \leq [\Phi_2]$ ,  $\min\Phi_3 \leq \Phi X_3 \leq [\Phi_3]$  to find the minimum value of the function

$$\min F = \min\{|\Phi_1 - \Phi X_1| + |\Phi_2 - \Phi X_2| + |\Phi_3 - \Phi X_3|\} \rightarrow 0.$$

We obtained the following two results, as shown in Figure 9.

347.65	160.72	150.53	71.101	615.56	1033.7	5.8363	NULL	NULL	NULL
-0.08232	-0.11768	-0.02322	-0.02678	-119.28	-30.72	-0.000252	-0.009748	0.	-15.058
$1.7997 \cdot 10^{-13}$	$1.3000 \cdot 10^6$	347.65	150.53	71.101	5.8363	NULL	NULL	NULL	NULL

328.86	148.37	142.93	77.319	603.76	1100.4	5.5216	NULL	NULL	NULL
-0.07020	-0.12980	-0.00428	-0.04572	-99.109	-50.89	-0.000056	-0.009944	0.	-19.149
$1.3776 \cdot 10^{-13}$	$1.3000 \cdot 10^6$	328.86	142.93	77.319	5.5216	NULL	NULL	NULL	NULL

**Figure 9.** Obtained solution in accordance with Table 6

The solution (20) is more optimized than the solution (19) at most of criteria, but it is only less at the criteria 5. However, the experts can estimate the importance of criteria 5 in comparison with the other 5 criteria to choose the solution (19) or (20). These two

solutions excel the purpose of the experts at the criteria 3.

Case 3: the experts estimate the threshold value of all of criteria, as present in Table 7. The requirement is to the vector suitable for all of the criteria simultanously.

**Table 7**

Case 3

$[\Phi_1]$	$[\Phi_2]$	$[\Phi_3]$	$[\Phi_4]$	$[\Phi_5]$	$[\Phi_6]$
$2 \times 10^{-13}$	$1.4 \times 10^6$	400	110	140	8

Similarly to the case 2, we add to the constraints six conditions  $\min\Phi_1 \leq \Phi X_1 \leq [\Phi_1]$ ,  $\min\Phi_2 \leq \Phi X_2 \leq [\Phi_2]$ ,  $\min\Phi_3 \leq \Phi X_3 \leq [\Phi_3]$ ,  $\min\Phi_4 \leq \Phi X_4 \leq [\Phi_4]$ ,  $\min\Phi_5 \leq \Phi X_5 \leq [\Phi_5]$ ,  $\min\Phi_6 \leq \Phi X_6 \leq [\Phi_6]$  to find the minimum vale of function  $\min F = \min \left\{ \sum_{i=1}^6 |\Phi_i - \Phi X_i| \right\} \rightarrow 0$ . We obtain the following three results, as shown in Figure 10.

300.05	135.81	100.00	100.29	637.82	1156.0	5.0245	NULL	NULL	NULL
-0.07144	-0.12856	-0.00557	-0.04443	-50.34	-99.66	-0.000042	-0.009958	0.01	-18.719
$1.4005 \times 10^{-13}$	$1.3501 \times 10^6$	300.05	100.00	100.29	5.0245	NULL	NULL	NULL	NULL

(21)

300.78	138.03	100.00	97.876	627.92	1162.5	4.9861	NULL	NULL	NULL
-0.07680	-0.12320	-0.00071	-0.04929	-48.656	-101.34	0.0000010	-0.010001	0.01	-16.874
$1.5995 \times 10^{-13}$	$1.3378 \times 10^6$	300.78	100.00	97.876	4.9861	NULL	NULL	NULL	NULL

(22)

333.66	143.07	99.998	84.891	690.76	1067.4	5.1848	NULL	NULL	NULL
-0.07590	-0.12410	-0.04288	-0.00712	-68.549	-81.45	0.0000014	-0.010001	0.01	-17.181
$1.8003 \times 10^{-13}$	$1.3204 \times 10^6$	333.66	99.998	84.891	5.1848	NULL	NULL	NULL	NULL

(23)

**Figure 10.** Obtained solution in accordance with Table 7

These solutions are Pareto solutions, because there are superior and inferior criteria when comparing one to another. The values of criteria at these solutions are much better than the requirements of the experts included in Table 7.

**4. Concluding remarks**

It is important to note that the solution for multi-objective optimization task applied to high performance concrete mixture proportioning is not unique. Because, the solution is a set of criteria values, but every

criteria has a different importance from one expert’s standpoint to another at the certain production circumstance. Therefore, the evaluation of one solution or another based on an equivalent function for all of criteria is not comprehensive.

Above all, 12 solutions have been found for the different cases in terms of criteria during the process of proportioning high performance concrete mixture. They are all Pareto solutions, which allow experts to choose in the proposed cases. The task can

also be extended with more variables, constraints, criteria when varying the amount, as well as the constituent material to make high performance concrete. Last but not least,

the multi-objective optimization would definitely provide an optimum solution for high performance concrete mix proportioning with high durability and reasonable cost ■

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