OPTIMIZATION OF DOUBLY REINFORCED BEAM DESIGN USING SIMULATED ANNEALING

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Abstract: The optimization of the doubly reinforced concrete beam was investigated in this paper using the simulated annealing. Materials costs are considered as the objective function. The variables are the width, depth, compression steel, tension steel and cost. The constraints are the ultimate moment of resistance, compression/tension-steel ratio, minimum and maximum area of reinforcements. At the concrete compressive strength of 25 MPa, it is demonstrated that simulated annealing method can be used to optimize the design of concrete beams.

Keywords: beams, doubly reinforced, optimum design, reinforced concrete, simulated annealing

1. INTRODUCTION

According to [1], compression reinforcement is reinforcement used in the compression zone when the ultimate moment of resistance is less than the applied moment. Hence, doubly reinforced concrete beams are those with both tensile and compressive reinforcements. Leonardo da Vinci and Galileo in the 1600s carried out full-scale structures test models to boost specific characteristics as one of the pioneers of structural design optimization [2]. Cohn and Dinovitzer [3] discovered the gulf between the theories and practical applications of structural optimization. A limited number of researches on the optimization of concrete structures were also highlighted. Sarma and Adeli [4] equally studied the concept of structural optimization. Structural optimization systems can be categorized into two main types namely the precise and the empirical methods. The precise method follows from the estimation of the optimum outputs of repetitive methods of liner computerization according to [5, 6]. Empirical or heuristic constitute the second type of optimization that involves mathematical or algorithm procedures based on the current development of artificial intelligence processes i.e. modeled annealing, genetic algorithms, tabu search, ant colonies etc. [7-10]. Areas outside structural engineering have also benefited from these optimization methods [11]. The substantial arithmetic task is needed in these methods however modest they appear in as much as they contain huge iterative procedures where the main objective function is assessed and constraints for structural systems are being regulated.

Rajeev S. and Krisnamoorthy C.S. [12] investigated using a genetic algorithm from the heuristic optimization process, the weight of steel structures. Coello et.al. [13] Investigated the optimization of the R.C beam using a genetic algorithm. Many applications of R.C structures obtained through optimization using genetic algorithms abound lately. Model structural optimization work was promulgated initially by Maxwell [14] whereas Schmit launched the maiden computerized optimization [15]. Arithmetical systems were immensely adopted in optimizing

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structures afterward due to the huge success recorded. Vital heuristic algorithms were generated in the 1980s heralding the standard of structural optimization for years to come. Simulated Annealing is a typical numerical instance which adopts numerical means to model the procedures of progression and collection. Darwin's concept of evolution became the background for the language of simulated annealing methods. "Individual" is the probable resolutions to a problem and a set of solutions referred to as "population".

Four main categories of algorithms have been established and refined over the years and they are the Genetic Algorithms (GAs) [16], Evolutionary Programming [17], Evolutionary Strategies [18] and Genetic Programming [19]. These four types of algorithms have been fused and reviewed leading to several hybrid optimization algorithms with similar sources. Elimination of unutilized resources from the design field formed the basis of evolutionary structural optimization (ESO) [20]. Genetic evolutionary structural optimization (GESO) resulted from the incorporation of genetic algorithms with evolutionary structural optimization [21]. Several variables being imposed by objective optimization problem formed the basis of the controlled evolution method. Elimination of unutilized resources from the design field formed the basis of evolutionary structural optimization (ESO) [20].

Structural optimization has been applied to structural works like pre-stressed concrete beams [22] and force-limiting floor anchorage systems [23]. The late 1990s saw the emergence of the particle swarm optimization algorithm. The recognition of oscillators by Van der Pol-Duffing was a by-product of this type of algorithm [24] and [25]. Diverse methods can be used to labeled structural optimization. Normally, the main aim is to generate a project which results in optimum cost and/or reduced weight or mass. Concrete, reinforcement, and formwork are the three main materials used for construction [26]. Structural design optimization can either be founded on cost apart from weight such as applicable in structural retrofitting. Reinforced concrete structures optimization under seismic loads were investigated by [27] and [28]. Plummeting the vibration and enhancing the dynamic structural characteristics of reinforced concrete structures using optimization algorithms methods should command study attention [29]. Investigated the optimization of structural passive control systems using a genetic algorithm while [30] attempted to optimize the concrete cable-stayed bridge under seismic action. The similarity to the physical method of metal annealing or hardening form the basis of simulated annealing which involves a stochastic relaxation technique. This system character can be related to the resolution of the absolute optimization procedure. The idea of energy is related to the cost of a project and the transiting of the state to the concept of transforming to original design variables. New formations by obtaining specimens from the system probability distribution are being produced by simulated annealing.

Transformations that lower the objective function and raise it with specific possibilities are being deployed as well. Lack of vulnerability to early conjunction near local optimal constitutes the main merit of simulated annealing algorithms. [31] and [32] were the first sets of the algorithm founders. Optimization problems of voluminous scopes are suitably resolved using the concept of simulated annealing [33]. The ability of SA to resolve difficult problems involving concealing universal optimum in the midst of numerous local optimal enhances its suitability as compared with other types of heuristics optimization means. The ideology is similar to the cooling and solidification of molten metal in the sense that the lowest energy phase is being achieved for gradually cooled system. Hence, attaining or realizing a minimum phase of energy is necessitated by a slow cooling process. Production of a random preliminary solution initiates the commencement of structural annealing (SA). The present outcome accommodates primarily minimal transformation at the early phase. The present solution is then related and estimated alongside the value of the objective function. Original solution is then selected supposing it has an improved value or if a larger value of the probability function actualized in SA is obtained compared with a randomly produced value otherwise, a fresh outcome is produced and assessed.
The probability of accepting an original outcome is stated as follows:

$$p = \begin{cases} 1 & if\ \Delta < 0 \\ e^{-\frac{\Delta}{t}} & if\ \Delta \geq 0 \end{cases} \tag{1}$$

The estimation of this probability depends on the parameter of temperature, T, which is called temperature, in as much as it simulates the temperature of the physical annealing procedure. The rate of lowering is retarded to prevent getting arrested at a local lowest point. The following process of temperature reduction can be used:

$$T_{i+1} = cT_i \quad i = 0,1,\dots and\ 0.9 \leq c \leq 1 \tag{2}$$

Most derogating movements may be accommodated at the commencement of SA but the most enhanced ones are likely to be permitted. This can assist the process to rise out of a local minimum. The attainment of a specific volume fraction or pre-determined run time of structures may signal the end of the algorithm process. Several authors have investigated the optimization of the design of doubly reinforced concrete beams in recent times. An attempt was made to optimize beams dimensions by [34] using algorithms. The process employed the ultimate strength design and stress block in its entirety. The deadweight of the beam was used as a variable and the materials cost i.e. concrete, steel, formwork were measured. The comparison of the economy between the singly and doubly reinforced concrete beams respectively resulted in the display of a normal equation. The design procedures were further expatiated using a numerical example. A simply supported doubly reinforced beam with both uniform and point loads was adopted for optimization by [35] factoring the parabolic stress block, serviceability variables, moments, etc. Optimization method using MATLAB, internal point algorithm, and generalized reduced gradient system constitute a comparative study carried out. Limit state design according to IS: 456-2000 was adopted to generate the initial outcome. The outcome of the optimization of reinforced concrete beams using simulated annealing was investigated by [36] and as recommended by the American Building Code Requirements for structural concrete (ACI 318-05). The objective function contained the costs of concrete, reinforcement, and formwork. Flexural beam strength, width-height ratio, minimum width and deflection variables were selected as the optimization constraints. MATLAB was used to generate an optimization problem. Several examples were resolved with the aid of the generated program and proved to give economical, productive, effectual and resourceful design.

2. MATERIALS AND METHODS

The purpose of optimization is the minimization of the objective function subjected to certain constraints for constrained case. This can be presented as: Min f(x) subjected to constraints g(x)<0.

In case of reinforced concrete beam, the cost of the reinforced concrete beam production is the objective function to be minimized subjected to both the flexural and geometric constraints. Thus, the objective function is given as:

$$f = C_c[b(d + d') − (A_{st} + A_{cs})] + C_s[A_{st} + A_{sc}] + C_f[b + 2(d + d')]$$

(3)

Let the tensile steel/cross section area $\rho = A_{st}/bd$.

Let the compression/tensile steel ratio is $\alpha = A_{sc}/A_{st}$.

Thus equation (1) becomes:

$$f = C_c[b(d + d') − \rho bd(1 + \alpha)] + C_s \rho bd[1 + \alpha] + C_f[b + 2(d + d')] \tag{4}$$

Where the width of the beam b is x_1, the depth of the beam d is x_2, a is x_3, r is x_4, Cc is the cost/unit length of concrete, Cs is the cost unit/length of steel, C_f is the cost /unit area of the formwork.

Substituting those variables into equation (4) yields:

$$f(x_1, x_2, x_3, x_4) = [x_1(x_2 + d') − x_1 x_2 x_4(1 + x_3)]C_c + x_1 x_2 x_4[1 + x_3]C_s + [x_1 + 2(x_2 + d^1)]C_f \tag{5}$$

The design flexural constraint is derived from the Figure 1.

$$C_c = T − C_s \tag{6}$$

$$0.405 f_{cu}bx = 0.87 f_y A_{st} − 0.87 f_y A_{sc} \tag{7}$$

Fig. 1. Section analysis.

$$x = \frac{2.1481\rho bd f_y (1-\alpha)}{f_{cu}} \qquad (8)$$

$$z = d - \frac{0.87 f_y \rho d (1-\alpha)}{0.9 f_{cu}} \qquad (9)$$

The moment of resistance M_r is given as:

$$M_r = (C_c + C_s)z \qquad (10)$$

$$M_r = \left(0.203 f_{cu} bd + 0.87 f_y \alpha \rho bd\right)z \qquad (11)$$

The flexural constraint is:

$$M_a - M_r \leq 0 \qquad (12)$$

Other constraints are:

$$\alpha - 1 \leq 0$$

(13)

$$\frac{0.85}{f_y} - \rho \leq 0$$

(14)

$$b - 4d \leq 0$$

(15)

The whole process is programmed in Java using all the equations presented overleaf. The simulated annealing procedure is presented in the pseudo-code in Figure 2.

The pseudo-code was developed into a Java program and tested to ascertain its accuracy. The program was then used to generate the parametric data presented in Figures 3 to 7 in the next sections.

```
Start the system
Get the initial values, T and so
Previous Cost = object Function (so)
Loop over cooling {
Loop over temperature {
obtain the new variables sn = get Variable ()
current Cost = objective Function(sn)
diff = current Cost – previous Cost
if (diff<=0 && constraints ()) {
accept sn
}
else {
If (r <= p=e-diff/T && constraints ()) {
accept sn
}
}
}
T = aT   //0=<a<1
}
```

Fig. 2. Pseudo-code for the simulated annealing.

3. RESULTS AND DISCUSSION

The results presented in this section are the variation of width, depth, compression reinforcement, tension reinforcement and cost with the moment. Figure 3 is the optimized beam width variation with increasing bending moment. It is evident from the plot that there are various minimum widths between 300 and 302 mm up to 500 kNm moment before the width continues to increase continuously. This is an indication that at a certain level of loading there must be commensurable geometry.
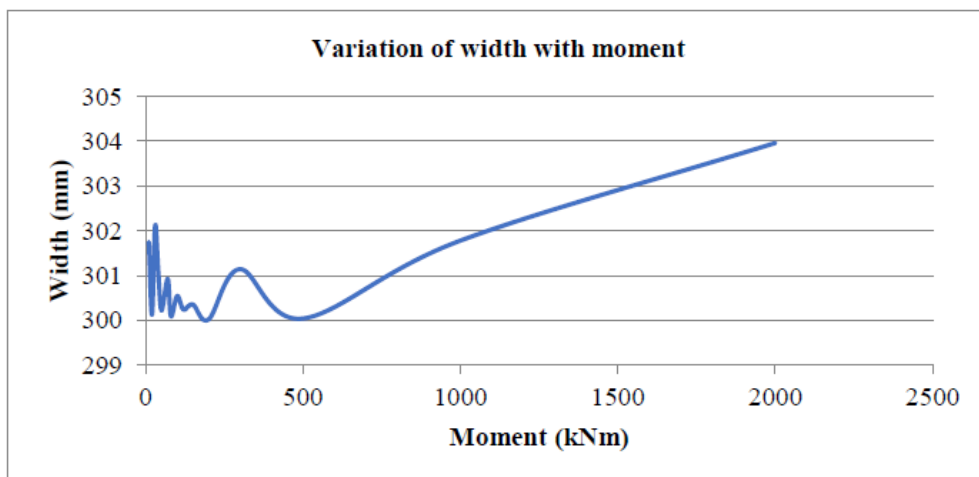

Fig. 3. Optimized beam width at various moment.

Figure 4 is the optimized beam depth variation with the bending moment. It is evident from the plot that the depth remains constant at 400 mm up to the moment of 300 kNm before a steady increase to over 1000 mm. The geometrical requirement increases with an increase of the moment. With the geometry considered reinforcement influence on the resistance is limited to about 300k Nm.

Figure 5 is the plot of the optimized compression reinforcement variation with moment. The compression reinforcement increases steadily up to maximum of 1000 mm^2 at 400 kNm before the reduction to about 520 mm^2

at about 500 kNm. The compression reinforcement starts to increase steadily from about 500 kNm at the compression reinforcement of 520 mm^2 to more than 2000 mm^2.
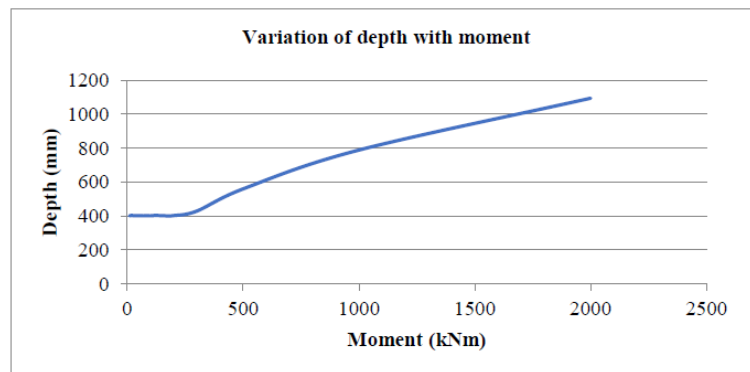

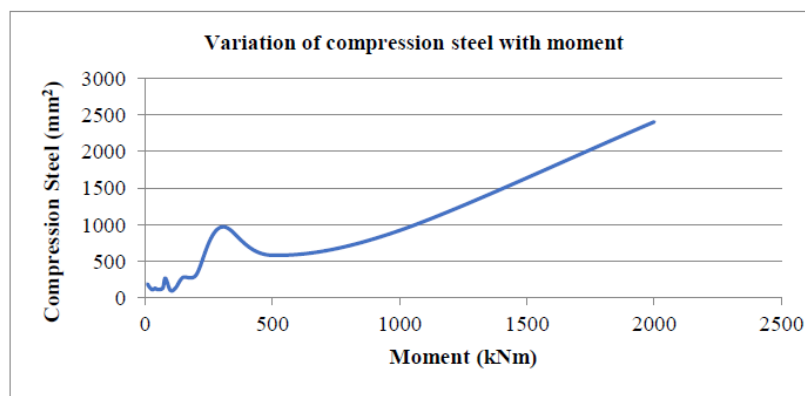Fig. 4. Optimized beam depth at various moment.

Fig. 5. Optimized compression reinforcement variation with moment.

Figure 6 is the plot of the optimized tension reinforcement variation with moment. The same trend as observed for the compression reinforcement is exhibited in this plot. The range of tension reinforcement between 900 mm^2 and 3200 mm^2 is required for the moment of up to 400 kNm before the decline to 2000 mm^2 at 500 kNm. The tension reinforcement required starts to increase from 2000 mm^2 to about 8000 mm^2 between 500 kNm and 2000 kNm. The same conclusion of geometry dictates the optimum cost rather than reinforcement.
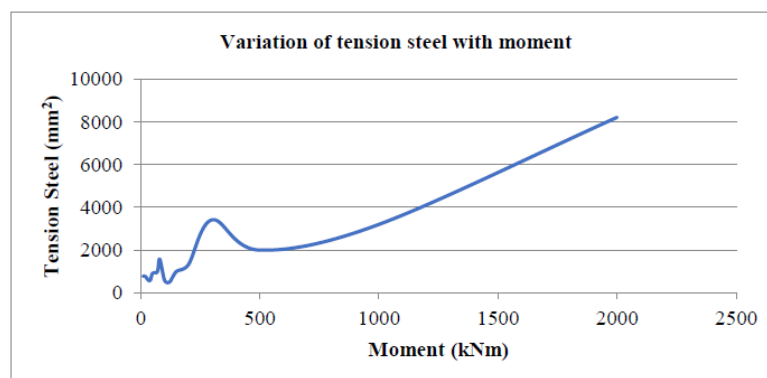

Fig. 6. Optimized tension reinforcement variation with moment.

Figure 7 is the plot of the optimized cost variation with moment. The cost remains constant at about N3000 for up to about 300 kNm. The cost steadily increases with moment. The behavior might be that at low moment the behavior is more of a singly reinforced beam. Once the concrete capacity in bending is surpassed, the contribution of compression reinforcement is required.
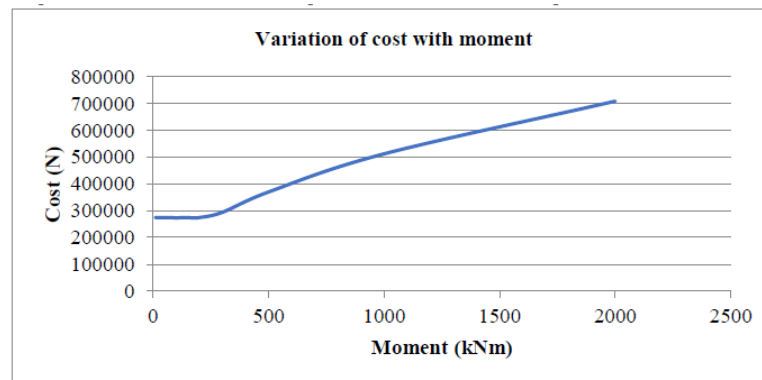
Fig. 7. Optimized cost variation with moment.

4. CONCLUSIONS

The simulated annealing method of structural optimization was successfully utilized in this research as a substitute for the ESO technique. Simulated annealing can handle constrained optimization of structural design. The optimized cost of reinforced concrete structural members such as beam can easily be accomplished using simulated annealing. The method competes well with other methods such a genetic algorithm, artificial neural network and fuzzy logic to mention but few.

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