DOI: 10.17626/dBEM.ICoM.P01.2012.p014

MEASURING THE PERFORMANCE OF STUDENTS IN A PRODUCTION SIMULATION GAME WITH DATA ENVELOPMENT ANALYSIS

Tamás Koltai, Judit Uzonyi-Kecskés
Budapest University of Technology and Economics
Department of Management and Corporate Economics, Budapest, Hungary
E-mail: koltai@mvt.bme.hu

Summary: Data Envelopment Analysis (DEA) is a quantitative tool to measure the performance of independent decision-making units (DMU). Several application of DEA can be found in the service sector to compare the performance of banks, restaurants, call centers and academic institutions. In this paper a two-phase CCR input model is used to compare the performance of student groups in a production simulation game. The simulation game is part of a course in a master program in the area of management. As a consequence of the complex nature of the simulation game there is not a single measure to compare the performance of student groups. On one hand, the results of DEA have created the basis of grading student performance, but have also provided information about the effectiveness of teaching the related topics.

Keywords: performance evaluation, data envelopment analysis, training, higher education

1. INTRODUCTION

The comparison of the performance of several production and/or service units is a general problem, which managers frequently have to face. In most cases, there is no any single parameter, which can be used for this evaluation. The compared production and/or service systems provide similar outputs (services or products) and they can independently decide on the amount of inputs used. Simply, we call these production and/or service systems as decision-making units (DMU). The comparison of the performance of several branches of a bank, several units of a restaurant chain, or several production lines of the same plant is typical cases of this problem.

Charnes, Cooper and Rhodes (1978) suggested a linear programming model in 1978, which compared DMUs using relative efficiency measures. Based on the suggested model relative efficiency analysis, or data envelopment analysis (DEA) became an important research area and a useful tool for practitioners. Several applications of DEA are reported in the literature in the service and in the production sector as well (see for example Panayotis, 1992; Sherman and Ladino, 1995; Markovits-Somogyi et al., 2011). A frequently applied area of DEA is the efficiency analysis of higher educational institutions. Jones (2006) compared more than 100 higher educational institutions in England using a nested DEA model. Sinuany-Stern et al. (1994) analyzed the relative efficiency of several departments within the same university. We apply DEA in higher education, but we concentrate on the efficiency analysis of student performance and on teaching efficiency.

In this paper we show, how DEA was applied for the evaluation of the performance of student groups in a production simulation game. This simulation game is part of a course in a master program in the area of management. As a consequence of the complex nature of the simulation game there is not any single measure which can be used for the comparison. There are two objectives of the application of DEA in this case:

– An evaluation method considering the production and financial results and the efficient utilization of the applied resources is needed for deciding on the ranking of student groups.
– Information about how the methods of the production management, financial management and marketing were mastered by the students and used in the simulation game is required.

In the following part of this paper first a review of the applied DEA models is provided. Next, the application environment is presented and some important results of the application of DEA are explained. Finally, conclusions are drawn, and the possibilities of the refinement of the presented evaluation and the areas of future research are summarized.

2. THE BASIC MODELS OF DEA

The objective of DEA is to determine the most efficient decision making units relative to each other, and to assign efficiency measures to each unit. By definition, efficiency is measured as a ratio of weighted output and weighted input. The highest value of efficiency is equal to 1 and the lowest value is equal to 0.

Assume that we have N DMUs with M inputs and T outputs. Notations used in the paper are summarized in Table 1.

Table 1: Notation

Indices:
- j - indice of decision making units, j=1, …, N,
- i - indice of inputs, i=1, … M,
- r - indice of outputs, , r=1, …, T.

Parameters:
- Y - matrix containing the output values of each DMU,
- Y_0 - vector containing the output values of the DMU examined,
- Y_j - vector containing the output values of DMU j,
- X - matrix containing the input values of each DMU,
- X_0 - vector containing the input values of the DMU examined,
- X_j - vector containing the input values of DMU j,
- e - unit vector,

Variables:
- u - vector containing the weights of outputs,
- v - vector containing the weights of inputs,
- λ - the ratio of inputs and the ratio of outputs in the optimal composition,
- λ_j - the ratio of inputs and the ratio of outputs of DMU j in an efficient DMU,
- θ - relative efficiency score,
- θ^* - the optimal value of the relative efficiency score,
- s^- - vector containing the input surplus values of each DMU,
- s^+ - vector containing the output shortage values of each DMU.

Vector Y_j contains the values of outputs of unit j (j=1,…, N), and vector X_j contains the values of inputs of unit j (j=1,…, N). The elements of variable vector u are the weights of the different outputs. The elements of variable vector v are the weights of the different inputs. Our objective is to find those values of the u and the v vectors, which maximize the efficiency of a specific DMU indicated by index 0. The constraints are imposed by the definition of efficiency, that is, at the selected weights the weighted output per weighted input ratio must be less than or equal to 1. The mathematical programming model describing these constraints and goals are the following,

$$\begin{aligned} &\text{Max} \quad \frac{uY_0}{vX_0} \\ &\text{DMU}: \quad \frac{uY}{vX} \leq 1 \\ &u, v \geq 0 \end{aligned} \tag{1}$$

Model (1) has no unique solution. It is easy to see that multiplying the numerator and the denominator as well with the same number we get different but equally optimal solutions. Fixing, however, the weighted inputs at value 1 and rearranging (1) by eliminating the ratio of variables, we get the primal input model of efficiency. This model is also called multiplier form of the input oriented CCR model after Carnes, Cooper and Rhodes. The multiplier CCR input model is the following,

$$\begin{aligned} &\text{Max} \quad uY_0 \\ &\text{DMU}: \quad uY - vX \leq 0 \\ &\text{Input}: \quad vX_0 = 1 \\ &u, v \geq 0 \end{aligned} \tag{2}$$

Linear programming problem (2) consists of N+1 constraints and M+T variables. The optimal solution of model (2) consists of the relative efficiency value of DMU 0, and of the optimal values of the input and output weights (u, v). In case of N DMUs, N number of LP models must be solved, to get the relative efficiency of each DMU. In practice, for mathematical and for management reasons the solution of the dual form of (2) is used. If θ is the dual variable of the input normalization equation and λ_j are the dual variables belonging to the inequality of DMU j, then the dual form of (2) is as follows,

$$\begin{aligned} &\text{Min} \quad \theta \\ &\text{Output}: \quad \lambda Y \geq Y_0 \\ &\text{Input}: \quad -\lambda X + \theta X_0 \geq 0 \\ &\lambda \geq 0; \\ &\theta \geq 0; \theta \leq 0 \end{aligned} \tag{3}$$

Linear programming problem (3) consists of M+T constraints and N+1variables. The optimal solution of (3) consists of the efficiency score (θ) of DMU 0, and of the optimal values of the dual variable vector λ. The optimal solution of (3) tells the decision maker how much the input of non-efficient DMUs should be reduced to achieve the efficiency of the best DMUs. It also tells the decision maker the optimal composition of inputs. Those DMUs, for which $\lambda_j \geq 0$ create the reference set of DMU 0. If the input of the DMUs in the reference set are combined according to the values of λ_j the highest efficiency can be achieved.

The results of model (2) or (3) provide information about the proportional change of all inputs. It is assumed that all inputs must be decreased by the same proportion (θ^*). Sometimes, however, it is possible to decrease some inputs independently of the other inputs without influencing the outputs. Similarly, sometimes some outputs can be increased independently of the other output without requiring more inputs. These possibilities can be explored by the introduction of the input surplus (s^-) and the output shortfall (s^+) vector variables. The model, which determines the input surpluses and output shortfalls, is called the slack model. The slack model for the dual input oriented CCR model is as follows,

$$\begin{array}{rlllll} \text{Max} & & es^{-} & + & es^{+} & \\ \text{Output:} & \lambda Y & & - & s^{+} & = Y_0 \\ \text{Input:} & -\lambda X & - s^{-} & & & = -\theta^{*} X_0 \end{array} \quad (4)$$

First model (3) and next model (4) must be solved. The optimal efficiency score provided by model (3) for DMU 0 is θ^*. The difference between the reduced input of DMU 0 ($\theta^* X_0$) and the optimal composition of inputs (λX) is the input surplus. The difference between the output of DMU 0 (Y_0) and the optimal composition of outputs (λY) is the output shortfall. Model (4) determines the maximal values of the independent input reduction for each inputs and the independent output increase for each output. Note, that first all inputs are decreased according to θ^*, and next inputs are further decreased according to s^-.

Several other models can be found in the literature for the calculation of relative efficiency. If in model (1) the weighted output is fixed then an output oriented models is defined. If the scaling effect between input and output is considered, then a variable return to scale (VRS) model is given. If input surplus and output shortage is maximized directly, without determining first the relative efficiency score, then the group of additive models is determined. A good review of the existing models is given by Cooper et al (2007).
In the following, we will show, how DEA can be used for the evaluation of the performance of students groups in a simulation game. The presented analysis is based on the results of models (3) and (4).

3. APPLICATION ENVIRONMENT

The applied production simulation game is developed by Ecosim to support the education and training in the production management area. We applied this simulation game in the Decision Making in Production and Service Systems course of the Management and Leadership Master Program for students specialized in Production and Operations Management at the Budapest University of Technology and Economics.

The objective of the game is to simulate production management decision making in a car engine manufacturing factory. The factory produces three different car engines for five different markets. Each market has its own demand characteristics. The car engines are assembled from parts on assembly lines operated by workers. The following decisions must be made by each student group for the next production period (year):
– *Production quantities of the three car engines*. Based on demand information forecasts must be prepared about the expected demand. The expected demand, the available production capacity and the final product inventory information are used to determine the production quantities of the next year.
– *Price and paying conditions*. Demand can be stimulated by selling price changes and by favourable payment conditions. Decision must be made on the purchase price of the next production period and on the allowed payment delay percentages.
– *Ordered quantities of parts*. Based on the planned production quantities, on the bill-of-material of the car engines and on inventory and financial information, order quantities of the different part groups must be determined.
– *Number of workers, number of shifts, and quantity of overtime*. Production quantity is determined by machine capacity and by the number of workers. On short term, capacity can

be changed by hiring or firing workers and by changing the number of production shifts or by applying overtime.
– *Investments in production line and in space*. On long term, production capacity can be increased by investments in new production lines and in space available for production and for inventory. Decision must be made in each production period about the number of new production line installations and about the number of square meters of space extensions.
– *Launch of efficiency improvement projects*. It is possible to launch projects which may improve production conditions. The predefined projects have different effects and different launch and maintenance costs. Decision must be made on which projects to launch in a given production period.
– *Application for credits*. There are three different credit types available for financing the operation of the factory. Each type of credit has different conditions. Decision must be made about the amount used of each credit type and about the payback of earlier credits.
After submitting the decisions, the simulation program generates the results of the actual production period. The results are summarised in two reports:
– *Production report*. The production report summarises the decisions for the actual production period. It summarises the quantity of engines produced and soled, the quantity of parts used and the engine and part inventories at the end of the production period. The number of workers, machine capacities, number of production lines and space, available for the next production period are also listed.
– *Financial report*. The financial report provides the balance sheet, the revenue report and the cash flow report valid at the end of the actual production period.
Concluding the seventh production period the student groups are evaluated. Evaluation, however, is very difficult even if only the financial situation of the plants is considered. Pure financial analysis can be misleading. Here are some examples:
– Short term success may not necessarily lead to long term success. The plant may accumulate high profit in the first 7 periods, but if production resources (production lines, production space, improvement projects) do not support production increase for the future, financial performance may decrease.
– A group may follow a cautious strategy. They may decide on low production quantity, financed by their available own financial sources. In this case small profit, slow but steady growth can characterise the plant.
– Long term strategic thinking may provide unfavourable financial results on the short run. Heavy investments can be made at the beginning using credits in order to secure capacity for future growths. If all this is paired with demand stimulating marketing policy and with efficiency improvement projects, profit will be low at the beginning, but steep growth can be expected in the future.
Evaluation is further complicated by the fact, that the simulation game is used not only for deciding the winner according to a specific financial measure. We also wanted to know how students mastered the different areas of production management. It may occur that students made poor financial decisions, but they made good inventory management and/or capacity management decisions.
The next section shows, how DEA helped to evaluate the results of student groups considering jointly and separately financial and operational aspects as well.

4. ANALYSIS OF THE RESULTS

We used relative efficiency analysis (DEA) for evaluating the performance of student groups at the end of the seventh period of the simulation game. In the analysis, two outputs and four inputs were considered. The two outputs are the following:

– *Net cumulated profit*. The profit integrates the effect of marketing, production and financial decisions.
– *Cumulated production quantity*. The production quantity reflects the effect of production management decisions related to machine and worker capacity, to material requirement planning and to inventory management.
The four inputs represent the resources used in the production process, that is,
– The *cumulated number of workers* represents the amount of human resources.
– The *cumulated number of machine hours* represents the amount of technical resources.
– The *cumulated sum of money spent on raw materials and on parts* represents the amount of material resources.
– The *cumulated value of credits* represents the amount of financial resources.

The performance of 9 student groups is compared using a two-phase input oriented CCR model. The results are summarised in table 2, 3 and 4.
Table 2 shows the case when the cumulated production quantity is the *only* output and the previously indicated four inputs are considered. These results help to evaluate the application of production management knowledge in the decision making process. Column 2 shows the total quantity of engines produced during seven production periods. Column 3 shows the relative efficiency scores. We can see that the highest quantity is found at group 8, although, the efficiency score of this group is not the highest. This group should have produced this output using less input. An efficient unit should use 1.14% less of all the inputs of group 8. Furthermore, excess machine capacity and overly high credit was used, as indicated by column 5 and 7. The last column shows, that if this group wants to increase efficiency, they should implement a mixture of the production practices of group 3 and 7.
Table 2 also shows, that group 1, 3, 5 and 7 have the maximum efficiency. We can see that the production quantity of group 5 and 7 is among the highest, the production quantity of group 3 is around the average, and the production quantity of group 1 is below the average. These groups have applied different but equally efficient production practices. In case of group 1 and 3 smaller quantities were produced, but the quantity of resources used was smaller as well.

Table 2: DEA results with production quantity output

1	2	3	4	5	6	7	8
Team	Output Prod. Quant.	Efficiency θ^*	Workers $s(1)^-$	Machine cap. $s(2)^-$	Material $s(3)^-$	Credit $s(4)^-$	Reference set
Group 1	2 793 305	**1,0000**	0	0	0	0	-
Group 2	2 779 163	0,9454	1 555	140 739	0	1 308 838	7
Group 3	2 899 000	**1,0000**	0	0	0	0	-
Group 4	2 889 423	0,9906	0	237 219	0	79 023	3, 5
Group 5	3 054 527	**1,0000**	0	0	0	0	-
Group 6	2 940 133	0,9838	0	375 767	0	524 956	3, 7
Group 7	3 057 918	**1,0000**	0	0	0	0	-
Group 8	3 130 992	0,9886	0	104 360	0	839 646	3, 7
Group 9	1 621 135	0,8753	2 372	317 291	0	2 030 721	7

Table 3 shows the case when the cumulated net profit is the *only* output and the previously indicated four inputs are considered. These results help to evaluate the joint application of marketing, production management and finance related knowledge in the decision making process. The highest possible efficiency is indicated at group 1 and 3. Note that these groups were among the most efficient groups in table 2 as well. The efficiency of group 7 is, however, among the worst, although it produced the second highest quantity. The reason for this is that high production quantity was not pared with efficient utilization of resources. An efficient group could have produced this output using 23.17% less of all resources. Furthermore, overly high number of workers and too many materials were used, as indicated by column 4 and 6 of Table 3. The last column shows, that if this group wants to increase efficiency, it should implement a mixture of the production practices of group 1 and 3.

Table 3: DEA results with profit output

1	2	3	4	5	6	7	8
Team	Output Profit	Efficiency θ^*	Workers $s(1)^-$	Machine cap. $s(2)^-$	Material $s(3)^-$	Credit $s(4)^-$	Reference set
Group 1	1 578 563	**1,0000**	0	0	0	0	-
Group 2	0	0,0000	0	0	0	0	-
Group 3	1 759 553	**1,0000**	0	0	0	0	-
Group 4	1 538 303	0,9856	0	521 462	334 359 868	0	1, 3
Group 5	1 410 080	0,9005	0	378 256	383 806 582	0	1, 3
Group 6	1 182 609	0,6588	538	38 959	0	194 838	3
Group 7	1 259 507	0,7683	1 155	0	247 415 155	0	1, 3
Group 8	632 569	0,3466	149	0	0	298 208	1, 3
Group 9	0	0,0000	0	0	0	0	-

Finally, Table 4 considers together the cumulative production quantity and the cumulative profit as outputs. The differences among the groups are smoothed out in this case. Five groups are considered the most efficient. Group 4 was not the most efficient in any of the previous two cases, but their efficiency scores were very near to one (0,99 and 0,98). If we evaluate together production quantity and profit, this group joins the set of most efficient groups.

Table 4: DEA results with production quantity and profit outputs

1	2	3	4	5	6	7	8
Team	Output Prod. Quant.	Output Profit	Efficiency θ^*	Workers $s(1)^-$	Machine cap. $s(2)^-$	Material $s(3)^-$	Credit $s(4)^-$
Group 1	2 793 305	1 578 563	**1,0000**	0	0	0	0
Group 2	2 779 163	0	0,9454	0	0	0	0
Group 3	2 899 000	1 759 553	**1,0000**	0	0	0	0
Group 4	2 889 423	1 538 303	**1,0000**	0	0	0	0
Group 5	3 054 527	1 410 080	**1,0000**	0	312 424	357 210 274	0
Group 6	2 940 133	1 182 609	0,9838	803	58 175	0	290 940
Group 7	3 057 918	1 259 507	**1,0000**	458	0	20 933 541	196 759
Group 8	3 130 992	632 569	0,9886	0	168 424	0	894 769
Group 9	1 621 135	0	0,8753	1 465	235 195	0	1 267 252

5. SUMMARY AND CONCLUSIONS

In this paper, the application of DEA is presented for the performance evaluation of student groups in a production simulation game. Relative efficiency of the groups is evaluated based on two different outputs. Cumulated production quantity is used for the evaluation of production management related decisions. Cumulated net profit is used for the evaluation of the joint effect of production, financial and marketing related decisions. Four major resources (human, machine, material financial) are used as inputs in the analysis. The quantity of these inputs used for production is decided exclusively by the student groups, therefore the student groups can be considered as DMUs.

An input oriented two-phase CCR model is used for the analysis. The results correctly reflect the performance of the student groups, however, some further refinement of the analysis is recommended:

– We applied large group sizes (5-6 students) in the simulation game and consequently the number of student groups was relatively small. The small group number smoothed out the differences in performance. The application of smaller group size and higher group number is recommended in future applications.

– The same initial conditions were given for each group at the beginning of the simulation. As a result of different growth strategies, however, scaling effect may appear after some production periods. Consequently, the application of a variable return to scale model might be appropriate.

– There was not any specific rule for student group formation. As a result, very different composition of groups concerning the study results and the interest area of students were formed. The consideration of the composition of students as non-discretionary variable may further refine the results.

– Finally, the analysis of the dynamic change of performance of student groups during the simulation may highlight some interesting mechanisms of the learning process.

The presented application of DEA completed with the proposed extensions might be a useful tool for student evaluations in higher education, but can also be applied for the evaluation of participants in any management training program as well.

REFERENCES

1. Charnes, A., Cooper, W.W., Rhodes, A. (1978): Measuring the efficiency of decision making units. *European Journal of Operations Research*, 2, pp.429-444.
2. Cooper, W.W., Seiford, L.M., Tone, K. (2007): *Data envelopment analysis*. Springer.
3. Doyle, J.R., Green, R.H. (1991): Comparing products using data envelopment analysis. *Omega International Journal of Management Sciences*, 19(6), pp.631-638.
4. Johnes, J. (2006): Data envelopment analysis and its application to the measurement of efficiency in higher education. *Economics of Education Review*, 25(3), pp.273-288.
5. Markovits-Somogyi. R., Gecse, G., Bokor, Z. (2011): Basic efficiency measurement of Hungarian logistics centres using data envelopment analysis. *Periodica Polytechnica Social and Management Sciences*, 19(2), pp.97-101.
6. Panayotis, A.M. (1992): Data envelopment analysis applied to electricity distribution districts. *Journal of the Operations Research Society*, 43(5), pp.549-555.
7. Sherman, H.D., Ladino G. (1995): Managing bank productivity using data envelopment analysis (DEA). *Interfaces*, 25(2), pp.60-73.
8. Sinuany-Stern, Z., Mehrez, A., Barboy, A. (1994): Academic departments efficiency via DEA. *Computers & Operations Research*, 21(5), pp.543-556.