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Controllability of Nonlocal Impulsive Stochastic Quasilinear Integrodifferential Systems

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Abstract

Sufficient conditions for controllability of nonlocal impulsive stochastic quasilinear integrodifferential systems in Hilbert spaces are established. The results are obtained by using evolution operator, semigroup theory and fixed point technique. As an application, an example is provided to illustrate the obtained result.

Keywords: Controllability, Impulsive stochastic quasilinear integrodifferential systems, Fixed point.

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1 Introduction

The concept of controllability plays an important role in many areas of applied mathematics. Random differential and integral equations play an important role in characterizing numerous social, physical, biological and engineering problems. Stochastic differential equations are important from the viewpoint of applications since they incorporate randomness into the mathematical description of the phenomena and therefore provide a more accurate description of it. Impulsive effects exist widely in many evolution processes in which states are changed abruptly at certain moments of time involving fields such as medicine, biology, economics, electronics and telecommunications etc., (see [26, 33]). Besides impulsive effects, stochastic effects also exist in real systems. Most of the dynamical systems have variable structures subject to stochastic abrupt changes, which may result from abrupt phenomena such as stochastic failures and repairs of the components, sudden environment changes and changes in the interconnections of subsystems.

Mathematical modelling of real life problems usually results in functional equations, like ordinary or partial differential equations, integral equations, integrodifferential equations and stochastic equations. Integrodifferential equations play an important role in many branches of linear and nonlinear functional analysis and their applications in the theory of engineering, mechanics, physics, chemistry, astronomy,

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biology, economics, potential theory and electrostatics. Various mathematical formulation of physical phenomena contain integrodifferential equations, these equations arises in fields such as fluid dynamics, biological models and chemical kinetics. The nonlocal condition which is a generalization of the classical initial condition was motivated by physical problems. The pioneering work on nonlocal conditions is due to Byszewski [10].

Quasilinear evolution equations are encountered in many areas of science and engineering. It forms a very important class of evolution equations as many time dependent phenomena in physics, chemistry and biology can be represented by such evolution equations. For more details on the theory and applications of quasilinear evolution equations we refer to [25]. Several authors have studied the existence of solutions of abstract quasilinear evolution equations in Banach space [1, 5, 15, 17, 18, 34]. Bahuguna [3], Oka [28], Oka and Tanaka [29] discussed the existence of solutions of quasilinear integrodifferential equations in Banach spaces. Kato [16] studied the nonhomogeneous evolution equations and Chandrasekaran [11] proved the existence of mild solutions of the nonlocal Cauchy problem for a quasilinear integrodifferential equation. Dhakne and Pachpatte [14] established the existence of a unique strong solution of a quasilinear abstract functional integrodifferential equation in Banach spaces. Recently, the study on controllability of quasilinear systems has gained renewed interests and only few papers have appeared (see [6, 8, 9]).

Also, the controllability and stability of nonlinear stochastic systems in finite and infinite-dimensional spaces have been studied by several authors [2, 13, 27, 30]. Many extensive results on stochastic controllability were investigated by Jerzy Klamka in [19]-[24]. Balachandran and Karthikeyan [4] and Balachandran et al. [7] derived the sufficient conditions for the controllability of stochastic integrodifferential systems in finite dimensional spaces. We refer to the paper of Sakthivel et al. [32] who derived the controllability of nonlinear impulsive stochastic systems. Subalakshmi and Balachandran [35, 36] studied the controllability of semilinear stochastic functional integrodifferential systems and approximate controllability of nonlinear stochastic impulsive integrodifferential systems in Hilbert spaces. Moreover, controllability of impulsive stochastic quasilinear integrodifferential systems has not yet studied in the literature. Motivated by this consideration, in this paper we study the controllability of nonlocal impulsive stochastic quasilinear integrodifferential systems described by

$$\begin{aligned} dx(t) &= \Big[A(t,x)x(t) + Bu(t) + f(t,x(t)) + \int_0^t g\Big(t,s,x(s),\int_0^s \kappa\big(s,\eta,x(\eta)\big)d\eta\Big)ds\Big]dt \\ &\qquad + \sigma(t,x(t))dw(t), \quad t \in J := [0,a], \quad t \neq \tau_k, \\ \triangle x(\tau_k) &= x(\tau_k^+) - x(\tau_k^-) = I_k(x(\tau_k^-)), \qquad k = 1,2,\cdots,m, \\ x(0) &+ h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big) = x_0. \end{aligned} \tag{1.1}$$

where $0 < t_1 < t_2 < \cdots < t_p \leq a$ $(p \in N)$. Here, the state variable $x(\cdot)$ takes values in a real separable Hilbert space H with inner product $(\cdot,\cdot)$ and norm $\|\cdot\|$

and the control function $u(\cdot)$ takes values in $L^2(J,U)$, a Banach space of admissible control functions for a separable Hilbert space U. Also, $A(t,x)$ is the infinitesimal generator of a C_0-semigroup in H and B is a bounded linear operator from U into H. Let K be another separable Hilbert space with inner product $(\cdot,\cdot)_K$ and the norm $\|\cdot\|_K$. Suppose $\{w(t) : t \geq 0\}$ is a given K-valued Wiener process with a finite trace nuclear covariance operator $Q \geq 0$. We employ the same notation $\|\cdot\|$ for the norm $\mathcal{L}(K,H)$, where $\mathcal{L}(K,H)$ denotes the space of all bounded linear operators from K into H. Further, $f : J \times H \to H$, $g : \Lambda \times H \times H \to H$, $\kappa : \Lambda \times H \to H$, $\sigma : J \times H \to \mathcal{L}_Q(K,H)$ are measurable mappings in H-norm and $\mathcal{L}_Q(K,H)$ norm respectively, where $\mathcal{L}_Q(K,H)$ denotes the space of all Q-Hilbert-Schmidt operators from K into H which will be defined in Section 2 and $\Lambda = \{(t,s) \in J \times J : s \leq t\}$. Here, the nonlocal function $h : \mathcal{PC}[J^p \times H : H] \to H$ and impulsive function $I_k \in C(H,H)$ $(k = 1,2,\cdots,m)$ are bounded functions. Furthermore, the fixed times τ_k satisfies $0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_m < a$, $x(\tau_k^+)$ and $x(\tau_k^-)$ denote the right and left limits of $x(t)$ at $t = \tau_k$. And $\triangle x(\tau_k) = x(\tau_k^+) - x(\tau_k^-)$ represents the jump in the state x at time τ_k, where I_k determines the size of the jump.

2 Preliminaries

For more details in this section refer [12]. Let $(\Omega, \mathcal{F}, P; \mathbf{F})$ $\{\mathbf{F} = \{\mathcal{F}_t\}_{t\geq 0}\}$ be a complete filtered probability space satisfying that $\mathcal{F}_0$ contains all P-null sets of $\mathcal{F}$. An H-valued random variable is an $\mathcal{F}$-measurable function $x(t)\colon \Omega \to H$ and the collection of random variables $S = \{x(t,\omega) : \Omega \to H \setminus t \in J\}$ is called a stochastic process. Generally, we just write $x(t)$ instead of $x(t,\omega)$ and $x(t) : J \to H$ in the space of S. Let $\{e_i\}_{i=1}^{\infty}$ be a complete orthonormal basis of K. Suppose that $\{w(t) : t \geq 0\}$ is a cylindrical K-valued wiener process with a finite trace nuclear covariance operator $Q \geq 0$, denote $Tr(Q) = \sum_{i=1}^{\infty} \lambda_i = \lambda < \infty$, which satisfies that $Qe_i = \lambda_i e_i$. So, actually, $\omega(t) = \sum_{i=1}^{\infty} \sqrt{\lambda_i}\omega_i(t)e_i$, where $\{\omega_i(t)\}_{i=1}^{\infty}$ are mutually independent one-dimensional standard Wiener processes. We assume that $\mathcal{F}_t = \sigma\{\omega(s) : 0 \leq s \leq t\}$ is the σ-algebra generated by ω and $\mathcal{F}_a = \mathcal{F}$. Let $\Psi \in \mathcal{L}(K,H)$ and define

$$\|\Psi\|_Q^2 = Tr(\Psi Q \Psi^*) = \sum_{n=1}^{\infty} \|\sqrt{\lambda_n}\Psi e_n\|^2.$$

If $\|\Psi\|_Q < \infty$, then Ψ is called a Q-Hilbert-Schmidt operator. Let $\mathcal{L}_Q(K,H)$ denote the space of all Q-Hilbert-Schmidt operators $\Psi : K \to H$. The completion $\mathcal{L}_Q(K,H)$ of $\mathcal{L}(K,H)$ with respect to the topology induced by the norm $\|\cdot\|_Q$ where $\|\Psi\|_Q^2 = \langle \Psi, \Psi \rangle$ is a Hilbert space with the above norm topology. $L_2^{\mathcal{F}}(J,H)$ is the space of all $\mathcal{F}_t$ - adapted, H-valued measurable square integrable processes on $J \times \Omega$.

Denote $J_0 = [0,\tau_1]$, $J_k = (\tau_k, \tau_{k+1}]$, $k = 1,2,\cdots,m$, and define the following class of functions:

$\mathcal{PC}(J, L_2(\Omega, \mathcal{F}, P; H)) = \{x : J \to L_2 :$x(t) is continuous everywhere except for some τ_k at which $x(\tau_k^-)$ and $x(\tau_k^+)$ exists and $x(\tau_k^-) = x(\tau_k), k = 1, 2, 3, \cdots, m\}$ is the Banach space of piecewise continuous maps from J into $L_2(\Omega, \mathcal{F}, P; H)$ satisfying the condition $\sup_{t\in J} E\|x(t)\|^2 < \infty$. Let $\mathcal{Z} \equiv \mathcal{PC}(J, L_2)$ be the closed subspace of $\mathcal{PC}(J, L_2(\Omega, \mathcal{F}, P; H))$ consisting of measurable, $\mathcal{F}_t$ - adapted and H-valued processes $x(t)$. Then $\mathcal{PC}(J, L_2)$ is a Banach space endowed with the norm

$$\|x\|_{\mathcal{PC}}^2 = \sup_{t\in J} \ \big\{E\|x(t)\|^2 : x \in \mathcal{PC}(J, L_2)\big\}.$$

Let H and Y be two Hilbert spaces such that Y is densely and continuously embedded in H. For any Hilbert space $\mathcal{Z}$ the norm of $\mathcal{Z}$ is denoted by $\|\cdot\|_{\mathcal{PC}}$ or $\|\cdot\|$. The space of all bounded linear operators from H to Y is denoted by $B(H, Y)$ and $B(H, H)$ is written as $B(H)$. We recall some definitions and known facts from [31].

Definition: 2.1 *Let S be a linear operator in H and let Y be a subspace of H. The operator $\tilde{S}$ defined by $D(\tilde{S}) = \{x \in D(S) \cap Y : Sx \in Y\}$ and $\tilde{S}x = Sx$ for $x \in D(\tilde{S})$ is called the part of S in Y.*

Definition: 2.2 *Let Q be a subset of H and for every $0 \le t \le a$ and $q \in Q$, let $A(t, q)$ be the infinitesimal generator of a C_0 semigroup $S_{t,q}(s), s \ge 0$, on H. The family of operators $\{A(t, q)\}$, $(t, q) \in J \times Q$, is stable if there are constants $M \ge 1$ and ω such that*

$$\begin{aligned} \rho(A(t,q)) &\supset (\omega, \infty) \qquad for\ (t,q) \in J \times Q, \\ \Big\|\prod_{j=1}^{k} R(\lambda : A(t_j, q_j))\Big\| &\le M(\lambda - \omega)^{-k} \ \ for \ \ \lambda > \omega \end{aligned}$$

and every finite sequences $0 \le t_1 \le t_2 \le \cdots \le t_k \le a$, $q_j \in Q, 1 \le j \le k$. The stability of $\{A(t, q)\}$, $(t, q) \in J \times Q$, implies [31] that

$$\Big\|\prod_{j=1}^{k} S_{t_j,q_j}(s_j)\Big\| \le M \exp\ \{\omega \sum_{j=1}^{k} s_j\} \ \ for\ s_j \ge 0$$

and any finite sequences $0 \le t_1 \le t_2 \le \cdots \le t_k \le a$, $q_j \in Q, 1 \le j \le k$. $k = 1, 2, \cdots$.

Definition: 2.3 *Let $S_{t,q}(s)$, $s \ge 0$ be the C_0 semigroup generated by $A(t, q), (t, q) \in J \times Q$. A subspace Y of H is called $A(t, q)$- admissible if Y is invariant subspace of $S_{t,q}(s)$ and the restriction of $S_{t,q}(s)$ to Y is a C_0- semigroup in Y.*

Let $Q \subset H$ be a subset of H such that for every $(t, q) \in J \times Q$, $A(t, q)$ is the infinitesimal generator of a C_0-semigroup $S_{t,q}(s), s \ge 0$ on H. We make the following assumptions:

(E1) The family $\{A(t,q)\}, (t,q) \in J \times Q$ is stable.

(E2) Y is $A(t,q)$ - admissible for $(t,q) \in J \times Q$ and the family $\{\tilde{A}(t,q)\}, (t,q) \in J \times Q$ of parts $\tilde{A}(t,q)$ of $A(t,q)$ in Y, is stable in Y.

(E3) For $(t,q) \in J \times Q$, $D(A(t,q)) \supset Y$, $A(t,q)$ is a bounded linear operator from Y to H and $t \to A(t,q)$ is continuous in the $B(Y,H)$ norm $\|\cdot\|$ for every $q \in Q$.

(E4) There is a constant $L > 0$ such that

$$\|A(t,q_1) - A(t,q_2)\|_{Y \to H} \le L\|q_1 - q_2\|_H$$

holds for every $q_1, q_2 \in Q$ and $0 \le t \le a$.

Let Q be a subset of H and let $\{A(t,q)\}$, $(t,q) \in J \times Q$ be a family of operators satisfying the conditions $(E1)-(E4)$. If $x \in \mathcal{PC}(J, L_2)$ has values in Q then there is a unique evolution system $U(t,s;x), 0 \le s \le t \le a$ in H satisfying (see [31])

(i) $\|U(t,s;x)\| \le Me^{\omega(t-s)}$ for $0 \le s \le t \le a$, where M and ω are stability constants.

(ii) $\frac{\partial^+}{\partial t}U(t,s;x)y = A(s,x(s))U(t,s;x)y \quad$ for $\; y \in Y,\ 0 \le s \le t \le a.$

(iii) $\frac{\partial}{\partial s}U(t,s;x)y = -U(t,s;x)A(s,x(s))y \quad$ for $\; y \in Y,\ 0 \le s \le t \le a.$

Further we assume that

(E5) For every $x \in \mathcal{PC}(J, L_2)$ satisfying $x(t) \in Q$ for $0 \le t \le a$, we have

$$U(t,s;x)Y \subset Y, \quad 0 \le s \le t \le a$$

and $U(t,s;x)$ is strongly continuous in Y for $0 \le s \le t \le a$.

(E6) Closed bounded convex subsets of Y are closed in H.

(E7) For every $(t,q) \in J \times Q$, $f(t,q) \in Y$, $((t,s),q_1,q_2) \in \Lambda \times Q \times Q, g(t,s,q_1,q_2) \in Y$ and $(t,q) \in J \times Q$, $\sigma(t,q) \in Y$.

Definition: 2.4 *[13] A stochastic process x is said to be a mild solution of* (1.1) *if the following conditions are satisfied:*

(a) $x(t,\omega)$ is a measurable function from $J \times \Omega$ to H and $x(t)$ is $\mathcal{F}_t$ -adapted,

(b) $E\|x(t)\|^2 < \infty$ for each $t \in J$,

(c) $\triangle x(\tau_k) = x(\tau_k^+) - x(\tau_k^-) = I_k(x(\tau_k^-)),\ k = 1,2,\cdots,m,$

(d) *For each $u \in L_2^{\mathcal{F}}(J,U)$, the process x satisfies the following integral equation*

$$\begin{aligned} x(t) =& U(t,0;x)\big[x_0 - h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big)\big] + \int_0^t U(t,s;x)[Bu(s)+f(s,x(s))]ds \\ &+\int_0^t U(t,s;x)\Big[\int_0^s g\Big(s,\eta,x(\eta),\int_0^\eta \kappa\big(\eta,\gamma,x(\gamma)\big)d\gamma\Big)d\eta\Big]ds \\ &+\int_0^t U(t,s;x)\sigma(s,x(s))dw(s) + \sum_{0<\tau_k<t} U(t,\tau_k;x)I_k(x(\tau_k^-)), \textit{for a.e. } t \in J, \\ x(0) \;+&\;\; h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big) = x_0 \in H. \end{aligned} \tag{2.1}$$

Definition: 2.5 *The system* (1.1) *is said to be controllable on the interval J, if for every initial condition x_0 and $x_1 \in H$, there exists a control $u \in L^2(J,U)$ such that the solution $x(\cdot)$ of* (1.1) *satisfies $x(a) = x_1$.*

Further there exists a constant $\mathcal{N} > 0$ such that for every $x, y \in \mathcal{PC}(J, L_2)$ and every $\tilde{y} \in Y$ we have

$$\|U(t,s;x)\tilde{y} - U(t,s;y)\tilde{y}\|^2 \leq \mathcal{N}a^2\|\tilde{y}\|_Y^2\|x-y\|_{\mathcal{PC}}^2.$$

In order to establish our controllability result we assume the following hypotheses:

(H1) $A(t,x)$ generates a family of evolution operators $U(t,s;x)$ in H and there exists a constant $\mathcal{C}_U > 0$ such that

$$\|U(t,s;x)\|^2 \leq \mathcal{C}_U \quad \text{for } 0 \leq s \leq t \leq a,\ x \in \mathcal{Z}.$$

(H2) The linear operator $W : L^2(J,U) \to H$ defined by

$$Wu = \int_0^a U(a,s;x)Bu(s)ds$$

is invertible with inverse operator W^{-1} taking values in $L^2(J,U) \setminus kerW$ and there exist a positive constant $\mathcal{C}_W$ such that

$$\|BW^{-1}\|^2 \leq \mathcal{C}_W.$$

(H3) The nonlinear function $f : J \times \mathcal{Z} \to \mathcal{Z}$ is continuous and there exist constants $\mathcal{C}_f > 0$, $\tilde{\mathcal{C}_f} > 0$ for $t \in J$ and $x, y \in \mathcal{Z}$ such that

$$E\|f(t,x) - f(t,y)\|^2 \leq \mathcal{C}_f\|x-y\|^2$$

and $\tilde{\mathcal{C}_f} = \sup_{t\in J}\|f(t,0)\|^2$.

(H4) The nonlinear function $g : \Lambda \times \mathcal{Z} \times \mathcal{Z} \to \mathcal{Z}$ is continuous and there exist positive constants $\mathcal{C}_g$, $\tilde{\mathcal{C}_g}$, for $x_1, x_2, y_1, y_2 \in \mathcal{Z}$ and $(t,s) \in \Lambda$ such that

$$E\Big\|g(t,s,x_1,y_1) - g(t,s,x_2,y_2)\Big\|^2 \leq \mathcal{C}_g(\|x_1 - x_2\|^2 + \|y_1 - y_2\|^2)$$

and $\tilde{\mathcal{C}_g} = \sup_{(t,s)\in\Lambda} \|g(t,s,0,0)\|^2$.

(H5) The function $\kappa : \Lambda \times \mathcal{Z} \to \mathcal{Z}$ is continuous and there exist positive constants $\mathcal{C}_\kappa$, $\tilde{\mathcal{C}_\kappa}$ for $(t,s) \in \Lambda$ and $x, y \in \mathcal{Z}$ such that

$$E\Big\|\int_0^t \Big[\kappa(t,s,x) - \kappa(t,s,y)\Big]ds\Big\|^2 \leq \mathcal{C}_\kappa \|x - y\|^2$$

and $\tilde{\mathcal{C}_\kappa} = \sup_{(t,s)\in\Lambda} \big(\big\| \int_0^t \kappa(t,s,0)ds\big\|^2\big)$.

(H6) The function $\sigma : J \times \mathcal{Z} \to \mathcal{L}_Q(K,H)$ is continuous and there exist constants $\mathcal{C}_\sigma > 0$, $\tilde{\mathcal{C}_\sigma} > 0$ for $t \in J$ and $x, y \in \mathcal{Z}$ such that

$$E\|\sigma(t,x) - \sigma(t,y)\|_Q^2 \leq \mathcal{C}_\sigma \|x - y\|^2$$

and $\tilde{\mathcal{C}_\sigma} = \sup_{t\in J} \|\sigma(t,0)\|^2$.

(H7) The nonlocal function $h : \mathcal{PC}(J^p \times \mathcal{Z} : \mathcal{Z}) \to \mathcal{Z}$ is continuous and there exist constants $\mathcal{C}_h > 0$, $\tilde{\mathcal{C}_h} > 0$ for $x, y \in \mathcal{Z}$ such that

$$E\|h(t_1,t_2,\cdots,t_p,x(\cdot)) - h(t_1,t_2,\cdots,t_p,y(\cdot))\|^2 \leq \mathcal{C}_h\|x - y\|^2,$$
$$E\|h(t_1,t_2,\cdots,t_p,x(\cdot))\|^2 \leq \tilde{\mathcal{C}_h}.$$

(H8) $I_k : \mathcal{Z} \to \mathcal{Z}$ is continuous and there exist constants $\beta_k > 0$, $\tilde{\beta_k} > 0$ for $x, y \in \mathcal{Z}$ such that

$$E\|I_k(x) - I_k(y)\|^2 \leq \beta_k\|x - y\|^2, \quad k = 1, 2, \cdots, m$$

and $\tilde{\beta_k} = \|I_k(0)\|^2$, $k = 1, 2, \cdots, m$.

(H9) There exists a constant $r > 0$ such that

$$7\Big\{\mathcal{C}_U(\|x_0\|^2 + \tilde{\mathcal{C}_h}) + a^2\mathcal{C}_U\mathcal{G} + 2a^2\mathcal{C}_U(\mathcal{C}_f r + \tilde{\mathcal{C}_f}) + 2a^3\mathcal{C}_U\Big[\mathcal{C}_g\big((1+2\mathcal{C}_\kappa)r + 2\tilde{\mathcal{C}_\kappa}\big) + \tilde{\mathcal{C}_g}\Big]$$
$$+ 2a\ \mathcal{C}_U\ Tr(Q)\big(\mathcal{C}_\sigma r + \tilde{\mathcal{C}_\sigma}\big) + 2m\mathcal{C}_U\Big[\textstyle\sum_{k=1}^m \beta_k r + \sum_{k=1}^m \tilde{\beta_k}\Big]\Big\} \leq r$$

and

$$\nu = 7\Big\{(1 + 12a^2\mathcal{C}_U\mathcal{C}_W)(N_1 + N_2 + N_3 + N_4 + N_5) + 2a^3\mathcal{N}\mathcal{G}\Big\}$$

where

$$\begin{aligned}
N_1 &= \mathcal{N}a^2\|x_0\|^2 + 2(\mathcal{N}a^2\tilde{\mathcal{C}_h} + \mathcal{C}_U\mathcal{C}_h)\\
N_2 &= 2a^2\big[2\mathcal{N}a\big(\mathcal{C}_f r + \tilde{\mathcal{C}_f}\big) + \mathcal{C}_U\mathcal{C}_f\big]\\
N_3 &= 2a^3\big[2\mathcal{N}a\big(\mathcal{C}_g\big((1+2\mathcal{C}_\kappa)r + 2\tilde{\mathcal{C}_\kappa}\big) + \tilde{\mathcal{C}_g}\big) + \mathcal{C}_U\mathcal{C}_g(1 + \mathcal{C}_\kappa)\big]\\
N_4 &= 2a\big[2\mathcal{N}a\ Tr(Q)\big(\mathcal{C}_\sigma r + \tilde{\mathcal{C}_\sigma}\big) + \mathcal{C}_U\ Tr(Q)\mathcal{C}_\sigma\big]\\
N_5 &= 2m\Big[2\mathcal{N}a^2\Big(\sum_{k=1}^m \beta_k r + \sum_{k=1}^m \tilde{\beta_k}\Big) + \mathcal{C}_U\sum_{k=1}^m \beta_k\Big].
\end{aligned}$$

3 Controllability Result

Theorem: 3.1 *If the conditions* $(H1)-(H9)$ *are satisfied and if* $0 \leq \nu < 1$, *then the system* (1.1) *is controllable on* J.

Proof: Using the hypothesis $(H2)$ for an arbitrary function $x(\cdot)$, define the control

$$\begin{aligned} u(t) &= W^{-1}\Big[x_1 - U(a,0;x)\big[x_0 - h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big)\big] - \int_0^a U(a,s;x)f(s,x(s))ds \\ &\quad - \int_0^a U(a,s;x)\Big[\int_0^s g\Big(s,\eta,x(\eta),\int_0^\eta \kappa\big(\eta,\gamma,x(\gamma)\big)d\gamma\Big)d\eta\Big]ds \\ &\quad - \int_0^a U(a,s;x)\sigma(s,x(s))dw(s) - \sum_{0<\tau_k<a} U(a,\tau_k;x)I_k(x(\tau_k^-))\Big](t). \qquad (3.1) \end{aligned}$$

Let $\mathcal{Y}_r$ be a nonempty closed subset of $\mathcal{PC}(J,L_2)$ defined by

$$\mathcal{Y}_r = \{x : x \in \mathcal{PC}(J,L_2) | E\|x(t)\|^2 \leq r\}.$$

Consider a mapping $\Phi : \mathcal{Y}_r \to \mathcal{Y}_r$ defined by

$$\begin{aligned} (\Phi x)(t) &= U(t,0;x)\big[x_0 - h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big)\big] + \int_0^t U(t,s;x)BW^{-1}\Big[x_1 - \\ &\quad U(a,0;x)\big[x_0 - h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big)\big] - \int_0^a U(a,s;x)f(s,x(s))ds \\ &\quad - \int_0^a U(a,s;x)\Big[\int_0^s g\Big(s,\eta,x(\eta),\int_0^\eta \kappa\big(\eta,\gamma,x(\gamma)\big)d\gamma\Big)d\eta\Big]ds \\ &\quad - \int_0^a U(a,s;x)\sigma(s,x(s))dw(s) - \sum_{0<\tau_k<a} U(a,\tau_k;x)I_k(x(\tau_k^-))\Big](s)ds \\ &\quad + \int_0^t U(t,s;x)\Big[\int_0^s g\Big(s,\eta,x(\eta),\int_0^\eta \kappa\big(\eta,\gamma,x(\gamma)\big)d\gamma\Big)d\eta\Big]ds \\ &\quad + \int_0^t U(t,s;x)f(s,x(s))ds + \int_0^t U(t,s;x)\sigma(s,x(s))dw(s) \\ &\quad + \sum_{0<\tau_k<t} U(t,\tau_k;x)I_k(x(\tau_k^-)). \end{aligned}$$

We have to show that by using the above control the operator Φ has a fixed point. Since all the functions involved in the operator are continuous therefore Φ is continuous. For our convenience we take

$$\begin{aligned} V(\mu,x) &= BW^{-1}\Big[x_1 - U(a,0;x)\big[x_0 - h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big)\big] - \int_0^a U(a,s;x)f(s,x(s))ds \\ &\quad - \int_0^a U(a,s;x)\Big[\int_0^s g\Big(s,\eta,x(\eta),\int_0^\eta \kappa\big(\eta,\gamma,x(\gamma)\big)d\gamma\Big)d\eta\Big]ds \\ &\quad - \int_0^a U(a,s;x)\sigma(s,x(s))dw(s) - \sum_{0<\tau_k<a} U(a,\tau_k;x)I_k(x(\tau_k^-))\Big](\mu). \end{aligned}$$

From our assumptions we have

$$\begin{aligned}
E\|V(\mu,x)\|^2 \ \le\ & 7\mathcal{C}_W\Big\{\|x_1\|^2+\mathcal{C}_U(\|x_0\|^2+\tilde{\mathcal{C}_h})+2a^2\mathcal{C}_U(\mathcal{C}_f r+\tilde{\mathcal{C}_f})+2a^3\mathcal{C}_U\Big[\mathcal{C}_g\big((1+2\mathcal{C}_\kappa)r\\
&+2\tilde{\mathcal{C}_\kappa}\big)+\tilde{\mathcal{C}_g}\Big]+2a\mathcal{C}_U Tr(Q)\big(\mathcal{C}_\sigma r+\tilde{\mathcal{C}_\sigma}\big)+2m\mathcal{C}_U\Big[\sum_{k=1}^m\beta_k r+\sum_{k=1}^m\tilde{\beta_k}\Big]\Big\}\\
&:=\mathcal{G}.
\end{aligned}$$

and

$$\begin{aligned}
E\|V(\mu,x)-V(\mu,y)\|^2 \ \le\ & 6\mathcal{C}_W\Big\{\mathcal{N}a^2\|x_0\|^2+2(\mathcal{N}a^2\tilde{\mathcal{C}_h}+\mathcal{C}_U\mathcal{C}_h)+2a^2\Big[2\mathcal{N}a\big(\mathcal{C}_f r+\tilde{\mathcal{C}_f}\big)\\
&+\mathcal{C}_U\mathcal{C}_f\Big]+2a^3\Big[2\mathcal{N}a\Big(\mathcal{C}_g\big((1+2\mathcal{C}_\kappa)r+2\tilde{\mathcal{C}_\kappa}\big)+\tilde{\mathcal{C}_g}\Big)\\
&+\mathcal{C}_U\mathcal{C}_g(1+\mathcal{C}_\kappa)\Big]+2a\Big[2\mathcal{N}aTr(Q)\big(\mathcal{C}_\sigma r+\tilde{\mathcal{C}_\sigma}\big)+\mathcal{C}_U Tr(Q)\mathcal{C}_\sigma\Big]\\
&+2m\Big[2\mathcal{N}a^2\Big(\sum_{k=1}^m\beta_k r+\sum_{k=1}^m\tilde{\beta_k}\Big)+\mathcal{C}_U\sum_{k=1}^m\beta_k\Big]\Big\}\|x-y\|^2\\
\le\ & 6\mathcal{C}_W\Big(N_1+N_2+N_3+N_4+N_5\Big)\|x-y\|^2.
\end{aligned}$$

First we show that the operator Φ maps $\mathcal{Y}_r$ into itself. Now

$$\begin{aligned}
E\|(\Phi x)(t)\|^2 \ \le\ & 7\Big\{E\Big\|U(t,0;x)\big[x_0-h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big)\big]\Big\|^2\\
&+E\Big\|\int_0^t U(t,\mu;x)V(\mu,x)d\mu\Big\|^2+E\Big\|\int_0^t U(t,s;x)f(s,x(s))ds\Big\|^2\\
&+E\Big\|\int_0^t U(t,s;x)\Big[\int_0^s g\Big(s,\eta,x(\eta),\int_0^\eta \kappa\big(\eta,\gamma,x(\gamma)\big)d\gamma\Big)d\eta\Big]ds\Big\|^2\\
&+E\Big\|\int_0^t U(t,s;x)\sigma(s,x(s))dw(s)\Big\|^2+E\Big\|\sum_{0<\tau_k<t}U(t,\tau_k;x)I_k(x(\tau_k^-))\Big\|^2\Big\}\\
\le\ & 7\Big\{\mathcal{C}_U(\|x_0\|^2+\tilde{\mathcal{C}_h})+a^2\mathcal{C}_U\mathcal{G}+2a^2\mathcal{C}_U(\mathcal{C}_f r+\tilde{\mathcal{C}_f})+2a^3\mathcal{C}_U\Big[\mathcal{C}_g\big((1+2\mathcal{C}_\kappa)r\\
&+2\tilde{\mathcal{C}_\kappa}\big)+\tilde{\mathcal{C}_g}\Big]+2a\mathcal{C}_U Tr(Q)\big(\mathcal{C}_\sigma r+\tilde{\mathcal{C}_\sigma}\big)+2m\mathcal{C}_U\Big[\sum_{k=1}^m\beta_k r+\sum_{k=1}^m\tilde{\beta_k}\Big]\Big\}\\
\le\ & r.
\end{aligned}$$

From $(H9)$ we get $E\|(\Phi x)(t)\|^2\le r$. Hence Φ maps $\mathcal{Y}_r$ into $\mathcal{Y}_r$. Let $x,y\in\mathcal{Y}_r$, then

$$\begin{aligned}
E\|(\Phi x)(t)-(\Phi y)(t)\|^2 \ \le\ & 7\Big\{E\Big\|U(t,0;x)\big[x_0-h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big)\big]\\
&-U(t,0;y)\big[x_0-h\big(t_1,t_2,\cdots,t_p,y(\cdot)\big)\big]\Big\|^2\\
&+E\Big\|\int_0^t\Big[U(t,\mu;x)V(\mu,x)-U(t,\mu;y)V(\mu,y)\Big]d\mu\Big\|^2\\
&+E\Big\|\int_0^t\Big[U(t,s;x)f(s,x(s))-U(t,s;y)f(s,y(s))\Big]ds\Big\|^2
\end{aligned}$$

$$
\begin{aligned}
&+E\Big\|\int_0^t\Big[U(t,s;x)\Big[\int_0^s g\Big(s,\eta,x(\eta),\int_0^\eta\kappa\big(\eta,\gamma,x(\gamma)\big)d\gamma\Big)d\eta\Big]\\
&-U(t,s;y)\Big[\int_0^s g\Big(s,\eta,y(\eta),\int_0^\eta\kappa\big(\eta,\gamma,y(\gamma)\big)d\gamma\Big)d\eta\Big]\Big]ds\Big\|^2\\
&+E\Big\|\int_0^t\Big[U(t,s;x)\sigma(s,x(s))-U(t,s;y)\sigma(s,y(s))\Big]dw(s)\Big\|^2\\
&+E\Big\|\sum_{0<\tau_k<t}\Big[U(t,\tau_k;x)I_k(x(\tau_k^-))-U(t,\tau_k;y)I_k(y(\tau_k^-))\Big]\Big\|^2\Big\}\\
\leq\ & 7\Big\{(1+12a^2\mathcal{C}_U\mathcal{C}_W)(N_1+N_2+N_3+N_4+N_5)+2a^3\mathcal{N}\mathcal{G}\Big\}\|x-y\|^2\\
\leq\ & \nu\|x-y\|^2.
\end{aligned}
$$

Since $\nu<1$, the mapping Φ is a contraction and hence by Banach fixed point theorem there exists a unique fixed point $x\in\mathcal{Y}_r$ such that $(\Phi x)(t)=x(t)$. This fixed point is then the solution of the system (1.1) and clearly, $x(a)=(\Phi x)(a)=x_1$ which implies that the system (1.1) is controllable on J.

4 Stochastic Quasilinear Delay Integrodifferential System

In this section we consider the following class of impulsive stochastic quasilinear delay integrodifferential system with nonlocal conditions

$$
\begin{aligned}
dx(t)&=\Big[A(t,x)x(t)+Bu(t)+f(t,x(\alpha(t)))+\int_0^t g\Big(t,s,x(\beta(s)),\int_0^s\kappa\big(s,\eta,x(\gamma(\eta))\big)d\eta\Big)ds\Big]dt\\
&\quad+\sigma(t,x(\rho(t)))dw(t),\quad t\in J:=[0,a],\quad t\neq\tau_k,\\
\triangle x(\tau_k)&=x(\tau_k^+)-x(\tau_k^-)=I_k(x(\tau_k^-)),\qquad k=1,2,\cdots,m,\\
x(0)&+h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big)=x_0.
\end{aligned}\tag{4.1}
$$

where A,B,f,g,κ,h,σ are as before and α,β,γ,ρ are continuous on J. Assume the following additional condition

(H10) The function $\alpha,\beta,\gamma,\rho:J\to J$ are absolutely continuous and there exist constants $\delta_1,\delta_2,\delta_3,\delta_4>0$ such that $\alpha'(t)\geq\delta_1$, $\beta'(t)\geq\delta_2$, $\gamma'(t)\geq\delta_3$, $\rho'(t)\geq\delta_4$ for $0\leq t\leq a$.

(H11) There exists a constant $r>0$ such that

$$
7\Big\{\mathcal{C}_U(\|x_0\|^2+\tilde{\mathcal{C}}_h)+a^2\mathcal{C}_U\mathcal{G}^*+2a^2\mathcal{C}_U(\mathcal{C}_f^*r+\tilde{\mathcal{C}}_f)+2a^3\mathcal{C}_U\Big[\mathcal{C}_g^*\big((1+2\mathcal{C}_\kappa^*)r+2\tilde{\mathcal{C}}_\kappa\big)+\tilde{\mathcal{C}}_g\Big]
+2a\mathcal{C}_UTr(Q)\big(\mathcal{C}_\sigma^*r+\tilde{\mathcal{C}}_\sigma\big)+2m\mathcal{C}_U\Big[\textstyle\sum_{k=1}^m\beta_kr+\sum_{k=1}^m\tilde{\beta}_k\Big]\Big\}\leq r
$$

and

$$\nu^* = 7\Big\{(1+12a^2\mathcal{C}_U\mathcal{C}_W)(N_1+N_2^*+N_3^*+N_4^*+N_5)+2a^3\mathcal{N}\mathcal{G}^*\Big\}$$

where

$$\begin{aligned}
N_1 &= \mathcal{N}a^2\|x_0\|^2+2(\mathcal{N}a^2\tilde{\mathcal{C}}_h+\mathcal{C}_U\mathcal{C}_h)\\
N_2^* &= 2a^2\Big[2\mathcal{N}a\big(\mathcal{C}_f^*r+\tilde{\mathcal{C}}_f\big)+\mathcal{C}_U\mathcal{C}_f^*\Big]\\
N_3^* &= 2a^3\Big[2\mathcal{N}a\Big(\mathcal{C}_g^*\big((1+2\mathcal{C}_\kappa^*)r+2\tilde{\mathcal{C}}_\kappa\big)+\tilde{\mathcal{C}}_g\Big)+\mathcal{C}_U\mathcal{C}_g^*(1+\mathcal{C}_\kappa^*)\Big]\\
N_4^* &= 2a\Big[2\mathcal{N}a\ Tr(Q)\big(\mathcal{C}_\sigma^*r+\tilde{\mathcal{C}}_\sigma\big)+\mathcal{C}_U\ Tr(Q)\mathcal{C}_\sigma^*\Big]\\
N_5 &= 2m\Big[2\mathcal{N}a^2\Big(\sum_{k=1}^m\beta_kr+\sum_{k=1}^m\tilde{\beta}_k\Big)+\mathcal{C}_U\sum_{k=1}^m\beta_k\Big]\\
\mathcal{G}^* &= 7\mathcal{C}_W\Big\{\|x_1\|^2+\mathcal{C}_U(\|x_0\|^2+\tilde{\mathcal{C}}_h)+2a^2\mathcal{C}_U(\mathcal{C}_f^*r+\tilde{\mathcal{C}}_f)+2a^3\mathcal{C}_U\Big[\mathcal{C}_g^*\big((1+2\mathcal{C}_\kappa^*)r\\
&\quad +2\tilde{\mathcal{C}}_\kappa\big)+\tilde{\mathcal{C}}_g\Big]+2a\ \mathcal{C}_U\ Tr(Q)\big(\mathcal{C}_\sigma^*r+\tilde{\mathcal{C}}_\sigma\big)+2m\mathcal{C}_U\Big[\sum_{k=1}^m\beta_kr+\sum_{k=1}^m\tilde{\beta}_k\Big]\Big\}\\
\mathcal{C}_f^* &= \frac{\mathcal{C}_f}{\delta_1^2},\ \mathcal{C}_g^*=\frac{\mathcal{C}_g}{\delta_2^2},\ \mathcal{C}_\kappa^*=\frac{\mathcal{C}_\kappa}{\delta_3^2},\ \mathcal{C}_\sigma^*=\frac{\mathcal{C}_\sigma}{\delta_4^2}.
\end{aligned}$$

The mild solution of the system (4.1) is given by

$$\begin{aligned}
x(t) &= U(t,0;x)\big[x_0-h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big)\big]+\int_0^t U(t,s;x)[Bu(s)+f(s,x(\alpha(s)))]ds\\
&\int_0^t U(t,s;x)\Big[\int_0^s g\Big(s,\eta,x(\beta(\eta)),\int_0^\eta\kappa\big(\eta,\xi,x(\gamma(\xi))\big)d\xi\Big)d\eta\Big]ds\\
&+\int_0^t U(t,s;x)\sigma(s,x(\rho(s)))dw(s)+\sum_{0<\tau_k<t}U(t,\tau_k;x)I_k(x(\tau_k^-)),\ t\in J. \quad (4.2)
\end{aligned}$$

Theorem: 4.1 *If the conditions from $(H1)-(H8)$, $(H10)$ and $(H11)$ are satisfied and if $0\le\nu^*<1$, then the system (4.1) is controllable on J.*

Proof: Using the hypothesis $(H2)$ for an arbitrary function $x(\cdot)$, define the control

$$\begin{aligned}
u(t) &= W^{-1}\Big[x_1-U(a,0;x)\big[x_0-h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big)\big]-\int_0^a U(a,s;x)f(s,x(\alpha(s)))ds\\
&-\int_0^a U(a,s;x)\Big[\int_0^s g\Big(s,\eta,x(\beta(\eta)),\int_0^\eta\kappa\big(\eta,\xi,x(\gamma(\xi))\big)d\xi\Big)d\eta\Big]ds\\
&-\int_0^a U(a,s;x)\sigma(s,x(\rho(s)))dw(s)-\sum_{0<\tau_k<a}U(a,\tau_k;x)I_k(x(\tau_k^-))\Big](t).
\end{aligned}$$

Let $\mathcal{Y}_r$ be a nonempty closed subset of $\mathcal{PC}(J,L_2)$ defined by

$$\mathcal{Y}_r=\{x:x\in\mathcal{PC}(J,L_2)|E\|x(t)\|^2\le r\}.$$

Consider the nonlinear operator $\psi : \mathcal{Y}_r \to \mathcal{Y}_r$ defined by

$$\begin{aligned}
(\psi x)(t) &= U(t,0;x)\big[x_0 - h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big)\big] + \int_0^t U(t,s;x)BW^{-1}\Big[x_1 - \\
&\quad U(a,0;x)\big[x_0 - h\big(t_1,t_2,\cdots,t_p,x(\cdot)\big)\big] - \int_0^a U(a,s;x)f(s,x(\alpha(s)))ds \\
&\quad - \int_0^a U(a,s;x)\Big[\int_0^s g\Big(s,\eta,x(\beta(\eta)),\int_0^\eta \kappa\big(\eta,\xi,x(\gamma(\xi))\big)d\xi\Big)d\eta\Big]ds \\
&\quad - \int_0^a U(a,s;x)\sigma(s,x(\rho(s)))dw(s) - \sum_{0<\tau_k<a} U(a,\tau_k;x)I_k(x(\tau_k^-))\Big](s)ds \\
&\quad + \int_0^t U(t,s;x)\Big[\int_0^s g\Big(s,\eta,x(\beta(\eta)),\int_0^\eta \kappa\big(\eta,\xi,x(\gamma(\xi))\big)d\xi\Big)d\eta\Big]ds \\
&\quad + \int_0^t U(t,s;x)f(s,x(\alpha(s)))ds + \int_0^t U(t,s;x)\sigma(s,x(\rho(s)))dw(s) \\
&\quad + \sum_{0<\tau_k<t} U(t,\tau_k;x)I_k(x(\tau_k^-)).
\end{aligned}$$

Obviously ψ maps $\mathcal{Y}_r$ into itself by $(H11)$ and

$$\begin{aligned}
E\|\psi x(t) - \psi y(t)\|^2 &\le 7\Big\{(1+12a^2\mathcal{C}_U\mathcal{C}_W)(N_1+N_2^*+N_3^*+N_4^*+N_5)+2a^3\mathcal{N}\mathcal{G}^*\Big\}\|x-y\|^2 \\
&\le \nu^*\|x-y\|^2.
\end{aligned}$$

Since $\nu^* < 1$, the mapping ψ is a contraction and hence by Banach fixed point theorem there exists a unique fixed point $x \in \mathcal{Y}_r$ such that $(\psi x)(t) = x(t)$. This fixed point is then the mild solution of the system (4.1) and clearly, $x(a) = (\psi x)(a) = x_1$ which implies that the system (4.1) is controllable on J.

5 Example

Consider the following partial integrodifferential equation of the form

$$\begin{aligned}
\partial z(t,y) &= \Bigg(\frac{\partial^3}{\partial y^3}z(t,y) + z(t,y)\frac{\partial}{\partial y}z(t,y) + \mu(t,y) + \frac{1}{4}(1+e^{-t})\sin z(t,y) \\
&\quad + \frac{1}{t(1+t)(1+t^2)}\Bigg[\int_0^t \Big[\sin z(s,y) + z(s,y)\int_0^s e^{-z(\eta,y)}d\eta\Big]ds\Bigg]\Bigg)\partial t \\
&\quad + \frac{1}{4}e^{-2t}(t+2)z(t,y)dw(t), \qquad y \in R,\ t \in J := [0,1],\ t \neq \tau_k, \\
z(0,y) &+ \sum_{i=1}^p \frac{1}{k_i}\int_{t_i}^{t_i+k_i} h_i\ z(\eta,y)d\eta = z_0(y), \\
\triangle z|_{t=\tau_k} &= I_k(z(y)) = (\alpha_k|z(y)| + \tau_k)^{-1}, \quad k = 1,2,\cdots,m. \qquad (5.1)
\end{aligned}$$

where $k_i,\ h_i, 1 \leq i \leq p$ are constants such that $k_i > 0, t_i + k_i \leq 1$ and the constants $\alpha_k, k = 1, 2, \cdots, m.$ are small.

For every real s we introduce a Hilbert space $H^s(R)$ as follows [31]. Let $z \in L^2(R)$ and set

$$\|z\|_s = \Big(\int_R (1+\xi^2)^s |\widehat{z}(\xi)|^2 d\xi \Big)^{1/2},$$

where $\widehat{z}$ is the Fourier transform of z. The linear space of functions $z \in L^2(R)$ for which $\|z\|_s$ is finite is a pre-Hilbert space with the inner product

$$(z, y)_s = \Big(\int_R (1+\xi^2)^s \widehat{z}(\xi) \overline{\widehat{y}(\xi)} d\xi \Big)^{1/2}.$$

The completion of this space with respect to the norm $\|\cdot\|_s$ is a Hilbert space which we denote by $H^s(R)$. It is clear that $H^0(R) = L^2(R)$.

Take $H = U = K = L^2(R) = H^0(R)$ and $Y = H^s(R), s \geq 3$. Define an operator A_0 by $D(A_0) = H^3(R)$ and $A_0 z = D^3 z$ for $z \in D(A_0)$ where $D = d/dy$. Then A_0 is the inifinitesimal generator of a C_0-group of isometries on H. Next we define for every $v \in Y$ an operator $A_1(v)$ by $D(A_1(v)) = H^1(R)$ and $z \in D(A_1(v))$, $A_1(v)z = vDz$. Then for every $v \in Y$ the operator $A(v) = A_0 + A_1(v)$ is the infinitesimal generator of C_0 semigroup $U(t, 0; v)$ on H satisfying $\|U(t, 0; v)\| \leq e^{\beta t}$ for every $\beta \geq c_0 \|v\|_s$, where c_0 is a constant independent of $v \in Y$. Let $\mathcal{Y}_r$ be the ball of radius $r > 0$ in Y and it is proved that the family of operators $A(v), v \in \mathcal{Y}_r$, satisfies the conditions $(E1)-(E4)$ and $(H1)$ (see [31]). Put $x(t) = z(t, \cdot)$ and $u(t) = \mu(t, \cdot)$ where $\mu : J \times R \to R$ is continuous,

$$f(t, x(t)) = \frac{1}{4}(1+e^{-t}) \sin z(t, y) \quad , \quad \sigma(t, x(t)) = \frac{1}{4} e^{-2t}(t+2) z(t, y),$$

$$h(t_1, t_2, \cdots, t_p, x(\cdot)) = \sum_{i=1}^{p} \frac{1}{k_i} \int_{t_i}^{t_i+k_i} h_i \ z(\eta, y) d\eta,$$

$$\int_0^t g\big(t, s, x(s), \int_0^s \kappa(s, \eta, x(\eta)) d\eta\big) ds = \frac{1}{t(1+t)(1+t^2)} \times$$
$$\times \Big[\int_0^t \Big[\sin z(s, y) + z(s, y) \int_0^s e^{-z(\eta, y)} d\eta \Big] ds \Big].$$

With this choice of $A(v)$, I_k, f, g, h, σ, $B = I$, the identity operator and $w(t)$, one dimensional standard wiener process, we see that (5.1) is an abstract formulation of the system (1.1). Further we have

$$\Bigg\| \frac{1}{t(1+t)(1+t^2)} \Bigg[\int_0^t \Big[\sin z(s, y) + z(s, y) \int_0^s e^{-z(\eta, y)} d\eta \Big] ds \Bigg] \Bigg\| \leq \frac{1}{1+t^2} \|z\|.$$

Assume that the operator $W : L^2(J, U)/KerW \to H$ defined by

$$Wu = \int_0^1 U(1, s; x) \mu(s, \cdot) ds$$

has an inverse operator and satisfies condition $(H2)$ for every $x \in \mathcal{Y}_r$.
Further other assumptions $(H3) - (H9)$ are obviously satisfied and it is possible to choose k_i, h_i, α_k in such a way that the constant $\nu < 1$. Hence, by Theorem 3.1, the system (5.1) is controllable on J.

6 Conclusion

Our paper contains some controllability results for impulsive stochastic quasilinear systems. The result proves that the Banach fixed point theorem can effectively be used in control problems to obtain sufficient conditions. We can extend the controllability result for neutral impulsive stochastic quasilinear systems with different types of delays in our subsequent papers.

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References

[1] H. Amann, Quasilinear evolution equations and parabolic systems, *Trans. Amer. Math. Soc.*, 29 (1986), 191-227.

[2] A. Anguraj and A. Vinodkumar, Existence, uniqueness and stability results of impulsive stochastic semilinear neutral functional differential equations with infinite delays, *Electron. J. Qual. Theory Differ. Equ.*, 67 (2009), 1-13.

[3] D. Bahuguna, Quasilinear integrodifferential equations in Banach spaces, *Nonlinear Anal.*, 24 (1995), 175-183.

[4] K. Balachandran and S. Karthikeyan, Controllability of stochastic integrodifferential systems, *Int. J. Control*, 80 (2007), 486-491.

[5] K. Balachandran and K. Uchiyama, Existence of solutions of quasilinear integrodifferential equations with nonlocal condition, *Tokyo J. Math.*, 23 (2000), 203-210.

[6] K. Balachandran, P. Balasubramaniam and J. P. Dauer, Controllability of quasilinear delay systems in Banach spaces, *Optim. Contr. Appl. Meth.*, 16 (1995), 283-290.

[7] K. Balachandran, S. Karthikeyan and J. H. Kim, Controllability of semilinear stochastic integrodifferential systems, *Kybernetika*, 43 (2007), 31-44.

[8] K. Balachandran, J. Y. Park and E. R. Anandhi, Local controllability of quasilinear integrodifferential evolution systems in Banach spaces, *J. Math. Anal. Appl.*, 258 (2001), 309-319.

[9] K. Balachandran, J. Y. Park and S. H. Park, Controllability of nonlocal impulsive quasilinear integrodifferential systems in Banach spaces, *Rep. Math. Phys.*, 65 (2010), 247-257.

[10] L. Byszewski, Theorems about the existence and uniqueness of continuous solution of nonlocal problem for nonlinear hyperbolic equation, *Appl. Anal.*, 40 (1991), 173-180.

[11] M. Chandrasekaran, Nonlocal Cauchy problem for quasilinear integrodifferential equations in Banach spaces, *Electron. J. Differential Equations*, 33 (2007), 1-6.

[12] G. Da Prato and J. Zabczyk, *Stochastic Equations in Infinite Dimensions*, Cambridge University Press, Cambridge, 1992.

[13] J. P. Dauer and N. I. Mahmudov, Controllability of stochastic semilinear functional differential equations in Hilbert spaces, *J. Math. Anal. Appl.*, 290 (2004), 373-394.

[14] M. B. Dhakne and B. G. Pachpatte, On a quasilinear functional integrodifferential equations in a Banach space, *Indian J. Pure. Appl. Math.*, 25 (1994), 275-287.

[15] Q. Dong, G. Li and J. Zhang, Quasilinear nonlocal integrodifferential equations in Banach spaces, *Electron. J. Differential Equations*, 19 (2008), 1-8.

[16] S. Kato, Nonhomogeneous quasilinear evolution equations in Banach spaces, *Nonlinear Anal.*, 9 (1985), 1061-1071.

[17] T. Kato, Quasilinear equations of evolution with applications to partial differential equations, *Lecture Notes in Math.*, 448 (1975), 25-70.

[18] T. Kato, Abstract evolution equation linear and quasilinear, revisited, *Lecture Notes in Math.*, 1540 (1993), 103-125.

[19] J. Klamka, Stochastic controllability of linear systems with state delays, *Int. J. Appl. Math. Comput. Sci.*, 17 (2007), 5-13.

[20] J. Klamka, Stochastic controllability of linear systems with delay in control, *Bull. Pol. Ac.: Tech.*, 55 (2007), 23-29.

[21] J. Klamka, Stochastic controllability of systems with variable delay in control, *Bull. Pol. Ac.: Tech.*, 56 (2008), 279-284.

[22] J. Klamka, Constrained controllability of semilinear systems with delayed controls, *Bull. Pol. Ac.: Tech.*, 56 (2008), 333-337.

[23] J. Klamka, Stochastic controllability and minimum energy control of systems with multiple delays in control, *Appl. Math. Comput.* , 206 (2008), 704-715.

[24] J. Klamka, Stochastic controllability of systems with multiple delays in control, *Int. J. Appl. Math. Comput. Sci.*, 19 (2009), 39-47.

[25] G. E. Ladas and V. Lakshmikhantham, *Differential Equations in Abstract Spaces*, Academic Press, New York, 1972.

[26] V. Lakshmikantham, D. D. Bainov and P. S. Simeonov, *Theory of Impulsive Differential Equations*, World Scientific, Singapore, 1989.

[27] N. I. Mahmudov, Controllability of semilinear stochastic systems in Hilbert spaces, *J. Math. Anal. Appl.*, 288 (2003), 197 -211.

[28] H. Oka, Abstract quasilinear Volterra integrodifferential equations, *Nonlinear Anal.*, 28 (1997), 1019-1045.

[29] H. Oka and N. Tanaka, Abstract quasilinear integrodifferential equations of hyperbolic type, *Nonlinear Anal.*, 29 (1997), 903-925.

[30] J. Y. Park, P. Balasubramaniam and N. Kumerasan, Controllability for neutral stochastic functional integrodifferential infinite delay systems in abstract space, *Numer. Funct. Anal. Optim.*, 28 (2007), 1369–1386.

[31] A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer, Berlin, 1983.

[32] R. Sakthivel, N. I. Mahmudov and Sang-Gu Lee, Controllability of nonlinear impulsive stochastic systems, *Int. J. Control*, 82 (2009), 801-807.

[33] A. M. Samoilenko and N. A. Perestyuk, *Impulsive Differential Equations*, World Scientific, Singapore, 1995.

[34] N. Sanekata, Abstract quasilinear equations of evolution in nonreflexive Banach spaces, *Hiroshima Math. J.*, 19 (1989), 109-139.

[35] R. Subalakshmi and K. Balachandran, Approximate controllability of nonlinear stochastic impulsive integrodifferential systems in Hilbert spaces, *Chaos, Solitons and Fractals*, 42 (2009), 2035-2046.

[36] R. Subalakshmi, K. Balachandran and J. Y. Park, Controllability of semilinear stochastic functional integrodifferential systems in Hilbert spaces, *Nonlinear Anal. Hybrid Syst.*, 3 (2009), 39-50.

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