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Secondary Teachers' Noticing of Students' Mathematical Thinking as They Participate in a Professional Development Program Centered on Task-Based Student Interviews

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Secondary Teachers' Noticing of Students' Mathematical Thinking as They Participate in a Professional Development Program Centered on Task-Based Student Interviews

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ABSTRACT

SECONDARY TEACHERS' NOTICING OF STUDENTS' MATHEMATICAL THINKING AS THEY PARTICIPATE IN A PROFESSIONAL DEVELOPMENT PROGRAM CENTERED ON TASK-BASED STUDENT INTERVIEWS

by Gurkan Kose

Teacher's noticing of students' mathematical thinking has been an important focus of research in the past two decades (e.g., Jacobs et al., 2010; Sherin et al., 2011). Noticing matters, but it is not an end in itself (Schoenfeld, 2011). It is operationalized within the context of teachers' dispositions and knowledge which shape decisions teachers make while responding to student thinking and planning the next instructional steps. In order for teachers to adapt productive beliefs about how children learn and shift to student-centered instruction, they need to acknowledge the importance of understanding students' existing conceptions of mathematical ideas (Carpenter & Lehrer, 1999). A professional development (PD) program with a central focus of task-based student interviews can potentially improve their noticing of student thinking. In this study, I report on the experiences of three middle school mathematics teachers who participated in such a program. I used multiple-case study methodology and examined teachers' written responses to video-based noticing prompts as well as their discussions of selected interview clips shown during the PD sessions. The results indicated that teachers initially focused on interviewer actions and over time, attended to and interpreted students' thinking in a more comprehensive manner. This study supported others' findings (e.g., Krupa et al, 2017) that teachers did not score high in responding to students' mathematical thinking. Lastly, my study underlined the critical role of the PD facilitator.

Keywords: Teacher noticing, task-based student interviews, professional development.

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Chapter 1: Introduction

Calls for reform in mathematics education highlight the importance of using student thinking as a basis for instruction (Ferrini-Mundy & Martin, 2000; Hiebert, 2003). More recently, the National Council of Teachers of Mathematics (NCTM) suggested the following in its landmark publication, Principles to Actions (2014):

> Effective teaching involves finding the mathematics in students' comments and actions, considering what students appear to know in light of intended learning goals and progressions, and determining how to give the best response and support to students on the basis of their current understanding. (p.56)

Being responsive to student thinking requires intentional effort from teachers. This includes first attending to students' mathematical thinking and solution strategies. Teachers then interpret what the students do and do not understand. Based on those interpretations, teachers offer questions and tasks that further probe or extend students' thinking. Jacobs and colleagues (Jacobs et al., 2010) define this set of three interrelated skills, attending, interpreting, and deciding how to respond, as *professional noticing of children's mathematical thinking*.

Previous research suggests that teacher noticing is "trainable" (Krupa et al., 2017; Sherin et al., 2011a). Nonetheless, teachers need genuine windows into individual students' thinking, and this is not always possible in the presence of other students, as in the classroom (Heng & Sudarshan, 2013). One way to address this challenge is by conducting one-on-one interviews with students as they engage in solving mathematics problems (Buschman, 2001). During those interviews, students' conceptions of mathematics and their thinking can surface (Copeland, 1984). As a result, teachers can better understand how students approach problems and make solution attempts (Heng & Sudarshan, 2013). Furthermore, Jacobs and colleagues (Jacobs et al., 2006) found that when teachers conduct student interviews and also share and discuss them with other teachers, they tend to improve their noticing and begin to adopt the productive beliefs suggested by reform documents.

Supporting teachers as they develop noticing skills is a challenging task. Generally, participation in carefully designed, coherent, content-focused professional development (PD) that provides active learning opportunities over long periods of time may result in changes in teacher practice and improve student learning outcomes (Darling-Hammond et al., 2017; Desimone, 2009). With these tenets in mind and with the goal of engaging teachers in student interviews and then discussions about those interviews, I designed a PD program centered on task-based student interviews. This study explores the engagement of teachers as they participate in this PD program.

Professional Development on Teacher Noticing

One way to support teachers in noticing is to have them participate in PD programs that give them the opportunity to see and acknowledge students' thinking. There have been efforts to investigate the complex ways in which teachers perceive, interpret, and respond to students' thinking in PD settings (Sherin et al., 2011a). While some of these studies used video-based discussions in their PD programs as a tool to improve teacher noticing (Star & Strickland, 2008; van Es & Sherin, 2006), others used classroom artifacts to offer teachers windows into student thinking (Goldsmith & Seago, 2011).

Additionally, some studies have investigated the use of student interviews to support teachers' as they hone their ability to notice students' thinking. Some results have shown that through task-based interviews, students' conceptions of mathematics can surface and as a result, teachers can better understand the ways in which students approach problems and make solution attempts (Copeland, 1984; Heng & Sudarshan, 2013). There has also been research on the use of student interviews to improve the noticing skills of pre-service teachers, but to date, this is limited to only two studies that took place in methods courses (Krupa et al., 2017; Lesseig et al., 2016).

In terms of using task-based interviews in a PD setting, some researchers have used student interviews to help in-service teachers (ISTs) assess student understanding in depth (Jacobs et al., 2006). My study aimed to extend these studies on in-service mathematics teachers using videos of their own task-based interviews. My goal was to determine how middle school teachers engaged with a PD program that used task-based interviews as a central focus.

Significance

This study aims to fill a gap in the literature related to the secondary in-service mathematics teachers' noticing of students' mathematical thinking. To conduct the research, I first developed a PD program that had teachers conduct task-based interviews. This PD program involved three middle school mathematics teachers who met over five weeks to discuss videos of their task-based interviews. I modeled the research on professional noticing of children's mathematical thinking framework and used metrics to relate to attending, interpreting, and responding to help determine patterns in teachers' noticing. My aim was to potentially inform other researchers and practitioners in developing effective programs to support teachers' noticing.

I worked with three teachers and the small size of the group allowed me to focus my attention on how they noticed student thinking. The case study methodology allowed me to dive deeper into the specific experiences of each teacher and use qualitative tools that helped me understand beyond simple cause and effect relationships. The goal of the research was to investigate the role of task-based student interviews and collaborative discussions of the interview clips on in-service middle school mathematics teachers' situation-specific skills, namely, attending to, interpreting, and deciding how to respond to students' mathematical thinking.

In their meta-analysis of studies on teachers' noticing, Stahnke and colleagues (2016) found that about a third of the studies investigated secondary school teachers' noticing. My study added to that limited body of research on secondary teachers by offering another glimpse into the use of the professional noticing framework in a professional development setting. The objective of this study was to investigate the following research question: "How does a professional development program centered on task-based student interviews foster secondary in-service teachers' noticing of students' mathematical thinking?"

In the following chapters, I first review the literature on PD, teacher noticing, and the previous efforts to support teachers' noticing of students' thinking. Second, I present the methods I used to collect and analyze data. Next, I describe the patterns in each teachers' noticing and describe the results of a cross-case analysis that presents an overall picture of those patterns. Lastly, I outline the significance and limitations of the study in the context of the existing literature by outlining its implications for research and practice.

Chapter 2: Review of the Literature

This literature review is based on my interest in teacher noticing of students' mathematical thinking, which has been an important focus of research in the past two decades. Noticing matters, however it is not an end in itself (Schoenfeld, 2011). It is operationalized within the context of teachers' dispositions and knowledge that shapes the nature of the decisions teachers make while responding to student thinking and planning the next instructional steps. In order for teachers to adapt productive beliefs about how children learn and shift to student-centered instruction, they need to acknowledge the importance of understanding students' existing conceptions of mathematical ideas (Carpenter & Lehrer, 1999). This review is an attempt to present the methods, theories, and findings of studies on professional development and teacher noticing in order to inform the design of a professional development program that might potentially complement the efforts to improve teacher noticing. The preliminary research question investigated is "How can a professional development program centered on task-based student interviews foster secondary in-service teachers' noticing of students mathematical thinking?"

Since the publication of the report A Nation at Risk (NCEI, 1983) several calls have been made about the importance of rigor in mathematics classrooms (Crosswhite et al., 1989; Ferrini-Mundy & Martin, 2000; Hiebert, 2003). For instance, the widespread adoption of the Common Core State Standards in Mathematics (CCSSM) in recent years brings with it some essential shifts in the instructional practices of mathematics teachers (Kanold & Larson, 2012). The standards call for teaching mathematics at a conceptual level rather than mere knowledge transmission portrayed as the common "script" by Hiebert and Stigler (2009) in which the teacher is at the center of instruction and the interactions are limited to knowledge transmission. Calls for reform, including the CCSSM, push teachers to reconsider their assumptions about the teaching and learning of mathematics. In light of a growing body of research, teachers are expected to leverage students' intuitive approaches to solving problems to build the desired knowledge about them (Hiebert, 2003; NCTM, 2014). Teachers must acknowledge the intuitive ways in which students approach problems and build their instruction on those (Carpenter & Lehrer, 1999).

In this literature review, I first synthesize relevant research on effective professional development (PD). Next, I present an overview of professional vision (Goodwin, 1994) with a focus on teacher noticing. Although I list different ways in which noticing is defined, for this study, I use the professional noticing of children's mathematical thinking framework. Jacobs and colleagues (2010) define professional noticing as "a set of three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings" (p.172). Third, I describe different PD efforts that are designed to improve teacher noticing. I conclude the review by describing a conceptualization of teacher competence that links teachers' knowledge to their performance in the classroom.

Teacher Professional Development

Teachers play a central role in student learning (Nye et al., 2004). The effects of well-prepared teachers on student achievement can be stronger than the influences of student background factors, such as poverty, language background, and minority status (Lynch et al., 2019). Teachers are at the center of all reform movements and their training is essential for the desired goals of those movements to find their place in the classrooms (Darling-Hammond et al., 2017).

Darling-Hammond and colleagues (Darling-Hammond et al., 2017) reviewed an extensive range of literature published in the past 30 years and coded each one to identify the elements of effective PD models. They define effective PD as “structured professional learning that results in changes in teacher practices and improvements in student learning outcomes” (p.2). Despite the importance and value of teacher learning, the effectiveness of professional development programs offered to teachers is questionable. There is no consensus on how to measure effectiveness of professional development programs. I will talk more about this later. The Teaching and Learning International Survey of 2013 (TALIS) reports on the perspectives of teachers and principals in schools from 34 jurisdictions across the globe (Burns & Darling-Hammond, 2014). The report provides important data on the current status of teaching in different schools and implications for the future regarding the quality of education, support and development of the teaching force, and effective design of policy. Teachers surveyed in TALIS mentioned two major barriers to their professional learning: (1) lack of time built in the school day, (2) lack of professional development that is relevant to their daily work. Many schools organize information sessions, bring experts, and expect teachers to improve their content knowledge as well as their instructional practices. These typical, one-shot workshops usually are centered around knowledge transmission and sharing best practices. These events are far from providing meaningful and sustained opportunities for professional learning that can lead to instructional improvement (Edwards et al., 2015). There is a growing consensus about the potential change to teacher practices as a result of collaborative, sustained, content-focused teacher professional development (Bailey, 2015; Kanold & Larson, 2012). Drawing on her extensive work on teacher learning and careful review of the literature, Desimone (2009) defines the critical features of professional development: (1) content-focus, (2) active learning, (3) coherence, (4) duration, and (5) collective participation. Below, I discuss each of these features in light of supporting literature.

Content Focus

Content focus has been listed as an influential feature of high quality professional development (e.g., Desimone, 2009; Park et al., 2018). Training that focuses on the teaching and learning of specific content is important for teachers to find professional development relevant to their daily work. Focusing on subject matter content, especially on how students learn it, increases teacher knowledge and skills that lead to improvements in instruction, and eventually results gains in student achievement (Carpenter & Lehrer, 1999; Cohen & Hill, 1998; Desimone et al., 2002).

Cohen and Hill (1998) analyzed the survey results of 975 elementary teachers and the statewide assessment results of their students. The survey questions were designed to examine the ways in which professional development, assessment, and curriculum influences student achievement and teacher practice. Their findings indicate that students of teachers who attended extensive professional development on teaching particular mathematics content outperformed their peers whose teachers did not participate in similar professional development. Also, participation in professional development on the generic features of teaching and pedagogy was not related to student achievement, further making the case for content-specific PD.

Doppelt and colleagues (Doppelt et al., 2009) conducted a two-year quasi-experimental study with 23 8th-grade science teachers to investigate the impact of a PD program on teacher practice and student achievement. Teachers attended five content-based collaborative inquiry (CBCI) sessions supporting their implementation of a new science curriculum during year 1. There were two sessions prior to the implementation, two sessions during, and one final reflection session after the implementation. During those sessions, the participating teachers engaged in the curriculum materials just as their students would. They analyzed student work, discussed students' understandings, and devised instructional strategies to address issues related to student learning. The researchers measured student achievement using two knowledge tests, administered at the end of each year. They used qualitative methods to analyze the PD session videos and the field notes from the informal observations of class activities. The results indicated that students whose teachers participated in the PD outperformed those whose teachers did not This confirms others' findings about the importance of content-focused professional development (Bailey et al., 2015; Briars, 2012; Cohen & Hill, 1998). While the emphasis on content is crucial, the ways in which training sessions are facilitated impacts the effectiveness of professional development and the enactment of ideas suggested. Next, I discuss active learning as one of the important characteristics of effective professional development.

Active Learning

Drawing on the constructivist perspective of cognition (e.g., Fosnot, 2005) for children, one may infer that adults also need meaningful experiences to create their own learning. There is also evidence that the teachings of the constructivist school of thought applies to adults (National Research Council, 2000). This idea implies that participants of professional development must be actively engaged in their own learning as opposed to listening to a presenter, typically portrayed as passive learning.

Banilower and Shimkus (2004) conducted a professional development study that included the observation and analysis of over 2,100 professional development sessions between the years 1997 and 2003. The sessions were part of a larger project entitled Local Systemic Change (LSC) funded by the National Science Foundation. Each participating teacher attended 130 hours of professional development over the course of the project's funding. The focus of the trainings was to improve instruction in science, mathematics, and technology. The researchers used the Professional Development Observation Protocol (PDOP) (Horizon Research, 2001), an instrument that was developed to measure the quality of an observed K-12 science or mathematics professional development session. It examines the following components of the session: design, implementation, mathematics/science content, exploring pedagogy/instructional materials, leadership content, and culture. The protocol was developed and is used as part of the core evaluation of the Local Systemic Change through Teacher Enhancement program (Heck et al., 2008). The researchers used quantitative methods to analyze the PD session observation data. One of the questions they investigated was "What professional development strategies appear to be most effective?" (p.1). The sessions in which teachers were engaged in problem solving and discussions received higher ratings compared to the sessions that did not include either of the two of those activities. Based on quantitative analysis of the observation data, the researchers confirmed active engagement of teachers as a characteristic of effective professional development. Although they deemed these features to contribute to effective PD, they did not define effectiveness in their research.

Kennedy (2016) investigated the effectiveness of 28 professional development programs. I note here too, that this study never defined the term “effective”. Kennedy categorized the programs by the main learning goals for the participating teachers (such as portraying curricular content, containing behavior, enlisting participation, exposing student thinking) and the strategy of enactment (suggested plan for teachers to implement the ideas presented in their classrooms such as giving them prescriptions, strategies, or expecting them to draw on their insights). For the purpose of the current literature review, I will focus on the enactment methods of the programs Kennedy reviewed. Fifteen of the 28 programs were listed under the category of portraying curricular content. Of these 15, the ones that relied on teacher insights or sharing of strategies had greater impact on student learning compared to those that prescribed steps or merely presented bodies of knowledge. This supports the idea of active learning. When teachers are actively engaged in their own learning, they can fit the new ideas into their existing systems of practice with more fidelity in ways that result in higher student achievement gains. It is also important to note that active engagement of teachers in professional learning that is not coherent may not produce the desired goals for teachers and students. In other words, active engagement and coherence are both necessary components of effective PD (Darling-Hammond et al., 2017; Desimone, 2009; Park et al., 2018). In the next section, I discuss what the literature brings up about coherence in greater detail.

Coherence

Coherent professional development can be defined as purposefully designed series of learning events that cover fewer topics more in depth (Desimone, 2009; Firestone et al., 2005). The opposite would be a training that aims to address many areas at once and that does not help teachers develop the knowledge and skills necessary to improve their teaching (Darling-Hammond & McLaughlin, 2011).

Firestone et al. (2005) studied PD programs in three urban school districts in New Jersey. They helped district officials collect and use PD data in order to develop PD programs that sought to be impactful. The data came from interviews with teachers, school administrators, and district administrators. Each interviewee shared their experiences with the PD programs in detail. The researchers also collected individualized records of PD participation by reviewing school improvement plans, budgets, and other documents related to PD offerings and attendance. They analyzed the data qualitatively and found that teachers learned more through their engagement in content-focused, coherent PD in two of the three districts compared to the teachers who attended sessions that were not coherent and targeted generic features of teaching without a particular focus on content. Working on lessons aligned to state standards, the teaching and learning of specific mathematics content, and learning authentic, engaging instructional strategies were among the aspects of PD valued by the teachers. One of the districts was identified as having the least clear district-level focus. In that district some teachers attended random PD sessions on using manipulatives in math, others have learned about group work. As the PD programs consisted of scattered, independent events with limited coherence, less than one third of the teachers reported learning the implementation of any authentic pedagogy and/or strategies. As with some of the other studies (e.g., Cohen & Hill, 1998), this study was based on self-reported data and did not measure the impact of the PD on teachers' practices.

The case for coherent PD has been supported by researchers and policy documents for decades. As an example, Garet et al. (2001) collected survey responses from a nationally representative sample of teachers. The survey was conducted as part of the national evaluation of Eisenhower Professional Development Program, a federal program which supports professional development for teachers, mainly in mathematics and science (p. 918). The authors conceptualized coherence in three ways: (1) connections with goals and other activities, (2) alignment with state and district standards and assessments, and (3) professional communication with other teachers. Survey items that target these three aspects of coherence led to an overall index of coherence. Analyzing the survey data of 1,027 teachers using advanced quantitative methods, the researchers found that coherence had a substantial positive impact on teachers' knowledge and skills. Professional learning activities that are purposefully sequenced and connected to other PD experiences of teachers are more likely than disconnected, isolated events to enhance the knowledge and skills of teachers. In order for PD activities to show coherence, they have to be planned and spread across extended periods of interaction time.

Duration

Research suggests a positive relationship between the duration of PD and depth of change in teaching practice (Shields et al., 1998; Stigler & Hiebert, 2009) This can be tied to two reasons. One aspect of reform-based activities such as lesson study, video clubs, or course-based professional learning communities is that they allow teachers to engage in deep discussions of content, pedagogy, and student thinking. While such engagement is shown to improve teacher practice (Darling-Hammond et al., 2017), it requires extended amounts of time. Another reason why duration matters is that while engaging in such activities, teachers have opportunities to pilot the new ideas in their classrooms and either self-reflect or receive feedback from others on the efficiency of those practices (Kanold & Larson, 2012). This also requires the PD activities to be extended over time.

Research on PD also suggests that programs that offer 30-100 hours of contact time with teachers in a school year can produce positive change in teacher practice and as a result, improve student achievement (Darling-Hammond et al., 2009). This is consistent with the findings of others that time span and contact hours can provide significant opportunities for active learning by teachers (Desimone, 2009; Garet et al., 2001). These studies note that providing teachers with the structural arrangements to participate in PD for longer periods of time does not intrinsically improve the quality of their learning experience. This enables teachers to collaborate with others in order to discuss, plan, test, revise, and retest new ideas before they can make sustained changes on their teaching practices.

Collective Participation

Teachers can learn the content they teach by exploring conceptual foundations and interconnections (Althauser, 2015) of the content, as well as the content specific pedagogy by engaging in PD over extended periods of time. However, implementing this knowledge and set of skills requires practice and collaboration (Kanold & Larson, 2012). Teachers need to be given opportunities to collaboratively plan, analyze, and reflect on lessons while implementing the ideas suggested in the professional development. Teacher collaboration is identified as a critical component of professional learning by the authors of TALIS (Burns & Darling-Hammond, 2014) as it positively influences teachers' job satisfaction and self-efficacy. As an indirect result, this improves teacher retention and effectiveness. Collaborative learning opportunities such as team teaching, peer observation, coaching, and mentoring are highlighted for their potential to improve student achievement.

The ways in which PD is facilitated is shown to be a significant predictor of how PD influences visions for instruction and teaching practices based on the extensive analysis of data from PD sessions studied by Banilower and Shimkus (2004). Their work suggests that collaborative learning activities such as study groups, mentoring, team teaching, and coaching create a better vision for effective instruction when compared to traditional workshops. From an organizational standpoint, the collaborative participation of a group of teachers from the same school can support a vision for effective instruction even if some teachers leave the school and others join the group (Garet et al., 2001). The common understandings, planning, and evaluation of lessons can form a shared professional knowledge base (Morris & Hiebert, 2011) that can result in improvements in teaching.

Taken together, all of these efforts and emphasis on teacher learning have one goal: improving classroom instruction to improve student achievement. The constructivist approach to learning has two important implications for teaching: (1) learners create their own knowledge, (2) learners are not empty vessels expecting to be filled with knowledge simply as a result of teachers' telling, rather, learners come to school with intuitive ways to approach mathematics concepts and problems (Copeland, 1984; Skemp, 1987). Hence, PD programs must have the five critical characteristics described thus far, but they must also be informed by research-based strategies of teaching and learning. As an example, while teachers learn the content they teach by exploring conceptual foundations and interconnections (Althauser, 2015) they must also take potential student conceptions into account. Teachers must understand that when students are introduced to a novel, unfamiliar problem situation, they use their intuitions, prior experiences, and knowledge to begin working on solutions. In this way, teachers can acknowledge and elicit them to make connections in order to develop students' understanding of more formal methods and concepts (van Galen & Gravemeijer, 2003). This is unlikely to happen if teachers do not notice students' mathematical thinking.

Teacher Noticing

The concept of teacher noticing (Mason, 2002) has been studied intensively in the past two decades. Mathematics teacher noticing is also considered to be a critical aspect of effective mathematics instruction (Sherin et al., 2011a). In the following section, I elaborate on the construct of teacher noticing the professional development efforts that aim to support teacher noticing of students' mathematical thinking.

Teacher noticing has been studied from a range of perspectives that includes teachers' attention to the complex events of the classroom and students' thinking. Star and Strickland (2008) focused on the aspects of classroom activity that are perceived by teachers. Based on this perspective, when teachers view a classroom lesson, what they attend to and what they miss constitutes the heart of noticing. Like in many aspects of teaching, there are differences between the ways in which experts and novices view a classroom. While experts focus on the relevant features of instruction, novices tend to struggle with narrowing down their attention to the crucial aspects of teaching and student thinking (Sherin et al., 2011b). While Star and Strickland (2008) focus on mostly about classroom events, Sherin and colleagues (2011b) emphasize expert teachers' attention to student thinking. For example, novices attend primarily to teacher's actions and words while experts take into account how those actions impact student understanding. Also, experts pay attention to what students say while the novices are focused mostly on teacher actions. Following the methods and frameworks from the expertise research (Berliner, 1992), Star & Strickland (2008) conceptualize noticing as perception without including other aspects of it such as interpreting and deciding how to respond.

Sherin and van Es (2002) define noticing in their Learning to Notice Framework as follows:

> (a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom events. (p. 573)

Their perspective entails the interpretation of classroom situations in addition to the initial filtering of activities. Drawing on their knowledge, beliefs, and dispositions, teachers make sense of what they notice about student thinking and the content being taught. Since teachers' beliefs and knowledge influence how they perceive and interpret classroom situations (Conner et al., 2011; Stigler & Hiebert, 2009), it is important to consider interpretation as a critical aspect of teacher noticing.

Another group of researchers considered teachers' decisions in responding to what's perceived and interpreted. Kaiser and colleagues (2015) describe their PID-model as follows: (P) Perceiving particular events in an instructional setting, (I) Interpreting the perceived activities in the classroom and (D) Decision-making, either as anticipating a response to students' activities or as proposing alternative instructional strategies (p. 374). This model takes the concept of noticing beyond the perception and interpretation of particular incidents like error detection or finding gaps in students' thinking.

Jacobs et al. (2010) define *professional noticing of students' mathematical thinking* as a combination of three interrelated skills: "attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings" (p. 172). Their perspective is strictly on the noticing of students' mathematical thinking. They note that what teachers attend to and how they interpret students' mathematical thinking are consequential to the decisions they make in their instructional practice. Some researchers conceptualized this last component of teacher noticing as teachers' next steps and expected teachers to come up with hypothetical actions they would have taken after attending to and interpreting students' thinking (e.g., Krupa et al, 2017).

Above, I reviewed different perspectives on teacher noticing. I note that for this study, I will be using the professional noticing of children's mathematical thinking framework (Jacobs et al., 2010). In the next section, I summarize the efforts for supporting pre-service and in-service teachers' (PST and IST, respectively) noticing more broadly and later visit the framework by Jacobs et al., (2010) in greater detail for the purposes of my study.

Efforts to Support Teacher Noticing

A growing body of research confirms the idea that teacher noticing is trainable (Jacobs et al., 2010; Star & Strickland, 2008). Researchers have designed programs to improve the noticing skills of PSTs and ISTs and reported improvements in teachers' noticing skills. While these efforts vary in their designs, methods, and tools, each of them contributes to the literature in its own ways.

Stahnke and colleagues (2016) systematically reviewed empirical research on mathematics teachers' situation-specific skills, namely, perception, interpretation, and decision making. They selected studies published between the years 1995 and 2016 that report empirical data on the above-mentioned skills of mathematics teachers. Thus, theoretical articles, literature reviews, empirical studies that focus on student variables (such as gender or ethnicity), studies that deal with teaching of subjects other than mathematics, reform, policy, and other pedagogical topics such as classroom management are excluded. This elimination resulted in 60 studies in total that met the review criteria. In the light of this systematic review of literature, as well as my other readings, I will summarize the programs designed to improve teacher noticing by highlighting their design and methods, including the sample and school level of participants, and materials used to improve teacher's noticing skills.

Video as a Tool to Improve Teacher Noticing. An increasing number of researchers use video of mathematics lessons as a means to improve teacher noticing. One example is Star and Strickland's work (2008) with 28 secondary PSTs who participated in a semester-long methods course designed to improve their noticing of critical features of instruction. A pre-assessment coupled with a framework to observe mathematics classroom videos made teachers aware of the features of the lesson they failed to notice. Participating in more video viewings and discussions, PSTs started noticing the aspects of instruction that are critical to the teaching and learning of mathematics. Upon viewing the initial clip for a second time, PSTs became more attentive to the critical features of the lesson as outlined in the framework mentioned. Although the researchers characterized noticing as perception only and did not focus on interpretation and decision making, this study reports gains in PSTs noticing skills after their participation in the video viewings and related discussions.

More recently, Jacobs and colleagues (2018) provide an account of a group of secondary mathematics teachers that participated in a video-in-the-middle professional development program. The authors planned the viewing of purposefully selected video clips from teachers' classrooms in between pre- and post-viewing activities. They embedded video within a broader professional development program by intentionally sequencing the viewing of videos in between designated activities. Each sequence, including the pre-viewing, viewing of the video, and post-viewing activities is defined as a video case. The teachers first solved the tasks that are highlighted in the video, then watched the video, and finally were engaged in a discussion facilitated by a researcher. The tasks and the video excerpts were selected by the researchers during this 10-session workshop. Although this article does not report any empirical data, based on their extensive prior work with using video for teacher development, the authors found the design to be promising in terms of promoting "a detailed and focused examination of complex mathematical content, the relationship between pedagogical decisions and practices, and an unpacking of students' mathematical thinking" (p. 44). Depending heavily on the facilitator of the video cases, this kind of content-focused and coherent professional development program (Desimone, 2009) can contribute to teachers' situation-specific noticing skills.

Van Es and Sherin (2006) compared two different video club designs in terms of their supporting of teacher noticing. Video club is a specific type of PD that consists of a group of teachers who meet to watch and discuss one another's teaching videos. Both video clubs, consisting of elementary teachers, met ten times over the course of the study. The first group was facilitated by a researcher who selected video clips of participating teachers' classrooms in which student thinking was prominent. The facilitator maintained the focus of discussions around student thinking and often asked teachers to support their claims with evidence from the videos. The second group, not facilitated by a researcher, consisted of three elementary classroom school teachers and two teachers who taught Spanish, music, and physical education. These teachers videotaped their own lessons and one of them (the focus teacher) shared a short clip at each session. The focus teacher introduced an issue or topic prior to the viewing of the clip and then they watched it together. After the viewing, they discussed a variety of issues and topics concerning teaching and learning independent from specific content. The goal of the second group meetings was to create space for novice and experienced teachers to discuss general issues on instruction. Researchers interviewed each teacher before and after the video club meetings. During the pre- and post-interviews, teachers viewed 2- to 5-minute video segments from elementary mathematics classrooms and responded to questions getting at their noticing. Based on qualitative analysis of the field notes and video transcripts, the researchers found that in the first group, teachers' noticing became more focused on students' mathematical thinking. This may seem obvious considering the focused design of the video club, however, since teachers do not always respond to the goals of professional development sessions (Burns & Darling-Hammond, 2014), it is important to note that a video club with a central focus on student thinking has the potential to improve teacher noticing. Teachers in the second group adapted new ways to comment on and discuss what they noticed in the video viewings. Their views on issues of teaching and learning broadened over time as they participated in the video club discussions. Overall, despite the difference in design and structure, teachers who participated in the above-mentioned studies changed in their noticing, supporting the idea that engaging in video clubs (or other kinds of video-based PD) can improve teacher noticing.

Studying Student Work and Lesson Artifacts to Improve Noticing. Another line of research is related to how PD can incorporate student work and classroom artifacts to improve teacher noticing. The premise is that classroom artifacts reflect an authentic sense of the complex classroom environment and the practices of teaching and learning (Hiebert & Morris, 2012). Despite the multiple channels of information offered by video, it takes intentional effort from the facilitators to direct teachers' attention to the goals of the PD (Kaiser et al., 2015). Studying classroom artifacts and student work can be preferable from this point of view. For example, when teachers look at a student's work on a given problem, there is not much on the paper that might divide their focus. On the other hand, watching a video, teachers may get distracted by factors such as a student's inappropriate behavior or a teacher move that is not necessarily relevant to the goals of the PD.

Goldsmith and Seago (2011) designed a comparative study with 49 high school teachers participating in four seminars facilitated by researchers. 25 additional teachers from the same district participated in the pre- and post-assessments as a comparison group. During each seminar, teachers discussed sample student work with a central focus on noticing students' mathematical thinking. The facilitators encouraged teachers to base their claims on the actual artifacts rather than simply providing their opinions and making judgements about the students. During the second, third, and fourth seminars, the facilitators needed to give fewer reminders compared to the first one to direct teachers' attention to student thinking. Over the course of the PD the discourse was grounded more in particular details from the artifacts. For instance, during the first session teachers made comments like “Students do not read what the question says” This was generalization that wasn’t substantiated by a particular feature of the artifact discussed. Through their participation in the PD sessions, the same teachers started making comments like “With the *a minus 1 for a = 1*, maybe they were referring to zero/no adults”. In other words, teachers started referring to the strategies employed by the students evidenced in the artifacts. Over the course of the program, teachers’ interpretations of student thinking improved as they supported their claims by evidence from the artifacts. While at the beginning of the PD most teachers relied on their prior experience with students and made bold judgements about students’ work and the logic behind them. As they participated in the program, they started acknowledging and trying to understand student’s lines of reasoning.

Overall, studying classroom artifacts and student work can potentially improve teachers’ noticing of student thinking. It is important to note that artifacts do not intrinsically improve noticing. Purposeful selection and use of classroom artifacts potentially help teachers become more aware of their students’ thinking.

Using Both Video and Classroom Artifacts to Improve Noticing. Some researchers have taken a different approach and used both video and classroom artifacts to study teacher noticing. This approach is promising as it, in a sense, allows teachers to make claims on student thinking that are backed by multiple sources of evidence.

Jacobs et al. (2010) worked with 131 PSTs and ISTs with 4-33 years of teaching experience. Teachers read research, solved mathematics problems, analyzed video segments of students’ problem solving, and evaluated written student work. These were artifacts selected by the researchers with particular learning goals in mind. The goal of the PD sessions was to help teachers make sense of students' mathematical thinking. Teachers watched a video clip and/or explored a set of written student work before they provided written responses to prompts targeting how they attend to, interpret, and how to respond to students' mathematical thinking. They were expected to provide evidence to their claims from the video and artifacts. Based on qualitative analysis of teachers written responses, the authors reported gains in all three aspects of teacher noticing. Through their participation in the PD sessions, teachers improved in their ways of attending to students' mathematical thinking and made claims backed by specific evidence from the artifacts and/or videos. Their interpretation of student thinking also evolved in productive ways. Rather than simply making "snap evaluations based on minimal evidence" (p.173), teachers started valuing student thinking and showed efforts to understand different stances they take as they solve problems. Although the gains in deciding how to respond were not as high as in other aspects of noticing, teachers improved in selecting the next task that would potentially progress the students in their path towards the given learning goal. One caveat is that this study only measured teachers' skills on selection of the next task as an indicator of responding to student thinking. Broader conceptualization of "teachers' next instructional move" in a given situation might yield different results in this category. For example, a teacher may not be able select the next task for a student due to her limited experience in task selection, however, she might use effective follow up prompts to help her student progress in understanding the concept involved.

Using Student Interviews to Understand Student Thinking. There is evidence in the literature that through clinical interviews conducted by teachers, students' conceptions of mathematics can surface and as a result, teachers can better understand the ways in which students approach problems and make solution attempts (Copeland, 1984; Heng & Sudarshan, 2013). Although not all are strictly about teacher noticing, since I use the conceptualization of Jacobs and colleagues (2010) that emphasizes noticing of student thinking, I review the studies below to show what others have done related to interviews and student thinking.

Buschman (2001) reported the findings from an action-research study designed to investigate how conducting student interviews might enable teachers to better understand students' mathematical thinking. Teachers at an elementary school wanted to see how student interviews influenced the teaching and learning of mathematics. Particularly, they were wondering if the interviews helped teachers adopt a more student-centered instructional practice. A small group of elementary school teachers volunteered to participate in workshops to learn how to conduct effective clinical interviews. Teachers then conducted one-on-one interviews with each student while the rest of their students participated in school wide activities planned by the school administration. Teachers kept reflection journals over the course of the study. The qualitative data analysis came with results that supported the group's initial conjectures. Teachers were able to identify students' strengths and areas of improvement with more detail and accuracy. This helped them adjust their plans according to each student's needs. Teachers also acknowledged the need to plan mathematics instruction in ways that allow students to share out and discuss different ways to approach problems rather than just following the strategies suggested in the textbooks or the teacher. Lastly, teachers felt confident in their students' intuitive abilities to solve problems. This also propelled the shifts in teachers' planning to allow more airtime in the classroom for students' sharing of their strategies and related discussions.

Along the same lines, Jacobs and colleagues (2006) used teacher-produced videotapes of student interviews to facilitate discussions among a group of 18 elementary school teachers participating in a study group facilitated by one of the authors. Throughout the school year, each teacher recorded their interviews with students at least three times. They were asked to select one of those videos to be shared with the whole group. All student interviews discussed in the group presented a student working on a problem guided by teacher prompts. While the teachers viewed and discussed the videos, they were able to generate ideas about existing understandings of students, as well as next steps in instruction to support students' learning. As they participated in the meetings, they adopted an inclusive approach to welcome a variety of ways in which students reason about mathematics. During the interviews, teachers were able to observe the different approaches students have taken as they solved problems. As a result, they learned to discuss the advantages and disadvantages of different solution strategies instead of focusing on what is right or wrong. This program provided teachers with opportunities to develop productive beliefs about teaching and learning of mathematics (NCTM, 2014) as they were engaged in a "dynamic, personally meaningful process of collaboratively exploring their own questions about their own students" (p. 280).

More recently, Krupa et al. (2017) designed a structured curriculum module centered around a student interview and used it to investigate secondary mathematics PSTs' noticing of student thinking. Thirty-two PSTs completed the activities in the curriculum module over the course of three weeks. First, they took a video assessment aimed to capture their initial state of noticing skills. This included watching a purposefully selected videos of student thinking and responding to targeted noticing prompts. Next, they were prepared for an interview assignment by reading an article on effective questioning and discussing it in the class meeting. After that, PSTs interviewed a secondary student using a task-based interview protocol. The interview assignment asked PSTs to rate their students' problem-solving, versatility, and adaptability levels by analyzing the student's verbal and written responses according to rubrics provided. Lastly, PSTs submitted a reflection paper on the interview experience. At the end of the three weeks, PSTs took a post-assessment similar to the pre-assessment that consisted of watching a video clip and answering related questions on the three nested components of noticing (Jacobs et al., 2010). Results indicated that PSTs improved in attending to and interpreting student thinking. As they completed the student interview assignment, PSTs became aware of the fact that they had to base their interpretation on student thinking. However, consistent with other research on noticing, the authors did not observe change in PSTs' decision making in response to student thinking. This component of teacher noticing has been found to show the least improvement, both with PSTs (Krupa et al., 2017) and ISTs (Jacobs et al., 2010). To address this issue, Krupa and colleagues created additional materials to focus on responding and saw improved results (Casey et al., 2018; Monson et al., 2020).

Researchers who used student interviews to support teachers' situation-specific skills note that interviewing students resulted in other benefits than just understanding student thinking. Teachers who incorporated student interviews into their practice started organizing their instruction centered around student thinking and discussion of different ways to solve problems (Buschman, 2001; Jacobs et al., 2006). Additionally, these studies found that teachers who used interviews began to reconsider what they thought about how students learn.

Theoretical Framework

Here I describe the various theories that are related to my work. I have attempted to give an overview of each theory as well as how each one is linked to my inquiry into teacher noticing. I start with presenting the professional noticing of children's mathematical thinking framework (Jacobs et al, 2010). Then I go over professional knowledge of teachers as it connects to teacher noticing. Lastly, I present a model that conceptualizes teacher competence as a continuum and how task-based student interviews can feed in the process to support the model for supporting teacher noticing.

Professional Noticing of Children's Mathematical Thinking

The core of my theoretical framework is the professional noticing of children's mathematical thinking (Jacobs et al, 2010). This conceptualization of noticing consists of three intertwined situation-specific skills: (1) attending to children's thinking and strategies, (2) interpreting children's understandings, and (3) deciding how to respond (p. 172). Teachers must pay attention to the particular ways in which students approach mathematical concepts by observing their written or verbal work. Then, what is noticed must be interpreted in meaningful ways to guide teachers in taking the next instructional steps and responding to students' thinking.

Professionals attend to important aspects of complex phenomenon in particular ways. Goodwin (1994) uses the term highlighting and describes how it facilitates the relevant details about the work of professionals to stand out. The highlighting process makes what's important visible while fading other, irrelevant things into the background. In the context of teacher noticing, this translates into teachers' attention to children's strategies. Understanding these complex strategies is important as they provide gateways into children's mathematical understandings (Carpenter & Lehrer, 1999; Hiebert, 2003). Those intuitive understandings of children can be used as a foundation for instruction. That's why noticing the details in children's strategies matters (Jacobs, et al., 2010).

Interpretation of children's strategies is also important, as mentioned earlier. However, according to Jacobs et al. (2010), interpretation deals with "the extent to which teacher's reasoning is consistent with evidence and details of the particular child's strategies as well as with the relevant research on children's mathematical development" (p.172). It is not about expecting teachers to produce a single perfect interpretation. The authors adopt the productive and unproductive interpretations as defined by Mason (2002). Productive interpretation is strictly evidence-based as opposed to unproductive interpretation that includes snap evaluations based on minimal evidence. Consistent with others' work (e.g., van Es, 2011), in the professional noticing of children's mathematical thinking framework, interpretation is separate from evaluation. In other words, teachers are expected to base their interpretations of students' thinking on evidence from student's written or oral work rather than making judgements that are not supported by any tangible evidence.

The third component of the framework deals with the reasoning used by teachers when deciding how to respond to students' mathematical thinking and strategies. Rather than focusing on a single best response, this conceptualization of noticing values the extent to which teachers use what they learned about children's strategies to decide on the next instructional moves. It also focuses on the consistency between teachers' responses to student thinking and the research on children's mathematical development. It is important to note that the framework by Jacobs et al. (2010) puts emphasis on the intended response rather than the actual execution of the response. This aligns with others' work on teachers' consideration of next instructional steps (e.g., Santagata, 2011) while analyzing lessons.

Noticing matters as it shapes what teachers do and don't do. What's equally important is the fact that noticing cannot be isolated from teachers' professional knowledge since teachers' knowledge and views about students, mathematics, teaching, and learning impact their observable behavior in the classroom. As mentioned above, teachers perceive crucial situations in the classroom and interpret them according to broader principles of teaching and learning. This requires the use of their professional knowledge.

Professional Noticing of Children's Mathematical Thinking and Professional Knowledge

Noticing is connected to the different components of teachers' professional knowledge. Schoenfeld (2011) argues that noticing cannot be viewed as an end in itself. Rather, he suggests the following:

Noticing is essential, but it does not suffice by itself. It takes place within the context of teachers' knowledge and orientations; and the decisions that teachers make regarding whether and how to follow up on what they notice are shaped by the teachers' knowledge (more broadly, resources) and orientations. (p. 233)

This means there is a connection between teachers' noticing and their views and knowledge. The question then becomes "what constitutes this relationship?" Although this is an important question, it is far-ranging and broad. It is nonetheless clear that as teachers attend to student thinking, interpret it, and decide how to respond to it, they draw on their corresponding knowledge and views. For example, attending to students' mathematical thinking is closely related to "teachers' knowledge of what is mathematically significant and skill in finding those mathematically significant indicators in children's messy, and often incomplete, strategy explanations" (Jacobs et al., 2010, p. 194).

Since Shulman (1986) distinguished the three areas of content knowledge in teaching as (1) content knowledge, (2) pedagogical content knowledge, and (3) curricular knowledge, many researchers have studied teacher knowledge in depth and contributed to our understanding of teacher knowledge. While some have taken a cognitive perspective on teacher knowledge, others have studied the phenomenon through a more enacted or situated perspective.

Ball and colleagues (2008) took the cognitive perspective and conceptualized mathematical knowledge for teaching (MKT) and its specific components, refining Shulman's (1986) work. They argued that teachers' profound content knowledge base was an important factor in determining the quality of instruction. Defining one subdomain under MKT, namely knowledge of content and students (KCS), they wrote the following:

It is the knowledge that combines knowing about students and knowing about mathematics. Teachers must anticipate what students are likely to think and what they will find confusing. Teachers must also be able to hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language. (p. 401)

This is closely related to the first two components of noticing (attending to and interpreting student thinking). However, here the authors define it as knowledge at the intersection of students and content. For example, teachers must be familiar with common student errors about particular mathematics content. While the MKT framework has moved the field forward in terms of understanding teacher knowledge, it was also criticized for conceptualizing it as something that is independent from the situations in which it is used (Petrou

& Goulding, 2011). They cite another pitfall of the framework which is that it ignores teacher beliefs and views about mathematics teaching.

The other line of research on teachers' professional knowledge that took the situated perspective also contributed to our understanding of teacher knowledge. Researchers using this lens (e.g., Borko et al., 2000; Cobb & Bowers, 1999) argued that the situated approach was more appropriate for understanding the complex work of teaching and what knowledge really mattered in teaching, because it highlighted knowing how to act over factual knowledge about teaching. These scholars adapted frameworks and methods from expertise research (Berliner, 1992). They mostly compared expert teachers to novices in terms of their perception and interpretations of teaching situations. This is generally done via using approximate representations of teachers' such as classroom artifacts (e.g., student work) and/or video clips from teaching and learning scenarios (Goldsmith & Seago, 2011; Van Es & Sherin, 2002).

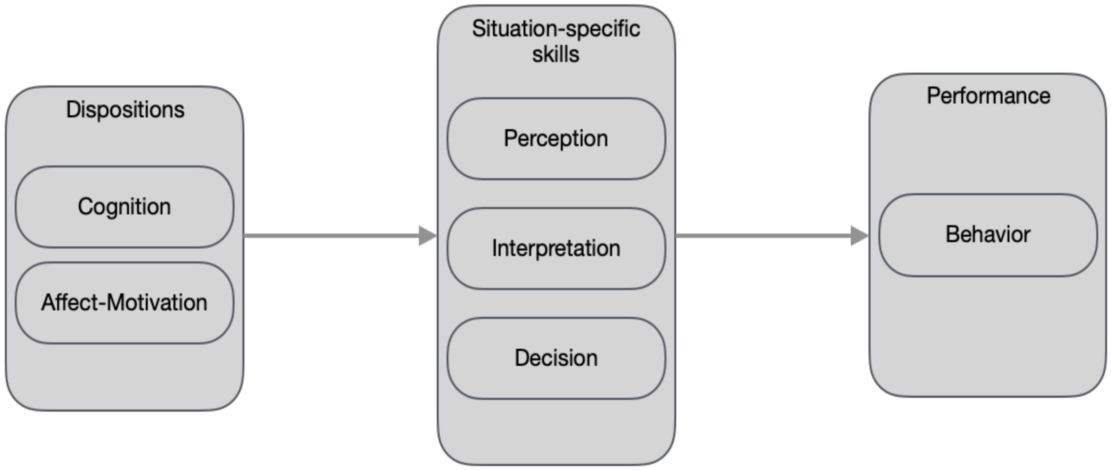
In summary, noticing is connected to the different components of teachers' professional knowledge. Teachers draw on their professional knowledge as they attend to, interpret, and respond to students' mathematical thinking. For example, teachers must be knowledgeable about common student misconceptions about the content they are teaching. This is crucial for them to be able to anticipate what students are likely to think and do as Ball and colleagues (2008) suggest. On the other hand, teachers also need to know how to enact that knowledge into their teaching. In the daily lives of teachers, the acts of attending, interpreting, and responding occur almost simultaneously. Considering these skills independently may seem "too removed from teachers' everyday work" (Jacobs et al, 2010, p. 197). Thus, teacher noticing can be examined more effectively from a perspective that integrates the cognitive and situated approaches to teachers' professional knowledge.

Integrated Perspective on Teacher Competencies and Task-Based Student Interviews

Competence as a Continuum. For my study, I will be drawing on a conceptualization of teacher competence that links teachers' knowledge to their performance in the classroom. Although my study is about examining teachers' noticing of students' mathematical thinking and I focus mainly on the situation-specific skills, I present an integrated perspective that claims to resolve the dichotomy between cognitive and situated perspectives in studying teacher knowledge. Blömeke and colleagues (2015) define competence as a continuum that starts from cognitive and affect-motivation (disposition) aspects that extend to situation-specific skills leading to observable behavior (performance) as shown on Figure 1. In their earlier work, Blömeke et al. (2008) define cognition as the "cognitive abilities and skills—in terms of teachers' professional knowledge—to solve certain problems" (p.720). They also claim that this knowledge is learnable. For affect-motivation, Blömeke and colleagues (2008) refer to "the motivational, volitional and social willingness and skills to apply solutions successfully and responsibly in variable situations—in terms of teachers' professional beliefs" (p. 720). The authors refer to teachers' practices in and out of classroom for the performance category in the figure.

Figure 1

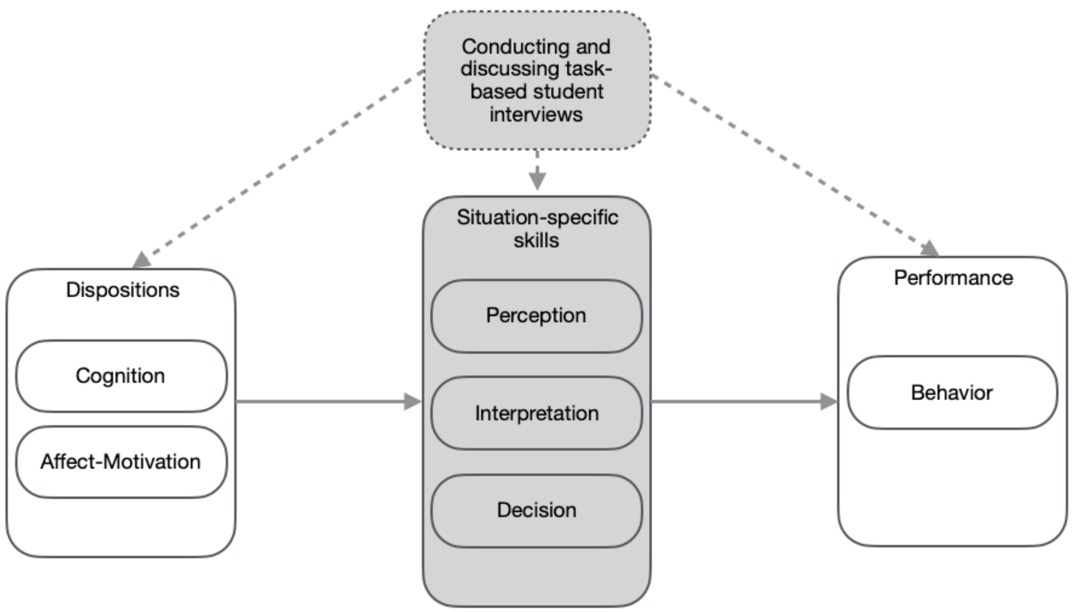
Modeling competence as a continuum (based on Blömeke et al., 2015)



In this model, the situation-specific skills mediate between disposition and performance. Knowledge is conceptualized as an important factor underlying performance, however, the relationship between the two is mediated via the situation-specific skills. Blömeke et al. (2015) define these skills as the PID Model as follows: (P) perceiving relevant events in a teaching environment; (I) interpreting the perceived activities in the classroom; and (D) decision making, either as anticipating a response to students' activities or as proposing alternative instructional strategies. This aligns with the three-layered noticing framework by Jacobs et al. (2010) that is mentioned in the beginning of this section. One notable difference is that the work of Jacobs and colleagues focuses on children's mathematical thinking, while the PID model in the model presented in Figure 1 comprises other aspects of teaching such as classroom management, cognitive activation of learners, and individual learner support (Santagata & Yeh, 2016). While I acknowledge that beliefs and dispositions play a role in teachers' noticing, as Jacobs et al. (2010) did, in this study, I keep the focus on teachers' noticing of students' mathematical thinking. Additionally, although I am curious to see how teachers notice students' mathematical thinking in their classrooms after participating in my PD program, I am not focusing on performance for this study.

Figure 2

Integrated perspective on teacher competencies and how they can be influenced by the use of task-based student interviews (adapted from Blömeke et al., 2015)



The Use of Task-based Student Interviews in a PD Setting to Support Teacher Noticing. As mentioned earlier, among the efforts to improve teacher noticing, task-based student interviews can be used to direct teachers' focus on student thinking (Buschman, 2001). Figure 2 illustrates the integrated perspective on teacher competencies (Blömeke et al., 2015) and how they can be influenced by the use of task-based student interviews. The solid lines and arrows come from the original model proposed by Blömeke et al. (2015) and the dashed lines and arrows are added to illustrate how different design characteristics of a PD program centered on task-based interviews can influence it.

In order to develop the cognitive foundation necessary to notice and understand student thinking, certain schema and knowledge base have to be developed. Conducting interviews can operationalize affective-motivational dimension since they can see authentic windows into student thinking through the interviews. Research has shown that conducting one-on-one student interviews can help boost teachers' motivation to understand student thinking (Buschman, 2001; Heng & Sudarshan, 2013; Jacobs et al., 2006). These relationships are presented in Figure 2 with the dashed arrow that goes into dispositions indicating a potential influence. Again, as others who studied professional noticing did (e.g., Jacobs et al., 2010; Krupa et al, 2017), I acknowledge teacher beliefs and dispositions play a role in teachers' noticing, but in this study, I keep the focus on teacher noticing of students' mathematical thinking. My hypothesis includes that engaging in task-based interviews and collaboratively discussing the interview clips may motivate teachers understand the importance of noticing students' mathematical thinking. These discussions can invite teachers to deeply analyze student thinking that surfaces during the interviews. The use of video has been praised as it fosters a reflection cycle that helps teachers learn from their practice (Jacobs et al., 2006; Seago et al., 2018). Engaging in video-based discussions also helps teachers to improve their situation-specify skills of attending to, interpreting, and deciding how to respond to student thinking as defined by the framework proposed by Jacobs et al. (2010). The discussion questions can have a goal of focusing teachers' attention on productive interpretations of student thinking (Mason, 2002) based on evidence from the videos as mentioned earlier. They can also provide the coding schemes (Goodwin,

1994) necessary in order to highlight the relevant features of the videos stand out. As teachers engage in discussions around the video vignettes, they can use what they learned about children's strategies to consider the next instructional steps (deciding how to respond). The discussions around the videos of task-based student interviews is presented with the dashed arrow going into the Situation-specific skills in Figure 2.

Main Ideas Presented

The main ideas presented in this review inform my study. First, teachers are at the center of any reform movement and their professional learning matters. Research indicates that when teachers actively engage in content-focused and coherent PD that spans over extended periods of time and provides collaboration opportunities, they start adopting productive beliefs and start enacting effective instructional strategies (Desimone, 2009; Garet et al., 2001).

Second, professional noticing is an important component of teacher knowledge (Sherin et al., 2011a) that is situated in instructional practice. There is consensus among researchers that teacher noticing is trainable (Jacobs et al., 2010; Stahnke et al., 2016). Teachers can get better at all of the three nested components of professional noticing of students' mathematical thinking, namely, attending to, interpreting, and responding to students' thinking (Jacobs et al., 2010). I will be using the professional noticing of students' mathematical thinking framework in my study as it seems to be more comprehensive compared to the earlier conceptualizations of noticing. Through well designed, effective PD programs focused on teacher noticing, teachers may be able to better identify students' needs and plan instruction in the most productive ways that result in student achievement gains.

Gaps That Remain

There has been a great deal of research on teacher PD and professional noticing. However, there are a number of gaps that remain. First of all, considering the critical value of content-focused PD (Desimone, 2009) and the role of secondary school mathematics teachers as gatekeepers for students' academic and career choices (Remillard et al., 2017) the field needs more work on the noticing of secondary school teachers.

Second, the literature reviewed here collectively suggests that teachers, overall, are struggling with deciding how to respond to students' mathematical thinking. It is important for teachers to notice and interpret students' mathematical thinking, but these actions are not ends in themselves. Rather, they are gateways for effectively responding to students in ways that help them progress in their mathematical understandings (Jacobs et al., 2011). Therefore, teachers must improve in all three components of noticing. Despite the consensus on teacher noticing as a trainable phenomenon, both PSTs and ISTs show the lowest improvement in deciding how to respond to student thinking compared to attending to and interpreting. Ironically, most studies do not use measures to study responding. About half of the studies systematically reviewed by Stahnke and colleagues (2016) reviewed address responding and the majority of them focus on elementary and upper elementary teachers. There is a need for studies that explore this important component at the secondary level.

Third, student interviews have been used to help ISTs assess student understandings in depth (Jacobs et al., 2006). Research also is emerging in the use of student interviews to improve the noticing skills of PSTs. There has also been research on the use of student interviews to improve the noticing skills of pre-service teachers, but to date, this is limited to only two studies

(Krupa et al., 2017; Lesseig et al., 2016). Therefore, more studies are needed to further investigate the use of task-based interviews as a means to improve ISTs noticing.

Lastly, teacher noticing is a multi-faceted phenomenon. Similar to many other areas of education research, noticing takes place in contexts that are surrounded by parameters and events that cannot be completely controlled. Also, it is hard to clearly define the interactions between different components of noticing and other variables such as teachers' beliefs, attitudes, and self-efficacy. In such situations, case study method is recommended (Yin, 2009) as an effective methodology. Unfortunately, there is very limited case study research that explores secondary school ISTs' noticing.

In summary, this review suggests that there is a need for research on supporting secondary school teachers' noticing through task-based student interviews. The methods, theories, and findings from the elementary and middle school level studies inform the design of such work. The promising efforts at the lower grade levels can be complemented with more qualitative inquiry at the secondary level.

Chapter 3: Methodology

Introduction and Rationale for the Study

This study is based on my interest in teachers' noticing of students' mathematical thinking; an important research focus in the past two decades (Jacobs et al., 2010; Sherin et al., 2011). The objective of this study was to investigate the following research question: "How does a professional development program centered on task-based student interviews foster secondary in-service teachers' noticing of students' mathematical thinking?" To answer this question, I designed a professional development program centered on task-based student interviews and conducted a qualitative study that involved teachers conducting task-based student interviews and discussing clips of those collaboratively.

This is a multi-case study of three teachers that participated in a professional development (PD) program centered around task-based student interviews over five weeks in spring 2021. As part of the PD program, teachers interviewed 1-2 students one-on-one for four rounds and shared their noticing experiences with other teachers engaged in the same process. Each teacher interviewed students as they solved two tasks, one at each sitting. As a pre-assessment, teachers watched a video of a teacher-student interview as the student solved a math problem. This was a video of a researcher interviewing a middle school student. They then answered some questions regarding what they noticed while watching the video. As part of the five-week PD program, teachers conducted one-on-one student interviews, watched selected clips of their own and others' videos, and discussed their observations collaboratively during the sessions. The same video-based assessment was given at the end of the PD program to see the changes (if any) in the teachers' noticing skills. Data came from these pre-and post-video assessments of teachers' noticing skills, their written responses to weekly video-based noticing prompts, and their discussions captured in the PD sessions' video recordings.

Connection to the Theoretical Framework

As indicated in the theoretical framework, teachers' situation-specific skills (namely perception, interpretation, and decision making) mediate between their disposition and performance. It is important to note here that although I acknowledge the role dispositions play in teacher noticing, the focus of my study is on the situation-specific skills. These skills correspond to the three interrelated components of professional noticing; attending to, interpreting, and deciding to respond (Jacobs et al., 2010). I hypothesized that learning about and conducting task-based student interviews can operationalize the cognitive and affective-motivational dimensions of their dispositions as the interviews present genuine windows into student thinking (Buschman, 2001). With this, teachers also can be motivated to understand student thinking in a more profound way. When they discuss their interview experiences with other teachers that engage in the same practice, teachers may improve their situation-specific skills of attending to, interpreting, and deciding how to respond to student thinking as defined by the framework proposed by Jacobs et al. (2010). Each of the three teachers had different teaching backgrounds, taught in various settings, and potentially represented a range in noticing skills (determined via the pre-interview video assessment). I anticipated that there would be differences among their experiences with noticing students' mathematical thinking both initially and throughout the study as they conducted interviews and shared experiences.

The Context for the Study

The research took place at three schools in an urban public charter school district located in Northern New Jersey, USA. All school names and teacher names are pseudonyms. These schools serve diverse student populations in low-SES neighborhoods. Mathematics teachers at the schools report to their local campus administrators. The mathematics program is overseen by a leadership team that consists of a K-12 program director and six mathematics instructional coaches. This leadership team is not involved in formal classroom observations or performance evaluations of the teachers. Coaches meet with each teacher individually every week to set and follow up on professional development goals, plan and co-teach lessons, debrief, and discuss instructional strategies. Coaches also attend the weekly course-based professional learning community meetings (PLC). These PLCs have been the propeller of the professional development of the mathematics teachers at the district for the past five years. Teachers teaching the same course at different campuses meet at a common location every week for 75 minutes during regular school hours. They reflect on their teaching experiences, study curricula, share best practices, create common lessons and assessments, discuss student performance and other course-related matters.

For the past five years, the mathematics program leadership team has been pushing teachers for more reform-based teaching practices centered around student thinking. While some progress has been reported, due to the high teacher turnover rate, the rapid growth of the network, and other possible reasons, the instruction quality has not been consistent.

Participants

The study participants were three middle school mathematics teachers and a selected group of their students from the three schools described above. All three teachers taught sixth-grade mathematics. The most senior teacher was Mary from School 1, with ten years of teaching experience at the secondary level. Linda had four years of teaching experience at New York City Public Schools, where she taught seventh and eighth-grade mathematics. She started teaching at School 2 as a leave replacement teacher in March 2019. Nick, a career-changer, worked as an engineer for over twenty years and then decided to start teaching. He began teaching sixth-grade mathematics at School 3 in Fall 2018, his first full-time teaching experience. The three teachers who volunteered to participate in this study had a collegial relationship built during the grade six mathematics PLC meetings and other district events. There was one more teacher in the grade 6 mathematics PLC; however, he could not participate in the study due to health-related reasons. Linda, Mary, and Nick agreed to participate in the PD program voluntarily. Each teacher received a $200 gift card for their time and participation.

Teachers identified a total of 24 sixth-grade students who they thought would communicate their ideas effectively. There were no other criteria, such as math performance or work ethic, used in the selection. I was able to collect consent and assent forms from seven of the 24 students: three students from Linda’s school and two students from each of Mary’s and Nick’s schools. Since I did not have any direct relationship with the students, there was no coercion risk. The interviews took place during hours agreed upon by the teachers and each student. The work students completed during the interviews was not part of their regular school assignments; thus, they were not graded and the interviews did not occur during class. The interviews' duration varied from 15-25 minutes (with one exception of 42 minutes). Initially, only the child’s written work and hands would have been visible in the videos; however, due to the COVID-19 Pandemic, all interviews were held through Zoom (an online videoconferencing tool), and students' faces were visible. Teachers asked the students to hold their whiteboards and notebooks to the camera when they wanted to see students' written work. Teachers did this consistently throughout the interviews, which helped the discussions as we viewed the clips during the PD sessions. I kept the recordings in a secure, password-protected folder. Only the research team (I, the three teachers, and the second-rater) had access to the videos.

My Role as a District Administrator

I am currently the Director of Curriculum and Instruction at iLearn Schools. I lead the team of mathematics instructional coaches that provide coaching and PD support for all mathematics teachers. I have been working with Mary for three years and with the other two teachers for the past year. They have been seeing me in their schools and classrooms throughout the school year. I have been leading districtwide PD sessions, sharing my plans and views about mathematics teaching and learning. I have also been attending PLC sessions and other PD events with them. During the PLCs, I usually listen and, on occasion, answer questions or add my comments to the discussions.

The PD events I had previously attended with these teachers were not explicitly focused on the professional noticing of students' mathematical thinking; however, I might have emphasized the importance of recognizing students' intuitive ideas and building instruction on those ideas. For this study, I designed the PD sessions and facilitated the discussions as the group watched the interview videos.

Aligning to the Guiding Principles of Effective PD

The PD program I designed included the critical features of effective PD as proposed by Desimone (2009): it was (1) content-focus, (2) active learning, (3) coherence, (4) duration, and

(5) collective participation. Rather than discussing general topics on education, teachers focused on the content they taught daily, the sixth-grade mathematics curriculum. Hence, the content-focus element was at the core of the PD program.

Teachers conducted student interviews and shared their experiences on noticing student thinking with their colleagues. They were actively engaged in understanding students' mathematical thinking during the interviews and as they collaboratively watched and discussed video clips of those interviews. These activities provided teachers active learning experiences, as highlighted by Desimone (2009). Thus, active learning was a central design element of the PD program.

The content and activities of the PD program consisted of a purposefully designed series of learning events that covered fewer topics more in-depth. The PD program did not address many areas at once as this does not help teachers develop the knowledge and skills necessary to improve their teaching (Darling-Hammond & McLaughlin, 2011). Teachers focused on noticing students' mathematical thinking through collaboratively watching and discussing videos of student interviews. Throughout the program, teachers' focus was on students' mathematical thinking. During each PD session, teachers were engaged in activities designed to support their noticing skills. Thus, the program was coherent.

Although the PD program was designed for five weeks, it gave teachers ample time to pay closer attention to student thinking and then reflect on their own noticing and the noticing of others. Ideally, the program could have been scaled up and extended to more extended periods. However, this was not possible due to the COVID-19 Pandemic and the school closures that followed. Despite these challenges, as designed, the program's duration could support teachers' noticing skills.

Lastly, collective participation was embedded in the PD program's design as the three teachers met biweekly and discussed their experiences. They had opportunities to share takeaways with their colleagues regularly and collaborate on improving the interview process.

Description of Professional Development Activities

Over the five weeks of the PD program, the three sixth-grade mathematics teachers conducted one-on-one task-based student interviews to notice students' mathematical thinking. Teachers then came together weekly for the PD to discuss their experiences with noticing students' mathematical thinking. All activities over the five weeks were tied to the sixth-grade mathematics content taught (See Task 1 and Task 2 Protocols in Appendix B). The central feature of the PD program was the task-based interviews.

The PD program was initially intended to be completed in person. Each teacher was to interview 1-2 students per week during morning tutorial hours and attend PD sessions after their weekly professional learning community meetings. Although intended to be in-person, schools shifted to online instruction a few weeks before the PD program's intended start due to the Covid-19 Pandemic. As a result, the student interviews and teacher PD sessions had to be held online through Zoom, a web-based video conferencing tool. Teachers attended the PD sessions and completed the activities outlined in Table 1. Each session was planned for 60 minutes.

Table 1

Schedule of the PD Program

Session, Date	Teacher Activities	Researcher Activities
Before Session 1	• Read the questioning article and got ready for a discussion. • Completed the Pre-Interview Video Assessment via Google Forms.	• Assigned the questioning article. • Administered the Pre-Interview Video Assessment. • Recruited a pool of students and had the consent/assent forms signed. • Created rubric for the pre-/post-video assessment. • Shared Task 1 Protocol with teachers.
Session 1 4/29/20	• Participated in the questioning article discussion. • Watched and discussed a clinical interview sample video. • Reviewed Task 1 Protocol.	• Facilitated and recorded the session.
Between Sessions	• Interviewed 1-2 students following Task 1 Protocol. • Uploaded interview recordings to the shared folder.	• Reviewed the interview videos and identified "rich segments" using the video selection rubric. • Prepared clips for online viewing.
Session 2 5/6/20	• Individually viewed the first selected clip for the week, submitted the AIR form, and participated in clip discussion. • Repeated this process for 1-2 additional clips.	• Facilitated the session. • Assigned clips and monitored the collection of AIR data. • Recorded the session.
Between Sessions	• Interviewed 1-2 additional students following Task 1 Protocol. • Uploaded recordings to the shared folder. • Reviewed Task 2 Protocol.	• Reviewed the interview videos and identified "rich segments" using the video selection rubric. • Prepared clips for online viewing. • Shared Task 2 Protocol.
Session 3 5/13/20	• Individually viewed the first selected clip for the week, submitted the AIR form, and participated in clip discussion. • Repeated this process for 1-2 additional clips.	• Facilitated and recorded the session.
Between Sessions	• Interviewed 1-2 students following Task 2 Protocol. • Uploaded recordings to the shared folder.	• Reviewed the interview videos and identified "rich segments" using the video selection rubric. • Prepared clips for online viewing.
Session 4 5/20/20	• Individually viewed the first selected clip for the week, submitted the AIR form, and participated in clip discussion. • Repeated this process for 1-2 additional clips.	• Facilitated and recorded the session.

Session, Date	Teacher Activities	Researcher Activities
Between Sessions	• Interviewed 1-2 additional students following Task 2 Protocol. • Uploaded recordings to the shared folder.	• Reviewed the interview videos and identified "rich segments" using the video selection rubric. • Prepared clips for online viewing.
Session 5 5/27/20	• Individually viewed the first selected clip for the week, submitted the AIR form, and participated in clip discussion. • Repeated this process for 1-2 additional clips.	• Facilitated and recorded the session.
After Session 5	• Completed the Post-Interview Video Assessment.	• Administered the Post-Interview Video Assessment.

Before Session 1, I assigned the teachers to read the article "Questioning our Patterns of Questioning" (Herbel-Eisenmann & Breyfogle, 2005) to prepare them for appropriate questions during their one-on-one interviews with students. Each teacher read it individually and was asked to come prepared to discuss it during Session 1. During Session 1, I also introduced Task 1 Protocol to the teachers. I selected Task 1 and Task 2 based on the guidelines provided in NCTM's landmark publication "Principles to Actions" (NCTM, 2014). In the section dedicated to selecting tasks that promote reasoning and problem solving, the authors of Principles to Actions underline the importance of selecting tasks that do not suggest or imply a particular solution method. Instead, by design, these tasks invite students to use a variety of approaches and solution strategies. I needed such tasks for us to have windows into student thinking. When I shared Tasks 1 and 2 with the participating teachers, they agreed about both tasks' cognitive demand and rigor. We also collectively confirmed this during the PD sessions as we watched the clips of interviews. Teachers had worked on the mathematics of the tasks individually, anticipated the different approaches students would have taken and potential difficulties they would have encountered (common issues). The tasks were revised slightly with teacher input and finalized to be used during the interviews.

During the first session, there was a group discussion about effective questioning techniques around the article assigned. Teachers generated some strategies for asking focusing questions to elicit and build upon student thinking (Krupa et al., 2017). Next, I showed a clinical interview video similar to the one each teacher was getting ready to conduct. It was a 3-minute video of fifth-grade students working on a card-match activity about interpreting multiplication and division. One student was making a match and explaining their thinking in the video, matching a card with six circles all split into thirds. The other two students were prompting her to clarify her thinking and guide her in making a match. After watching the video, the group discussed how they could implement the strategies used in the sample.

After Session 1, each teacher interviewed one student via Zoom following the Task 1 Protocol. Each interview was about 15-25 minutes (with one exception of 42 minutes). During the interviews, the teachers' main objective was to understand the students' mathematical thinking rather than to teach or help them produce correct answers. To help capture student thinking, teachers asked students to think aloud as they worked on the tasks and held their papers to the camera. After each interview, teachers uploaded the video recordings to a folder that was shared only with me. I watched each video fully and identified "rich segments" to be discussed during Session 2. I used the video selection criteria shown in Table 2 to select these rich segments.

I selected short clips (ranging from 1:36 to 5:39) from the teachers' interviews of the students based on some interesting patterns in students' mathematical thinking. Depending on the interviews' content, I brought three clips to each session representing different student approaches from different parts of each task. We watched and discussed two clips at each session, except for Session 3, during which we watched three clips. I assigned a score of 1-3 to each clip and recorded those scores on a chart shown in Table 3. All of the nine clips shown during the PD sessions scored either a 2 or a 3 in the category of windows into student thinking. Students were instructed to talk through their solutions and occasionally, teachers asked them to show their work to the camera. Only in three of the nine clips, the conversations were limited to teacher initiation, student response, and teacher evaluation of the response (IRE). Those clips were scored 2. Similarly, all clips scored either a 2 or a 3 in depth of student thinking and clarity with one exception. Session 4 Clip 1 and as a result that clip scored a 1 in clarity.

Table 2

Video selection criteria, adapted from (Sherin et al., 2009)

Criteria for Characterizing Video Clips of Student Mathematical Thinking

		Level		
Criteria	Key Question	Low	Medium	High
Windows into student thinking	Is there evidence of student thinking in the video clip?	Little evidence of student thinking from any source (e.g., very few comments from students)	One or more sources of information exist, but little detail provided (e.g., IRE exchanges dominate[a])	Detailed information from one or more sources (e.g., student narrates and provides written account of solution strategy)
Depth of student thinking[b]	Are students exploring substantive mathematical ideas?	Task is routine for student; calls for memorization or recall on part of student (e.g., student applies known algorithm)	Some sense making applied to routine task (e.g., student questions step in known algorithm)	Student engages in math sense making, works on task at conceptual level (e.g., student devises invented strategy)
Clarity of student thinking	How easy is it to understand the student thinking shown in the video?[c]	Student thinking not transparent (e.g., "What is that student talking about?")	Much of student thinking transparent, though some ideas may be unclear (e.g., "I think I understand, but what did she mean by 'straight?'"	Student thinking transparent; viewer sense making not called for or single interpretation obvious (e.g., "She gives a very clear explanation.")

During the second session, we discussed the two clips I selected from the previous week's interviews. Teachers answered a question that targeted their general noticings upon watching the video clip for the first time and then two more questions that target interpretation and deciding to respond upon watching the video for the second time. Teachers submitted their responses to these prompts in writing via an online form. After a brief introduction and unstructured conversation, I paused the session and muted all participants. I shared the link to a Google Form (see Appendix C) that prompted teachers to watch the video and answer, "After watching the video for the first time, what do you notice (Question A)?" Then, they were asked to watch the video one more time and answer the questions I "Now that you watched the video a second time, how would you describe what this student understands (Question I)?" and "Describe some ways you might respond to this student if you were her teacher and explain why you chose those responses (Question R)." I will be referring to this set of questions as AIR, which stands for attending (A), interpreting (I), and responding (R).

Table 3

Summary of scores for the selected clips

Summary of Scores for the Selected Video Clips									
	Session 2		Session 3			Session 4		Session 5	
Criteria	Session 2 Clip 1	Session 2 Clip 2	Session 3 Clip 1	Session 3 Clip 2	Session 3 Clip 3	Session 4 Clip 1	Session 4 Clip 2	Session 5 Clip 1	Session 5 Clip 2
Windows into student thinking	3	3	2	3	3	2	3	3	3
Depth of student thinking	2	2	2	2	2	2	2	3	3
Clarity of student thinking	2	3	3	3	3	1	2	3	2
Totals	**7**	**8**	**7**	**8**	**8**	**5**	**7**	**9**	**8**

Once I received submissions from each of the participants, we reconvened and discussed the clip. I set the meetings' flow like this because I did not want the teacher's initial noticings to be influenced by the discussions. During group discussions, I asked the two teachers who were not in the clip selected, to take the lead on each clip's discussion. The teacher who was the interviewer in the clip commented last. I did this to reduce preconceived notions regarding the student's mathematical thinking in each clip

The interviewer also had the chance to hear from others some of the critical details in the clip that opened windows into the student's mathematical thinking. A sample session protocol can be found in Appendix D. After Session 2; each teacher interviewed 1-2 additional students using the Task 1 Protocol. I identified three more rich segments to be discussed during Session 3 in the same format described above. Between Session 2 and Session 3, I shared the Task 2 Protocol with the teachers, and the teachers conducted interviews using that for two more rounds of interviews. Session 4 and Session 5 followed the same format as the previous two sessions.

Throughout the five weeks (including the pre- and post-assessments), I completed the tasks listed under the "Researcher Activities" column in Table 1. These included student recruitment, facilitating the sessions, recording and collecting session data (such as videotaping the session and collecting the pre-and post-assessment responses), reviewing the interview videos, selecting and identifying "rich segments." Student recruitment was mostly my task. I asked each teacher to nominate 5-6 students to have a pool to choose from. Students did not have to be at a certain academic level in math (or other subjects); however, I asked explicitly for students who could clearly talk about their work. After we had the pool, I contacted each parent and shared assent and consent forms. Upon collection of the forms, I informed teachers, and they scheduled interviews with the students. I planned the recruitment process in this way to minimize the risk of coercion.

Research Design

Drawing on Jacobs et al.'s (2010) professional noticing of children's mathematical thinking framework and the literature on student interviews to support teacher noticing (Buschman, 2001; Krupa et al, 2017), I designed this study to investigate the research question "How does a professional development program centered on task-based student interviews foster secondary in-service teachers' noticing of students mathematical thinking?". I employed case study methodology to explain the complex nature of teacher noticing and how it evolves as teachers engage in student interviews and collaboratively discuss them in PD sessions. In my study, each of the three participating teachers is a case.

The Use of Case Studies

Case study methodology was appropriate for my research question as case studies usually uncover why and how certain things happen. Especially when there are no clear boundaries between the context and the questions studied, case studies help by providing detailed records of participants and their interactions with the focus of inquiry. Yin (2009) defines a case study as "an empirical inquiry that investigates a contemporary phenomenon (the 'case') within its real-life context, especially when the boundaries between phenomenon and context may not be clearly evident (p. 13)" Particularly in my inquiry, it was hard to know how teachers' noticing of students' mathematical thinking would have evolved as they started interviewing students one-on-one and participated in the PD. This could be accomplished if the phenomenon of noticing was investigated within its context. Thus, I built a case for each of the three teachers and observed the changes in noticing patterns within each case and eventually across three cases.

I used a multiple-case design due to this approach's analytical benefits over single-case designs. The act of teaching requires many important decisions, and a teacher's actions are shaped by those decisions (Ball et al., 2005). Teacher noticing of students' mathematical thinking is not an exception to this (Sherin et al., 2011a). Teachers draw on their knowledge, beliefs, and experiences as they are engaged in the complex phenomena of noticing students' mathematical thinking. As a result, the ways in which teachers notice varies based on the factors listed above. Working with a single teacher and studying teacher noticing through a single-case design did not seem sufficient to me. Hence, I decided to design a multiple-case study design. This enabled me to compare and contrast the different ways in which three teachers noticed student thinking and how these ways may have changed over time.

In my study, I followed the order of Mary, Nick, and Linda as I developed the individual cases. Analyzing the patterns in one teacher's noticing skills and comparing them with another teacher's case helped me validate some of my first case findings. In other words, when I saw similarities between Mary's and Nick's patterns in noticing through the sessions, I was able to confirm some of my initial thoughts about Mary's case. It also pushed me to question my initial observations and review Mary's data with a new focus, something I learned from analyzing another teacher's case. Based on Yin's (2009) categorization, my study fits the definition of an explanatory case study. My goal was to answer a "how" question, and I do not have control over the occurrence of events. I focused on phenomena within the context of real-life situations of student-teacher and teacher-teacher interactions. I traced the operational links between teachers' interviewing, watching and discussing clips of interviews, and the patterns in how they noticed students' mathematical thinking. As highlighted by Yin (2009), the cross-case analysis I made was appropriate for the multi-case explanatory study I designed.

Data Collection

Data for case studies come from a variety of sources. Yin (2009) lists the most commonly used sources of evidence as documents, archival records, interviews, direct observation, participant-observation, and physical artifacts. Each of these requires knowledge and skills of different methodological procedures. The goal of using these sources for case study research design is to examine one unit of analysis at a time deep enough to find answers to the research question.

My study's first set of data came from the pre-and post-program video assessments (Appendix A). Before the first session, teachers completed the Pre-Interview Video Assessment individually through a Google Form before the first session. Each teacher individually watched the video once and answered the first prompt: "After watching the video for the first time, what do you notice?" This prompt was intended to capture teachers' general noticing (attending to). After the second viewing of the same video, they answered, "Now that you watched the video a second time, how would you describe what this student understands?" This prompt targeted the "interpreting" component of professional noticing. Lastly, they were asked to "describe some ways you might respond to this student if you were her teacher and explain why you chose those responses." This question was given to address the "responding to" component of professional noticing.

After the last session, each teacher completed post-assessments in a similar fashion. The post-assessment was identical to the pre-assessment, but three questions were added to collect teachers' overall reflections about the PD program. During the seven-week study, the clip and the prompts from the pre-and post-assessments were not discussed to minimize bias on teachers' responses. These two assessments constituted the primary source of data I collected. These are the written responses teachers submitted, and I considered each response separately as I rated them.

The second primary data source was the teachers' written responses to the AIR prompts for each video clip watched during the PD sessions. After the group watched a video together during a PD session, each teacher provided a written response to three questions (AIR) independently on a Google Form (Appendix C). These responses are rated by a second-rater and me.

The third primary source of data came from the video recordings of the PD sessions. I attended each session, reviewed, transcribed each session verbatim, and analyzed these sessions' video recordings to explore the shifts in teachers' noticing skills as they viewed their own and others' interview clips.

I anticipated seeing some shifts in their noticing patterns throughout their participation in the PD program. I recorded my observations in a research journal. The journal entries included my general thoughts about the PD sessions, the study's execution, meetings with the second-rater, revisions made on the tools and instruments used (such as the video selection tool, rubric for video-based teacher noticing assessments), and slight changes made in the overall study design. I kept this separate from the three primary data sources to minimize my own biases and other subjectivities.

My goal in using these three primary sources was to triangulate the data to better support my findings and arrive at more credible conclusions (Yin, 2009). Crosschecking these data sources provided me with a better understanding of the extent to which each teacher attended to, interpreted, and responded to students’ mathematical thinking. Most of the data analysis process was shaped as I continuously met with my advisor. I describe the analytical strategies used in the next section.

Data Analysis

Having clear theoretical propositions is an important starting point for case studies (Yin, 2009). These initial statements shape the data collection plan and the specific analytical techniques chosen. It also helps the researcher focus on some data while ignoring others. My central proposition for this study was that through teachers’ participation in the PD program, there would be changes in their’ noticing of students’ mathematical thinking. Throughout the data analysis process, I looked for evidence that supported that proposition and some rival explanations that may be related to the changes in teachers’ noticing skills. In the following paragraphs, I describe the analytical process starting with teachers’ written responses. I then provide details about the analysis of the PD session transcripts. Finally, I present the case development and cross-case analysis processes.

Analyzing Teachers’ Written Responses

As mentioned earlier, the teachers’ written responses constitute the majority of data collected during the study. Each teacher provided written responses to the AIR prompts given on the pre-and post-assessments, and the weekly prompts to the nine clips viewed during the PD sessions. There were a total of 33 entries, 11 for each teacher. In addition to these questions about their noticings, on the post-assessment, each teacher responded to three questions that asked about their general takeaways and opinions about the study. Teachers' written responses were scored based on the AIR Rubric shown in Figure 3. The rubric was created and used by Krupa et al. (2017) to rate the written responses of pre-service teachers' video-based assessments. I adapted it for the current

Figure 3

Rubric for Video-based Teacher Noticing Assignments. Adapted from Krupa et al, 2017.

Rubric for Video-based Teacher Noticing Assessments

	No evidence (0)	Limited (1)	Emerging ability (2)
Attending to	Responses attend to non-mathematical aspects of the interview (i.e., student confidence or demeanor, interviewer/teacher questions) or make general observations with no mathematical details, or refer to events that are not in the clip.	Responses attend to elements of the student's mathematical thinking and/or solution strategy, but with less detail and structure. These responses often entail disjointed observations.	Written responses attend to significant mathematical ideas, describe strategies employed by the student in a coherent and comprehensive manner, and refrain from evaluative comments.
Interpreting	The students' thinking is interpreted in a non-mathematical manner, incorrectly, or too vaguely to be meaningful.	Responses interpret elements of the student's mathematical thinking and/or solution strategy based on the evidence provided, but are not in a connected or comprehensive manner. Responses often include over generalizations without direct connections to the data within the video-recorded interview or focus upon only one aspect.	Written responses need to incorporate strengths and/or weaknesses of the student's thinking and connect interpretations to specific mathematics ideas and relationships evidenced in the video.
Responding to	There is neither a connection to the mathematics the student understands nor a rationale for the suggested next steps. These responses are generally vague and discuss larger concepts without identifying specific next steps based on the student's thinking. Responses may also list topics the teacher might cover next, but remain unclear on how topics would be addressed and why.	Responses are connected to the teachers' response given in the previous prompt regarding what the student understands and no rationale for next steps is given, or a rationale for next steps is given, but the response is not connected to what the student understands.	Written responses need to offer questions to further probe or extend student thinking based on what the student understood. Responses could include additional tasks, new representations, questions, or instructions to confront a misconception in student understanding evidenced in the video. It is important that responses include a rationale for furthering the students' thinking and is connected to how the teacher answered the previous prompt about what the student understands.

study and made some minor revisions to meet my research needs. I first organized the paragraph text in a table, as shown in Figure 3. Then, I changed the wording of some identifiers slightly. For example, the original paragraph from the attending to category for the no evidence (0) rating did not have the part "…or refer to events that are not in the clip". I added this section because some teachers referred to events that were not in the clip. As another example, under the interpreting category, the original paragraph for the emerging ability (2) rating read as follows: "Written responses need to incorporate both strengths and weaknesses of the student's thinking and connect interpretations to specific mathematics ideas and relationships evidenced in the video." (Krupa et al., 2017, p. 68). I changed it to "Written responses need to incorporate strengths and/or weaknesses of the student's thinking and connect interpretations to specific mathematics ideas and relationships evidenced in the video." This was a result of my conversation with the second-rater. We decided that some teacher entries described weaknesses of student thinking while others referred to the strengths of student thinking. We did not have a case that described both. Hence, we decided to make that minor revision to the rubric.

Before starting the analysis, I organized and indexed the raw data from the written responses for easy access. In a master spreadsheet, I added the pre-assessment, post-assessment, and weekly prompts responses in separate tabs. Next to each response, I added a column that helped me add a score of 0-2 (no evidence, limited ability, emerging ability). I duplicated that master sheet for the second-rater to allow us to score independently. Each of us assigned a single rating based on the quality and details to each of the responses as described on the rubric. In the following paragraphs, I describe my work with the second-rater.

Working with the Second Rater. I started working with Alan (pseudonym) after organizing and indexing all of the written response data. He was a doctoral candidate in the same program as me, and teacher noticing was one of his research interests. During our first meeting, I introduced my study and went over some essential documents such as the AIR rubric, the master sheet of teachers' written responses, and the task protocols. Additionally, I shared the Krupa et al. (2017) article for him to get a sense of how we could rate teachers' written entries using the AIR rubric. Alan spent a week studying the project documents and rated teachers' responses (three each) on Session 2 Clip 1. I also rated the same responses, and we met a week later. Out of the nine entries rated, Alan and I only agreed on five scores. We discussed our differences; however, we could not come to a consensus for some of the ratings. I firmly believed in interrater reliability and did not want to continue rating the written responses until we reached an agreement on at least 80% of the scores assigned, which is the generally agreed-upon benchmark for interrater reliability (Syed & Nelson, 2015). As a result, Alan and I decided to meet again and watched a clinical interview video that was not part of the current study. For each of the three noticing prompts (tied to AIR), we made a list of possible teacher responses that would have earned 0, 1, and 2 (See Appendix E). For instance, "the student struggles to solve the problem" was an example of "no evidence (0)" in the attending to category as it only provided a general observation and did not specify anything evidenced in the clip. In contrast, we made up the following teacher response as an example for a score of "emerging ability (2)":

"The student demonstrates conceptual understanding about division. She makes 3 groups of 5 and identifies 1 as the remainder. When asked, she can interpret each number in the context of the problem. Ex: There is 1 extra dollar. One cube represents a dollar. She also is able to show five cubes together represent the cost of a book." (Sample Score-2 response, Meeting with Alan, October 3, 2020).

This entry describes the student's critical mathematical ideas and strategies, and it refrains from evaluative comments. Similarly, we created at least one possible teacher response for each of the three categories of professional noticing that would have earned a score of 0, 1, and 2. This process helped us understand the AIR rubric and document we frequently referred to while scoring teachers' written responses. After the scoring alignment session, I met with Alan for four weeks to go over the scoring of the responses. We had disagreements on at most two out of the nine scores each of us assigned. These were mostly about minor nuances about how each of us interpreted the AIR rubric. We discussed our differences and came to a consensus on all nine after the discussion. Hence, the interrater reliability was 100% after discussion for all scores.

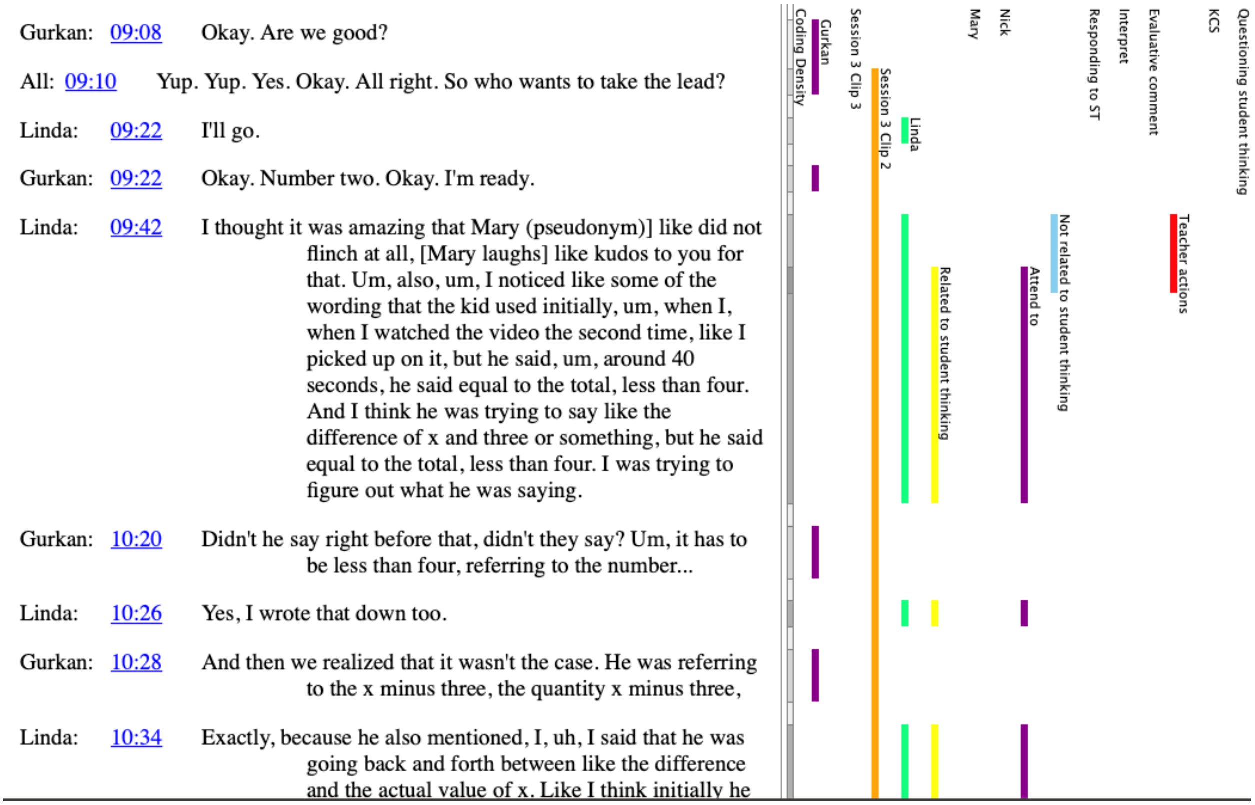
Analyzing the Session Transcripts

I transcribed the recorded videos (screen captures of the Zoom meetings) of the PD sessions and replaced the teacher and student names with pseudonyms to protect their identities. After that, I printed all session transcripts and read them to gain familiarity with the data. Having the research question and the theoretical framework in mind, I highlighted sessions that were relevant to my research questions and the theoretical framework and did some preliminary coding. Some of these initial codes were "teacher actions," "evaluative comment," "off-clip," "student thinking," "interviews." After having a general idea about the PD session discussions, I transferred all transcript data into NVivo, a Qualitative Data Analysis (QDA) computer software package produced by QSR International (2020), to reduce manual tasks and analyze the data more thoroughly. It was my first time using NVivo, and I had to spend a considerable amount of time reading user manuals and watching video tutorials. I invested my time learning how to use NVivo because I knew it promised to deliver many advantages. In the end, I did not regret that decision. The software helped me keep all the data in one place, and some of its features reduced the amount of time on coding, analyzing, and retrieving data as I built my cases.

First and foremost, I labeled all the data by speaker name. The software helped me achieve this task by auto-label all four session transcripts by each speaker. As a result, I was able to filter each speaker's verbal comments with a few clicks. Then I checked the timestamps on the transcripts and separated the sections with informal conversations from the ones with actual clip discussions. As a result, I created categorical labels for each of the nine clips. For example, on the transcript of Session 3, the section from 09:10 through 14:49 was labeled as “Session 3 Clip 1”. This helped me access the parts of the data relevant to my research question quickly and easily. Especially while comparing a teachers’ written comments to their verbal remarks about a particular clip, this setup on NVivo helped significantly. While reviewing the data, I highlighted specific sections of the data using the color-based highlighting features coupled with the coding strips, as shown in Figure 4.

Figure 4

Sample NVivo screenshot that shows the coding strips.



After I became more familiar with NVivo and set up my data as described above, I started reading each session's transcript for the second time and added codes as I noticed some themes. The codes I used on NVivo, along with the number of references for each code, are shown in Table 4. Some of these came from my initial reading of the data, while I identified others later. When possible, I tried to assign one of the three components of professional noticing (attending to, interpreting, and responding to) to the comments made during the collaborative clip discussions to compare teachers' written entries to their verbal comments about each clip.

Table 4

Codes used on the PD session transcript analysis.

Code	Example
Not related to student thinking	
Teacher (interviewer) actions	"I also liked how Linda (pseudonym), she did not guide her, but she reminded her of when she was stuck."
Interview-classroom comparison	"Um, and I felt that she, as teachers, we are not getting this window, if you will, to enough, I should say we're not getting enough opportunity to really see how one thinks."
Reflection (On teaching/interviewing)	"It was hard for me to like take away that teacher moment and just not say anything."
PD Reflections	"Overall, it was something that, it was an eye-opening experience."
Related to student thinking	
Attend to	"She said that it works for certain numbers because um, the negative numbers less than four. Uh-huh. But then she didn't test other numbers to see that it's not working in certain cases."
Interpret	"It seemed like the kid did have an understanding of what a sale price is and like how to get from the original to the sale price."
Responding to student thinking	"I would try to help her what proportion, um, to set up a proportion where she compares the original price to the discounted price and uses that 80%, 100%."
Evaluative comment (About student thinking)	"It's obvious that she knows what she's talking about, and I am assuming she's a good student."
(Teachers' own) Knowledge of Curriculum and Students (KCS)	"But, um, it's, it's the thing that most of the kids don't jump to fractions and decimals. They, you know, would dive into the negatives, but won't touch, um, the in-betweens."
Questioning student thinking	"I'm curious to know if she was kind of sparked to do that because she saw in question three that there were inequalities."

After coding all data, I grouped them into two major categories: (1) Not related to student thinking and (2) Related to student thinking, as shown in Table 4.

Codes Not Related to Student Thinking. The first code under this category was teacher (interviewer) actions. I used this code to highlight the discussions where teachers referred to the teacher actions in the clip rather than the students' mathematical thinking. The following is a comment that I marked as teacher actions: "I noticed how Linda was able to guide the students' thinking." It is strictly about a non-mathematical aspect of the interview, and it does not provide any specifics about the student's thinking.

Another code I used under this category was the *interview-classroom comparison*. During the PD sessions, teachers compared the interview setting to their everyday classroom interactions with students. During Session 5, Linda said the following:

The part where that kid said $25 and you take the 5 away, and you get 20, like being in a class, you know, like with your timeframe and everything. And you're not really able to probe that one specific student. I feel like in that setting, I would just move on." (Linda, Session 5, May 27, 2020).

Here, Linda shares her appreciation of the one-on-one setting to be able to understand a students' mathematical thinking. I used this code to compile similar comments that reflect teachers' thoughts about the PD program.

The code *reflection (on teaching/interviewing)* refers to the comments in which teachers reflected on their interviewing practice during the PD sessions. A statement like "It was hard for me to like take away that teacher moment and just not say anything." (Linda, Session 5, May 27, 2020). I used this code to mark how teachers opened up about their teaching practices during the clip discussions. Related to reflections, I coded teachers' direct comments about the PD program under *PD reflections*. These comments were not too numerous, and all of them occurred during

Session 5, the last PD session. I used this code to access teachers' takeaways from the PD program easily.

The majority of the codes were related to student thinking. Next, I describe those codes with examples.

Codes Related to Student Thinking. Teachers made many comments related to student thinking, the first of which was attend to. I coded the sections from the discussions where teachers paid specific attention to how students approach problems and use solution strategies. One example of this was the following quote: "Um, I think that he, um, spoke about 20 and then take away five, which makes it 15. And he thinks that 20 is also the price, not the percent." (Nick, Session 5, May 27, 2020). The teacher described a strategy employed by the student and provided some details from the clip.

Another code related to student thinking was *interpret*. Throughout the PD sessions, teachers interpreted some elements of student thinking, often without supporting their claims with evidence from the interview clips. Consider the following comment from Nick: "I mean I, as you see the video, you see that she understands, you see that she knows what she's talking about" (Nick, Session 2, May 6, 2020). The teacher makes a bold claim that the student understands what she is talking about; however, he does not provide any direct evidence from the clip. On the other hand, the following quote provides a comprehensive interpretation of the students' strategies with evidence from the clip:

When she was saying like 20% of five, I don't think she was trying to figure out 20% of five. I think it was more of an example because she also said, um, the 20% of the $10 shirt would be $8. So I don't think she was confused or maybe I, maybe I just didn't take it that way. Um, I don't think she was confused that it was 20% of $5. I think she was just like practicing the calculation. (Linda, Session 5, May 27, 2020).

In her comment, Linda interpreted the student's thinking with direct evidence from the clip. She could dig into the students' words about calculating 20% of 5 and why she might be doing that. I marked both of the comments listed above with code interpret. This helped compare teachers' written responses to the interpretation prompts with their comments about the same clips during the PD sessions.

Another code that was related to student thinking was evaluative comment (about student thinking). Teachers made many general comments that were not necessarily about the student's mathematical thinking in the clip viewed. Instead, they shared their judgment of the student vaguely without providing any evidence from the clip. Comments like "the student is brilliant", "She is really great" are coded as evaluative comments.

Next, I have *knowledge of content and students* (Ball et al., 2005) which I refer to as KCS. I chose this code because teachers were making references to how students learn certain content or some misconceptions about specific math topics. This seems consistent with Jacobs et al. (2010)'s conception of interpreting students' mathematical thinking:

On the basis of a single problem, we do not expect a teacher to construct a complete picture of a child's understandings, but we are interested in the extent to which the teacher's reasoning is consistent with both the details of the specific child's strategies and the research on children's mathematical. (p.172)

On the other hand, I used the code KCS in this study for teachers' verbal comments that were not necessarily linked to the specific strategies evidences in the clips. The following quote is an example of KCS: “But, um, it's, it's thing that most of the kids don't jump to fractions and decimals. They, you know, would dive into the negatives, but won't touch, um, the in-betweens.” (Linda, Session 3, May 13, 2020). This claim did not provide any specifics evidenced in the clip that was being discussed. Instead, Linda made an interpretation based on her knowledge of content and her past experiences with students. I wanted to differentiate such comments from those that were explicitly about the interview clips viewed, and that’s why I came up with the code KCS. That is why this code was not merged with *interpretation*.

Teachers made occasional comments about *responding to student thinking*. All of those comments came out as teachers reflected on their own interview clips. The following is a good example of a segment that was coded responding to student thinking: “I was trying to use different numbers and also to guide him to use a different presentation, which is, in a sense helped him.” (Nick, Session 4, May 20, 2020). The teacher offered questions to further probe the student’s thinking in ways that could have potentially enhanced his understanding.

Lastly, I coded some comments as *questioning student thinking*. These were the times I could not categorize what teachers said about student thinking under any of the three components of professional noticing (AIR). They were not attending to a particular aspect of the student’s thinking, nor were they interpreting it in any specific way. Those comments also were not about responding to students’ mathematical thinking. The following illustrates an occasion in which Linda questioned a student’s mathematical thinking: “I'm curious to know of like what, how he would have done with fractions or decimals.” (Linda, Session 3, May 13, 2020). Here, Linda raised a question that could not have been answered with the limited information viewed in the clip. She questioned the student's thinking, and thus it is related to student thinking. However, it does not fall under one of the AIR components of professional noticing.

Developing Individual Cases

I first reviewed Mary's data and started developing her case. This process started with organizing Mary's written response data in a single file. This helped me filter and sort her responses chronologically (by session date) and by each of the AIR categories. I initially focused on the pre-and post-assessment results on each of the three AIR categories and made comparisons. I reported any changes I saw and supported my observations with direct quotes from Mary's responses.

Next, I reviewed Mary's data from the weekly AIR prompts and created a visual representation of the ratings of her responses. As I described the data, I provided excerpts from her written entries as I deemed necessary. When possible (because she did not comment on every clip during the discussions), I reviewed the weekly prompts data side-by-side with the session transcripts. Before reporting on my observations, I crosschecked Mary's written responses to the weekly AIR prompts with her comments about the clips in order to gain a deeper insight into her noticing skills.

Lastly, I reviewed Mary's written responses to the three questions related to her reflections on the post-assessment. I compared those to the occasional comments she made about the PD program and her experiences with interviewing and weekly clip discussions during the PD sessions. Using all this data, I developed the case of Mary. I used a similar approach as I wrote the other two cases. However, the ways in which teachers engaged with the PD was different. As a result, each teachers' case was unique in many ways. I report this under the cross-case analysis section.

Cross-case Analysis

After developing the cases for each teacher, I reexamined the data to understand the similarities and differences between teachers'' experiences with the PD program and the patterns in their noticing skills. I organized and indexed the data in ways that helped me sort and view it from different perspectives. For instance, I was able to see the ratings of written responses to noticing prompts by teacher while developing individual teacher cases. In contrast, during the cross-case analysis, I sorted the data by session and clip numbers. This helped me see the patterns in teacher performance across different clips. For instance, all three teachers scored "no evidence (0)" on Session 2 Clip 1.

Similarly, I reviewed the session transcripts from an angle different from how I did it the first time while developing each teacher's case. As an example, instead of focusing on Mary's attending, I reviewed all sections coded as "attending to" independent of the teacher who commented. I looked for patterns, similarities, and differences in teachers' noticing of students' mathematical thinking across different clips viewed and discussed.

Positionality and the Paradigms I Find Myself In

Theoretical perspectives and how one understands research process guide the procedures they use to acquire knowledge (Silver & Herbst, 2007). Although it is hard to classify myself in any school of thought, I know that I belong in some and do not agree with others. To begin with my ontological orientation, I am a critical realist. I believe there are many factors that interfere between my research and reality. I cannot imagine reality (or many aspects of reality) without recognizing those factors such as my own biases, subjectivities, and the deceptive nature of complex systems such as schools and classrooms.

From an epistemological standpoint, I see myself closer to the constructivist and interpretivist school of thought. I do not believe in an objective truth that is waiting for me to discover it. I believe that meaning is constructed in people's minds (Mertens, 2014). Researchers have to try staying neutral and minimizing biases as positivists suggest; however, I do not believe it is possible to completely eliminate those subjectivities. Within the limited research encounters, projects, and most recently, during my dissertation study, I experienced many benefits of acknowledging my own subjectivity and transparently writing about it. As opposed to the positivist belief that perceives researcher bias as a deficiency in the validity and value of research findings, I argue that the insights and personal experiences the researcher brings illuminates deeper understanding of the phenomenon studied. I shared these in Chapter 1.

As a district administrator, I noted some of my biases and pre-conceived notions about the three teachers. Since I knew them before starting the study, I had some predictions about how each of the teachers would have progressed through the different parts of the study. I only knew the teachers within my capacity as they taught lessons and participated in PLC meetings. Despite the fact that I have not observed them as they interviewed students one-on-one, I had some informal ideas on how each teacher would have carried out the interviews. Similarly, I made some assumptions that one teacher would have noticed and interpreted student thinking in more comprehensive and meaningful ways than the other two. In the end, once I analyzed the data, some of these assumptions seemed to be invalid or incomplete. I was able to confront with my preconceived notions and tried to be as objective as possible while reporting on my observations and findings.

Chapter 4: Research Findings

The Case of Mary

At the time of the study, Mary had ten years of teaching experience. Her school administrators identified Mary as one of the strongest teachers at School 1. She consistently scored highly effective, the highest level of performance based on the Danielson Framework for Teaching (Danielson, 2007). According to her official evaluation results, she planned effective lessons with students' cognitive engagement in mind and delivered them with fidelity. She had a good rapport with students, and as a result, she rarely experienced classroom management problems. She actively participated in the weekly sixth-grade mathematics professional learning community (PLC) meetings and was considered one of the middle school grade band leaders. She showed immediate interest in being part of the current study and carried out the interviews with great enthusiasm.

Throughout the PD program, she was attentive and engaged in the conversations. Her spoken comments about the clips were mainly consistent with her written responses to the noticing prompts. On occasion, she provided additional details about what she noticed (in addition to what she wrote) as the group discussed the clips. Mary contributed very little to the discussions during Session 2 (the first session in which interview clips were discussed). She might have been trying to figure out the expectations and routines of the program during Session 2. However, Mary provided her insights in great detail during the other sessions. There were some patterns in Mary's engagement with different components of professional noticing of students' mathematical thinking as she participated in the PD program. In the following sections, I report on her experiences with noticing throughout the PD, starting with the pre-and post-assessments. I then present the details in her noticing of students' mathematical thinking during the PD sessions. I conclude the section with Mary's reflections about her experience participating in the PD program.

Pre- and Post-assessment Comparison

Table 5 shows Mary's scores on each noticing component (AIR) on the two assessments (See Appendix A), rated as described in the methods section.

Table 5

Mary's Pre-and Post-assessment AIR Scores

Assessment	AIR Components		
	Attending to	Interpreting	Responding to
Pre-assessment	0	2	1
Post-assessment	1	2	2

Note. On this table, 0 represents No Evidence, 1 represents Limited Ability, and 2 represents Emerging Ability.

From the pre-assessment to the post-assessment, Mary showed signs of improvement in all three components of professional noticing. Although the numbers show no progress in interpretation, the quality of her response to earn a score of "emerging ability (2)" on the post-assessment was higher when compared to her answer from the pre-assessment. I provide more details about this in the paragraphs below.

Attending to Students' Mathematical Thinking on the Pre-and Post-assessments. At the outset of the PD program, Mary started with relatively low scores in attending to student thinking. On the pre-assessment, after watching the video once, she wrote, “I noticed that students are engaged in constructing viable arguments regarding a matching problem. I also noticed that student 2 is driving the discussion by prompting student 1 in the correct direction” (Mary, Pre-assessment, April 28, 2020). This statement outlines little to no specifics about student thinking. Instead, Mary provides some general observations with little mathematical detail.

At the end of the PD program, she answered the same question about the same clip as follows:

> I think student A understands the concept of grouping/division process. Student A was able to rectify her thinking process by reasoning. Student B asked meaningful questions to spark and redirect student's A thinking in the right direction without funneling Student A's thinking. (Mary, Post-assessment, May 31, 2020)

In the first sentence above, she referred to some elements of the student's mathematical thinking compared to her initial response. That is why her response here was rated as “limited ability (1)”. Though not significant, there was a slight improvement in her attending to students’ mathematical thinking based on the two results.

Interpreting Students’ Mathematical Thinking on the Pre-and Post-assessments. Mary scored “emerging ability (2)” in interpreting on both the pre-and post-assessments. However, this does not mean she did not improve in this component of professional noticing. On the pre-assessment, she wrote, “I think (Student A) is looking at the model and identifying the 6 halves, and thinks since there are 6 half pieces, that 6 is divided into 1/2 pieces which gives 12 total” (Mary, Pre-assessment, April 28, 2020). She was able to identify the strengths in the student's thinking and connect them to specific ideas evidenced in the clip without making any overgeneralizations. On the post-assessment, she wrote:

I think Student A thinks that the model corresponds to the expression 6/3 because the model has 6 parts, and she is confusing it by 6 groups given in the expression. She is unable to identify the dividend and the quotient in the problem. In her thinking, she is considering the 6 (dividend) to the 6 (one-half) shown in the model. (Mary, Post-assessment, May 31, 2020)

This response includes more details about the student's thinking, and it provides a better connection to the student's written and oral work in the clip compared to her response on the pre-assessment. It is evident in this response what the student in the clip understands and does not understand. Mary's knowledge of content and students (Ball et al., 2005) is displayed better in the part where she wrote, "She (the student) is unable to identify the dividend and the quotient in the problem." Hence, Mary's interpreting skills seemed to have changed from qualitatively from the pre- to the post-assessment, although the numerical ratings do not reflect it. But, as I will describe below, during the five weeks of the PD, her scores on interpreting fluctuated.

Responding to Students' Mathematical Thinking on the Pre-and Post-assessments. When it came to responding to student thinking, Mary score changed from 1 to 2 between the two assessments. Her response on the pre-assessment was as follows:

I would start by redirecting the student to what the question is asking. It is asking to find each part, not the total number of parts. From there, I will prompt Student A to think about what is the whole in this question and what we are dividing it by. (Review vocabulary terms with Student A). Since Student A believes that the model matches 6/3, I will have the student model it for me using circles. 6 circles divided into three groups will give 2 in each group. From here, I will guide the student to analyze the two models and match the correct model to the question. (Mary, Pre-assessment, April 28, 2020)

Although it is detailed, this response does not provide a clear rationale for how the teacher moves described can enhance student learning. The next steps given are vaguely connected to Mary's response to the previous question regarding what the student understands. Therefore, this response was rated "limited ability (1)" by the two raters. Mary's response to the same prompt on the post-assessment was as follows:

I would have her draw a model to explain her thinking. Give her another question or even ask her to come up with a division question and draw a model to explain her answer. I think this will give her a better understanding of the concept as now she is illustrating her thinking instead of making sense out of a given model. I will also prompt her to reason her thinking, clarify why she believes her model matched the expression. Once she is able to justify, I will ask the student to find the match for the question given. As she finds the match, I will ask her to share her thinking process along the way. By saying out loud, students are able to hear themselves and can easily identify their mistakes. (Mary, Post-assessment, May 31, 2020)

Here, Mary made more specific references to the student's mathematical thinking. The next steps suggested here are connected to Mary's previous response about what the student understands and does not understand (the interpretation prompt). Mary also provided a rationale for furthering students' thinking about identifying the dividend and quotients in a division problem. This response received a rating of "emerging ability (2)". Hence, there may be evidence for the improvement in Mary's responding to students' mathematical thinking. I will discuss what happened during the PD sessions later.

Fluctuating Scores in Mary's Noticing During the PD Sessions

I analyzed Mary's written responses to the weekly noticing prompts and the PD session transcripts of group discussions to seek evidence of variations in her noticing skills throughout the PD sessions. Table 6 shows Mary's AIR scores based on the written responses she provided to the weekly noticing prompts for each video clip. The interview clips viewed during Sessions 2 and 3 show students working on Task 1, while the ones viewed during Sessions 4 and 5 show students working on Task 2. As described in the methods section, the scores for each component of noticing (AIR) represent the evaluation of two raters through independent rating and discussions to arrive at a consensus. I report my findings in the following sections. Although the duration of the professional development program and the ratings preclude me from making any claims about changes to Mary's noticings over time, below, I point out some patterns in her noticings and attempt to relate them to the professional development program.

Table 6

Mary's Scores Based on Written Responses to the Weekly AIR Prompts

Task	Session	Clip	AIR Components		
			Attending to	Interpreting	Responding to
Task 1	Session 2	Clip 1	0	0	0
Task 1	Session 2	Clip 2[a]	1	1	0
Task 1	Session 3	Clip 1	0	1	1
Task 1	Session 3	Clip 2[a]	0	1	1
Task 1	Session 3	Clip 3	2	1	0
Task 2	Session 4	Clip 1	1	2	1
Task 2	Session 4	Clip 2	0	1	0

Task 2	Session 5	Clip 1[a]	1	1	0
Task 2	Session 5	Clip 2	1	2	1

Note. On this table, 0 represents No Evidence, 1 represents Limited Ability, and 2 represents Emerging Ability.

[a] Indicates the clips in which Mary was the interviewer.

Inconsistencies in Attending to Students' Mathematical Thinking. As shown in Table 5, there are some differences in Mary's attending to students' mathematical thinking between the earlier sessions (2&3) and the later ones (4&5). Her scores in this category were not consistent. One example from the earlier sessions is her written response to the attending to prompt for Session 2 Clip 1 where she wrote, "(I noticed) how the teacher was able to guide student's thinking in the right direction without giving out too much information" (Mary, Session 2 Clip 1 response, May 6, 2020). This response describes teacher actions instead of student's thinking and, as a result, was scored "no evidence (0)". Since this was the first clip that was viewed and written about, I cross-checked this with her comments about the same clip during the PD session to see if she misunderstood something on the written prompt. During the discussion of this clip, referring to the statement and the given inequalities in Task 1 (Appendix B), Mary said, "I really liked how she (the student in the clip) was able to connect her answer to the statement" (Mary, Session 2, May 6, 2020). This was the only comment Mary made about what she noticed in Session 2 Clip 1 during the discussion. Besides that, she talked about the interviewers' actions guiding the student. One comment was, "I noticed something that I should not be doing would be giving her response after every sentence that she was saying. So, I think that was kind of like a disruptive little bit" (Mary, Session 2, May 6, 2020). This and other similar remarks were coded as "not related to student thinking."

For the rest of the clips, Mary's written responses to the attending to prompts were rated as 0's as she provided general comments about the interviewer actions not necessarily related to the students' mathematical thinking. This is in line with literature that indicates that teachers tend to focus on teacher actions rather than student thinking when reviewing video of a teaching or interviewing scenario (Star et al., 2011). She also received some 1's as she attended to some elements of the students' solution strategy, though it was with less detail and structure. One exception to this was her response to Session 3 Clip 3 attending to prompt for which she wrote, "Student uses substitution to find the correct inequality. She refers to the original questions to justify her thinking" (Mary, Session 3 Clip 3 response, May 13, 2020). She attended to a significant mathematical idea (using substitution). Mary described a strategy employed by the student (referring to the original questions to justify their thinking) coherently and comprehensively. She also did not make any evaluative comments about the student while describing her thinking. This was the single highest score, "emerging ability (2)", Mary received in attending to students' mathematical thinking. Her comments during the session about this clip were as follows:

I noticed how she (Linda) made the student make a connection back to the statement every time to justify her thinking. Um, every time she gave, and she went back and she kind of like, justified it by saying, Oh, this is why my inequality is correct or is incorrect because the original statement is asking for this. Um, so she was kind of like very clear in her thinking, where she's going and how she's going to, why her answer is correct. Um, and also, I liked how she used the elimination process in the beginning too. Like these two are not going to be my answers and then focus on the other two, and then went back and proved why those two are not her choices. (Mary, Session 3, May 13, 2020)

This comment aligns with her written response and provides a more in-depth explanation of the student's mathematical thinking.

However, it is interesting to note that on the one clip for which she scored a 2 for attending, she did not score very high on interpreting or responding. She received a 1 for interpreting and a 0 for responding. This reveals that while she was able to articulate the students' thinking, her interpretation read, "Student makes a clear connection to the original statement by referring it back in the end to justify her answer choice" (Mary, Session 3 Clip 3 response, May 13, 2020). With this entry, Mary interpreted some elements of the student's thinking, but she left out some details and focused upon only one aspect of the student's strategy. In her response to the attending to prompt, she included the student's strategy of using substitution. In the interpreting entry, she did not refer to that. Instead, she provided a general overview of what the student did throughout the clip. As a result, her interpretation scored a 1. When it comes to responding, Mary wrote, "Linda did an awesome job in probing the student for a deeper understanding of the question. I would have done the same" (Mary, Session 3 Clip 3 response, May 13, 2020). She did not link the suggested next steps to what she thought the student understood. Nor did she provide a rationale for the next steps she offered. As a result, her responding score for this particular clip was a 0.

Based on the evidence provided above, I can conclude that Mary's attending to students' mathematical thinking was not consistent. Through her participation in the PD program, she started providing more details about what she noticed and supporting her observations with references from the clip rather than making evaluative comments and overgeneralizations. However, this was not definitive since the data is limited to only four weeks of PD sessions, interviewing, and the viewing of nine clips.

The inconsistencies in Mary's attending may be attributed to many factors, one of which can be how the PD program was structured. From the outset of the program, I did not want to funnel the teachers' noticing of students' thinking in any way. That is why I did not provide specific instructions on how they should respond to each of the AIR prompts for the videos shown. I also did not share the AIR rubric I was going to use to rate their responses. For example, the question that asks them to report on their attending to reads, "After watching the video for the first time, what do you notice?" (Appendix C). This prompt does not instruct the teachers to focus on student thinking and refrain from evaluative comments. As a result, Mary might have provided general observations not necessarily related to the students' mathematical thinking or shared her noticings about the non-mathematical aspects of the interviews. Had she known those would have been scored 0s, she might have focused more on student thinking in her written responses. Nonetheless, I do not see that as a limitation of the study. It is valuable to see teachers' noticings emerge naturally as the way the program structure is set up. I enacted many moves listed in the framework for facilitation of video-based discussions (van Es et al., 2014) such as orienting the group to the video analysis task, sustaining an inquiry stance by highlighting noteworthy comments, redirecting focus on the video and the mathematics when the conversations were off tracked, and supporting group collaboration by sometimes standing back and distributing participation. However, I did not share the AIR rubric with the teachers nor did I give them feedback on their written responses. I intentionally avoided a funneling-interaction pattern as this kind of approach might have limited what teachers were able to contribute. This is because I did not want to direct their noticings in a predetermined path based only on how I as the facilitator would have attended to, interpreted, and responded to student thinking in the ideal way outlined on the AIR rubric. With this, I also modeled the focusing approach suggested in the questioning article (Herbel-Eisenmann & Breyfogle, 2005) we read and discussed with the teachers during Session 1.

From Vague Interpretations to Specific Ideas Evidenced in Clips. At the beginning of the PD program, Mary scored a "no evidence (0)" interpreting Session 2 Clip 1 with this response: "The student was able to connect her answer to the original statement by reasoning her thinking." (Mary, Session 2 Clip 1 response, May 6, 2020). She interpreted the students' thinking in a way that is too vague to be meaningful. This was the only time she scored a 0 in interpreting. On the other eight clips, she averaged a score of 1.25/2, scoring either a 1 or 2 on her responses.

Two of her nine responses in the interpretation category were rated "emerging ability (2)". One of those responses was as follows:

> The student was able to figure out the sale price, and he understood that by adding the $5 to the sale price, he will be able to get the original price. However, was unable to make a connection between the $5 and 20% given in the question. (Mary, Session 4 Clip 1 response, May 20, 2020)

In her response, Mary mentioned both the strengths and weaknesses of the student's mathematical understandings. As evidenced in the video, the student was able to see that he had to add $5 to the sale price to find the original price; however, he did not demonstrate any evidence that he was able to connect that $5 and the 20% discount. She made this interpretation in a non-judgmental way and provided only the mathematically relevant details about the student's thinking in her response. As a result, this entry was scored as "emerging ability (2)" in interpretation by both raters.

The other response that was rated "emerging ability (2)" was from Session 5 and it read as follows:

The student goes back and forth, trying to make sense of her thinking. The student insisted on showing a number line to explain her thinking. It made sense to her that if 20% is the $5, then by adding $5 each time to get to 100%, she will get the original price. (Mary, Session 5 Clip 2 response, May 27, 2020)

Here, Mary interpreted the student's mathematical thinking in several ways. First, she had a clear grasp of the student's strategy to solve the problem. She provided a general observation that the student was showing an effort to make sense of her thinking. Then, she referred to the student's choice of representation (using a number line) to explain how she visualized the problem. Finally, she shared the fact from the clip that the student understood the equivalence between the 20% and the $5 given in the problem and used this to get to 100%, the original price. This comprehensive response was scored as "emerging ability (2)". As a side note, Mary's attending to score (1) is lower than her interpreting score (2) for this particular clip. Her written entry to the attending to prompt was not detailed and structured enough to score a 2. Nonetheless, the way she interpreted the students' thinking was comprehensive and evidence-based. This is interesting as usually, the opposite occurred in the data set. Teachers scored higher in attending than they did in interpreting a given clip.

During the PD sessions, Mary interpreted students’ mathematical understandings in great detail on some clips. One example of this is her comments on Session 5 Clip 1:

He knows how to break down the problem. When he started the question, it looked like he got it, but then as I, the deeper I figured out, Oh, okay. So, he's basically just taking random numbers and trying to make sense out of it, but I did like that he was using a little bit of guesswork and a little bit of estimation to figure out if he was in the right direction, but then he was unable to get there. (Mary, Session 5 Clip 1 discussion, May 27, 2020)

Here, Mary started with a general observation and then provided more insight into her noticing. While watching the clip, she assumed the student had a clear understanding of the problem given. However, she later realized that the student was just trying out random numbers to find the answer. This was a verbal comment, and we only scored the written entries. While her verbal comment on this clip was strong, her written entry was not as robust. Her comment shows that Mary was able to interpret the student’s mathematical understandings based on the mathematical ideas and relationships evidenced in the clip.

Overall, Mary moved from vague interpretations to specific ideas evidenced in clips. She tended to do better on interpreting in the latter half of the PD while interviewing and watching clips about Task 2. For example, she only scored 2’s in the latter half during Sessions 4 and 5. This may be attributed to the differences between the two tasks which I will describe in the cross-case analysis. Alternatively, by this point in the PD program, she completed more interviews, watched more clips, and engaged in more discussions with her colleagues during the PD sessions. Those experiences might have given Mary windows into student thinking and opportunities to observe students’ attempts to solve problems.

As discussed in the literature review, noticing matters only if teachers' observable behavior changes and act on what they noticed and interpreted (Jacobs et al., 2011, Schoenfeld, 2011). In the next section, I report on Mary's responding to students' mathematical thinking.

Room for Improvement in Responding to Students' Mathematical Thinking. Mary's performance in responding to students' mathematical thinking fluctuated between 0 and 1 based on her written responses to the weekly noticing prompts shown in Table 6. She scored a "no evidence (0)" for five of the nine clips shown during the PD sessions. In all five cases, the "next steps of action" she suggested were not tied to the student's understandings. For example, for Session 2 Clip 1, she wrote, "The student was able to connect her answer to the original statement by reasoning her thinking" for the interpreting prompt, which was scored 0 as it was too vague to be meaningful. It also discounted the fact that the student seemed to be unsure about her choice of inequality while testing out numbers in the other inequalities. When asked to describe some ways she would have responded to this student, she wrote, "I would also have her work out other answer choices, so that she is clear with her thinking" (Mary, Session 2 Clip 1 response, May 6, 2020). This response was hardly connected to the mathematics the student did or did not understand. The comment was focused more on the questioning techniques used by the interviewer, which is a form of responding. This might have been because in the previous session, teachers read and discussed an article related to questioning (Cite). In order to score higher on this, she needed to have provided more details on what was or was not clear in the student's thinking. She also needed to provide a rationale for the suggested next steps.

For another clip, she wrote, "(I would) have him dig a little bit deeper" (Mary, Session 5 Clip 1 response, May 27, 2020). Again, it was not clear how this action was related to the student’s thinking and how it could have helped the student with the concept. In both of these instances, Mary’s written feedback was in line with having the student “keep trying” or “try harder”, which are not connected to what the student understands, and there is no rationale provided for the next steps. This kind of feedback is also vague enough to apply to any situation. In fact, she didn’t really need to watch the clip to provide this kind of feedback.

Responding to student thinking on other clips, Mary gave answers less related to student thinking and more related to the interviewer's actions. She wrote, “Linda did an awesome job in probing the student for a deeper understanding of the question. I would have done the same” (Mary, Session 3 Clip 3 response, May 13, 2020). Along the same lines, for another clip, she wrote, “I like how Linda had her review her multiplication and inquired about how the student can make a connection to the 1 she found” (Session 4 Clip 2 response, May 20, 2020). Both of these responses lacked specificity in relation to the students’ understandings evidenced in the clip. She also did not provide any clear rationale behind those suggested next steps. As a result, those answers were scored “no evidence (0)”.

For two of the clips, Session 4 Clip 1 and Session 5 Clip 2, Mary provided slightly better strategies that could potentially enhance student understanding. She scored a 1 in attending and 2 in interpretation for both clips. In other words, she did not write her initial noticings in detail after watching the videos once. Upon the second viewing, she shared her interpretation of what the students understood in greater detail. Those were the clips on which she scored highest in the interpretation category. Her entry on responding, however, only scored a 1. Her written entry to the students’ mathematical thinking (for the respond to category) for the first clip of the two clips read as, “I would probe a little deeper to get a better understanding of why the student feels that the sale price is $20" (Mary, Session 4 Clip 1, May 20, 2020). Here, the strategy Mary suggested is connected to what she thought the student understood (the $5 difference between the sale price and the original price) and did not understand (the connection between the $5 and the 20% given). However, she did not provide a rationale for the suggested next step. Thus, it is not clear how this teacher move (probing a little deeper) would have enhanced the student's understanding. "Probing a little deeper" is also not providing many specifics on the nature of the probing. As a result, she scored a "limited evidence (1)" on this entry.

For the second clip, Mary similarly scored a 1 on attending to and 2 on interpreting, but a 1 on responding to. She wrote the following to describe her response to the students' thinking:

The student explained her thinking quite clearly once she got hold of the concept using the number line. I would have also asked her to use her understanding of the number line to see if the equation she was trying to set up made any sense. (Mary, Session 5, Clip 2 response, May 27, 2020)

Similar to her entry to the previous clip mentioned above, the strategy Mary suggested here is connected to her response about the students' understandings. However, Mary did not provide any details about why and how she thought her proposed strategy would have worked. This response also was scored as "limited evidence (1)" in the responding to category.

Both of these clips point to room for improvement in responding. Mary's written feedback on these clips indicated that although she did not provide much detail on what she attended to, she was able to give robust interpretations. But, on the other hand, although she was able to interpret the student's thinking, when it came to using that interpretation to suggest next steps, she again did not provide clear guidance in response to the student's understandings.

All in all, based on Mary's performance in responding to students' mathematical thinking fluctuated between the scores of 0 and 1. The evidence presented above suggests that there is room for improvement for Mary in this category of noticing.

Mary's Reflections on the PD Program

After the last PD session (Session 5), I sent the post-assessment (see Appendix A) to the teachers. As mentioned earlier in the methods section, it was identical to the pre-assessment with one difference. The post-assessments included the following additional items that were intended to capture teachers' takeaways and feedback about the overall PD program and study design:

1. In what ways do you think this study supported you as a math teacher?
2. How do you think the PD Program can be improved?
3. How do you think the study can be scaled up to support more teachers?

For now, I will only discuss her response to the first question as it is relevant to Mary's reflections on the PD program. Describing how the program supported her as a math teacher, she wrote, "I really appreciate all the feedback given to me during this study which actually helped me to rethink my questioning skills" (Mary, Post-assessment response, May 31, 2020). Here, she referred to the article that was read and discussed during Session 1 (Herbel-Eisenmann & Breyfogle, 2005) and also the main focus of the PD on student thinking. This feedback she referred to was not about her noticing skills though. Instead, she referred to the feedback she received from Linda and Nick about her questioning and interviewing skills. She went on to write, "As teachers, we get stuck in our teaching routines, this study encouraged me to listen and participate with other fellow teachers, learn from their perspectives and taught me to identify my weakness" With this comment, Mary underlined the importance of gaining different perspectives from the discussions with other teachers and reflecting on her own teaching practices. This can be tied to the collective participation (Desimone, 2009) aspect of the PD program. She finished her entry by writing, “It showed me the importance of spending extra few minutes or even seconds to understand a student's thinking process and directing them accordingly. This study enabled me to understand what it means to encourage productive struggle in the classroom” Based on these words, it can be concluded that the study helped Mary value students’ mathematical thinking and understand why teachers should encourage productive struggle. There are hints in her response that she is motivated to create more opportunities in her classroom for students to develop their thinking about mathematical concepts without getting interrupted by the teacher. The active learning aspect of the PD program (Desimone, 2009) might have contributed to these gains Mary mentioned in her comments since she was actively interviewing students and discussing interview clips with her colleagues.

During the PD sessions, Mary’s comments about the PD program were consistent with her response above. She talked about the opportunities for paying attention to student thinking and discussions with her colleagues and reflecting on her actions as a teacher. As the group was discussing one of her interview clips, she said:

So now like I'm watching them like, oh my God, I should not do that because it's like, I'm like, she has a thought process that's going on, and I'm like, kind of like putting mine. Okay. So that is something that I think, um, for me, um, something that I need to work on. (Mary, Session 2, May 6, 2020)

In her response, Mary acknowledged and valued the mathematical ideas and strategies presented by the student. It is also evident that Mary was not happy about how she interfered with the student’s thought process. Mary openly shared with her colleagues that she wanted to improve in that area. Additionally, while she was talking about her experiences watching her own interview video, she said, “Something that I should not be doing would be, should be, um, not kind of like giving her response after every sentence that she was saying. So I think that was kind of like, disruptive little bit” (Mary, Session 2, May 6, 2020). Again, she did not want to disrupt the student’s thought process.

Towards the end of Session 5, Mary commented on her experience participating in the PD program as follows:

I think we were just getting the hang of this, and now it's ending. I mean, this last session was the richest of all, one of those sessions that we were like, oh, like, okay, this is what we have to do, this is how it's going to work. It was a learning experience. It was an eye-opening experience. I actually think that we are so used to doing it in our classroom, and then you look at a different perspective. This is something that you should not be doing. This is something we should be focusing on more. Thank you so much for letting us be a part of this. (Mary, Session 5, May 27, 2020)

So, this “eye-opening learning experience” helped Mary calibrate her priorities and reconsider her moves as a teacher with the perspectives gained during the program. She learned to acknowledge student thinking and build instruction on that. When coupled with her written response on the post-assessment, it is evident that Mary learned to acknowledge student thinking and strategies. She saw the value in the details of what students do, write, or say. Mary appreciated watching her own videos as well as the videos of other teachers working with students. She developed some awareness about her teacher moves through her participation in the collaborative discussions during the PD sessions. These comments are general about her questioning and teaching and not specifically about noticing, which gives me feedback about the PD program. I touch upon these comments further in the discussion.

Summary

Overall, Mary showed differences across sessions and between the pre- and post-assessments in all three components of professional noticing of students' mathematical thinking (Jacobs et al., 2010). Although she scored a 2 on both assessments in the interpreting category, the amount of detail she provided on the post-assessment interpretation response supported the improvement claim.

During the PD sessions, Mary's contribution to the discussions gradually increased. She was quiet and hesitant at first, her entries on attending and interpreting moved from general to more specific details evidenced in the clips. Starting with low initial scores, Mary's attending to students' mathematical thinking fluctuated. However, she started saying more about what the student in a given clip understood or did not understand. It was not just the number of comments she made but also the quality of those comments in relation to the AIR rubric's (Figure 3) interpretation category. This was evident in her interpretation entries from the weekly responses. So, she interpreted what she noticed in a more comprehensive manner and connected those interpretations to specific strategies students used in the clips.

Responding to students' thinking seemed to be a challenge for Mary. Although she went from a score of 1 on the pre-assessment to a score of 2 on the post-assessment in this category, most of her scores during the PD sessions were 0's and she did not score any 2's. Based on her written responses as well as her comments during the PD sessions, my interpretation is that she could improve in responding to students' mathematical thinking. For the most part, her "suggested next steps" were either not connected to the students' understandings or were not provided with a rationale for how those actions could have potentially enhanced student understanding. This is consistent with the Krupa et al., (2017) that although the prompt about responding to specifically asked teachers to provide a rationale for their suggested next steps, most did not.

From her own perspective, Mary enjoyed participating in the PD program. She appreciated how interviewing students helped her realize that students have important ideas. When teachers pay close attention to those ideas, interpret them effectively, and guide students accordingly, they can better serve their students' learning needs. She also liked the discussions that focused on student thinking during the PD sessions. Watching interview clips together and the discussions that followed helped her gain a variety of perspectives that she would not have developed otherwise.

The Case of Nick

After working as an engineer for over twenty years, Nick started teaching in the Fall of 2018. He has been teaching sixth-grade mathematics at School 3 since then. Nick struggled with classroom management during his first year of full-time teaching and received extra support in that area from his school administrators. Based on the official teacher evaluation scores and his supervisors' observations, Nick improved his teaching performance significantly in the past two years. He contributed to the weekly sixth-grade mathematics PLC meeting discussions and shared many ideas and best practices. He defined himself as a life-long learner and agreed to participate in my study with enthusiasm. During the PD, as Nick progressed through interviewing and participating in the clip discussions with the other teachers, he started paying attention to more specific details of students' thinking and interpreted his noticings more productively. Based on the information I gathered from his school administrators and colleagues and my professional observations in the past two years, Nick is known to have a strong repertoire of teaching ideas. He is not shy to share those ideas in PLC meetings and other district-level PD events. This might have led him to score higher than the other two teachers in responding to students' mathematical thinking. Although he did not always base the next instructional steps he suggested to the students' understandings, he shared ideas and actions that could potentially extend students' thinking. Starting with the pre-and post-assessment results, I provide more details about Nick's experiences with professional noticing in the paragraphs below.

Pre- and Post-assessment Comparison

Table 7 shows Nick's scores on each noticing (AIR) component on the two

Table 7

Nick's Pre-and Post-assessment AIR Scores

Assessment	AIR Components		
	Attending to	Interpreting	Responding to
Pre-assessment	1	1	2
Post-assessment	2	2	2

Note. On this table, 0 represents No Evidence, 1 represents Limited Ability, and 2 represents Emerging Ability.

assessments (See Appendix A), rated as described in the methods section. Based on the data above, Nick showed improvement in attending to and interpreting students' mathematical thinking between the pre-and post-test. He started with a high score in responding to students' mathematical thinking and remained at that high level in that component of professional noticing. In the paragraphs below, I will be supporting these general results with details from Nick's own written responses to the noticing prompts on the pre-and post-test. I will then go into more detail about what happened during the PD program as Nick conducted interviews with students, engaged in watching interview clips, and discussions with his colleagues.

Attending to Students' Mathematical Thinking on the Pre-and Post-assessments. Compared to the other two teachers, Nick started with relatively high noticing scores at the beginning of the study. When asked to report his initial noticings on the pre-assessment, Nick first provided a narrative description of what he saw in the clip with not many mathematical details. He wrote, "The girl in the pink shirt tried to explain why she chose the two cards but was having trouble reasoning her selection/solution" (Nick, Pre-assessment, April 28, 2020). He went on to add some details about the thinking process of the two students in the clip by saying, "The boy in the black shirt pointed out that the two cards that the girl selected do not match, and he was trying to prove her wrong by asking her to provide the answer to each question". He finished his response with the following sentences that included more details about the mathematical processes students engaged in:

At the beginning, the girl in the pink shirt could not find the answer 2 on the other piece of paper that showed 3 circles divided into 2 parts but thought that she had the right match because she saw the number 6 on both, 6:3 and 6 half circles. (Nick, Pre-assessment, April 28, 2020)

In this last part, Nick provided direct evidence from the clip and some details about the girl's thought process. Overall, he referred to specific evidence from the video clip about how the student (the girl in the pink shirt) approached the problem initially and then changed her mind. With these elements, Nick's response was rated as "limited evidence (1)" but not an "emerging ability (2)" because he did not clarify why the student thought she had the right match, so Nick's comment there is unsubstantiated.

When given the same prompt at the end of the PD program, Nick provided a narrative description of what he saw, similar to his pre-assessment response. He then wrote, "She counts the parts and sees 6 pieces in all, she says if you take 3 and divide it into halves, you will get six, then says that the matching math problem is 6:3" (Nick, Post-assessment, May 31, 2020). Here, he described the mathematical strategy employed by the student and then provided more details by writing, "The girl says at some point 6 divided into half is 12, so it seems that she understands the math but finds it hard at first to reason the correct visual representation." He supported this claim with evidence from the clip by writing, "The girl says at some point six divided by half is 12." Nick described these details about the mathematical thinking of the student coherently and comprehensively. He also did not make any evaluative comments about the student or the strategies she used. As a result, this response was rated as "emerging ability (2)".

Interpreting Students' Mathematical Thinking on the Pre-and Post-assessments. Nick interpreted some elements of the student's mathematical thinking on the pre-assessment by providing some details in the given clip. He wrote, "She understands that dividing 6 into 3 ends up being a smaller number; however, I think that the student could not explain that dividing 3 into halves gives you six pieces." (Nick, Pre-assessment, April 28, 2020). While this response highlights some of the critical details about what the student understood, it does not conjecture about the possible cause of the student's confusion. For example, Nick did not clarify why the student was unable to explain the operation "3 divided by half is equal to 6" and did not include a reference from the clip to support that claim. As a result, Nick's response was rated as "limited evidence (1)".

For the same prompt on the post-assessment, Nick made a similar interpretation of the student's understanding of the problem by writing, "She had problems to relate the visual to the math problem. She said at some point, "I see 6 divided by 3" instead of 3 divided by half. She seems to have challenges identifying the dividend and the divisor" (Nick, Post-assessment, May 31, 2020). Notice that this time, Nick provided what he saw in the clip and then interpreted that observation. So, the interpretation was supported by a direct reference from the clip. He went on to write the following to say more about the student's understandings and giving a hint about a potential next step:

> She ends up selecting the correct card and agrees that 3 divided by 6 is half which is a variation of the math question 3 divided by half is 6. I do not know if 3: 1/2 was a card available on the table. It would be interesting to know if she would pick this card instead. (Nick, Post-assessment, May 31, 2020)

In the paragraph above, again, Nick finalized his interpretation of what the student understood. This time, he added his thoughts about showing the student an alternative card (3: 1/2) to see if she could identify the divisor and the dividend. This interpretation could have led to a next instructional step, and it paved the way for Nick's responding to the student's mathematical thinking. As a result, this response as a whole was rated "emerging ability (2)". Compared to his response on the pre-assessment, Nick's entry on the post-assessment included more robust evidence about his interpretation of the student's mathematical thinking. As described above, he supported his claims about the students' thought process with direct evidence from the clip shown.

Responding to Students' Mathematical Thinking on the Pre-and Post-assessments. At the beginning of the PD program, Nick's performance in responding to students' mathematical thinking was rated higher than the other two teachers. When asked on the pre-assessment what he would do next with the student in the clip, he described a list of steps he would have taken with the student:

I might start with one circle. Ask her to divide the whole circle into 2 and to tell me how many parts she sees. I am expecting her to say two. Then I would ask her to add another circle and cut it into two as well, then count the parts that she sees--I am expecting her to say 4. Finally, I would go back to the original drawing and ask her to count the parts that she sees--I am expecting her to tell me that she sees 6 pieces. (Nick, Pre-assessment, April 28, 2020)

Here, Nick did provide the questions he would have asked and provided anticipated student responses to his questions. He went on to write, "At that point, I will go over the basic parts of a division problem with whole numbers. I will ask her what 6 : 3 = 2 means. Together we will try (to) identify the dividend, the divisor, and the quotient" (Nick, Pre-assessment, April 28, 2020). After supporting the student's conceptual understanding of division with the visual, he connected the concept to the current problem. In other words, he provided a rationale as to why he started with the visual approach. He continued with additional sentences to describe the justification for the next instructional steps he chose by writing, “The idea is to help her understand the problem and also to make sure that she gets the different representations. I chose this answer because I think that the student can come up with the general rule, by deduction.” He concluded his entry by suggesting the use of manipulatives, again with the same rationale of supporting the student’s visual understanding of the concept. Overall, Nick’s response above is connected to what he believed the student understood. He described that in his response to the interpretation prompt, and here, he provided the next steps connected to the strengths and weaknesses of the student’s mathematical thinking. He suggested questions and a manipulative as an alternative representation that could potentially further the students’ thinking. Again, these were based on the understandings and misconceptions of the student evidenced in the clip. Moreover, Nick provided a rationale for each of the questions and tasks he suggested as the next steps. As a result, this response was rated “emerging ability (2)”.

Similar to the pre-assessment, on the post-assessment, Nick’s response included clear evidence that he considered the student’s existing strategy and suggested next steps based on that. He started by writing, “I think that I would ask her to identify the components of a division problem, in other words, divided, divisor and quotient and remind her that multiplication and division are very much related” (Nick, Post-assessment, May 31, 2020). Here, he set up the stage for the student to understand the conceptual relationship between multiplication and division. He went on and described how he would have started with whole numbers by writing, “I would start with whole numbers and ask her to create a visual representation then follow up with a multiplication triangle. If she said 10 : 5 = 2 we can talk about 10 : 2 = 5 and 2 x 5” Nick’s suggested next step here provides evidence of his knowledge about the progression of the CCSSM (2010) as the standards recommend students to “apply and extend previous understandings of multiplication and division to multiply and divide fractions” (p. 42). He went on and suggested that he would ask the student to create visual representations for both the whole number and fraction versions of the multiplication and division problems. Again, he aimed to build the student’s conceptual understanding with the questions and tasks he suggested. The strategies he suggested were linked to how he interpreted the student’s understanding of the problem evidenced in the video. Nick’s response was rated as “emerging ability (2)” since it met the expectations outlined in the AIR rubric.

Based solely on the pre-and post-assessment scores, it is not possible to conclude that Nick improved in responding to students’ mathematical thinking. Instead, it would be fair to say he already was at a high level in this aspect of professional noticing, according to the AIR rubric. Although he scored 2 on both the pre-and post-test, as I will describe below, he did not maintain this level of noticing during the PD sessions.

Patterns in Nick’s Noticing During the PD Sessions

I analyzed Nick’s written responses to the weekly noticing prompts along with the PD session transcripts of group discussions to examine the patterns in his noticing skills throughout the PD sessions. Table 8 shows Nick’s AIR scores based on the written responses he provided to the weekly noticing prompts for each video clip. As described in the methods section, the scores for each component of noticing (AIR) represent the evaluation of two raters through independent rating and discussions to arrive at a consensus.

Mixed Results in Attending to Students' Mathematical Thinking. As shown in Table 8, there are some differences in Nick's attending to students' mathematical thinking between the earlier sessions (2&3) and the later ones (4&5). I note here that students in the clips shown during Sessions 2 and 3 worked on Task 1. In the clips shown during Sessions 4 and 5, students worked on Task 2. Nick's scores in attending were not consistent. Occasionally, he attended to some elements of student's thinking during Sessions 2 and 3. As an example, Nick wrote the following:

The student gave the correct answer. For the 2nd inequality, the student said that it's not working and asks the teacher to present the slide with Abigail's thoughts. The teacher guides the student to reason her choice by plugging numbers that she found before. (Nick, Session 2 Clip 1 response, May 6, 2020)

Table 8

Nick's Scores Based on Written Responses to the Weekly AIR Prompts

Task	Session	Clip	AIR Components		
			Attending to	Interpreting	Responding to
Task 1	Session 2	Clip 1	1	0	1
Task 1	Session 2	Clip 2	1	1	1
Task 1	Session 3	Clip 1[a]	0	1	1
Task 1	Session 3	Clip 2	2	0	1

Task 1	Session 3	Clip 3	1	0	1
Task 2	Session 4	Clip 1[a]	0	1	0
Task 2	Session 4	Clip 2	1	2	2
Task 2	Session 5	Clip 1	2	1	1
Task 2	Session 5	Clip 2	2	2	1

Note. On this table, 0 represents No Evidence, 1 represents Limited Ability, and 2 represents Emerging Ability.

[a] Indicates the clips in which Nick was the interviewer.

In this response, while there is evidence that Nick attended to some elements of the student's mathematical thinking, he did not provide much detail. For example, he noticed that the student chose the correct inequality that matched Abigail's statement in the problem (See Task 1 in Appendix B). He also noticed that the student proved how the second inequality did not match Abigail's statement. These two elements partially captured the student's thought process. Nick missed out on some details, such as how the student was unsure about her choice of inequality while testing numbers out in the other inequalities given. Nonetheless, Nick's last sentence was focused on teacher actions rather than the student's thinking. Since Nick did not capture the student's thinking comprehensively and partially focused on the interviewer's actions, Nick's response was rated "limited evidence (1)". Similar to the conjecture I made in Mary's case, Nick's attention to the teacher actions may be related to the questioning article discussed during

Session 1. I provide more insights into the impact of the article on teachers' noticing in the cross-case analysis.

Nick's performance on the AIR prompts fluctuated significantly in sessions 3 and 4. For some clips, he wrote statements that were not related to student thinking. One example reads as follows: "I noticed that my intonation and I wonder if I pushed the student to change his mind." (Nick, Session 3 Clip 1 response, May 13, 2020). This was one of his own clips, and he was focused on his own actions as the interviewer rather than providing details about his noticings of the students' mathematical thinking. Along the same lines, he wrote the following for the other clip in which he was the interviewer:

(The) audio was bad :) ... I liked how I repeated the answers. I gave the student time to think about the question. I also liked my no reaction when he said $25 (which is the correct answer) and the no reaction when he said I chose 20 because of the 20%. (Nick, Session 3 Clip 1 response, May 13, 2020)

With this entry, Nick attended to the non-mathematical aspects of the interview. The focus was not on the student's thinking. Instead, it was on Nick's actions as the interviewer. This response was rated "no evidence (0)" due to the lack of details about student thinking. Along the same lines, for the other clip in which he was the interviewer, he wrote the following:

The audio was bad :) ... I liked how I repeated the answers. I gave the student time to think about the question. I also liked my no reaction when he said $25 (which is the correct answer) and the no reaction when he said I chose 20 because of the 20% (Nick, Session 4 Clip 1 response, May 20, 2020)

These two clips are the only two for which Nick scored 0's in attending. Nick's attention to his own actions as the interviewer is consistent with the previous research (Goldsmith & Seago, 2011) as teachers tend to focus on their own behavior as they watch self-recorded clips of classroom situations. They are less accustomed to focusing on students' learning. I also see Nick trying to improve his interviewing skills, as exemplified in the questioning article (Herbel-Eisenmann & Breyfogle, 2005) we read and discussed together during Session 1. I noticed the impact of this article on the overall attitude of the teachers as they commented on the clips. I discuss this further in Chapter 5.

In contrast to the entries above, during the same session, but for someone else's clip, Nick described his noticings in a clear and concise manner for a clip shown in Session 3:

The student was able to reason his selection and show various numbers that can work, and I was also impressed by the choice of a negative number, but he did not mention, in that part of the video, which numbers do not work, e.g., that 7 is not a solution, in other words, he concentrated on the solution but did not talk much about the numbers that will make this statement false. (Nick, Session 3 Clip 2 response, May 13, 2020)

There is evidence that Nick attended to significant ideas employed by the student in the clip. He also described the students' strategies in detail by saying that while refraining from evaluative comments. This response was rated "emerging ability (2)" since it satisfied that rating on the AIR rubric. It is important to note here that although Nick attended to significant details of the students' mathematical thinking, he did not score high on interpreting or responding. While he was able to articulate the student's thinking, his interpretation read as follows:

I now noticed that he mentioned the number 7 as not being an option, but no other numbers that make it false like 8 or above. I liked the fact that he said the total of … as if to say the number minus 3 needs to be less than 4. (Nick, Session 3 Clip 2 response, May 13, 2020)

With this entry, Nick touched upon additional elements of the student's thinking that he did not mention in his response to the attending prompt. He did not elaborate on what the student did or did not understand. From an interpretation standpoint and considering the expectations on the AIR rubric, this entry is too vague to be meaningful. As a result, it scored a 0. When it comes to responding, Nick wrote, "I would try to push him to find more solutions that do not work and talk about the concepts of false and true; what makes it false and or true" (Nick, Session 3 Clip 2 response, May 13, 2020). Although this entry is somewhat linked to what Nick wrote in response to the interpretation prompt, it did not include a rationale for the suggested move. As a result, it scored a 1. The scores of the three entries for the AIR prompts for this particular clip indicate that, as was the case with Mary, Nick did not maintain his high score in attending to with interpreting and responding. I will present my further interpretation of this situation in the discussion.

Despite the inconsistencies in his noticing performance during sessions 2-4, during Session 5, Nick scored "emerging ability (2)" in attending to students' mathematical thinking on both clips shown during Session 5. For one of those clips, he wrote, "The student tried different approaches, one was guessing and checking (20 and 15 then 25 and 20 as an example), the other one was an equation to find 20% of 5 which was 1" (Nick, Session 5 Clip 1 response, May 27, 2020). This was consistent with his comments during the discussion of this clip in Session 5 as far as the mathematical details evidenced in the clip went. He said, "Um, I think that he, um, spoke about 20 and then take away five, which makes it 15. And he thinks that 20 is also the price, not the percent" (Nick, Session 5, May 27, 2020). In both his written response and the verbal comments, Nick provided essential details about the student's thought process and the strategies employed by the student.

In his response to the attending to prompt for Session 5 Clip 2, Nick was able to pinpoint the confusion that the student was experiencing. He was specific in his speculation as to why she was confused. He wrote, "She confused again the 20% of $5 which is 4, and then she said that the $5 represent 20% of the price, which in this case works as the whole, in this case, would be 5 x 5 = 25" (Nick, Session 5 Clip 2 response, May 27, 2020). Not only did Nick notice the student's error, but he was also able to offer one reason why her answer would not have yielded the correct answer. In this way, he was noticing her error but also interpreting and beginning to respond to it.

His written response continued with his noticing of the student's strategy as she attempted to solve the problem. He wrote, "I liked the fact that she used a visual and the fact that when she thought about adding $5, she kept adding them to the model and the 10% was the original price" (Nick, Session 5 Clip 2 response, May 27, 2020). This part of the response showcases Nick's ability to note the student's use of a visual to reason through the problem and his acknowledgment that this was a sound approach. In this way, he was able to follow the student's reasoning and describe the strategy employed by the student. Similar to Nick's response to the first clip shown during Session 5, this response is rated "emerging ability (2)".

During the discussion of these two clips, Nick made comments that supported and extended his written responses. He described in great detail what he noticed coherently and comprehensively. In Nick's written responses and verbal comments about the two clips shown during Session 5, there is evidence that he noticed important mathematical ideas and strategies used by the students. Nick described what he attended to in great detail and did not make evaluative comments about how students approached the problems.

All in all, I can conclude that Nick's attending to the students' mathematical thinking throughout the PD program was not consistent based on the evidence provided above. He occasionally scored higher in this category, but he was not able to maintain those high scores. He did poorly on his own clips as he was focused on his own actions as the interviewer.

From Overgeneralizations to Specific Student Understandings. During the first two sessions, Nick scored "no evidence (0)" for three of the five clips shown and "limited evidence (1)" on the other two. Later on in Session 4, he scored his first "emerging ability (2)". Also, he did not score any zeros after Session 3. I describe more details about his experiences with the interpretation component of professional noticing with examples in the paragraphs below.

After watching Session 2 Clip 1, Nick described what he thought the student understood by writing, "The teacher is not tempted to tell the student the $3 - x$ is different than $x - 3$" (Nick, Session 2 Clip 1 response, May 6, 2020). This response focuses on the interviewing teacher's actions rather than the student's thinking. As a result, it was rated "no evidence (0)". During the discussion of this clip in Session 2, Nick made comments like "it's obvious that she knows what she's talking about" and "she knows the entire procedure, she knows how to solve such problems" (Nick, Session 2, May 6, 2020). These comments align with his written response as far as the lack of details and direct connections to the clip. His written responses to the other clips shown in Session 3 and his interpretations of student thinking during the group discussion of those clips were not much different. Nick's interpretation of students' mathematical thinking during the earlier sessions was mostly overgeneralized.

During Sessions 4 and 5, there were some changes in Nick's written responses and verbal comments about the clips shown about the interpretation of students' mathematical thinking. Nick scored his first "emerging ability (2)" when he wrote, "I think that the student wants to find what 20% of 5 is, and she realized it is 1. Not sure what the student meant by, 'I would do the inverse operation'" (Nick, Session 4 Clip 2 response, May 20, 2020). Here, Nick highlighted an idea and solution strategy employed by the student. He also questioned the student's thinking about the use of inverse operations. He linked his interpretation to the student's ideas evidenced in the video without any overgeneralizations.

Another example is from Session 5. For the last clip shown in Session 5, Nick wrote the following:

> I like how the student said, "the sale price is missing, let's call it *m*, and so the original price is *m + 5*." Her thinking was a bit interrupted, but she realized that her model works now and the $5 might represent 20% and so she started adding groups of 5 till 100% showed $25. (Nick, Session 5 Clip 2 response, May 27, 2020)

In this response, Nick described the student's solution strategy of assigning a variable to the unknown and defining what each number represented. Next, he wrote about how the student started adding groups of five to arrive at $25 for the original price. This entry was scored "emerging ability (2)" since it addressed the strengths of the student's thinking by providing evidence from the clip in a comprehensive manner. Again, in this response, Nick refrained from making any overgeneralizations.

During the collaborative clip discussions in the PD sessions, Nick moved from comments like “it's obvious that she knows what she's talking about” (Nick, Session 2, May 6, 2020) and started interpreting students’ thinking by saying things like the following:

He (the student in the clip) couldn't figure out what is the right answer because there were a few options when you see 15 versus 25 and 20, so he wasn't able to convince himself. And when he did the math, he was able to convince himself that he has something to work with, that he has a solution. (Nick, Session 5, May 27, 2020)

With his comment, Nick referred to the student’s initial confusion and how he was able to clarify his thinking after carrying out the mathematical procedure for solving the problem. In alignment with his written comments for the clips shown during Session 5, his verbal comments targeted the students’ mathematical thinking and connected to data within the video clips.

All in all, Nick moved from overgeneralizations to providing specific details of student’s understandings. He tended to do better after Session 3 and did not score any 0’s in interpreting. One explanation for this might be that student thinking was more visible on the interview clips in which students solved Task 2. All three teachers scored slightly better interpreting those clips. I discuss this further in Chapter 5. An alternative explanation may be that Nick completed more interviews by Sessions 4 and 5. He also attended more PD sessions, watched and discussed more clips with his colleagues. Those experiences might have provided more chances for him to observe and critique students’ solution strategies and thought processes.

As evidenced above, Nick’s written responses and verbal comments included more direct links to the clips, and he avoided overgeneralizations as he progressed through the five weeks of PD sessions.

Relatively More Robust Decisions on Responding to Students' Mathematical Thinking. Compared to Linda and Mary, Nick scored higher in responding to students' mathematical thinking throughout the PD program. Overall, Nick suggested the next steps to enhance student learning based on what he noticed and how he interpreted his noticings about students' thinking. Out of the nine clips shown during the sessions, he scored a "no evidence (0)" on only one clip and "emerging ability (2)" on another clip. His score was "limited evidence (1)" on the other seven clips.

Although Nick scored 2's in both the pre-and post-assessments, he did not score that high during the PD sessions. Nick scored his only "no evidence (0)" during Session 4, watching his own clip. He wrote, "I advised the student that in this case $20 might work but then changed the discount to 40% and told him to try again. This example shows that 40% and $40 won't work" (Nick, Session 4 clip 1 response, May 20, 2020). First, this response is not connected to Nick's interpretation of the student's thinking. When asked to describe what he thought the student understood, he wrote, "The student understands that the original price and the discounted price are different numbers. Not sure what he meant when he divided the %" (Nick, Session 4 Clip 1 response, May 20, 2020). The next step Nick suggested is more a response to the student's random guess (as confirmed by the student's own words in the video) of $20 as the sale price. It was not linked to the student's understanding of the original price and the sale price being different numbers. Nick also did not provide a rationale for the next steps he suggested here. It is not clear in his entry how changing the numbers in the problem can potentially enhance the student's understanding of the discount concept. It is also interesting that this is one of Nick's own clips. Similar to his lower performance in attending on self-recorded interview clips, Nick scored poorly on this clip in the responding category.

In order for a teacher to score a “limited evidence (1)” in the responding category, their written entry had to include at least one of the following two elements: (1) connection to the teacher’s response to the interpretation prompt, (2) clear rationale for the suggested next steps (See the AIR Rubric in Figure 3 for more information). Nick’s responses to the seven clips shown during the PD sessions missed one of the two elements above and were scored as “limited evidence (1)”. For example, during Session 5, he described how he would have responded to what he interpreted about the student’s thinking:

I would suggest using a visual, ask him to explain the problem with a visual model, then work on the 3 types of questions, find the % when part and whole are given, find the part when the % and the whole are given and finally find the whole when the part and % are given. (Nick, Session 5 Clip 1 response, May 27, 2020)

This response is connected to Nick’s interpretation of the student’s understandings but lacks the rationale for how this suggestion might further the student’s thinking.

Nick scored “emerging ability (2)” only in Session 4 when he wrote, “I would go back to the conceptual understanding, use visuals to discuss various % cases then focus on discount, have the student show what a discount means then work on this problem again” (Nick, Session 4 Clip 2 response, May 20, 2020). This is the only score of 2 in responding. Neither Mary nor Linda was able to score a 2 in this category. Here, Nick made a connection between what he thought the students understood, and he wanted to show the student what discount meant. For this, he suggested the use of visuals on different percent scenarios. So, this response includes both a connection to the teacher's response to the interpretation prompt and a clear rationale for the suggested next steps.

In conclusion, Nick scored relatively higher than the other two teachers in this component of professional noticing, although he only scored a 2 once on the nine clips shown during the PD sessions.

Nick's Reflections on the PD Program

On the post-assessment (See Appendix A), Nick wrote the following when asked how his participation in the study supported him as a math teacher:

It helped me to focus even more on the student. As teachers, we have the urge to teach, and many times we fail to listen. Sometimes we even imagine that we heard certain things which serve as cues for us to continue. In our interactions, l learned to listen and pay attention to details. (Nick, Post-assessment, May 31, 2020)

Based on this response, Nick commented on how he became more conscious about the importance of listening as he participated in the PD program. He referred to learning to pay attention to details of (presumably) students' thinking, and he gave credit to the PD program for giving him this opportunity.

During the PD sessions, Nick made comments about his engagement with the PD program and how it helped him see students' thinking in a more focused way. One example was, "As teachers, we are not getting this window, we're not getting enough opportunity to really see how one thinks" (Nick, Session 2, May 6, 2020). Here he might be referring to the interviews he conducted and/or the video viewing and discussion experiences. After watching a clip during the same session, he said, "I just felt that there was an incredible opportunity to see how somebody thinks about a problem" (Nick, Session 2, May 6, 2020). These comments indicate that Nick appreciated the perspectives he gained about students' thinking.

Along the same lines, towards the end of the last session, Nick said, "Teaching can be a very lonely profession because you're most of the day in the classroom with your students. So, although there are lots of interactions with the kids, there's not much or enough interactions with adults, like-minded" (Nick, Session 5, May 27, 2020). Here, he appreciated the collaborative discussions of the interview clips with his colleagues. The five-week-long interactions with the same group of professionals might have contributed to the sense of community Nick is referring to.

Summary

All in all, there were some patterns of change in Nick's noticing skills. He seemed to pay closer attention to students' mathematical thinking and provided more details about it in his written entries compared to the beginning of the PD program. His interpretations of students' thinking moved from overgeneralizations to more specific details linked to the clips. Nick suggested relatively more robust next steps to address student misunderstandings compared to his colleagues though he only scored a 2 once on the nine clips shown during the PD sessions in responding to students' mathematical thinking.

Throughout the course of the PD program, Nick shared his opinions on the various things he noticed about the video clips shown and discussed. During the PD sessions, Nick shared his ideas openly and contributed to almost all discussions. There were times when he made some general evaluative comments without providing specifics evidenced in the clips. Other times, he focused on interviewer actions rather than student thinking. Nonetheless, during the last two sessions, he started noticing and sharing mathematically important details from the clips. In his interpretations, he started providing links to the specific details evidenced in the clips. As mentioned earlier, Nick's broad repertoire of teaching strategies helped him score relatively higher than the other two teachers in responding to students' mathematical thinking. In past PD programs, Nick stood out as an active participant, as evidenced by the ease with which he shared his ideas and his eagerness to engage in discussions with others. This disposition, as one who is enthusiastic about PD in general, may have contributed to his being more detailed in every category, which was most pronounced in the responding category since teachers are expected to suggest next steps based on students' understandings.

Based on his own comments and written responses, Nick benefited from participating in the PD program mainly in two ways. First, he appreciated the window through which he was able to see students' thinking about mathematics problems. He stated that as he interviewed students one-on-one, viewed the clips of himself and those of others, and collaboratively discussed the clips, he gained interesting insights into students' mathematical thinking and solution strategies. Second, he commented on the collaborative structure in which he and his colleagues were able to discuss their experiences with interviewing students and how he enjoyed the discussions with like-minded colleagues.

The Case of Linda

Linda has been teaching middle school mathematics for five years. Based on the official observations of School 2's administration, she was identified as an effective teacher (Danielson, 2007). Her administrators defined Linda as an organized and knowledgeable teacher who was able to create a safe and effective learning environment. During my classroom visits outside of this study, I noted that she was teaching mathematics through cognitively demanding tasks and using high-level questions to engage students in learning. At the PLC meetings, she contributed to the discussions and shared her ideas clearly and professionally.

During the PD sessions, Linda actively participated in the clip discussions and openly shared what she noticed about students' thinking. She made comments that were consistent with her written responses to the weekly AIR prompts. In some instances, she extended her written responses and shared more comprehensive accounts of what she noticed. Her scores on the written responses were mostly 1's and 0's until Session 5. In the following sections, I report on her engagement with the PD program, starting with the pre-and post-assessment comparisons. I then present the patterns in her noticing of students' mathematical thinking during the PD sessions based on her written responses to the weekly AIR prompts as well as her comments during the clip discussions. I conclude the section with Linda's reflections about her participation in the PD program.

Pre- and Post-assessment Comparison

Linda's scores on each component of professional noticing (AIR) on the pre-and post-assessments (see Appendix A) are shown in Table 9. Her written responses to the AIR prompts on the assessments were scored by two raters as described in the methods section. As shown in Table 9, Linda fluctuated between 0s and 1s and did not score any 2s on any component. Although this data is inconclusive, I point out more details about her entries below.

Attending to Students' Mathematical Thinking on the Pre-and Post-assessments. Linda started the PD program with a score of "no evidence (0)" in attending to students' mathematical thinking. She watched the given clip on the pre-assessments and wrote the following:

I noticed that the girl picked her answer and began justifying it. She defended her answer despite the questions being asked. The boy was asking her how it makes sense. He asked her to tell him what the answer was for each card. Then she realized that the answers were different, which meant they did not match (Linda, Pre-assessment, April 29, 2020).

In this response, Linda attended to the non-mathematical aspects of the clip shown. She shared her general observations with little to no mathematical details.

Table 9

Linda's Pre-and Post-assessment AIR Scores

Assessment	AIR Components		
	Attending to	Interpreting	Responding to
Pre-assessment	0	1	1
Post-assessment	1	0	1

Note. On this table, 0 represents No Evidence, 1 represents Limited Ability, and 2 represents Emerging Ability.

After the last session, Linda was given the same clip, and she provided slightly more details about the student's thinking:

I noticed that the student decides the answer and justifies it. I also notice another student probing questions of clarity to the student. The student answers the question justifying her work;

however, the other student asks for more. This prompts her to realize she made an error (Linda, Post-assessment, May 31, 2020).

Linda's response above attends to some elements of the student's mathematical thinking. She acknowledged that the student realized the problem with her strategy. Compared to her response on the pre-assessment here, Linda provided slightly more details, such as the student realizing the error in her strategy. As a result, this response was scored as "limited evidence (1)" by the two raters.

Interpreting Students' Mathematical Thinking on the Pre-and Post-assessments. Linda scored "limited evidence (1)" in the interpretation category of the pre-assessment with the following written entry:

I would say this student understands the reasoning behind the answer. I think the student understands that although the numbers in her original answer corresponded, once questioned, the answers to each card did not match. She understood that her choice was not logical. She understood which number was being broken up by 6. (Linda, Pre-assessment, April 29, 2020)

Linda provided hints about what the student understood, but she did not make specific references to the clip. Also, her last sentence about what she thought the student understood was not clear. This response was scored "limited evidence (1)". On the post-assessment, Linda answered the same prompt as follows:

I notice the student rationalizing the statement with the picture. The other student prompts her to talk more about it. She starts to solve 6 divided by 3 mathematically. The student points out the order of the numbers and what the answer would be if it is 3 divided by 6. He is showing her the diagram. The boy student is asking her to solve them mathematically. He asks her to solve it, and then she realizes the answers do not match. (Linda, Post-assessment, May 31, 2020)

Instead of providing details about what she thought the student understood, she described the actions of the students in the clip. In other words, Linda provided a narrative description of what she saw happening in the video clip. She did not interpret the mathematical thinking of the student. Her response was too vague to be a meaningful interpretation and thus was scored as “no evidence (0)”. I will provide more details about Linda’s interpretation, presenting her weekly session data in the following pages.

Responding to Students’ Mathematical Thinking on the Pre-and Post-assessments. Linda maintained a score of “limited evidence (1)” in responding to students’ mathematical thinking on the two assessments. On the pre-assessment, she wrote the following in response to the thinking of the student shown in the clip:

I would ask her how reasonable her answer is. I would ask if her answer choice makes sense. I would ask her to tell me what one shaded area represents in the picture. I would also ask her to draw/diagram what 6 divided by 3 looks like. I would want her to see the difference between 6 divided by 3 drawn out and compare it to the picture given. (Linda, Pre-assessment, April 29, 2020)

Here, Linda suggested a visual representation to address the student’s misunderstanding. She also provided a rationale for the suggested next steps, but it is not clear how those actions are linked to Linda’s response to the previous question regarding her interpretation of the student’s mathematical thinking. Hence, this response was rated “limited evidence (1)”.

When asked how she might have responded to the student in the clip on the post-assessment, Linda wrote the following:

I would prompt the student to solve it. I would prompt the student to draw a diagram of 3 divided by 6 and 6 divided by 3 along with a representation of the answer and see which one compares to the diagram that was originally provided. (Linda, Post-assessment, May 31, 2020)

This entry is almost identical to Linda's response from the pre-assessment. Again, she did not provide a link to how she interpreted the student's mathematical thinking. Thus, this was rated "limited evidence (1)". Her written responses to the weekly AIR prompts and her comments during the clip discussions provide more insights into her noticing skills. I describe these with evidence from the study data in the following sections.

Patterns in Linda's Noticing During the PD Sessions

I observed some interesting patterns in Linda's noticing skills in her written responses to the weekly noticing prompts and in the PD session transcripts of group discussions. Table 8 shows Linda's AIR scores based on the written responses she provided to the weekly noticing prompts for each video clip.

As described in the methods section, the scores for each component of noticing (AIR) represent the evaluation of two raters through independent rating and discussions to arrive at a consensus. One pattern that was evident is that Linda only scores 0's and 1's (with one exception of a 2 in Session 3 Clip 3) on all clips until Session 5. In Session 5, on the other hand, she scored 2's on attending and interpreting on both clips and 1 and 0 on responding. Another way to consider this is that she scored 9 out of 12 possible points on the two clips shown during Session 5. This is markedly different from her scores on the seven clips shown during the previous three sessions. Therefore, as I report my findings about Linda's scores in the following sections, I consider Session 5 seperately.

Scores Fluctuated Between 0 and 1 on All Components During Sessions 2-4. As shown in Table 10, Linda scored low on all AIR components during Sessions 2-4 with the exception of a 2 in attending on Session 3 Clip 3. I present some of her written entries during those sessions, along with some possible explanations in the next sections.

Low Scores in Attending. During Sessions 2-4, Linda scored 1's in attending on four out of seven clips. A typical response that was scored "limited evidence (1)" was from Session 3 as she wrote, "I noticed their shift in thinking or at least in expressing the answer. They shifted from saying the number (value of x) has to be less than 4 to then the difference has to be less than 4" (Linda, Session 3 Clip 2 response, May 13, 2020). Here, Linda attended to some aspects of the student's thinking but did not provide many specifics in a coherent and comprehensive way. One thing she noticed was the shift in the student's choice of words from saying "the number", which referred to x, to "the difference," which referred to $x - 4$. During the discussion of this clip in Session 3, Linda made additional comments regarding what she noticed about the student's thinking. As an example, she said, "I also liked that he went into the negatives, and he made his own conjecture that it could be any of the negative numbers as high as you can go" (Linda, Session 3, May 13, 2020). Since she did not include this observation in her written response, it is likely that this was something she realized after submitting her response. Alternatively, she might have remembered this as Nick and Mary were sharing their noticings about the clip. Linda's responses to the other three clips for which she received a score of 1 were similar to this one in terms of the amount of detail Linda provided.

Table 10

Linda's Scores Based on Written Responses to the Weekly AIR Prompts

Task	Session	Clip	AIR Components		
			Attending to	Interpreting	Responding to
Task 1	Session 2	Clip 1[a]	1	1	1
Task 1	Session 2	Clip 2	1	1	0
Task 1	Session 3	Clip 1	1	0	1
Task 1	Session 3	Clip 2	1	0	1
Task 1	Session 3	Clip 3 [a]	2	0	0
Task 2	Session 4	Clip 1	0	1	0
Task 2	Session 4	Clip 2 [a]	0	0	0
Task 2	Session 5	Clip 1	2	2	1
Task 2	Session 5	Clip 2 [a]	2	2	0

Note. On this table, 0 represents No Evidence, 1 represents Limited Ability, and 2 represents Emerging Ability.

[a] Indicates the clips in which Linda was the interviewer.

During Session 4, Linda scored “no evidence (0)” on both clips. One was, “Teacher was neutral in responses. The student picked 20 because of percent” (Linda, Session 4 Clip 1 response, May 20, 2020), and the other was, “Teacher reaction of "are you sure?" student made errors in calculations" (Linda, Session 4 Clip 2 response, May 20, 2020). In both entries, she attended to non-mathematical aspects of the interviews and provided only some general observations with no specific details. In both responses, it is evident that Linda's focus was on the interviewer. In line with the conjecture I made in the other two cases, Linda's attention to teacher actions may be attributed to the questioning article read and discussed during Session 1. I provide more insights into the impact of the article on teachers' noticing in the cross-case analysis.

It is also important to note that Session 4 Clip 2 was one of Linda's own interview clips and the only one for which she scored 0's in all AIR categories. The poor score in attending may be explained by the findings of previous research (Goldsmith & Seago, 2011) that teachers tend to focus less on students' learning and more on their own actions as they watch self-recorded clips of teaching. Nonetheless, I cannot make definitive conclusions here since Linda also scored 2's and 1's in attending on other self-recorded clips.

Vague Interpretations. Linda scored three 1's in interpreting during Sessions 2-4 and 0's on the other four clips. In general, her interpretations provided little detail about students' understandings. One example of her poor scores is, "I wonder what could have been said if the student expanded on the idea of a negative number" (Linda, Session 3 Clip 1 response, May 13, 2020). This response is too vague to be meaningful. It does not interpret any elements of the students' thinking, nor does it provide any direct connections to what was shown in the clip. As a result, it was rated "no evidence (0)".

Another example that scored a 0 in interpreting from the same session read, "I noticed little prompting or reaction from Mary. It was entirely the students thinking. They went as far as testing numbers negative and positive and made the conjecture that it could be all negative numbers too" (Session 3 Clip 2 response, May 13, 2020). Linda was focused on the interviewer actions rather than the student's thinking. She did not provide any details on what the student did and did not understand.

Linda also scored a 1 on interpreting three clips during Sessions 2-4. For one of those clips, she wrote, "The student understands the idea of the sale price, discount, and original price" (Linda, Session 4 Clip 1 response, May 20, 2020). Here, Linda provided a list of mathematical concepts she believed the student understood. Compared to her previous responses above, this is slightly different in terms of interpreting some aspects of the student's thinking. On the other hand, it still lacks any direct links to the mathematical ideas evidenced in the clip. Hence, this response was rated "limited evidence (1)".

Responses Lacking Specificity. Linda scored 0's in responding to student's mathematical thinking on five of the nine clips shown during the PD sessions. Similar to Mary, she did not score any 2's in this category. One example of Linda's low performance in responding to students' thinking is her written response to a clip shown in Session 3, which was, "I would push for rational numbers and negatives for the first option" (Linda, Session 3 Clip 3 response, May 13, 2020). This response is not connected to Linda's interpretation of what mathematics the student understood. Also, it is not clear why and how the push for rational numbers and negatives for the first option can further probe or extend the student's thinking. There is no rationale provided for this. Thus, this response was rated "no evidence (0)".

For another clip shown during Session 4, Linda wrote, "Prompt her to write an equation based on the words" (Linda, Session 4, Clip 2 response, May 20, 2020). Similar to the response she gave above, this one is not linked to Linda's response to the interpretation prompt. It is not clear what she meant by "write an equation based on words", which is technically the whole point of the task. It is a learning goal rather than a concrete next instructional step that can move the student's thinking further. As a result, this response was rated a 0.

One of the three entries that scored a 1 in responding read, "I would ask them to elaborate more on the negative number. Why did they feel the other wouldn't work because the results were negative" (Linda, Session 3 Clip 1 response, May 13, 2020). This response is not directly linked to Linda's interpretation of the student's understandings. Nonetheless, she provided a rationale on why elaborating more on negative numbers could potentially help the student make the right choice. Negative results, all which are less than 4 make the first inequality true (See Task 1 in Appendix C). When it comes to the second inequality, $3 - x < 4$ produces negative results for positive values of x that are greater than 3. If the student elaborated more on different negative values of x, she might have seen that for $x \leq -1$, the inequality is false. So, for providing this rationale, Linda's response was rated a 1.

Lowest Scores in Session 4. Out of the 12 possible points for the two clips shown during Session 4, Linda was able to collect only 1 point, which was for interpreting Session 4 Clip 1. The scores were 0's for both clips due to her focus on the interviewer actions instead of student thinking as described earlier. She scored a 1 with her interpretation, "The student understands the idea of sale price, discount, and original price" (Linda, Session 4 Clip 1 response, May 20, 2020), since she described what she thought the student understood, though it was not very specific. For the second clip shown during the same session, Linda wrote, "The student understands how to set up the problem. Student didn't understand how to initially solve" (Linda, Session 4 Clip 1 response, May 20, 2020). This was too vague to be meaningful and lacked the specificity and detail to be a comprehensive interpretation. Thus, it scored a 0. She also was not able to suggest meaningful next steps that are linked to the students' understandings. Her entries in the responding category were rated a 0.

There may be several reasons why Linda's scores were low in Session 4. One reason might be about the clips shown during that session which were scored relatively lower than the others. This was mainly because of two reasons: (1) the lack of evidence of student thinking in the clip, (2) the lack of student sense-making in the clip. Session 4 Clip 1 also lacked the clarity needed for teachers to attend to students' mathematical thinking. As a result, this clip scored a 5 out of 9 points on the video selection criteria (Figure 5). Linda's low score may be attributed to the low quality of the videos shown during Session 4. Nonetheless, compared to Mary, who scored a 5 out of 12 points, and Nick, who scored a 6 out of 12 points in Session 4, Linda's score of 1 out of 12 points is still very low.

Highest Scores in Session 5. After scoring very low during Session 4, interestingly, Linda scored 9 out of the 12 possible points for the clips shown during Session 5, with 2s on all attending to and interpreting responses. I present her written entries during Session 5, along with some possible explanations in the next sections.

Attending to Significant Mathematical Ideas. In Session 5, Linda shared many details about what she noticed in regard to the students' mathematical thought processes, both in her written responses and verbal comments during clip discussions. These included not only direct references from the clips, such as numbers and solution strategies but also how the students reasoned and made sense of the problems. One example is her response to the attending to prompt for Session 5 Clip 1:

I noticed he took the number 20 and subtracted the 5 dollars and got 15 dollars. He said the original price is 5 dollars. He had the answer 25 and just talked his way in a hole. The student guessed 25, and it worked, but they did not verify that when asked. He made sense of it but did not do it mathematically until prompted. The student was able to make sense of the problem and the reason that it couldn't be 1 dollar for a shirt. (Linda, Session 5 Clip 1 response, May 27, 2020)

Linda provided a detailed account of the student’s solution strategy and how his thinking progressed in a comprehensive manner. She also did not make any assumptions or evaluative comments about the student’s thinking. This response was rated “emerging ability (2)”. Her comments during the discussion of this clip were along the lines of her written response:

Towards the end, when, um, they tried to mathematically figure it out, like logically, it made sense in terms of him guessing and choosing a number and then taking the $5 off. But then when he tried to do it mathematically, he got a dollar because he did 20% of five, and Mary said, does that make sense? And he knew that it didn't make sense. You know, like at least he was able to, you know, um, stop for a second and acknowledge, like, that's not right. (Linda, Session 5, May 27, 2020)

Linda did provide not only the details of what she noticed in the clip but also described the student’s thinking and sense-making process.

Robust Interpretations. During Session 5, as she did with attending to students' mathematical thinking, Linda scored 2's in the interpretation component of professional noticing. One example of that is her response below:

> The student understands how to calculate a discount. The student understands the idea of trying to find the unknown amount and adding on the $5 to this unknown (sale price). The student is aware of the parts of this problem. The student can separate the idea of sale and original and how to go from sale to original using the amount they provided. The student realizes their original error. (Linda, Session 5 Clip 2 response, May 27, 2020)

Here, Linda provided a detailed explanation about the strengths of the student's thinking and connected her interpretations to specific ideas evidenced in the clip. She refrained from overgeneralizations and included multiple aspects of the student's understandings. Linda scored "emerging ability (2)" with this response. During the discussion of this clip, Linda extended her interpretation with additional points to be considered:

> When she was saying like 20% of five, I don't think she was trying to figure out 20% of five. I think it was more of an example because she also said, um, the 20% of the $10 shirt would be $8. So, I don't think she was confused, or maybe I, maybe I just didn't take it that way. Um, I don't think she was confused that it was 20% of $5. I think she was just practicing the calculation. (Linda, Session 5, May 27, 2020)

Linda's comment above brought an important perspective to the discussion and led to an important conversation. She was the interviewer in the clip, and she misheard something the student said. In the clip, the student said, "20% *off* of a $10 shirt is $8". Linda most likely did not hear the word "off" and assumed the student said, "20% of $10 is $8". She interpreted this as the student practicing the calculation. In the end, she acknowledged her misunderstanding, and we all learned from the situation. It is crucial that teachers pay close attention to what students say and ask clarifying questions as needed.

Struggling with Responding. Although Linda scored 2's in attending to and interpreting students' mathematical thinking in Session 5, she did not score high in deciding how to respond. For example, for the first clip shown during that session, she wrote, "I would ask the student to justify their response mathematically. When they picked numbers, I would ask them to show the percent of whatever number they chose. I would prompt the student to explain the logic behind their answer" (Linda, Session 5 Clip 1 response, May 27, 2020). This response did not give any rationale for the suggested next steps, but it was thinly connected to Linda's interpretation of what the student understood, but it did not provide a clear rationale as to how those steps could have furthered the student's thinking. Linda ended her entry by writing, "It is hard to respond to that without teaching", which means she found it hard to decide how she could respond to students' mathematical thinking in a hypothetical situation. This could mean she might have come up with more "next steps" had she been the teacher working with the student in the clip and that for her, viewing an interview episode did not allow her to connect completely with the scenario. Considering all these elements in Linda's response, her interpretation was rated "limited evidence (1)".

For the second clip shown during Session 5, Linda wrote, "I would try to help prompt the student to map out their work. I would prompt them to check the math as well" (Linda, Session 5 Clip 2 response, May 27, 2020). The suggested next steps here are not connected to Linda's interpretation of the student's understandings. She also did not provide a rationale about how these actions could have enhanced the student's progress with the problem. This entry scored "no evidence (0)".

One possible explanation for this marked difference between Linda's scores during Session 2-4 and Session 5 might be related to clip selection. All three teachers scored relatively higher during Session 5 in comparison to the previous sessions. However, the difference was the largest in Linda's scores. She also was very engaged in the discussion during Session 5. Compared to the 14 comments Linda made during Session 4, she made 42 comments during Session 5. When I asked the group if it was alright with them to run over our scheduled ending time, Linda said, "I got all the time in the world for this" (Linda, Session 5, May 27, 2020). Linda was much more engaged, as evidenced by her frequency of participation, in Session 5 when compared to previous sessions. It may be that there was more to say about the clips shown in Session 5, and it may point to the importance of teacher buy-in when it comes to their participation.

In summary, although Linda's AIR scores were generally inconsistent throughout the PD sessions, her scores in Session 5 were promising. I should note here that although Linda ended the PD program with many 2's, her scores on the post-assessment were 0's and 1's.

Linda's Reflections on the PD Program

When asked how participating in the PD program supported her as a math teacher, Linda wrote, "I believe this study helped shed light on how easy it is to funnel and focus thoughts and not allow a student to develop their ideas fully" (Linda, Post-assessment, June 1, 2020). It is evident that Linda acknowledged the value of student's thoughts and the negative results of teachers' funneling those thinking processes. This is also related to the questioning article

(Herbel-Eisenmann & Breyfogle, 2005) and the discussions we had about the funneling and focusing questioning patterns. She believes students cannot develop their ideas fully when teachers constantly guide them in a certain way of thinking. She went on to say, “The study helped me to be more cautious of trying to understand what students do and don't know. Sometimes it may appear that they know the material, but in reality, they do not, and we often miss that” (Linda, Post-assessment, June 1, 2020). She reiterated the value of students’ thinking and underlined the importance of focusing on students’ true understandings.

Along the same lines, during Session 5, she said, “I'm thinking about it now, it's like, wow, I probably have done that so often and lost my kids in the process because just one minor thing that they shared out might've been right. But there's no understanding” (Linda, Session 5, May 27, 2020). She was referring to the student in the clip shown who randomly found the correct answer, but when probed further, it was clear that he did not understand the concept in the problem. Her comment indicates that teachers need to pay close attention to what students do and say.

During the same session, Linda made another comment comparing the PLC meetings to the PD program described in this study. She said, “It's nice to see. Um, I mean, in PLCs we always talk about what we do, but it's nice to see what the kids in other schools do, um, and how we can learn from that” (Linda, Session 5, May 27, 2020). This comment highlights the importance of engaging in a PD with teachers from other schools and observing how their students engage in problem solving.

Summary

All in all, Linda's scores in attending to, interpreting, and responding to students' thinking were not consistent. During the last session, she provided what she noticed about student thinking in greater detail. Her interpretations of what she saw in the clips as to what the students understand and do not understand were very strong. She supported her claims about student thinking with specific examples evidenced in the clips. This was not the case in the responding category. The next instructional steps she suggested were not tied to her interpretation of students' understandings. She also did not provide much rationale as to why and how those actions could have enhanced student's progress with the concepts studied.

During the PD sessions, Linda commented mostly about what she noticed with student thinking and how she interpreted those noticings. She did not comment on possible next steps to further students' understandings, and when she did, it was only about the clips in which she was the interviewer. Her verbal comments were mostly consistent with what she wrote for the weekly AIR prompts. In general, she was very active during the clip discussions.

Per her own comments during the PD sessions and her written response on the post-assessment, Linda learned to focus her attention on the specific things students write and share. She learned not to rush and funnel students' thinking in order for them to develop a robust understanding of the concepts at hand.

Teachers' Noticing Themes: A Cross-case Analysis

After I analyzed the individual case of each mathematics teacher, I saw some common themes across their experiences with the PD program. Table 11 shows all scores for all three teachers on each component of the rubric. Some of the patterns that emerged were discussed in the individual cases. All in all, I found that the pre- and post-test scores did not tell the complete story, and that in most cases, they did not align with scores in the early or late sessions of the program. I also found that looking for patterns over time was not sufficient and that I needed to delve deeper into other factors (beyond time) that might have been contributing to differences in scores. In this section, I present a cross-case analysis of the three teachers with respect to two main categories: (1) Patterns in teachers' noticing, (2) PD facilitator's role in teachers' professional noticing.

Patterns in Teachers' Professional Noticing

As teachers conducted interviews and collaboratively discussed video clips from those interviews, there were some observable patterns in their noticing. Since expertise in professional noticing is a complex and multifaceted phenomenon (Jacobs et al., 2010), I was not expecting to see robust evidence of shifts within five weeks of PD engagement. In the following sections, starting with general observations, and then going into more specifics, I describe the patterns I observed in teachers' professional noticing with evidence from the data.

Table 11

Teachers' Scores Based on Written Responses

Teacher	AIR	Assessments and Clips										
		PR	S2C1	S2C2	S3C1	S3C2	S3C3	S4C1	S4C2	S5C1	S5C2	PO
Mary												
	A	0	0	1[a]	0	0[a]	2	1	0	1[a]	1	1
	I	2	0	1[a]	1	1[a]	1	2	1	1[a]	2	2
	R	1	0	0[a]	1	1[a]	0	1	0	0[a]	1	2
Nick												
	A	1	1	1	0[a]	2	1	0[a]	1	2	2	2
	I	1	0	1	1[a]	0	0	1[a]	2	1	2	2
	R	2	1	1	1[a]	1	1	0[a]	2	1	1	2
Linda												
	A	0	1[a]	1	1	1	2[a]	0	0[a]	2	2[a]	1
	I	1	1[a]	1	0	0	0[a]	1	0[a]	2	2[a]	0
	R	1	1[a]	0	1	1	0[a]	0	0[a]	1	0[a]	1

Note. On this table, 0 = No Evidence, 1 = Limited Ability, and 2 = Emerging Ability, A= Attending to, I=Interpreting, R=Responding, PR=Pre-assessment, PO=Post-assessment, S2C1=Session 2 Clip 1.

[a] Self-recorded clip

General Patterns. I categorize the general patterns I observed as follows: (1) focus on student thinking, (2) the disconnect between scores on individual AIR components, and (3) focus on teacher actions.

Focus on Student Thinking During Interviews. Conducting the one-on-one interviews with students and watching the video clips of those interviews helped some teachers reconsider their instructional priorities. Their focus shifted from "moving on" (Mary, Session 5, May 27, 2020) to pausing and making sure students understand the topic at hand. Teachers realized that this could happen if they increased their focus on what the students did and said. For example, on the post-assessment, Mary wrote, "It showed me the importance of spending extra few minutes or even seconds to understand a student's thinking process and directing them accordingly" (Mary, Post-assessment, May 31, 2020). During some interviews, teachers saw students giving correct answers, but when probed, students were not able to justify how they arrived at those answers. One example of that was when Nick's student randomly said the original price was $25 and the sale price was $20 (Session 4 Clip 1). When asked how he found the answer, this student said, "I saw the 20% and thought the sale price would have been $20. Then I added $5 and found the original price". The student did not have a mathematically valid strategy, and Nick admitted that had this happened in a classroom, he would have moved on after hearing the correct answer from the student. He believed the study gave him "incredible opportunities to see how students approached problems" (Nick, Session 2, May 6, 2020), and he acknowledged the importance of attending to student's mathematical thinking before moving on with the next instructional steps.

Another example that helped teachers understand the importance of focusing on student thinking was a case of teacher misunderstanding. Teachers may misunderstand something a student says or writes. Without attending to or interpreting student thinking adequately, those may cause problems for student learning. There was an example of this situation during Session 5. One of Linda's students was explaining how she thought about the concept of discount. She said, "20% *off of* $10 is $8." Linda asked, "20% *of* $10 is $8? Are you sure?" The student was confused as she thought her explanation was inaccurate. It was Linda who did not hear the word "off" in the student's comment. Later in the conversation with the student, Linda realized her mistake, and the student continued to share how she solved the problem. As the teachers discussed this clip, they learned how easy it could be to miss out on the things students do and say. Linda found herself guilty of not realizing the importance of attending to student's thinking by saying, "I probably have done that so often and lost my kids in the process, because just one minor thing that they shared out might've been right" (Linda, Session 5, May 27, 2020).

Additionally, teachers reflected on their interviewing practices as they watched the video clips. Especially while watching their own interview clips, they made comments that supported their increased focus on student thinking. One example was Mary's comment on acknowledging a student's thought process. She said, "I was giving her (the student interviewed) a response after every sentence that she was saying. So, I think that was kind of like a disruption" (Mary, Session 2, May 6, 2020). It is important that Mary realized this was interrupting the student's thinking process. She went on to say, "She (the student interviewed) has a thought process that's going on, and I am putting mine. This is something I need to work on" (Mary, Session 2, May 6, 2020). Here, Mary verbalized her realization that students' thought processes should not have been disrupted. In the next section, I provide a more specific account of the evidence of teachers' noticing on the three nested components of professional noticing. These changes align with the growth indicators highlighted by Jacobs and colleagues (2010). I shared these indicators earlier in the methods section. I support my claims about any changes with examples from teachers' written responses to the AIR prompts.

The Disconnect Between Scores on Individual AIR Components. Jacobs and colleagues (2010) suggest that there is a nested relationship among the three components of professional noticing of children's mathematical thinking. This means that teachers attend to significant mathematical details in students' thinking, then interpret them in a meaningful way, and then decide how to act on those interpretations in ways that can further probe or extend student thinking. In other words, this relationship can be viewed almost in a dependent fashion such that if a teacher does not score well in attending, they will most likely score poorly on the other two components of professional noticing. On the other hand, if a teacher scores high in responding, that is because they scored high in attending and interpreting. The results of my study show that these assumptions are not always true. In the following paragraphs, I present some evidence from the data that support this disconnect in teachers' scores on the three components.

For example, as mentioned in her case, Mary scored a 2 for attending to significant details of the student's mathematical thinking and idea (using substitution) in her entry (Mary, Session 3 Clip 3 response, May 13, 2020). While she was able to articulate the students' thinking, her interpretation left out some details and focused upon only one aspect of the student's strategy. When it came to responding, Mary scored a 0 because she did not link the suggested next steps to what she thought the student understood nor did she provide a rationale for the next steps she offered. Hence, Mary's high score in attending did not predict high scores in interpreting and responding. Similarly, Linda's scores for the three entries for the AIR prompts for Session 3 Clip

3 indicate that, similarly to Mary, she did not follow her high score in attending with high scores in interpreting and responding (for which she received 0's). Table 11 shows other examples of this situation. In all, there were 14 out of 27 times where high scores on "previous" components coincided with low scores on "subsequent" ones.

On the contrary, for some clips, teachers' scores in interpreting were higher than their scores in attending. This is perhaps more surprising than scoring lower on subsequent components because it is hard to imagine someone interpreting something they didn't attend to. For example, Mary scored a 1 in attending on Session 4 Clip 1 (attending to prompt), because she did not provide much detail. On the other hand, she made her interpretation in a non-judgmental way and provided only the mathematically relevant details about the student's thinking in her response. As a result, her interpretation entry was scored as "emerging ability (2)" by both raters. As evidenced above, Mary's score of 1 in attending did not predict her higher score in interpreting. Again, Table 11 shows more examples of this situation. In fact, teachers had higher scores in interpreting than attending on a total of 8 out of 27 occasions.

In summary, although the professional noticing framework (Jacobs et al., 2010) envisions the existence of a nested relationship between the three components of noticing, the evidence provided above shows that scores in one component do not necessarily predict scores in the others.

Focus on Teacher Actions. One common theme across the cases was that teachers frequently focused on the actions of the interviewing teacher instead of focusing on student thinking. For example, for an attending prompt Mary wrote, "I noticed how the teacher made the student refer back to the original statement" (Mary, Session 3 Clip 1 response, May 13, 2020).

For the same clip, Nick wrote, “I noticed that my intonation and I wonder if I pushed the student to change his mind” (Nick, Session 3, Clip 1 response, May 13, 2020) and Linda wrote, “I noticed the student going back and rethinking their answer. Nick also reiterated what the student was saying. I am curious to know if Nick's response, although as subtle as it was (around 1:17), prompted the student to change their mind” (Linda, Session 3 Clip 1 response, May 13, 2020). In all of these entries, it is evident that teachers focused more on the actions of the interviewing teacher rather than focusing on students’ thought processes.

Teachers’ attention to interviewer actions may be attributed to the article titled “Questioning our Patterns of Questioning” (Herbel-Eisenmann & Breyfogle, 2005), which was read and discussed during Session 1. I chose that article intentionally because I wanted to prepare the teachers for the task of interviewing students. As we discussed the article in Session 1, we compared the differences between a funneling questioning pattern and a focusing question pattern. Funneling pattern refers to a sequence of questions posed by the teacher in order to help the student reach a particular answer. In contrast, with focusing questions, the teacher probes the student based on the answers or solution strategies he or she presents. The goal of focusing is to build the conversation upon what the student does or says. During our discussion, we highlighted the importance of focusing questions for the teachers to effectively conduct the interviews and for student thinking to surface. This emphasis on questioning might have shifted teachers’ focus to the actions of the interviewing teacher rather than on student thinking. Also, it is quite possible that the teachers wanted to improve their interviewing techniques, and as a result, paid more attention to the interviewers than the students.

To examine this pattern further, I paid specific attention to each teacher's scores on their self-recorded interview clips. Although it is difficult to make definitive conclusions about this, there is a common theme for Nick and Linda. Each teacher scored a 0 in attending on their own clips. For example, Nick focused on his own actions as the interviewer on two self-recorded clips instead of sharing what he noticed about students' mathematical thinking. These two clips are the only two for which Nick scored 0's in attending. Along the same lines, Linda wrote, "(I noticed) the teacher (Linda) reaction of "are you sure?" (I also noticed) student made errors in calculations" (Linda, Session 4 Clip 2 response, May 20, 2020) for one of her own clips. In this entry, she attended to non-mathematical aspects of the interview and provided only some general observations with no specific details. It is evident that Linda's focus was on the interviewer. In line with the conjecture I made in the other two cases, Linda's attention to teacher actions may be attributed to the questioning article read and discussed during Session 1.

Nick's and Linda's attention to their own actions as the interviewer are consistent with the previous research (Goldsmith & Seago, 2011) as teachers tend to focus on their own behavior while watching self-recorded clips of classroom situations. They are less accustomed to attending to, interpreting, or responding to students' learning. I also see Nick trying to improve his interviewing skills as exemplified in the questioning article (Herbel-Eisenmann & Breyfogle, 2005) we read and discussed together during Session 1. I noticed the impact of this article on the overall attitude of the teachers as they commented on the clips. I discuss this further in Chapter 5.

Mary, on the other hand, provided some details about students' thinking on her self-recorded clips and scored 1's. Only on Session 3 Clip 2 did she score a 0 in attending, but this was not because she focused on her own actions as the interviewer. Instead, she scored poorly for not providing enough details about the students' thinking. On the other two self-recorded clips, she was able to write the details of the students' thinking as she did for the clips of other teachers. There was no mention of herself or her actions in those responses.

Patterns Specific to Each AIR Component. In addition to the general patterns listed above, there were some patterns specific to each AIR component. It is important to note that the mathematical context changed from Task 1 (Sessions 2 and 3) to Task 2 (Sessions 4 and 5). Both tasks had a high cognitive demand and both invited students to use a variety of approaches and solution strategies. Task 2 however, had a more tangible real-life context (shirt sale scenario that includes sale price and discount) compared to the artificial context of Task 1 (Abigail has a number). This difference might have resulted in better surfacing of student thinking as the students tended to talk more during the Task 2 interviews. I discuss the impact of task selection on teacher noticing further in Chapter 5. Regardless, through their participation in the PD program, I noticed that teachers showed some evidence of minor shifts in different components of professional noticing.

From General Strategy Descriptions to Specific Mathematical Details. Initially, when prompted about what they noticed in a given interview clip, teachers tended to provide responses that described some general strategies employed by the students. Some entries also included non-mathematical aspects of the interviews, such as the students' general demeanor or interviewer actions. As the teachers conducted more interviews and participated in the discussion of interview clips in the PD sessions, they started attending to more specific mathematical details evidenced in the clips. Table 12 shows examples of these differences in teachers' written responses. Teachers provided minimal to no details about the students' mathematical thinking in Session 3 Clip 1. Linda made a weak reference to the clip while the other two teachers completely ignored the student's thinking. They wrote about the interviewer's actions that are not related to the student's thought process in the clip. This is a typical example of teachers' responses that were scored low in the attending to category. On the other hand, all three teachers showed that they attending to some mathematical details and describing what they noticed in a detailed manner in the second clip from session five. Their descriptions of what they noticed in this clip include more details about students' mathematical ideas and solution strategies in a non-evaluative and comprehensive manner.

Table 12

Some signs of changes in attending to students' mathematical thinking between session 3 and session 5.

	General Strategy Description	**More Specific Mathematical Details**
Clip	**Session 3 Clip 1**	**Session 5 Clip 2**
Linda	I noticed the student going back and rethinking their answer. Nick also reiterated what the student was saying. I am curious to know if Nicks response, although as subtle as it was (around 1:17), prompted the student to change their mind **(Score: 1-Limited Evidence)**	The student is trying to make sense of the work with algebra as in 20% of the original price should be 5. Something prompted the student to think of the double line/bar method. They were able to identify that 100 would be related to the total cost of the shirt. The 5 represented the 20% and thats how they were able to match up the numbers. **(Score: 2-Emerging Ability)**
Mary	I noticed how the teacher made the student refer back to the original statement. **(Score: 0-No Evidence)**	Student uses different strategies to make sense of the problem. She initially thought that $5 is the original price. **(Score: 1-Limited Evidence)**
Nick	I noticed that my intonation and I wonder if I pushed the student to change his mind. **(Score: 0-No Evidence)**	The student understands the concepts of discount. She confused again the 20% of $5 which is 4, then she said that the $5 represent 20% of the price, which in this case works as the whole in this case would be 5 x 5 = 25. I liked the fact that she used a visual and when she thought about adding $5 she kept adding them to the model and the 10% was the original price. I do have a feeling that she did not fully understand the problem (that the $5 was the difference (delta) between the original price and the discounted price. **(Score: 2-Emerging Ability)**

It is evident from the above table that teachers had more to say in Session 5. A simple word count would reveal this. Here, I note that one possible explanation is that the mathematical context changed from Task 1 to Task 2 after Session 3. The difference in tasks may have led to the notable differences presented in Table 12. I compare and contrast the two tasks with further detail under the heading PD Facilitator's Role.

Another interesting phenomenon evident when comparing the responses to the attending prompt in these two sessions is that of perspective. All three teachers started their responses with "I noticed" in Session 3 Clip 1. On the other hand, they started with "the student" for Session 5 Clip 2. This may point to the fact that teachers seemed to move from "me as the observer" to "the student as the learner or thinker".

Overgeneralizing Students' Understandings to Forming Ideas Evidenced in the Clips.

During the earlier sessions, teachers' interpretations of students' mathematical thinking included overgeneralizations without direct connections to the situation in the video clips as shown in Table 13.

Table 13

Some signs of changes in interpreting students' mathematical thinking between session 2 and session 5.

	Overgeneralizing Students' Understandings	Specific Ideas Evidenced in the Clips
Clip	**Session 2 Clip 1**	**Session 5 Clip 2**
Linda	I notice that she is struggling to make sense of the fact that mathematically, the inequality is true for Abigail's numbers but if she were to write an inequality based on words, it doesn't seem to make sense how it matches. **(Score: 1-Limited Evidence)**	The student understands how to calculate discount. The student understands the idea of trying to find the unknown amount. and adding on the $5 to this unknown (sale price). The student is aware of the parts of this problem. The student can separate the idea of sale and original and how to go from sale to original using the amount they provided. The student realizes their original error. **(Score: 2-Emerging Ability)**
Mary	The student was able to connect her answer to the original statement by reasoning her thinking. **(Score: 0-No Evidence)**	Students goes back and forth trying to make sense of her thinking. Student insisted on showing a number line to explain her thinking. It made sense to her that if 20% is the $5, then by adding $5 each time to get to 100%, she will get the original price. **(Score: 2-Emerging Ability)**
Nick	The teacher is not tempted to tell the student the 3 - x is different than x - 3 . **(Score: 0-No Evidence)**	I like how the student said the sale price us missing lets call it m and so the original price is m + 5. Her thinking was a bit interrupted but she realized that her model works now and the $5 might represent %20 and so she started adding groups of 5 till 100% showed $25. **(Score: 2-Emerging Ability)**

In some cases, teachers did not even provide any details about student understandings. Instead, they chose to comment on interviewer's actions, which may be attributed to the questioning article as discussed earlier. In later sessions, teachers based their interpretations on specific ideas evidenced in the clips. This difference may indicate that they became more comfortable conducting interviews (and therefore focused less on the process), became more focused on student thinking through their participation in the discussions or that the two tasks led to different responses and that the mathematical contexts of them were somewhat different. Alternatively, Task 2 might have allowed student thinking to surface in a more concrete way and as a result, teachers might have noticed more and shared more about the clips shown during Session 5.

Only one teacher (Linda) scored a score other than 0 on Session 2 Clip 1. On the other hand, all three teachers scored 2's on interpreting in Session 5 Clip 2. Linda and Mary's responses for Session 2 Clip 1 were somewhat connected to the student's understanding of writing an inequality based on a verbal description. Nonetheless, this connection was weak and not supported with any evidence from the video clip shown. On the other hand, Nick's response was mainly focused on the interviewer's actions although he was specifically asked to describe the student's understandings.

Although it is hard to make definitive conclusions since the mathematical context was different between the two sessions and the data spans only four weeks of PD participation, teachers provided more detailed accounts of what students understand along with stronger links to the mathematical ideas evidenced in the interview videos in Session 5. These interpretations included the strengths (or sometimes weaknesses) of the students' thinking and were connected to specific mathematical work the students have demonstrated in the videos. More robust conclusions perhaps can be made over longer periods of time as teachers conduct more interviews and discuss more clips together.

Challenges in Responding to Students' Mathematical Thinking. Teachers' attending and interpretations of students' mathematical thinking matter in order for them to base important instructional decisions on what they believe students do or do not understand (Sherin et al., 2011a). At this point, responding to students' mathematical thinking becomes crucial. However, as stated earlier in the literature review, this component of professional noticing can be challenging. Even if teachers attend to students' mathematical thinking and interpret them effectively, they may or may not be able to respond on the basis of their attending and interpretations (Jacobs et al., 2011). In my study, although teachers attained some scores of “2” in attending to and interpreting students’ mathematical thinking, this score was only given to one response (Nick, Session 4 Clip 2) out of 27 during the entirety of the PD program.

The AIR rubric (See Figure 3) called for two elements in order for teachers to score “emerging evidence (2)” in responding to students’ mathematical thinking: (1) connection to the teacher’s response to the interpretation prompt, (2) clear rationale for the suggested next steps. From the earlier sessions, all three teachers suggested next steps that satisfied *at most* one of the two elements mentioned above. There was no evidence in teachers’ written responses to indicate otherwise. Teachers either suggested next steps that were not connected to students’ understandings, or they did not provide a rationale, although it was clearly indicated in the AIR prompt (see Appendices A and C). There was no significant difference in teachers’ deciding how to respond to students’ thinking between clips or sessions. In fact, all scores from the PD sessions except for one (Nick, Session 4 Clip 2) were 0’s and 1’s in responding.

Hence, in alignment with the previous studies that investigated teachers’ responding to students' mathematical thinking (Jacobs et al., 2011; E. E. Krupa et al., 2017), I report that teachers’ scores in responding were over the course of the PD program were lower than their attending or interpreting scores. Again, this pattern might have been different had the teachers conducted more interviews, discussed interview clips together over more extended periods of time, or if the PD had been structured differently. As mentioned earlier, I did not share the AIR rubric with the teachers, nor did I give them feedback on their written entries. The findings I report are in the context of teacher-driven discussions. Although I facilitated the discussions, I did not lead teachers to discuss possible responses. These decisions might have resulted in how the teachers scored on different components of professional noticing. I discuss these further in the following paragraphs.

PD Facilitator's Role

As the PD facilitator in this study, I played an important role in selecting the tasks for the student interviews and the interview clips to be viewed during the PD sessions. I also moderated the conversations as the clips were discussed. I summarize my findings about these important elements of the PD program in the following sections.

Facilitating the Clip Discussions. Teachers' scores on different components of professional noticing may be related to many factors such as the design of the PD program, the role of the facilitator, and how the expectations are set. One of the choices I made about facilitating the clip discussions was that I did not ask teachers to share their entries about each AIR component. The teachers watched each video independently and answered the attending prompt. They then watched each video for a second time and answered the interpreting and responding prompts. Once everyone was done, we started talking about our noticings collectively. I tried to avoid funneling teachers' thinking and noticings toward a desired response I wanted to get from them. I wanted teachers' noticings to emerge naturally from their discussions. I enacted many moves listed in the framework for facilitation of video-based discussions (van Es et al., 2014) such as orienting the group to the video analysis task, sustaining an inquiry stance by highlighting noteworthy comments, redirecting focus on the video and the mathematics when the conversations were off tracked, and supporting group collaboration by sometimes standing back and distributing participation. I knew from the outset that it would have taken time for teachers to understand the expectations fully. This may be the reason why teachers had a more positive attitude during the last session. Mary said the following during Session 5:

I think we were just getting the hang of this, and now it's ending. I mean, this last session was the richest of all, one of those sessions that we were like, oh, like, okay, this is what we have to do, this is how it's going to work. (Mary, Session 5, May 27, 2020)

Mary liked “getting a hang of” the interviewing process and seemed to understand the expectations more clearly by the end of the program.

Another choice I made was about giving feedback to teachers based on their written entries. I did not share the AIR rubric with the teachers, nor did I give them feedback on their written responses. I intentionally avoided a funneling-interaction pattern as this kind of approach might have limited what teachers were able to contribute. This is because I did not want to direct their noticings in a predetermined path based only on how I as the facilitator would have attended to, interpreted, and responded to student thinking in the ideal way outlined on the AIR rubric. With this, I also modeled the focusing approach suggested in the questioning article (Herbel-Eisenmann & Breyfogle, 2005) we read and discussed with the teachers during Session 1. This is in line with other studies that examined patterns in teacher noticing via video viewing (Goldsmith & Seago, 2011; Jacobs et al., 2010). Similar to the actions the PD facilitators in those studies taken, I shifted the discussions to maintain focus on the task of video analysis, prompted participants to elaborate on their ideas, restated and revoiced when necessary to ensure common understanding of an idea, offered alternative points of view, and directed attention to noteworthy moments in the videos if teachers did not bring them up already. The PD facilitators in the above-mentioned studies also did not provide feedback on teachers’ written responses, nor did they share the rubrics they used to rate teachers' written entries. These moves helped the group keep the focus on student thinking and the potentially minimized unsubstantiated generalizations during the sessions in my study. Nonetheless, since the discussions took place after teachers submitted their written responses for a given clip, it is hard to make a definitive conclusion that those discussions helped teachers with the subsequent clips watched within the same session. I discuss the role of the facilitator in greater detail in the following sections.

Task Selection. The task-based student interviews conducted in the current study aimed to help teachers gain insights into students' mathematical thinking. I considered this while selecting the tasks for the interviews. In alignment with the guidelines provided in "Principles to Actions" (NCTM, 2014, p. 17-24), I chose tasks that do not suggest or imply a particular solution method. Rather, by design, these tasks invite students to use a variety of approaches and solution strategies. I needed such tasks for us to have windows into student thinking. Although both tasks were rigorous and open to many solution methods, there were some differences between them. I present these differences as they are related to the teachers' noticing of students' mathematical thinking.

Task 1 (Appendix B) calls students to reason about inequalities and match a verbal description to a symbolic representation. It has some context (e.g., Abigail's number), and it is designed to help the student understand the concept of inequality and make sense of the symbolic representation of inequalities. On the other hand, Task 2 (Appendix B) is a real-life problem about the concept of discount. It is designed to help students use reasoning about rates and ratios to solve real-world mathematical problems. It has a relatively more concrete context compared to Task 1.

These features of the two tasks reflected on how the students approached the problems during the interviews. For example, Andrea (pseudonym), the student in Session 2 Clip 1, struggled to share her thinking about Task 1. She could not communicate her ideas clearly as she was not comfortable with the context and all the terminology involved. At one point, she said, "(referring to an inequality in the task) It is correct but not correct at the same time because it doesn't mean Abigail's statement." (Andrea, Session 2 Clip 1). Linda, the interviewing teacher, might have had some idea about what the student meant since she conducted the whole interview. Nonetheless, this may not have been the case for Mary and Nick, who watched the clip for the first time during Session 2. Teachers scored an average of 0.67 out of 2 points on the attending to prompt for this clip. Another reason might be that this was the first time teachers watched a clip and submitted their responses to the AIR prompts. Teachers might not have been clear about the expectations as to what they should have written for each of the prompts.

Although it's not related to the task, another reason to explain is that teachers, who had recently (one week before Session 2) read an article on questioning patterns, might have been over-focused on the actions of the interviewer instead of paying attention to the student's thinking. Because this article was assigned and read by the teachers just prior to Session 2, it may have influenced them to focus on the teachers' questioning techniques.

Linda interviewed Andrea again for Task 2. The same student was much more open with her thinking while solving Task 2. She had more to say about the concept of discount. She gave examples from real-life discount scenarios with smaller numbers and then used that thinking with the numbers given in the problem. These instances were shown in Session 5 Clip 2. Compared to the clip mentioned in the previous paragraph, it was evident that Andrea's thinking was more visible and accessible to the viewers. This might have contributed to the average attending to score of 1.67 for the clip. Again, other factors such as Session 5 being the last session in the PD program, teachers' experiences with the AIR prompts by Session 5, and Linda’s improved interviewing skills might have been in play. Regardless, it seemed that a concrete, real-life context contributed to the surfacing of the student’s mathematical thinking.

Studies on a larger scale and careful examination of the relationship between task selection and teachers’ noticing skills may provide more robust findings about this phenomenon. Next, I report my findings about the selection of video clips and how this may have changed the ways in which teachers noticed student thinking during PD sessions.

Clip Selection. I selected the clips (for teachers to discuss) as using the rubric (Sherin et al., 2009) described in the methods section. Here, I re-present the scores for each of the clips I selected with the score descriptors for each category. Analyzing Figure 5, I noticed some patterns when I cross-checked the average teacher score on each clip across the AIR components with the chart above. There was a tendency for clips with higher scores to elicit higher scores on noticing. For example, Session 3 Clip 1 and the two clips shown during Session 4 were scored 7, 5, and 7 points, respectively, out of a possible 9 points. On those clips, teachers’ average attending to scores were the lowest among all clips (0.33 out of 2 points). The above-mentioned clips were scored relatively lower than the others mainly because of two reasons: (1) the lack of evidence of student thinking in the clip, (2) the lack of student sense making in the clip. Session 4 Clip 1 also lacked the clarity needed for teachers to attend to students’ mathematical thinking. As a result, this clip scored 5 out of 9 points. Unfortunately, among all the interview clips considered to be viewed for Session 4, this was the best one based on the video selection criteria. That is why I chose it over the others despite its low score. The inclusion of the clip, however, did lead to an indication of the importance of using clips that meet the criteria set forth in the video selection criteria for eliciting teacher noticings.

Figure 5

Summary of scores for the selected video clips

Criteria	Key Question	Level			Session 2		Session 3			Session 4		Session 5	
		Low (1)	Medium (2)	High (3)	Session 2 Clip 1	Session 2 Clip 2	Session 3 Clip 1	Session 3 Clip 2	Session 3 Clip 3	Session 4 Clip 1	Session 4 Clip 2	Session 5 Clip 1	Session 5 Clip 2
Windows into student thinking	Is there evidence of student thinking in the video clip?	Little evidence of student thinking from any source (e.g., very few comments from students)	One or more sources of information exist, but little detail provided (e.g., IRE exchanges dominatea)	Detailed information from one or more sources (e.g., student narrates and provides written account of solution strategy)	3	3	2	3	3	2	3	3	3
Depth of student thinking	Are students exploring substantive mathematical ideas?	Task is routine for student; calls for memorization or recall on part of student (e.g., student applies known algorithm)	Some sense making applied to routine task (e.g., student questions step in known algorithm)	Student engages in math sense making, works on task at conceptual level (e.g., student devises invented strategy)	2	2	2	2	2	2	2	3	3
Clarity of student thinking	How easy is it to understand the student thinking shown in the video?	Student thinking not transparent (e.g., "What is that student talking about?")	Much of student thinking transparent, though some ideas may be unclear (e.g., "I think I understand, but what did she mean by 'straight?'"	Student thinking transparent; viewer sense making not called for or single interpretation obvious (e.g., "She gives a very clear explanation.")	2	3	3	3	3	1	2	3	2
				Total	**7**	**8**	**7**	**8**	**8**	**5**	**7**	**9**	**8**

I acknowledge there are many factors that contribute to teachers' professional noticing of students' mathematical thinking. At the same time, I saw a pattern between the quality of the interview clips shown and the ways in which teachers attended to the mathematical details about students' thinking.

When students engaged in math sense making and worked on the given task at a conceptual level, more of their mathematical thinking surfaced in the interview, and this was caught by the viewers. Similarly, the clarity of student thinking in the selected clip helped viewers understand the student's ideas and described the strategies employed by the student in a coherent and comprehensive manner. This led to higher performance in teachers' attending to students' mathematical thinking.

Lastly, I observed that longer clips tended to yield higher scores in attending. Teachers performed an average score of 1.67 only on Session 3 Clip 3, Session 5 Clip 1, and Session 5 Clip 2. The duration of those clips was the longest among all nine clips shown (3 minutes and 53 seconds, 5 minutes and 8 seconds, 5 minutes and 39 seconds, respectively). Again, among many other factors, the duration of a clip may be associated with the amount of material it provides in regard to students' mathematical thinking.

This concludes my cross-case analysis. Teachers' professional noticing of students' mathematical thinking is a complex phenomenon and it is hard to make definitive conclusions with a limited amount of data collected in five weeks. My analysis of the three cases together point to the possible influences of the mathematical and real-life contexts in which the tasks are embedded, the quality and duration of the clips to be analyzed, the readings assigned to participants, and the role of the facilitator in guiding the discussion. All of these findings have implications for future offerings of PD programs that focus on noticing and on the research related to them. These are discussed in the following chapter.

Chapter 5: Discussion, Limitations, Implications, and Conclusion

In this study, I examined the patterns in teachers' noticing of students' mathematical thinking as they engaged in a PD program centered around task-based student interviews and collaborative discussions of interview clips. I employed a case study methodology, and each case consisted of a teacher's engagement with the PD program along with a cross-case analysis. This was important as the nature of the particular case study methodology I chose for this study. In particular, the explanatory multi-case study method allowed me to first examine each teachers' noticing patterns. After having a clear idea about each teacher's case, I was able to explain some commonalities and differences in those patterns across the three cases. In this chapter, I present the meaning, importance, and relevance of my findings. I start with discussing the main findings and contextualizing each finding within previous research and presenting the study's contributions. Then, I present the limitations of the study. Finally, I discuss the study's implications for research and practice.

In the results section, I pointed to some patterns in the teachers' noticing of students' mathematical thinking. One of the patterns was that the teachers tended to notice their actions over student thinking (especially in the earlier sessions). They scored the lowest in suggesting or responding with the next instructional steps based on their attending and interpretation of students' understandings. It is also notable that, despite the nested nature of the three components of professional noticing (Jacobs et al., 2010), high scores in one component did not always predict high scores in the other two. Lastly, I found that PD structure, the PD facilitator's role, and the choices they make may influence how teachers notice students' mathematical thinking.

As such, task selection, clip selection, and the facilitation of the PD have some critical implications on participating teachers' noticing.

Focus on Student Thinking

The task-based student interviews presented authentic windows into students' thinking (Jacobs et al., 2006), and based on what the teachers said about the PD program, they were drawn into trying to understand how students approached problems. This is significant because acknowledging the importance of understanding students' existing conceptions of mathematical ideas is an essential first step for teachers to adopt productive beliefs regarding how students learn (Carpenter & Lehrer, 1999). In this way, teachers are likely to shift from teacher-centered to student-centered instruction (Van de Walle et al., 2014).

As teachers participated in the PD program, they recognized some unwarranted effects of their actions on students' progress with mathematical concepts and reflected on them during the PD sessions. This ranged from misunderstanding a student's mathematically valid statement to disrupting a student's thought process by frequent responses and overuse of scaffolding. From this perspective, my findings support Heng and Sudarshan's (2013) study with grade 1 and 2 mathematics teachers who confronted their lack of understanding of students' thinking and realized how that could prevent students from fully developing conceptual understanding of mathematical concepts. The middle school teachers in my study shared similar concerns about their actions and recognized the need for improvement in their teaching practices. The use of video fostered a productive reflection cycle that helped teachers learn from their own practice (Jacobs et al., 2006; Seago et al., 2018). As the teachers engaged in video-based discussions of student interviews, at times, they tended to score higher in attending to and interpreting students' mathematical thinking. Additionally, teachers developed awareness about their actions that might have interrupted students' thought processes. Teachers seemed to understand the importance of slowing down and letting students share their conceptions without intervening and funneling their thinking.

The benefits of increased focus on student thinking are well-documented in the research that has studied the impact of one-on-one student interviews on teachers noticing of student thinking (Buschman, 2001; Heng & Sudarshan, 2013). The current study extended those findings from the elementary school level to a PD program designed for secondary school in-service mathematics teachers. It showed the applicability of the ideas from those studies to higher grade-level settings. Additionally, the current study included the collaborative viewing and discussion of the interview videos as part of its key design elements similar to other studies (e.g., Star & Strickland, 2008; van Es & Sherin, 2006). Those discussions contributed to the teacher's development in focusing on students' mathematical thinking.

Focus on Teacher Actions

One finding from the data was that teachers frequently focused on the actions of the interviewing teacher instead of focusing on student thinking. I linked this to the article titled "Questioning our Patterns of Questioning" (Herbel-Eisenmann & Breyfogle, 2005) which was read and discussed during Session 1. Perhaps teachers wanted to become better interviewers, as outlined in that article. As a result, their focus was centered on the actions of the interviewing teachers in the clips shown. Teachers have a natural tendency to notice pedagogy, in other words, the actions of the teacher in an instructional setting rather than focusing on students' thinking (Sherin & van Es, 2005). Assigning the article, I might have unintentionally conditioned teachers to notice interviewer actions, although my goal was to prepare them for the task of interviewing. Examining this further, I realized that two of the teachers scored low in attending on self-recorded clips. They wrote about the non-mathematical aspects of the clips and mainly talked about their own actions as interviewers. Teachers may have acted differently had I reminded them about the role of the interviews as tools for student thinking to surface.

The Disconnect Between Scores on Individual AIR Components. By definition, the professional noticing of children's mathematical thinking framework consists of three nested components: (1) attending to, (2) interpreting, (3) deciding how to respond to students' mathematical thinking. Jacobs and colleagues (2010) describe this relationship between the components as follows:

In conceptualizing the construct of professional noticing of children's mathematical thinking, we envisioned the existence of a nested relationship among the three component skills such that deciding how to respond on the basis of children's understandings can occur only if teachers interpret children's understandings and these interpretations can be made only if teachers attend to the details of children's strategies. Given this nested relationship, one could conclude that professional development should focus exclusively on attending before interpreting and interpreting before deciding how to respond. We worry that an approach that addresses these skills only independently and sequentially may seem too removed from teachers' everyday work. Instead, we argue that professional developers can focus on all three skills in integrated ways but be aware of the component skills and their growth indicators (p.197).

Based on this conceptualization and the design of other studies that examined similar phenomena (Krupa et al, 2017), I planned a PD program that incorporated all three components of professional noticing. I collected data on all three components and observed patterns in how the teachers scored on each. I assumed that this nested relationship could have been viewed in a dependent manner. For example, if a teacher did not score well in attending, they would most likely score poorly on the other two components of professional noticing. On the other hand, if a teacher scored high in responding, that would be because they scored high in attending and interpreting. Nonetheless, the results showed that this was not always the case. The evidence provided that scores in one component do not necessarily predict scores in the other two.

This is in line with the findings of (van Es, 2011) that there is no linear progression of development in attending, then in interpreting. Instead, she suggests several pathways in which teachers' noticing is developed. In the video club setting she designed, van Es suggests that teachers may move back and forth between their previous noticing practices and their emerging practices. Walkoe and colleagues (2020) indicated that attending and interpreting have aspects inseparable as cognitive processes. This makes assessment and evaluation of these components harder. The field needs to come up with new tools and methodologies to understand the development of the noticing components to inform future research on noticing. This may imply that the AIR prompts and the associated rubric needs to be reexamined. The goal can be integrating all three components while identifying the complex relationship between the development of each one. This may also help with the design of effective PD programs aimed to develop teachers' noticing with a specific focus (e.g., interpreting students' mathematical thinking).

From General Observations to Specifics of Student Understanding

At the outset of the PD program, teachers shared shallow descriptions of what they saw in the interview video clips. These observations ranged from students' general demeanor to the actions of the interviewer. In other words, teachers' attention and focus were more on the non-mathematical aspects of the interviews. As they progressed through the PD program, teachers started seeing and describing more specific mathematical details about the students' thinking. Their descriptions were comprehensive and non-evaluative. Despite not being able to report on changes to the quantitative scores, these results are promising. The question on the weekly AIR prompts regarding attending to read, "After watching the video for the first time, what do you notice?", which did not guide the teachers in any way to write things that are related to students' mathematical thinking. Also, during the PD sessions, I did not guide the teachers or funneled their focus in any way besides directing their attention to student thinking when the conversations were sidetracked to irrelevant topics. The level of detail in teachers' entries in attending to students' thinking emerged naturally as they participated in the PD program.

Teachers' tendency to provide more specific details of student thinking is consistent with the previous body of research that aimed to improve teacher noticing via student interviews. For instance, Bushman's (2001) action research study reported that student interviews helped elementary school teachers notice students' strengths and areas of improvement with more detail and accuracy. In Bushman's design, there was no videotaping and collaborative video viewing of the interviews. Regardless, by solely interviewing students one-on-one, teachers tended to notice students' mathematical thinking in more productive ways (Bushman, 2001). Interviews provided authentic windows into students' thinking. Unlike their regular interactions with students during normal school hours, teachers were not concerned about teaching or grading students' work at the interviews. This might have contributed to the changes in teachers' attending to students' thinking in their study.

When collaborative video viewing was incorporated in a similar design, Jacobs and colleagues (2006) reported similar results. The discussions helped teachers not only attend to important mathematical strategies employed by the students but also interpret those strategies in a productive manner. The role of discussions in my study was not different. Teachers seemed to benefit from each other's perspectives as they discussed video clips of self-recorded interviews as well as the videos of other teachers' interviews. As they conducted a handful of interviews and engaged in four 1-hour sessions of clip discussions, teachers started to notice their students' thought processes and shared those noticings and interpretations in more comprehensive and substantiated ways

Additionally, it is important to note that this study added to the body of work on secondary in-service mathematics teachers' noticing. The previous studies investigated the noticing of either elementary in-service teachers (Buschman, 2001; Jacobs et al., 2006) or secondary pre-service teachers (Krupa et al, 2017). There may be unique patterns in secondary school teachers' progress with any PD program as the mathematics content in middle school builds upon elementary content. Students are required to draw on their foundational skills as they approach problems at the secondary level as a natural result of the way mathematics content standards progress (Kanold & Larson, 2012). As a result, relatively more sophisticated thought processes may surface as middle school mathematics teachers interview their students. This may impact the ways in which middle school teachers attend to their students' thinking. In that regard, the current study adds to the body of research on secondary school mathematics teachers' noticing and may serve as a starting point for future studies.

From Overgeneralizations to Evidence-based Interpretations

Another important finding of the study is the patterns in teachers' interpretations of students' mathematical thinking. It is important to note here that the second rater and I were not looking for a single best way teachers could have interpreted students' understandings. Instead, we examined the extent to which teachers' interpretations were aligned with the students' solution strategies evidenced in the clips. This is underlined in the professional noticing framework (Jacobs et al., 2010), and we assigned our independent ratings to the teachers' written responses with this idea in mind.

Although not definitive, each of the three teachers tended to avoid overgeneralizing without direct connections to the video clips. Instead, they started to connect their interpretations to specific student ideas evidenced in the clips. In the framework van Es and colleagues developed for learning to notice students' mathematical thinking (2011), this corresponds to a shift from baseline evaluations to more focused and extended interpretations. The authors suggested that productive interpretation has to be strictly evidenced-based rather than snap evaluations based on minimal evidence. The growth indicators identified by Jacobs and colleagues (2010) also highlight a "shift from overgeneralizing children's understandings to interpretations to specific details of the situation" (p. 196). All three teachers in the study seemed to fit in this shift in varying degrees on this component of professional noticing.

Teachers' interpretation of students' mathematical thinking is important as it shapes the instructional decisions they make. Teachers act on what they notice and how they interpret what students do and do not understand (Schoenfeld, 2011). By putting aside general observations and basing their interpretations on specific evidence about what students do or say, teachers have a better chance to accurately diagnose and address misconceptions (Heng & Sudarshan, 2013). Conducting interviews and collaboratively discussing video clips of the interviews enables teachers to improve their interpretations of students' mathematical thinking. Considering the nested relationship among the three components of professional noticing (Jacobs et al., 2010), this is a crucial step towards teachers' advancing in how they make instructional decisions to further students' understandings.

Challenges in Responding to Students' Mathematical Thinking

The three teachers who participated in this study scored low in responding to students' mathematical thinking. In alignment with the professional noticing framework (Jacobs et al., 2010), the second rater and I focused on the teachers' reasoning as they decided how to respond. We did not look for a specific next step. Regardless, teachers either did not connect the suggested next steps to their interpretations of student understandings or did not provide a rationale for their suggestions. As a result, they did not score well in deciding how to respond, despite some high scores in attending to and interpreting students' mathematical thinking. Krupa and colleagues (2017) found that teachers do not necessarily improve in this category of professional noticing with more experience. Instead, they suggested that teachers participate in PD that is focused specifically on responding to students' thinking.

One interesting result of my study was about the role of teaching experience in responding to students' thinking. While Linda and Mary, with five and ten years of teaching experience respectively, consistently scored low in this area, Nick, who had only two years of teaching experience, demonstrated relatively higher performance. Nick tended to suggest next steps that could have potentially enhanced student learning. His responses were more connected to what he noticed and how he interpreted students' understandings. Though not consistent, Nick tended to avoid reasoning about instructional next steps in the abstract to reasoning that was based on students' existing understandings. On occasion, he provided a rationale for the next steps he suggested. He scored the single 2 (out of 27 responses) in this category. One reason for this might be related to Nick's strong interest in teaching, as evidenced by his enthusiasm during other PD events and meetings. Based on the reports by his school administrators and my professional evaluation of him, Nick is an outspoken teacher who often has opinions about the teaching and learning of mathematics. He shares his ideas during the PD events and the weekly PLC meetings. This might have contributed to his relatively stronger performance in deciding how to respond. Although it seems surprising since expertise in teaching is usually associated with years of experience (Burns & Darling-Hammond, 2014), this finding aligns with previous research in teacher noticing. Jacobs et al. (2011) worked with teachers with a wide range of teaching experience and examined their professional noticing skills. Among other findings, they reported that teaching experience alone was not associated with teachers' deciding how to respond on the basis of students' understandings. In this regard, Nick's relatively higher performance in responding supports the findings of previous research.

The Potential Influence of Task and Clip Selection

Task Selection. One of the critical features of effective PD is the content focus (Desimone, 2009). Increased focus on subject matter content, particularly on how students learn it, supports teachers' knowledge and situation-specific skills, which then lead to gains in student achievement (Cohen & Hill, 1998; Desimone et al., 2002). Teachers in the current study interviewed students using two task-based protocols that were focused on sixth-grade mathematics content. The end goal of the study, however, was not about the interviews. Instead, the interviews served as mediums through which student thinking surfaced. Through analyses of the interview videos, teachers had opportunities to notice students' thought processes that emerged in the interviews.

The differences in the two tasks used in the interviews may have resulted in differences in how students approached problems and talked through their strategies. I found that tasks that presented mathematics in real-life context had greater potential for the surfacing of student thinking. Students usually struggle with talking about abstract concepts that are represented with mathematics symbology (Stein & Smith, 1998) as in their interaction with Task 1 (Appendix B). Since the interviews push students to communicate their ideas and strategies verbally, there may be a need for problems that students can relate to. The context in Task 2 (Appendix B), a task that involved sales and discount, was more relatable for the students. They talked and reasoned about the original price, sale price, and the reasonableness of their answers in a more concrete way. This might have led to more explicit math talk during the interviews and provided more for teachers to discuss.

It is important to note that as I designed the PD program, my goal was to select tasks that did not suggest or imply a particular solution method. Rather, I chose tasks that invited students to use a variety of approaches and solution strategies. In that regard, the two tasks were not different from each other. On the other hand, the real-life connection element in Task 2 resulted in better surfacing of students' mathematical thinking. This is an important contribution of my study. Researchers must not only focus on selecting cognitively demanding tasks for student interviews but also consider the potential of the tasks as to how they invite students to talk about their ideas.

Clip Selection. Another factor that shaped the teachers' noticing of students' mathematical thinking was the selection of clips to be viewed during the PD sessions. Using the video selection criteria (Table 2) developed by Sherin and colleagues (2009), I assigned a score of 1-3 to each clip. Before the PD sessions, I watched each teachers' interview videos and tried to select rich segments based on the criteria mentioned above. Among the ones that made it to the summary of scores shown in Figure 5, some clips were scored lower than others due to two main reasons: (1) the lack of evidence of student thinking in the clip, (2) the lack of student sense-making in the clip. Comparing the scores of those clips with teachers' AIR scores in the attending to category, I observed some patterns. On the clips with the lowest overall scores, teachers' average attending to scores were the lowest among all clips.

Validating their video selection instrument on 26 clips, Sherin et al. (2009) found tha*t the windows into student thinking* and *depth of student thinking* were the most important factors that contributed to the quality of video discussions and the extent to which teachers attended to students' mathematical thinking. For example, if a clip was rich in both windows into and depth of student thinking, regardless of the *clarity*, teachers spent time to understand the rich content in the video. In the video selection criteria, clarity refers to the answer to a key question that is, "How easy is it to understand the student thinking shown in the video?" (Figure 5). It means the transparency of student thinking in the video and the extent to which viewers are able to understand what the student is doing and saying. If a clip was low in those two components, clarity did not contribute to the quality of the discussions.

The results of my study support the work of Sherin and colleagues (2009). Relatively lower scores in windows into and depth of student thinking resulted in lower scores in attending to students' mathematical thinking regardless of the clarity of student thinking demonstrated in the clip. Session 3 Clip 1 and Session 4 Clip 1 both scored a 2 in the categories of windows into and depth of student thinking. Student thinking in the first clip was very transparent and scored a 3. The second clip, however, scored a 1 due to the bad audio quality and lack of transparency of the students' thinking. This difference in the clarity of the two clips did not result in any major differences in the teachers' attending scores. Thus, clarity did not contribute to the noticing performance of the teachers.

The Role of the PD Facilitator

A methodological choice I made was about facilitating the discussions. I hypothesized that teachers would talk about what they noticed, how they interpreted their noticings, and how they decided to respond to students' mathematical thinking in a natural way without the need for specific guidance for each of those components of professional noticing. I avoided guiding and funneling their thinking in order to capture their noticings as they emerged naturally. After coding the PD session transcripts, I noticed that there was not enough discussion material for each of the AIR components per clip. For instance, if I wanted to cross-check a teachers' interpretation score on their written response to a given clip with their comments about the same clip during the PD session, I was unable to find enough material in the session discussions. This was mainly because not every teacher necessarily commented about each component of AIR for each clip.

On the other hand, I made a deliberate choice about not giving feedback to teachers on their written responses. I intentionally avoided funneling teachers' thinking and noticings toward a desired response. I wanted teachers' noticings to emerge naturally from their discussions. I made a list of things as the facilitator, including, but not limited to orienting the group to the video analysis task, highlighting noteworthy comments, redirecting focus on the video when lost focus, and supporting group collaboration by sometimes standing back and distributing participation (van Es et al., 2014). Nonetheless, I did not guide the teachers to write or say anything specific to the videos viewed. This is in alignment with other research that studied the role of the facilitator in video-based PD (Goldsmith & Seago, 2011; Jacobs et al., 2010). The facilitators in the cited studies implemented the actions listed in the framework suggested by van Es and colleagues (2014); however, they did not give teachers feedback on their written entries, nor did they share their scoring instruments. I believe this choice helped me avoid directing teachers' noticings in a predetermined path based only on how I, as the facilitator, would have attended to, interpreted, and responded to student thinking in the ideal way outlined on the AIR rubric.

In conclusion, the ways in which I facilitated the interview clip discussions might have impacted the nature of teachers' noticing of students' mathematical thinking. Since I did not analyze the session transcripts focusing on my moves as the PD facilitator, I cannot make definitive conclusions about the role of facilitation on teachers' noticing experiences.

Overall Contributions of the Study

The results of my study contributed to the teacher noticing research in several ways. First, it was an attempt to examine teachers' engagement in a PD program based on the professional noticing of children's mathematical thinking framework (Jacobs et al., 2010). The goal of the PD program was to investigate the role of task-based student interviews and collaborative discussions of the interview clips on in-service middle school mathematics teachers' situation-specific skills, namely, attending to, interpreting, and deciding how to respond to students' mathematical thinking. About one-third of the studies systematically reviewed by Stahnke and colleagues (2016) investigated secondary school teachers' noticing, and my study added to that body of research by offering another glimpse into the use of the professional noticing framework in a professional development setting.

Second, the methods of my study have the potential to inform future research in this area. I worked with a small group of teachers, and that allowed me to focus my attention on their progress with noticing. The case study methodology allowed me to dive deeper into the specific experience of each teacher and use qualitative tools that helped me understand beyond simple cause and effect relationships. Also, the program, in its entirety, was conducted fully remotely due to the Covid-19 Pandemic. The data collection tools I used, such as the structure of the web-based weekly video viewing and AIR prompts response collection system (Appendix C), can inform the design of future research based on the professional noticing framework. Additionally, the analytical process, particularly the rater consistency assurance practice tool (Appendix E), can be useful for researchers that want to use the AIR rubric to assign ratings to teachers' written or verbal responses to AIR prompts.

Lastly, this study aimed to meet the five critical features of effective PD defined by Desimone (2009) as content focus, coherence, active learning, collective participation, and duration. The methodological choices and the findings indicate that it met four of the five critical features. It was *content-focused* as the mathematics content was limited to two sixth-grade mathematics tasks that promoted reasoning and problem-solving. It was *coherent* since the PD program did not address many areas at once as this does not help teachers develop the knowledge and skills necessary to improve their teaching (Darling-Hammond & McLaughlin, 2011). Teachers conducted student interviews and shared their experiences on noticing student thinking with their colleagues. They were the main figures in both of those activities, and they *actively learned* about student thinking. Teachers focused on noticing students' mathematical thinking through collaboratively watching and discussing videos of student interviews, so *collective participation* was also a key feature of the PD program. Nonetheless, the *duration* of the program was limited to only five weeks, including the first session that prepared the teachers to conduct the interviews. Hence, teachers only were able to do a handful of interviews and discuss only nine clips in four 60-minute sessions. Research on effective PD suggests that programs that offer 30-100 hours of contact time with teachers in a school year can produce positive change in teacher practice and, as a result, improve student achievement (Darling-Hammond et al., 2009). This is consistent with the other findings that time span and contact hours can provide significant opportunities for active learning by teachers (Desimone, 2009; Garet et al., 2001). Based on the teachers' comments and the data from the PD, I conclude that even if the other four key features of effective PD are met, a program must be designed in ways that allow teachers to discuss, plan, test, revise, and retest new learning over a long period of time before sustained changes can be observed in their teaching practices.

Limitations

The last component mentioned above is certainly a limitation of the study. The program was designed for five weeks, and I initially hypothesized that it could give teachers ample time to pay closer attention to student thinking and then reflect on their own noticing and the noticing of others. The results indicate that stronger and more plausible conclusions could have been made if the program were scaled up and extended in time. That would have allowed for more interviews to be conducted by the teachers and the discussion of more interview clips (maybe for multiple viewing of the same clip during a session). That way, the teachers could have spent more time focusing on students' mathematical thinking. Furthermore, it might have allowed me more time to focus on each clip and facilitate discussions around each of the AIR components. Additionally, if the duration of the program had been longer, teachers could have used a variety of tasks in an alternating fashion. That way, we could have collected representative data from the earlier and later phases of the PD program for each task. This would have enhanced the findings about the impact of mathematical context on teachers' noticing. For example, during Session 5, Mary said, "I think we were just getting the hang of this, and now it's ending" (Mary, Session 5, May 27, 2020). This implies that teachers could have benefitted from participating in the PD program for longer periods of time.

Another limitation was about the use of the AIR rubric (Figure 3) used to evaluate teachers' written responses. This 2-point rubric was not fine-grained enough to rate the teachers' written responses. It is hard to observe significant differences quantitatively when the scoring is limited to a 0-2 range. For example, consider the following responses to the same clip:

1. The student understands substitution to test if the value they solved for or picked makes the inequality true. The student understands how to make sense of the inequality (Linda, Session 2 Clip 2 response, May 6, 2020).

2. I like how the student is using math terms: e.g., to substitute, what makes the statement true, expression, inequality ... the student is describing in length her thinking ... it is as if she gives the teacher an opportunity to see exactly what she knows and where might she have a problem ... if we, as teachers, know this we can better address the student's needs (Nick, Session 2 Clip 2 response, May 6, 2020).

Both of the responses received “limited evidence (1)” in the category of interpreting students’ mathematical thinking as they both interpreted only one aspect of the students’ thinking. However, the responses are qualitatively different. Linda’s response is more precise and to the point. In other words, Linda describes what she thinks the student understands directly. Nick’s response has more material, but he barely refers to what he thinks the student understands. With that, he scores a 1 since 0 is reserved for completely vague or incorrect interpretations. A more fine-grained rubric might have helped identify these little nuances between teachers’ responses. Perhaps another indicator for the amount of detail in addition to the metric for content would parse out those that lacked details and those that did not address the thinking adequately. In other words, the quality of what teachers write and the amount of writing can be evaluated separately. That way, more robust conclusions can be drawn from the data. The

AIR rubric can be improved, and more levels of performance can be defined and added to it based on the teacher responses.

During the PD sessions, I paused the Zoom recordings as we watched the clips independently. It was the early days of the Covid-19 Pandemic, and I was just learning about using video conferencing tools, particularly Zoom. I hesitated to share my screen with the video to be watched due to potential technical issues and the low audio quality that might have produced. For those reasons, we watched the clips independently, I paused the Zoom recording during that time, and we reconvened after everyone was done. In other words, we did not watch the videos together, nor did we view certain parts of the video multiple times. If we were able to watch them synchronously, I could have paused and rewound them as teachers wished. This might have limited teachers' access to the student thinking shown in the video clips. Additionally, teachers might have lost focus as they disconnected from the virtual PD environment temporarily.

Furthermore, the study was conducted during the rise of the Covid-19 Pandemic. The design of the PD program had to be changed to meet the unprecedented conditions under which teaching and learning was taking place. Teacher-student interactions were limited to Zoom meetings and lacked the face-to-face connection. It was harder to see students' written work at times. Additionally, some of the benefits of face-to-face interaction with students are the availability of students' gestures, facial expressions, and other cues that allow teachers to understand students' thinking. These were somewhat not available through Zoom meetings. Similar situations occurred with the teacher-to-teacher and teacher-to-researcher interactions. The lack of face-to-face interaction between the participants might have changed the dynamics of the PD program. Also, both the teachers and I had to deal with a number of technical issues ranging from bad audio of a student during an interview to internet connection problems during the PD sessions. Additionally, everyone was distracted as we heard about the news regarding the rate of spread, hospitalizations, death rates, and the decline in people's morale was inevitable.

Lastly, as I looked for teachers' direct comments about the PD, I struggled to find detailed information. The data were limited to teachers' written responses on the post-assessment (Appendix A) and some comments made during the PD sessions. I could have incorporated an exit interview with each teacher to discuss their experiences with the PD program.

It is important to consider the limitations mentioned above as one contextualizes the findings of the current study. Regardless, the study has many important implications for research and practice. I summarize some of those in the next section.

Implications for Research

Teacher noticing is an important phenomenon that needs to be studied in order to improve the teaching and learning of mathematics (Schoenfeld, 2011). The current study confirms the growing body of research that claims teacher noticing to be "trainable" (Krupa et al., 2017; Sherin et al., 2011a; Star & Strickland, 2008). Future research can examine the changes in teachers' noticing as they engage in a similar PD program over extended periods of time. Researchers can then compare and contrast the findings that emerge from short-term and long-term participation in this kind of PD.

The study can be extended by the addition of a teacher observation component into the design. Teachers can be observed in their classrooms as they participate in the PD program. This might yield important information about their noticing skills within the complex dynamics of the everyday mathematics classroom. It may also help researchers understand how teachers' performance on written AIR prompts or during the interviews compares to their noticing of students thinking in the classroom where there are many other parameters in place.

Additionally, further research is needed to thoroughly examine the impact of task and clip selection on the ways in which teachers notice students' thought processes. The findings of the current study are limited to two tasks and nine clips that have been used by a small group of teachers and students. Also, in the current study, teachers watched clips of students interviewed on Task 1 for Sessions 2 and 3, and then Task 2 for Sessions 4 and 5. As a result, the changes in teachers' noticing scores during the last two sessions may be attributed to some differences in the design of the tasks. To address this, the tasks can be alternated between the sessions. That way, we can have interviews from the earlier and later sessions that are conducted using both tasks.

Research that incorporates a larger pool of tasks and clips may support or challenge the findings of the current study. Future research can also provide deeper and more detailed interpretations about some key features of tasks and videos and their impact on how teachers notice students' thinking.

The role of the facilitator also is crucial in any PD program (Borko et al., 2014) As mentioned earlier, I used an open-ended and semi-structured approach as I facilitated the discussions during the PD sessions. While this was helpful to encourage teachers' noticings to emerge naturally, it might have led to discussions that lacked the specificity about each of the three nested components of professional noticing. Future research can be designed in a way that pushes the participants to share about their attending, interpretation, and responding.

Furthermore, researchers can examine the ways in which different styles of facilitation lead to different noticing patterns among the participants.

In terms of the analysis, I did not analyze the session transcripts with the idea of how my facilitation might have impacted teachers' noticing. It is well documented in previous research that the facilitator plays a key role in professional development (e.g., Borko et al., 2014; Doppelt et al., 2009). I focused my analysis mostly on teachers' written responses to the AIR prompts and what *they* said about student thinking during the PD sessions rather than looking at the possible impact of my facilitation moves on teachers' noticing. Future studies can look into the role of the facilitator in similar PD.

Another important implication of the study is about the AIR rubric (Figure 3). As indicated earlier, the rubric lacked the specificity to properly address the range of responses collected for the written AIR assessments (pre-and post-assessments and the weekly AIR prompts) during the course of the current study. Researchers can develop a more fine-grained and adaptable rubric using more examples of teacher responses. This may improve the ways in which the patterns of change regarding teachers' noticing emerge from the data. Additionally, we used the holistic rubric to rate each written entry. Future researchers can create a rubric and list the criteria for each clip that will be shown during PD sessions. Those rubrics coupled with sample responses for each score category can result in more accurate and reliable ratings for teacher entries. This may eventually become a repository of video clips that each have a specific rubric. The field can benefit from such a library of resources.

Additionally, the current study supported the challenges reported by other scholars about the responding component of professional noticing. Yet, this is the component that can potentially lead to a significant change in the teaching and learning of mathematics since it is about teachers' acting on what they notice and how they interpret their noticings (Schoenfeld, 2011). Researchers must find ways to examine this important component of professional noticing through carefully designed studies that aim to understand the factors that lead to a change in teachers' deciding how to respond to students' mathematical thinking. Further investigation is needed to understand how teachers act on their noticings.

Lastly, as mentioned earlier, teachers' beliefs and dispositions may play a role in their noticing of students' mathematical thinking. Following other research (Jacobs et al., 2010; Krupa et al., 2017) what others did, my focus in this study was solely on teachers' professional noticing of students' mathematical thinking as shown in my theoretical framework (Figure 2). Further research into issues of equity related to teachers' beliefs and dispositions around noticing is warranted.

Implications for Practice

This study supported the previous research on the benefits of student interviews as they provide authentic windows into student thinking (Buschman, 2001; Heng & Sudarshan, 2013). The task-based student interviews can be incorporated into classroom teaching. With careful planning and time management, teachers can complete short interviews with each of their students. This may help teachers gain insights into students' thinking while allowing students to talk through their ideas and conceptions without the fear of being evaluated, graded, or judged. Teachers can interview one student while the rest of the students work on some other tasks independently.

PD organizers can take ideas from the current study and incorporate them into their PD planning. For example, some of the student interviews can be videotaped and can be used as springboards to start discussions about student thinking. Video selection can be made either by the teachers themselves or the PD facilitators using the selection criteria developed by Sherin et al. (2009). Video-based PD has been proven to be an effective tool for teacher learning (Rodríguez et al., 2018; Sherin & van Es, 2005; Sherin et al., 2009). Hence, the current study's design can inform the planning of video-based PD that is centered around task-based student interviews and the discussion of video vignettes in a collaborative environment, especially over a period of time to allow for camaraderie to build among the participants.

Lastly, since the study was conducted in the midst of the Covid-19 Pandemic, all interactions had to be done remotely via web-based tools such as Zoom and Google applications. Although this brought some unexpected challenges, as mentioned in the limitations, there were some logistical advantages as well. For example, teachers were able to schedule interviews with students via Zoom without the need to be in the same room with them. Originally, they were going to arrange a quiet room during regular school hours and the recording equipment to capture the interviews, which might have brought logistical challenges. Instead, teachers simply recorded the Zoom sessions and shared the videos with me on a secure drive. Similarly, teachers did not have to travel to attend the PD sessions since we used Zoom to run our meetings. I was able to capture clear video and audio of the sessions by simply recording the meetings. Even after we resume in-person teaching, these technology tools will not be taken away from us. Therefore, teachers may still use these technologies to interview their students and attend PD sessions remotely, allowing teachers from different locations across the globe to participate in

PD programs similar to the one designed in the current study. The methods and design of this study have proven that despite many logistical challenges, effective PD programs that aim to improve teacher noticing can be implemented successfully with strategic use of available technologies and applications.

Conclusion

Conducting task-bask student interviews, teachers were able to gain insights into students' thought processes. Additionally, they were able to observe how students demonstrate their knowledge and skills, and how they apply those to novel problem situations. Teachers reported that they were drawn into trying to understand what they saw through the authentic windows into student thinking provided by the interviews they conducted and the clips they watched. Acknowledging the importance of understanding students' existing knowledge and intuitive conceptions about mathematical content is crucial for teachers to adopt productive beliefs on how students learn (Carpenter & Lehrer, 1999). This is promising since teachers tend to shift from teacher-centered to student-centered instruction once they adopt those productive beliefs (Van de Walle et al., 2014).

Supporting teachers in their noticing of students' mathematical thinking is not an easy task. Teachers need to participate in carefully designed, coherent, content-focused professional development (PD) that provides active learning opportunities over long periods of time. This study added to the body of research on secondary in-service teachers' noticing of students' mathematical thinking by offering another glimpse into the use of the professional noticing framework in a professional development setting. I described the experiences of three middle school mathematics teachers as they conducted task-based student interviews and collaboratively discussed the video clips of those interviews. I provided steps towards improving PD programs to support teachers' noticing of students' mathematical thinking. More research is needed to better understand this multifaceted phenomenon.

References

Althauser, K. (2015). Job-embedded professional development: its impact on teacher self-efficacy and student performance [Article]. *Teacher Development, 19*(2), 210-225. https://doi.org/10.1080/13664530.2015.1011346

Bailey, T. R. (2015). *Redesigning America's community colleges: A clearer path to student success*. Harvard University Press.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide?

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special. *Journal of Teacher Education, 59*(5), 389-407.

Banilower, E., & Shimkus, E. (2004). Professional development observation study. *Chapel Hill, NC: Horizon Research*.

Berliner, D. C. (1992). The nature of expertise in teaching. *Effective and responsible teaching: The new synthesis*, 227-248.

Blömeke, S., Gustafsson, J., & Shavelson, R. (2015). Beyond dichotomies: Viewing competence as a continuum. *Zeitschrift für Psychologie, 223*(1), 3-13.

Borko, H., Koellner, K., & Jacobs, J. (2014). Examining novice teacher leaders' facilitation of mathematics professional development. *The Journal of Mathematical Behavior, 33*, 149-167.

Borko, H., Peressini, D., Romagnano, L., Knuth, E., Willis-Yorker, C., Wooley, C., Hovermill, J., & Masarik, K. (2000). Teacher education does matter: A situative view of learning to teach secondary mathematics. *Educational Psychologist, 35*(3), 193-206.

Burns, D., & Darling-Hammond, L. (2014). Teaching around the world: What can TALIS tell us. *Stanford Center for Opportunity Policy in Education, Stanford, CA*.

Buschman, L. (2001). Using student interviews to guide classroom instruction: An action research project. *Teaching Children Mathematics, 8*(4), 222.

Carpenter, T. P., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. *Mathematics classrooms that promote understanding*, 19-32.

Casey, S., Lesseig, K., Monson, D., & Krupa, E. E. (2018). Examining Preservice Secondary Mathematics Teachers' Responses to Student Work to Solve Linear Equations. *Mathematics Teacher Education and Development, 20*(1), 132-153.

Cobb, P., & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher, 28*(2), 4-15.

Cohen, D. K., & Hill, H. C. (1998). Instructional policy and classroom performance: The mathematics reform in California.

Conner, A., Edenfield, K. W., Gleason, B. W., & Ersoz, F. A. (2011). Impact of a content and methods course sequence on prospective secondary mathematics teachers' beliefs. *Journal of mathematics teacher education, 14*(6), 483-504.

Copeland, R. W. (1984). *How children learn mathematics : teaching implications of Piaget's research* (4th ed. ed.). Macmillan;

Collier Macmillan.

Crosswhite, F. J., Dossey, J. A., & Frye, S. M. (1989). NCTM standards for school mathematics: Visions for implementation. *Journal for Research in Mathematics Education, 20*(5), 513-522.

Danielson, C. (2007). *Enhancing professional practice: A framework for teaching*. ASCD.

Darling-Hammond, L., Hyler, M. E., & Gardner, M. (2017). Effective teacher professional development. *Learning Policy Institute*.

Darling-Hammond, L., & McLaughlin, M. W. (2011). Policies that support professional development in an era of reform. *Phi delta kappan, 92*(6), 81-92.

Darling-Hammond, L., Wei, R. C., Andree, A., Richardson, N., & Orphanos, S. (2009). Professional learning in the learning profession. *Washington, DC: National Staff Development Council, 12*.

Desimone, L. M. (2009, 2009/04/01). Improving Impact Studies of Teachers' Professional Development: Toward Better Conceptualizations and Measures. *Educational Researcher, 38*(3), 181-199. https://doi.org/10.3102/0013189X08331140

Desimone, L. M., Porter, A. C., Garet, M. S., Yoon, K. S., & Birman, B. F. (2002, 2002/06/01). Effects of Professional Development on Teachers' Instruction: Results from a Three-year Longitudinal Study. *Educational Evaluation and Policy Analysis, 24*(2), 81-112. https://doi.org/10.3102/01623737024002081

Doppelt, Y., Schunn, C. D., Silk, E. M., Mehalik, M. M., Reynolds, B., & Ward, E. (2009, 11/01/). Evaluating the Impact of a Facilitated Learning Community Approach to Professional Development on Teacher Practice and Student Achievement. *Research in Science & Technological Education, 27*(3), 339-354.

Education, N. C. o. E. i. (1983). A nation at risk: The imperative for educational reform. *The Elementary School Journal, 84*(2), 113-130.

Edwards, A. R., Sandoval, C., & McNamara, H. (2015). Designing for improvement in professional development for community college developmental mathematics faculty. *Journal of teacher education, 66*(5), 466-481.

Ferrini-Mundy, J., & Martin, W. (2000). Principles and standards for school mathematics. *Reston: National Council of Teachers of Mathematics (NCTM)*.

Firestone, W. A., Mangin, M. M., Martinez, M. C., & Polovsky, T. (2005). Leading coherent professional development: A comparison of three districts. *Educational Administration Quarterly, 41*(3), 413-448.

Fosnot, C. T. (2005). *Constructivism : theory, perspectives, and practice* (2nd ed. ed.). Teachers College Press.

Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American educational research journal, 38*(4), 915-945.

Goldsmith, L. T., & Seago, N. (2011). Using classroom artifacts to focus teachers' noticing. *Mathematics teacher noticing: Seeing through teachers' eyes*, 169-187.

Goodwin, C. (1994). Professional vision. *American anthropologist, 96*(3), 606-633.

Heck, D. J., Banilower, E. R., Weiss, I. R., & Rosenberg, S. L. (2008). Studying the effects of professional development: The case of the NSF's local systemic change through teacher enhancement initiative. *Journal for research in mathematics education, 39*(2), 113-152.

Heng, M. A., & Sudarshan, A. (2013). "Bigger number means you plus!"—Teachers learning to use clinical interviews to understand students' mathematical thinking [research-article]. *Educational Studies in Mathematics*(3), 471. https://doi.org/10.1007/s10649-013-9469-3

Herbel-Eisenmann, B. A., & Breyfogle, M. L. (2005). Questioning Our Patterns of Questioning. *Mathematics Teaching in the Middle School, 10*(9), 484-489.

Hiebert, J. (2003). What research says about the NCTM standards. *A research companion to principles and standards for school mathematics*, 5-23.

Horizon Research, I. (2001). *Local Systemic Change through Teacher Enhancement Professional Development Observation Protocol*. Retrieved 5/4/21 from http://www.horizon-research.com/local-systemic-change-through-teacher-enhancement-professional-development-observation-protocol

Jacobs, V. R., Ambrose, R. C., Clement, L., & Brown, D. (2006). Using Teacher-Produced Videotapes of Student Interviews as Discussion Catalysts [Article]. *Teaching Children Mathematics, 12*(6), 276-281.

Jacobs, V. R., Lamb, L. L., Philipp, R. A., & Schappelle, B. P. (2011). Deciding how to respond on the basis of children's understandings. *Mathematics teacher noticing: Seeing through teachers' eyes*, 97-116.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional Noticing of Children's Mathematical Thinking. *Journal for research in mathematics education, 41*(2), 169-202.

Kaiser, G., Busse, A., Hoth, J., König, J., & Blömeke, S. (2015). About the complexities of video-based assessments: Theoretical and methodological approaches to overcoming shortcomings of research on teachers' competence. *International Journal of Science and Mathematics Education, 13*(2), 369-387.

Kanold, T. D., & Larson, M. R. (2012). *Common core mathematics in a PLC at work Leader's guide*. Solution Tree Press.

Kennedy, M. M. (2016, 2016/12/01). How Does Professional Development Improve Teaching? *Review of Educational Research, 86*(4), 945-980. https://doi.org/10.3102/0034654315626800

Krupa, E., Huey, M., Lesseig, K., Casey, S., & Monson, D. (2017). Investigating Secondary Preservice Teacher Noticing of Students' Mathematical Thinking. In *Teacher noticing: bridging and broadening perspectives, contexts, and frameworks* (pp. 49-72). Springer.

Krupa, E. E., Huey, M., Lesseig, K., Casey, S., & Monson, D. (2017). Investigating Secondary Preservice Teacher Noticing of Students' Mathematical Thinking. In E. O. Schack, M. H. Fisher, & J. A. Wilhelm (Eds.), *Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks* (pp. 49-72). Springer International Publishing. https://doi.org/10.1007/978-3-319-46753-5_4

Lesseig, K., Casey, S., Monson, D., Krupa, E. E., & Huey, M. (2016). Developing an Interview Module to Support Secondary PST's Noticing of Student Thinking. *Mathematics Teacher Educator, 5*(1), 29-46. https://doi.org/10.5951/mathteaceduc.5.1.0029

Lynch, K., Hill, H. C., Gonzalez, K. E., & Pollard, C. (2019). Strengthening the research base that informs STEM instructional improvement efforts: A meta-analysis. *Educational Evaluation and Policy Analysis, 41*(3), 260-293.

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. Routledge.

Mertens, D. M. (2014). *Research and evaluation in education and psychology: Integrating diversity with quantitative, qualitative, and mixed methods*. Sage publications.

Monson, D., Krupa, E., Lesseig, K., & Casey, S. (2020). Developing secondary prospective teachers' ability to respond to student work. *Journal of mathematics teacher education, 23*(2), 209-232.

Morris, A. K., & Hiebert, J. (2011). Creating shared instructional products: An alternative approach to improving teaching. *Educational Researcher, 40*(1), 5-14.

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Nye, B., Konstantopoulos, S., & Hedges, L. V. (2004). How large are teacher effects? *Educational evaluation and policy analysis, 26*(3), 237-257.

Park, M. S., Kim, Y. R., Moore, T. J., & Wyberg, T. (2018). Professional Development Framework for Secondary Mathematics Teachers. *International Journal of Learning, Teaching and Educational Research, 17*(10).

Petrou, M., & Goulding, M. (2011). Conceptualising teachers' mathematical knowledge in teaching. In *Mathematical knowledge in teaching* (pp. 9-25). Springer.

Remillard, J. T., Baker, J. Y., Steele, M. D., Hoe, N. D., & Traynor, A. (2017). Universal Algebra I policy, access, and inequality: Findings from a national survey. *education policy analysis archives, 25*, 101.

Santagata, R. (2011). From teacher noticing to a framework for analyzing and improving classroom lessons. *Mathematics teacher noticing: Seeing through teachers' eyes*, 152-168.

Santagata, R., & Yeh, C. (2016). The role of perception, interpretation, and decision making in the development of beginning teachers' competence. *ZDM, 48*(1-2), 153-165.

Schoenfeld, A. H. (2011). Noticing matters. A lot. Now what? In *Mathematics teacher noticing* (pp. 253-268). Routledge.

Seago, N., Koellner, K., & Jacobs, J. (2018). Video in the middle: Purposeful design of video-based mathematics professional development. *Contemporary Issues in Technology and Teacher Education, 18*(1), 29-49.

Sherin, M., Jacobs, V., & Philipp, R. (2011a). *Mathematics teacher noticing: Seeing through teachers' eyes*. Routledge.

Sherin, M., Jacobs, V., & Philipp, R. (2011b). Situation awareness in teaching: What educators can learn from video-based research in other fields. In *Mathematics teacher noticing* (pp. 81-95). Routledge.

Sherin, M. G., Linsenmeier, K. A., & van Es, E. A. (2009). Selecting video clips to promote mathematics teachers' discussion of student thinking. *Journal of teacher education, 60*(3), 213-230.

Sherin, M. G., & van Es, E. A. (2005). Using Video to Support Teachers' Ability to Notice Classroom Interactions. *Journal of Technology and Teacher Education, 13*(3), 475-491.

Shields, P. M., Marsh, J. A., & Adelman, N. E. (1998). Evaluation of NSF's Statewide Systemic Initiatives (SSI) program: The SSIs' impacts on classroom practice. *Menlo Park, CA: SRI*.

Silver, E., & Herbst, P. (2007). Theory in mathematics education scholarship. *Second handbook of research on mathematics teaching and learning, 1*, 39-67.

Skemp, R. R. (1987). *The psychology of learning mathematics* (Expanded American ed. ed.). L. Erlbaum Associates.

Stahnke, R., Schueler, S., & Roesken-Winter, B. (2016, April 01). Teachers' perception, interpretation, and decision-making: a systematic review of empirical mathematics education research [journal article]. *ZDM, 48*(1), 1-27. https://doi.org/10.1007/s11858-016-0775-y

Star, J. R., Lynch, K. H., & Perova, N. (2011). Using video to improve mathematics' teachers' abilities to attend to classroom features: A replication study. *Mathematics teachers' noticing: Seeing through teachers' eyes*.

Star, J. R., & Strickland, S. K. (2008). Learning to observe: Using video to improve preservice mathematics teachers' ability to notice. *Journal of mathematics teacher education, 11*(2), 107-125.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268-275.

Stigler, J. W., & Hiebert, J. (2009). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. Simon and Schuster.

Syed, M., & Nelson, S. C. (2015). Guidelines for establishing reliability when coding narrative data. *Emerging Adulthood, 3*(6), 375-387.

Van de Walle, J. A., Lovin, L. H., Karp, K. S., & Bay-Williams, J. M. (2014). Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades PreK-2 (Volume I)(Teaching Student-Centered Mathematics Series).

van Es, E. A. (2011). A framework for learning to notice student thinking. In *Mathematics teacher noticing* (pp. 164-181). Routledge.

Van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education, 10*(4), 571-596.

van Es, E. A., & Sherin, M. G. (2006). How different video club designs support teachers in "learning to notice". *Journal of computing in teacher education, 22*(4), 125-135.

van Galen, F., & Gravemeijer, K. (2003). Facts and algorithms as products of students' own mathematical activity. In *A research companion to principles and standards for school mathematics* (pp. 114-122). NCTM.

Walkoe, J., Sherin, M., & Elby, A. (2020, 08/01). Video tagging as a window into teacher noticing. *Journal of mathematics teacher education, 23*. https://doi.org/10.1007/s10857-019-09429-0

Yin, R. K. a. (2009). *Case study research design and methods Robert K. Yin* [Book]. http://ezproxy.montclair.edu:2048/login?url=https://search.ebscohost.com/login.aspx?direct=true&db=edswao&AN=edswao.381652467&site=eds-live&scope=site

Appendix A

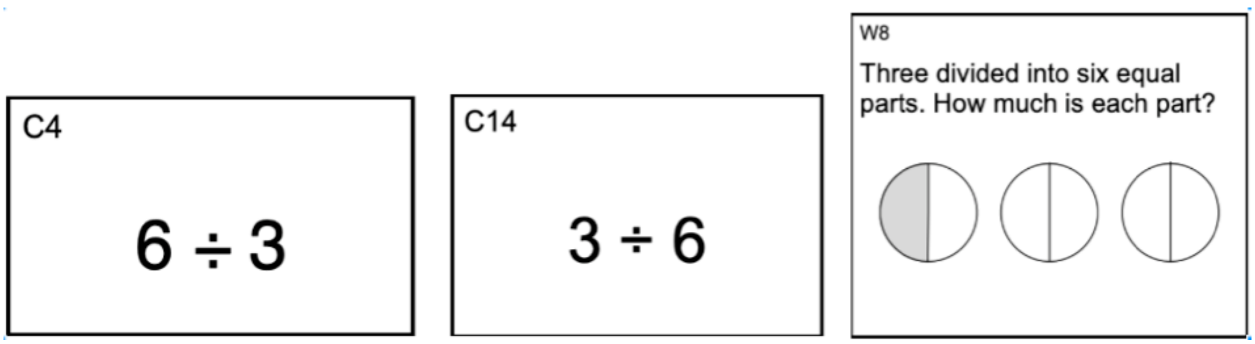
Pre/Post Assessment of Teacher Noticing*

Context

You will view a video of fifth-grade students working on a card-match activity about interpreting multiplication and division. In the video, one student is making a match and explaining their thinking, matching a card with 6 circles all split into thirds. The other two students prompt her to clarify her thinking and guide her in making a match.

Do the math:

Do the Math: Take 3 minutes to match card W8 with one of C4 and C14. Explain your reasoning.



Watch the video:

Watch the video, and feel free to take notes during the viewing if you think that will be helpful. Make sure to devote your attention to the video.

Please note that you can find the video and the accompanying video supplement (with the conversations' transcript) at this link: https://drive.google.com/...

(A) After watching the video for the first time, what do you notice?

Watch the video again:

Now watch the video one more time!

Again, you can find the video and the accompanying video supplement (with the conversations' transcript) at this link: https://drive.google.com/...

(I) Now that you watched the video a second time, how would you describe what this student understands?

(R) Describe some ways you might respond to this student if you were her teacher and explain why you chose those responses.

Note: The post-assessment had the following additional questions:

1. In what ways do you think this study supported you as a math teacher?

2. How do you think the study can be improved?

3. How do you think the study can be scaled up to support more teachers?

Appendix B

Task 1 Protocol: Number Operations with Inequalities[1]

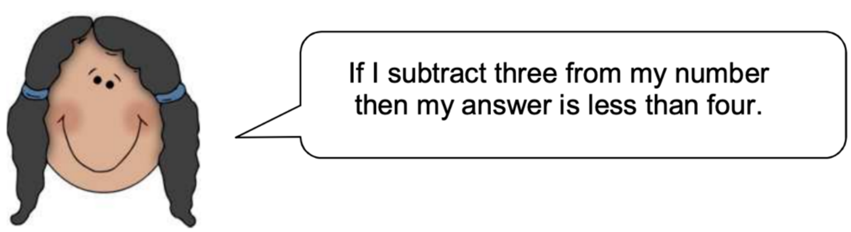
CCSSM 6.EE:

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.

Hello ___________________. Thank you for participating in this study. As you know, I am very interested in understanding how students like you think about ideas in mathematics. We have learned that if we understand how you see things, we can learn more about teaching students well. Therefore, I will ask you to explore an idea by working with a task I give you. I will ask you to talk aloud as you work. Now, letting me work with you is very kind of you. If at any time you wish to stop, just say so, and we will stop. And remember, there are no right or wrong answers here, so feel free to share your ideas with me. Lastly, this is not for a grade. Do you have any questions before we begin?

Ok. Here is a math task for you:

Abigail is thinking of a number:



1. What numbers could she be thinking of? Describe them all.

2. Could Abigail be thinking of 8? Explain your answer.

---

[1] Adapted from the Evaluating Statements About Number Operations task by MARS Formative Assessment Lessons.Original Lesson URL: https://www.map.mathshell.org/lessons.php?unit=6225&collection=8 .

3. Which of the following means the same as Abigail's statement? Check all that apply.

$x - 3 < 4$	$3 - x < 4$	$x - 3 > 4$	$3 - x > 4$
----------	----------	----------	----------

Task 2 Protocol: Shirt Sale[2]

CCSSM 6.RP.3:

- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Hello ___________________. Thank you for participating in this study. As you know, I am very interested in understanding how students like you think about ideas in mathematics. We have learned that if we understand how you see things, we can learn more about teaching students well. Therefore, I will ask you to explore an idea by working with a task I give you. I will ask you to talk aloud as you work. Now, letting me work with you is very kind of you. If at any time you wish to stop, just say so, and we will stop. And remember, there are no right or wrong answers here, so feel free to share your ideas with me. Lastly, this is not for a grade. Do you have any questions before we begin?

Ok. Here is a math task for you:

Selina bought a shirt on sale that was 20% less than the original price. The original price was $5 more than the sale price. What was the original price? Explain or show work.

Notes for the Teacher:

- Make sure to capture the student's written work. This can be done by pinning students' screens during the video recording as they hold their paper towards the

[2] Adapted from the Shirt Sale task by Illustrative Mathematics. Original Lesson URL: https://tasks.illustrativemathematics.org/content-standards/6/RP/A/3/tasks/54.

camera. This process can be repeated as the student produces essential mathematics on their paper.

- There are several different ways to reason through this problem. Try not to guide the student and/or funnel their thinking. Our goal is not to have students produce the correct answer. We want to understand their conceptions even their thinking/methods are not valid.

Appendix C

Weekly Noticing Prompts-Google Form

Noticing Project Weekly Prompts

Context: You will view video clips of students working on math tasks through one-on-one interviews with their teachers.

* Required

1. Email address *

____

Teacher Name

2. Teacher Name *

Mark only one oval.

- ◯ Mary
- ◯ Linda
- ◯ Nick

3. Session Date/Session Number *

Mark only one oval.

- ◯ 5/6/20 (Session 2)
- ◯ 5/13/20 (Session 3)
- ◯ 5/20/20 (Session 4)
- ◯ 5/27/20 (Session 5)

4. Select Clip *

Mark only one oval.

- Clip #1
- Clip #2
- Clip #3

5. Please certify: I watched the entire video, took my notes, and I am ready to start answering questions about it. *

Check all that apply.

- Yes

Noticing Prompt 1 (A)

6. (A) After watching the video for the first time, what do you notice? *

Stop and Watch Again!

Now watch the video one more time!

Again, the links to the video clips can be found on the session meeting notes document that was shared with you earlier (https://docs.google.com/document/d/15Txqj-h6L2DOTElptHBx7ehBgzM0bDr0D8lY7E4wniY/edit?usp=sharing)

7. Please certify: I watched the entire video one more time and I am ready to answer additional questions. *

Mark only one oval.

- Yes

Noticing Prompts 2&3 (I&R)

Now please answer the prompts below.

8. (I) Now that you watched the video a second time, how would you describe what this student understands? *

Noticing Prompts 2&3 (I&R)

Now please answer the prompts below.

8. (I) Now that you watched the video a second time, how would you describe what this student understands? *

9. (R) Describe some ways you might respond to this student if you were her teacher and explain why you chose those responses. *

Certification of Confidentiality

10. Please certify: I completed this form by myself and will not share the content and/or my answers with anyone else. *

Check all that apply.

☐ Yes

Appendix D

Sample PD Session Protocol

Noticing Project Session 2 (May 6, 2020)

Present: Linda, Mary, Nick, Gurkan

Disclaimer: These meeting sessions will be recorded, and Gurkan and his advisors will only access the recordings.

1. Reflecting on the interviews
    a. What went well?
    b. What was a challenge?
    c. Tips and tricks
        i. Pinning the student shows their work
        ii. Copying the task to eliminate back and forth between slides.
        iii. Other?
2. Watching Session 2-Clip#1 (Originally hyperlinked for easy access)
    a. Visit the “Noticing Project Weekly Prompts” form (Originally hyperlinked for easy access) and follow the instructions. Be sure to select “**Clip #1**” from the dropdowns.
    b. (After we all submit the form for the given clip) Let’s discuss!
3. Watching Session 2-Clip#2 (Originally hyperlinked for easy access)
    a. Visit the “Noticing Project Weekly Prompts” form (Originally hyperlinked for easy access) and follow the instructions. Be sure to select “**Clip #w**” from the dropdowns.
    b. (After we all submit the form for the given clip) Let’s discuss!

Appendix E

Rater Consistency Assurance Practice (October 3, 2020)

1. Watch Direct Modeling Clip together and anticipate teacher responses that would get 0,1,2 on each of the AIR categories.
    1. (A) After watching the video for the first time, what do you notice?
        1. 0-No evidence response might look like this:
            1. The student was confident but hesitant to work with big numbers.
            2. The student struggles to solve the problem.
            3. The student has a lack of foundational knowledge/skills. The student could not use paper/pencil, jumped to use manipulatives.
            4. The student can divide 16 by 5 and find 3. She is confused about the remainder of 1 (The student was not confused, response referring to something that is not in the clip)
        2. 1-Limited evidence response might look like this:
            1. The student found the correct answer (The response does not provide specific information or details; instead, it is a general/global comment).
        3. 2-Emerging ability response might look like this:
            1. The student demonstrates a conceptual understanding of division. She makes 3 groups of 5 and identifies 1 as the remainder. When asked, she can interpret each number in the context of the problem. Ex: There is 1 extra dollar. One cube represents a dollar. She also is able to show five cubes together to represent the cost of a book.

b. (I) Now that you watched the video a second time, how would you describe what this student understands?

1. 0-No evidence response might look like this:
    1. The student can use cubes to solve the problem and get the correct answer.
    2. The student is very slow; she could have just used a paper and pencil to divide 16 by 5.
2. 1-Limited evidence response might look like this:
    1. The student seems to understand how to solve division problems. The student divided the cubes into three groups to solve the problem. (Response captures student's strategy, but does not connect it to the problem/context)
3. 2-Emerging ability response might look like this:
    1. The student can make sense of the problem using the cubes. She can interpret each number in the context of the problem using cubes. Ex: 1 cube = $1. This shows that she can attach meaning to the strategy she chose. She did not simply work with the numbers in the problem to get a random number. She is intentional in creating three groups of 5 within 16.

c. (R) Describe some ways you might respond to this student if you were her teacher and explain why you chose those responses.

1. 0-No evidence response might look like this:
    1. I would have given her similar problems and drills on division within 20 (Very general suggestion is made. No rationale provided why the student

should be doing drills in division. Also, no references to what's observed in the clip).

2. I would have asked the student to do calculations using a pencil and paper, then show me their steps.

2. 1-Limited evidence response might look like this:
    1. I would have had the student start solving the problem on pencil and paper. I would have then allowed them to use cubes. (The response is connected to student thinking interpretation; however, no rationale is provided why this is a good move).
3. 2-Emerging ability response might look like this:
    1. I wonder how the student would have solved this problem using paper and a pencil. I would do this to confirm student's conceptual understanding of division.
    2. I would have asked the student the following question: "What if the cost of each book was $4 instead of $5?". The student's response to this question would help me confirm their conceptual understanding of division (Whether she can transfer what she knows with the current problem to a slightly different scenario). Lastly, I would have told the students the original problem with number 22. My rationale for this is I want to see if she can do division problems with or without remainders.