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Quantum Imaging of High-Dimensional Hilbert Spaces with Radon Transform

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Abstract: We introduce a post-processing method for quantum imaging that is based on the Radon transform. We show that the entropic separability bound is violated considerably more strongly in comparison to the standard setting.
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1. Introduction

High-dimensional Hilbert spaces possess large information encoding and transmission capabilities. Characterizing exactly the real potential of high-dimensional entangled systems is a cornerstone of tomography and quantum imaging. The accuracy of the measurement apparatus and devices used in quantum imaging is physically limited, which allows no further improvements to be made. To extend the possibilities, we introduce a post-processing method for quantum imaging that is based on the Radon transform and the projection-slice theorem. The proposed solution leads to an enhanced precision and a deeper parameterization of the information conveying capabilities of high-dimensional Hilbert spaces. We demonstrate the method for the analysis of high-dimensional position-momentum photonic entanglement. We show that the entropic separability bound in terms of standard deviations is violated considerably more strongly in comparison to the standard setting and current data processing. The results indicate that the possibilities of the quantum imaging of high-dimensional Hilbert spaces can be extended by applying appropriate calculations in the post-processing phase.

The Radon transform is a useful tool in medical imaging and particularly in the processes of medical tomography. This transform consists of the integral transform of several pieces of an unknown function (e.g., a physical object), from which the unknown density can be recovered by the inverse Fourier transform. A well-known medical application of Radon transform is X-raying, where several parallel lines (rays) each from a different angle convey information about an unknown internal density function, and each ray captures and characterizes a different piece of the unknown target. The aim of Radon transform in these traditional applications is to collect together these information slices, and then to apply an appropriate inverse transformation that is able to recover the unknown internal function from the gathered slices. In our quantum imaging scenario we explicitly do the same thing to reach several advantageous features. However, instead of physically emitted rays and spatial rotations (such as is the case in X-raying), our model is interpreted by “abstracted” lines in the high-dimensional Hilbert space, whose “lines” are defined by the coincidence measurements and convey information about the position and momentum components of the analyzed high-dimensional quantum system. Similarly, the rotation does not mean a physical rotation in the spatial space, but a unitary transformation in the phase.

2. Measurement setup

The schematic view of the measurement setup for the Radon transform-based high-dimensional quantum imaging is summarized in Fig. 1. The source is assumed to be a collimated laser beam that has undergone a spontaneous parametric down-conversion (SPDC) at a nonlinear crystal. The outputs of the BS are sent to micro-mirror devices, at the Fourier plane and the image plane. The unitary phase rotation of ϕ is implemented by a phase modulator (PM) in the image plane path. Other supplementary devices (focusing lens, quarter wave plates, polarizing beam splitters) of the experimental setting are not depicted in the figure and are not part of our discussion. The detectors are characterized by their dimension (measurement space), d, and the capacity of the quantum system is measured in $\mathrm{bits}/\mathrm{photon}$, which is quantified by the joint detection events in the coincidence measurement. (*Note*: A general setup [1,6] contains no PM, i.e., $\phi = 0$, and this kind of setup is referred to as the *standard model* throughout.)

In the post-processing phase only the coincident photon detections are taken into account to derive the mutual information. The output of the nonlinear crystal is fed into a beamsplitter (BS). The outputs of the BS are measured in the Fourier plane and in the image plane. The image plane path also contains a phase modulator (PM) for the unitary phase rotation. The measurements are taken for $0 \le \phi < \pi$.

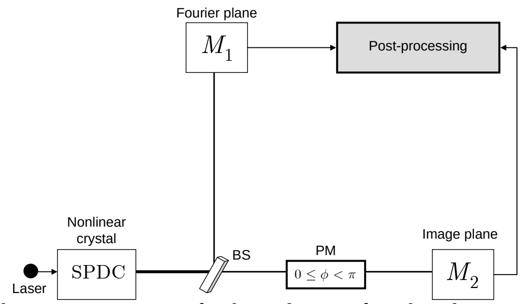

Figure 1. The measurement setup for the Radon transform-based quantum imaging.

3. Results

In the numerical analysis we use the system parameterization of [1,6], i.e., the position-momentum entanglement is characterized as follows. The input laser source has a wavelength of 325 nm, and $\sigma_s = 1500\ \mu m$ and $\sigma_C = 40\ \mu m$. Based on these parameters, the optimal mutual information function $I_0(A:B)$ of the standard model in the position basis at $\phi = 0$ can be exactly evaluated by the joint detection events at $d \to \infty$ as $I_0(A:B) \approx 10\ \text{bits}/\text{photon}$. In the Radon transform the optimum is different; $I_{\mathcal{R}}(A:B) \approx 13\ \text{bits}/\text{photon}$, for the range of $0 \le \phi < \pi$. Hence, the optimal amount of the extractable mutual information can be increased in the asymptotic limit of the measurement space. The same connections hold for the momentum basis. Using the position basis, these quantities are first depicted in Fig. 2(a) for a fixed dimension, $d = 900$. In Fig. 2(b), the quantities are depicted as a function of the dimension, $0 \le d \le 1000$.

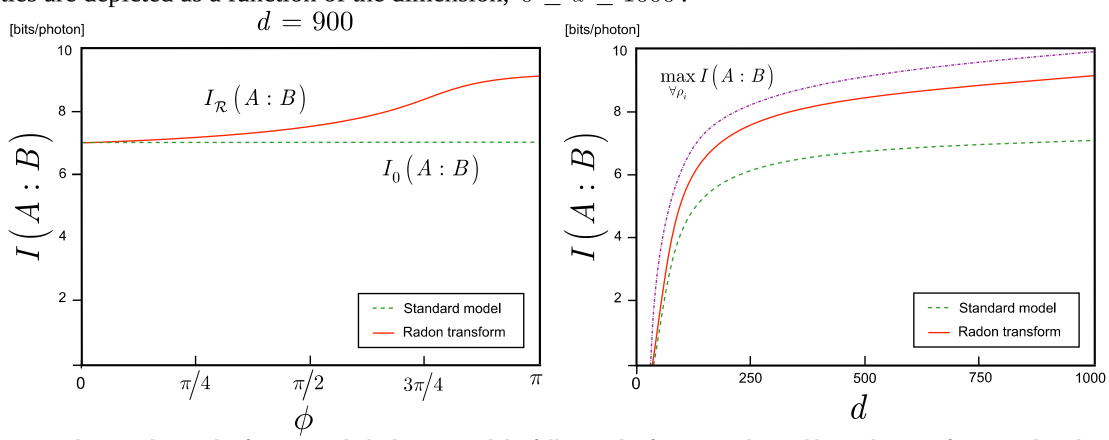

Figure 2. (a): The partial mutual information (dashed green) and the full mutual information obtained by Radon transform (single red) at a fixed dimension. (b): The mutual information of the standard model and the Radon transform as a function of the dimension. The theoretical maximum, $\log_2(d)$, is depicted by the dash-dotted line.

The results show that the Radon transform-based model offers higher extractable mutual information at an arbitrary dimension. The quantity $I_{\mathcal{R}}(A:B)$ of the Radon transform approximates more precisely the theoretical upper bound $\log_2(d)$ than the mutual information of the standard model [6].

In this work, we introduced a Radon transform-based post-processing for quantum imaging and quantum tomography, which uses the raw data of the coincidence measurements to enhance the accuracy of the study of information transmission capabilities of high-dimensional position-momentum entangled quantum states. The proposed post-processing phase provides several benefits for us to get a sharper and considerably deeper picture from the internal life of high-dimensional Hilbert spaces.

4. References

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