

Original Article



# Remarks on the Logic of Irreversible Actions

KOME – An International Journal of Pure  
Communication Inquiry  
Volume 2 Issue 1 p. 37-68  
© The Author(s) 2014  
Reprints and Permission:  
[kome@komejournal.com](mailto:kome@komejournal.com)  
Published by the Hungarian Communication  
Studies Association

## Márton Demeter

Károli Gáspár University of the Reformed Church, Institute of Liberal Arts, Hungary

**Abstract:** This paper proposes a logical system in which reversible and irreversible processes could be uniformly handled. This attempt originates in the observation that the question of reversibility not essentially emerges in logic, nor yet in temporal logic in spite of the fact that, in principle, it should be an eminent question as regards any action (where time is the vehiculum and presupposition of change). The situation is quite similar with communication theories, where the expression 'communication' usually refers to an individual process or action in spite of the fact that any process presupposes a given, timeless type for this process. This timeless or time-indifferent types are the prototypes of pure communication.

**Keywords:** reversibility, irreversibility, tense logic, change, action, pure communication

## Philosophical Propedeutics as regards the Idea of Separate Change

The early Wittgenstein wrote in his *Tractatus* that *«Eines kann der Fall sein oder nicht der Fall sein und alles übrige gleich bleiben»*<sup>1</sup> (Wittgenstein 1921; 1.21.) and, I think, the consequence of this statement is a real philosophical mess-up. Let's call this view the Idea of Separate Change (ISC). In this paper I try to show that ISC is an idea as regards our *models* of the world, but there can be no separate changes in the world itself.

---

<sup>1</sup> Translated by D. F. Pears and B. F. McGuinness as „Each item can be the case or not the case while everything else remains the same” (Wittgenstein 2001)

*Address for Correspondence:* Marton Demeter, email: [demeter\[at\]komejournal.com](mailto:demeter[at]komejournal.com)

Article *received* on the 06th March, 2014. Article *accepted* on the 17th April, 2014

*Conflict of Interest:* The author is the co-EIC of KOME.

Let's suppose, for example, that there is an ISC event, namely,  $A\langle aTb\rangle$ , which is an event member of the world  $W_1$ , where  $a$  and  $b$  are individual states of affairs and  $a$  transforms to  $b$  while no other transformations happens in  $W_1$ . Similarly, an event  $B\langle bTa\rangle \in W_2$  could be postulated. Now suppose the conjunction of transformations in A and B as (1) shows:

$$(1) \quad W_1\langle A\langle aTb\rangle\rangle T W_2\langle B\langle bTa\rangle\rangle$$

Now we obtain from (1) by condensation that

$$(2) \quad W_1\langle aTb\rangle T W_2\langle bTa\rangle$$

and then

$$(3) \quad \langle W_1a\rangle T \langle W_2a\rangle$$

But from the definition of ISC it follows that no distinction could be made between  $W_1$  and  $W_2$ : the succession of the transformations A and B results in no transformation, which is absurd. The plain fact that  $W_1$  and  $W_2$  could be indexed as different worlds entails the presupposition that there are at least one difference between them apart from  $a$ . Moreover, its counterintuitive enough to say that successive transformations could be identical with no transformations. Therefore I think that ISC applies to notations, and not to the events of the world.

To buttres up this argumentation many examples could be cited; if the window is open and I close it, and then I open the window again, who could say that I did nothing? Of course I used energy, wasted time and so on. But if a model of the possible states of the window describes the opened-window-state as A, the closed-window-state as B, it could be said that A could be transformed to B and then it could be transformed to A: then this is a reductive notation of the actual processes with identical input and output notation. Similarly, a pure dialogue (with the possibility of any discussion) is the type for every possible (individual) dialogues.

### Handlig events and notations

The relation between events and notations is not self-evident so it has to be analyzed itself. It's trivial that notations are notations of *something*, say events, states, processes and others, so it seems plausible to say that the relation between events and notations is simply that notations are representations of events. The fact that notations are events themselves is far less evident and this recognition leads to infinite regress unless the well known restrictions for object language sentences (notations). Suppose that A is an event and B is the notation of A; but since notations are events, too, B should be represented as C, and than C as D and so on. We can say, of course, that B could be represent both the event B and the notation of this event but in this case events and notations became indistinguishable. Escaping from this situation, logicians started to demarcate object language sentences from meta language sentences, but this effort proved to be unaccomplishable without the aim of further notations, as in the case of (3).

(3) It's raining.

(3b) 'It's raining.'

It could be seen that, as (3b) shows, single quotes notate the notation of the event (3). Notating (3b) requires additional quotation marks and so on, and the problem of infinite regress remains. Obviating this situation logicians started to use the expression 'notation' for its result only, which, at least at first sight, did not seem to be an event. But as a matter of fact, not just the noting, but the reading of a notation is also an event *per se*, since both noting and writing happens in time, and none of them are irreversible. Note, that any communication presupposes a notation, so what holds for 'notation' also holds for 'communication'.

### Basal considerations as regards ISC

Based on the above discussed questions we should ascertain some basal considerations as regards time, change and notations.

- (I) Let the fact that ISC applies to notations and not to the events of the world be our first consideration.

But since noting or reading (so: communicating) a notation is an event too, and notations are in the world, we have to say that, strictly speaking, ISC applies only to the *idea* of notations, and not to the notations themselves.

- (II) Let the fact that ISC applies only to transcendental objects be our second consideration.

But it's hard to say that logic is evidently transcendental. We should skip the ontological questions of mathematics and logics here, and we could still say that whether logic is transcendental or ISC not hold for logic itself.

### Remarks on the concept of notations as events

Of course not Wittgenstein nor later logicians thought that logical representations of any structures refer to events, so the distinction between notations as events and notations referring to events should be made. For example, noting or reading a tautology is always an event, but no tautology refers to any event. In spite of the fact that the ordinary language transcriptions of logical connectives may contain event-words like 'if', 'and' or 'then', there is nothing event-characteristic in their logical equivalents. The above mentioned characteristic of tautologies could be easily apprehended by examining the following formulas.

$$(4) \quad A \equiv B$$

The literal translation of (4) is that A is identical with B.

$$(4a) \quad A \equiv B \supset B \equiv A$$

The literal translation of (4a) is that if A is identical with B then B is identical with A.

$$(4b) \quad A \equiv B \supset \langle A \supset B \rangle \wedge \langle B \supset A \rangle$$

The literal translation of (4b) is that if A is identical with B then if A then B and if B then A.

$$(4c) \quad \langle\langle A \supset B \rangle \wedge \langle B \supset C \rangle\rangle \supset \langle A \supset C \rangle$$

The literal translation of (4c) is that if A then B and if B then C then A then C.

But, unlike ordinary language sentences, logical formulas could be conceived as diagrams. When a formula is conceived like this, the reading of the formulae could be undirected. The following examples are diagrammatic interpretations of (4) – (4c).

$$(4) \quad A \equiv B$$

The undirected diagrammatic interpretations of (4) could be either

that A is identical with B.  
or  
that B is identical with A.

$$(4a) \quad A \equiv B \supset B \equiv A$$

The undirected diagrammatic interpretations of (4a) could be either

that if A is identical with B then B is identical with A.  
or  
that if B is identical with A then A is identical with B.  
or  
that if A is identical with B then A is identical with B.  
or  
that if B is identical with A then B is identical with A.

$$(4b) \quad A \equiv B \supset \langle A \supset B \rangle \wedge \langle B \supset A \rangle$$

The undirected diagrammatic interpretations of (4b) could be either

that if A is identical with B then if A then B and if B then A.  
or  
that if B is identical with A then if A then B and if B then A.  
or  
that if A is identical with B then if B then A and if A then B.  
etc.

$$(4c) \quad \langle\langle A \supset B \rangle \wedge \langle B \supset C \rangle\rangle \supset \langle A \supset C \rangle$$

The undirected diagrammatic interpretations of (4c) could be either

that if A then B and if B then C then if A then C.  
or  
that if B then C and if A then B then if A then C  
or  
that if A then C then it is impossible that if A then B then if C then not-B

Construing undirected diagrammatic interpretations, of course, presupposes precursory knowledge on logical languages, but the situation is the very same with ordinary and conventionally interpreted logical languages, and, as it could be seen by (5) –(5b), with arithmetic itself.

$$(5) \quad 1 = 1$$

$$(5a) \quad 1 + 1 = 2$$

$$(5b) \quad 1 + 1 + 1 + 1 = 2 + 2 = 4$$

Interpreters have to know that addition and equality are reversible and they have to be adept in, at least, some naive number theory (NNT<sup>2</sup>). Then undirected diagrammatic interpretations of the above run as follows.

$$(5) \quad 1 = 1$$

The undirected diagrammatic interpretations of (5) could be either

that 1 equals 1  
or  
that equal-sign connects equals

so (5) could be interpreted as the definition of 1 and the definition of equal-sign alike.

$$(5a) \quad 1 + 1 = 2$$

The undirected diagrammatic interpretations of (5a) could be either

that 1 plus 1 equals 2  
or  
that 2 equals 2  
or  
that 1 plus 1 equals 1 plus 1  
or  
that 2 minus 1 equals 1

$$(5b) \quad 1 + 1 + 1 + 1 = 2 + 2 = 4$$

The undirected diagrammatic interpretations of (5b) could be either

that 1 plus 1 plus 1 plus 1 equals 2 plus 2 equals 4  
or  
that 4 equals 4 equals 4  
or  
that that 1 plus 1 plus 1 plus 1 equals that 1 plus 1 plus 1 plus 1 equals that 1 plus 1 plus 1 plus 1  
etc.

---

<sup>2</sup> Let NNT refer here to adequate knowledge on relational properties of basal arithmetical operations and ordinary knowledge on natural numbers.

Interpreting arithmetical formulas in this diagrammatic way presupposes the following assumptions only:

- (III) Let the fact that the interpreters of arithmetical formulas have to be acquainted with NNT be our third consideration.
- (IV) Let the fact that any formula includes itself, and any concept A includes the concept of A be our fourth consideration.
- (V) Let the fact that the concept of natural numbers includes the concept of any natural number with all the operations on them be our fifth consideration.

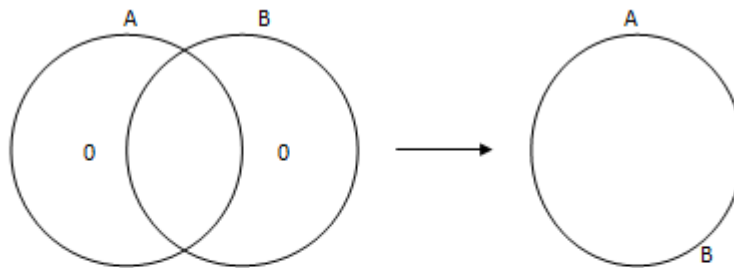
Based on the considerations (III) – (V) it has to be said, that any natural number and any valid operation on them asserts the concept of natural numbers. In the same way, any valid structure of logical symbols asserts the concept of the corresponding logical system.

- (VI) Let the fact that any valid logical formulae is a tautology which asserts the concept of the corresponding logical system be our sixth consideration.

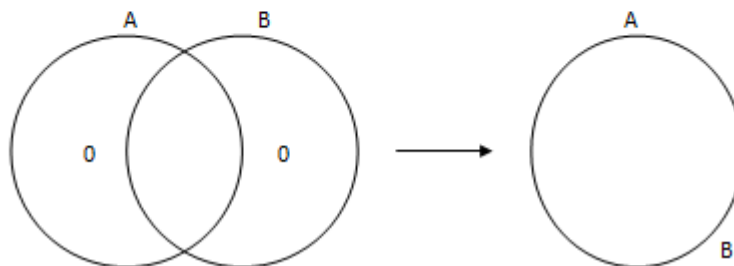
### Diagrammatic representations of logical concepts

At this point we could represent the above mentioned tautological structures diagrammatically. The most perspicuous, contemporary way of representing logical structures is the Venn-II system of Sun-Joo Shin (Sun-Joo Shin, 1994, 2006). The following representations harmonize with her system but a few differences will be introduced and elucidated.

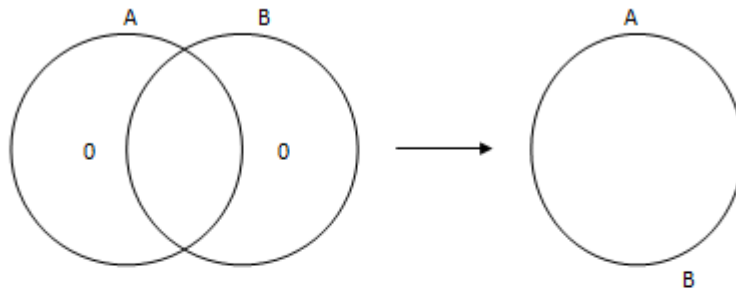
(4)  $A \equiv B$



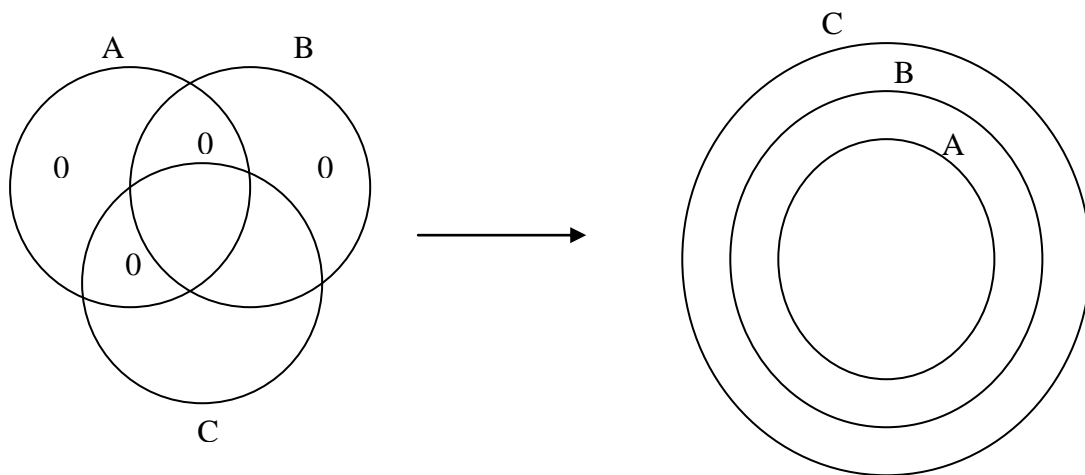
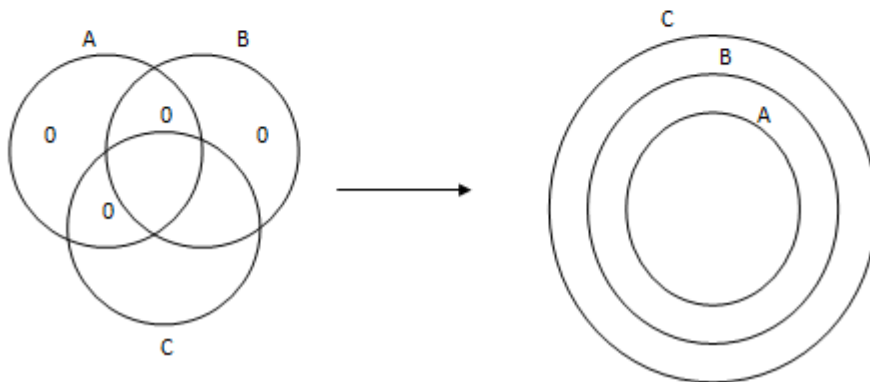
(4a)  $A \equiv B \supset B \equiv A$



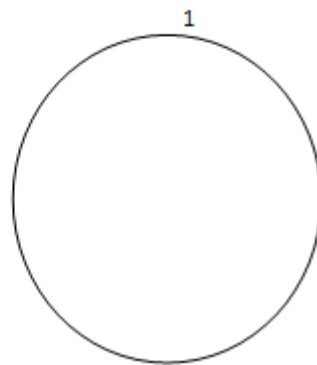
(4b)  $A \equiv B \supset \langle A \supset B \rangle \wedge \langle B \supset A \rangle$



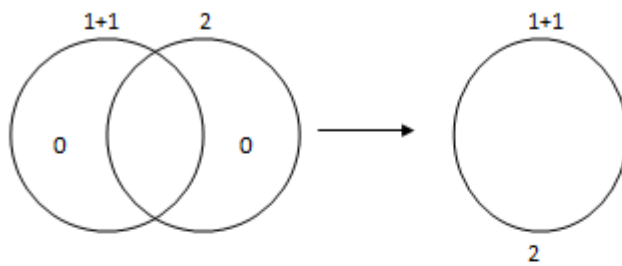
(4c)  $\langle \langle A \supset B \rangle \wedge \langle B \supset C \rangle \rangle \supset \langle A \supset C \rangle$



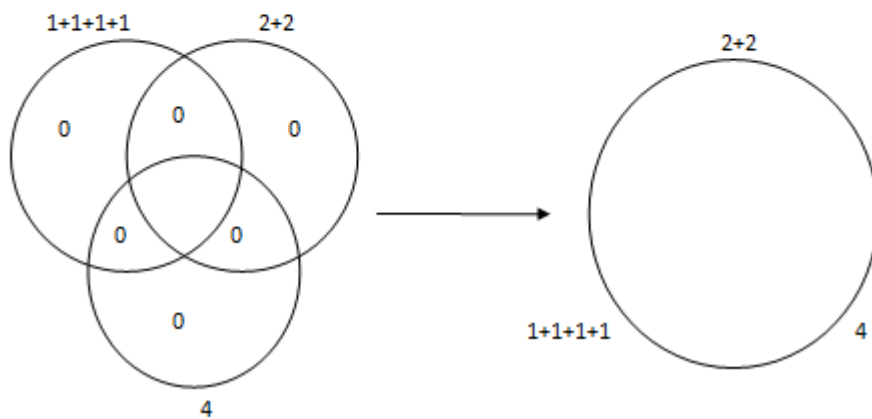
(5)  $1 = 1$



(5a)  $1 + 1 = 2$



(5b)  $1 + 1 + 1 + 1 = 2 + 2 = 4$





We should interpose some axioms as regards the representations stand above.

- Axiom 1.* Each closed curve represents a set.  
*Axiom 2.* Any closed intersection of sets is a set.  
*Axiom 3.* When a set C is an intersection of sets A and B, then A and B are the parent sets of C.  
*Axiom 4.* Each set should be labelled.  
*Axiom 5.* Labels should be written outside the set.  
*Axiom 6.* Intersections inherit all the labels of their parent sets.  
*Axiom 7.* The label of a set is its name.  
*Axiom 8.* When a set has multiple names, then multiple names signify the same set.  
*Axiom 9.* Any set includes all its members, so a closed curve without any symbol signifies all its members.  
*Axiom 10.* A closed curve with the symbol '0' signifies the empty set.  
*Axiom 11.* Empty sets signifies the same set without respect to their names.  
*Axiom 12.* Empty sets could be deleted without changing the reference of the corresponding representation scheme.

As it could be seen, this representational method can handle the problem of extensional identity that could be stated in spite of intensional differences. Its opposite, viz. the intensional identity despite extensional differences should not be occurred in formal languages.

### **Some additional remarks on ISC**

As we could now delineate, diagrammatic representation systems show that logical expressions need not, and could not refer reversible or irreversible processes or actions, because they do not refer to actions at all. Instead, logical expressions refer to states, which are the preconditions of processes and actions. As against physical and mental phenomena, where the observer encounters with processes alone, the logical sphere consists of states.

(VI) Let the fact that logical expressions (tautologies) are the types of processes be our sixth consideration.

That being the case, we have to differentiate logical types from physical or mental exemplars: the first refer to states, and the second, being temporal derivatives of the first, refer to processes. For example, opening or closing a window is always a physical process; similarly, the intention to close or open a window is always a mental process. But the logical precondition of all the above mentioned processes is a state, i.e. that the core definition of being a window includes that it could be opened or closed. None but this differentiation can lead us to the correct questions as regards reversibility: we should consider that the questions pertain to the exemplars of logical states, namely, physical and mental processes and not to logical expressions themselves.

## Representing types and exemplars: states become processes

In the recent paper we will represent types as hypergraphs, and exemplars will be represented as directed graphs derived from the corresponding type.<sup>3</sup> Note that hypergraph-based representational systems describe *concrete* (non-discrete or analogue) tense logic, while representation systems operating on directed graphs describe *discrete* tense logic. This remark should be considered later, when non-logical expressions like 'next', 'until' or 'now' will be analyzed. Since there are no temporal expressions apart from 'always' and 'at least once' that could be handled as logical expressions<sup>4</sup>, the above mentioned terms belong to physical and mental languages: consequently, they will be represented as directed graphs.<sup>5</sup>

Before constructing a graph system based on the above mentioned considerations it should be mentioned that, contrary to the usual graph constructions which derive hypergraphs from normal graphs, the basis of our recent system will be the hypergraphs themselves, and normal graphs will be the derivatives. This sequentiality corresponds to the consideration that exemplars (processes and actions) always derive from possible states i.e., from types. Let's call this type-exemplar graph system TEGS hereunder. As it could be conjectured, a TEGS consists of types and exemplars where types are the basal entities. A simple representative of TEGS could be constructed as follows.

*Definition 1.* A  $G = (V,E)$  graph consists of the sets  $V$  and  $E$ ; the elements of  $V$  are the  $v \in V$  *vertices*, the elements of  $E$  are the  $e \in E$  *edges*.

*Definition 2.* For every edge  $e$ , there is a set  $C$  contains  $c$  vertices which are the *end vertices* of  $e$ .

*Definition 3.* If  $v$  is the end vertex of  $e$ , then  $v$  *suits*  $v$  and  $e$  *suits*  $v$ .

*Definition 4.* Let's call the  $n$  number of edges between vertices *multiplicity*.

*Definition 5.* Let's call the  $c$  number of vertices suits  $e$  the *cardinality* of  $e$ .

*Definition 6.* Let's call the  $k$  number of edges suits  $v$  the *degree* of  $v$  [ $\varphi(v)$ ].

*Definition 7.* If  $e$  suits  $n$  vertices and  $n = 1$ , then  $e$  is a *knot*; if  $e > 1$  then  $e$  is a *proper edge*.

*Definition 8.* The set of minimum 2 proper edges or knots suit the same end vertices is a *multiedge*.

*Definition 9.* A graph wherein  $c$ ,  $k$  and  $n$  are unspecified is a *hypergraph*.

*Definition 10.* A *simple graph* could be derived from a hypergraph, wherein  $c \leq 2$  and  $k$ ,  $n$  are unspecified.

*Definition 11.* The alternate sequence of edges and vertices in a  $G$  graph is the  $W$  *walk*.

$$W = \{v_0, e_1, v_1, e_2, \dots, e_n, v_n\}$$

*Definition 12.* If there is a walk between  $v_1$  and  $v_2$  then  $v_1$  and  $v_2$  are *accessible* vertices.

*Definition 13.* A graph is *connected* when all its vertices are accessible from each other.

*Definition 14.* Both edges and vertices could be *labeled*. Labels are represented with colours. Edges and vertices with the same colour belong to the same *label class* (LC).

<sup>3</sup> In this paper we treat types and exemplars from a basal logical point of view. In the case of our diagrammatic system, types are labelled with capital letters (A), and exemplars are labelled with lower cases (a). We will later specify the logical properties of types and exemplars, but for the sake of an anticipatory understanding one could correspond them with the sentential calculus; then exemplars correspond to individual propositions (p,q), and types correspond to general propositions ( $\varphi, \phi$ ).

<sup>4</sup> Where 'always' is the temporal equivalent of the universal, and 'at least once' is the temporal equivalent of the existential quantifier.

<sup>5</sup> According to Tarski, no logical expressions may refer to temporality (future, past) at all.

Consider a representative of TEGS on Fig1.

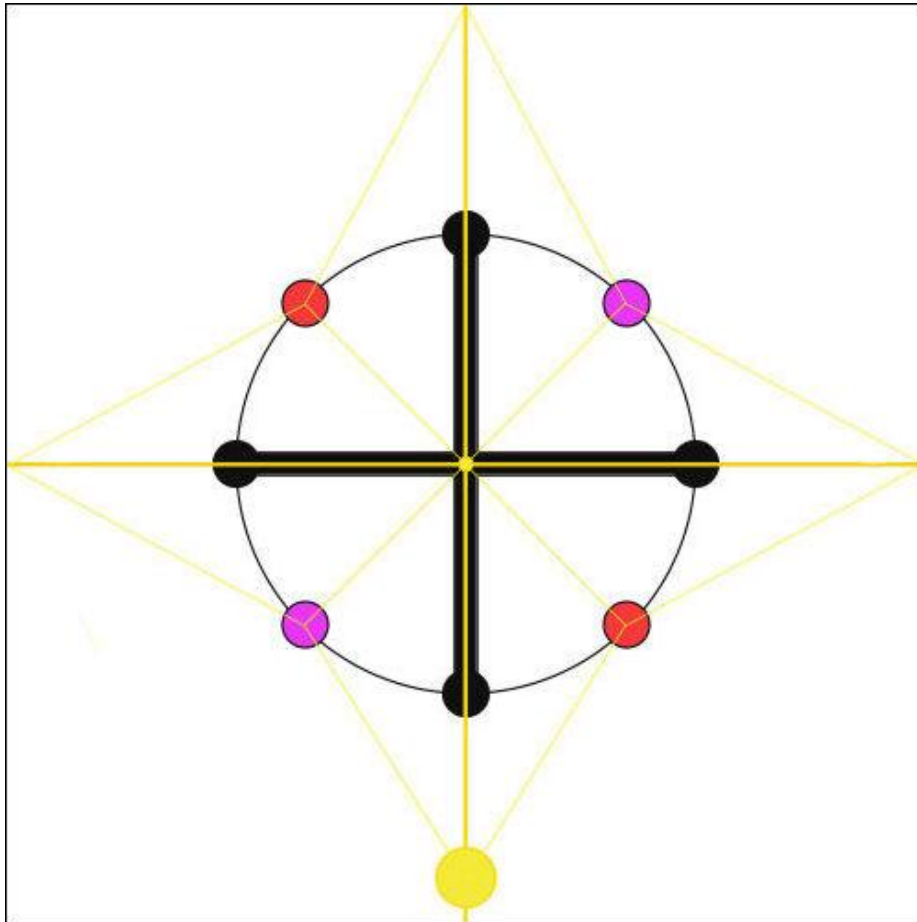


Fig 1. A representative of TEGS . Different colours represent different LCs.

The TEGS represented by Fig 1. is a connected simple graph with hypergraphs as its vertices. All its edges and vertices are labelled. Nevertheless this paper will be concerned with graphs with only one LC, because it focuses on tense logical problems where different LCs are not needed.

*Definition 15.* The vertices and edges belong to the same LC are the *fragments* of that LC ( $\uparrow$ LC)

*Definition 16.* If fragments  $x,y$  have an edge that  $e: (x,y)$  and  $x,y \ni LC$ , than  $x$  and  $y$  are accessible from each other *simply*.

*Definition 17.* If fragments  $x,y$  aren't accessible from each other simply, but they have an edge that  $e: (x,y)$  and  $x \ni LC \wedge \neg y \ni LC$  or  $y \ni LC \wedge \neg x \ni LC$  than  $e$  is a bilateral edge, and  $x,y$  are accessible from each other *synthetically*.

By definition 17. it follows that fragments of the same LC are accessible from each other simply, fragments of different Lcs are accessible from each other synthetically. Different LCs are represented by different colours on TEGS and they could be indexed as  $LC_1, LC_2$  and so on. Any representative of TEGS could be constructed as a complex of LC(s) so a TEGS is a recursively constructed class of hypergraphs, which are the basis graphs of TEGS.

To represent exemplars, types need to be dissolved as follows.

*Definition 18.* The  $P_2$  structure of an  $H: (V,E)$  hypergraph induces a 2-vertices walk on  $H$ ; now  $P_2H$  shows the  $v \ni V$  vertices induces the fitting  $e \ni E$  edge.

*Definition 19.* Dissolving an  $H: (V,E)$  hypergraph results in the  $P_2H \subset H$ .

*Definition 20.* The  $R: \{V=( \dots, v_0, v_1, v_2, \dots), E=( \dots, v_0v_1, v_1v_2, v_2v_3, \dots)\}$  subgraph of the  $H: (V,E)$  hypergraph is its *radius*.

Now states and processes could be represented by TEGS as follows. The first relation is the most simple as it represent that any part of the LC could be access from its basal LC (fig2). Note that, as *Definition 20.* states,  $R$  is a subgraph of the dissolved hypergraph labelled with the corresponding LC. In our undermentioned examples LCs will be represented with colours (black, red etc.).

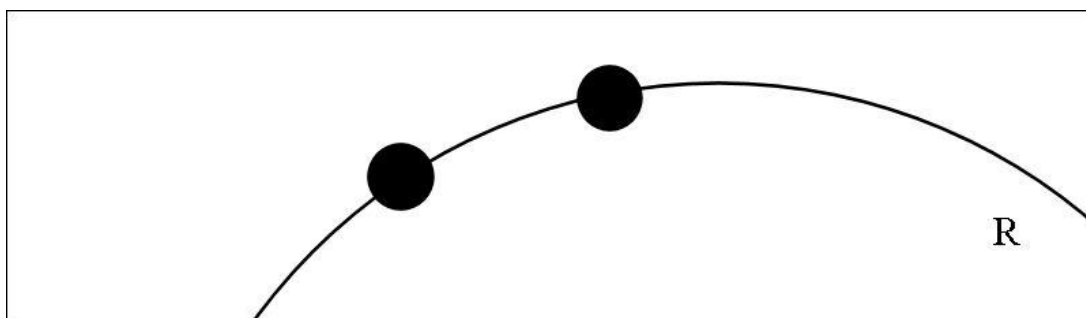


Fig 2. Any part of an LC could be simply access from the LC.

The relation that Fig 2. shows is the diagrammatic equivalent of inclusion. For a conceptual example, suppose that an LC carries the label 'Number'. Now any part of that LC, say, 'rational number' carries the label 'Number' at the same time. Without direction, Fig 2. could represent the state that, for example, A and B are the parts of the  $R \langle LC \rangle$ . For an everyday language example that had been formerly quoted, suppose that the LC represented with black on Fig 2. carries the label „Window”. Then Fig 2. could represent the state of being an opened window and the state of being a closed window are parts of being a window. It means, conceptually, that the concept of window includes the possibilities of being opened and being closed: these possibilities are determined by the basal concept.

The second relation on Fig 3. shows that any fragment of an LC could be simply access from the LC.

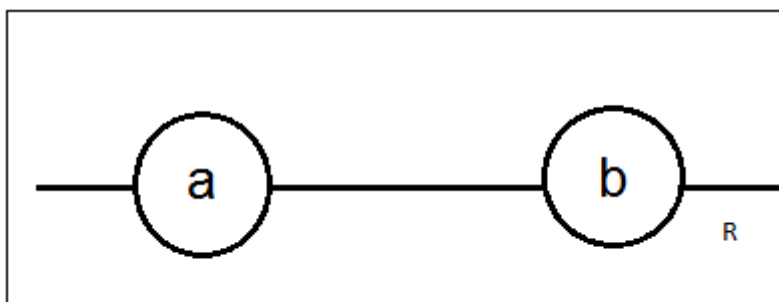


Fig 3. Any fragment of an LC could be simply access from that LC.

As it could be seen, on Fig 3. the fragments of the LC are named (as A and B), so the graph shows the relations between  $LC : A : B$ . But naming fragments is not necessary: without additional marks, a blank closed curve means 'a fragment of that LC include it'. Some logicians use the symbol 'O' representing the empty set, viz. types without fragments but this

representational tradition could not be harmonized with our conception of graphical logic from causes we could only touch here.

In the case of diagrammatic logic, iconicity is a very important factor, so the so-called negative fact should not be represented, if it is possible. For example, to represent the fact that A is an empty set, we have better ignore the representation of the set A instead of labeling a set with A, and then mark it with a 'O'. This could be considered as the extensional criteria for TEGS. An intensional criteria should be formulated as well, because one could state that even if a set hasn't got any elements, we should represent the set itself as it could be labelled lucidly. For example, the set of round squares are empty, but the label 'round squares' could be considered intelligible (even if I think that it is not the case). As regards diagrammatic representation, the situation is very simple here because only that fact has to be represented that the set of round things and the set of squares are disjuncts. A slightly more difficult problem may emerge with labels like 'centaur' in spite of the fact that many logicians thought that they should be analysed in the same way that round squares: centaurs should be represented as an empty intersection of the sets 'man' and 'horse'. I think that an analysis of that kind may violate our conception of normal language usage, because 'centaur' does not seem like a complex expression at all. So I propose that empty sets, which are not to be represented as intersections could be represented as types without exemplars. In this case, the representation refers to the existence of a label, for example, it could show that we have the conception of centaurs (but we don't actually know exemplars falling into this category). The issues of the above mentioned considerations are shown at Fig 4.

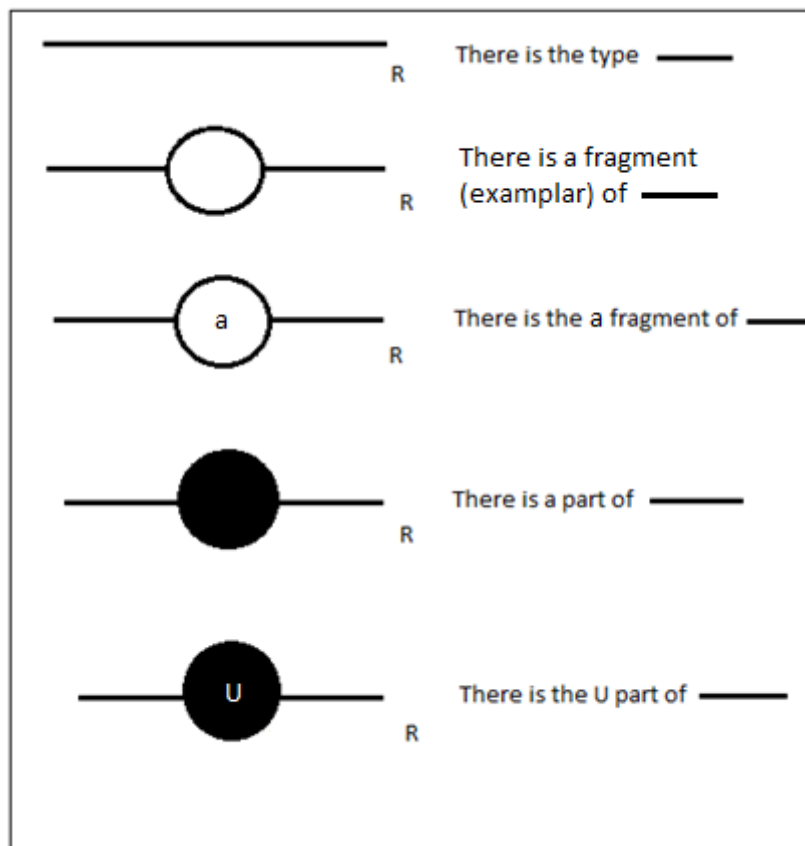


Fig 4.

In addition, Fig 5. shows the relations of universality, uniqueness and definite uniqueness in TEGS.

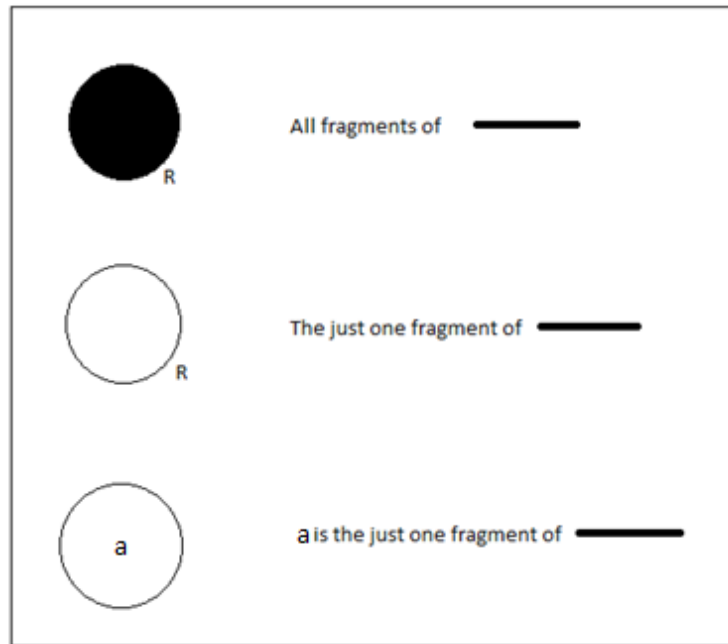


Fig 5.

Now the well known logical relations could be represented by TEGS as Fig 6. shows.

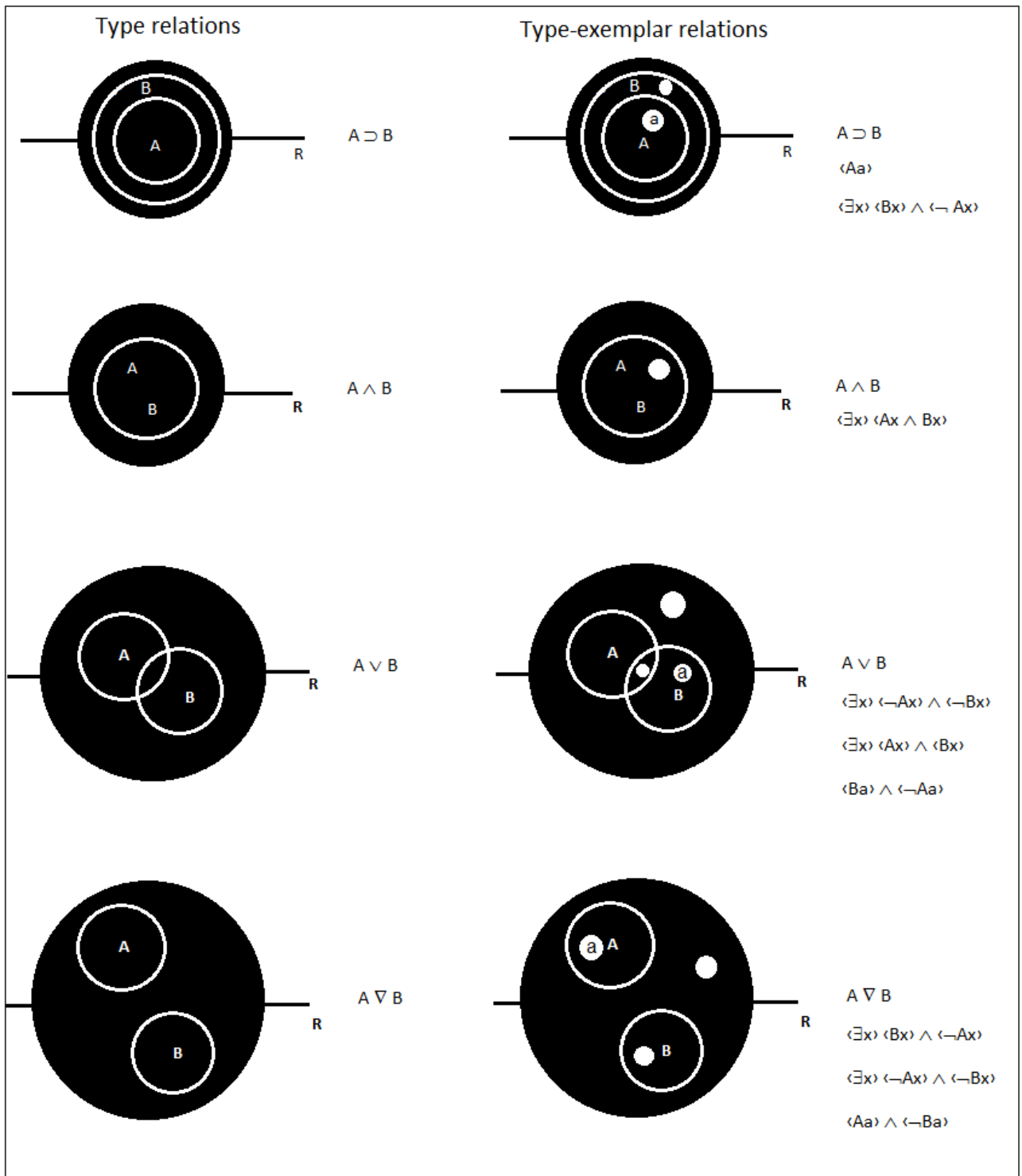


Fig 6.

For a better understanding consider the following examples.

*Example 1.*

Let  $R$  be the part of the LC with the label 'Man'; let „ $A$ ” represent the type 'Male', let „ $B$ ” represent the type 'Female' and let „ $a$ ” represent 'Adam'. Now consider Fig 7.

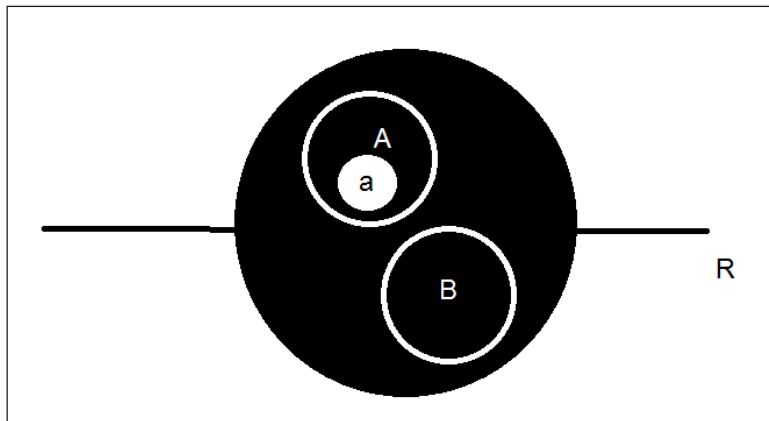


Fig 7.

*Example 2.*

Let  $R$  be the part of the LC with the label 'Window'; let „ $A$ ” represent the type 'Opened thing', let „ $B$ ” represent the type 'Closed thing' and let „ $a$ ” represent a definite description of an individual window at a given time. Now consider Fig 7. again.

Of course there can be logical states where we have to operate on different LCs. Then, as it had been mentioned earlier (see Definition 17.) the accessibility of the constituents is synthetical. But in this paper we won't need the representations of synthetical accessibilities because we will focus on the problem of time and change, and this could be considered as a one-dimensional problem as regards LCs. Of course there are philosophical problems that could not be handled without different LCs, and the most obvious of them are presumably the so-called body-mind problems, or the problem of propositional attitudes, where physical and mental states have to be differentiated by different LCs. It stands to reason that the above mentioned problems could not be discussed in this paper.

As regard our original problem, we earlier said that the questions of states and processes could be handled by representing only one LC, because we only consider here physical changes in time. Now we have the representations of logical relations and we said that they representing logical states, so they could not, and should not represent processes at all, but they are the presuppositions of any process. So hereunder we should deal with processes as they are derivated from states.

It seems obvious, that types are not subjects to change, because they are the bases to any change could be conceived. For example, if  $A$  is the type of a closed window and  $B$  is the type of an opened window,  $A$  could not became  $B$ , but an individual window could fall under  $A$  in  $t_1$ , and fall under  $B$  in  $t_2$ .



*Definition 21.* An  $e: (v_1, v_2)$  edge is *directed*, when its end vertices are ordered as  $v_1 \rightarrow v_2$ . Here  $v_1$  is the *initial verticle* of  $e$ , and  $v_2$  is the *terminal verticle* of  $e$ .

*Definition 22.* An edge with ordered vertices is the  $a: (v_1 \rightarrow v_2)$  *arc*.

*Definition 23.* The  $W = \{v_0, e_1, v_1, e_1, \dots, e_n, v_n\}$  walk is a *directed walk* when it has an arc in its set of edges. Then  $v_0$  is the initial verticle,  $v_n$  is the terminal verticle, and any other vertices in  $W$  are *internal verticles*.

Now directed TEGSs could be drawn with initial, internal and terminal vertices. The most basal interpretation of  $\rightarrow$  is undeniably the temporal interpretation: then  $\rightarrow$  means temporal succession, so  $v_1 \rightarrow v_2$  should be interpreted as 'v<sub>1</sub> precedes v<sub>2</sub>' or 'v<sub>2</sub> follows v<sub>1</sub>'. A more problematic interpretation of  $v_1 \rightarrow v_2$  could be that 'if v<sub>1</sub> then v<sub>2</sub>'. Since the latter interpretation suggests implication, some remarks should have been taken here as regards inclusion and implication.

According to the philosophy of this paper, inclusion (or entailment) is a logical relation between types, and it evokes logical necessity. For example, the type 'Number' includes the type 'Rational number' so it is logically impossible to be a rational number and not to be a number. When  $a$  is a rational number then it must be a number by logical necessity. On the other hand, implication evokes factual necessity, like in the case of physical laws. The fact that if I lose hold of an apple then it will fall down is governed by physical, but not logical laws, so it cannot be represented by inclusion. Finally, the weakest sense of implication could be called hypothetical implication, like in the case of the statement 'If you don't go I'll call the police'.

With regard to the above mentioned differences we denote inclusion (logical consequences) with closed curves as before. Now let's introduce the sign  $\Rightarrow$  for factual implication, so  $v_1 \Rightarrow v_2$  means that 'v<sub>1</sub> causes v<sub>2</sub>'. Finally, let's maintain the sign  $\rightarrow$  for temporal succession.

Now basal temporal questions could be represented starting with the most simple one viz. when a process is a simple change of states without any agent (see fig 8.)

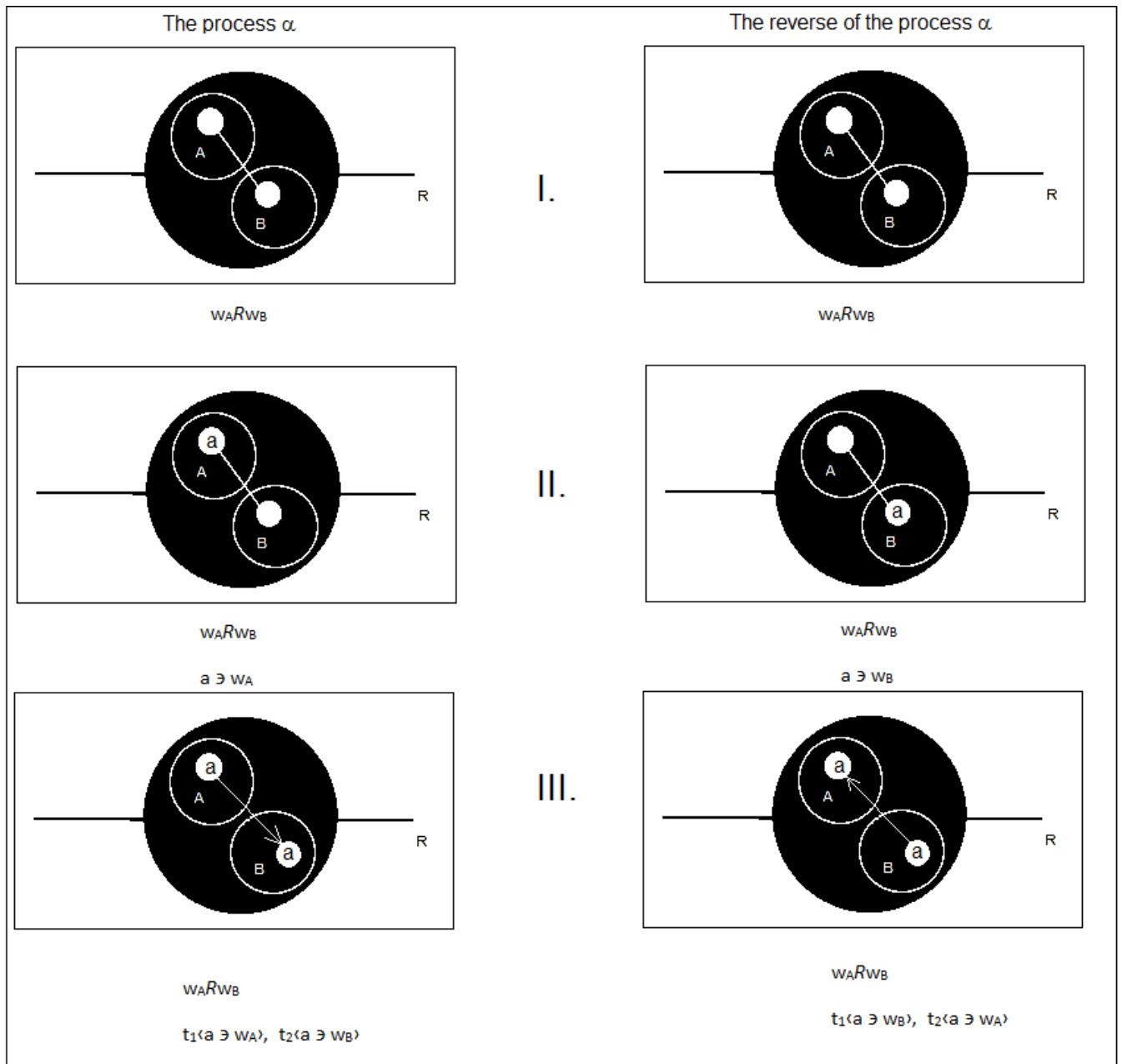


Fig 8.

Fig 8. shows a simple change of states without the indication of an agent and it also shows the reverse process. Figures in the line I. show that the presupposition of both processes is the condition that there are exemplars of type A and type B and they are accessible from each other. Symbolically it could be expressed as accessibility relations between possible worlds  $w_A$  and  $w_B$ . Figures in the line II. show the position of individual  $a$  at  $t_1$ . Finally, figures in the line III. show the process that  $Aa$  becomes  $Ba$ , and its reversal, when  $Ba$  becomes  $Aa$ .

### Processes become actions

Actions are processes with agent(s), who cause(s) change(s) in time. A typical action could be represented by TEGS as figure 9. shows.

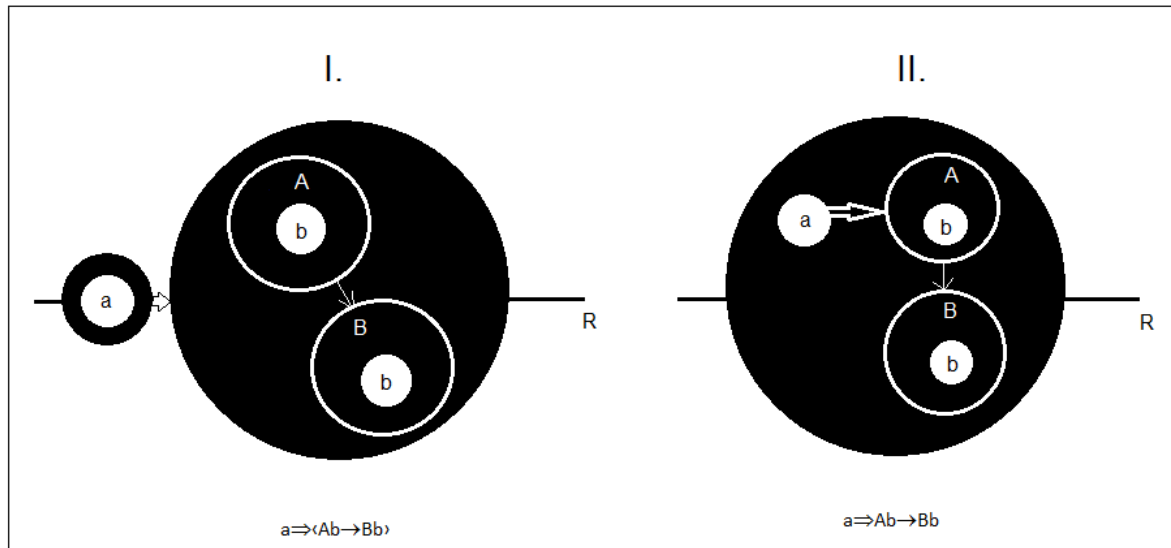


Fig 9. Two ways of representing actions in TEGS

As it could be seen by Fig 9. even elementary actions can lead to alternative explanations. First, as I. shows,  $\langle$ the change of  $b$  from  $A$  to  $B$  by  $a$  $\rangle$  could be conceived as  $a$  causes the process that  $Ab$  becomes  $Bb$ . Second, as II. shows, it could be interpreted as  $a$  causes the state  $Ab$ , and if  $Ab$ , then  $Bb$ . In any way soever, diagramatization could clarify the situation. Note that some logicians suggest constructing separate diagrams for every state (we will refer to this idea as 'separate-diagrams-tradition' or SDT), so we have to defend our position that we use only one diagram for representing processes and actions. For this reason we should mention one another consideration.

(VII) Let the fact that any individual could take up only one (logical) place at a given time be our 7th consideration.

Now it follows from (VII) that, normally, a given exemplar has to itinerate during change, so initial vertices has to be deleted as their contents (the individuals) become terminal vertices. Nevertheless, as Fig 10. shows, taking notice of (VII), our representational method TEGS (I.) and SDT (II.) are in correspondence.

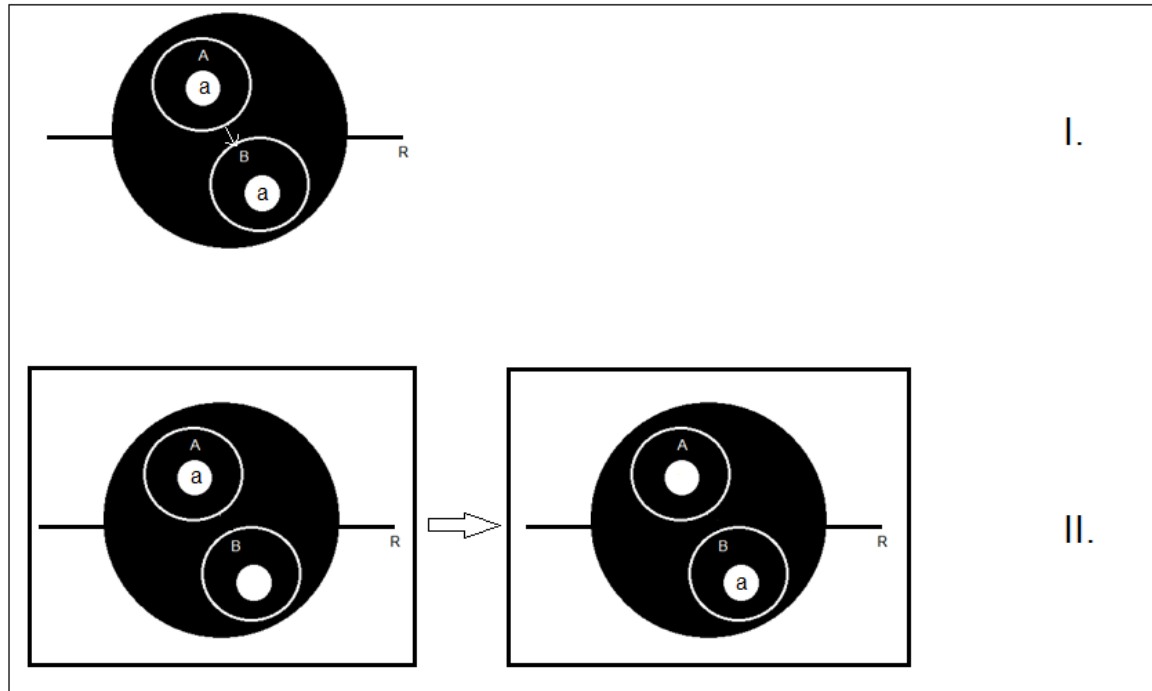


Fig 10. The diagrammatization of change with TEGS (I) and SDT (II).

However, beside simple change of states, there are at least two typical processes, namely *addition* and *abstraction*. In the case of addition, an entity broadens and acquires new properties, while in the case of abstraction an entity narrows and loses properties. Now we can symbolize addition as  $Aa \rightarrow Aa \wedge Ba$ , and abstraction as  $Aa \wedge Ba \rightarrow Aa$ , and Fig 11-12. show their diagrammatization by TEGS and SDT.

Note that (VII) still applies on addition and abstraction because both the process  $\langle Aa \rightarrow Aa \wedge Ba \rangle$  and  $\langle Aa \wedge Ba \rightarrow Aa \rangle$  change the logical position of  $a$ . The illustration of addition and abstraction by traditional Venn-methods (signed with III. on both Fig 11. and Fig 12.) make the situation apparent.

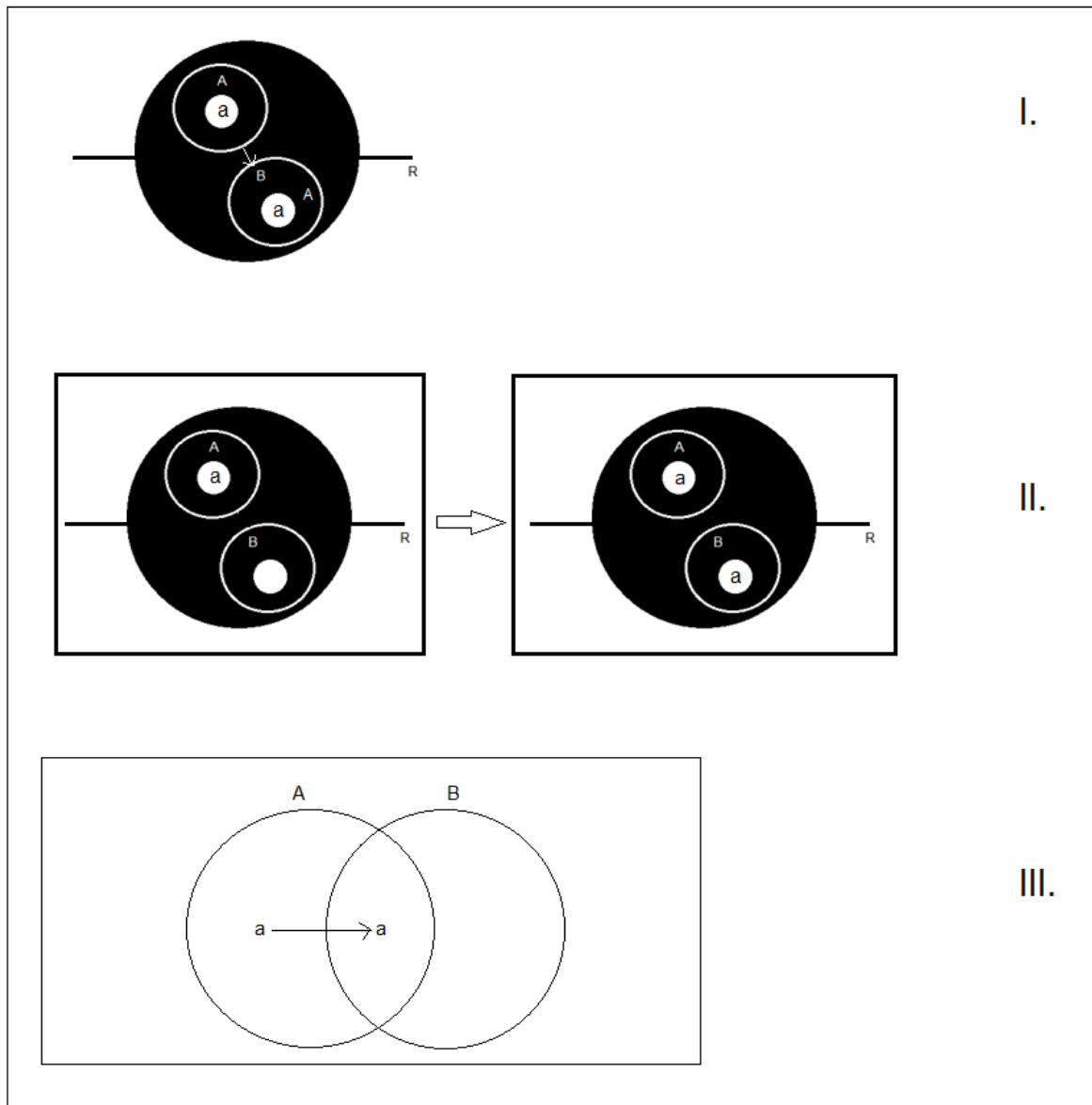


Fig 11. Addition, represented by TEGS (I) SDT (II) and Venn-diagramms (III).

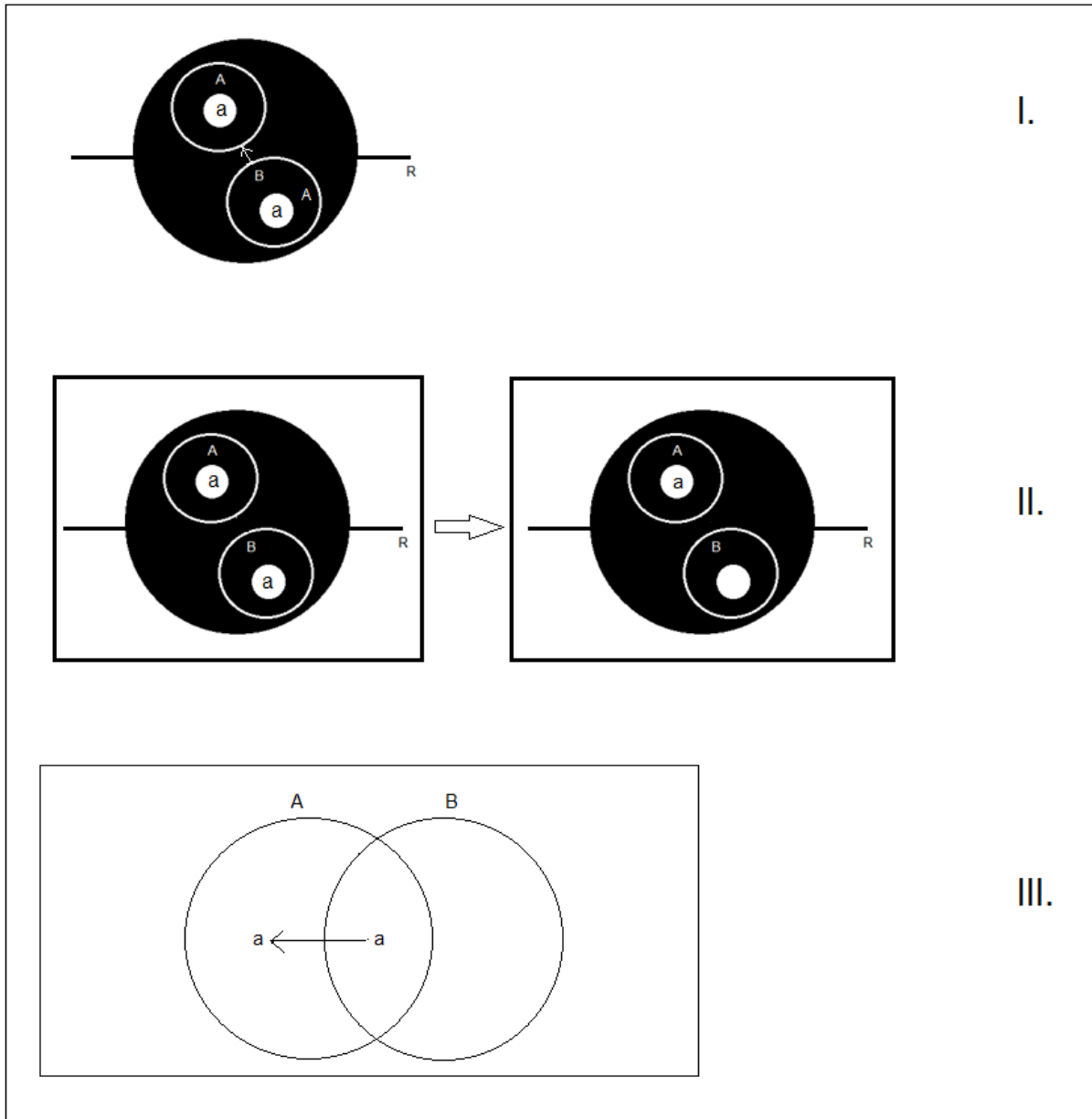


Fig 12. Abstraction, represented by TEGS (I) SDT (II) and Venn-diagramms (III).

**Interpreting related systems of temporal logics in TEGS: T-calculi**

The first system we would like to interpret in TEGS will be Von Wright’s T-calculi as it was presented in his Norm and Action (Wright 1963). Note that we could only mention the basal ideas and notation (Fig 13.)

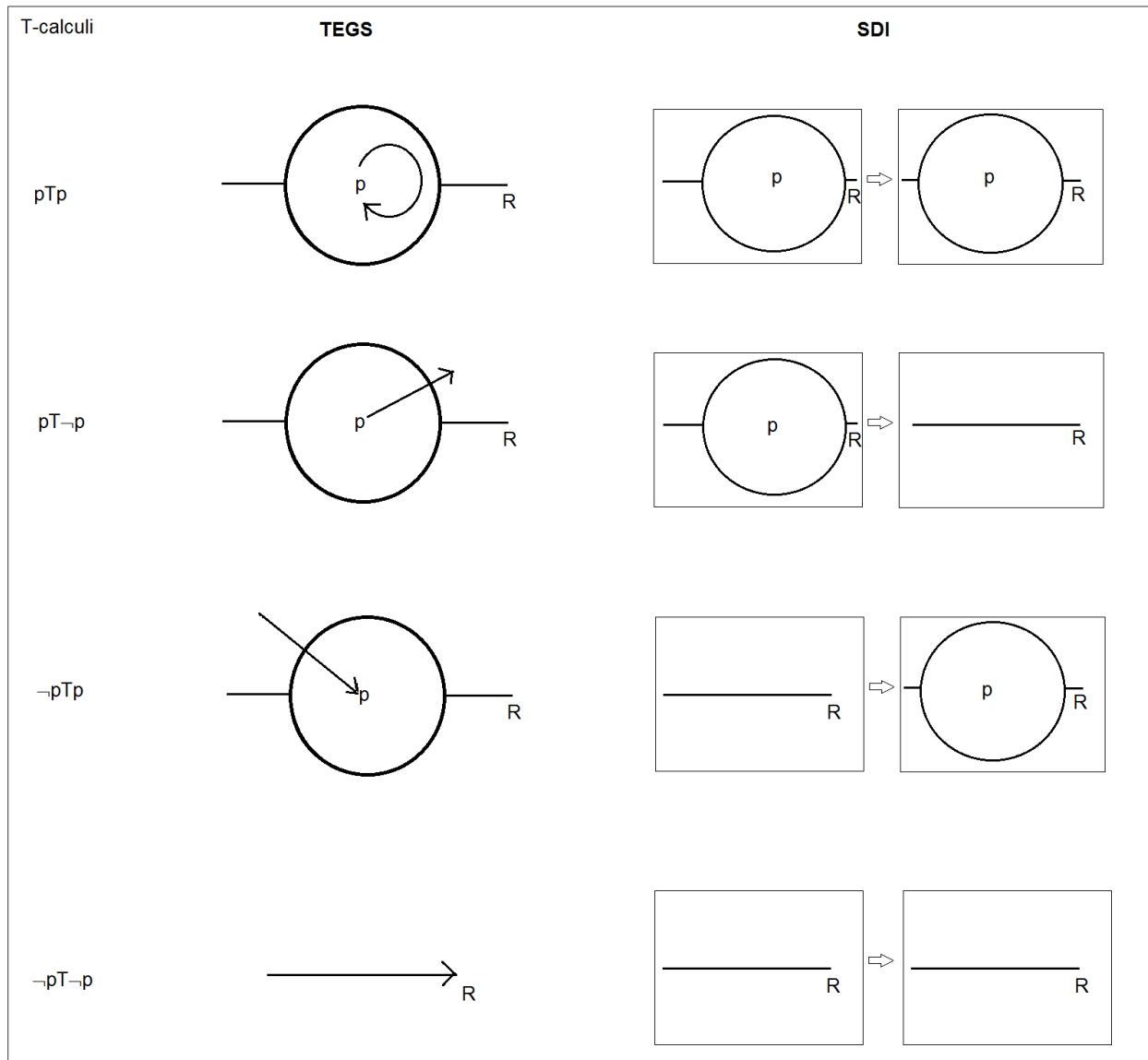


Fig 13. Primary relations in T-calculi, TEGS and SDT.

The first line expresses no change but time with a given content  $p$ . Note, that only *types* are constants; on the other hand, exemplars *have to* change at least inasmuch as they exist at *different* moments, while types exist timelessly. The second line expresses termination while the third line shows formation. Finally, the fourth line (like the first one) expresses no change but time without any content. Note that without positive facts, tense-operator expresses only the change of time, viz. the pure container of any possible change.

A more problematic application of T-calculi will be represented on Fig 14. with the original scheme of Von Wright. Note that, from a diagrammatical point of view, his scheme seem to be elliptic in the sense that they should be augmented with presuppositions. For example, his first scheme runs as  $pTq$ , but it can be conceived in many ways. Based on the complex formula, we could extricate two states (S1,S2) and the relation T. The complex formula contains information about two propositions (p,q) but elementary formulas (S1,S2) affirm only one of them: so the position of the suppressed proposition should be presupposed. Then  $pTq$  could be interpreted 4 ways (explicit information is marked with bold).

- a.  $p \wedge q \mathbf{T} p \wedge q$
- b.  $p \wedge q \mathbf{T} \neg p \wedge q$
- c.  $p \wedge \neg q \mathbf{T} p \wedge q$
- d.  $p \wedge \neg q \mathbf{T} \neg p \wedge q$

Now Fig 14. shows the interpretations of T-calculi with two propositions.

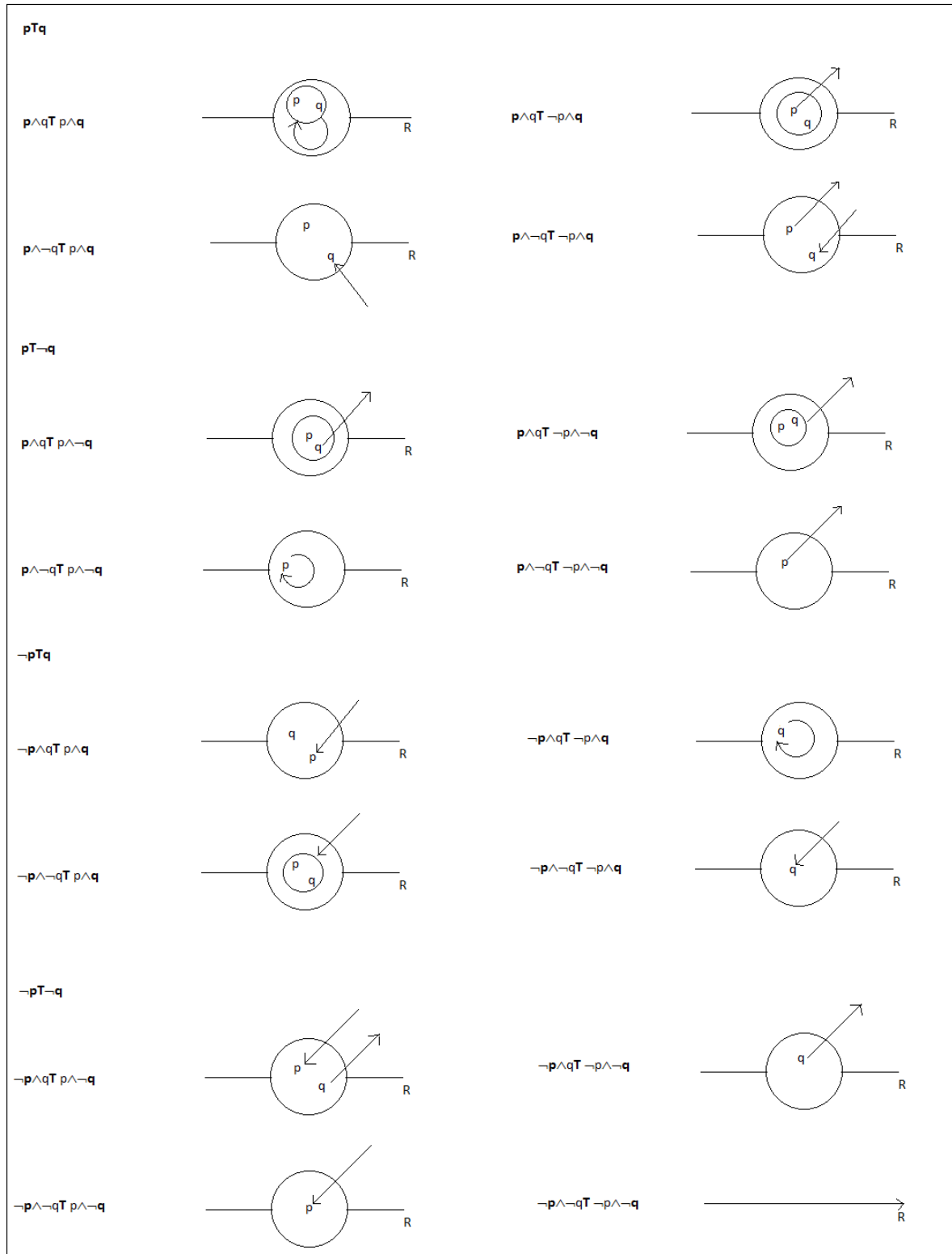


Fig 14. T-calculi and TEGS with 2 propositions



**IBT**

Our second system is based on the idea of branching time (IBT) as it has been interpreted by Prior and Kripke (Prior 1967, Ohrstrom – Hasle 1995). The main problem as regards IBT is indisputably the question of determinism, but we won't be concerned with it as our interests are the structural properties of IBT here. But since in IBT the direction of time is structurally decisive, we should analyse its representative methods first.

According to Galton (Galton 1987) there are at least four ways of representing the topology of time, namely the linear, the parallel, the branching and the circular representational systems. Note that the above mentioned four concept could be reduced to linear and nonlinear ones, since linear time could be considered as hapax linear (single linear), as bilinear (parallel) as multilinear and as branching time. But they are not distinct topological systems, because, as Fig 15 shows, the more complex topologies could include simpler ones. Let us represent the topology of the different time concepts as follows.

- [A] The topology of nonlinear time
- [B] The topology of hapax time
- [C] The topology of bilinear time
- [D] The topology of multilinear time
- [E] The topology of branching time

Let the index TC signify the topological complexity of different topological structures. Topological complexity means that representations with more TC could include representations with less TC. Now Fig 15 shows that  $[A]_{TC} < [B]_{TC} < [C]_{TC} < [D]_{TC} < [E]_{TC}$

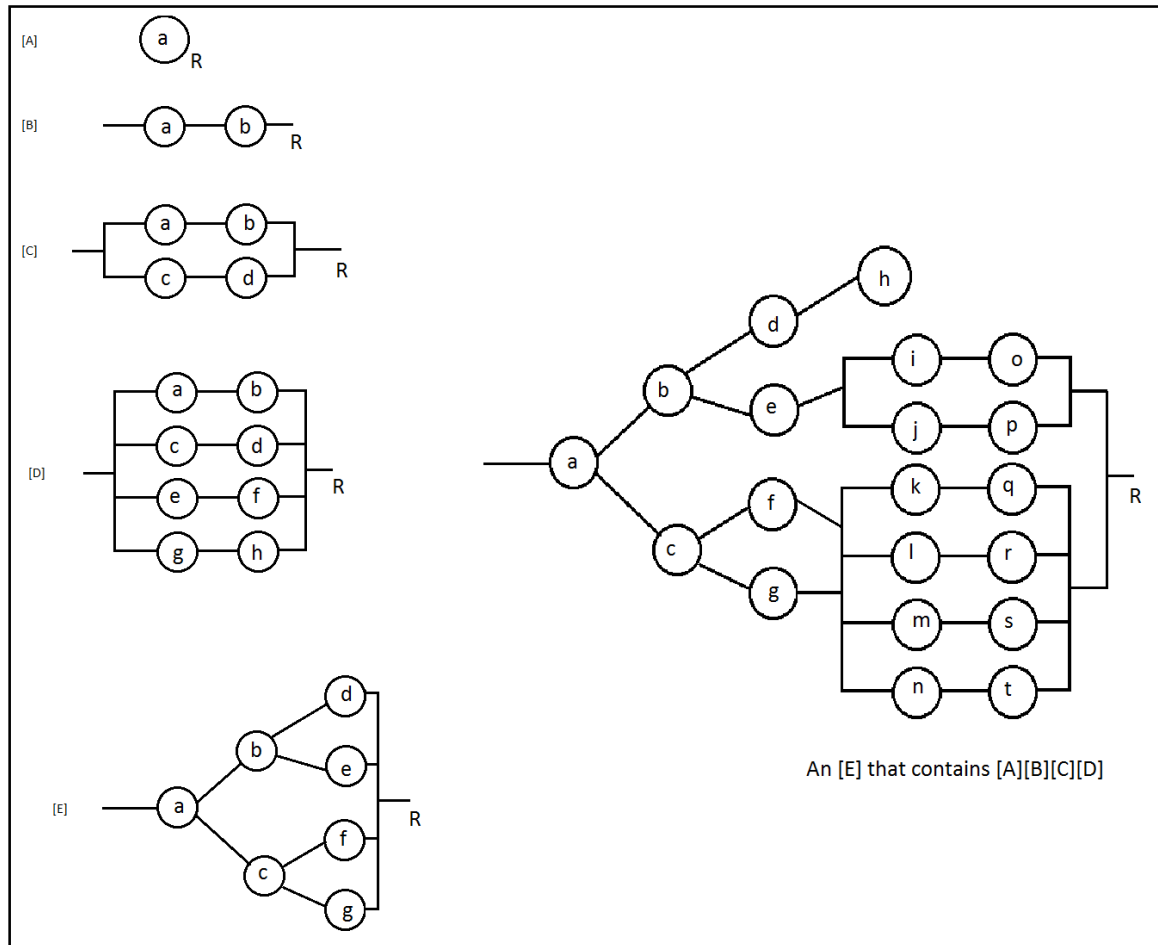


Fig 15.

Fig 15. shows that [E] could include [A] or [B] or [C] or [D]; the diagram on the right shows that an [E] contains an [A] (see unit 'a'), a [B] (see, for example units 'a-b'), a [C] (see units 'i -j-o-p') or a [D] (see units 'k-l-m-n-q-r-s-t'). But it also shows that logical connections as regards parallel and branching time are different; in the case of the former the connection is 'AND', while in the case of the latter it is „XOR". Parallel time concepts could be interpreted at least two ways. The first is the simpler one because it only suggests that different propositions could be true at the same time (as in the case of [C] on Fig 15). It simply means that parallel states could exist simultaneously. But the second interpretation could suggest a more problematic situation that could be represented as Fig 16 shows.

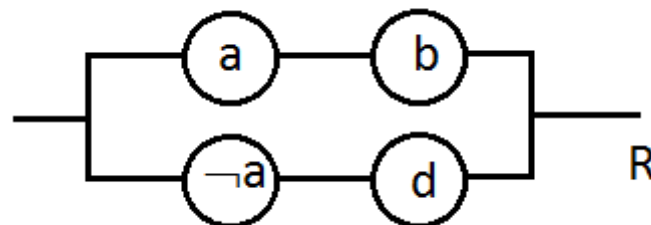


Fig 16.

As opposed to the first interpretation Fig 16. shows a situation where a and not-a could be simultaneously true in parallel times. But this curiosity is not important for us here, so let us get on to our original subjects, namely types and exemplars.

The simpler challenge is apparently the question of exemplars: we plainly have to direct the edges of the appropriate graphs.

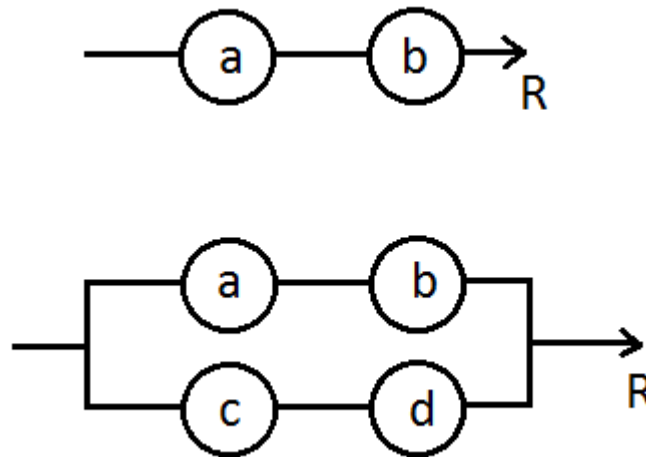


Fig 17.

Note that we need not direct all edges because the sequential order could be indicated by directing R only. Consider moreover that this way of representing temporal sequences shares problems with the T-calculi as it is elliptic too. For example, 'the change from a to b' could be interpreted either as 'the change from  $\langle a \rangle$  to  $\langle b \text{ and not-}a \rangle$ ' or as 'the change from  $\langle a \rangle$  to  $\langle a \text{ and } b \rangle$ '. Even though this kind of ambiguity significantly increases in the case of IBT we won't deal with it because the thickening here is basically simply combinatorial so it could be handled by means of the considerations as regards the interpretation of T-calculi.

Representing types is far more interesting because we have to show that type-structures should not contain directed edges. For this end we will show that *structural* succession is not a question of time, but a question of encompassment.

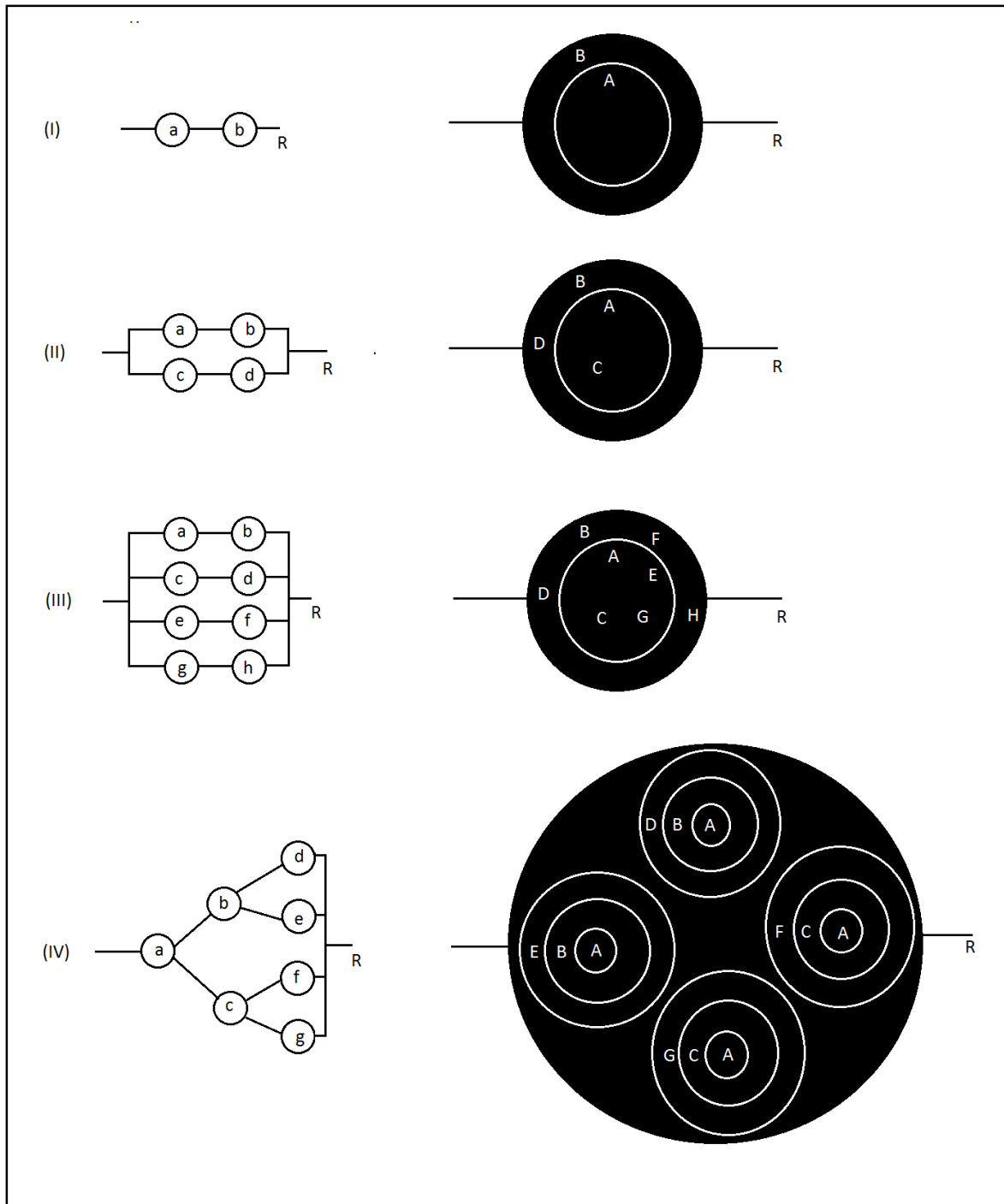


Fig 18.

Now Fig 18. shows that parallel and branching time-concepts could be represented without directed edges when we represent types instead of exemplars. Note that while IBT representations show temporal relations, TEGS interpretations show the logical structure itself. So representations on Fig 18. should be interpreted as follows.

- (I) If A then B
- (II) If A and C then B and D
- (III) If A and C and E and G then B and D and F and G
- (IV) If A, then either B or C  
If B, then either D or E  
If C, then either F or G  
So If A, then either D or E or F or G

Note, that in the case of IBT 'then' should be conceived in a temporal sense while in the case of TEGS 'then' signifies logical implication.

### Some remarks on additional expressions of temporal logics

As it is well known, temporal logics are usually considered as extensions of modal logics which operate on temporal expressions of the ordinary language: Prior's system is an early example (Prior 1957). In our days, the main disciplines studying temporal questions are AI and Computer Science which have adopted the modal and linguistic conception of temporal logic (note that Prior's original expression was 'tense logic' which directly refers to a linguistic approach). In this paper I tried to show that some apparent temporal expressions of natural languages can be substituted with timeless (logical) expressions or, more precisely, temporal diagrammatizations could be substituted with atemporal ones. In spite of the fact that many so-called temporal expressions could have been analysed (for example: 'until', 'eventually', 'unless', 'at the next moment', 'precedes' see LAMPORT 1980, JOSKO 1987) we will concentrate here only two of them, namely 'always' and 'never'.

Note that, in the case of temporal logics, originally timeless expressions like 'always' are usually analyzed as they occur in time. For example, expressions like  $\langle \Box p \rangle$  that means 'p is true throughout every possible future' or 'p was, is and will be true forever' suggest a temporal interpretation of being always true. But I think these interpretations misunderstand the logical structure of originally atemporal expressions since the word 'always' simply states logical inclusion between individuals, concepts, propositions or functors. Consider the following examples.

- a. Peter will always be a human.
- b. Romeo will always love Juliet.
- c. 2 will always be an even number.

Its easy to see that none of these statements express temporality, since they could be transcribed as follows.

- a'. Being Peter entails being human.
- b'. Being Romeo entails loving Juliet.
- c'. The concept of '2' entails being an even number.

The situation is slightly different when inclusion applies to functors as in the case of d.

- d. A bachelor will always be unmarried.
- d'. The functor 'x is a bachelor' entails the functor 'x is unmarried'.

Inclusion is far more apparent in the case of functors since the sentence 'A bachelor will always be unmarried' could be transcribed as 'All bachelors are unmarried', and it is well known that this latter expression doesn't express a statement on  $\langle \text{all bachelors} \rangle$ , but it expresses that  $\langle \text{if } x \text{ is a bachelor then } x \text{ is unmarried} \rangle$ . So 'always' should be analyzed so much as 'all'. Then, for example, the sentence 'Romeo will always love Juliet' does not express a temporal statement that Romeo will love Juliet tomorrow, the day after tomorrow etc. but it states that if someone is Romeo then he loves Juliet:  $\langle \forall x \rangle \langle Fx \supset Gx \rangle$

In the same way, we don't have to refer to temporality as regards expressions contains the word 'never' because the point is that, logically, 'never' refers to disjunctive individuals, propositions, concepts or functors.

- a. Peter will never be a spider.
  - b. Romeo will never love a dinosaur.
  - c. 2 will never be an odd number.
  - d. A bachelor will never be a bigamist.
- a' 'Being Peter' and 'being a spider' are disjuncts.
  - b' 'Being Romeo' and 'loving a dinosaur' are disjuncts.
  - c' The concept '2' and the concept of 'odd number' are disjuncts.
  - d' The functor 'x is a bachelor' and the functor 'x is a bigamist' are disjuncts.

Fig 19. shows some TEGS representations of 'always' and 'never'.

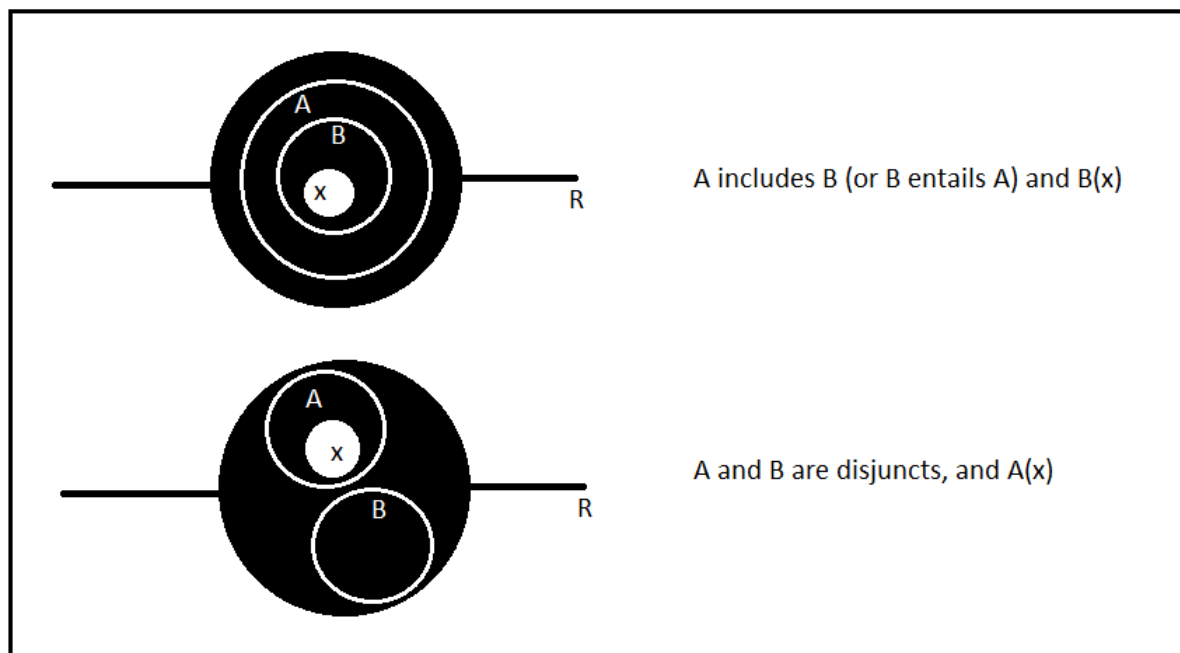


Fig 19.

### Concluding remarks

At first sight, reversibility presupposes temporality, but I hope that I could show that logical types and their relations are atemporal. So reversibility and irreversibility are irrelevant categories as regards logical types. Nevertheless, exemplars and their relations are subject of change; but the possibility of these changes are preordained by the logical types they could be

occurred in. In a few words, the boundary conditions of any possible change are always atemporal type-relations. Pseudo-temporal expressions like 'always' and 'never' could be transcribed as expressions with atemporal structure. The idea of 'reversible process' is not a temporal, but a logical idea which refers to symmetric structures. In other words: notations could be mirrored as long as they are symmetrical. Logically symmetrical structures are primarily conjunction, disjunction and exclusive disjunction (Fig 20.)

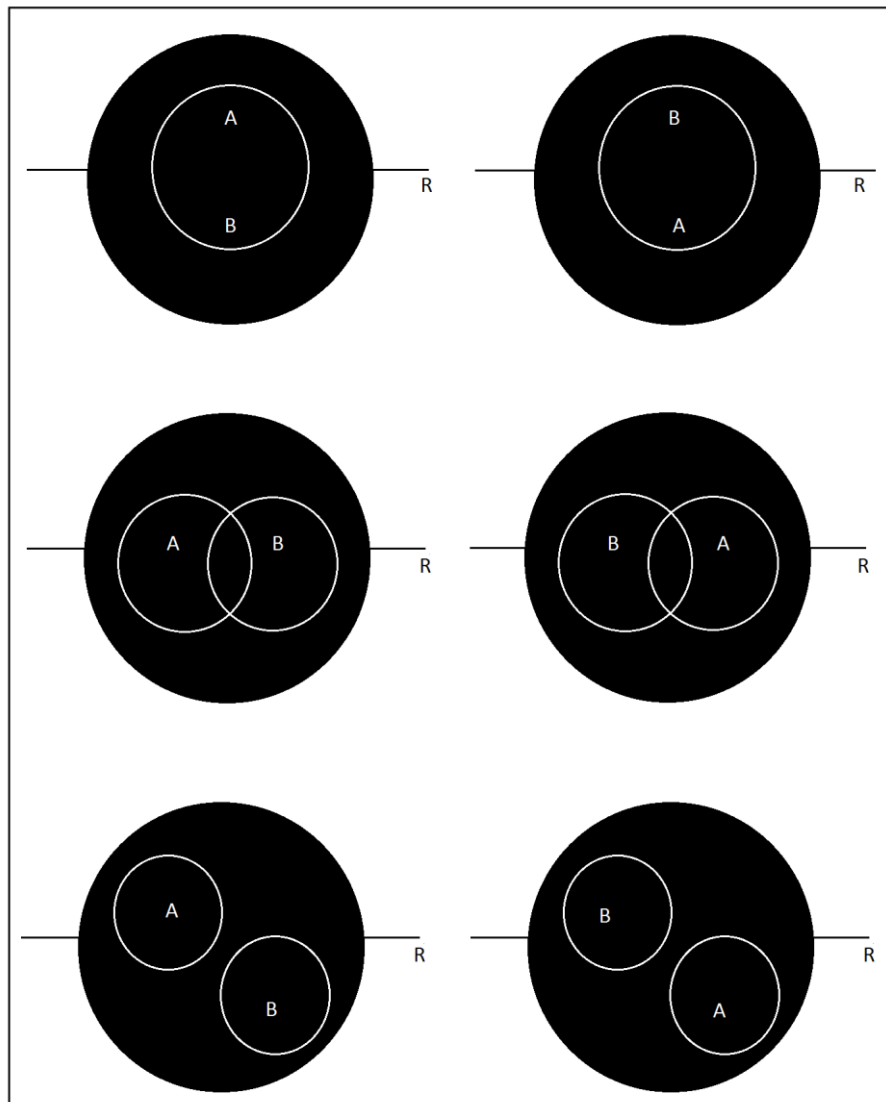


Fig 20.

As for communication, we could now express three considerations. First, communication (as an event, process or action) could not be reversed. Second, the type of a given communication could be structurally symmetric (which does not mean that it is reversible). And third, the possibility of a given communication is *a priori* to its individual occurrence. Let Wittgenstein's Tractatus re-emerge: *„In der Logik ist nichts zufällig: Wenn das Ding im Sachverhalt vorkommen kann, son muß die Möglichkeit des Sachverhaltes im Ding bereits präjudiziert sein<sup>6</sup>.“* (Wittgenstein 1921:2.012).

<sup>6</sup> Translated by D. F. Pears and B. F. McGuinness as „In logic nothing is accidental: if a thing *can* occur in a state of affairs, the possibility of the state of affairs must be written into the thing itself” Wittgenstein 2001: 2.012.

## References

- Ben-Ari, M.; Pnueli, A.; Manna, Z.: 1981, 'The Temporal Logic of Branching Time', *Proc. 8th ACM Syrup. on Principles of Programming Languages*, 1981, pp. 164-176.
- Benthem, J. F. A. K. van: 1983, *The Logic of Time*, London
- Burgess, J. P.: 1980, 'Decidability for Branching Time', *Studia Logica*, p. 203ff.
- Fisher, M (2011): *An Introduction to Practical Formal Methods Using Temporal Logic*. John Wiley & Sons.
- Galton, Antony (editor): 1987, *Temporal Logics and Their Applications*. Academic Press, London.
- Goos, G. – Hartmanis, J. (eds); 1987, *Temporal Logic in Specification*. Springer Verlag.
- Greiner, W. – Brendt, M.; 1994: *Quantum Mechanics. Symmetries*. 2nd ed., Springer.
- Josko, Bertrand: MCTL - AN Extension of CTL for Modular Verification of Concurrent Systems in Goos, G. – Hartmanis, J. (eds); 1987, *Temporal Logic in Specification*. Springer Verlag.
- Lamport, L. (1980): '„Sometime” is sometimes „not never”: on the temporal logic of programs.' *Proceedings of the 7th ACM Symp. on Principles of Programming Languages*, pp. 174-185.
- McDermott, D.: 1982, 'A Temporal Logic for Reasoning about Processes and Plans', *Cognitive Science* 6, pp. 101-55.
- Nishimura, Hirokazu: 1979, 'Is the Semantics of Branching Structures Adequate for Non-metric Ockhamist Tense Logics', *Journal of Philosophical Logic* 8, pp. 477-478.
- Ohrstrom, Peter – Hasle, Per F.V.: 1995, *Temporal Logic. From Ancient Ideas to Artificial Intelligence*. Kluwer Academic Publishers, Dordrecht, Boston, London.
- Prior, A. N., 1957, *Time and Modality*, Oxford: Clarendon Press.
- Prior, A. N.: 1967, *Past, Present and Future*, Oxford: Clarendon Press.
- Rescher, N. – Urquhart, A.: 1971: *Temporal Logic*. Springer-Verlag, Wien – N.Y..
- Sachs, R. G.: 1987: *The Physics of Time Reversal*. University of Chicago Press.
- Stirling, Colin: Comparing Linear and Branching Time Temporal Logics in Goos, G. – Hartmanis, J. (eds); 1987, *Temporal Logic in Specification*. Springer Verlag.
- Sun-Joo Shin: *The Logical Status of Diagrams*. Cambridge University Press, 1994, 2006.
- Wille, R. – Drechsler, R.: 2010: *Towards a Design Flow for Reversible Logic*. Springer.
- Wittgenstein, Ludwig (1921): *Logisch-Philosophische Abhandlung. Annalen der Naturphilosophie, Band 14, 1921, S. 185–262*.
- Wittgenstein, Ludwig, 2001. *Tractatus Logico-Philosophicus*. Routledge Classics, 2001.
- Wolper, Pierre: On the Relation of Programs and Computations to Models of Temporal Logic in Goos, G. – Hartmanis, J. (eds); 1987, *Temporal Logic in Specification*. Springer Verlag.
- Wright, Georg Henrik von, 1963. *Norm and Action. A logical Enquiry*. Routledge – Kegan Paul, London. .