A thesis submitted for the degree of Doctor of Philosophy

Essays in Family Economics

Alexander Vickery

Royal Holloway, University of London

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Declaration of Authorship for Co-authored Work

Name of candidate: Alexander Vickery

Thesis title: Essays in Family Economics

I confirm that the thesis I am presenting has been co-authored with Professor Dan Anderberg. Within this partly co-authored work, I declare that the following contributions are entirely my own work:

I declare that the first chapter of my thesis is not co-authored and it is entirely my own work. I downloaded the data from the Korean Research Institute for Vocational Education and Training, and have conducted all of the analyses independently.

I declare that the second chapter of my thesis is co-authored. I downloaded the data from the UK data service repository, the subsequent analysis was conducted under the supervision of Professor Dan Anderberg.

Alexander Vickery

Abstract

Understanding the patterns of household formation and decision-making is vital for the design of policies to successfully improve welfare. This is the central theme of my thesis which consists of two chapters, the connecting theme across the two chapters is the of use high-powered structural econometric modelling of complex equilibria in household settings.

In the first chapter, I study the development of child academic skills through adolescence in anticipation of entry to university in South Korea. I look at how heterogeneity in initial household income and child academic skills affects parents' decisions to invest in private education for their children, and how the resulting choices contribute to inequality in university admissions, and also to lower social mobility in terms of lifetime earnings. I allow for interactions between the academic skills that enter the human capital production functions. Understanding these skill complementarities is crucial for policy design. I then place the estimated human capital production functions within an equilibrium framework in order to account for the fact that places at the top universities are highly attractive but also limited. As a result, parents' decisions depend on their expectations about the investments made by other households. Competition is particularly detrimental to financially-constrained low-income households.

In the second chapter, we exploit the post-war immigration-induced regional variation in ethnic composition among British-born individuals to study inter-ethnic marriages in the UK. Black and Asian individuals are more likely to marry intra-ethnically in regions where the own ethnicity share is relatively large. In order to disentangle the relative roles played by supply effects, preferences and local social norms we estimate a structural equilibrium marriage market model that allows for conformity behaviour. Using the estimated model, we make predictions for a set of more recent cohorts whose marital choices are still to be completed.

Acknowledgements

In the spring of 2011 I was diagnosed with Ulcerative Colitis. I became severely unwell to the extent that, by the summer of 2011, I required life saving emergency surgery. Looking back, I never would have imagined that I would be in the position that I am currently, and I extremely grateful for the support that I have received over that period.

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Lastly, a special thank you to my fiancee (and soon to be wife) Jingling. She has kept me sane during the most difficult periods and I am so thankful for her support. I am also glad that (hopefully) she will no longer be the only one in our household with a PhD.

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Chapter 1

Reaching for the SKY: Parental Investments in Academic Skills and Competition for University Places

Alexander Vickery

Abstract

I study the development of child academic skills throughout adolescence in anticipation of entry to university. The setting is South Korea which is internationally unique in terms of spending on private tuition and competition for entry into top (SKY) universities. I look at how heterogeneity in initial household income and child skills affects parents' decisions to invest in private education for their child, and how the resulting choices contribute to inequality in university admissions in the first instance, and also to lower social mobility in terms of lifetime earnings. I use a non-linear factor approach to estimate human capital production functions that are placed within an equilibrium framework to account for the fact that places at the top universities are highly attractive but also limited. I find that there are strong complementarities between different academic skills and that private education in one subject can have strong spillover effects on the accumulation of skills in cognate academic subjects. The implied equilibrium competition for limited places at top universities strongly contributes to low inter-generational social mobility, and, conversely, policies that would limit competition could improve equality and the allocation of talent.

1.1 Introduction

During adolescence, the stock and the accumulation of a child's academic skills has important consequences for their performance in high-stakes examinations. As children approach adulthood, their examination performance becomes more salient, influencing university admission decisions, and consequently, labour force participation (Keane and Wolpin, 1997), and their future earnings (Neal and Johnson, 1996). However, evidence suggests the existence of a large achievement gap in test-scores between rich and poor students, even within the upper percentiles of the distribution of test-scores.[1] Since test-scores are related to earnings, the achievement gap between rich and poor students can potentially limit opportunities for upward social mobility and presents a source of lasting income inequality across generations (Hanushek and Woessmann, 2008).

Existing evidence attributes the achievement gap to a number of factors such as: variation in parental ability and parental investments (Todd and Wolpin, 2007), differences in high school quality between rich and poor students (Rivkin, Hanushek and Kain, 2005), differences in household income (Manski and Wise, 1983), (Cameron and Heckman, 2001), (Ellwood, Kane et al., 2000), (Belley and Lochner, 2007), and contrasting effort choices among students (Myong, 2016). Understanding the achievement gap depends crucially on the underlying technology that describes the formation and accumulation of a child's academic skills, and the choice of empirical specification for this technology can lead to vastly different results, even when using the same data sets (Krueger, 2003), (Todd and Wolpin, 2003). By convention, authors define the skill production technology as a function of the child's initial skill endowment, and additional inputs that include parental investments (Ben-Porath, 1967), (Leibowitz, 1974). Estimation of this technology is challenging because skill endowments are latent, inputs are chosen endogenously, and outputs (e.g. test-scores) are noisy measures of the child's true, underlying, skills. In addition, there is a growing consensus on the existence of important dynamic complemen-

[1] Myong (2016) uses data from the US Educational Longitudinal Survey 2002 and finds that for students within the upper 10th percentile of standardized scores, students from the top quintile of the income distribution achieve SAT scores that are 95 points higher than those from the bottom quintile of the income distribution.

tarities and interactions between the different skills and inputs in the production technology.[2] In order to precisely quantify the achievement gap, and to evaluate the effectiveness of policies designed to reduce the achievement gap between rich and poor students, it is crucial that the interactions and complementarities between different academic skills, and parental investments in academic skills, are fully taken into account. For example, how do complementarities between academic skills compare in magnitude to the self-productivity of academic skills? Do academic skills accumulate in unison, or are certain academic skills substitutes for other skills? Will interventions designed to affect academic skill development persist into the future or will the effect of the intervention exhibit fade-out? How do parental investments in one academic skill affect the accumulation of another cognate academic skill? This question is particularly salient because, for example, if academic skill complementarities are large, providing low-income households with financial support for investing in one academic skill could potentially have large positive spillover effects on the promotion of other academic skills, which, in turn, could help to reduce the observed achievement gap. Lastly, what effect does competition for places at top universities have on parental decisions to invest in academic skills, the subsequent accumulation of academic skills, and how does this effect link to the achievement gap between rich and poor students?

In this paper, I answer these questions by studying the dynamic process of child academic skill development throughout adolescence in anticipation of entry to university. Academic skills are multi-dimensional (e.g math skill, English skill, etc ...) and I model the development process from when a child enters their penultimate year of middle-school (age 14) until they complete high-school and reach the age of university entry (age 18). I allow for dynamic interactions between all academic skills, and investments in academic skills that are made by the child's parents. The academic skill technologies are placed within an equilibrium framework to account for the fact that places at the top universities are highly attractive but also limited. I estimate the parameters of the model using data from a South Korean child development study that contains rich information on academic skills, academic performance, and parental investments. Using the

[2]For further reference see: (Cunha and Heckman, 2007), (Heckman, 2007), (Currie and Almond, 2011), (Cunha, Heckman and Schennach, 2010), (Del Boca, Flinn and Wiswall, 2014), (Attanasio, Meghir and Nix, 2020).

estimated model I analyse the effectiveness of parental investments in academic skill development across a range of academic subjects, the existence of complementarities between academic skills and parental investments, and the implications for the timing and targeting of policies aimed at reducing the achievement gap between rich and poor children. I focus on parental investments because competition for places at top universities generates an education 'arms race' among households in which higher income households have a comparative advantage (Ramey and Ramey, 2009). If parental investments are indeed effective in promoting the accumulation of a child's academic skills, in absence of intervention, there are limited opportunities for upward social mobility. In addition, there are likely to be significant complementarities between parental investments in specific academic skills and the development of cognate skills, these cannot be estimated by considering academic skills and parental investments independently.

To estimate the technology that describes the accumulation of child academic skills I use the non-linear latent factor approach developed by (Cunha, Heckman and Schennach, 2010), and recently extended by (Agostinelli and Wiswall, 2016), (Attanasio, Meghir and Nix, 2020). Full details of the econometric method are provided in section (1.6), but the premise behind this approach is that if the researcher has rich enough data that includes detailed measures of parental investments and child outcomes (e.g. test-scores), at various stages of childhood, the joint distribution of these measures can be used to estimate the joint distribution of latent academic skills, as well as the parameters of the underlying academic skill production functions. One key feature of this approach is that it allows for interactions between the inputs that enter the production functions (i.e. academic skills and parental investments). I structurally estimate the remaining parameters of the model using the method of simulated moments (MSM).

My results provide a set of important findings that contribute to the existing literature on government intervention and educational attainment (Cameron and Taber, 2004), (Cameron and Heckman, 1998), (Lochner and Monge-Naranjo, 2002), the impact of university admission processes on the assignment of students to universities (Arcidiacono, 2004), (Hickman, 2013), (Kapor, 2015), and the impact of need-based vs merit-based aid policies on university enrollment (Kane, 2003), (Dynarski, 2010), (Van der Klaauw, 2002). First, I find that there are strong complementarities between all academic skills included in the model. Academic skills are highly self-productive, as in (Cunha, Heckman and Schennach, 2010) and (Attanasio, Meghir and Nix, 2020), with estimated self-elasticities in the range 0.2 - 0.5 for all ages. However, the estimated academic skill complementarities produce elasticities that are of similar magnitude to the self-elasticities. This result suggests that, throughout adolescence, academic skills accumulate predominantly in unison. As a result, while this suggests that children can reduce deficiencies in certain academic skill areas through increased levels of academic skills in cognate subject areas, the existence of deficiencies in a certain academic skill also restricts the accumulation of academic skills in cognate subject areas.

Second, I find that academic skills become more self-productive over time. For all academic skills, the estimated self-elasticities at the end of the high-school are estimated to be almost twice the magnitude of the corresponding self-elasticities that are estimated at the end of middle-school. This suggests that an intensive intervention designed to enhance child academic skills prior to middle-school will exhibit the fade out property that has been observed in a number of youth development studies (Walker et al., 2005), (Andrew et al., 2018), however, eventually there will be an increasing persistence of academic skill accumulation. This finding also implies that if interventions are left too late (i.e. until the end of high-school), a student's academic skills will not be affected to the same extent as they would have been in the counterfactual where the same intervention had occurred earlier on in the academic skill development process.

Third, I find that parental investments in private education are productive for the accumulation of all academic skills, at all ages, but parental investments in private education have the largest effect in the earlier stages of the academic skill development process. I also find that, during middle-school, the estimated elasticities with respect to parental investments in private education for math are larger that the estimated elasticities for parental investments in all other skills, this is true for the accumulation of all academic skills. In other words, during middle-school, parental investments in private education for math are not only more productive for the accumulation of a child's math skill than are private education investments in other skills, private education investments in math are also more productive for the accumulation of a child's other academic skills. This, combined with the finding that academic skills become more self productive over time, has several important implications. First, is that an intervention targeting a child's math skill during middle-school should be effective at promoting the accumulation of academic skills in a range of other academic subject areas as the the child enters and progresses through high-school. Second, is that households, in general, could achieve a higher return on their investments by targeting their investment portfolio into specific productive skill areas such as math.

Finally, I find that the implied equilibrium competition for places at top universities strongly contributes to the achievement gap between rich and poor students, and hence, contributes to lower inter-generational social mobility. This is because places at top universities are awarded primarily based on a child's performance in their end of high-school examination. The child's test-scores are affected by parental investments in private education in different academic skill areas, and higher income households have a comparative advantage with respect to these investments. This result is consistent with recent findings showing that interventions designed to limit competition can help to improve equality and the allocation of talent (Arcidiacono, 2004), (Kapor, 2015).

To demonstrate the implications of my findings I conduct a series of counterfactual policy simulations using the estimated structural model. The results of the policy simulations suggest that providing a cash transfer to low-income households in order to increase their relative spending power is effective at reducing the achievement gap between rich and poor students and hence, the gap between the proportion of rich and poor students that are accepted to the top universities. However, cash transfers are most effective if the transfer is received earlier in the academic skill development process and are designed to target specific academic skills, for example, a child's academic skill in math. This result is due to the presence of academic skill complementarities and dynamic complementarities with respect to parental investments in academic skills that exist within the model. Cash transfer policies are, however, expensive to implement. My results therefore suggest that policies designed to reduced the effect of equilibrium competition for places at the top universities are a more cost effective way at reducing the achievement gap between rich and poor students. Specifically, I find that a policy that introduces a limit on the total amount that parents can invest in private education for their child is equally as effective at reducing the achievement gap between rich and poor students and hence, the gap between the proportion of rich and poor students that are accepted to the top universities, but in absence of the high implementation cost.

The setting of this paper is South Korea (Korea). Korea is a country that is internationally unique in terms of parental investments in private education and competition for entry into top universities. Korea therefore provides an ideal setting for tackling the key research questions. I use data from the Korean Employment and Education Panel (KEEP) which is an annual child development study that follows a sample of 2,000 middle school students from 2004 until the present day (12 years of data are currently available).[3] In addition to rich information on academic skills, academic performance, and parental investments, the data is also linked to national records of students results in the national university entrance examination. Due to the time horizon of the data, I am able to observe the university that a child subsequently attended (or did not) due to their performance in the entrance examination. I also observe their future labour market outcomes over a short time horizon.

My paper is closely related to the literature that estimates production technologies for child skill development such as (Cunha and Heckman, 2007), (Cunha, Heckman and Schennach, 2010), (Williams, Heckman and Schennach, 2010), (Todd and Wolpin, 2007), (Bernal, 2008), and (Attanasio, Meghir and Nix, 2020). Specifically, I use the non-linear latent factor approach developed by (Cunha, Heckman and Schennach, 2010) (see also (Schennach, 2004), (Hu and Schennach, 2008)), and recently extended by (Agostinelli and Wiswall, 2016), (Attanasio, Meghir and Nix, 2020). (Cunha, Heckman and Schennach, 2010) use this methodology to estimate the technology of cognitive and non-cognitive skill formation for young children aged 0-14 in the US, using data from the NLSY. They use their estimated model to evaluate the optimal targeting of interventions designed to help disadvantaged children. They find that substitutability in the

[3]the KEEP data also follows an additional 4,000 students over the same time horizon. However, in 2004 these students were already in their penultimate year of high-school. As a result, there is not enough time variation within that sample to be useful for this study.

production of cognitive skills decreases as the child gets older, this finding suggests that interventions targeting cognitive skills are more effective at earlier stages of childhood. (Attanasio, Meghir and Nix, 2020) estimate production functions for cognition and health for young children in India. They allow for interactions between cognition and health and find that the early impact of child health is important for the accumulation of a child's cognitive skills. They also find that parental investments affect the development of cognitive skills, but that the magnitude of the effect is more pronounced in early childhood. Compared to these studies, the children in my study are aged 14/15 at the beginning of the sample window, so are much older and further along in the development process. Also, instead of general cognitive skill development, I focus on the development of specific academic skills. I do not include non-cognitive skills or health as inputs in my model as I do not observe rich enough information or measures on these inputs in my data. However, I do allow for interactions between all academic skills and parental investments in academic skills within my production technology.

My paper is also related to the literature on pre-university parental investments and the gap between rich and poor students, such as: (Becker and Tomes, 1994), (Kinsler and Pavan, 2011), (Hoxby, Turner et al., 2013), (Myong, 2016). One paper is close to mine: In her paper, (Myong, 2016), focuses on the role of need-based aid from selective universities and the effect that this has on the achievement gap between rich and poor students, using data from the US. She develops a structural model of students learning, and university application and admission decisions, where the model also includes competition for places at selective universities. She finds that the amount of financial aid offered by universities has a significant effect on students effort choices and this, in turn, contributes to the achievement gap between rich and poor students. In her model, test-scores are determined by the type of high-school that a student attends, the number of AP classes they take, their initial ability, and an unobserved characteristic. However, her definition of a child's initial ability is uni-dimensional and each test-score in her model is determined by a unique learning technology that does not allow for interactions and complementarities between inputs, despite a growing consensus that skill interactions and complementarities are important (Cunha and Heckman, 2007), (Heckman, 2007), (Currie and Almond, 2011), (Cunha, Heckman and Schennach, 2010), (Del Boca, Flinn and Wiswall, 2014), (Attanasio, Meghir and Nix, 2020). I build upon her contribution by decomposing a child's initial ability into ability in multiple academic skill areas, all of which could potentially be important for a child's future test-scores. I estimate the technology that describes the accumulation of academic skills, fully taking into account interactions and complementarities between all academic skills and parental investments that enter the production technology. Because I place the academic skill technologies within an equilibrium framework I also take into account the equilibrium effect that competition for places at top universities has on parental choices and, in turn, accumulation of a child's academic skills.

The layout for the rest of the paper is as follows: in section (1.2), I provide a brief summary of Korea's institutional background over recent decades in order to highlight the intense competition for places at top universities in Korea. In section (1.3), I provide an introduction to the KEEP data and highlight the descriptive features of child academic skill development and parental investment in academic skills. I also provide descriptive evidence of how heterogeneity in household income and parental investments in private tuition is linked to the achievement gap between rich and poor students. In section (1.4), I introduce the structural model that describes the accumulation of a child's academic skills and outline the key mechanisms of the model. I also provide a simple step-by-step algorithm that can be used for computing the model equilibrium. In section (1.5), I outline the non-linear factor model approach that I use to estimate the production technology. I also explain how I identify and estimate the remaining parameters of the model. In section (1.6), I present the main results and discuss how to interpret the estimates. Section (1.7) presents the results of the counterfactual policy experiments and section (1.8) concludes.

1.2 Institutional Background

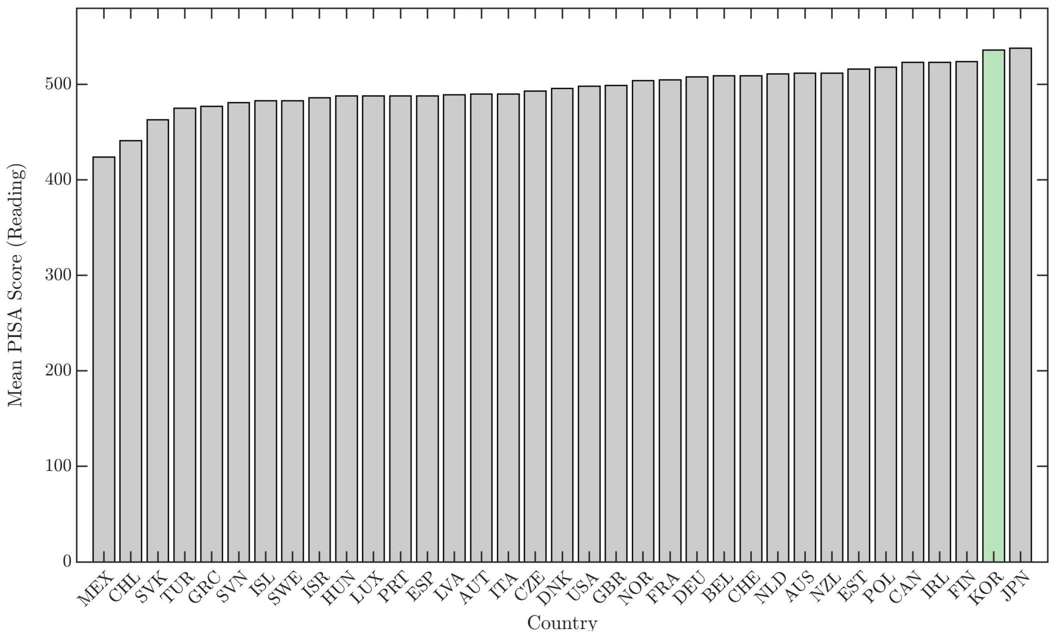
Since the early 1960s, South Korea (Korea) has seen unprecedented economic growth that has taken it from being one of the poorest nations in the world, to one of the top 10 economies in terms of nominal GDP (US$1.6 trillion), with a nominal per capita GDP of US$30,644 (IMF, 2020). Much of Korea's success over the last 50 years has been attributed to government-regulated long-term education plans and policies. As a result of the government's vast investment in education, Korea has now produced one of the world's most educated labour forces. It is estimated that in Korea, on average, over 70% of 25-24 year old individuals have completed tertiary education, this proportion is higher than the corresponding proportion for any other OECD country (OECD, 2015). In addition, Korean children are also highly educated relative to international standards. Figure (1.1) shows a comparison of PISA scores for Reading (panel 1.1a) and math (panel 1.1b) for 15 year old children across OECD countries in 2012. The tests primarily measure problem solving and cognition, and Korean 15 year-old children continue to score highly on these tests when compared to children from other OECD countries.

While achieving a high level of education is necessary to climb the social hierarchy and to elevate one's social status in Korea, it is not a sufficient condition. The sufficient condition is to obtain a degree from one of the top three tertiary institutions, namely Seoul National University, Korea University, and Yonsei University. Together these institutions are commonly referred to as the 'SKY' universities. Graduating from a SKY university is extremely valuable for an individual's social status, as doing so facilitates entry into employment in government offices, or in one of the large multi-national corporations (known in Korea as 'chaebols') such as LG, Samsung, and Hyundai. Graduating from a SKY university therefore also provides individiuals with an unparalled social network as a result. The financial reward from chaebol employment is long-term regular employment with an estimated initial wage premium of approximately 50% and higher rates of wage growth (KEF, 2017). Since their emergence in the early 1960s, chaebols continue to dominate Korean markets, and their growth is co-linear with the growth in Korean GDP, consequently, in 2016, Korea's chaebols together constituted over 58% of Korea's GDP (Chiang, 2016).

However, in recent years, due to a series of image damaging scandals, and sustained periods of economic uncertainty, growth within chaebols has been largely attributed to growth in productivity and to globalisation. For example, in a recent study by the Bank of Korea, they estimated that the annual productivity increase of the largest corporations was due to labour saving technological advances and by outsourcing 20% of production to overseas (BankofKo-

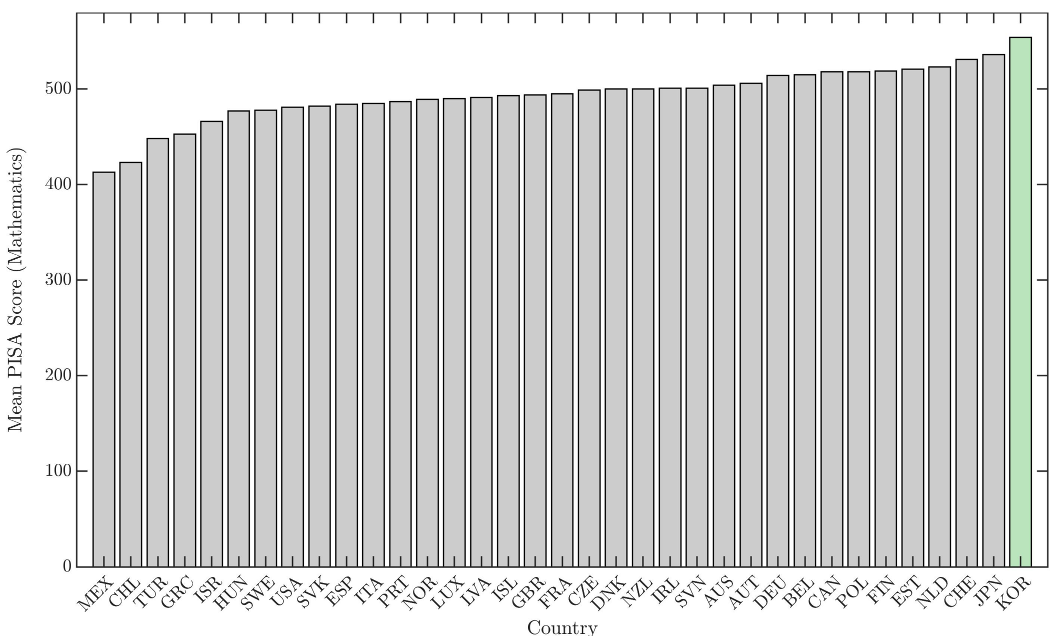
Figure 1.1: PISA scores in reading and math, at age 15, for OECD countries

(a) Reading



Source: OECD (2020), Mathematics performance (PISA), https://data.oecd.org/pisa/mathematics-performance-pisa.htm.
Notes: Mathematical performance, for PISA, measures the mathematical literacy of a 15 year-old to formulate, employ and interpret mathematics in a variety of contexts to describe, predict and explain phenomena, recognising the role that mathematics plays in the world.

(b) Math



Source: OECD (2020), Reading performance (PISA), https://data.oecd.org/pisa/reading-performance-pisa.htm.
Notes: Reading performance, for PISA, measures the capacity to understand, use and reflect on written texts in order to achieve goals, develop knowledge and potential, and participate in society.

rea, 2012). As a result, chaebols are hiring fewer workers domestically and have responded by recruiting less new university graduates in general. It has become preferable for chaebols to instead turn their focus toward hiring workers that have high levels of prior market experience and/or non-regular workers that have fixed short-term contracts. These contracts are preferable because they require no severance or bonus payment to the employee upon completion of the fixed term. Accordingly, by the first quarter of 2017, youth unemployment in Korea had reached 8.5 percent (StatisticsKorea, 2017) and individuals that were employed on non-regular contracts earned 38% lower wages than their regular counterparts, even for equivalent tasks and working hours (OECD, 2016).

Due to chaebol firms reducing the number of regular contracts being offered to university graduates, competition for the existing vacancies has increased. Obtaining a degree from a SKY university is seen as a key determinant of an applicants success rate and so, simultaneously, the value of a degree from a SKY university has increased substantially in recent years. The spillover effect is that now more emphasis is being placed on the admission criteria to SKY universities than ever before. The entrance criteria to SKY universities is set at the institution level and contains multiple dimensions related to a child's academic background, their performance in aptitude tests, and their responses during interviews. However, a necessary condition for entry to SKY university is a near perfect performance on the College Scholastic Ability Test (CSAT, also know as 'suneung').[4] The CSAT is a national standardised test that is created and administered by the Korean Institute of Curriculum and Education in November of each year. The test is taken by students when they are in the process of graduating from high-school (age 18), it is split into six sections that are designed to mirror the subjects that were studied by the students throughout their high-school lives.

Korean parents are aware of how important their child's performance on the CSAT is for their future university and labour market prospects. As a result, there exists a large and competitive market of for-profit private tutoring academies (known as 'Hagwons') across the country.[5] Parents are willing to spend a large proportion of their monthly income to send their children to these private academies as they promise a high return on the parent's investment, in

[4] Full details regarding the layout, timing, and content of the CSAT are provided in section (1.2.2)

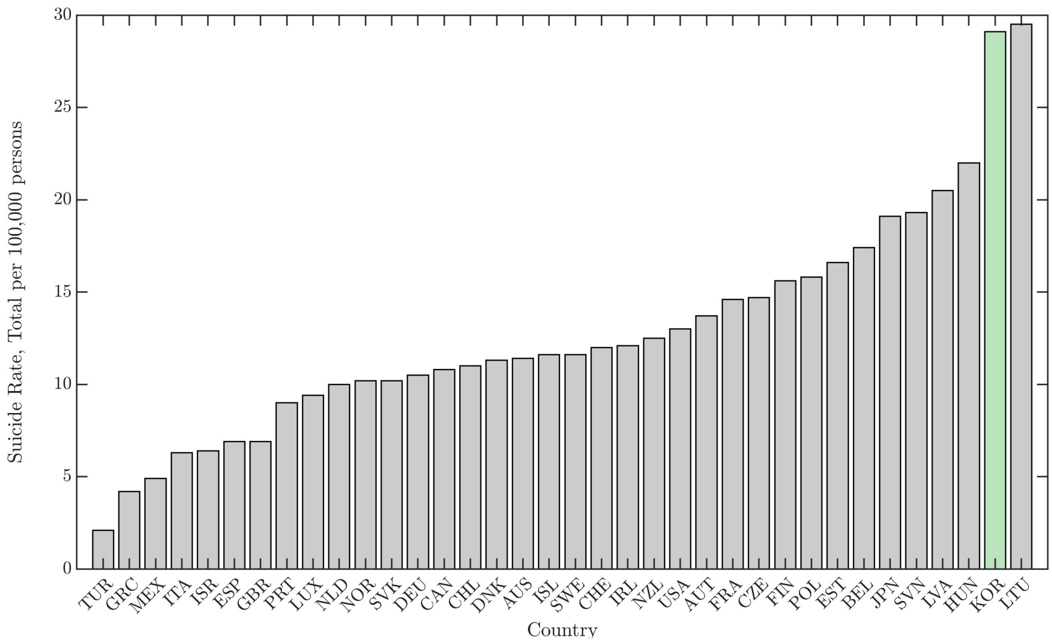
[5] By 2007 it was estimated that there were over 50,000 hagwons across Korea and nearly 3.5 million students were enrolled in them, this number corresponds to a participation rate of approximately 45% of the student population (KMOEHRD, 2007).

the form of increased CSAT scores and an increased probability of entrance to a SKY university. However, Korean parents are also aware that in an equilibrium where all households pay for their child to attend a private academy, the effect that sending their child to the academy has on their own child's probability of entrance to a SKY university will be diminished. As a result, parents have responded by sending their children to academies at younger ages and by increasing their spending in order to send their child to more prestigious academies that have higher rates of success. Indeed, in 2016, in a survey conducted by the Korea Institute of Child care and Education it was estimated that 84% of five year old children and 36% of two year old children were enrolled in private academies (Chung, 2017). It was also estimated that, in 2015, total private expenditures by households were equal to 17.8 trillion won and that, on average, each household was spending approximately 250,000 won (∼\$220) per month on private tuition for their children (StatisticsKorea, 2015).

The increased spending on and presence of private tuition academies in Korea has not occurred not without consequences. Long hours of studying under intense stress has led to negative impacts on the mental health of Korean children and has contributed to suicide being the primary cause of death among Korean youth (McKinsey, 2013). Figure (1.2) shows that the sucide rate per 100,000 persons in Korea is over double the rate observed in the US and over five times as large as the rate observed in the UK. In addition, spending on private academies has become an increased financial burden for low and middle income households with 84% of surveyed households stating that spending on private tuition represented a 'significant' burden on their finances (Kim and Park, 2010). Over half of Korean middle and low income households indicate that they are cash-flow constrained, are unable to save, and have accumulated large debts (BankofKorea, 2017). As a result, participation and spending on private tuition at hagwons is highly correlated with household income (Bray and Lykins, 2012). A recent survey by Statistics Korea indicates that the gap in participation rates between households in the top and bottom deciles of the earnings distribution is equal to approximately 50 percentage points, and that the equivalent gap in spending is 350,000 won (∼\$300) per month (StatisticsKorea, 2015).[6]

[6]The particpation rate for households in the top decile of the earnings distribution is 83% and households in this decile of the earnings distribution spend, on average, 420,000 won (∼\$380) per month on private tuition for

Figure 1.2: Suicide Rates, Total per 100,000 persons, for OECD countries



Source: OECD (2020), Suicide rates, https://data.oecd.org/pisa/suicide-rates.htm.
Notes: Suicide rates are defined as the deaths deliberately initiated and performed by a person in the full knowledge or expectation of its fatal outcome. Rates have been directly age-standardised to the 2010 OECD population.

Therefore, if private tuition at hagwons is indeed effective for the accumulation of the academic skills that are necessary for achieving high performance in the CSAT exam, then a household's ability to invest in private tuition could be an important determinant of the achievement gap in CSAT scores between rich and poor students. Due to the importance of CSAT scores for admission to university in Korea, this, in turn, could also contribute to the gap in probabilities of admission to SKY universities between rich and poor students and therefore limit opportunities for upward social mobility. In order to evaluate whether this is the case, it is crucial that we understand how academic skills accumulate, in general, and how the accumulation of academic skills is affected by parental investments in private tuition whilst fully taking into account the interactions and complimentarities between all skills and parental investments. Given the unique nature of parental investments in private tuition in Korea and the fierce competition for places at the top SKY universities, Korea provides an ideal setting to tackle these issues.

One potential concern when evaluating the effect of tuition at private academies on the accumulation of academic skills, and its contribution to the achievement gap between rich and

their child.

poor students, is how to control for heterogeneity in the quality of formal schooling received by students. Indeed, heterogeneity in high school quality has been shown to have a significant effect on the achievement gap between rich and poor students (Rivkin, Hanushek and Kain, 2005). However, there is a unique feature of the Korean education system that will alleviate most of these concerns. Specifically, in the late 1960s and early 1970s the Korean government introduced a series of school equalisation policies. The next subsection (1.2.1) provides a brief summary of the key features of the equalisation policies and how they eliminate heterogeneity in formal schooling quality across Korean children.

1.2.1 The Korean School Equalisation Policy

Following liberation from Japanese colonial rule and the conclusion of the Korean war in 1953, the education system in Korea was inadequate by international standards. Less that 70% of eligible children were enrolled in primary school and the corresponding enrollment rates for high-school and higher education were 20% and 2% respectively. As a result, the government quickly pushed for the establishment of universal primary schooling (McGinn et al., 1980), (Adams and Gottlieb, 2017). Through a series of aggressive policies such as reducing the training period for teachers, increasing classroom sizes, and constructing large numbers of classrooms, universal primary schooling was achieved in Korea by the 1970s (KMOE, 1998). However, during this period of primary school expansion, the capacity of public middle and high-schools remained largely unchanged. In order to fill the shortage in supply of public middle and high-schools, a growing number of private schools were formed. The fees charged and curriculum offered by private schools were regulated by the government and so, initially, private schools had little financial support and had inferior facilities.

Enrollment at middle-schools and high-schools, irrespective of whether they were public or private, was determined by a competitive entrance examination. A ranking of middle-schools and high-schools was soon established and naturally, parents wanted their children to attend the schools that had achieved the highest ranks. This intense competition for places created a number of negative spillover effects. Firstly, the stress of studying long hours under intense pressure hindered the mental and physical well-being of Korean eleven-year old children. Secondly, large numbers of children failed to enter their preferred school and chose to retake the entire academic year in order to prepare and take the school entrance exam again one year later. Finally, there existed large variation in the quality and the demand for schooling across regions.

As a result, in 1961 when General Park came to power, his government quickly tried to resolve the situation by introducing the school equalisation policy (Kim and Lee, 2002). This policy stated that all schools, whether public or private, could no longer select their incoming students. Instead, all incoming students would be allocated to middle schools and high schools by a random lottery that would be drawn by the Korean Ministry of Education. In addition, the policy ensured that there was no difference in the curriculum being taught or the salaries of teachers between public and private schools, the government was even willing to subsidise the operating costs for financially constrained schools to ensure that this policy was enacted. Essentially, the policy made sure that the only tangible difference that existed between public and private schools was in the way that they were governed, but even this remained closely monitored. The school equalisation policy was met with some opposition, but by the year 2000, it was estimated that all middle-school students in Korea and 60% of Korean high-school students were covered by the equalisation policy.

The main consequences of the Korean school equalisation policy for this paper are twofold. The first consequence is that although the children in my sample attend different middle-schools and subsequent high-schools, there should not be any meaningful differences in the accumulation of a child's academic skills throughout middle and high-school that can be attributed solely to attending different schools. In other words, since the quality of formal schooling is constrained by the government to be homogeneous across students, I assume that the effect of formal schooling in Korea on the accumulation of the child's academic skills is identical across students. This assumption means that I am able to separately identify the effect of private schooling on the accumulation of a child's academic skills from the corresponding effect of formal schooling. The second consequence is that when parents invest in private tuition for their child, I assume that their motivation for doing so is that they believe their investment will help develop the academic skills required to increase their child's CSAT scores above the level that would be expected under formal schooling alone. That is, I assume that parents are not investing in private tuition for their child as a catch up mechanism to account for deficiencies in the quality of formal schooling that their child has received relative to other children at different schools. I justify this assumption because under the school equalisation policy, these deficiencies, if any, should be minimal.

As the demand for private tuition in korea is primarily driven by competition for places at SKY universities and places are allocated based on a child's CSAT scores, the next subsection (1.2.2) briefly outlines the layout, timing, and content of the CSAT exam.

1.2.2 The College Scholastic Aptitude Test (CSAT)

The college scholastic aptitude test (CSAT) is a national standardised test for students that are in the process of graduating from high-school (age 18) in Korea. It is the key factor that decides whether a student can enter university in Korea and, in turn, also determines the quality of university that the student is eligible to attend. The test is created and administered each November by the Korean Institute of Curriculum and Evaluation and is designed to closely reflect the subjects that were studied by Korean students throughout high-school. The test is taken very seriously to the extent that, on the day of the exam, students are often ushered to test sites by police officers, air traffic is prohibited from flying over test sites, and stock markets and certain other business open late in order to ease congestion around test sites within major cities. In addition, the professors and teaching staff involved with the creation of the test are forced to sign a non-disclosure agreement directly with the Korean Institute of Curriculum and Evaluation.

The test is split into six broad categories that are labeled as: National Language (Korean), Mathematics, English language, Korean History, Social Studies/Science/Vocational Education, and Foreign language. All categories are optional, except for Korean History which was recently made mandatory. Mathematics is further split into two sub-categories 'Ga' and 'Na' where the former is generally considered to be more difficult than the later, students taking the mathematics part of the exam are therefore required to chose which of these two sub-categories they would like to be examined on. The Social Studies/Science/Vocational Education section is split into numerous subjects and students can select up to two subjects, provided that the selected subjects are both from the same section. For example, students cannot select one Science subject and one Social Studies subject. This means that students are unable to combine Economics with Physics, for example, but can combine Physics with Biology. The 'Science' subjects are also further split into two subjects i.e. there is no 'Physics' subject, instead there is 'Physics I' and 'Physics II', 'Biology I' and 'Biology II', etc ... and students can select both of these subjects if they want to. The foreign language section consists of a range of different languages and students can select one of these to be examined on. Therefore, in total, students can select up to seven subjects to be examined on throughout the day. However, the modal number of subjects is six subjects because students typically elect not to be examined on an additional foreign language. In order to meet the criteria required for acceptance at SKY universities, the selection of subjects that a student chooses for their CSAT exam is important. For example, taking the 'Ga' sub-section of the mathematics exam is a necessary condition for entry. For students that hope to study STEM subjects at degree level it is also necessary to choose two subjects from within the Science section and their selections need to be diverse, for example, selecting Biology I and Biology II would not be allowed.

In addition to the students subject selections, the most important factor for acceptance at a SKY university is the students exam scores. Students receive their grade, their percentile, and their standard score for each subject on their exam transcript. Grades are allocated into a nine-point standard scale according to a reversed Stanine curve. Stanines are obtained by ranking the students' results from lowest to highest then giving the lowest 4% of grades a Stanine of nine, the next lowest 7% of Grades a Stanine of eight, and so on. The premise behind the approach to obtaining the proportions of students in each Stanine group is that the normal distribution of standard scores is split into nine intervals, where each interval has a width of 0.5 standard deviations, excluding the first and last intervals which make up the tails of the distribution. The students standard scores are calculated by the following formula:

$$S = z\sigma + m \tag{1.1}$$

where S is the standard score, z is the students Z-score, σ is the standard deviation of the students' standard scores, and m is the average of the students' standard scores. σ and m are fixed exogeneously by the Korean Institute of Curriculum and Evaluation. For the mathematics, English language, and national language sections the parameters are 20 and 100 respectively, for all other subject areas the parameters are set to 10 and 50. As an example of possible test-scores, a student's CSAT transcript for mathematics could report: 2, 93, and 130. The interpretation of this transcript is that the student's standardised score for mathematics is 130, this standardised score puts the student in the 93rd percentile of the distribution of mathematics scores, and gives the student a Stanine grade of 2. The criteria for acceptance at SKY universities is extremely high and varies according to the major that a student is applying for. However, based on the criteria outlined in Yonsei university's undergraduate prospectus, a general rule is that all six of a students subject selections must be at least above Stanine grade 2 (i.e. equal to 2 or to 1). In other words, the student must achieve test scores that are in at least the 90th percentile for all of the subjects that he or she has entered.

Since the subjects taken in the CSAT exam are designed to mirror the subjects a student studies throughout middle-school and high-school, the academic skills that a child develops throughout middle and high-school will remain highly productive for their performance in the CSAT exam. Therefore, if private tuition at hagwons is indeed effective for the accumulation of the academic skills, then a household's ability to invest in private tuition could be an important determinant of the achievement gap in CSAT scores between rich and poor students. In addition, because SKY universities require test-scores in all subjects to be above a certain threshold, if the development of academic skills exhibits complementarities, households that are unable to invest in private tuition to facilitate the development of academic skills will fall further behind in terms of their child's SKY acceptance probability and their opportunities for upward social mobility. As a result, in order to fully understand the achievement gap in CSAT scores between rich and poor students, and to evaluate policies designed to reduce the achievement gap, it is essential that we estimate the production technology of academic skills whilst fully taking into account interactions and complementarities between all inputs, and the effect that competition for places has on parental choices of inputs.

In the next section (1.3), I provide a brief introduction to the KEEP data-set that I use in this paper. I use the selected sample to highlight the descriptive features of child academic skill development and parental investment in academic skills in Korea. I also provide descriptive evidence of how heterogeneity in household income and parental investments in private tuition is linked to the achievement gap between rich and poor students.

1.3 Data: The Korean Education and Employment Panel

The data I use in this paper is from the Korean Employment and Education Panel (KEEP). The KEEP is a longitudinal child development study that follows a representative sample of Korean middle-school and high-school children from the year 2004 on-wards. The study was developed and administered by the Korea Research Institute for Vocational Education and Training (KRIVET) and is ongoing, currently 12 waves of data are available for use. The study initially randomly selected a representative sample of 6,000 students, where 2,000 students were in their penultimate year of middle-school, 2,000 students were in their penultimate year of high-school, and the remaining 2,000 students were in their penultimate year of vocational high-school. In addition to the student surveys, the KEEP team also surveyed the child's parents each year to obtain background information on the child's household, the team also surveyed school administrators and the child's homeroom teacher.[7] One key feature of the KEEP study is that the individual data is linked to student records of their performance in the College Scholastic Aptitude Test (CSAT). As a result, by following the middle-school cohort of the KEEP data, I am able to observe the path of child academic skill development throughout middle-school and high-school, and how academic skill developments relates to the child's subsequent performance on the CSAT exam. The middle-school cohort typically enter university in the fifth survey year (2008), and, due to the length of the study, I am able to observe which university the child attended as a result of their performance on the CSAT exam. Lastly, I observe whether the child graduated from university and their early labour market outcomes.

In order to fully characterise the development of a child's academic skills, and to maximise

[7]School administrators and the child's homeroom teacher were only surveyed in the first survey year (2004) and in the fourth survey year (2007).

my sample size, I use a liberal sample selection criteria that constrains the sample only by ensuring that, for each child, relevant information is available. Specifically, I require that the student is from the middle-school cohort and was present in all available survey waves, has available information on their educational attainment throughout middle-school and high-school, available information on their CSAT scores, their university enrollment outcome, their background demographics, their household demographics, their parents' educational attainment, and importantly, information on the investments in private education made by the child's parents. As a result, I obtain a final sample that includes 7,224 person-year observations with just over 600 students in each year. Table (1.1) highlights the key descriptive statistics of the KEEP sample when the child was observed in the first survey wave, i.e. when the child was in their penultimate year of middle-school.[8]

The first column of table (1.1) highlights descriptive statistics for the sample as a whole. The sample contains marginally more girls than boys, households have, on average, just over 2 children and approximately half of the children in the sample have an older sibling. Urban is a dummy variable that is equal to 1 if the household lives in an urban area, for example, Seoul, Busan, Daegu, etc ... and is equal to 0 otherwise. The majority of households in the sample live in urban areas. On average, the fathers of the children in the sample have graduated from high-school with just over 18 years of completed schooling and the average monthly household income is 3,273,000 South Korean Won (∼\$2,600 per month). The final three rows in table (1.1) use information on the students performance in their middle-school exams, specifically, the child's percentile in the subject specific distribution of standard exam scores. This information is provided by the survey of the child's homeroom teacher. I focus on three academic subjects: math, English, and Korean. This is because these subjects are taken by all students throughout middle-school and high-school, they are also key subject areas on the CSAT exam and nearly all of CSAT exam takers select these three subjects. As a result, for the remainder of the paper, I focus on the development of academic skills in these three subject areas. Trivially, the average middle-school exam score percentile is close to the 50th percentile for the sample as a whole, for

[8]While variables such as 'Urban', 'Number of Kids', and 'Household Income' can all potentially exhibit time variation, the aggregate descriptive statistics are robust across survey waves.

Table 1.1: Descriptive Statistics of the KEEP Sample

	Full Sample	*Income Quartiles*			
		Quartile 1	*Quartile 2*	*Quartile 3*	*Quartile 4*
Female	0.55	0.69	0.51	0.49	0.50
	(0.49)	(0.46)	(0.50)	(0.50)	(0.50)
Number of Kids	2.24	2.41	2.25	2.09	2.18
	(0.64)	(0.75)	(0.64)	(0.44)	(0.58)
Older Sibling	0.48	0.51	0.48	0.48	0.48
	(0.49)	(0.50)	(0.49)	(0.49)	(0.49)
Father Years of Education	18.70	17.10	18.54	18.85	20.86
	(2.90)	(2.58)	(2.60)	(2.49)	(2.36)
Urban	0.59	0.38	0.59	0.66	0.77
	(0.49)	(0.49)	(0.49)	(0.47)	(0.42)
Household Income	327.3	158.4	274.1	374.1	586.3
	(171.7)	(46.8)	(27.6)	(26.8)	(168.0)
	Middle-School Test-Score Percentiles				
Math Percentile	51.17	51.56	50.05	49.79	53.79
	(27.69)	(26.77)	(28.15)	(27.57)	(28.28)
English Percentile	51.06	46.51	52.01	53.71	52.79
	(26.95)	(26.47)	(26.74)	(26.57)	(27.83)
Korean Percentile	51.43	49.29	51.25	51.39	54.43
	(28.10)	(27.71)	(27.71)	(26.76)	(28.28)
N	602	155	178	150	119

Notes: The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education. Standard deviations in parenthesis.

all subjects, with a standard deviation between 27 and 28 for each subject.

The remaining columns (2)-(5) in table (1.1) highlight variation in the descriptive statistics across income quartiles.[9] In terms of demographics, the sample is mostly homogeneous across income quartiles. The lowest income quartile contains slightly more girls than boys relative to the entire sample. However, the number of children within the household and the proportion of children with an older sibling exhibits little variation across income quartiles. Fathers from the highest income households are more educated, on average, than fathers from the poorest

[9] In the data, households typically round their household income to the nearest ten-thousand South Korean Won, as a result, the number of students from the second quartile of the household income distribution is larger than other quartile groups.

households, and the wealthiest households disproportionately live in urban areas relative to the poorest households. The gap in monthly income between the richest and poorest households in just over 4,000,000 South Korean Won (∼\$3,500). However, the most interesting feature of table (1.1) is that there is little variation in middle-school exam score percentiles across income quartiles, and this is true for all academic subjects. Specifically, while the lowest income households perform slightly worse in English and Korean relative to the highest income households, this difference is not statistically significant. Also, the lowest income households have higher average math score percentiles than children from income quartiles 2 and 3. Since test-scores are key measures of a child's academic skills, this suggests that heterogeneity in household income does not generate significant heterogeneity in academic skills among Korean middle-school children. There are a number of reasons why this may be the case, one of which could be attributed to the success of the Korean School Equalisation Policy. However, investigating this observation is not the purpose of this paper. This observation is however important as it means that, with respect to household income, the children in my sample, on average, have similar initial levels of academic skills. Therefore, any future differences in academic skills that emerge with respect to household income, will be fully characterised within my sample period.

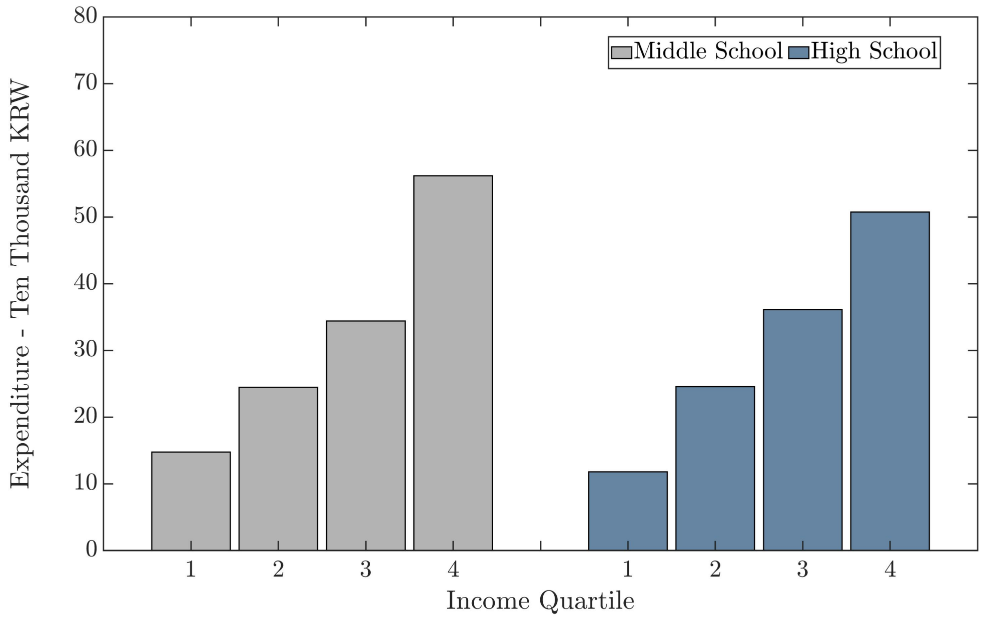
1.3.1 Parental Investment in Private Education

Section (1.2) highlighted that a key feature of academic skill development in Korea is that Korean parents invest heavily in private tuition for the children. Private tuition can take different forms such as one-to-one tutoring, group tutoring, and Internet tutoring. However, in Korea, by far the most common type of private tuition is at private tutoring academics known as 'hagwons'. Figure (1.3a) shows the variation in private education investments made by the parents in the KEEP sample across income quartiles, while figure (1.3b) shows the variation in private education participation by students in the KEEP sample across income quartiles.

The grey columns on the left of figure (1.3) correspond to expenditure and participation in private education during the first survey year, when children were in their penultimate year of middle-school. The blue columns on the right of figure (1.3) correspond to aggregate expenditure and participation in private education during the next three survey years (when the children

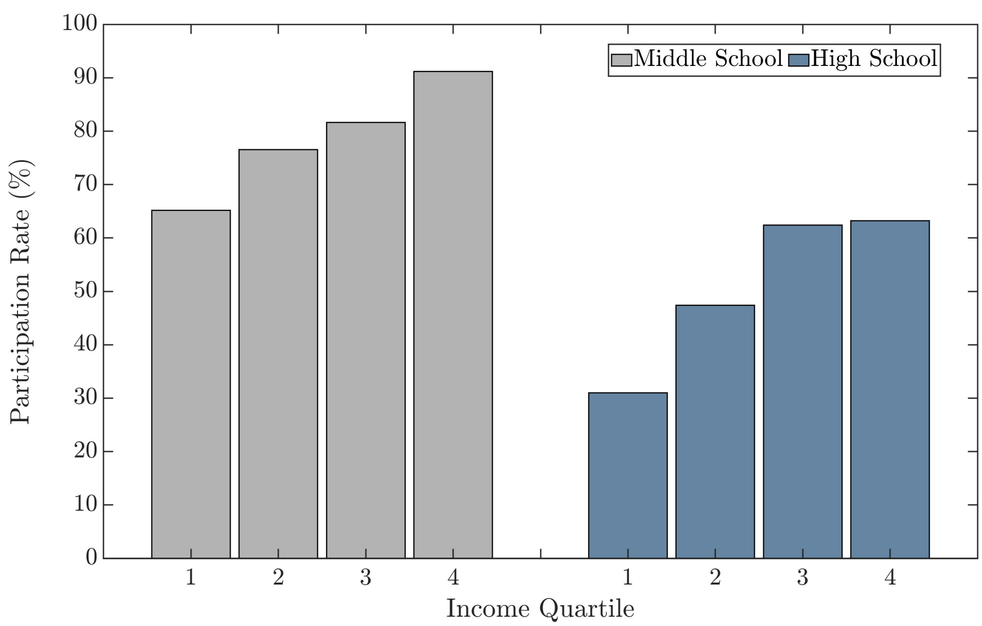
Figure 1.3: Private Education Expenditure and Participation by Income Quartile and School Level

(a) Expenditure - Ten Thousand KRW



Notes: The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

(b) Participation - %



Notes: The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

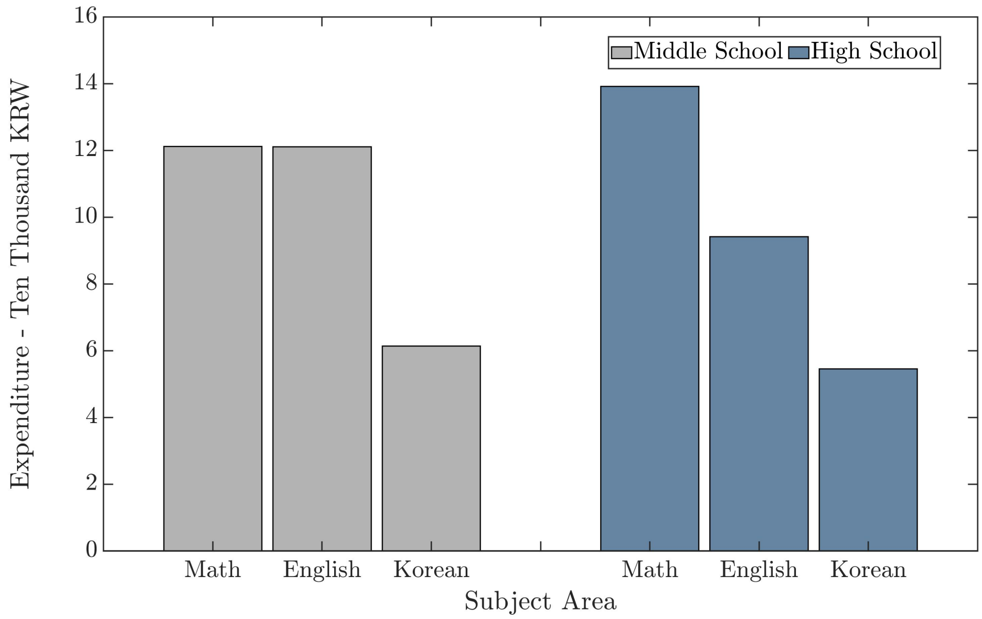
were enrolled in high-school). There are a two notable features in figure (1.3). First is that participation rates in private education are high for all income quartiles. In the penultimate year of middle-school the lowest participation rate is observed for the lowest income children, but their participation rate is still well above 60 percent. During high-school, participation rates fall relative to the rates observed during middle-school, but for the three highest income quartiles, the participation rate is still above 50 percent. The second feature of figure (1.3) is that while variation in the participation rate is less pronounced across the distribution of income, variation in private education expenditure is highly significant across the income distribution. Households in the lowest income quartile spend, on average, 150,000 KRW (∼$120) per month on private education for their child compared to 500,000 KRW (∼$400) per month spent by the richest households. Therefore, in order to match the spending of the richest households, the poorest households would instead be required to spend over a third of their monthly income on private education for their child. The large expenditures on private tuition for children in the highest income quartiles can be attributed to two factors. First, relative to children in the lowest income quartiles, high-income parents could be sending their children to higher quality, higher fee-charging private academies (the extensive margin). Second, is that, relative to children in the lowest income quartiles, high-income parents are more able to afford to send their child to the same academy, but for an increased number of hours (the intensive margin).

While figure (1.3) shows aggregate private education expenditure and participation in private tuition, a key feature of academic skill development in Korea is that entry into top universities requires proficiency in multiple academic subject areas. As a result, figure (1.4a) shows the variation in private education investments made by parents in the KEEP sample across academic subject areas, and figure (1.4b) shows the variation in private education participation by students in the KEEP sample across academic subject areas.

The notable aspects of figure (1.4) are, firstly, that participation rates are high for all subjects and, secondly, that private education spending and participation are concentrated mainly towards the subject of math. During middle-school, participation rates are above 50 percent in all subjects and the corresponding proportion remains above 30 percent throughout high-school.

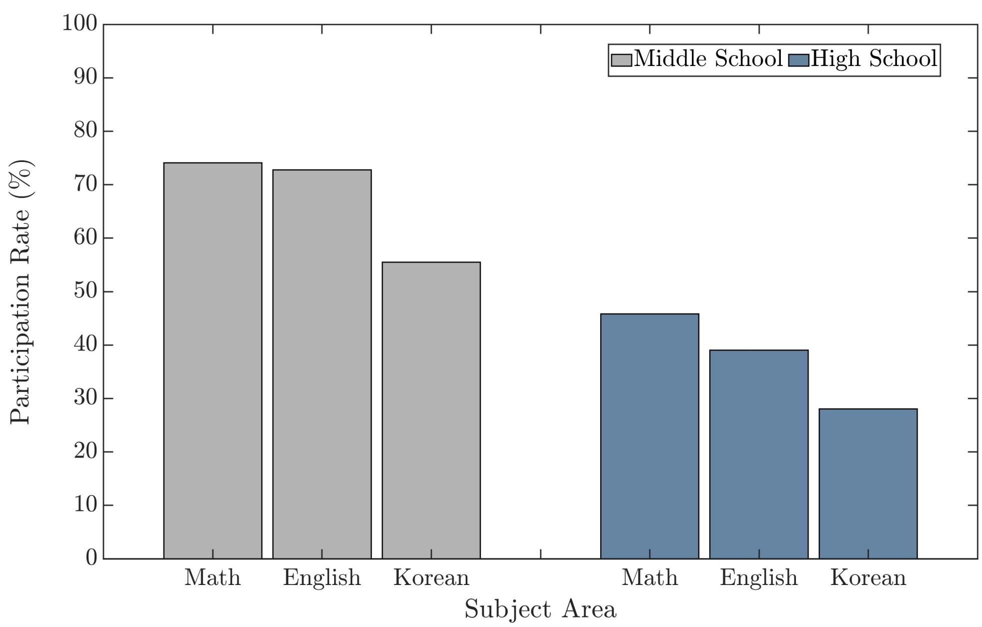
Figure 1.4: Private Education Expenditure and Participation by Academic Subject and School Level

(a) Expenditure - Ten Thousand KRW



Notes: The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

(b) Participation - %



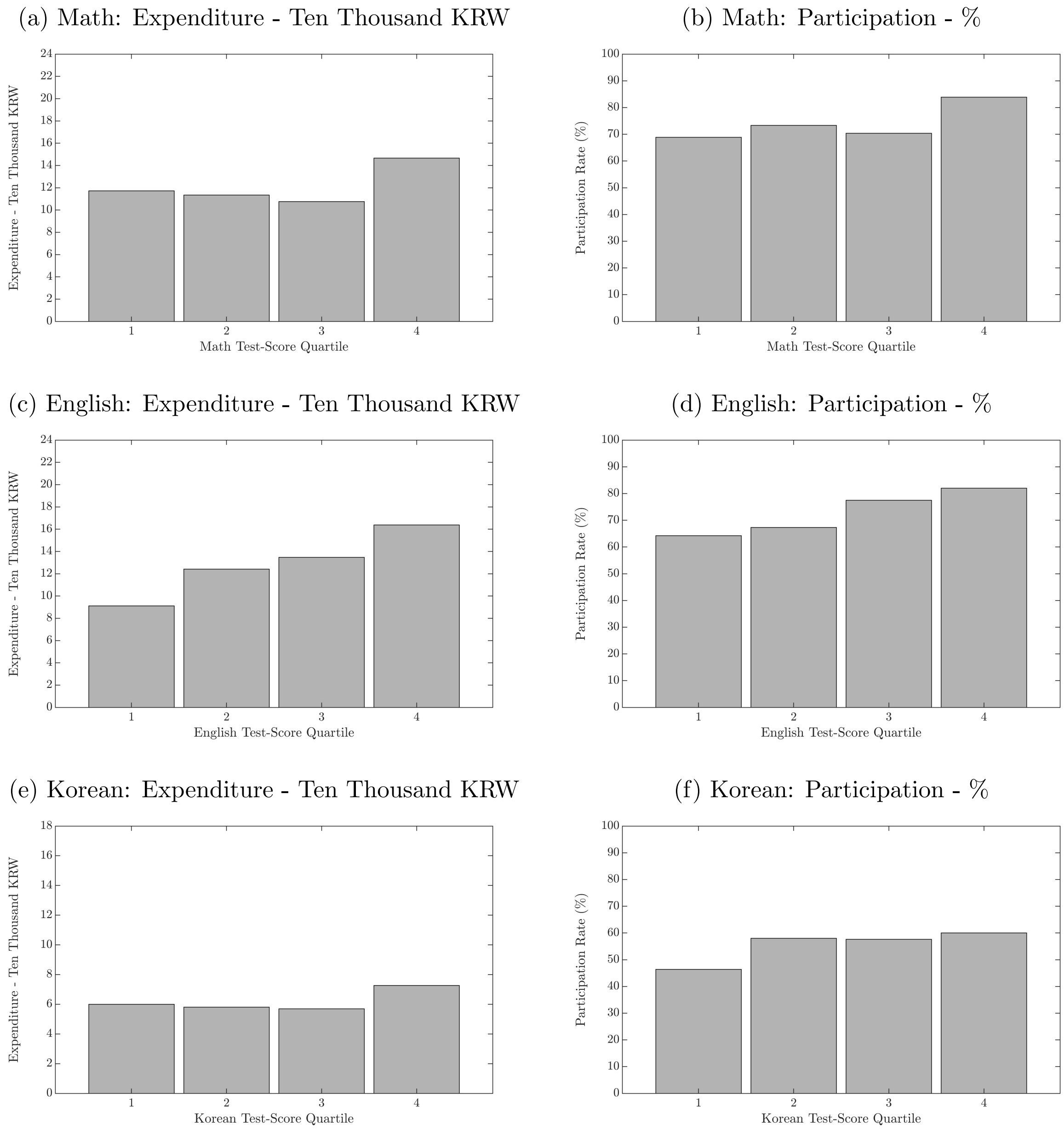
Notes: The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

Math is the subject with the highest participation rate overall. In terms of monthly expenditure, households spend the largest amount on private education in math and in English. Math expenditures are, on average, twice the amount of expenditures on Korean. Given that the average monthly household income in the KEEP sample is approximately 3,300,000 KRW, figure (1.4) suggests that households spend nearly 4 percent of their monthly income on math private tuition alone. These spending patterns are interesting because it suggests that parents value investments in math above investments in other subjects. This could be because parents consider math to be the most difficult academic subject and therefore prioritise tuition in this subject area as a result. However, it could also be that investments in private tuition for math might not only be productive for the development of a child's math skill, but also that investments in private tuition in math may exhibit spillover effects for the development of other cognate skills. Understanding whether or not this is the case is a key research question that will be answered by this paper.

The final feature of child academic skill development and parental investment in private education in Korea I highlight in this section is the extent to which competition for places at SKY university affects parental investment choices. It is often conjectured that private tuition provides a mechanism for children that are performing badly to 'catch up' with other students in their academic cohort. While this mechanism may certainly exist in Korea, figure (1.5) shows that private tuition in Korea is equally used as a mechanism to 'stay ahead' of other students. Figure (1.5) plots private education expenditure and participation rates in private tuition across middle-school test-score quartiles for each academic subject.

The first striking property of figure (1.5) is that, there is no significant variation in participation rates across middle-school test-score quartiles. In other words, for example, students that performed poorly in their middle-school math exam subsequently participate in private education in a similar magnitude to the students that achieved the highest test-scores on their middle-school math exam. This observation also remains true for English and for Korean subject areas. This observation suggests that the proportion of middle-school students that are using private tuition to 'catch up' to their peers is not dramatically different from the proportion that

Figure 1.5: Private Education Expenditure and Participation by Test-Score Quartile



Notes: The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

are using private tuition to 'stay ahead' of their peers. Indeed, participation rates for private tuition in math for the lowest and highest middle-school math test-score quartiles are both above 68 percent and the gap in participation rates between these quartiles is not statistically significant. The second important property of figure (1.5) is that for all subjects, parents of children in the highest middle-school test-score quartiles spend the largest amount on private tuition for their children. This further highlights the influence that competition for places at the top universities has on the investment choices of parents. After learning that their child is in the upper quartile of middle-school test-scores parents respond by continuing to send their child disproportionately to private education academies throughout high-school.

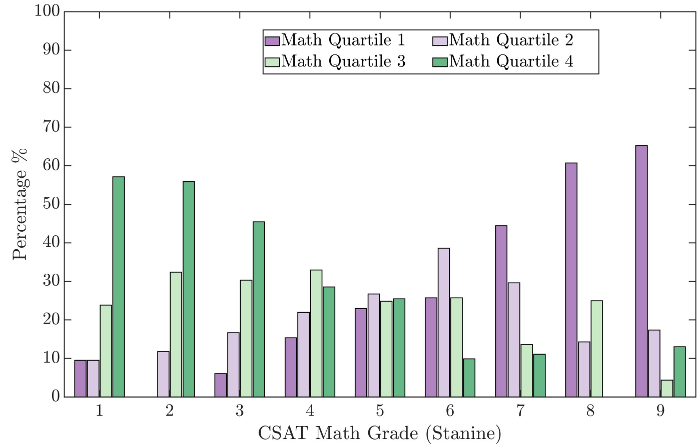
1.3.2 Academic Skill Development

In this sub-section I highlight an important feature of academic skill development which is how initial academic skills, measured by middle-school test-scores, correlate with terminal academic skills, measured by a child's CSAT test-scores. Both sets of test-scores are noisy measures of the child's true underlying academic skills. However, these measures are both observable to parents and it is important for the parent's private education investment choices to understand the implications of how heterogeneity in middle-school test-scores is linked to CSAT outcomes. For example, if my child performed badly in their middle-school exams, is it still possible for them to subsequently achieve the CSAT scores that are necessary for entry to a SKY university? Standard CSAT scores are categorised into nine groups based on a Stanine curve. Students in group 1 represent the highest standard CSAT scores and students in group 9 represent the lowest standard CSAT scores. Figure (1.6) plots, for each math, English and Korean CSAT score group, the proportion of students from each middle-school test-score quartile.

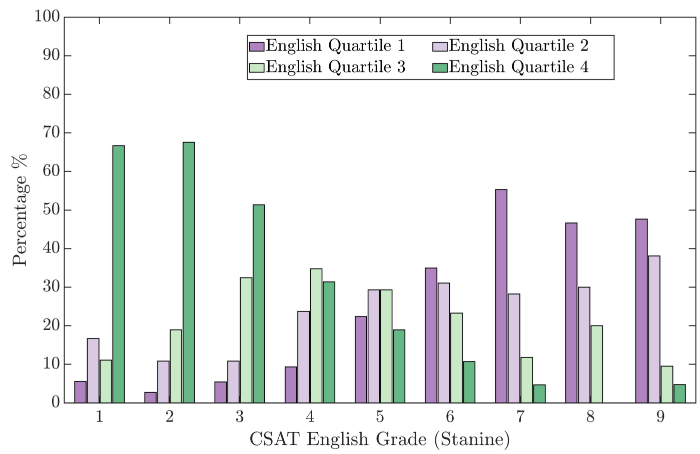
For example, in figure (1.6a), the interpretation of the first set of bars is that for all students who achieved a CSAT math grade of 1, 10 percent of these students were in the lowest quartile of middle-school math test-scores, 10 percent were in the second lowest quartile of middle-school math test-scores, 24 percent were in the second highest quartile of middle-school math test-scores, and 56 percent were in the highest quartile of middle-school math test-scores. Overall, figure (1.6) shows that there is a strong correlation between middle-school test-score performance and CSAT test-score performance. Students who were in the upper quartile of middle-school test-scores are disproportionately represented in the top 3 CSAT grades, while students who were in the lowest quartile of middle-school test-scores are disproportionately represented in the bottom 3 CSAT grades. This is interesting for a number of reasons. Firstly, although figure (1.6) suggests that a child's academic skills are, on average, persistent throughout high-school, it remains

Figure 1.6: Correlation Between Middle-School Test-Scores and CSAT Test-Scores

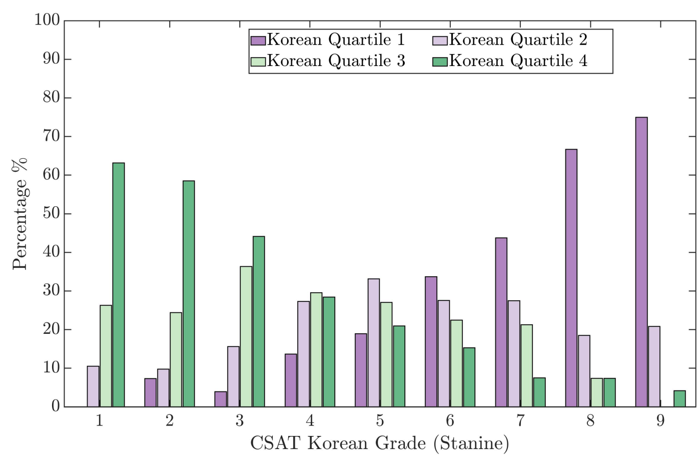
(a) Math



(b) English



(c) Korean



Notes: The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

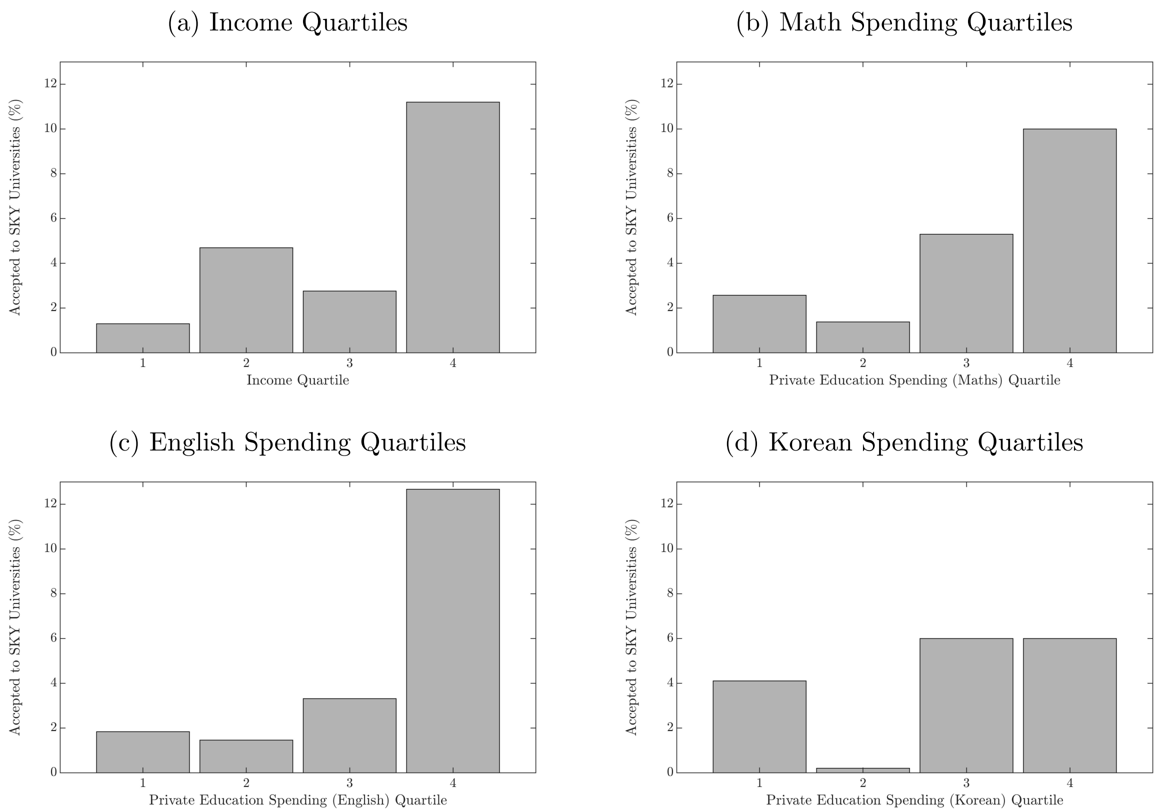
unclear which factors are driving the persistence in academic skills. In figure (1.5) we observe that the highest scoring middle-school children also invest disproportionately in supplementary private education, therefore, it could be that the persistence in academic test-scores can be attributed to the positive effect of parental investments in private tuition. Alternatively, it could be that a child's academic skills are highly self productive and that parental investments in private education only have a marginal effect on development of academic skills and subsequently on CSAT scores. Disentangling these effects is also a key research question that this paper will answer. The second interesting feature of figure (1.6) is that, although middle-school test-scores and CSAT test-scores are highly correlated, it remains possible for students to improve or to regress along the distribution of grades. For example, in math, 10 percent of students in CSAT grade 1 originally achieved middle-school math test-scores that were in the bottom quartile of the distribution. Similarly, around 15 percent of students in CSAT math grade 9 originally achieved middle-school math test-scores that were in the highest quartile of the distribution. It is this uncertainty that provides parents of children that achieved poorly during middle-school with hope that, through their investments in private education, the dream of their child attending a SKY university may still be possible.

1.3.3 Reaching for the 'SKY' and the Achievement Gap Between Rich and Poor Students

In this subsection I highlight the dimensions of heterogeneity that exist with respect to the types of students that are accepted to SKY universities, and subsequently highlight the achievement gap in test-scores between rich and poor students. Figure (1.7) shows the variation in the proportion of students from each income quartile that were accepted to SKY universities, it also shows the proportion of students from each quartile of the private education expenditure distribution that were accepted to SKY universities.

Figure (1.7a) shows that acceptance to SKY universities is highly correlated with household income. The capacity of SKY universities in total is just below 5 percent of the student population. Therefore, if household income is uncorrelated with acceptance at SKY univer-

Figure 1.7: Proportion Accepted to SKY Universities



Notes: The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

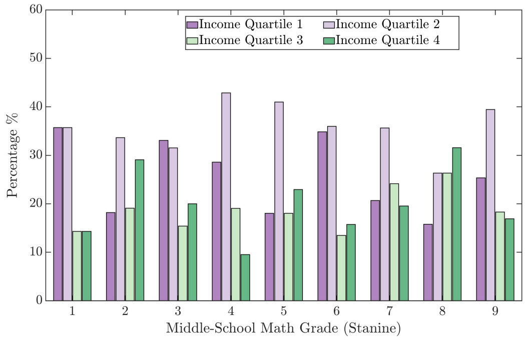
sities we would expect each bar in figure (1.7a) to equal approximately 5 percent. However, for students from the highest quartile of the income distribution, 12 percent were accepted to SKY universities. The corresponding proportion for students from the lowest quartile of the income distribution was just above 1 percent. Looking at quartiles of the private education expenditure distribution across subjects in figures (1.7b), (1.7c), and (1.7d) I observe a similar pattern. Children of parents whose expenditure on private education is in the highest quartiles of the expenditure distribution are disproportionately likely to be accepted to SKY universities, for example, 10 percent of children in the highest quartile of private educating expenditure on math are accepted to SKY universities, compared to just over 2 percent of children in the lowest quartile of private education expenditure on math. However, for private education expenditures in Korean there seems to be no significant differences in the proportion of children accepted to SKY university across spending quartiles. Overall, despite the positive correlation between expenditures on private education and acceptance at SKY universities, given that students from the upper end of the academic skill distribution are disproportionately investing in private education it remains unclear the extent to which private education expenditures are driving this pattern relative to the self-productivity and complementarities between academic skills.

In table (1.1), I showed that, with respect to household income, there were no significant differences in students middle-school test-scores across income quartiles, and that this was true for all academic subjects. Therefore, in order to show the existence of the achievement gap between rich and poor students and how the achievement gap has arisen after students graduate from middle-school, I compare the relationship between middle-school test-scores and household income, with the relationship between CSAT test-scores and household income. Figure (1.8) plots (on the left hand side) the distribution of students across income quartiles for each middle-school test-score grade, and (on the right hand side) the distribution of students across income quartiles for each CSAT test-score grade.

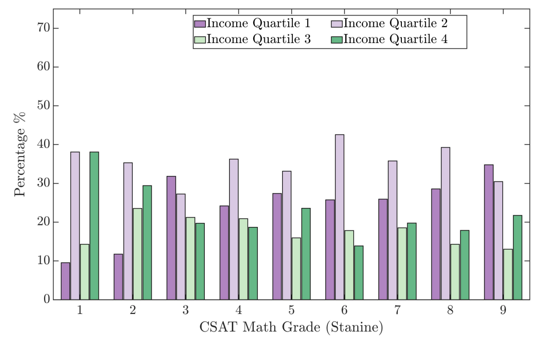
For example, looking at panel (1.8a), the interpretation is that, for students who achieved the highest middle-school math grade (grade 1), 35 percent of these students were from the lowest quartile of the income distribution, this is compared to only 15 percent of students from the highest quartile of the income distribution. Therefore, in panel (1.8a), there is no positive achievement gap between rich and poor students, in fact, the poorest students disproportionately achieved the highest grade in math during middle-school. However, when contrasting this observation with panel (1.8b) I observe that, for students who achieved the highest CSAT math grade, only 10 percent of students were from the lowest income quartile, compared to almost 40 percent of students from the highest income quartile. Therefore, over this four year period, the gap in math performance has reversed and widened dramatically. Given that SKY universities typically require all of a student's CSAT scores to be above grade 2, I also examine the achievement gap between rich and poor students with respect to grade 2 middle-school test-scores, and with respect to grade 2 CSAT test-scores. Specifically, for math, during middle-school, 18 percent of grade 2 students were from the lowest quartile of the income distribution, compared to almost 30 percent of grade 2 students that were from the highest quartile of the income

Figure 1.8: The Achievement Gap Between Rich and Poor Students

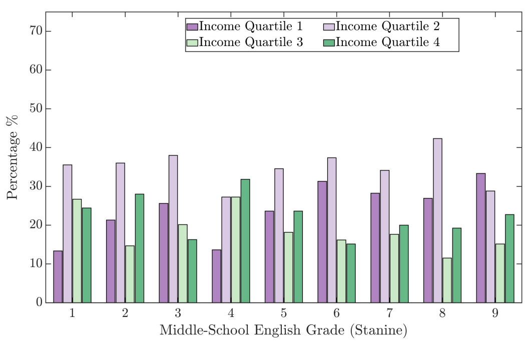
(a) Middle-School Math



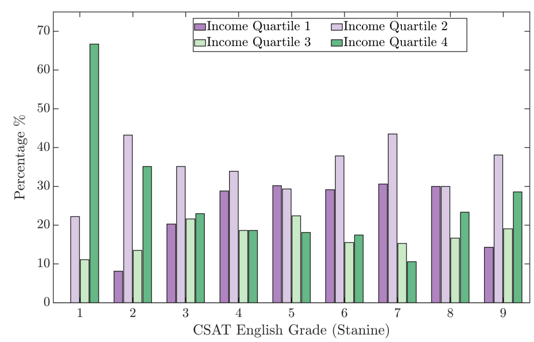
(b) CSAT Math



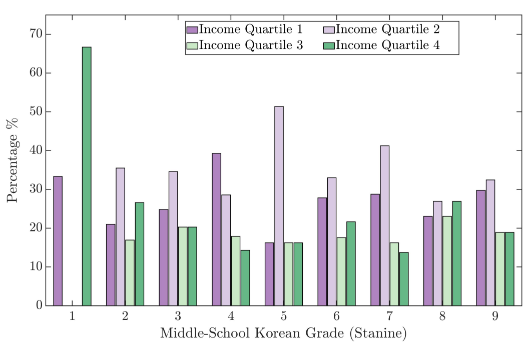
(c) Middle-School English



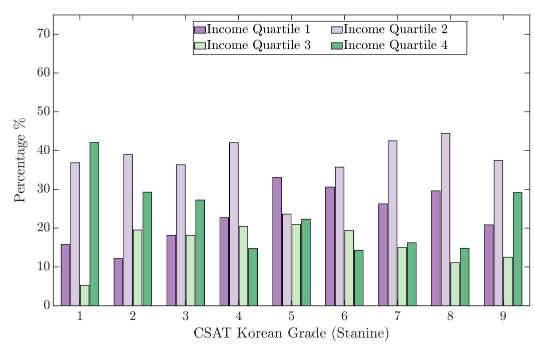
(d) CSAT English



(e) Middle-School Korean



(f) CSAT Korean



Notes: The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

distribution. However, contrasting this with CSAT math scores, just over 10 percent of grade 2 students were from the lowest quartile of the income distribution, compared to around 30 percent of grade 2 students that were from the highest quartile of the income distribution. As a result, the appearance of the achievement gap between rich and poor students is not limited to just those students in the very top percentiles of the test-score distribution, it also exists throughout all percentiles of the distribution that are crucial for entry to SKY universities. Panels (1.8c), (1.8d), (1.8e), and (1.8f) show that this observation remains true for English test-scores and also for Korean test-scores. In particular, the achievement gap between rich and poor students is most pronounced when comparing CSAT English scores.

In order to fully characterise the existence of the achievement gap between rich and poor students and to show how the gap has developed after students graduate from middle-school, table (1.2) shows, for students that achieved grades 1 and 2 on their middle-school exams and for students that achieved grades 1 and 2 on their CSAT exams, the exact proportion of students that were from the upper and lower quartiles of the income distribution.

The first panel of table (1.2) shows, for students that achieved grade 1 and grade 2 in their CSAT scores, the proportion that are from the highest quartile and the proportion that are from lowest quartile of the income distribution. For example, for CSAT math grade 1, the proportion of students from the highest quartile of the income distribution was 38 percent, whereas the proportion of students from the lowest quartile of the income distribution was only 10 percent, this generates an achievement gap between the richest and poorest students of 28 percentage points. Importantly, the first panel of table (1.2) shows that the achievement gap between the richest and poorest students' CSAT scores is statistically significant for all subjects and for all exam grades. The largest achievement gap, of 67 percentage points, is observed in CSAT English grade 1.

The second panel of table (1.2) shows, for students that achieved grade 1 and grade 2 in their middle-school test-scores, the proportion that are from the highest quartile and the proportion that are from lowest quartile of the income distribution. For example, for middle-school math grade 2, the proportion of students from the highest quartile of the income distribution was 29 percent, whereas the proportion of students from the lowest quartile of the income distribution was only 18 percent, this generates an achievement gap between the richest and poorest students of 11 percentage points. The key observation from the second panel of table (1.2) is that the achievement gap between the richest and poorest students during middle-school was not statistically significant for any subject or any test-score grade. Therefore, table (1.2), provides

Table 1.2: Quantifying the Achievement Gap Between Rich and Poor Students

	CSAT Grades					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.10	0.12	0	0.08	0.16	0.12
	(0.06)	(0.10)	(0)	(0.06)	(0.08)	(0.07)
Income quartile 4	0.38	0.29	0.67	0.35	0.42	0.29
	(0.11)	(0.10)	(0.11)	(0.11)	(0.11)	(0.10)
quartile 4 - quartile 1	0.28**	0.17**	0.67***	0.27***	0.26*	0.17*
	(0.12)	(0.08)	(0.11)	(0.09)	(0.14)	(0.09)
	Middle-School Grades					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.36	0.18	0.13	0.21	0.33	0.21
	(0.13)	(0.10)	(0.05)	(0.06)	(0.27)	(0.24)
Income quartile 4	0.14	0.29	0.24	0.28	0.66	0.27
	(0.09)	(0.12)	(0.06)	(0.07)	(0.27)	(0.26)
quartile 4 - quartile 1	-0.22	0.11	0.11	0.07	0.33	0.05
	(0.16)	(0.16)	(0.08)	(0.07)	(0.38)	(0.05)
	Accepted to SKY University (%)					
Income quartile 1	0.01					
	(0.00)					
Income quartile 4	0.11					
	(0.03)					
quartile 4 - quartile 1	0.10***					
	(0.03)					

Notes: The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education. Standard errors in parenthesis. (* $\Rightarrow p < 0.1$), (** $\Rightarrow p < 0.05$), (***$\Rightarrow p < 0.01$).

descriptive evidence, firstly, of the existence of an achievement gap between rich and poor students in terms of CSAT exam-scores, and secondly, that the achievement gap developed in the four year period after the students graduated from middle-school. The final panel of table (1.2) shows the gap in the proportion of students that were accepted to SKY universities between students in the highest and lowest income quartiles of the income distribution. The estimated gap is equal to 10 percentage points and is statistically significant.

In the next section (1.4), I introduce the structural model that I use to study the dynamic process of child academic skill development throughout adolescence in anticipation of entry to university. In the model the multi-dimensional academic skills that I include directly correspond to the academic subjects outlined in this section (i.e. math, English, and Korean).

1.4 The Model

The model begins when a child is in their penultimate year of middle-school. Each household $i \in I$ contains an adult parent and her child, and the model progresses through three distinct stages as the child transitions from adolescence into adulthood.

1.4.1 Stage 1: Development

The first stage of the model is the '*development stage*' which consists of $t \in T$ development periods that correspond to the years a child spends in middle-school and in high-school. In each development period, $t \in T$, heterogeneity across households is fully characterised by the vector of state variables, Ω, which contains the child's vector of *academic skills* $\Theta_{i,t}$, and household income $y_{i,t}$. A typical element of child i's vector of academic skills corresponds to their stock of an academic skill in a specific subject area, in development period t. This subject-specific stock of an academic skill is denoted $\theta^k_{i,t}$, where $k \in K$ indexes the subject area. For example, in the empirical application, $K = 3$, and θ^1 corresponds to the stock of a child's academic skill in math, θ^2 corresponds to the stock of a child's academic skill in English, and θ^3 corresponds to the stock of a child's academic skill in Korean. Academic skills are unobserved by the child's parent, instead, in each development period t, the child's parent observes measures of their child's true, latent, academic skills. For example, the child's parent will observe a vector of their child's test-scores, $S_{i,t}$, where each element, $s^k_{i,t}$, corresponds to child i's test-score in subject area k. Although academic skills are unobserved by the child's parent, I assume, however, that a child's parent knows the system that maps the observed measures into latent academic skills. Let $m_{j,k,t}$ denote measure j for academic skill θ^k observed in development period t. The measurement system is then given by (the i subscript is removed to ease notation):

$$m_{j,k,t} = g(\theta^k_t) + \varepsilon^\theta_{j,k,t} \tag{1.2}$$

where $\varepsilon^{\theta}_{j,k,t}$ are normally distributed measurement errors that are independent of the academic skills θ^k_t and of each other. As a result, when a child's parent observes measures of their child's true academic skills, such as their child's test-scores, through their understanding of the measurement system they can formulate expectations about their child's true academic skills, in each development period t. The parent's expectations about their child's true academic skills are important because, in each development period t, in addition to choosing the amount of household income, $y_{i,t}$ they would like to consume, denoted $c_{i,t}$, the child's parent can also choose to supplement the accumulation of her child's academic skills by investing in additional private schooling in each academic subject, k. The monetary amount invested in private schooling in academic subject k, in development period t, is denoted $I^{p,k}_{i,t}$. I assume that the child's parent has full control over private education investments in the academic skills of her child, that is, the child both accepts and fully cooperates with the investment decision of her parent. The child's parent also has the opportunity to borrow funds in each development period t. Borrowed funds can be used to help smooth consumption across development periods but can be also used by the child's parent to investment additional resources into private education for her child. The amount of loans that a child's parent chooses to borrow is denoted $L_{i,t}$. I assume that the child's parent can only borrow up to a certain percentage of household income given by the parameter λ^y. Specifically, the borrowing constraint for household i is given by:

$$L_{i,t} \leq \lambda^y y_{i,t} \tag{1.3}$$

In addition, I assume that if the child's parent takes a loan in development period t they are required to pay back the full amount of the loan in period $t+1$, and the child's parent is unable to take out a loan in the final development period ($t = T$). Once the parent's consumption, borrowing, and investment decisions, in all development periods, have been realised, at the end of the final development period ($t = T$) the child graduates from high-school and obtains a vector of *terminal academic skills*, denoted $\Theta_{i,T}$:

$$\Theta_{i,T} = \left(\theta^1_{i,T}, \theta^2_{i,T}, \ldots, \theta^K_{i,T}\right) \tag{1.4}$$

In order to understand how terminal academic skills, $\Theta_{i,T}$, are formed, we need to understand how its elements evolve throughout the middle-school and high-school development periods. I follow Cunha, Heckman and Schennach (2010) and Attanasio, Meghir and Nix (2020) and express the evolution of a child's academic skills by a series of production functions over each development period. In each development period t, the evolution of academic skills depends on the current stock of academic skills, parental investments in private education and other environmental variables (the i subscript is removed to ease notation).

$$\begin{aligned} \theta^1_{t+1} &= f^1_t(\Theta_t, I^{p,1}_t, I^{p,2}_t, \ldots, I^{p,K}_t, X_t) \\ \theta^2_{t+1} &= f^2_t(\Theta_t, I^{p,1}_t, I^{p,2}_t, \ldots, I^{p,K}_t, X_t) \\ \vdots \;\; &= \qquad\qquad \vdots \\ \theta^K_{t+1} &= f^K_t(\Theta_t, I^{p,1}_t, I^{p,2}_t, \ldots, I^{p,K}_t, X_t) \end{aligned} \tag{1.5}$$

where the vector X_t contains demographic characteristics of household i that may affect the development of a child's academic skills and also includes shocks to the academic skill development process. The technology f^k_t is strictly increasing, concave in $I^{p,k}_{i,t}$ and, twice continuously differentiable in all of its arguments. The academic skill production technologies in (1.5) define the dynamics of the academic skill accumulation process and the effect that parental investments in private schooling have on its path. Specifically, how private education investments in academic skills made by a child's parents in period t affect private education investments in academic skills in a subsequent development periods $s > t$. From Cunha and Heckman (2007) we know that *dynamic complementarity* with respect to a particular academic skill occurs when the following comparative static holds:

$$\frac{\partial^2 f^k_t(\theta^k_t, I^{p,k}_t, X_t)}{\partial \theta^k_t \partial I'^{p,k}_t} > 0, \tag{1.6}$$

Equation (1.6) says that dynamic complementary exists when an increase in the stock of an academic skill θ^k_t makes parental investments in private education in that academic skill, $I^{p,k}_t$, in development period t more productive. Academic skills will be at their highest level towards the end of the development stage. Therefore, the presence of dynamic complementarities suggests that investments in private education will be most productive for parents of children that have high stocks of academic skills towards the end of the development stage. Because low-income parents are constrained by the amount that they can invest in private education relative to high-income parents, the presence of dynamic complementarities could be an important contributor to the achievement gap in CSAT scores that I observe between rich and poor Korean children in section (1.3), particularly towards the top of the distribution of test-scores. From Cunha and Heckman (2007) we also know that *self-productivity* exists when:

$$\frac{\partial f_t(\theta_t^k, I_t^{p^k}, X_t)}{\partial \theta_t^k} > 0, \tag{1.7}$$

this says that, a increased stock of a particular academic skill in one period creates a higher stock of that academic skill in the next period. Existence of self-productivity is important in the Korean context when paired with the existence of dynamic complementarities. This is can help to explain, for example, the correlation between middle-school test-scores and CSAT test-scores in figure (1.8). In other words, the existence of dynamic complementarities provides a mechanism that can describe why investments in private education in academic skills made by parents of children at the lower end of the academic skill distribution were not effective at increasing their child's future test-scores relative to the investments in private education in academic skills made by parents of children at the upper end of the academic skill distribution.

1.4.2 Stage 2: University Admission

At the end of the final development period, once the vector of terminal academic skills $\Theta_{i,T}$ is realised, and the child has graduated from high-school, the household progresses to the second stage of the model. The second stage of the model is the 'University Admission' stage. In this stage the child takes the university entrance exam. The university entrance exam is a necessary condition for entrance to university and the child's vector of university entrance exam test-scores, S^{uni}, will determine the quality, $q \in \{h, l\}$, of the university to which a child is allocated. The notation, $q = h$, corresponds to 'high-quality' universities and the notation, $q = l$, corresponds to 'low-quality' universities. I assume that there exists a series of increasing output functions, $f^{uni}(\cdot)$, that map a child's terminal vector of academic skills, $\Theta_{i,t}$, into a vector of university entrance exam test-scores, S^{uni}, with typical element s_k^{uni}. The output functions are denoted:

$$\begin{aligned}
s_1^{uni} &= f_1^{uni}(\Theta_{i,T}, X) \\
s_2^{uni} &= f_2^{uni}(\Theta_{i,T}, X) \\
\vdots &= \qquad \vdots \\
s_K^{uni} &= f_K^{uni}(\Theta_{i,T}, X)
\end{aligned} \tag{1.8}$$

where X includes a vector of, correlated, university entrance exam test-score shocks, with variance-covariance matrix Σ^{uni}. The interpretation of the university entrance exam test-score shocks is that, for example, the child could became unwell during the exam and this would affect her performance, if this is indeed the case, then the effect of the shock on her test-scores will be correlated across academic subjects.

While obtaining a vector of test scores S^{uni} is a necessary condition for entry to a high-quality university, it is not a sufficient condition for entry. Since all children take the university entrance exam, high-quality universities will only accept the children that obtain the highest university entrance exam test-scores. This means that entry into a high-quality university arises endogenously in the model equilibrium. Specifically, in order for a child to obtain entry to a high-quality university, each element of the child's vector of test scores, S^{uni}, must satisfy:

$$s_k^{uni} \geq \bar{s}^{uni} \quad \text{for all} \quad k \in K \tag{1.9}$$

where $\bar{s}^{uni}$ is the test-score threshold that is determined in equilibrium by the following condition:

$$\bar{s}^{uni} = s \quad \text{s.t} \quad \sum_i \mathcal{I}(s_k^{uni} \geq \bar{s}^{uni} \quad \forall \quad k \in K) = N_h \tag{1.10}$$

where $\mathcal{I}(\cdot)$ is an indicator function equal to one if child i's subject specific university entrance exam score, s_k^{uni}, is greater than the threshold $\bar{s}^{uni}$ for all of her academic subjects $k \in K$, and is equal to zero otherwise. N_h is the capacity of high quality universities and is a fixed exogenous parameter.[10] I assume that the capacity of low-quality universities, N_l is equal to $I - N_h$ so that low-quality universities can accept all students that were not accepted to the high-quality universities.

Due to the equilibrium competition for places at the high-quality universities, during the development stage, when a parent is evaluating whether to invest in the development of their child's academic skills (the extensive margin), and how much to invest in the development of their child's academic skills (the intensive margin), a key determinant of their choice is the probability of acceptance into a high-quality university, $\mathbb{P}_i^h$. The crucial probability is the following joint probability:

$$\mathbb{P}_i^h = Prob\Big(s_k^{uni} + \varepsilon_k^{uni} \geq \bar{s}^{uni}\Big) \quad \text{for all} \quad k \in K \tag{1.11}$$

in other words, the joint probability that, given the realization of university entrance exam test-score shocks ε_k^{uni}, child i scores sufficiently high university entrance exam test-scores in all of her $k \in K$ subjects. Since university entrance exam test-scores are determined by a child's terminal academic skills, $\Theta_{i,T}$, a parent can seek to influence their child's probability of acceptance by making investments in private education during the development stage.

1.4.3 Stage 3: Adulthood

After all students have taken the university entrance exam, their test-scores have been realised, and they have been sorted into either high-quality or low-quality universities, the students then progress to the third stage of the model. The third stage is the 'adulthood' stage of the model. In this stage, each student graduates from the university that they attended and enters the labour market for $T^{Adult} < \infty$ periods. There are no decisions made by either the student or her parent in the adulthood stage. The household jointly consumes the labour market income that the student earns in each period. The labour market income that a student receives in the first period of adulthood is a function of their university quality q, their vector of university entrance exam test-scores S_{uni}, other environmental variables that are important for determining labour

[10] In the empirical application N_h will correspond directly to the fixed capacity of SKY universities in Korea.

market earnings, X, and an earnings shock, ε^y. A student's expected earnings in the first period of adulthood, $\mathbb{E}\big[W_1\big]$, are therefore given by:

$$\mathbb{E}\big[W_1\big] = y(q, S^{uni}, X) + \varepsilon^y \tag{1.12}$$

I assume that graduating from a high-quality university generates higher first-period earnings than graduating from a low-quality university for a student with equivalent university entrance exam scores, S^{uni} and demographics X. Specifically, in the first period of adulthood I assume that:

$$y(h, S^{uni}, X) > y(l, S^{uni}, X) \tag{1.13}$$

Then, for the remaining $T^{Adult} - 1$ periods of adulthood, I assume that the earnings profile of a student evolves according to an AR(1) process so that:

$$W_{t+1} = W_t + \varepsilon_t^w \tag{1.14}$$

Where ε_t^w are a series of i.i.d earnings shocks that are uncorrelated with W_t and with each other. Therefore, the expected lifetime-earnings for a student over the T^{Adult} periods of adulthood is given by:

$$\mathbb{E}\Big[Y(q, S^{uni}, X)\Big] = \sum_{t=1}^{T^{Adult}} \frac{W_t + \varepsilon_t^w}{(1+r)^{(t-1)}} \tag{1.15}$$

where r is the interest rate. As a result, any parental investments in private education that a parent makes during the development stage occur specifically due to altruistic preferences of the child's parent. If the child's parent chooses to invest in private education for her child during the development stage, she is choosing to trade-off her current consumption for a higher expected stock of terminal academic skills, $\Theta_{i,T}$, and subsequently, higher expected adulthood lifetime-earnings for her child, $\mathbb{E}\Big[Y(q, S^{uni}, X)\Big]$.

1.4.4 The Household's Problem

By combining the three separate stages of the model together I can construct household i's lifetime maximisation problem from the perspective of the initial development period. First, I define the set of choice variables: $X_i = \Big\{c_{i,t}, L_{i,t}, I_{i,t}^{p,1}, I_{i,t}^{p,2}, \ldots, I_{i,t}^{p,K}\Big\}_{t=1}^{T}$, then household i's problem is given by:

$$\max_{X_i} \int \Bigg\{ \textstyle\sum_{t=1}^{T} \beta^{t-1} u(c_{i,t}) + \mathbb{P}_i^h \cdot \Big[\beta^{T+1} u(c_{i,T+1}^h)\Big] + (1 - \mathbb{P}_i^h) \cdot \Big[\beta^{T+1} u(c_{i,T+1}^l)\Big] \Bigg\} d\Gamma \tag{1.16}$$

subject to

$$\begin{aligned}
&c_{i,1}(\Gamma) + \textstyle\sum_{k=1}^{K} I_{i,1}^{p,k}(\Gamma) \leq y_{i,1} + L_{i,1}(\Gamma) \\
&c_{i,t}(\Gamma) + \textstyle\sum_{k=1}^{K} I_{i,t}^{p,k}(\Gamma) \leq y_{i,t} + L_{i,t}(\Gamma) - (1+r)L_{i,t-1}(\Gamma) \quad \text{for} \quad t > 1 \\
&L_{i,t}(\Gamma) \leq \lambda^y y_t \ , \qquad L_{i,T} = 0 \\
&c_{i,T+1}^q(\Gamma) = \mathbb{E}\Big[Y(\Gamma)\Big]
\end{aligned} \tag{1.17}$$

for all $\Gamma = \{S^{uni}, \varepsilon^y, \varepsilon_t^w\}$, where $q \in \{h, l\}$, and households take as given the measurement system in equation (1.2), the academic skill production technologies in equation (1.5), the university entrance exam score output technologies in equation (1.8), and the acceptance criteria for entry to high-quality universities given in equation (1.10). In other words, household i chooses consumption, loans, and private education investments in each development period, in order to maximise their expected lifetime utility by integrating over all possible realisations of their university entrance exam test-scores and adulthood earnings shocks. I will now outline the empirical specification for each element of the strucutal model.

1.4.5 Household Preferences

I assume that the utility the household receives from consumption takes logarithmic form, specifically I assume that the utility household's recieve from consumption in period t is given by:

$$u(c_{i,t}) = \beta^{t-1} \ln(c_{i,t}) \tag{1.18}$$

where the structural parameter β corresponds to the per-period discount factor.

1.4.6 The Measurement System

Following the factor analytic approach of (Schennach, 2004), (Hu and Schennach, 2008), which was recently developed by (Cunha, Heckman and Schennach, 2010), (Attanasio, Meghir and Nix, 2020), and (Agostinelli and Wiswall, 2016), if $m_{j,k,t}$ denotes measure j for academic skill θ^k observed in development period t. I assume a semi-log relationship between the measures that are observed by the child's parents and their child's true academic skills (the i subscript is removed to ease notation).

$$m_{j,k,t} = a_{j,k,t} + \lambda_{j,k,t} \ln(\theta_t^k) + \varepsilon_{j,k,t}^{\theta} \tag{1.19}$$

where $\lambda_{j,k,t}$ is the factor loading, and $\varepsilon_{j,k,t}^{\theta}$ are normally distributed measurement errors that are independent of the academic skills θ_t^k and of each other. (Cunha, Heckman and Schennach, 2010) derive the assumptions that are necessary for the non-parametric identification of academic skills, θ_t^k, including the most general case when the mapping I assume in equation (1.19) is not known. As a result, the measurement system I employ with separable measurement errors and with a dedicated measurement system where only one academic skill loads onto a particular measure is more restrictive than is necessary for identification.

1.4.7 Academic Skill Production Technologies

I assume that a child's academic skills develop according to a CES production function. That is, the stock of a given academic skill in period $t+1$, θ_{t+1}^k is a CES function of the previous level of academic skills Θ_t, parental investments in private education in each of the child's academic skills $I^p_{k,t}$, a TFP term A_t, and a random shock $\varepsilon^f_{k,t}$.

$$\begin{aligned}
\theta^1_{t+1} &= \left[\sum_{i=1}^{K}\delta^{i,1}_{\theta,t}(\theta^i_t)^{\rho_{1t}} + \sum_{i=1}^{K}\delta^{i,1}_{I,t}(I^{p,i}_t)^{\rho_{1t}}\right]^{\frac{1}{\rho_{1t}}} A_{1,t} \\
\theta^2_{t+1} &= \left[\sum_{i=1}^{K}\delta^{i,2}_{\theta,t}(\theta^i_t)^{\rho_{2t}} + \sum_{i=1}^{K}\delta^{i,2}_{I,t}(I^{p,i}_t)^{\rho_{2t}}\right]^{\frac{1}{\rho_{2t}}} A_{2,t} \\
\vdots &= \qquad\qquad \vdots \qquad\qquad\qquad \vdots \\
\theta^K_{t+1} &= \left[\sum_{i=1}^{K}\delta^{i,K}_{\theta,t}(\theta^i_t)^{\rho_{Kt}} + \sum_{i=1}^{K}\delta^{i,K}_{I,t}(I^{p,i}_t)^{\rho_{Kt}}\right]^{\frac{1}{\rho_{Kt}}} A_{K,t}
\end{aligned} \tag{1.20}$$

where

$$A_{k,t} = exp(\delta^k_{0,t} + \delta^k_{X_t} X_t + \varepsilon^f_{k,t}) \qquad \text{for all} \qquad k \in K$$

and the following constraint on the δ parameters holds for each development period t, and for all academic skills $k \in K$:

$$\sum_{i=1}^{K}\delta^{i,k}_{\theta,t} + \sum_{i=1}^{K}\delta^{i,k}_{I,t} = 1 \tag{1.21}$$

The parameters of the CES technology (1.20) are all time dependent. $\varepsilon^f_{k,t}$ is interpreted as a shock to the accumulation of academic skills in subject k (for example, the child became unwell during the academic year). The total factor productivity term, $A_{k,t}$, depends on the demographic characteristics of household i, X_t, which, includes information on household composition, birth-order, the child's gender and parents's education, these variables are included to capture additional heterogeneity between children in the accumulation of academic skills. The ρ parameters in (1.20) determine the elasticity of substitution between the inputs in each academic skill production function. The value of this parameter is important because it determines the extent to which the productivity of parental investments in private education in academic subject k, vary with respect to parental investments in private education in subject $k' \neq k$. For example, if the ρ parameters are equal to one, the interpretation is that the academic skill production function is linear and the inputs are perfect substitutes. This means that, for example, investments in private education for a child's math skill can be perfectly substituted with investments in private education for a child's English skill, with respect to the accumulation of academic skills in subject k. Alternatively, if the ρ parameters are equal to zero, in this case, the the academic skill production functions are Cobb-Douglas and the elasticity of substitution is equal to unity. Lastly, if the ρ parameters are greater than one then the inputs in the the academic skill production functions are complementary. In this case the interpretation would be, for example, that parental investments in private education for a child's math skill has a spillover effect on parental investments in private education in English, with respect to the accumulation of skills in subject k.

1.4.8 University Entrance Exam Output Technologies

I assume that a child's university entrance exam test-scores are also determined by a CES technology. That is, the child's university entrance exam test-scores, S^{uni}, are a CES function of the child's terminal academic skills Θ_T, a TFP term B, and a vector of university entrance exam test-score shocks ε^{uni}.

$$\begin{aligned} s_1^{uni} &= \left[\sum_{i=1}^{K} \delta_{uni}^{i,1} (\theta_T^i)^{\gamma_1} \right]^{\frac{1}{\gamma_{1t}}} B_1 \\ s_2^{uni} &= \left[\sum_{i=1}^{K} \delta_{uni}^{i,2} (\theta_T^i)^{\gamma_2} \right]^{\frac{1}{\gamma_2}} B_2 \\ \vdots &= \qquad\qquad \vdots \\ s_K^{uni} &= \left[\sum_{i=1}^{K} \delta_{uni}^{i,K} (\theta_T^i)^{\gamma_K} \right]^{\frac{1}{\gamma_K}} B_K \end{aligned} \tag{1.22}$$

where:

$$B_k = exp(\delta_{0,uni}^k + \delta_{X,uni}^k X + \varepsilon_k^{uni}) \qquad \text{for all} \qquad k \in K$$

and the following constraint on the δ parameters holds for for all academic skills $k \in K$:

$$\sum_{i=1}^{K} \delta_{uni}^{i,k} = 1 \tag{1.23}$$

ε^{uni} is a vector of correlated university entrance exam test-score shocks across academic subjects, with variance-covariance matrix Σ^{uni}. The total factor productivity term, B_k, depends on demographic characteristics of household i that could also be related to the child's performance on the university entrance exam.

1.4.9 The Acceptance Probability

If university entrance exam test-scores, S^{uni}, were uni-dimensional and therefore based on a single academic subject, $(K = 1)$, the acceptance probability, $\mathbb{P}_i^h$, reduces to:

$$\mathbb{P}_i^h \;=\; Prob\Big(s_k^{uni} + \varepsilon_k^{uni} \geq \bar{s}^{uni}\big) \;=\; 1 - F\Big(\bar{s}^{uni} - s^{uni}\Big) \tag{1.24}$$

where $F(\cdot)$ is the cumulative distribution function of the test-score shocks ε^{uni}. The closed form solution for the university acceptance probability in equation (1.24) is identical to the university acceptance probability that (Myong, 2016) uses to determine whether a student is accepted to a selective university in her model. However, in my model, academic skills, and subsequently, test-scores, are multi-dimensional. Since $K > 1$, it is no longer trivial to obtain a closed form solution for the probability of acceptance to high-quality universities. In order to obtain the probability of acceptance to high-quality universities when the number of academic subjects K is greater than one, I use Sklar's Theorem (Sklar, 1959). First, I define:

$$s_k^{uni} + \varepsilon_k^{uni} \equiv \hat{s}_k^{uni} \tag{1.25}$$

Sklar's theorem (Sklar, 1959) says that for random variables $\hat{s}_1^{uni}, \hat{s}_2^{uni}, \dots, \hat{s}_K^{uni}$ with joint cumulative distribution function:

$$F(\hat{s}_1^{uni}, \hat{s}_2^{uni}, \dots, \hat{s}_K^{uni}) = Prob\Bigg(\bar{s}^{uni} \leq \hat{s}_1^{uni},\; \bar{s}^{uni} \leq \hat{s}_2^{uni},\; \dots,\; \bar{s}^{uni} \leq \hat{s}_K^{uni}\Bigg) \tag{1.26}$$

and marginal cumulative distribution functions:

$$F_k(\hat{s}_k^{uni}) = Prob\Big(\bar{s}^{uni} \leq \hat{s}_k^{uni}\Big), \qquad k = 1, 2, \dots, k \tag{1.27}$$

there exists a copula $C(\cdot)$ such that:

$$F(\hat{s}_1^{uni}, \hat{s}_2^{uni}, \dots, \hat{s}_K^{uni}) = C\Big[F_1(\hat{s}_1^{uni}), F_2(\hat{s}_2^{uni}), \dots, F_K(\hat{s}_K^{uni})\Big] \tag{1.28}$$

this result means that I can describe the joint distribution of $\hat{s}_1^{uni}, \hat{s}_2^{uni}, \dots, \hat{s}_K^{uni}$ by the marginal distributions $F_k(\hat{s}_k^{uni})$, and their inter-dependencies, given by the copula $C(\cdot)$. As a result, I obtain a closed form solution for the acceptance probability, $\mathbb{P}_i^h$, for the general case when the the number of academic subjects K is greater than one. Therefore, the acceptance probability for an arbitrary number of academic subjects ($K > 1$) is given by:

$$\mathbb{P}_i^h = 1 - C\Big[F_1(\hat{s}_1^{uni}), F_2(\hat{s}_2^{uni}), \dots, F_K(\hat{s}_K^{uni})\Big] \tag{1.29}$$

1.4.10 Adult Income

I assume that student i's expected log earnings in the first period of adulthood, $E[\ln(W_1)]$, are given by the following function:

$$E[\ln(W_{i,1})] = \zeta_0 + \zeta_1 \cdot \mathcal{I}\big(q = h\big) + \sum_{k=1}^{K} \zeta_{k+1} \cdot \ln(s_{i,k}^{uni}) + \zeta_X X + \varepsilon_i^y \tag{1.30}$$

where $\mathcal{I}\big(q = h\big)$ is an indicator function equal to one if the student graduated from a high-quality university and is equal to zero otherwise. The parameter ζ_1 therefore captures the initial wage premium a student gains by graduating from a high-quality university. $s_{i,k}^{uni}$ is the student's university entrance exam test-score in academic subject k. X is a matrix of demographic controls that are important for explaining differences in expected earnings which includes the child's gender and the region where they work. ε_i^y is the initial adulthood earnings shock for student i.

1.4.11 Model Equilibrium

Taking the capacity of high-quality universities N_h as given, the criteria used to determine the university entrance exam test-score threshold for acceptance to high-quality universities, $\bar{s}^{uni}$, is given by equation (1.10). Define the set of choice variables for each household $i \in I$

as $X_i = \Big\{c_{i,t}, L_{i,t}, I^{p,1}_{i,t}, I^{p,2}_{i,t}, \ldots, I^{p,K}_{i,t}\Big\}_{t=1}^{T}$ and define Π as the vector of structural parameters included in the model. Taking the vector of structural parameters, Π, as given, the set of all choices of all households, $\Big\{X_i\Big\}_{i=1}^{I}$, combined with the university entrance exam test-score threshold, $\bar{s}^{uni}$, is an equilibrium of the model if, all households $i \in I$ solve the following maximisation problem:

$$\max_{X_i} \quad \mathbb{E}\Bigg[\sum_{t=1}^{T+1} \beta^{t-1} u\Big(c_{i,t}(\Gamma)\Big)\Bigg] \tag{1.31}$$

subject to

$$\begin{aligned}
& c_{i,1}(\Gamma) + \textstyle\sum_{k=1}^{K} I^{p,k}_{i,1}(\Gamma) \leq y_{i,1} + L_{i,1}(\Gamma) \\
& c_{i,t}(\Gamma) + \textstyle\sum_{k=1}^{K} I^{p,k}_{i,t}(\Gamma) \leq y_{i,t} + L_{i,t}(\Gamma) - (1+r)L_{i,t-1}(\Gamma) \quad \text{for} \quad t > 1 \\
& L_{i,t}(\Gamma) \leq \lambda^y y_t \ , \qquad L_{i,T} = 0 \\
& c_{i,T+1}(\Gamma) = Y(\Gamma)
\end{aligned} \tag{1.32}$$

and where the constraint in equation (1.10), repeated below, is satisfied.

$$\bar{s}^{uni} = s \quad \text{s.t} \quad \sum_{i} \mathcal{I}(s^{uni}_k \geq \bar{s}^{uni} \quad \forall \quad k \in K) = N_h$$

I will now provide a simple algorithm that can be used to solve for the model equilibrium.

Step 1: Guess the optimal choices for each household i. This generates a set of choices that I denote $\omega_1 = \Big\{X^{\omega_1}_i\Big\}_{i=1}^{I}$ and a test-score threshold $\bar{s}^{uni}_{\omega_1}$.

If the set of choices, ω_1, and the threshold $\bar{s}^{uni}_{\omega_1}$ indeed constitute an equilibrium, in addition to the choices ω_1 generating the university entrance exam test-score threshold $\bar{s}^{uni}_{\omega_1}$, it must be also be true that, given the threshold $\bar{s}^{uni}_{\omega_1}$, the choices ω_1 solve the maximisation problem in (1.31) and satisfy the constraints given in (1.32).

Step 2: Given the test-score threshold $\bar{s}^{uni}_{\omega_1}$, I solve the maximisation problem in (1.31) subject to the constraints given in (1.32), for each household i, to find their optimal choices, denoted ω_1^*. Using the optimal choices ω_1^*, I re-calculate the university entrance exam test-score threshold, $\bar{s}^{uni}_{\omega_1}$, and the new threshold is given by $\bar{s}^{uni}_{\omega_1^*}$.

If the new university entrance exam test-score threshold $\bar{s}^{uni}_{\omega_1^*}$ is equal to the original university entrance exam test-score threshold $\bar{s}^{uni}_{\omega_1}$ i.e.

$$|\bar{s}^{uni}_{\omega_1^*} - \bar{s}^{uni}_{\omega_1}| < \epsilon \tag{1.33}$$

where ϵ is sufficiently small. This means that the choices ω_1^* and the threshold $\bar{s}^{uni}_{\omega_1^*}$ constitute an equilibrium because the optimal choices, ω_1^*, generate the threshold, $\bar{s}^{uni}_{\omega_1^*}$, and the threshold, $\bar{s}^{uni}_{\omega_1^*}$, generates optimal choices, ω_1^*. If, however, the convergence condition in (1.33) is not satisfied, then I move to step 3.

Step 3: Given the updated test-score threshold $\bar{s}^{uni}_{\omega_1^*}$, I solve the maximisation problem in (1.31) subject to the constraints given in (1.32), for each household i, to find their optimal choices, denoted ω_2^*. Using the optimal choices ω_2^*, I re-calculate the university entrance exam test-score threshold, $\bar{s}^{uni}_{\omega_1^*}$, and the new threshold is given by $\bar{s}^{uni}_{\omega_2^*}$.

If the new university entrance exam test-score threshold $\bar{s}^{uni}_{\omega_2^*}$ is equal to the university entrance exam test-score threshold in the previous step $\bar{s}^{uni}_{\omega_1^*}$ (according to the convergence criteria in (1.33)), then the choices, ω_2, and the threshold, $\bar{s}^{uni}_{\omega_2^*}$, constitute an equilibrium. If the convergence condition in (1.33) is not satisfied, then I repeat step 3 a finite number of times, N, until an equilibrium is obtained.

1.5 Estimation

The model is estimated in three distinct steps. In this section I outline the precise details of each step and how the combination of all steps allows me to identify the key parameters of the model.

1.5.1 Step 1: Academic Skill Production Technologies and University Entrance Exam Output Technologies

I estimate the parameters of the academic skill production technologies and the university entrance exam output technologies in an exogenous first step. In the model, a child's academic skills, θ_t^k, are latent and are only observed with measurement error through measures of the child's academic skills such as their subject specific test-scores. The academic skills that I consider are a child's academic skills in math, English, and in Korean.[11] As a result, all other inputs into the academic skill production technologies, for example, parent's investments in private education, $I_{i,t}^{p,t}$, are assumed to be measured without error.

The the measurement system that I use to relate measures of academic skills, $m_{j,k,t}$, with latent academic skills, θ_t^k, follows the factor analytic approach of (Schennach, 2004), (Hu and Schennach, 2008), which was recently developed by (Cunha, Heckman and Schennach, 2010), (Attanasio, Meghir and Nix, 2020), and (Agostinelli and Wiswall, 2016). The empirical specification for the measurement system that I use is shown in equation (1.19). In order to to identify the scale and the location of the latent academic skills I need to impose normalisation restrictions. A child's academic skills evolve dynamically over time and understanding any differences in the stock of academic skills that appear throughout high-school and upon taking the university entrance exam are a key outcome of interest in this paper. As a result, I normalise all academic skills only in the first development period. By normalising only in the first period I can identify the mean of a child's academic skills in all subsequent periods by assuming that growth in measures, for example, a child's test-score in math, only occurs due to growth in the child's academic skill in math. In addition, more generally, allowing for growth in latent factors, such as the academic skills used in this paper, eliminates a source of bias when estimating the parameters of the academic skill production technologies (Agostinelli and Wiswall, 2016). The scale of each academic skill is determined by setting the factor loading, $\lambda_{j,k,t}$, of a particular measure, $m_{j,k,t}$, equal to one. It is important to normalise each academic skill on the same measure in each period in order to ensure that the scale of each academic skill is comparable over time (Agostinelli and Wiswall, 2016).[12] As a result, due to the richness of the KEEP data, I am able to normalise each child's academic skills on their subject specific test-score percentile,

[11] These subjects are studied by all children throughout middle-school, high-school, and nearly all students select these subject areas on the CSAT exam, as a result, this selection of academic subjects was chosen to maximise the sample size.

[12] Therefore I am required to assume that the there is no variation in the mapping between academic skills and measures over development periods.

in each development period.

I estimate the parameters of the academic skill production technologies and the university entrance exam output technologies in three parts, following (Attanasio, Meghir and Nix, 2020). The first part involves estimating the joint distribution of all observed measures in the KEEP data, I assume that the estimated joint distribution is a mixture of normals. As mentioned in (Attanasio, Meghir and Nix, 2020), assuming that the distribution of log academic skills is a mixture of normals is important because, for example, alternatively assuming joint normality imposes that the academic skill production technologies are Cobb-Douglas. The joint distribution of log academic skills, F_θ, across all development periods t, is therefore given by:

$$F_\theta = \tau \cdot \phi(\mu_A, \Omega_A) + (1 - \tau) \cdot \phi(\mu_B, \Omega_B) \tag{1.34}$$

where $\tau \in [0, 1]$ is the mixture parameter, and $\phi(\mu, \Omega)$ is the CDF of a normal distribution with mean vector μ and variance covariance matrix Ω. Because academic skills, θ, are latent, it is not possible to directly estimate the joint distribution in equation (1.34). However, it is possible to recover the parameters of (1.34) by utilising the mapping between measures and latent academic skills. In matrix form the measurement system is given by:

$$M = A + \Lambda \ln \theta + \Sigma \varepsilon^\theta \tag{1.35}$$

where A is a vector of constants. Λ is a matrix of factor loadings that defines the mapping from specific measures to specific academic skills and also includes the normalisation restrictions required to identify the scale and metric of each academic skill. Σ is a diagonal matrix of standard deviations for the measurement errors and ε^θ is a vector of mutually independent standard normal errors. Combining the assumption that there is a semi-log relationship between measures and the child's true academic skills, the assumption that the measurement errors are normally distributed, and the assumption that the joint distribution of log academic skills is a mixture of normals, together imply that the joint distribution of measures is also a mixture of normals. The joint distribution of measures is given by:

$$F_M = \tau \cdot \phi(\Pi_A, \Psi_A) + (1-\tau)\cdot \phi(\Pi_B, \Psi_B) \tag{1.36}$$

where:

$$\begin{aligned} \Psi_A &= \Lambda^T \Omega_A \Lambda + \Sigma \; ; \qquad \Psi_B = \Lambda^T \Omega_B \Lambda + \Sigma \\ \Pi_A &= A + \Lambda \mu_A \; ; \qquad\quad \Pi_B = A + \Lambda \mu_B \end{aligned} \tag{1.37}$$

and in order to identify growth in academic skills I need to impose the initial period normalisation restriction, specifically:

$$\tau \cdot \mu_{A,t=1} + (1-\tau)\cdot \mu_{B,t=1} = 0 \tag{1.38}$$

Estimation of the parameters of the academic skill production technologies and the university entrance exam output technologies is then achieved through three simple parts.

1) Use maximum likelihood estimation (MLE) to estimate the parameters of the joint distribution of measures in equation (1.36). Specifically, τ, Π_A, Π_B, Ψ_A, and Ψ_B, can be recovered by using all available data on measures that are observed in the corresponding survey waves of the data.

2) Recover the parameters of the joint distribution of academic skills in equation (1.34). Specifically, the parameters in A, Λ, Σ, μ_A, μ_B, Ω_A, and Ω_B can be recovered from the parameters Π_A, Π_B, Ψ_A, and Ψ_B that were estimated in part 1. This is achieved through using a minimum distance algorithm that imposes the constraints in (1.37) and (1.38), in addition to the age-invariance assumptions, initial period normalisations and zero restrictions in Λ.

3) Use τ, and the recovered parameters, μ_A, μ_B, Ω_A, and Ω_B to draw a synthetic data set according to the joint distribution in equation (1.34). The joint distribution contains information on all academic skills that enter the production technologies and the university entrance exam output technologies. Using the synthetic data set it is then possible to estimate the parameters of the academic skill production technologies and the university entrance exam output technologies by non-linear least-squares regression. For the synthetic

data set, I draw 10,000 observations in order to reduce simulation error.

In the maximum likelihood estimation in the first part I use the expected maximisation (EM) algorithm of (Dempster, Laird and Rubin, 1977) which has been more recently characterised by (Peel and McLachlan, 2000) and (Arcidiacono and Jones, 2003). The EM algorithm proceeds in two steps, starting with initial values for the component means, the covariance matrices, and the mixing proportions. Then, for each observation, the algorithm computes posterior probabilities of component memberships. This step generates an n-by-k matrix, where element (i, j) corresponds to the posterior probability that observation i belongs to component j. This process is the E-step of the EM algorithm. In the second step, using the component-membership posterior probabilities as weights, the algorithm then estimates the component means, covariance matrices, and mixing proportions through maximum likelihood estimation. This process is the M-step of the EM algorithm. The algorithm iterates over these steps until convergence is achieved.

In addition to academic skills, the academic skill production technologies and university entrance exam output technologies also include parental investments in private education for each academic subject, and additional demographic controls such as fathers years of education, number of children in the household, the child's gender, and whether or not the child has an older sibling. In order to reflect the dependencies between academic skills, parental investments in private education, and demographic controls in the academic skill production technologies and the university entrance exam output technologies, the joint distribution I estimate in equation (1.36) needs to include all relevant variables. As a result, following (Attanasio, Meghir and Nix, 2020) I extend the measurement system to include the parental investments and control variables that are assumed to be measured without error. To reflect that these variables are measured without error, I set their factor loading, λ, in each development period t, to equal one, and I set the corresponding standard deviation in Σ to equal zero. The augmented joint distribution is given by:

$$F_{\theta,X} = \tau \cdot \phi\Big(\mu_A^{\theta,X}, \Omega_A^{\theta,X}\Big) + (1 - \tau) \cdot \phi\Big(\mu_B^{\theta,X}, \Omega_B^{\theta,X}\Big) \tag{1.39}$$

where X corresponds to parental investments in private education for each academic subject, and additional demographic controls that are measured without error. I obtain confidence intervals for all parameter estimates by non-parametric bootstrap over all three parts.

1.5.2 Step 2: Adulthood Income

I estimate the parameters of the adulthood earnings function in an exogenous second step. The empirical specification for student i's expected log earnings in the first period of adulthood, $E[\ln(W_1)]$, is given by equation (1.30). In order to identify student i's expected log earnings in the first period of adulthood, I use ordinary least squares (OLS) to estimate the following wage regression:

$$\ln(W_{i,1}) = \zeta_0 + \zeta_1 \cdot \mathcal{I}\big(q = h\big) + \sum_{k=1}^{K} \zeta_{k+1} \cdot \ln(s_{i,k}^{uni}) + \zeta_X X + \varepsilon_i^y \tag{1.40}$$

where $\mathcal{I}\big(q = h\big)$ is an indicator function equal to one if the student graduated from a high-quality university and is equal to zero otherwise. $s_{i,k}^{uni}$ is the student's university entrance exam test-score in academic subject k. X is a matrix of demographic controls which includes the child's gender and the region where they work, and ε_i^y is the initial adulthood earnings shock for student i. As a robustness check, I also estimate regression (1.40) by using corresponding information from the 2,000 students that were originally selected into the high-school cohort of the KEEP data set. Specifically, I use corresponding information on CSAT test-scores, university quality, and demographic controls, for students in the high-school cohort of the KEEP data set during the survey wave that they graduated from university and entered the labour market.

1.5.3 Step 3: Method of Simulated Moments

Taking the results of the first two estimation stages as given, I structurally estimate the remaining parameters of the model using the method of simulated moments (MSM). The criteria function for the method of simulated moments (MSM) estimator $\hat{\Pi}$ is given by:

$$\hat{\Pi} = \Pi : \quad \arg\min_{\Pi} \quad \left[\sum_{i=1}^{n}(m_i - \hat{m}_i(\Pi))\right]' \hat{\Sigma}'^{-1} \left[\sum_{i=1}^{n}(m_i - \hat{m}_i(\Pi))\right] \tag{1.41}$$

where

$$\hat{m}_i(\Pi) = \frac{1}{ns}\sum_{s=1}^{ns} m_i^s(\Pi). \tag{1.42}$$

$\hat{m}_i^s$ is the simulated moment for individual i in simulation s, while m_i is the direct data counterpart. The moments are constructed based on the identification arguments that I outline in the next sub-section (1.5.4) and based upon the orthogonality assumption. The weighting matrix $\hat{\Sigma}$ that I use is the two-step variance-covariance estimator of $\hat{\Sigma}$. In total there are 15 structural parameters to be estimated and I identify the parameters using 30 moment conditions.

1.5.4 Identification

I now outline the identification arguments that I use to identify the structural parameters in Π. The first parameter in Π is β, the per-period discount factor. To identify β I use household income, consumption, and investment choices. Since all households in the model have the same inter-temporal preferences, I identify β from the choices of high-income households that will not be credit constrained, specifically, by using variation in household consumption and investment choices across income quartiles.

The next parameter is λ^y, the proportion of household income that households can borrow in each development period t. Firstly, the borrowing constraint disproportionately affects the choices of low-income households and, secondly, for households that borrow the maximal amount, as the value of the parameter λ^y increases, their consumption and investment will be further reduced in subsequent development periods. As a result, in absence of the borrowing constraint, there should be no correlation between the household income and the inter-temporal marginal rate of substitution (MRS) of consumption between development periods. Consequently, I use the correlation between household income and the inter-temporal MRS as a moment condition to identify λ^y. The borrowing constraint in the model also limits the ability of low-income households to invest in private education for their child. As a result, if private education investments are effective for a child's academic skill development and increasing their subsequent university entrance exam test-scores, then the extent of borrowing constraints could potentially drive selection bias into high-quality universities across income quartiles. If this is the case, I would observe a disproportionate amount of low-income, high-initial-academic-skill students among those that did not make it to high-quality universities, relative to high-income, high-initial-academic-skill students that did not make it to high-quality universities. Therefore, I use the corresponding proportions as an additional moment condition that helps to identify λ^y.

The next set of parameters are the variance terms for the distribution of shocks that affect the accumulation of academic skills. Specifically, each academic skill accumulation shock follows a normal distribution, $\varepsilon^f_{k,t} \sim N(0, \sigma^{2,f}_{k,t})$, with mean zero and variance $\sigma^{2,f}_{k,t}$. I identify each variance parameter by using data on the child's measures of their academic skills, such as the child's subject specific test-score percentiles, in each development period. Since measures map into academic skills via the measurement system, and since the parameters of the measurement system are estimated in an exogenous first step (and taken as given), in the model the $\varepsilon^f_{k,t}$ shocks shift students' predicted levels of academic skills. As a result, in turn, I can therefore use the $\varepsilon^f_{k,t}$ shocks to minimise the distance between the measures of academic skills that I observe in the data, and the measures of academic skills that are predicted by the model, through their relationship that is given by the measurement system.

For the shocks that affect the mapping of academic skills into university entrance exam test-scores, ε^{uni}_k. I assume that the vector of shocks follows a multivariate normal distribution with mean vector, μ^{uni}, with each element equal to zero, and variance-covariance matrix, Σ^{uni}. In order to identify the variance and covariance parameters in Σ^{uni} I use data on the child's university entrance exam test-scores in each academic subject area. In the model, the ε^{uni}_k shocks shift the students' predicted university entrance exam test-scores, as a result I use the ε^{uni}_k shocks to minimise the distance between the subject specific CSAT exam test-scores that I observe in the data, and the university entrance exam test-scores that are predicted by the model.

The final parameter that I estimate is the variance of the earnings shocks, $\sigma^{2,w}$. I assume that the earnings shocks, in each period of adulthood, $t^{adult} \in T^{adult}$, are drawn from a normal distribution, specifically, $\varepsilon^w_t \sim N(0, \sigma^{2,w})$, with mean equal to zero and variance equal to $\sigma^{2,w}$. To identify $\sigma^{2,w}$ I use available information on students' earnings after they graduate from university. For the majority of students in the sample, I observe their labour market outcomes for the first three years after they graduate. The parameter $\sigma^{2,w}$ therefore helps me to fit the model predicted distribution of earnings to the distribution of earnings that I observe in the data.

Lastly, given the available data I am unable to precisely identify the interest rate parameter r. Consequently, I choose to set the parameter r equal to the level that was set by the Korean Central Bank during the time period that corresponds to the development periods in the model. This is the period 2004-2008, during this time the interest rate in Korea fluctuated between 3.25% and 5%, as a result, I set the parameter r to equal to 4.5%.

1.6 Results

1.6.1 Model Fit

Figures (A.1) - (A.7) in appendix (2.7) show the model fit based on the model predictions of the corresponding descriptive statistics that were presented in section (1.3). In general, the model fits the data reasonably well. In terms of aggregate expenditure on private education across income quartiles, the model slightly over-predicts the amount spent by households during the middle-school period and slightly under-predicts the amount spent by households during the high-school period. This is due to the combination of the self-productivity of academic skills and the presence of dynamic complementarities with respect to parental investments in academic skills that exist in the model. However, the model predicts aggregate participation in private education across income quartiles well. The model also does a good job of capturing the expenditure and participation patterns in private education across academic subjects and across the distribution of initial middle-school test-scores.

The correlation between initial test-scores and university entrance exam test-scores is slightly more pronounced in the model than I observe in the data. Specifically, in the model, all children that achieved the lowest university entrance exam test-score grade also initially achieved the lowest middle-school test-score grade. This is because, in the model, investments in private education are highly dependent on a child's probability of acceptance to high-quality universities, if this probability is close to zero, the child's parents will find it optimal to not invest heavily in private education for their child. Therefore, it becomes less likely for their child to move up the test-score distribution as a result. In addition, generally, the children that achieved the lowest initial middle-school test-score grades will disproportionately have the lowest initial values of the acceptance probability.

Lastly, the model does a good job of matching the achievement gap between rich and poor students, both in terms of university entrance exam test-scores, and in terms of the proportion of children that are accepted to high-quality universities. Table (B1) in appendix (2.7) shows that the model generates a statistically significant gap between the proportion of students from the lowest quartile of the income distribution and the proportion of students from the highest quartile of the income distribution that achieved grade 1 in the university entrance exam. This gap occurs in the model for all academic subjects. The model also generates a statistically significant gap between the proportion of students from the lowest quartile of the income distribution and the proportion of students from the highest quartile of the income distribution that achieved grade 2 in the university entrance exam for math and for English subjects. The size of the gap generated by the model is, in general, slightly lower than the equivalent gap that I observe in the data, and this is true for nearly all academic subjects and test-score grades. This result can also be attributed to the probability of acceptance to high-quality universities included in the model. The model assumes that all parents can infer this probability perfectly, as a result, parents in my model face less uncertainty regarding their investment choices than would be true in reality. Therefore, low-income parents are slightly more likely to invest in private education for their children than I observe in the data, and they also invest more in private education than the low-income parents that I observe in the data. This also explains why the gap between the proportion of children accepted to SKY universities from the lowest and highest income quartiles predicted by the model is marginally lower than the equivalent gap that is observed in the data.

1.6.2 Academic Skill Production Functions

In order to estimate the academic skill production functions I assign the measures that I observe in the data to latent academic skills. Specifically, I use a dedicated measurement system which states that each measure can only be dependent on a particular academic skill. Table (1.3) shows the assignment of measures to academic skills. In each cell I report the signal to noise ratio which, given the measurement system in equation (1.2), captures the information content of each measure. The signal to noise ratio is given by:

$$\xi^{\ln(\theta_{k,t})} = \frac{(\lambda_{j,k,t})^2 \cdot Var(\ln \theta_{k,t})}{(\lambda_{j,k,t})^2 \cdot Var(\ln \theta_{k,t}) + Var(\varepsilon^{\theta}_{j,k,t})} \tag{1.43}$$

I observe the same measures for each academic skill in each development period. Specifically, for each academic skill the measures that I observe and subsequently use are: the child's subject specific test-score percentile in development period t, the child's subject-specific, self-reported, self-study hours in development period t, and the child's self-reported aptitude, or skill level, in the specific academic subject in development period t. The child's subject specific test-score percentile is reported in the data by the child's school homeroom teacher. The child's subject-specific, self-reported, self-study hours do not include hours spent in private education, therefore this is a measure that specifically captures any additional study hours that the child has devoted to academic subject k. This measure could be linked to a child's academic skills because children with increased self-study hours could have higher levels of academic skill in that specific subject area, relative to children with no self-study hours in that subject. Alternatively, it could be that children use increased self-study hours as a mechanism to catch up on a particular academic skill relative to their peers. The child's self-reported skill in the specific academic subject is answered on a scale of 1-5, where 1 indicates that the child is 'low-skilled' and 5 indicates that the child is 'high skilled' in that subject area. In order to make the comparisons of latent academic skills consistent over time I follow (Agostinelli and Wiswall, 2016) and normalise academic skills on the same measure in each development period. The measure that I use for the normalisation is the child's subject specific test-score percentile in development period t.

Table (1.3) indicates that there is significant variation in the information content of measures both within and across academic skills. The signal to noise ratio for a child's subject specific test-score percentile is above 71% for all academic skills in all development periods. This highlights that the child's subject specific test-score percentile contains a large proportion of information

Table 1.3: The Information Content of Measures of Academic Skills

	Development period		
	Initial period	*Middle-school*	*High-school*
		Math Skill - θ_t^1	
Test-score percentile	79%	82%	85%
Hours of self study	5%	7%	20%
Self-reported skill	28%	22%	25%
		English Skill - θ_t^2	
Test-score percentile	71%	85%	89%
Hours of self study	10%	7%	9%
Self-reported skill	20%	14%	24%
		Korean Skill - θ_t^3	
Test-score percentile	79%	82%	81%
Hours of self study	1%	1%	3%
Self-reported skill	12%	5%	8%

Notes: The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education. Test-score percentiles are in the range (1,99), and are reported by the child's homeroom teacher. Hours of self study are subject-specific and are self-reported by the child. Self-reported skills are on a scale of 1-5 and is self reported by the child.

regarding a child's academic skill in that subject area, however, it also shows how important it is to take into account the presence of measurement error. For example, if I were to use the child's test-score percentile alone to estimate the academic skill production function, this could introduce a serious bias to the parameter estimates. In addition, the child's self-reported hours of self-study are extremely noisy measures of their academic skills, specifically, for a child's Korean skill, the information content of the measure in the initial and middle-school periods is only 1%. The child's self-reported skill is a less noisy measure and contains an information content between 20% and 30% for both math and English academic skills.

After the assignment of measures to latent academic skills I use maximum likelihood estimation (MLE) to estimate the parameters of the joint distribution of the measures that is shown in equation (1.36). The mixing parameter τ is estimated to be 0.59, with a 90% confidence interval of $[0.56, 0.62]$. The value of the estimated mixing parameter, τ, and the variation in estimated means, variance, and covariance terms across the mixtures suggests that the joint distribution is significantly different from a normal distribution.[13] Next, I use the parameter estimates to recover the parameters of the joint distribution of academic skills in equation (1.34). Finally, I use τ, and the recovered parameters, μ_A, μ_B, Ω_A, and Ω_B to draw a synthetic data set according to the joint distribution in equation (1.34). The joint distribution contains information on all academic skills that enter the production technologies and the university entrance exam output technologies. Using the synthetic data I then estimate the parameters of the academic skill production technologies and the university entrance exam output technologies by using non-linear least-squares regression.

The empirical specification for the academic skill production technologies is shown in equation (1.20). The parameter estimates in the academic skill production technologies characterise the development of academic skills throughout middle-school and high-school, allowing for complementarities between all inputs. The parameter estimates for the academic skill production technologies are shown in table (1.4). The academic skill production functions are all close to Cobb Douglas and the estimated substitution elasticity is not significantly different from one (except for math skill during high-school) which suggests that the inputs into the academic skill technologies are complements. In addition to implying complementarity of the inputs, this also suggests that the coefficients of the inputs can be interpreted as elasticities.[14]

Math Skill. The estimated elasticities for a child's math skill suggest that math, in general, is highly self-productive, this result was also found by (Cunha, Heckman and Schennach, 2010) and (Attanasio, Meghir and Nix, 2020) for a child's cognitive skills in general. The child's math skill also becomes more self-productive during high-school than it was during the initial middle-

[13] The estimates of the mean for each mixture and the variance-covariance matrix are omitted due to their size but are, however, available upon request. These will be published in an online appendix with the data replication files.

[14] I have also estimated the model relaxing the assumption of constant returns to scale (CRS). However, in all cases the scale coefficient was estimated to be very close to one. I maintain the CRS specification as a result.

Table 1.4: Academic Skill Production Technology Parameter Estimates

	Math Skill - θ_t^1		English Skill - θ_t^2		Korean Skill - θ_t^3	
Age	Middle-school	High-school	Middle-school	High-school	Middle-school	High-school
			Lagged Skills			
Math Skill - θ_{t-1}^1	0.231	0.553	0.121	0.208	0.133	0.165
	[0.216,0.280]	[0.537, 0.572]	[0.078,0.136]	[0.161,0.214]	[0.115,0.165]	[0.151,0.194]
English Skill - θ_{t-1}^2	0.139	0.114	0.234	0.426	0.158	0.209
	[0.056,0.158]	[0.097, 0.132]	[0.208,0.256]	[0.395,0.440]	[0.149,0.210]	[0.190,0.242]
Korean Skill - θ_{t-1}^3	0.126	0.156	0.151	0.287	0.242	0.490
	[0.109,0.192]	[0.146, 0.171]	[0.138,0.208]	[0.250,0.297]	[0.199,0.257]	[0.480,0.522]
			Parental Investments - Private Education			
Maths - $I_t^{p,1}$	0.207	0.039	0.199	0.033	0.188	0.019
	[0.181,0.229]	[0.013,0.056]	[0.180,0.258]	[0.008,0.067]	[0.164,0.228]	[0.004,0.030]
English - $I_t^{p,2}$	0.159	0.045	0.170	0.016	0.130	0.033
	[0.077,0.180]	[0.012,0.065]	[0.140,0.190]	[0.002,0.058]	[0.108,0.155]	[0.003,0.049]
Korean - $I_t^{p,3}$	0.138	0.093	0.120	0.031	0.149	0.083
	[0.093,0.179]	[0.067,0.117]	[0.094,0.133]	[0.006,0.057]	[0.134,0.164]	[0.032,0.099]
			TFP - Demographic Controls			
Log TFP	3.931	0.674	3.617	3.587	3.720	0.522
	[1.090,3.980]	[0.546,0.831]	[1.626,4.335]	[0.118,4.967]	[0.866,4.091]	[0.352,5.145]
Rank	-0.007	0.002	-0.006	0.002	-0.007	0.004
	[-0.010,-0.006]	[0.001,0.003]	[-0.007, -0.005]	[0.001,0.003]	[-0.008,-0.006]	[0.002,0.005]
Income	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
	[-0.001,-0.000]	[-0.000,-0.000]	[-0.000, -0.000]	[-0.000,0.000]	[-0.000,-0.000]	[-0.000,-0.000]
Number of Kids	0.058	-0.079	0.008	-0.038	0.036	-0.039
	[0.007,0.094]	[-0.105,-0.046]	[-0.025, 0.045]	[-0.073,-0.008]	[-0.000,0.061]	[-0.071,-0.004]
Older Sibling	-0.105	0.120	-0.043	0.147	-0.102	0.117
	[-0.160,-0.069]	[0.085,0.145]	[-0.081,0.013]	[0.117,0.176]	[-0.134,-0.057]	[0.071,0.155]
Urban	-0.189	0.012	-0.220	-0.041	-0.175	-0.068
	[-0.240,-0.143]	[-0.015,0.057]	[-0.263,-0.150]	[-0.064,0.025]	[-0.215,-0.115]	[-0.096,-0.007]
Father yrsed	0.003	-0.016	0.008	-0.002	0.004	-0.008
	[-0.020,0.016]	[-0.023,-0.011]	[-0.000, 0.019]	[-0.010,0.005]	[-0.006,0.015]	[-0.015,0.000]
Female	-0.032	-0.038	0.177	-0.001	0.186	-0.032
	[-0.088,-0.001]	[-0.075,-0.014]	[0.142,0.209]	[-0.049,0.030]	[0.141,0.231]	[-0.064,0.000]
			Production Function Structure			
ρ	-0.000	0.375	-0.000	-0.000	-0.000	0.189
	[-0.000,0.000]	[0.335,0.402]	[-0.000,0.000]	[-0.000,0.000]	[-0.000,0.000]	[-0.000,0.216]
Subst. Elast.	1.000	1.599	1.000	1.000	1.000	1.233
	[1.000,1.000]	[1.504,1.672]	[1.000,1.000]	[1.000,1.000]	[1.000,1.000]	[1.000, 1.276]

Notes: 90% confidence intervals based on 100 bootstrap replications in square parentheses. "Subst. Elast.": Elasticity of Substitution $\frac{1}{1-\rho}$. All skills are normalised on standard test score percentiles in each development period (standard test score percentiles are between 0-99). The unit of measure for parental investments in private education is 10,000 South Korean Won (KRW). The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

school period. This means that interventions that occur during middle-school, or even prior to middle-school, will exhibit some form of fade out as the child progresses throughout high-school, but it also means that their academic skill in math will become increasingly persistent over time.[15] As a result, the timing of interventions designed to aid the accumulation of academic skills is highly important and I will highight this further in the counterfactual policy analysis in section (1.7).

In addition to the self-productivity of the child's math skill, one of the key results of this paper is the effect that the child's cognate academic skills have on the accumulation of their math skill. Understanding and taking into account these academic skill complementarities is crucial for the understanding and the implications of policy interventions. Specifically, for the child's math skill, the complementarities with respect to other academic skills are quite large. The estimated elasticites for the accumulation of a child's math skill with respect to their English skill and with respect to their Korean skill are approximately half the magnitude of the estimated own elasticitiy of the math skill itself. This suggests that while a child's math skill is indeed self-productive, having increased levels of cognate academic skills will facilitate the development of a child's math skill further. This, therefore, also suggests that interventions designed to target, for example, a child's academic skill in English, will actually exhibit a non-trivial degree of spillover onto the accumulation of the child's academic skill in math. Importantly, the magnitude of the academic skill complementarities for math remain similar across both development periods.

Next I outline the effect that parental investments in private education in specific subject areas have on the accumulation of a child's math skill. The key result is that parental investments in private education do indeed have a large effect on the accumulation of a child's academic skill in math. This provides an explanation for why private education investments are commonplace in Korea. Unsurprisingly, parental investments specifically in a child's math skill have the largest effect with an estimated elasticity close to 0.2, however the estimated elasticities with respect to investments in private education in the child's English skill and investments in private education in the child's Korean skill are only marginally smaller in magnitude. Therefore,

[15]See (Walker et al., 2005) and (Andrew et al., 2018) for examples of early interventions that exhibit fade out with age of the child.

due to the presence of complementarities that private education investments in English and Korean skills have with respect to the accumulation of a child's academic skill in math, this means that when parents invest in private education for their child in a cognate subject, their investment also spills over and aids the accumulation of their child's academic skill in math. The productivity of parental investments in private education both directly, through investments in math, and indirectly, through investments in cognate subjects, is a critical result in terms of understanding the achievement gap between rich and poor students in Korea, especially since high-income households have a comparative advantage with respect to the investments that they can make in private education for their child. As a result, interventions that, for example, help to subsidise private education investments for low-income households could be effective at narrowing the observed achievement gap. However, it is important to note that parental investments in private education in Korea are also driven by the equilibrium competition for places at the top universities, as a result, interventions such as subsidisation could turn out to be very costly for policymakers and have reduced effectiveness if high-income households respond by proportionately increasing the private education investments that they make. Understanding and evaluating the extent to which this is the case is a key objective of section (1.7). It is also important to note that the elasticities with respect to private education investments are lower during the high-school period than in the middle-school period, this means that, firstly, parents will allocate their investment spending disproportionately to the middle-school period (this is true both in the data and in the model), and secondly, that parents of children that have fallen behind during middle-school are required to spend disproportionately more during the high-school period if they would like their child to catch up.

English Skill. For a child's English skill the results largely mirror the results that are observed for the child's math skill. The child's English skill is highly self-productive and is observed to be more self-productive during high-school than during the initial middle-school period. While the estimated own elasticities of the child's English skill are approximately 0.2 and 0.4 in the middle-school and high-school periods respectively, the estimated elasticities with respect to cognate academic skills are approximately half in magnitude. As a result, a child's English skill also exhibits a significant degree of complementarity with respect to their math and Korean academic skills.

Parental investments in private education are also highly productive for the accumulation of a child's English skill, however, surprisingly, the estimated elasticity with respect to private education investments in English, 0.170, is smaller than the estimated elasticity with respect to math, 0.199. This means that for a proportionate increase in private education investments, the return on investments in math is higher than the return on investments in English for the accumulation of a child's English skill. This result is important because households spend similar amounts on private education in math and as they do in English. Therefore, it suggests that interventions that target specific academic skills, for example, a subsidy to low-income households that can only be used for private education investments in a child's math skill, could be more effective, both in terms of cost and productivity, than a general blanket subsidy to low-income households that they can then allocate across investments in academic skills themselves. This is another key hypothesis that will be evaluated in section (1.7).

Korean Skill. Lastly, the general results that I observe for a child's Korean skill are highly similar to the results that I observe for the child's academic skills in math and in English, both in terms of the self-productivity of the child's Korean skill and the estimated complementarities with respect to other academic skills. Parental investments in private education for the child's Korean skill are also effective and have estimated elasticities that are similar in magnitude to those observed for the other academic skills. During the high-school period the estimated elasticities with respect to private education investments in a child's Korean skill are higher than the corresponding elasticities for private education investments in math and in English, this is true not only for the accumulation of a child's Korean skill, but also for the accumulation of their skill in math. However, whilst this suggests that the return on a proportionate investment will be highest for private education investments in Korean, it is worth noting that, during the high-school period, households invest significantly less in Korean than they invest in other academic skills.

1.6.3 University Entrance Exam Output Functions

The empirical specification for the university entrance exam output technologies is shown in equation (1.22). The parameter estimates in the university entrance exam output technologies characterise the relationship between a child's terminal academic skills and their university entrance exam test-scores, allowing for complementarities between all terminal academic skills. The parameter estimates for the university entrance exam output technologies are shown in table (1.5). The university entrance exam output functions are all close to Cobb Douglas with an estimated substitution elasticity just above one, this suggests that the inputs into the university entrance exam output technologies are highly complementary. Interestingly, the coefficients on the child's terminal academic skill in math are highest not only for the child's math score itself, but also for the child's scores in all other subject areas. Given that all academic skills are normalised on the same measure in each development each period, there is not significant variation in the aggregate average level of terminal skills across academic subjects. This means that, for a child with average levels of terminal skills in each academic subject area, a proportionate increase in her math skill will be more productive in terms of increasing her test-scores in all subjects, than would be a proportionate increase in her other academic skills.

The other important result is in degree of complementarity between the terminal academic skills. We know that in order for a child to be accepted to a high-quality university, they are required to obtain sufficiently high test-scores in all academic subject areas. Given that the estimated parameters on terminal academic skills are similar in magnitude across subject areas, this suggests that in order for a child to indeed perform well, in terms of test-scores, in all of her academic subjects, this can be facilitated by entering the exam with a portfolio that contains high levels of terminal academic skills in all subject areas. Whilst this result may seem trivial from an ex-ante point of view, it is important to note that the reason behind this result is not simply because high-levels of an academic skill, in a specific subject area, translate into a high test-scores in that same subject area, the result is instead driven by the estimated complementarities that exist between terminal academic skills with respect to university entrance exam test-scores.

Table 1.5: University Entrance Exam Production Technology Parameter Estimates

	Math Score - s_1^{uni}	English Score - s_2^{uni}	Korean Score - s_3^{uni}
		Terminal Academic Skills	
Math Skill - θ_T^1	0.389	0.380	0.412
	[0.374,0.399]	[0.372,0.396]	[0.399,0.422]
English Skill - θ_T^2	0.316	0.317	0.318
	[0.312,0.334]	[0.304,0.329]	[0.309,0.332]
Korean Skill - θ_T^3	0.295	0.303	0.271
	[0.278,0.303]	[0.286,0.313]	[0.257,0.279]
		TFP - Demographic Controls	
Log TFP	2.064	2.062	2.089
	[1.580,2.827]	[1.611,2.826]	[1.604,2.847]
Rank	0.008	0.008	0.009
	[0.007,0.009]	[0.008,0.010]	[0.008,0.010]
Income	-0.000	-0.000	-0.000
	[-0.000,-0.000]	[-0.000,-0.000]	[-0.000,-0.000]
Number of Kids	-0.025	-0.013	-0.008
	[-0.004,0.040]	[-0.023,0.041]	[-0.014,0.047]
Older Sibling	0.004	-0.015	-0.016
	[-0.011,0.031]	[-0.031,0.013]	[-0.032,0.018]
Urban	0.019	0.012	0.044
	[0.008,0.028]	[0.004,0.051]	[0.026,0.084]
Father yrsed	-0.018	-0.020	-0.022
	[-0.012,-0.000]	[-0.032,-0.002]	[-0.035,-0.005]
Female	-0.067	-0.056	-0.089
	[-0.082,-0.022]	[-0.091,-0.013]	[-0.102,-0.053]
		Production Function Structure	
γ	0.189	0.202	0.098
	[0.181,0.217]	[0.186,0.225]	[0.087,0.099]
Subst. Elast.	1.233	1.253	1.096
	[1.221,1.277]	[1.229,1.290]	[1.095, 1.097]

Notes: 90% confidence intervals based on 100 bootstrap replications in square parentheses. "Subst. Elast.": Elasticity of Substitution $\frac{1}{1-\rho}$. All skills are normalised on standard test score percentiles in each development period (standard test score percentiles are between 0-99). The unit of measure for parental investments in private education is 10,000 South Korean Won (KRW). The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

1.6.4 Adulthood Income

The empirical specification for student i's expected log earnings in the first period of adulthood is given in equation (1.30). The parameter estimates characterise the relationship between a child's university entrance exam test-scores, university quality, and other demographic controls on the child's expected earnings. The parameter estimates for student i's expected log earnings in the first period of adulthood are shown in table (1.6). The estimated elasticities for the child's

Table 1.6: Adulthood Earnings Parameter Estimates

Dependant Variable - Log Monthly Wage					
Log Standard KSAT Scores					
Math - s_1^{uni}		English - s_2^{uni}		Korean - s_3^{uni}	
0.101**		0.201***		0.010	
[0.022, 0.179]		[0.113, 0.288]		[-0.069, 0.090]	
Demographic Controls					
SKY University		Male		Constant	
0.100*		0.134***		3.792***	
[0.009, 0.192]		[0.109, 0.160]		[3.488, 4.097]	
Regional Controls					
Busan	Daegu	Incheon	Gwangju	Daejeon	Ulsan
-0.063*	-0.047	-0.025	-0.066	0.019	-0.000
[-0.119, -0.006]	[-0.113, 0.019]	[-0.090, 0.041]	[-0.137, 0.004]	[-0.041, 0.078]	[-0.078, 0.077]
Gyeonggi	Gangwon	Chungbuk	Chungnam	Jeonbuk	Jeonnam
-0.018	-0.161**	-0.012	-0.052	-0.090**	-0.046
[-0.057, 0.021]	[-0.280, -0.043]	[-0.074, 0.050]	[-0.117, 0.013]	[-0.148, -0.031]	[-0.111, 0.020]
Gyeongbuk	Gyeongnam	Jeju	Sejong	Foreign	
-0.029	-0.012	-0.022	0.098***	0.299***	
[-0.086, 0.028]	[-0.066, 0.041]	[-0.179, 0.136]	[0.068, 0.130]	[0.141, 0.457]	
Other Statistics					
N = 602		$R^2 = 0.11$		$\bar{R}^2 = 0.10$	

Notes: 90% confidence intervals based on 1,000 bootstrap replications in square parentheses. The unit of measure for monthly wage is 10,000 South Korean Won (KRW). For regional controls Seoul is the excluded region. (* $\Rightarrow p < 0.1$), (** $\Rightarrow p < 0.05$), (***$\Rightarrow p < 0.001$). The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

university entrance exam test-scores are positive for all subject areas and are significantly different from zero for math and for English. The estimated coefficient on a child's math university entrance exam test-scores suggests that a 10 percent increase in a child's standard university entrance exam test-score in math should lead to an increase in earnings of around 1 percent. The equivalent elasticity for a child's English exam test-score is approximately 2 percent. The effect of an increase in a child's university entrance exam test-scores in Korean on earnings is much smaller and is not significantly different from zero.

The other important coefficient in table (1.6) is the coefficient on the dummy variable that indicates whether or not the child attended a SKY (high-quality) university. The estimated coefficient suggests that there is a SKY wage premium of approximately 10.5 percent. This estimated SKY wage premium is significantly lower than the wage premium that was estimated by the (KEF, 2017). However, the KEF estimates were obtained using data from a later time period, specifically, after the financial crisis, whereas the children in my sample entered the labour market while the Korean economy was in a period of recovery. In order to check the robustness of these parameter estimates, I also estimate the same regression but instead use corresponding information on the 2,000 students that were originally selected into the high-school cohort of the KEEP data set. Specifically, I use corresponding information on CSAT test-scores, university quality, and demographic controls, for students in the high-school cohort of the KEEP data set during the survey wave that they graduated from university and entered the labour market. These students entered the labour market approximately 4 years earlier than the students in my sample, specifically, at the beginning of the great recession. The parameter estimates are shown in table (B2) in appendix (2.7). Overall, the parameter estimates that I obtain for the high-school cohort are not drastically different from the estimates I obtained for the middle-school cohort.

1.6.5 Other Parameters

The estimates for the remaining structural parameters are shown in table (1.7). These include the variance terms for the academic skill production technology shocks, $\varepsilon^{f}_{k,t} \sim N(0, \sigma^{2,f}_{k,t})$, the variance and covariance terms for the university entrance exam output technology shocks, $\varepsilon^{uni} \sim N(\mu^{uni}, \Sigma^{uni})$, the inter-temporal preference parameter, β, the proportion of income that households can borrow, λ^{y}, and the variance of the earnings shock, $\sigma^{2,w}$. In total, there are

Table 1.7: Structural Parameter Estimates

Academic skill production shocks: $\varepsilon^{f}_{k,t} \sim N(0, \sigma^{2,f}_{k,t})$					
$\sigma^{2,f}_{1,1}$	$\sigma^{2,f}_{2,1}$	$\sigma^{2,f}_{3,1}$	$\sigma^{f,2}_{1,2}$	$\sigma^{2,f}_{2,2}$	$\sigma^{2,f}_{3,2}$
0.224	0.215	0.238	0.249	0.268	0.251
(0.033)	(0.052)	(0.044)	(0.058)	(0.073)	(0.069)
University entrance exam shocks: $\varepsilon^{uni} \sim N(\mu^{uni}, \Sigma^{uni})$					
$\sigma^{2,uni}_{1}$	$\sigma^{2,uni}_{2}$	$\sigma^{2,uni}_{3}$	$\sigma^{uni}_{1,2}$	$\sigma^{uni}_{1,3}$	$\sigma^{uni}_{2,3}$
9.941	7.445	5.182	0.820	0.446	0.366
(3.881)	(2.026)	(1.877)	(0.342)	(0.283)	(0.191)
Other parameters					
	β	λ^{y}	$\sigma^{2,w}$	r	
	0.702	0.442	0.011	0.045	
	(0.157)	(0.228)	(0.005)	(-)	

Notes: Standard errors in parentheses. The sample consists of all Korean middle-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

15 structural parameters that were estimated using 30 moment conditions. In general, all of the parameters are precisely estimated. The estimate for λ^y imposes that households can only borrow up to 44 percent of their household income in each development period, and the estimate for β imposes that households value consumption in future development periods at 70 percent of the value of equivalent consumption in the initial development period.

The most interesting feature of table (1.7) is that the estimates for the covariance terms for the university entrance exam output technology shocks are all positive and significantly different from zero. This suggests that university entrance exam test-score shocks are indeed correlated, and so, if a child were to experience a significant negative shock to one of her test-scores, for example, becoming ill on the day of the university entrance exam, the effect of this shock would also spill over onto the test-scores that are obtained in her other academic subjects.

Since I assume the value for the interest rate r, I also test the robustness of all structural parameter estimates to small perturbations of the interest rate r, within the reasonable range for the given time period. The resulting parameter estimates did not change significantly and so I maintain the value of r equal to 4.5 percent as a result.

1.7 Policy Experiments

In order to test the implications of section (1.6) I conduct a series of counterfactual policy experiments using the estimated structural model. For each policy experiment I fix the parameters values of the academic skill production technologies at the levels shown in table (1.4), I fix the values of the university entrance exam output technologies at the levels shown in table (1.5), I fix the parameters of the adulthood earnings equation to the levels shown in table (1.6), and the remaining structural parameters are fixed at their level shown in table (1.7).

1.7.1 The Timing of Interventions

To highlight the importance of the timing of policy interventions I start by evaluating the impact of an identical policy that is imposed during alternate periods of a child's academic skill development. Specifically, I evaluate the impact of a monetary transfer, made by the government, to households in the bottom quartile of the income distribution during middle-school, or alternatively, during high-school. I choose the size of the transfer equal to the gap in spending on private education investments, on average, between households in the top quartile of the income distribution and households in the bottom quartile of the income distribution that I observe in the data. As a result, the size of the transfer that I choose is equal to approximately 400,000 KRW ($\sim$350 USD). The premise behind this policy experiment is to understand what the model predicts would happen to the achievement gap between rich and poor students in a counterfactual world where the lowest income households have a similar spending capacity to the highest income households, firstly, if the transfer is made during middle-school, and secondly, if the transfer is made during high-school.

The results for this policy experiment are shown in table (1.8). The first panel of table (1.8) shows the standard model predictions for the achievement gap between children in the highest quartile of the income distribution and children in the lowest quartile of the income distribution, this panel is therefore used as a reference point for the subsequent policy analysis. The second

Table 1.8: Policy Experiment - Cash Transfer to Low-Income Households

	CSAT Grades - Model Predicted					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.16	0.16	0.17	0.09	0.16	0.17
	(0.04)	(0.04)	(0.05)	(0.04)	(0.05)	(0.05)
Income quartile 4	0.29	0.28	0.29	0.41	0.30	0.30
	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)	(0.06)
quartile 4 - quartile 1	0.13**	0.12**	0.12*	0.32***	0.14**	0.13
	(0.06)	(0.05)	(0.07)	(0.12)	(0.07)	(0.12)
	CSAT Grades - Cash Transfer to Low-Income Households during Middle-School					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.20	0.10	0.16	0.12	0.16	0.17
	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.06)
Income quartile 4	0.27	0.26	0.32	0.27	0.30	0.29
	(0.05)	(0.08)	(0.06)	(0.09)	(0.06)	(0.09)
quartile 4 - quartile 1	0.07	0.16*	0.16**	0.13	0.14**	0.12
	(0.07)	(0.09)	(0.07)	(0.11)	(0.07)	(0.12)
	CSAT Grades - Cash Transfer to High-Income Households during High-School					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.14	0.21	0.13	0.24	0.14	0.24
	(0.07)	(0.08)	(0.05)	(0.08)	(0.05)	(0.07)
Income quartile 4	0.25	0.21	0.24	0.21	0.22	0.32
	(0.06)	(0.08)	(0.06)	(0.08)	(0.06)	(0.08)
quartile 4 - quartile 1	0.11	0.00	0.11	-0.03	0.08	0.08
	(0.07)	(0.11)	(0.07)	(0.11)	(0.08)	(0.11)

	Accepted to SKY University (%)		
	Model Predicted	*Middle-School Cash Transfer*	*High-School Cash Transfer*
Income quartile 1	0.02	0.03	0.01
	(0.01)	(0.01)	(0.01)
Income quartile 4	0.08	0.08	0.05
	(0.02)	(0.02)	(0.02)
quartile 4 - quartile 1	0.06***	0.05*	0.04**
	(0.02)	(0.03)	(0.02)

Notes: The sample consists of all Korean middle school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education. Standard errors in parenthesis. (* $\Rightarrow p < 0.1$), (** $\Rightarrow p < 0.05$), (***$\Rightarrow p < 0.01$).

panel of table (1.8) shows the results for the cash transfer to low-income households during the middle-school development period. There are a number of key insights that materialise from this policy experiment. First is that the cash transfer increased the proportion of low-income children that achieved the highest grade in math on the university entrance exam, it also increased the overall number of low-income children represented in the highest two grades of university entrance exam scores in English. In addition, the gap between the proportion of students from the lowest income quartile and the proportion of students from the highest income quartile that achieved math grade 1 and English grade 2 is no longer statistically significant. This suggests that the policy does help, relative to the baseline, to reduce the achievement gap between rich and poor students in certain academic subjects in certain parts of the grade distribution. However, the second insight is that while the policy helped to reduce the achievement gap in certain areas, it did not eliminate the achievement gap, in fact, as a result of the policy the achievement gap between the proportion of students from the lowest income quartile and the proportion of students from the highest income quartile that achieved math grade 2 and English grade 1 remained statistically significant and widened relative to the baseline. In addition, the policy had no effect at all on the achievement gap for Korean, and the gap between the proportion of students from high-income households and the proportion of students from low-income households that were accepted to SKY universities remains statistically significant and similar in magnitude to the baseline case.

These insights can be explained by the contrasting mechanisms that exist within the structural model. First, the direct effect of the cash transfer within middle-school is that it increases the spending capacity of low-income households. Investments in private education are relatively more productive during the middle-school period, therefore if the low-income parents invest the cash transfer they receive in private education for their child, their child will enter high-school with higher levels of academic skills. Academic skills are more self-productive during the high-school period and so the productivity of the investments made during middle-school, via the cash transfer, increase further. Also, due to the existence of academic skill complementarities, the investments made during middle-school will spill over and aid the development of the child's cognate academic skills. Therefore the, ex-ante, direct effect of the cash transfer, if low-income households choose to invest it in private education for their child, should lead to a reduction in the achievement gap between rich and poor students. However, the reduction in the achievement gap that I observe as a result of the cash transfer is only partial, and this is due to the indirect effect of the policy. The indirect effect of the policy is the effect that the increase in private education investments made by low-income households have on the equilibrium choices of high-income households. Specifically, in equilibrium, all households are competing for limited places at the top universities, and when low-income households receive a cash transfer and increase their investments in private education, all else equal, this will reduce the acceptance probability for some high-income households. For some of these high-income households, their best response to the reduction in their acceptance probability will be to reallocate a proportion of their spending from consumption in order to increase private education investments in their child. As a result, the direct effect that the cash transfer has on reducing the achievement gap is crowded out by the indirect effect of the cash transfer because the increased competition for places that the cash transfer generates provides an incentive for some high-income households to increase their spending on private education further, the extent to which crowding out occurs of course depends on the size of the initial cash transfer.

The third panel of table (1.8) shows the results for the cash transfer to low-income households during the high-school development period. From the estimates it is clear that the cash transfer helps to reduce the achievement gap between rich and poor students. Specifically, the gap between the proportion of students from the lowest income quartile and the proportion of students from the highest income quartile that achieved grade 1 and grade 2 on the university entrance exam is not statistically significant for any subject. However, despite the reduction in the achievement gap, the proportion of low-income students that were accepted to SKY universities is lower than when the cash transfer was provided during middle-school and even lower than in the baseline case. In other words, the cash transfer during high-school helps to increase the performance of low-income children on the university entrance exam, in general, but this does not translate into an increased presence of low-income children at the top universities.

This result seems somewhat paradoxical, however it can be explained by the mechanisms that exist within the structural model. Specifically, investments in private education are less effective during the high-school period, relative to the middle-school period, both in terms of the accumulation of a specific academic skill, and the spill over effect that the investments have on the accumulation of cognate academic skills. This means that unless the size of the cash transfer is very large, it is unlikely that the investments in private education that low-income parents make during the high-school period, as a result of the cash transfer, will be sufficient to enable their child's university entrance exam test-scores to be above the required threshold, in all subject areas, that is necessary for entry to the top universities. As a result, low-income households instead choose to narrow their investment portfolio by investing the cash transfer in specific academic subjects, e.g. math and English, that are correlated with increased future labour market earnings for their child even if their child does not make it to the top universities. This is why I observe, for example, a much larger proportion of low-income students achieving university entrance exam test-scores of grade 2 in math and English relative to the baseline.

Overall, this policy experiment highlights how the timing of the intervention is crucial for increasing the presence of low-income children at the top universities. Due to the presence of complementarities between academic skills and the existence of dynamic complementarities with respect to parental investments in private education, policy interventions will be more effective if they occur earlier in the academic skill development process. It is also worth noting however that while a cash transfer to low-income households during middle-school can help to increase the presence of low-income children at the top universities, the policy will be extremely costly.

1.7.2 Targeting Specific Academic Skills

A key issue with the cash transfer policy in section (1.7.3) is that low-income households that receive the transfer have the freedom to allocate the cash transfer in any way that they thought was optimal. For example, if their child had a low endowment of academic skills, and hence, a low acceptance probability, it is possible that the child's parents found it optimal to instead consume the cash transfer and not invest the transfer in private education for their child. Therefore, in this section, I contrast the results of the general cash transfer policy of section (1.7.3) with an alternative cash transfer policy that forces a child's parents to invest the transfer in their child's academic skills. Specifically, I transfer parents from the lowest quartile of the income distribution the same amount as before, 400,000 KRW (∼350 USD), during the middle-school period, but in this experiment, the child's parents are forced to invest the entire amount of the transfer in private education in a specific academic subject area.

The results for this policy experiment are shown in table (1.9). Each panel of table (1.9) corresponds to a specific academic skill. For example, the first panel corresponds to a cash transfer policy where the child's parents are forced to invest the full amount of the cash transfer in private education in math. Since the value of the cash transfer in this policy experiment is equal in value to the cash transfer in section (1.7.1), it is interesting to first contrast the results between these respective policies. In general, a cash transfer that targets specific academic skills is much more effective than a generic cash transfer that parents have the freedom to invest themselves in reducing the achievement gap between rich and poor students. In table (1.9) the cash transfer that targets specific academic skills eliminates the gap between the proportion of students from the highest quartile of the income distribution and the proportion of students from the lowest quartile of the income distribution that achieved grade 1 or grade 2 on their university entrance exam, for all subject areas. The cash transfer that targets specific academic skills also eliminates the gap between the proportion of students from the highest quartile of the income distribution and the proportion of students from the lowest quartile of the income distribution that were accepted to SKY universities. Importantly, both of these results remain true irrespective of the academic skill that is targeted by the policy.

There are a few key explanations for this result. First is that households place a high value on consumption, in the case of the generic cash transfer in section (1.7.1), if households invest the transfer in private education for their child it is because the expected future return on their investment exceeds the value of consuming the transfer immediately. There will be some low-income households where this inequality does not hold and, as a result, will prefer to consume a proportion of the cash transfer. This diminishes the overall effectiveness of the policy and is a contributing factor for why the achievement gap remains even after the cash

Table 1.9: Policy Experiment - Cash Transfer that Targets Specifc Academic Skills

	CSAT Grades - Policy Targetting Math Skill					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.32	0.27	0.30	0.31	0.32	0.25
	(0.06)	(0.08)	(0.06)	(0.09)	(0.06)	(0.08)
Income quartile 4	0.19	0.23	0.18	0.17	0.20	0.19
	(0.05)	(0.08)	(0.05)	(0.07)	(0.05)	(0.07)
quartile 4 - quartile 1	-0.13*	-0.04	-0.12	-0.14	-0.12	-0.06
	(0.07)	(0.11)	(0.08)	(0.11)	(0.08)	(0.11)
	CSAT Grades - Policy Targetting English Skill					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.29	0.24	0.33	0.26	0.32	0.21
	(0.06)	(0.08)	(0.08)	(0.06)	(0.06)	(0.07)
Income quartile 4	0.18	0.17	0.17	0.23	0.21	0.15
	(0.05)	(0.07)	(0.05)	(0.08)	(0.05)	(0.06)
quartile 4 - quartile 1	-0.11	-0.07	-0.16**	-0.03	-0.11	-0.06
	(0.07)	(0.11)	(0.08)	(0.11)	(0.08)	(0.09)
	CSAT Grades - Policy Targetting Korean Skill					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.28	0.16	0.13	0.27	0.27	0.13
	(0.06)	(0.06)	(0.07)	(0.08)	(0.05)	(0.07)
Income quartile 4	0.22	0.25	0.22	0.25	0.23	0.17
	(0.05)	(0.08)	(0.05)	(0.08)	(0.05)	(0.08)
quartile 4 - quartile 1	-0.06	0.09	0.09	-0.02	-0.04	-0.04
	(0.08)	(0.10)	(0.08)	(0.11)	(0.08)	(0.10)

	Accepted to SKY University (%)		
	Targetting Math Skill	*Targetting English Skill*	*Targetting Korean Skill*
Income quartile 1	0.05	0.05	0.05
	(0.02)	(0.02)	(0.01)
Income quartile 4	0.05	0.05	0.05
	(0.02)	(0.02)	(0.02)
quartile 4 - quartile 1	0.00	0.00	0.00
	(0.02)	(0.03)	(0.02)

Notes: The sample consists of all Korean middle school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education. Standard errors in parenthesis. (* $\Rightarrow p < 0.1$), (** $\Rightarrow p < 0.05$), (***$\Rightarrow p < 0.01$).

transfer to low-income households. The second explanation is due to the presence of academic skill complementarities and dynamic complementarities with respect to parental investments. In the targeted cash transfer policy, parents are forced to invest the transfer in private education in a specific academic skill area in the middle-school period. The productivity of private education investments is highest during middle-school and the investments also spill over and aid the accumulation of cognate academic skills. Therefore, the children of the parents that receive the cash transfer will enter high-school with higher stocks of academic skills, and, due to the presence of dynamic complementarities, the investments in private education made by parents during high-school will be more productive as a result. Lastly, during high-school, academic skill complementarities are at their highest level, this means that children that enter high school with, for example, a high stock of academic skill in math, this high level of academic skill in math will spill over and aid the accumulation of cognate academic skills during the high-school period. This is why the targeted cash transfer policy is effective at reducing the achievement gap irrespective of the academic skill that is targeted by the policy.

The differences in outcomes between the specific academic skill that is targeted by the policy are due to differences in the self-productivity of academic skills, differences in the magnitude of the complementarities between academic skills, and differences in the spill over effect of private education investments in academic skills. For example, private education investments in math are generally more productive than private education investments in other academic skill areas. As a result, by comparing the respective panels of table (1.9), the most effective targeted cash transfer policy would be one that forces the child's parents to invest the full amount of the transfer in private education in math.

There are some important caveats however, the first is that in this experiment the size of the cash transfer was large enough to negate any crowding out effect that may have occurred due to the equilibrium responses of high-income households. In other words, the transfer was sufficiently large so that, despite the equilibrium competition for places at the top universities, some high-income households did not find it optimal to disproportionately reallocate their spending from consumption in order to increase private education investments for their child. Therefore, for lower values of the cash transfer it is likely that the reduction in the achievement gap may be crowded out. The second caveat is that this policy, like the policy in section (1.7.1), would be very expensive to implement. Therefore, in the next section, (1.7.3), I propose more realistic policy alternatives that achieve a similar result but with much lower cost.

1.7.3 Restricting Private Education Investments

While cash transfers to low-income households can be effective at reducing the achievement gap and increasing the relative proportion of low income households at SKY universities, such policies are very costly. The key factor that raises the level of the transfer required to be effective, and therefore the overall cost of the policy, is the equilibrium competition for places at SKY universities. This is because high-income households observe the increased investments in private education made by low-income households due to the cash transfer and can respond by increasing their own investments accordingly. As a result, in this section I evaluate the effectiveness of alternative policies that are designed to reduce the effect of equilibrium competition. Specifically, I first evaluate the effect of prohibiting private education investments entirely, this policy completely eliminates the effect of competition. I then contrast this with an alternative policy that imposes a spending cap on the total amount invested in private education. Specifically, I impose the spending cap on all private education investments to equal 5% of the median household income (150,000 KRW, ∼130 USD). While this level seems somewhat arbitrary, I chose this amount as it is approximately equal to the average amount spent on private education investments by households in the lowest quartile of the income distribution. Consequently, the results of this experiment will give some indication of the equilibrium that materialises in the counterfactual world where all households have the same spending capacity as the lowest income households.

The results for this policy experiment are shown in table (1.10). The first panel of table (1.10) shows the standard model predictions for the achievement gap between children in the highest quartile of the income distribution and children in the lowest quartile of the income distribution, this panel is therefore used as a reference point for the policy analysis. The second panel of table (1.10) shows the corresponding achievement gap for the case where private education

Table 1.10: Policy Experiment - Restricting Private Education Investments

	CSAT Grades - Model Predicted					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.16	0.16	0.17	0.09	0.16	0.17
	(0.04)	(0.04)	(0.05)	(0.04)	(0.05)	(0.05)
Income quartile 4	0.29	0.28	0.29	0.41	0.30	0.30
	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)	(0.06)
quartile 4 - quartile 1	0.13**	0.12**	0.12*	0.32***	0.14**	0.13
	(0.06)	(0.05)	(0.07)	(0.12)	(0.07)	(0.12)
	CSAT Grades - Prohibit Private Education Investments					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.28	0.07	0.22	0.21	0.18	0.26
	(0.08)	(0.03)	(0.06)	(0.05)	(0.05)	(0.05)
Income quartile 4	0.19	0.21	0.23	0.23	0.27	0.28
	(0.06)	(0.08)	(0.06)	(0.07)	(0.07)	(0.07)
quartile 4 - quartile 1	-0.09	0.14	0.01	0.02	0.09	0.02
	(0.09)	(0.10)	(0.07)	(0.09)	(0.10)	(0.08)
	CSAT Grades - Limit Priv. Ed. Investments to 5% of Median Houshold Income					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.27	0.15	0.27	0.16	0.26	0.26
	(0.05)	(0.07)	(0.05)	(0.07)	(0.05)	(0.08)
Income quartile 4	0.13	0.15	0.08	0.26	0.10	0.22
	(0.04)	(0.07)	(0.05)	(0.08)	(0.03)	(0.08)
quartile 4 - quartile 1	-0.12**	0.00	-0.19***	0.10	-0.16***	-0.04
	(0.06)	(0.10)	(0.06)	(0.10)	(0.06)	(0.12)

	Accepted to SKY University (%)		
	Model Predicted	*Prohibit Priv. Ed. Inv.*	*Priv. Ed. Inv. Ceiling*
Income quartile 1	0.02	0.04	0.05
	(0.01)	(0.02)	(0.02)
Income quartile 4	0.08	0.04	0.02
	(0.02)	(0.02)	(0.01)
quartile 4 - quartile 1	0.06***	0.00	-0.03
	(0.02)	(0.02)	(0.02)

Notes: The sample consists of all Korean middle school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education. Standard errors in parenthesis. (* $\Rightarrow p < 0.1$), (** $\Rightarrow p < 0.05$), (***$\Rightarrow p < 0.01$).

investments are prohibited. By prohibiting investments in private education, differences in university entrance exam test-scores across children are caused by initial differences in academic skills that are compounded throughout the development periods due to the self-productivity of academic skills and the complementarities that exist between academic skills. The second panel of table (1.10) shows that when private education investments are prohibited, the difference between the proportion of students in the highest income quartile and the proportion of students in the lowest income quartile that achieved grade 1 or grade 2 on the university entrance exam closely mirrors the corresponding differences that were observed during middle-school, for all academic subjects. Specifically, there is no significant achievement gap between the university entrance exam test-scores of children from the richest and poorest households in all subject areas. There is also no significant gap between the proportion of children that are accepted to SKY universities from the richest households and from the poorest households. Therefore, the model predicts that a policy prohibiting private education investments by parents will achieve the same desired outcome as the targeted cash transfer policy but in absence of the significant cost. However, in reality, if such a policy were to be implemented it is likely that it would be met with discontent and that, collectively, the parents would advocate for its removal. Indeed, during the 1980s, when President Chun Doo-Hwan banned private education in Korea, this was met with such civil unrest that by 1990 the ban was ruled unconstitutional. The other issue with prohibiting private education is that such a policy is likely to create a black market for private education. If children from high-income households disproportionately partake in such a market, the outcome of prohibiting private education could actually amplify the achievement gap between rich and poor children.

The third panel of table (1.10) highlights the results for the policy experiment where I impose a spending cap on all private education investments equal to 5% of the median household income. In this experiment, the difference between the proportion of students in the highest income quartile and the proportion of students in the lowest income quartile that achieved grade 1 becomes negative. In other words, due to the private education spending cap, the lowest income students disproportionately achieve the highest university entrance exam test-score grade in all academic subject areas and the gap in proportions is statistically significant. In addition, there is no significant difference in the proportion of students in the highest income quartile and the proportion of students in the lowest income quartile that achieve university entrance exam test-score of grade 2, and this is true for all academic subjects. As a result, due to the reversal of the achievement gap between the proportion proportion of students in the highest income quartile and the proportion of students in the lowest income quartile that achieved university entrance exam test-scores in grade 1, for all academic subjects, the private education spending cap also led to students from low-income households being disproportionately accepted to SKY universities relative to students from high-income households.

The reason for this result is that, when a spending cap on private education investments is introduced, high-income parents are no longer able to overcome the negligible initial heterogeneity in academic skills through out-investing parents of low-income children in private education. Specifically, in the data, there is no significant heterogeneity in the levels of academic skills of children from different income quartiles during the first development period. The achievement gap that I observe, for all academic subjects, in panel 1 of table (1.10) is established between the first development period and when the child enters university. This gap is driven by competition for places at the top universities by encouraging parents to invest in private education for their child, in which high-income households have a comparative advantage and therefore disproportionately invest. However, when a spending cap is introduced, some high-income parents will realise that the probability of their child being accepted to a SKY university becomes sufficiently low so that they would instead prefer to consume their income and reduce the investments that they make in private education for their child. Moreover, for low-income households, the spending cap is less likely to bind, and, due to the reduction in investments in private education made by high-income households, their child's acceptance probability is likely to have increased. It is this mechanism that leads to a reversal of the achievement gap under a spending cap relative to the achievement gap that is predicted by the standard model and an increased presence of low-income children at SKY universities.

1.8 Conclusion

In this paper, I study the dynamic process of child academic skill development throughout adolescence in anticipation of entry to university. Academic skills are multi-dimensional and I model the development process from when a child enters their penultimate year of middle-school until they complete high-school and reach the age of university entry. I allow for dynamic interactions between all academic skills, and investments in academic skills that are made by the child's parents. The academic skill technologies are placed within an equilibrium framework to account for the fact that places at the top universities are highly attractive but also limited.

I find that academic skills become more self-productive over time and that there are strong complementarities between all academic skills included in the model. This suggests that, throughout adolescence, academic skills accumulate predominantly in unison. I also find that parental investments in private education are productive for the accumulation of all academic skills, at all ages, but parental investments in private education have the largest effect in earlier stages of the academic skill development process. The effect of parental investments in private education also spills over and aids the accumulation of cognate academic skills. Finally, I find that the implied equilibrium competition for places at top universities strongly contributes to the achievement gap between rich and poor students, and hence, contributes to lower inter-generational social mobility.

To demonstrate the implications of my findings I conduct a series of counterfactual policy simulations using the estimated structural model. The results of the policy simulations suggest that providing a cash transfer to low-income households in order to increase their relative spending power is effective at reducing the achievement gap between rich and poor students and hence, the gap between the proportion of rich and poor students accepted to the top universities. However, cash transfers are most effective if the transfer is received earlier in the academic skill development process and are designed to target specific academic skills, for example, a child's academic skill in math. This result is due to the presence of academic skill complementarities and dynamic complementarities with respect to parental investments that exist within the model. Cash transfer policies are, however, expensive to implement. My results therefore suggest that policies designed to reduced the effect of equilibrium competition for places at the top universities are a more cost effective way at reducing the achievement gap between rich and poor students. Specifically, I find that a policy that introduces a limit on the total amount that parents can invest in private education for their child is equally as effective at reducing the achievement gap between rich and poor students and hence, the gap between the proportion of rich and poor students accepted to the top universities, but in absence of the high implementation cost.

There remain however some important considerations for future research. First, in this paper, I do not include factors such as the child's non-cognitive skills and their mental and physical health as inputs into the academic skill production technologies. Existing research on the formation of human capital shows that there are important interactions between a child's cognitive skills and non-cognitive skills (Cunha, Heckman and Schennach, 2010) and health (Attanasio, Meghir and Nix, 2020), during childhood. It is likely that the complementarities between cognitive skills, non-cognitive skills, and mental and physical health are also important during adolescence, specifically in the Korean context, due to concerns that the competition culture has created negative mental health consequences and contributes to suicides being the primary cause of death among Korean youth (McKinsey, 2013). Second, while I model the intense competition for places at the top universities, I do not model the problem from the perspective of the universities. In reality, the top universities could respond to competition for places by changing their entrance criteria. This would have important consequences for the investment decisions of the child's parents and, in turn, the child's academic skill development, I leave this analysis open to future research.

Chapter 2

The Role of Own-Group Density and Local Social Norms for Ethnic Marital Sorting: Evidence from the UK

Dan Anderberg, Alexander Vickery

Abstract

We exploit the post-war immigration-induced regional variation in ethnic composition among British-born individuals to study inter-ethnic marriages in the UK. Black and Asian individuals are more likely to marry intra-ethnically in regions where the own ethnicity share is relatively large. In order to disentangle the relative roles played by supply effects, preferences and local social norms we estimate a structural marriage market model that allows for conformity behaviour. Using the estimated model, we make predictions for a set of more recent cohorts whose marital choices are still to be completed.

2.1 Introduction

As western economies are growing increasingly ethnically diverse through immigration, how minorities integrate is crucial not only to their own experiences of their host countries, but also to public opinion on immigration. Indeed, the very concept of integration remains controversial and a variety of indicators and measures have been proposed (Ager and Strang, 2004). A commonly held view is that marriages between ethnic-minority immigrants and natives is the ultimate proof of integration, stirring the ethnic melting pot and diminishing the significance of group differences (Blau, Beeker and Fitzpatrick, 1984). From a social acceptance perspective, inter-ethnic marriages have been seen as the breaking of the "last taboo" in ethnic and race relations (Qian, 2005). The role of inter-ethnic marriages raises interesting dynamic questions. In particular, as ethnic minorities grow in relative size do inter-ethnic marriages become more or less frequent? What role in this process is played by changing marital opportunities and by evolving social norms respectively?

The sharing of a common culture and identity contributes to the value of marriage, rationalizing observed assortative matching on, for instance, race, religion, and nationality. Indeed, marrying across ethnic lines remains a relatively rare event. In his survey of US marriage trends, Fryer (2007) notes that inter-racial marriages account for approximately 1 percent of White marriages, 5 percent of Black marriages, and 14 percent of Asian marriages. Similarly, the empirical marriage pattern suggests that members of large dominant religious groups – Protestants, Catholics and Jews – have strong preferences for having their children identify with their own religious beliefs, thereby creating strong incentives for intra-group marriages (Bisin, Topa and Verdier, 2004).

In this paper we study inter-ethnic marriages in the UK among White, Black and Asian British-born individuals, exploiting a strong regional variation in ethnic composition stemming directly from the settlement patterns of the post-war immigrants.[1] We find that inter-ethnic marriages are more common among Blacks than among Asians. More importantly we find that

[1] There is no single agreed international definition of ethnicity and race or of the distinction between them. In general, ethnicity has many dimensions which include or combine nationality, citizenship, race, colour, language, and religion. Our broad categorization of ethnicity into White, Black and Asian people is a condensed version of the categorization used by the Office of National Statistics.

Black and Asian individuals are more likely to be married within their own ethnicity in regions where the density of their own ethnicity is relatively high.

In order to interpret these findings and to identify the separate roles played by population supplies, preferences, and local social norms we estimate a structural marriage market model. The seminal work of Choo and Siow (2006) has provided a workhorse model for empirically implementing Becker's transferable utility model of the marriage market (Becker, 1973). Their framework recasts marriages as choices among discrete "types" and with marital surpluses systematically depending on a couple's type-profile. The framework developed by Choo and Siow (2006) has, over the past decade, been implemented and extended in a variety of directions and applied in a variety of contexts.[2] One key recent extension by Mourifié and Siow (2017) and Mourifié (2019) has been towards incorporating social preferences. Conceptually there are strong parallels to the literature on discrete choice with social interactions (Brock and Durlauf, 2001), a literature that has been instrumental in empirically operationalizing the notion of social influence and endogenous norms of behaviour (Glaeser and Scheinkman, 2014). Marital choices, in particular in the context of inter-ethnic marriages, may reflect endogenous social norms and conformity behaviour: marrying inter-ethnically may be less of a taboo when others do the same. Hence the current paper will study marital choices within and across ethnic boundaries in the UK using a Choo-Siow style model extended to incorporate endogenous conformity preferences. Such an extension requires a strong source of identification (Galichon and Salanié, 2015) which in our case is provided by the rich regional demographic variation. We adopt a multi-market approach where each region is treated as a separate marriage market and where an individual's type is given by her ethnicity-qualification profile. We specify the form of the joint systematic marriage utility to have two key components (i) a "principal preferences" component – common across all marriage markets – that varies with a couple's type profile, and (ii) an endogenous social norm component that depends the strength of conformity preferences and on how common that particular marriage choice is locally among individuals of the own type.

Our paper makes three contributions. First, we highlight that the ethnic composition of UK-

[2] Key contributions include *inter alia* Galichon and Salanié (2015), Choo (2015), Dupuy and Galichon (2014), Chiappori, Salanié and Weiss (2017), Graham (2013), and Brandt, Siow and Vogel (2016).

born individuals varies substantially across regions, directly reflecting the settlement pattern of the first generation of immigrants arriving to the UK between the mid-1950s and the mid-1970s. We document the marriage patterns within our estimating cohorts of UK-born individuals born between 1965 and 1989, both in the aggregate and across regions.

Second, we estimate a structural marriage model with conformity preferences, identified through the regional variation in population supplies. We find that the principal preferences exhibit significant complementarity in ethnicity as well as in academic qualifications. We also find evidence of strong conformity preferences which in turn implies strong variation in local social norms. We show that the estimated model naturally fits the regional pattern with respect to inter-ethnic marriage frequencies: in areas with relatively larger ethnic minority groups, Whites naturally more frequently marry inter-ethnically, but critically, individuals from the ethnic minorities less frequently marry Whites.

Third, we use the estimated model to predict marriage patterns among individuals born between 1990 and 2006. In these recent cohorts the ethnic Asian minority in particular and the Black minority to a lesser extent are substantially larger than in the estimating cohorts. We find that, as the ethnic minorities grow in terms of their shares of the population, Whites will become more likely to marry inter-ethnically. However, Blacks and Asians will themselves become less likely to marry inter-ethnically. These effects are amplified by the endogenously evolving social norms.

A number of recent papers have adopted structural marriage market modelling in order to study marriages across cultures and borders. Relevant to the UK setting, Marini (2019) draws on Dupuy and Galichon (2014) to allow for multiple continuous traits. One of the traits included in her analysis is a measure of identity, specifically an ethnolinguistic fractionalization (ELF) index based on the country of origin of the mother. The author finds that strong evidence for matching on ELF, which is consistent with strong ethnic matching. However, comparing to the current study, Marini does not consider matching directly on ethnicity as a discrete characteristic and does not account for regional demographic variation or social conformity preferences. Two further recent papers use structural marriage market models to study marriages between natives and migrants. Focusing on the case of Italy, Adda, Pinotti and Tura (2019) use the enlargement of the European Union as a natural experiment to highlight the role played by both cultural distance and legal status. Ahn (2020) studies selection into cross-border marriages between Taiwan and Vietnam, predominantly between Taiwanese men and Vietnamese women.

The rest of the paper is organized as follows. Section 2.2 gives a brief overview of post-war immigration into the UK. Section 2.3 describes the data that we use and the marriage pattern among the estimating cohorts. Section 2.4 set up the model and outlines how the model is identified. Section 2.5 presents the estimation results while section 2.6 highlights our predictions for the more recent cohorts. Section 2.7 concludes.

2.2 UK Post-War Immigration

Following the conclusion of the second world war the UK had a non-White population of around 30,000 people. By the end of the 20^{th} century this figure was over 3 million.[3] The post-war rise in immigration can be attributed to a combination of government policy and labour demand. In 1948, the British Nationality Act granted individuals from the British Empire the freedom to live and work in the UK. These extensive rights were in place until the early 1960s when, in response to a perceived heavy influx of immigrants, regulation was significantly tightened. Meanwhile, the post-war Labour government embarked on an extensive nationalization policy, promising full employment, fair wages, and homes for all. Recognizing that reconstruction of the British economy required a large influx of immigrant labour, appeals for new workers were aimed at both Europeans and non-European – mainly from the Caribbean and from the Indian subcontinent. The symbolic starting point for immigration from the Caribbean was the journey of the *SS Empire Windrush* from Kingston, Jamaica, to Tilbury, Essex, in June 1948, carrying close to 500 West Indians to the UK. This began a large wave of migration now referred to as the "Windrush generation". The majority of immigrants from the Indian subcontinent arrived to Britain during the 1950s and 1960s following Britain's relinquishing in 1947 of its Indian empire in 1947 and the turbulent partition of India into what is known today as the Republic of India

[3] See Hansen (2000) for a comprehensive outline of post-war immigration.

and the Islamic Republic of Pakistan.

One of the key features of the settlement of ethnic minority immigrants arriving to the UK between the 1950s and the 1970s was its particular geographical pattern. As the post-war immigration was driven primarily by a shortage of labor, both skilled and unskilled immigrants settled in areas where the shortages were perceived to be the largest. Many of the Asian immigrants were attracted to the industrial towns in the East and West Midlands and to London's East end, as well as to the traditional textile producing towns in Yorkshire and the North West. The Caribbean immigrants settled predominantly in London – in areas such as Brixton and Notting Hill – filling labor shortages in London's hospitals and transportation.

Our interest will be in UK-born individuals and we will use the substantial spatial variation in ethnicity across regions, reflecting the settlement pattern of the post-war arrivals.

2.3 Data

Our aim is to study the marital choices of UK-born individuals primarily in terms of ethnicity. We will further include educational attainment in our analysis to explore if there is a relationship between education and ethnic marital sorting.

For our analysis we want to focus on a set of cohorts who (i) exhibit a substantial and geographically varied presence of UK-born ethnic minorities, and (ii) are old enough to have made their marital choices. To this end, we focus on the birth cohorts 1965-1989. We will use data on individuals, aged 25 or above, from the Quarterly Labour Force Survey (QLFS) – the largest household study in the UK – for the survey years 1996-2015. Our choice of geography is based on the Statistical Regions, specifically Wales, Scotland, and the nine statistical regions of England.[4]

2.3.1 Sample Population

The QLFS allows us to characterize each individual's ethnicity and educational attainment. Moreover, as the survey interviews all adults in each household, it allows us to measure the

[4]We will omit Northern Ireland as the proportions of ethnic minorities there are too small for any meaningful analysis.

same characteristics for partners. The ethnic groups that we will consider are Whites (W), Blacks (B), and Asians (A), based on standard classification by the Office for National Statistics. Educational attainment is broadly classified into two groups: "Low" (L) and "High" (H). We classify an individual as having "low" educational attainment if their highest academic qualification is a GCSE (General Certificate of Secondary Education) or no academic qualification at all. In contrast, we classify an individual as having "high" educational attainment if their highest academic qualification is an A-level (Advanced Level) or higher including a university degree.[5]

For our sample, we will focus on UK-born individuals, born between 1965-1989 and aged 25 and above. The age cutoff ensures that the individuals in the sample have had the necessary time to complete full time education and reach a marriage age.[6] Pooling the 20 years of the QLFS we then obtain a sample of $203,802$ individuals, distributed across ethnicity, qualification and gender as shown in Table 2.1.

Table 2.1: Descriptive Statistics of the QLFS Sample

	White		**Black**		**Asian**	
	Low Qual	**High Qual**	**Low Qual**	**High Qual**	**Low Qual**	**High Qual**
Males	44,903	49,316	447	579	963	1,737
Females	45,815	55,456	519	845	1,179	2,043

Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born between 1965-1989, living in England, Scotland or Wales, and aged 25 or above when observed, and with available information on gender, ethnicity, educational attainment, marital status.

As the QLFS also identifies cohabiting couples, we treat cohabitors as married. Hence being "married" in our context includes both formal marriage and live-in partnerships. In total, 67 percent of the individuals in the sample population are, by this definition, married. It should be noted that we do not impose the same cohort and age-restrictions on partners, though the vast majority of partners do satisfy them. While our key focus will be on marriages between UK-born individuals, marriages to non-UK-born spouses are also observed and are known to

[5]The GCSE is the first tier of academic qualifications in the UK, obtained at the end of the academic year in which the individual turns 16 (which also corresponds to the end of compulsory education for the cohort in question). The A-level degree is obtained at the age of 18 – after two years of post-compulsory schooling – and is the standard qualification for entry to university.

[6]Below we will also check on the sensitivity of our results to this cutoff age.

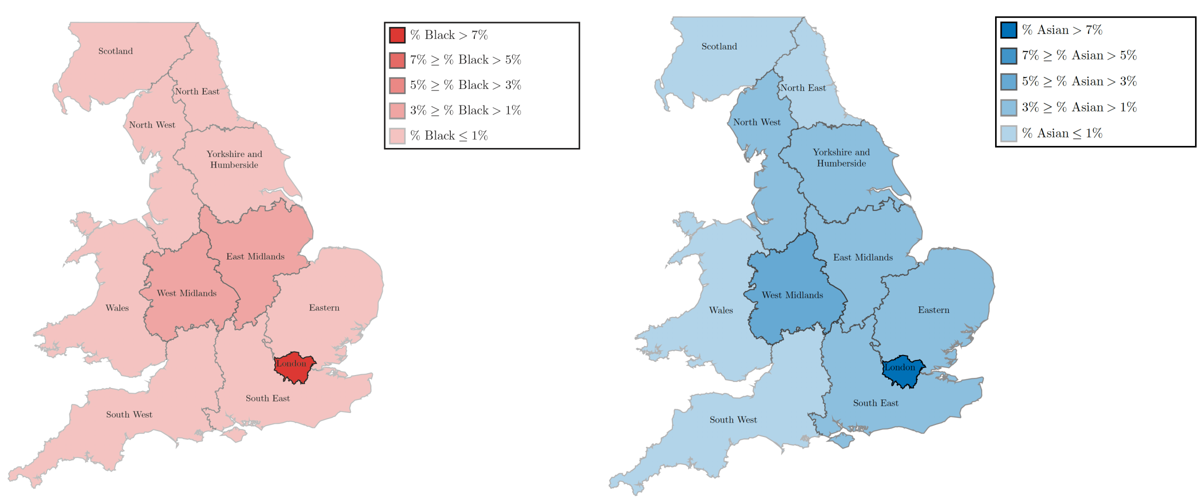
be particularly common among the low qualified Asians (Charsley et al., 2012). Hence we will include such marriages in our analysis below.

2.3.2 Geographical Variation

Figure 2.1 highlights the uneven presence of ethnic Black and Asian across the UK in the estimating cohorts. Panel (a) shows how the ethnic Black are heavily concentrated in the London region, and to a lesser extent in the East and West Midlands. Panel (b) shows how the ethnic Asians are heavily concentrated in London and the West Midlands, and, to a lesser extent, in the North West and in eastern regions. Conversely, this of course implies that proportion Whites (not shown) is relatively low in London, and in the East and West Midlands, and particularly high in regions such as the South West and the North East. As our estimating sample includes only UK-born individuals, this illustrates how the geographical variation of ethnicity among our sample population reflects the settlement patterns of the first generation discussed in Section 2.2. This gives us reassurance that the location choices of the second generation individuals were, in effect, determined exogenously by their parents.

Figure 2.1: Ethnic and Qualification Composition by Region

(a) Prop. Blacks   (b) Prop. Asians



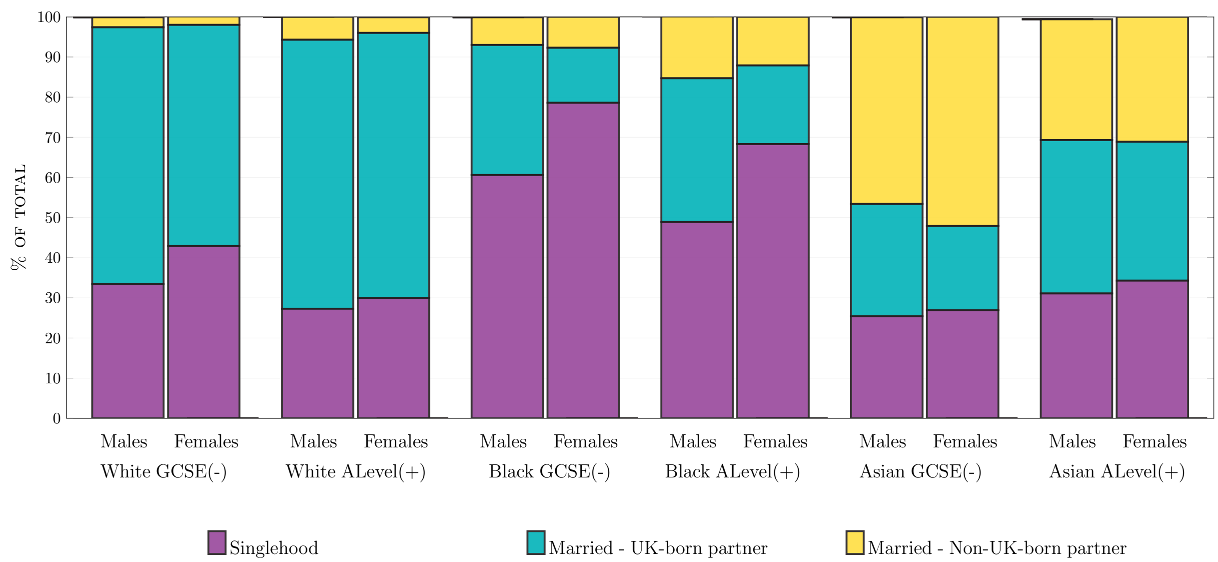
Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born between 1965-1989, living in England, Scotland or Wales, and aged 25 or above when observed, and with available information on gender, ethnicity, educational attainment, marital status.

While not shown here, there is also some geographical variation in the distribution of educational attainment with London, the southern regions of England, and England having higher levels of attainment than the rest of England and Wales.[7]

2.3.3 Empirical Marital Choices

We will define "marital status" to have three categories: single, married to a UK-born partner, and married to a non-UK-born partner.[8] Figure 2.2 shows the distribution of marital status by gender, educational attainment and ethnicity for our sample population.

Figure 2.2: Distribution of Marital Status by Own Type and Gender



Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born between 1965-1989, living in England, Scotland or Wales, and aged 25 or above when observed, and with available information on gender, ethnicity, educational attainment, marital status.

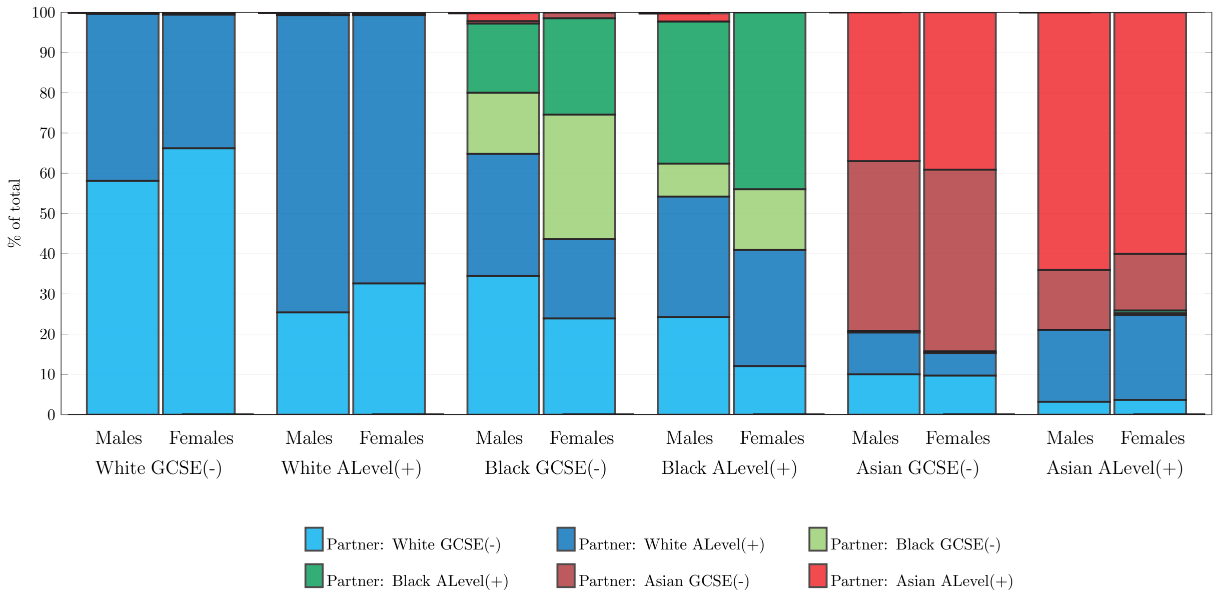
Two key features with respect to ethnicity stand out. First, the rate of singlehood is substantially higher among Blacks than among Whites and Asians, mirroring findings from the US (Ellwood and Crane, 1990; Saluter, 1994; Brien, 1997; Seitz, 2009). Second, the proportion of individuals who are married to non-UK-born partners is particularly high among Asians. In contrast, only a very small proportion of White individuals – irrespective of gender and education – marry partners who are born outside the UK. With respect to educational attainment, it

[7] As expected, the qualification rate is also generally higher among females than among males and higher among the ethnic minorities than among Whites. These patterns are in line with findings from the literature, for instance Burgess (2014).

[8] Our classification is based on current marital status as, for married individuals, we need to measure the characteristics of their partners. This means that we classify divorced, separated and widowed individuals as single.

is interesting that, for both the Black and the White ethnic groups, being more educated also makes you more likely to be married, whereas the opposite is true for Asians.

Figure 2.3: Distribution of Partner Type by Own Type and Gender



Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born between 1965-1989, living in England, Scotland or Wales, aged 25 or above, married to a UK-born partner at the time of the survey, and with available information on gender, ethnicity, and educational attainment.

Figure 2.3 shows the distribution of partner type by own "type" – defined by ethnicity and qualification – and gender.[9] A few things stand out. The vast majority of married White individuals are married to White partners. In contrast, among married Black males, White partners are as common as Black partners. Around 80 percent of married Asians are married to Asian partners.[10] The figure also highlights that there is educational homogamy: for any ethnicity and gender, a qualified individual is more likely to have qualified spouse than an unqualified individual. Overall, 58 (66) percent of low qualified married males (females) are married to low qualified partners, whilst around 75 (67) percent of high qualified married males (females) have high qualified partners.

More central to our analysis is how the prevalence of intra- v inter-ethnic marriages varies

[9] For this we restrict the sample to those married to UK-born partners for two reasons. First, doing so directly ties in with the modelling approach below where we model the supply of UK-born prospective partners, but not the non-UK-born supply. Second, doing so avoids having to classify in particular the qualifications of the non-UK-born partners.

[10] Note that this is conditional on the partner also being UK-born.

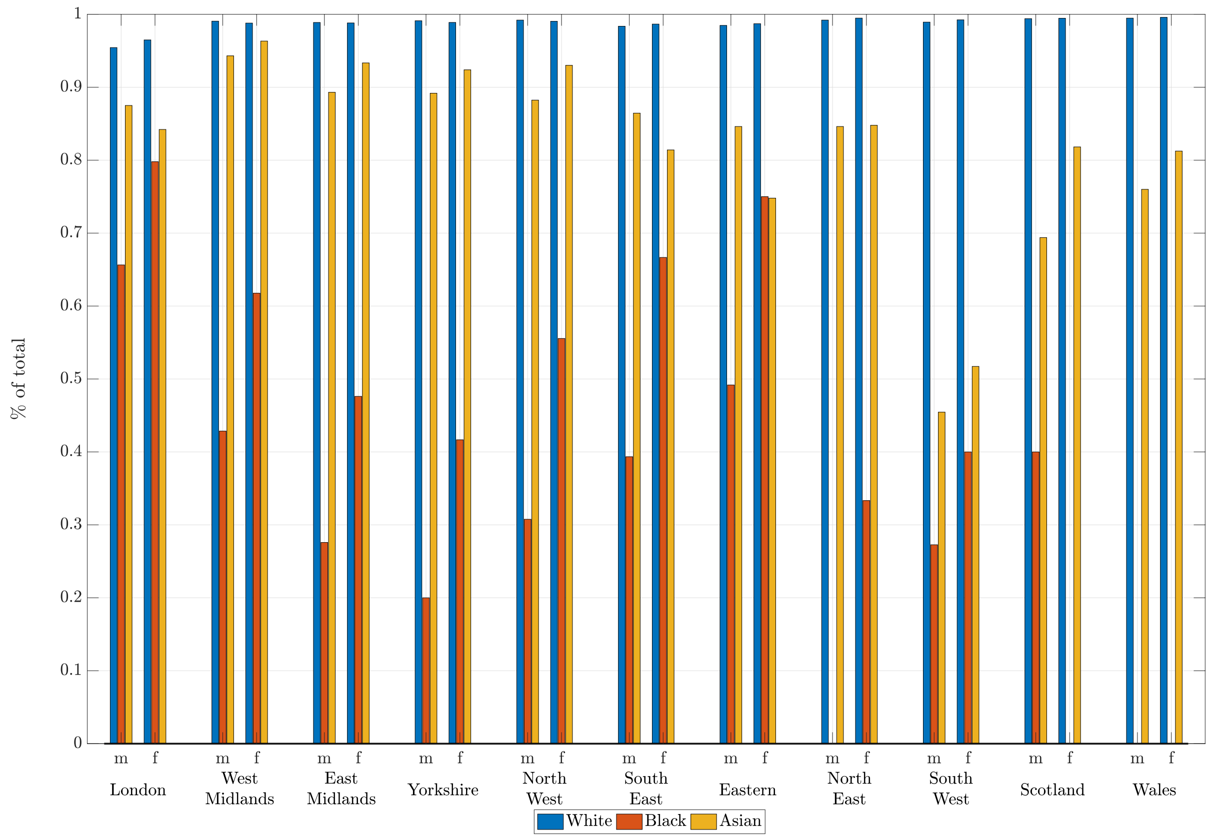
with the ethnic composition. To give a first indication of this, Figure 2.4 shows the proportion of intra-ethnic marriages by region, gender and ethnicity. In order to make the figure more interpretable, the regions have been ordered in ascending order in terms of the proportion Whites, starting with London as the most ethnically diverse region through to Wales as the most homogenously ethnically White. Naturally, the proportion of intra-ethnic marriages for Whites is close to 100 percent in regions where the Black and Asian ethnic minorities are very small. The more central feature highlighted by the figure is how the prevalence of intra-ethnic marriages among the ethnic minorities varies systematically across regions. Specifically, the figure suggests that, for both Blacks and Asians, intra-ethnic marriages are more common in areas where each respective ethnic minority is comparatively large. As we will see below, this will be a central feature of the model that we will estimate and also for the predictions about future marriage behaviour.[11]

In order to assess the robustness of this stylized observation Table 2.2 presents a set of simple probit regressions. In each reported regression we use the relevant subsample of individuals married to UK-born partners, and regress a dummy for the individual being intra-ethnically married on the proportion of marriage market peers who are of the own ethnicity. As we have defined an individual's relevant marriage market as comprising all UK-born individuals, born between 1965-1989 and in the own region, we use our full estimating sample to characterize the proportion of marriage market peers who are of the own ethnicity.

The estimated marginal effects from these basic probit regressions are shown for each gender and ethnicity. For Whites the estimated marginal effects of 0.065 for males and 0.074 for females imply that a 15 percentage point reduction in the proportion Whites in the local marriage market (roughly corresponding to the difference between London and Wales) is associated with a one percentage point reduction in the likelihood of the partner to a married White person being White. For Blacks the relationship between the proportion intra-ethnically married and the local proportion Blacks is much more dramatic. A ten percentage points increase in the local

[11] In a few regions we observe no intra-ethnic marriages – only a small number of inter-ethnic marriages to White partners – due to low prevalence of the ethnic minority in question. This applies to Black males in the North East and in Wales, and to Black women in Scotland and Wales.

Figure 2.4: Proportion Intra-Ethnic Marriages by Region and Gender



Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born between 1965-1989, living in England, Scotland or Wales, aged 25 or above, married to a UK-born partner at the time of the survey, and with available information on gender, ethnicity, and educational attainment.

share of Blacks (again roughly corresponding to the difference between London and a typical area with less than one percent Blacks), is associated with around a 40 percentage points increase in the likelihood of the partner to a married Black person being Black. Similarly, ten percentage points increase in the share of Asians in the marriage market is associated with around a 20-30 percentage points increase in the likelihood of the partner to a married Asian person being Asian. These simple regressions thus strongly indicate that, as a stylized fact, the prevalence of intra-ethnic marriages among the ethnic minorities is markedly increasing in the share of the own ethnicity in the local marriage market.

A further benefit to these regression is that they allow us to mitigate the potential concern that the current population structure may be affected by selective migration. To this end we create an instrument that draws on the logic that the settlement pattern of the postwar immigrants was a strong determinant of the local ethnic compositions of our estimating cohorts.

Table 2.2: The Effect of the Own Ethnicity Share on the Probability of Being Intra-Ethnically Married

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
	Males: White		**Males: Black**		**Males: Asian**	
Prop. Own. Ethn.	0.065***	0.065***	4.235***	4.840***	3.234***	3.417***
	(0.006)	(0.006)	(0.579)	(0.554)	(0.531)	(0.568)
Obs.	61,760		352		933	
	Females: White		**Females: Black**		**Females: Asian**	
Prop. Own. Ethn.	0.074***	0.074***	3.683***	4.377***	2.200***	1.960***
	(0.007)	(0.007)	(0.772)	(0.777)	(0.539)	(0.585)
Obs.	61,844		237		955	
Est. Spec.	Probit	IV Probit	Probit	IV Probit	Probit	IV Probit

Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born between 1965-1989, living in England, Scotland or Wales, aged 25 or above and married to a UK-born partner, and with available information on gender, ethnicity, educational attainment, marital status. The proportion of the local population who are of the own ethnicity ("Prop. Own Ethn.") is measured using the same data but without conditioning on being married. The instrument used for the own ethnicity proportion is the log proportion of own ethnicity in the local population among all (UK- and non-UK-born) individuals born 1940-1960 as observed in the QLFS 1996-2015.

Specifically we measure the share of the own ethnicity among individuals born between 1940 and 1960 and living in the same region. This way we create an instrument for the ethnic spatial pattern among our UK-born estimating sample that is based on that of, effectively, the parent generation.[12] Instrumenting in this way leaves the estimated marginal effects of the own ethnicity share on the likelihood of a marriage being intra-ethnic completely unaffected for Whites and largely unaffected for Blacks and Asians. This analysis thus suggests that the empirical relationship between the share of own ethnicity in the local population and the prevalence of intra-ethnic marriages is robust to potential internal migration. Hence, in the structural modelling below, we will take the spatial distribution of ethnic types among the UK-born sample as exogenously given and reflecting the post-war settlement pattern.

2.4 The Model

Our model builds on Choo and Siow (2006), whose seminal work showed how the static, frictionless, transferable utility equilibrium model of the marriage market could be recast and estimated

[12] Note that the individuals used in the create of the instrument can be either UK- or non-UK-born. This way we capture the first generation immigrants as potential parents to the UK-born estimating sample cohorts.

as a set of discrete choice problems, connected via equilibrium constraints, that collectively identify the marital surplus structure. Their key innovation was to assume that all individuals belong to some discrete set of observable types and that individuals have preferences over partner type with these preferences having both a deterministic and a random component.

The framework introduced by Choo and Siow (2006) has been subsequently generalized in a variety of directions. Our model is closest in spirit to the generalization by Mourifié and Siow (2017) and Mourifié (2019) who allow for "peer effects". The notion that peers may influence individuals' marital choices is one that has previously been explored in the literature. For instance, Adamopoulou (2012) uses panel data on friendship networks and shows that direct friends influence individuals' partnership formation choices. But peer effects also encompass wider social norms, including what choices are socially approved or, conversely, what behaviours are considered to be taboos. What makes social norms particularly interesting and challenging to study is that they are endogenous equilibrium concepts that can vary across groups and geographical areas. Indeed, one of our innovations is to implement a marriage market model with social preferences in a multi-market setting where social norms vary across regions (Burke and Young, 2011).

But before turning to the full empirical model, we will start here by presenting a simple special case that has a full analytical solution. This will provide a set of key insights that will be useful going forward.

2.4.1 An Illustrative Special Case

The special case considered here imposes three simplifying assumptions relative to the full model below. First, we assume that there are only two types of individuals; hence for now we let the type-space be $X = \{x_1, x_2\}$, where we can think of x_1 as a "majority" ethnic group and x_2 as a "minority" ethnic group. Second, there are two genders, males m and females f, and for now we assume that everyone marries someone of the opposite sex. Third, we impose complete gender symmetry, both in terms of population supplies and in terms of preferences. Let $h(x)$ be the proportion of the population (of either gender) that is of ethnicity-type $x \in X$.[13] As x_1 is the majority type, $h(x_1) > 1/2$ and $h(x_2) < 1/2$.

Utility is assumed to be transferable, and we postulate a "principal" joint marital utility function that maps husband-wife type profiles $(x', x'') \in X \times X$ into joint utility $\sigma_{x'x''}$. We can collect all such σ-terms in a matrix Σ, which, in this simple case, has dimension 2×2. Furthermore, gender-symmetry implies that $\sigma_{x_1 x_2} = \sigma_{x_2 x_1}$: in a mixed-ethnicity marriage, the joint utility is the same irrespective of whether it is the husband or the wife who is the majority/minority type. In contrast, there is no restriction on the relationship between $\sigma_{x_1 x_1}$ and $\sigma_{x_2 x_2}$. In addition to the preference parameters in Σ, there is a conformity-preference parameter ϕ parameterizing the strength of peer effects.

For convenience we will use probability notation to describe the equilibrium.[14] Hence let $\mu^k_{x''|x'}$ denote the equilibrium probability that an individual of gender k and type x' marries a partner of type x''. In the gender-symmetric equilibrium, these probabilities will be gender-independent, but we include the gender superscript for now so as to help characterize the equilibrium. The joint systematic utility when a male of type x' marries a female of type x'', inclusive of peer effects, is, for any type profile $(x', x'') \in X \times X$, assumed to take the form

$$W_{x'x''} \equiv \sigma_{x'x''} + \phi \log\left(\mu^m_{x''|x'}\right) + \phi \log\left(\mu^f_{x'|x''}\right). \tag{2.1}$$

In other words, in addition to the principal utility $\sigma_{x'x''}$, the joint utility depends on the equilibrium proportions of males and females, respectively, of the own type who make the same choice.[15] It will be useful to establish some terminology. We will refer to ϕ as the "social preferences" (parameter) and the $\phi \log\left(\mu^k_{x'|x''}\right)$ terms as the equilibrium "social norms".

In addition, any given individual is assumed to have additional additively separable random utility shocks over possible partner types $x \in X$ which, following the literature, we take to

[13] Gender symmetry and the assumption that everyone marries someone of the opposite gender, means that we here assume that the male and female populations are equally large.

[14] It is otherwise common, and completely equivalent, to use a population measure notation.

[15] We are assuming that social preferences enter utility in log form; this is contrary to Brock and Durlauf (2001). In principle, there is no obvious economic *ex ante* justification for either the linear or log specification. The log specification allows a semi-closed form solution.

be extreme value distributed and independent across individuals and partner types (see below for details). A well-known feature of the equilibrium, driven by the assumption of additively separable utility shocks over partner type (Chiappori, Salanié and Weiss, 2017), is that the joint marital utility $W_{x'x''}$ will be split into a male part, $U_{x'x''}$, and a female part, $V_{x'x''}$. The extreme-value distribution of preference shocks in turn implies that the equilibrium choice frequencies will take the standard logit form. Specifically, the relative choice frequencies will satisfy $\log\left(\mu^m_{x''|x'}/\mu^m_{x'|x'}\right) = U_{x'x''} - U_{x'x'}$ for males and $\log\left(\mu^f_{x'|x''}/\mu^f_{x''|x''}\right) = V_{x'x''} - V_{x''x''}$ for females, which, when added together, gives that

$$\log\left(\frac{\mu^m_{x''|x'}}{\mu^m_{x'|x'}}\right) + \log\left(\frac{\mu^f_{x'|x''}}{\mu^f_{x''|x''}}\right) = W_{x'x''} - \frac{W_{x'x'} + W_{x''x''}}{2}, \tag{2.2}$$

where we used that $U_{x'x''} + V_{x'x''} = W_{x'x''}$ and that, due to gender-symmetry, the utilities from intra-ethnic marriages will be equally shared, $U_{xx} = V_{xx} = W_{xx}/2$ for either type $x \in X$. We can substitute for the joint systematic utilities in (2.2) using (2.1) and simplify using that the equilibrium marriage probabilities are gender-independent.[16] Doing so, and rearranging, yields

$$\log\left(\frac{\mu_{x'|x'}}{\mu_{x''|x'}}\frac{\mu_{x''|x''}}{\mu_{x'|x''}}\right) = \frac{\Delta}{2} + \phi\log\left(\frac{\mu_{x'|x'}}{\mu_{x''|x'}}\frac{\mu_{x''|x''}}{\mu_{x'|x''}}\right), \tag{2.3}$$

where $\Delta \equiv \sigma_{x'x'} + \sigma_{x''x''} - 2\sigma_{x'x''}$ and is strictly positive under type-complementarity in the principal joint utilities. Let $r \equiv h(x_2)/h(x_1) \in (0, 1/2)$ denote the relative frequency of the ethnic minority type. Market balance implies that, in equilibrium, the inter-ethnic marriage frequency for the majority type will be directly proportional to the inter-ethnic marriage frequency for the minority type, $\mu_{x_2|x_1} = r\mu_{x_1|x_2}$. Moreover, since everyone marries someone, $\mu_{x'|x'} = 1 - \mu_{x''|x'}$, for either type. Hence we can re-write (2.3) as a single equation characterizing the inter-ethnic marriage rate among the minority type, $\mu_{x_1|x_2}$, as follows,

$$\frac{r\left(\mu_{x_1|x_2}\right)^2}{\left(1 - r\mu_{x_1|x_2}\right)\left(1 - \mu_{x_1|x_2}\right)} = \exp\left(-\frac{\Delta}{2(1-\phi)}\right). \tag{2.4}$$

[16]Specifically, $\mu^m_{x''|x'} = \mu^f_{x''|x'}$ for any $x', x'' \in X$.

Equation (2.4) has a unique solution that characterizes $\mu_{x_1|x_2}$ in terms of Δ, ϕ and r, and provides a number of insights, starting with some basic comparative statics.[17]

First, complementarity in the principal preferences, $\Delta > 0$, generates positive marital sorting. To see this, note that random matching would imply, $\mu_{x_1|x_2} = h(x_1)$, and would be an equilibrium if and only if $\Delta = 0$. In contrast, any $\Delta > 0$ implies $\mu_{x_1|x_2} < h(x_1)$. Second, equation (2.4) highlights how conformity preferences ϕ "amplify" the complementarity from the principal preferences: for a given $\Delta > 0$, a higher value of ϕ reduces $\mu_{x_1|x_2}$.[18] Third, as the left hand side of (2.4) is increasing in both r and $\mu_{x_1|x_2}$ it follows that $\mu_{x_1|x_2}$ is decreasing in r. Hence the model predicts that, as the minority grows as a share of the population, they become less likely to marry inter-ethnically. This latter feature underlies the model's ability to replicate the stylized fact observed above that ethnic minority individuals are less likely to marry inter-ethnically in areas where they make up a larger share of the population.

Three further points are worth noting before we move to the full empirical model. The first point relates to scaling of the preference parameters. Equation (2.4) highlights that rescaling Δ to $\widetilde{\Delta} = \lambda\Delta$ and ϕ to $\widetilde{\phi} = 1 - \lambda(1 - \phi)$ by some arbitrary $\lambda > 0$ would leave the equilibrium unaffected. This feature will return in the full model where we show that the preference parameters are only identified up to a scaling factor, implying a need for a normalization.

The second point relates to over-identification restrictions imposed by the model. In particular, note that the relative type frequency r does not feature in equation (2.3). This means that, even as r varies, the *pairwise relative marriage frequencies* in the log-term (same on both sides) do not vary. In the full model this property translates into a key specification test.

Third, and finally, the log-terms on two sides of (2.3) being the same reflects the static nature of the model, in particular the assumption that social norms adjust contemporaneously. But what if norms were more "sticky" or "backward-looking"? Consider for instance the possibility that the current marrying cohort use the marriage behaviour of a previous cohort – when the

[17] Existence and uniqueness follows trivially since the left hand side is strictly increasing in $\mu_{x_1|x_2}$ over $[0, 1]$, and limits to 0 and ∞ at 0 and 1 respectively, and whereas the right hand side is positive constant.

[18] We can assume here that $\phi < 1$. The generalization of this assumption will feature in the condition for existence of an equilibrium in the full model below. Indeed, in the limit where $\phi \to 1$, any $\Delta > 0$ would lead to complete positive sorting, $\mu_{x_1|x_2} = \mu_{x_2|x_1} = 0$.

relative type composition r was different – as reference behaviour. In that case, the choice probabilities on the left hand side of (2.3) represents the current equilibrium behaviour whereas the corresponding choice probabilities on the right hand side would be reference behaviour of the previous cohort. Perhaps surprisingly, in this simplified case, whether social norms adjust instantaneously or with a "lag" does not matter. This directly reflects the aforementioned testable property: as long as the preferences are stable, the *pairwise relative marriage frequencies* will also remain stable along any sequence of evolving economics under either norm-formation.[19] This invariance result with respect to norm formation will not carry over to the generalized environment below as there we will also have singlehood as a choice, and we will also have gender differences in both principal and social preferences (and between marriage and singlehood). Indeed we will consider both contemporaneous and backward-looking norms when we simulate the predicted future marriage behaviour in Section 2.6.

2.4.2 General Setup

Our estimating model generalizes the above simple illustrative case by having more than two types, by dropping gender symmetry, allowing for singlehood, and by having multiple markets. Thus consider an economy consisting of a continuum of men and women. The individuals differ in observable type $x \in X$, where X is a discrete set with N elements. The individuals in the economy are further partitioned into a discrete set of groups, denoted G with typical element g. Let $h(x, k, g)$ denote the probability mass function describing the population distribution across types $x \in X$, genders $k = m, f$, and groups $g \in G$.

Each individual faces a choice between marrying and remaining single ("option 0"). Marriage between a male of type x' and a female of type x'' in group g generates a principal systematic

[19]Specifically, consider a sequence of economies indexed by time $t = 0, 1, 2, ..., T$, where T could be infinite, and let $\{r_t\}_{t=0}^{T}$ be the corresponding sequence of relative type supplies (while Δ and ϕ remain constant). Assume that for the initial cohort, $t = 0$, the reference behaviour is the own cohort behaviour (i.e. the social norms are "instantaneous"), but for all subsequent cohorts $t \geq 1$, the reference behaviour is the behaviour of the previous cohort, $t - 1$. It is easy to show that the sequence of equilibria in this case will be identically the same as the sequence of equilibria that would obtain if all cohorts formed instantaneous social norms: $\frac{\mu^t_{x''|x'}}{\mu^t_{x'|x'}} \frac{\mu^t_{x'|x''}}{\mu^t_{x''|x''}} = \exp\left(-\frac{\Delta}{2(1+\phi)}\right)$ will emerge endogenously in the initial cohort, and will then be replicated across all subsequent cohorts.

utility denoted $\sigma^g_{x'x''}$, and we can collect these terms in group-specific $N \times N$ matrices, Σ^g, for $g \in G$. The principal utility from remaining single is normalized to zero. As noted above, choices also reflect additive individual random preferences over possible partner types $\varepsilon(x)$ and singlehood $\varepsilon(0)$. Following Choo and Siow (2006), these random utilities are all taken to be i.i.d. extreme value distributed across individuals and choice options.[20]

As above, we use a probability notation to describe equilibrium choices. As there is no interaction between the groups, an equilibrium occurs group-by-group. Hence let $\mu^{g,k}_{x''|x'}$ be the probability that a person from group g and of gender k and type x' chooses $x'' \in X \cup \{0\}$.

In this multi-market environment, we assume that the relevant reference group for a given individual is the set of individuals of the same gender, type and group. Hence we generalize (2.1) the joint systematic marriage utility to

$$W^g_{x'x''} \equiv \sigma^g_{x'x''} + \phi^m_1 \log\left(\mu^{g,m}_{x''|x'}\right) + \phi^f_1 \log\left(\mu^{g,f}_{x'|x''}\right). \tag{2.5}$$

Note that (2.5) generalizes (2.1) also by allowing the social preference parameter ϕ to differ between men and women and to be specific to marriage. As we now model also singlehood as a choice, we allow for gender-specific social preferences also with respect to this choice, shifting the systematic utility from (the normalized) zero to

$$U^g_{x0} \equiv \phi^m_0 \log\left(\mu^{g,m}_{0|x}\right), \quad \text{and} \quad V^g_{0x} \equiv \phi^f_0 \log\left(\mu^{g,f}_{0|x}\right), \tag{2.6}$$

for males and females of type x in group g respectively. In this general setup we thus have four social preference parameters which we can collect in a vector $\Phi \equiv \left\{\phi^k_s\right\}^{k=m,f}_{s=0,1}$.

As utility is transferable in marriage, it can be shared in any way between the partners. As above, in the equilibrium (in group g), $W^g_{x'x''}$ will be split into a male part $U^g_{x'x''}$ and a female part $V^g_{x'x''}$. A given male of type x' from group g will then make the choice, $x'' \in X \cup \{0\}$, that maximizes $U^g_{x'x''} + \varepsilon(x'')$ that, in addition to $U^g_{x'x''}$ accounts for his idiosyncratic utility

[20] Note that individuals in this economy do not hold preferences over specific individuals of the opposite gender, only over their types. This assumption on preferences effectively rules out sorting on any unobserved personal characteristics (Galichon and Salanié, 2015).

shocks. With the random preferences being i.i.d. extreme value distributed, it follows that the choice frequencies take the standard logit form. Specifically, relative to singlehood, for males, $\log\left(\mu^{g,m}_{x''|x'}/\mu^{g,m}_{0|x'}\right) = U^g_{x'x''} - U^g_{x'0}$, while for females, $\log\left(\mu^{g,f}_{x'|x''}/\mu^{g,f}_{0|x''}\right) = V^g_{x'x''} - V^g_{0x''}$.

2.4.3 Identification

We will here outline how variation in population supplies across groups – in our case regions – is central to identification. However, we will start by noting that the model as specified is too general in two key ways: (i) the inclusion of peer effects on all choices means that the preference parameters are only identified up to a scale factor, thus requiring a further normalization to be imposed, and (ii) allowing the principal preferences to be unrestricted across groups is too general to be identified.

To see the first part, we can use the logit forms and the fact that $U^g_{x'x''} + V^g_{x'x''} = W^g_{x'x''}$. Substituting using (2.5) and (2.6) we obtain that, in equilibrium,

$$\sigma^g_{x'x''} = (1-\phi^m_1)\log\left(\mu^{g,m}_{x''|x'}\right) + \left(1-\phi^f_1\right)\log\left(\mu^{g,f}_{x'|x''}\right) - (1-\phi^m_0)\log\left(\mu^{g,m}_{0|x'}\right) - \left(1-\phi^f_0\right)\log\left(\mu^{g,f}_{0|x''}\right). \tag{2.7}$$

Equation (2.7) highlights the need for a normalization as it can be multiplied through by an arbitrary constant. Specifically, consider the rescaling by some arbitrary $\lambda > 0$ (as highlighted above), whereby $\widetilde{\sigma}^g_{x'x''} = \lambda\sigma^g_{x'x''}$ for each group $g \in G$ and type profile $(x', x'') \in X \times X$, and $\widetilde{\phi}^k_s = 1 - \lambda\left(1-\phi^k_s\right)$ for each gender $k = m, f$ and marital status $s = 0, 1$. Such a rescaling would leave the equilibrium in every group unaffected, reflecting that the preference parameters are only identified up to a scaling factor. A natural approach is to impose a normalization on Φ and we will return to this below.

The second part reflects the well-known property that the unrestricted Choo and Siow (2006) model with a single market and no peer effects is exactly identified. Specifically, the observed marital choice frequencies across all the groups and types could be perfectly replicated by unrestricted group-specific joint principal marital utility matrices $\{\Sigma^g\}_{g\in G}$ and no peer effects. But that is too general to allow for any meaningful testing and would preclude us from exploring the role of peer effects. Hence it is useful to impose some form of restriction on how the principal preferences vary across groups. The natural first assumption is that they do not vary at all across groups.

Assumption 1. (Common principal preferences) $\Sigma^g = \Sigma$ for all $g \in G$ for some $N \times N$ matrix Σ.

This assumption is testable. To see this, rearrange (2.7) and use that, in equilibrium, the market is balanced. This gives us the following form for the matching equations:

$$\mu^{g,m}_{x''|x'} = \left[\exp\left(\sigma^g_{x'x''}\right) \left(\mu^{g,m}_{0|x'}\right)^{1-\phi^m_0} \left(\mu^{g,f}_{0|x''}\right)^{1-\phi^f_0} \left(\frac{h(x'', f, g)}{h(x', m, g)}\right)^{1-\phi^f_1} \right]^{\frac{1}{2-\phi^m_1-\phi^f_1}}, \tag{2.8}$$

for males and

$$\mu^{g,f}_{x'|x''} = \left[\exp\left(\sigma^g_{x'x''}\right) \left(\mu^{g,m}_{0|x'}\right)^{1-\phi^m_0} \left(\mu^{g,f}_{0|x''}\right)^{1-\phi^f_0} \left(\frac{h(x', m, g)}{h(x'', f, g)}\right)^{1-\phi^m_1} \right]^{\frac{1}{2-\phi^m_1-\phi^f_1}}, \tag{2.9}$$

for females. Equations (2.8) and (2.9) give the following generalization of (2.3), for the pairwise relative marriage rates,

$$\log\left(\frac{\mu^{g,k}_{x'|x'}}{\mu^{g,k}_{x''|x'}} \frac{\mu^{g,k}_{x''|x''}}{\mu^{g,k}_{x'|x''}}\right) = \frac{\sigma^g_{x'x'} + \sigma^g_{x''x''} - \sigma^g_{x'x''} - \sigma^g_{x''x'}}{2 - \phi^m_1 - \phi^f_1} \text{ for } x', x'' \in X \text{ and } k = m, f. \tag{2.10}$$

Under the assumption of common principal preferences, the expression on the right hand side of (2.10) does not vary across groups. Since the probabilities on the left hand side have observable counterparts this has, as noted above, a testable implication, and we will present tests below.

As our estimation involves solving for the equilibrium at each trial value of the parameter, existence and uniqueness is central to our approach. For this we will draw heavily on Mourifié (2019). Recall also that, within any given group $g \in G$, adding-up holds: $\mu^{g,k}_{0|x} + \sum^{g,k}_{x' \in X} \mu^{g,k}_{x'|x} = 1$ for all $x \in X$ and $k = m, f$. Substituting in these adding-up equation using (2.8) and (2.9) generates $2N$ equations – one for each male and female type – in the $2N$ unknown singles rates. Mourifié (2019) used this approach to define a mapping from the space of singles rates to itself, which is continuous and where a fixed point corresponds to an equilibrium, allowing the author to prove equilibrium existence using Brouwer's fixed-point theorem. As the current model is a special case, equilibrium existence is guaranteed also in our case. Mourifié (2019) further provides conditions for equilibrium uniqueness. In our special case, uniqueness is guaranteed if

$$\frac{1-\phi_0^k}{2-\phi_1^m-\phi_1^f} > 0 \text{ for } k = m, f. \tag{2.11}$$

We will assume that (2.11) holds, and will also check it at the end of the estimation.[21]

Having imposed the assumption of common principal preferences, relative marriage frequencies will vary across groups fundamentally due to variation in the relative supply of types. Hence such variation is central to the identification strategy.

Assumption 2. (Variation in population supplies) There are at least two groups, $g, g' \in G$, with different relative population supplies, $h(x, k|g) \neq h(x, k|g')$ for some $x \in X$ and $k = m, f$.

Using Assumptions 1 - 2 we can now turn to identification. To this end, we can collect all preference parameters in a vector, denoted $\theta \equiv \{\Sigma, \Phi\}$, which includes a normalization on Φ. Proposition 1 focuses on the simplest two-by-two case (two types and two groups) and notes that θ is identified in this case. The proof, which is provided in the Appendix, shows that in two-by-two case (where θ has eight parameters) there are eight matching equations that form a linear equation system of the form $\mathbf{A}\theta = \mathbf{B}$, where $\mathbf{A}$ and $\mathbf{B}$ are a matrix and a vector, respectively, containing only marriage and singles rates (observable in the limit). However, reflecting the need for a normalization, the matrix $\mathbf{A}$ has one less than full rank. Consequently, one parameter can be normalized (for instance, setting $\phi_0^f = 0$) and one equation dropped, before uniquely solving for the remaining parameters.

Proposition 1. ***(Identification)*** *Suppose $N = 2$ and $|G| \geq 2$, and that Assumptions 1 and 2 hold. Then θ is identified from observable marriage and singles rates.*

Proposition 1 shows that two groups is sufficient for identification under the assumption of common principal preferences (Assumption 1). With three or more groups, it becomes possible to relax this assumption. In our empirical application, our groups are the eleven regions and we

[21]Note that the condition in (2.11) is unaffected by the permissible parameter rescaling highlighted following equation (2.7).

will introduce one particularly simple generalization to Assumption 1 that allows us to account in particular for the feature that singles rates are higher in some regions than in others. Specifically, we will introduce group-specific fixed-effects in the marriage utilities such that $\Sigma^g = \Sigma + \psi_g$ for some ψ_g (with $\psi_g = 0$ for a reference group). This thus introduces a further $|G| - 1$ parameters.[22]

As a further generalization that is specific to the current application we include as a further choice option, denoted -1, marrying a non-UK-born partner. We model this option as having a type- and gender-specific systematic utility – denoted $\sigma_{x,-1}$ for males and $\sigma_{-1,x}$ for females – along with its own individual i.i.d. extreme values distributed additive utility component. This thus adds a further $2N$ parameters. As marrying a partner from outside the economy is – just like singlehood – an observable unilateral choice, the identification of these parameters follows directly from the frequency of this choice relative to the frequency of singlehood.

2.4.4 Empirical Types and Specification Tests

An individual's "type" in our setting is given by their ethnicity and qualification profile. There are three possible ethnicities $Z \equiv \{W, B, A\}$ and two qualification levels $Q \equiv \{L, H\}$. Hence an individual's type is a pair $x = (z, q) \in X \equiv Z \times Q$, and there are $N = 6$ types in total. For instance a type $x = (W, L)$ is a White low-qualified individual. Our set of groups G are the eleven regions – Wales, Scotland and the nine statistical regions of England – as outlined in Section 2.3. The group-specific population distributions $h(x, k|g)$ are taken as given by the observable relative frequencies of ethnicity-qualification-gender types by region.

Before turning to model-estimation we will consider the specification tests implied by (2.10) under common principal preferences (Assumption 1) with additive group fixed-effects. When $\Sigma^g = \Sigma + \psi_g$, the right hand side of (2.10) will be independent of g, and hence (log) pairwise relative marriage frequencies on the left hand side, which we can denote by $\log\left(\zeta^{g,k}_{x'x''}\right)$, should also be independent of g. As these marriage frequencies have directly observable counterparts, a testable implication of Assumption 1 is that $\log\left(\zeta^{g,k}_{x'x''}\right)$ is constant across groups for all type profiles $(x', x'') \in X \times X$.

[22]The appendix also contains details of how the identification of the model extends to the case of more than two types and to more than two groups, including group fixed-effects. Note also that the additive group-fixed-effects cancel out in (2.10) and hence the specification tests based on this equation remains valid.

The proposed specification test has an interesting parallel in the current literature. Recent work by Chiappori, Costa Dias and Meghir (2020), building on Choo and Siow (2006), explores how alternative measures of homogamy proposed in the literature agree with, or not, the notion of stronger preferences for assortative matching as represented by the degree of complementarity in the joint marital utility function. The authors note that many available measures – including popular ones based on random matching as benchmark – fail to separate out the effect of population changes from changes in preferences. The application that Chiappori, Costa Dias and Meghir (2020) focuses on is the much-debated question of whether educational homogamy has increased or decreased over recent decades. The empirical counterpart to the left hand side of (2.10), which we can denote $\log\left(\widetilde{\zeta}^{g,k}_{x'x''}\right)$, can be viewed as a measure of homogamy, not for a single type but for a pair of types, x' and x'', involving their relative propensities to marry *within-* versus *across-type*. As (2.10) shows, this measure identifies potential variation across groups in the degree of type complementarity. However, whereas Chiappori, Costa Dias and Meghir (2020) are interested in *measuring changes* in the preferences for assortative matching across cohorts, our interest here lies in *testing stability* of preferences across regions.

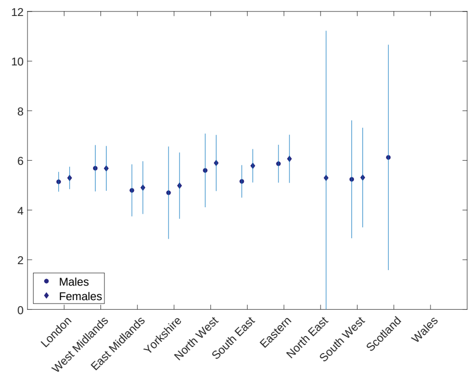
As there are $N = 6$ types in our application, there are $N^2 = 36$ possible type-pairs. However it is also clear that some type-pairs will, in many regions, have low frequencies leading the tests to have low power. For this reason, we will consider here, as a simple diagnostic, how $\log\left(\widetilde{\zeta}^{g,k}_{x'x''}\right)$ varies across regions when we focus on ethnicity and educational attainment separately as types/characteristics. Hence, consider first ethnicity, and Whites and Blacks in particular. For this ethnic pair, the measure is the (log of) the product of (i) the proportion of Whites who are intra-ethnically married relative to the proportion of Whites who are married to Black partners, and (ii) the proportion of Blacks who are intra-ethnically married relative to the proportion of Black individuals who are married to White partners. At population level it would be equivalent to construct the measure using either male- or female-marriage frequencies.[23] At sample level there will be some gender-variation as some individuals have partners who do not meet the sample cohort and age-restrictions. Hence, for completeness we present the homogamy

[23] Using that, in equilibrium, $\mu^{g,m}_{x''|x'} = \left(h\left(x'', f, g\right)/h\left(x', m, g\right)\right)\mu^{g,f}_{x'|x''}$ it immediately follows that $\log\left(\zeta^{g,m}_{x'x''}\right) = \log\left(\zeta^{g,f}_{x'x''}\right)$.

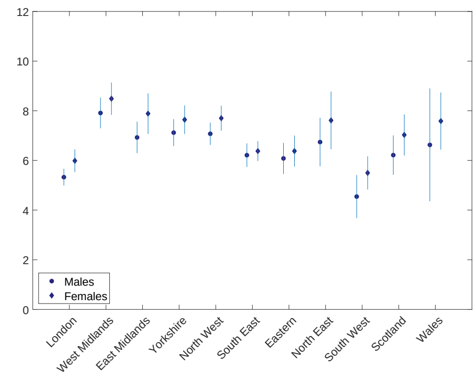
measure calculated both using male and female empirical marriage frequencies.

Figure 2.5: Tests for Constant Type-Complementarity Across Regions

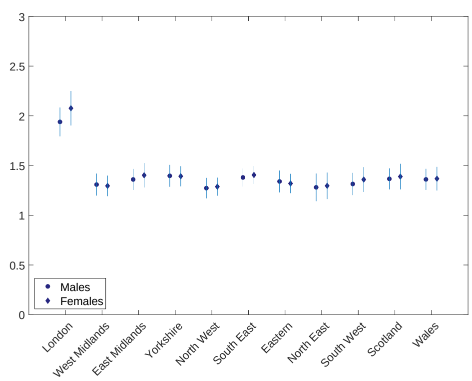
(a) Whites and Blacks



(b) Whites and Asians



(c) Qualifications



Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born between 1965-1989, living in England, Scotland or Wales, and aged 25 or above when observed, and with available information on gender, ethnicity, educational attainment, marital status. The figure plots the empirical counterpart to the left hand side expression in equation (2.10) using the observed intra- and inter-marriages between Whites and Blacks (panel a), Whites and Asians (panel b), and low- and high-qualified individuals (panel c), for male and female sample members.

Panel (a) in Figure 2.5 shows the results for Whites and Blacks. The figure shows that, despite there being substantial variation in the prevalence of Blacks across the regions, the homogamy measure is very stable. The value of the measure is missing for males in Wales and the North-East, and for females in Wales and Scotland. These are cases where we do not observe any intra-ethnic marriages involving Black individuals (see Figure 2.4). Panel (b) shows the results for Whites and Asians. The homogamy measure here is slightly higher, on average, than for Whites and Blacks, indicating a higher degree of complementarity. The measure is again fairly constant, though London and possibly the South-West stand out as being less homogamous.

London stands out more clearly when we consider educational attainment in Panel (c). Here London exhibits substantially more educational homogamy than the other regions. In contrast, the educational homogamy is strikingly consistent across the remaining ten regions. The analysis based on full ethnicity-qualification types has been placed in the Appendix as it provides little in way of additional insights.[24]

The main upshot from this analysis is that our assumption of common principal preferences appears to be broadly supported by the data with London being a possible exception. For this reason, when we explore the robustness of our results, we will present estimates that leave out London.

2.4.5 Maximum Likelihood Estimator

The model is structurally estimated using maximum likelihood. The ML-estimation takes the region-specific population distributions as given and solves for the local equilibria, denoted $\mu^{g,k}_{x'|x}\left(\widehat{\theta}\right)$, for every trial value of the parameter vector $\widehat{\theta} \in \mathbf{\Theta}$, where $\mathbf{\Theta}$ is the set of possible (normalized) parameter vectors. In the data we observe $M^{g,k}(x)$ individuals from group $g \in G$, of gender $k = m, f$, and of type $x \in X$, and their empirical choice frequencies $\widetilde{\mu}^{g,k}_{x'|x}$ for $x' \in X \cup \{0, -1\}$. The likelihood contribution of this group, given $\widehat{\theta}$, is

$$L^{g,k}_{x}\left(\widehat{\theta}\right) =_{x' \in X \cup \{0,-1\}} \left[\mu^{g,k}_{x'|x}\left(\widehat{\theta}\right)\right]^{M^{g,k}(x)\widetilde{\mu}^{g,k}_{x'|x}}, \tag{2.12}$$

and the overall log-likelihood at $\widehat{\theta}$ takes the log of (2.12) and sums over group, genders and types.[25]

[24] The analysis based on full type does not find any evidence of regional variation in complementarities involving Whites and Blacks for any qualification profile. There is some evidence of regional variation in complementarities involving Whites and Asians, in all cases for high-educated Asians and, in the majority of cases, involving London. There is some evidence that educational homogamy is stronger among Asians than among Whites or Blacks, and that sorting on ethnicity is slightly lower among Asians and Whites when both types are high-qualified.

[25] Further details about the algorithm used to solve for the local equilibria are presented in the Appendix.

2.5 Results

One of the benefits of using the ML-estimator is that we can use likelihood ratio (LR) tests to test parameter restrictions.[26] Above we have outlined how the estimated model can include regional ("group") fixed effect $\{\psi_g\}_{g\in G}$ and marriage and singlehood social preferences $\{\phi_s^k\}_{s=0,1}^{k=m,f}$. We will first report on the LR-tests used for model selection before presenting the detailed results for the preferred specification.

Each column in Table 2.3 corresponds to a specification, and each expanding on previous one. Specification (i) includes neither regional fixed effects or social preferences. Specification (ii) adds regional fixed effects. Unsurprisingly, allowing regional fixed effects greatly improves the fit and the LR-test reported rejects their exclusion from the model. Specification (iii) add gender-specific social preference over marriage, ϕ_1^m and ϕ_1^f, as free parameters while setting $\phi_0^m = \phi_0^f = 0$.[27] This further improves model fit and the LR-test rejects the no-social-preferences specification (ii) in favour of specification (iii). Finally, specification (iv) estimates all four social preferences (under a normalization). However, this final addition does not substantially improve the fit to the data and the restricted specification (iii) is not rejected by the LR-test. Hence we will proceed with (iii) as our preferred specification.

Table 2.3: Alternative Empirical Model Specifications

	Specification			
	(i)	**(ii)**	**(iii)**	**(iv)**
Log Likelihood Value	-242,775	-242,029	-242,020	-242,020
LR Test Statistic		1,490.5	19.2	0.2
p-value		<0.001	<0.001	0.920
Regional Effects	N	Y	Y	Y
Marriage Social Preferences	N	N	Y	Y
Full Social Preferences	N	N	N	Y

Notes: The LR test statistic and associated p-value reported for each specification (ii) - (iv) tests whether the constrained model in the previous column is statistically rejected.

[26] See Chen et al. (2019) for a discussion of algorithms for ML estimation of matching equilibria under existence and uniqueness.

[27] Note that one of these equalities serves as normalization and one as constraint. This constraint is then relaxed in specification (iv).

2.5.1 Model Fit

The model easily fits the aggregate data on marital status and partner type. In the Appendix, we present model-predicted versions of Figure 2.2 and Figure 2.3 which are both very close to the empirical versions. Thus the model naturally replicates the empirical homogamy both on ethnicity and on qualifications at the aggregate level.

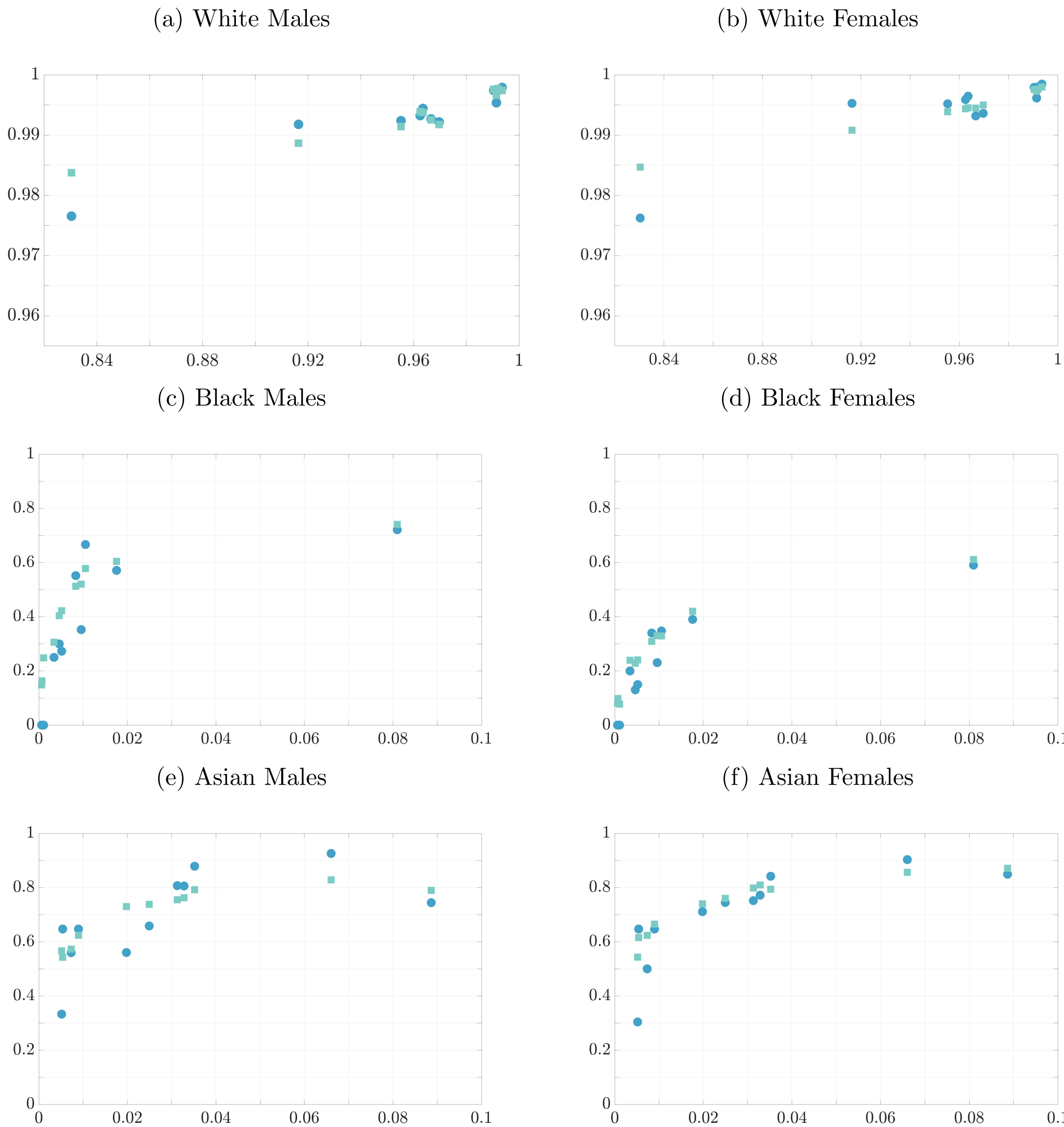
More critical for our purposes is how the model fits the local marriage patterns, in particular how marriage choices vary with the local ethnic composition. Figure 2.6 plots, for each ethnicity and gender, the observed and model-predicted proportions of marriages that are intra-ethnic against the share of own ethnicity in the local population.

The two top panels highlight how, trivially, for White males and females the proportion of intra-ethnic marriages approaches unity as the share of Whites in the local population approaches unity. The figures however also show that the model provides a realistic prediction for the one area (London) where the share of Whites in the local population is substantially lower (82 percent). Turning to Blacks, the two middle panels highlight how the proportion of Blacks who are married to Blacks increases with the share of Blacks in the local population, and how the model correctly predicts this relationship. The bottom two panels do the same for Asians, again highlighting how both in the data and in the model predictions, the proportion of marriages that are intra-ethnic increases with the share of the own ethnicity in the local population.

2.5.2 Parameter Estimates

Our preferred specification has the following set of free parameters: (i) 36 principal joint utility terms $\sigma_{x'x''}$ in Σ, (ii) two gender-specific social preferences over marriage, ϕ_1^k for $k = m, f$, (iii) ten (after one normalization) regional marriage utility fixed effects, ψ_g, and (iv) twelve gender and type-specific utilities from marrying a non-UK-born partner, $\sigma_{x,-1}$ for men and $\sigma_{-1,x}$ for women. Hence the preferred specification has a total of 60 estimated free parameters.

Figure 2.6: Observed and Predicted Proportion of Intra-Ethnic Marriages across Regions by Ethnicity and Gender



Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born between 1965-1989, living in England, Scotland or Wales, aged 25 or above, married to a UK-born partner at the time of the survey, and with available information on gender, ethnicity, and educational attainment.

Principal Preferences and Complementarities

Table 2.4 presents the estimated principal utility terms. The preference parameters are fairly precisely estimated with exceptions relating to some very rare marriages between Blacks and Asians. As the model assumes transferable utility assortative matching is fundamentally driven by complementarities in the joint marital utility function. Hence it is instructive to explore the strength of complementarities with respect to ethnicity and with respect to qualifications. A complication when doing so is that, when looking for instance at complementarities in ethnicity, this can occur for alternative qualification profiles etc.

Table 2.4: Estimates of Principal Joint Marital Utility by Ethnicity-Qualification Profile

		Female Type					
		White, GCSE(-)	White, A-Level(+)	Black, GCSE(-)	Black, A-Level(+)	Asian, GCSE(-)	Asian, A-Level(+)
Male Type	White, GCSE(-)	0.81*** (0.14)	0.69*** (0.20)	-4.49*** (0.78)	-4.39*** (0.77)	-3.82*** (0.80)	-4.19*** (0.82)
	White, A-Level(+)	0.23 (0.24)	1.71*** (0.11)	-4.59*** (0.82)	-3.28*** (0.66)	-4.25*** (0.89)	-2.03*** (0.57)
	Black, GCSE(-)	-3.51*** (0.62)	-3.40*** (0.65)	-2.38*** (0.46)	-2.37*** (0.46)	-5.47*** (1.22)	-4.74*** (0.93)
	Black, A-Level(+)	-3.34*** (0.64)	-2.85*** (0.62)	-2.51*** (0.51)	-0.99*** (0.32)	-32.6 (>200)	-3.99*** (0.82)
	Asian, GCSE(-)	-4.18*** (0.78)	-3.84*** (0.78)	-5.52*** (1.21)	-32.43 (>200)	0.15 (0.33)	-0.39 (0.37)
	Asian, A-Level(+)	-4.93*** (0.86)	-2.66*** (0.62)	-33.0 (>200)	-33.3 (>200)	-0.48 (0.39)	0.74** (0.21)

Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born between 1965-1989, living in England, Scotland or Wales, aged 25 or above when observed, and with available information on gender, ethnicity, educational attainment, marital status. Standard errors in parenthesis.

Hence consider first complementarity with respect to ethnicity. Fix a male-female qualification profile $(q_m, q_f) \in Q \times Q$ and a pair of ethnicities $z, z' \in Z$, $z \neq z'$, and define

$$\Delta_{z,z'|q_m,q_f} \equiv \sigma_{z,q_m;z,q_f} + \sigma_{z',q_m;z',q_f} - \sigma_{z,q_m;z',q_f} - \sigma_{z',q_m;z,q_f}, \tag{2.13}$$

as the ethnicity complementarity between z and z' conditional on (q_m, q_f). As there are four possible male-female qualification profiles and three possible ethnicity pairs, there are a total of twelve conditional ethnicity complementarities.

Similarly, consider complementarity with respect to qualification. Now fix a male-female ethnicity profile $(z_m, z_f) \in Z \times Z$ define,

$$\Delta_{q,q'|z_m,z_f} \equiv \sigma_{z_m,q;z_f,q} + \sigma_{z_m,q';z_f,q'} - \sigma_{z_m,q;z_f,q'} - \sigma_{z_m,q';z_f,q}, \tag{2.14}$$

as the ethnicity complementarity between q and q' conditional on (z_m, z_f). In this case, there (L, H) is the only possible qualification-pair. As there are nine possible ethnicity profiles, there are there a total of nine qualification complementarities.

Table 2.5: Complementarities in Ethnicity and Qualifications

	Panel A: Ethnicity Complementarity		
	Ethnicity Pair		
Qual. Profile	White, Black	White, Asian	Black, Asian
Low, Low	6.44***	8.97***	8.76***
	(0.45)	(0.72)	(1.16)
High, Low	7.32***	9.95***	64.2
	(0.59)	(0.91)	(>200)
Low, High	7.55***	10.7***	36.1
	(0.58)	(1.01)	(>200)
High, High	6.85***	7.13***	36.99
	(0.54)	(0.59)	(>200)
	Panel B: Qualification Complementarity		
	Wife Ethnicity		
Husb. Ethnicity	White	Black	Asian
White	1.60***	1.22***	2.58***
	(0.06)	(0.10)	(0.10)
Black	0.37***	1.51***	27.88
	(0.04)	(0.14)	(>200)
Asian	1.93***	26.67	1.76***
	(0.07)	(>200)	(0.07)

Notes: Panel A presents the estimates of $\Delta_{z,z'|q_m,q_f}$ while Panel B presents the estimates of $\Delta_{q,q'|z_m,z_f}$. See text for definitions. Standard errors in parenthesis.

Table 2.5 presents all ethnicity complementarity measures in Panel A and all qualification complementarity measures in Panel B.

All measures in Panel A are positive, indicating complementarity in ethnicity for every qualification profile and ethnicity combination. The estimates for Asian-Black marriages are naturally very imprecise and included mainly for completeness. There are two noteworthy features of the estimates in Panel A. First, the ethnicity complementarities do not show any strong qualification patterns, except for possibly being lower for lower for high-qualified individuals in the case of Whites and Asians. Second, $\Delta_{W,A|q_m,q_f} > \Delta_{W,B|q_m,q_f}$ for every male-female qualification profile (q_m, q_f); this suggests that the impetus towards positive ethnic sorting is uniformly stronger between Asians and Whites than among Blacks and Whites.

Similarly, all measures in Panel B are positive indicating complementarity in qualifications for every ethnicity profile. The numbers on the main diagonal – thus involving intra-ethnic marriages – are of similar magnitude. Marriages between Whites and Asians generally exhibit stronger qualification complementarities, whereas the opposite holds for marriages between Whites and Blacks.

Social Preferences

Our preferred specification includes the marriage social preferences, ϕ_1^m and ϕ_1^f, as free parameters while ϕ_0^m and ϕ_0^f are set to zero. The estimated values of the marriage social preferences parameters were $\phi_1^m = 0.47$ (0.07) and $\phi_1^f = 0.38$ (0.08). Both parameters are strongly positive and fairly precisely estimated, indicating that conformity preferences with respect to marital choices contribute to shaping how the equilibrium marriage patterns respond to variation in population supplies. As the (common) principal preferences in Σ feature strong complementarities – making cross-group marriages relatively rare – social norms tend to reinforce type-based positive sorting.[28]

Henceforth we will use $\eta^g_{x'x''}$ to denote the joint marital utility arising from local social norms regarding marriages of husband-wife type profile (x', x''), defined as

$$\eta^g_{x'x''} \equiv \phi_1^m \log\left(\mu^{g,m}_{x''|x'}\right) + \phi_1^f \log\left(\mu^{g,f}_{x'|x''}\right). \tag{2.15}$$

It is worth noting that, while the social preferences parameters, ϕ_1^m and ϕ_1^f, are common to all men and women respectively, the equilibrium social norms vary both across partner type-profiles and across regions.

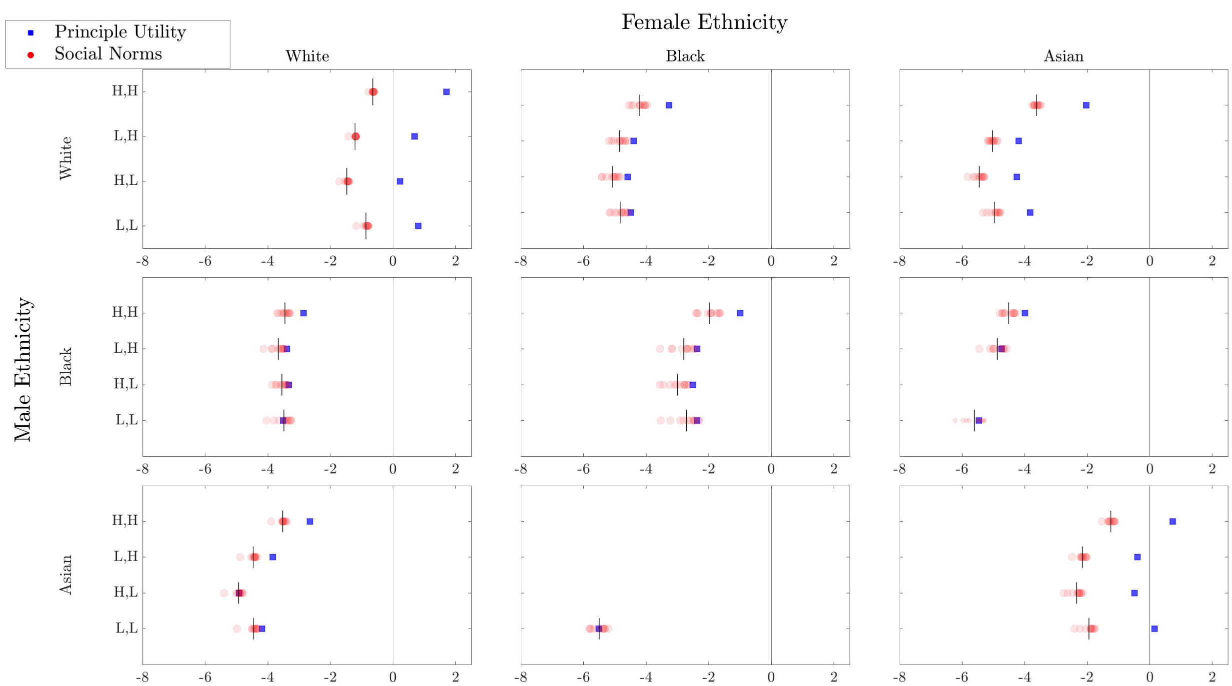
The estimates thus indicate that a marrying couple experience a utility penalty if their particular type profile occurs infrequently within their relative reference groups. One might intuitively expect social norms to be more favourable towards inter-ethnic marriages in areas where a given ethnic minority is larger. However, that is not necessarily the case as the two components of (2.15) tend to move in opposite directions: as the minority group gets larger,

[28] Note also that the estimated social preferences satisfy the condition (2.11) for uniqueness of the equilibrium.

members of the majority group will generally more often marry inter-ethnically, but at the same time, the members of the minority group will themselves tend to less frequently marry inter-ethnically.

Recall that the total systematic marriage utility is $W^g_{x'x''} = \sigma_{x'x''} + \eta^g_{x'x''}$. In order to visualize the location and spread of social norms relative to the underlying principal preferences Figure 2.7 plots the principal marital utilities $\sigma_{x'x''}$ and the distribution of the corresponding social norms, $\eta^g_{x'x''}$.

Figure 2.7: Principal Marriage Utility and Distribution of Social Norms by Marriage Type-Profile



Notes: The "principal utility" depicts the estimated $\sigma_{x'x''}$ for each husband-wife type profile while the "social norms" depicts the distribution of $\eta^g_{x'x''}$ across husband-wife type profiles and regions.

The figure is organized into nine panels, each panel corresponding to a husband-wife ethnicity profile.[29] Within each panel there are four rows, each corresponding to a qualification profile. The $\sigma_{x'x''}$ terms, which are replicated from Table 2.4, are highlighted in blue squares in Figure 2.7. For marriage profiles where both partners belong to the same ethnic group (the panels on the lead diagonal), there is a distinctive U-shaped pattern in the $\sigma_{x'x''}$ terms. This reflects the

[29]For ease of comparison, the figure uses the same scale in each panel. This means that a few the marital utility terms for marriages between Blacks and Asian are "below the scale" and hence not shown.

aforementioned qualification-complementarities.

More of interest here is the distribution of the social norm terms $\eta^g_{x'x''}$. For each husband-wife type-profile (x', x'') there is a distribution (across regions) of values of $\eta^g_{x'x''}$. These are illustrated using red dots, with a vertical black line indicating their mean value. A few things are worth noting. First, there is distinct variation in $\eta^g_{x'x''}$ for some key marriage profiles, for instance for marriages between Asian men and White women. Second, while the principal preferences $\sigma_{x'x''}$ are generally quite similar for White-Asian and White-Black inter-ethnic marriages, social norms are, on average, somewhat stronger against White-Asian marriages than against White-Black marriages.

Other Preferences

The remaining estimated parameters include the regional fixed effects and the preferences for marrying a non-UK-born spouse. Here we will briefly mention what these estimates show. The estimated values are presented in the Appendix.

The region fixed-effects ψ_g – as they are type-profile-independent additive terms – mainly acts as preference shift term for marriage versus singlehood. These are effects generally precisely estimated. While most areas have similar, London stands out as having a substantially lower regional marriage utility. This thus allows the model to capture the higher singles rate in capital. The preferences for marrying a non-UK-born spouse are also fairly precisely estimated. Moreover, they show the expected pattern of being substantially higher among Asians than among Whites or Blacks, and particularly high among low-qualified Asians.

2.5.3 Robustness to Sample Selection

Before moving to using the estimated model for prediction purposes, we will mention here two robustness analysis. First, we explored the robustness of the estimated preferences with respect to lowering the age threshold in the sample selection criteria.[30] In the analysis above, we only included individuals aged 25 or above in order to ensure that nearly everyone would have completed their education. In the robustness check, we lowered this threshold to 20. Doing

[30]The estimated preference parameters Σ and ϕ^m_1 and ϕ^f_1 from these two cases are available in the Appendix.

so naturally led to higher observed singles rates. Consequently the estimated marriage utilities were generally smaller. Moreover, the estimated social preferences in marriage were marginally increased. Nevertheless, the qualitative features – for instance, complementarity in education and similarity in the principal preferences associated with White-Black and White-Asian intra-ethnic marriages. Second, we re-estimated the model omitting London. This generally led to less variation in the principal preferences, in particular somewhat less negative utilities from inter-ethnic marriages. But it also led to stronger estimated social preferences to amply those principal preferences. The same qualitative features remained.

2.6 Predicted Future Ethnic Homogamy

In this final section we will use our estimated model to predict future ethnic homogamy in the UK. Specifically, we will use the parameters from the model – estimated on cohort born between 1965 and 1989 – alongside the observed demographic shifts to predict marriage patterns among individuals in a set of "more recent" cohorts, born between 1990 and 2006. In doing so we will also explore what role endogenously evolving social norms can be expected to play in this process.

In the Appendix we present a detailed description of the demographic changes. Most notably, both ethnic minorities have grown as a share of the population with the Asian population growing faster (from just below 3 percent to over 6 percent) than the Blacks (from around 1.2 percent to about 2.4 percent). The rate of holding a high qualification has also increased in all ethnic groups and both for males and females.[31]

Some of the key mechanisms involved are by now fairly clear. From Figure 2.6 we know that, for both Asians and Blacks, as an empirical stylized fact, intra-ethnic minority marriages are more common the larger is the own ethnicity as a share of the local population. In contrast, inter-ethnic marriages were observed to be more common among Whites when the minority shares are larger. The same figure shows that the estimated model naturally replicates these empirical patterns. As a consequence, in terms of projections for the recent cohorts, as the shares

[31] Noting the some of these recent cohorts have yet to complete their education, we impute predicted distributions of completed education by gender, ethnicity and region. Details of this are provided in the Appendix.

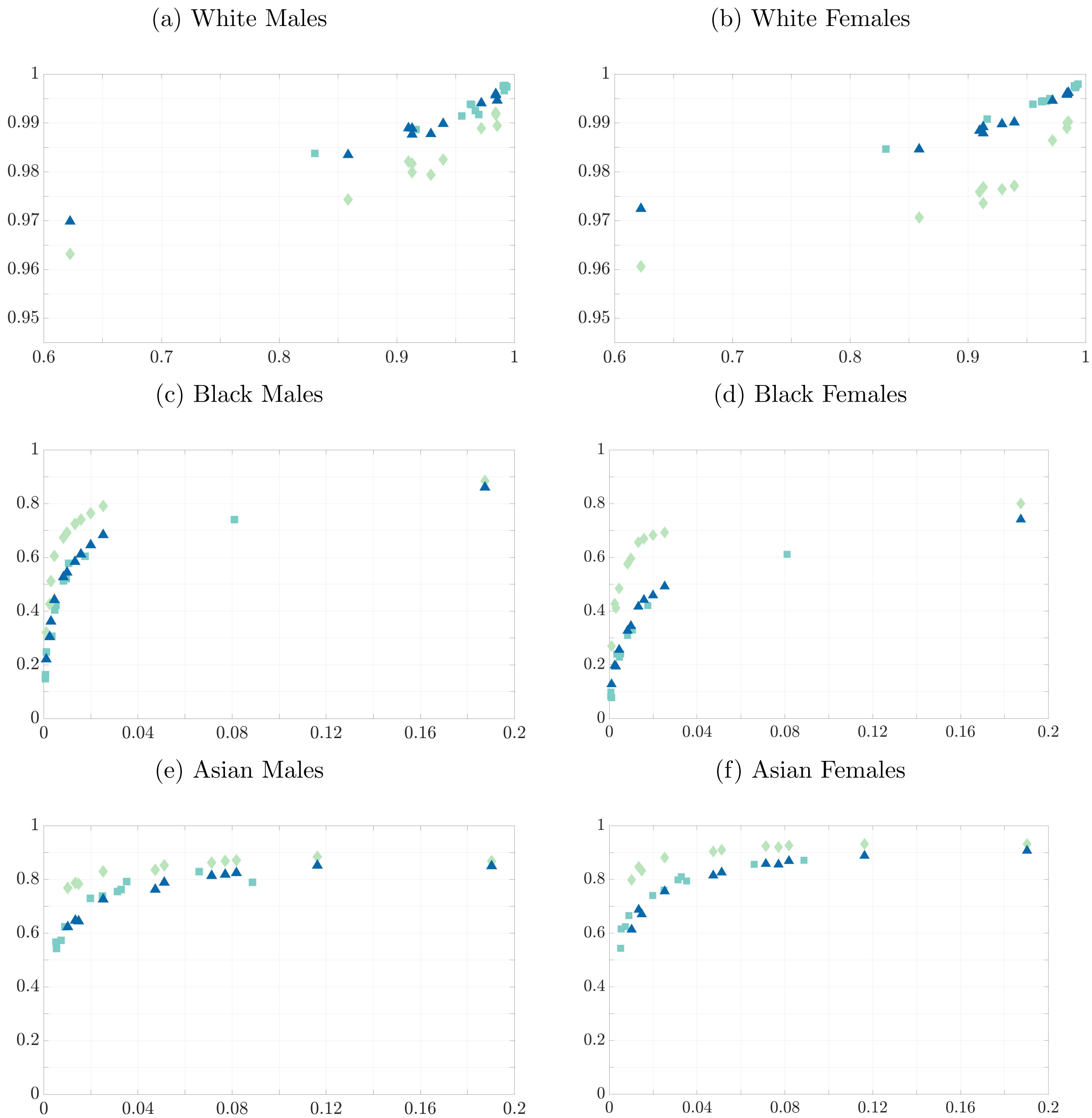
of Asians and Blacks have grown , the model will naturally predict that the two ethnic minorities will marry intra-ethnically more frequently whilst the majority Whites will inter-marry more frequently. These opposing direct effects will in turn have an ambiguous effect on social norms. Hence evolving social norms may either boost future integration through inter-ethnic marriages or reduce it. With respect to qualifications, our estimated model did not uncover any marked differences in the strength of preferences for ethnic marital sorting across qualification levels (see Table 2.5). Consequently, the increase in the rate of holding high qualifications between the estimating and recent cohorts can be expected to have at most a minor effect on the future proportion of inter-ethnic marriages.

In Figure 2.8 we plot predicted shares of intra-ethnic marriages across ethnicity, gender and regions. Indeed, each subfigure presents three sets of predictions. The first set, using *square markers*, illustrates the model predictions for the estimating cohorts and are thus carried forward from Figure 2.6 as benchmark. The second set, using *diamond-shaped* markers, illustrates the predictions for the recent cohorts. Notably, these include predicted equilibrium future social norms. Specifically to highlight the role of the evolving social norms, we also present also a third set of predictions, using *triangular* markers. This set of predictions is computed holding each social norm term $\eta^{g}_{x'x''}$ fixed at the value for the estimating cohorts.

Looking first at the Asians (panels e and f), we see that, due to their increasing population shares there is a general movement to the right between the estimating and the recent cohorts. For instance, in the estimating cohorts, the share of Asians was below four percentage points in the majority of areas. In contrast, in the recent cohorts only four out of the eleven regions now have less than four percent Asians. Similarly, in London the share of Asians nearly doubled from around nine percent to around nineteen percent.

There is a marked positive direct impact – holding social norms constant – on the predicted rate of intra-ethnic marriages among Asians. This predicted direct effect is then further boosted by evolving social norms to the extent that the model predicts that the proportion of intra-ethnic marriages among Asians, both men and women, in the recent cohorts will be 75 percent or higher in every region.

Figure 2.8: Predicted Shares of Intra-Ethnic Marriages across Regions by Ethnicity and Gender in Recent Cohorts Compared to Estimating Cohorts



Notes: The square-shaped markers indicate the model predictions for the "estimating cohorts" born 1965-1989 (carried forward from Figure 2.6). The diamond-shaped markers indicate the model predictions for the recent cohorts, born 1990-2006, and described in detail in the Appendix. The triangle-shaped markers indicate the model predictions for the recent cohorts but with social norms held constant at the levels estimated for the estimating cohorts.

Turning to the Blacks, Figure 2.8 (panels c and d) shows that the growth of their population share has been much more modest in all areas outside of London, but nevertheless systematic. Consequently, in terms of predicted shares of intra-ethnic marriages among Black, are also predicted to increase. Moveover, as the these shares increase from relatively low levels, the amplification via evolving social norms becomes relatively large. For London, there has been a sharp increase in the share of Blacks in the local population – from just over 8 percent in the estimating cohorts to over 18 percent in the recent cohorts. But for London, the predicted share of intra-ethnic marriages among Blacks was high – around 80 percent – already for the estimating cohorts, and is predicted to further increase only relatively marginally in the recent cohorts.

Finally, for the Whites (panels a and b), their population shares were 95 percent or higher in all areas except two in the estimating cohorts. In contrast, in the recent cohorts the share of Whites is at 95 percent or higher only in four areas. As a result, their predicted local rates of intra-ethnic marriage decrease, and this is further amplified by predicted evolving social norms. Nevertheless, the predicted rate of intra-ethnic marriage remains above 95 percent for both men and women in all regions, including London.

Table 2.6: Predicted Shares of Intra-Ethnic Marriages by Gender, Ethnicity, and Qualification in the Estimating and Recent Cohorts

	Panel A: Males					
	White		**Black**		**Asian**	
	Low Qual.	**High Qual.**	**Low Qual.**	**High Qual.**	**Low Qual.**	**High Qual.**
Born 1965-1989	0.994	0.993	0.551	0.592	0.837	0.734
Born 1990-2006	0.984	0.983	0.741	0.821	0.933	0.868
Difference	-0.010	-0.010	0.190	0.229	0.096	0.134
	Panel B: Females					
	White		**Black**		**Asian**	
	Low Qual.	**High Qual.**	**Low Qual.**	**High Qual.**	**Low Qual.**	**High Qual.**
Born 1965-1989	0.997	0.993	0.320	0.435	0.800	0.794
Born 1990-2006	0.989	0.978	0.597	0.741	0.925	0.919
Difference	-0.008	-0.015	0.277	0.306	0.125	0.125

Notes: Predictions for the estimating cohorts (born 1965-1989) and for recent cohorts (born 1990-2006).

Table 2.6 aggregates the predictions over regions to show predicted shares of marriages that are intra-ethnic by gender, ethnicity and qualification level both within the estimating cohorts and the recent cohorts. This highlights how the model predicts that the rate of intra-ethnic marriages will increase in both ethnic minority groups, with a relatively larger increase and starting from a lower level among Blacks.

Finally, if we aggregate over all marriages in the estimating and the recent cohorts respectively, we find that the predicted fraction of all marriages (between UK-born partners) that are inter-ethnic doubles from about 1.3 percentage points to about 2.5 percentage points. Hence overall as a proportion of all marriages, inter-ethnic marriages are predicted to increase. However, this is still consistent with a predicted decrease in the rates of inter-ethnic marriages among both Blacks and Asians. From Table 2.6 it is clear that most of this aggregate increase in inter-ethnic marriages comes from changes in marital behaviour for both low- and high-qualified individuals rather than from the increase qualifications between the two cohorts.

2.7 Conclusions

As Western economies become more ethnically diverse, will inter-ethnic marriages act as a force for long-term integration, breaking down barriers between natives and immigrants? The UK is a natural example to consider due to its distinct history of post-war Black and Asian immigration. That history created a particular and persistent regional variation in ethnic composition that has carried over to the second and beyond generations of immigrants. Most of the second generation immigrants who were born through the 1960s and 1970s have now gone on to marry and we can study how their marital choices reflect the ethnic composition of their local populations. For example, London is the UK region with by far the largest population shares of both Blacks and Asians. It is then striking that the rate of inter-ethnic marriage is lower among Blacks and Asians in London compared to other regions of the UK; this suggests that higher ethnic minority densities may not foster higher rates of inter-ethnic marriage among members of the minority groups. Furthermore marrying inter-ethnically can also be argued to go against social norms, raising the question of whether, as the ethnic minorities grow in relative size, social norms can be expected to change and contribute to the integration process.

To answer these question we have set up and estimated a structural model of the marriage market, building on the workhorse model of Choo and Siow (2006), and extended with endogenous social norms modelled as conformity preferences. Using that the geographical distribution of UK-born ethnic minorities reflects the settlement patterns of the first generation post-war immigrants, we found that Blacks and Asians are systematically less likely to marry inter-ethnically if they live in regions where their own ethnicity is comparatively larger. The estimates suggest strong ethnic complementarities – stronger than for qualifications – reflected in strong ethnic homogamy. We also found that allowing for endogenous social norms provided a better fit to the data.

We used the estimated model to predict marital patterns among a set of more recent cohorts in order to consider whether more integration through marriage can be expected going forward. Here we found that, although inter-ethnic marriages can be expected to grow as a share of all marriages, members of the ethnic minority groups are predicted to become less likely to marry inter-ethnically, a result that is also amplified by evolving endogenous social norms. In this sense, the model does not suggest that the Black and Asian populations will become increasingly integrated with the White majority group through the formation of marriages.

A caveat to the current approach is of course the assumption of stable preferences: at the heart of the model is a set of principal preference parameters that describe how the systematic joint marriage utility relate to partner type profile, and these preferences are assumed not to change. Viewed from this perspective, the results presented above can be argued to show that future integration through marriage will require that a change of fundamental preferences. Indeed, the recent work by Merlino, Steinhardt and Wren-Lewis (2019), using quasi-random variation in ethnic exposure during childhood, show that exposure to Black peers when young lead whites to have more relationships with Blacks as adults. The authors argue that their results reflect underlying effects on attitudes which, translated to our setting, is more akin to a change in the principal preferences than to a change social norms. It should however be noted that our specification tests detected no systematic differences across the regions in the underlying primitive preferences. This is notable since the members of the estimating cohorts will have had very different exposure to ethnic variation in their youth.

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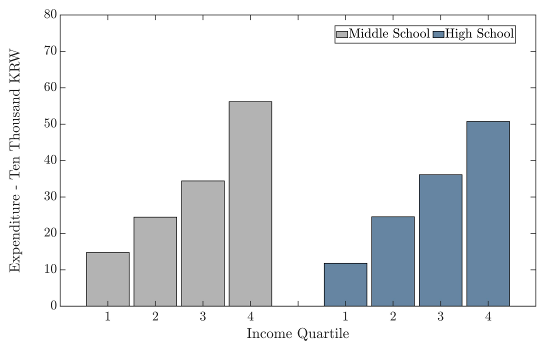
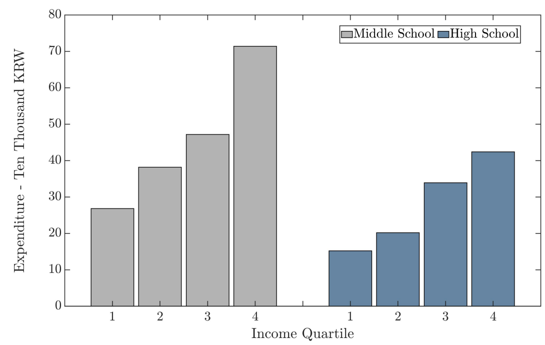
Appendix A: Additional Figures (Chapter 1)

Figure A.1: Model Fit: Private Education Expenditure and Participation by Income Quartile and School Level

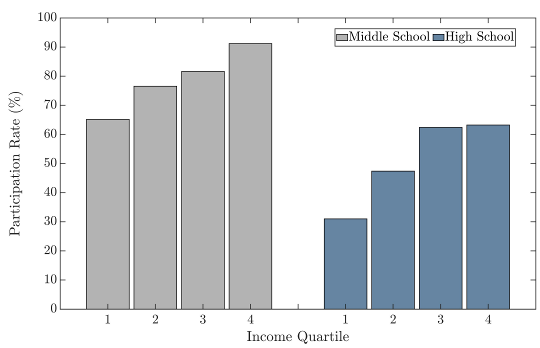
(a) Data: Expenditure - Ten Thousand KRW



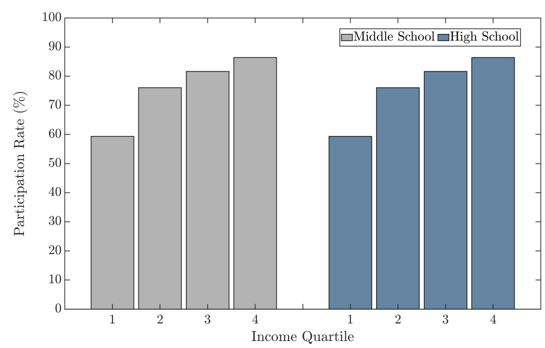
(b) Model: Expenditure - Ten Thousand KRW



(c) Data: Participation - %



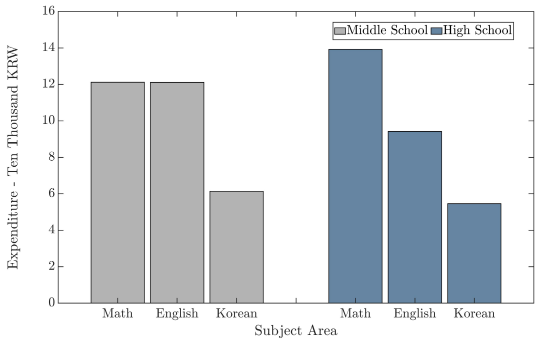
(d) Model: Participation - %



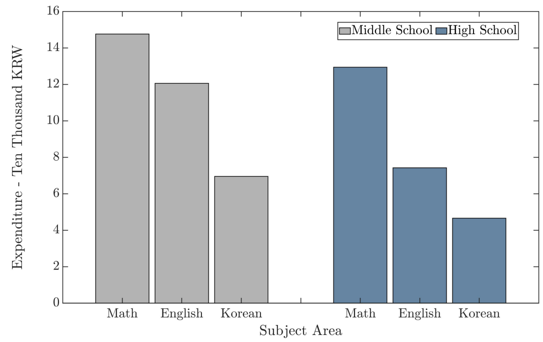
Notes: The sample consists of all Korean middle school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

Figure A.2: Model fit: Private Education Expenditure and Participation by Academic Subject and School Level

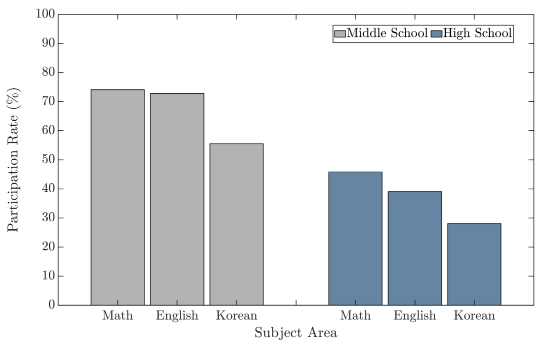
(a) Data: Expenditure - Ten Thousand KRW



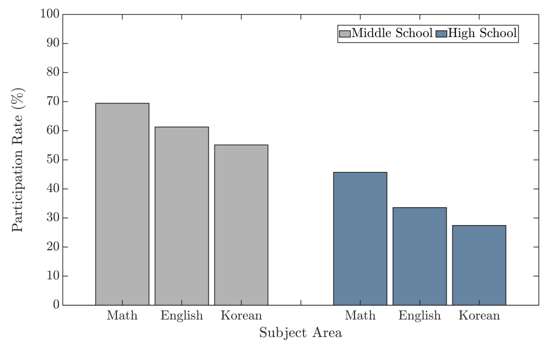
(b) Model: Expenditure - Ten Thousand KRW



(c) Data: Participation - %

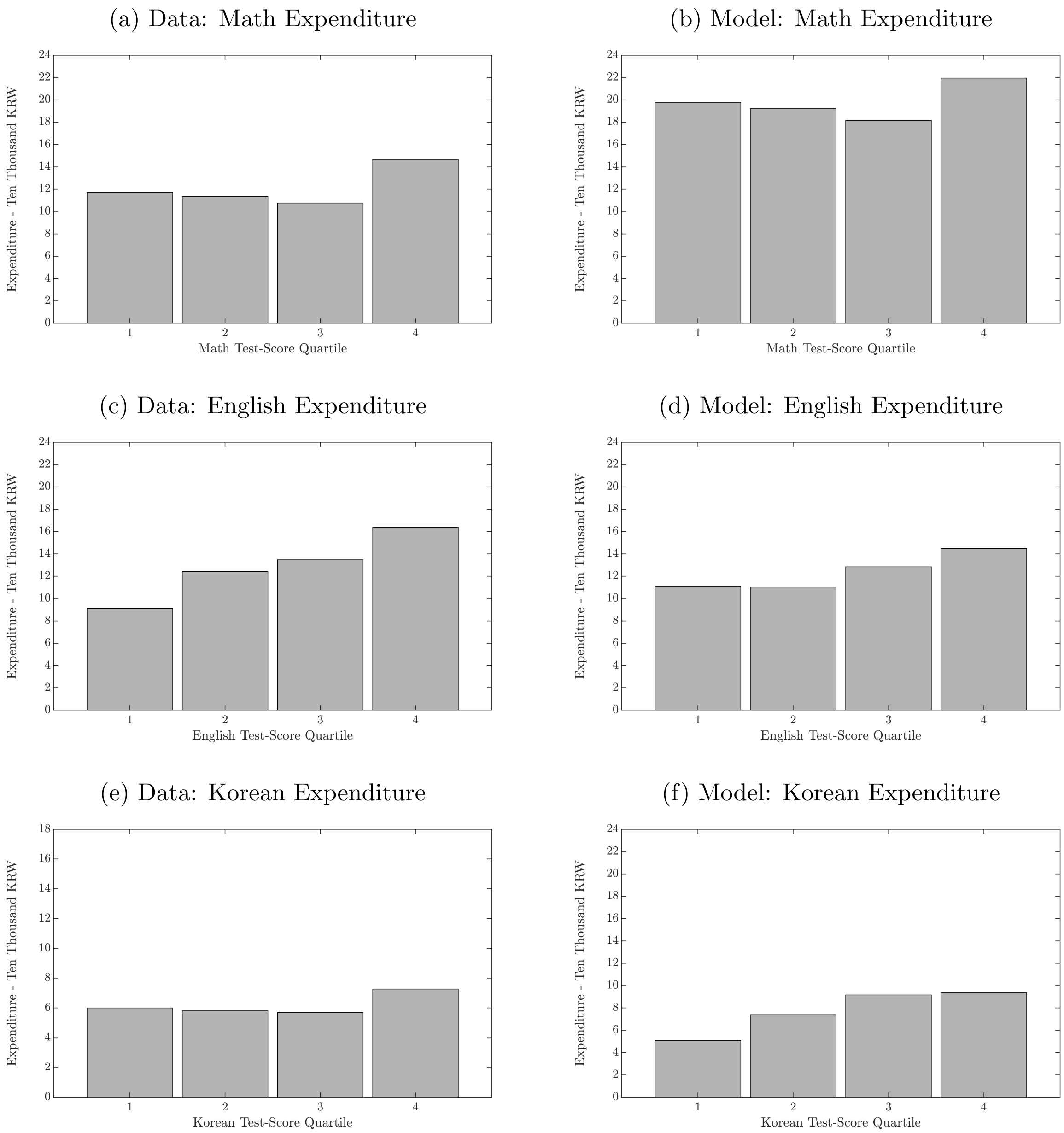


(d) Model: Participation - %



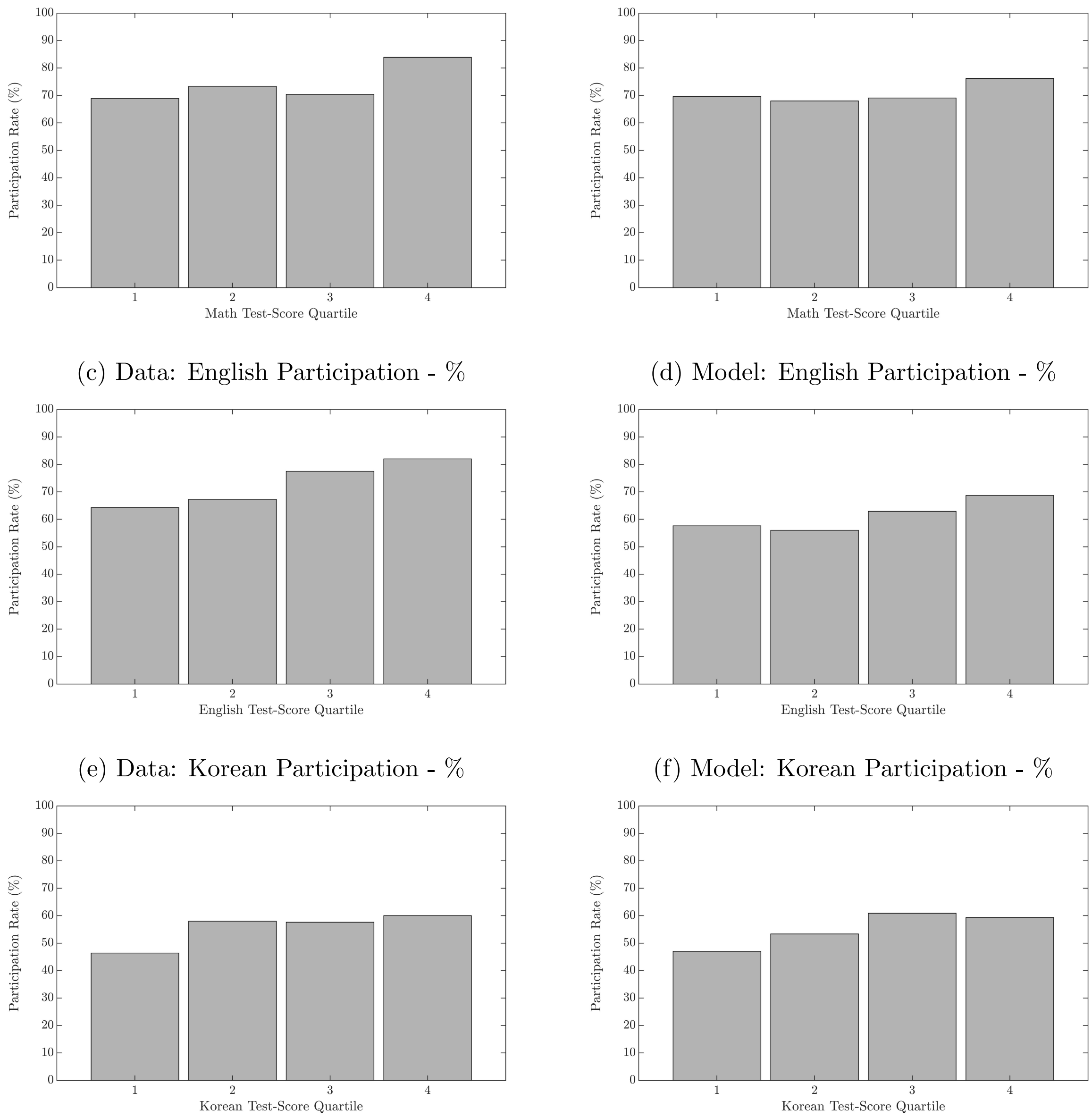
Notes: The sample consists of all Korean middle school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

Figure A.3: Model fit: Private Education Expenditure by Test-Score Quartile



Notes: The sample consists of all Korean middle school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

Figure A.4: Model fit: Private Education Participation by Test-Score Quartile



(a) Data: Math Participation - %

(b) Model: Math Participation - %

(c) Data: English Participation - %

(d) Model: English Participation - %

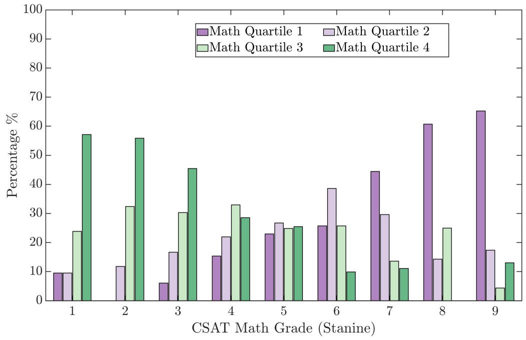
(e) Data: Korean Participation - %

(f) Model: Korean Participation - %

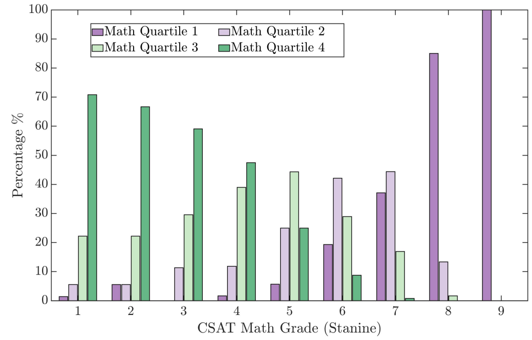
Notes: The sample consists of all Korean middle school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

Figure A.5: Model fit: Correlation Between Middle-School Test-Scores and CSAT Test-Scores

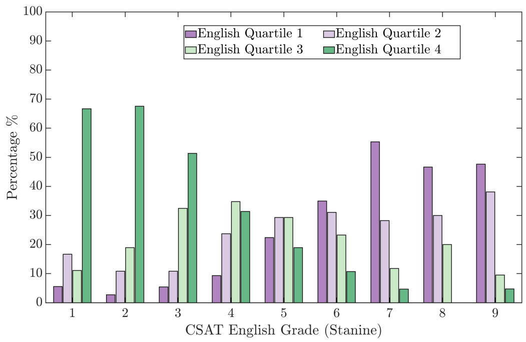
(a) Data: Math



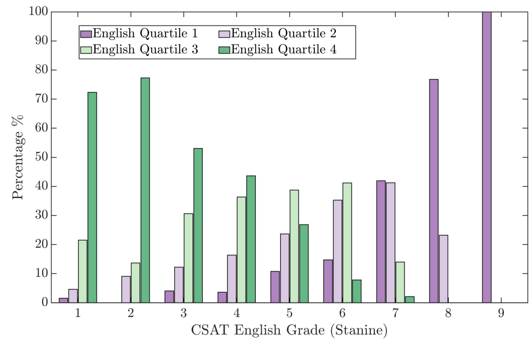
(b) Model: Math



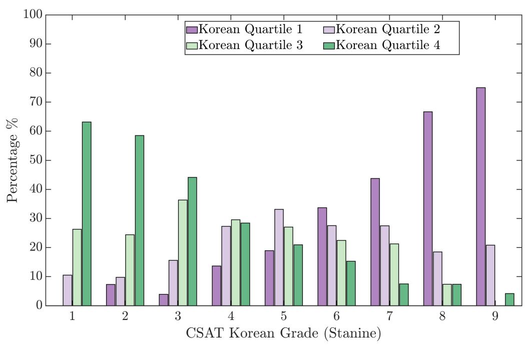
(c) Data: English



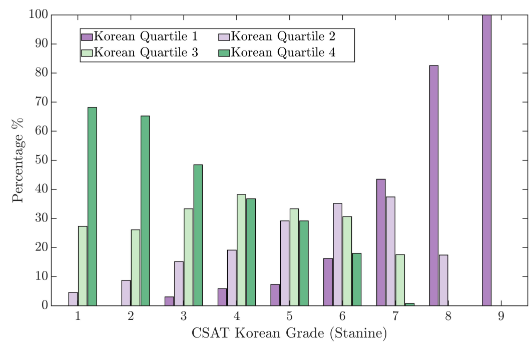
(d) Model: English



(e) Data: Korean



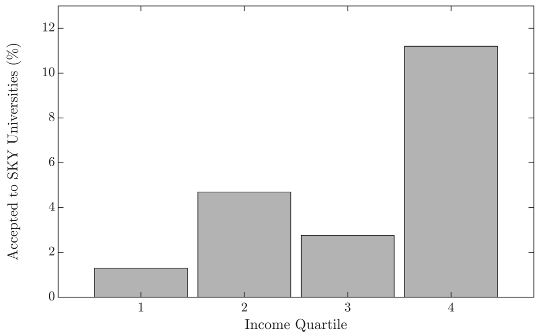
(f) Model: Korean



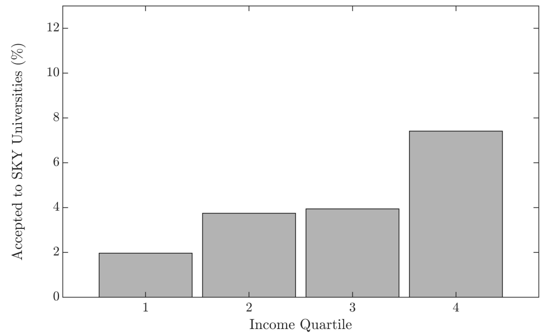
Notes: The sample consists of all Korean middle school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

Figure A.6: Model fit: Proportion Accepted to SKY Universities

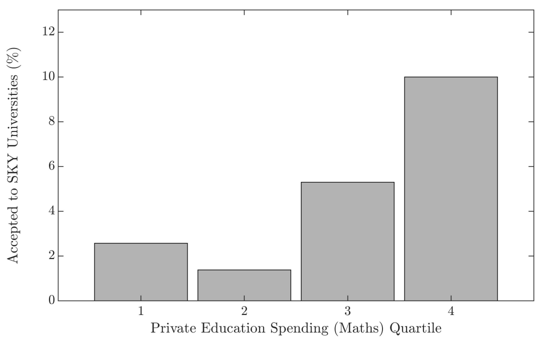
(a) Data: Income Quartiles



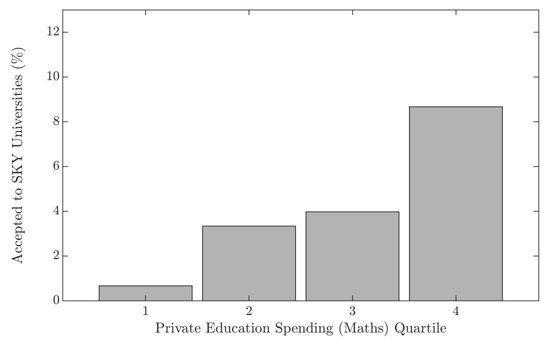
(b) Model: Income Quartiles



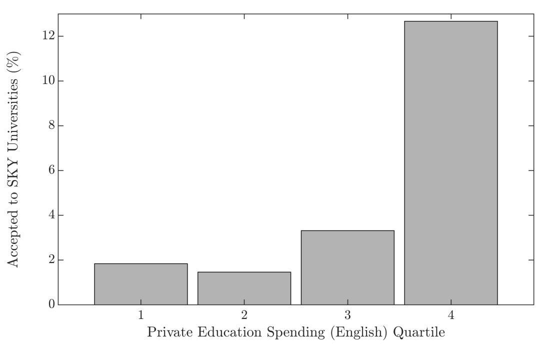
(c) Data: Math Spending Quartiles



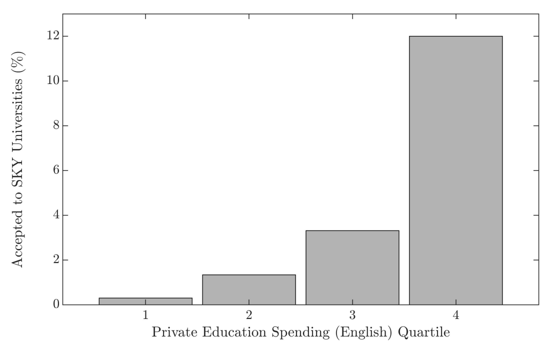
(d) Model: Math Spending Quartiles



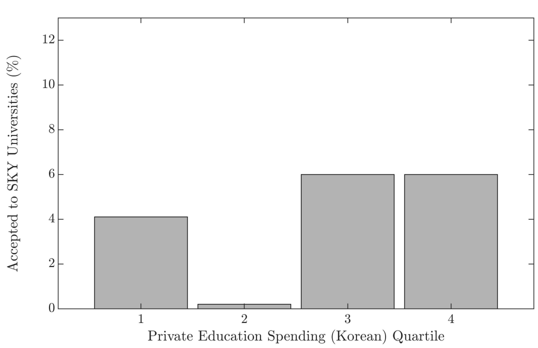
(e) Data: English Spending Quartiles



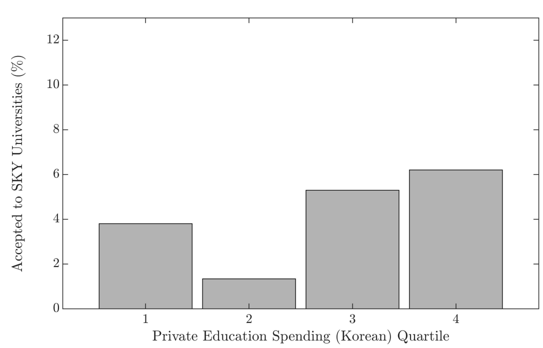
(f) Model: English Spending Quartiles



(g) Data: Korean Spending Quartiles



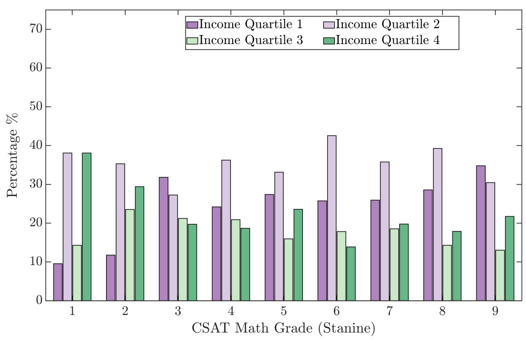
(h) Model: Korean Spending Quartiles



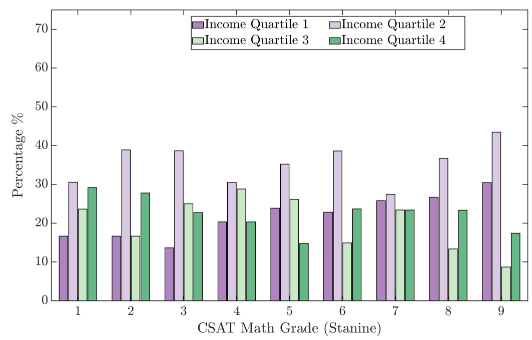
Notes: The sample consists of all Korean middle school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

Figure A.7: Model fit: Achievement Gap Between Rich and Poor Students

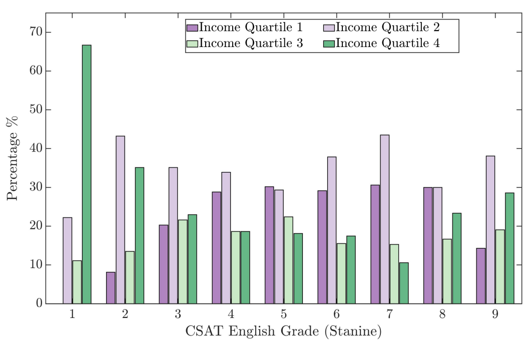
(a) Data: CSAT Math



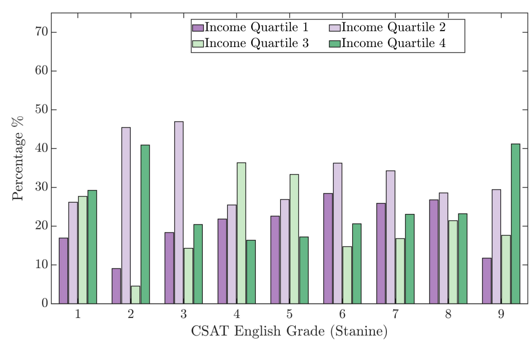
(b) Model: CSAT Math



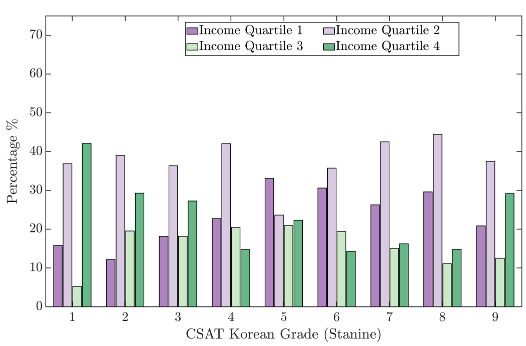
(c) Data: CSAT English



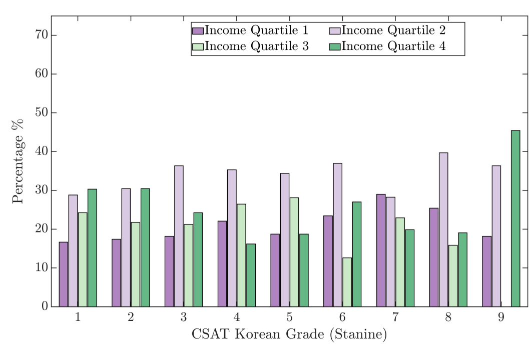
(d) Model: CSAT English



(e) Data: CSAT Korean



(f) Model: CSAT Korean



Notes: The sample consists of all Korean middle school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

Appendix B: Additional Tables (Chapter 1

Table B1: Model fit: Quantifying the Achievement Gap Between Rich and Poor Students

	CSAT Grades - Data					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.10	0.12	0	0.08	0.16	0.12
	(0.06)	(0.10)	(0)	(0.06)	(0.08)	(0.07)
Income quartile 4	0.38	0.29	0.67	0.35	0.42	0.29
	(0.11)	(0.10)	(0.11)	(0.11)	(0.11)	(0.10)
quartile 4 - quartile 1	0.28**	0.17**	0.67***	0.27***	0.26*	0.17*
	(0.12)	(0.08)	(0.11)	(0.09)	(0.14)	(0.09)
	CSAT Grades - Predicted					
	Math		*English*		*Korean*	
	grade 1	*grade 2*	*grade 1*	*grade 2*	*grade 1*	*grade 2*
Income quartile 1	0.16	0.16	0.17	0.09	0.16	0.17
	(0.04)	(0.04)	(0.05)	(0.04)	(0.05)	(0.05)
Income quartile 4	0.29	0.28	0.29	0.41	0.30	0.30
	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)	(0.06)
quartile 4 - quartile 1	0.13**	0.12**	0.12*	0.32***	0.14**	0.13
	(0.06)	(0.05)	(0.07)	(0.12)	(0.07)	(0.12)

	Accepted to SKY University (%) - Data	*Accepted to SKY University (%) - Predicted*
Income quartile 1	0.01	0.02
	(0.00)	(0.01)
Income quartile 4	0.11	0.08
	(0.03)	(0.02)
quartile 4 - quartile 1	0.10***	0.06***
	(0.03)	(0.02)

Notes: The sample consists of all Korean middle school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education. Standard errors in parenthesis. (* $\Rightarrow p < 0.1$), (** $\Rightarrow p < 0.05$), (***$\Rightarrow p < 0.01$).

Table B2: Robustness Check: Aduluthod Income Parameter Estimates

Dependant Variable - Log Monthly Wage		
Log Standard KSAT Scores		
Math - s_1^{uni}	English - s_2^{uni}	Korean - s_3^{uni}
0.127**	0.162***	0.008
[0.048, 0.194]	[0.101, 0.245]	[-0.091, 0.047]
Demographic Controls		
SKY University	Male	Constant
0.134*	0.158***	4.223***
[0.007, 0.184]	[0.120, 0.189]	[3.998, 4.573]

Regional Controls					
Busan	Daegu	Incheon	Gwangju	Daejeon	Ulsan
-0.048*	-0.043	-0.029	-0.066	0.021	-0.000
[-0.113, -0.004]	[-0.108, 0.017]	[-0.093, 0.048]	[-0.139, 0.008]	[-0.047, 0.081]	[-0.072, 0.084]
Gyeonggi	Gangwon	Chungbuk	Chungnam	Jeonbuk	Jeonnam
-0.011	-0.152**	-0.007	-0.043	-0.087**	-0.058
[-0.061, 0.024]	[-0.276, -0.051]	[-0.068, 0.057]	[-0.118, 0.017]	[-0.141, -0.025]	[-0.112, 0.023]
Gyeongbuk	Gyeongnam	Jeju	Sejong	Foreign	
-0.033	-0.018	-0.027	0.091***	0.324***	
[-0.082, 0.034]	[-0.061, 0.047]	[-0.171, 0.143]	[0.065, 0.134]	[0.142, 0.459]	

Other Statistics		
N = 1,406	$R^2 = 0.14$	$\bar{R}^2 = 0.12$

Notes: 90% confidence intervals based on 1,000 bootstrap replications in square parentheses. The unit of measure for monthly wage is 10,000 South Korean Won (KRW). For regional controls Seoul is the excluded region. (* $\Rightarrow p < 0.1$), (** $\Rightarrow p < 0.05$), (***$\Rightarrow p < 0.001$). The sample consists of all Korean high-school cohort students observed in the Korean Education and Employment Panel (KEEP), survey years 2004-2016, with available information on CSAT scores, educational attainment, university enrollment, gender, household demographics, parental background, parental and investments in private education.

Appendix C: Proof of Identification (Chapter 2)

By Assumption 2, we have access to (at least) two groups with non-identical relative population supplies. Without loss of generality, we can focus on exactly two non-identical groups, $g \in G = \{g_1, g_2\}$. With two types $X = \{x_1, x_2\}$, there are four possible husband-wife type-profiles $(x', x'') \in X \times X$. Hence with two groups, there is a total of $2N^2 = 8$ matching equations of the form (2.7). Using the assumption of common principal preferences, $\sigma^g_{x'x''} = \sigma_{x'x''}$ (Assumption 1) we can conveniently rewrite (2.7) as follows

$$\sigma_{x'x''} + \phi_1^m \log \mu^{g,m}_{x''|x'} + \phi_1^f \log \mu^{g,f}_{x'|x''} - \phi_0^m \log \mu^{g,m}_{0|x'} - \phi_0^f \log \mu^{g,f}_{0|x''} = \log \left(\frac{\mu^{g,m}_{x''|x'} \mu^{g,f}_{x'|x''}}{\mu^{g,m}_{0|x'} \mu^{g,f}_{0|x''}} \right). \quad \text{(C1)}$$

We can collect these matching equations in matrix form by defining, for each group $g \in G$, the following 4×4 matrix,

$$A^g \equiv \begin{bmatrix} \log\left(\mu^{g,m}_{x_1|x_1}\right) & -\log\left(\mu^{g,m}_{0|x_1}\right) & \log\left(\mu^{g,f}_{x_1|x_1}\right) & -\log\left(\mu^{g,f}_{0|x_1}\right) \\ \log\left(\mu^{g,m}_{x_2|x_1}\right) & -\log\left(\mu^{g,m}_{0|x_1}\right) & \log\left(\mu^{g,f}_{x_1|x_2}\right) & -\log\left(\mu^{g,f}_{0|x_2}\right) \\ \log\left(\mu^{g,m}_{x_1|x_2}\right) & -\log\left(\mu^{g,m}_{0|x_2}\right) & \log\left(\mu^{g,f}_{x_2|x_1}\right) & -\log\left(\mu^{g,f}_{0|x_1}\right) \\ \log\left(\mu^{g,m}_{x_2|x_2}\right) & -\log\left(\mu^{g,m}_{0|x_2}\right) & \log\left(\mu^{g,f}_{x_2|x_2}\right) & -\log\left(\mu^{g,f}_{0|x_2}\right) \end{bmatrix}, \quad \text{(C2)}$$

and the following 4×1 column vector,

$$B^g \equiv \left[\log\left(\frac{\mu^{g,m}_{x_1|x_1}}{\mu^{g,m}_{0|x_1}} \frac{\mu^{g,f}_{x_1|x_1}}{\mu^{g,f}_{0|x_1}}\right) \quad \log\left(\frac{\mu^{g,m}_{x_2|x_1}}{\mu^{g,m}_{0|x_1}} \frac{\mu^{g,f}_{x_1|x_2}}{\mu^{g,f}_{0|x_2}}\right) \quad \log\left(\frac{\mu^{g,m}_{x_1|x_2}}{\mu^{g,m}_{0|x_2}} \frac{\mu^{g,f}_{x_2|x_1}}{\mu^{g,f}_{0|x_1}}\right) \quad \log\left(\frac{\mu^{g,m}_{x_2|x_2}}{\mu^{g,m}_{0|x_2}} \frac{\mu^{g,f}_{x_2|x_2}}{\mu^{g,f}_{0|x_2}}\right) \right]' \text{ for } g \in G. \quad \text{(C3)}$$

The $2N^2$ matching equations can be stacked and written succinctly as $\mathbf{A}\theta = \mathbf{B}$ where,

$$\mathbf{A} \equiv \begin{bmatrix} I_4 & A^1 \\ I_4 & A^2 \end{bmatrix}, \qquad \text{and} \qquad \mathbf{B} \equiv \begin{bmatrix} B^1 \\ B^2 \end{bmatrix}, \quad \text{(C4)}$$

and where I_n is the $n \times n$ identity matrix, and where the parameter vector θ in this case is

$$\theta \equiv \left[\sigma_{x_1x_1}, \sigma_{x_1x_2}, \sigma_{x_2x_1}, \sigma_{x_2x_2}, \phi_1^m, \phi_0^m, \phi_1^f, \phi_0^f\right]'. \quad \text{(C5)}$$

Note that the dimensions of $\mathbf{A}$ is 8×8 and $\mathbf{B}$ is 8×1 and that $\mathbf{A}$ and $\mathbf{B}$ only contain (log-values of) population marriage and singles rates. These rates all have empirical counterparts, which by standard arguments, converge in probability to their population values as the sample size grows. Hence, $\mathbf{A}$ and $\mathbf{B}$, can be taken to be known.

Remark 1. It is instructive to note how the current setting generalizes the original Choo and Siow (2006) setting with just one group and no peer-effects. In that case the reduced parameter vector would be $\theta_{CS} \equiv [\sigma_{x_1x_1}, \sigma_{x_1x_2}, \sigma_{x_2x_1}, \sigma_{x_2x_2}]'$ and the matching equations written in matrix form would be $\mathbf{I}_2\theta_{CS} = \mathbf{B}$, which trivially just notes that $\theta_{CS} = \mathbf{B}$.

This then suggests that θ could be obtained by inverting $\mathbf{A}$. However, $\mathbf{A}$ does not have full rank. This is consistent with θ being identified only up to a scaling factor: as it not possible to identify the scale of θ, one parameter can be arbitrarily fixed (e.g. $\phi_0^f = 0$). We will correspondingly show that, generically, $rank\,(\mathbf{A}) = 7$. That is, we will show that there is one linear dependence among the eight matching equations.

To do this, we can use basic Gauss-Jordan elimination. As a first step, we can subtract the upper half of $\mathbf{A}$ from the lower half to obtain

$$\mathbf{A}' \equiv \begin{bmatrix} I_4 & A^1 \\ 0_{4\times 4} & \Delta_A \end{bmatrix}, \tag{C6}$$

where $\Delta_A \equiv A^2 - A^1$, contains all group log-differences in marriages and singles rates and where $0_{n\times m}$ is the $n\times m$ matrix with only zeros. Fully written out,

$$\Delta_A = \begin{bmatrix} \log\left(\frac{\mu^{2,m}_{x_1|x_1}}{\mu^{1,m}_{x_1|x_1}}\right) & -\log\left(\frac{\mu^{2,m}_{0|x_1}}{\mu^{1,m}_{0|x_1}}\right) & \log\left(\frac{\mu^{2,f}_{x_1|x_1}}{\mu^{1,f}_{x_1|x_1}}\right) & -\log\left(\frac{\mu^{2,f}_{0|x_1}}{\mu^{1,f}_{0|x_1}}\right) \\ \log\left(\frac{\mu^{2,m}_{x_2|x_1}}{\mu^{1,m}_{x_2|x_1}}\right) & -\log\left(\frac{\mu^{2,m}_{0|x_1}}{\mu^{1,m}_{0|x_1}}\right) & \log\left(\frac{\mu^{2,f}_{x_1|x_2}}{\mu^{1,f}_{x_1|x_2}}\right) & -\log\left(\frac{\mu^{2,f}_{0|x_2}}{\mu^{1,f}_{0|x_2}}\right) \\ \log\left(\frac{\mu^{2,m}_{x_1|x_2}}{\mu^{1,m}_{x_1|x_2}}\right) & -\log\left(\frac{\mu^{2,m}_{0|x_2}}{\mu^{1,m}_{0|x_2}}\right) & \log\left(\frac{\mu^{2,f}_{x_2|x_1}}{\mu^{1,f}_{x_2|x_1}}\right) & -\log\left(\frac{\mu^{2,f}_{0|x_1}}{\mu^{1,f}_{0|x_1}}\right) \\ \log\left(\frac{\mu^{2,m}_{x_2|x_2}}{\mu^{1,m}_{x_2|x_2}}\right) & -\log\left(\frac{\mu^{2,m}_{0|x_2}}{\mu^{1,m}_{0|x_2}}\right) & \log\left(\frac{\mu^{2,f}_{x_2|x_2}}{\mu^{1,f}_{x_2|x_2}}\right) & -\log\left(\frac{\mu^{2,f}_{0|x_2}}{\mu^{1,f}_{0|x_2}}\right) \end{bmatrix}. \tag{C7}$$

Remark 2. Generically, $rank\,(A^g) = 4$ for each group $g \in G$. If the two groups $g \in \{g_1, g_2\}$ had not only common principal preferences (Assumption 1), but also identical relatively population supplies (violating Assumption 2), the equilibrium marriage and singles-rates would be identical in the two groups for each gender, type, and type-profile, whereby $A^1 = A^2$ and $B^1 = B^2$. It then trivially follows that $\mathbf{A}$ would only have $rank(\mathbf{A}) = 4$ and the model with peer effects would not be identified.

As $\mathbf{A}'$ was obtained from $\mathbf{A}$ via row subtraction, $rank\,(\mathbf{A}) = rank\,(\mathbf{A}')$ and we will prove that the lower half of $\mathbf{A}'$ does not have linearly independent rows. Specifically, we will demonstrate that $rank\,(\Delta_A) = 3$. For notational simplicity, let δ_{ij} denote the (i,j)-th element of Δ_A. Two rounds of Gauss-Jordan elimination, pivoting on rows 1 and 2 of Δ_A in turn yields,

$$\Delta'_A = \begin{bmatrix} 1 & 0 & \frac{\delta_{13}}{\delta_{11}} - \frac{\delta_{12}}{\delta_{11}} \frac{\delta_{23}-\delta_{21}\frac{\delta_{13}}{\delta_{11}}}{\delta_{22}-\delta_{21}\frac{\delta_{12}}{\delta_{11}}} & \frac{\delta_{14}}{\delta_{11}} - \frac{\delta_{12}}{\delta_{11}} \frac{\delta_{24}-\delta_{21}\frac{\delta_{14}}{\delta_{11}}}{\delta_{22}-\delta_{21}\frac{\delta_{12}}{\delta_{11}}} \\ 0 & 1 & \frac{\delta_{23}-\delta_{21}\frac{\delta_{13}}{\delta_{11}}}{\delta_{22}-\delta_{21}\frac{\delta_{12}}{\delta_{11}}} & \frac{\delta_{24}-\delta_{21}\frac{\delta_{14}}{\delta_{11}}}{\delta_{22}-\delta_{21}\frac{\delta_{12}}{\delta_{11}}} \\ 0 & 0 & \begin{matrix}\left(\delta_{33} - \delta_{31}\frac{\delta_{13}}{\delta_{11}}\right) \\ -\left(\delta_{32} - \delta_{31}\frac{\delta_{12}}{\delta_{11}}\right)\frac{\delta_{23}-\delta_{21}\frac{\delta_{13}}{\delta_{11}}}{\delta_{22}-\delta_{21}\frac{\delta_{12}}{\delta_{11}}}\end{matrix} & \begin{matrix}\left(\delta_{34} - \delta_{31}\frac{\delta_{14}}{\delta_{11}}\right) \\ -\left(\delta_{32} - \delta_{31}\frac{\delta_{12}}{\delta_{11}}\right)\frac{\delta_{24}-\delta_{21}\frac{\delta_{14}}{\delta_{11}}}{\delta_{22}-\delta_{21}\frac{\delta_{12}}{\delta_{11}}}\end{matrix} \\ 0 & 0 & \begin{matrix}\left(\delta_{43} - \delta_{41}\frac{\delta_{13}}{\delta_{11}}\right) \\ -\left(\delta_{42} - \delta_{41}\frac{\delta_{12}}{\delta_{11}}\right)\frac{\delta_{23}-\delta_{21}\frac{\delta_{13}}{\delta_{11}}}{\delta_{22}-\delta_{21}\frac{\delta_{12}}{\delta_{11}}}\end{matrix} & \begin{matrix}\left(\delta_{44} - \delta_{41}\frac{\delta_{14}}{\delta_{11}}\right) \\ -\left(\delta_{42} - \delta_{41}\frac{\delta_{12}}{\delta_{11}}\right)\frac{\delta_{24}-\delta_{21}\frac{\delta_{14}}{\delta_{11}}}{\delta_{22}-\delta_{21}\frac{\delta_{12}}{\delta_{11}}}\end{matrix} \end{bmatrix}. \tag{C8}$$

To prove that $rank\,(\Delta_A) = 3$ we will show that the last two rows of Δ'_A are identically the same. We start with the two final terms in column 3 of Δ'_A. A simple rearranging shows that these terms are identical if and only if

$$\frac{\left(\delta_{42} - \delta_{41}\frac{\delta_{12}}{\delta_{11}}\right) - \left(\delta_{32} - \delta_{31}\frac{\delta_{12}}{\delta_{11}}\right)}{\delta_{22} - \delta_{21}\frac{\delta_{12}}{\delta_{11}}} = \frac{\left(\delta_{43} - \delta_{41}\frac{\delta_{13}}{\delta_{11}}\right) - \left(\delta_{33} - \delta_{31}\frac{\delta_{13}}{\delta_{11}}\right)}{\delta_{23} - \delta_{21}\frac{\delta_{13}}{\delta_{11}}}, \tag{C9}$$

holds. We will show that (C9) indeed holds as both sides are equal to unity.

Consider first the left hand side of (C9) and note that this is unity if and only if

$$\frac{\delta_{31} + \delta_{21} - \delta_{41}}{\delta_{11}} = \frac{\delta_{32} + \delta_{22} - \delta_{42}}{\delta_{12}}. \tag{C10}$$

But (C10) holds as both sides are equal to unity: using the definition of terms in Δ_A in (C7), equation (2.10) and Assumption 1 immediately imply that $\delta_{11} - \delta_{21} = \delta_{31} - \delta_{41}$ and, directly through their definitions, $\delta_{12} = \delta_{22}$ and $\delta_{32} = \delta_{42}$.

Consider then the right hand side of (C9) and note that this is unity if and only if

$$\frac{\delta_{33} + \delta_{23} - \delta_{43}}{\delta_{13}} = \frac{\delta_{31} + \delta_{21} - \delta_{41}}{\delta_{11}}. \tag{C11}$$

But (C11) also holds as both sides are unity: as already confirmed above, $\delta_{11} - \delta_{21} = \delta_{31} - \delta_{41}$, while equation (2.10) and Assumption 1 imply that $\delta_{13} - \delta_{23} = \delta_{33} - \delta_{43}$. A corresponding set of steps show that the final two terms in column 4 of Δ'_A are identical.

Equality of the final two rows of Δ'_A implies that $rank\left(\Delta'_A\right) = 3$. Generically, with the two groups having different relative population supplies, there is no further linear dependence between the rows of A^1 and $A^2 - A^1$, and hence $rank\left(\mathbf{A}\right) = rank\left(\mathbf{A}'\right) = 7$.

Remark 3. The demonstrated equality of the two final rows of Δ'_A implies that the *row echelon form* of $\mathbf{A}$ is

$$ref\left(\mathbf{A}\right) = \begin{bmatrix} I_7 & \widetilde{A}_{7\times 1} \\ 0_{1\times 7} & 0_{1\times 1} \end{bmatrix},$$

where $\widetilde{A}$ is a non-zero vector with indicated size.

The one-less-than-full rank of $\mathbf{A}$ reflects that the equilibrium is invariant to a rescaling of all parameters by a scaling factor $\lambda >$ as outlined in the text. Consequently, one could impose $\phi_0^f = 0$ as normalization, drop one equation, and solve the remaining seven equations for the remaining seven parameters.

Remark 4. Extending to N types expands the dimensions of A^g to $N^2 \times 4$ and hence expands the dimension of $\mathbf{A}$ to $2N^2 \times \left(N^2 + 4\right)$. Still, as the parameters can be identified only up to a scaling factor, $rank\left(\mathbf{A}\right) = N^2 + 3$. In particular, in this generalization, the echelon form of $\mathbf{A}$ is

$$ref\left(\mathbf{A}\right) = \begin{bmatrix} I_{N^2+3} & \widetilde{A}_{(N^2+3)\times 1} \\ 0_{(N^2-3)\times(N^2+3)} & 0_{(N^2-3)\times 1} \end{bmatrix},$$

allowing for the identification of Σ and, after normalization, three peer effects parameters. At the same time, expanding to more types leads to more testable restrictions based on (2.10).

Remark 5. Extending to $|G| \geq 3$ groups and allowing for group fixed-effects such that $\Sigma^g =$

$\Sigma + \psi_g$ further modifies the matching equation system. With (for simplicity) $N = 2$ types and $|G|$ groups, the extended parameter vector can be written as $\theta_{FE} \equiv \left[\sigma_{x_1x_1}, \sigma_{x_1x_2}, \sigma_{x_2x_1}, \sigma_{x_2x_2}, \phi_1^m, \phi_0^m, \phi_1^f, \phi_0^f, \psi_1, ..., \psi_{|G}\right.$ and the matching equations can be written in matrix form as $\mathbf{A}\theta_{FE} = \mathbf{B}$ where

$$\mathbf{A} \equiv \begin{bmatrix} I_4 & A^1 & C_1 \\ \vdots & \vdots & \vdots \\ I_4 & A^{|G|} & C_{|G|} \end{bmatrix}, \quad \text{and} \quad \mathbf{B} \equiv \begin{bmatrix} B^1 \\ \vdots \\ B^{|G|} \end{bmatrix},$$

where C_g is the $4 \times |G|$ matrix which has ones in column g and zeros everywhere else. In this case, the dimensions of $\mathbf{A}$ is $4\,|G| \times (8 + |G|)$. But the rank of A is $rank\,(A) = N^2 + 2 + |G|$. For instance, with $|G| = 3$ groups, we have 12 matching equations, nine of which are linearly independent, and we correspondingly have three testable restrictions of the form (2.10).

Appendix D: Specification Tests Based on Full Type (Chapter 2)

In this appendix we will present results from specification tests based on full types, $x \in X$. Recall that, under "common principal preferences" (Assumption 1) and its extension to regional fixed-effects, $\Sigma^g = \Sigma + \psi_g$, the right hand side of (2.10) is independent of g. To test this, we define the empirical counterpart to the left hand side as measure of assortative matching among types x' and x'',

$$\log\left(\widetilde{\zeta}^{g,k}_{x'x''}\right) = \log\left(\frac{\widetilde{\mu}^{g,k}_{x'|x'}}{\widetilde{\mu}^{g,k}_{x''|x'}}\frac{\widetilde{\mu}^{g,k}_{x''|x''}}{\widetilde{\mu}^{g,k}_{x'|x''}}\right), \tag{D1}$$

by region g, gender k and by type-profile $(x', x'') \in X \times X$, and we compute standard errors by bootstrapping.

For a number of cells it is not possible to compute the measure as one or more of the empirical frequencies in (D1) is zero due to small numbers.

This is generally the case for type-pairs involving Blacks and Asians. Hence we will not present any results for Black-Asian pairs of types. But there are also a set of cells where the ethnic minorities are too small for measure to be computed. Hence, for any given type-pair (x', x'') and gender k, there will be maximum of G estimated measures available.

Under our assumptions, $\log\left(\widetilde{\zeta}^{g,k}_{x'x''}\right)$ should be independent of g for every (x', x'') and k. A simple and instructive way to test for equality across groups is to consider every available comparisons over pairs of regions, g and g'. An advantage to this approach is that it allows a more direct inspection of where any failure may be coming from.[32]

The results from this exercise are presented in Table D1. The results above the diagonal are those obtained from using the male marriage frequencies, $k = m$, while the corresponding numbers below the diagonal are those obtained using the female marriage frequencies, $k = f$. In general, these will of course be very similar, but can vary as some spouses to sample members do not themselves meet the sample selection criteria.

For each type-pair (and gender), the table presents three numbers. The first is an overall mean of the measure $\log\left(\widetilde{\zeta}^{g,k}_{x'x''}\right)$. Here we see that, for all type-pairs involving Whites and

[32] An alternative would have been an ANOVA test.

Blacks, the value of the measure is around 6, whereas for type-pairs involving Whites and Asians, the value of the measure is generally around 8, though slightly lower when both types are high qualified. This indicates that, generally, there is more marital mixing between Whites and Blacks than among Whites and Asian. The table also shows that there is generally stronger assortative matching on education among Asians than among Whites and Blacks.

Table D1: Specification Tests by Type-Profile

		GCSE(-)			**A-Level(+)**		
		White	**Black**	**Asian**	**White**	**Black**	**Asian**
GCSE(-)	**White**	(-)	5.26 (0/15)	7.80 (0/45)	1.41 (10/55)	6.47 (0/28)	8.48 (0/45)
	Black	5.74 (0/15)	(-)	*	6.09 (0/21)	0.91 (0/10)	*
	Asian	8.50 (0/45)	*	(-)	7.85 (0/45)	*	1.73 (0/45)
A-Level(+)	**White**	1.42 (10/55)	6.30 (0/10)	9.55 (0/21)	(-)	5.79 (0/36)	5.70 (10/55)
	Black	6.57 (0/28)	1.42 (0/15)	*	5.95 (0/28)	(-)	*
	Asian	8.91 (0/55)	*	1.64 (3/55)	6.31 (4/55)	*	(-)

Notes: The table reports the overall mean of the measure $\log\left(\widetilde{\zeta}^{g,k}_{x'x''}\right)$. In parenthesis, it then reports the results from specification tests based on pairwise comparisons across geographical areas for given type-pair. The first number in each parenthesis is the number of failures while the second number is the number of valid pairwise tests. With $G = 11$ regions, the maximum number of pairwise tests in any given cell is $(G \times (G-1))/2 = 55$. The results for the male sample are above the diagonal while the results for the female sample are below the diagonal.

The numbers in parenthesis then state the number of pairwise comparisons with statistically significant differences over the total number of pairwise comparisons available. If the measure $\log\left(\widetilde{\zeta}^{g,k}_{x'x''}\right)$ can be computed for $G^* \leq G$ regions, the number of available pairwise comparisons will be $\sum_{g=1}^{G^*-1} g = \left[G^*\left(G^*-1\right)\right]/2$. Hence if the measure is available for all regions, 55 pairwise comparisons can be made, while, if it is missing for one region, 45 comparisons can be made etc.

Overall 782 pairwise tests for equality could be made, with a total of 37 (= 4.7 percent) being rejected.

Moreover, the pattern follows that outlined in the text. 20 of the 37 rejections relate to educational sorting (among the majority Whites) and all involve London. Most of the remaining rejections involve Whites and Asians, and close inspection of these reveal that about half of these also involve London. Finally, a small number of rejections relate to regional variation in educational sorting among Asians. Hence we conclude that the assumption of “common principal preferences” generally has strong support in the data, possibly with the exception of the London region.

Appendix E: Estimation (Chapter 2)

The estimated parameters include Σ and Φ (normalized). As noted in Section 2.4 we further include as a choice option - denoted -1 – that an individual marries a spouse who is not UK-born and we allow for gender-type-specific preferences for this option. This adds $2N$ additional parameters. Finally, we allow for group-fixed-effects which adds one further parameter, ψ_g, for each group (except for the reference group).

The available data is taken to be a random sample from the population. The data on individuals from group $g \in G$ is used to characterize the group-specific type-distributions $h(x, k|g)$ which are taken as given throughout the estimation. The data further characterizes *the empirical choice-frequencies* $\widetilde{\mu}^{g,k}_{x'|x}$ for all groups $g \in G$, both genders $k = m, f$, all own-types $x \in X$, and all choice-options, $x' \in X \cup \{0, -1\}$. Implementing the ML-estimator requires computing the corresponding *model-predicted equilibrium choice-frequencies* at any candidate parameter vector $\widehat{\theta}$, denoted $\mu^{g,k}_{x'|x}\left(\widehat{\theta}\right)$. Hence we next outline the procedure used for solving for these equilibrium choice-frequencies given $\widehat{\theta}$.

Computing the Model-Predicted Choice Frequencies

Consider a trial value $\widehat{\theta}$ of the parameter vector. In order to compute the likelihood value associated with $\widehat{\theta}$ we solve for the equilibrium choice frequencies of all types in all groups. Since groups do not interact, the equilibria are solved group-by-group. Below we describe the simple Newton algorithm used.

Within-Group Algorithm

Fix a group $g \in G$.

Step 0. Guess a single rate for each gender and type, $\mu^{g,k}_{0|x}$, and place in a $2N$-vector $\mu^g_0 = \left[\mu^{g,m}_{0|x_1}, ..., \mu^{g,m}_{0|x_N}, \mu^{g,f}_{0|x_1}, ..., \mu^{g,f}_{0|x_N}\right]'$.

Step 1. Given $\widehat{\theta}$, and given μ^g_0, compute the candidate gender-specific marriage rates of all types to all types using (2.8) and (2.9) and also the rates of marriages to non-UK-born partners using

$$\mu^{g,m}_{-1|x_i} = \left[\exp \sigma_{x_i,-1} \left(\mu^{g,m}_{0|x_i}\right)^{1-\phi^m_0}\right]^{\frac{1}{1-\phi^m_1}}, \text{ and } \mu^{g,f}_{-1|x_j} = \left[\exp \sigma_{-1,x_j} \left(\mu^{g,f}_{0|x_j}\right)^{1-\phi^f_0}\right]^{\frac{1}{1-\phi^f_1}}, \quad \text{(E1)}$$

for males and females respectively.

Step 2. Compute the implied gender-type-specific "excess demands" as the deviation from the adding-up conditions,

$$\Delta^{g,m}_{x_i} = \sum_{x_j \in X \cup \{0,-1\}} \mu^{g,m}_{x_j|x_i} - 1, \text{ and } \Delta^{g,f}_{x_j} = \sum_{x_i \in X \cup \{0,-1\}} \mu^{g,f}_{x_i|x_j} - 1. \quad \text{(E2)}$$

Stack the excess demands in a $2N$-vector $\Delta^g = \left[\Delta^{g,m}_{x_1}, ..., \Delta^{g,m}_{x_N}, \Delta^{g,f}_{x_1}, ..., \Delta^{g,f}_{x_N}\right]'$. The equilibrium is characterized by zero excess demand for all types and both genders, $\Delta^g = 0$. Let $\|\cdot\|_\infty$ denote the uniform norm. If the candidate marriage/singles rates involve no excess demand, i.e. $\|\Delta^g\|_\infty \leq \epsilon$ for a sufficiently small ϵ (in our case $\epsilon = 10^{-5}$), terminate the algorithm and take the candidate marriage/singles rates to be the equilibrium rates in group g. Otherwise, go to step 3.

Step 3. Update the guess for the singles rates, μ^g_0, using a simplified Newton step. Let J denote the diagonal $2N \times 2N$ matrix where the diagonal terms are the partial derivatives of each excess demand terms $\Delta^{g,k}_x$ with respect to the "own" singles rate $\mu^{g,k}_{0|x}$, that is

$$\frac{\partial \Delta^{g,k}_x}{\partial \mu^{g,k}_{0|x}} = 1 + \sum_{x' \in X \cup \{-1\}} \frac{\partial \mu^{g,k}_{x'|x}}{\partial \mu^{g,k}_{0|x}}, \quad \text{(E3)}$$

with $\partial \mu^{g,k}_{x'|x} / \partial \mu^{g,k}_{0|x}$ and $\partial \mu^{g,k}_{-1|x} / \partial \mu^{g,k}_{0|x}$ obtained from differentiating (2.8) and (2.9) and (E1) respectively. Hence J is the Jacobian of Δ^g with respect to μ^g_0 but ignoring cross-partials. The singles rates are then updated using the Newton step

$$\mu^{g\prime}_0 = \mu^g_0 + \kappa J^{-1} \Delta^g \quad \text{(E4)}$$

where $\kappa \in (0, 1)$ is a "dampening" factor. Go back to step 1 with the updated guess, and iterate on steps 1 - 3 until the excess demands satisfy the criteria $\|\Delta^g\|_\infty \leq \epsilon$.

Once the equilibrium singles rates of men and women and of all types have been found, equations (2.8) and (2.9) along with (E1), can be used to obtain the equilibrium marriages rates, including to non-UK-born partners.

This thus provides us with the model-predicted equilibrium choice frequencies $\mu_{x'|x}^{g,k}\left(\widehat{\theta}\right)$, for $k = m, f$, $x \in X$ and $x' \in X \cup \{0, -1\}$ given the trial vector $\widehat{\theta}$. Repeating for all groups, $g \in G$, gives the full set of model-implied equilibrium choice frequencies at $\widehat{\theta}$.

The Likelihood Function

Let $M^{g,k}(x)$ be the observed number of individuals of gender $k = m, f$ from group $g \in G$ of type $x \in X$, and recall that $\widetilde{\mu}_{x'|x}^{g,k}$ for $x' \in X \cup \{0, -1\}$ are the empirical choice frequencies for this observed set of individuals. The likelihood contribution of this group, at the parameter vector $\widehat{\theta}$, is hence given by

$$L_x^{g,k}\left(\widehat{\theta}\right) =_{x'\in X\cup\{0,-1\}} \left[\mu_{x'|x}^{g,k}\left(\widehat{\theta}\right)\right]^{M^{g,k}(x)\widetilde{\mu}_{x'|x}^{g,k}}, \tag{E5}$$

The overall log-likelihood function is obtained by summing $\log L_x^{g,k}\left(\widehat{\theta}\right)$ over groups, gender, types, to obtain

$$\log L\left(\widehat{\theta}\right) = \sum_{g\in G}\sum_{k=m,f}\sum_{x\in X}\sum_{x'\in X\cup\{0,-1\}} M^{g,k}(x)\,\widetilde{\mu}_{x'|x}^{g,k}\log\mu_{x'|x}^{g,k}\left(\widehat{\theta}\right). \tag{E6}$$

The ML-estimator is hence $\widehat{\theta}_{ML} = \arg\max_{\widehat{\theta}\in\boldsymbol{\Theta}}\left(\log L\left(\widehat{\theta}\right)\right)$. Regularity conditions and standard arguments implies that $\widehat{\theta}_{ML} \xrightarrow{p} \theta$ and that $\widehat{\theta}_{ML}$ is asymptotically normal distributed. Reported standard errors are based on the estimated asymptotic variance-covariance matrix of $\widehat{\theta}_{ML}$, approximated, in standard ways, using the Hessian matrix of the log-likelihood function.

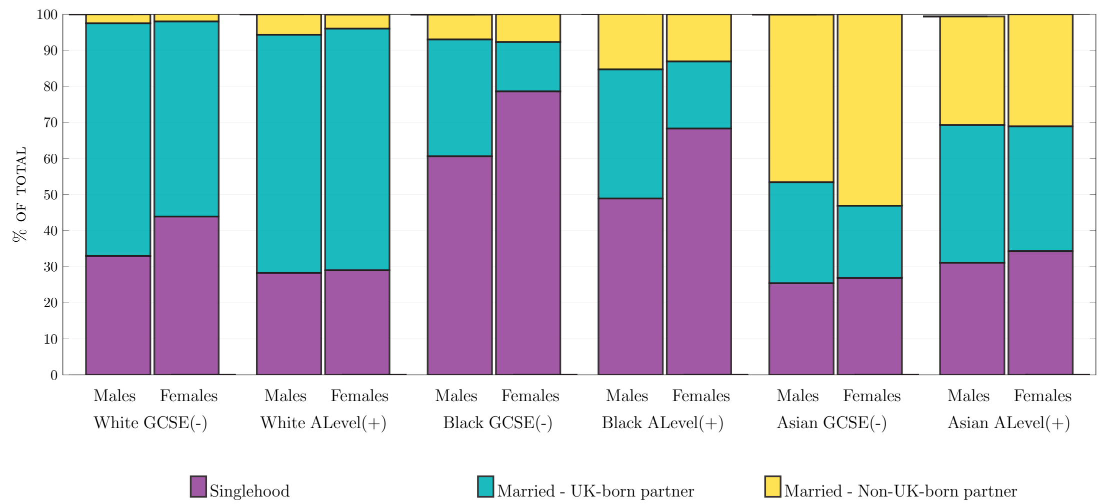
Appendix F: Further Estimation Results (Chapter 2)

Aggregate Fit

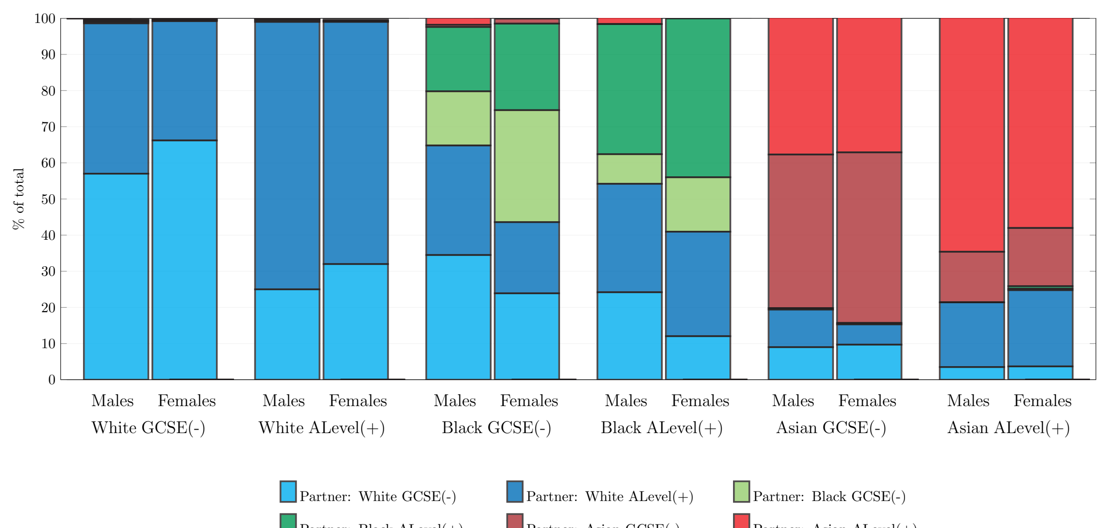
Panels (a) and (b) of Figure F.1 show the model-predicted versions of Figures 2.2 and 2.3.

Figure F.1: Predicted Distribution of Marital Status and Partner Type by Own Type and Gender

(a) Marital Status



(b) Partner Type



Notes: Predictions based on the preferred model specification (iii) in Table 2.3.

Remaining Preference Parameters from Main Specification

Table F1 presents the remaining estimated preference parameters from the main specification.

These include the regional fixed effects ψ_g for $g \in G$, and the gender-type-specific preferences for marrying a non-UK-born spouse, $\sigma_{x,-1}$ for males and $\sigma_{-1,x}$ for females.

Table F1: Estimates of Remaining Preference Parameters

Panel A: Regional-Specific Preferences					
ψ_{London}	$\psi_{\text{West Midl.}}$	$\psi_{\text{East Midl.}}$	$\psi_{\text{South East}}$	$\psi_{\text{North West}}$	$\psi_{\text{East Engl.}}$
-0.99***	0.10***	0.36***	0.02	-0.13***	0.14***
(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
$\psi_{\text{Yorks.\&Humb.}}$	$\psi_{\text{South West}}$	$\psi_{\text{North East}}$	ψ_{Wales}	ψ_{Scotland}	
0.12***	0.03	0 (ref)	0.02	-0.22***	
(0.04)	(0.04)	-	(0.04)	(0.04)	
Panel B: Preferences for Non-UK-Born Spouse					
Males ($\sigma_{x,-1}$)					
White, GCSE(-)	White, A-Level(+)	Black, GCSE(-)	Black, A-Level(+)	Asian, GCSE(-)	Asian, A-Level(+)
-2.59***	-1.57***	-2.17***	-1.16***	0.60**	-0.01
(0.03)	(0.02)	(0.19)	(0.12)	(0.08)	(0.06)
Females ($\sigma_{-1,x}$)					
White, GCSE(-)	White, A-Level(+)	Black, GCSE(-)	Black, A-Level(+)	Asian, GCSE(-)	Asian, A-Level(+)
-2.79***	-1.93***	-2.09***	-1.42***	0.72***	-0.03
(0.03)	(0.02)	(0.15)	(0.10)	(0.06)	(0.04)

Notes: See notes to Table 4. Standard errors in parenthesis.

Robustness to Sample Selection

Here we report the results from re-estimating the model using two alternative samples. In case (i) we reduce the age threshold for inclusion in the sample from 25 to 20. In case (ii) we eliminate London region from the sample.

Table F2 presents the estimated principal preferences in case (i). Similarly, Table F3 presents the estimated principal preferences in case (ii). In the first, case the estimated Σ-matrix now has slightly lower values as a rule, reflecting that we include more young people who are more often single. There is also some suggestion that mixed marriages have less negative values, but the overall structure of the matrix is nevertheless unchanged. The estimated social preferences in this case were $\phi_1^m = 0.54$ (0.06) and $\phi_1^f = 0.38$ (0.06) and hence marginally larger than for the main sample.

In the second case, the utilities from intra-ethnic marriages remain largely unchanged, possibly with the exception of a higher utility from marriages between low-qualified Asians. But more generally, after leaving out London the estimated utilities from inter-ethnic marriages between

Table F2: Estimates of Principal Joint Marital Utility by Ethnicity-Qualification Profile with an Age 20 Sample Threshold

		Female Type					
		White, GCSE(-)	White, A-Level(+)	Black, GCSE(-)	Black, A-Level(+)	Asian, GCSE(-)	Asian, A-Level(+)
Male Type	White, GCSE(-)	0.62*** (0.13)	0.51*** (0.17)	-3.99*** (0.66)	-3.90*** (0.66)	-3.42*** (0.68)	-3.84*** (0.70)
	White, A-Level(+)	0.09 (0.21)	1.34*** (0.10)	-4.11*** (0.70)	-2.98*** (0.56)	-3.82*** (0.75)	-2.01*** (0.49)
	Black, GCSE(-)	-3.11*** (0.53)	-3.03*** (0.56)	-2.19*** (0.39)	-2.19*** (0.40)	-4.83*** (1.04)	-4.31*** (0.79)
	Black, A-Level(+)	-2.92*** (0.54)	-2.53*** (0.52)	-2.28*** (0.44)	-0.98*** (0.28)	-33.9 (>200)	-3.64*** (0.70)
	Asian, GCSE(-)	-3.82*** (0.66)	-3.54*** (0.67)	-5.06*** (1.03)	-34.54 (>200)	0.15 (0.29)	-0.74*** (0.33)
	Asian, A-Level(+)	-4.52*** (0.73)	-2.59*** (0.53)	-34.9 (>200)	-35.5 (>200)	-0.74*** (0.34)	0.19 (0.20)

Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born between 1965-1989, living in England, Scotland or Wales, aged 20 or above when observed, and with available information on gender, ethnicity, educational attainment, marital status. Standard errors in parenthesis.

either Whites and Blacks or Whites and Asians are somewhat less negative. The estimated social preferences in this case were $\phi_1^m = 0.61$ (0.06) and $\phi_1^f = 0.54$ (0.06) and hence also marginally larger than for the main sample.

Table F3: Estimates of Principal Joint Marital Utility by Ethnicity-Qualification Profile without London

		Female Type					
		White, GCSE(-)	White, A-Level(+)	Black, GCSE(-)	Black, A-Level(+)	Asian, GCSE(-)	Asian, A-Level(+)
Male Type	White, GCSE(-)	1.11*** (0.12)	1.13*** (0.17)	-3.33*** (0.69)	-3.23*** (0.69)	-2.16*** (0.68)	-2.65*** (0.71)
	White, A-Level(+)	0.74*** (0.20)	1.91*** (0.09)	-3.16*** (0.70)	-2.38*** (0.60)	-2.75*** (0.80)	-1.03*** (0.51)
	Black, GCSE(-)	-2.62*** (0.54)	-2.27*** (0.54)	-2.25*** (0.46)	-2.01*** (0.44)	-3.50*** (0.95)	-2.90*** (0.71)
	Black, A-Level(+)	-2.36*** (0.54)	-1.98*** (0.54)	-2.21*** (0.51)	-0.72*** (0.29)	-31.4 (>200)	-3.56*** (0.97)
	Asian, GCSE(-)	-3.39*** (0.64)	-2.09*** (0.65)	-31.8 (>200)	-30.9 (>200)	0.84*** (0.28)	0.41 (0.32)
	Asian, A-Level(+)	-3.03*** (0.72)	-1.31*** (0.52)	-31.2 (>200)	-30.5 (>200)	0.45 (0.32)	1.20*** (0.19)

Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born between 1965-1989, living in England, Scotland or Wales, (excluding London) aged 25 or above when observed, and with available information on gender, ethnicity, educational attainment, marital status. Standard errors in parenthesis.

Appendix G: The 1990-2006 Cohorts (Chapter 2)

Here we provide details of the construction the type distribution in the "recent cohorts" by region. To this end we use the QLFS 1996-2015 and keep all individual born in the UK between 1990 and 2006. As ethnicity is directly observable, we can directly characterize the ethnic distribution by gender and region for these cohorts. However, as many of the individuals included in this sample had not completed their education by the time they were observed, we impute the proportion with a high qualification by gender, region, and ethnicity in these recent cohorts.

To do so, we estimate a linear probability model for holding a high qualification (A-level+) using the estimation cohort sample observed in the QLFS 1996-2015, born in the UK and aged 25 or higher when observed. The estimated specification models the educational attainment (dummy for having an A-level+ qualification) of individual i, of gender $k = m, f$, living in region $g \in G$, and of ethnicity $z \in \{W, B, A\}$ and birth cohort c_i, as

$$q_{ikgz} = \alpha + \beta_k + \gamma_g + \delta_z + \kappa_{kgz} c_i + \varepsilon_{ikgz}. \tag{G1}$$

The model thus includes gender-, region- and ethnicity fixed-effects, and models a linear growth in rate of holding an A-level+ qualification that is also gender-, region, and ethnicity-specific. Based on the estimated model we then impute an average qualification rate by gender, region and ethnicity for the recent cohorts using the distribution of c_i within the cell.

Table G1 shows aggregate type distribution for the estimating- and the recent cohorts. The table highlights an (i) increase in the rate of hold a high qualification in each ethnic group and gender, and (ii) an increase in the population share for each of the two ethnic minorities with Asian population increasing proportionately more.

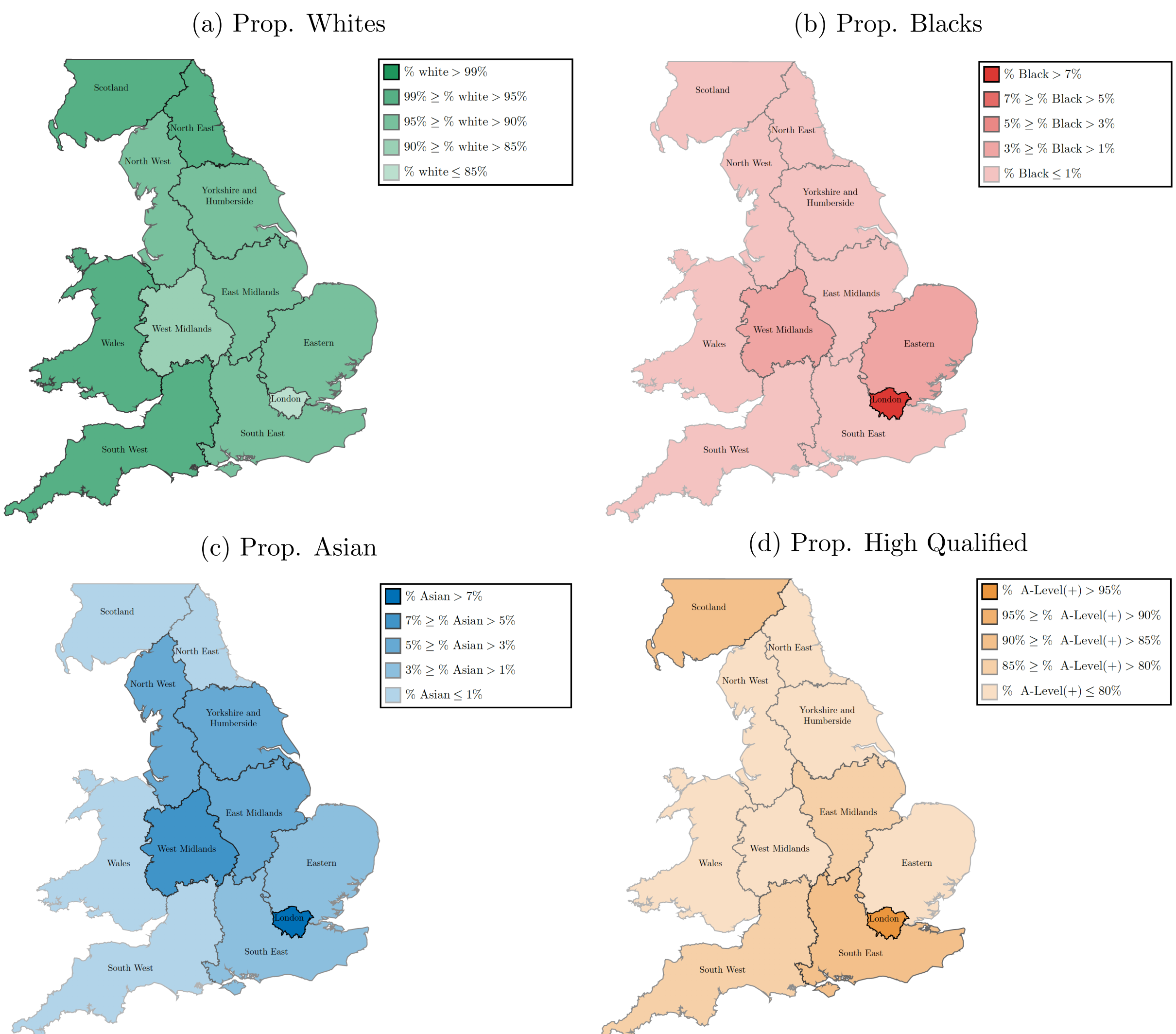
Figure G.1 (panels a-c) shows the distribution of ethnicity across regions in the recent cohorts. Panel (d) shows the (imputed) rate of holding a high qualification by region.

Table G1: The Distribution of Ethnicity and Qualifications in the Estimating and Recent Cohorts

		Birth Cohorts			
		1965-1989		**1990-2006**	
		Males	**Females**	**Males**	**Females**
White	GCSE(-)	0.458	0.433	0.173	0.158
	A-Level(+)	0.503	0.524	0.739	0.754
	Total	0.961	0.957	0.912	0.912
Black	GCSE(-)	0.005	0.005	0.003	0.002
	A-Level(+)	0.006	0.008	0.022	0.022
	Total	0.011	0.013	0.025	0.024
Asian	GCSE(-)	0.010	0.011	0.007	0.006
	A-Level(+)	0.018	0.019	0.056	0.058
	Total	0.028	0.030	0.063	0.064

Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born in the indicated cohorts, living in England, Scotland or Wales, and with available information on gender, ethnicity, and educational attainment.

Figure G.1: Demographic Composition by Region Cohorts 1990-2006



Notes: The sample consists of all UK-born individuals observed in the QLFS 1996-2015, born between 1990 and 2006 cohorts, living in England, Scotland or Wales, and with available information on ethnicity.