Noncommutative causality in algebraic quantum field theory

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Abstract

In the paper it will be argued that embracing noncommuting common causes in the causal explanation of quantum correlations in algebraic quantum field theory has the following two beneficial consequences: it helps (i) to maintain the validity of Reichenbach's Common Causal Principle and (ii) to provide a local common causal explanation for a set of correlations violating the Bell inequality.

Key words: Common cause, noncommutativity, algebraic quantum field theory, Bell inequality.

1 Introduction

Algebraic quantum field theory (AQFT) is a mathematically transparent quantum theory with clear conceptions of locality and causality (see (Haag, 1992) and (Halvorson, 2007)). In this theory observables are represened by a net of local C^*-algebras associated to bounded regions of a given spacetime. This correspondence is established due to the axioms of the theory such as isotony, microcausality and covariance. A state ϕ in this theory is defined as a normalized positive linear functional on the quasilocal observable algebra $\mathcal{A}$ which is the inductive limit of local observable algebras. The representation $\pi_\phi \colon \mathcal{A} \to \mathcal{B}(\mathcal{H})$ corresponding to the state ϕ transforms the net of C^*-algebras into a net of von Neumann observable algebras by closures in the weak topology.

In AQFT *events* are typically represented by projections of a von Neumann algebra. Although due to the axiom of microcausality two projections A and B commute if they are contained in local algebras supported in spacelike separated regions, they can still *be correlating* in a state ϕ, that is

$$\phi(AB) \neq \phi(A)\phi(B) \tag{1}$$

in general. In this case the correlation between these events is said to be *superluminal.* A remarkable characteristics of Poincaré covariant theories is that there exist "many" normal states establishing superluminal correlations (for the precise meaning of "many" see (Summers, Werner 1988) and (Halvorson, Clifton 2000)). Since spacelike separation excludes direct causal influence, one may look for a causal explanation of these superluminal correlations in terms of *common causes.*

The first probabilistic definition of the common cause is due to Hans Reichenbach (1956). Reichenbach characterizes the notion of the common cause in the following probabilistic way. Let (Σ, p) be a classical probability measure space and let A and B be two positively correlating events in Σ that is let

$$p(A \wedge B) > p(A)\,p(B). \tag{2}$$

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Definition 1. An event $C \in \Sigma$ is said to be the *common cause* of the correlation (A, B) if the following conditions hold:

$$
\begin{aligned}
p(A \wedge B|C) &= p(A|C)p(B|C) &(3)\\
p(A \wedge B|C^{\perp}) &= p(A|C^{\perp})p(B|C^{\perp}) &(4)\\
p(A|C) &> p(A|C^{\perp}) &(5)\\
p(B|C) &> p(B|C^{\perp}) &(6)
\end{aligned}
$$

where $C^{\perp}$ denotes the orthocomplement of C and $p(\,\cdot\,|\,\cdot\,)$ is the conditional probability.

The above definition, however, is too specific to be applied in AQFT since (i) it allows only for causes with a *positive* impact on their effects, (ii) it excludes the possibility of a *set* of cooperating common causes, (iii) it is silent about the spatiotemporal *localization* of the events and (iv) most importantly, it is *classical*. Therefore we need to generalize Reichenbach's original definition of the common cause. For the sake of brevity, we do not repeat here all the intermediate steps of the entire definitional process (for this see (Hofer-Szabó and Vecsernyés, 2012a)), but jump directly to the most general definition of the common cause in AQFT.

Let $\mathcal{P}(\mathcal{N})$ be the non-distributive lattice of projections (events) in a von Neumann algebra $\mathcal{N}$ and let $\phi\colon \mathcal{N} \to \mathbb{C}$ be a state on it. A set of mutually orthogonal projections $\{C_k\}_{k\in K} \subset \mathcal{P}(\mathcal{N})$ is called a *partition of the unit* $\mathbf{1} \in \mathcal{N}$ if $\sum_k C_k = \mathbf{1}$. Such a partition defines a *conditional expectation*

$$E\colon \mathcal{N} \to \mathcal{C},\ A \mapsto E(A) := \sum_{k\in K} C_k A C_k, \tag{7}$$

that is a unit preserving positive surjection onto the unital C^*-subalgebra $\mathcal{C} \subseteq \mathcal{N}$ obeying the bimodule property $E(B_1 A B_2) = B_1 E(A) B_2; A \in \mathcal{N}, B_1, B_2 \in \mathcal{C}$. We note that $\mathcal{C}$ contains exactly those elements of $\mathcal{N}$ that commute with $C_k, k \in K$. Recall that $\phi \circ E$ is also a state on $\mathcal{N}$.

Now, let $A, B \in \mathcal{P}(\mathcal{N})$ be two commuting events correlating in state ϕ in the sense of (1). (We note that in case of projection lattices we will use only algebra operations (products, linear combinations) instead of lattice operations $(\vee, \wedge)$. In case of commuting projections $A, B \in \mathcal{P}(\mathcal{N})$ we have $A \wedge B = AB$ and $A \vee B = A + B - AB$.)

Definition 2. A partition of the unit $\{C_k\}_{k\in K} \subset \mathcal{P}(\mathcal{N})$ is said to be a *common cause system* of the correlation (1) if

$$\frac{(\phi \circ E)(ABC_k)}{\phi(C_k)} = \frac{(\phi \circ E)(AC_k)}{\phi(C_k)} \frac{(\phi \circ E)(BC_k)}{\phi(C_k)} \tag{8}$$

for $k \in K$ with $\phi(C_k) \neq 0$. If C_k commutes with both A and B for all $k \in K$ we call $\{C_k\}_{k\in K}$ a *commuting* common cause system, otherwise a *noncommuting* one. A common cause system of size $|K| = 2$ is called a *common cause*. Reichenbach's definition (without the inequalities (5)-(6)) is a commuting common cause in the sense of (8).

Some remarks are in place here. First, in case of a commuting common cause system $\phi \circ E$ can be replaced by ϕ in (8) since $(\phi \circ E)(ABC_k) = \phi(ABC_k), k \in K$. Second, using the decompositions of the unit, $\mathbf{1} = A + A^{\perp} = B + B^{\perp}$, (8) can be rewritten in an equivalent form:

$$(\phi \circ E)(ABC_k))(\phi \circ E)(A^{\perp}B^{\perp}C_k) = (\phi \circ E)(AB^{\perp}C_k)(\phi \circ E)(A^{\perp}BC_k),\ k \in K. \tag{9}$$

One can even allow here the case $\phi(C_k) = 0$ since then both sides of (9) are zero. Third, it is obvious from (9) that if $C_k \leq X$ with $X = A, A^{\perp}, B$ or $B^{\perp}$ for all $k \in K$, then $\{C_k\}_{k\in K}$ serves as a (commuting) common cause system of the given correlation independently of the chosen state ϕ. Hence, these solutions are called *trivial common cause systems*. If $|K| = 2$, triviality means that

$\{C_k\} = \{A, A^\perp\}$ or $\{C_k\} = \{B, B^\perp\}$. Obviously, for superluminal correlation one looks for nontrival common causal explanations.

In AQFT one also has to specify the spacetime localization of the common causes. They have to be in the past of the correlating events. But in which past? One can define different pasts of the bounded regions V_A and V_B in a given spacetime as:

$$\begin{aligned} \textit{weak past:} \quad & wpast(V_A, V_B) := I_-(V_A) \cup I_-(V_B) \\ \textit{common past:} \quad & cpast(V_A, V_B) := I_-(V_A) \cap I_-(V_B) \\ \textit{strong past:} \quad & spast(V_A, V_B) := \cap_{x \in V_A \cup V_B} I_-(x) \end{aligned}$$

where $I_-(V)$ denotes the union of the backward light cones $I_-(x)$ of every point x in V (Rédei, Summers 2007). Clearly, $wpast \supset cpast \supset spast$.

With all these definitions in hand we can now define six different common cause systems in local quantum theories according to (i) whether *commutativity* is required and (ii) whether the common cause system is localized in the *weak*, *common* or *strong* past. Thus we can speak about *commuting/noncommuting (weak/strong) common cause systems.*

To address the EPR-Bell problem we will need one more concept. In the EPR scenario the real challenge is to provide a common causal explanation *not* for one *single* correlating pair but for a *set* of correlations (typically three or four correlations). Therefore, we also need to introduce the notion of the so-called joint[1] common cause system:

Definition 3. Let $\{A_m; m = 1, \dots M\}$ and $\{B_n; n = 1, \dots N\}$ be finite sets of projections in the algebras $\mathcal{A}(V_A)$ and $\mathcal{A}(V_B)$, respectively, supported in spacelike separated regions V_A and V_B. Suppose that all pair of spacelike separated projections (A_m, B_n) correlate in a state ϕ of $\mathcal{A}$ in the sense of (1). Then the set $\{(A_m, B_n); m = 1, \dots M; n = 1, \dots N\}$ of correlations is said to possess a commuting/noncommuting (weak/strong) *joint* common cause system if there exists *a single* commuting/noncommuting (weak/strong) common cause system for *all* correlations (A_m, B_n).

Since providing a *joint* common cause system for a set of correlations is much more demanding than simply providing a common cause system for a *single* correlation, therefore we keep the question of the common causal explanation separated from that of the *joint* common causal explanation. In Section 2 we will investigate the possibility of a common causal explanation for a *single* correlation—or in the philosophers' jargon, the status of Reichenbach's famous Common Cause Principle in AQFT. In Section 3 we will address the more intricate question as to whether EPR correlations can be given a *joint* common causal explanation. The crucial common element in both sections will be *noncommutativity.* We will argue that embracing *noncommuting* common causes in our causal explanation helps us in both cases: (i) in the case of common causal explanation it helps to maintain the validity of Reichenbach's Common Causal Principle in AQFT; (ii) in the case of *joint* common causal explanation it helps to provide a local, *joint* common causal explanation for a set of correlations violating the Bell inequalities. We conclude the paper in Section 4.

2 Noncommutative Common Cause Principles in AQFT

Reichenbach's Common Cause Principle (CCP) is the following metaphysical claim: If there is a correlation between two events and there is no direct causal (or logical) connection between the correlating events, then there exists a common cause of the correlation. The precise definition of this informal statement that fits to AQFT is the following:

[1] In (Hofer-Szabó and Vecsernyés, 2012a, 2013a) called *common* common cause system.

Definition 4. A local quantum theory is said to satisfy the Commutative/Noncommutative (Weak/Strong) CCP if for any pair $A \in \mathcal{A}(V_A)$ and $B \in \mathcal{A}(V_B)$ of projections supported in spacelike separated regions V_A, V_B and for every locally faithful state $\phi \colon \mathcal{A} \to \mathbb{C}$ establishing a correlation between A and B in the sense of (1), there exists a *nontrivial* commuting/noncommuting common cause system $\{C_k\}_{k\in K} \subset \mathcal{A}(V)$ such that the localization region V is in the (weak/strong) common past of V_A and V_B.

What is the status of these six different CCPs in AQFT?

The question as to whether the Commutative CCPs are valid in a Poincaré covariant local quantum theory in the von Neumann algebraic setting was first raised by Rédei (1997, 1998). As a positive answer to this question, Rédei and Summers (2002, 2007) have shown that the Commutative Weak CCP holds in algebraic quantum field theory with locally infinite degrees of freedom in the following sense: for every locally normal and faithful state and for every superluminally correlating pair of projections there exists a weak common cause, that is a common cause system of size 2 in the weak past of the correlating projections. They have also shown that the localization of a common cause cannot be restricted to $wpast(V_A, V_B) \setminus I_-(V_A)$ or $wpast(V_A, V_B) \setminus I_-(V_B)$ due to logical independence of spacelike separated algebras.

Concerning the Commutative (Strong) CCP less is known. If one also admits projections localized only in *un*bounded regions, then the Strong CCP is known to be false: von Neumann algebras pertaining to complementary wedges contain correlated projections but the strong past of such wedges is empty (see (Summers and Werner, 1988) and (Summers, 1990)). In spacetimes having horizons, e.g. those with Robertson–Walker metric, there exist states which provide correlations among local algebras corresponding to spacelike separated bounded regions such that the common past of these regions is again empty (Wald 1992). Hence, CCP is not valid there. Restricting ourselves to *local* algebras in Minkowski spaces the situation is not clear. We are of the opinion that one cannot decide on the validity of the (Strong) CCP without an explicit reference to the dynamics.

Coming back to the proof of Rédei and Summers, the proof had a crucial premise, namely that the algebras in question are *von Neumann algebras of type III.* Although these algebras are the typical building blocks of Poincaré covariant theories, other local quantum theories apply von Neumann algebras of other type. For example, theories with locally finite degrees of freedom are based on von Neumann algebras of type I. This raised the question as to whether the Commutative Weak CCP is generally valid in AQFT. To address the problem Hofer-Szabó and Vecsernyés (2012a) have chosen a specific local quantum field theory, the local quantum Ising model having locally finite degrees of freedom. It turned out that the Commutative Weak CCP does *not* hold in the local quantum Ising model and it cannot hold either in theories with locally finite degrees of freedom in general.

But why should we require commutativity between the common cause and its effects at all?

Commutativity has a well-defined role in any quantum theories. In standard quantum mechanics observables should commute to be simultaneously measurable. In AQFT the axiom of microcausality ensures that observables with spacelike separated supports—roughly, events happening 'simultaneously'—commute. But cause and effect are typically *not* such simultaneous events! If one considers ordinary QM, one well sees that observables do not commute even with their own time translates in general. For example, the time translate $x(t) := U(t)^{-1}xU(t)$ of the position operator x of the harmonic oscillator in QM does *not* commute with $x \equiv x(0)$ for generic t, since in the ground state vector ψ_0 we have

$$[x, x(t)]\, \psi_0 = \frac{-i\hbar \sin(\hbar\omega t)}{m\omega} \psi_0 \not\equiv 0. \tag{10}$$

Thus, if an observable A is not a conserved quantity, then the commutator $[A, A(t)] \neq 0$ in general. So why should the commutators $[A, C]$ and $[B, C]$ vanish for the events A, B and for their common cause C supported in their (weak/common/strong) past? We think that commuting common causes are only unnecessary reminiscense of their classical formulation. Due to their relative spacetime localization, that is due to the time delay between the correlating events and the common cause, it is also an unreasonable assumption.

Abandoning commutativity in the definition of the common cause is therefore a desirable move. The first benefit of allowing noncommuting common causes is that the noncommutative version of the result of Rédei and Summers can be regained. This result has been formulated in (Hofer-Szabó and Vecsernyés 2013a) in the following:

Proposition 1. The *Noncommutative* Weak CCP holds in local UHF-type quantum theories. Namely, if $A \in \mathcal{A}(V_A)$ and $B \in \mathcal{A}(V_B)$ are projections with spacelike separated supports V_A and V_B correlating in a locally faithful state ϕ on $\mathcal{A}$, then there exists a common cause $\{C, C^\perp\}$ localized in the weak past of V_A and V_B.

Now, let us turn to the more complicated question as to whether a *set* of correlations violating the Bell inequality can have a *joint* common causal explanation in AQFT. Since our answer requires some knowledge of the main concepts of the Bell scenario in AQFT and some acquaintance with the model in which our results were formulated, we start the next section with a short tutorial on these issues (for more details see (Hofer-Szabó, Vecsernyés, 2012b, 2013b).

3 Noncommutative joint common causal explanation for correlations violating the Bell inequality

The Bell problem is treated in AQFT in a subtle mathematical way (Summers and Werner, 1987a,b, Summers 1990); here we introduce, however, only those concepts which are related to the problem of common causal explanation (for more on that see (Hofer-Szabó, Vecsernyés, 2013b)).

Let $A_1, A_2 \in \mathcal{A}(V_A)$ and $B_1, B_2 \in \mathcal{A}(V_B)$ be projections with spacelike separated supports V_A and V_B, respectively. We say that in a locally faithful state ϕ the Clauser–Horne-type *Bell inequality is satisfied* for A_1, A_2, B_1 and B_2 if the following inequality holds:

$$-1 \leqslant \phi(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) \leqslant 0 \tag{11}$$

otherwise we say that the *Bell inequality is violated*. (Sometimes in the EPR-Bell literature another inequality, the so-called Clauser–Horne–Shimony–Holte-type Bell inequality is used as a constraint on the expectation of (not *projections* but) self-adjoint *contractions*. Since these two inequalities are equivalent, in what follows we will simply use (11) as *the* definition of the Bell inequality.)

In the literature it is a received view that if a set of correlations violates the Bell inequality, then the set cannot be given a joint common causal explanation. The following proposition proven in (Hofer-Szabó and Vecsernyés 2013b) shows that this view is correct *only if* joint common causal explanation is meant as a *commutative* joint common causal explanation:

Proposition 2. Let $A_1, A_2 \in \mathcal{A}(V_A)$ and $B_1, B_2 \in \mathcal{A}(V_B)$ be four projections localized in spacelike separated spacetime regions V_A and V_B, respectively, which correlate in the locally faithful state ϕ. Suppose that $\{(A_m, B_n); m, n = 1, 2\}$ has a joint common causal explanation in the sense of Definition 3. Then the following Bell inequality

$$-1 \leqslant (\phi \circ E_c)(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) \leqslant 0. \tag{12}$$

holds for the state $\phi \circ E_c$. If the joint common cause is a *commuting* one, then the original Bell inequality (11) holds for the original state ϕ.

Proposition 2 states that in order to yield a *commuting* joint common causal explanation for the set $\{(A_m, B_n); m, n = 1, 2\}$ the Bell inequality (11) has to be satisfied. This result is in complete agreement with the usual approaches to Bell inequalities (see e.g. (Butterfield 1989, 1995, 2007)).

But what is the situation with *noncommuting* common cause systems? Since—apart from (12)—Proposition 2 is silent about the relation between a *noncommuting* joint common causal explanation and the Bell inequality (11), the question arises: Can a *set* of correlations violating the Bell inequality (11) have a *noncommuting* joint common causal explanation?

In (Hofer-Szabó, Vecsernyés, 2012b, 2013b) it has been shown that the answer to the above question is positive: the violation of the Bell inequality does *not* exclude a joint common causal explanation *if* common causes can be noncommuting. Moreover, these common causes turned out to be localizable just in the 'right' spacetime region (see below). For this result, we applied a simple AQFT with locally finite degrees of freedom, the so-called local quantum Ising model (for more details see (Hofer-Szabó, Vecsernyés, 2012b, 2013b); for a Hopf algebraic introduction of the model see (Szlachányi, Vecsernyés, 1993), (Nill, Szlachányi, 1997), (Müller, Vecsernyés)).

Consider a 'discretized' version of the two dimensional Minkowski spacetime $\mathcal{M}^2$ covered by minimal double cones $V^m_{t,i}$ of unit diameter with their center in (t,i) for $t,i \in \mathbb{Z}$ or $t,i \in \mathbb{Z}+1/2$ (see Fig. 1). A non-minimal double cone $V_{t,i;s,j}$ in this covering can be generated by two minimal double

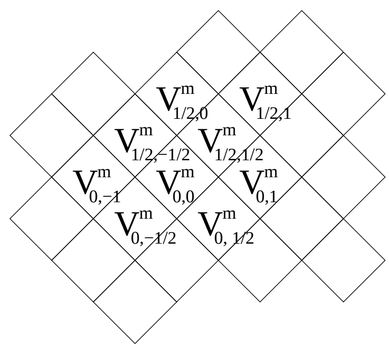

Figure 1: The two dimensional discrete Minkowski spacetime covered by minimal double cones.

cones in the sense that $V_{t,i;s,j}$ is the smallest double cone containing both $V^m_{t,i}$ and $V^m_{s,j}$. The set of double cones forms a directed poset which is left invariant by integer space and time translations.

The 'one-point' observable algebras associated to the minimal double cones $V^m_{t,i}$ are defined to be $\mathcal{A}(V^m_{t,i}) \simeq M_1(\mathbb{C}) \oplus M_1(\mathbb{C})$. By introducing appropriate commutation and anticommutation relations between the unitary selfadjoint generators of the 'one-point' observable algebras (which relations respect microcausality) one can generate the net of local algebras. Since there is an increasing sequence of double cones covering $\mathcal{M}^2$ such that the corresponding local algebras are isomorphic to full matrix algebras $M_{2^n}(\mathbb{C})$, the quasilocal observable algebra $\mathcal{A}$ is a uniformly hyperfinite (UHF) C^*-algebra and consequently there exists a unique (non-degenerate) normalized trace $\mathrm{Tr}\colon \mathcal{A} \to \mathbb{C}$ on it.

Now, consider the double cones $V_A := V^m_{0,-1} \cup V^m_{\frac{1}{2},-\frac{1}{2}}$ and $V_B := V^m_{\frac{1}{2},\frac{1}{2}} \cup V^m_{0,1}$ and the 'two-point' algebras $\mathcal{A}(V_A)$ and $\mathcal{A}(V_B)$ pertaining to them (see Fig. 2). It turns out that all the minimal projections in $A(\mathbf{a}) \in \mathcal{A}(V_A)$ and $B(\mathbf{b}) \in \mathcal{A}(V_B)$ can be parametrized by unit vectors $\mathbf{a}$ and $\mathbf{b}$, respectively in $\mathbb{R}^3$. Now, consider two projections $A_m := A(\mathbf{a^m}); m = 1,2$ localized in V_A, and two other projections $B_n := B(\mathbf{b^n}); n = 1,2$ localized in the spacelike separated double cone V_B.

Let the state of the system be the singlet state ϕ^s defined in an appropriate way (by a density operator composed of specific combinations of generators taken from various 'one-point' algebras). It turns out that in state ϕ^s the correlation between A_m and B_n will the one familiar from the EPR situation:

$$corr(A_m, B_n) := \phi^s(A_m B_n) - \phi^s(A_m)\,\phi^s(B_n) = -\frac{1}{4}\,\langle \mathbf{a^m}, \mathbf{b^n} \rangle \tag{13}$$

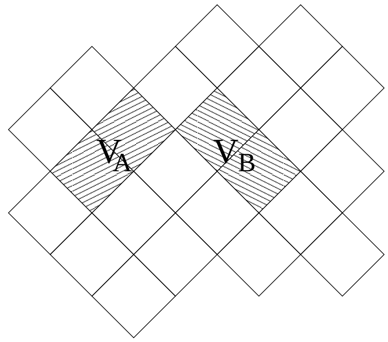

Figure 2: Correlations between events in V_A and V_B.

where $\langle\ ,\ \rangle$ is the scalar product in $\mathbb{R}^3$. In other words A_m and B_n will correlate whenever $\mathbf{a^m}$ and $\mathbf{b^n}$ are not orthogonal. To violate the Bell inequalitity (11) set $\mathbf{a^m}$ and $\mathbf{b^n}$ as follows:

$$\mathbf{a^1} = (0,1,0) \tag{14}$$
$$\mathbf{a^2} = (1,0,0) \tag{15}$$
$$\mathbf{b^1} = \frac{1}{\sqrt{2}}(1,1,0) \tag{16}$$
$$\mathbf{b^2} = \frac{1}{\sqrt{2}}(-1,1,0) \tag{17}$$

With this setting (11) will be violated at the lower bound since

$$\phi^s(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) =$$
$$-\frac{1}{2} - \frac{1}{4}\left(\langle \mathbf{a^1}, \mathbf{b^1}\rangle + \langle \mathbf{a^1}, \mathbf{b^2}\rangle + \langle \mathbf{a^2}, \mathbf{b^1}\rangle - \langle \mathbf{a^2}, \mathbf{b^2}\rangle\right) = -\frac{1+\sqrt{2}}{2} \tag{18}$$

Now, the question as to whether the four correlations $\{(A_m, B_n); m, n = 1, 2\}$ violating the Bell inequality (11) have a joint common causal explanation was answered in (Hofer-Szabó, Vecsernyés, 2012b) by the following

Proposition 3. Let $A_m := A(\mathbf{a^m}) \in \mathcal{A}(V_A), B_n := B(\mathbf{b^n}) \in \mathcal{A}(V_B); m, n = 1, 2$ be four projections parametrized by the unit vectors *via* (14)-(17) violating the Bell inequality in the sense of (18). Then there exist a noncommuting join common cause $\{C, C^\perp\}$ of the correlations $\{(A_m, B_n); m, n = 1, 2\}$ localizable in the common past $V_C := V_{0,-\frac{1}{2};0,\frac{1}{2}}$ of V_A and V_B (see Fig. 3).

Observe that C is localized in the *common past* of the four correlating events that is in the region which seems to be the 'physically most intuitive' localization of the common cause.

Proposition 2 and 3 together show that the relation between the common causal explanation and the Bell inequality in the noncommutative case is different from that in the commutative case. In the latter case the satisfaction of the Bell inquality is a necessary condition for a set of correlations to have a joint common causal explanation. In the noncommutative case, however, the violation of the Bell inequality for a given set of correlations does *not* exclude the possibility of a joint common causal explanation for the set. And indeed, as Proposition 3 shows, one can find a common cause even for a set of correlations violating the Bell inequality. To sum it up, taking seriously the noncommutative character of AQFT where events are represented by not necessarily commuting projections, one can provide a common causal explanation in a much wider range than simply sticking to commutative common causes.

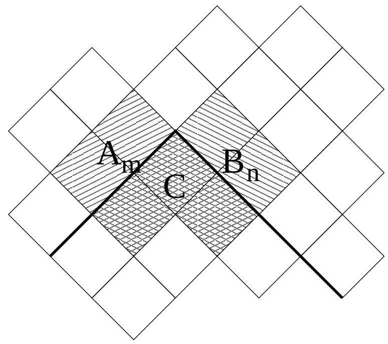

Figure 3: Localization of a common cause for the correlations $\{(A_m, B_n)\}$.

4 Conclusions

In the paper we were arguing that embracing noncommuting common causes in our explanatory framework is in line with the spirit of quantum theory and it gives us extra freedom in the search of common causes for correlations. Specifically, it helps to maintain the validity of Reichenbach's Common Causal Principle in the context of AQFT and it also helps to provide a local, *joint* common causal explanation for a set of correlations even if they violate the Bell inequalities.

Using noncommuting common causes naively to address the basic problems of the causal explanation in quantum theory in a formal way is no use whatsoever, if it is not underpinned by a viable ontology on which the causal theory can be based. This is a grandious research project. I conclude here simply by posing the central question of such a project:

Question. What ontology exactly is forced upon us by using noncommuting common causes in our causal explanation?

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