EPR correlations, Bell inequalities and common cause systems

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Abstract

Standard common causal explanations of the EPR situation assume a so-called joint common cause system that is a common cause for all correlations. However, the assumption of a joint common cause system together with some other physically motivated assumptions concerning locality and no-conspiracy results in various Bell inequalities. Since Bell inequalities are violated for appropriate measurement settings, a local, non-conspiratorial joint common causal explanation of the EPR situation is ruled out. But why do we assume that a common causal explanation of a set of correlation consists in finding a joint common cause system for *all* correlations and not just in finding *separate* common cause systems for the different correlations? What are the perspectives of a local, non-conspiratorial *separate* common causal explanation for the EPR scenario? And finally, how do Bell inequalities relate to the weaker assumption of *separate* common cause systems?

Key words: EPR correlations, common cause, Bell inequality

1 Introduction

In the history of probabilistic causation Reichenbach's definition (Reichenbach, 1956) was the first formal grasp of the notion of common cause. The conceptual novelty of the Reichenbachian definition has attracted immense interest among philosophers of science from the very beginning (Salmon, 1975; van Fraassen, 1982). From the physical side, the need for a common causal explanation of the EPR situation called attention to the definition of the common cause, even though in standard hidden variable strategies a slightly different common causal concept than the Reichenbachian has been applied (Bell, 1971; Jarrett, 1984; van Fraassen 1989). An important step in the conceptual clarification of the common cause in the EPR-Bell situation was the paper of Belnap and Szabó (1996) in which the difference between the so-called *joint* and *separate* common cause had been first recognized. Belnap and Szabó pointed out that in standard common causal explanations of the EPR correlations common cause is actually meant as a joint common cause accounting for *all* correlations.

Concerning the algebraic-probabilistic features of the Reichenbachian common cause Hofer-Szabó, Rédei and Szabó (1999) proved the following proposition. Classical (and also non-classical) correlations can be given a probabilistic common causal explanation in the sense that any classical probability measure space with correlating pairs of events can be extended such that the extension contains a Reichenbachian *separate* common cause for each correlation. (For the precise definitions see below.) Then in (Hofer-Szabó, Rédei, Szabó, 2002) it was proven that this proposition does not apply if Reichenbachian *separate* common causes are replaced with Reichenbachian *joint* common causes. In other words, classical probability measure spaces containing correlating pairs of events generally cannot be extended such that the extension contains a Reichenbachian *joint* common cause for *all* correlations. Thus, being a joint common cause of a set of correlations turned out to be a much stronger demand than being a common cause of a single correlation.

The first to apply the concept of separate common cause to the EPR situation was Szabó (2000). Since factorizability, locality and no-conspiracy together entail various types of Bell inequalities, EPR correlations cannot be given a local, non-conspiratorial, joint common causal model. Now, Szabó's idea was to replace the joint common causes with separate common causes and thus to give a separate common causal model for the EPR correlations. He constructed a number of separate common causal models which were local and non-conspiratorial in the usual sense that the measurement settings were statistically independent of the different common causes. However, the models were conspiratorial on a deeper level. The measurement settings statistically correlated with various algebraic combinations of the separate common causes. This fact called attention to the subtle but important difference between the so-called *weak* no-conspiracy where statistically independence is required only from the measure settings and the common causes *themselves*, and *strong* no-conspiracy where statistically independence is required from *any Boolean combination* of the measure settings and *any Boolean combination* of the common causes. After numerous computer simulations aiming to remove the unwanted conspiracies Szabó concluded with the conjecture that EPR cannot be given a local, *strongly* non-conspiratorial, separate common causal model.

The conjecture of Szabó has been first proven by Grasshoff, Portmann and Wüthrich (2005). The proof consisted in deriving some Bell inequality from the same assumptions that Szabó intended to apply in his separate common causal models for the EPR correlations. A crucial premise of this derivation was that the (anti)correlation between some events be *perfect*. Assuming perfect anticorrelations, however, turned the *separate* common causal explanations into a *joint* common causal explanation. This fact has been shown in (Hofer-Szabó, 2008). In the same paper Hofer-Szabó eliminated the assumption of perfect anticorrelations and presented a separate common causal derivation of some Bell-like inequalities (Bell(δ) inequalities). At the same time Portmann and Wüthrich (2007) presented a very similar result for the Clauser-Horne inequality replacing separate common causes with the more general notion of the so-called separate common cause systems (see below). Finally, in Hofer-Szabó (2011, 2012) a general recipe has been given how to derive any type of Bell(δ) inequality provided that the original Bell inequality can be derived from a set of perfect anticorrelations.

Although due to the above proofs the separate common causal explanation of the EPR scenario has been excluded, there is a sense in which Szabó's conjecture is still not decided. Szabó's original conjecture referred to the so-called Clauser–Horne set that is a set of four correlations violating the Clauser–Horne inequality. His question was as to whether the Clauser–Horne set can be given a local, strongly non-conspiratorial, separate common causal model. Interestingly enough—in the face of the above results—this question is still open.

In Section 2 we make explicit the concepts and propositions introduced informally in the Introduction. In Section 3 the standard *joint* common causal explanation of EPR correlations will be recalled. In Section 4 and 5 we explicate what has been and what has *not* been proven in the local, non-conspiratorial, *separate* common causal explanation of the EPR scenario. We conclude the paper in Section 6.

2 Joint and separate common cause systems

Let us start the common causal explanation with Reichenbach's (1956) definition of the common cause. Let (Σ, p) be a classical probability measure space and let $A, B \in \Sigma$ be two positively correlating events, i.e.

$$p(A \cap B) > p(A)p(B) \tag{1}$$

Reichenbach then defines the common cause of the correlation as follows:

Definition 1. An event $C \in \Sigma$ is said to be the *Reichenbachian common cause* of the correlation between A and B, if the events A, B and C satisfy the following relations:

$$\begin{aligned} p(A \cap B|C) &= p(A|C)p(B|C) && (2)\\ p(A \cap B|\overline{C}) &= p(A|\overline{C})p(B|\overline{C}) && (3)\\ p(A|C) &> p(A|\overline{C}) && (4)\\ p(B|C) &> p(B|\overline{C}) && (5) \end{aligned}$$

where $\overline{C}$ denotes the complement of C and the conditional probability is defined in the usual way. Equations (2)-(3) are referred to as "screening-off" properties and inequalities (4)-(5) as "positive statistical relevance" conditions. (Here we do not discuss the problem as to whether conditions (2)-(5) are necessary or sufficient conditions for an event C to be a common cause and simply take them to be the *definition* of the common cause.)

Physicists use the notion of 'common cause' in a different meaning. We obtain this meaning if (i) we drop the positive statistical relevance conditions (4)-(5) from the definition, and (ii) we do not restrict the screening-off properties (2)-(3) to the partition $\{C, \overline{C}\}$ of Σ:

Definition 2. Let (Σ, p) be a classical probability measure space and let (A, B) be a correlating pair of events in Σ. A partition $\{C_k\}$ $(k \in K)$ of Σ is said to be the *common cause system* of the pair (A, B) if for all $k \in K$ the following conditions are satisfied:

$$p(A \cap B|C_k) = p(A|C_k)p(B|C_k) \tag{6}$$

The cardinality $|K|$ (the number of events in the partition) is called the *size* of the common cause system. We will refer to a common cause system of size 2 (that is of the form $\{C, \overline{C}\}$) as a *common cause.* (Sometimes we will also refer to C as a common cause.)

Now, let (Σ, p) be a classical probability measure space as before and let (A_1, B_1) and (A_2, B_2), respectively be two positively correlating pairs of events in Σ, i.e. for $i = 1, 2$

$$p(A_i \cap B_i) \neq p(A_i)p(B_i) \tag{7}$$

In order to give a common causal explanation for *both* correlating pairs we have two options. Either we assume that the two correlations arise from the same causal source or we attribute different causal sources to the correlations. In the first case we explain the correlation by a so-called *joint* common cause system, in the second case we employ two *separate* common cause systems. The definition of joint and separate common cause systems, respectively are the following:

Definition 3. A partition $\{C_k\}$ $(k \in K)$ of Σ is said to be the *joint common cause system* of correlations (A_i, B_i) $(i = 1, 2)$, respectively if for $i = 1, 2$ and $k \in K$ the following relations are satisfied:

$$p(A_i \cap B_i|C_k) = p(A_i|C_k)p(B_i|C_k) \tag{8}$$

Definition 4. Two different partitions $\{C^i_k\}$ $(i = 1, 2;\ k(i) \in K(i))$ of Σ are said to be *separate common cause systems* of the correlations (A_i, B_i) $(i = 1, 2)$, respectively if for $i = 1, 2$ and $k(i) \in K(i)$ the following relations hold:

$$p(A_i \cap B_i|C^i_k) = p(A_i|C^i_k)p(B_i|C^i_k) \tag{9}$$

Having defined different common causal structures let us turn to the procedure of causal explanation. A common causal explanation of a given correlation is realized mathematically by the extension of the probabilistic measure space in such a way that for the original correlation there exists a common cause system in the extended probabilistic measure space. In the case of two (or more) correlations we can extend the algebra in two different ways according to our causal intuition. In order to model a joint common causal source of the correlations we extend the algebra such that in the extended algebra all correlations have a joint common cause system. On the other hand to account for separate causal mechanisms we extend the algebra such that in the extended algebra different correlations have *separate* common cause systems.

The extendability of the probabilistic measure spaces by joint respectively separate common causal structures crucially depends on the size of the common cause system. In the case of a common cause system of size 2 that is in the case of a common cause there is a great difference between joint and separate common cause extensions as it is shown in the following two propositions:

Proposition 1. (Hofer-Szabó, Rédei, Szabó, 1999) Let (Σ, p) be a classical probability measure space and let (A_1, B_1) and (A_2, B_2), respectively be two correlating pairs of events in Σ. Then there always exists a (Σ', p') extension of (Σ, p) such that for the correlation (A_1, B_1) there exists a common cause C^1 and for the correlation (A_2, B_2) there exists a common cause C^2 in (Σ', p').

Proposition 2. (Hofer-Szabó, Rédei, Szabó, 2002) There exists a (Σ, p) classical probability measure space and two correlating pairs (A_1, B_1) and (A_2, B_2), respectively in Σ such that there is no (Σ', p') extension of (Σ, p) which contains a joint common cause C in (Σ', p') for both correlations.

Proposition 1 claims that for two correlating pairs a separate common causal explanation is always possible by extending the probability measure space in an appropriate way. (Moreover, if Σ contains $n \in \mathbb{N}$ correlating pairs, *each* correlation can be given a *separate* common causal explanation.) However, according to Proposition 2 this strategy does not work generally if we are going to obtain the same common cause for the two (or more) correlating pairs. Thus, being a joint common cause imposes much stronger demand on C than simply being a separate common cause.

However, strangely enough this difference between the common and separate common causal extendability of a probability measure space disappears if the size of the common cause system is *not* specified. In other words, to find a joint common cause system *of arbitrary size* for a set of correlations is *not* a stronger demand than to find separate common cause systems for the same set. To see this, let (A_1, B_1) and (A_2, B_2) be two arbitrary correlating pairs in Σ. Then the partition

$$\{A_1 \cap B_1, A_1 \cap B_2, A_2 \cap B_1, A_2 \cap B_2, \}$$

is always a joint common cause system in Σ for both correlations. Obviously, this partition can be regarded only as a *trivial* joint common cause system of the correlations. This makes it clear that without further specification a joint common causal explanation is not more compelling than a separate common causal explanation. In the following sections we will see how these two types of explanations diverge due to extra requirements.

3 No local, non-conspiratorial joint common cause system for the EPR

Consider the standard EPR-Bohm experimental setup with a source emitting pairs of spin-$\frac{1}{2}$ particles prepared in the singlet state $|\Psi_s\rangle$. Let $p(a_i)$ denote the probability that the spin measurement apparatus is set to measure the spin in direction $\vec{a}_i$ $(i \in I)$ in the left wing and let $p(b_j)$ denote the same for direction $\vec{b}_j$ $(j \in J)$ in the right wing. Furthermore, let $p(A_i)$ stand for the probability that the spin measurement in direction $\vec{a}_i$ in the left wing yields the result $+1$ ('up') and let $p(\overline{A}_i)$ denote the probability of the result -1 ('down'). Let $p(B_j)$ and $p(\overline{B}_j)$ be defined in a similar way in the right wing for direction $\vec{b}_j$. (See Fig. 1) Quantum mechanics then yields the following conditional probabilities for the events in question:

$$p(A_i \cap B_j|a_i \cap b_j) = Tr(W_{|\Psi_s\rangle}\,(P_{A_i} \otimes P_{B_j})) \quad = \quad \frac{1}{2}\sin^2(\frac{\theta_{a_i b_j}}{2}) \tag{10}$$

$$p(A_i|a_i \cap b_j) = Tr(W_{|\Psi_s\rangle}\,(P_{A_i} \otimes I)) \quad = \quad \frac{1}{2} \tag{11}$$

$$p(B_j|a_i \cap b_j) = Tr(W_{|\Psi_s\rangle}\,(I \otimes P_{B_j})) \quad = \quad \frac{1}{2} \tag{12}$$

where $W_{|\Psi_s\rangle}$ is the density operator pertaining to the pure state $|\Psi_s\rangle$; P_{A_i} and P_{B_j} denote projections on the eigensubspaces with eigenvalue $+1$ of the spin operators associated with directions $\vec{a}_i$ and $\vec{b}_j$, respectively; and $\theta_{a_i b_j}$ denotes the angle between directions $\vec{a}_i$ and $\vec{b}_j$.

Thus, for non-perpendicular directions $\vec{a}_i$ and $\vec{b}_j$ there is a conditional correlation

$$p(A_i \cap B_j|a_i \cap b_j) \quad \neq \quad p(A_i|a_i \cap b_j)p(B_j|a_i \cap b_j) \tag{13}$$

and for parallel directions there is a perfect anticorrelation between the outcomes:

$$p(A_i \cap B_j|a_i \cap b_j) \quad = \quad 0 \tag{14}$$

Now, consider a set $\{(A_i, B_j)\}_{(i,j)\in I\times J}$ of EPR correlations in the sense of (13). A full-fledged common causal explanation of the set $\{(A_i, B_j)\}_{(i,j)\in I\times J}$ must comply with three demands on the statistical level. Firstly, all the correlations must be screened-off by a joint common cause system. Secondly, statistical relations among the measurement outcomes and the measurement settings must reflect the spacetime location of these events in the sense that spatially separated events have to be statistically independent. Thirdly, the measurement settings and the common cause should not influence each other, they have to be statistically independent. We refer to these requirements in turn as 'joint common cause system', 'locality' and 'no-conspiracy'. In the case of 'no-conspiracy' we will distinguish two types: the 'weak' and the 'strong no-conspiracy'. The precise probabilistic formulation of these demands is the following:

1. *Joint common cause system:* There exists a partition $\{C_k\}$ of Σ such that for every A_i, B_j, a_i and b_j in Σ $(i \in I, j \in J)$ and for any $k \in K$ the following factorization holds:

$$p(A_i \cap B_j|a_i \cap b_j \cap C_k) = p(A_i|a_i \cap b_j \cap C_k)p(B_j|a_i \cap b_j \cap C_k) \tag{15}$$

2. *Locality:* For every A_i, B_j, a_i, b_j and C_k in Σ $(i \in I, j \in J, k \in K)$ the following screening-off relations hold:

$$p(A_i|a_i \cap b_j \cap C_k) = p(A_i|a_i \cap C_k) \quad p(B_j|a_i \cap b_j \cap C_k) = p(B_j|b_j \cap C_k) \tag{16}$$

3. *a. Weak no-conspiracy:* For every a_i, b_j and C_k in Σ $(i \in I, j \in J, k \in K)$ the following independence holds:

$$p(a_i \cap b_j \cap C_k) = p(a_i \cap b_j)p(C_k) \tag{17}$$

b. Strong no-conspiracy: Consider two Boolean subalgebras $\mathfrak{A}$ and $\mathfrak{C}$ of Σ such that $\mathfrak{A}$ is generated by the partition of the different measurement choices $\{a_i b_j\}$ $(i \in I, j \in J)$ on the opposite wings, and $\mathfrak{C}$ is generated by the partition of the common cause system $\{C_k\}$ $(k \in K)$. Then for any element $E \in \mathfrak{A}$ and $F \in \mathfrak{C}$ the following independence holds:

$$p(E \cap F) = p(E)p(F) \tag{18}$$

It is straightforward to see that in the case of *joint* common cause systems (17) and (18) are equivalent, the probabilistic independence of the *Boolean combinations* of common causes and the measurement settings does not demand more than simply the probabilistic independence of the common causes and the measurement settings *themselves*. Thus, in the case of the joint common cause system type explanations equation (17) will suffice as a no-conspiracy requirement.

However, as it is well-known (15)-(17) result in various Bell inequalities which are violated for special measurement settings in the EPR experiment. For the simplest set of correlations, namely for the Clauser–Horne set $\{(A_i, B_j)\}_{(i,j)\in CH}$ where $CH = I \times J$ with $I = \{1, 2\}$ and $J = \{3, 4\}$ the Bell theorem is the following:

Proposition 3. (Clauser, Horne, 1974) For some measurement directions $\vec{a}_1, \vec{a}_2$ and $\vec{b}_3, \vec{b}_4$ there cannot exist extension of the probability space (Σ, p) such that the extension contains local, (weakly or strongly) non-conspiratorial joint common cause systems for all EPR correlations of $\{(A_i, B_j)\}_{(i,j)\in CH}$.

Consequently, EPR correlations fall short of a local, non-conspiratorial, joint common cause system type explanation. One premise has to be given up.

4 Local, weakly non-conspiratorial separate common cause systems do exist for the EPR

Strategies aiming to avoid Bell inequalities and to give a common causal explanation for the EPR correlations can be grouped according the abandoned premise. The first group consists of approaches abandoning locality and preserving the joint common causal background and no-conspiracy. Bohmian mechanics is an eminent representative of this group. The second group consists of less attractive models in which no-conspiracy is given up. Examples of this approach are Brans' and Szabó's models (Brans, 1988; Szabó, 1995). In these models the authors relinquished no-conspiracy and provided a local, deterministic but conspiratorial joint common cause system type explanation for the EPR. (For the problem of free will and no-conspiracy see (SanPedro, 2013.) In this paper, however, we will follow a third strategy which gives up the hypothesis of a joint common cause system. The key idea here is to replace the concept of joint common cause system with that of separate common cause systems and to provide a local, non-conspiratorial, *separate* common cause system type explanation for the EPR. A *separate* common cause system type explanation for a set $\{(A_i, B_j)\}_{(i,j)\in I\times J}$ consists in finding for every $(i, j) \in I \times J$ index pair a *separate* partition $\{C_k^{ij}\}$ $(k(ij) \in K(ij))$ such that screening-off, locality, and (weak or strong) no-conspiracies holds in the following sense:

1. *Separate common cause systems:* For every A_i, B_j, a_i and b_j in Σ $(i \in I, j \in J)$ there exists a separate partition $\left\{C_k^{ij}\right\}$ of Σ such that for any $k(ij) \in K(ij)$ the following factorization holds:

$$p(A_i \cap B_j | a_i \cap b_j \cap C_k^{ij}) = p(A_i | a_i \cap b_j \cap C_k^{ij}) p(B_j | a_i \cap b_j \cap C_k^{ij}) \tag{19}$$

2. *Locality:* For every $i \in I, j \in J$ and $k(ij) \in K(ij)$ the following screening-off relations hold:

$$p(A_i | a_i \cap b_j \cap C_k^{ij}) = p(A_i | a_i \cap C_k^{ij}), \quad p(B_j | a_i \cap b_j \cap C_k^{ij}) = p(B_j | b_j \cap C_k^{ij}) \tag{20}$$

3. *a. Weak no-conspiracy:* For every a_i, b_j and $C_k^{i'j'}$ in Σ $(i, i' \in I;\ j, j' \in J;\ k(i'j') \in K(i'j'))$ the following independence holds:

$$p(a_i \cap b_j \cap C_k^{i'j'}) = p(a_i \cap b_j) p(C_k^{i'j'}) \tag{21}$$

b. Strong no-conspiracy: Consider again two Boolean subalgebras $\mathfrak{A}$ and $\mathfrak{C}$ of Σ such that $\mathfrak{A}$ is generated by the partition of the different measurement choices $\{a_i b_j\}$ $(i \in I, j \in J)$ and $\mathfrak{C}$ is generated by the partition of *all the different* common cause systems $\left\{\cap_{ij} C_k^{ij}\right\}$ $(k \in K)$. Then for any element $E \in \mathfrak{A}$ and $F \in \mathfrak{C}$ the following independence holds:

$$p(E \cap F) = p(E)p(F) \tag{22}$$

Here, requirement (21) does *not* entail (22), that is the independence of the separate common cause systems of the choice of the measurement settings does *not* assure that *any Boolean combination* of the common causes will also be independent of *any Boolean combination* of the measurement settings. Thus, in the case of separate common cause system type explanations one has to take into consideration two different versions of no-conspiracy.

The idea to replace the concept of a joint common cause system with that of separate common cause systems and to provide a local, non-conspiratorial separate common cause system type explanation for the EPR was first raised by Szabó (2000). Actually, Szabó replaced the joint common cause system with separate common cause systems of size 2 that is with separate common causes. Szabó provided a number of separate common causal models for the Clauser–Horne set $\{(A_i, B_j)\}_{(i,j)\in CH}$ such that the models were local and non-conspiratorial in the *weak* sense of (22). In a precise form, Szabó's proposition was the following:

Proposition 4. (Szabó, 2000) Let $\{(A_i, B_j)\}_{(i,j)\in CH}$ be the Clauser–Horne set of correlations in (Σ, p). Then for any measurement directions $\vec{a}_1, \vec{a}_2$ and $\vec{b}_3, \vec{b}_4$ there exists an extension of the probability space (Σ, p) such that the extension contains local, *weakly* non-conspiratorial separate common causes for the correlations of $\{(A_i, B_j)\}_{(i,j)\in CH}$.

The common causal models provided by Szabó, however, were all conspiratorial in the *strong* sense of (22). After numerous computer simulations aiming to remove the unwanted conspiracies Szabó finally concluded with the conjecture that EPR *cannot* be given any local, separate common causal model free from *all* type of conspiracies.

5 Local, strongly non-conspiratorial separate common cause systems for the EPR?

Szabó's conjecture is then the following:

Conjecture 1. For some measurement directions $\vec{a}_1, \vec{a}_2$ and $\vec{b}_3, \vec{b}_4$ there cannot exist extension of the probability space (Σ, p) such that the extension contains local, *strongly* non-conspiratorial separate common cause systems for the correlations of $\{(A_i, B_j)\}_{(i,j)\in CH}$.

Although a lot has happened since 2000 in understanding the status of the separate common causal explanation of the EPR scenario, Szabó's conjecture in its original form is *still an open question*. What has actually been excluded, is not a local, strongly non-conspiratorial separate common causal explanation of the the Clauser–Horne set $\{(A_i, B_j)\}_{(i,j)\in CH}$, but that of *another set*. Let $I = J = \{1, 2, 3, 4\}$ and let PA be the following subset of $I \times J$:

$$PA = \{(1,1), (2,2), (3,3), (4,4)\}$$

Then one can prove the following proposition:

Proposition 5. For some measurement directions $\{\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4\}$ and $\left\{\vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4\right\}$ there cannot exist extension of the probability space (Σ, p) such that the extension contains local, *strongly* non-conspiratorial separate common cause systems for all EPR correlations of $\{(A_i, B_j)\}_{(i,j)\in PA}$.

The above proposition was first proved by Grasshoff, Portmann and Wüthrich (2005). They have shown that no local, *strongly* non-conspiratorial separate common cause systems are possible for all correlations of $\{(A_i, B_j)\}_{(i,j)\in PA}$, *if* for any index pair $(i,j) \in PA$ there is a *perfect anticorrelation* (hence the denotation 'PA') in the sense of (14).

The assumption of perfect anticorrelations, however, was unsatisfactory in two respects. The first problem concerns experimental testability. Since perfect anticorrelations cannot be tested experimentally with absolute precision, the proof of Grasshoff, Portmann and Wüthrich did not provide an experimentally verifiable refutation of a separate common causal explanation of the EPR.

The second problem was more conceptual. Standard derivations of the Bell inequalities assume a joint common cause system. The chief virtue of the proof of Grasshoff, Portmann and Wüthrich was that it avoided this strong concept of a *joint* common cause system and used the weaker concept of *separate* common cause systems instead. However, in the perfect anticorrelation case the assumptions of separate common cause systems turned out to be reducible to the assumptions of the standard joint common cause system as it was shown in the following proposition:

Proposition 6. (Hofer-Szabó, 2008) Let $\left\{C_k^{ij}\right\}_{(i,j)\in PA}$ be local, strongly non-conspiratorial *separate* common cause systems for the correlations of $\{(A_i, B_j)\}_{(i,j)\in PA}$. Then the partition $\{D_l\} := \left\{\cap_{ij} C_k^{ij}\right\}$ generated by the intersections of the different separate common cause systems is a local, non-conspiratorial *joint* common cause system of the same correlations of $\{(A_i, B_j)\}_{(i,j)\in PA}$.

The assumption of perfect anticorrelations, however, turned out not to be indispensable in the proof of Proposition 5. Portmann and Wüthrich (2007) and Hofer-Szabó (2008) have shown that Proposition 5 also holds if one only assumes that the correlations to be explained form an *almost perfect anticorrelation* set, $\{(A_i, B_j)\}_{(i,j)\in PA(\delta)}$, in the sense that there exists a δ of some small but not zero value such that

$$p(A_i \cap B_j | a_i \cap b_j) \quad \leqslant \quad \delta \tag{23}$$

for any index pair $(i,j) \in PA(\delta)$.

Finally, Hofer-Szabó (2011, 2012) generalized this proof by deriving arbitrary Bell(δ) inequality—that is to say, an inequality differing from the corresponding Bell inequality in a term of order δ. The recipe of this derivation is roughly the following. Consider a Bell inequality resulting from the local, non-conspiratorial *joint* common causal explanation of a given set of correlations $\{(A_i, B_j)\}_{(i,j)\in I\times J}$ (not necessarily $\{(A_i, B_j)\}_{CH}$). Now, define the set PA for $\{(A_i, B_j)\}_{(i,j)\in I\times J}$ as follows: let PA contain all the index pairs (k,k) in $(I \cup J) \times (I \cup J)$ that is all indices appearing either on the left or the right hand side of the correlations in $\{(A_i, B_j)\}_{(i,j)\in I\times J}$.

Now consider the set $\{(A_i, B_j)\}_{PA(\delta)}$ of almost perfect anticorrelations and suppose that it has a local, strongly non-conspiratorial separate common causal explanation. This assumption results in a Bell(δ) inequality differing from the original Bell inequality in a term of order of δ where the exact magnitude of this term is the function of the approximation. Choose the setting which violates the Bell inequality maximally. If the δ term is smaller than the violation of the original Bell inequality, then the Bell(δ) inequality will also be violated, excluding a local, strongly non-conspiratorial separate common causal explanation of the set $\{(A_i, B_j)\}_{PA(\delta)}$.

6 Conclusions

In the paper, first, different common causal concepts ranging from Reichenbach's definition to the most general concept of the common cause system have been listed. Then the role of the different causal notions in the common causal explanation of the EPR scenario has been exposed. It was said that a completely satisfactory common causal explanations of the EPR would consist in finding a *joint* common causal source for all correlations which is local and non-conspiratorial. Since these assumptions together entail various Bell inequalities one assumption has to be abandoned. The ambition of the separate common cause system type approach of the EPR was to preserve the latter two physically motivated assumptions of locality and no-conspiracy at the expense of replacing the strong concept of the joint common cause system with the weaker concept of separate common cause systems. It has been shown, however, that the weakening of the common causal concept does not provide a solution to this problem since the weakened assumptions still entail some Bell and Bell(δ) inequalities. Consequently, there exists neither a local, (weakly or strongly) non-conspiratorial *separate* common causal explanation of the EPR.

A weakness of all the above no-go theorems, however, is that they are all based on either *perfect* or *almost perfect* EPR correlations. As it was made clear in Proposition 6 the separate common causal explanation of such correlations is always parasitic on some joint common causal explanation. Therefore it would be highly desirable to derive some Bell inequality form a local, strongly non-conspiratorial separate common causal explanation of a set of *genuine* (not almost perfect) EPR correlations. For example it would be widely wanted to prove or falsify Szabó's original conjecture (Conjecture 1)—that is for the set $\{(A_i, B_j)\}_{(i,j)\in CH}$ violating the Clauser–Horne inequality

(i) either to derive the Clauser–Horne inequality (or some other constraint) from the assumption that $\{(A_i, B_j)\}_{(i,j)\in CH}$ has a local, strongly non-conspiratorial separate common causal explanation;

(ii) or to show up local, strongly non-conspiratorial separate common cause systems for the set $\{(A_i, B_j)\}_{(i,j)\in CH}$.

Neither option seems to be a trivial task.

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