Solution of periodic review inventory model with general constrains

Prof. Dr. J. Benkő

SZIU, Gödöllő

Summary

Reasons for presence of inventory (stock of goods) are physical and economical constraints in business. However we would like, according to our present knowledge, inventory can not be eliminated from production and distribution of goods, at most we can reduce and minimize it. The name of the science that deals with determination of the optimal lot size is inventory theory. Inventory theory formulates mathematical models to help decision makers. One groups of inventory models is called periodic review that allows to vary the required amounts from period to period. This study deals with such a model in that maximum quantity produced or ordered for period, maximum inventory after charging and minimum inventory at the end of period are given and takes also a suggestion for the solution.

Introduction

The presence of stocks in the economy is caused by physical and economic constraints. Even if we would like to obtain stocks from the production and distribution processes, according to our present knowledge the best we can do is to decrease or minimize stocks. The science that determines the optimal size of stocks is called stocking theory that helps the people to make decisions by using mathematical models.

Preliminaries

The first, classical stock managing model that became known as Optimizing Economic Order Quantity (EOQ) model was published in 1915 [3]. The original usability conditions of the model or formula that is able to compute the optimal order quantity are so strict that they can hardly be realized in life, but in spite of this it is still the most often mentioned and used model. The reason for it's wide use is not only because it can be used by mathematically less trained people, but because the solution is not so sensitive to the accuracy of the input parameters. The new models, developed from the original form are the continuous stock watching models [1]. These models are able to consider the shortage, the limited refilling capacity, the ordering expenses that is depending on the size of the amount, but they still assume that the rates of consumption or the decrement are equal in every period.

The continuous stock watching models in the first place can be used at companies that produce great amounts of products in almost equal rate without hurting the starting conditions. But these conditions are not full-filled in case of production, that show seasonal fluctuation or in case of production that follows the changes of orders. In this case if we do not want to make too big mistake, we are forced to use models that are more complicated, need more preparing and more work. One group of these kinds of models is the Periodic Review Inventory Models that are able to write down the predictable, changing demands over periods. The first model was published by Wagner and Within in 1958 who used dynamic programming to solve the problem. Later Within himself developed the algorithm [4].

In the end of the 50's and in the beginning of the 60's the relative backwardness of computer science forced the mathematicians and researchers to develop algorithms that use the less memory and counting in computers. This ambition can be discovered in the Wagner-Within algorithm too, where the creators have made the calculations easier, but the components and

results became less punctual. This means, that they have set up bounds and cost function that needed less calculations. In the past time the developing of computer technology that was showed in the increase of the speed, besides others, the difficulties have passed off. Now there is the possibility to create models that are more accurate and closer to reality.

Description of the problem

This study is also engaged in using and solving the developed Wagner-Within model. The great leap forward in the new model is that in this calculation the upper and lower bounds that are needed in practice can be prescribed and the concept of stock in the cost function is closer to reality. The constraints are the following: the periodical purchase or production and the stock after refilling can not exceed a limit, the stock can't fall below the prescribed amount.

Let us first examine the cost unit considered in the model. Let K be the constant cost that occurs at the beginning of the periods ($i = 1, 2, \dots n$), at the starting of ordering or production. We call this ordering or setting cost. The cost of purchase or production (c_i) per piece is generally constant, in the new model it can change through periods, or it may be based on the amount of the order. The amount of stock per piece (h_i) can also be constant or changing through period or based on the amount of the stock. In the original model they considered the left amount at the end of the period as the stock that generates the stock keeping cost. In the developed model, assuming linear changes, we consider stock as the average of the countable amount in the beginning and at the end of the period. The amount can be optional or limited. Limiting the minimum amount of stock in practice is usually because of safety reasons and that means setting a minimal amount (s) and the stock can never go below this limit. The upper limit of the stock can be determined as the amount that can be purchased or produced (Z) or the accessible capacity of the warehouse (8).

It is obvious that the amount that is left at the end of period i (stock) is the starting stock of the next period ($i+1$). We assume that the starting stock is known in the beginning of the first period and the stock decreases to 0 to the end of the period n. In the model we describe the change of stock by the demand of the period i: r_i (i=1, 2, ... ,n), instead of the rate of consumption (the changing of stock per time unit).

The aim of solving the stock problem written above is to determine the amount of products that must be ordered or produced in the beginning of a certain period (z_i (i=1,2, ... ,n)) and to minimize the total cost at one time. We will use dynamic programming to solve this problem that's variates are the following in connection with storing. The phase i is identified as the period i, and the conditions are ordered to answer the possible stock that step into period i, indicated by x_i (i=1,2, ... ,n). The amount of the x_i stock is known at the beginning of the first period and at the end of the last period the same is O, in other words: $x_{n+1} = 0$. Let the decision variable be the ordered or produced amount (z_i) in the beginning of period i. And the demand in period i is r_i.

The solution of the problem

The cost in period i is $B_i(x_i, z_i)$ that is function of the starting stock (x_i) and the produced amount (z_i). As we indicated before, we consider the constant or setting cost (K), the c_i purchasing or producing cost that is proportional with the produced pieces and the stock-keeping cost h_i.

The amount of the stock in period i is the average of the stock after the refilling $x_i + z_i$ and the remaining stock $x_{i+1} = x_i + z_i - r_i$ that is:

$$\frac{x_i + z_i + x_i + z_i - r_i}{2} = x_i + z_i - r_i/2 \, ,$$

using this the cost of period i:

$$B_i(x_i, z_i) = \begin{cases} K + c_i(z_i)z_i + h_i(x_i + z_i - r_i/2), & \text{ha a } z_i > 0, \\ h_i(x_i - r_i/2), & \text{ha a } z_i = 0. \end{cases}$$

In the $B_i(x_i, z_i)$ cost function the c_i purchasing and production and the h_i stock-keeping cost can change from one period to another, furthermore $B_i(x_i, z_i)$ is not necessarily must be a linear function. For example it is very common in life that we get discount if we order bigger amount of a certain product, that means the price per piece (c_i) is depending on the ordering amount (z_i).

Because the nature of the problem the possible constraints are the following:

The maximum of purchasing or producing is limited: $Z \geq z_i$,

The stock after production or refilling is maximized: $S \geq z_i + x_i$,

We do not allow shortage: $x_i + z_i - r_i \geq 0$,

The stock is minimized at the end of the period: $s \leq x_i$.

Because we chose z_i as decision variable, we can arrange the constraints as it follows:

$$z_i \leq Z,$$
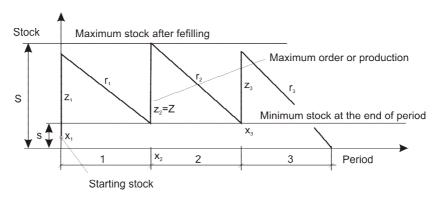$$z_i \leq S - x_i,$$
$$z_i \geq r_i - x_i,$$
$$x_i \geq s \, .$$

Figure 1 A three periods model

If we transform the last constraint, from Figure 1 we can write:

$$x_{i+1} = x_i + z_i - r_i$$

from this equation:

$$x_i = x_{i+1} + r_i - z_i.$$

Substituting into the last constraint and arranging it we get:

$$x_{i+1} + r_i - z_i \geq s \, ,$$

$$z_i \leq x_{i+1} + r_i - s \, .$$

Contracting the conditions:

$$\min\{Z, (x_{i+1} + r_i - s),(S - x_i)\} \ge z_i \ge \max\{r_i - x_i\}.$$

Let $C_i(x_i,z_i)$ the total costs of the subpolicies from the beginning of the i period to the end of period n that are as appropriate, depending on the ingoing stock and the produced amount. Furthermore in case of x_i starting stock we indicate the minimal value of the $C_i(x_i,z_i)$ set with $C_i^*(x_i)$.

Considering the limitations, the best subpolicies can be calculated with the following recurrent formula in every $i= 1,2,\dots,n$ period

$$C_i^*(x_i) = \min_{\substack{z_i \le \min\{Z, (x_{i+1}r_i - s),(S-x_i)\} \\ z_i \ge \max\{r_i - x_i\}}} \{C_i(x_i,z_i)\} = \min_{\substack{z_i \le \min\{Z, (x_{i+1}r_i - s),(S-x_i)\} \\ z_i \ge \max\{r_i - x_i\}}} \{B_i(x_i,z_i) + C_{i+1}^*(x_i + z_i - r_i)\},$$

where C_{n+1}^* is definitionally equal zero and:

$$x_{i+1} = x_i + z_i - r_i.$$

Further conditions of the solution:

$$\sum_{i=1}^{n} r_i \le nZ + x_1$$

That means that the sum of the products that can be purchased or produced and the amount of the starting stock can not be less than the total demand. And the demand of the first period can not be more than the sum of the starting stock of the first period and the maximum amount of the products that can be purchased or produced in a single period:

$$r_1 \le Z + x_1.$$

At the end of the planning horizon the $x_{n+1}=0$ condition can only be true if:

$$s \le r_n,$$

that means that the minimal stock can not be more than the demand of the last period.

The application of the results

To show the computer program based on the algorithm (the screen that shows the input of the data can be seen in Figure 2.) let us examine the following example:

Table 1

Period (i)	1	2	3	4	5	6
Purchasing price (c_i) [thousand Forints /tons]	11	18	13	17	20	10
Demand (r_i) [tonn]	8	5	3	2	7	4
Stock keeping cost (h_i) [thousand Forints/tons]	1	1	1	1	1	1

A company's Material Supply Department must ensure the basic material for producing in the amount (r_i) that is given in Table 1. to guarantee it's annual producing plan. The purchasing price (c_i) changes periodically and the capacity of the warehouse is limited: $S=9$ tons. The starting stock in the first period is: $x_1=2$ tons. The ordering cost is $K=2$ thousand Forints per order, the specific stock keeping costs are equal in every period, $h_i=1$ thousand Forints per tons. Let us determine the optimal stocking policy! The question is what amounts (z_i) do we have to order in the certain periods if we do not limit the stock, and after this we examine what is the cost increase if we change the minimal stock to 1 tons.

After running the program the results we get: the starting stocks of the periods (x_i), the purchased amounts in the periods (z_i) and the stock after refill (x_i+z_i) at zero minimal stock level can be seen on Figure 2 and at 1 tons minimal stock level can be seen on Figure 3. In the first case the total purchasing and stocking cost is 395,5 thousand Forints. This amount grows to 414,5 thousand Forints when increasing the minimal stock level to 1 ton. This means that the 1 ton increase in the minimal stock level drops the costs by 19 thousand Forints.
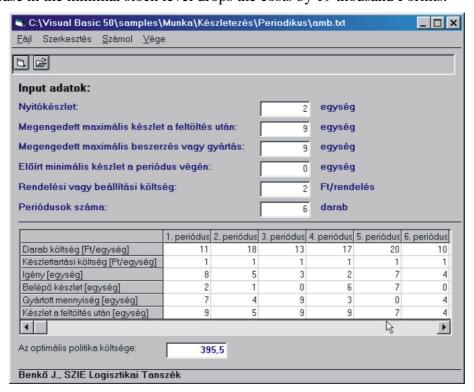

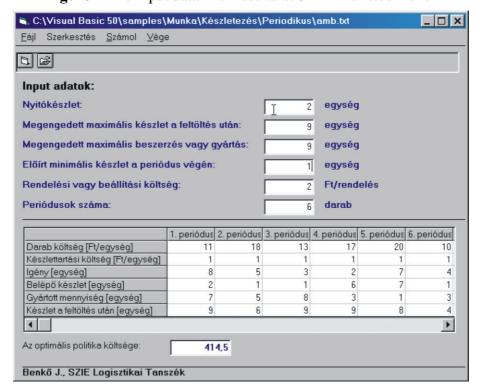**Figure 2** The input data with results at 0 minimal stock level

Figure 3 The input data with results at 1 minimal stock level

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