Investment allocation model at ABC Gelderland

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ABSTRACT

The author shows a decision making model for an investment allocation problem in Holland which was developed from the base model of transportation problem. Former the problem in the case study in general was solved by mixed integer programming (MIP). The suggested new model has some essential advantages against the MIP regarding the memory demand and computation time.

INTRODUCTION

Scope of ABC Gelderland

ABC Gelderland is a centre for Dutch agrarian interest. Its activities comprise the supply of an extensive package of products and services attuned to modem farming. Marketing is done in Holland and abroad.

The major activity is the production and sale of mixed feeds. For this ABC Gerderland commands four modem and well-equipped production companies. Total mixed feed production and sales amounts to more than 725 000 tones annually and consists of 30% beef feed, 58% pig feed, 12% poultry feed. More than 85% of these feed are produced in complete form, 95% of delivery is in bulk.

Production involves a large number of mixed feeds fine tuned to specific use in practice. A broad assortment makes it possible to choose the right mixed feed for any type of animal under different operating conditions. Figures from practice show that these mixed feeds produce outstanding results. Outstanding quality coupled to attractive prices means that mixed feeds from ABC Gelderland make an important contribution to achieving a maximum profit at livestock management company.

As was indicated above ABC Gelderland has four plants located in Lochem, Doetinchem, Aalten and Winterswijk. The last is specialized on one product line, while the others produce the whole assortment. The assortment of the products is very wide included 225 products.

PROBLEM AND METHODS

ABC Gelderland is planning to expand its production capacity to satisfy the increasing demand of clients. Another reason of the investment is that the plant located in Doetinchem has obsolete production line. The question is, what is the most advantageous allocation of the investment. The possible places are Lochem and Doetinchem because both of them can be reached by ship.

Management of ABC Gelderland needs to make decision on which place to choose. The basis for the decision is the minimal total logistical and production costs. Since the production cost is independent of the location (the fixed and variable cost of the production are the same on both of places) it is sufficient to consider only the logistical cost.

This problem can be described in very simple way as a transportation problem. The advantages of this method will be shown in the next part of the report.

Transportation problem

Probably the most important special type of linear programming problem is the so called transportation problem. Special solution procedure can be used for it which is a kind of streamlining of simplex method and also can be obtained by exploiting the special structure in the problem.

To describe the general model for transportation problem, we need to use terms. In particular, the general transportation problem is concerned with distributing any commodity from any group of supply centers, called sources, to any group of receiving centers, called destination, in such a way as to minimize the total distribution cost.

Thus, in general source i *(* i= 1, 2,...,n*)* has a supply of s_i units to distribute to destination, and destination j (j= 1, 2,..., m) has demand for d_j units to be received from the sources. A basic assumption is that the cost of distributing unit from source i to destination j is directly proportional to the number distributed, where c_{ij} denotes the cost per unit distributed. These data can be summarized conveniently in the cost and requirements table shown in *Table 1.*

Table 1.

Cost and requirements table

		Destination				
		1	2	…	m	Supply
	1	c_{11}	c_{12}	…	c_{1m}	s_1
Source	2	c_{12}	c_{22}	…	c_{12}	s_2
	…	…	…	…	…	…
	n	c_{n1}	c_{n1}		c_{nm}	s_n
	Demand	d_1	d_2		d_m	

Letting z be total distribution cost and x_{ij} (i= 1, 2, ... ,n;j= 1,2, ... ; m) be the number of units to be distributed from source i to destination j, the linear programming formulation of this problem becomes:

$$\sum_i \sum_j c_{ij} \cdot x_{ij} = z \to \min$$

subject to

$$\sum_j x_{ij} = s_i \qquad \text{for } i = 1, 2, \ldots, n$$

$$\sum_i x_{ij} = d_j \qquad \text{for } j = 1, 2, \ldots, m,$$

and

$$x_{ij} \geq 0 \qquad \text{for all } i \text{ and } j.$$

Moreover, a necessary and sufficient condition for transportation problem to have any feasible solution is that

$$\sum_i s_i = \sum_j d_j$$

For many applications, the supply and demand quantities in the model (the s_i and d_j) have integer values, and implementation will require that the distributed quantities (the x_{ij}) also have integer value. Fortunately, because of the special structure the problems have the following property: for transportation problem where every s_i and d_j has and integer value, all the basic variables (allocations) in every basic feasible solution also have integer values. Therefore, it is unnecessary to add a constraint to the model that the x_{ij} must have integer value.

Remarkable, in almost every case can be reached by multiplication s_i and d_j to be integer, and even, in this way it can be obtained c_{ij} to be integer also. As a result, we can save dramatic computational time and also memory place.

Mathematical model of allocation problem

The allocation problem can be drafted: Given a company with *j* plants which produce *i* sorts of products for *k* number of customers. The question is how to split the products between the plants in such a way that demand of the customers will be satisfied and the total logistical cost will be minimal.

At the same time, we have to insure that two kinds of options can be available:

- permit product splitting, where the same product is produced in more than one places,
- prohibit product splitting.

This problem can be described as a so-called *more steps transportation problem.*

To understand the model, we need to introduce some new terms.

To obtain the most advantageous product splitting, we have to use two types of capacity:

$CAPACITY_{ij}$ the potential capacity of *jth* plant from *ith* product,

$CAPACITY_j$ the total real capacity of *jth* plant.

With $CAPACITY_{ij}$ we can reach that *ith* product will be prohibited or restricted at *jth* plant. If $CAPACITY_{ij}=0$, it means the *ith* product is prohibited at *jth* plant. If $CAPACITY_{ij} \leq CAPACITY_j$, it means the *ith* product is restricted.

In this way, in the model

$$\sum_i CAPACITY_{ij} > \sum_j CAPACITY_j ,$$

therefore we have to provide that the difference

$$FDEMAND_j = \sum_i CAPACITY_{ij} - \sum_j CAPACITY_j$$

will be contracted, to avoid overloading of *jth* plant. $FDEMAND_j$ is fictitious quantity. To get a feasible solution for the problem, necessary and sufficient condition is that

$$\sum SOURCE = \sum DEMAND .$$

This condition can be obtained with introducing.

SURPLUS capacity surplus,

LACK lack of capacity, what can be calculated the next way. Let it be.

$$\rho = \sum_i \sum_j CAPACITY_{ij} - \sum_i \sum_k DEMAND_{ik}$$

than

$$SURPLUS = \begin{cases} 0, & \text{if } \rho \le 0 \\ \sum_j CAPACITY_{ij} - \sum_i \sum_k DEMAND_{ik}, & \text{if } \rho > 0 \end{cases}$$

$$LACK = \begin{cases} 0, & \text{if } \rho \ge 0 \\ \sum_i \sum_k DEMAND_{ik} - \sum_j CAPACITY_{ij}, & \text{if } \rho < 0 \end{cases}$$

After this introduction let see the model (*Table 1* and *Table 2*).

Variables:

x_{ijk} production from *ith* product at *ith* product at *jth* plant for *kth* c1ient,

y_{ij} fictitious production from *ith* product for *jth* fictitious c1ient,

α_{ij} surplus of capacity from *ith* product at *jth* plant,

β_{ik} lack of capacity from *ith* product for *kth* c1ient.

Coefficients:

c_{ijk} cost matrix

c_{ijk}=(average supply cost)$_{ij}$+(average distribution cost)$_{ijk}$

$CAPACITY_{ij}$ potential production capacity from *ith* product at *jth* plant,

$CAPACITY_j$ total real production capacity of *jth* plant,

$DEMAND_{ik}$ demand of *kth* client from *ith* product,

$FDEMAND_j$ fictitious demand of *jth* fictitious client,

SURPLUS capacity surplus,

LACK lack of capacity.

Object function:

$$\sum_i \sum_j \sum_k c_{ijk} \cdot x_{ijk} \rightarrow \min$$

Conditions:

$x_{ijk} \ge 0$; $y_{ij} \ge 0$; α_{ij}; $\beta_{ik} \ge 0$

$\sum_k x_{ijk} + y_{ij} + \alpha_{ij} = CAPACITY_{ij}$ for i=1,2,…I and j=1,2,…,J

$\sum_i \sum_k \beta_{ik} = LACK$ for k=1,2,…K, where

$$LACK = \begin{cases} 0, & \text{if } \rho \ge 0 \\ \sum_i \sum_k DEMAND_{ik} - \sum_j CAPACITY_{ij}, & \text{if } \rho < 0 \end{cases}, \text{ and}$$

$$\rho = \sum_i \sum_j CAPACITY_{ij} - \sum_i \sum_k DEMAND_{ik}$$

COST AND REQUIREMENTS TABLE OF INVESTMENT ALLOCATION PROBLEM

			PRODUCT(1)			PRODUCT(i)			PRODUCT(I)			FICTITIOUS CLIENT			FICT. CLI-ENT	SUPPLY
			CLIENT			CLIENT			CLIENT							
			(1)	(k)	(K)	(1)	(k)	(K)	(1)	(k)	(K)	(1)	(j)	(J)		
PRO-DUCT (1)	PLANT	(1)	c(1,1,1)	c(1,1,k)	c(1,1,K)	M	M	M	M	M	M	0	M	M	0	CAPACITY(1,1)
		(j)	c(1,j,1)	c(1,j,k)	c (1,j, K)	M	M	M	M	M	M	M	0	M	0	CAPACITY(1,j)
		(J)	c (1, J, 1)	c (1,J,k)	c (1, J, K)	M	M	M	M	M	M	M	M	0	0	CAPACITY(1,J)
PRO-DUCT (i)	PLANT	(1)	M	M	M	c(i,1,1)	c(i,1,k)	c(i,1,K)	M	M	M	0	M	M	0	CAPACITY(i,1)
		(j)	M	M	M	c(i,j,1)	c(i,j,k)	c(i,j,K)	M	M	M	M	0	M	0	CAPACITY(i,i)
		(J)	M	M	M	c(i,J,l)	c(i,J,k)	c(i,J,K)	M	M	M	M	M	0	0	CAPACITY(i,J)
PRO-DUCT (I)	PLANT	(1)	M	M	M	M	M	M	c(I,1,1)	c(I,1, k)	c(I,1,K)	0	M	M	0	CAPACITY(I,1)
		(j)	M	M	M	M	M	M	c(I,j,1)	c(I,j,k)	c(I,j,K)	M	0	M	0	CAPACITY(I,j)
		(J)	M	M	M	M	M	M	c(I,J,l)	c(I,J,k)	c(I,J,K)	M	M	0	0	CAPACITY(I,J)
FICT. PRODUCT			0	0	0	0	0	0	0	0	0	M	M	M	M	LACK
DEMAND			Dem(1,1)	Dem(1,k)	Dem(1,K)	Dem(i,1)	Dem(i,k)	Dem(i,K)	Dem(I,1)	Dem(I,k)	Dem(I,K)	Fd(1)	Fd(j)	Fd(J)	SURP	

Table 3

SOLUTION TABLE OF INVESTMENT ALLOCATION PROBLEM

			PRODUCT (1)			PRODUCT (i)			PRODUCT (I)			FICTITIOUS CLIENT			FICT. CLI-ENT	SUPPLY
			CLIENT			CLIENT			CLIENT							
			(1)	(k)	(K)	(1)	(k)	(K)	(1)	(k)	(K)	(1)	(j)	(J)		
PRO-DUCT (1)	PLANT	(1)	x(1,1,1)	x(1,1,k)	x(1,1,K)	M	M	M	M	M	M	y(1,1)	M	M	α(1,1)	CAPACITY(1,1)
		(j)	x(1,j,1)	x(1,j,k)	x(1,j, K)	M	M	M	M	M	M	M	y(1,j)	M	α(1,j)	CAPACITY(1,j)
		(J)	x(1, J, 1)	x(1,J,k)	x(1, J, K)	M	M	M	M	M	M	M	M	y(1,J) 0	α(1,J)	CAPACITY(1,J)
PRO-DUCT (i)	PLANT	(1)	M	M	M	x(i,1,1)	x(i,1,k)	x(i,1,K)	M	M	M	y(i,1)	M	M	α(i,1)	CAPACITY(i,1)
		(j)	M	M	M	x(i,j,1)	x(i,j,k)	x(i,j,K)	M	M	M	M	y(i,j)	M	α(i,j)	CAPACITY(i,i)
		(J)	M	M	M	x(i,J,l)	x(i,J,k)	x(i,J,K)	M	M	M	M	M	y(i,J)	α(i,J)	CAPACITY(i,J)
PRO-DUCT (I)	PLANT	(1)	M	M	M	M	M	M	x(I,1,1)	x(I,1,k)	x(I,1,K)	y(I,1)	M	M	α(I,J)	CAPACITY(I,1)
		(j)	M	M	M	M	M	M	x(I,j,1)	x(I,j,k)	x(I,j,K)	M	y(I,j)	M	α(I,j)	CAPACITY(I,j)
		(J)	M	M	M	M	M	M	x(I,J,1)	x(I,J,k)	x(I,J,K)	M	M	y(I,J)	α(I,J)	CAPACITY(I,J)
FICT. PRODUCT			β(1,1)	β(1,k)	β(i,K)	β(i,1)	β(i,k)	β(i,K)	β(I,1)	β(I,k)	β(I,K)	M	M	M	M	LACK
DEMAND			Dem(1,1)	Dem(1,k)	Dem(1,K)	Dem(i,1)	Dem(i,k)	Dem(i,K)	Dem(I,1)	Dem(I,k)	Dem(I,K)	Fd(1)	Fd(j)	Fd(J)	SURP	

$$\sum_j x_{ijk} + \beta_{ik} = DEMAND_{ik} \quad \text{for } i=1,2,\ldots I \text{ and } k=1,2,\ldots,K$$

$$\sum_j \beta_{ik} = FDEMAND_j \quad \text{for } j=1,2,\ldots J, \text{ where}$$

$$FDEMAND_j = \sum_i CAPACITY_{ij} - \sum_j CAPACITY_j$$

$$\sum_i \sum_j \alpha_{ij} = SURPLUS \text{ , for } i=1,2,\ldots I \text{ and } j=1,2,\ldots,J, \text{ where}$$

$$SURPLUS = \begin{cases} 0, & \text{if } \rho \le 0 \\ \sum_j CAPACITY_{ij} - \sum_i \sum_k DEMAND_{ik}, & \text{if } \rho > 0 \end{cases}$$

$$\sum_i \sum_j CAPACITY_{ij} + LACK = \sum_i \sum_k DEMAND_{ik} + \sum_j FDEMAND_j + SURPLUS$$

DISCUSSION

Clustering Of products and clients

To decrease the number of variables, the clustering of products and clients is recommended. Since the large number of products and clients may result into too large effectiveness matrix. Therefore the products and clients were aggregated into clusters. This operation resulted into 9 products clusters including more than 200 animal feeds and 20 clients clusters with more than 5000 customers. The clients clusters are assumed as average customer who is located in the center of the geographical cluster.

Complements to usage of the model

In order to get answer for our question, as to what is the most advantageous allocation or allocations of the investment, we need to run the model three times with different boundary conditions. The next possible varieties of allocation have to be analyzed:

a) plants: Lochem with expanded capacity
Aalten
Winterswijk

b) plants: Lochem
Doetinchem with expanded capacity
Aalten
Winterswijk

c) plants: Lochem with expanded capacity
Doetinchem with expanded capacity
Aalten
Winterswijk

The value of the object function will give the answer of which is the most beneficial solution.

At the same time, the values of variables (x_{ijk}, α_{ij} and β_{ik}) indicate the following:

x_{ijk} shows satisfaction in demand of the *kth* client for *ith* product from *jth* plant, α_{ij} or β_{ik} informs us on, how much capacity of *jth* plant is under utilized or how much the lack of capacity of *jth* plant.

To apply the model we need to use special algorithm and software. Besides the simplex method there are also two special methods which give exact solution of transportation problem: the *transportation simplex* and the *Hungarian* methods. Both methods can be used but in our case, where the model consists of a lot of *M* (infinite) elements the Hungarian method is more beneficial. The Hungarian method does not require the storage the infinite elements since during the iterative solution procedure these elements have never been allocated. But in this case special matrix handling method have to be used.

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