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ON THE DIFFUSION APPROXIMATION OF OPERATING SYSTEMS II.

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I. INTRODUCTION

In the first part of the paper (in hungarian) we gave a brief account of our experiments on CDC – 3300. In this part we give the mathematical description of a diffusion approximation of the operating systems.

First we try to indicate the intuitive reasons of the diffusion approximation. It is well known that the Brownina – motion process $\omega(t)$ $(0 \leqslant t < \infty)$ may be obtained as the limit of the following process (see e.g. Feller [6]). Let $N(t)$ denote a Poisson process, with parameter λ, i.e. the distribution of the time interval between two events be exponential and the random variables are independent.

After an event the particle moves up or down with the value a, with probability 1/2. Let $\omega_{\lambda,a}$ (t) denote the displacement of a Brownian particle at time t, then

$$\omega_{\lambda,a}(t) = \sum_{n=0}^{N(t)} \eta_n$$

where

$$\eta_n = \begin{cases} a, & P(\eta_n = a) = 1/2 \\ -a, & P(\eta_n = -a) = 1/2 \end{cases}$$

The following diagram indicates a possible realization:

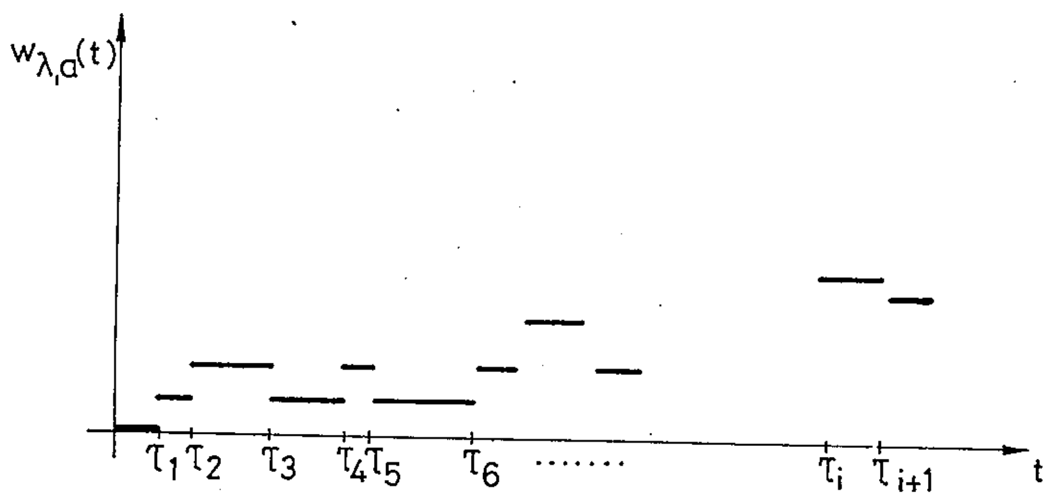

Fig. 1.

τ_i denotes the i-th jump time of the particle, where

$$P(\tau_i - \tau_{i-1} < t) = 1 - e^{-\lambda t}.$$

With standard methods it can be proved that

$$\log E\, e^{i\omega_{\lambda,a}(t)\cdot s} = 1/2\, s^2 \sigma^2 t + a\Theta |s|^3 \sigma^2 t$$

where $\sigma^2 = a^2\lambda$, $|\Theta| \leq 1$. Assuming that $\lambda \to \infty$, $a \to 0$ and $a^2 \cdot \lambda = \sigma^2 =$ const we obtain

$$\log E\, e^{i\omega_{\lambda,a}(t)\cdot s} \to -\frac{1}{2} s^2 \sigma^2 t$$

This means (heuristically) that the limit process $\omega(t) = \lim \omega_{\lambda,a}(t)$ is a Brownian motion. The exact interpretation and proofs of such type of theorems are found in Gihman–Skorohod's [10] (§3 ch. 9) or Borovkov's [4] books (§24). In the latter the "heavy traffic" conditions of queuing theory are investigated.

1. The CPU utilization problem.

First let us assume that there are one CPU and two DTU-s and there are 2 jobs in the computer. For the first job, the CPU and DTU service times η_{1i} (resp. ξ_{1i}) have the distribution

$$P(\eta_{ni} < t) = P(\xi_{1i} < t) = 1 - e^{-\lambda_1 t}$$

and they are independent. For the second job, the distributions are

$$P(\eta_{2i} < t) = P(\xi_{2i} < t) = 1 - e^{-\lambda_2 t}.$$

It seems that one of the most interesting utilization problems is connected with the absolute priority rule. This means that if job 1 has absolute priority then, upon an I/0 interrupt, the CPU is assigned to job 1 if it wants to take it* and the second job is waiting or it is in the DTU. As the system has two DTU-s, there is no queue before them. If, however, job 2 had absolute priority then it would no wait for the CPU. The first priority rule will be denoted by $\upsilon(1,2)$ and the seconed by $\upsilon(2,1)$.

In the sequel we assume that the mean service time of job 1 is much less than of job 2. This requirement is usually satisfied in practice and, for this case, we prove the following statement.

* The so-called pre-emptive priority for CPU

Theorem 1. *We assume that* $1/\lambda_1 << 1/\lambda_2 << 1$, *then the priority rule* $\nu(1,2)$ *gives the CPU utilization time* ζ_t *at absolute time* t *as*

$$(1.1)\qquad \zeta_t^{\nu(1,2)} = \sum_{1}^{N(t)} \eta_{1i} + \sum_{1}^{N(t')} \eta_{2i},$$

where

$$t'' = t - \sum_{i=0}^{N(t)} \eta_{1i} < t,$$

which is asymptotically normally distributed with parameters.

$$(1.2)\qquad E\zeta_t^{\nu(1,2)} \approx \frac{1}{2}t + \frac{1}{2}\cdot\frac{2}{3}t = \frac{5}{6}t$$

$$(1.3)\qquad D^2\zeta_t^{\nu(1,2)} \approx t\left(\frac{1}{\lambda_1} + \frac{2}{3}\cdot\frac{1}{\lambda_2}\right)$$

Further, the priority rule $\nu(2,1)$ gives the CPU utilization time

$$(1.4)\qquad \zeta_t^{\nu(2,1)} = \sum^{N(t)} \eta_{2i} + \sum^{N(t'')} \eta_{1i},$$

where

$$t'' = t - \sum_{i=1}^{N(t)} \eta_{2i} < t,$$

which asymptotically has also Gaussian distribution, wich parameters

$$(1.5)\qquad E\zeta_t^{\nu(2,1)} \approx \frac{1}{2}t + \frac{1}{2}\cdot\frac{1}{2}t = \frac{3}{4}t$$

$$(1.6)\qquad D^2\zeta_t^{\nu(2,1)} \approx t\left(\frac{1}{\lambda_2} + \frac{1}{2}\cdot\frac{1}{\lambda_2}\right)$$

Proof. The random variable $N(t)$ (resp. $N\ (t)$) has Poisson distribution with parameter $\frac{\lambda_1}{2}\cdot t$ (resp. $\frac{\lambda_2}{2}\cdot t$). First we prove that the logarithm of the characteristic function of the stochastic process

$$x_1(t) = \zeta_{t,1}^{\nu(1,2)} = \sum_{i=1}^{N(t)} \eta_{1i}$$

has the from

$$(1.7)\qquad \log Ee^{isx_1(t)} = ist\cdot\frac{1}{2} - \frac{1}{2}s^2 t\frac{1}{\lambda_1} + \Theta|s|^3 t\frac{\lambda_1}{2}\,\frac{3!}{\lambda_1^3}\,.$$

This follows from the relations

$$Ee^{isx_1(t)} = Ee^{is\sum_{k=1}^{N(t)}\eta_{1k}} = EE[e^{is\sum_{k=1}^{n}\eta_{1k}}|N(t)=n] =$$

$$= \sum_{n=0}^{\infty} Ee^{is\sum_{k=1}^{n}\eta_{1k}} \cdot \frac{e^{-\frac{\lambda_1}{2}t}\left(\frac{\lambda_1}{2}t\right)^n}{n!} =$$

$$= \sum_{n=0}^{\infty} (Ee^{is\eta_{11}})^n \frac{e^{-\frac{\lambda_1}{2}t}\left(\frac{\lambda_1}{2}t\right)^n}{n!} =$$

$$= e^{\frac{\lambda_1}{2}}[Ee^{is\eta_{11}} - 1]$$

and (using the moments of the exponential distribution and the expected value)

$$E(e^{is\eta_{11}} - 1) = isE\eta_{11} - \frac{1}{2}s^2E\eta_{11}^2 + \Theta|s|^3E|\eta_{11}|^3 =$$

$$= is\frac{1}{\lambda_1} - \frac{1}{2}s^2\frac{\quad}{\lambda_1^2} + \Theta|s|^3\frac{3!}{\lambda_1^3},$$

where $|\Theta| \leqslant 1$.

In the same way

$$\text{(1.8)} \qquad \log Ee^{isx_2(t)} = ist\frac{1}{2} - \frac{1}{2}s^2t\frac{1}{\lambda_2} + \Theta|s|^3t\frac{1}{\lambda_2^2}$$

where

$$x_2(t) = \sum_{i=1}^{N(t)} \eta_{2i}$$

In the second step, there is a great difference between the priority rules ν (1,2) and ν (2,1) as the sums

$$\sum_{i=1}^{N(t')} \eta_{2i} \quad \text{and} \quad \sum_{i=1}^{N(t'')} \eta_{1i}$$

behave in different ways. See Fig.2.

In case $\nu(1,2)$ the variables η_{2i} remain further exponentially distributed with parameter λ_2, but on the CPU time axis the DTU periods ξ_{2i} run quicker and are exponentially distributed with parameter

$$\tilde{\mu}_2 = \lambda_2\frac{2\lambda_1}{\lambda_1} = 2\lambda_2$$

(the reason is that the compute time of the first job overlaps with the DTU time of the second job).

With a similar argument as used to prove (7), one can also prove that for process

$y_1(t) = \sum_{i=1}^{N(t'')} \eta_{2i}$ (note that t'' is also a random variable)

$$(1.9)\qquad \log Ee^{isy_1(t)} = ist\,\frac{1}{2}\cdot\frac{\tilde{\mu}_2}{\mu_2+\lambda_2} - $$

$$-\frac{1}{2}s^2t\,\frac{1}{2}\,\frac{2\tilde{\mu}_2}{\mu_2+\lambda_2}\cdot\frac{1}{\lambda_2} + \Theta|s|^3\,\frac{t}{2}\,\frac{3}{\lambda_2^2} =$$

$$= ist\,\frac{1}{2} - \frac{1}{3}s^2t\,\frac{2}{3}\,\frac{1}{\lambda_2} + \Theta|s|^3\,\frac{t}{2}\,\frac{3}{\lambda_2^2}$$

(7) and (9) prove (2) and (3). In case of $\nu(2,1)$, η_{1i} is further exponentially distributed with parameter λ_1. The DTU periods ζ_{1i} remain the same during a DTU period of job 2 (here are only end effects), and during the CPU period of the second job both the CPU and DTU periods of job 1 are staying (waiting). Only end effects may be at the beginning of the CPU period. This means that with a similar argument as in proof (7) we obtain for process

$y_2(t) = \sum_{i=1}^{N(t'')} \eta_{1i}$ (note again t'' is a random variable)

$$(1.10)\qquad \log Ee^{isy_2(t)} = ist\,\frac{1}{2}\cdot\frac{1}{2} - \frac{1}{2}s^2t\,\frac{1}{2}\,\frac{1}{\lambda_1} + \Theta|s|^3\,\frac{t}{2}\,\frac{3}{\lambda_1^2}$$

Relations (8) and (10) prove (5) and (6). The Theorem is proved.

Heuristically Theorem 1 involves that process $\zeta_t^{\nu(1,2)}$ and $\zeta_t^{\nu(2,1)}$ converge (weakly) to Brownian motion processes and have different local parameters.

REMARK 1. Let the CPU and DTU periods for job 1 be exponentially distributed with parameters λ_1 and μ_1 and let they be for the second job λ_2, μ_2. With a similar argument as in Theorem 1 we may prove that, if $\mu_1 \gg \mu_2$ priority $\nu^{(2,1)}$ leads to the CPU utilization time $\zeta_t^{\nu(1,2)}$, which has an appoximate normal distribution with parameters

$$(1.11)\qquad E\zeta_t^{\nu(1,2)} \approx t\,\frac{\mu_1\lambda_1}{\mu_1+\lambda_1}\,\frac{1}{\lambda_1} + t\left(1-\frac{\mu_1}{\mu_1+\lambda_1}\right)\cdot\frac{\tilde{\mu}_2}{\mu_1+\lambda_2}\,,$$

$$(1.12)\qquad D^2\zeta_t^{\nu(1,2)} \approx t\,\frac{2\mu_1}{\mu_1+\lambda_1}\,\frac{1}{\lambda_1} + t\left(1-\frac{\mu_1}{\mu_1+\lambda_1}\right)\frac{2\tilde{\mu}_2}{\mu_2+\lambda_2}\,\frac{1}{\lambda_2}\,.$$

where

$$\tilde{\mu}_2 = \mu_2\,\frac{\mu_1+\lambda_1}{\mu_1}\,.$$

When priority rule $\nu(2,1)$ is in force we have

$$(1.13) \qquad E\xi_t^{\nu(2,1)} = t\,\frac{\mu_2\lambda_2}{\mu_2+\lambda_2}\,\frac{1}{\lambda_2} + t\left(1-\frac{\mu_2}{\mu_2+\lambda_2}\right)\frac{\mu_1}{\mu_1+\lambda_1}$$

and

$$(1.14) \qquad D^2\zeta_t^{\nu(2,1)} = t\,\frac{2\mu_2}{\lambda_2+\mu_2}\,\frac{1}{\lambda_2} + t\left(1-\frac{\mu_2}{\mu_2+\lambda_2}\right)\frac{2\mu_1}{2\mu_1+\lambda_1}\cdot\frac{1}{\lambda_1}$$

It is remarkable that, with $\mu_2 \leqslant \mu_1$ the priority rule $\nu(1,2)$ is always better then $\nu(2,1)$, as (with the notation)

$$p_i = \frac{\mu_i}{\lambda_i+\mu_i} \qquad (i=1,2),$$

$$(1.15) \qquad p_1 + (1-p_1)\frac{\dfrac{p_1\mu}{p_1\mu_2+\lambda_2}}{p_1\mu_2+\lambda_2} > p_2 + (1-p_2)p_1 = p_1+p_2-p_1p_2$$

REMARK 2. Theorem 1 suggests that there should be an analogue of it for more general service distributions. Suppose, for example, that the independent, not necessarily indentically distributed random variables η_{ki} and ξ_{ki}, $k = 1,2$ $i = 1,2,\dots$, have finite second moments then it is possible to deduce the statement of Theorem 1 assuming only that $E\xi_{1i} \leqslant E\xi_{2i}$ (for all is).

In case $E\xi_{1i} = b_1 \sim E\xi_{2i} = b_2$ there are examples, with non exponental distributions, when priority rule $\nu(2,1)$ is better then $\nu(1,2)$ and $b_1 < b_2$ fulfils (see Tomkó [21]).

REMARK 3. It is possible to deduce the Brownian motion approximation for more than 2 DTU-s. Here we shall not give the results.

Theorem 1 above is a useful aid for giving the time interval where the Brownian motion approximation holds. In order to use the result for exponential service times, we need to assume $t \gg \lambda_2$.

The major result of Theorem 1 is in the relation between the priority rules. The theorem estimates "how much" time the CPU must spend in service for the different priority rules. Using these estimates for instants $t_1 < t_2 < \dots < t_n < \dots$, where $t_i - t_{i-1} \sim c\,\lambda_2$ (with $c \gg 1$) it is possible to construct a stochastic control model with Gaussian random variables, as it was proposed in our paper [3].

A further approximation is the following. The discrate time process $\zeta_{t1}, \zeta_{t2}, \dots \zeta_{tn}$ may be handled as a continuous process, if n is large enough. In this case the statistical investigation of the CPU utilization process ζ_t means the following. Let ζ_t denote the CPU utilization time in absolute time t, then it satisfies the equation

$$(1.16) \qquad d\zeta_t \doteq m\,dt + \sigma\,d\omega(t)$$

where m and σ depend on the priority rule $\nu(i_1,\dots,i_k)$. To estimate these parameters, one has to know the number of interrupts for every job and their own CPU utilization time, which depends on the priority rule. To guarantee CPU service, for every job, the computer has a cyclic service with quantum lenght (with round–robin service rule). The round–robin rule ensures the short turn around of short jobs and, in our description, the possibility to measure the CPU utilization time for every job.

2. An approximation of the cyclic queue model

By using the cyclic queue model for one CPU and one DTU (see Fig.1. in part I.), Gaver and Shedler [8], [9] and independently Kobayashi [13], [14] examined the distribution of the number of programs $N_c(t)$ present at the CPU at time t, including those queued in addition to the program currently being serviced. Throughout this section it will be assumed that we have one CPU and one DTU.

Let $A(t)$ represent the number of arrivals at the CPU in $(0,t)$ and $D(t)$ the number of CPU departures in $(0,t)$. We assume that $N_c(0) = 0$ and

$$N_c(t) = A(t) - D(t)$$

The $A(t)$ and $D(t)$ processes are approximately normally distributed with means $E(A(t)) = t/E(\xi_1)$, $E(D(t)) = t/E(\eta_1)$ and variances $D^2(A(t)) = tD^2(\xi_1)/[E(\xi_1)]^3$, $D^2(D(t)) = tD^2(\eta_1)/[E(\eta_1)]^3$. Here ξ_i means the service time in DTU, η_i the same in the CPU. We assume that $E\xi_i > E\eta_i$ and that the ξ_i-s are identically distributed independent random variables and the same is supposed for η_i, $i = 1,2,\ldots$. The $A(t)$ and $D(t)$ processes are not independent: for the special case $E(\xi)/E(\eta) \approx 1$ ("heavy traffic" condition) it can be proved by the method of Borkovov [4] (§ 24) that $N_c(t)$ is approximately a Brownian–motion process and satisfies the equation

$$dN_c(t) = \mu dt + \sigma^2 d\omega(t) \tag{2.1}$$

where $\omega(t)$ is the standard Brownian–motion process with drift

$$\mu = 1/E(\xi) - 1/E(\eta) \tag{2.2}$$

and infinitesimal variance

$$\sigma^2 = D^2(\xi)/[E(\xi)]^3 + D^2(\eta)/[E(\eta)]^3$$

The $N_c(t)$ process (neglecting the boundary effects at 0 and K which cause the dependence of $A(t)$ and $D(t)$ too) has to be in the interval $[0,K]$, $0 \leqslant N_c(t) \leqslant K$, i.e. 0 and K are reflecting barriers. This means

$$P(N_c(t) < 0) = 0 \qquad \text{and} \qquad P(N_c(t) \leqslant K) = 1. \tag{2.3}$$

If the number K of programs is unlimited, the behaviour of a cyclic queue model is well approximated with an ordinary sigle–server system, in which there is no restriction upon the number of waiting customers (see Feller [5], 194–198)

(2.4) $$M_n = \max(0, S_1, S_2, \ldots S_n)$$

where

$$S_n = X_1 + \ldots + X_n, \qquad X_i = \eta_i - \xi_i, \qquad E(\eta_i - \xi_i) = -\delta < 0,$$
$$D^2(\eta_i - \xi_i) = \sigma_0^2 .$$

Now, and this is new in our treatment (not used by Gaver–Shedler [9]), we use the well known formulas of sequential hypothesis testing for the Brownian motion process. Let $\omega(t)$ denote the standard Brownian motion process with $\omega(0) = x$, $E\omega(t) = 0$, $E\omega^2(t) = 1.t$, and two barriers $A < x < B$. Let

(2.6) $$\lambda^x(t) = x + \frac{r}{\sigma^2}(\sigma\omega(t) - \frac{r}{2}t), \quad r > 0, \quad \sigma > 0$$

be a process, and let $\tau^x_{A,B}$ be the first time point (Markov–point) where $\lambda^x(t)$ goes out from $[A,B]$ i.e.

(2.7) $$\tau^x_{A,B} = \inf\{t \geqslant 0 : \lambda^x(t) \bar{\in} (A,B)\}$$

Then (see Shiryayev [19] p. 182)

(2.8) $$P\{\lambda^x(\tau^x_{A,B}) = B\} = \frac{e^x - e^A}{e^B - e^A} .$$

From this formula we obtain, if $A \longrightarrow \infty$.

(2.9) $$P\{\lambda^x(\tau^x_{-\infty,B}) = B\} = e^{x-B}$$

and further, if $x = 0$,

(2.10)

$$P\{\lambda(\tau_{-\infty,B}) = B\} = e^{-B} .$$

Using the Brownian–motion approximation for S_n and M_n we obtain that, for $n \longrightarrow \infty$ (using theorem 2.ch.IX.§3 in Gihman–Skorohod's book [10]).

$$P\{W_n > \sqrt{n}\,B\} = P\{M_n > \sqrt{n}\,B\} =$$

$$= P\left\{\max_{1 \leqslant i \leqslant n}\left(\frac{\delta_i + i\,\delta}{\sqrt{n}\;\sigma_0} - \frac{i}{\sqrt{n}}\,\frac{\delta}{\delta_0}\right) > \frac{B\sqrt{n}}{\sigma_0\sqrt{n}}\right\} \approx$$

$$\approx P\left\{\sup_t \omega_t > t\frac{\delta}{\sigma_0} + \frac{B}{\sigma_0}\right\} \approx$$

$$\approx P\left\{\omega_\tau - \tau\,\frac{\delta}{\sigma_0} = \frac{B}{\sigma_0}\right\} =$$

$$= P\left\{\frac{2\delta}{\sigma_0}\left(\omega_\tau - \tau\frac{\delta}{\sigma_0}\right) = \frac{2\delta}{\sigma_0^2}\,B\right\} = e^{-\frac{2\delta}{\sigma_0^2}B} .$$

Further the proof is the same as in the cited paper [7]. The number Q of customers in the queue is the number that arrive during the waitting time of an arbitrary customer. We investigate the stationary distributions of both W_n and Q. If $G(x)$ is the distribution of ξ and *represents Stieltjes convolution, then

(2.11) $\quad P\{Q \geqslant k | W = x\} = G^{k^*}(X)$

and

(2.12) $\quad P\{Q \geqslant k\} = \int_0^\infty G^{k^*(x)}\, e^{-cx}\, c dx = C[\hat{G}(c)]^k$

where $\hat{G}(c)$ is the Laplace–Stieltjes transform of G, evaluated at c. Relation (12) includes the following theorem:

Theorem 2. *The number Q of jobs in the CPU queue has asymptotical exponential distribution;*

(2.13) $\quad P\{Q \geqslant x\} \approx Ce^{x \ln G(c)}$,

where

$$c = \frac{2\delta}{\sigma_0^2}$$

This means that, under heavy traffic conditions $(\rho = E(\xi)/E(\eta) \sim 1)$, the stationary distribution of the number of costumers in the system is exponential. This result is quite the same as in Gaver–Shedler's [8] diffusion approximation, where they obtained

(2.14) $\quad P\{Q \geqslant x\} = C' e^{\frac{2\mu}{\sigma^2} x}$

where μ, σ are given by (2).

When $\sigma \sim 0$ then with the Taylor series expasion, we can easily prove that (12) and (14) give the same approximation. In this case $-\delta \sim \mu$ and $\sigma_0^2 \sim \sigma^2$.

The reader may find numerical results in Gaver and Shedler's cited papers.

In our paper [3] we have used diffusion type processes (first and second order autoregressive processes) for the approximation of the number of I/0 interrupst over a long time period of the CPU. The above model shows the legality of such heuristic approximations.

The same first order, Gaussian, autoregressive model with discrete time parameter may be used for the working set size $\omega(t,T)$ introduced by Denning [5]. This means that $\omega(t,T)$ is normally distributed and it satisfies the stochastic difference equation

$$\omega(t+1,T) = \rho\,\omega(t,T) + \epsilon(t+1)$$

where $\epsilon(t)$ is an independent, normal random sequence. T is the window and it is fixed. In the sequence $r_{t-\tau+1}, \ldots, r_t$ the number of different page numbers (where r_i means the number of page at time i) is denoted by $\omega(t,T)$. The working set principle for memory management means a dynamical page treatment in the main memory. Here we do not elaborate on this problem.

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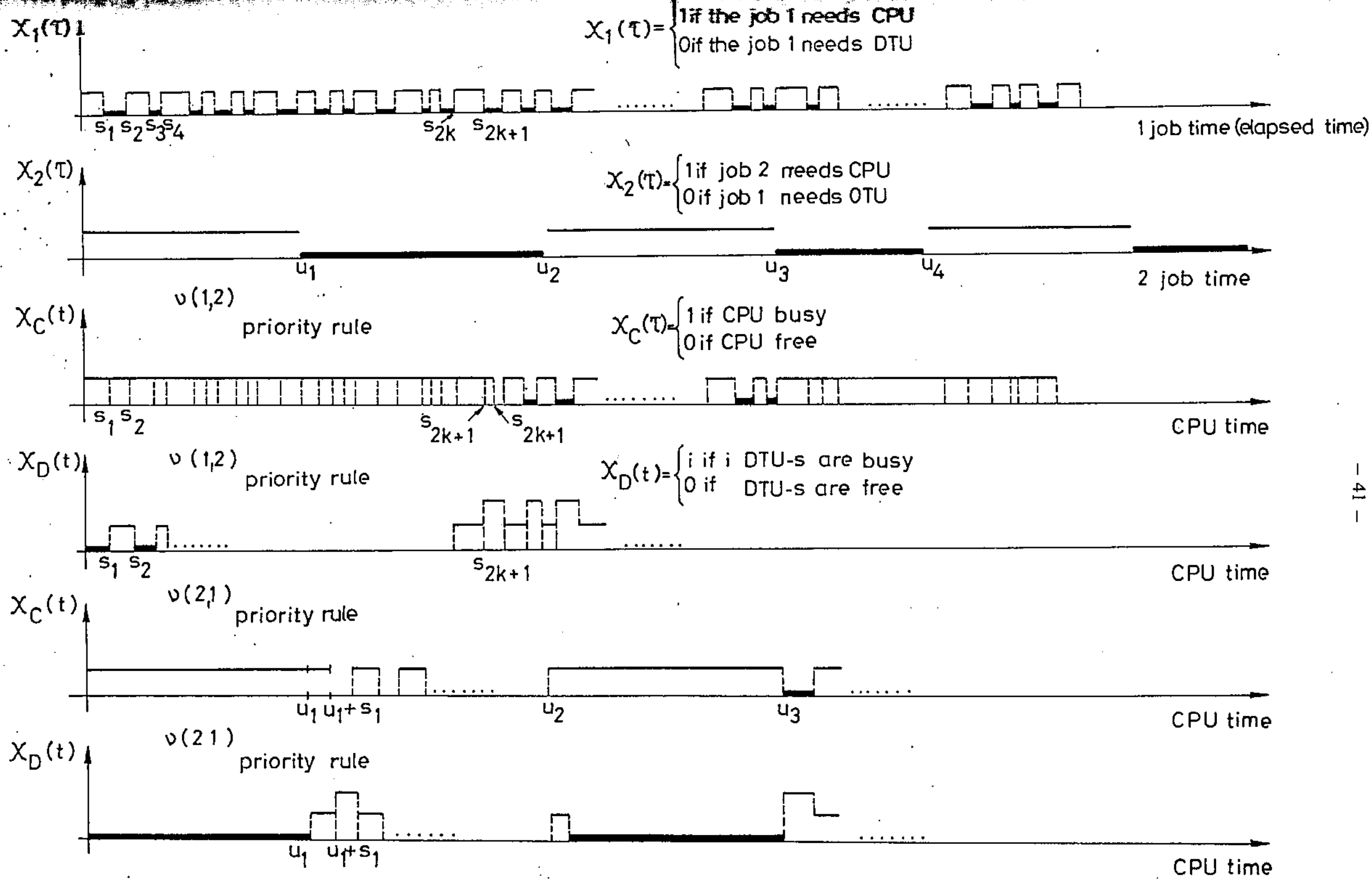
Р е з ю м е

О диффузионном приближении операционных систем I-II

Матиаш Арато

В статьях даётся общее описание опыта статистических измерений на машине CDC 3300 в Институте Вычислительной Техники и Автоматизации ВАН. Показывается, что приближение работы операционной системы с помощью диффузионных процессов является естественным в длинном периоде работы машины. Такое приближение использовалось раньше в статьях Gaver-Shedler и Kobayashi для моделей теории очереди.



X1(τ)
X1(τ)= 1 if the job 1 needs CPU
0 if the job 1 needs DTU
s1 s2 s3 s4
s2k s2k+1
1 job time (elapsed time)
X2(τ)
X2(τ)= 1 if job 2 needs CPU
0 if job 1 needs DTU
u1 u2 u3 u4
2 job time
XC(t)
ν(1,2)
priority rule
XC(τ)= 1 if CPU busy
0 if CPU free
s1 s2
s2k+1 s2k+1
CPU time
XD(t)
ν(1,2)
priority rule
XD(t)= i if i DTU-s are busy
0 if DTU-s are free
s1 s2
s2k+1
CPU time
XC(t)
ν(2,1)
priority rule
u1 u1+s1
u2
u3
CPU time
XD(t)
ν(21)
priority rule
u1 u1+s1
CPU time

Fig. 2.