ROOT COUNTING IN PRODUCT HOMOTOPY METHOD

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To my beloved family and friends.

Thanks for all the efforts, guidance, tender support and blessings that shower on me.

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ABSTRACT

Product Homotopy method is used to solve dense multivariate polynomial systems for finding all isolated solutions (real or complex). There are two stages in the computation of Homotopy method which are root counting and root finding. This study focuses on root counting which involves the computation of multi-homogeneous Bézout number (MHBN). This value determines the number of solution path in the second stage. Homogenization of partition each gives its own MHBN. Therefore, it is crucial to have minimum MHBN. The computation of minimum MHBN using local search method, fission and assembly method and genetic algorithm had become intractable when the system size gets larger. Hence, this study applied recent heuristic method, Tabu Search. Other than that, the computation of estimating MHBN is of exponential time. For large size system, the usage of row expansion with memory becomes impossible, hence, this study focus on implementing General Random Path algorithm (GRPA). This study implements Tabu search method and GRPA into several systems of different sizes. Tabu search is effective since the global minimum is obtained instead of the local minimum. Other than that, the number of visited partition is much smaller compared with the previous method. Although GRPA gives estimated value, it helps for large size system. We implement two accuracy level in the computation and in the result, the N=1000 gives more accurate result. Hence, GRPA is important when it comes to solve estimated MHBN for large size system.

ABSTRAK

Kaedah homotopi produk digunakan untuk menyelesaikan sistem padat polinomial berbilang pembolehubah untuk mencari semua penyelesaian terpencil (sebenar atau kompleks). Terdapat dua peringkat dalam pengiraan kaedah homotopi iaitu pengiraan akar dan penemuan akar. Kajian ini memberi tumpuan kepada pengiraan akar yang melibatkan pengiraan nombor Bézout berbilang-homogen (MHBN). Nilai ini menentukan bilangan jalan penyelesaian dalam peringkat kedua. Setiap pengkelasan memberikan MHBN yang berbeza. Oleh itu, penting untuk mempunyai MHBN minimum. Pengiraan MHBN minimum dengan menggunakan kaedah carian tempatan, kaedah pemecahan dan penggabungan serta algoritma genetik tidak boleh digunakan bagi menyelesaikan sistem bersaiz besar. Oleh itu, kajian ini menggunakan kaedah heuristik yang terbaru iaitu 'Tabu Search'. Selain daripada itu,pengiraan MHBN mempunyai pola meningkat secara eksponen dimana nilai MHBN sukar ditentukan apabila berdepan dengan sistem bersaiz besar Oleh itu, penggunaan 'Row Expansion with Memory' menjadi mustahil. Fokus kajian ini adalah menggunakan algoritma 'General Random Path' (GRPA). Kajian ini melaksanakan kaedah Tabu search dan GRPA terhadap beberapa sistem yang berlainan saiz. Tabu search berkesan kerana minimum global diperolehi bukannya minimum tempatan. Selain daripada itu, bilangan pengkelasan yang terlibat adalah jauh lebih kecil berbanding dengan kaedah sebelumnya. Walaupun GRPA cuma memberikan nilai anggaran, ia membantu untuk sistem saiz besar. Kami melaksanakan dua tahap ketepatan dalam pengiraan GRPA dan keputusan N = 1000 memberikan hasil yang lebih tepat. Oleh itu, penggunanan GRPA adalah sangat penting untuk sistem yang bersaiz besar.