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Completely Simple and Regular Semi Hypergroups

[1]HOSSEIN MOUSA JAFARABADI, [2]NOR HANIZA SARMIN
AND [3]MOHAMMAD REZA MOLAEI

[1,2]Department of Mathematics, Faculty of Science,
Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Malaysia
[3]Department of Mathematics, University of Kerman (Shahid Bahonar), Kerman, Iran
[1]hmjafarabadi@gmail.com, [2]nhs@fs.utm.my, [3]mrmolaei@mail.uk.ac.ir

Abstract. In this paper the notion of simple and completely simple semi hypergroups are introduced. Basic properties of these algebraic structures are considered. Some methods for constructing new kinds of these hyperstructures are presented. The regularity of semi hypergroups is considered and three structural results are proved.

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1. Introduction

The origin of hypergroup can be traced back to the time of the rise of group theory in 1900 with the work of Frobenious. In the mid of 30's, Marty [11] and Wall [16] introduced the concept of an algebraic hypergroup, mainly within the theory of Non-abelian groups and related structures of spaces of conjugacy classes and double cosets. Now this field of modern algebra is widely studied from the theoretical and applied viewpoints because of their applications to many subjects of pure and applied mathematics. This theory has been subsequently developed by Corsini [1, 2], Davvaz [4, 5, 6], Mittas [12], Vougiouklis [15] and by various authors. The basic notions and results of the object can be found in [1]. In 2003, Corsini and Leoreanu presented numerous applications of hyperstructure theory [2]. These applications can be used in the following areas: geometry, graphs, fuzzy sets, cryptography, automata, lattices, binary relations, codes, and artificial intelligence. By an analogue to semi group theory, semi hypergroups can be considered from two points of view: algebraic and harmonic analysis. The theory of hypergroups was introduced into harmonic analysis in the 70's by the papers of Dunkl [7], Jewett [10], and Spector [14]. Up to now many researchers have been studying in this field of applications of hyperstructures theory. Norbert Youmbi has studied completely simple semi hypergroups from harmonic analysis point of view [17].

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He considered a semi hypergroup S that is also a locally compact Hausdorff space, and hyperoperation in S defines a probability measure with the compact support contained in S. In 2006, Chaopraknoi and Triphop introduced the regular semi hypergroups and they proved some results on the regularity of semi hypergroups of infinite matrices [3]. This paper is concerned to a generalization of Rees theorem for semi group theory [8, 13] to semi hypergroups theory. It has proved that Rees matrix semi hypergroup is a completely simple semi hypergroup. The outline of this paper is as follows: Section 2 is a brief overview of some basic notions and results on hyperstructures theory related to this research. Simple semi hypergroups and some properties of this algebraic structure are presented in Section 3. In Section 4, some methods for constructing new simple semi hypergroups are presented. Completely simple semi hypergroups and some properties of this algebraic structure are presented in Section 5. Section 6 is concerned with three methods for constructing new completely simple semi hypergroups. In this paper, it will be proved that the product of two simple semi hypergroups and also two completely simple semi hypergroups are simple and completely simple semi hypergroup, respectively. The quotients of semi hypergroups are considered and it is shown that if S is a simple semi hypergroup and ρ is a regular equivalence relation then S/ρ is a simple semi hypergroup. It is shown that in this case if S is a regular hyperhroup then S/ρ is a completely simple semi hypergroup. In Section 7 three structural results on the regularity of semi hypergroups are presented.

2. Basic notions and preliminaries

We recall the following terminologies from [1, 2, 12, 15]. Let H be a nonempty set and $P^*(H)$ be the set of all nonempty subsets of H. An *n-hyperoperation* on H is a map $\circ : H^n \to P^*(H)$ and a set H endowed with a family Γ of hyperoperations, is called a *hyperstructure* (multivalued algebra). If Γ is a singleton, that is $\Gamma = \{f\}$, then the hyperstructure is called *hypergroupoid*. The hyperoperation is denoted by "$\circ$" and the image of (a,b) of H is denoted by $a \circ b$ and is called the *hyperproduct* of a and b. If A and B are nonempty subsets of H then $A \circ B = \bigcup_{a\in A, b\in B} a \circ b$. A *semi hypergroup* is hypergroupoid $(H,\circ)$ such that:

$$\forall (a,b,c) \in H^3, (a \circ b) \circ c = a \circ (b \circ c).$$

A *hypergroup* is a semi hypergroup $(H,\circ)$ such that :

$$\forall a \in H, a \circ H = H \circ a = H.$$

For example, let $(G,\circ)$ be a group and H be a normal subgroup of G. Then $(G,*)$ with the following hyperoperation is a hypergroup:

$$\forall (x,y) \in H^2, x * y = Hx \circ y.$$

Let $(H,\circ)$ be a hypergroupoid. An element e is called an *identity* or *unit* if

$$\forall a \in H, a \in a \circ e \cap e \circ a.$$

Let $(H,\circ)$ be a hypergroup endowed with at least an identity. An element $a' \in H$ is called an *inverse* of $a \in H$ if there exists an identity $e \in H$ such that:

$$e \in a \circ a' \cap a' \circ a.$$

A hypergroup is called *regular* if it has at least one identity and each element has at least one inverse. The element a in hypergroup $(H,\circ)$ is called *scalar* if

$$\forall x \in H, |a \circ x| = |x \circ a| = 1.$$

Let $(H,\circ)$ and $(K,*)$ be two hypergroups, and $F : H \to K$ be a map. Then:

(i) f is a *homomorphism* if

$$\forall (a,b) \in H^2, f(a\circ b) \subseteq f(a) * f(b).$$

(ii) f is a *good homomorphism* if

$$\forall (a,b) \in H^2, f(a\circ b) = f(a) * f(b).$$

Let $(H,\circ)$ a hypergroupoid and ρ be an equivalence relation on H. We say that ρ is *regular on the right* if the following implication holds: $a\rho b \to \forall u \in H, \forall x \in a\circ u, \exists y \in b\circ u : x\rho y$ and $\forall y' \in b\circ u, \exists x' \in a\circ u : x'\rho y'$. Similarly, the *regularity on the left* can be defined. The equivalence relation ρ is said to be *regular* if it is regular on the right and on the left.

3. Simple semi hypergroups

In this section the concept of simple in the context of semi hypergroups is introduced. The results of this section give an important characterization of simple semi hypergroups.

If a semi hypergroup S with at least two elements contains an element 0 such that $0s = 0s = \{0\}$, for all s in S, then 0 is said to be a *zero scalar element* (or just zero scalar) of S and S is called a *semi hypergroup with zero scalar*. The concept of *hyperideal* in semi hypergroups theory is given as the following: A nonempty subset I of a semi hypergroup S is called *left hyperideal* if $SI \subseteq I$, a *right hyperideal* if $IS \subseteq I$, and (two-sided) *hyperideal* if it is both left and right hyperideal [4]. Using the terminologies of semi group theory, the following definitions for 0-simple and simple semi hypergroups are presented.

Definition 3.1. *[9] A semi hypergroup without zero scalar is called* simple *if it has no proper hyperideals. A semi hypergroup S with zero scalar is called* 0-simple *if it has the following conditions:*

i) 0 *and S are only its hyperideals.*
ii) $S^2 \neq 0$.

Example 3.1. The set of real numbers R with the hyperoperation

$$a.b = \begin{cases} (a,b), & \text{if } a<b; \\ (b,a), & \text{if } b<a; \\ \{a\}, & \text{if } a=b; \end{cases} \quad \text{for all } \ a,b \in R$$

is a simple semi hypergroup.

Proposition 3.1. *Semi hypergroup S is 0-simple if and only if $SaS = S$ for all $a \in S$. It means for every $a,b \in S-\{0\}$ there exist x,y in S such that $b \in xay$.*

Proof. The scheme of the proof is similar to the proof of the same result in semi group theory [8]. First, suppose that S is a 0-simple semi hypergroup. It is easy to see that S^2 is a hyperideal of S. By Definition 5.1, $S^2 \neq 0$. Thus $S^2 = S$, and it follows that $S^3 = S$. Now consider an element $a \in S$ that is not a zero scalar element. It is clear that SaS is a hyperideal of S. In the case of $SaS = \{0\}$ the set $I = \{s \in S, SsS = \{0\}\}$ is a nonempty subset of S since $a \in I$. If x is an element of SI, then there exist elements s in S and i in I such that $x \in si$ and hence $SxS \subseteq SsiS \subseteq SiS = \{0\}$.

This implies $SxS = \{0\}$ and $x \in I$. In a similar way, it can be shown that IS is also a subset of I. It follows that I is a hyperideal of S, hence $I = S$. Thus for all s in S, $SsS = \{0\}$, that is $S^3 = \{0\}$, which is a contradiction to $S^3 = S$. Therefore, $SaS = S$ for every $a \neq 0$ in S. Conversely, assume that $SaS = S$ for all $a \neq 0$ in S, then S^2 is not equal to $\{0\}$ ($S^2 = \{0\}$ means $SaS = \{0\}$ for all $a \neq 0$ in S). Now, suppose that a is an element of S that is not a zero scalar and I is a hyperideal of S containing a. Then $S = SaS \subseteq SIS \subseteq I$ or $S = I$. Thus S is 0-simple. ∎

Proposition 3.1 leads to the following important corollary that can be used for a characterization of simple semi hypergroups.

Corollary 3.1. *A semi hypergroup S is simple if and only if for all a in $S, SaS = S$.*

Proposition 3.2. *Every hypergroup is a simple semi hypergroup.*

Proof. If H is a hypergroup, then for all a in H , $aH = Ha = H$. Hence, $H = aH \subseteq HH \subseteq H$ and so $HH = H$. On the other hand, $aH = H$, thus $HaH = HH = H$, that is H is simple. ∎

4. Construction of simple semi hypergroups

This section is concerned with some methods to construct new simple semi hypergroups. Let $(S,*)$ and $(T,\circ)$ be two semi hypergroups. It has been proved that the Cartesian product of these two semi hypergroups is a semi hypergroup with the following hyperoperation [1]: $(s_1,t_1) \otimes (s_2,t_2) = (s_1 * s_2) \times (t_1 \circ t_2)$.

Theorem 4.1. *Let $(S,*)$ and $(T,\circ)$ be two simple semi hypergroups. Then the product $S \times T$ with the above hyperoperation is a simple semi hypergroup.*

Proof. Suppose that (a,b) is an arbitrary element of $S \times T$ that is not a zero scalar. It is clear that $SaS = S$ and $TbT = T$, and so $S \times T = SaS \times TbT$. Now, consider $z = (x,y)$ as an element of $S \times T$. It follows that there exist (c,d) in $Sa \times Tb$ and (s,t) in $S \times T$ such that $(x,y) \in cs \times dt = (c,d) \otimes (s,t)$.

There exists also (s',t') in $S \times T$ such that $(c,d) \in (s',t')(a,b)$, and so $(x,y) \in (c,d) \otimes (s,t) \subseteq S \times T(a,b)(s,t) \subseteq S \times T(a,b)S \times T$. Hence $S \times T \subseteq S \times T(a,b)S \times T$. It is clear that $S \times T(a,b)S \times T \subseteq S \times T$, and so $S \times T(a,b)S \times T = S \times T$. That is $S \times T$ is simple. ∎

Let $(H,\circ)$ be a semi hypergroup and ρ an equivalence relation on H. Then:

(i) If ρ is regular, then H/ρ is a semi hypergroup, with respect to the following hyperoperation $\forall(\bar{x},\bar{y}) \in (H/\rho)^2, \bar{x} \otimes \bar{y} = \{\bar{z}, z \in x \circ y\}$.

(ii) Conversely, if hyperoperation "$\otimes$" is well-defined on H/ρ , then ρ is regular.

(iii) The canonical projection $\pi : H \to H/\rho$ is a good epimorphism and when $(H,\circ)$ is a hypergroup, then $(H/\rho,\otimes)$ is also a hypergroup, denoted by H/ρ [2].

New simple semi hypergroups by using of the above result and the next proposition can also be constructed.

Proposition 4.1. *Let $(H,\circ)$ be a simple semi hypergroup and ρ be a regular equivalence relation on H. Then H/ρ is a simple semi hypergroup with respect to the following hyperoperation $\forall(\bar{x},\bar{y}) \in (H/\rho)^2, \bar{x} \otimes \bar{y} = \{\bar{z}, z \in x \circ y\}$.*

Proof. As mentioned above in this case H/ρ is a semi hypergroup. Suppose that $\bar{a}$ and $\bar{b}$ are two arbitrary elements of H/ρ. There exist elements a and b in H such that $\bar{a}$ and $\bar{b}$ are the images of a and b in H/ρ with respect to the canonical projection respectively. Since H

is simple, then there exist elements x and y in H such that $b \in xay$. Thus $\overline{b} \in \overline{xay}$, and hence $\overline{b} \in \overline{x} \otimes \overline{a} \otimes \overline{y}$ (in this case canonical projection is a good homomorphism [1, 2]), that is H/ρ is simple. ∎

Example 4.1. Consider the set of integer numbers Z with the hyperoperation $i \circ j = \{i, j\}$. By Proposition 3.2, this hyperstructure is a simple semi hypergroup. Let the equivalence relation ρ be the congruence modulo 2, that is a regular equivalence relation. Then $\overline{Z} = Z/\rho = \{\overline{0}, \overline{1}\}$ with respect to the following hyperoperation is a simple semi hypergroup: $\overline{i} \otimes \overline{j} = \{\overline{k}, k \in i \circ j = \{i, j\}\}$. In fact, $\overline{Z} \otimes \overline{i} \otimes \overline{Z} = \overline{Z}$ for $\overline{i} = \overline{0}, \overline{1}$.

The following theorem is an approach to a generalization of Rees theorem in semi group theory. By using of this theorem new simple semi hypergroups can also be constructed.

Theorem 4.2. *Let H be a regular hypergroup, and I, Λ be nonempty sets. Let $P = (p_{ij})$ be a $\Lambda \times I$ regular matrix (it has no row or column that consists entirely of zeros) with entries from H.Then $S = I \times H \times \Lambda$ (*Rees Matrix Semi hypergroup*)with respect to the following hyperoperation is a simple semi hypergroup: $(i, a, \lambda)(j, b, \mu) = \{(i, t, \mu), t \in ap_{\lambda j} b\}$.*

Proof. In a direct verification, the associativity of the hyperoperation can be proved. Let $(i, a, \lambda), (j, b, \mu)$ and (k, c, ψ) be arbitrary elements of S, and z is an element of the following set:

$$(i, a, \lambda)[(j, b, \mu)(k, c, \psi)] = \bigcup_{t \in bp_{\mu k} c} \{(i, x, \psi), x \in ap_{\lambda j} t\}.$$

There exists t' in $bp_{\mu k} c$ and x' in $ap_{\lambda j} t'$ such that $z = (i, x', \psi)$. This means there exists v in $p_{\mu k} c$ and u in $ap_{\lambda j}$ such that $t' \in bv$ and $x' \in ut'$, and so $x' \in ubv$. It follows that there exists s in $ap_{\lambda j} b$ such that $x' \in sp_{\mu k} c$. Thus

$$z = (i, x', \psi) \in \bigcup_{s \in ap_{\lambda j} b} \{(i, y, \psi), y \in sp_{\mu k} c\} = [(i, a, \lambda)(j, b, \mu)](k, c, \psi).$$

In a similar way, it can be shown that every element of $[(i, a, \lambda)(j, b, \mu)](k, c, \psi)$ is an element of the set $(i, a, \lambda)[(j, b, \mu)(k, c, \psi)]$. Therefore, S is a semi hypergroup.

To verify that S is simple, suppose that (i, a, λ) and (j, b, μ) are two elements of S that are not zero scalar. Since H is a hypergroup, then there exist elements x and y in H such that $b \in xay$, and due to the regularity of H, there exists an identity element e in H such that $b \in xeaey$, and so $b \in xp_{vi}{}^{-1} p_{vi} a p_{\lambda k} p_{\lambda k}{}^{-1} y$ (by the regularity of matrix P the elements v in Λ and k in I can be chosen such that $p_{\lambda k}$ and p_{vi} are not zero scalar). It follows that there exists t in $xp_{vi}{}^{-1} p_{vi} a$ such that $b \in tp_{\lambda k} p_{\lambda k}{}^{-1} y$, hence

$$(j, b, \mu) \in \{(j, e, v), e \in xp_{vi}{}^{-1}\}(i, a, \lambda)\{(k, f, \mu), f \in p_{\lambda k}{}^{-1} y\}$$

$$= \bigcup_{t \in xp_{vi}{}^{-1} p_{vi} a} \{(j, s, \mu), s \in tp_{\lambda k} p_{\lambda k}{}^{-1} y\}.$$

It means that there exists e' in $xp_{vi}{}^{-1}$ and f' in $p_{\lambda k}{}^{-1} y$ such that

$$(j, b, \mu) \in (j, e', v)(i, a, \lambda)(k, f', \mu).$$

Thus S is simple. ∎

5. Completely simple semi hypergroups

In this section completely simple semi hypergroup is introduced. By using of the results in Sections 3 and 4 new results about completely simple semi hypergroups are proved. For notational simplicity, hereafter we identify a singleton with its element.

Definition 5.1. *An element e in a semi hypergroup S is called an* idempotent *if* $e \in e^2$.

In the set of all scalar idempotent elements of semi hypergroup S we can define an order $e \leq f$ if and only if $e = ef = fe$ It is easy to show that this relation is an order relation.

Definition 5.2. *A scalar idempotent e in the set of all scalar idempotent elements of semi hypergroup S is called* primitive scalar idempotent *(or just primitive) if it is minimal within the set of all nonzero scalar idempotent elements of S. Thus a primitive scalar idempotent has the following property: If* $0 \neq f = ef = fe$ *then* $e = f$.

Definition 5.3. *A semi hypergroup is called* completely simple semi hypergroup *if it is simple and has primitive idempotent.*

Example 5.1. Consider semi hypergroup $S = \{p, q, r, t\}$ with respect to the following Cayley table: In this semi hypergroup p and t are idempotent elements. The idempotent p is

$*$	p	q	r	t
p	p	q	r	t
q	q	$\{p,r\}$	$\{q,r\}$	t
r	r	$\{q,r\}$	$\{p,q\}$	t
t	t	t	t	S

Table 1. Cayley table for a completely simple semi hypergroup

the only scalar idempotent, that is a primitive idempotent. This semi hypergroup is clearly simple $(SaS = S, \forall a \in S)$ so it is a completely simple semi hypergroup.

Example 5.2. Semi hypergroup of Example 3.1 is a completely simple semi hypergroup. In fact, every element of this semi hypergroup is a primitive idempotent.

Lemma 5.1. *Every scalar idempotent of a regular hypergroup is a scalar identity.*

Proof. Let H be a regular hypergroup and a be a scalar idempotent of H. Then $a^2 = a$, so $a^{-1}a^2 = a^{-1}a$ (due to regularity of H, a^{-1} as an inverse of a exists), it means there exists an identity element such as e in H such that $ea = e$ (a is scalar). On the other hand, e is the identity, thus $ea = ae = a$, and so $a = e$. ∎

Proposition 5.1. *Every regular hypergroup is a completely simple semi hypergroup.*

Proof. Let H be a regular hypergroup. Assume that e and f are two scalar idempotent elements of H, and $f = ef = fe$, By Lemma 1, f is an identity element of H, thus $e = ef = fe$, and so $e = f$. It means every scalar idempotent of H is primitive and this conduces that H is a completely simple semi hypergroup. ∎

6. Construction of completely simple semi hypergroups

In this section, three methods to construct new completely simple semi hypergroups are presented. To construct new completely simple semi hypergroups by using of the quotient semi hypergroups the following theorem will be used.

Theorem 6.1. *Let $(H,*)$ be a regular hypergroup and ρ be a regular equivalence relation on H. Then H/ρ with respect to the following hyperoperation is a regular hypergroup: $\forall(\bar{x},\bar{y}) \in (H/\rho)^2, \bar{x}\otimes\bar{y} = \{\bar{z}, z \in x*y\}$.*

Proof. It is proved that if H is a hypergroup then H/ρ is also a hypergroup [1, 2]. Suppose that e is an identity element of H. Then $x \in xe \cap ex$, for all $x \in H$, so$\bar{x} \in \overline{xe} \cap \overline{ex}$. Canonical projection $\pi : H \to H/\rho$ is a good epimorphism [1, 2], thus $\bar{x} \in \bar{x}\,\bar{e} \cap \bar{e}\,\bar{x}$. It means $\bar{e}$ is an identity element of H/ρ. On the other hand, if $\bar{x}$ is an arbitrary element of H/ρ, then there exists x in H such that $\pi(x) = \bar{x}$. By the regularity of H, there exist elements such as e (identity) and x' in H such that $e \in xx' \cap x'x$ and so $\bar{e} \in \overline{xx'} \cap \overline{x'x} = \bar{x}\,\overline{x'} \cap \overline{x'}\,\bar{x}$. It means $\overline{x'}$ is an inverse for $\bar{x}$. ∎

Example 6.1. Consider the set of integers $(Z,+)$ as a regular hypergroup. Let the equivalence relation ρ be the congruence modulo 2, that is a regular equivalence relation. Then it is easy to show that $\overline{Z} = Z/\rho = \{\bar{0},\bar{1}\}$ with respect to the following hyperoperation is a regular semi hypergroup: $\forall\left(\bar{i},\bar{j}\right) \in \overline{Z}^2, \bar{i}\otimes\bar{j} = \{\bar{k}, k = i+j\}$.

Corollary 6.1. *Let $(H,*)$ be a regular hypergroup and ρ be a regular equivalence relation on H. Then H/ρ with the following hyperoperation is a completely simple semi hypergroup. $\forall(\bar{x},\bar{y}) \in (H/\rho)^2, \bar{x}\otimes\bar{y} = \{\bar{z}, z \in x*y\}$.*

Proof. The proof follows from Theorem 6.1 and Proposition 5.1. ∎

Proposition 6.1. *Every finite hypergroup with at least one scalar element is a completely simple semi hypergroup.*

Proof. Let H be a finite hypergroup with at least one scalar element. Then there exists an identity element of H such as e such that the set of all scalar elements of H is a group with the identity e [2, page 9]. It is clear that e is a scalar idempotent. It now follows that the primitive scalar idempotent exists. Consider a descending chain $e_1 \geq e_2 \geq e_3 \geq ...$ of scalar idempotent elements in H. This chain has to be finite because H is finite. It means H has primitive scalar idempotent, and so it is completely simple semi hypergroup. ∎

Theorem 6.2. *Let $(S,*)$ and $(T,\circ)$ be two completely simple semi hypergroups. Then the product $S\times T$ with the following hyperoperation is a completely simple semi hypergroup: $(s_1,t_1)\otimes(s_2,t_2) = (s_1 * s_2)\times(t_1\circ t_2)$.*

Proof. In Theorem 4.1, it is proved that $S\times T$ with the above hyperoperation is a simple semi hypergroup. So it is sufficient to prove that $S\times T$ has primitive scalar idempotent element. Suppose that e_s, e_t are primitive scalar idempotent in S and T respectively. Then $e_s = e_s * e_s$ and $e_t = e_t \circ e_t$ so $(e_s,e_t) = e_s * e_s \times e_t \circ e_t = (e_s,e_t)\otimes(e_s,e_t)$.

It means (e_s,e_t) is a scalar idempotent element of $S\times T$. On the other hand, if (f_s,f_t) is another scalar idempotent element in $S\times T$ and $(f_s,f_t) = (e_s,e_t)\otimes(f_s,f_t) = (f_s,f_t)\otimes(e_s,e_t)$, then $(f_s,f_t) = e_s * f_s \times e_t \circ f_t = f_s * e_s \times f_t \circ e_t$. And so $f_s = e_s * f_s = f_s * e_s$ and $f_t = e_t \circ f_t = f_t \circ e_t$. Hence $(f_s,f_t) = (e_s,e_t)$, and (e_s,e_t) is a primitive scalar idempotent for $S\times T$. ∎

Theorem 6.3. *Let H be a regular hypergroup, and I,Λ be nonempty sets. Let $P=(P_{ij})$ be a $\Lambda\times I$ regular matrix (it has no row or column that consists entirely of zeros) with entries from H. Then $S=I\times H\times\Lambda$ (*Rees Matrix Semi hypergroup*) with respect to the following hyperoperation is a completely simple semi hypergroup: $(i,a,\lambda)(j,b,\mu)=\{(i,t,\mu),t\in ap_{\lambda j}b\}$.*

Proof. The associativity of the hyperoperation and to verify that S is simple are already proved in Theorem 4.2. By analogues to semi group theory, it is easy to show that every scalar idempotent of S is primitive idempotent. Therefore, S is a completely simple semi hypergroup. ∎

7. On the regularity of semi hypergroups

In this section, three results on the regularity of semi hypergroups are proved. An element s in a semi hypergroup $(S,\circ)$ is called *regular* if there exists an element x in S such that $s\in s\circ x\circ s$. A semi hypergroup is called *regular semi hypergroup* if all of its elements are regular. For example let S be a semi group, P be a nonempty subset of S. Then S with the hyperoperation $x\circ y=xPy(x,y\in S)$ is a regular semi hypergroup [3].

Theorem 7.1. *Let $\varphi:S\to T$ be a good homomorphism from a regular semi hypergroup S into semi hypergroup T. Then $\Im\varphi$ is a regular semi hypergroup.*

Proof. Assume that t is an arbitrary element in $\Im\varphi$. There exists an element such as s in S such that $\varphi(s)=t$. By the regularity of S there exists an element x in S such that $s\in sxs$, and so $t\in\varphi(s)\in\varphi(sxs)=\varphi(s)\varphi(x)\varphi(s)=t\varphi(s)t$. Thus $\Im\varphi$ is a regular semi hypergroup. ∎

Corollary 7.1. *Let $(S,\circ)$ be a regular semi hypergroup and ρ be a regular equivalence relation on s. Then S/ρ with respect to the following hyperoperation is a regular semi hypergroup: $\forall(\bar{x},\bar{y})\in(S/\rho)^2,\bar{x}\otimes\bar{y}=\{\bar{z},z\in x\circ y\}$.*

Proof. It is proved that with the above hyperoperation, S/ρ is a semi hypergroup and canonical projection from S into S/ρ is a good epimorphism [3]. Now, by using of Theorem 7.1 it is clear that S/ρ is a regular semi hypergroup. ∎

Theorem 7.2. *Let $(S,*)$ and $(T,\circ)$ be two regular semi hypergroups. Then the product $S\times T$ with respest to the following hyperoperation is a regular semi hypergroup: $(s_1,t_1)\otimes(s_2,t_2)=s_1*s_2\times t_1\circ t_2$.*

Proof. As mentioned in Section 4, it is proved that $S\times T$ with the above hyperoperation is a semi hypergroup. Let (s,t) be an arbitrary element of $S\times T$. Then there exist elements x in S and y in T such that $s\in sxs$ and $t\in tyt$. Thus $(s,t)\in sxs\times tyt=(s,t)\otimes(x,y)\otimes(s,t)$. It means $S\times T$ is a regular semi hypergroup. ∎

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References

[1] P. Corsini, *Prolegomena of Hypergroup Theory*, Aviani Editore, Tricesimo, 1993.

[2] P. Corsini and V. Leoreanu, *Applications of Hyperstructure Theory*, Advances in Mathematics (Dordrecht), 5, Kluwer Acad. Publ., Dordrecht, 2003.

[3] S. Chaopraknoi and N. Triphop, Regularity of semihypergroups of infinite matrices, *Thai J. Math.* **4** (2006), no. 3, Special issue, 7–11.

[4] B. Davvaz, Intuitionistic hyperideals of semihypergroups, *Bull. Malays. Math. Sci. Soc.* (2) **29** (2006), no. 2, 203–207.
[5] B. Davvaz, Isomorphism theorems of polygroups, *Bull. Malays. Math. Sci. Soc.* (2) **33** (2010), no. 3, 385–392.
[6] S. O. Dehkordi and B. Davvaz, Γ-semihyperrings: Approximations and rough ideals, *Bull. Malays. Math. Sci. Soc.* (2), accepted.
[7] C. F. Dunkl, The measure algebra of a locally compact hypergroup, *Trans. Amer. Math. Soc.* **179** (1973), 331–348.
[8] J. M. Howie, *Fundamentals of Semigroup Theory*, London Mathematical Society Monographs. New Series, 12, Oxford Univ. Press, New York, 1995.
[9] H. M. Jafarabadi, N. H. Sarmin and M. R. Molaei, Simple semi hypergroups, *Aust. J. Basic Appl. Sci.* **5** (2011), no. 4, 51–55.
[10] R. I. Jewett, Spaces with an abstract convolution of measures, *Advances in Math.* **18** (1975), no. 1, 1–101.
[11] F. Marty, *Sur uni generalization de la notion de group*, in: 8th Congress Math. Scandenaves, Stockholm, 1934, pp. 45-49.
[12] J. Mittas, Hypergroupes canoniques, *Math. Balkanica* **2** (1972), 165–179.
[13] D. Rees, On semi-groups, *Proc. Cambridge Philos. Soc.* **36** (1940), 387–400.
[14] R. Spector, Mesures invariantes sur les hypergroupes, *Trans. Amer. Math. Soc.* **239** (1978), 147–165.
[15] T. Vougiouklis, *Hyperstructures and Their Representations*, Hadronic Press Monographs in Mathematics, Hadronic Press, Palm Harbor, FL, 1994.
[16] H. S. Wall, Hypergroups, *Amer. J. Math.* **59** (1937), no. 1, 77–98.
[17] N. N. Youmbi, *Probability Theory on Semihypergroups*, ProQuest LLC, Ann Arbor, MI, 2005.