EXPLORING MATHEMATICS LEARNERS' PROBLEM-SOLVING SKILLS IN CIRCLE GEOMETRY IN SOUTH AFRICAN SCHOOLS

(A case study of a high school in the Northern Cape Province)

by

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ABSTRACT

This study examined "problem solving skills in circle geometry concepts in Euclidean Geometry. This study was necessitated by learners' inability to perform well with regards to Euclidean Geometry in general and Circle Geometry in particular. The use of naturalistic observation case study research (NOCSR) study was employed as the research design for the study. The intervention used for the study was the teaching of circle geometry with Polya problem solving instructional approach coupled with social constructivist instructional approach. A High School in the Northern Cape Province was used for the study. 61 mathematics learners (grade 11) in the school served as participants for the first year of the study, while 45 mathematics learners, also in grade 11, served as participants for the second year of the study. Data was collected for two consecutive years: 2018 and 2019. All learners who served as participants for the study did so willingly without been coerced in any way. Parental consent of all participants were also obtained.

The following data were collected for each year of the research intervention: classroom teaching proceedings' video recordings, photograph of learners class exercises (CE), field notes and the end-of-the- Intervention Test (EIT). Direct interpretations, categorical aggregation and a problem solving rubric were used for the analysis of data. Performance analysis and solution appraisal were also used to analyse some of the collected data. It emerged from the study that the research intervention evoked learners' desire and interest to learn circle geometry. Also, the research intervention improved the study participants' performance and problem solving skills in circle geometry concepts. Hence, it is recommended from this study that there is the need for South African schools to adopt the instructional approach for the intervention: Polya problem solving instructional approach coupled with social constructivist instructional approach, for the teaching and learning of Euclidean geometry concepts.

Key terms: Advanced mathematical thinking; Euclidean Geometry; van Hiele levels of geometric leaning model, Problem Solving; Polya problem-solving approach

DECLARATION

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Problem Solving Skills in Circle Geometry Concepts: A Case Study of a High School in the Northern Cape Province (South Africa).

I declare that the above dissertation/thesis is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

28-10-2019

(SIGNATURE) DATE

(FITZGERALD ABAKAH)

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I also want to extend my acknowledgement to all personnel in the Northern Cape Department of Education, especially to all the principals and SGB's of all the schools in the District Departments of education, who allowed some of their learners to serve as participants for this study. This research study would have been fruitless without your assistance.

DEDICATION

This research work is dedicated to all my past and present students, and also, to all my past and present teachers and lecturers. My interaction and association with them is what keeps me elevating academically and professionally.

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LIST OF ABBREVIATIONS

AACU – Association of American Colleges and Universities

ANA – Annual National Assessment

CAPs - Curriculum and Assessment Policy Statement

CE- Class Exercises

DBE- Department of Basic Education

DOE - Department of Education

EIT- End-of-Intervention Test

FET - Further Education and Training

GET - General Education and Training

GSP - Geometer’s Sketch Pads

NCS – National Curriculum Statement

NCTM- National Council of Teachers of Mathematics

NSC – National Senior Certificate

NOCSR - Naturalistic Observation Case Study Research Design

PPSH – Polya Problem Solving Hypothesis

PSSS – Problem Solving Strategy Steps

TIMSS – Trends in Mathematics and Science Study

TLMs - Teaching and Learning Materials

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CHAPTER ONE

BACKGROUND OF THE STUDY

1.0 INTRODUCTION

This study investigates learners' problem solving skills in circle geometry. The trend of poor learners' performance in mathematics in South African Schools informs the need for this study (Department of Basic Education Diagnostic report, 2017). I am the researcher, I am a mathematics teacher and I am also a marker of the mathematics matric examinations. I teach grade 12 mathematics in my school. I know how badly learners in my school mystify mathematics as a subject. My experiences as a grade 12 mathematics teacher reveals how difficult it is to teach mathematics in grade 12, more especially the aspect of Euclidean geometry.

My classroom mathematics teaching experience makes me not to wonder how badly the learners answer the questions in the Euclidean geometry aspect of matric examinations. I, anecdotally, found a sustainable link between the classroom learners' reluctance in the learning of Euclidean Geometry aspect of mathematics and their answers approach to Euclidean geometry in the matric examinations. The DBE (2015) diagnostic report also informs that learners' lack good problem solving skills in Euclidean Geometry.

As a result of this matric examination insight, the researcher is inspired to interrogate deeply into the cause of poor learners' problem solving approaches in Euclidean Geometry, while the main study focus will be circle geometry. Circle geometry was chosen because the learning of circle geometry comprehensively entails almost every concept in Euclidean geometry: abstract thinking, connecting abstract thinking to physical representations and applications.

Problem solving, according to the National Council of Teachers of Mathematics (NCTM), is an important part of mathematics learning, and should not be an isolated part of the mathematics curriculum. It demands students to apply their existing knowledge to solving a problem in which the answer to the problem is not known in advance. Making efforts to solve problems stimulates mathematical reasoning and understanding, which can lead to new mathematical knowledge. The extent that a student can analyse and interpret a mathematical problem, in conjunction with logical and deductive proofs of mathematical concepts can determine the extent that a student can solve a problem convincingly and acceptably.

As an appointed marker for the DBE National Senior Certificate (NSC) examinations for mathematics paper 2 for the past five consecutive years, I greatly marvel at the number of pages I have to skip because the candidates normally write the question numbers of Euclidean Geometry questions in their answer booklet, leaving the pages blank. Other candidates also end up providing irrelevant answers to the questions of Euclidean Geometry, with just a handful of candidates giving proper responses to the Euclidean Geometry questions.

My classroom teaching experiences corroborate what I see in the matric examination marking. Anytime I begin to solve level 3 (complex procedure questions) and level 4 (problem solving questions), as rated by bloom's taxonomy, in class with my learners the most common reactions I do receive from my learners include: "eish" which in literal terms, is used to express dismay or discomfort; I wish mathematics questions demands for only "Yes "or "No" responses; I wish mathematics questions demands for "True" or "False" responses. Is it to say that circle geometry concepts in Euclidean Geometry is a monster in the Curriculum of Mathematics or not an interesting content to be studied?

Learners' poor problem solving skills in geometry might have greatly contributed to their high failure rates in mathematics, which eventually affect the placement and admission of learners into their preferred, respective courses at the tertiary institutions since mathematics is considered to be one of the very important subjects required for admissions into the major university courses such as: Medicine, Accounting, Actuarial Science, Physical Sciences and careers in the built environment (engineering, architecture, building construction, et cetera). Without a very good pass in mathematics, the chances of a candidate being admitted into the above listed University courses becomes very limited or impossible.

The teaching of Euclidean Geometry needs to be taught properly in schools to build the learners conceptual understanding and confidence in this unfortunately difficult-to-learn area of mathematics, but there are no enough professionally qualified mathematics teachers in our schools (Mosses & Mji, 2006). Perhaps, the low teacher-learners ratio might explain why non-mathematics specialist teachers (like economics, chemistry, geography, and et cetera teachers) are made to be teaching mathematics in South African schools. Qualified teachers' attrition might have been informed by documentation-deluged, mountain-sized portfolios coupled with the recent wave of school crimes.

However, at the inception of democratic governance in South Africa, efforts were made to build a strong education foundation. The complete dismantling and reorganization of the national education system, through the establishment of new national and provincial education departments after a series of demonstrations by South Africa's black populace after the country's first democratic elections in 1994, which also purged the apartheid curriculum of its offensive racial content paved the way for the essential alterations to school subject matter to be made. Particular attention was paid to obtaining quality, equity and efficient education system on the basis of developing an egalitarian foundation of education for South Africa's citizenry (White paper on Education, 1995).

In her article "Third International Mathematics and Science Study-Repeat (TIMSS – R): What has changed in South African pupils' performance in mathematics between 1995 -1998?". Howie (2003) concludes with a prescient remark about the state of South Africa's education system:

"Now that access to education and the right to learn has been established for the majority in the country, it is time to set key priorities for the country's future. If South Africa is to succeed in a rapidly changing and competitive technological world, it will need to develop and protect its capacity to produce well-qualified teachers. In order to improve the lack of change detected by TIMSS-R from developing further, resources have to be put into a variety of well- designed, planned and effective programmes promoting and implementing mathematics and science. Greater collaboration within and between government and the private sector will be required to optimise energies and resources."

In addition, over the years, South Africa has had a number of factors which have detrimentally influenced effective curriculum practice and the mathematics curriculum is no exception. One of these factors is incessant unexpected curriculum changes, which have been dominated by aimless trial-and-error curriculum practices, with interpretive murkiness which existed, which also represented an apogee of curriculum debates and the value of such curricula remained contentious and extremely unpleasant. The Outcome Based Education (OBE) and the National Curriculum Statement (NCS) which were respectively discarded, can be mentioned as examples in this regard (Carl, 2012; Berlach, 2004).

The mathematics curriculum went through a series of metamorphosis which eventually came to the conclusion that South Africa's mathematics curriculum misses the needed spice to cater for the mathematical needs of its citizenry, hence, a meritorious attempt to redress the vacuum in the mathematics curriculum created the birth of Euclidean Geometry. This gives the mathematics curriculum a raison d'etre. Though as it was introduced, the teachers however, hung their heads in despondency, as they were not well abreast with the concept of Euclidean Geometry, they latter accepted that its introduction is for a good course and a reasonable adventure.

Moreover, most pedagogues of mathematics, especially at the grade 12 level contend that Euclidean Geometry is a concept introduced into the curriculum not long ago. A lack of resonance exists between curriculum expectations, teachers' content knowledge and pedagogical competences in Euclidean Geometry. The teachers' content knowledge appears to be out of sync with their pedagogical competences. More Mathematics Educators may have good knowledge of the content to be taught but employing the appropriate pedagogical presentation skills might be a problem. While some educators might also have developed good approaches at teaching this concept, there might be lapses in their content knowledge, hence they teach the areas of the content they know and jump the areas they encounter difficulties in, to the disadvantage of learners.

Internationally, South Africa is gaining recognition for lacking behind in the field of Mathematics and their low efforts in Mathematics Education becomes evident when they participate in TIMSS (Trends in Mathematics and Science Study) where South Africa is mostly ranked last or almost last, when the scores of their participating learners are compared with the scores of learners of other participating countries. Not only do South Africa fall behind hierarchically in the TIMSS examinations ,but also all other factors associated with the teaching and learning of mathematics such as the environment for teaching and learning , tools (equipment and materials) needed and noticeably the provision of well trained personnel for mathematics instructions, were not adequately provided.

The National School Certificate (NSC) examination for mathematics is partitioned into two papers: paper 1 and paper 2. The duration for each paper is three hours. Paper 1 covers the following broad topics: Algebra, Equations and Inequalities; Patterns & Sequence; Finance, Growth and Decay; Functions and Graphs; Differential Calculus and Probability, while paper 2 constitute the following topics: Trigonometry, Statistics, Analytical Geometry, and Euclidean Geometry.

Euclidean geometry is a mathematical system attributed to Euclid, a Greek mathematician. Euclid's methods consists of assuming a set of intuitive axioms and deducing many other theorems (propositions) by constructive proofs involving Space and shapes: one dimensional object, two dimensional objects and three dimensional objects, (coxeter, 1961). One of the prominent use of Euclidean Geometry is in optics. It is also notably used extensively in art and architecture.

Euclidean Geometry is an integral part of South Africa's mathematics curriculum as spelt out in the Curriculum and Assessment Policy Statement (CAPs) document , which is currently in use across all grades of its educational hierarchy, (that is, from grades R-12). Perhaps, this was introduced as a thinking tool which may help the development of deductive and logical mathematical thinking, as well as strengthening the interpretation and evaluation of mathematical arguments. According to the curriculum, learners are required to critically make an analysis of geometric figures, measurements and deductive proofs, to recognize visually relevant geometrical properties, et cetera, which sharpens students' mathematical thinking skills (Driscoll, 2010).

1.1 CLASSROOM TEACHING OF MATHEMATICS (FROM PERSONAL PERSPECTIVES)

Teaching mathematics is a difficult endeavour, which requires adequate planning with regards to the content to be taught, and more importantly, adapting the desired pedagogical presentation skills (NCTM, 2000). I am an ambassador of the constructivists' teaching and learning approach. In my classroom teaching of mathematics, presenting my lesson in a sequential and logical order to promote understanding of the content to avoid ambiguity and misconception of the content is my utmost priority.

As mentioned above, learners mystify Euclidean geometry. Most learners contend that the sight of the diagrammatical representations of Euclidean Geometry makes them to panic and hence makes them to lose focus and their interest in answering the questions fade away. Most learners end up writing the questions they are to solve as solutions to the questions and hence, they end up not scoring any mark. The responses of the learners (which are always very few), who attempt to solve the questions are not encouraging, with the very few of learners getting the solutions to the questions right. I spend a lot of time in class, motivating and encouraging my learners not to be dismayed at the diagrams. Instead, they must channel their energies into learning how to interpret those diagrams.

In addition, as a marker for the NSC mathematics examinations, I found a positive correlation between learners' classroom attitudes during the teaching of Euclidean Geometry and the general learners' performance in the NSC mathematics examinations. The reader should note that I am not particularly referring to the NSC performance of the learners that I taught, this is my general observation from the NSC scripts that I have marked. French (2009) informs that learners' performance in geometry gives an idea of how the learners will perform in the other areas of mathematics.

The figure below shows the performance trends of candidates for the NSC mathematics examinations for the past four consecutive years (2014- 2017), as indicated in the diagnostic report of 2017, by the DBE:

Figure 1.1: Overall Achievement Rates in Mathematics

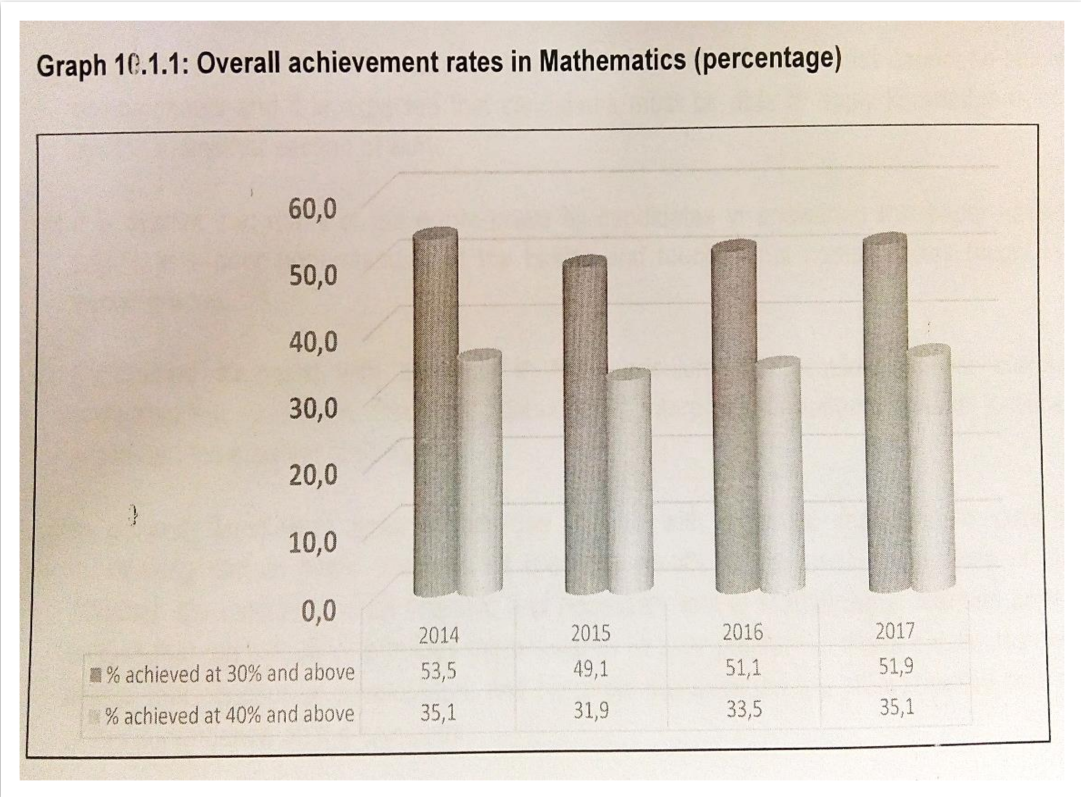


Figure 1.2 Diagnostic Question analysis for paper 2 (2015)

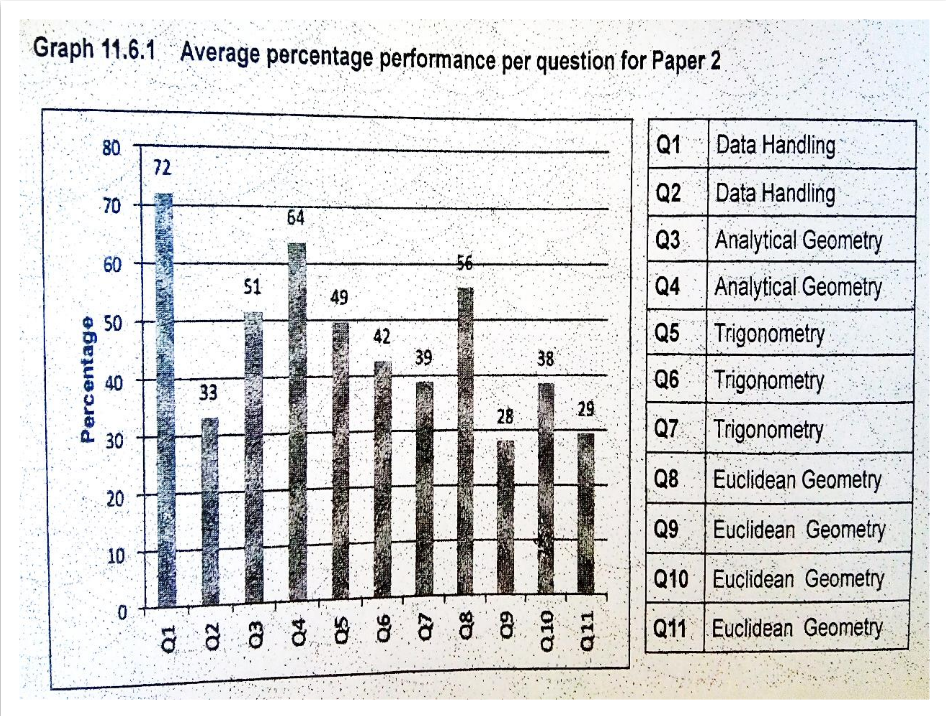


Figure 1.3 Diagnostic Question analysis for paper 2 (2016)

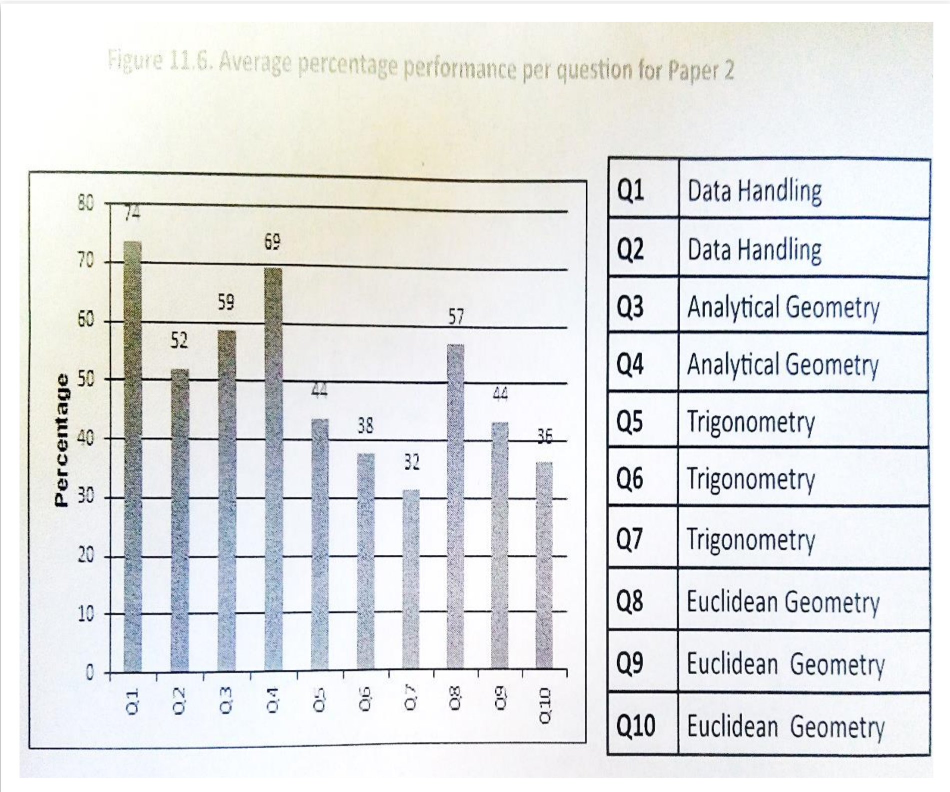


Figure 1.4 Diagnostic Question Analysis for paper 2 (2017)

Figure 10.6.1 Average percentage performance per question for Paper 2

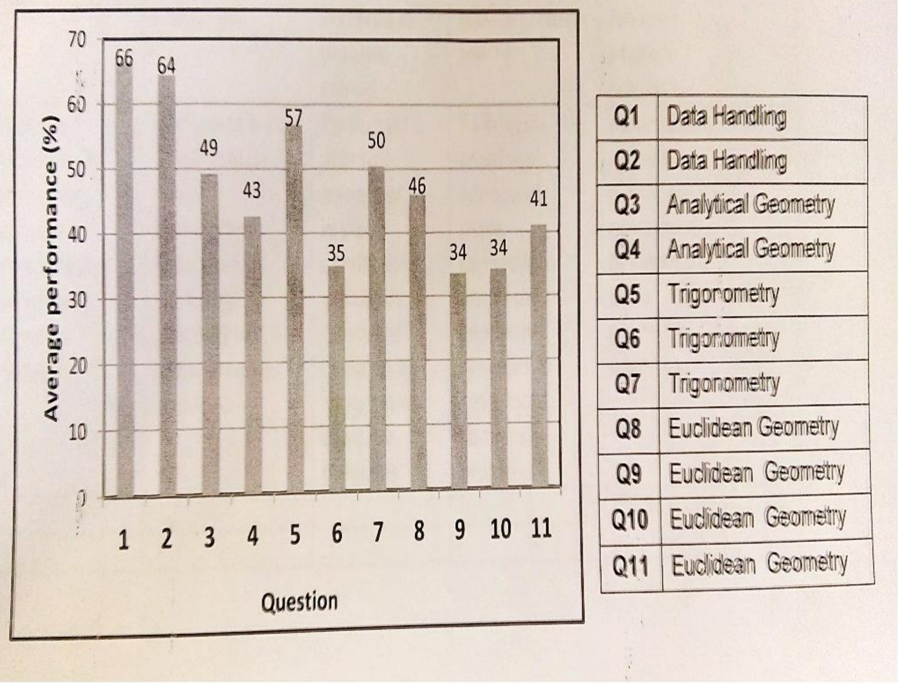


Figure 1.1 above gives the comparison in learners' performance pass rate when it is calculated from 30% and above, and when it is calculated from 40% and above , from 2014-2017. The pass rate generally went down when 40% pass was applied. Table 1.2 (page 12) shows that Euclidean Geometry has the highest mark allocation in paper 2 (50 marks), while figure 1.2 to figure 1.4 give the learners' performance in each area of mathematics. It will be observed that for the consecutive years of 2015 to 2017, learners performance in Euclidean Geometry were the lowest.

Since Euclidean Geometry carries the highest mark allocation in mathematics paper 2 coupled with the findings of French (2004) which shows that learners' conceptual ability in other areas of mathematics depends on their ability in geometry, may be the major factor why learners are failing mathematics at the NSC examinations. Hence, the researcher consider this study to be very important part of the efforts to improving learners' performance in Mathematics in South African Schools.

1.2 THE TEACHING OF EUCLIDEAN GEOMETRY IN SOUTH AFRICA

The teaching of Euclidean geometry in South Africa is not without some challenges, some of which are enumerated below.

1.2.1 HISTORY OF EUCLIDEAN GEOMETRY IN SOUTH AFRICA

From 1994 until 2007, the mathematics curriculum in South Africa was partitioned into two: the standard grade and higher grade. Learners were required to do either standard grade mathematics or higher grade mathematics. This was in use from 1994, until 2007 when the last batch of learners wrote the national examination under this curriculum. During this period Euclidean geometry was a compulsory content to be learnt by all mathematics learners in South Africa's mathematics curriculum. However, in 2006 at the inception of the NCS, it was removed from South Africa's mathematics curriculum as a compulsory content. From 2006, learners in grade 10 to grade 12 learnt Euclidean geometry, optionally. It was voluntarily written as paper 3, which continued until July 2014, when the structure of the grade 12 matric examination was amended, which made the optional paper 3 obsolete.

In the year 2010, the universities raised concerns about the inability of students pursuing engineering and other mathematics related programmes to cope because they did not do Euclidean geometry at high school, which is a pre-requisite for those university modules (Jansen & Dardagan, 2014). The introduction of the Curriculum and Assessment Policy statement (CAPs) curriculum in 2012, directed Euclidean geometry to be offered by all mathematics learners from grade 10 to grade 12, when the new structure of the grade 12 matric examinations (paper 1 and paper 2) began. Hence, from 2014 the grade 12 mathematics examination was structured into two papers: paper 1 and paper 2. Euclidean geometry which used to be in the optional paper 3, was made to be part of paper 2, to be written by all mathematics learners compulsorily, which was written for the first time in 2014, until now.

1.2.2 GEOMETRY CURRICULUM IN SOUTH AFRICAN SCHOOLS UNDER THE CAPs CURRICULUM

1.2.2.1 GEOMETRY CURRICULUM OVERVIEW

The underpinning concepts of Euclidean geometry in the earlier grades from grade R to grade 9, as the foundation to the FET band is categorized into two main sub-areas: “space and shape” geometry and “measurement”. The concept of Euclidean geometry at the FET band, especially in grade 12, is a combination of all the concepts: measurement and space geometry as outlined in the lower grades. A deep understanding of shapes, geometric figures, angles and line geometry as progressed from the early grades through to grade 10 is required to comprehend the concepts in grade 11 and grade 12, as elaborated below:

Table 1.1 **:** Geometry Curriculum

GRADES	**MEASUREMENT (CAPS OVERVIEW)**	**SPACE AND SHAPE GEOMETRY (CAPS OVERVIEW)**
7	1. Areas and perimeters of 2D shapes 2. Surface area and volume of 3D objects	1. Geometry of 2D shapes 2. Geometry of 3D objects 3. Geometry of straight lines 4.Transformation geometry 5. Construction of geometric figures
8	1. Areas and perimeters of 2D shapes 2. Surface area and volume of 3D objects 3. The theorem of Pythagoras	1. Geometry of 2D shapes 2. Geometry of 3D objects 3. Geometry of straight lines 4.Transformation geometry 5. Construction of geometric figures
9	1. Areas and perimeters of 2D shapes 2. Surface area and volume of 3D objects 3. The theorem of Pythagoras	1. Geometry of 2D shapes 2. Geometry of 3D objects 3. Geometry of straight lines 4.Transformation geometry 5. Construction of

		geometric figures
10	1.Properties of quadrilateral: kite, rectangle, rhombus, parallelogram, square and trapezium 2. Solve problems and prove riders using the properties of parallel lines, triangles and quadrilaterals 3. The midpoint theorem	1. Revise basic results established in earlier grades regarding lines, angles and triangles, especially the similarity and congruence of shapes. 2. Revise the volume and surface areas of right prisms, cylinders, spheres, right pyramids and right cones
11	1. Revision of properties of quadrilaterals	1.CIRCLE GEOMETRY
12		1. Conditions for polygons to be similar 2. Similarity and proportionality of shapes 3.Pythagoras theorem by similar triangles

As shown in the table above, circle geometry is a topic to be taught in grade 11 as indicated in the CAPs mathematics curriculum.

Table 1.2 Mathematics Paper 2 Marks composition

TOPICS	MARKS COMPOSITION
1. STATISTICS	20
2. ANALYTICAL GEOMETRY	40
3. TRIGONOMETRY	40
4. EUCLIDEAN GEOMETRY	50

Euclidean Geometry covers about 50 marks out of a total of 150 marks, which is about one-third of the total marks allocation for paper 2 of the National Senior Certificate grade 12 mathematics examination. This implies that Euclidean Geometry is one of the very important aspects of the content of the mathematics curriculum. However, the marks that mathematics candidates at the NSC examinations obtain from Euclidean Geometry questions, particularly questions from circle geometry concepts are relatively low and fall below expectations as indicated in the diagnostic question analysis for paper 2, for 2015, 2016 and 2017 respectively as shown in subsection

1.1 above.

An overview of the diagnostic report from the Department of Basic Education over the years, as presented below, shows how learners are performing poorly with regards to questions related to Euclidean Geometry in general and circle geometry in particular, juxtaposed with other questions from other concepts. From the diagnostic report by the Department of Basic Education (DOE, 2015) on questions related to circle geometry concepts in Euclidean Geometry that is questions 8-11, it was revealed that Euclidean Geometry questions really poses much treat to students' success in mathematics as shown below:

Table 1.3**:** DOE Diagnostic report, 2015

Euclidean Geometry questions	**Comments on learners performance**
8	Many candidates answered this question fairly well
9	This question was poorly answered by the majority of candidates
10	Performance in question was fair
11	This question was very poorly answered

How learners respond to questions in the Euclidean Geometry aspect of the NSC mathematics examination determine their performance in the other various aspects as shown in figures 1.2-1.4 in subsection 1.1 above. Hence the focus of this study is learners' problem solving skills in Euclidean Geometry.

1.3.0 PROBLEM SOLVING

The Association of American Colleges and Universities (AACU), defined problem solving as the process of designing, evaluating and implementing a strategy to achieve a desired goal (AACU, 2009). It provides opportunities for support and reflection on both the content learnt and the learning process and encourages the testing of ideas among alternative viewpoints. To corroborate the above, Smith and

Stepelman (2010), concluded from the research they conducted that Problem solving becomes much effective if: the necessary important contents are covered, useful mathematical techniques are developed and sufficiently practised, and classes of problems achieve coherence so that the associated concepts and relationships can be constituted at an abstract level.

1.3.1 PROBLEM SOLVING STRATEGY

Davidson, Deuser, and Sternberg (2012), defined a problem solving strategy as the ability to identify the nature of a problem, deconstruct it (break it down) and develop an effective set of actions to address the challenges related to it. In view of this, Cooper (2017) informs that problem solving strategy in geometry may include making a diagram, intelligent, guessing and testing strategy, brainstorming, Using known information, simplifying the problem. The teacher should not be afraid of problems for learners to learn from him/her and that learners may become expert problem solvers if they get lots of practice in solving problems.

1.3.2 RECOGNISED PROBLEM SOLVING STRATEGIES

A lot of research studies in mathematics education elaborated on mathematics problem solving strategies in general. Some of the problem solving work are discussed below:

(1) <u>Polya's (1945) problem solving model</u>

Polya's work about how a mathematics problem can be approached and solved has gained much recognition globally and his problem solving procedure is used in a lot of fields in Mathematics. Polya (1945) suggested a four-step heuristic approach that is useful to solve mathematical problems, which requires problem solvers to: Understand the problem, Devise a plan, Carry out the plan, Look back and review the solution.

(2) <u>The Van Hiele Instructional model</u>

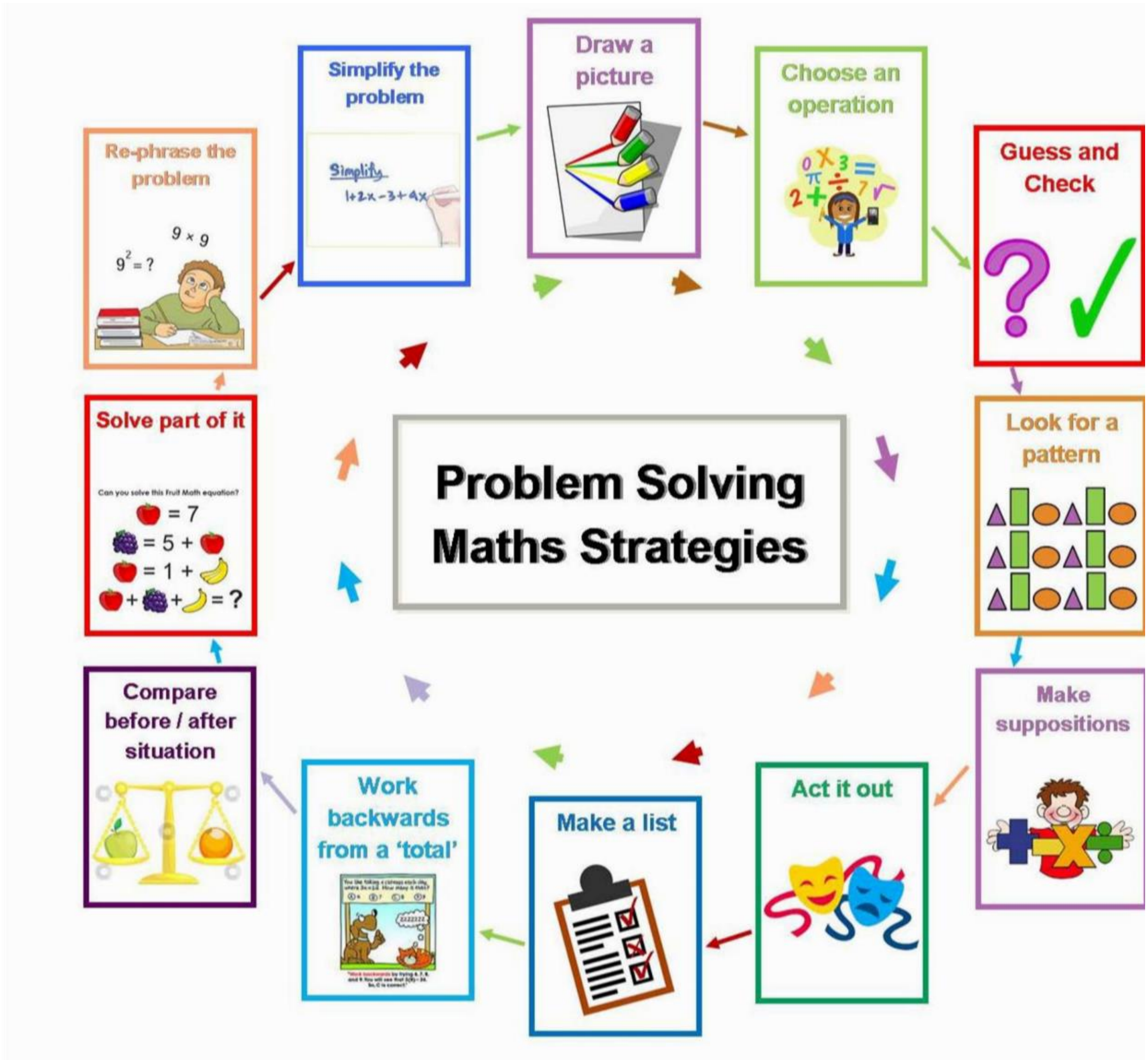
This strategy for solving geometry problems provides a focus on the student's levels of thinking in geometry and the role of formal instruction in helping students move from one level of thought to the next, hence van Hiele proposed the instructional approach which may help the geometry learners in learning and problem solving. It also gives a clear description of how geometry learners think, the nature of geometric ideas learners think about and highlights the differences in learners' geometrical concepts, and geometric thinking and how these differences came to be. The instructional model consists of five levels of thought in geometry. "Level 0" according to Van Hiele (1995) is **visualization**, "Level 1" is **analysis**, "Level 2" of geometric thought is **informal deduction,** "Level 3" of thought is **Deduction**, and **Rigor** is "level 4", according to Van Hiele. (Oladosu, 2014).

(3) Hierarchical Problem solving strategy

Hierarchical problem solving strategy emphasize the use of abstraction approach in problem solving. This involve breaking up the problem into smaller sub-problems (Knoblock, 1990). The problem is first solved in abstract space and the abstract solution is used to guide the successive solution space in hierarchy form.

Singapore adopted and adapted this problem solving strategy and adapted it to be from concrete representations to pictorial representations, before abstract aspect of mathematical ideas can be presented. This is the foundation of the problem solving strategies employed in Singapore's mathematics classrooms. A pictorial view of some of Singapore's problem solving strategies is illustrated below:

Figure 1.5 Hierarchical problem solving approach adopted by Singapore



1.3.3 HOW PIAGET'S PERSPECTIVE OF LEARNING INCORPORATES PROBLEM SOLVING

Piaget envisaged that people are born with the tendency to organize their thinking processes into psychological structures and these psychological structures are our systems for understanding and interacting with the world, and described learning as the passive formation of associations. Piaget emphasize schemas, as the building blocks of thinking which aids in mental representation of mathematical objects and concepts (Woolfolk, 2014). Problem solving, like Piaget's perspective considers a student's prior /existing knowledge as paramount for learning new mathematical concepts and new mathematical knowledge can only be learnt by fitting them into existing schemas (assimilation) and existing schemas need to be altered or new ones created in response to new information (accommodation) and the student assumes a passive role in the learning process (Woolfolk, 2014).

1.3.4 HOW VYGOTSKY'S PERSPECTIVE OF LEARNING INCORPORATES PROBLEM SOLVING

The zone of proximal development, according to Vygotsky is the area between the child's current level of development and the level of development the child can achieve. It also represents the phase at which a child can master a task if given appropriate help and envisages learning as an active process. Juxtaposing problem solving to Vygotsky's zone of proximal development, this may imply that, problem solving relies on what the learner knows with the help of more knowledgeable peers, leading to better mathematical conceptual understanding and structural formation. The problem solving process also exhibits a region between what the problem solver knows and the level the problem solver can achieve (Woolfolk, 2014).

1.3.5 HOW THE SITUATED PERSPECTIVE OF LEARNING INCORPORATES PROBLEM SOLVING

The situated perspective links learning to real life situations in context, and knowledge is situated and constructed in new ways with individual cognitive development, conceived of as a new pathway through which students gradually increase knowledge and competence. The situated perspective approach characterizes a student's development as a social learning experience and process which gradually increases with the desire to participate and their feelings of acceptance and belonging in the situation (environment). Problem solving like the situated perspective is linked to real life contexts and requires the problem solver to construct his/her existing knowledge about the situation or context intuitively and conceptually in a bid to be able to present his/her existing knowledge in a new dimension to suit current purpose and from which predictions about parameters can be made, which are characteristics of problem solving inherent in the situated perspective. (Woolfolk, 2014).

1.3.6.0 TEACHERS' ROLE IN PROBLEM SOLVING

One of the most relevant roles of the teacher in a problem solving situation is to choose appropriate tasks/ problems to be solved and to sequence the problems in the right order within their mathematical structures. The teacher, assumes the responsibility as a role model within the scene of the learning environment by solving problems himself/herself so that learners will be able to know what is expected of them in the problem solving process and to teach a variety of problem solving strategies to learners for learners to learn from them and to encourage learners to think divergently, explain what they are thinking , share strategies, think of other ways the same problem could be asked / answered and to discover different problems that can be solved using the same approach.

The teacher is also expected to choose problems that required a considerable amount of time for students to be able to think through the solution, to give a variety of problems, give similar or the same problems in different ways and more importantly, make learners to articulate their own problem solving process. In a problem solving classroom, the teacher may offer problem solving heuristics to learners and may give suggestions (not answers) to learners in the problem solving process, especially, when the learners appears to be frustrated at getting solutions to the circle geometry problems as they appear to have ran out of ideas.

In addition, the teacher is expected to help learners identify concepts and misconceptions, and particularly, to help learners identify their own problem solving errors. The teacher is also expected to provide encouragement and appreciation in the efforts posed by learners in the problem solving process and the teacher is also expected to make learners aware that the process of obtaining the solution is much important than the answer in the learning situation (Smith & Stepelman, 2010).

1.3.6.1 CHARACTERISTICS OF TEACHERS IN PROBLEM SOLVING

Teachers as pioneers and ambassadors of the problem-solving approach must bear the following characteristics to promote the efficacy of this paradigm in the classroom:

1. The teacher must be well vested in pedagogical and didactical skills, problem-solving strategies and skills, the curriculum, mathematics epistemology and particularly well vested in the content knowledge of mathematics.

A strong teacher efficacy of all these factors will ensure that the learner obtains the right guidance and pedagogy at the needed time and also ensures that the sequencing of problems to be solved do not fall out of line with the envisaged mathematics epistemology and within the cognitive level of the learner, after all the main aim of learning is to add value to the society using the acquired mathematical ideas.

2. The teacher must be knowledgeable about the characteristics and nature of learners, learners' behaviour and more importantly, how learners learn mathematics.

This would assist the teacher to adapt the best mathematical practices for the class and guide the teacher to know the nature and level of problem solving tasks to be introduced to each group of learners. Whether the tasks must be accelerated (for talented learner groups) or must be task-synthesised (slow learner groups) or a group of learners who need special attention because they have unique challenges all fall within the domain of the ability of the teacher to be well vested in learners behaviour, nature of learners and how learners learn.

3. The teacher must be able to assume the role of a role model

The teacher must be able to solve problems with mastery and uniquely using a variety of strategies in an interesting manner which attracts learners to learn them and evokes their quest and curiosity to learn more and to perform problem solving like the teacher since some learners learn best through observation and practice. This will guide learners to develop their competences in problem solving. Learners will learn and would love to do things, the way the teacher does them. Learners would want to imitate the actions of the teacher and by so doing, helps learners to learn the problem solving strategies better.

4. The teacher must possess the attribute of a strong motivator

For effective delivery of the problem solving lessons which is recognised to be a time consuming mode of learning, learners need to be regularly motivated by encouraging learners not to give up when the solutions to the problems are becoming difficult to obtain and also the teacher can also motivate learners by appreciating every constructive effort the learner puts up in the problem solving process to help learners know that they are on the right track of getting the solution and also to help learners have the believe that they can make it.

1.3.7 LEARNERS' ROLE IN PROBLEM SOLVING

The learner is expected to perceive the teacher as his/her mentor (role model). From this the learner is expected to observe, imitate and needs to have the quest of developing his/her problem solving skills to be as good as the teacher in problem solving situations. Learners are expected to abide by the guidance of the teacher and to try as hard as possible to get the expected solution to the problem and that they need to try, try and try again in their quest for getting an acceptable solution to the problems. The learner is expected to understand that problem solving is a time consuming process and that adequate skills ,techniques and strategies are required in the process and also the process of the problem-solving is much important than the product (the solution).(Smith & Stepelman, 2010).

1.3.8.0 MATERIALS USED IN PROBLEM SOLVING

Students demonstrate conceptual understanding in mathematics problem solving when they use diagrams and instructional materials to demonstrate solution path (Rose & Arline 2009). The diagrams aid students to be able to interpret the problem better. The mental representations and images of the diagrams enable the problem solver to be able to make connections across different directions of thought.

The nature of the problem to be solved dictates the type of instructional materials that may be needed in solving the problem in the classroom. For investigation and formulation of patterns and relationships: concrete objects, manipulative objects, geometers sketchpad, ruler, compass, protractor, T-square ,set square, drawing board, graph board/sheet ,et cetera may be used. For establishing number concepts: manipulative objects, abacus, created tables for arranging constructed data, et cetera may be used. Cognitively, mind pictures (visual images) may be very helpful in the problem solving process.

1.3.8.1 SELECTION AND SEQUENCING OF PROBLEMS FOR PROBLEM SOLVING

Good mathematics problems give students the chance to solidify and extend what they know and, when well chosen, can stimulate mathematics learning, according to NCTM's standards for problem solving (NCTM, 2000). The selection and sequencing of problems is dependent on the nature of the content and the nature of learners in the classroom: level of mastery of the basic mathematical concepts and learners' environment. When problems are selected appropriately and sequenced in the right order, learners will be able to depend on their existing knowledge to make generalisations, inventions, to make sense and assign meanings and more importantly, helps them to interact mathematically.

The tasks/problems to be selected must encourage learners to think divergently, explain how they are thinking, think of other ways that same problem could be solved, must be from the learners mathematical structures, well-planned, and must fall in line within the appropriate cognitive level of the learners. A learner's cognitive abilities may favour investigation problems, word problems, formulation of patterns and relationships or abstract construction but weak in other problem areas, thus, if necessary the learners must be allowed to master these areas that best fits their mathematical structures and cognitive abilities; like a footballer played out of position and being criticized of underperforming (Cuoco, 2000).

1.4 THEORETICAL CONSIDERATION FOR THE STUDY

This study is underpin by the Polya problem Solving Hypothesis (PPSH). The hypothesis states that: "in problem solving, there are four steps to be taken. (1) to understand the problem ; (2) plan for completion; (3) solve the problem according to plan; and (4) to re-examine to ensure all steps are done", Polya (1985) in Mushlihuh and Sugeng (2017). PPSH emphasize that the mathematics problem solver should first understand the problem to be solved, then device a plan on how the problem may be solved, proceed in solving the problem according to the stated plan and lastly, evaluate all the steps to ensure that all the steps are done.

Polya problem solving hypothesis was employed in this study in view of the constructivism learning theory. Lee (2009) informs that constructivism theory provides learners' centre learning. The theory is about making learners to be actively involved in their learning by creating, interpreting and reorganising new knowledge individually. Hence, the researcher believe that coupling Polya's mathematics problem solving hypothesis with constructivism learning theory will facilitate the study participants' construction of mathematics problem solving skills.

The researcher noticed that the proposed theories are best suitable for problem solving in geometry concepts, especially in circle geometry. A typical circle geometry problem will require the problem solver to first read and comprehend the nature of the problem and the associated diagram, critically think of the plan to solve the problem which might involve interpretations of relevant theorems and proves, use the theorems and proves to step-wisely solve the problem and lastly, go over the proposed solution to ensure that appropriate theorems and proves are applied and calculations are done correctly. All these are better achieved where there is peer-to-peer interaction as it is when learners sit in groups and discuss during knowledge construction.

1.5 OBJECTIVE OF THE STUDY

Literatures and yearly results of the matric examinations in mathematics show that learners always perform poorly in geometry aspect of mathematics examinations in South Africa. Thus, this research is to investigate learners problem-solving skills in mathematics with a focus on circle geometry in one of the secondary schools in South Africa, this was coupled with introducing Polya problem-solving teaching approach. The objective of these approaches is to improve the learners' problem-solving skills in the teaching and learning of circle geometry, improved learning indicators or learning deteriorating indicators like improved interest in the learning of circle geometry, good classroom dynamics during the teaching and learning circle geometry and so on will be looked out for in the results of the data analysis.

1.6 SIGNIFICANCE OF THE STUDY

The envisaged significance for conducting this research include contributing to circle geometry teaching and learning by giving learners and the pedagogues of mathematics adequate information about the problem solving strategies and techniques that may improve the teaching and learning of Euclidean geometry, especially circle geometry, based on the findings emanating from this study.

The findings of the study can serve as a reference tool for policy makers in general and Mathematics curriculum developers in particular for future effective curriculum planning and implementation of the content part of Euclidean Geometry to eradicate current problems associated with it.

1.7 THE PROBLEM OF THE STUDY

The problem of this study is to investigate the learners' solution approach, their thinking processes and solution steps in giving answers to questions in circle geometry during and after the intervention in an attempt to explore the study participants' problem-solving skills in circle geometry. The researcher hope to achieve these by finding answers to the research questions below:

(1) How does the intervention influence the study participants' conceptual understanding of circle geometry?
(2) How does the intervention influence the study participants' problem solving skills in the learning of circle geometry?

(3) How does the intervention influence the study participants' performance in the learning of the concepts of circle geometry?

1.8 DEFINITIONS OF KEY TERMS, CONCEPTS AND VARIABLES

The following terms are used throughout this research report and they are defined here for readers' of this dissertation to concisely understand them:

Shapes- Mathematics geometric forms such as a rectangle, triangle, a cone, cube, et cetera.

Space- A set of points that have geometric properties which are governed by axioms, such as Euclidean space.

Measurement- The size, length, quantity or rate of something that has been measured.

Geometry- The branch of mathematics that is concerned with the properties and relationships of points, lines, angles, curves, surfaces and solids.

Circle geometry – A branch of Euclidean geometry which deals with the application of important properties (theorems) of the circle for problem solving.

Euclidean geometry- Geometry according to the principles of Euclid, as described in his Elements.

Problem Solving- The process of working through details of a problem to reach a solution.

Problem Solving Strategies- Methods and procedures that can be employed in problem solving situations.

1.9 ORGANISATION OF THE DISSERTATION

This dissertation is subdivided into six distinct chapters. The first chapter provides the details about the need to conduct the study, the research problem, how the dissertation is organized. All the parameters detailed in chapter one serves as the background of the research under investigation, and also, serve as the focus of the research. Finally, the chapter gives explanations of key terms about the topic under investigation.

The second chapter is about literature review, which informs about past similar research studies conducted. The third chapter gives details about the methodology employed in conducting the research: the research design; data collection techniques, issues of reliability and validity, as well as the sampling techniques adapted for choosing the needed participants and the data analysis techniques employed in the study. Presentation of Data Analysis results are discussed under chapter four. Chapter five was about the presentation of research findings and discussions. The conclusions and recommendations from this research study are as well, discussed in chapter five.

CHAPTER TWO
LITERATURE REVIEW

This chapter discusses the theories in which mathematics problem solving is located. Various theories that span mathematics problem-solving were discussed and emerging teaching strategies about it are presented. In addition, literatures that are relevant to this study are summarised.

2.1 Theoretical Framework

Mathematics is a subject which seeks to understand patterns that permeate both world around us and within us (Grouws, 1992). This is based on rules that mathematics students must understand. For example, the concept of addition, multiplication and division are basic arithmetic concepts that follows certain rules and patterns. Mathematics students right from the primary school are expected to have sound grip on these rules and patterns. The main focus of every mathematics instruction is to find solutions to the given mathematics problems by using some mathematics rules and exploring patterns that might facilitate the resolution of the given problems.

Perhaps it was in view of the above that Baykul & Tertemiz (2004) and Decorte, (2004) in Vasif, Ünal, Hasan, Ali and Ayça (2015) describe the learning of mathematics as acquiring mathematics knowledge and skills which are important to mathematics problem solving. This implies that acquisition of the requisite mathematical knowledge involves being grounded in the mathematical rules and skills, having the technical-know-how to explore appropriate patterns for a specific mathematics problem. Mason, Stacy and Burton, (2015) referred to this as mathematics thinking. Mathematics thinking is a complex activity which involves specializing, conjecturing and generalisation, (Kashefi, Ismail & Yusof, 2013). Mental complex activities such as specializing, conjecturing and generalisation are manifestations of students' cognitive abilities.

Students' ability in problem solving depends on their level of cognitive ability to be able to think as they explore solution to mathematics problems. Many education researchers (Piaget, 1980; Vygostsky 1978; Bandura, 1977; Fox, 2001) have studied students' cognitive ability in learning. Piaget (1980) informs that students acquire cognitive abilities through assimilation, accommodation and equilibrium. He explained that assimilation refer to making connections between new knowledge and the existing knowledge (prior knowledge), accommodation is the process of adapting the cognitive structure to the new structure of knowledge, while equilibrium is being open to new knowledge that bring growth. Piaget's cognitive theory formed the bases for the theory of constructivism.

Many scholars (Confrey, 1990; Dewey, 1916; Piaget, 1970; Vygostsky 1978; Bandura, 1977; Fox, 2001) have studied the theory of constructivism. Constructivism learning theory informs that knowledge is a product of learners' cognitive acts, the theory describes knowledge as a process of construction (Confrey, 1990; Piaget, 1978, 1980; Bandura, 1977). Knowledge is constructed as learners make efforts to organise his/her experiences in terms of pre-existing mental structures. Constructivism theory also helps us to understand the process involved in learners' mathematical thinking. As mentioned above, the learning of mathematics is all about learning how to solve mathematical problems using mathematical rules and patterns. Mathematics problem solvers explore appropriate pattern in solving a particular mathematics problem. The process of exploring involves mental cognitive activities in an attempt to construct a pattern (plan) to solve the problem. Swartz and Perkins, (1990) refer to this as metacognition, that is, the part that links the thinking skill to the thinking process.

Polya (1985) in Chang Chili-Chi (2012) opined that mathematics problem solving theory is situated in the theory of constructivism. Polya put the process of mental construction (exploring appropriate pattern) in mathematics problem solving in steps, hence Polya proposed that there are four steps involved in the mathematical problem solving. These are (1) understanding the problem, (2) device a plan action, (3) carry out the plan, and (4) review the steps.

The nature and quality of mental construct in mathematical problem solving by the students is another thing to consider, since all students may not be able to reason in the same way or have the same depth of mental cognitive capability. Cobb, Yackel and Wood, (1992) infer that though students construct their knowledge but the central issue is the quality of those mental constructs. For example, in a mathematical class, if students were taught mathematical problem solving and afterwards a test is given, the performance of each student in the test, will attest to his or her individual mental cognitive ability. This might have informed Ernest's research in (Ernest, 1991), who reported that mathematical knowledge construct by students is firstly an individual and secondly a social activity. Ernest concluded that social interaction provides contribution in cognitive development. Ernest's research further informs that social interaction in a mathematics class create opportunity for students to discuss their thinking and the discuss encourage reflection. Glasserfield (1991) also reported that reflective ability is a major source of knowledge on all levels of mathematics education. Again, through classroom social interaction, teachers and students construct a consensual domain (Richards, 1991) which allow students to actively participate in the community of negotiation and institutionalisation of mathematical knowledge. This is social constructivism.

The theory of social knowledge construction proposes that knowledge is socially and culturally constructed, and not transmitted (Vygotsky, 1978; Borko & Putnam, 1998). In the process of knowledge construction, social constructivism places less emphasis on individual but more on group discuss. Students that are learning in a group are able to compare their knowledge construct while at the same time slow learners are able to be helped to understand mathematics concepts faster than when they are alone. Vygotsky, (1978) enlightens that a level of cognition is attained when students engage in social interaction. Vygotsky believe that students can improve their level of intelligence with the help of a more competent peer, and he called this "zone of proximal development" (ZPD), (Daniels, 2001). That is, students learn better in a community of discuss. Perhaps, this is why in a social constructivism mathematics instructional class students are sited in groups. This type of siting arrangement allow for easy social interaction among the group members, hence it is easy for the group members to compare their solution plans to each other.

Despite all the studies that are carried out to facilitate mathematics conceptual understanding among mathematics learners, it is a common knowledge that students still find problem solving in mathematics learning a hard nut-to-crack and hence mystify mathematics, resulting in poor students' performance in mathematics examinations. In his contribution to research in mathematics problem solving, Newman in Abdul, (2015) studied the error sources in mathematics problem solving and came up with an error check hypothesis in mathematics problem solving. Newman highlighted the following as the common error students commit in mathematics problem solving: (1) reading error, students' error in his inability to read the given mathematics problem and identify sentences and mathematical symbols used in setting the mathematics problem; (2) comprehension error, students' inability to understand mathematics problems; (3) transformation error, students' inability to be able to determine the appropriate method of mathematical solution; (4) the process skill error, is the students' inability to correctly process the solution to mathematics problem and (5) students' inability to write the encoding error according to the given question. These error factors are meant to be used in analysing students' mathematics problem solving approaches with a view to improve their mathematics problem solving skills.

However, the role of good mathematics problem solving instruction strategy, combine with the teachers' sound relevant content knowledge cannot be over emphasized in the teaching and learning of mathematics.

2.2 Problem Solving and Instructional Approach

As mentioned in 2.1 above, the teaching and learning of mathematics entails solving various types of mathematical problems at different cognitive levels. The teacher or instructor transfers the mathematical problem solving technical-know-how to the learner. During the skill transfer process, the learner seeks to acquire basic mathematical understanding that are pertinent for mathematical thinking. The method used by the instructor to facilitate the skill transfer process is referred to as instructional approach.

Traditionally, the mathematical instructional approach involve the mathematics instructor first solving some set of mathematical problems (which he/she may refer to as examples), next put forward similar problems to be solved by the learners, who are expected to use the same mathematical rules and pattern used by the instructor in solving the problems used as examples. Carlos and Joan (2014), refer to this method as "analogical transfer". The duo explained that the success in this method is when the learners are able to establish suitable analogy between the given problems and the examples. In case the problems are dissimilar a bit, then transfer becomes difficult. Some other scholars (Mestre, 2003; Hammer, Elby, Scherr & Redish, 2005; Polotskaia, Savard & Freiman, 2015) refer to traditional mathematics instruction as route learning. In that, the learner needs to copy the problem solving pattern in the example (step by step) to solve a similar problem.

The researcher noted that the traditional instructional approach does not include the metacognitive aspect of problem solving. In that the learners are not giving the opportunity to do any mathematical thinking. They only copy a procedure used in solving a mathematical problem in solving another one. Perhaps, this is where difficulties crept in, in the learning of mathematics. The learners may not have properly been taken through the steps of understanding the basic mathematical rules, and solution exploration method when they are just to copy a certain solution pattern in solving another one. Definitely, solving a similar mathematical problem which is structured in another form will be difficult because this will require some form of mathematical thinking to comprehend the differences. Unfortunately, mathematical thinking is limited in the traditional instruction approach.

Over the decades, mathematics researchers (Piaget, Vygotsky, Polya, Van Hiele and some others) are concerned with effective instruction for mathematical problem solving. Piaget proposed that meaningful learning takes place as a result of mental construct (Piaget, 1980). Vygotsky, while building on Piaget's learning hypothesis proposed that learners learn better in the company of more competent learners, hence the learners sit in groups in a mathematics social constructivism classroom. It should be noted that coupling traditional and social constructivism instructional approaches did not explicitly bring mathematical thinking process into the mathematics problem solving continuum. This only brought in some sort of discussions into the teaching and learning continuum.

For the teachers to deliver Polya's mathematics problem solving instructional strategy, the teacher should teach the students how to understand the problem, explore solution pattern, solve the problem as planned, and review the solution process. This implies that the mathematics instructor needs to inculcate the proposed four steps into his instructional approach.

Van Hiele's (1999) work focused on mathematics geometrical problem solving and recognised that though learning involves that learners should be able to construct their own knowledge but the process and the level of knowledge construction in geometry varies from one learner to the other. The study proposed five levels of geometrical mathematical thinking learners must attain before becoming a good geometrical mathematics problem solver: (1) visualization; (2) analysis; (3) informal deduction (4) deduction and (5) rigor. For learners to be able to acquire these levels of geometric metacognitive ability, Van Hiele proposed the following instructional approach: (1) the instructor to start with interview, that is, the instructor to find out the learners' prior knowledge in the geometry concept to be taught; (2) direct orientation, that is, the instructor to give exercises that may link the learners prior knowledge to the current concept to be taught; (3) explanation; (4) free performance, where at this stage the instructor gives the learners activities in the current concepts to be learnt and (5) integration, the learners are allowed to summarize and internalize the concepts learnt, and see how to apply the concepts to solve any other similar problem in other situations.

There are other studies that are investigating other types of mathematics instructions in a way to facilitate and simplify the problems associated with the learning of mathematics. In the current study, the researcher coupled Polya's problem solving instructional approach and the social constructivism instructional approach in teaching of circle geometry. The coupled Polya-social constructivist instruction approach deviate from the traditional mathematics teaching approach, in that, the mathematics instructor taught the learners with full cognisance of Polya's learning steps while learners always sit in groups and discuss knowledge construction in the mathematics class during the intervention.

2.3 The Review of Similar Literatures

It was mentioned in subsection 2.1 above that the learning of mathematics involve understanding of mathematical rules and patterns, as well as, being able to use the rules and patterns in solving mathematical problems. Mathematics education curriculums should contain the learning of mathematical rules, from the early childhood education. If learners failed to conceptualise these rules, they may not be able to catch-up with it at all throughout their education life. For example, habitual use of calculator from the early age may deprive mathematics learners from correctly conceptualising basic mathematical rules (Faleye & Mogari, 2009).

The duo found that university mathematics students were having difficulties in basic arithmetic calculations (multiplication and fraction operation) in mathematics problem solving because of habitual use of calculators. Perhaps, that is why the South Africa's Annual National Assessment (ANA) emphasized that mathematics poor performance start from primary school. This implies that learners' poor mathematics problem solving skills start from the primary school.

Chien Lee carried out a research on using Polya's mathematics instructional approach to strengthen the mathematics problem solving skills of learners in an elementary school in Taiwan (Chien, 2015). The semi-experimental research work involved mathematics learners in two elementary schools in Taiwan: one is an experimental school, while the other one was used as a control group. In another study, Elena, Annie and Viktor, (2015) focused on the difficulties learners in elementary schools were having in mathematics words problem. Elena et al (2015) primarily focused on the learners' ability on "reversibility of arithmetic operations and their flexibility of mathematics thinking". According to Elena et al (2015), these two concepts are key elements in elementary mathematics problem solving skills. By using "The Three Little Pigs" tale, the researchers were able to analyse the difficulties learners were experiencing while learning to solve word problems involving addition and subtraction. The researchers as part of the results in their study explained learners' difficulties and suggested an appropriate teaching principle to help the instructor in the teaching of these basic arithmetic rules.

The research work by Faleye and Mogari (2009), provided insight into how learners may continue to battle with conceptualisation of basic arithmetic rules. In support of Faleye and Mogari (2009), Adler and Sfard (2015) revealed that in South Africa, two third of the mathematics learners in the secondary schools in Gauteng leave primary school while they did not have enough mathematics knowledge that could help them succeed in the secondary school mathematics.

Carlos and Joan (2014), targeted the learners' mathematics thinking process. The aim of the study is to analyse the learners' cognitive difficulties at the early stage of knowledge transfer process in problem solving. The study involved participants' solving a target mathematical problem (exercise) in view of the similar source problem (example). The participants think aloud in the process of solving the target problem. The study was used to develop "Think-Aloud problem solving instruction".

Tugba and Bulent (2016) carried out a case study with five study participants. The study was carried out to evaluate the belief of high school students about problem solving. Clinical interview and three mathematical problems were used as data collection instruments. It emerged that students who thought that problem solving should be a short process and that they could approach it by memorization of rules also belief problem solving is difficult. It also emerged that belief and personal factors affect mathematical problem solving.

Another study was carried out by Jinwen and Bikai (2007). They investigated problem solving in Chinese mathematics education. The duo informs that problem solving can be viewed as both as an instructional goal and as an instructional approach. In addition, it was found that problem solving research in China has been more of content and experience-based, rather than been cognitive and empirical-based. In China, there are: "one problem multiple solutions activity", multiple problems one solution activity", and "one problem one solution activity". Tolga (2014) reported on the effects of Problem Solving Strategy Steps (PSSS) on students' achievement skills, and confidence. The study was conducted in physics but in Newtonian mechanics which is more of mathematics. The study was carried out in two years with 70 study participants from two different groups: one group is experimental while the other is a control group. The result indicated that the experimental group participants gained critical and analytical skills.

As may be observed in this literature review, that the researcher is trying to substantiate that students are having mathematical conceptual difficulties from the primary school right through to the tertiary school level. Sevda and Zehra (2013), investigated the relationship between the metacognitive awareness of 70 first year university mathematics students and their solutions in problem solving. The data collection instruments were Metacognitive Awareness, Inventory Scale and

Mathematics Problem Types Test Scale. The study showed a positive correlation between the students' metacognitive awareness levels and their problem solving levels regarding routine and non-routine problems. In addition, it was found that metacognitive awareness significantly predicted problem solving levels. Also, Nourooz, Mohd, Hamidreza and Mahani (2015) presented the difficulties experienced by university mathematics students in problem solving in calculus modules. A learning strategy which involve mathematics thinking and generalization was designed as an intervention in this study. Prompts and Test instrument was used to collect the required data. The intervention was found to have increased the students' problem solving ability.

Mushlihah and Sugeng (2017) reported on problem solving analysis in mathematics. This study investigated the types of mistakes students commit in problem solving and the factors that are responsible for these mistakes using Newman's theory. There were 147 study participants from Indonesia secondary school. The data collection instrument was a structured test. Newman's theory error finding procedure was used to analyse the data collected. Newman's theory procedure include reading errors, comprehension errors, transformation errors, process skill error and encoding error. The results that emerged from the data analysis showed that students were: not able to absorb information well, not properly understanding the transformation of the problem, not following the material thoroughly, and not comprehending the needed mathematical concepts well.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1.0 RESEARCH DESIGN

This study employed a qualitative research approach which involved the use of naturalistic observation case study research design, (NOCSR). NOCSR was considered ideal for this study, since an intensive study and observation of the study participants over a period of time in their natural classroom setting was required to be able to give a detailed explanation of events at the research field which was needed, for effective data collection. Social constructivist instructional approach coupled with Polya problem solving instructional approach was used as an intervention in the NOCSR design. The study was carried out in a two-year duration.

In addition, the researcher narrated how dry, non-responsive and lack of motivation attitude his students always were whenever he had to teach geometry in the classroom as a mathematics teacher himself (see paragraphs 2, 4 and 5 of section 1.0). The mathematics teacher of the study participants confirmed that he experience just exactly the same classroom attitude from the study participants whenever he teaches them geometry.

3.1.1 CLASSROOM SETTING IN NOCSR DESIGN

As mentioned above, social constructivist instructional approach was coupled with Polya problem solving instructional approach as an instructional intervention in this study. The social constructivism classroom instruction require that learners sit in groups to facilitate knowledge construction discuss, while Polya problem solving instructional approach gives four steps to be followed in the teaching of mathematics, see page 22, subsection 1.4 of this dissertation. In the light of the above, throughout the intervention period, learners were arranged to sit in groups of 4s, while spaces were provided for the teacher to move around freely in the classroom to interact with each group.

The teaching content focus of this study is circle geometry. Previously, circle geometry was taught in the third term of the academic year but it was taught in term one with effective from the 2019 academic year in accordance with the new mathematics work schedule, released to all schools prior to the 2019 academic year by the Northern Cape Department of Education. It is taught for three weeks. Hence, in the first year of this study in 2018, circle geometry was taught in term 3 while in 2019, which is the second year of this study, it was taught in term one. In the research field, all the learners go to the class of the teacher teaching mathematics for mathematics lessons. In 2017, the mathematics teacher involved in this study was trained on how to teach according to the proposed instructional intervention and he continued to teach according to this proposed instructional approach throughout the 2017 academic year for him to perfect the use of the instructional approach in his mathematics class before he had to use it during the research period. By the time the research started in 2018, the teacher was efficacious with the proposed instructional approach.

3.1.2 RESEARCH PROCEDURE

As mentioned in 3.1.0, the study was carried out in two years. Intervention for the first year started on the 2nd of August, 2018 and ended on the 24th of August, 2018 (term 3). In the second year of the intervention, the intervention started on 12th of February, 2019 and ended on the 5th of March, 2019 (term 1). In the first year of the intervention, 15 classroom observations were conducted, while 16 classroom observations were also carried out in the second year of the intervention. It is important to note that the set of study participants for the first year were the 2018 grade 11 mathematics learners, while in the second year, the participants were the 2019 grade 11 mathematics learners.

This study was carried out in the grade 11 mathematics classroom. The research field had only two grade 11 classes: 11A and 11B. One period for teaching at the research field is 30 minutes, while mathematics period for grade 11 is double period, except Fridays which is a period (30 minutes). In a week, the grade 11 classes were having

9 periods (four hours, 30 minutes) for mathematics lessons, hence, the researcher had 5 observations per week. This did not change throughout the duration of the research.

In each of the research intervention year (2018 and 2019 school academic year), two weeks to the start of the research intervention, the teacher introduced the research and the researcher to the study participants. The teacher also discussed the classroom sitting arrangements and other intervention logistics like discussing solutions to examples among the group members, questions raised by the teachers and how the teacher will only be giving a leading question that will be propelling the mathematics lessons as a form of mathematics knowledge construction processes. On the first day, all the study participants were given two research consent letters (one for each study participant and the other one for their parent or guardian) to sign, see Appendix G and Appendix H.

As a result of the above, each time the learners came for the mathematics class during the intervention, they already knew classroom ethics and the expected teaching procedure. Before the study participants arrived, for each classroom mathematics lesson to be observed, the researcher would have prepared ready, note pad and video recorder, which is used for both video and taking photographs when necessary.

3.1.3 INTERVENTION INSTRUCTIONAL PROCEDURE

As mentioned in subsection 3.1.0 above, the intervention is Polya problem solving approach coupled with social constructivism instructional approach in the teaching of circle geometry. The intervention is structured for teaching problem solving using Polya problem solving instructional hypothesis in the constructivism mathematics classroom settings.

The intervention was presented in the following order:

(1) The teacher gives a leading question (in the case of a new concept) or write a problem to be solved on the board (in the case of continuation of the previous concept).

(2) The study participants start to discuss the solution in view of the Polya problem solving approach steps enumerated in subsection 1.4 of this dissertation and

which the teacher took the study participants through in the first class of this intervention.

(3) The teacher goes round each group to moderate or correct the groups' discussion

(4) The teacher stops the discussion and allow the study participants to present their solutions and allow other groups to criticise each other's solutions.

(5) The teacher finalised the solution by accepting or correcting the solution proposed by the study participants and give more detailed explanation on the problem(s) before introducing another problem to be solved to the study participants.

(6) At the end of the class, the teacher gives homework to the study participants.

3.2 STUDY POPULATION AND SAMPLING

The study population are the Grade 11 learners in high schools in South Africa, while the sample population are the grade 11 learners in the Northern Cape Province high schools. South Africa is divided into nine provinces. The researcher chose the Northern Cape Province for some proximity logistics. In the Northern Cape, there are 139 high schools divided into five education districts. The chosen research field is in the education district two. Convenient sampling method was used to choose the research field as a result of some selection factors like research consent from the school, financial consideration, and suitability of the school for the research. Intact group of grade 11 learners in the chosen school constitute the study participants. The study was carried out in two years (2018 and 2019). In the first year of the study (2018), the study participants were 61 (27 boys and 34 girls), while the sample size for the second year was 45 (20 boys and 25 girls). The school had only two grade 11 mathematics classes: grade 11A and 11B.

3.3 DATA COLLECTION

3.3.1 Data collection procedure

The nature of the data in this study is qualitative. As mentioned in 3.1.0, the research was two-year in duration, hence two sets of qualitative data were collected: one set of data for each year. Data were collected through classroom observations, video recordings, classroom exercises (CE), field notes and end-of-the- Intervention Test EIT). Learners' work in the exercises and class test were photographed during the lesson. Hence, after each lesson, the researcher collected the following data: classroom teaching proceedings' video recordings, photograph of learners class exercises and field notes. At the end of the interventions, the researcher collected the scripts of the study participants after the end-of-the- Intervention Test.

3.3.2 INSTRUMENTATION

As mentioned in 3.1.0, this study is a qualitative study that involved majorly classroom observations, thus the instruments used are mainly the instruments used in qualitative data collection. The instruments used are video recorder, camera, note pads, class exercises (CE), end-of-the- Intervention Test (EIT) and the problem solving rubric (PSR). The PSR was used as a guide during data collection and in the data analysis which is in subsection 3.6.

Instruments like the video recorder, camera, note pads, and class exercises (CE) need not to be developed and as such, need not to be validated nor reliability checked. The CE are given according to the concepts to be learnt and the teacher chose them at random from the textbook. The video recorder and the camera were ensured to be in good condition before the start of the intervention. However, the EIT and PSR instruments needed to be developed and hence needed to be validated and reliability checked. To this end, the remaining discussion and instrumentation shall focus on the EIT and PSR instruments.

3.3.2.1 End-of- the- Intervention Test Instrument (EIT)

(i) Development of EIT

The EIT instrument consists of eight (8) written questions. The questions were structured in line with the mathematics curriculum and the research focus. The researcher needed to observe and analyse the solution approach expressed by the study participants. In constructing the EIT instrument, each question item was to provide information for a particular research question, also the level of difficulties in each question varies. For example item 1 on the instrument was to test the participants on basic circle geometry concepts like circle terminologies; item 2 on the instrument was meant to provide information for research question 1 and research question 2, it was meant to reveal the study participants' solution approach skills as they express their ability in geometrical properties and axioms and it was of a higher difficulty compared to item 1 on the EIT instrument; items 3 to 8 on the instrument were meant to provide information for research question 2 and research question 3, they were meant to reveal the study participants' solution approach skills on different difficulty level scales.

The data analysis rubric (that is PSR) was formulated to assist the researcher in this regard. The researcher identified four important steps in problem solving (see Appendix C) and the items in EIT (see Appendix A) were formulated such that the steps would be expressed from the expected answers to the problems in the EIT instrument. Therefore item 1 on the EIT instrument seeks to test learners' ability to identify circle geometry theorems with correct terminologies. Item 2 test learners' ability in geometrical properties and axioms, while items 3-8 require more deductive reasoning in circle geometry.

ii) Validity of EIT instrument

Validity of the EIT instrument is the extent to which the test instrument can measure the study participants' ability in the circle geometry concept learnt. Face validity test and content validity test are the most common validity tests that researchers carry out on a test instrument (Alias, 2005). Face validity test is to test if the test instrument is properly structured, that is in terms of grimness and the correctness of the mathematical statements, while content validity ensures that the test covers the intended content area/scope it is supposed to test. Content validity also ensures that the content level of the test is appropriate for the level of the learners that the test is meant for.

Five teachers with more than 10 years mathematics teaching experience, were appointed to help with the face and content validation of the EIT instrument. They were given validation forms for this purpose. Separate validation forms were drafted for face and content validation (see Appendix D and Appendix E). Though the EIT instrument was rated well to be used for the research during validation, the exercise resulted in some items been removed while some were restructured according to the comments from the validation judges.

iii) Reliability of EIT instrument

Reliability of a test instrument is the measure of the consistency of the results obtained when used in more than one occasion. In this study, the reliability of the EIT instrument was measured. The test-retest reliability technique was used. In 2017, before the pilot study, a different school which was not used to conduct this research study was selected for this reliability test. Intact group of 30 grade 12 learners, who had been taught circle geometry from the selected school, were used for the reliability test. After administering the research instrument as a test on a particular day, the retest was conducted a week later, without creating any awareness of the retest to them. The test items for the first test were reshuffled to minimize participants identifying the test items to ascertain if the participants' responses to the respective items of the test and retest are consistent.

The degree of strength of the relationship between the test scores and the scores of the re-test was then determined by drawing a scatter diagram to determine how the scores are wide apart or close to each other. The scatter diagram drawn indicated a positive correlation due to the direction of flow of the scores on the diagram. The plotted scores were within a correlation coefficient range of 0,6 – 0,7 with regards to how close the plotted scores were found to be around the regression line of best fit, when drawn through the plotted scores on the scatter plot diagram. This implied that the test instrument was acceptably reliable (Please see Appendix I).

3.3.2.2 The Problem Solving Rubric (PSR)

The problem solving rubric was used for the data analysis in this study. However, since it is an instrument used in this study, the researcher presented its development, validity, and reliability as discussed below:

(i) Development of PSR

The PSR rubric used for this study was generated using the knowledge dimension in learning and van Hiele's learning hierarchy. The knowledge dimensions are factual knowledge; conceptual knowledge; procedural knowledge and meta-cognitive knowledge, which are represented in the rubric as geometric terminology; identify appropriate axioms; geometric properties and theorems; follow appropriate skills and techniques in problem solving and connect various geometric concepts to solve more complex problems respectively.

(ii) Validity of PSR Instrument

Face validity was carried out on the PSR instrument. Three university lecturers whose research interest is in connection with the Bloom's Taxonomy and Van Hiele's learning theory were approached to help in validating this instrument (see Appendix F). The judges were to rate the instrument on how well the PSR instrument is structured. One of the instrument validity judges raised a concern about the PSR instrument. This led to the restructuring of the instrument and it was sent back to the judges. The result from the judges were unanimous as it was rated as very well structured to be used for the research study.

(iii) Reliability of PSR Instrument

The rubric was content validated after it was generated. 7 experienced mathematics educators, who were teaching in schools outside Education District two of the Northern Cape Department of Education were randomly selected to serve as judges for the reliability process. One of the judges used for the validity was requested to carry out the reliability test, he was requested to use the rubric to rate the EIT instrument on whether the instrument was good for grade 11 on three different occasions, while the question items were reshuffled on the three occasions. The comments made by the judge on the reliability indicated that the items on the problem solving rubric are worthy enough to determine the participants' problem solving abilities.

3.4 DATA ANALYSIS

As mentioned in subsection 3.3.1, the following data were collected: classroom teaching proceedings' video recordings, photograph of learners class exercises and field notes, and learners' scripts on learners' responses to the EIT instrument.

3.4.1 Data Analysis Procedure

The study was carried out in two years: 2018 and 2019, hence in each year complete set of data listed in 3.4 above were collected. The data were analysed year by year and the results from each year were compared. The reason for carrying out the study in two years is to determine if there will be similar/dissimilar data analysis results. This is very important as similarity or dissimilarity in the yearly results trend will strengthen the conclusion of the study.

On the set of data collected for each year, data analysis was carried out in two folds due to the nature of the collected data. Data from the video recordings and data from the note pads were compatible, hence they were analysed together. In the same reasoning, the data from the classroom exercises (CE) and the data from the learners' EIT scripts were compatible though they were not analysed together but they were analysed separately using the same method.

(i) Analysis of Video Recordings of Classroom Procedures and Note Pads Data

The data recorded by the video recorder was transcribed (the researcher repeated this three times to avoid data killing). The transcribed data was analysed together with the data from note pad. The combined data from video recordings and note pad were partitioned into categories with reference to the research questions. Each identified category was then coded so that they can be tabulated for easy comparison. The emerging themes were noted. The procedure of the data analysis is put in a symbolic format below for easy comprehension.

Raw Transcribed data → Categorizing → Coding→ Tabulation → Developing themes

(ii) Analysis of CE and EIT Data

(1) The CE Data

The data is a random photograph of some of the study participants' solution to the classroom exercises or homework given during the intervention. This set of data was analysed using solution appraisal method. The solution appraisal method involves the use of Van Hiele's proposed geometric conception level hierarchy'. This include visualization, analytical, abstract, deduction and regor, which are named level 0, 1, 2, 3, 4 respectively (see chapter 1, page 15). In this data analysis, the researcher scrutinized each data and categorized the study participants' knowledge construct on a particular Van Hiele geometric conception hierarchy level.

(2) End-of-the-Intervention Test (EIT)

This set of data was analysed in two folds: (a) performance Analysis (b) solution appraisal.

(a) Performance Analysis

The scripts were marked according to the prepared memorandum for the EIT instrument, the Curriculum and Assessment Policy Statement (CAPs) grading code (presented below) was used to analyse the study participants' performance.

Table 3.1 Codes and Percentages for Performance Analysis (adapted from CAPs)

RATING CODE	DESCRIPTION OF COMPETENCE	PERCENTAGE
7	Outstanding achievement	80 – 100
6	Meritorious achievement	70 - 79
5	Substantial achievement	60 - 69
4	Adequate achievement	50 - 59

3	Moderate achievement	40 - 49
2	Elementary achievement	30 - 39
1	Not achieved	0 - 29

The researcher used table 3.1 to analyse the number of study participants that passed the EIT very well, averagely or poorly.

(b) Solution Appraisal using the PSR

First, the items on the rubric were categorised as A, B, C, D according to each item's cognitive level, where

A- Study participants were able to recognise/visualize visual geometric figures and use appropriate terminology

B- Study participants were able to conceptualize geometric properties, appropriate axioms and theorems

C- Study participants were able to follow appropriate techniques and skills in problem solving

D- Study participants were able to make connections across the geometry concepts to solve more complex problems

How the PSR items were categorised is summarised in the table below:

Table 3.2 Learning Concepts Categories for Metacognitive analysis

Category	Van Hiele's Level	Concept identification	EIT item number
A	0	Identify and using correct geometric terminology	1
B	1	Conceptualize appropriate relevant properties, axioms and theorems	2

C	2 and 3	Follow appropriate techniques and skills in problem solving	3 and 4
D	4	Making connections across the geometric concepts to solve more complex problems	5,6,7 and 8

Each study participants' marked EIT script was analysed using the information contained in table 3.2 above as well as scrutinising the solution steps provided in each script. This was carried out after the performance analysis had been carried out on each EIT script. This second round of analysis on the EIT scripts involved determination of the number of study participants that belong to category A, B, C, or D according to how they answered the questions that fall into each category.

3.5 PILOT STUDY

The pilot study was carried out in one of the secondary schools in educational district 5 in the Northern Cape Province school district. It will be recalled that the research field was in the district two of the same Northern Cape Province educational district (see subsection 3.2) but the pilot study field and the research field are far apart. The pilot study field was chosen this way in order not to compromise the study.
In addition, the pilot study was necessary for testing the efficacy of the research instruments, the intervention and to try out the scheduled research procedure. The pilot study was carried out from 1st of August to 22nd of August in the 2017 academic year in grade 11 of the chosen school. 36 grade 11 mathematics learners, all in one class (11A), in the chosen school served as participants for the pilot study. The pilot study intervention was carried out in three weeks after which the EIT was administered.

3.5.1 Pilot Study Procedure

The researcher presented the intervention instead of the mathematics teacher in a grade 11 mathematics class. This was necessitated as a result of the fact that the trained mathematics teacher at the research field could not leave his school to present the intervention at the pilot study field. Besides, he was only trained in the use of the intervention in the teaching of geometry, hence, he might not have been well conversant with the instructional method at that time.

During the study, the exact research procedure plan was implemented: from classroom setting to the last day of the research when the EIT instrument was administered (see subsection 3.1.2). All the necessary data were collected as stated in subsection 3.3.2. The result of the pilot study is presented in chapter 4.

3.6 Ethical Considerations

3.6.1 Informed Consent

The Institute of Science and Technology Education was contacted to demand for a written letter, permitting the research to be conducted. Also, a permission letter was requested from Education District two under the Northern Cape Department of Education. Permission letter was also requested from the School Governing Body of the school in which the research study was conducted. Parental consent of all the participants and individual participant's consent was also obtained.

3.6.2 Confidentiality

The identities of all individuals who participated in this research were concealed to ensure anonymity by not demanding any form of identification from the research participants.

3.6.3 Voluntary participation

The research instrument was only given to participants who were willing to participate in the research. All the respondents participated voluntarily without being forced or blackmailed in any way to participate in the research.

CHAPTER FOUR

PRESENTATION OF DATA ANALYSIS RESULTS

Before the results of the intervention data analysis are presented, the researcher will like to acquaint the reader with the research field by giving details of the research field profile.

4.1 Research Field Profile

The research field is a big school which consists of thirty-six (36) classrooms and one library. There are enough spaces for learners' recreation, there is a standard football field and a netball field. The classrooms are of standard classroom size. In each of the classrooms, learners' chairs and lockers are arranged in a way that anyone will be able to move around the classroom freely. In each of the classrooms, there is a chalkboard, a white board for power point presentation, a cupboard, a teacher's table and chair (including visitors chair), as well as a notice board. Most (but not all) classrooms have teaching aids like charts and pictures.

The school's population is 1096, out of which there are 37 teaching staff and 5 non-teaching staff. Each teaching staff is allocated a classroom. Each teacher is the class teacher of the class allocated to him/her. The learners attend the teacher's lesson in his or her class. English language is the medium of instruction of all the subjects (except home language subjects) in the school. The school covers 11 periods each school day (Monday to Friday) for all subjects. The duration of each period is 30 minutes. In accordance with the CAPs mathematics curriculum, Grade 11 mathematics is taught for four and half hours (9 periods) per week in the school. Lessons (intervention) on circle geometry theorems were conducted for three weeks in each of the two consecutive years of the study. This is the timeframe for teaching circle geometry allocated in South Africa's Mathematics curriculum, as indicated on the pacesetter for grade 11 mathematics.

4.1.1 The School's Mathematics Department

All the staff members in the mathematics department are teaching staff, the department does not have its own non-teaching staff. The mathematics department consist of eight (8) staff members, of which there are seven (7) male teachers and one (1) female teacher. All the staff members are qualified mathematics teachers. The head of the department, a male staff member has a master's degree (Master of Science) in Mathematics Education, five (5) other teachers have Bachelor of Education degree in Mathematics Education, while the other two (2) teachers have Diploma qualifications, majoring in Mathematics Education.

The particular mathematics teacher that participated in this research study is a male mathematics teacher with a Bachelor of Education degree in Mathematics Education. He has eight (8) years teaching experience and he is always ready to learn new teaching concepts. The teacher embraced the pedagogy involved in the intervention. He availed himself for the intervention training and after the intervention training, he kept to the newly acquired classroom instructional procedure.

4.1.2 The Teaching of Mathematics in Grade 11 before the Intervention

The teacher participant has been teaching mathematics in the research field for the past eight years. He had taught grade 10 and grade 11 mathematics. A week before the intervention, the researcher had an unscheduled, unstructured interview with the teacher. The intention of the researcher is to have an insight into the teaching and learning of mathematics in the research field, more importantly how the learners respond to the teaching of mathematics in the research field.

The outcome of the unofficial interview revealed that since the teacher has been teaching mathematics in the school, majority of the learners have not been showing much interest in the learning of mathematics, the teacher has being teaching with some difficulties like learners not responding in the class, mathematics class truancy, not doing homework and so on. The teacher said he thinks that all these cumulate into mass failure in mathematics, even at 30% pass rate.

4.2 Presentation of the Data Analysis Results

The result of the pilot study is presented first, then the result of the data analysis of the main study. In presenting the main study's data analysis results, the results of the 2018 intervention year is presented first, followed by the 2019 intervention year data analysis results. The classroom observation transcribed data analysis result is presented first, followed by the CE and EIT data analysis results.

4.2.1 Results of the Pilot Study Data Analysis

The intention of carrying out the pilot study was to try out the research intervention and the instruments meant for the main study, whether they are proper and reliable to be used for the main study. As mentioned in subsection 3.5.1, the researcher presented the intervention in the pilot study school. The intervention presentation went well for the three week intervention. No problem was noted in carrying out the pilot study, hence this indicated that the researcher does not anticipate any problem in carrying out the main study. During the pilot study intervention, CE data were successfully collected and the EIT data was also collected at the end of the pilot study intervention. The results of the data analysis of the classroom observation and the EIT data collected show that they were able to answer the research questions.

In all, the researcher was satisfied with the pilot study. Hence, the researcher was assured of the research design and the planned data collection procedure.

4.2.2 Presentation of the data Analysis of the main study

As mentioned in subsection 4.2 above, in presenting the data analysis results of the main intervention, the results for the 2018 intervention year is presented first, then followed by the 2019 intervention year results.

4.2.2.1 2018 Intervention year Data Analysis Results

The set of data collected during the 2018 intervention year and analysed were the transcribed data from the video recordings and note pads, the classroom exercises

(CE) and the end-of- the- intervention test (EIT). The presentation of the data analysis results starts with the presentation of the results for the data analysis of the transcribed data, followed by the results of CE data analysis and lastly, the results of the EIT data analysis.

(i) Results of the classroom observations

For the first year of the intervention (2018), the intervention started on the 2nd of August, 2018 and ended on the 24th of August, 2018. The intervention was the use of Polya's problem solving instructional approach to teach circle geometry in social constructivism classroom settings. The researcher was always present in the research field from the start of the intervention to when it ended, from the start of each school day to the time the school day ended. This allowed the researcher to thoroughly study the participants before, during and after the mathematics lessons. The study participants were sixty-one (61) in total but they were divided into two different classes: grade 11A with 31 study participants and grade 11B with 30 study participants. Grade 11A and grade 11B were scheduled to attend mathematics lessons together in the class of the mathematics teacher who is the teacher participant in this research. Each lesson is 30 minutes duration but grade 11 mathematics lessons were two periods consecutively except on Fridays when it is only one period. All the study participants were briefed on the intervention teaching approach including classroom sitting arrangements. The study participants were divided into the classroom sitting groups before the first lesson of the intervention.

The results of the classroom data analysis showed that the teacher taught circle geometry according to the intervention in all the lessons presented in the classroom. The results of the data analysis also revealed that in the first two days of the intervention, the study participants were making unnecessary noise during the lesson, hence during the second day of the intervention the teacher re-explained the expected classroom conduct of the study participants during each mathematics lesson. He emphasised that the study participants should only discuss the mathematics focus concepts whenever they are in the mathematics lesson. The data analysis results showed that from the 3rd day, the lessons went very well.

It also emerged that in most of the sitting groups, the study participants imbibed the Polya Problem Solving Hypothesis: that is to understand the problem, plan for the solution procedure, solve the problem according to the plan and lastly review the solution. It emerged that many of the study participants were enthusiastic, concentrative and were actively exploring solution plans while in each group. However, it also emerged from the note pad data that a group at the back of the class was always discussing things that were not of the mathematics concepts which they were expected to discuss. While in one other group, they could not pass through the solution exploration levels easily and they were always going to other groups for assistance. When the teacher puts a problem on the chalkboard, he leaves the study participants to discuss the solution in each group, while he goes round to each group for possible correction or contribution. The study participants were always active and were ready to present their solution(s) on the classroom chalkboard. The researcher gives unedited classroom procedure of one of the classroom lessons observed (this lesson was the first lesson in the third week of the intervention: 13/08/2018.

Teacher: "Good morning class"

Study participants: "Good morning sir"

Teacher: "Today we shall continue with the learning of circle geometry". He paused for few minutes, looked around the class. "I noticed that three learners are not in the classroom". He pointed to a group at the middle of the class." That group, where is Tebogo?"

Study participants: "I saw him in school before we came here for this lesson, maybe he will still come". Answered one of the study participants in the group. The teacher continued.

Teacher: "Remember how we have been approaching finding solutions to any given problem in circle geometry. I also told you that henceforth, this is how we shall be solving our mathematics problems". He paused, and then continued. "By the way, who can remind us of this new method?" One of the study participants in one of the groups at the back raised up her hand to answer the teacher's question.

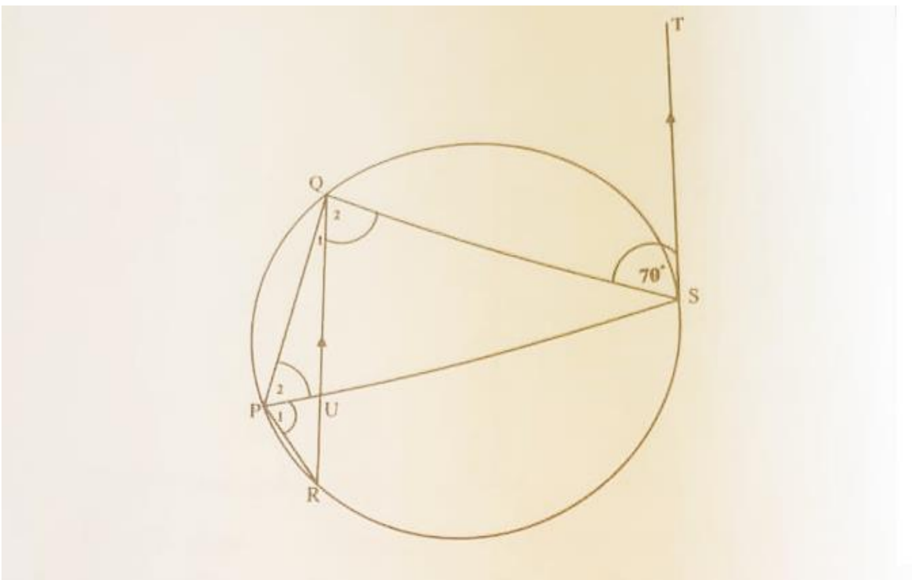
Teacher: "Yes, Sibongile, you want to answer the question?"

Sibongile: “Yes sir, we must first read and understand the question, plan for how to find solution to the question, plan how to find solution and continue to work it out”.
Teacher: “Thank you Sibongile, but who can put it in a better way”. Another girl from a group at the front stood up to answer the question.
Teacher: “Yes Thandi, tell us”
Thandi: “We need to understand the question, plan for the solution, follow our plan to solve the problem, and review our solution” She was reading from her mathematics class notes book. She continued, “I wrote it down from the first day you (the teacher) introduced the method to us”.
Teacher: “Thank you, Thandi. That is very good of you”. The teacher continued “Based on what we learnt last week, we shall solve this problem” He moved towards the chalkboard and started writing the problem from the mathematics textbook he was holding as shown in the figure below:

QUESTION 8

Circle RPQS is given below.ST∥ RQ and ST is a tangent to the circle at S. Chords PQ, QS, PS and PR are drawn. T$\hat{S}$Q = 70^0.

Figure 4.1- A classroom exercise question



8.1 Determine with reasons, the size of the following angles:

8.1.1 $\widehat{Q_2}$ (2)

8.1.2 $\widehat{P_2}$ (2)

8.2 Prove that PS bisects Q$\hat{P}$R. (2)

8.3 Determine, with reasons, the size of $\widehat{Q_1}$ such that PS will be a diameter of the given circle. (2)

[8]

As the teacher was writing on the chalkboard, the groups started copying and murmuring (discussing the problem). The whole class became active. By the time, the teacher finished writing the problem on the chalkboard; all the groups had started discussing how to solve the problem. In about three minutes into solving the problem, a study participant from the group at the middle of the class stood up to ask a question.

Teacher: "Williams, yes"

Williams: "Those arrows on those two lines" pointing at the diagram on the chalk board, "means". He appeared not to know what to say further. The teacher came to his rescue.

Teacher: the teacher moved to the chalkboard, he was touching the lines, and the arrows contained in the question. "You mean these lines and these arrows", "Who can answer that question? A male study participant stood up to answer the question. The teacher continued, Offense, yes!! Offense!"

Offense: "That tells us that the two lines are parallel to each other"

Teacher: "Good, Offense" the teacher looked round the classroom and continued, "did you all hear that, the arrows indicates that the two lines are parallel to each other and that have implication on the problem you are solving?"

Study participants: "Yes, sir"

The teacher then emphasised to the study participants why it is always important to read and understand a question before an attempt can be made to get its solution. The teacher went to the chalkboard again, and pointed to the part of the question that states that ST∥ RQ. The teacher emphasised again why it is relevant to read and understand a question, which is one of the solution paths, been used in the classroom circle geometry problem solving. Then the teacher, started going round the class, to see how each group was progressing in solving the problem he had put on the chalkboard. After 10 minutes the study participants had been working, the teacher announced that the learners should stop work and it was time for them to present their solutions on the chalkboard. Most of the participants shouted that they had not finished, and that, they will need about five minutes more.

Teacher: "Okay, I will give you all five more minutes to finish, you know we have to solve more problems. I want us to look at more difficult problems, class".

The teacher continued to go round the class, attending to each group as the need arose, while he continued to check his phone for time (he was not wearing a wrist watch). When 5 minutes had elapsed, he announced:

Teacher: "Okay class, stop, it's time to look at your solutions. We want each group to come to the chalkboard to present their work. The teacher stopped talking for about 2 minutes obviously thinking, then he continued talking again.

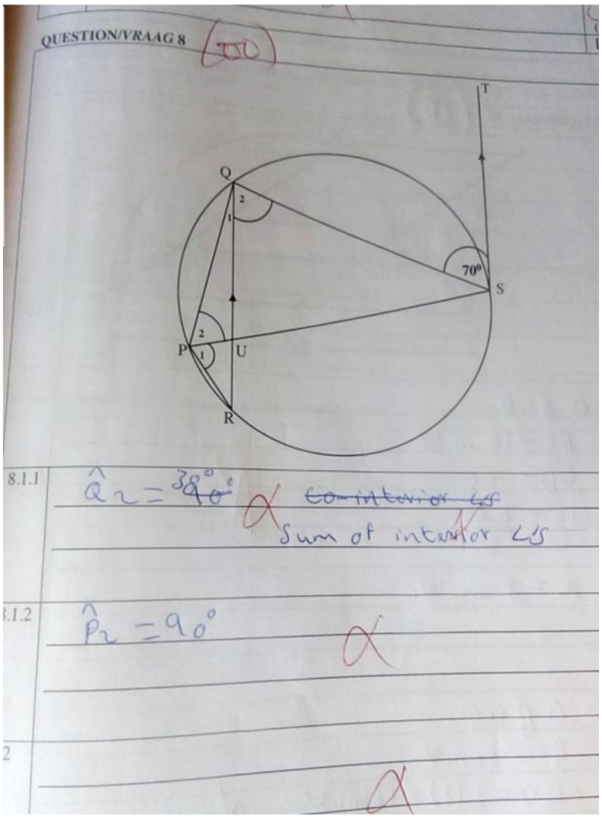
Teacher: "Before presentation, I want to quickly mark your work". At this stage the researcher started taking sample pictures of the study participants work (the CE data). After about 7 minutes, the teacher had gone round all the groups, marking their answers to the problem on the chalkboard.

Teacher: "I want that group to present their solution" pointing to one group at the back. The teacher had noted while he was going round that, the group was struggling with exploring appropriate solution plan for the problem. The group did not want to come to the chalkboard to present their solution initially but the teacher persuaded them to comply. The group eventually appointed one member of the group to do the presentation on behalf of the other group members. Thus, a male study participant, by the name Andile, came to the chalkboard for the presentation.

Andile, "For 8.1.1 $\widehat{Q_2} = 30^0$, the reason: sum of interior" He stopped talking, apparently not knowing what to say again.

The sample of the group work is shown in the figure below:

Figure 4.2 – A sample of a group's work



Teacher: "Okay, thank you". The teacher asked the presenter to go and sit down. He asked one of the groups at the front to give their presentation. A male member of the group came out and presented their solution. His name was given as Mokwa. "In our group, we first studied the problem and discussed the solution; this is what we have:

8.1.1 $\widehat{Q_2} = 70^0$ QR∥ TS - Alternate angles

8.1.2 $\widehat{P_2} = 70^0$ - tanθ – chord

8.3 $\widehat{Q_1} + \widehat{Q_2} = 90^0$ - angles of semi- circle

$\widehat{Q_1} + 70^0 = 90^0$

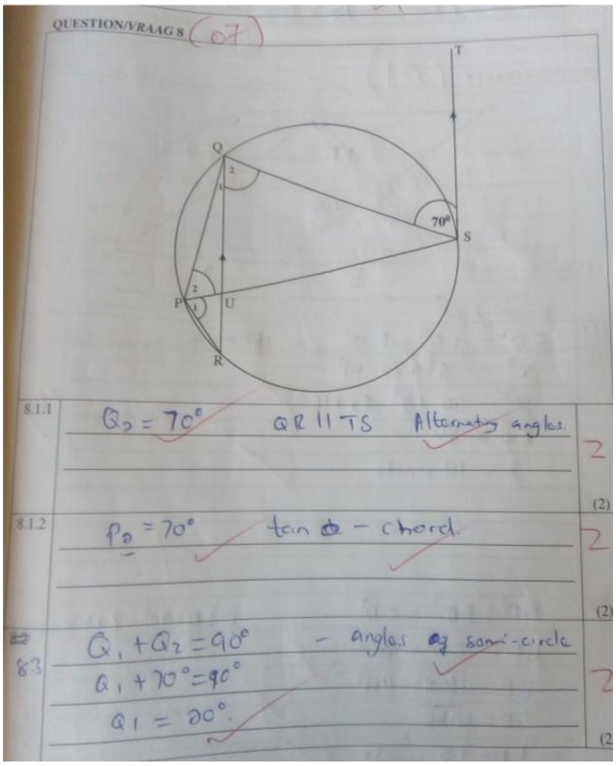
$\widehat{Q_1} = 20^0$

He stopped and looked at the teacher. Teacher: "That's good, go and sit down".

The teacher said that the group tried but they did not give enough reasons for their answers. He gave each group the opportunity to present their solution on the chalkboard but each group did not answer Q8.2. The teacher noted that no group could solve Q8.2 correctly. He then started with the correction, as he was asking leading questions in solving the problem from the study participants.

Figure 4.3 below is a sample of the work done by the group that presented.

Figure 4.3 – A Sample of a group's work

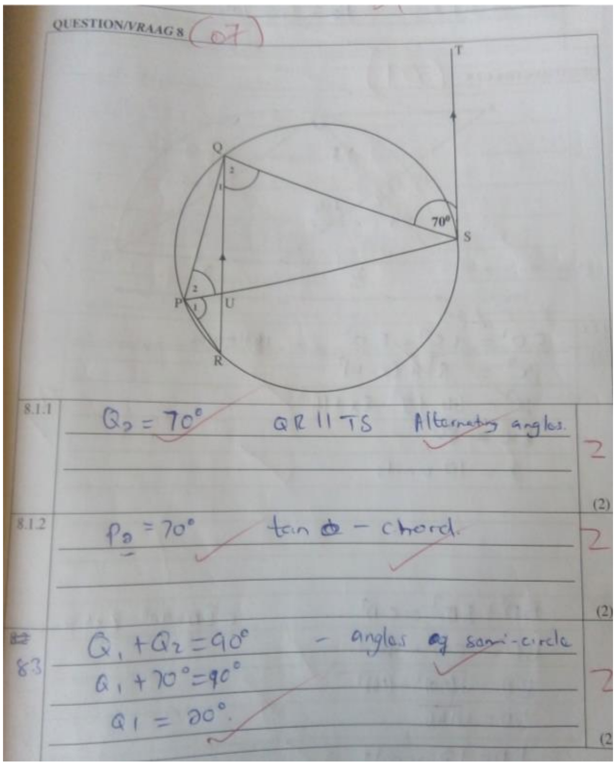


The teacher presented two more problems for the day's lesson, using the same teaching procedure before the siren for subject change sounded.

(ii) Results of the Class Exercises(CE) Data Analysis

The CE data were the photographs, video recordings and field note data of the relevant pages which showed step-wisely how the study participants provided answers to the classroom exercises in their notes book. The purpose of this data is to be able to analyse the study participants' knowledge construction when they were building their circle geometry conceptual understanding as they were introduced to new circle geometry concepts. Van Hiele's geometric learning levels hypothesis was used to measure the study participants' level of geometric knowledge demonstrated in the CE. With reference to the problems that are at Van Hiele's levels 1 and 2, the field note data analysis informs that most of the study participants were able to recognise the shape(s) and they discussed the properties of geometric shape(s). It also shows on the video recording how majority of the study participants were able to recognise and discuss the circle geometry shapes and their associated properties, though they could not do that convincingly. See an example of one of the classroom exercises that belong to Van Hiele's levels 1 and 2, and how it was approached, as shown in figure 4.4 below.

Figure 4.4 A sample of participants' work



In figure 4.4 the study participants were able to use appropriate properties to solve some of the questions but made mistakes in some other questions. This exercise was given in the third lesson of the first week of the intervention.

The result of the CE indicated that as the intervention progressed, many of the study participants were becoming more comfortable in the use of Polya's problem solving instructional approach, which is the intervention. Many participants were able to solve more complex problems, the problems that are categorised as level 4. That is, they were able to engage in abstract reasoning and deductive proofs to solve problems. This is demonstrated in figure 4.5 shown below.

Figure 4.5 A sample of a participant's work

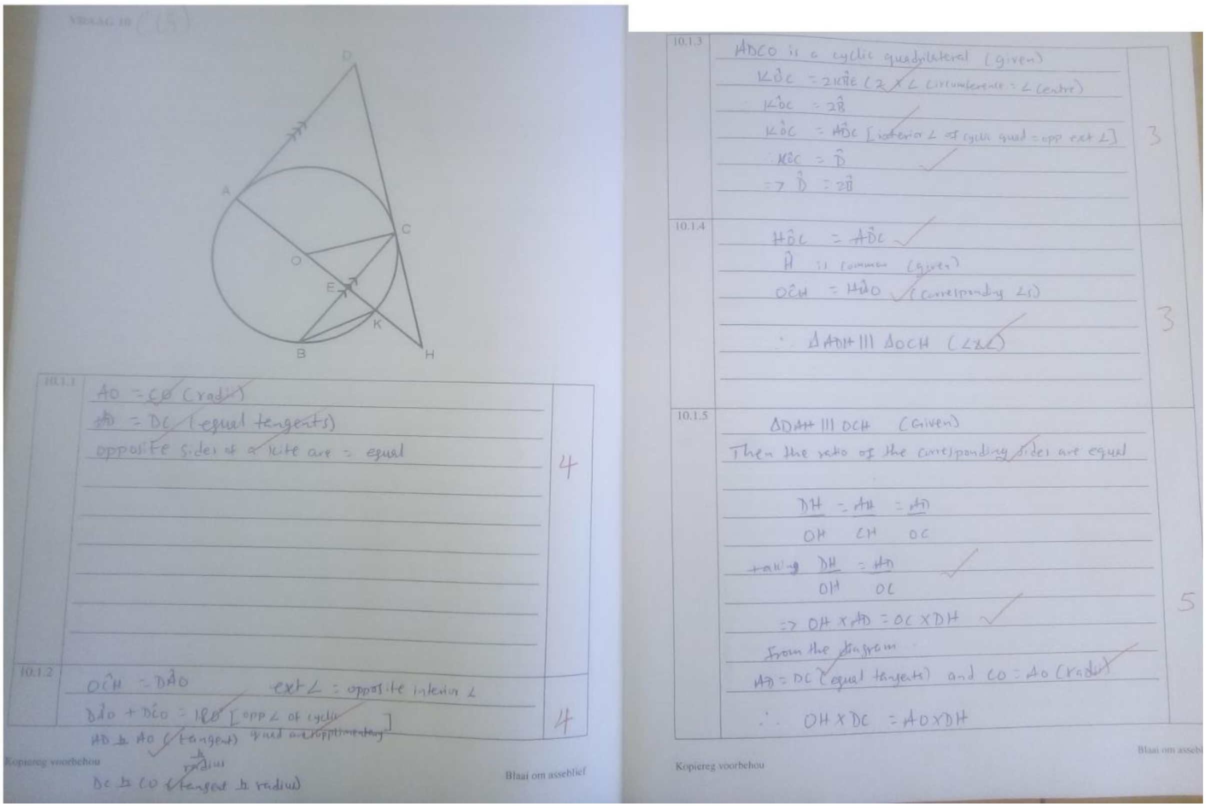
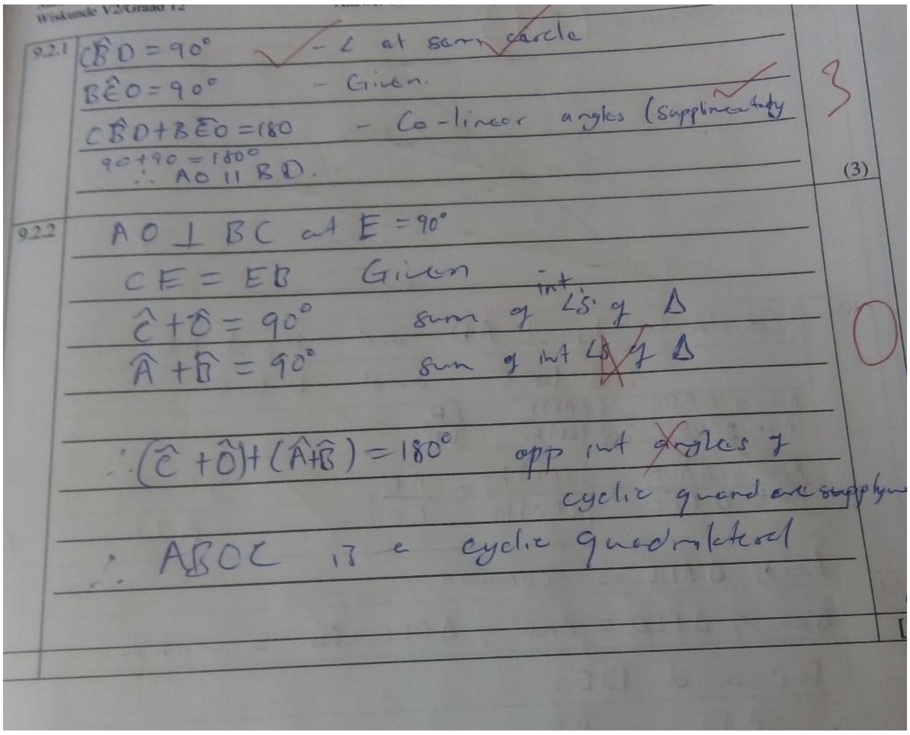


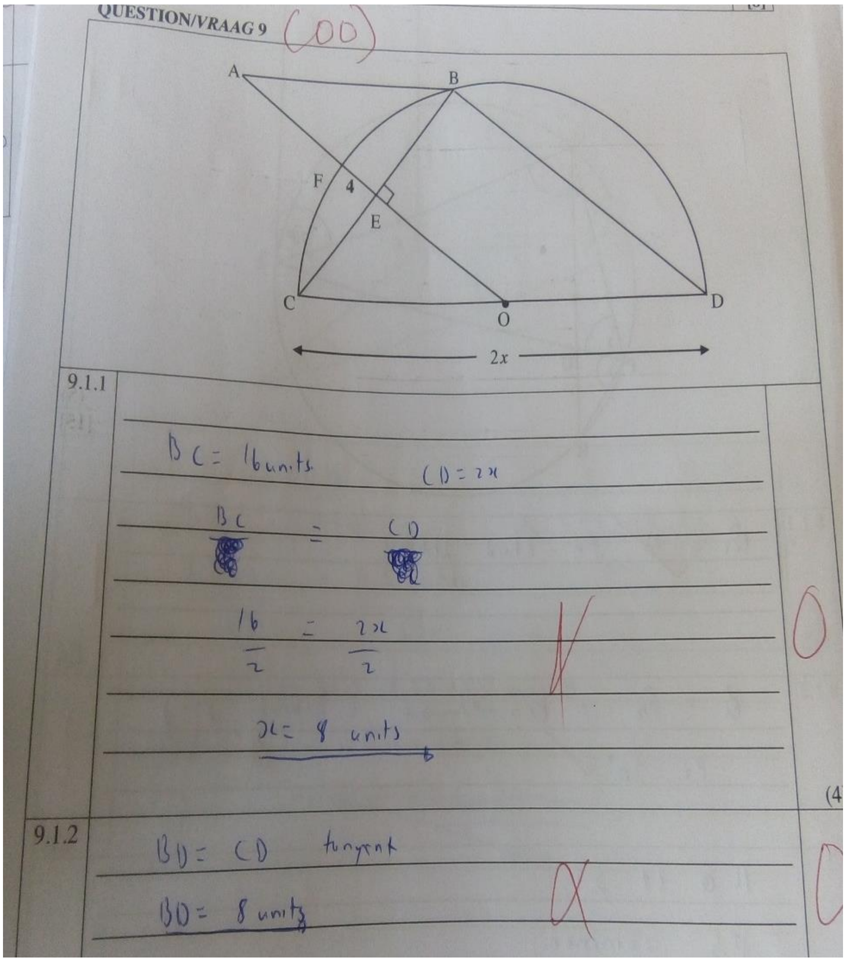
Figure 4.5 above shows how the study participants were able to solve the given problems using appropriate deductive reasoning. However, it was also discovered that in two groups (these groups were sitting at the back), they were really finding it difficult to solve problems that require deep deductive reasoning. The figure below gives an example of the work from one of the groups that were finding Van Hiele's level 4 difficult to conceptualize.

Figure 4.6 A Sample of a group's work



The example given in figure 4.6 above shows that in the group that produced the work, none of the four members of the group could reason to the level of the problem given. Another example is shown in figure 4.7 below:

Figure 4.7 A Sample of a group's work



The data analysis of the CE also show that some of the study participants were able to progress from Van Hiele's level 1 or 2 to Van Hiele's level 4. For instance, the figure 4.2 (the first CE example), almost all the study participants were still grabbling with the circle geometry concepts, while in the last example (figure 4.5), majority of the study participants were already comfortable with solving problems involving deductive reasoning which is Van Hiele's level 4.

(iii) EIT (2018) Data Analysis Results

At the end of the 2018 intervention, a comprehensive test (called EIT) that covered what was taught during the intervention was administered to the study participants. The purpose of the EIT instrument was to test how the intervention had impacted on the study participants' performance in solving circle geometry problems. The analysis was carried out in two parts: performance analysis and solution appraisal. The results of the data analysis on the study participants performance in the EIT test is presented first followed by the solution appraisal of how the study participants answered the question items in the EIT test.

(a) Results of the study participants' performance in the EIT test

In the 2018 intervention year, the study participants were 61 in number. Table 3.1 was used to analyse the study participants' performance in the EIT test, see the result of the analysis in appendix k. It emerged from the analysis that 1 (2%) of the study participants scored between 80% to 100%, 1 (2%) of the study participants scored between 70% to 79%, 4 (7%) of the study participants scored between 60% to 69%, 8 (13%) of the study participants scored between 50% to 59%, 12 (20%) of the study participants scored between 40% to 49%, 19(31%) of the study participants scored between 30% to 39%, and 16 (26%) of the study participants scored between 0% to 29%.

Further analysis revealed that 2 (3%) of the study participants scored between 70% to 100% which can be regarded as distinction, 12 (20%) of the study participants scored between 50% to 69% which can be regarded as average performance and 47(77%) of the study participants scored between 0% to 49% which may be regarded as failed in the EIT test. The extended analysis further show that 14(23%) of the study participants scored between 50% to 100% which is regarded as the number of the study participants that actually passed the EIT test, while 47(77%) of the study participants scored between 0% to 49% may be considered as failed the EIT test.

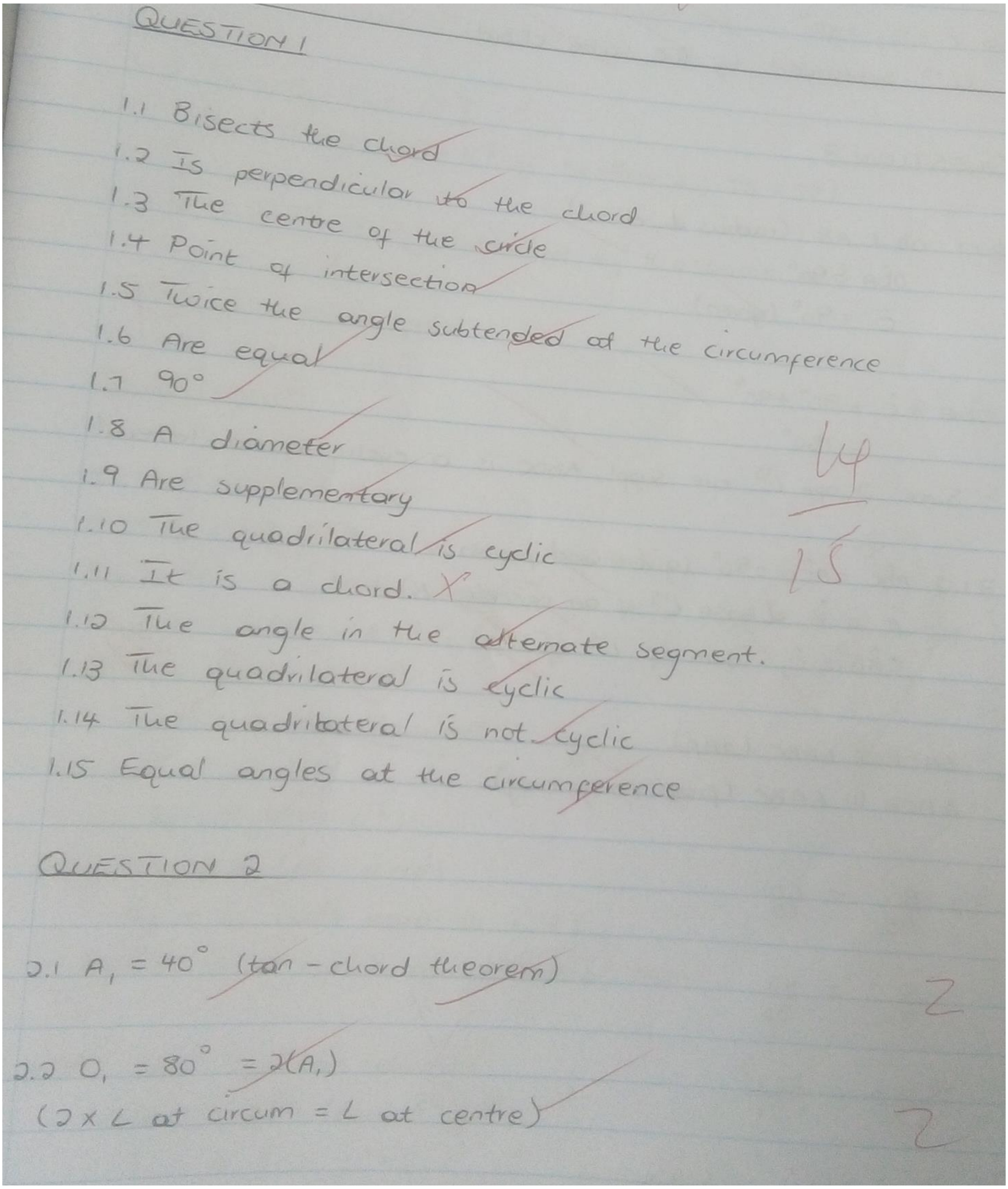
(b) Results of the solution Appraisal Data Analysis

How the study participants answered each question item in the EIT test was further scrutinised. Table 3.2 was used to analyse the difficulty level of each question item on the EIT instrument and how the study participants were able to answer each of these question items by grouping them.

In table 3.2, category A is the category of study participants' scripts which show evidence that they can identify and use correct geometric terminology in problem solving. Category B are the scripts which demonstrate that the study participants can conceptualize appropriate relevant properties, axioms and theorems in problem solving. Category C scripts implies that the study participants know how to follow appropriate techniques and skills in problem solving. Category D scripts have evidence that the study participants are capable of making connections across the geometric concepts to solve problems that are more complex.

The results from the data analysis show that 39(64%) of the study participants were categorized in category A, and 14(23%), 6(10%) and 2(3%) were found to belong to category B, C and D respectively. Figure 4.8 below, is a sample of how one of the study participants approached the EIT test. The study participant scored 62%.

Figure 4.8 Sample Script



QUESTION 1

1.1 Bisects the chord
1.2 Is perpendicular to the chord
1.3 The centre of the circle
1.4 Point of intersection
1.5 Twice the angle subtended at the circumference
1.6 Are equal
1.7 90°
1.8 A diameter
1.9 Are supplementary
1.10 The quadrilateral is cyclic
1.11 It is a chord.
1.12 The angle in the alternate segment.
1.13 The quadrilateral is cyclic
1.14 The quadrilateral is not cyclic
1.15 Equal angles at the circumference

QUESTION 2

2.1 $A_1 = 40°$ (tan - chord theorem)

2.2 $O_1 = 80° = 2(A_1)$
(2 × ∠ at circum = ∠ at centre)

(1)

2.3 $A\hat{B}E = 90°$
($\angle$ in a semi-circle)

2.4 $\hat{C} = E_2 = 50°$
($\angle$'s subtended by the same chord)

QUESTION 3

3.1.1 $OD \perp AB$ (radius $\perp$ chord)
$O\hat{D}A = 90°$
$\hat{C} = 90°$ (given)

$O\hat{D}A + \hat{C} = 90° + 90°$
$= 180°$
$\therefore$ Since 2 opp $\angle$'s are suppl, ADOC is a cyclic quadrilateral.

3.1.2 $A\hat{D}O = A\hat{C}O = 90°$ (given and proven)
$\hat{A}_1 = \hat{B}$ (base $\angle$'s of an isosceles Δ)
$C\hat{A}D = \hat{O}_2$ (sum of $\angle$'s of a Δ)

$\therefore \Delta AOD \equiv \Delta ABC$ (AAA)
$\therefore \Delta AOD ||| \Delta ABC$ (from congruency)

3.2 $\frac{BC}{AB} = \frac{AD}{OD}$

$\frac{BC}{8x} = \frac{4x}{5x}$

$\frac{BC\,5x}{5x} = \frac{32x^2}{5x}$

$\therefore BC = \frac{32x^2}{5x} = 6,4x$

(2)

QUESTION 4

4.1 $x = \hat{R}$ (tan-chord theorem)
$x = Q_1$ (base ∠'s of an isosceles Δ)
$x = T_2$ (corr ∠'s, PT ∥ SR)

6/7 4

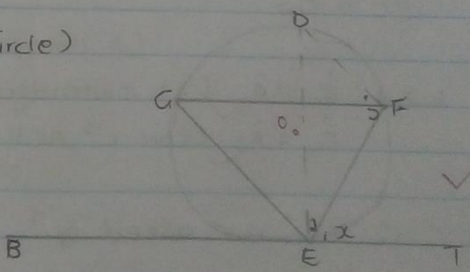
4.2 $T_2 = S_3$ [alt ∠'s]
$S_3 = R = x$ [base ∠'s of isosceles Δ ; TS = TR)
∴ $T_2 = x$
$Q_1 = T_2 = x$ (∠'s subtended by the same arc) 2

∴ PQTS is a cyclic quad

QUESTION 5

5.1 let $F\hat{E}T = \hat{E}_1 = x$
$F_1 + F_2 = 90°$ [∠'s in a semi-circle)
$E_2 = 90° - x$ [Radius ⊥ tan]
$\hat{D} = x$ (sum of ∠'s of a Δ)
$\hat{G} = x$ (tan-chord)
∴ $F\hat{E}T = \hat{G}$; as required

13/21 7

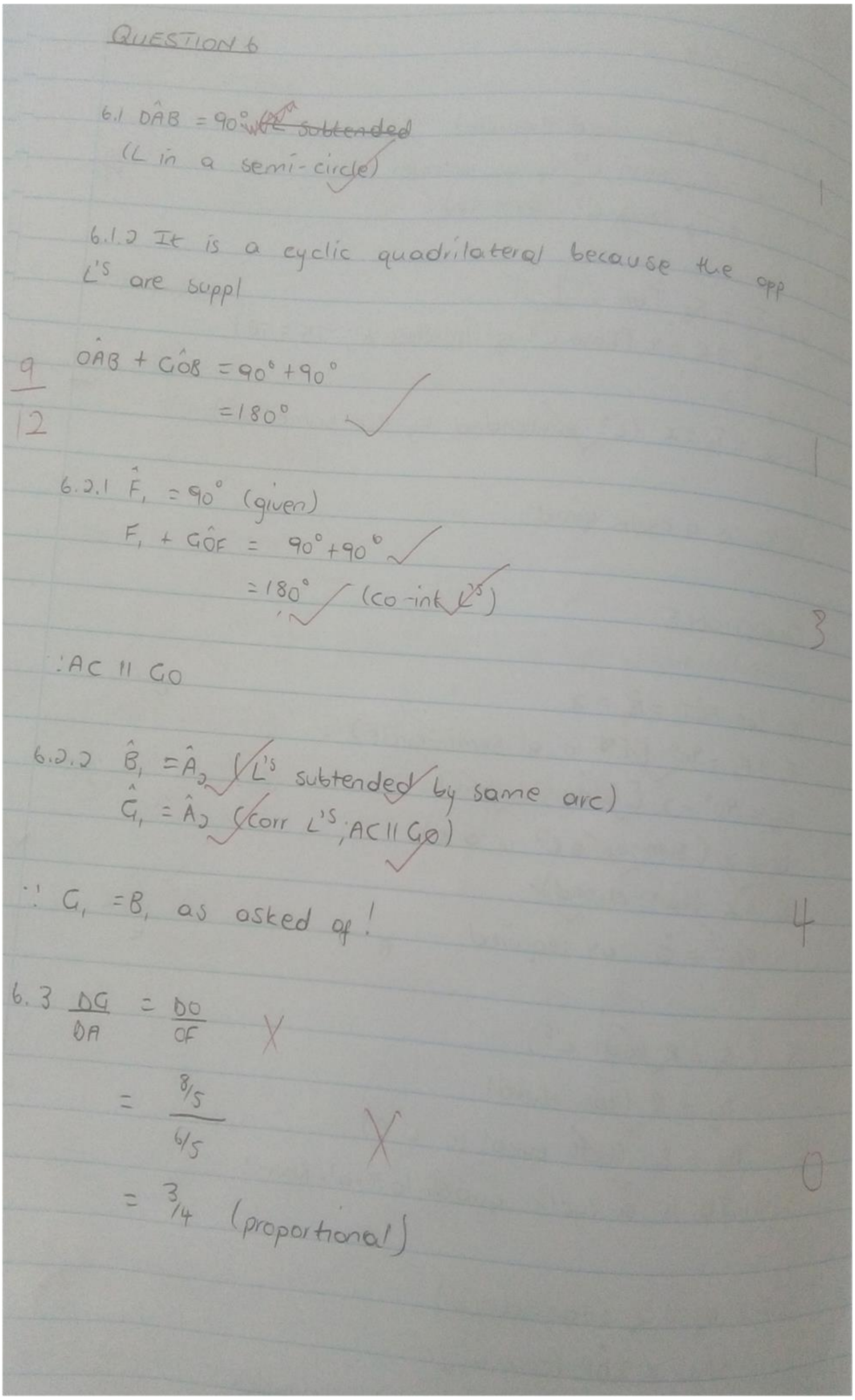


5.2.1 $S_4 = R$ (corr ∠'s)
$D_3 = R$ (tan-chord)
∴ $D_3 = S_4$ (both equal to ∠ R) 3
∴ SWTD is a cyclic quadrilateral. Reason?

5.2.2 $D_3 = S_2$ (tan-chord)
$S\hat{P}W = R\hat{D}P$ (corr ∠'s)
$S_2 = D_1$ (alt ∠'s) Incomplete 3

5.2.3 x x x 0

(3)



QUESTION 6

6.1 $D\hat{A}B = 90°$ ~~w.r.t subtended~~
($\angle$ in a semi-circle) 1

6.1.2 It is a cyclic quadrilateral because the opp $\angle$'s are suppl

$O\hat{A}B + G\hat{O}B = 90° + 90°$
$= 180°$ 1

9/12

6.2.1 $\hat{F}_1 = 90°$ (given)
$F_1 + G\hat{O}F = 90° + 90°$
$= 180°$ (co-int $\angle$'s) 3

$\therefore AC \parallel GO$

6.2.2 $\hat{B}_1 = \hat{A}_2$ ($\angle$'s subtended by same arc)
$\hat{G}_1 = \hat{A}_2$ (corr $\angle$'s; $AC \parallel GO$)

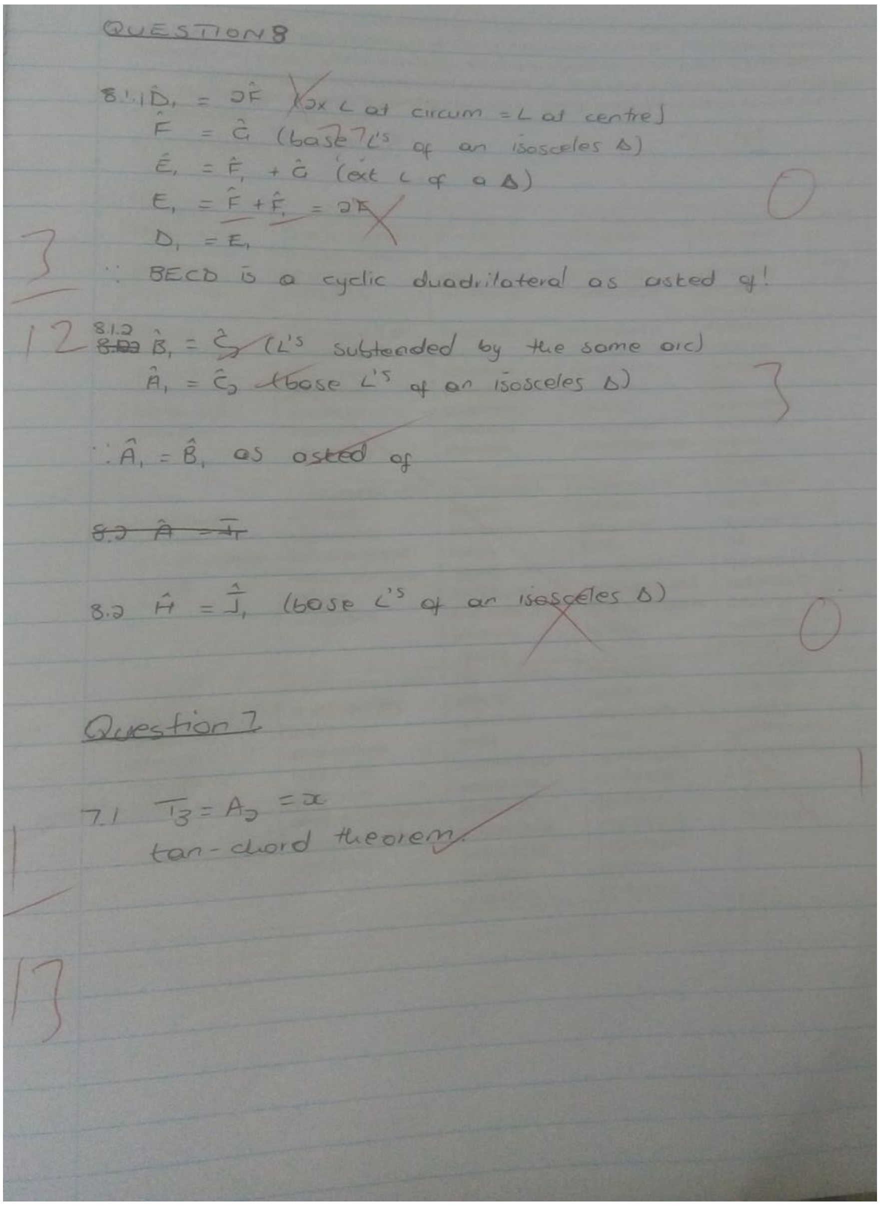
$\therefore G_1 = B_1$ as asked of! 4

6.3 $\frac{DG}{DA} = \frac{DO}{OF}$ ✗

$= \frac{8/5}{6/5}$ ✗

$= 3/4$ (proportional) 0

(4)


QUESTION 8

8.1.1 $\hat{D}_1 = 2\hat{F}$ (2x ∠ at circum = ∠ at centre)
$\hat{F} = \hat{G}$ (base ∠'s of an isosceles Δ)
$\hat{E}_1 = \hat{F}_1 + \hat{G}$ (ext ∠ of a Δ)
$E_1 = \hat{F} + \hat{F}_1 = 2\hat{F}$
$D_1 = E_1$
∴ BECD is a cyclic quadrilateral as asked of!

8.1.2 $\hat{B}_1 = \hat{C}_2$ (∠'s subtended by the same arc)
$\hat{A}_1 = \hat{C}_2$ (base ∠'s of an isosceles Δ)

∴ $\hat{A}_1 = \hat{B}_1$ as asked of

~~8.2 $\hat{A} = \hat{J}_1$~~

8.2 $\hat{H} = \hat{J}_1$ (base ∠'s of an isosceles Δ)

Question 7

7.1 $T_3 = A_2 = x$
tan-chord theorem

(5)

The study participant was analysed to have achieved van Hiele level 3 which means that the study participant is in abstract stage in geometric knowledge construction. It was analysed that the study participant provided answers to EIT question items 1, 2, 3 and 4 very well which earned the study participant $\frac{14}{15}$, $\frac{9}{15}$, $\frac{7}{11}$ and $\frac{6}{7}$ respectively (see figure 4.8 numbered 1 and 2). However, from question items 5 upward the study participant could not show much understanding of the concepts. The study participant scored $\frac{13}{21}$, $\frac{9}{12}$, $\frac{1}{13}$ and $\frac{3}{13}$ in question items 5, 6, 7 and 8 respectively (see figure 4.8 numbered 3 and 5).

4.2.2.2 2019 Intervention year Data Analysis Results

In this research, the researcher used intervention repetition method in the same research field, the same teacher participant but different set of study participants. It should be noted that the only change in 2019 intervention is the set of participants. Thus, all the set of data collected in 2018 were also collected for 2019. For the purpose of emphasis, the data collected were: classroom teaching proceedings' video recordings, photograph of learners' class exercises and field notes and the end-of-the-Intervention Test. Results of the data analysis follow the same format as the researcher presented the 2018 intervention year data analysis results in subsection 4.2.2.1 above.

(i) Results of the classroom observation

The intervention for the 2019 school year started on the 12th of February, 2019 and ended on the 5th of March, 2019. The researcher will like to note that the 2018 intervention was conducted in the 3rd term of the 2018 school year while the 2019 intervention was conducted in the 1st term of 2019 school year. This was because, the Northern Cape Department of Education shifted the teaching of circle geometry to the first term of each school year, which was to take effect from the 2019 academic year, as contained in the work schedule for FET (grades 10-12) mathematics released to all schools prior to the beginning of the 2019 academic year. Another major difference in the 2019 intervention was that the total number of study participants were 45 (grade

11 A had 23 learners while grade 11B had 22 learners) as compared to 61 for the 2018 intervention year. Besides the differences mentioned above between the 2018 and the 2019 intervention year, all other occurrences were similar. Thus the researcher shall not bore the readers with presenting the classroom procedures again. The results of the data from the video recordings showed that the study participants were very enthusiastic, showed a lot of interest and commitment in the learning of circle geometry.

(ii) Results of the Classroom Exercises(CE) Data Analysis

As was done in the 2018 intervention year, the teacher always pose a leading question for the study participants to discuss and solve while the teacher goes round the groups to improve on the groups' discussion or to correct their knowledge construct. It emerged from the CE data analysis that most of the learners were able to show good problem solving approach in providing solution to the classroom exercises given in the class as a result of the intervention. They were able to recognise and discuss circle geometry shapes and their associated properties. Figure 4.9 and 4.10 are sample CE scripts.

Figure 4.9 Sample CE Scripts

Question 1

1.1 Bisect the chord

1.2 perpendicular to the chord

1.3 The midpoint of AB

1.4 point of intersation

1.5 Half the angle of the centre

1.6 Are equal

1.7 90°

1.8 is a diameter

1.9 Supplemetary

1.10 is a cyclic quadrileteral

1.11 The line is a chord

1.12 To the opposite intirior angla

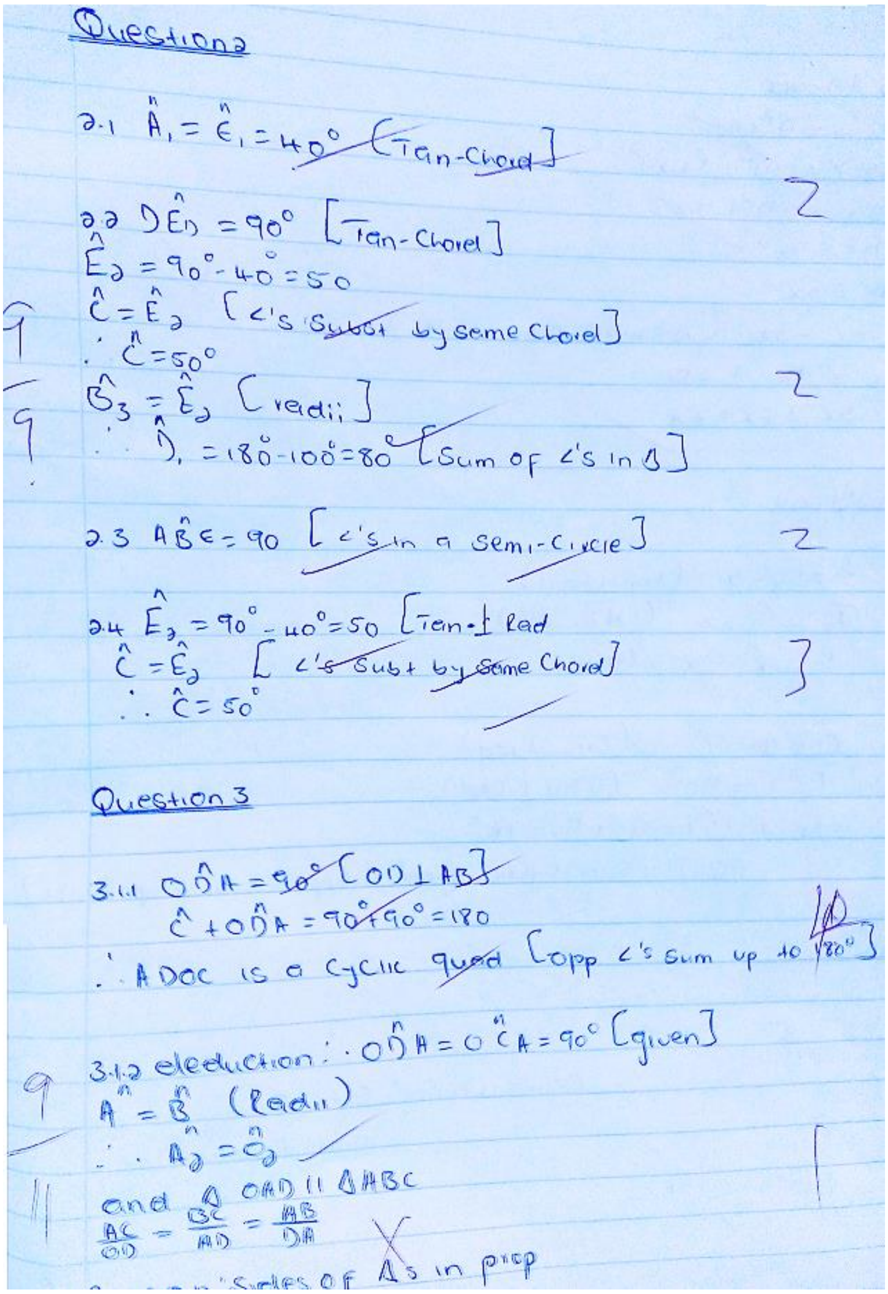
1.13 is a cyclic quadrilateral

1.14 is not a cyclic Quadrileteral

1.15 eqaul angles

12/15

Figure 4.10 Sample CE Scripts


Question 2

2.1 $\hat{A}_1 = \hat{E}_1 = 40^\circ$ [Tan-Chord]

2.2 $D\hat{E}D = 90^\circ$ [Tan-Chord]
$\hat{E}_2 = 90^\circ - 40^\circ = 50$
$\hat{C} = \hat{E}_2$ [∠'s Subt by same Chord]
$\therefore \hat{C} = 50^\circ$
$\hat{B}_3 = \hat{E}_2$ [radii]
$\therefore \hat{D}_1 = 180^\circ - 100^\circ = 80^\circ$ [Sum of ∠'s in Δ]

2.3 $A\hat{B}E = 90$ [∠'s in a semi-circle]

2.4 $\hat{E}_2 = 90^\circ - 40^\circ = 50$ [Tan ⊥ Rad
$\hat{C} = \hat{E}_2$ [∠'s Subt by same Chord]
$\therefore \hat{C} = 50^\circ$

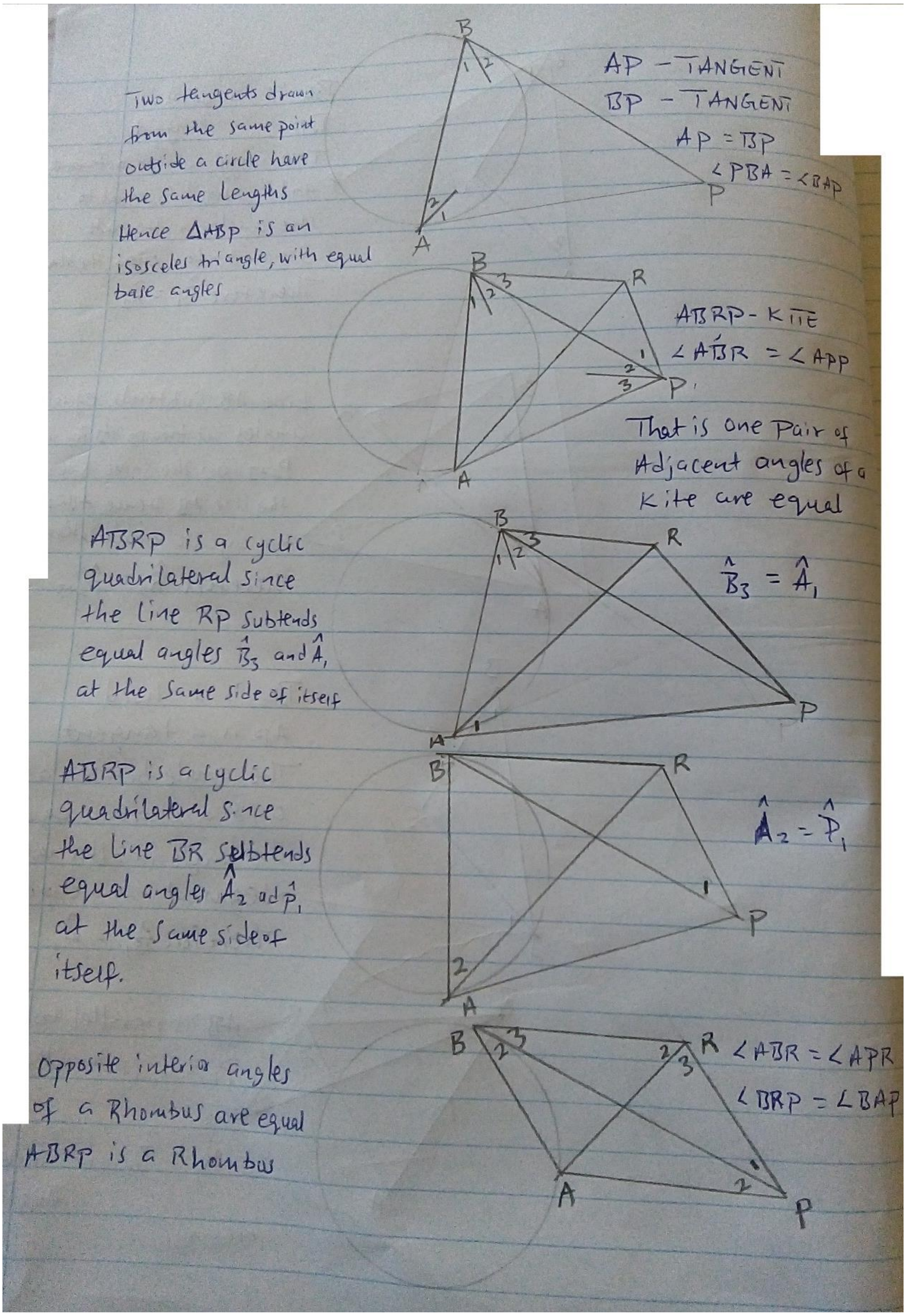
Question 3

3.1.1 $O\hat{D}A = 90^\circ$ [OD ⊥ AB]
$\hat{C} + O\hat{D}A = 90^\circ + 90^\circ = 180$
∴ ADOC is a cyclic quad [opp ∠'s sum up to 180°]

3.1.2 deduction: ∴ $O\hat{D}A = O\hat{C}A = 90^\circ$ [given]
$\hat{A} = \hat{B}$ (radii)
$\therefore \hat{A}_2 = \hat{O}_2$
and Δ OAD ||| Δ ABC
$\frac{AC}{OD} = \frac{BC}{AD} = \frac{AB}{DA}$
sides of Δs in prop

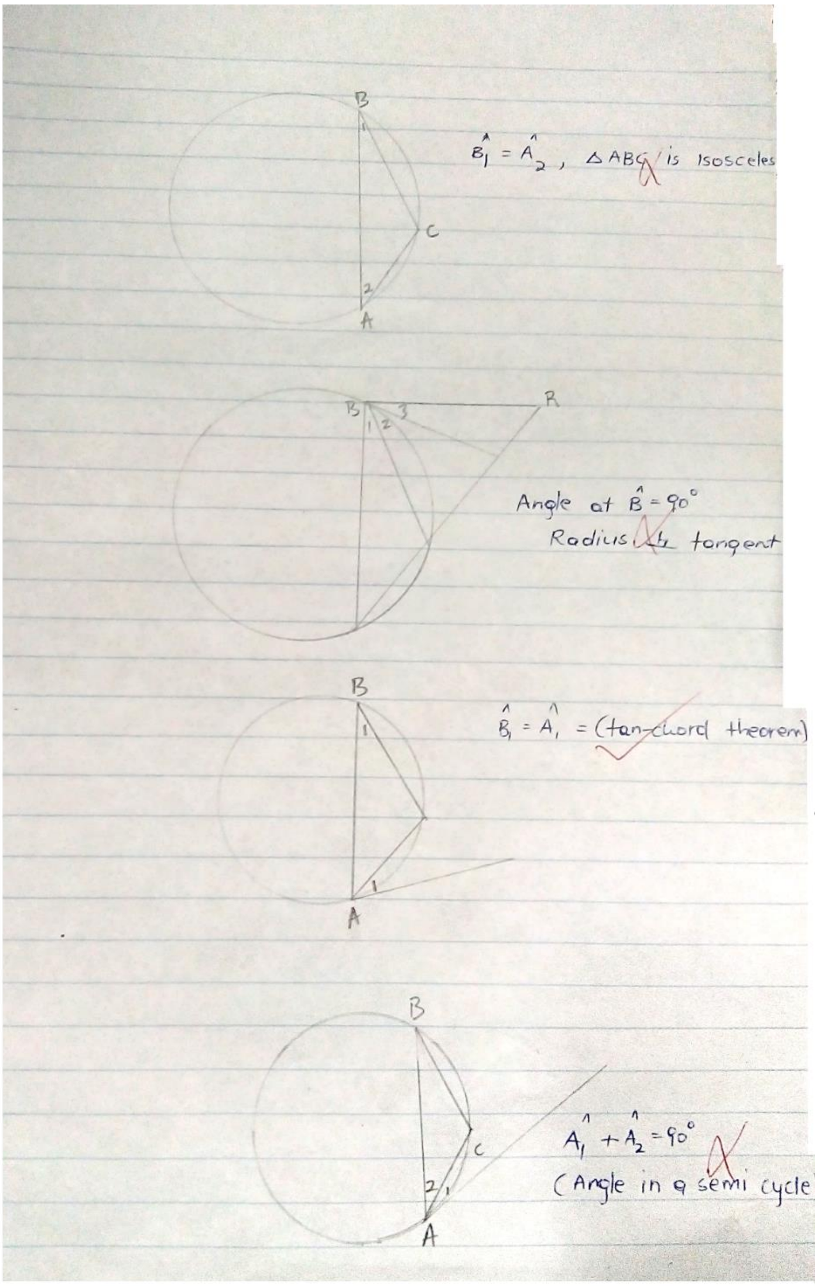
It can be seen in figures 4.9 and 4.10 above that the study participants could appropriately conceptualize Van Hiele's levels 1 and 2. This was presented in the second lesson of the first week of intervention. The results of the analysis of the CE data shows that the study participants were progressing in their circle geometry knowledge acquisition. This was supported by the data analysis of the video recording of the fourth lesson in the second week of the intervention. The teacher presented a problem on the chalkboard and figures 4.11 & 4.12 below are some of the students' responses.

Figure 4.11 Best group work



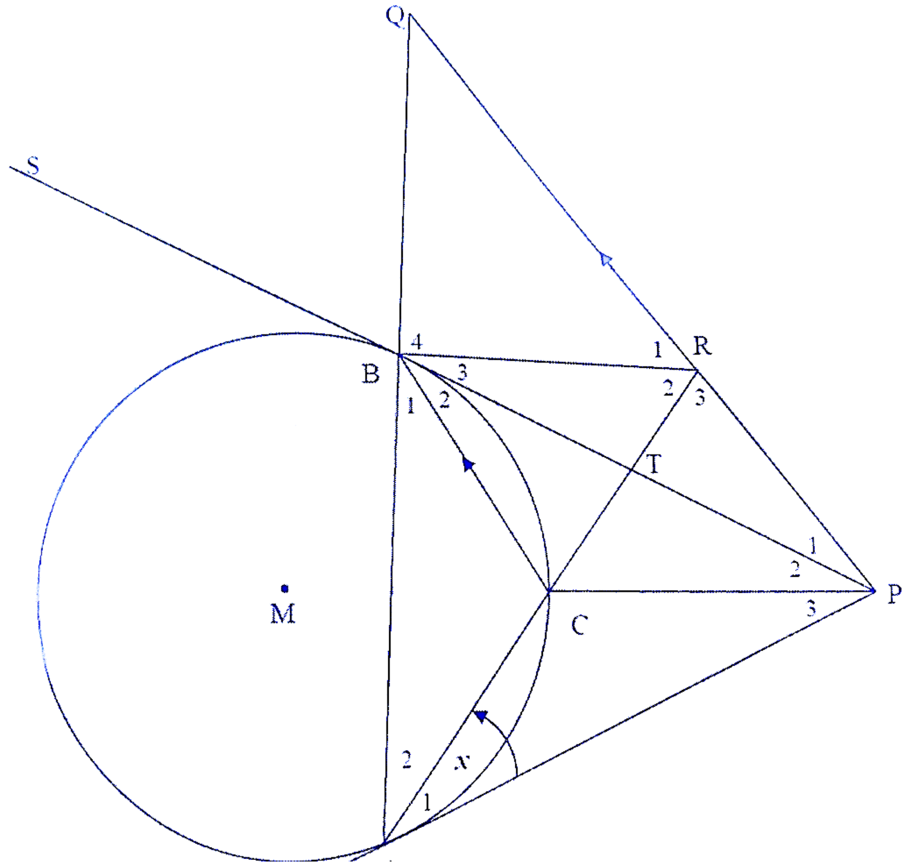
As presented above, the best group work provided nine circle geometry theorems or concepts. Averagely, the other groups provided five circle geometry theorems or concepts. The worst group work provided four circle geometry theorems or concepts, with only one of the four theorems provided, which was correct, as illustrated below:

Figure 4.12 Worst group work



However, it also emerged as the intervention progressed, and as the complexity level of the circle geometry increased, some of the study participants in various groups and the entire group members in one of the groups, were getting confused and demotivated. For example, in the fifth lesson of 2nd week, the problem in figure 4.13 below was given to be solved.

Figure 4.13 Problem solving question



Questions

1.1 Prove that $\widehat{P_2} = \widehat{P_3}$ (6)

1.2 ABRP is a cyclic quadrilateral (4)

1.3 PRQ is a tangent to circle BCR at R (4)

The results of the data analysis of the study participants approach showed that they were struggling to answer the questions in this problem. They could not even write anything before the siren sounded for the change of period, hence, the solution to this problem was not conclusive. It was on a Friday, mathematics lesson was a single period (30 minutes).

(iii) EIT (2019) Data Analysis Results

At the end of the 2019 intervention, the EIT test was administered to the study participants. The results of the data analysis on the study participants' performance in the EIT test is presented first, followed by the solution appraisal of how the study participants approached the EIT test items, as they were presented for the 2018 intervention results.

(a) Results of the study participants' performance in the EIT test.

The presentation of the results of performance from the data analysis for the 2019 study participants is similar to that of 2018 intervention data analysis presentation as shown in subsection 4.2.2.1. The number of study participants that took the EIT test were 45 and the data were analysed the same way as the performance data for the 2018 intervention year.

The results showed that 0 (0%) of the study participant scored between 80% to 100%, 2 (4%) of the study participants scored between 70% to 79%, 6 (13%) of the study participants scored between 60% to 69%, 5 (11%) of the study participants scored between 50% to 59%, 9 (20%) of the study participants scored between 40% to 49%, 13(29%) of the study participant scored between 30% to 39%, and 10 (22%) of the study participants scored between 0% to 29%, (see appendix J).

More in-depth analysis showed that 2 (4%) of the study participants scored between 70% to 100% which can be regarded as distinction, 11 (24%) of the study participants scored between 50% to 69% which is regarded as average performance while 32(71%) of the study participants scored between 0% to 49% which may be regarded as failed in the EIT test. More detailed analysis informs that 13(29%) of the study participants scored between 50% to 100% which is regarded as the number of the study participants that passed in the EIT, and 32 (71%) of the study participants scored from 0% to 49%, which is regarded as the number of study participants that failed the EIT test. See appendix k for detailed analysis result.

(b) Results of the Solution Appraisal Data Analysis

This part of the results scrutinised how the study participants were able to answer each question item in the EIT instrument. This is done by scrutinizing the solution approach and also, following the thinking between the presentations of each answer by each of the study participants. The EIT instrument items were categorised base on the difficulty level of each instrument item (see table 3.2). Table 3.2 composition was enumerated under 4.2.2.1. The results showed that 27(60%) of the study participants belong to category A, 13 (29%) of the study participants belong to category B, 4(9%) of the study participants belong to category C, and 1 (2%) of the study participant belong to category D.

CHAPTER FIVE

DISCUSSION OF THE FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

This chapter presents the findings in this study. The findings were discussed against in view of the research questions and the existing literature. However, before presenting the findings, the researcher summarises the study in other to remind the readers of the processes that led to the findings in this study.

5.1 Summary of study

This study investigates South Africa's mathematics learners' problem solving skills in Euclidean geometry with focus on circle geometry. The study employed NOCSR design to carry out the study. The intervention used for the study was the teaching of circle geometry with social constructivist instructional approach coupled with Polya's problem solving instructional approach. The research field is a secondary school in the Northern Cape Province in South Africa. The investigation was carried out in two years: 2018 and 2019. The study participants were 61 (27 boys and 34 girls) and 45 (20 boys and 25 girls) in the 2018 and 2019 intervention years respectively. In each year of the research intervention, the following data were collected: classroom teaching proceedings' video recordings, photograph of learners class exercises (CE), field notes and the end-of-the- Intervention Test (EIT). The classroom observations and field notes data analysis involved transcribing the video recorded data and merging it with the field notes data, categorised, coded and themes were developed. The themes that emerged were noted. The CE data were analysed through solution appraisal and the EIT data were analysed through performance analysis and solution appraisal. The results of the various analysis were presented in chapter four of this dissertation. The findings that emerged from this study are presented in 5.2 below.

5.2 Research Findings

The results of the data analysis for both intervention years (2018 and 2019) are comparatively similar. This gives some credibility to the study. Thus, the researcher may not necessarily discuss the findings from the results separately. The following findings emerged from the results of the data analysis of the study:

(1) The intervention arouse the study participants' interest in the learning of circle geometry.

(2) The intervention improved the classroom dynamics during the mathematics lessons.

(3) The study participants' individual problem solving skills was improved.

(4) The study participants' individual performance in circle geometry was improved.

However, the following sub-findings also emerged from the results of the data analysis:

(1) Study participants were demotivated when only academically weak study participants formed a group.

(2) Unruly classroom activities were encouraged when only notorious study participants formed a group.

The reader should note that the researcher considered learning motivation, improved interest in learning, and improved classroom dynamics as improved learning indicators.

5.3 Discussion of the Findings

The findings shall be discussed in view of the research questions stated in subsection 1.7 of this dissertation.

(i) How does the intervention influence the study participants' conceptual understanding of circle geometry?

The intervention motivated the study participants to learn circle geometry. This was demonstrated during the classroom observations as it arouse the study participants' interest and improved classroom dynamics. The "interest" and "classroom dynamics" dimensions were used to show the influence of the intervention on the learning of circle geometry.

The Interest Dimension

The findings from the study show that the intervention arouse the study participants' interest in the learning of circle geometry. The classroom observation results show how the study participants were always enthusiastic and their facial expression implied that they were happy each time they sat in their group(s) during the intervention. This type of countenance shows that the study participants were showing interest in the learning of the circle geometry concepts.

Fischer, Dobbs – Oates, Doctoroff & Arnold (2012) inform that learners' interest in the core learning improves academic achievement. This might be the reason the study participants could participate in their group problem solving discussion and presentation of solutions on the chalkboard. The classroom exercise (CE) results show that most of the groups' attempts in solving circle geometry problems were good and some of them were very good, see figure 4.3, page 57. This kind of interest in the learning of geometry was contrary to what the teacher participant said he usually experience in his mathematics classroom before the intervention, subsection 4.1.2. In subsection 4.1.2 the teacher expressed how the learners have been showing disinterest in the learning of mathematics, especially geometry concepts. They were not kin in attending mathematics classes. He mentioned that this might be contributing to their poor performance in mathematics, most especially the geometry aspect of mathematics.

The Classroom Dynamics Dimension

The findings from the study also show that during intervention there were always good classroom dynamics. The classroom observation results show that the study participant were always involved in group knowledge construction. Each of the study participants put up ideas as a result of their individual mathematical reasoning. Mason et al (2015) refer to this as mathematical thinking and Kashefi et al (2013) informs that mathematics thinking is a complex activity in the process of acquiring mathematical knowledge and skills. Perhaps that is why Piaget (1980) and Vygotsky (1978) informs that students' ability in problem solving depends on their level of cognitive ability. Study participants discussing and comparing their mathematical mental reasoning might have also helped them to understand and for them to be able to solve the circle geometry problems both in the class as a group and in the EIT test as individual. The findings are in line with the literature, Vygotsky (1978) refer to this as zone of proximal development (ZPD). The improved classroom dynamics witnessed in this study is also contrary to the traditional mathematics instruction classroom, including the previous mathematics lessons the teacher participants taught before the intervention, where the learners only sit to listen to the teacher, with little or no contribution to whatever the teacher say. Khan (2012) informs that in traditional mathematics classrooms, mathematics students are like empty knowledge seekers while the teacher is the direct and unilateral instructor. Kahn mentioned that in such a mathematics class, students do not have the opportunity to initiate, question, to argue their personal thought or interact with other learners. Stofflett (1998) described a traditional mathematics instructor as a body of knowledge that must be taken without question. The traditional mathematics instruction classroom dynamic is very poor. Maybe that is why traditional mathematics learners show no interest in the learning of mathematics and always perform very poor in geometry.

However, despite the findings that study participants were motivated to learn circle geometry, it also emerged in the findings that grouping academically weak study participants in the same group may demotivate them to learn circle geometry. The results (part of the 2018 results) show that from the group this was noted, the study participants in this group were not showing much interest nor been enthusiastic in the learning of circle geometry. During one of the classroom presentations, the teacher made the group to present their solution to the given problem on the chalkboard. This was shown in figure 4.2. Though, Vygotsky and Piaget advocate peer learning, Vygotsky emphasis peers that are more knowledgeable. In addition, demotivation also emerged in another group where the group members were behaving unruly. This came up in the 2018 results. The members of this group were usually not discussing circle geometry concepts but other things. The video-recorded data captured a moment as the teacher was approaching this group, all the group members pretended to be working on the problem given to be solved and as soon as the teacher left their group they put aside their class mathematics book and continued with their private discussion. In this type of grouping, the group members can learn nothing because learning might have been impaired.

(ii) How does the intervention influence the study participants' problem solving skills in the learning of circle geometry?

The study participants' individual problem solving skills improved. The intervention, which coupled the Polya problem solving instructional approach with the constructivism instructional approach, improved how the study participants approach solving circle geometry problems. The researcher compared the classroom learning attitude of the study participants (as testified by the study participants' mathematics teacher, see subsection 3.1.0 paragraph two). For example see figure 4.8, most of the reasoning and the problem solving skills demonstrated were quite good. This is just one of many of such good works. The study participants were sitting in groups and this gave them the opportunity to interact and compare their individual knowledge constructs. In going about solving a circle geometry problem, participants individually try to understand the problem to be solved, and go into mathematical reasoning on the solution approach, then present individual mental construct on the solution approach within their groups. The group discuss improved how each of them might have thought of solving the problem. Again, the impact of these communal solution approach was evident in the analysis of the solution approach in the EIT test where each study participant wrote the test individually. This is a great improvement on the learning of the geometry in this research field, subsection 4.1.2 gave a picture of how the teacher participant's previous mathematics classroom learners will only sit and watch him do everything, while they were reluctant to come to the board to present solution to any problem.

The researcher wish to remind readers that table 3.2 was used to analyse the study participants' level of solution approach skills acquired during intervention in the EIT test. The table categorised the EIT instrument according to the level of complexity of the EIT items. This category runs from A to D, where A is at the visual level and D is being able to solve complex problems. From the 2018 study participants, they were categorised as follows: 64%, 23%, 10% and 3% of the study participants for categories A, B, C, and D respectively, while for 2019, it was 60%, 29%, 9% and 2% of the study participants for categories A, B, C, and D respectively.

The skill levels (expressed as percentage) acquired in both 2018 and 2019 are comparable. If Van Hiele's geometric learning model is applied, then this will translate to more than 80% of the study participants in each year acquiring between Van Hiele's levels 2 and 3, which means that more than 80% of the study participants in each year understand basic properties of geometric figures, can form abstract definitions, and sufficient conditions for circle geometry concepts. About 9% of the study participants can be placed between Van Hiele's Levels 3 and 4, which means that 9% of the study participants in each year can conceptualize formal proofs, axioms and theorems. Also, about 3% of the study participants in each year achieved problem solving skills between Van Hiele's levels 4 and 5, which implies that this set of study participants were able to solve complex circle geometry problems. This was achieved within the three week intervention period. Perhaps if the intervention period was longer, we might have witnessed better improvement in the individual study participants circle geometry problem solving skills. A sample of the study participants' script is given in figure 4.8. The figure shows that, the study participants demonstrated good skills in solving question items 1 to 6 but failed to demonstrate good problem solving skills in question items 7 and 8. Though there is no evidence of any research in which Polya and social constructivism instructional approaches were coupled in a single research, this finding is in line with Chien (2015), reported in subsection 2.3 of this dissertation, in which Polya's instructional approach was used to strengthen learners' problem solving skills.

(iii) How does the intervention influence the study participants' performance in the learning of the concepts of circle geometry?

The study participants' individual performance in circle geometry improved. The previous performance of learners in mathematics was examined by the researcher before the start of the intervention as stated in subsection 4.1.2 of this dissertation. Though it was difficult to measure the performance of learners in circle geometry for the period of eight years the teacher participant have been teaching mathematics at the research field, the general performance in mathematics have been very poor.

The summary of the study participants' performance in the EIT test which was conducted after the intervention shows that for the 2018 intervention year, 14(23%) of the study participants passed the EIT test with the marks ranging from 50% to 100% and 47 (77%) of the study participants failed the EIT test with the marks ranging from 0% to 49%. Further analysis on the number of study participants that failed the EIT test shows that 16(26%) of the study participants scored marks ranging from 0% to 29%, while 31(51%) of the study participants scored marks ranging from 30% to 49% in the EIT test.

Similarly, for the 2019 intervention year, 13(29%) of the study participants passed the EIT test with marks ranging from 50% to 100%, while 32(71%) of the study participants failed the EIT test with marks ranging from 0% to 49%. Further analysis on the study participants that failed the EIT test revealed that 10(22%) of the study participants scored marks ranging from 0% to 29% in the EIT test, while 22(49%) of the study participants scored marks ranging between 30% to 49% in the EIT test.

The first observation the researcher made from these results is the close correlation in the 2018 and 2019 marks scored (expressed as percentage) by the study participants. It should be noted that the two groups were taught the same contents by the same teacher. The only variable that changed were the set of study participants. This gives a level of credibility to the findings emanating from this study. Secondly, it was noted that the number of study participants that scored between 0% to 29% in the 2018 and 2019 intervention years were 16(26%) and 10(22%) respectively. This translate to about 80% of the study participants who acquired levels 2 and 3 of Van Hiele's learning levels in circle geometry problem solving skills that was reported in subsection 5.3.

It was also noted that 31(51%) and 22(49%) of the study participants scored marks between 30% to 49% in the EIT test for the 2018 and 2019 intervention years. This implies that most of the study participants are in this category. Perhaps, with more time, they may improve beyond the 30% to 49% category. Also, some of the study participants had distinctions in the EIT test: these are 2(3%) and 3(4%) of the study participants for the 2018 and 2019 intervention years respectively.

In subsection 4.1.2, the teacher participant illustrated how learners in grade 11 were failing mathematics for the past eight years he had been teaching mathematics in the research field before the intervention particularly the geometry aspect of the mathematics curriculum. Hence, the results enumerated above is an improvement compared to the previous performance of learners in mathematics.

5.4 Conclusion

It was explained in chapter one of this dissertation how South African learners have been demonstrating poor understanding of mathematics concepts, more particularly in Euclidean geometry. It is against this background this study was conceptualised. The study investigated the influence the use of Polya's problem solving instructional approach coupled with social constructivism instructional approach may have on the learning of Euclidean geometry with a particular focus on circle geometry. The findings that emerged from the study show that the study intervention motivated the study participants in the learning of circle geometry. In addition, it also emerged among other findings that the study participants' performance in circle geometry was improved.

As mentioned in section 1.0, the Department of Basic Education (DBE) have been changing the educational curriculum frequently over the years, of which the concern of this study is mathematics curriculum, obviously searching for a curriculum that may facilitate the learning of mathematics concepts, more particularly the teaching and learning of geometry. The researcher belief that the instructional approach used in the study may be an antidote for the poor mathematics understanding of Euclidean geometry concepts. The DBE may want to adopt the instructional approach proposed in this study for the teaching of geometry in South African Schools.

5.5 Recommendation

As a result of the findings that emerged from this study, the researcher wish to recommend as follows:

- The researcher noted that further research is needed to investigate the effect of the coupled Polya problem solving instructional approach and social constructivism instructional approach on the teaching of other aspect of mathematics.

- However, in implementing the coupled Polya problem solving instructional approach and social constructivism instructional approach, teachers should allocate learners to groups by themselves. If learners are allowed to pick and choose their group members, it may result in grouping academically weak learners in the same group. It was revealed in this study that such groups may not be able to conceptualise mathematical concepts properly as a result of possible anti-progressive academic knowledge construction. In addition, notorious learners in the class may group themselves together. This will similarly lead to anti-progressive academic knowledge construction.

- This innovative tested teaching approach is proposed for the teaching of geometry in South African schools, in place of the traditional teaching approach that is prevalent in teaching mathematics in South African schools.

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APPENDIX A

EIT INSTRUMENT

QUESTION 1

Complete the following statements by filling in the missing words:

1.1 The line drawn from the centre of a circle , perpendicular to the chord …
1.2 The line drawn from the centre of a circle to the midpoint of a chord …
1.3 If PQ is a perpendicular bisector of chord AB, then PQ passes through …
1.4 If PQ and JK are the perpendicular bisectors of any two non-parallel chords on the same circle, then PQ and JK will intersect each other and the centre of that circle will lie on their …
1.5 The angle subtended by a chord at the centre of the circle is …
1.6 The angles subtended by a chord in the same segment of the circle …
1.7 The angle subtended by a diameter on the circumference of a circle is always equal to …
1.8 If a chord subtends a right angle on the circumference of a circle, then the chord is …
1.9 The opposite angles of a cyclic quadrilateral …
1.10 If the opposite angles of a quadrilateral are supplementary, then …

1.11 If a line subtends equal angles at two points on the same side of itself, then …

1.12 The angle between a chord and a tangent is equal to …

1.13 If the exterior angle of a quadrilateral is equal to the interior opposite angle, then …

1.14 If the exterior angle of a quadrilateral is not equal to the interior opposite angle, then …

1.15 Equal chords subtends … (1 ×15 = 15)

QUESTION 2

In the diagram, O is the centre of circle ABEC. AOE is a straight line. DE is a tangent to the circle at E. $\widehat{E_1} = 40^0$.

Determine, with reasons, the magnitude of the following:

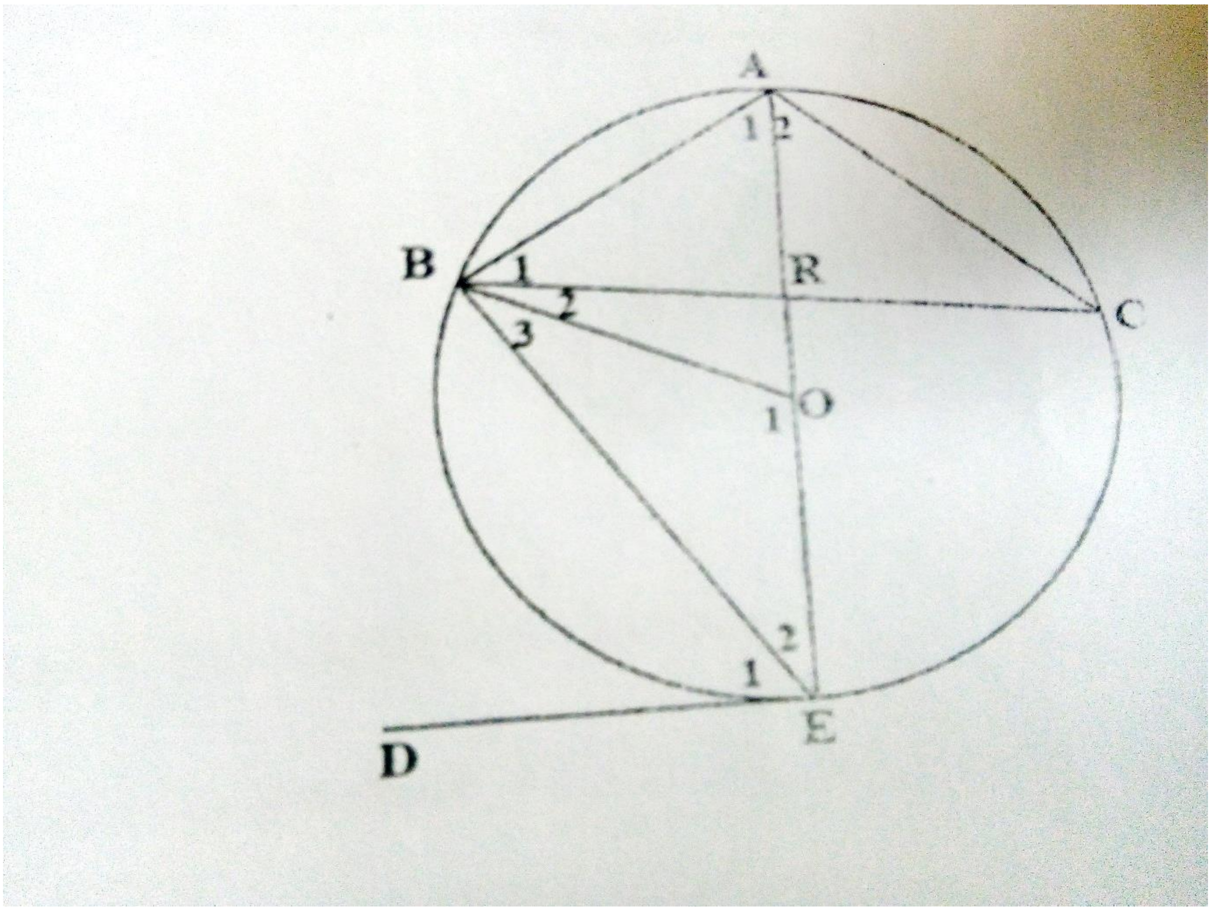
2.1 $\widehat{A_1}$ (2)

2.2 $\widehat{O_1}$ (2)

2.3 A$\hat{B}E$ (2)

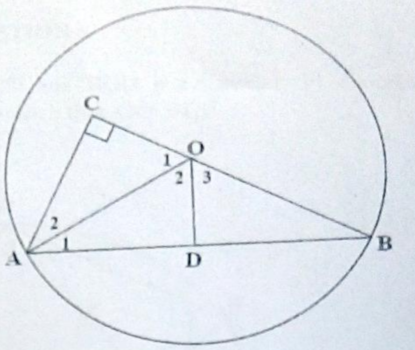
2.4 $\hat{C}$ (3)

[9]



QUESTION 3

In the diagram alongside, AB is a chord of the circle with centre O. BO is produced to C such that AC ⊥ BC and AD = DB



3.1 Prove that:

3.1.1 ADOC is a cyclic quadrilateral (4)

3.1.2 $\Delta OAD \parallel\!| \Delta ABC$ (3)

3.2 If the radius of the circle is $5x$ units and the length of chord AB is $8x$ units, determine the length of BC in terms of x. (4)

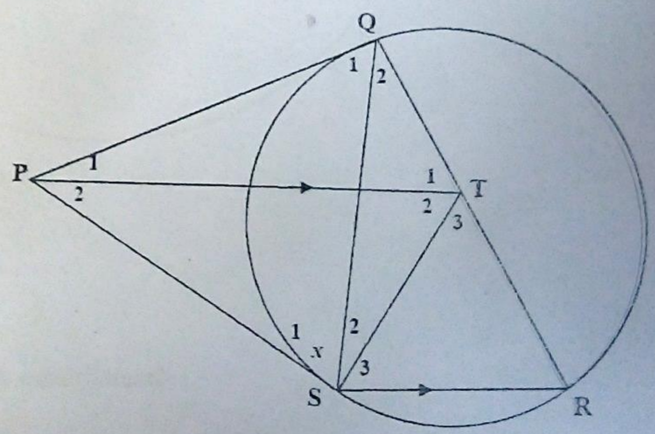
[11]

QUESTION 4

In the diagram alongside, circle QRS has tangents PQ and PS.

PT || SR with T on QR

$\hat{S}_1 = x$



4.1 Name, with reasons, THREE other angles each equal to x. (5)

4.2 Prove that: PQTS is a cyclic quadrilateral (2)

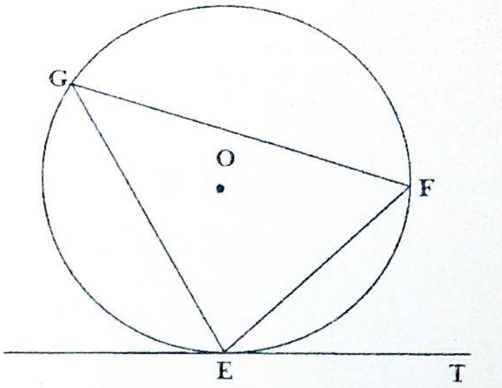
[7]

QUESTION 5

5.1 In the diagram alongside, O is the centre of circle GEF.

Use the diagram on DIAGRAM SHEET 2 or redraw the diagram in your ANSWER BOOK to prove the theorem which states that:

If ET is a tangent to the circle, then $\hat{FET} = \hat{G}$



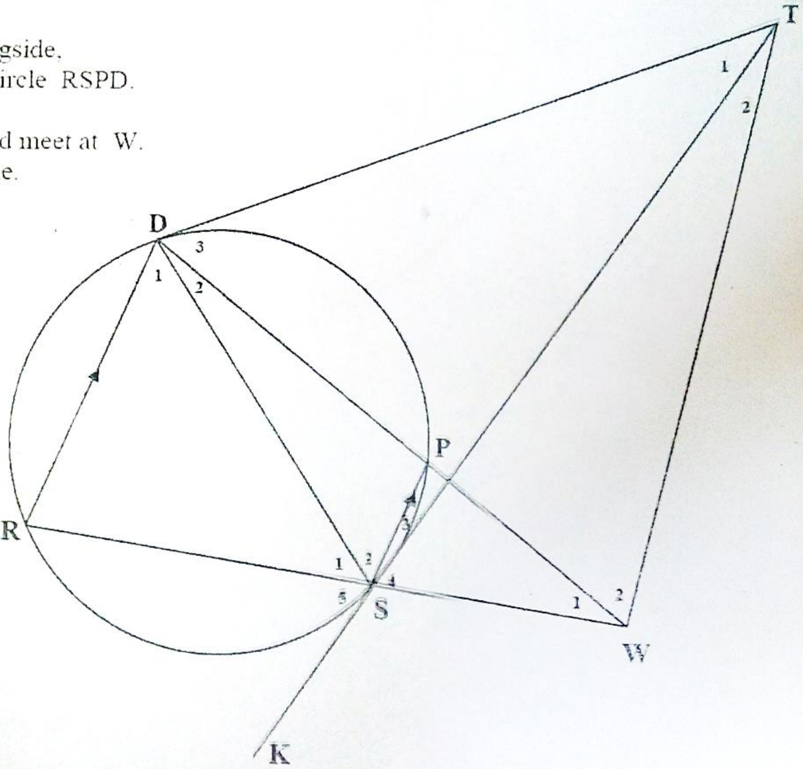
(7)

5.2 In the diagram alongside, TD is a tangent to circle RSPD.

RS and DP produced meet at W.
KST is a straight line.

$\hat{S}_4 = \hat{S}_3$

DR || PS



Prove that:

5.2.1 SWTD is a cyclic quadrilateral (4)

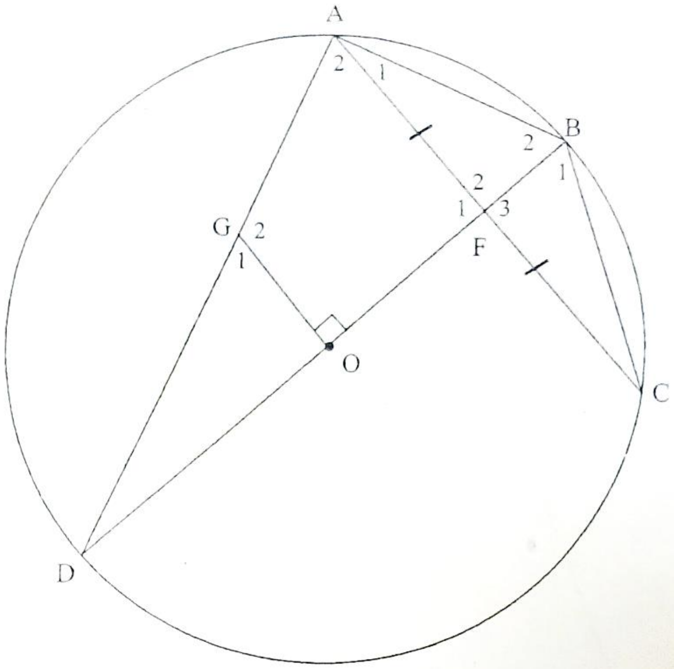
5.2.2 TSK is tangent to circle RSPD (4)

5.2.3 TW || PS (6)

[21]

QUESTION 6

In the diagram, O is the centre of circle ABCD and BOD is a diameter. F, the midpoint of chord AC, lies on BOD. G is a point on AD such that GO ⊥ DB.



6.1 Give a reason why:

6.1.1 $D\hat{A}B = 90°$ (1)

6.1.2 AGOB is a cyclic quadrilateral (1)

6.2 Prove that:

6.2.1 AC || GO (3)

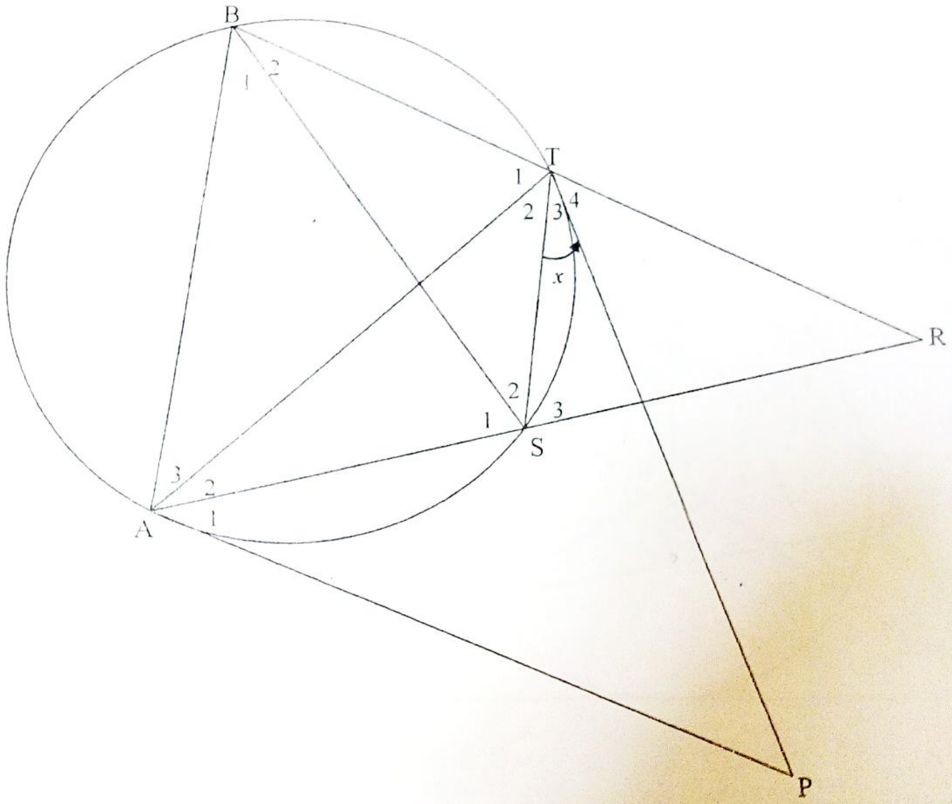
6.2.2 $\hat{G}_1 = \hat{B}_1$ (4)

6.3 If it is given that $FB = \frac{2}{5}r$, where r is the radius of the circle, determine, with reasons, the ratio of $\frac{DG}{DA}$. (3)

[12]

QUESTION 7

In the diagram, PA and PT are tangents to a circle at A and T respectively. B and S are points on the circle such that BT produced and AS produced meet at R and BR = AR.
BS, AT and TS are drawn. $\hat{T}_3 = x$.



7.1 Give a reason why $\hat{T}_3 = \hat{A}_2 = x$. (1)

7.2 Prove that:

7.2.1 AB || ST (5)

7.2.2 $\hat{T}_4 = \hat{A}_1$ (5)

7.2.3 RTAP is a cyclic quadrilateral (2)

[13]

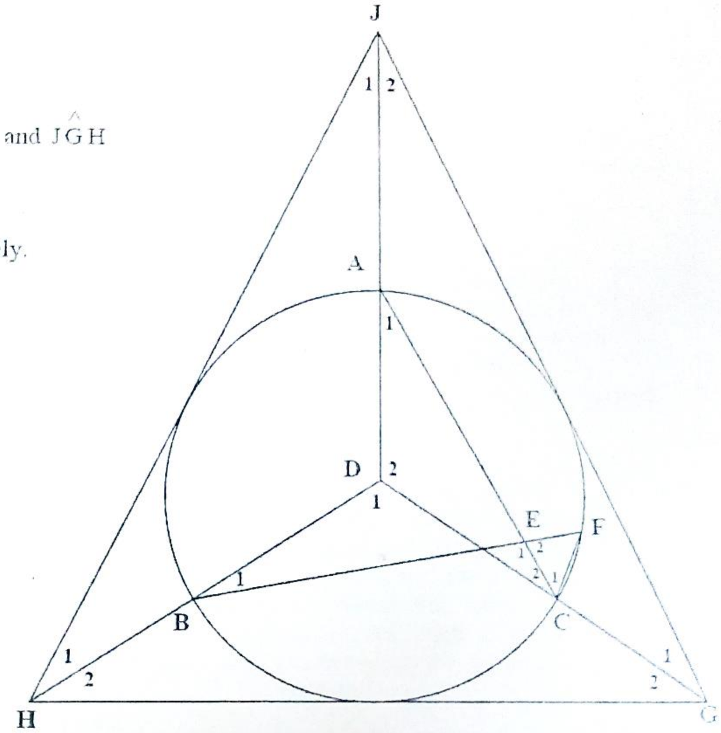
QUESTION 8

In the diagram alongside, circle BCFA with centre D is inscribed in Δ JHG.

The bisectors of angles $J\hat{H}G$ and $J\hat{G}H$ intersect at D.

JD, HD and GD intersect the circle at A, B and C respectively.

Chord AC intersects chord BF at E such that EF = EC



8.1 Prove that:

8.1.1 BCED is a cyclic quadrilateral (5)

8.1.2 $\hat{B}_1 = \hat{A}_1$ (3)

8.2 If $\hat{H}_1 = x$ and $\hat{G}_1 = y$, express $\hat{J}_1$ in terms of x and y. (4)

[12]

TOTAL MARKS: 100

APPENDIX B

MEMORANDUM FOR QUESTION 1 OF THE EIT INSTRUMENT

QUESTION 1

1.1 Bisects the chord
1.2 Becomes perpendicular to the chord/ meets the chord at 90^0
1.3 The centre of the circle
1.4 Point of intersection
1.5 Twice the angle at the circumference of the circle
1.6 Are equal
1.7 90^0
1.8 A diameter
1.9 Are supplementary/ add up to 180^0
1.10 The quadrilateral is cyclic

1.11 The quadrilateral is cyclic

1.12 Angle in opposite/ alternate segment or interior opposite angle

1.13 The quadrilateral is cyclic

1.14 The quadrilateral is not cyclic

1.15 Equal angles at the circumference of the circle

APPENDIX C

PROBLEM SOLVING RUBRIC (PSR) INSTRUMENT

This rubric was generated using the knowledge dimension in learning and van Hiele's learning hierarchy. The knowledge dimensions are factual knowledge; conceptual knowledge; procedural knowledge and meta-cognitive knowledge, which are represented in the rubric as geometric terminology; identify appropriate axioms; geometric properties and theorems; follow appropriate skills and techniques in problem solving and connect various geometric concepts to solve more complex problems respectively.

STUDENT NAME……………………………….. DATE………………………………….

SKILLS	Rating = 4	Rating = 3	Rating = 2	Rating = 1	Score
Identify and using correct geometric terminology	Participant is able to apply correct terminology adequately	Participant is fairly able to apply correct terminology	Participant apply terminology inadequately	Participant could not remember geometry terminology	
Identify appropriate relevant properties, axioms and theorems	Participant is able to identify the appropriate properties, axioms or theorem adequately	Participant is able to somehow identify the properties, axioms or theorem	Participant is inadequately able to identify the properties, axioms or theorem but mixed them up	Participant could not remember the appropriate properties, axioms or theorem	
Follow appropriate techniques and skills in problem solving	Participant is able to follow appropriate techniques and skills in problem solving adequately	Participant is able to somehow follow appropriate techniques and skills in problem solving	Participant is not able to adequately follow appropriate techniques and skills in problem solving	Participant could not remember the techniques and skills to follow in problem solving	
Making connections across the geometry concepts to solve more complex problems	Participant is able to adequately make connections across the geometry concepts to solve more complex problems	Participant is fairly able to make connections across the geometry concepts to solve more complex problems	Participant could not adequately make connections across the geometry concepts to solve more complex problems	Participant is not able to make any connection between the circle geometry concepts	
				TOTAL SCORE	…../16

APPENDIX D

FACE VALIDATION FORM FOR THE EIT INSTRUMENT

The research instrument under consideration is designed to measure participants' problem solving skills in circle geometry concepts. In order to have the appropriate items on the research instrument so that our desired aim of determining the study participants' problem solving skills would be achieved, your assistance is highly needed in this regard.

Please rate each item on the EIT instrument on how well, the items are structured, using the scale below:

1= Not well structured   2= somewhat well-structured   3= very well structured

Further comment(s) if any

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Personal information of Evaluator

Qualification: ………………………………   Status: …………………………………

Signature: …………………………………   Date: ………………………………….....

APPENDIX E

CONTENT VALIDATION FORM FOR THE EIT INSTRUMENT

The research instrument under consideration is designed to measure participants' problem solving skills in circle geometry concepts. In order to have the appropriate items on the research instrument so that our desired aim of determining the study participants' problem solving skills would be achieved, your assistance is highly needed in this regard.

Please judge each item on the EIT instrument on its level of relevance and level of appropriateness to grade 11 learners as a test in circle geometry. In addition, please judge the EIT instrument on how well it covers the grade 11 circle geometry content. You are required to use the scale below:

Level of Relevance

1= Low/not relevant 2= somewhat relevant 3= highly relevant

Level of Appropriateness

1= Not appropriate 2= somewhat appropriate 3= highly appropriate

Level of content covered

1= Not well covered 2= somewhat well covered 3= Very well covered

Further comments(s) if any …………………………………………………….................

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Personal information of Evaluator:

Qualification: ………………………. Status: ………………………………

Signature: …………………………… Date: ……………………………….

APPENDIX F

VALIDITY FORM FOR THE PSR INSTRUMENT

This study examined “problem solving skills in circle geometry concepts in Euclidean Geometry”. To answer the research questions, a problem solving rubric was developed to measure participants’ problem solving abilities. The rubric was developed using the knowledge dimension in learning and van Hiele’s learning hierarchy.

Your assistance is highly needed to ensure that the description of the ratings for each problem solving skill indicated on the rubric can serve its desired purpose of determining the extent of competence of each problem solving strategy on the rubric. Please judge each item on the PSR instrument on how well the item is well structured by using the scale below:

1= Not well structured　　2= somewhat well-structured　　3= very well structured

Further comment(s) if any

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Personal information of Evaluator

Qualification: ………………………………… Status: ………………………………………..

Signature: …………………………………. Date: ……………………………………..

APPENDIX G

LEARNER'S CONSENT FORM

Title of study: Problem solving skills in circle geometry concepts: A Case Study of a High School in the Northern Cape Province (South Africa).

Dear respondents,

The researcher for this research study is FITZGERALD ABAKAH, a postgraduate student of the Institute of Science and Technology Education of the University of South Africa. This research study is mainly for academic purposes and all forms of information regarding your identification would be handled confidentially. We therefore require you to participate in this research as it would in the long run, contribute significantly in your understanding of Circle Geometry concepts.

Please kindly note that, you are not been forced in any way to take part in this research study. You may voluntarily, participate in this research, though we recommend to you to take part in this academic research and we assure you that your decision to participate in this research would never be in vain.

TO BE COMPLETTED BY RESPONDENTS

I …………………………………………..willingly agree/disagree to serve as a participant for this research study. I understand that this is an academic research and thus, I voluntarily participate in this research at my own risk, with any form of indemnity involved. Hence I would not hold the researcher or authorities of the school responsible in lieu of any damages or unforeseen circumstances that may occur. As I have willingly accepted to take part in this research, I pledge to be of good behaviour and cooperate fully to achieve the desired outcomes of this research study.

……………………………………………. ……………………………….

(Signature of respondent) Date

APPENDIX H

PARENT'S CONSENT FORM

Title of study: Problem solving skills in circle geometry concepts: A Case Study of a High School in the Northern Cape Province (South Africa).

Dear Parent/Guardian,

The researcher for this research study is FITZGERALD ABAKAH, a postgraduate student of the Institute of Science and Technology Education of the University of South Africa. This research study is mainly for academic purposes and all forms of information regarding your identification would be handled confidentially. We therefore require you to allow your child/dependant to participate in this research as it would in the long run, contribute significantly in their understanding of mathematics.

Please kindly note that, you are not been forced in any way to permit your child/dependant to take part in this research study.

TO BE COMPLETTED BY PARENT/GUARDIAN

I …………………………………… willingly agree/disagree to permit my child/dependant to serve as a participant for this research study. I understand that this is an academic research and thus, I would not hold the researcher or authorities of the school responsible in lieu of any damages or unforeseen circumstances that may occur. As I have willingly accepted to allow my child/dependant to take part in this research, I pledge to encourage my child/dependent to be of good behaviour and cooperate fully to achieve the desired outcomes of this research study.

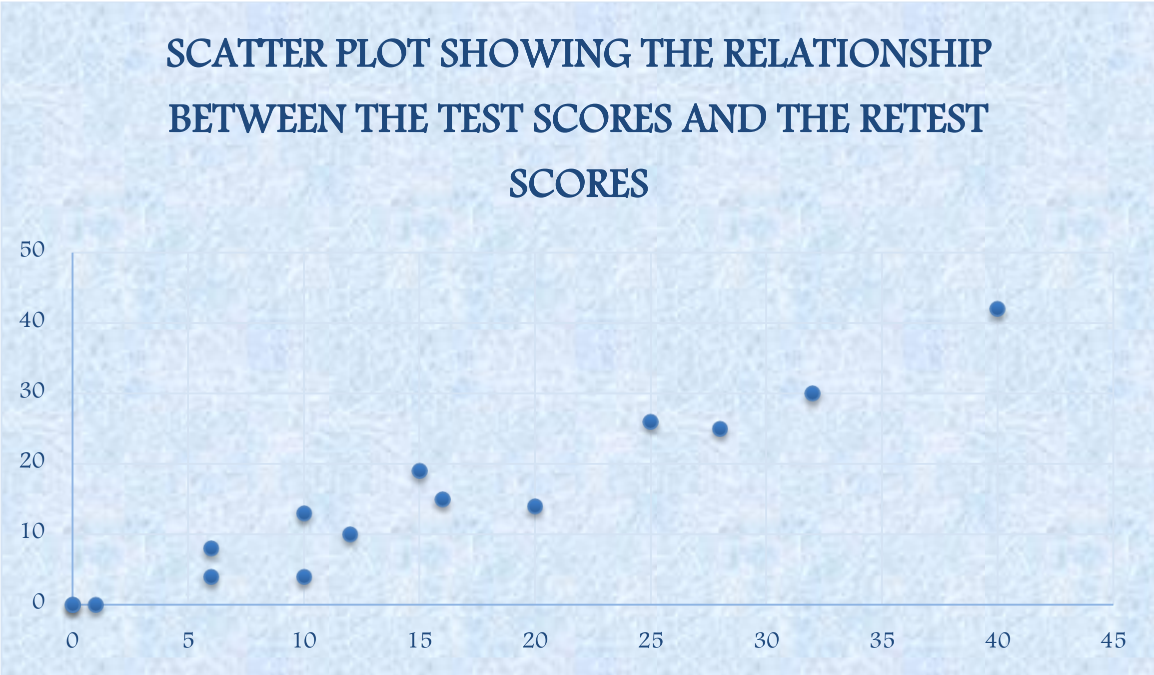
…………………………………………….   ……………………………….

(Signature of parent/guardian)   Date

APPENDIX I

RESULT OF THE TEST AND RE-TEST RELIABILITY TEST

TEST SCORES	32	15	0	6	12	25	40	16	20	10	1	0	6	10
RETEST SCORES	30	19	0	8	10	26	42	15	14	13	0	0	4	4



TEST SCORES

The upward direction flow of the scores plotted on the scatter diagram above indicates a positive correlation between the test scores and the re-test scores. A regression line drawn through the plotted points, will have the plotted scores cluster around it, not far apart from the regression line, which indicates a strong correlation between the test scores and the re-test scores.

By estimation, with regards to how close the plotted scores are on the regression line or close to the regression line, the scores fall within a correlation coefficient range of 0,6 – 0,7, which indicates a strong correlation between the test scores and the re-test scores. Hence, it can be concluded that the measuring research instrument can be considered as reliable and can serve the desired purpose.

APPENDIX J

PRESENTATION OF LEVEL OF PERFORMANCE

FIRST YEAR – 61 Participants

RATING CODE	DESCRIPTION OF COMPETENCE	PERCENTAGE	NUMBER OF LEARNERS WHO ACHIEVED
7	Outstanding achievement	80 – 100	1
6	Meritorious achievement	70 - 79	1
5	Substantial achievement	60 - 69	4
4	Adequate achievement	50 - 59	8
3	Moderate achievement	40 - 49	12
2	Elementary achievement	30 - 39	19
1	Not achieved	0 - 29	16

73, 77 % - achieved 26, 22% - not achieved

SECOND YEAR – 45 Participants

RATING CODE	DESCRIPTION OF COMPETENCE	PERCENTAGE	NUMBER OF LEARNERS WHO ACHIEVED
7	Outstanding achievement	80 – 100	0
6	Meritorious achievement	70 - 79	2
5	Substantial achievement	60 - 69	6
4	Adequate achievement	50 - 59	5
3	Moderate achievement	40 - 49	9
2	Elementary achievement	30 - 39	13
1	Not achieved	0 - 29	10

77.78 % - achieved 22, 22% - not achieved