Nonlinear Ensemble Transform Filter (NETF) for Ocean Assimilation

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Motivation

In nonlinear systems, the analysis moments of the *local ensemble transform Kalman filter* (LETKF)[1] are biased due to the Gaussian assumption for prior density and observation.
The *particle filter* (PF) performs a non-parametric and Bayesian analysis, but suffers from weight divergence.

Approach: Nonlinear Ensemble Transform Filter (NETF) [2]

→ Creates new, equally-weighted analysis ensemble such that its mean and covariance *exactly* match the Bayesian estimators
→ Deterministic square root filter as the ETKF
→ Domain localization as in the LETKF
→ Outperforms (L)ETKF in Lorenz63/96 tests with small ensembles [2]

NETF Analysis Step: Analogy to the (L)ETKF

Transform forecast ensemble into analysis ensemble with *exactly* specified mean and covariance:

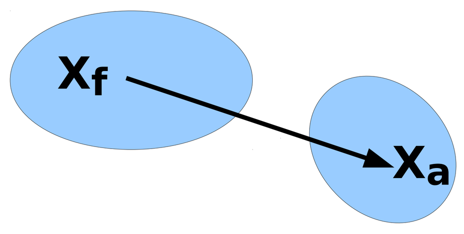
1. Update mean with **weight vector w**:

$$\overline{\mathbf{x}}_a = \overline{\mathbf{x}}_f + \mathbf{X}'_f\mathbf{w}$$

2. Update perturbations with **transform matrix T**:

$$\mathbf{X}'_a = \mathbf{X}'_f\mathbf{T}\mathbf{\Lambda}$$

3. Compose final ensemble:



$$\mathbf{X}_a = \overline{\mathbf{X}}_a + \mathbf{X}'_a$$

NETF

Analysis ensemble with Bayesian moments [2] (Monte Carlo estimators)

$$\mathbf{w} = (w^1, \ldots, w^m)^T$$

Usual PF weights: $w^i \propto p(\mathbf{y}|\mathbf{x}^i_f)$

$$\mathbf{T} = \sqrt{m}\left(\mathrm{diag}(\mathbf{w}) - \mathbf{w}\mathbf{w}^T\right)^{1/2}$$

→ Identical update mechanism: NETF & ETKF only differ by the explicit entries in **T** & **w**!

ETKF

Analysis ensemble with KF moments (Gaussian assumption)

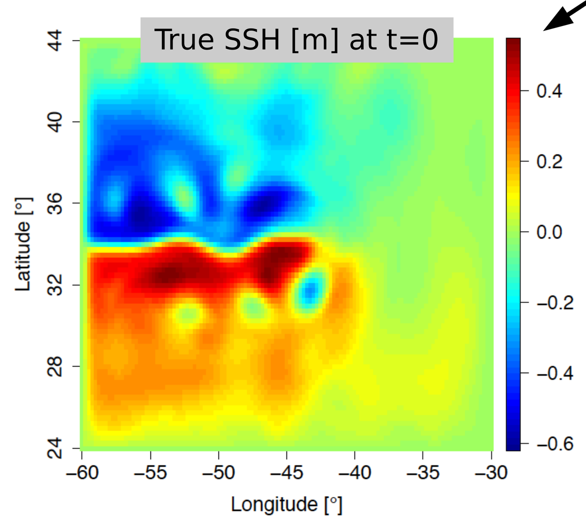
$$\mathbf{w} = \frac{1}{m-1}\mathbf{T}\mathbf{T}^T\mathbf{Y}'^T_f\mathbf{R}^{-1}(\mathbf{y} - \overline{\mathbf{y}}_f)$$

$$\mathbf{T} = \left(\mathbf{I} + \mathbf{Y}'^T_f\mathbf{R}^{-1}\mathbf{Y}'_f/(m-1)\right)^{-1/2}$$

Notation
x = state vector
m = ensemble size
$\mathbf{X}_{f/a}$ = forecast/analysis ens. matrix = [$\mathbf{x}^1$, ..., $\mathbf{x}^m$]
X' = ens. perturbations
y = observation vector
R = obs. error covariance
H = observation operator
Y = ***H*X**, $\overline{\mathbf{y}}$ = mean(**Y**)
p(**y**|**x**) = likelihood density
Λ = random rotation matrix

High-Dimensional Ocean Twin Experiment

Model: NEMO v3.3
- Closed square basin, 0.25°, 5km depth
- Driven by zonal wind
- 74 years spin-up
- DA exp. in year 75

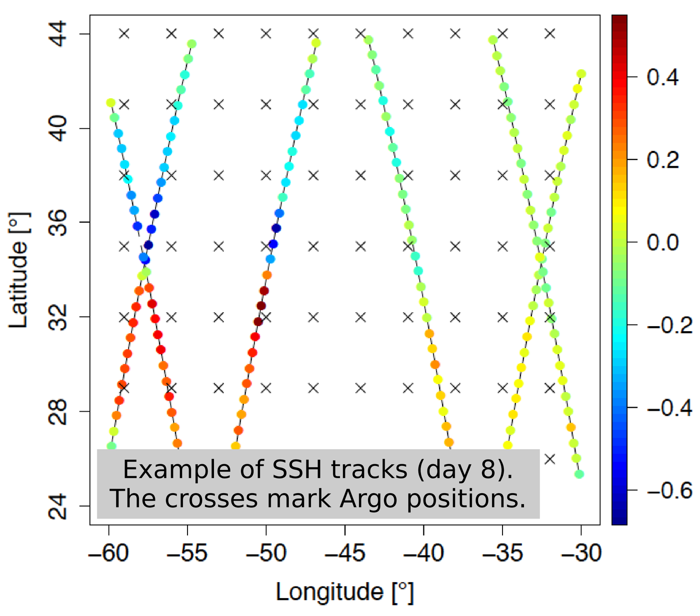


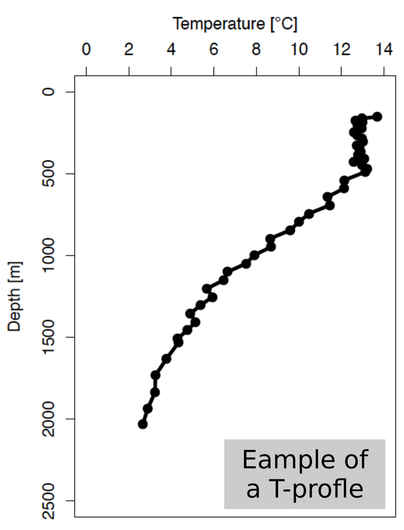
State vector
T, U, V, SSH
(on 121x81x11 grid)
→ **dim(state)≈3.3·10^5**

Dynamics
- Double gyre circulation
- Central jet
- Mesoscale eddies
- as e.g. in North Atlantic

→ Realistic & challenging assimilation experiment

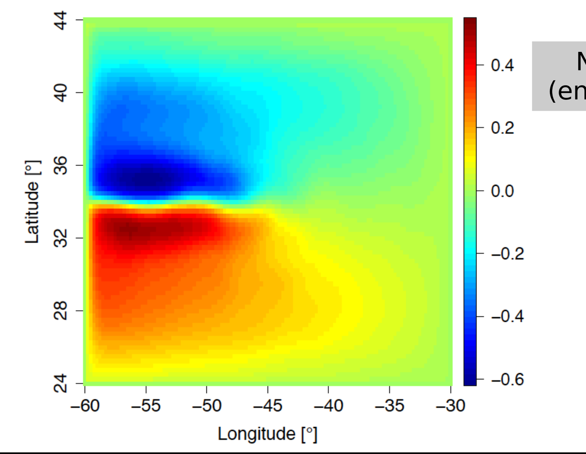
Artificial observations[3]
- each 2nd day → 180 analysis steps
- SSH on *Envisat* tracks
- *Argo* temperature profiles on 3°x3° grid
→ **dim(obs)≈3300**





Filter setup
- Localization radius: 2.5° (on average 100 observations per ocean column)
- Inflation factor: 1.025

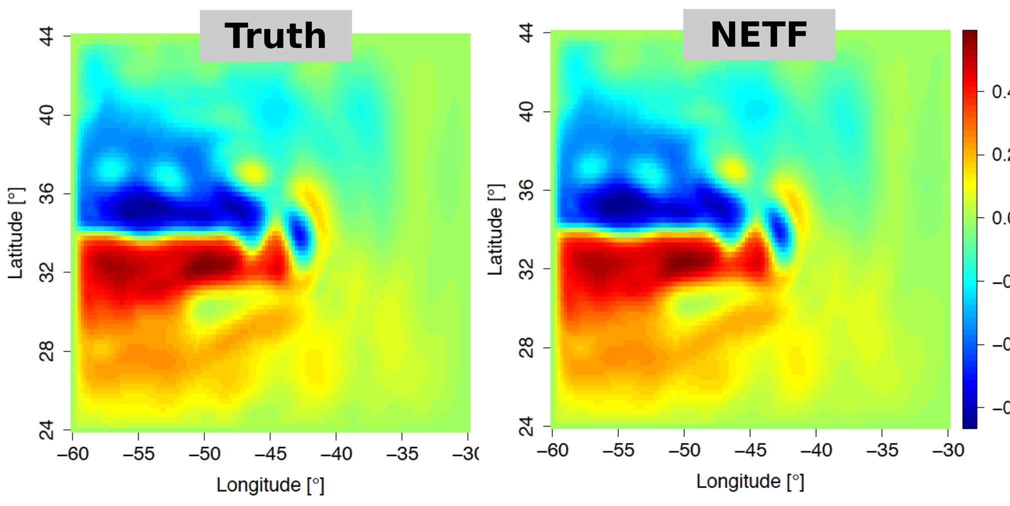
Initial ensemble
- **dim(ens)=120**
- from model climatology
→ no information about true flow at t=0



→ Generic NETF, no model-dependent changes

Results and Evaluation [4]

Qualitative evaluation
Snapshots of SSH [m] on day 260:

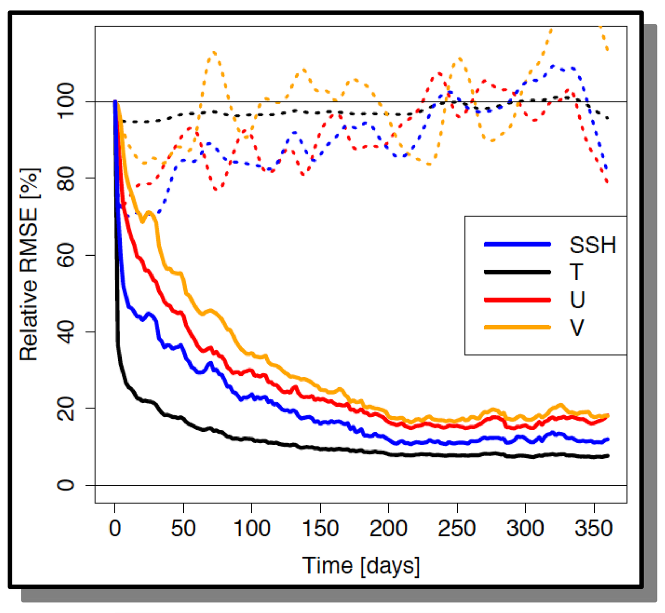


→ NETF reproduces the true circulation

Quantitative evaluation
RMSEs (normalized at t=0):

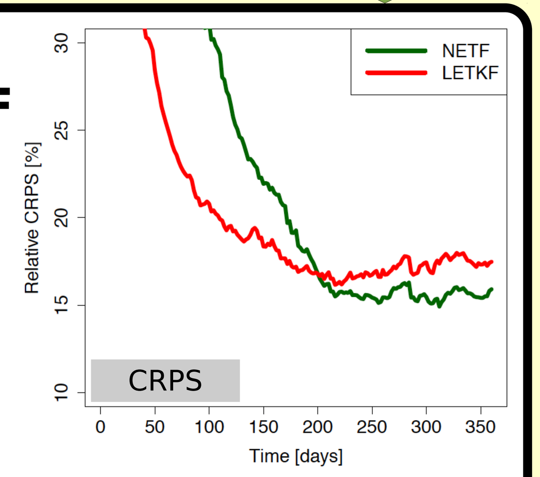
- strong error reduction with time compared to free run
- holds for observed (T, SSH) and hidden variables (U, V)
- filter remains stable

→ NETF successfully assimilates the observations



Relative RMSEs for all variables for NETF (full) and free run (dashed)

Comparison to LETKF
with CRPS (averaged over T,U,V,SSH)
Considers entire ensemble distributions



- NETF requires a longer spin-up phase than LETKF
- But: better score after convergence

→ Potential benefits of nonlinear analyis

Conclusions and Outlook

Conclusions
→ Promising nonlinear filter for high-dim. assimilation
→ Simple implementation: analog to (L)ETKF
→ Works well in Lorenz to ocean models with small ensemble sizes: *overcomes curse of dimensionality*

Successful development
Theoretical justification → Toy models → Realistic large-scale models →

Future work
▸ More large-scale applications
▸ Comparison to EWPF
▸ Extension to nonlin. smoother
▸ ...

References
[1] Hunt, B. R., E. Kostelich, I. Szunyogh (2007): Efficient data assimilation for spatiotemporal chaos: A local ensemble transform Kalman filter. Physica D, 230, 112-126.
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[3] Yan, Y., A. Barth, J. M. Beckers (2014): Comparison of different assimilation schemes in a sequential Kalman filter assimilation system. Ocean Modelling, 73, 123-137.
[4] Tödter, J., P. Kirchgessner, L. Nerger, B. Ahrens (2015): Assessment of a nonlinear ensemble transform filter for high-dimensional data assimilation. MWR, under review.

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