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Overshooting Tipping Point Thresholds in A Changing Climate

Paul D. L. Ritchie[1], Joseph J. Clarke[1], Peter M. Cox[1], Chris Huntingford[2]

[1] College of Engineering, Mathematics and Physical Science, University of Exeter, EX4 4QF, UK
[2] UK Centre for Ecology and Hydrology, Wallingford, Oxon OX10 8BB, UK

Paleo-records suggest that the climate system has tipping points, where small changes in forcing cause substantial and irreversible alteration to Earth system components called tipping elements. As atmospheric greenhouse gas concentrations continue to rise due to fossil fuel burning, human activity could also trigger tipping. These would be difficult for society to adapt to. Previous studies report low global warming thresholds above pre-industrial conditions for key tipping elements such as ice-sheet melt. If so, high contemporary rates of warming imply that the exceedance of these thresholds is almost inevitable. It is widely assumed that this means we are now committed to suffering these tipping events. We show that this conventional wisdom may be flawed, especially for slow onset tipping elements in our rapidly changing climate. Recently developed theory indicates that a threshold may be temporarily exceeded without prompting a change of system state, if the overshoot time is short compared to the effective timescale of the tipping element. To demonstrate this, we consider transparently simple models of tipping elements with prescribed thresholds, driven by global warming trajectories that peak before returning to stabilise at 1.5℃ of global warming.

Introduction

Multiple strands of evidence indicate that components of the Earth System, called Tipping Elements[1], are capable of large and rapid changes in response to relatively small changes to forcings[2]. Tipping Elements are often irreversible over multiple human generations: the original system state is not recovered when the forcing is brought back to its original value. The point beyond which a Tipping Element changes state is called a Tipping Point[3,4]. Tipping points are evident in paleoclimate records[5,6], as well as in future projections made with Earth System Models[7]. Tipping points are normally characterised by the global warming levels at which they occur. These tipping point thresholds (hereafter thresholds) are often estimated to be at low levels of global warming[8-12], and it is these assessments that in part have led to the societal aspiration to restrict global warming to low levels such as 2.0℃ or even 1.5℃ above the pre-industrial period[10,13,14]. However, current emission levels and measured warming rates suggest that keeping below these global warming levels will be difficult for society to achieve[15,16]. It is therefore important to ask if thresholds could be briefly overshot without triggering the transition to an alternative state. Despite the pressing requirement to answer this question, very few assessments exist of whether the overshoot of thresholds is possible, and if so what magnitudes and timescales are safe[17-19]. Nevertheless, dynamical systems theory shows that temporary tipping point overshoot is both possible and can be quantified[20,21]. We combine this theory with transparently simple models of four potential tipping elements to determine the possible global warming trajectories that allow for a safe overshoot of the prescribed thresholds.

The importance of timescales

Of greatest relevance here are tipping points that may occur in response to a change in global temperature. Such warming might be over long paleo periods, or as of interest here, at a faster decade-to-century timescale, largely driven by the human burning of fossil fuels and rising atmospheric greenhouse gas concentrations[22]. Once global warming passes a threshold for the system, the current state of the system starts to undergo a transition to an alternative state, where such a state might be vastly different. This transition may occur relatively quickly - we refer to these as having a fast tipping onset, and hypothetical examples include Amazon forest dieback[23] and disruption to monsoons[24]. Other transitions may take much longer, and these slow onset cases include ice sheet loss[25] and the collapse of the Atlantic Meridional Overturning Circulation (AMOC)[26]. A transgression of a tipping point threshold does not necessarily cause an instantaneous transition, especially for slow onset tipping elements, as illustrated schematically in Figure 1 (and animated with stability landscapes in **Video 1**). Instead the system lives on borrowed time[27] before tipping occurs, and such inertia might allow for a temporary 'safe' global warming overshoot.

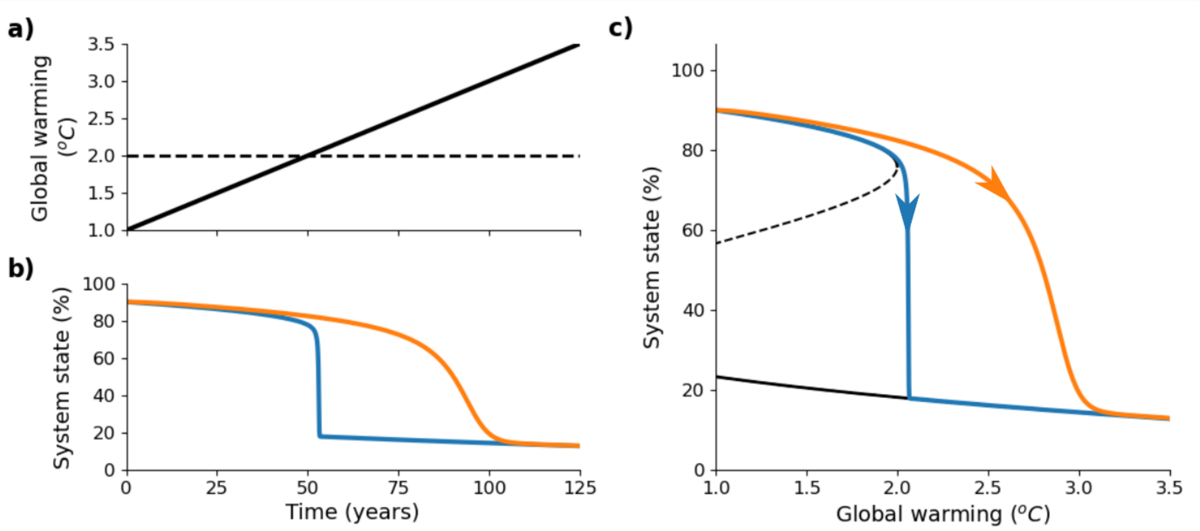

Figure 1: Comparison between slow and fast onset tipping elements. **a**) Idealised time series of a linear increase in global warming above pre-industrial levels that crosses an illustrative threshold of 2°C (dashed line). **b**) Time series of system state for a fast onset tipping system (blue) and slow onset tipping system (orange), with the same threshold and the same global warming forcing as in **a**). **c**) System state vs global warming for the fast onset tipping element (blue) and slow onset tipping element (orange). Both systems have a desired state, which is the upper solid black curve and represent contemporary conditions. An undesired stable state, given by the lower solid black curve, coexists with the desired state for warming levels below the 2°C threshold, separated by an unstable state (black dashed curve). Above the threshold, only the undesired equilibrium state remains.

Figure 1a considers a scenario where global warming increases linearly with time. We assume two tipping elements of the climate system which have the same threshold of 2°C of global warming, denoted by the horizontal black dashed line. Figure 1b displays the time series response of the system state to this linear warming increase for a fast-onset tipping element (blue) and a slow-onset tipping element (orange). Despite both tipping elements having the same threshold, which is transgressed after 50 years, only the fast system experiences rapid tipping. In contrast, the slow system maintains the initial system state for much longer, with full tipping not occurring until about year 100. Figure 1c presents these trajectories, and the equilibrium states, as a function of global warming. Stable states are represented by black solid curves and the unstable state by the black dashed curve. For warming levels below the threshold, the system is bistable, with a desirable upper stable branch (representing current conditions) and an undesirable lower stable system state coexisting. Importantly, beyond the threshold, only the undesirable state persists. The trajectories of both the fast and slow systems closely track the equilibrium state initially. Once the equilibrium state disappears at the threshold, the fast system tips nearly instantaneously, whereas the slow system at first appears unaware of the disappeared state. For the parameters in this illustrative example, it is not until the warming has exceeded 2.5°C that the slow system begins to tip. We assess if this delay in tipping can be exploited to enable safe overshoots of thresholds that do not result in the system tipping to the undesired state. Similar delayed tipping phenomena has been observed in numerical runs of an Energy Balance Model[28]. A ghost state, also known as an attractor relic, can be another reason for tipping to be delayed[29,30]. Features not included here such as internal variability and seasonal cycles will also have an influence on the size of delay.

In our analysis we consider global warming overshoot trajectories, ΔT, introduced by Huntingford et al. (2017)[31]:

$$\Delta T = \Delta T_0 + \gamma t - (1 - e^{-\mu t})[\gamma t - (\Delta T_{Lim} - \Delta T_0)], \qquad \mu(t) = \mu_0 + \mu_1 t. \tag{1}$$

These temperature trajectories contain five parameters. Parameter ΔT_0, is the absolute warming since the preindustrial period. Parameter ΔT_{Lim}, is an eventual stabilisation temperature, set to be the long-term Paris target of 1.5°C[14]. An exponential decay term characterises the transition that moves away from the current linear growth (and towards stabilisation). The transition timescale, $\mu(t)$, can change linearly in time, described by the parameters, μ_0 and μ_1. Such a time-dependency captures if society places more effort to lower global warming rates in the near term, or instead many decades ahead. For relatively slower transitions, the profiles initially overshoot ΔT_{Lim}, as used in this study. The parameter, γ, is chosen to ensure a realistic rate of global warming in the recent past, given the chosen values of the other parameters. These global temperature trajectories are designed to allow varying levels of temperature overshoot for varying periods of time, while also matching the contemporary rate of global warming and asymptoting to the long-term Paris target of 1.5°C.

Steffen et al. (2018)[11] characterised global warming ranges at which the climate tipping elements would undergo state changes if exceeded for sufficiently long. However, the models for the climate tipping elements we consider in this study (detailed in Boxes 1 and 2) have their individual forcing parameters and thresholds that cause the systems to undergo state changes if exceeded for sufficiently long. For clarity, these forcing parameters are: local temperature for forest dieback; solar constant for ice cap loss; planetary albedo for monsoon disruption; and freshwater forcing for AMOC collapse. We assume that each of these forcing parameters p are proportional to the global warming T, therefore

$$T = T_{TP} + (p - p_{TP})\frac{T_{TP} - T_{ref}}{p_{TP} - p_{ref}},$$

where the subscript TP refers to the threshold (for temperature these are chosen at the centre of the ranges given by Steffen et al. (2018)[11]). The subscript ref is a reference level related to the current climate.

We now move to specific examples of tipping elements. Figure 2 (animated with stability landscapes in **Video 2**) demonstrates the concept of overshooting a threshold for a model of the AMOC[32-34] – see Box 2 for details of the AMOC model. The potential collapse of the AMOC is one example of a slow tipping onset where the transitional timescale is assumed to be of the order of centuries[1]. Steffen et al. (2018)[11] have characterised the critical global warming for the collapse of the AMOC to be in the range of 3-5°C. In our model, we set the threshold to be at the centre of this warming range, namely at 4°C of global warming above pre-industrial.

In Figure 2a we consider two different overshoots of the AMOC threshold; the first trajectory (blue curve) represents a relatively small peak overshoot of 1°C but takes approximately 3,500 years to stabilise at 1.5°C. The second trajectory (orange curve) overshoots the threshold by 2°C but stabilises much faster (1,150 years) at the same level. Conventional wisdom suggests that both trajectories would lead to the AMOC tipping to the 'off' state, because in both cases the threshold has been exceeded. However, these trajectories have very different and rather counterintuitive risks of tipping when timescales are taken properly into account.

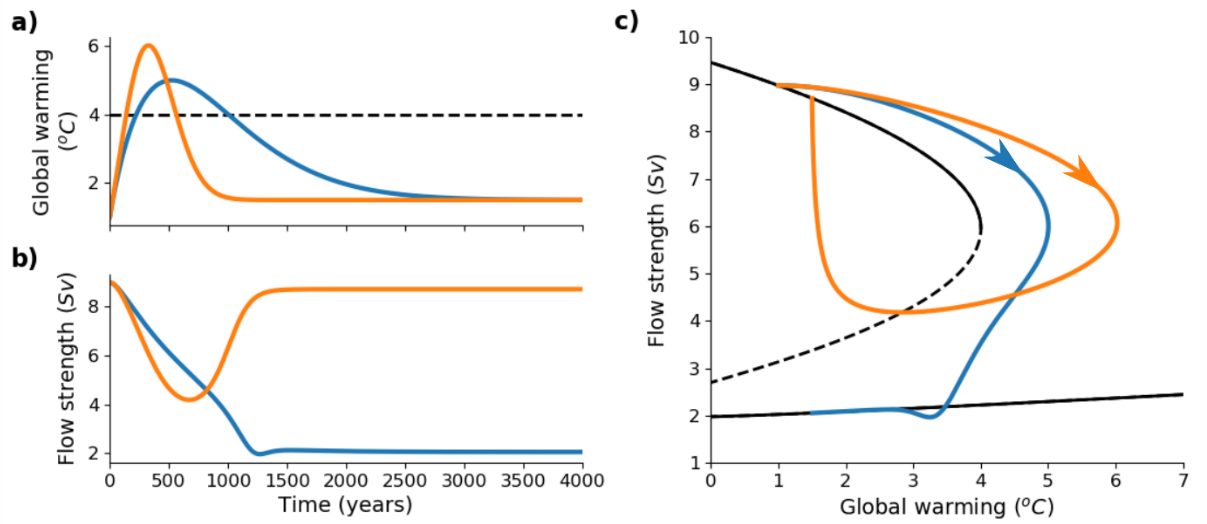

Figure 2: Illustration of overshooting a threshold in a model for the Atlantic Meridional Overturning Circulation (AMOC). **a**) Two time series of contrasting sample overshoot trajectories are shown in global warming given by Equation (1). The blue curve (parameter values: $\mu_0 = 1.8 \times 10^{-3}, \mu_1 = 2.0 \times 10^{-7}\ per\ year, \gamma = 0.0191°\mathrm{C}\ per\ year$) is a small and long overshoot, while the orange curve (parameter values: $\mu_0 = -1.3 \times 10^{-3}, \mu_1 = 7.0 \times 10^{-6}\ per\ year, \gamma = 0.02065°\mathrm{C}\ per\ year$) is a much larger yet quick overshoot. The black dashed line indicates the threshold, above which if global warming is fixed, the AMOC would eventually collapse. **b**) Time series response of ocean flow strength corresponding to the warming overshoot trajectories presented in **a**). **c**) Flow strength vs global warming for short, long overshoot and big fast overshoot (colours as in **a)** and **b)**). An AMOC 'on' state (upper solid black curve) and an AMOC 'off' state (lower solid black curve) both coexist for warming levels below the threshold of 4°C and are separated by an unstable state (black dashed curve). Above the threshold only the AMOC 'off' state remains.

The AMOC is characterised by its north-south flow strength, measured in Sverdrups (Sv). In Figure 2b the blue time series shows that a small but long-lasting overshoot does not prevent the system tipping, and instead causes the flow strength to drop from 9Sv to only 2Sv, indicating a sustained collapse of the circulation. The flow rate remains severely weakened, even as global warming decreases. In contrast, the larger but shorter overshoot allows the flow strength to recover after a strong initial weakening (orange time series). The AMOC has been able to recover in this scenario because the reversal in global warming has been sufficiently fast.

Plotting this as a function of warming (Figure 2c), reveals a clearer picture of the underlying dynamics taking place. For the small overshoot (blue trajectory), the substantial amount of time spent over the threshold means that once global warming is back below the threshold, the circulation has almost reached its collapsed state. Hence the circulation does not recover and is destined to remain 'off' regardless of how far warming is further reduced. For the faster reversal in global warming (orange trajectory), resulting in less time over the threshold once warming is brought back below 4°C the collapsed state has yet to be reached. Therefore, with a continued fast reduction in warming, the trajectory is able to cross the unstable state (black dashed curve) – also known as a melancholia state[28], after which the flow strength begins to recover, preventing the tipping. It is important to note, in Figure 2c, that such crossing occurs at a warming below 4°C. Hence safely returning to the initial state requires a period of time when global warming is below the threshold.

Theoretical basis

We now provide a more theoretical basis to the numerical findings presented in Figure 2. For a fixed stabilisation level, there are two attributes that determine whether a system can safely overshoot a threshold. These are the amount by which the threshold is exceeded relative to the difference between the threshold and the stabilisation level, χ, and the time spent over that threshold, t_e. For symmetric parabolic overshoots, it has been shown that a system will not tip if $\chi < 16\,\tau^2/{t_e}^2$, where τ is the effective timescale of the system[20]. Here we define the effective timescale of a tipping element as the recovery time from perturbations in the equilibrium state at 1.5°C global warming. The effective timescale depends on the distance to the threshold and can be determined from the lag-1 autocorrelation statistic (see Ritchie et al., 2019[20] for further details). In Figure 3 we show how the theory compares against four tipping elements of the climate system[1,11]: collapse of the AMOC[32]; melting of the ice cap[35]; disruption to the Indian Summer Monsoon[24]; and forest dieback[23]. The simple models used to represent these fast tipping elements and slow tipping elements are presented in Boxes 1 and 2 respectively.

Figure 3a shows the boundaries of safe and unsafe overshoots for each of these potential tipping elements, in a regime diagram defined by the peak overshoot and exceedance time of each threshold. There is a clear separation between tipping elements that can be classified as fast onset and those classified as slow onset tipping elements. The slower onset tipping elements are ice cap melt and AMOC collapse, and it is possible to safely overshoot their thresholds for multiple centuries before returning and stabilising at the 1.5°C level. In contrast, for the faster onset tipping elements of monsoon disruption and Amazon forest dieback, overshoot is possible only for decades or even just years before tipping would be induced. In Figure 3a, we present the boundaries derived numerically from the temperature overshoots defined by Huntingford et al. (2017)[31] (solid curves) and analytically from the inverse square theory (dashed curves). Normalising the time over the threshold with the effective timescale of each system, and also the peak warming overshoot with the distance from the threshold at 1.5°C, collapses the theoretical curves onto a single curve (Figure 3b). This panel shows a

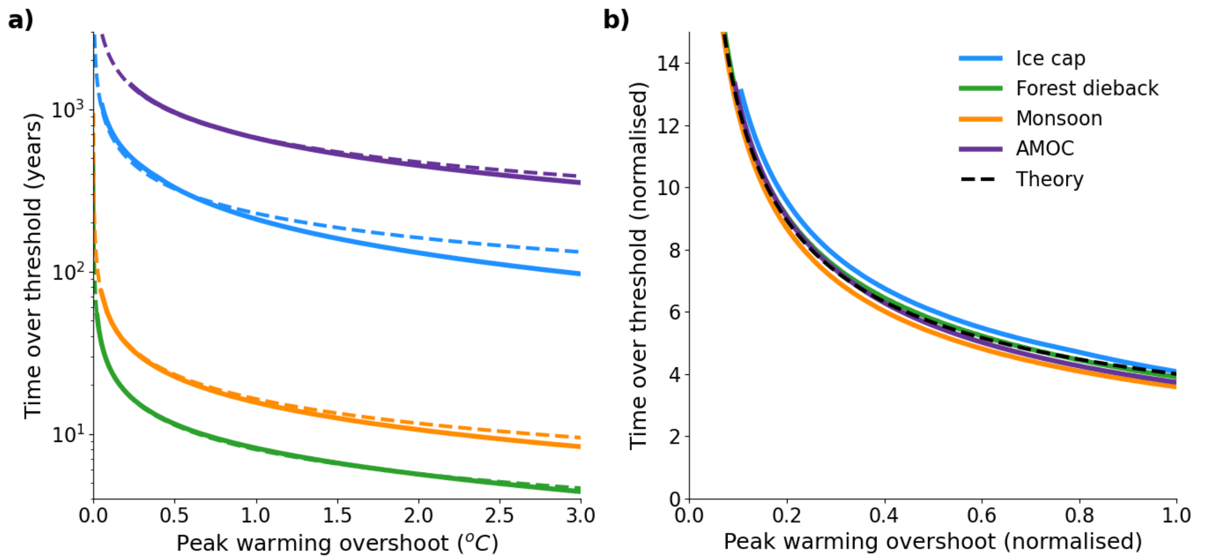

Figure 3: Boundary curves separating safe and unsafe overshoots that start at current warming levels and return to stabilise at the 1.5°C Paris Climate Agreement Target. **a**) Four climate tipping elements are shown (as marked), based on the peak warming overshoot and time over individual thresholds. Above and right of boundary curves represents where tipping occurs while below and left provides the safe overshoots of the threshold for each tipping element. Solid boundary curves are calculated numerically and give the exact boundary, and dashed curves represent the inverse square law theory. **b**) Same as **a**) but with the peak warming overshoot normalised by the threshold distance beyond the 1.5°C Paris Target and the time over the threshold normalised by the effective timescale of the individual systems. Presented in this way, the theoretical curves for each tipping element collapse onto one curve given by the black dashed curve.

good agreement between the boundary calculated numerically for our temperature overshoots and the theoretical inverse square boundary (which assumes a parabolic overshoot), for all four climate tipping elements.

Safe and unsafe overshoots

Instead of considering the peak and time over the specific thresholds, Figure 4a displays the numerical boundaries of safe overshoots for absolute peak global warming and time to stabilisation at the Paris Climate Agreement target of 1.5°C. The grey shaded region indicates all overshoots of the 1.5°C target that would not result in tipping for any of the four chosen tipping elements of the climate system. Figure 4a indicates that the ice cap could be preserved if peak warming is limited to 3°C instead of the more conventional 2°C threshold (i.e. an overshoot of 1°C) if the time taken to stabilise at 1.5°C is under 400 years. It is important to note that, overshooting slightly more than intended can be offset by stabilising at some lower level. Similarly, the AMOC can be maintained up to a peak warming of 6°C provided the time to converge to 1.5°C is less than 1,200 years, despite the assumed 4°C threshold. Significantly, for the faster onset tipping elements, there is little opportunity for safe overshoot.

We present a more detailed comparison between a slow onset tipping element (ice cap melt) with a low threshold, and a fast onset tipping element (Amazon forest dieback) with a higher threshold in Figure 4c. We consider two overshoot scenarios, as marked by a cross and a circle in Figure 4a, of the 1.5°C target before returning to stabilise at that level. The time evolution of these scenarios is shown in Figure 4b. One scenario considers a peak global warming of 3.5°C that stabilises at 1.5°C after 350 years (solid curve with circles) and the other has a higher peak warming of 5°C but takes 120 years to converge to 1.5°C (dotted curve with crosses). The responses to these overshoot scenarios are displayed in Figure 4c, where colour differentiates between the latitude of the ice cap boundary (blue) and vegetation fraction characterising forest dieback (green), and the line style and marker separates the overshoot scenarios. For a small but long overshoot of the 1.5°C target, the ice cap tips to an ice-

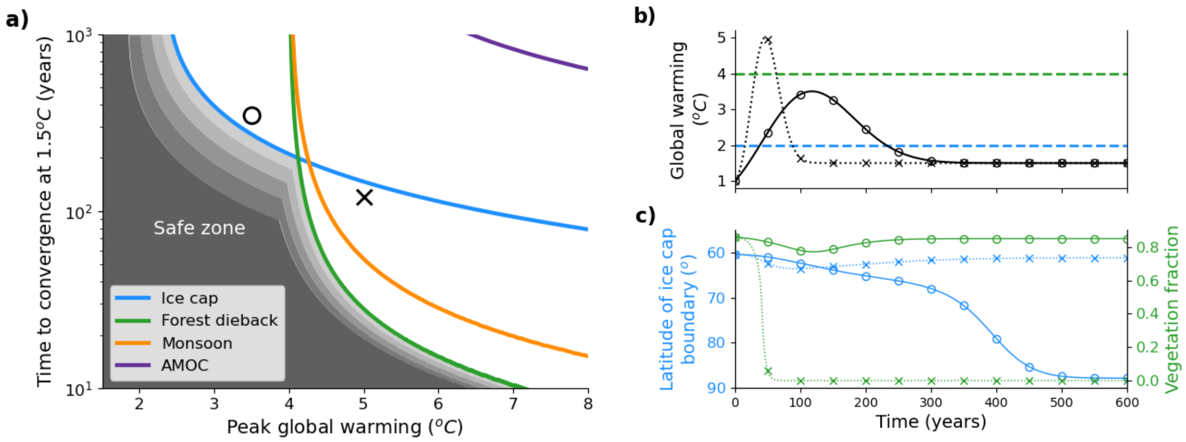

Figure 4: Boundaries of safe overshoots for multiple tipping points. a) Boundary curves separating safe and unsafe overshoots for four climate tipping elements considered (see Boxes 1 and 2), based on the peak global warming and time to stabilisation at 1.5°C warming. Above and right of the individual coloured boundary curves represents where tipping is not avoided, whilst below and left provides the safe zone for each particular tipping element. The grey shaded region indicates the safe zone for all tipping elements. Each different grey shade indicates the boundary of this safe zone if the threshold for all tipping elements were 0.1°C lower. Cross and circle markers indicate parameter values of contrasting sample warming overshoot trajectories considered in **b**) and **c**). **b**) Time series of sample overshoot trajectories in global warming, differentiated by line type and marker. Horizontal blue and green dashed lines denote thresholds for the ice cap and forest dieback respectively. **c**) Time series of ice cap boundary (blue) and Amazon vegetation fraction (green) response to the two overshoot trajectories presented in **b)**. Line type and symbols identical between **b**) and **c**).

free state because its threshold has been transgressed for a sufficiently long time. However, in this scenario, there has been no forest dieback simply because its threshold has not been crossed. However, the opposite behaviours are observed for a higher peak global warming with a quick convergence to 1.5°C. In this scenario the ice cap boundary is maintained at close to 60°N. Although there is a large exceedance of the threshold it is possible to return to the initial state because the time over that threshold is sufficiently short compared to the effective timescale of the system. Forest dieback is induced due to the fast onset of the tipping permitting only very limited overshoots of the threshold. Scenarios with a high peak warming and a fast convergence to the 1.5°C stabilisation level do however imply a fast reduction in temperature, which may not be possible with available technologies. The 'point of no return' is therefore also practically constrained by the rate at which global warming can be reduced.

Discussion

Our analysis reveals that for many climate tipping points it is possible to cross a threshold temporarily without triggering tipping to a new state. This finding is particularly relevant for potential slow-onset tipping elements such as ice-sheet melt or collapse of the AMOC. Hence, the 'point of no return' for a slow onset tipping element is not the threshold but a point beyond the threshold. How far this point is beyond the threshold is determined by three factors: 1) the effective timescale of the system 2) how fast global warming can be reduced and 3) the level at which warming stabilises.

In this study we have used transparently simple models for four climate tipping elements, with a prescribed choice of threshold in each case based-on previous studies[11]. In particular, with the exception of the monsoon model, the models used are first order with a single timescale. Further work is required to demonstrate that similar behaviour is present in more complex models, in which multiple variables act on multiple timescales and the dynamics around the threshold is much more complex. While the thresholds for climate tipping points are highly uncertain, we have assumed the central estimates as published by Steffen et al. (2018)[11] and focussed specifically on the impact of timescales on top of these prescribed thresholds. The tipping element with the earliest threshold is also the one with the longest timescale (ice sheet melt), whereas faster tipping elements tend to have higher thresholds (e.g. tropical forest dieback). This means that safe levels of peak warming are set by faster onset tipping points, but the safe stabilisation warming level is set by the slower onset tipping points.

In our analysis we have considered the safe and unsafe boundaries for each tipping element individually. However, recently it has been proposed that tipping elements are coupled and that tipping cascades are possible[36,37]. The timescales and coupling strength between tipping elements, could result in the overall threshold being lower. Therefore, further research is required to calculate 'safe' overshoots for coupled tipping elements.

This study highlights the importance of timescales for possible tipping points in a changing climate. Slow onset tipping elements permit temporary overshoots of a threshold without triggering tipping to a new state. Both the approach rate to any tipping point threshold, and actions taken to reverse warming once over that threshold, will therefore determine if climate remains safe from unwelcome state changes. state

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Box 1: Models of fast tipping elements

Forest dieback model

Forest dieback is represented by a modified version of the TRIFFID model[23] for a single vegetation type. Vegetation fraction v is modelled by a Lotka-Volterra equation:

$$\frac{dv}{dt} = gv(1 - v) - \gamma v,$$

where γ is a disturbance rate and g is a growth term which is assumed to be parabolic in the local temperature, T_l:

$$g = g_0 \left[1 - \left(\frac{T_l - T_{opt}}{\beta}\right)^2\right].$$

There is an optimal temperature T_{opt} for which growth is maximal and equal to g_0. The parameter β determines the temperature half-width for which vegetation grows. A negative growth-rate implies additional tree mortality. However, there is an additional feedback on the local temperature, T_l, a decline in vegetation results in an increase in temperature:

$$T_l = T_f + (1 - v)\alpha.$$

The temperature T_f is used as the forcing parameter and defines the temperature if there was total forest cover. The temperature difference between total forest and bare soil is defined by the parameter α. Table 1 lists all the parameters and their values used in the forest dieback model.

Table 1: Table of parameters used in vegetation dieback model

Parameter	Value	Unit	Description
$\boldsymbol{\alpha}$	5	ºC	difference between surface temperature of bare-soil and forest
$\boldsymbol{\beta}$	10	ºC	half-width of the growth versus temperature curve
$\boldsymbol{g_0}$	2	yr^{-1}	maximum growth-rate
$\boldsymbol{\gamma}$	0.2	yr^{-1}	disturbance rate
$\boldsymbol{T_{opt}}$	28	ºC	optimal temperature for plant growth

Indian Summer Monsoon model

The summer temperature gradient generates monsoon winds over the Indian subcontinent which hold moisture having emanated from the Indian Ocean. Once over the land the moisture falls as precipitation, which in turn releases heat, amplifying the temperature gradient and generating stronger winds[38]. This key feedback mechanism of the monsoon is captured in a reduced form model introduced by Zickfeld et al. (2005)[24]. Zickfeld et al. (2005)[24] identified a threshold in the planetary albedo A_{sys}, such that if it was exceeded for sufficiently long, the monsoon season would be disrupted. Ritchie et al. (2019) made further reductions to the model though retained the key mechanisms of the monsoon. We adopt the same version of the model as Ritchie et al. (2019)[20], which models the time evolution of the land temperature T, and specific humidity Q:

$$\frac{dQ}{dt} = \frac{E(Q,T) - P(Q) + A_v(Q,T)}{I_Q},$$

$$\frac{dT}{dt} = \frac{\mathcal{L}\big(P(Q) - E(Q,T)\big) + F_{\downarrow}\big(1 - A_{sys}\big) - F_{\uparrow}(T) + A_T(Q,T)}{I_T}.$$

For further details of variables and a table of the parameter values used, we ask the reader to refer to Ritchie et al. (2019)[20]. The increase in planetary albedo required to cause a tipping in the monsoon is arguably more likely to occur due to increases in reflective anthropogenic aerosols rather than increases in greenhouse gases. We follow Steffen et al. (2018)[11] in assuming that this threshold corresponds to about 4ºC of global warming.

Box 2: Models of slow tipping elements

Ice cap model

North (1984)[39] devised a model to offer an interpretation of the small ice cap instability. It was found that simple climate models, which employ heat diffusion and the ice-albedo feedback, show that ice caps smaller than a finite size are unstable. Specifically, for a given range in the solar constant S_0 (a proxy for temperature) a large ice cap state and an ice-free state could coexist. The small ice cap instability is assumed to play a key role in the formation of large ice sheets such as Greenland. In this study we use a model, introduced by Herald et al. (2013)[35], that models the sine of the latitude of the ice cap boundary x, and captures the key mechanisms of the North model, namely the bistable regime:

$$\frac{dx}{dt} = -0.003 + (1-x)\left[\frac{S_0}{4} - 355 + 3\left(\frac{x-0.89}{0.09}\right)^2\right].$$

Atlantic Meridional Overturning Circulation (AMOC) model

Analysing the flow between two boxes of water connected by an overflow and a capillary tube, Stommel (1961)[34] devised the first model of the AMOC. The model measures the evolution of salinity and temperature fluxes between the two boxes, where one box was used to represent the cold salty waters of the North Atlantic and the other the warm fresh waters of the Tropics. Cessi (1994)[32] made the additional assumption that the diffusion timescale is much larger than the temperature restoring timescale. This assumption means that temperature can be assumed constant and so the model can simply be written in terms of a rescaled salinity flux, y:

$$\frac{dy}{dt} = F - y[1 + \mu^2(1-y)^2],$$

where μ^2 is the ratio of the diffusive and advective timescales. The AMOC is a temperature and salinity driven circulation and hence, the rescaled salinity flux y acts as a proxy for the strength of the flow Q:

$$Q = \frac{\eta V[1 + \mu^2(1-y)^2]}{t_d}.$$

See Table 2 for a description of the parameters and their values, which are the same as those used in Dijkstra (2013)[33]. The freshwater forcing F represents freshwater added to the North Atlantic and has a threshold such that maintained freshwater forcing above this level will eventually lead to a collapse of the AMOC.

Table 2: Table of parameters used in AMOC model.

Parameter	Value	Unit	Description
$\boldsymbol{\mu^2}$	6.2	1	Ratio of diffusive and advective timescales
$\boldsymbol{t_d}$	180	yr	Diffusion timescale
$\boldsymbol{V}$	$300 \times 4.5 \times 8{,}250$	km^3	Ocean volume
$\boldsymbol{\eta}$	3.17×10^{-5}	$Sv\ yr\ km^{-3}$	Scaling factor

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Author Contributions

P.D.L.R., and P.M.C. designed and directed the research. All authors helped to shape the research and drafted the manuscript through weekly virtual meetings. P.D.L.R. performed the analysis and produced the figures and animations.

Competing interests

The authors declare no competing interests.