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An expected value of sample information (EVSI) approach for estimating the payoff from a variable rate technology.

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Abstract

This paper examines the payoff to variable rate technology (VRT) using a Bayesian approach following literature on the expected value of sample information (EVSI). In each cell within a field, we compare the expected payoff from an optimal variable rate conditioned on a signal from that cell, with the expected payoff from a uniform rate technology (URT) that is optimal for all cells in the field. This comparison, when evaluated across the theoretical distribution of signals, provides an estimate of the expected gross benefit from VRT relative to URT. Under plausible assumptions, a closed-form algebraic solution relates this expected benefit to field and nitrogen response characteristics. We apply our approach to data from on-farm field-level experiments conducted by the Data-Intensive Farm Management Project (DIFM) (Bullock, et al. 2019), which examined nitrogen (N) response across cells for which soil electroconductivity (EC) served as the signal related to nitrogen response. We calculate the expected gross benefits to be about $1.81/ac, insufficient to support costs of VRT implementation. Our model provides quantitative estimates of the extent to which this poor outcome could be improved by a higher correlation between the EC signal and the state of nature of interest, by higher variability of the state of nature across cells, and by a sharper curvature of yield response to N.

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Introduction

Variable rate technology (VRT) for agricultural inputs refers to a technology that adjusts the application rate for cells within a field, based on information that is unique for each cell. The technology requires an observable signal at each cell that conditions input response in that cell, and a combination of software and hardware capable of changing the application rate across subunits.

Computer-controlled VRT technologies have been commercially available in the U.S. since the late 1980s. Economic analyses of VRT beginning in the early 1990s have shown that VRT for fertilizer on grain crops is seldom profitable. Despite these adverse profitability findings, Lowenberg-DeBoer and Erickson (2019) report studies suggesting that by 2017, across states in the U.S. from 43% to 73% farmers had adopted VRT for fertilizer application. Adoption rates this high challenge the results of economic studies showing the practice to have little if any economic benefit. This suggests that further economic analysis is warranted.

Intuitively, there are several underlying factors that would affect the value of VRT versus a uniform rate technology (URT) on a given field. (1) The value of VRT should increase with the variability of the critical soil characteristics in cells across the field, and would be zero on a perfectly uniform field. (2) The value of VRT should increase with the curvature of the response function (because the cost of the wrong decision on individual cells within the field becomes higher). (3) The value of VRT increases as the effect of the soil characteristic on yield response increases. (4) The value of VRT should increase with the correlation of the signal and the critical soil characteristic(s). (5) The value of VRT should increase with the price ratio of output price to input price. An understanding of the relative importance of these underlying factors in determining the value of VRT from a given trial would be helpful in determining whether results from a given analysis can be extrapolated to other fields.

The modelling approach of this paper employs the concept of the Expected Value of Sample Information (EVSI) from information theory to examine the expected value of VRT. We assume that the response to the input is related to an *unobservable* soil characteristic distributed with some prior density across the field. We further assume that there is a characteristic that we *can* observe, correlated with the unobservable characteristic, which we refer to as a signal. Observation of this signal at a given point changes expectations about the unobservable characteristic at that point, and thus changes the optimal application rate at that point. We apply this approach to examine the expected payoff to nitrogen application, using soil electroconductivity (EC) as the signal, which we compare to expected payoff to a uniform rate technology (URT).

Some relevant economic studies of VRT for nitrogen application

Scores of economic studies have examined the benefits of adopting VRT for fertilizer application[1]. Ex-post evaluations of results on individual fields have shown widely

[1] Good literature reviews are provided by Lambert and Lowenberg-DeBoer (2000), Bullock and Lowenberg-DeBoer (2007), Lowenberg-DeBoer and Erickson (2019)

varying estimates of payoff[2]. While a majority of these studies found a positive payoff, the average of estimated payoff levels for VRT have been small.

Three previous studies estimated what we consider to be the ex-ante expected payoff to VRT versus URT. By ex-ante, we mean the expected payoff from employing VRT on a field prior to actually obtaining information about cells within that field. Babcock, Carriquiry and Stern (1996 - BCS hereafter) and Bullock, Ruffo, Bullock and Bollero (2009 - BRBB hereafter) both explicitly cast the value of VRT in a value of information context, i.e., they use information theory to examine the value of observing some information about each cell and adjusting the rate accordingly, versus applying a common rate across all plots in a field. Liu, Swinton and Miller (2006 - LSM hereafter) do not explicitly appeal to the literature on the value of information, but their analysis is similar in that they compare expected payoffs from VRT versus URT over a wide range of outcomes using Monte Carlo simulation.

BCS explicitly use a traditional Bayesian information theory, with a prior density function describing expectations about the field characteristic, updated to a posterior density function using the observed signal from a cell. They apply this approach to traditional experimental data from a single site over six years to estimate a linear response and plateuau (LRP) production function, using the applied nitrogen fertilizer rates to estimate the slope, and soil nitrate test levels (μ) to estimate the plateau. The optimal URT rate is that which maximizes expected payoff using a prior probability distribution on μ. They specify this prior alternatively as a uniform prior or a three-

[2] Examples include Hurley, Malzer and Kilian (2004); Anselin, Bongiovanni and Lowenberg-DeBoer (2004); Hurley, Oishi and Malzer (2005); Lambert, Lowenberg-DeBoer and Malzer (2006).

parameter gamma prior. For VRT, a plot-specific N rate is calculated as that which maximizes expected payoff using a Bayesian posterior density function that is conditional on the observed signal, μ. They then calculate the expected value of VRT as the difference between these two expected payoffs, using the posterior distribution of soil nitrate levels. Under the uniform prior, they found an expected VRT payoff of $10.03 per acre (gross payoff - without deducting soil test and extra application costs), compared to $2.93 using a gamma prior distribution. The BCS approach models the value of VRT very well using classic Bayesian theory, but the posterior density functions are not analytically tractable and were evaluated numerically, using Gauss-Legendre quadrature. Because of this complication, the approach does not appear to be operationally feasible for general use, and it offers no insights as to the contribution of underlying factors that determine the expected value of VRT.

LSM examine results from whole-field experiments with nitrogen on fourteen Michigan corn fields (only eight in their final evaluation) over a three-year period. Nitrogen treatments were allocated to plots of size 0.2 or 0.4 acres, and plot characteristics such as organic matter, electrical conductivity and wetness index (a proxy for weather) were included in response function estimates. They conceptualize the variance in the response function coefficients of these characteristics to represent variance in response across time and space. They estimate this variance with the variance-covariance matrix of the parameter estimates . They determine the optimal URT by maximizing expected profit with respect to this probability density function; they use simulated values of the coefficients to estimate "site-specific" optimal rates. The simulation was achieved with a Monte Carlo experiment that generated 3,000 yield functions, then bootstrapped to estimate 80% confidence intervals for the value of VRT versus URT. For their preferred model, upper bounds of these ranges are less than \$0.50/ac for four of the fields, between \$1 and \$5 for three more, and \$23.90/ac for the eighth. We note that their analysis is consistent with estimating the expected value of *perfect* information about a given cell, rather than the value of *sample* information, since the simulated values of the random parameters are modeled as known with certainty when optimal rates and outcomes are calculated. This creates an upward bias in the estimate of value of VRT, because there is less error in estimating the optimal rate than would be the case if an imperfect signal is observed instead of the structural coefficients themselves. The approach also offers no insights as to the contributions of the various underlying factors that determine the value of VRT.

BRBB estimate a "meta response" function in which fertilizer response by county is a function of site characteristics and weather, using data from four fields in 2002 and four more fields in 2003. Plot characteristics included topographic indices and a soil nitrogen test, while weather included mean monthly temperature and precipitation. They identify URT optimal rates for each field as those that maximize expected payoffs given the average levels of characteristics and weather. VRT optimal rates are those that maximize average payoffs with respect to each of the observed characteristics-weather outcomes. Calculated expected profit levels from URT and VRT were also determined using the average of observed levels of characteristics and weather. Their framework allows them to distinguish between the value of information (about plot characteristics and weather) from the value of the variable rate technology itself. They report an average payoff (willingness to pay) of about \$2.50/ha for a package including both the site-specific information and the variable rate technology. As was the case for LSM, this is an estimate of the expected value of *perfect* information, because the optimal N rates and related profits were calculated using the assumption that characteristics were known constants. As noted above, this results in an upward bias in the estimated value of VRT. The BRBB approach offers no information about the relative contributions of factors underlying the expected value of VRT.

In this paper we elaborate an approach to estimating the expected value of VRT that has two important features relative to previous approaches: it is tractable in allowing us to analytically identify the effects of underlying factors on the value of VRT (variability of soil characteristics across the field, response curvature, etc), and it acknowledges the reduction in expected value caused by the imperfection of observable signals relative to underlying soil characteristics.

Theoretical approach

Our approach is to postulate that crop yield (expressed per acre) at each point on a field is a quadratic function of the quantity of fertilizer applied and an unobservable soil characteristic at that point. A prior density function describes the decision-maker's beliefs about the frequency distribution of the unobservable characteristic across the field. There exists an observable signal at each point on the field, the distribution of which is correlated with the unobservable characteristic.

At each point in the field, there is a choice of observing the signal so as to apply a rate that maximizes expected profit conditional on the signal (a variable application rate), or simply applying the rate that maximizes expected profit conditional on the prior density of the characteristic (a uniform application rate). We define the expected payoff from observing the signal as the difference between the two expected payoffs, which in the decision theoretic literature is known as EVSI – the Expected Value of Sample Information[3]. We follow the approaches of Kihlstrom (1976) and Lawrence (1999) to model EVSI.

In this analysis, VRT is a package consisting of hardware capable of varying the application rate across the field, hardware capable of monitoring the signal s, and software supporting them[4]. At issue is the expected value of VRT relative to URT for a field, prior to knowing the values of the signal, s, that would be observed at points within that field. This difference we refer to in this paper as the *ex-ante expected payoff from adoption*, which is the same as the expected value of s, or the Expected Value of Sample Information, EVSI.

A simple Bayesian decision making framework

At every point on the field, profit[5] is determined by a variable input x and an unknown and unobservable state of nature γ :

$$\pi(x,\gamma) = p.f(x,\gamma) - w.x \,, \tag{1}$$

[3] Perfect information would be knowledge of the characteristic itself, whereas the observed signal is merely correlated with the characteristic.
[4] Bullock, et al (2009) examine the value of these components separately rather than as a package.
[5] We express profit, output and variable input on a per-acre basis, though it occurs at points in the field.

where $f(x, \gamma)$ is the production function (yield response function), p is the price of output, and w is the input price. In the absence of information about γ at a given point, the profit-maximizing choice of x is obtained from:

$$\max_x E_\gamma[\pi(x, \gamma)] = \int_G \pi(x, \gamma)\, g(\gamma) d\gamma , \qquad (2)$$

where $g(\gamma)$ is the density function representing the *prior* probability distribution of γ, and G is its range. We denote as x' the level of input that maximizes expected profits in equation (2). If the signal is not observed at any point, x' is optimal for the entire field and is thus the optimal uniform rate under URT. Following Kihlstrom (1976), we introduce the possibility of obtaining some soil information (a *signal*) $s \in S$, that is correlated with the true unknown soil characteristic γ. Having observed *s,* the expected profit maximization problem becomes:

$$\max_x E_{\gamma|s}[\pi] = \max_x \int_G \pi(x, \gamma)\; h(\gamma|s) d\gamma , \qquad (3)$$

where $h(\gamma|s)$ represents the *posterior* probability distribution of γ given *s*, obtained by Bayes' rule[6]. We denote $x''(\boldsymbol{s})$ as the application rate that maximizes expected profits when *s* is observed. It can be interpreted as a contingency plan describing the input decision in response to any message *s* that might be observed.

We assume that prior to adopting VRT on a given field, the prior distribution of γ and the response function are known. When a decision maker decides whether to obtain the

[6] Bayes' rule: the posterior density function describing the probability distribution of γ is

$$h(\gamma|s) = \frac{v(s|\gamma) \cdot g(\gamma)}{\phi(s)} \text{ where } \phi(s) = \int_G v(s|\gamma)\; g(\gamma) d\gamma$$

where $v(s|\gamma)$ represents the sampling distribution of the signal and $\phi(s)$ is the marginal density function of the signal.

signal, she does not know what message the signal will provide. The decision is thus based on an expected profit maximization in which the profit associated with each possible message provided by the signal is weighted by the probability of receiving that message. EVSI is the extra expected profit from observing *s*, with expectations taken with respect to the density functions of both γ and *s*:

$$EVSI = \int_S \int_G \pi(\boldsymbol{x}''(s), \gamma)\ h(\gamma|s)d\gamma\ \phi(s)ds - \int_G \pi(\boldsymbol{x}', \gamma)\ g(\gamma)d\gamma, \qquad (4)$$

where $\phi(s)$ is the marginal probability density function of the signal. The first expression on the right-hand side identifies the expected payoff from first observing the signal *s* and then applying the rate that maximizes expected profit given that signal, *evaluated prior to actually observing the signal*[7]. The expectation is taken with respect to the density of *s* across the field, $\phi(s)$, and since the profit is scaled to the level of one acre, it is the expected profit per acre using VRT across the field. Similarly, the second expression is the comparable expected profit per acre if the optimal uniform rate is applied across the field. Thus equation (4) is the expected extra profit per acre from observing and using the signal compared to a uniform rate. Note that this is the *gross value* of observing the signal, from which cost of adopting the VRT package must be deducted to determine the net benefit of VRT relative to URT.

A specification with a quadratic response function and bivariate Normal distributions.

[7] This is known as a preposterior analysis (see for example Raiffa and Schlaifer (1961))

Following Lawrence (1999), we specify a quadratic response function[8] $f(x, \gamma)$ and a bivariate Normal distribution of γ and s. The quadratic yield function is:

$$y = \alpha + \beta_1 x + \beta_2 x^2 + \beta_3 \gamma + \beta_4 \gamma x \qquad (5)$$

where $\beta_2 < 0$. The density functions g and h are Normal with means μ_γ and $\mu_{\gamma|s}$, respectively, and correlation ρ. The solutions to the maximization problems in (2) and (3) are, respectively,

$$x'(\mu_\gamma) = x(w, p, \mu_\gamma) = \frac{w/p - \beta_1 - \beta_4 \mu_\gamma}{2\beta_2} \quad \text{and} \qquad (5a)$$

$$x''(\mu_{\gamma|s}) = x(w, p, \mu_{\gamma|s}) = \frac{w/p - \beta_1 - \beta_4\, \mu_{\gamma|s}}{2\beta_2}.$$

The two optimal rates in (5a) yield two expected maximum profit functions, $\mathrm{V}(\mu_\gamma)$ and $\mathrm{V}(\mu_{\gamma|s})$. The theoretical expected value of sample information (EVSI) in (4) can be expressed in terms of these expectations rather than integrals :

$$\text{EVSI} = \mathrm{E}_s\left[\mathrm{V}(\mu_{\gamma|s})\right] - \mathrm{V}(\mu_\gamma) \qquad (6)$$

where $\mathrm{E}_s[.]$ indicates the expectation over the distribution of the signal. As Lawrence (1999) has demonstrated, for the quadratic yield function, these maximum expected profit functions can be expressed as:

$$\mathrm{V}(\mu_\gamma) = c_1 \mu_\gamma^2 + c_2 \mu_\gamma + c_3$$

[8] The quadratic yield specification is required for Lawrence's closed form analytical results. While some studies of alternative specifications have shown others to be preferable for nitrogen response (i.e., Bullock and Bullock, 1994), other studies have not (Perrin, 1976; Liu, 2006). In any case, the quadratic is probably the most commonly used in economic studies (see Anselin, Bongiovanni and Lowenberg-DeBoer, 2004; Liu et al., 2006; and Bullock, et al, 2009).

$$\mathrm{V}(\mu_{\gamma|s}) = c_1\mu_{\gamma|s}^2 + c_2\mu_{\gamma|s} + c_3 \qquad (7)$$

where $c_1 = -\frac{p\beta_4^2}{4\beta_2}$, $c_2 = p\beta_3 + \frac{p\beta_4}{2\beta_2}\left(\frac{w}{p} - \beta_1\right)$ and $c_3 = p\alpha + \frac{1}{2\beta_2}\left(w\beta_1 - \frac{w^2}{p} - \frac{p\beta_1^2}{2}\right)$, combinations of the known output and input prices and the parameters of the quadratic yield function. Plugging (7) into (6) yields:

$$\mathrm{EVSI} = c_1\left[\mathrm{E_s}\left(\mu_{\gamma|s}^2\right) - \mu_\gamma^2\right] \qquad (8)$$

Lawrence (1999), equation 5.7, pp118-119, shows that using the law of iterated expectations, equation (8) can also be presented in terms of the variances as:

$$\mathrm{EVSI} = c_1\left[\sigma_\gamma^2 - \mathrm{E_s}\left(\sigma_{\gamma|s}^2\right)\right] \;, \qquad (9)$$

where σ_γ^2 and $\sigma_{\gamma|s}^2$ are the prior and the posterior variances of γ, respectively. The expected value of obtaining information s about the unknown γ is proportional to the reduction in uncertainty about γ (ignoring the cost of the VRT technology package).

When γ and s are bivariate normally distributed with correlation ρ[9], the posterior variance of the distribution of γ is $\sigma_{\gamma|s}^2 = \sigma_\gamma^2(1-\rho^2)$. Lawrence then shows (his equation 5.8) that equation (9) above can in this case be expressed as:

$$\mathrm{EVSI} = c_1\left[\rho^2\sigma_\gamma^2\right] = -\frac{p\beta_4^2}{4\beta_2}\left[\rho^2\sigma_\gamma^2\right] \;. \qquad (10)$$

Equation (10) is a fundamental contribution of this analysis. It expresses the value of VRT as an explicit function of parameters representing the underlying factors we intuitively identify as affecting the value of VRT: the variance of the state of nature

[9] i.e. $(\gamma, s) \sim Bivariate\ Normal\ (\mu_\gamma,\ \sigma_\gamma^2;\ \mu_s,\ \sigma_s^2; \rho)$.

across the field, σ_γ^2; the correlation between the signal and the state of nature, ρ; the curvature of the response function[10] β_2; the effect of the state of nature on the response, β_4; and the price of output, *p*.

The sensitivity of EVSI to underlying parameters

While we cannot observe underlying parameters ρ and σ_γ, the assumed bivariate normal distribution of γ and *s* allows us to derive an approximation of equation (10). By Bayes' rule, the posterior mean of the distribution of γ, given s, is given by:

$$E(\gamma|s) = \mu_{\gamma|s} = \mu_\gamma + \rho\frac{\sigma_\gamma}{\sigma_s}(s - \mu_s) \tag{11}$$

Taking the expectation of (4) with respect to the prior distribution of γ, we obtain the following expression, for given values of *x*:

$$E_\gamma(y) = \alpha + \beta_1 x + \beta_2 x^2 + \beta_3\mu_\gamma + \beta_4\mu_\gamma x. \tag{12}$$

Similarly, we take the expectation of (4) with respect to the *posterior* distribution of γ to obtain:

$$E_{\gamma|s}(y) = \alpha + \beta_1 x + \beta_2 x^2 + \beta_3\left(\mu_\gamma + \rho\frac{\sigma_\gamma}{\sigma_s}(s - \mu_s)\right) + \beta_4\left(\mu_\gamma + \rho\frac{\sigma_\gamma}{\sigma_s}(s - \mu_s)\right)x. \tag{13}$$

[10] The value of VRT varies inversely with the curvature of the response function, β_2, i.e., the flatter the response curve, the smaller is the potential loss from a suboptimal application rate and therefore the less is the value in knowing the optimum rate.

Substituting $\tilde{s} = \frac{(s-\mu_s)}{\sigma_s}$, so that $\tilde{s} \sim Normal\ (0, 1)$, re-arranging terms and adding a random error term ϵ , we obtain the following estimating equation:

$$E_{\gamma|s}(y) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 \tilde{s} + \theta_4 \tilde{s}\ x + \epsilon\,, \qquad (14)$$

where $\theta_0 = \alpha + \beta_3 \mu_\gamma$, $\theta_1 = \beta_1 + \beta_4 \mu_\gamma$, $\theta_2 = \beta_2$, $\theta_3 = \beta_3 \rho \sigma_\gamma$ and $\theta_4 = \beta_4 \rho \sigma_\gamma$.

Given that the expected value of $\tilde{s}$ is zero, the first order condition for the expected profit maximizing application rate without observing s, the optimal uniform rate applied to all cells is

$$x'(\mu_\gamma) = \frac{w}{2\theta_2 p} - \frac{\theta_1}{2\theta_2}\,, \qquad (15)$$

and the first order condition for the expected profit maximizing application rate conditional on having observed s (the variable rate for a given cell) yields

$$x''(\tilde{s}) = \frac{w}{2\theta_2 p} - \frac{\theta_1}{2\theta_2} - \frac{\theta_4 \tilde{s}}{2\theta_2} \qquad (16)$$

Notably, the EVSI measure of the value of VRT in equation (10) becomes

$$EVSI = -\frac{p\beta_4^2}{4\beta_2}\ [\rho^2 \sigma_\gamma^2] = -\frac{p\theta_4^2}{4\theta_2} \qquad (17)$$

Sensitivity

Equation (17) reveals some of the determinants of the payoff from VRT. In brackets, we see that EVSI increases with the correlation between the signal and the state of nature, and with the variability of the state of nature across the field. There is no benefit to variable rate application if the field is perfectly uniform, or if the correlation between the signal and the state of nature is zero. From equation (17), the elasticity of EVSI with respect to the correlation ρ, is

$$\frac{\partial lnEVSI}{\partial ln\rho} = 2 \quad . \qquad (18)$$

Similarly, elasticity of EVSI with respect to variance of the state of nature, σ_γ^2 is 1.0, and the elasticity with respect to curvature β_2, is -1.0, while the elasticity with respect to the interaction coefficient, β_4, is 2.0.

Data

We use a rich set of experimental data from the Data-Intensive Farm Management (DIFM, 2018) project at University of Illinois[11]. To estimate the response function (14), we pooled data from 10 farm fields, 4 in Illinois in 2016, 5 in Illinois and 1 in Ohio in 2017, with a total of 7,294 cells[12]. The data consists of corn yields, applied nitrogen (N) and soil electroconductivity (EC), which is the observed signal for each cell *i* in field *j*. EC is usually associated with the availability of nitrate in the soil in which high levels are expected to increase yields (Johnson et al., 2003, Liu et al., 2006). It is a soil signal that correlates with some soil properties as: texture, drainage, cation-exchange capacity and subsoil characteristics (Grisso et al., 2005). Data on EC can be obtained in a shorter

[11] See https://publish.illinois.edu/data-intensive-farm-managment/2016/02/23/hello-world/

[12] We refer to sub-units within fields as "cells", but in the precision agriculture literature they are also referred to variously as "management zones", "sites", "plots", or "grids", depending somewhat on how the subunits are identified.

period and is more cost-efficient than traditional grid-based soil testing. Figure 1 depicts the whole-field experimental layout of a field typical of the DIFM project. Figure 2 illustrates a scatter plot of pooled yields vs N rates. Figure 3 illustrates the location of the 2017 trials. Descriptive statistics for the 10 fields used in this study are in Table 1.

Results

Estimated response function

To estimate equation (14) the signal $\tilde{s}$ is reprented as standardized observed ECs, referred to as $\widetilde{EC}$. Table 2 presents the estimate of the pooled yield response function for all 10 fields, 7,294 observations. The nitrogen and the nitrogen squared coefficients are both statistically significant at 10%. We included dummy variables for fields 2 to 10 so coefficients of these dummies are intercept changes relative to field 1. From the variable *N* response equation (14), we conclude that, because estimates of θ_2 and θ_4 are both negative, there is an inverse relationship between $\widetilde{EC}$ and optimal *N* rate. In other words, $\widetilde{EC}$ is a substitute for *N*.

Table 3 presents for each field the mean, standard deviation, minimum and maximum of EC and optimal VRT rates across the cells within each field, calculated using estimated equations (12) and (14), evaluated at the values of *x* and *s* for each cell in each field. Ordering from lowest to highest average EC, we observe the clear inverse relationship between optimal VRT rate and EC, as implied by the negative estimate of the interaction coefficient for N and EC, θ_4. For example, fields 7 and 10, which have the lowest average EC, have the largest average optimal VRT applications of 180.80 and 183.23 lbs/ac.

Fields 1 and 4, with the highest average EC, have the lowest average optimal VRT application of 143.62 and 130.77 lbs/ac. The relationship between the standard deviation of EC and the standard deviation of optimal VR application is similarly monotonic.

Using equation (17) with the parameter estimates for the pooled sample of 10 fields, we estimate EVSI to be 1.81/ac:

$$\widehat{\text{EVSI}} = -\frac{p\hat{\theta}_4^2}{4\hat{\theta}_2} = -\frac{3*(-.0494)^2}{4(-.00101)} = \$1.81/ac \quad . \tag{19}$$

We also use equation (14) to estimate EVSI for each individual cell, using the observed EC. Table 4 presents the average and range of cell-level EVSI estimates by field, using observed EC and estimated optimal VRT by cell from Table 3. These estimates range from \$0 to \$38.21/ac, but average \$1.82/ac; essentially the same as the estimate in equation (19).

The \$1.81/ac from equation (19) is our best ex-ante estimate of gross return to VRT for fields drawn from a distribution of fields similar to those in our sample. It is the expected return from using the signal from each cell to generate an optimal rate, ignoring the costs of the VRT technology package. While it is below estimates in the literature of those costs, this result is similar to the ones in other empirical studies. BCS (1996) estimated higher comparable payoffs at \$2.93-10.03 per acre. Bullock, et al (2009) found comparable payoffs to be a dollar per acre or less, and concluded that prospects for VAR "are generally dim".

Sensitivity

The estimated EVSI of \$1.81/ac is too low to warrant adoption. We examine here changes in various parameters that would be sufficient to achieve an EVSI of (arbitrarily) \$10/ac, which is about 5.6 times higher than the estimated level. We solve equation (17) for the various parameters to derive estimates of the changes in individual parameters that would be sufficient to increase the EVSI by about 5.5 times, to \$10/ac. Solving (17) for dlnρ, for example

$$dln\rho = \frac{5.52}{2} = 2.76. \tag{20}$$

We estimate that a ρ 2.76 times larger than the ρ of these fields would be sufficient to raise EVSI to \$10/ac. But of course, we do not have an estimate of ρ. Judging from the generally low VRT payoff measured here, this correlation must be low – perhaps ρ=0.1 or as little as ρ=0.01. If ρ=*0.1,* then from the equation above, ρ would need to increase from 0.1 to 0.376. If ρ=*0.01,* ρ would need to increase from 0.01 to 0.038. However, if $\rho \geq 0.362$, apparently there is no increase that would yield an EVSI of \$10/ac or more. In any case, if a 2.8-fold increase in ρ is required it seems that electrical conductivity is not sufficiently correlated with N response on these fields to be a profitable signal.

Considering now the variance of the state of nature over the field, σ_γ^2, from equation (11) and given that the elasticity of EVSI with respect to σ_γ^2 is 1.0, the necessary percentage increase in variance to achieve an EVSI of \$10/ac is 10/1.81= 5.52. If the distribution of γ is similar to that of EC, this would imply an increase of σ_γ^2 from about 87.6 (the variance of EC across all fields) to 484, which is much higher than the EC variance of 132 in field 7, the most variable of any of the fields. . Similarly, the curvature coefficient,

β_2, would need to increase from an absolute value of 0.001 to 0.0065, which is an indication that profits in our sample are not highly sensitive to the level of N applied. The interaction coefficient, β_4, would need to change from -0.0494 to -0.186, a further indication that N response in this sample is not greatly affected by the level of EC.

Conclusions

In this paper we have adapted insights from the decision theory literature on the value of information to provide an economic model of the value of VRT (variable rate technology) as the expected value of sample information (EVSI). The sample information in our case is the observed electroconductivity (EC) of the soil as a signal for an unobservable soil characteristic affecting nitrogen response. Our theoretical results provide an estimate of the expected value of VRT as an explicit function of parameteers representing five underlying factors: variability of soil characteristic across the field; curvature of the response function; effect of the soil characteristic on input response; correlation of the signal with the soil characteristic; and the ratio of crop price to input price. The expected value of VRT is taken with respect to the frequency distribution of the state of nature and the sampling distribution of the signal obtained across all cells in all eight field/years observed. This expected value can therefore be taken as the ex-ante expected payoff from adopting VRT on any field drawn from the same population of field/years as those observed.

To obtain tractable analytical results for this approach, we assume that the yield response to fertilizer is quadratic in applied nitrogen (N) and the state of nature, γ. The state of nature cannot be observed, but a signal s can be observed for each cell. A second assumption critical to our results is that γ and s are distributed bivariate normally across cells in these fields. Given this underlying structure, individual cell application rates can be adjusted to the level that maximizes the expected payoff conditional on the signal. The difference between this optimal expected payoff and the expected payoff from an optimal uniform application rate (UAR) provides the expected gross payoff from the adoption of VRT. In the decision theory literature, this is known as the expected value of sample information (EVSI)

We apply this approach to estimate the exante expected payoff to VRT using data from field-level experimental trials with nitrogen on 10 farmers' corn fields in Illinois and Ohio in 2017 and 2018, consisting of 7,294 gridded cells . The signal used to adjust the fertilizer rate for each cell is electrical conductivity. Our EVSI estimate of the ex-ante expected payoff of VRT is $1.81/ac (prior to subtracting VAR implementation costs). This is insufficient to warrant VAR implementation costs, which we believe to be in the range of $10/ac. Our analysis suggests that for VAR benefits to reach this level, the correlation between state of nature and signal would need to increase by roughly 2.8 times, though we are not able to estimate the level of that correlation. Alternatively, the same improvement in VRT value could be attained by an increase of similar size for either the curvature coefficient or the N times EC interaction coefficient, or a five-fold increase in the variance of the state of nature could reach the same outcome. Clearly, some of these changes could occur if we had a more robust measure than electrical conductivity (EC) of the state of nature affecting N response.

Our approach has obtained some results regarding the determinants of VRT payoff that were previously understood intuitively, but not analytically or quantitatively. Our claim for our results to be a measure of the expected payoff of VRT on a similar field, is that the $1.81/ac is a plausible estimate of the expected gross benefit of VRT across a field with cells drawn from the same distribution as the 7,294 cells in our sample. Perhaps such variables as soil classification, remote sensing data, etc., may provide coarser but cheaper signals for calibrating application levels.

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Figures and Tables

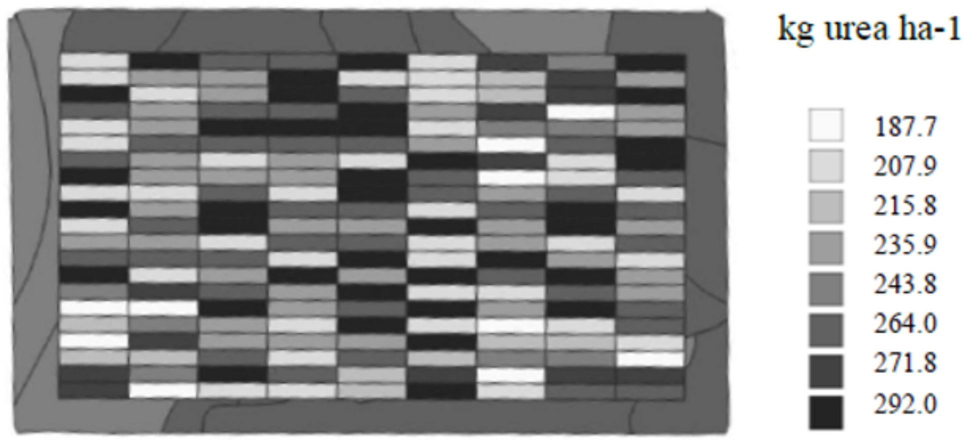

Figure 1. Layout of a 2018 on-farm N fertilizer trial conducted on a 32-ha field in Central Illinois. Source: DIFM data.

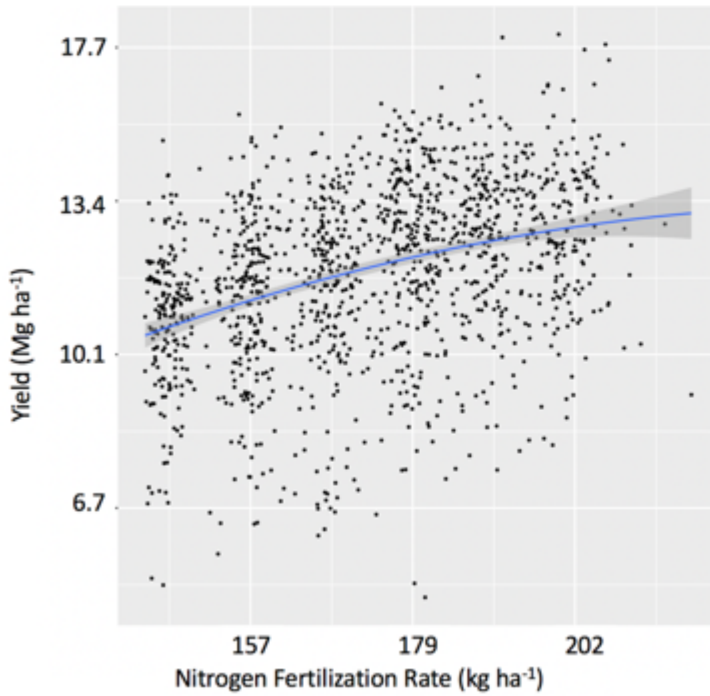

Figure 2. Scatter plot of (N, Corn Yield) data from 2017 trials. Source: DIFM.

Figure 3. Location of six 2017 trials used in the study. Source: DIFM.

Table 1: Descriptive statistics for cells within ten fields in Illinois (4 fields in 2016, 5 in 2017), and Ohio (1 in 2017).

Variable	Observations	Units	Mean	Std. Dev.	Min	Max
Corn dry yield	7,294	bushels/acre	232.65	31.45	0.0	315.67
Applied nitrogen (N)	7,294	pounds/acre	174.15	35.89	29.88	315.94
Soil EC	7,294	Veris EC scale	37.32	9.36	7.6	80.2
Standardized EC ($\widetilde{EC}$)	7,294	Veris EC scale	0.0	1.0	-3.17	4.58

Table 2: Estimated nitrogen response equation (equation 14), pooled data from within ten fields in Illinois (4 fields in 2016, 5 in 2017), and Ohio (1 in 2017), 7,294 cells.

Variable	Coefficient	Estimate (standard error)
N_{ij}	θ_1	0.455*
		(0.225)
N_{ij}^2	θ_2	-0.00101*
		(0.000547)
$\widetilde{EC}_{ij}$	θ_3	9.540***
		(2.182)
$\widetilde{EC}_{ij}.N_{ij}$	θ_4	-0.0494***
		(0.0113)
		77.75***
Fixed effect, field 2	d2	(0.455)
		87.53***
“	d3	(0.791)
		86.84***
“	d4	(0.831)
		38.82***
“	d5	(0.932)
		99.53***
“	d6	(1.435)
		86.84***
“	d7	(1.889)
		119.7***
“	d8	(1.281)
		109.9***
	d9	(0.767)
“		78.23***
	d10	(2.408)

Constant	90.80***
	(21.99)

Note: D2-D10 are dummy variables for fields 2 to 10. Standard errors in parentheses are clustered at the farm level. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 3: Soil electroconductivity (EC) and estimated optimal VRT Nitrogen application per field, Illinois (4 fields in 2016, 5 in 2017), and Ohio (1 in 2017).

		EC				Optimal VR N Application (estimated)[a]			
Field	Obs. (plots)	Mean	Std. Dev.	Min	Max	Mean	Std. Dev.	Min	Max
1	127	42.25	6.66	32.68	66.97	143.62	17.46	78.76	168.71
2	160	38.42	3.24	31.40	44.26	153.66	8.51	138.33	172.07
3	160	35.58	4.05	24.73	42.48	161.11	10.63	143.01	189.58
4	256	47.15	6.31	34.58	60.41	130.77	16.56	95.99	163.74
5	581	35.47	8.86	20.12	79.57	161.41	23.24	45.71	201.68
6	1,548	40.44	6.22	22.81	59.22	148.37	16.32	99.10	194.61
7	682	28.08	11.48	7.60	78.85	180.80	30.11	47.60	234.52
8	819	33.06	9.07	14.57	59.45	167.72	23.79	98.48	216.23
9	2,347	41.28	6.98	22.68	80.22	146.17	18.31	44.01	194.97
10	614	27.15	6.17	15.92	48.32	183.23	16.20	127.69	212.69
Pooled cells	7,294	37.32	9.36	7.6	80.2			44.01	234.52

[a]In lbs/a, using corn price=$3/bu and nitrogen price=$0.42/lb

Table 4: Estimated EVSI by cell, average and range per field in Illinois and Ohio, 2016-2017. (US$/a).

Field ID	Obs (cells).	Mean (Std. Dev)	Min	Max
1	127	1.42 (3.29)	0	18.25
2	160	0.24 (0.23)	0	0.99
3	160	0.40 (0.72)	0	3.29
4	256	2.83 (2.59)	0	11.06
5	581	1.70 (3.50)	0	37.07
6	1,548	1.00 (1.17)	0	9.95
7	682	4.50 (3.97)	0	35.81
8	819	2.08 (2.13)	0	10.75
9	2,347	1.33 (2.31)	0	38.21
10	614	2.94 (2.29)	0	9.52
Pooled cells	7,294	1.82 (2.64)	0	38.21