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The University of Akron Press

2020

Vol. 1 Ch. 5 Hopewell Topography, Geometry, and Astronomy in the Hopewell Core

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Recommended Citation

Hively, Ray and Horn, Robert, "Vol. 1 Ch. 5 Hopewell Topography, Geometry, and Astronomy in the Hopewell Core" (2020). Encountering Hopewell in the Twenty-first Century, Ohio and Beyond. 5. [https://ideaexchange.uakron.edu/encountering_hopewell/5](https://ideaexchange.uakron.edu/encountering_hopewell/5?utm_source=ideaexchange.uakron.edu%2Fencountering_hopewell%2F5&utm_medium=PDF&utm_campaign=PDFCoverPages)

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Chapter 5

Hopewell Topography, Geometry, and Astronomy in the Hopewell Core

Ray Hively and Robert Horn

The construction of large-scale, precise geometrical earthworks was a prominent practice in the Hopewell tradition associated with Native American populations in central Ohio from 200 BC to AD 500. The most spectacular and prominent practice in the Hopewell tradition associated with Native American populations in central Ohio from 200 BC to AD 500. The most spectacular and well-documented examples of this architecture are located in the valley of Raccoon Creek at Newark, Ohio, and in the valleys of the Scioto River and its tributary Paint Creek near Chillicothe, Ohio. The literature contains numerous references to the accuracy or coincident regularity of individual geometric figures in the various earthworks (Hively and Horn 1982, 2013; Romain 2004, 2005; Romain and Burks 2008a, 2008b; Thomas 1894). Surveys of these structures have established that they have a combination of geometrical precision and monumental scale unprecedented in the prehistoric world. The meticulous care and labor evident in these constructions strongly suggests that very careful planning was invested in their placement, design, and orientation.

Archaeological and historical evidence for prehistoric and ancient cultures across the world indicates that some monumental structures were plausibly sited and oriented with respect to topographical features and repetitive astronomical events such as the rising and setting of the sun and moon viewed against local horizons. The precision and scale of the Hopewell earthworks considered here provide a unique

opportunity for investigating the possibility that close alignment of these structures to astronomical events and local topographical features was a deliberate part of the plan. Structures built by few other prehistoric cultures have an ensemble of linear features or symmetry axes, which can be determined with sufficient accuracy to distinguish deliberate, precise alignment from approximate, accidental alignment.

Linear features and symmetry axes associated with Hopewellian earthworks have sufficient scale and precision that their azimuthal directions can be verified with an accuracy of better than 0.25°. This accuracy is required to build a persuasive case for deliberate alignments especially when supporting ethnographic evidence is sparse or absent. In such a case, repeated and likely deliberate regularity in design is the only and best evidence available for inferring the intentions of the builders. If these structures do exhibit a repetitive pattern of deliberate alignments and geometrical design, they may offer insight into the worldview of the builders not accessible in any other way. A thorough analysis of the geometrical, topographical, and astronomical context of these structures is a crucial step in the effort to understand them.

In this chapter, we present the results of our latest work in examining the geometry, topography, and astronomy evident in these earthworks. Our research shows that the siting, structure, and orientation of the earthworks can be understood in terms of three principles: (1) geometric designs based on exploration of circles and squares related by circumscription and inscription; (2) the discovery and implementation of a simple but accurate algorithm for constructing circles and squares with nearly equal areas; (3) deliberate siting and alignment of the structures along lines indicating the rise and set points of the sun at the solstices and the moon at the lunar standstills as viewed from prominent points in the local topography.

The ideas forming the hypothesis integrating topography, astronomy, and geometry (TAG) derive from our analysis of the Newark Earthworks (Hively and Horn 1982, 2006, 2013). We have since extended and tested this hypothesis by applying it to five earthwork sites in the Scioto and Paint Creek valleys near Chillicothe, Ohio. The results of this test show that TAG offers a consistent explanation for the design, structure, and siting of these additional earthworks.

Geometrical Studies of Hopewell Earthworks

Marshall (1987) and Romain (2000) have attempted to make the case that the builders utilized standard units of measurement, Pythagorean geometry, and a grid along cardinal directions. The statistical challenge of demonstrating that fits to such geometrical plans were intentional rather than fortuitous has been well

documented in the critical analysis of the work of Alexander Thom (1971) pertaining to the prehistoric Megalithic culture of Europe and Britain (Ruggles 1999, Part 1). Part of the difficulty in assessing the geometrical knowledge of the Hopewell builders is the fragmentary nature and uncertain accuracy of much of the survey data available for these earthworks. It is generally agreed that the most accurate and reliable survey data obtained for these geometric earthworks (when they could still be readily seen from the ground) was that obtained by James Middleton as part of the work of the Smithsonian Institution led by Cyrus Thomas (1894). With a small number of clearly stated exceptions, all of the quantitative data pertaining to earthwork surveys we have utilized in the formation of the TAG hypothesis comes exclusively from the Middleton surveys. The accuracy and reliability of those data has been discussed elsewhere (Hively and Horn 1982).

Our objective in analyzing the geometry of the earthworks is to extract a minimal and simple core of geometrical principles that explain the structure of the earthworks and that are directly tied to the shape and dimensions of the earthworks on the ground. We have sought to avoid any geometrical rationale that introduces hypothetical dimensions, grids, or geometries not immediately evident from the structure of the works themselves.

Geometry at the Newark Earthworks

First, we will summarize the relevant evidence for these geometrical principles at the Newark Earthworks. Figure 1 shows the major figures at Newark. We have previously discussed in some detail the impressive accuracy and precision associated with the construction of these figures (Hively and Horn 1982, 2013). Understanding the relationships underlying the geometry of the Newark figures begins with the observation that the most carefully constructed of the figures (the Observatory Circle connected to the Octagon) is a nearly perfect circle of diameter 1054 ft.

The geometrical structure of the Octagon shows that its design incorporates a combination of a square of 1054 ft (which would circumscribe the Observatory Circle) and a circle with a diameter of 1490 ft (which would circumscribe a 1054 ft square). As shown in Figure 2, the Octagon vertices ACEG form a square of 1054 ft. The remaining four intermediate vertices BDFH fall accurately on the perimeter of a circle of 1490 ft as shown in Figure 2. We also note for the first time here that the middle of each Octagon side falls on the perimeter of the 1490 ft circle. Using this design, the builders both insured that each pair of parallel sides in the Octagon would be separated by 1490 ft and that the area of the Octagon would be a close match to the area of the 1490 ft circle (equal to better than 4%).

Figure 1. This schematic map shows the main geometrical figures comprising the Newark Earthworks. Only the Observatory Circle, the Octagon, and the Great Circle still survive. The Observatory Circle has a diameter of 1054 ft. The earthworks contain about 7 million cubic ft of earth and cover 4 square miles.

Another equivalent way of describing the Octagon geometry (Figure 2) is that four vertices ACEG form a 1054 ft square and the intermediate vertices BDFH form a 1213 ft square. The 1054 ft square precisely circumscribes the Observatory Circle. The 1213 ft square side differs by only 1.7% from the diameter of the Great Circle (1185 ft) to the south of the Observatory Circle.

Previous studies have ignored the position of the mounds interior to the Octagon vertices shown in Figure 3 because they are not geometric points with exactly defined positions. Measurements taken from the original Middleton maps and contemporary aerial photos of the reconstructed mounds do allow the locations of their estimated centers to be located within a probable accuracy of around ±5 ft. These estimates strongly suggest the interior mounds were carefully placed to duplicate many of the geometrical features of the larger walled Octagon.

The eight interior mounds, if connected by straight lines, would form an octagon with sides parallel to the walls of the larger Octagon. The size of the mound-octagon repeats many of the regularities seen in the walled Octagon. Four

Figure 2. The geometry of the Newark Octagon. A dashed circle with a diameter of 1490 ft passes through vertices ACEG and the midpoint of each side. The Octagon vertices define vertices of two squares of sides 1054 ft and 1213 ft shown with dashed lines.

of the mounds interior to vertices BDFH form a square of 1054 ft and, therefore, fall on the perimeter of the same 1490 ft circle, which passes through vertices ACEG of the walled Octagon. The other four mound-octagon vertices ACEG in Figure 3 form an accurate square with an average side of 917 ft. This square's dimension differs by only 1.5% from the 931 ft dimension of the Wright Square to the south of the Octagon.

There is further evidence suggestive of deliberate geometrical relations between the Newark figures. While the Salisbury map (Salisbury and Salisbury 1862) is not as accurate as the Middleton data, it is generally superior to the map of Squier and Davis (1848), and so may be of some, although limited, value here. According to the Salisbury map, the average side length of the Salisbury Square in Figure 1 is equal within 1% to the side of a square that could be inscribed inside the Observatory Circle. The dimensions of the Cherry Valley Oval are imperfectly known but the

Figure 3. The original Middleton map (1888) of the Newark Octagon together with the interior flat-topped mounds inside the vertices. The dashed circle of 1490 ft is superimposed on the Octagon. The figure also shows that the eight interior mounds form two accurate squares (dashed lines) of 1054 ft and 917 ft.

best estimates for the major and minor axes of that structure (given by the Salisbury map as 1760 ft and 1460 ft, respectively) are approximately equal (within 3%) to the symmetry axes BF and AE (Figure 2) of the Octagon. The centers of the Observatory Circle and Great Circle and the centers of the Octagon and Wright Square are both separated by 6 x 1054 ft (within 0.4%; Hively and Horn 1982).

Squaring the Circle at Newark

The ability to construct squares and circles of nearly equal area is commonly referred to as squaring the circle. There is no archaeological or ethnographic evidence published to date that any prehistoric culture accomplished that feat. Nevertheless, we find that the geometrical evidence for that accomplishment by the Hopewell is sufficiently strong that it merits serious consideration.

Evidence at Newark for squaring the circle involves: (1) the area of the Observatory Circle is equal (within 0.6%) to the area of the major square (the precisely constructed Wright Square southeast of the Observatory Circle); (2) the area of the Great Circle is equal (within 0.8%) to the area of a square of side 1054 ft (formed by Octagon vertices ACEG); (3) the Octagon (interpreted as a symmetrically modified square) has an area equal (within 4%) to the area of a 1490 ft circle that circumscribes a square of side 1054 ft; (4) the interior-mound octagon has an area equal to that of a 1316 ft circle (within 0.5%) which would circumscribe the Wright Square; (5) a 1316 ft circle circumscribing the Wright Square has an area which matches that of a 1185 ft square circumscribing the Great Circle within 3%.

Several other area matches in the earthworks (which are consequences of these dimensions) should be recorded: (1) the sum of the areas of the Observatory Circle and Wright Square is equal (within errors of measurement) to the area of the 1490 ft circle which passes through the mid-walls of the Octagon; (2) the sum of the areas of the Great Circle and the 1054 ft square (formed by vertices ACEG) is similarly equal to the area of a 1490 ft square which circumscribes the Octagon by passing through the vertices ACEG (Figure 2); (3) the sum of the areas of the Observatory Circle and the interior-mound octagon is also equal (within 1%) to the area of a 1490 ft square; (4) the sum of the areas of the Great Circle and the walled Octagon is equal (within 1.2%) to twice the area of the 1213 ft square (formed by vertices BDFH; Figure 2).

Finally, it should be noted that the Salisbury map data for the dimensions of the Cherry Valley Oval show that it and its westward elliptical extension had an enclosed area which matched the area of a 1490 ft square or twice the area of the Great Circle (probably within 4%). Thus, the areas of the Great Circle and Oval have a relation similar to the areas of the Observatory Circle and the Octagon. Considering all the accurate area matches relating the Newark figures, the harmony of symmetry, dimension, and area is too repetitive and accurate to be dismissed as chance without very careful consideration.

The evidence suggests that the Hopewell intended to design figures with areas that were related and, thus, had some ability to compare and measure areas as well as lengths. If we are to contemplate the possibility that the Hopewell designers could accurately square the circle, we have to address two questions. Is there an algorithm that might have been plausibly discovered and utilized by the Hopewell for squaring the circle? Is there any additional evidence in support of circle squaring at other Hopewell sites? First, let us consider how the Hopewell might plausibly have done this.

The Hopewell were clearly concerned with circles and squares related by circumscription. This implies that they could likely compare the relative lengths of the sides and diagonals of squares. John Volker brought to our attention in an unpublished manuscript a very simple algorithm for squaring the circle. Volker noted that a circle of diameter d and a square of side s will have nearly equal areas (within 0.5%) if the ratio of the square diagonal to the circle diameter is 5/4, i.e., if $\sqrt{2s/d} = \frac{5}{4}$ (Volker 2003).

The geometry of the Octagon shows that the Hopewell were likely concerned with the relations between the dimensions of the sides and diagonals of squares together with the diameters of circles that could be inscribed within or circumscribed about a given square. It also seems plausible that they could compare areas in some quantitative fashion. Experimentation with these ideas could plausibly lead to the suggested algorithm. If the area duplication of circles and squares was a deliberate feature of the design of the Great Circle (with the area of a 1054 ft square) and the Wright Square (with the area of a 1054 ft diameter circle), we would expect as a geometrical consequence that the Great Circle and the Wright Square would have nearly equal perimeters. This expectation is confirmed within 0.2%.¹

The design principle motivating the construction of the Great Circle and the Wright Square does not appear, however, to be the equality of perimeters. First, we know of no other case where Hopewell circles and squares have common perimeters. Second, if perimeter equality was the primary intent, that aim alone does not predict the precise perimeter encountered here. If the intent was area duplication then the dimensions of the Great Circle and Wright Square follow directly from the area equivalents of a 1054 ft square and a 1054 ft circle. If constructing equivalent areas for circles and squares was not the intent of the Hopewell designers, then the dimensions of these figures remain unexplained.

If the suggested algorithm was actually utilized to construct the Great Circle and the Wright Square, we would expect that distances of one-fourth the diameter of the Great and Observatory Circles would have played an important role in their construction. Is there any evidence at the site that these distances of 296 ft (onefourth of the Great Circle diameter) and 264 ft (one-fourth of the Observatory Circle diameter) played an important role in the construction? The answer is yes in both cases. Middleton surveyed the length of the carefully constructed avenue connecting the Observatory Circle and Octagon as 295 ft Next, the Salisbury map (generally regarded as reasonably accurate) shows a 264 ft wall separated slightly from the Wright Square and trending northeast toward the Cherry Valley Oval

Figure 4. Arrows added to the Salisbury map (1862) show the 295 ft avenue connecting the Observatory Circle to the Octagon and the 264 ft wall trending northeast from the northeast side of the Wright Square. The Volker algorithm for squaring the Observatory Circle and the Great Circle predicts these lengths.

(Figure 4). Comparing the Salisbury map with that of Middleton, we have no reason to doubt this length has accuracy of about \pm 2 ft

These two otherwise unexplained but carefully constructed straight walls fit the lengths and placements predicted by the circle-squaring algorithm we have proposed. The 295 ft wall leads into the Octagon where the four vertices ADEG form a 1054 square with a diagonal, which, within errors of measurement, is five times the length of the adjoining wall. Similarly, the 264 ft wall next to the Wright Square is within measurement uncertainty of one-fifth of the diagonal of the Wright Square. These intriguing and otherwise thus far unexplained wall dimensions are consistent with the use of the suggested circle-squaring algorithm in the construction of these two squares. These two walls are indicated in the Salisbury map in Figure 4. Studies by Marshall and Romain have also concluded that a measurement unit of 264 ft played an important role in the design of Hopewell earthworks (Marshall 1987; Romain 2000).

When interpreting these data one must always be cautious of random patterns or dimensions, which appear simply by chance. One must also seek to avoid an intuitive application of this caution, which might preclude a fair evaluation of new or unexpected results. Ultimately, a critical analysis from a variety of perspectives will have to achieve a consensus on this point. By way of historical precedent, it should be noted that the earliest written record available of any culture squaring the circle comes from the Rhind Papyrus of ~1650 BC (Chace and Manning 1927:48–50). This document suggests that ancient Egyptians computed the area of a circle by approximating it with an octagon (although the octagon was neither regular nor equilateral), similar to what the Hopewell may have done. As early as 600 BC, literate cultures from the Mediterranean basin to China recognized this problem and offered solutions (Olson 2010:25–59).

When we suggest the Hopewell not only recognized the problem, but also displayed their good approximation on a grand scale, skepticism is inevitable. Seen in the context of formal mathematics, the Hopewell achievement is implausible. In the context of the empirical geometry from which the formal mathematics of the classical civilizations grew, it need be nothing surprising. Work with the basic shapes encountered in spinning, weaving, basket making, and construction demanded solutions such as how to achieve the space available in a circular dwelling, in a square one, or how to craft a coiled, circular cover for a basket with a rectilinear plaited base.² The evidence that the Hopewell more than met these demands is on the ground at Newark.

While some of the geometrical repetitions we have found may be accidental, the consistent and repetitive presence of area duplication and the 1054 ft dimension demands serious consideration as a deliberate and important part of the design. The regularities we have found involving combinations of squares and circles related by circumscription or area equivalence are what one might expect from a culture apparently concerned with the spatial properties of empirical geometry. If these regularities were indeed an important and conscious part of the design, we would expect to find confirming evidence elsewhere in the Hopewell core, which underscores the importance of the next section.

Geometry in the Scioto and Paint Creek Valleys

Accurate survey data, due to James Middleton, exist for parts of four geometrical earthworks in the Scioto and Paint Creek valleys (Figure 5):³ High Bank, Liberty, Baum, and Seip. The High Bank Circle-Octagon reveals much of the content, motivation, and methodology of Hopewell earthwork design and construction (Figure 5). A clear connection to the Newark geometry is established by

Figure 5. Squier and Davis maps of the five earthworks considered in this chapter: Liberty, Works East, Seip, Baum, and High Bank. These maps are known to be inaccurate in their quantitative details but do give a general idea of the earthwork features.

noting that the High Bank Circle is a very accurate circle with a radius of 526 ft, within errors of measurement identical to the Observatory Circle at Newark. The High Bank Octagon can be constructed using a method almost identical to one we earlier adopted for the Newark Octagon (Hively and Horn. 1982), that is, by drawing circular arcs (with 1052 ft radii) and centered on the vertices of a square (BDFH in Figure 6) with a side of 892 ft. The intersection of these circular arcs determines the location of the remaining vertices (ACEG in Figure 6).

Figure 6. This drawing shows the idealized High Bank Octagon with the High Bank circle (1052 ft in diameter) passing through the midpoint of each side. The isosceles triangle with a common side of 1052 ft that passes through the vertices A, D, and F shows the geometrical structure.

The geometric accuracy of the proposed method of construction is evident when one notes that the design predicts an octagon side of 453 ft compared to the measured average side length of 451 ft. The predicted interior angles at the vertices for the ideal design are 166.4° and 103.6° compared with the measured average values of 165.5° and 104.5°. Deviations from the ideal octagon plan are consistent with the intent to record astronomical alignments to the sun and moon (Hively and Horn 1984).

This 892 ft starting square for the octagon construction has a side length which would be a good first approximation for a designer concerned with squaring the High Bank Circle, i.e. the square has a side dimension about halfway between the squares which circumscribe and inscribe a circle with a diameter of 1052 feet. Indeed, an 892 ft square matches the area of the High Bank Circle within about 9%. The method of octagon construction illustrated in Figure 6 produces an octagon that matches the area of the High Bank Circle within 3.2% and for which opposite parallel sides are separated by 1052 ft (the diameter of the adjoining High Bank Circle). Both the scale of the square BDFH and the distance between parallel walls of the octagon can be understood if the attempt was to construct an octagon with an area equal to that of the High Bank Circle. At Newark, a circle with a diameter of 1490 ft and centered on the Octagon passes through the midpoints of each side. At High Bank, a circle with a diameter of 1052 ft and centered on that octagon passes through the midpoints of each side. These measurements highlight again the evident importance of the dimensions associated with the diagonal and side of a 1054 ft square.

One other intriguing regularity common to the two octagons should be noticed. At Newark the Octagon vertices ACEG form a 1054 ft square which has the same area (within 4%) as a 1213 ft circle inscribed within the square formed by the vertices BDFH. At High Bank, the vertices BDFH fall on a square of 892 ft, with an area equal (within 0.7%) to that of the 1007 ft circle, which circumscribes the square, formed by the remaining vertices ACEG. This commonality may be accidental, but it should be noted that these are the only two possible equilateral octagon shapes that permit this area duplication between circles and squares defined by the octagon vertices. These are precisely the two shapes chosen by the octagon designers. Other factors may govern specific details of the shape, specifically orientations to topographical features or astronomical events (Hively and Horn 1982, 1984).

Middleton was able to obtain survey data for nine sides of the three accurately constructed squares (Baum, Liberty, and Seip). These sides range in length from 1103 ft to 1129 ft with a mean of 1112 ft. The circle, which most closely circumscribes the High Bank octagon, has a diameter of 1261 ft (missing each vertex BDFH in Figure 6 by less than 6 ft). The square, which matches the area of this circumscribing circle, would have a side of 1115 ft (within 0.3% of the Middleton average). An equivalent statement of this relation is that an 1112 ft square has a diagonal which is equal (within 0.4%) to three times the radius of the High Bank Circle (i.e., 3x $(1052/2) = 1578$ ft).

Comparing this to the context of the Newark Octagon, it is noteworthy that a circle circumscribing that octagon (passing within six feet of vertices ACEG in Figure 2) matches the area of the 1490 ft square that circumscribes the octagon (passing through vertices BDFH) within 4%. In the Newark case squares and circles (with equal areas) circumscribing the Octagon were already implicit in the

design. This is not the case for the High Bank Octagon. If constructing an areamatching square for the octagon-circumscribing circle at High Bank was an important task for the builders, it would have to be introduced in a separate figure.

If Squier and Davis are to be trusted in their assertion that Works East and the Frankfort Earthworks had squares nearly identical to those of Baum, Seip, and Liberty, then the dimension of these five squares had a noteworthy significance for the Hopewell. The builders might well have attached great importance to the discovery that a square with a diagonal of three times the radius of the High Bank Circle would have the same area as a circle circumscribing the High Bank Octagon. It at least represents a rational and plausible reason why this particular square would be significant.

The Middleton survey of the Liberty Square (Figure 5) shows the positions of four mounds interior to the midpoints of the square sides and the dimensions of the small enclosure southwest of the Square. The mounds define the corners of a square with a side of 604 ft. The area of this smaller square added to the area of the High Bank circle exactly matches the area of the Liberty Square (within errors of measurement). This repeats a similar pattern found at Newark. There the area of the interior octagon formed by the interior mounds (when added to the area of the Observatory Circle) duplicates the area of a 1490 ft square.

The Middleton survey shows the small oval enclosure adjacent to the Liberty Square has an area of 511,000 ft². The sum of the oval area and the Liberty Square area is equal to twice the area of the High Bank Circle (within 1%). This further underscores the apparent importance to the builders of the relationship between the areas of major figures.

The Seip, Baum, Liberty, and Works East sites originally included earthen walls that formed partial circles or large arcs (Figure 5). We have Middleton data only for a small arc of the large Seip Circle. The only surveys available for the others involve Squier and Davis's surveys (often unreliable) and LiDAR, aerial imagery, and ground surveys of earthwork remnants not clearly visible from the ground. Still, it is useful to note that the best information available is consistent with the geometrical principles which we believe in part governed these constructions. The Squier and Davis survey, Marshall's ground survey, and the Romain-Burks LiDAR survey data agree in suggesting that the large incomplete circles had diameters of approximately 1700 ft This is consistent with a complete circle, which duplicates the area of a 1490 ft square or doubles the area of a 1054 ft square (within about 1%).⁴

Geometrical Conclusions

While it may never be possible to demonstrate the intentionality of a unique set of geometrical principles explaining these earthworks, the repetitive regularities found at Newark, together with the works in the Scioto-Paint Creek valleys, demonstrate with little doubt that a common core of geometrical ideas was employed at all of these sites.

The geometry evident from these studies suggests that the Hopewell builders possessed at least three skills related to empirical geometry: (1) the ability to design and build earthworks in the form of circles, squares, and octagons involving units of length derived from a standard square with a side of 1054 ft and a diagonal of 1490 ft; (2) the ability to compare the areas of geometrical figures such as squares, circles, and octagons; and (3) the ability to construct large-scale circles, squares, octagons, and ovals with nearly equivalent areas.

We do not believe the currently available data provide persuasive support for any speculation about the use of more elaborate numeric, algebraic, or geometrical systems. All of the information needed to construct the earthworks could be gathered (perhaps over several generations) by measuring the lengths and areas of geometrical figures using measuring cords or rods (Luecking 2004). Still the scale, precision, and the relations between the squares and circles of this Hopewell geometry are remarkable in that they have no clearly demonstrated precedent in the prehistoric world.

Topography, Astronomy, and Geometry in Scioto Hopewell

Here we examine the earthworks of the Scioto region for evidence to support or disconfirm the hypothesis derived from our work at Newark: that earthworks were designed and placed in part to record correspondences between local topographical features and astronomical phenomena. An understanding of the plausibility of the TAG hypothesis requires a description of the unique and special nature of the geological history that has shaped the topography of south-central Ohio.

The Hopewell built these earthworks in uncommonly broad river valleys that today channel the grossly under-fit and meandering Scioto River and Paint Creek. Low hills of the Appalachian Plateau line the punctuated horizon that borders the valleys. This unique and scenic topography is the product of two forces: (1) the preglacial and north flowing Nile-scale Teays River that dominated the Ohio landscape until it was destroyed by the advancing ice sheets of the Pleistocene era two million years ago; and (2) the effects of repeated glaciations until the last of the Wisconsinan glaciers retreated from Ohio (Hansen 1987). The result was a landscape that featured striking broad and fertile flood plains, well-defined valleys, and scenic vistas with distant horizons. These prominent valleys and distant hills established a commanding physical frame of reference and spatial orientation for the valley's inhabitants. The neighboring hills also provided a dominant and unchanging set of landmarks to reveal even small changes in the periodic cycles of the rise and set points of the sun and moon. The geophysical environment provided an unusually vivid stage for noticing geological and astronomical phenomena.

We begin our topographical analysis by noting the placement of three Hopewell sites in the Scioto Valley: Works East, High Bank, and the Liberty Works. Information about the location and shape of the earthworks comes from a variety of sources including: (1) the maps of Squier and Davis (Figure 5); $^{\text{s}}$ (2) aerial photos of some remnants of Liberty and High Bank; (3) survey data from the Smithsonian Thomas report for the internal geometry of the High Bank and Liberty sites (Thomas 1894:476–482); (4) maps prepared by James Marshall based on field surveys and aerial photos (Marshall 1987). Distinct traces of the High Bank site are still visible on the ground and on aerial photos. Based on all the available sources, we estimate that we know the location of Works East to an accuracy of 30 ft and the location of High Bank and Liberty with an accuracy of perhaps five to ten feet.⁶

The estimated locations of the centers of the large circles in these earthworks are shown on a map of the Scioto Valley provided in Figure 7. The centers of the three large circles associated with the three earthworks fall on a straight line with an azimuth of 143° within an accuracy of 1°. Figure 7 also shows that the hills defining the western edge of the Scioto Valley and the course of the Scioto River lie along an anomalous virtually straight linear feature with an azimuth of 143° (again with an accuracy of less than one degree) for a distance of six miles. Thus, it is plausible that these earthworks were intentionally aligned with the physical axis so compellingly defined by this linearity. The axis defined by the circle-avenue-octagon combination at High Bank also falls along the same azimuth (within 1 degree), which adds further weight to this conclusion. The uncertainty associated with the survey data and imperfection in the circularity of the large Works East and Liberty circles does not introduce an error of more than a small fraction of a degree (a negligible error <0.1°) to our proposed alignments.

As observed from prominent elevations in the local topography, the rise/set points of the moon (defined as lower limb tangency of the moon on a distant horizon) exhibit two cyclic motions. First the lunar rise/set positions on the

Figure 7. The centers of the black circles mark the location of the centers of the large circles in the Works East, High Bank, and Liberty earthworks. Note that a line passing through the centers of all three falls along an azimuth of 143° parallel to the black line showing the six-mile linear section of the Scioto Valley.

horizon move from a northerly extreme to a southerly extreme and then back to the northern extreme every sidereal month (27.3 days). Because of a precession of the moon's orbital plane, the azimuth of these extreme north and south rise/set points slowly oscillates (with a period of 18.6 years) between a maximum extreme and a minimum extreme value. When the moon is at a maximum or minimum extreme in this 18.6-year cycle, the extreme monthly northerly and southerly rise/ set points appear to *stand still* or remain close to the maximum or minimum extreme positions on the horizon for about two years. Hence, the term *standstill* is associated with the Moon being at either a maximum or minimum extreme position. Figure 8 illustrates this cycle.

Figure 8. This diagram shows the azimuths of the directions to the extreme north and south moonrises and moonsets associated with the major and minor positions of the lunar standstill cycle.

If these lunar extreme rise/set points were of significance to the builders of the earthworks, we might expect they would incorporate alignments to these directions in the design of the earthworks in an accurate and evident fashion, as we found at Newark. The alignments we propose here involve observations from high hills or bluffs over sightlines several miles long. In this case, the effects of vegetation on the azimuths of possible alignments are negligible (i.e., $<$ 0.1°).

The rise/set points of the sun move with an annual period of one year between a northern extreme at the summer solstice and a southern extreme at the winter solstice. At the solstices, the extreme rise/set points of the sun occur at the midway points between the maximum and minimum lunar extremes. The astronomical and algorithmic data associated with computing these rise/set points are detailed in previous papers (Hively and Horn 2006) and will not be repeated here. The astronomical alignments proposed for consideration in this paper have sub-degree accuracy (with only two exceptions of 1.0° accuracy) for lower limb tangency rise/set events. Our analysis of alignments at the Newark Earthworks led us to expect this level of accuracy. The precise numerical topographic and astronomic azimuths involved in this paper are shown in Table 1. All topographic azimuths were obtained from Global Mapper Software (version 14) using USGS topographic maps.

Topographical Azimuth	Astronomical Event Azimuth ^a
Rattlesnake Knob-Center of Mound City	Max North Moonset
307.7°	307.8°
Rattlesnake Knob-Center of Works East Circle and Adena Mound 300.5°	Summer Solstice Sunset 301.5°
Rattlesnake Knob-Center of Small Square in Hopewell Earthworks 293.1°	Min North Moonset 293.6°
Grandview-Center of Liberty Circle through notch 129.3° shown in Figure 9	Max South Moonrise 129.3°
Grandview-Center of High Bank Circle	Min South Moonrise
114.9°	114.9°
Spruce Hill-Center of Baum Circle	Max South Moonset
230.4°	230.4°
Spruce Hill-Center of Seip Circle	Min South Moonset
244.8°	245.0°
Spruce Hill-Jester Hill	Winter Solstice Sunset
$2.39.7^{\circ}$	$2.38.7^{\circ}$
Spruce Hill-Mount Logan	Summer Solstice Sunrise
59.1°	58.8°
Jester Hill-Center of Seip Circle	Max North Moonrise
51.7°	52.2°
Hill S1-Hill S3	Winter Solstice Sunrise
121.1°	121.1°
Hill S3-Hill S1	Summer Solstice Sunset
301.1°	301.5°
Hill S2-Hill S4	Summer Solstice Sunrise
58.3°	58.6°
Hill S4-Hill S2	Winter Solstice Sunset
238.3°	239.3°
Hill S1-South Corner of Liberty Square	Min South Moonrise
115.2°	115.0°
Hill S2-Northern Corner of Liberty Square	Min North Moonrise
66.1°	66.4°

Table 1. Topographical and Astronomical Azimuths.

a The negligible errors associated with alignments from Grandview and Spruce Hill result from our choice for the precise position of the backsights. The significance of this is that it was possible to choose the backsights at the prime overlooks of the valleys. The backsights on hilltops were chosen at the highest elevations. Astronomical alignments include small corrections for horizon altitudes. All rise/set events are for lower limb tangency to the horizon.

Our analysis of the siting and orientation of the Newark Earthworks led us to suggest that the Newark site was chosen, in part, because of a fortuitous correspondence between prominent topographical features and the extreme rise/set points associated with the lunar standstill cycle and the solar solstice cycle. This leads us to have the same expectation for the Scioto Valley earthworks. An analysis of the Scioto Valley topography confirms this expectation.

The remarkably straight six-mile segment of the river and the ancient valley along which the three earthworks are aligned is nearly perpendicular (within one degree) to the maximum extreme northern lunar moonrise (at the major standstill) as viewed from hills on the west edge of the valley (Figure 7). Given the obvious importance attached to the construction of perpendicular lines in Hopewell earthworks, the builders would plausibly notice this natural right angle between terrestrial and astronomical phenomena. Since the distant moon shows no observable parallax for local observers, anyone watching the major standstill northern extreme moonrise from the west side of the valley would see the moon rise at right angles to the river below, regardless of their position on the western ridge. This would be a striking visual effect, especially with moonlight reflected from the river.

The most obvious point of comparison between the Newark Earthworks and the Scioto sites is the azimuth of the circle-octagon axes in the Newark Earthworks and the High Bank Works. The circle-octagon axis at Newark aligns within 0.5° to the north maximum standstill rise point. The corresponding axis at High Bank (aligned with the linear section of the Scioto Valley) is perpendicular (with comparable sub-degree accuracy) to the same northern maximum standstill rise point. We suspect that the perpendicular relation between the northern lunar maximum extreme rise and the linear valley segment was a significant motivation for building earthworks at this site. This result leads us to expect that (as at the Newark site)

Figure 9. The three extreme northern set points associated with sun and moon as seen from Rattlesnake Knob, the most prominent peak in the Scioto Valley. Note the alignments pass through manmade features: the large circle of Works East (WE); the center of Mound City (MC), the Adena Mound (AM), and the eastern side of the Hopewell Earthworks (HO). Note too how all three alignments are framed by the Scioto Valley in striking fashion.

the Scioto earthworks would be located to mark additional coincidental correlations between topographical features and directions to the lunar standstills.

The terrain of the Scioto Valley south of Chillicothe highlights the standstill cycle in vivid fashion. Rattlesnake Knob, the most prominent peak in the valley, rises some 330 ft above the valley floor. As seen from the summit of Rattlesnake Knob, the broad valley formed by the ancient Teays River provides a visually arresting reference frame for the moonsets at the northern lunar standstills. The sightlines to the extreme northern moonset standstills as seen from Rattlesnake Knob are shown in Figure 9 (for coordinates, see Table 27).

Overlook Name	Latitude	Longitude
Rattlesnake Knob	39.28636° N	82.85647°W
Grandview Overlook	39.32284° N	82.98370° W
Mount Logan	39.35710° N	82.94656°W
Jester Hill	39.21154° N	83.26310°W
Spruce Hill Overlook	39.26945° N	83.13511° W
Hill S1	39.27929° N	82.93861°W
Hill S ₂	39.24613° N	82.92067° W
Hill S3	39.22748° N	82.82819°W
Hill S4	39.29624° N	82.81617°W

Table 2. Coordinates of Proposed Observing Sites.

As shown in Figure 9, an observer stationed at the top of Rattlesnake Knob would see the monthly extreme north moonset move back and forth from the east to west side of the valley during the standstill cycle. Such an observer would also see the summer solstice sunset occur down the center of the valley. Thus, the valley provides an impressive natural stage highlighting the extreme northern setting positions of the sun and moon as seen from the most prominent elevation in the valley.

Placement of Earthworks in the Scioto Valley

The first principle in aligning the three earthworks (Works East, High Bank, and Liberty) appears to have been to place the centers of their dominant circles on a line parallel to a linear segment of the Scioto Valley, i.e., a topographical alignment. What determines the location of these circles along this particular line? If one examines the landscape for the location which affords the most expansive and unobstructed view of the full southeastern extent of the Scioto Valley, one would most likely choose a point on the bluff overlooking the valley which is now occupied by Grandview Cemetery in Chillicothe. The view from this point 200 ft above the valley floor is so arresting and scenic that the area has not only been named Grandview, but it was placed on the National Register of Historic Places in 1978.

If lines are drawn through the centers of the High Bank Circle and the Liberty Circle perfectly aligned to the minor and major southern moonrise standstills, these lines converge at the edge of the bluff of Grandview cemetery offering what is arguably the most impressive possible view of the Scioto Valley toward the south. This point serves as a common backsight for the two standstill alignments through

Figure 10. The extreme southern moonrise points and minor and major standstill as viewed from the prominent Grandview overlook. The minor standstill alignment passes through the center of the High Bank circle; the major standstill alignment passes through the approximate "center" of the large Liberty Circle.

the circle centers. From this point, the observed southern extreme rise point of the moon at the standstills oscillates back and forth between the two circle centers every 18.6 years. Figure 10 shows the standstill alignments.

This proposed Grandview backsight also defines an alignment to the southern moonrise at the maximum standstill in a unique fashion. From this point, the extreme southern maximum moonrise will occur exactly in a notch (slightly wider than the full moon) that is by far the most prominent feature on the distant horizon. An observer watching the lunar cycle from the Grandview bluff would notice this striking correspondence between the moon and the local topography. This singular feature of this specific point on the Grandview bluff would explain its choice as the

Figure 11. The Scioto Valley as seen from the Grandview overlook. Black arrows show the horizon positions of the southern minor and major extreme moonrises. Notice the prominence of Rattlesnake Knob. Note also that the moonrise at the major standstill occurs in a very distinctive horizon notch .(Authors' photo)

backsight for the standstill alignments through the circular earthworks in the valley. Figure 11 shows this distant notch as seen from the suggested backsight.

The views of the southern extreme rise points for both the sun and moon as seen from Grandview are framed by the north and south sides of the Scioto Valley (and the associated earthworks) in much the same way as the northern extreme set points are framed as viewed from Rattlesnake Knob. As shown in Figure 10, an observer from the Grandview bluff sees the extreme southern moonrise move from the north side of the valley through the High Bank Circle to the south side of the valley through the Liberty Circle at the major standstill. The winter solstice sun would rise along a line passing through part of the High Bank earthworks (not shown in Figure 10) and along the center of the broad valley below.

At Newark, four such overlooks (each on a prominent bluff, peak, or ridge) served as backsights for all of the lunar standstills (with sub-degree accuracy) aligned through the centers or sides of major earthwork figures. The Newark example would lead us to predict that, if this pattern were deliberate, it would also appear in relations between the topography and earthworks in the Scioto and Paint Creek region. We find that the earthworks in these valleys fulfill these predictions.

Consider again Rattlesnake Knob, which provides a good view of the northern standstill moonsets and the summer solstice sunset (Figure 9). A line from the top of Rattlesnake Knob along the northern moonset at the major standstill passes directly through the center of Mound City, a notable Hopewell burial ground surrounded by a square-like earthwork with rounded corners. A line from the top of Rattlesnake Knob along the summer solstice sunset passes through the center of the large circle of Works East and over the top of the Adena Mound.⁸ Finally, a line from the top of Rattlesnake Knob along the direction of the northern moonset at the minimum standstill passes through the center of the eastern small square of the Hopewell Mound Group. Notice in this context that this last alignment to the minor standstill is between points that are not intervisible, although each is visible from an intervening ridge. All the proposed alignments have sub-degree accuracy.

The alignment between Rattlesnake Knob and Mound City to the extreme northern moonset at the major standstill repeats a very similar alignment found at Newark. There a line from the highest hill in the region (Coffman Knob) along the northern maximum moonset passes through the center of the Cherry Valley Oval earthwork containing the most prominent burials at the site and along the narrow Sharon valley.

The Placement of Earthworks in the Paint Creek Valley

Paint Creek joins the Scioto River south of Chillicothe near Works East. Flowing from the west, it meanders through another broad valley flanked by hills that punctuate the horizon.⁹ The Paint Creek Valley contains two major geometric earthwork complexes: the Seip Works and the Baum Works. Middleton survey data from the Thomas report are available for parts of the squares at the two sites. Good locations (known to within six feet) are provided by modern archival, aerial, and field surveys by Marshall (Marshall 1987) and by Romain and Burks (2008a, 2008b) using LiDAR and aerial photographs.

In assessing the possible astronomical and topographical correspondences for the Paint Creek valley, three facts stand out immediately: (1) Spruce Hill (the loca-

Figure 12. This map shows the location of Spruce Hill (a known Hopewell site with a stone wall) relative to prominent hills (Mount Logan and Jester Hill) which bracket the Paint Creek Valley. As viewed from Spruce Hill (at the point shown by the white circle) the summer solstice sunrise appears over Mount Logan. The winter solstice sunset occurs along a line through the center of Paint Creek Valley and over the top of Jester Hill.

tion of a unique Hopewell enclosure surrounded by a stone wall) provides the best overlook (toward the southwest) of the Paint Creek valley 350 ft below; (2) from a position on Spruce Hill the summer solstice sunrise occurs over Mount Logan (the highest elevation to the northeast) and winter solstice sunset occurs over Jester Hill (one of the highest elevations to the southwest)(Figure 12); (3) as seen from the southwest edge of the Spruce Hill bluff, the southern extreme moonset at the minor and major standstills is aligned with the edges of the valley (Figure 13). The impressive view of the valley and standstills from Spruce Hill is strikingly similar to the Grandview overlook noted previously in the Scioto Valley.

Figure 13. Alignments to the southern major and minor standstill extreme moonsets as seen from the Spruce Hill Overlook (SHO) through the centers of the large Seip and Baum circles. As viewed from Jester Hill the northern extreme moonrise at major standstill passes through the center of the large Seip circle. The minor northern standstill moonrise passes along the eastern edge of Paint Creek although it is otherwise unmarked.

Given the context provided by our analysis of the Newark and Scioto earthworks, the TAG hypothesis would predict that earthworks in the Paint Creek valley would likely be located to fall along lines from a prominent overlook on Spruce Hill toward standstill moonsets that were not marked in the Scioto Valley. The Seip and Baum earthworks in the Paint Creek valley both contain large arcs that, while not complete circles, nevertheless have reasonably well-defined centers. We have found that lines drawn through the estimated centers of the large Seip and Baum circles along the directions of the southern extreme moonsets at minor and major standstill converge at a suitable observation point near the southwest

edge of Spruce Hill (Figure 13 and Table 2). Thus the placement of these large circular earthworks relative to this Spruce Hill Overlook (SHO) mark the 18.6-year lunar standstill cycle in much the same fashion as we have found repeatedly at Newark and in the adjoining Scioto Valley.

As seen by an observer at SHO the southern maximum moonset moves between the large Seip and Baum circle centers during the standstill cycle. The summer solstice sunrise occurs over Mount Logan and the winter solstice sunset occurs over Jester Hill. All these alignments (with exception of the one-degree error for the Jester Hill alignment) are of sub-degree accuracy. The fact of these alignments occurring as seen from such a prime observation point appears to be either a rather improbable accident or a notable confirmation of the expectations of TAG.

If the Hopewell builders had deliberately incorporated six of the eight lunar standstill lines into their earthworks in this region (as seen from SHO, Grandview, and Rattlesnake Knob), then the TAG hypothesis suggests that they would have attempted to incorporate the remaining two standstill alignments into the scheme (i.e., the northern extreme standstill moonrises). We can test this prediction by asking what a hypothetical observer on the top of Jester Hill looking toward the northeast would see. An observer located at the high point of Jester Hill would see the northern extreme moonrise and major standstill occur (with sub-degree accuracy) over a line through the center of the large Seip Circle. Thus, our prediction for this sightline is confirmed.

This result then leads to the prediction that a similar earthwork would be located along a line from Jester Hill toward the northern extreme moonrise at the minor standstill. This minor standstill line from Jester Hill lies along the eastern edge of the Paint Creek valley. There is no space in which a major earthwork visible from Jester Hill could be constructed along this line. Moreover, this direction is effectively marked, without the aid of earthworks, by the eastern edge of the valley (Figure 13). An observer on Jester Hill would see the northern extreme moonrise at major standstill along the western edge of Paint Creek Valley and through the center of the Seip Circle. As the standstill cycle progressed, the monthly extreme northern moonrise would then sweep across Spruce Hill to the eastern side of the valley at the minor standstill.

From the four major observation points (Rattlesnake Knob, Grandview, Spruce Hill, and Jester Hill), all of the eight lunar standstill directions are plausibly marked by geometrical figures, topographical features, or both. In all these cases, the relevant extreme standstill rise/set events are confined within the boundaries of the

Figure 14. The four prominent peaks connected by solstice lines are labeled S1-S4. The 200 ft circle at High Bank is labeled HB, Rattlesnake Knob RK. Alignments from S1 to S3 and from S2 to S4 can be reversed to mark the summer solstice sunset and the winter solstice sunset.

area's most prominent valleys. These striking correspondences between the topography and the standstill and solstice events are rarely found and are possibly a primary reason for locating these works in these valleys. Supporting the intentional creation of the astronomical and topographical alignments we have found is the observation that, in each case, the intersection of two alignments determines the centers of the large circular earthworks at Works East, High Bank, Liberty, and Seip.

Placement and Orientation of the Liberty Square

The placement and orientation of the Liberty Square provides an important test of TAG. The Wright square at Newark has similar accuracy and an almost identical

Figure 15. Alignments from the peaks S1-S4 pass through the corners of the Liberty Square (shown as a black square). All four alignments mark lunar standstill moonrises or moonsets. Three of the alignments pass through the southern vertex, a rare occurrence which might have given added significance to that location for the Liberty Square.

orientation. The Wright Square is oriented so that lines running from the vertices toward lunar standstills pass directly through four high points which play an important role as observing stations for aligning the remaining parts of the Newark Earthworks (Hively and Horn 2013). We were surprised to find that lines between these same high points accurately indicated the solstice rise/set events of the sun. Therefore, it is important to ask if the orientation of the Liberty Square in the context of the different topography of the Scioto Valley can be explained in a comparable way.

We first looked at the high points on both sides of the Scioto Valley adjacent to the Liberty Square and found four $(S_1, S_2, S_3,$ and $S_4)$ that were connected by alignments with solstice rise/set events. An observer making long-term observations of the sun from hilltops adjacent to the valley would be likely to notice these alignments. All four high points provide prominent, accessible, and unobstructed views of the valley and river below. Figure 14 shows the four proposed observation points.

The view from S1 to S3 marks the winter solstice sunrise, while the reverse view from S3 to S1 marks the summer solstice sunset. In a similar fashion, the view from S2 to S4 marks the summer solstice sunrise, whereas the reverse view from S4 to S2 marks the winter solstice sunset. Figure 14 also shows that an observer from S1 sees the maximum north moonrise at the major standstill occurring perpendicular to the Scioto River and in alignment with a major tributary valley toward the northeast. This alignment also passes over the center of a 200 ft circle associated with the High Bank site (Burks 2013). An observer at S2 sees the same moonrise occurring over the prominent spire of Rattlesnake Knob, further accentuating the singular nature of this moonrise. The most striking aspect of the four observation points is that lines from all four of them pass through the vertices of the Liberty Square and align with standstill moonrises and moonsets. Three of these alignments converge on the southern vertex of the Liberty Square (Figure 15). It is also striking that the four standstill events aligned with the Liberty Square vertices are specifically the four not aligned with the vertices of the Wright Square at Newark.¹⁰ This result suggests the notion that the Chillicothe sites were complementary to the Newark site. Table 1 shows the astronomical alignments discussed here and their accuracy.

Lunar Standstills and the Fixed Stars

When discussing the observation of lunar standstills using horizon markers, the significance of the celestial background offered by the fixed stars is frequently overlooked or ignored. Observing the motion of the Moon carefully enough to notice the standstill cycle would likely reveal that the background stars form a reference frame of stellar positions for observing the lunar cycles and its rise and set points, which remain nearly constant over a period of many lunar standstill cycles.

A careful observer would learn that the standstill cycle could be observed and anticipated by watching the motion of the Moon through the stars and constellations. Observation would eventually show that, when the Moon passed through the Milky Way in the northern sky and past the bright star Pollux and into the constellation Cancer, the next moonrise and moonset would occur at a northern extreme. Similarly, when the Moon passed through the Milky Way in the southern sky and into the constellation Capricornus, the next moonrise and moonset would be at a southern extreme.

The progress of the Moon in the 18.6-year standstill cycle could easily be estimated by noting how closely the Moon passed by the star Pollux during its monthly

cycle. At the time of the major standstill (AD 256), the Moon passes below Pollux making a close approach of 2.0°. At the time of the minor standstill (AD 265), the Moon passed below Pollux at a significantly greater distance of 12° (a difference of twenty lunar diameters). Therefore, by estimating this distance between Pollux and the passing Moon each year, one could estimate the position of the Moon in the 18.6-year cycle. This standstill separation between the Moon and Pollux at closest approach does change slightly due to the effects of precession, but the change during the Hopewell era (100 BC–AD 500) is negligible $(\langle \circ, z^\circ \rangle)$.

Stellar markers could assist in the attempt to locate the extreme rise and set points of the Moon with 1° accuracy by noting the rise and set points of background stars such as Pollux. The rising of the Moon at its major standstill northern extreme was an especially striking sight as seen from central Ohio during the Hopewell era. When the two first magnitude stars Castor and Pollux had risen high enough to become distinctly visible (at altitudes of 6° and 2° , respectively), a vertical line through the two stars pointed within 0.5° to the horizon position of the major standstill northern extreme moonrise. The observation of the relation of the standstill cycle to the stellar landscape might well have contributed to the Hopewell motivation for seeking a similar relation of the standstill cycle to the local terrestrial landscape of the surrounding hills and valleys.

CONCLUSIONS

We conclude from our study of the five Scioto region earthworks that their placements and orientations relative to the local topography can successfully be understood in terms of the integration of the topographical, astronomical, and geometrical ideas we have described in this chapter. The ideas that emerged from our analysis of the Newark Earthworks are equally successful in explaining the layout of these Scioto and Paint Creek earthworks.

Specifically, the evidence is consistent with the TAG hypothesis: (1) the siting and geometry of the earthworks were in part intended to duplicate and record correspondences between the topographic and astronomic rise/set phenomena observed in this region, especially the 18.6 year cycle of the lunar standstills; and (2) the geometry of the earthworks reflect a precise method for constructing accurate squares, circles, and octagons of nearly identical areas using common dimensions extending throughout the Hopewell core. The embedding of observed astronomical and topographical regularities within the geometrical structure of the earthwork designs suggests a Hopewell tradition of recording lunar observations

that possessed a remarkable stability, organization, and motivation extending over many generations.

Accepting TAG as a plausible interpretation for the design of Hopewell geometrical earthworks encounters a number of challenges and objections: (1) there is no established precedent for a similar achievement of this magnitude in the prehistoric world; (2) the only ethnographic evidence for knowledge of the lunar standstill cycle by Native Americans is very tentative and confined to the Mississippian and Chacoan cultures;¹¹ (3) when topographic features are interpreted as astronomical backsights with no archaeological evidence that such backsights were actually noticed or used, it is easy to underestimate the likelihood of random chance alignments; and (4) the adjustable parameters associated with defining possible alignments are so numerous that the theory can be fit to virtually any earthworks structure.

The first two objections are notable, but in our opinion ultimately have little weight. The lack of precedent and sparse ethnographic supporting evidence pertains to any achievement which is either unique or being analyzed and recognized for the first time. Indeed, one could raise these objections about the reality of the claims for monumental precise earthwork geometry in Hopewell culture. Fortunately, for the case of geometry, the evidence on the ground overpowers doubts raised by precedence and the absence of ethnographic evidence. The evidence on the ground for the incorporation of topographic and astronomic alignment is also significant.

The most serious objections are questions about chance alignments and adjustable parameters undermining the credibility of TAG. These questions go to the heart of the matter and are simply too complex to be answered easily by either intuition or simple mathematical probability computations. Only a long-term immersion in the analysis of topographic, astronomic, and geometric data in the Hopewell context can educate our intuition on these questions. After conducting Monte Carlo studies of randomly constructed earthworks (Hively and Horn 2006), extending our analysis to several earthwork sites for which good survey data exists, and trying to conceive of alternatives to TAG, we remain convinced that it remains the most plausible and testable hypothesis thus far put forward for understanding the placement and geometrical structure of the earthworks. Still, the analysis of the signal to noise ratio in these data is an ongoing effort.

We do not believe that a priori speculation about the plausibility or motivation of Hopewell observations of the Moon offers much guidance at this stage of the

analysis. We might expect the Hopewell to regard the Moon with great awe and wonder. Perhaps a desire to follow the complex lunar cycle and connect with its power and regularity provided a strong motivation. We do not know. In any case, one must monitor a cycle like that of the standstills for an extended period before one can ascertain a pattern, let alone decide whether there is any practical utility to the effort. Perhaps the absence of practical utility was a factor in the ultimate decline of the TAG tradition associated with the Hopewell.

The construction of such precise and monumental earthworks with well-established and intricate geometrical relations is such a magnificent and unprecedented achievement for a prehistoric culture that we need to be open to the possibility that unprecedented knowledge and motivation may be part of the phenomenon. Even with all its uncertainties, the TAG hypothesis has enough supporting evidence and inherent plausibility that it deserves serious attention. It is the most developed and testable hypothesis currently available for understanding the Hopewell geometric earthwork tradition.

NOTES

3. Middleton survey data does exist for the Hopeton Earthworks in the Scioto Valley, but these earthworks are sufficiently irregular that the site will not be considered "geometric" for our purposes here.

4. Middleton's field notes on the survey of the Baum Earthworks cited in Thomas (Thomas 1894:483) indicate that he surveyed the large circle and found no significant discrepancy with the Squier and Davis survey of 1848. This gives some credibility to the estimate of a 1700 ft diameter.

5. Squier and Davis (1848) Plate XX (Liberty) at top left is rotated to correct the misprint in Ancient Monuments. Middleton's survey of the Liberty small circle showed it to be an 866 ft

^{1.} Romain (2000:40) first recorded this near equivalence of perimeter between the Wright Square and the Great Circle. Middleton's record of the side lengths of the Wright Square as 928, 926, 939, 951 feet is inconsistent with his documentation of its internal angles (Thomas 1894:466). Changing 951 to 931 gives a consistent result. We have chosen 931 as the more plausible length.

^{2.} Paulus Gerdes, Ethnogeometry: Awakening of Geometrical Thought in Early Culture (2003). Gerdes gives numerous examples of the elaboration or "working out" of practical craft problems into empirical geometry for the basket problem (see 2003:175–179). "In this context, an artisan could have noted that when the ratio of the radius of the circular mat to half the length of the side of the square mat is equal to 4:5 (under certain conditions of dimensions and plaiting pattern, this proportion is immediately visible) then the areas of the circle and of the smaller visible square that touches the bigger square at the midpoints of its sides are almost equal" (2003:177). Gerdes both illustrates this pattern, and analyzes it. His hypothesis is equivalent to Volker's and adds evidence from contemporary Africa that such a rule of thumb has been employed to approximate square and circular areas.

x 748 ft ellipse (Thomas 1894:482). Plate XXI, 3 at top center shows Works East. Seip (Plate XXI, 2) is at top right. Baum (Plate XXI, 1) is at bottom left. Plate XVI at bottom right shows High Bank. The images are resized to a common scale.

6. In 2010, we published a preliminary survey of our findings at Chillicothe Hopewell sites, including detailed accounts of lunar and solar alignments to the Logan Range at Shriver and Mound City, not repeated here. (Hively and Horn 2010). Here we apply the TAG hypothesis to the study of High Bank, Works East, and Liberty in the Scioto Valley, and Seip, Baum, and Spruce Hill in the Paint Creek Valley.

7. Coordinates for the locations referenced in this chapter are listed in Table 2.

8. See Lepper et al. 2014 for current radiocarbon results placing the construction of the Adena Mound between the end of the second century BC and the beginning of the first century AD.

9. See Hively and Horn (2010) for a description of the geology of the Paint Creek valley, and its relation to the ancient Teays River system.

10. Similar alignments are given by Hively and Horn (2013, Table 4) between elevated observation points and the vertices of the Wright Square. It should be noted that the alignment listed as E-H4 in that table should be E-H1. The alignments proposed in that paper go from vertex to high point i. e. the reverse of what is proposed here. The alignments for both the Liberty Square and the Wright Square could be assigned to standstill events in the opposite direction. This is because the eight standstill alignments occur in four closely parallel pairs which point in nearly opposite directions. Either assignment would provide comparable accuracy.

11. In recent years researchers have found Chaco (Malville 2004; Sofaer 2008) and Mississippian (Pauketat 2013) tradition sites that appear to have been planned to exhibit their alignment with both local topography and the horizon travels of the sun and moon. Recently Pauketat has suggested that possible lunar standstill alignments at the Mann and Angel sites in southern Indiana offer a bridge between the Hopewell core in Ohio and the Cahokia region: "Might knowledge of ancient Ohio history, if not the long lunar cycle, have been remembered via generations of experience at the Mann site after 400 BCE and up to the founding of Angel" (Pauketat 2013:158). If these claims hold up, it could bring knowledge of the long lunar cycle close to the horizon of historical memory. It also makes it even more puzzling that there is no convincing evidence for knowledge of it.

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