Science, dualities and the phenomenological map

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Abstract

We present an epistemological scheme of natural sciences inspired in Peirce's pragmaticist view, stressing the role of the *phenomenological map*, that connects reality and our ideas about it. The scheme has a recognisable mathematical/logical structure which allows to explore some of its consequences. We show that seemingly independent principles as the requirement of reproducibility of experiments and the Principle of sufficient reason are both implied by the scheme, as well as Popper's concept of falsifiability. We show that the scheme has some power in demarcating science by first comparing with an alternative scheme advanced during the first part of the XX century (which we call Popper-Einstein and has its roots in Hertz). Further, the identified differences allow us to focus in the construction of Special Relativity showing that it uses an intuited concept of velocity that does not satisfy the requirements of reality in Peirce. We track the problem to hidden hypothesis in Einstein's work. While the main mathematical observation has been known for more than a century it has not been investigated from an epistemological point of view, probably because the socially dominating epistemology in physics discourages so doing.

1 Introduction

Ever since the Greeks attempted to conceive an understanding of the world, a duality –a relation of two worlds– has been in the centre of the scene. In various Socratic dialogues (e.g., (Plato 2014)), Plato refers to the world of Forms, or Ideas, an eternal world of perfection, as well as to the world of imperfect copies, our material world. Plato's theory of forms is well known and has deserved extensive discussion[1], we will simply observe that the duality it introduces has been a substantial part of epistemology since then. We shall call *Ideal World*,

[1]See e.g., Standford Encyclopedia of Philosophy(accessed 2020-04-23)

IW, the World of Forms and *Sensible World*, SW, the material world, the world accessible with our senses. We credit Galileo for being one of the first in advancing that the IW was populated by a mental operation he called *idealisation* that produced perfect[2] (or at least perfected) models of the observable (Galilei 1914). Several authors have worked along this conception. One of the most remarkable has been Husserl 1983 who used the term *ideation* to indicate the process by which the observable was incorporated to our perception as ideas. Piaget and García 1989 made a clear distinction between the observed, that what reaches our senses, and the *facts*, the ideated, that what is incorporated to our knowledge as perceived. All these authors have in common not only the duality between both worlds but the existence of correspondences between elements in one and the other world. Plato emphasised the relation $IW \stackrel{\Gamma}{\longmapsto} SW$ while Galileo stressed the inverse relation $SW \stackrel{\Pi}{\longmapsto} IW$. We shall call the pair (Π, Γ) the *phenomenological map*. As far as we know, the properties and the consequences regarding the assumed existence of a phenomenological map have received little attention in the past. The matter was considered in Margenau and Mould 1957 and Dingle 1960a, who referred to the phenomenological map as "rules of correspondence" but they did not advance into the implied logical structure. In turn, (Feigl 1970) reminds us in his analysis of the "orthodox view of theories":

> In the picturesque but illuminating elucidations used, e.g., by Schlick, Carnap, Hempel, and Margenau, the "pure calculus," i.e., the uninterpreted postulate system, "floats" or "hovers" freely above the plane of empirical facts. It is only through the "connecting links," i.e., the "coordinative definitions" (Reichenbach's terms, roughly synonymous with the "correspondence rules" of Margenau and Carnap, or the "epistemic correlations" of Northrop, and only related to but not strictly identical with Bridgman's "operational definitions"), that the postulate system acquires empirical meaning.

and proposes a more strict correspondence, that he names "bridge laws":

> Let me emphasize once more that this manner of regarding theories is a matter of highly artificial reconstruction. It does not in the least reflect the way in which theories originate. Correspondence rules thus understood differ from bridge laws in that the latter make empirical assertions.

In the present work we intend to show that the phenomenological map is a key element in a traditional conception of science. Its construction implies foundational reasoning principles such as Leibniz' principle of sufficient reason (see e.g., Ballard 1960) and the related no-arbitrariness-principle (NAP) (H G Solari and M A Natiello 2018). We will show that other conceptions of science,

[2]Perfection must be understood here in the sense of being faultless and capturing/creating the essence of the World. Ultimately, the two worlds can only be understood as one in front of the other, reciprocally defining themselves as in all the fundamental dialectic relations.

such as that put forward by Popper (1959) and (Einstein 1936; Einstein 1940) go back to the platonic view, in the sense that only the map Γ (the interpretation) is (precariously) considered and, because of this, they are forced to introduce an independent symmetry principle (somehow borrowed from the traditional conception). Finally, we will show that the map Π cannot be completed for Special Relativity (SR), which results then in a theory that depends on previous epistemic decisions.

2 A pragmaticist view of the traditional[3] concept of science

Charles Peirce found convenient to rename his philosophical standpoint from pragmatism into pragmaticism, his argument being:

> So then, the writer, finding his bantling "pragmatism" so promoted, feels that it is time to kiss his child good-by and relinquish it to its higher destiny; while to serve the precise purpose of expressing the original definition, he begs to announce the birth of the word "pragmaticism," which is ugly enough to be safe from kidnappers.(Peirce 1955, p.255)

We will try to keep our view within Peirce's original view, knowing that every reader is a potential kidnapper of the term (the present authors included).

Peirce introduced the fundamental concept of **Reality** as follows:

> "Such is the method of science. Its fundamental hypothesis, restated in more familiar language, is this: There are Real things, whose characters are entirely independent of our opinions about them; those Reals affect our senses according to regular laws, and, though our sensations are as different as are our relations to the objects, yet, by taking advantage of the laws of perception, we can ascertain by reasoning how things really and truly are; and any man, if he have sufficient experience and he reason enough about it, will be led to the one True conclusion. The new conception here involved is that of Reality."(Peirce 1955, p. 18)

Since reality is independent of the subject we can say that a fundamental requirement of the real is to be objective or at least intersubjective. While the observations are prone of circumstances such as where and when, as well as the observer, the *facts* have been usually deprived of such elements. The *ideated*

[3]We leave to Section 3 a short explanation on how epistemology, particularly in physics, changed after the irruption in history of the Berlin school of physics with Helmholtz and Hertz among others. We designate as "traditional" the epistemological approach prevailing since Galileo until Maxwell (died 1879) and "new" the mature form that reached the evolution of the thoughts initiated at Berlin (to put a date, 1870 with Helmholtz becoming head of the physics department). The transition time was then the epoch in which the belief in a material ether was prevalent.

facts are the reality in Peirce, and these facts are the point upon what two observers can agree (Peirce 1955, p. 150). We also learn about hypothesis in Peirce, although we prefer to use the name **conjectures**:

> A hypothesis is something which looks as if it might be true and were true, and which is capable of verification or refutation by comparison with facts. The best hypothesis, in the sense of the one most recommending itself to the inquirer, is the one which can be the most readily refuted if it is false. (Peirce 1955, p.54)

For Peirce, predictions are predictions of facts since events/observations are haphazard (Peirce 1955, p.47). We will take a compatible but alternative view: we shall call **prediction** an expected observation based upon the known facts and logical/deductive elaborations. To produce a prediction we have to elaborate our known facts and, before verification, we have to provide the **particularities** [4](unpredicted elements) that move us back to SW from the IW. In mathematics, the map Π would be called a projection and the predictive mapping, Γ, is named a "lift"[5], while we will use the symbol ϕ to denote the rational elaboration of ideas. When dealing with spontaneous observations the need for Γ may appear unjustified, but if we are to conceive experiments to test a theory, Γ is a most relevant object that tells us what to expect, and it is this expectation what is really confronted against experimental results. [6]Figure 1 shows this outlined schema of science.

There are three conditions to be satisfied for the schema to be consistent. If we ideate a set of particular observations –name the particularism by α– to construct a theory, $\Pi(\{Obs\}_\alpha)$, when we interpret the ideas in the theory using the same particularism, $\Gamma_\alpha\left(\Pi(\{Obs\}_\alpha)\right)$, we must recover the observed,

$$\Gamma_\alpha\left(\Pi(\{Obs\}_\alpha)\right) = Obs_\alpha \tag{1}$$

[4]We use particularities in the sense given by Peirce in the following expression:

> But observed facts relate exclusively to the particular circumstances that happened to exist when they were observed. They do not relate to any future occasions upon which we may be in doubt how we ought to act. They, therefore, do not, in themselves, contain any practical knowledge. (Peirce 1955, p. 150)

[5]The word "lift" takes in mathematics a related but different meaning in the context of fibred spaces. Here Π is a projection that produces an abstract idea from observations, an idea that fits them all, and can rightly be called "the essence" of the phenomena. Thus, $\Pi : SW \mapsto IW$ is the structure of the fibred space with base IW. The fibre, what has been identified to a neutral element, 0, is the set of circumstances or arbitrariness, $\{\alpha\}$. Given the arbitrariness and the base, Γ produces a reconstruction of SW. In mathematics, the lift would connect SW with $(IW, \{\alpha\})$. We use the name in a more colloquial form as the collection of inverses of Π restricted to the observations with arbitrariness α.

[6]Hertz (1893, p.20) makes this matter clear:

> Notwithstanding the greatest admiration for Maxwell's mathematical conceptions, I have not always felt quite certain myself of having grasped the physical significance of his statements. Hence it was not possible for me to be guided in my experiments directly by Maxwell's book.

When testing theories, experimental scientists need Γ.

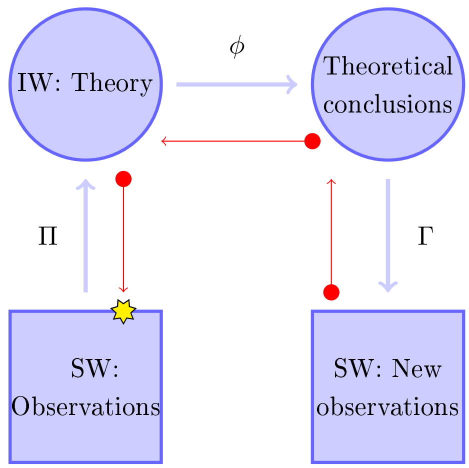

Figure 1: The proposed schema of Science. Π is a projection that produces the real out of the observed, ϕ stands for a theoretical elaboration (which eventually can be none, in such a case ϕ is the identity Id) and Γ is the interpretation that produces an expected observation.

Thus, $\Gamma_\alpha \circ \Pi$ acts as the identity within particularism α. Correspondingly, if we produce the theory out of an interpreted set of concepts and relations, $\Pi\left(\Gamma_\alpha(\tau)\right)$, we should get the original set of concepts and relations τ constituting the theory

$$\Pi\left(\Gamma_\alpha(\tau)\right) = \tau \tag{2}$$

The above conditions should hold for any set of particular observations. Furthermore, if our theory shall not be considered refuted, we should have that

$$\Gamma_\beta\left(\phi(\Pi(\{Obs\}_\alpha)\right) \equiv \Gamma_\beta \circ \phi \circ \Pi\left(\{Obs\}_\alpha\right) = \{Obs\}_\beta \tag{3}$$

i.e., that the theoretical conclusions elaborated by a set of observations with particularities α can be lifted to a corresponding set of observations Obs_β with particularity β. For the special case in which ϕ presents no elaboration (i.e., $\phi = Id$), eq.(3) represents the standard requirement of reproducibility of experiments. The set of functions $\{\phi\}$ are automorphisms of IW while the transformations $T_{\beta\alpha}$ among particular representations,

$$T_{\beta\alpha} = \Gamma_\beta \circ \Pi\left(\{\cdot\}_\alpha\right)$$

are automorphisms of SW.

The structure arising from the triple $\{\Pi, \phi, \Gamma\}$ corresponds to the idea of abduction-based reasoning, discussed in (Peirce 1955, p 150-156). Let us consider it in more detail.

The projection Π should not be mistaken for induction or inductive inference. Π conjectures and suggests which is the nature of relevant observations,

ϕ elaborates the ideation following the rules of mathematical logic and Γ confronts the elaborated ideas against the sensible world. In this sense, Γ can be called *interpretation*, connecting the ideas with the sensible world. If the confrontation is successful, we preliminarily accept the conjecture as a satisfactory explanation. If, on the contrary, the conjecture is refuted, we abandon it and proceed to generate a new, improved, conjecture in the light of the refutation. This last process is illustrated by the backwards path in Figure 1 indicating that disagreements between predictions and facts trigger improvements in Π. Also, if the conjecture does not contribute to organise and structure our views on the sensible world, it is rejected. Conjectures that are impossible to test must be rejected as well (Peirce 1955, p 150-151). Contemporary readers may recognise some of these ideas in Popper. Apart from precedence, Peirce goes further in his insight on Π.

Neither Π nor Γ are deductive or inductive. The only truly deductive part of the process lies in ϕ. Science cannot be reduced to logic only.

2.1 Subjective formulations and the no arbitrariness principle

According to Peirce (p. 150, 1955)

> All our knowledge may be said to rest upon observed facts. It is true that there are psychological states which antecede our observing facts as such.

Peirce found no reason to expect a relevant influence of such "psychological states" but such an idea must be reconsidered after Jean Piaget (1999), because at least the notion of space, fundamental to physics, is a notion produced in the early development of the child. Then, the space, an a-priori of knowledge in Kant, needs to be accounted for. Because the abducting map, Π, represents the process of ideation governed by intuition, the resulting theories quite often carry some degree of arbitrariness introduced by the subjective perception. Newton's mechanics, for example, is of such quality. Such subjective formulation of theories are in the same relation with the real than the observable. Correspondingly, the schema of Figure 1 applies as well.

It is at this point where the present discussion encounters Leibniz' principle of sufficient reason (Ballard 1960). Weyl (2015) introduced an elaboration based upon it as the principle of symmetry in relation to the duality subjective-objective, yet he failed to identify properly the automorphism that had to be considered (Nozick 1998; Catren 2009; Fortin and Lombardi 2015). The group of automorphisms that is relevant in this case is the *group of arbitrariness* with morphisms that map the expression of a natural law in terms of some arbitrary elections, to the corresponding expression under a different arbitrariness. Such is the case of Cartesian space which is often prescribed by a reference point and orthogonal frame. The transformations between such reference frames correspond to the elements in the Euclidean group in three dimensions, E^3, consisting of the semi-direct product of translations and orthogonal transformations (reflections and rotations). Thus, for example, the spatial relations between N bodies (assumed point-like) can be written as set of $3N$ Cartesian coordinates, R^{3N}, yet what is real in it is the projection made by modding away the group of arbitrariness, E^3, this is R^{3N}/E^3, Leibniz' relational or *objective space*.

The requirement of objectivity (or intersubjectivity) imposes the composition of transformations to be associative (hence automorphisms), since e.g.,

$$T_{\beta\alpha} \;\;=\;\; T_{\beta\delta}T_{\delta\alpha} = T_{\beta\delta}(T_{\delta\eta}T_{\eta\alpha}) = (T_{\beta\delta}T_{\delta\eta})T_{\eta\alpha} \tag{4}$$

regardless of any particularity ("opinions" in the wording of Peirce) of δ, η. Hence,

Theorem 1. *The transformations between arbitrary representations of SW form a group.*

Proof. This group is called the *group of arbitrariness* of the theory. The existence of identity transformations follows from eq.((1)), that of inverse transformations from eq.((2)) and the equivalence in front of arbitrary particularities, while associativity follows from eq.(4). □

The no arbitrariness principle (NAP) reads (H G Solari and M A Natiello 2018):

Principle 1. [No Arbitrariness Principle (NAP)] *No knowledge of Nature depends on arbitrary decisions.*

2.2 Symmetry in the perception of electromagnetic waves

In the same way in which an elephant is perceived differently from different perspectives while still being the same elephant [7], light is perceived in different forms (wavelength and frequency) under different circumstances of relative motion between source and observer (detector). This is known as the Doppler effect (Dingle 1960b; Mandelberg and Witten 1962) and it is not a case of arbitrariness: an observer moving with relative velocity, v, with respect to a source of electromagnetic waves perceives different colours according to the relative velocity. Here we understand as relative velocity what we construct intuitively and express in formulae in elementary texts of physics. It is worth to remember that relative velocities, in classical terms, correspond to an intuited objective concept than can measured. We select this example as it will make contact with Section 4.

The standard electromagnetic theory in physics textbooks relates the observation made by an observer at rest (relative to the source) to those made by observers in relative motion, applying a transformation to the action-fields associated to the source. The perceived fields can be obtained by the proper use of a Lorentz Transformation (LT) of the fields or associated potentials. This

[7] A metaphor taken from (Catren 2009)

theory has not been refuted so far although the constant value of the speed of light remains in a status of conjecture (Dingle 1960a; Moon and Spencer 1960; Bilbao 2016). Consider the perceptions of two observers that move with velocity u and v, respectively, with respect to the source. To predict the observation of the second one knowing the perception of the first one, we can go back to the perception of a privileged observer at rest with respect to the source and next transform the result to the perception of the second observer. In symbols we apply $L_v \circ L_{-u}$ (where L_v stands for a LT of parameter v) to the four-vector (A, V) of potentials in Lorenz gauge obtaining (E, B) following the general prescription. Although the composition of Lorentz transformations is not in general a Lorentz transformation, the operation is an element of the Poincaré-Lorentz group (Poincaré 1906).

3 The concept of science from Hertz to Einstein and Popper

Towards the second half of the XIX century, the force of the Enlightenment was declining in Europe while the second industrial revolution was developing. Important changes took place in philosophy, physics and mathematics.

The XIX century in philosophy experienced the entrance of materialism in the scene –with naturalism taking a dominant role after Darwin–, its struggles with metaphysics, and for the natural sciences began some sort of independisation from philosophy, a process that was intensified during the XX century (Beiser 2014).

On the side of physics, the success of electromagnetic theory raised the issue of the nature of light and electromagnetic phenomena as opposed to mechanical phenomena. While the Newtonian grounds of mechanics seemed solid, instantaneous action at a distance could not be harmonised with the fact that electromagnetic phenomena appear to propagate with finite velocity. At the same time, the way of conceiving electrodynamic phenomena shifted from *hypotheses non fingo* ("I frame no hypotheses"[8]) (Newton 1687, p. 506) to the *Bild* (Dieter 1998; Heidelberger 1998; Hoffmann 1998; D'Agostino 2004; Schiemann 1998) concept (see below for a brief explanation).

In contrast with the motion of bodies addressed by mechanics, electromagnetic phenomena reach us mostly in a form not apt for being directly intuited. We do not see currents, for example, what we see is rather the deflection of a needle, while the current is inferred. Scientists as Lord Kelvin and Maxwell supported their thoughts with analogies. Lord Kelvin writes

> I never satisfy myself until I can make a mechanical model of a thing.

8

> I frame no hypothesis; for whatever not deduced from the phenomena is to be called a hypothesis; and hypothesis, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy.

> If I can make a mechanical model I can understand it. As long as I cannot make a mechanical model all the way through I cannot understand; and that is why I cannot get the electromagnetic theory. (Thompson 2011, p.835).

while Maxwell says:

> Now we are unable to conceive propagation in time, except either as the flight of a material substance through space, or as the propagation of a condition of motion or stress in a medium already existing in space.[...] If something is transmitted from one particle to another at a distance, what is its condition after it has left the one particle and before it has reached the other? ([866], Maxwell 1873)

Indeed, light-travel is understood by analogies with bodies (we return to this idea in Section 4), still in our days, like a stone thrown by the source and captured by the detector. Also, electromagnetic waves were conceived mechanically and asked for a propagation medium that could sustain them after having abandoned the source and before reaching the detector.

A most decisive epistemological change was advanced by Hertz, a disciple of Helmholtz, who acted the idea that it is possible to separate the process of construction of a theory from the theory's mathematical content. Regarding Maxwell's electrodynamics, he states (Hertz 1893, p.21)

> "To the question: 'What is Maxwell's theory?'. I know of no shorter or more definite answer than the following:– Maxwell's theory is Maxwell's system of equations. Every theory which leads to the same system of equations, and therefore comprises the same possible phenomena, I would consider as being a form or special case of Maxwell's theory;[...]"

Hertz claimed that theories have to be provided with what he called *interpretation*, an element that runs alongside the mathematics and helps in constructing experiments related to theory (the action of Γ in Section 2) while at the same time theory can be detached from its construction. Several inequivalent interpretations can, in this form, be attached to a theory. Hertz' deep epistemological change has been highlighted by D'Agostino:

> ...by separating the mathematical structure of a theory from its modes of representation he [Hertz] has profoundly challenged the conception of a physical theory as an indivisible unity of the two – a conception accepted by Maxwell and other nineteenth century mathematical physicists." (D'Agostino 1968; D'Agostino 2004).

The Berlin school of Physics (see Footnote 3) made other profound contributions to epistemology. They introduced the *Bild* (Dieter 1998; Heidelberger 1998; Hoffmann 1998; D'Agostino 2004; Schiemann 1998) approach, a form of understanding supported in images. As a brief explanation:

> For Hertz, in contrast [with Helmholtz], representations of theories are signs of sensory impressions that are given to us. Only if we use theory to construct representations will it accomplish the most important task of natural knowledge, foresight of the future from experiences of the past. (Heidelberger 1998)

Conceptually, since mental images are the outcome of sensorial perception, using images to organise understanding implies to habilitate sensorial based intuition to be used by analogy in other realms.

According to Poincaré, there was a crisis in mathematical physics by 1904 (Poincaré 1913, original of September 1904). And indeed there was a deep crisis, the task of mathematicians at producing physical theories had been transferred to a new specialist: the theoretical physicist (Jungnickel and McCormmach 2017) and even more, philosophers no longer exercised critical thinking in matters of science, at least in Germany. From Kant, through Reinhold, Fichte until Hegel, the prevalent movement considered that philosophy was "'the guardian of the sciences,' their founder and systematizer" (Beiser 2014, p. 15). But the movements that emerged after Hegel were mostly antihegelian. Peirce comments:

> German universities for a whole generation turned the cold shoulder to every man who did not extol their stale Hegelianism, until it became a stench in the nostrils of every man of common sense. Then the official fashion shifted, and a Hegelian is today treated in Germany with the same arrogant stupidity with which an anti-Hegelian formerly was.(CP 1.77, Hartshorne and Weiss 1931)

For the new German philosophy the task of philosophy with respect to the sciences ranged from studying "the logic of the science" as acted by the scientists, to the extreme of the sciences declaring philosophy dead, as in "...neo-Kantian Jürgen Bona Meyer: 'The daughters now demand independence from their common mother, and they do not suffer it gladly when they are supervised or corrected; they would prefer that their old and morose mother lay herself to rest in her grave'" (quotations from (Beiser 2014, pp.18, 17)).

Poincaré made an attempt to rescue some of the learnings of the old science (as he called it) in terms of principles that have been established by the old science and appeared to him as foundational. Among them the Principle of relativity, the Principle of minimal action, conservation of energy, Carnot's 2nd law of thermodynamics and a few others. But he realised that these principles themselves were in crisis as well. It is interesting from the point of view of the present work to quote some of his words regarding the crisis of the Principle of relativity (Poincaré 1913, original in french of September 1904)

> Let us pass to the principle of relativity: this not only is confirmed by daily experience, not only is it a necessary consequence of the hypothesis of central forces, but it is irresistibly imposed upon our good sense, and yet it also is assailed. Consider two electrified bodies; though they seem to us at rest, they are both carried along by

> the motion of the earth; an electric charge in motion, Rowland has taught us, is equivalent to a current; these two charged bodies are, therefore, equivalent to two parallel currents of the same sense and these two currents should attract each other. In measuring this attraction, we shall measure the velocity of the earth; not its velocity in relation to the sun or the fixed stars, but its absolute velocity.
>
> I well know what will be said: It is not its absolute velocity that is measured, it is its velocity in relation to the ether. How unsatisfactory that is! Is it not evident that from the principle so understood we could no longer infer anything? It could no longer tell us anything just because it would no longer fear any contradiction. If we succeed in measuring anything, we shall always be free to say that this is not the absolute velocity, and if it is not the velocity in relation to the ether, it might always be the velocity in relation to some new unknown fluid with which we might fill space.

Here, Poincaré's criticism is aligned with Peirce's position, at least inasmuch he refuses to make non-refutable hypothesis.

The crisis identified by Poincaré proceeded with a new turn. By allowing free interpretations, the velocities involved in electromagnetism, that had been fully measurable relational velocities in Ampère, Faraday and Weber's experiments as well as in Maxwell's abduction, were reinterpreted in different ways. In their expressions of forces, and the derivation of them, Maxwell works with relative velocities between circuits (Maxwell 1873). Lorentz (1892) reinterpreted relative velocities as absolute velocities referring to the ether in its expression of the electromagnetic force. Later (Einstein 1905) kept Lorentz expression of the force while eradicating the ether, what implies another reinterpretation of the involved velocities, as velocities with respect to an inertial frame, and proposing to use Lorentz' transformations in place of Galilean boosts to restore the Relativity principle. In so doing, he was loyal to Hertz epistemological point of view, changing the interpretation of the glyphs of Maxwell's electrodynamics without changing the glyphs of the theory.

We have chosen Popper's "Logic of Scientific Research" to represent the alternative to the epistemology schematically presented in Section (2) that evolved from the Berlin school. Popper's view of the philosophy of sciences is well aligned with the post-Hegelians:

> I suggest that it is the task of the logic of scientific discovery, or the logic of knowledge, to give a logical analysis of this procedure; that is, to analyse the method of the empirical sciences.(Popper 1959, p.4)

From pages 3 to 7 he address the problem of induction, concluding that induction is not the support of trust in science. He next writes against psychologism,

> The initial stage, the act of conceiving or inventing a theory, seems to me neither to call for logical analysis nor to be susceptible of it.

> The question how it happens that a new idea occurs to a man—whether it is a musical theme, a dramatic conflict, or a scientific theory—may be of great interest to empirical psychology; but it is irrelevant to the logical analysis of scientific knowledge. (p. 7) [...]

returning recurrently to his main thesis:

> According to the view that will be put forward here, the method of critically testing theories, and selecting them according to the results of tests, always proceeds on the following lines. From a new idea, put up tentatively, and **not yet justified in any way**—an anticipation, a hypothesis, a theoretical system, or **what you will**—conclusions are drawn by means of logical deduction. These conclusions are then compared with one another and with other relevant statements, so as to find what logical relations (such as equivalence, derivability, compatibility, or incompatibility) exist between them. (p. 9, emphasis added)

We have highlighted two expressions that give the clear impression that for Popper theories come from nowhere, leaving the process of production as not belonging to science. In short, for Popper the phenomenological moment appears as non-scientific.

By rejecting induction, branding other elements in the process of the construction of theories as psychologism and ignoring abduction in its original form he comes into terms with Einstein's view

> "Physics constitutes a logical system of thought which is in a state of evolution, and whose basis cannot be obtained through distillation by any inductive method from the experiences lived through, but which can only be attained by free invention. The justification (truth content) of the system rests in the proof of usefulness of the resulting theorems on the basis of sense experiences, where the relations of the latter to the former can only be comprehended intuitively. Evolution is going on in the direction of increasing simplicity of the logical basis". (Einstein 1940)

The Einstein-Popper view disregards the abduction Π, which is replaced by "free invention" and put outside science, this is, outside logical examination. As free invention, the replacement of a concept in a formulae by another one must be admitted, although its immediate consequence is that the new theory must be put to test from scratch (see (Popper 1959, p. 63)). Popper does not address how we go from glyphs into experiments, he apparently ignores Γ as well. Einstein instead introduces "intuition" as part of the assessment of the "truth content". This intuition shall not be confused with Husserl's eidetic intuition, that goes from the observed to the facts, for this one moves in the opposite direction, from theory/Ideas into observations. In any case, to restore part of the coherence of the old science, such epistemology needs to be complemented with some principles such as the (intuited) Relativity principle and the requirement of reproducibility of experiments.

It must be noticed that the absence of logical conditions for the interpretation, as those that emerge from the abduction, makes free interpretation possible as well. The theories resulting from Popper's epistemology ought to be considered less simple (in his terms) than those supported by the present ("pragmaticist") approach since they can elude refutations by changing the interpretation, "saving" the core of mathematical relations.

4 A "pragmaticist" critic of Special Relativity

We want to illustrate in this section how the epistemological frame (Piaget and García 1989)[9] changes our appraisal of theories. We address the problem in practical terms considering Special Relativity, SR, one of the theories that prompted the need for a new epistemology as presented by Popper:

> [Referring to other epistemological approaches] They will hardly be ready to grant this dignity to modern theoretical physics in which I and others see the most complete realization to date of what I call 'empirical science'. [...] Thus I freely admit that in arriving at my proposals I have been guided, in the last analysis, by value judgments and predilections. (Popper 1959, p. 15)

Einstein as well as Poincaré (or any other scientist that we know about since them) have presented the Principle of Relativity as a truism, a self evident principle; Poincaré admitting, additionally, physicists' habituation to it by what he called the "old science" (Henri Poincaré 1913). The principle will make no sense to the lay man unless she/he has been initiated in the mathematics of classical mechanics. It is a common experience to find students from the humanities that believe they agree with Einstein as they profess philosophical relativism [10]. Habituation in classical mechanics is then the source of it, since without habituation the belief does not emerge.

The present pragmaticist view indicates that the Principle of Relativity in classical mechanics is not a new or independent principle, but rather the consequence of requiring a rational foundation for our understanding and therefore eliminating arbitrariness. It is supported in the relational view of mechanics that goes back to Leibniz (H G Solari and M A Natiello 2018) and we have called it the Non arbitrariness principle (NAP). In some sense, it integrates Newton's mechanics with Leibniz' objections, thus extending beyond both. This form of surpass imposes that the mappings connecting presentations of dynamical processes under different arbitrary conditions constitute a *group* (a mathematical

[9] Generally speaking, an epistemological frame provides an a-priori form of organising concepts, formulating questions, producing and presenting answers. As such, it can discard questions as unsuitable or lacking interest and even produce "blindness" as in the example presented in (Rolando García 1981). Moreover, the same question/answer might have completely different lectures depending on the epistemological frame.

[10] https://plato.stanford.edu/entries/relativism/ (accessed May 13 2021)

structure of associative binary operations having inverse and identity operation). In classical mechanics one of the groups relating arbitrary (subjective) choices is the group of Galilean transformations, eliminating the arbitrariness in the relative motion between reference systems. Relativity proposes to replace Galileo's transformations by Lorentz transformations (LT). Under the pragmaticist schema it is then necessary that the LT's constitute a group (which they don't) and that they eliminate the corresponding arbitrariness in the relative motion between reference systems. Nothing is gained by enlarging these transformations with the full Poincaré-Lorentz group (PL), the arbitrariness cannot be fully eliminated.

The questions are: which is the residual arbitrariness that remains? How was it introduced in Special Relativity?

These questions examine the axioms and the inference (abduction) leading to the axioms, i.e., they concern the projection Π, before refutation or verification can enter the discussion. Therefore its criticism lies outside Popper's epistemic approach and outside the "orthodox" approach (as described by Feigl (1970)) as well.

4.1 The algebraic structure associated to Lorentz's transformations

The analysis of the PL group in relation with Einstein's velocity addition is presented in e.g., (Gilmore 1974), while some results were known since the early days of SR (Silberstein 1914, p. 161-169). For our goal it is sufficient to realise that the consecutive application of two LT's is not in general another LT:

$$L_v L_u = L_{v\oplus u} R_{(v,u)} = R_{(v,u)} L_{u\oplus v}$$

Here L stands for the different LT's, $v \oplus u$ is Einstein's velocity addition and $R_{(v,u)}$ is a rotation named Wigner or Thomas rotation[11] depending on the source. This is not a property deduced through a pictorial presentation of the problem[12], but rather a fundamental property of the PL group, independent of the form in which we may want to present SR. The standard claim that the LT maps the results from one inertial system to another (originated in Einstein's paper) cannot be sustained, no matter the amount of mathematics thrown over the problem. To "save" the theory (the equations), the least that is needed is to reinterpret its symbols, but such a thing is alien to the "pragmaticist" epistemology. Thus, for the "pragmaticist" the theory is inconsistent with the abduction process used in its construction and must be abandoned or built from the grounds specifying the phenomenological map from the beginning.

To further grasp the issue we may try to solve the following problem: an observer, S, associates reference systems to two bodies, A, B, moving with velocities v_A and v_B relative to S along different directions. Given these measurable

[11] Wigner rotation (accessed 14 May 2021)

[12] Since pictures relate to our intuitive view of world, pictures may inadvertently introduce elements alien to the problem.

velocities we attempt to provide the velocity v_{BA} of B relative to A that allows a direct description by A of the motion of B and vice versa (as a function of quantities pertaining only to A and B, in order to preserve objectivity). This is, the velocity involved in the LT relating both systems.

According to the received wisdom we use successive LT's to change the description to a reference system S_A where the body A is at rest (or alternatively S_B where B is at rest). Further, the theory claims that the velocity of B as described by A, v_B^A, must relate to the velocity of A as described by B, v_A^B in the form $v_B^A = -v_A^B$ (a well-known property in everyday life, and a cornerstone of classical mechanics demanded as well by Einstein (1905) in his construction). However, what is computed via the PL group does not relate in that way, as noted by Silberstein (Silberstein 1914, p. 167). To save the formulas (the theory, in Popper's epistemology) we may want to claim that we can still rotate the frames so that the computed velocities satisfy the demanded property (although the theory has no prescription for this operation), but this would imply that the transformation from one system to another depends on the presence of other non-interacting bodies. Hence, rather than a transformation between the frames of A and B it is a transformation between the frames A and B *in the presence of the witness S.* The inconsistency detected by Silberstein cannot be removed.

At this point we must remark that the use we make of the Lorentz transformation in Subsection 2.2 is not subject to the present criticism. In that case, the PL group does not act as an "arbitrariness relating group" but rather as a group relating perceptions under different physical circumstances in which the frame of the source has a natural right to be distinguished.

Following critical thinking we must trace the problem backwards to uncover its roots. We undertake this task in the coming subsection.

4.2 Einstein's inference

Einstein's fundamental paper (Einstein 1905) rests on a few hypotheses that are almost taken for granted. In the first paragraph it is stated that physical phenomena depend on relative motion (of the interacting parts), suggesting that relative velocity is a well-defined concept. This idea is completed in Part I, §3 stating that the velocity v of A with respect to B is minus the velocity of B with respect to A, inasmuch the transformation between two reference systems in relative motion depends on the same function of v or $-v$, being the transformations inverse of each other. At the beginning of Part I §3, it is stated that these reference systems are parallel orthogonal Cartesian reference frames (throughout the motion) that can be chosen so that one of the axes lies along the direction of motion. No special notion of "parallel" is introduced, so we must accept it is the standard notion.

The above observations, although obvious for classical physics, conflict with the fundamental requirements of special relativity (SR). Indeed, the *relative velocity* of SR makes sense only for a given pair of reference frames (or particles/bodies). In the original wording: the "stationary" frame S and the "moving" frame A. When incorporating a third frame B, moving relative to the stationary frame along a direction not collinear with SA, the demand imposed by eq.(4) cannot be fulfilled. There is no way to establish the velocity of B relative to A by coordinate transformations without involving S, hence the concept is not objective, since a relational quantity between two objects can only be computed through the point of view of a third object; what we have is opinion (what we actually have is the subjective view of S about what v_B^A and v_A^B should be).

A second hypothesis, to which the vast majority of the scientific community has adhered over the years is that light "travels" or "propagates" through space, departing from the emitter and arriving at a later time to the detector. We call this the *body-like* conception of light propagation, since it corresponds to the motion of a material projectile. The idea of the photon inherits this conception. The body-like approach was mostly bound to the need for a propagating medium (the ether) for electromagnetic disturbances. However, (Einstein 1905) considers concepts such as the "luminiferous ether" or the need for an "absolute stationary space" superfluous[13]. In the second half of the XIX century, electrodynamics not resorting to the ether comes almost solely from the Göttingen school (Betti 1867; Riemann 1867; Neumann 1868) being most clearly exposed by Lorenz (Lorenz 1867).

In conclusion, Einstein (1905) rests upon at least four hypothesis:

1. There exists an objective velocity between reference frames (observers) (call it v). By objective we mean that can be measured one form or another by any observer, obtaining the same vector, i.e., the velocity is *real* in the terms of Peirce.

2. The same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good (Principle of relativity).

3. Light travels body-like (substantialist hypothesis). This contrasts with Lorenz electrodynamics, which is compatible with assuming that light does not "travel", but is the result of a time-delayed interaction between source and target.

4. Light is always propagated in empty space with a definite velocity C which is independent of the state of motion of the emitting body.

Hypothesis 1 and 3 are hidden, used as true in the argumentation but not made explicit. As a result of them, Einstein concludes that the transformations of the descriptions from one inertial frame to another one are those of Lorentz, which depend on the velocity v. The problem is then how to measure v? We have

[13]Einstein will later reject this idea, stating that every theory has its "ether" (Einstein 1924, p. 93), an issue which is seldom discussed in textbooks: "[...], we will not be able to do without the aether in theoretical physics, that is, a continuum endowed with physical properties; for general relativity, to whose fundamental viewpoints physicists will always hold fast, rules out direct action at a distance."

argued in the previous subsection that there is no such v consistent with the hypotheses.

Actually, hypothesis 1 and 2 are supported by the conception of science presented in Section 2; one of the fundamental tasks of a relativity principle is precisely to assure that v is objective and not an opinion. Hypothesis 3 is imposed by the epistemological frame[14], while hypothesis 4 is a conjecture supported in part by realism: if we cannot make difference between light as perceived when we are at rest or in motion relative to the source, we would expect the description of light to be the same in all systems where we place a set of detectors which are at rest from one another.

Since there is a contradiction, we should be forced to abandon at least one hypothesis. Abandoning hypotheses 1 or 2 means to leave realism behind. Abandoning 3 has little costs since it simply introduces an analogy, a relation between matters that we know are not equivalent but we decided to explore as such to get further insight. Analogies have only two possible final destinations, either they become equivalences or they are no longer used when we progress in our understanding. Hypothesis 4 should be put to experimental test.

The body-like conception of light refers to an epistemological frame different and incompatible with the relational conception. A neat example of the relevance of the epistemological frame is Michelson-Morley's experiment, that only makes sense if performed for electric disturbances travelling through the ether, opening for questions about velocities with respect to it. Within a relational view, this experiment makes little sense, since source and detector are at relative rest. Another example, also in this field, are the experiments (mainly by Graneau (P. Graneau and N. Graneau 1996)) attempting to decide between Ampère's and Lorentz' electromagnetic forces. The same experiments receive a different interpretation in each of the two epistemological frames and in the end the issue is still undecided. A comprehensive discussion can be found in (Assis 1994).

5 Maxwell and the propagation of light

The body-like conception of light propagation is present in Maxwell, as discussed in the previous section, and lies as a support for the ether, that would be the medium through which light is travelling, in analogy with mechanical waves. While the quoted paragraph from Maxwell (see Section 3) supports the idea of the ether, Maxwell and also Faraday entertained doubts in this respect. The context of the quoted paragraph is to claim the right to investigate this physical hypothesis, strongly opposed by the Göttingen school that he much admired. In the same form, Maxwell wrote for and against the Göttingen approach. In what can be considered the most balanced expression, he writes

[14]Einstein defines velocity as $\frac{distance}{time\ interval}$ which is in fact an objective (invariant, real) velocity in classical mechanics but he later uses $\frac{|X(t_1)-X(t_0)|}{t_1-t_0}$ which is a subjective velocity that changes with reference frame. For the difference between objective and subjective velocities see (H G Solari and M A Natiello 2018).

> That theories apparently so fundamentally opposed should have so large a field of truth common to both is a fact the philosophical importance of which we cannot fully appreciate till we have reached a scientific altitude from which the true relation between hypotheses so different can be seen.(Maxwell 2003, p. 228) (Address to the Mathematical and Physical Sections of the British Association. (Liverpool, September 15, 1870.))

To his disappointment, the existence of the ether was refuted by experiments while the relational theories were abandoned by adherence to substantialism. We defer to the Appendix a deeper discussion of this point.

It is important to notice that Maxwell's analogies using the ether produce some lasting contributions, like the "displacement current". However, the displacement current can be produced without invoking the ether through the strategy of the Göttingen school: if there is evidence that electromagnetic disturbances propagate as waves, write it in formulae, i.e., build the corresponding phenomenological map. In contrast, Maxwell's expression for the electromechanical force was rejected by experiments. The currently accepted electromagnetic force was produced by Lorentz arguing from the ether, but with a curiosity: the "virtual displacements" of the probe he used were at the same time displacements with respect to the ether and with respect to the remaining electromagnetic objects (Lorentz 1892, §71). Hence, his argument holds true not because of the ether (call it free thinking) but because it once again agrees with the relational point of view. A recent discussion contains the details of the calculation (M. Natiello and H. Solari 2021).

6 Concluding Remarks

We have discussed the logical requirements implied in the traditional conception of science and compared to new concepts socially dominant since the beginning of the XX century. We have further shown than the traditional concept has a certain unifying power with respect to beliefs such as the requirement of reproducibility of experiments and symmetry requirements of scientific laws (in particular of physical laws). Additionally, the concept of science put forward is operative since it allows us to check the construction of theories (as it has been shown in Section (4)), implying as well that the sciences cannot be considered autonomous and much less independent of philosophy.

It is worth to be noticed that the scheme of Figure 1 operates not only at the level of completed theories but during the production of them as well. In our experience, the construction of theories implies the scheme at every level of detail in what can be portrayed as a fractal structure.

The replacement of the phenomenological map by free invention disproves any attempt of studying the fundamentals. For example, we have to accept by habit or consensus the Relativity principle without having any control of its appropriateness or the conditions that it must satisfy. In such a form, the work of critical philosophy as a motion towards the fundamentals (as put by Hegel) is hindered, with the addition of gaining freedom for interpretations, thus making falsifiability harder to assess and helping to sustain dogmatic beliefs. Also, it made possible a continuous "progress" of science, without set backs, in as much as new interpretations and physical entities (if needed) are introduced, thus protecting the core beliefs.

The new form of science emerged in physics at the beginning of the XX century and appears to evolve from a need for legitimating analogies. In this sense, anthropologist Sharon Traweek in her observation of the high-energy physicists community says:

> Undergraduate physics students, to be successful, must display a high degree of intellectual skill, particularly in analogical thinking. The students learn from textbooks whose interpretation of physics is not to be challenged; in fact, it is not to be seen as interpretation. (Traweek 1992, p. 74)[...] Teachers show students how to recognize that a new problem is like this or that familiar problem; in this introduction to the repertoire of soluble problems to be memorized, the student is taught not induction or deduction but analogic thinking. (p. 77)

The observation gives a fair idea on how this form of thinking is trained and selected. This shows how the epistemic frame is socially reproduced.

The price to be paid when accepting analogies is high, as SR illustrates. We must leave behind universal time and relative positions accepting a new space-time. We have to accept a debilitated epistemology, hidden inconsistencies in the theory (the use of intuited velocities that cannot be measured) and we finally have to forbid natural questions concerning non-inertial frames. In contrast, if we accept that interactions are not bodies and hence, they do not need to have a place in space and consequently that analogies with bodies are of limited relevance, we not only maintain the unity of physics but strengthen it, showing that several seemingly independent principles all correspond to a conceptual unity. The question is unavoidable: what did we obtain paying this price? We conjecture that the gains were social, but it deserves investigation.

With respect to Peirce's complaint regarding that the word "pragmatism" has been kidnapped, we observe that kidnapping words is a frequent practice. Currently most of those claiming to adhere to a relational view actually adhere to a formal procedure that has emptied the word of meaning, since instead of seeking for reality they make room for a world of incommensurate subjective opinions (see (Margenau and Mould 1957) for a distinction between an "older meaning" and a "modern form" of relativity). Even worse, the same can be said of "critic". How can science be critic when it does not allow to search for its fundamentals but considers only its consequences? In particular, the consistency of the triple $\{\Pi, \phi, \Gamma\}$ is set aside, since both ends are debilitated or absent. Theories should be exposed to experimental analysis, but they may be rejected even earlier, if the phenomenological map is inadequate for the problem.

The phenomenological map connects and at the same time keeps as distinguished the world of ideas (IW) and the world of observations (SW). For critical philosophy, cognition requires both, as in Kant's dictum:

> Understanding cannot intuit, and the sensuous faculty cannot think. In no other way than from the united operation of both, can knowledge arise.

The phenomenological map that represents the dialogue between the sensuous faculty and our thoughts separates them and prevents hypostatisation, the reification of ideas. Not only Kant spoke against hypostatisation in his discussion of metaphysics. Faraday, a Natural Philosopher, wrote:

> But it is always safe and philosophic to distinguish, as much as is in our power, fact from theory; the experience of past ages is sufficient to show us the wisdom of such a course; and considering the constant tendency of the mind to rest on an assumption, and, when it answers every present purpose, to forget that it is an assumption, we ought to remember that it, in such cases, becomes a prejudice, and inevitably interferes, more or less, with a clear-sighted judgment. (Faraday 1844, p.285)

It is not surprising then that simple considerations regarding the largely forgotten phenomenological map cast a distinctive light on our educated beliefs. Against the concept of science in Einstein and Popper (Section 3) we must raise Kant's words (we invite the reader to substitute "pure reason" by "the sciences" in the next quotation)

> Reason must be subject, in all its operations, to criticism, which must always be permitted to exercise its functions without restraint; otherwise its interests are imperilled and its influence obnoxious to suspicion. There is nothing, however useful, however sacred it may be, that can claim exemption from the searching examination of this supreme tribunal, which has no respect of persons. The very existence of reason depends upon this freedom; for the voice of reason is not that of a dictatorial and despotic power, it is rather like the vote of the citizens of a free state, every member of which must have the privilege of giving free expression to his doubts, and possess even the right of veto.
>
> But while reason can never decline to submit itself to the tribunal of criticism, it has not always cause to dread the judgement of this court. Pure reason, however, when engaged in the sphere of dogmatism, is not so thoroughly conscious of a strict observance of its highest laws, as to appear before a higher judicial reason with perfect confidence. On the contrary, it must renounce its magnificent dogmatical pretensions in philosophy. (Kant 1787, p. 475)

In closing, we must return the credit to the "student of humanities", since every reference system has a right to its own opinion and there is no manner of putting them in correspondence (to "mod out" subjectivity, in mathematical terms), there is no real relative velocity, according to SR, but just opinions based upon previous beliefs.

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A On Maxwell's argument against the Göttingen theory

In a communication to the Royal Society read June 18, 1868 (Maxwell 2003, pp. 125–143), Maxwell comments and criticises the electromagnetic theories of Lorenz and Riemann. He would later in practice retract from this criticism in his treatise (Maxwell 1873), admitting that Lorenz' equations are equivalent to his system and, as we have seen in Section 4, also in a communication addressed to the Mathematical and Physical Sections of the British Association (Liverpool , September 15, 1870) (Maxwell 2003, pp. 215–229). Nevertheless, it is interesting to use his argument to show how interpretation and analogy irrupts in scientific argumentation even in the words of a celebrated scientist. Maxwell writes (Maxwell 2003, pp. 137–138):

> **For let two oppositely electrified bodies A and B travel along the line joining them with equal velocities in the direction AB, then if either the potential or the attraction of the bodies at a given time is that due to their position at some former time (as these authors suppose), B, the foremost body, will attract A forwards more than A attracts B backward.** Now let A and B be kept asunder by a rigid rod. The combined system, if set in motion in the direction AB, will pull in that direction with a force which may either continually augment the velocity, or may be used as an inexhaustible source of energy. I think that these remarkable deductions from the latest developments of Weber and Neumann's theory can only be avoided by recognizing the action of a medium in electrical phenomena. (emphasis added)

On a technical aspect, the "combined system" can be described easily with Lagrange's formulation. It results in a Lagrangian independent of time (despite the retarded interaction, which is nullified by the rigid rod), meaning that

Hamilton's function is constant in time. With the introduction of generalised coordinates it does not require an explicit dependence on time. Hamilton's function is the energy. The system does not create energy. Furthermore, since the interaction he considered was associated exclusively to Coulomb's potential, the acceleration of the system is strictly zero and the potential is constant along the trajectory. The matter illustrates the dangers in "talking physics".

The highlighted part deserves further analysis. Assume the described situation is possible without coercing it through a rod. The electrical action jumps to the space of the emitter and it delivers the action only when it is at a place in coincidence with the receiver (it sounds pretty much like a modern photon emitted from A towards B). Let t_B be the time when the signal that left A is sensed by B and t_0 the time of emission. The distance travelled by this signal would be then $X_B(t_B) - X_A(t_0)$ (as measured in the frame of A, a subjective distance –see (H G Solari and M A Natiello 2018)–). This coincides with the distance travelled by a body moving in a straight line from $X_A(t_0)$ to $X_B(t_B)$. In the same form, let t_A be the time when the signal that left B at t_0 is sensed by A. The corresponding distance is then $X_A(t_A) - X_B(t_0)$ (in the frame of B). But since B is moving along the straight line AB while the signal is travelling, we have $|X_B(t_B) - X_A(t_0)| > |X_A(t_A) - X_B(t_0)|$. This relationship between positions, which is obvious for material bodies, is hence assumed to hold for electromagnetic interactions as well. We must admit that this form of thinking is odd: how do we know the signal jumps to the space of the receiver at the end of the process and not at the beginning? More relevant: this interpretation breaks the original objective conception of the Göttingen school in terms of relative positions and velocities. The relative distance between A and B is the same as that between B and A, except for orientation. We have broken the contract, and Equation 1 is not satisfied, a tribute to the use of intuitions based upon images by analogy.

In any case, the observer that will measure the electromagnetic signal is associated to the detector and not to the emitter. Let us say that an array of detectors is placed in the frame of the receiver to track the electrical action. The receiver (observer) will register the distance between source and target at the time of emission. The action is tracked by the detectors of any observer since the moment of emission and on this basis we can compute the velocity of the signal. If τ is the time taken to reach the detector, we have $X_B(t_0) - X_A(t_0)$ which is an objective distance (equally registered from any reference system), as well as τ, both invariant in classical physics. Then, $\frac{X_B(t_0) - X_A(t_0)}{\tau}$ is an invariant velocity and there is no problem involved. All observers determine the same value. The price paid is to leave analogies behind.

The "travelled distance" is a concept pertaining to absolute space. In contrast, we usually measure distances travelled by vehicles using an odograph. Place a car in a freight train and deliver it to some distant place. The odograph of the train will indicate a distance between train stops and the odograph of the car will indicate zero. We know both readings are correct, distances travelled are relative as well as velocities. To pretend that electromagnetic action has a speed solely determined by its identity is the same that asserting that light moves in absolute space.