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Essays on Strategic Communication

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Contents

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Declarations

This thesis is submitted to the University of Warwick in accordance with the requirements of the degree of Doctor of Philosophy. I declare that the thesis is my own and original work. Chapter 1 develops the dissertation I submitted at the University of Warwick for the Master of Research. The basic set up of the model has changed, as well as most of the analysis. The material contained in this thesis has not been submitted for a degree to any other university.

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Abstract

This thesis consists of three essays on strategic communication. It deals with the motivations behind experts' incentives to transmit information to decision-makers. Large organizations, such as governments and global corporations, rely on expertise associated with multiple areas of knowledge. The necessary information is thus provided by many individuals or subunits who, in turn, may be interested in influencing decisions. Chapter 1 studies the interaction between a decision-maker who needs to take action on correlated issues, and experts who can communicate through costless, non-verifiable messages. Credible communication depends on how information relevant to one decision affects other decisions. The paper shows that a specialized expert can be trusted more than an expert whose knowledge extends to multiple areas. Even if the latter advises on a single discipline, information from other areas of knowledge may favour his interests, increasing his incentives to be dishonest. Chapter 2 expands this framework by introducing the strategic allocation of authority and the acquisition of information. The correlation between decisions affects the extent of the informational gains from delegation in three significant ways. First, there is a commitment value of delegation: giving up control over a controversial decision can motivate experts to transmit information relevant to less controversial decisions. Secondly, delegation hampers incentives to acquire information because it restricts the expected 'marginal return' of being informed. Lastly, restricting an expert's access to information he is not expected to communicate enhances his credibility because it reduces incentives to be dishonest. Chapter 3 studies in more depth the relationship between authority and information acquisition. It focuses on how much costly information a biased expert acquires. It shows that experts with intermediate bias acquire more information under centralization than delegation when costs are sufficiently high. In such cases, the principal prefers to retain decision-making authority.

Chapter 1

Information Aggregation in Multidimensional Cheap Talk

1.1 Introduction

This paper studies strategic communication in environments with three features: a decision-maker has to take action in multiple issues, the relevant information is dispersed among many agents, and what is relevant for one decision also influences other decisions. I analyse how this *informational interdependence* affects the decision-maker's ability to aggregate information. Such interdependence arises in many real-world scenarios including policy making, product development in multinational corporations, and diplomatic negotiations. Effective communication poses a challenge because each agent possesses imperfect information that cannot be transmitted as hard facts. Also, different players usually have conflicting preferences over decisions, raising the problem of credibility.

Consider energy policy as an example. The main objectives of such policy in most developed countries are to guarantee stable energy supply, at the minimum possible cost for consumers, and with the minimal environmental impact. To address these objectives, policy-makers usually implement a combination of policy instruments, such as financial and fiscal stimulus for projects associated to different sources of energy and measures to improve consumption efficiency. Now, depending on how each of these instruments addresses the different policy objectives, information about changes in voters' policy priorities, or new technologies associated to the different sources, or the risk of oil price disruptions will require re-calibrations of the different policy instruments.[1] Most of this information, in turn, is generally dispersed among government agencies, legislative committees, research institutions, and interest groups. Because providing hard evidence is typically difficult, political actors use their information strategically, taking into account how it will affect on the different decisions.

[1] A recent article from 'The Guardian' observes that "[s]witching just some of the huge subsidies supporting fossil fuels to renewables would unleash a runaway clean energy revolution, [...] significantly cutting the carbon emissions that are driving the climate crisis [...] but could cause short-term price rises and political difficulties, as the benefits of lower costs in the future and reduced air pollution are less obvious" (Carrington, 2019).

I construct a model of multidimensional decision-making under informational interdependence. A policy consists of two decisions and the policy-maker (receiver, she) needs information about two state variables. The information is related to how each decision addresses the different policy objectives, such that information about each state influences both decisions.[2] Two agents (senders) observe private signals about the states and send costless, non-verifiable messages to the policy-maker. Incentives for communication depend on how information affects decisions, and on each sender's preferences associated to each decision. I assume linear informational interdependence, which allows me to capture in a parsimonious way the observation that information about any state affects both decisions. To isolate the effect of interdependence, I also assume preferences are additively separable on decisions.[3]

My main result is that an agent's communication incentives depend on the nature of his information. Based on the analysis of two different information structures, I characterize two novel effects. First, because information affects both decisions, communication depends on how the informational interdependence aggregates decision-specific biases. When each agent observes information associated to one state, the single determinant of communication is the *aggregate conflict of interest* with the policy-maker. Because agents' information refer to different states and these are orthogonal, individual message strategies are independent of each other. The problem then becomes similar to the uniform-quadratic case in Crawford and Sobel (1982) with one difference: the conflict of interest aggregates decision-specific biases. I exploit the apparent simplicity of each senders' incentive compatibility constraint to characterize informational spillovers.

The second effect arises when each agent observes noisy information about both states. Imperfect information makes individual message strategies dependent on each other, similar to Morgan and Stocken (2008) and Galeotti et al. (2013) among others. Now, having information about both states leads to a *credibility loss* as compared to having information about only one. To see this note that agents observe one signal associated to each state (two in total). Because states are orthogonal and each decision depends on information about both states, some realizations of an agent's information are be ambiguous—i.e. one signal suggests a decision has to be adjusted in a particular direction relative to the prior beliefs, while the other signal suggests the opposite. In such cases, the agent in question has incentives to follow the signal that favours his own preferences on that decision. This means that revealing information about one state is incentive compatible for a smaller set of biases as compared to the case in which the agent observes information about that state only.

I also show that, due to the interaction between effects, the degree of informational interdependence affects communication in a non-monotonic way. More interdependence means that information about any state influences both decisions more similarly. Hence, the aggregate conflict of interest between an agent and the policy-maker puts similar weights on the decision-specific biases. This means that an agent with

[2] Milgrom (2008) and Dziuda (2011) present a similar interpretation of decision problems defined in terms of multiple 'issues' which, in turn, are associated to the information the decision-maker needs—see also Gong and Yang (2018).

[3] Chapter 9 in Baumgartner and Jones (2009) provides an extensive discussion on how different issue areas are home to different configuration of policy biases. Returning to the energy policy example, pro-environment groups may have strong preferences for renewables and against fossil fuels, with little willingness to trade between these. Corporations related to fossil fuels have recently started to invest in some renewable sources because they consider them as complement of their main business (see, for instance, Macalister, 2016).

strong biases in both dimensions fully reveals his information because biases compensate each other. But more interdependence also means that incentives to manipulate ambiguous information increase, and the credibility loss becomes more pervasive. As messages involving ambiguous information become less credible, communication tends to focus on information that reflects the interdependence—that is, signals realizations leading to decisions being adjusted in the same direction.

Finally, I study the possibility of beneficial congestion. When an agent has imperfect information about both states, having another agent revealing information to the policy-maker can alleviate his credibility loss. Suppose that an agent's preferences are aligned with the policy-maker in the first decision but feature a large bias in the second. Due to interdependence, the large bias limits his ability to transmit information, even when it is most relevant to the first decision. Now, suppose a *second agent* reveals information most relevant to the *second decision*. Any information transmitted by the first agent has now a weaker overall influence on decisions, but much weaker on the second dimension in which his bias is large. Therefore, effective communication from the first agent will be incentive compatible for a larger set of biases than without the second agent. Such beneficial congestion can have real-world implications in the composition of decision-making teams, such as government cabinets, legislative committees, and organization boards.

1.1.1 Related literature

The paper contributes to the theoretical literature on multidimensional cheap talk. When each sender observes both states perfectly, the receiver can extract all the information by restricting influence to the dimension of common interests (Battaglini, 2002).[4] In other words, the receiver (in equilibrium) can commit to ignore part of the information each sender provides because it will be provided by the other sender on path. Levy and Razin (2007) show that the receiver loses such (equilibrium) commitment power when senders are imperfectly informed and decisions are interdependent. Senders' incentives thus depend on how information affects both decisions, which can impede communication when the conflict of interest in one dimension is sufficiently large. My paper builds on Levy and Razin (2007) and presents a framework to fully characterize incentives for communication under linear interdependence.

Equilibrium communication in my model shares key elements with prior work on multidimensional cheap talk. The very notion of aggregate conflict of interest is somewhat similar to 'mutual discipline' in Goltsman and Pavlov (2011) since revealing information leads to utility gains in one dimension that compensate the utility losses in the other (see also Farrell and Gibbons, 1989; Chakraborty and Harbaugh, 2010). In addition, my framework features a class of strategies consisting in full revelation of some signal realizations and non-influential messages otherwise. A sender fully revealing ambiguous information (and nothing otherwise) is somewhat equivalent to providing rankings of the different attributes associated to decisions, as in Chakraborty and Harbaugh (2007).

The paper also contributes to the literature on organizational design. Strategic communication has important consequences on the organization of legislative institutions (Austen-Smith and Riker, 1987; Austen-

[4] Battaglini (2004) shows that this results is robust to imperfect signals when states are orthogonal. Ambrus and Takahashi (2008) show that it is not robust to restricted state spaces for large conflict of interests.

Smith, 1990), decision-making cabinets (Dewan et al., 2015), and political parties (Dewan and Squintani, 2015). Most of the existing literature restricts attention to uni-dimensional decision problems. My focus on multidimensional and multi-causal problems is thus a step forward to understand incentives in such complex environments. A very similar notion of complexity has been used by Baumgartner and Jones (2009) to study the effects of both problem prioritization and information on agenda setting and institutional evolution.

The notion of interdependent decisions is also important among firms, but the focus of this strand of literature has been on the trade-off between coordination and adaptation (Alonso et al., 2008; Rantakari, 2008). Incentives for communication in such contexts are characterized by non-separability of preferences (need for coordination), senders' informational advantage (need for adaptation), and the difference in issue salience among players (biases). However, the nature of some firms may lead to different trade-offs. For instance, product design in multi-product firms relies on consumers' preferences over different attributes, with different products having different combinations of those attributes. Such firms face the need to gather information about consumer preferences, technological innovations, and government regulations related to each attribute, potentially leading to informational spillovers that will affect the organization of project development teams.[5] My framework captures these issues and can be used to study how organizational structures respond to the effects of informational interdependence.

The rest of the paper proceeds as follows. Section 1.2 presents the basic set up. In section 1.3 I analyse the different information structures, and derive the two main effects shaping communication incentives. Section 1.4 provides comparative statics on how interdependence affects equilibrium information transmission, while section 1.5 presents the result on beneficial congestion. Section 1.6 concludes.

1.2 The Model

1.2.1 Set Up

A receiver has to decide on two issues, $\mathbf{y} = (y_1, y_2) \in \Re^2$, for which she needs information in hands of two senders. Each decision is affected by two states, θ_1, θ_2, such that information about any of them *affects both decisions*. I represent the (state-dependent) 'optimal calibration' of decisions as a vector $\boldsymbol{\delta} = (\delta_1(\theta_1, \theta_2), \delta_2(\theta_1, \theta_2))$. Player j's payoff is thus defined in terms of decisions, composite states, and biases as follows (for $j = \{R, 1, 2\}$):

$$U^j(\mathbf{y}, \mathbf{b}^j) = -\left(y_1 - \delta_1(\theta_1, \theta_2) - b_1^j\right)^2 - \left(y_2 - \delta_2(\theta_1, \theta_2) - b_2^j\right)^2$$

The vector $\mathbf{b}^j = (b_1^j, b_2^j) \in \Re^2$ represents j's bias in the two dimensions. I normalize $\mathbf{b}^R = 0$, such that $\mathbf{b}^i$ represents the conflict of interest between sender i (he) and the receiver (she). Optimal actions depend on

[5] See, for example, Andersson and Forsgren (2000); Gassmann and von Zedtwitz (1999); Boutellier et al. (2008).

the realization of states, as follows:

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \equiv \begin{bmatrix} w_{11}\,\theta_1 + w_{12}\,\theta_2 \\ w_{21}\,\theta_1 + w_{22}\,\theta_2 \end{bmatrix}$$

States are uniformly distributed in the unit square, $\theta_1, \theta_2 \sim U[0,1]$. This assumption is not without loss of generality, but is one that has been used extensively in the cheap talk literature (e.g. uniform-quadratic case in Crawford and Sobel, 1982). The weighting matrix $\mathbf{W}$ characterizes the informational interdependence, where all weights are weakly positive and $w_{11}, w_{22} > 1/2$, so that the index corresponding to the state also reflects which decision that state is more important for. Informational interdependence is thus linear and information about any state affects decisions in the same direction with respect to the ex-ante optimal calibration. Note that informational interdependence occurs despite the states are not correlated. It can be characterized by:[6]

$$\text{Corr}(\delta_1, \delta_2) = \frac{(w_{11}\,w_{21}\ + w_{12}\,w_{22})}{\left[(w_{11}^2 + w_{12}^2)(w_{21}^2 + w_{22}^2)\right]^{\frac{1}{2}}}$$

The receiver obtains information through private, cheap talk communication with each sender. Each sender observes a signal associated to each state, $\mathbf{S}^i = (S_1^i, S_2^i) \in \mathcal{S}$, with their precision given by σ_1^i and σ_2^i, respectively. I analyse two different information structures. In the first, the signal and state spaces coincide, $\mathcal{S} = [0,1]^2$. Senders are *specialists*, i.e. each of them observes a perfectly informative signal associated to a different state and has no information about the other state. Which state each sender specialises on is common knowledge. This information structure has been used in the organizational economics literature (see Alonso et al., 2008, 2015; Rantakari, 2008). Section 1.3.1 analyses this case.

Secondly, I analyse the case of senders observing noisy signals about both states. Formally, the signal space is given by $\mathcal{S} = \{0,1\}^2$ and the precision of a any given signal is such that $Pr(S_1^i = 1|\theta_1) = \theta_1$ and $Pr(S_2^i = 1|\theta_2) = \theta_2$. This information structure is commonly used to study policy debate (see Austen-Smith, 1990; Galeotti et al., 2013; Dewan et al., 2015) in which each legislator has information about the likely effects of decisions on the constituency he/she represents. I thus refer to senders in this case as *local representatives*.

1.2.2 Timing

First, senders privately observe their information; secondly, each of them sends a cheap talk message to the receiver; finally, the receiver chooses $\mathbf{y} = (y_1, y_2)$ and payoffs realize. The message space under each information structure coincides with the signal space, that is $\mathbf{m}^i \in \mathcal{M} = \mathcal{S}$.[7] I focus on pure-strategy Perfect Bayesian Equilibria (PBE). Communication incentives will depend on the information each sender has and, in some cases, on the other sender's equilibrium message strategy. Denote by $\mathbf{m}^i(\mathbf{S}^i) \in M$ the message strategy of sender i, where $M \in \mathcal{S}$ represents the message space. A PBE of this game is characterized by a decision vector, $\mathbf{y}^*$, and a collection of message strategies, $\mathbf{m}^* = \{\mathbf{m}^{1*}, \mathbf{m}^{2*}\}$, such that:

[6] Note that the assumptions on $\mathbf{W}$ lead to non-negative correlation, which is without loss of generality

[7] This assumption is without loss since the type of information each sender observes is common knowledge. In the case of binary signals, any message between 0 and 1 could, in principle, reflect a mixed strategy, but I restrict the analysis to pure strategies

- The receiver's decisions satisfy:

$$\mathbf{y}^* = \mathrm{W}\, E(\boldsymbol{\theta}|\mathbf{m}^*)$$

- Sender i's message strategy satisfies:

$$\mathbf{m}^{i*}\left(\mathbf{S}^i\right) \in \arg\max_{\mathbf{m}^i} \left\{E\left[-\left(\mathbf{y}^*(\mathbf{m}^i,\mathbf{m}^{-i}) - \mathrm{W}\,\boldsymbol{\theta} - \mathbf{b}^i\right)' \left(\mathbf{y}^*(\mathbf{m}^i,\mathbf{m}^{-i}) - \mathrm{W}\,\boldsymbol{\theta} - \mathbf{b}^i\right)\left|\mathbf{S}^i\right.\right]\right\}$$

1.3 Equilibrium Analysis

1.3.1 Senders as Specialists

In this section each sender is perfectly informed about one of the states but has no information about the other beyond the prior. This information structure is commonly used in applications of multidimensional cheap talk to organizations (Alonso et al., 2008; Rantakari, 2008). I also provides a useful benchmark to extend Levy and Razin (2007) characterization of informational spillovers and analyse their communication implications.

Let assume that sender 1 observe a perfectly informative signal about θ_1 and sender 2 about θ_2. Each sender's problem becomes a Crawford-Sobel (CS) problem in which his message affects two decisions according to the interdependence structure. Following CS, let $a^i_{\tilde{r}}$ denote sender i's generic 'boundary type' $\tilde{\theta}_r$. Also, let $\bar{y}_d(a^i_{\tilde{r}}, a^i_{\tilde{r}+1}) = \frac{a^i_{\tilde{r}+1} - a^i_{\tilde{r}}}{2}$ be the expected action on dimension $d = \{1, 2\}$ when reporting on the upper interval, and similarly with respect to $a^i_{\tilde{r}-1}$.

Lemma 1.1. *Suppose sender $i \in \{1, 2\}$ observes a perfectly informative signal about θ_i. Sender i's equilibrium message strategy involves noisy messages unless there is no conflict of interest with the receiver. Moreover, the set of actions induced in equilibrium is finite.*

The incentive-compatibility constraint leads to the following arbitrage condition:

$$a^i_{\tilde{r}+1} = 2a^i_{\tilde{r}} - a^i_{\tilde{r}-1} + 4\frac{(b^i_1 w_{1r} + b^i_2 w_{2r})}{(w^2_{1r} + w^2_{2r})} \tag{A}$$

Proof. All proofs can be found in the Appendix. □

The corollary below characterizes the more immediate effect of informational interdependence on communication incentives, which I denote by 'aggregate conflict of interest'.

Corollary 1.1. *Condition* (A) *coincides with the arbitrage condition in CS when the bias is equal to:*

$$B^i_r \equiv \frac{(b^i_1 w_{1r} + b^i_2 w_{2r})}{(w^2_{1r} + w^2_{2r})}$$

Information about θ_1 affects both decisions and the magnitude (and direction) of that influence is given by w_{11} and w_{12}. When there is no informational interdependence ($w_{11} = 1$ and $w_{12} = 0$), communication is characterized by CS. As long as information about θ_1 affects y_1 and y_2, incentives for communication depend on sender i's biases in both dimensions. Linear interdependence means that the aggregate conflict of interest is a weighted average of the decision-specific biases. I now proceed to characterize equilibrium communication.

Proposition 1.1. *Suppose sender $i \in \{1, 2\}$ observes a perfectly informative signal about θ_i. Any equilibrium message strategy consists of finite partitions of the state space unless $B^i_r = 0$. Moreover, the intervals are characterized by:*

$$a^i_{\tilde{r}} = a^i_1 \tilde{r} + 2\tilde{r}(\tilde{r} - 1)B^i_i$$

And the maximum number of intervals is given by:

$$N(B^i_i) = \left\lceil -\frac{1}{2} + \frac{1}{2}\left(1 + \frac{2}{B^i_i}\right)^{1/2} \right\rceil$$

Proof. Follows from Crawford and Sobel (1982) when $b = B^i_r$. □

Proposition 1.1 characterizes communication for each sender when they are specialists. The expressions concide with those in Crawford and Sobel (1982) when $b = B^i_r$. Note that the information at i's disposal is orthogonal to that of the other sender and, thus, he cannot infer the message strategy of his counterpart. This is why i's message strategy does not depend on the other sender's strategy.

Maximal Incentives for Communication. In order to get a graphical intuition of communication incentives and informational spillovers, I define the maximal incentives to reveal information which, in turn, is a function of the interdependence.

Definition 1.1. *Let $\lambda_r \equiv \left\{\mathbf{z} \in \Re^2 | \ \mathbf{z}' \mathbf{W}_r = 0\right\}$ be the locus with slope $-\frac{w_{1r}}{w_{2r}}$ related to θ_r. This locus represents the bias vectors for which incentives to reveal information about θ_r are maximal.*

Note that $\mathbf{b}^i \in \lambda_i$ implies that $B^i_i = 0$, meaning that i fully reveals his information to the receiver. Definition 1.1 helps characterize the set of bias vectors for which there is information transmission (influential messages) from any individual sender. Lemma 1.1 and Proposition 1.1 imply that equilibria with influential messages exist only if $B^i_i \leq 1/4$, which can be re-expressed as:

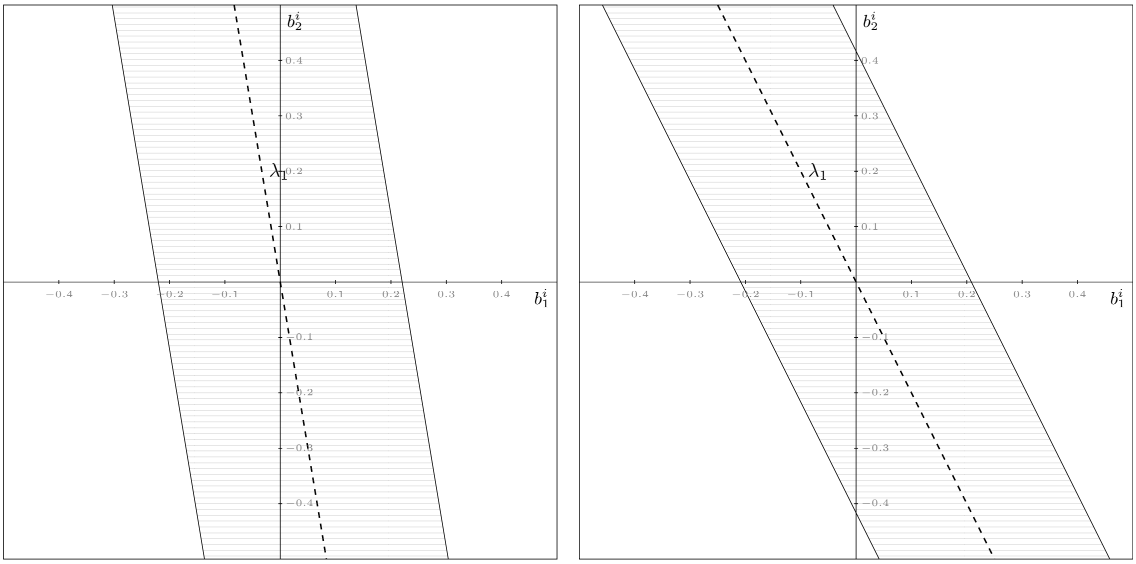
$$||\mathbf{b}^i - \text{Proj}_{\lambda_i}(\mathbf{b}^i)|| \leq \frac{\left[(w_{1i})^2 + (w_{2i})^2\right]^{\frac{1}{2}}}{4}$$

Here $\text{Proj}_{\lambda_r}(\mathbf{b}^i)$ is the projection of i's bias vector onto the locus λ_r.[8]

Figure 1.1 shows the set of bias vectors for which an influential equilibrium exists. Panel (a) illustrates the case of small interdependence, for which the maximal incentives to reveal information, λ_1, are close to the vertical axis. The small rotation indicates that revealing information about θ_1 will mostly influence y_1 but also has a small effect on y_2. Therefore, the incentive compatibility constraint weighs the sender's

[8]More precisely, the LHS of the expression above constitutes the module of the rejection of $\mathbf{b}^i$ on λ_i.

Figure 1.1: Bias vectors for which an influential equilibrium for Sender 1 exists



(a) $\mathrm{Corr}(\delta_1, \delta_2) = 0.3$

(b) $\mathrm{Corr}(\delta_1, \delta_2) = 0.8$

Notes: Shaded areas indicate bias vectors for which $B_1 \leq 1/4$, for different levels of interdependence.

biases in both dimensions. Positive correlation means that maximal incentives for communication happens when biases have different signs. As interdependence increases, maximal incentives take place when biases are more similar in magnitude, as Panel (b) shows. The overall width of the region for which $B_1 \leq 1/4$ is decreasing in interdependence, which reflecting the denominator of B_r^i: as interdependence increases, the overall influence of each sender is split between two decisions.

Informational Spillovers. Levy and Razin (2007) show how informational spillovers can kill communication when signals are imperfect. In their argument, a large bias in one dimension is a sufficient condition for communication breakdown even when preferences of the sender and the receiver are perfectly aligned in the other dimension. Unlike the Fully Revealing Equilibrium in Battaglini (2002), the receiver loses his equilibrium commitment power to ignore information across dimensions because senders are imperfectly informed. Levy and Razin define this communication breakdown as a negative informational spillover. In the next lemma, I present the possibility of positive informational spillovers. I define these by comparing the two-dimensional decision problem with two separate unidimensional problems with the same preferences and information structure.

When senders are specialists, I can define informational spillovers by comparing the two-dimensional problem with those associated to the decisions for which each sender's information is more important. The lemma below characterizes i's incentives for communication in the one-dimensional problem for which his information has the highest influence ($w_{dr} = w_{ii} \geq 1/2$), and determines the necessary conditions for negative and positive spillovers.

Lemma 1.2. *When sender $i = \{1, 2\}$ observes θ_i, the one-dimensional problem associated to decision $d = \{1, 2\}$ is characterized by:*

$$B^i_{di} \equiv \frac{b^i_d}{w_{di}}$$

Let $j = \{1, 2\}$ denote the decision for which sender i's information is less important ($w_{ji} < 1/2$). A necessary condition for negative informational spillovers is $B^i_{ii} < B^i_{ji}$; whereas, a necessary condition for positive informational spillovers is $B^i_{ii} > B^i_{ji}$.

Consider the case of sender 1, who is perfectly informed about θ_1. Negative spillovers occur when he would transmit some information in equilibrium if y_1 were decided separately, but the equilibrium of the two-dimensional problem involves no information transmission. Such harm to communication increases when b^1_2 is large relative to the influence of θ_1 on y_2 (w_{21}). Figure 1.2 illustrates these cases with a minus sign for bias vectors featuring small conflict of interest in the first dimension and large in the second.

Positive spillovers, on the other hand, arise when his incentives to reveal information are stronger in the two-decision problem as compared to the one-decision problem. This takes place when b^1_2 is small relative to b^i_1. As a consequence, 'adding' this second decision dilutes the conflict of interest in the first dimension. Figure 1.2 shows such bias vectors with a plus sign.

1.3.2 Senders as Local Representatives

In this section, I study the case in which each sender observes two noisy signals, one associated to each state. From a policy-making perspective, such information structure reflects the fact that political actors (senders) observe information from the constituencies they represent. From an organizational perspective, the information structure can be interpreted in terms of headquarters seeking local information from geographically dispersed subsidiaries. Formally, each sender observes one binary signal associated to each state, $\{S^i_1, S^i_2\} \in \mathcal{S} = \{0, 1\}^2$, such that $\Pr(S^i_1 = 1) = \theta_1$ and $\Pr(S^i_2 = 1) = \theta_2$. Recall that the message and signal spaces coincide: $(m^i_1, m^i_2) \in \mathcal{M} = \{0, 1\}^2$. After messages have been sent, the receiver updates beliefs according to a Beta-binomial process. Let $k_r \leq 2$ be the number of senders truthfully revealing their signals and ℓ_r the number of those signals that equals 1, where $r = \{1, 2\}$ indexes the states. Then, the updated expectation for each state is given by:[9]

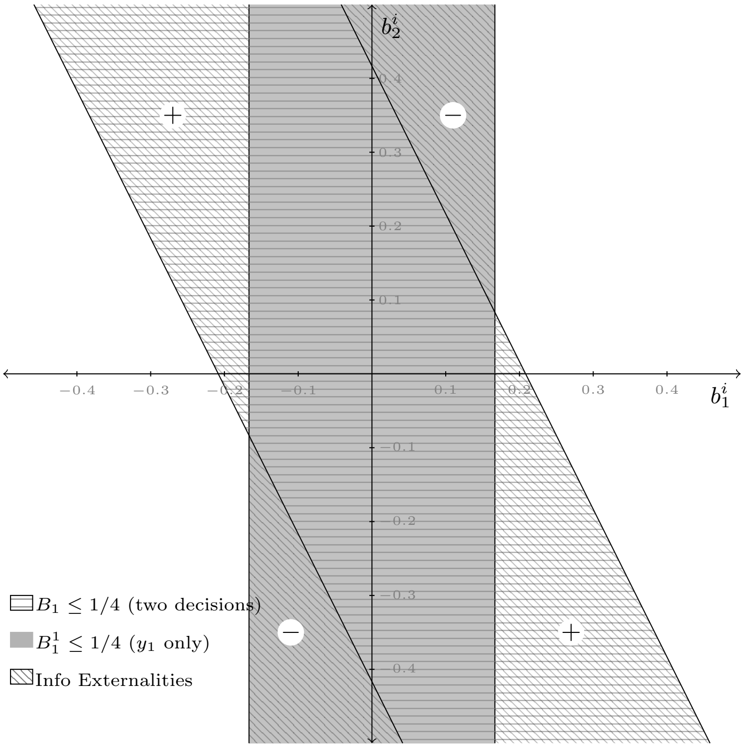
$$E(\theta_1|k_1, \ell_1) = \frac{(\ell_1 + 1)}{(k_1 + 2)} \qquad\qquad E(\theta_2|k_2, \ell_2) = \frac{(\ell_2 + 1)}{(k_2 + 2)}$$

Let k^*_1 and k^*_2 denote the number of truthful messages the receiver has in equilibrium; also, let ℓ^*_1 and ℓ^*_2 denote the number of "ones" reported in equilibrium. From now on, k and ℓ (no superscript) denote i's

[9] The conditional pdf and the receiver's posterior on state θ_r are, respectively:

$$f(\ell_r|\theta_r, k_r) = \frac{k_r!}{\ell_r!(k_r - \ell_r)!}\theta_r^{\ell_r}(1 - \theta_r)^{k_r - \ell_r} \qquad\qquad h(\theta_r|\ell_r, k_r) = \frac{(k_r + 1)!}{\ell_r!(k_r - \ell_r)!}\theta_r^{\ell_r}(1 - \theta_r)^{k_r - \ell_r}$$

Figure 1.2: Informational Spillovers for Sender 1 – Two decisions vs one-dimensional problem (y_1)



Note: Corr$(\delta_1, \delta_2) = 0.8$. $+$ $(-)$ indicates positive (negative) Info Spillovers.

conjecture about other senders revealing truthfully and reporting ones on the equilibrium path.[10]

Because of the possibility of multiple equilibria, I need to specify the selection criterion. As most papers in the cheap talk literature, I focus on the receiver-optimal equilibrium. This criterion chooses the message strategy that maximizes the receiver's ex-ante expected utility. Unlike the analysis in the previous section, here more messages do not always lead to more information transmitted to the principal. For a given $\mathbf{m}^*$ the receiver's equilibrium actions are given by:

$$y_1^* = w_{11}\frac{(\ell_1^* + 1)}{(k_1^* + 2)} + w_{12}\frac{(\ell_2^* + 1)}{(k_2^* + 2)} \qquad\qquad y_2^* = w_{21}\frac{(\ell_1^* + 1)}{(k_1^* + 2)} + w_{22}\frac{(\ell_2^* + 1)}{(k_2^* + 2)}$$

The above expressions show how a sender's information, if revealed, affects the receiver's beliefs and decisions. Revealing one signal affects decisions according to the importance of the associated state. Just as in the specialists case, incentives for communication depend on the way the interdependence aggregates decision-specific biases. But having information about both states makes each sender's incentives depend on the information the other sender is expected to reveal in equilibrium.[11] Moreover, independence of states can

[10]Sender i's conjecture will be correct on path, and whenever his equilibrium message strategy involves revealing the corresponding signal then $k_r^* = k_r + 1$, while $k_r^* = k_r$ otherwise.

[11]Indeed, this is caused by the fact that both senders observe information about the same states (see Krishna and Morgan, 2001b)

produce signals that, if fully revealed would move decisions *against* the informational interdependence. The possibility of such information raises some additional incentives to deviate for each sender, leading to a loss of credibility that harms communication. This 'credibility loss' is another important effect of informational interdependence.

In order to build up intuition, I start with sender i's incentives to reveal one signal only and then derive incentives for full revelation. Finally, I show that equilibrium communication is not only based on revealing information on separate dimensions independently. There are equilibrium strategies in which the amount of information a sender transmits depends on his signals realizations. Proposition 1.2 characterizes the receiver-optimal communication equilibrium for sender i.

Incentives to reveal one signal only. Revealing any single signal amounts to revealing information about one state only, which would move both decisions in the same direction due to the positive informational interdependence.[12] As in the previous section, this means that sender i's incentives depend on the aggregate conflict of interest. But having information about both states alters these incentives. Because states are orthogonal, a sender's information may consist in signals realizations that contradict each other in terms of policy recommendations. Take, for example, a sender who has positive bias in both dimensions and is expected to reveal information about θ_1 in equilibrium. Because of his biases, incentive compatibility demands that he is better-off revealing $S_1^i = 0$ than lying about this signal realization. These incentives for truth-telling are weakened when his other piece of information is $S_2^i = 1$, because it implies adjustments towards i's biases on both decisions. The fact that he cannot credibly transmit this piece of favourable information in the equilibrium under consideration undermines his incentives to reveal S_1^i truthfully. Therefore, the very possibility of such 'ambiguous information' harms credibility. Below I present the incentive compatibility (IC) constraint for this message strategy.

Lemma 1.3. *Consider an equilibrium $(\mathbf{y}^*, \mathbf{m}^*)$ in which $\{S_r^i\} \in \mathbf{m}^{i*}$. Revealing information about state θ_r is incentive compatible for sender i if:*

$$|B_r^i| \leq \frac{1}{2}\left[\frac{1}{(k_r+3)} - \frac{C_r}{(k_{\tilde{r}}+3)}\right] \tag{1.1}$$

where $C_r = \frac{(w_{11}w_{12}+w_{21}w_{22})}{(w_{1r}^2+w_{2r}^2)} \in [0,1]$; such that when $w_{11} = w_{22} = w$, then $C_1 = C_2 = Corr(\delta_1, \delta_2)$.

Sender i's incentives to reveal information about $\theta_r = \{\theta_1, \theta_2\}$ depend on the other sender's strategy for two reasons. First, information is noisy and this imposes an upper bound on how much 'additional precision' sender i can induce by revealing his signal. Since both senders have binary signals, i can predict the marginal effect of revealing his signal on the corresponding posterior— captured by k_r in equation (1.1).[13] The first term in square brackets shows how i's incentives depend on whether sender j reveals information about the

and that information is noisy (see Austen-Smith, 1993; Galeotti et al., 2013).

[12]Recall that this stems from the assumptions on $\mathbf{W}$, which are without loss of generality.

[13]If the other sender is not expected to reveal S_r^j on path, the set of possible posteriors is given by $E(\theta_r|m_1^i = S_1^i) = \{1/3, 2/3\}$; while if he is expected to reveal, the set becomes $E(\theta_r|m_1^i = S_1^i, m_1^j = S_1^j) = \{1/4, 1/2, 3/4\}$.

same state, similar to the unidimensional decision problems in Austen-Smith (1990); Morgan and Stocken (2008); Galeotti et al. (2013).

The second effect relates to the fact that senders have information about both states. I have already described how i experiences increased incentives to lie when information about one state is favourable and that about the other is unfavourable. Recall that, due to positive interdependence, revealing information about one state moves decisions in the same direction. Ambiguous information hence arises for signal realizations $(S_1^i, S_2^i) = \{(0,1),(1,0)\}$.[14] The second term of the right-hand side in expression (1.1) captures the credibility loss. Note that it is increasing in the informational interdependence, which is captured by C_r (for given k_1 and k_2).[15]

Incentives to reveal both signals. The way in which i's information affects decisions depends on its realization. Fully revealing $(S_1^i, S_2^i) = \{(0,0);(1,1)\}$ moves decisions in the same direction with respect to the prior, whereas fully revealing $(S_1^i, S_2^i) = \{(0,1);(1,0)\}$ moves decisions in opposite directions. Therefore, the set of $\mathbf{b}^i$ for which revealing both signals is incentive compatible depends on their realizations.

Lemma 1.4. *Consider the equilibrium $(\mathbf{y}^*, \mathbf{m}^*)$ in which $\{S_1^i, S_2^i\} \in \mathbf{m}^{i*}$. Sender i finds full revelation incentive compatibility if:*

- *he does not have incentives to lie on any signal individually; that is, for $\theta_r = \{\theta_1, \theta_2\}$:*

$$|B_r^i| \leq \frac{1}{2\,(k_r+3)} \tag{1.2}$$

- *he does not have incentives to lie on both signals when $\mathbf{S}^i = \{(0,0);(1,1)\}$:*

$$0 \leq \frac{1}{(k_1+3)}\left[\frac{1}{(k_1+3)} + \frac{C_1}{(k_2+3)} \pm 2\,B_1^i\right] + \frac{1}{(k_2+3)}\left[\frac{1}{(k_2+3)} + \frac{C_2}{(k_1+3)} \pm 2\,B_2^i\right] \tag{1.3}$$

- *he does not have incentives to lie on both signals when $\mathbf{S}^i = \{(0,1);(1,0)\}$:*

$$0 \leq \frac{1}{(k_1+3)}\left[\frac{1}{(k_1+3)} - \frac{C_1}{(k_2+3)} \pm 2\,B_1^i\right] + \frac{1}{(k_2+3)}\left[\frac{1}{(k_2+3)} - \frac{C_2}{(k_1+3)} \mp 2\,B_2^i\right] \tag{1.4}$$

The operators $\pm$ in (1.3) mean that the expression must hold when B_1^i and B_2^i are both added and when they are subtracted, keeping their signs. Incentive compatibility thus requires that the RHS is non-negative in both cases, which in turn indicates the way in which decision-specific biases aggregate. Similarly, the operators $\pm$ and $\mp$ in (1.4) mean that the expression must hold when B_1^i is added and B_2^i subtracted, and vice-versa. The latter means that this IC constraint is slack when b_1^i and b_2^i have the same sign, as opposed to expressions (1.1), (1.2), and (1.3) that are slack when b_1^i and b_2^i have different signs.

[14] In section 1.5 I show that for those signals realizations, sender i's incentives to deviate from fully revealing one of his signals are lower when j reveals information about the other state in equilibrium.

[15] Alternatively, note that when $\text{Corr}(\delta_1, \delta_2) = 0$, i's information about θ_2 does not affect y_1.

Fully revealing $(S_1^i, S_2^i) = (0,0)$ or $(S_1^i, S_2^i) = (1,1)$ moves both decisions in the same direction with respect to the prior. Note that each signal reinforces the influence of the other, which is captured by the the terms C_1 and C_2 having the same sign in (1.3). Note also that the expressions within square brackets correspond to the incentives to reveal each signal individually (equation (1.1)). To see this consider the case in which i fully reveals both signals and j is expected to reveal information about θ_1 only. In this case, i's message strategy has a smaller influence on y_1 than on y_2 because the receiver is expected to be better informed about θ_1 (from j equilibrium message strategy). As a consequence, i's IC constraints put more weight on B_2^i than B_1^i. The following corollary summarizes this intuition:

Corollary 1.2. *When sender i reveals both signals, the larger (smaller) k_1 relative to k_2 the smaller (larger) the influence on y_1 relative to y_2.*

Incentives to fully reveal signals $(S_1^i, S_2^i) = \{(0,1); (1,0)\}$ are somewhat different. Not only decisions moves in different directions, but also the net influence on each decision is smaller than the influence of revealing any of the signals individually. Moving decisions in opposite directions leads to a different aggregation of decision-specific biases. As a consequence, the RHS of equation (1.4) is less restrictive when when B_1^i and B_2^i have the same sign. The smaller overall influence makes this IC constraint relatively more restrictive as compared to the others in Lemma 1.4. Finally, Corollary 1.2 also applies to this IC constraint, meaning that the relative importance of each piece of information depends on whether the other sender is revealing the same information or not.

Dimensional non-separable message strategies. The equilibrium characterization in this game is not just based on communication on separate dimensions independently. In fact, it is possible that senders credibly transmit information for some signals realizations but not for others. This is the case when, for example, i's biases are $b_1^i > 0$ and $b_2^i < 0$, and they are large but similar in magnitude ($|b_1^i| \sim |b_2^i|$). If such a sender truthfully announces $\mathbf{m}^i = \{(1,1)\}$, the receiver should believe him because his utility gains from increasing y_1 compensate the utility losses from decreasing y_2. But announcing $\mathbf{m}^i = \{\{(0,1)\}\}$ or $\mathbf{m}^i = \{\{(1,0)\}\}$ does not have the same compensation across dimensions. If these messages were believed, i would *always* announce $\mathbf{m}^i = \{(1,0)\}$, which cannot be an equilibrium. As a result, a sender with such biases fully reveals signals that coincide and transmits no information when they do not (meaning that he plays the corresponding babbling strategy). I call such message strategies dimensional non-separable (DNS, henceforth). In Appendix A1, I show that the only strategies of this class arising in the receiver-optimal equilibrium has the structure of the example: full revelation for two combinations of signal realizations and babbling for the other two. The lemma below characterizes communication incentives when the message strategy includes both babbling and influential messages, for the class of DNS strategies arising in equilibrium:

Lemma 1.5. *Consider an equilibrium $(\mathbf{y}^*, \mathbf{m}^*)$ in which $\mathbf{m}^{i*}$ includes a babbling strategy and full revelation of some signal realizations. Then, full revelation is incentive compatible for sender i when:*

- *he does not have incentives to lie on both signals when $\mathbf{S}^i = \{(0,0);(1,1)\}$:*

$$\frac{1}{(k_1+3)}\left[\frac{1}{(k_1+3)}+\frac{C_1}{(k_2+3)}\pm 4\,B_1^i\right]+\frac{1}{(k_2+3)}\left[\frac{1}{(k_2+3)}+\frac{C_2}{(k_1+3)}\pm 4\,B_2^i\right]\geq 0 \quad (1.5)$$

- *he does not have incentives to lie on both signals when $\mathbf{S}^i = \{(0,1);(1,0)\}$:*

$$\frac{1}{(k_1+3)}\left[\frac{1}{(k_1+3)}-\frac{C_1}{(k_2+3)}\pm 4\,B_1^i\right]+\frac{1}{(k_2+3)}\left[\frac{1}{(k_2+3)}-\frac{C_2}{(k_1+3)}\mp 4\,B_2^i\right]\geq 0 \quad (1.6)$$

The similarities between conditions in Lemmas 1.4 and 1.5 reflect the fact that the influential messages fully reveal the sender's information. The main difference between the two is the weight of the aggregate conflict of interests: DNS message strategies put more weight on B_1^i and B_2^i, meaning that conditions (1.5) and (1.6) hold for a smaller set of biases. This is due to the influence of deviations to non-influential messages. When any of these messages are announced, the receiver does not update decisions. However, if i announces one of the influential messages, both decisions move away from the prior. This deviation features a lower overall influence than lying on both signals. Because of this lower influence, non-influential types[16] are more tempted to deviate and, thus, the IC constraints hold for a small set of biases.

I now characterize the receiver-optimal equilibrium of this game.

Proposition 1.2. *The receiver-optimal Perfect Bayesian Equilibrium for sender i consists of the following message strategies:*

1. ***Revealing both signals**, if $\mathbf{b}^i$ satisfies conditions in Lemma 1.4 with respect to both states.*
2. ***Revealing one signal only**, if $\mathbf{b}^i$ satisfies conditions in Lemma 1.3 with respect to one state only and does not satisfy those in Lemma 1.4.*
3. ***Dimensional non-separable message strategies** in the following cases:*
   (a) *Fully revealing $(S_1^i, S_2^i) = \{(0,0);(1,1)\}$ if $\mathbf{b}^i$ satisfies condition (1.5) only;*
   (b) *Fully revealing $(S_1^i, S_2^i) = \{(0,1);(1,0)\}$ if $\mathbf{b}^i$ satisfies condition (1.6) only.*
4. ***No communication** (babbling strategy), if none of the above holds.*[17]

First note that the set of strategies that constitute an equilibrium is a strict subset of the strategy space. Proposition 1.2 implies that equilibrium communication depends on the profile of biases and the informational interdependence. The system of beliefs that supports each strategy is characterized in Appendix A1.

[16] A sender's type is given by his signals realization. By non-influential type I mean a sender whose signals lead to babbling message strategies on path.

[17] Formally, the message strategy in (1) is $\mathbf{m}^i = \Big\{\{(0,0)\};\{(0,1)\};\{(1,0)\};\{(1,1)\}\Big\}$; in (2) when revealing S_1^i is $\mathbf{m}^i = \Big\{\{(0,0);(0,1)\};\{(1,0);(1,1)\}\Big\}$, and when revealing S_2^i is $\mathbf{m}^i = \Big\{\{(0,0);(1,0)\};\{(0,1);(1,1)\}\Big\}$; in (3).$a$ is $\mathbf{m}^i = \Big\{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\Big\}$, and in (3).$b$ is $\mathbf{m}^i = \Big\{\{(0,0);(1,1)\};\{(0,1)\};\{(1,0)\}\Big\}$; and in (4) is $\mathbf{m}^i = \Big\{\{(0,0);(1,1);(0,1);(1,0)\}\Big\}$.

Intuitively, when the strategy involves some sort of pooling between types (Parts 2 and 3), there are at least two messages that induce the same actions and, upon hearing any of such messages, the receiver assigns equal probability to the types involved. Note also that the non-influential messages in DNS strategies provide no information at all to the receiver. This is due to the fact that the signals realizations involved perfectly compensate each other (and the receiver puts equal probability on each pair).

The Fully Separating Equilibrium arises only if the aggregate conflict of interest between i and the receiver is sufficiently small. The shape of the different correspondences reflects how each message strategy affects decisions. Recall that λ_1 and λ_2 represent the maximal incentives to reveal information about each state. Positive informational interdependence then leads to the negative slopes of λ_1 and λ_2[18] because incentives to reveal any single signal are greater when decision-specific biases have different signs. Condition (1.1) can thus be re-expressed as follows:

$$||\mathbf{b}^i - \text{Proj}_{\lambda_r}(\mathbf{b}^i)|| \leq \frac{[w_{1r}^2 + w_{2r}^2]^{\frac{1}{2}}}{2} \left[\frac{1}{(k_r + 3)} - \frac{C_r}{(k_{\tilde{r}} + 3)} \right]$$

This formulation allows the comparison with the specialist case. The main difference relates to the effects of ambiguous information on incentives for communication: the set of biases for which there is any information transmission is strictly smaller than when senders are specialists.[19]

Figure 1.3 illustrates the set of biases for which the different strategies arise in equilibrium, for a given set of parameter values. The blue and red regions correspond to DNS message strategies. Note that the blue region consists mainly of biases in the II and IV quadrants, meaning that b_1^i and b_2^i have different signs. When i's biases lie on this region and his signals are $\mathbf{S}^i = \{(0,0); (1,1)\}$, fully revealing them moves both decisions in the same direction and, as a consequence, i's utility gains associated to one decision compensate the utility losses associated to the other. This interaction between interdependence, biases, and messages makes the aggregate conflict of interest relatively small, and the message credible. On the contrary, if the receiver were to believe any of the messages given by $\mathbf{m}^i = \{\{(0,1)\}; \{(1,0)\}\}$, i will announce the message that moves decisions according to the biases. Similar intuition applies when biases are in the red region (quadrants I and III), for full revelation of signals $\mathbf{S}^i = \{(0,1); (1,0)\}$—decisions move in different directions in this case.

Next I present a formal discussion on the effects of informational interdependence on communication.

1.4 How Informational Interdependence Affects Communication

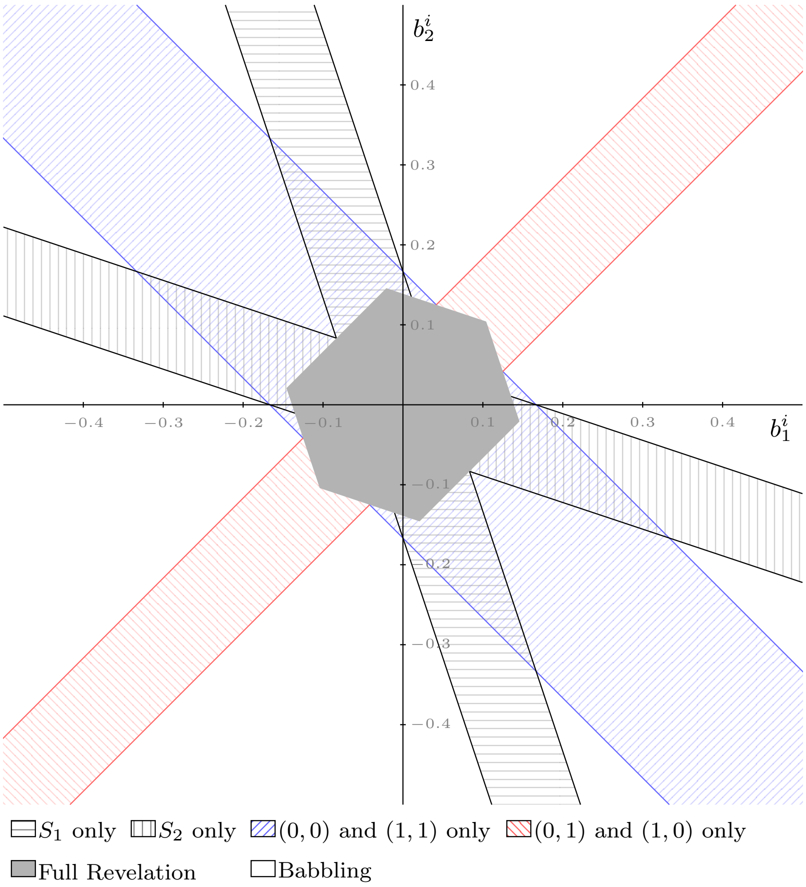
To provide a more intuitive interpretation of the results, I assume $w_{11} = w_{22} = w$ and $w > \frac{1}{2}$. The analysis looks at how sender i's IC constraints change when w changes, given sender j's message strategy. Note that a higher w means less informational interdependence.

Proposition 1.3. *Let $1/2 \leq w \leq 1$. Increasing interdependence affects communication through the effect of the credibility loss and the aggregate conflict of interest. The effect through the credibility loss associated to*

[18] The different slopes relate to the fact that incentives to reveal information about θ_1 (θ_2) weighs more the bias on y_1 (y_2).

[19] Here I only refer to whether there is communication or not, leaving aside the *amount* of information transmitted in each case.

Figure 1.3: Equilibrium communication for a sender with information about both states (Prop. 1.2)



Note: $w_{11} = w_{22} = 3/4$ (Corr$(\delta_1, \delta_2) = 0.6$) and $k_1 = k_2 = 0$

state $\theta_r = \{\theta_1, \theta_2\}$ is given by:

$$CL_r = \frac{\partial C_r}{\partial w} < 0$$

The effect through the aggregate conflict of interest associated to state $\theta_r = \{\theta_1, \theta_2\}$ is given by:

$$ACI_r = \frac{\partial |B_r^i|}{\partial w}$$

Interdependence changes the extent to which the credibility loss affects communication: more interdependence increases the effect of credibility loss on incentives. To see the intuition, consider the case in which i is expected to reveal information on θ_1. When interdependence increases, the opportunity costs of not revealing favourable information about θ_2 increases as well—revealing information about the latter has a relatively higher influence on both decisions due to the stronger interdependence. Whenever i observes ambiguous information, his incentives to deviate to sending a favourable message associated to θ_1 increase. A similar intuition holds for the incentives to reveal both signals, since equations (1.3) and (1.4) require than revealing at least one signal individually is incentive compatible.

On the other hand, interdependence also affects the aggregate conflict of interest. Below I show the cases in which more interdependence reduces the aggregate conflict of interest.

Lemma 1.6. *A necessary condition for interdependence to reduce the aggregate conflict of interest is* $sign(B_1^i) = sign(b_1^i - b_2^i)$*, which implies that:*

1. *if* $sign(b_1^i) = sign(b_2^i)$*, the necessary condition becomes:* $|b_1^i| > |b_2^i|$*;*
2. *if* $sign(b_1^i) \neq sign(b_2^i)$*, the necessary condition becomes:* $w|b_1^i| > (1-w)|b_2^i|$*.*

Suppose, again, that sender i is expected to reveal information about θ_1 in equilibrium. Higher interdependence loosens the corresponding IC constraint when b_2^i is sufficiently small relative to b_1^i.[20] In other words, information in hands of i has a higher influence on a decision of smaller conflict of interest between him and the principal.

Figure 1.4 depicts the intuitions behind Proposition 1.3 and Lemma 1.6, illustrating the two channels described above. The change in the aggregate conflict of interest can be seen in the rotation of λ_1 and λ_2, which represent the maximal incentives to reveal information about θ_1 and θ_2, respectively. Higher interdependence means that information about θ_1 (θ_2) affects both decisions more similarly, then the weights of b_1^i and b_2^i on the corresponding IC constraint will be more similar, i.e. λ_1 (λ_2) rotates towards the -45^o line.

The second effect relates to the credibility loss. More interdependence exacerbates the negative implications of ambiguous information. Take the incentives to reveal one signal. As interdependence increases, such strategy is incentive compatible for a smaller set of biases, which is represented by how the corresponding IC constraints narrow in Figure 1.4. The same effect is also present for the DNS strategy involving full revelation of $\mathbf{S}^i = \{(0,1),(1,0)\}$. Recall that in such case the overall influence on decisions is smaller than revealing each signal individually. Higher interdependence means each signal counteracts the effect of the other and, thus, the overall influence decreases.

Note that the DNS strategy involving full revelation of $\mathbf{S}^i = \{(0,0);(1,1)\}$ is not affected by the credibility effect. The influence that revealing each signal has on decision reinforces that of the other in such strategy. As a consequence, sender i's incentives to deviate from truth-telling are driven only by his bias and not his information.

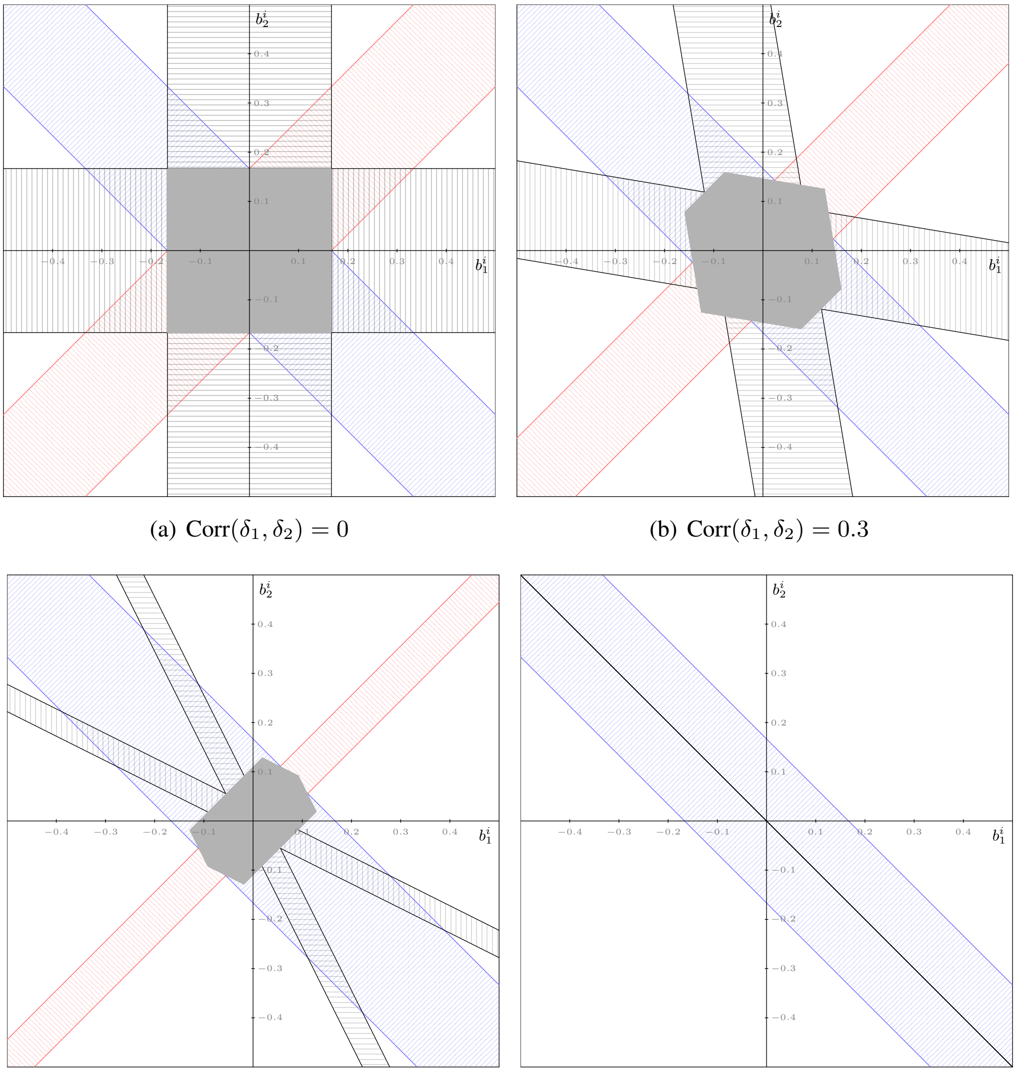
In the next section I explore the possibility of beneficial congestion, which is related to the role of k_1 and k_2 on i's incentives to fully reveal his signals (at least for some realizations).

1.5 Beneficial Congestion

The cheap talk literature has identified many instances in which an additional source of information negatively affects the incentives of the original source. For perfectly informed senders, Krishna and Morgan (2001b) show that if their biases go in the same direction the receiver cannot do better than consulting the sender with the lowest bias. When senders observe imperfect information, Morgan and Stocken (2008);

[20] Note that b_2^i corresponds to the decision for which θ_1 is less important, i.e. $w_{12} < w_{11}$.

Figure 1.4: Equilibria in Proposition 1.2 for different levels of interdependence



(a) $\text{Corr}(\delta_1, \delta_2) = 0$ (b) $\text{Corr}(\delta_1, \delta_2) = 0.3$

(c) $\text{Corr}(\delta_1, \delta_2) = 0.8$ (d) $\text{Corr}(\delta_1, \delta_2) = 1$

Notes: $k_1 = k_2 = 0$

Galeotti et al. (2013) among others, show that individual incentives for communication decrease with the number of truthful messages transmitted in equilibrium. In one-decision problems, therefore, congestion harms individual incentives for communication.[21,22]

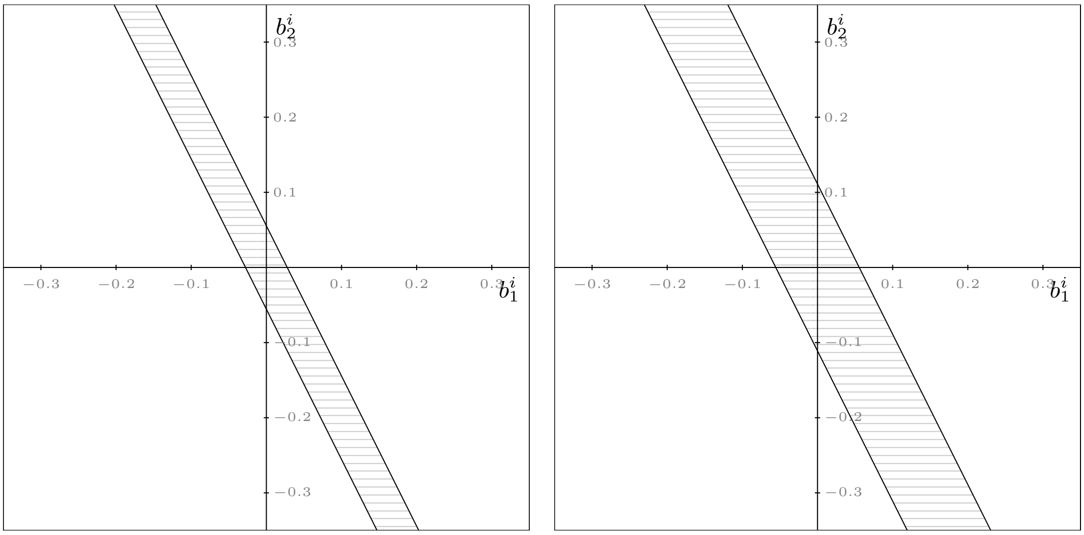
When senders are imperfectly informed and decisions are correlated, the possibility of ambiguous information harms communication incentives. In particular, sender i's incentives to reveal information about

[21]This also holds for multidimensional problems in which decisions are not correlated (Dewan et al., 2015).

[22]Krishna and Morgan (2001b) show that the receiver can extract more information from senders with opposing biases. As I shall show, the mechanism is completely different to the case of multiple decisions and imperfectly informed senders.

θ_1 also depend on the information the receiver gets about θ_2 in equilibrium. If she is expected to be well informed about θ_2, sender i's signal associated to this state would represent a small proportion of the receiver's information. Sender i's credibility loss is therefore weaker, i.e. his incentives to deviate from revealing S_1^i are less affected by ambiguous information. Figure 1.5 shows how the set of biases for which revealing S_1^i is incentive compatible expands as the number of sender revealing information about θ_2 increases.

Figure 1.5: Beneficial Congestion effect on incentives to reveal S_1^i



(a) $k_1 = 0,\ k_2 = 0$ (b) $k_1 = 0,\ k_2 = 1$

Note: $\text{Corr}(\delta_1, \delta_2) = 0.8$.

This beneficial congestion effect becomes stronger with interdependence, that is:

$$\frac{\partial IC_{(1.1)}}{\partial k_{\tilde{r}}} = \frac{C_r}{(k_{\tilde{r}} + 3)^2}$$

Beneficial congestion also arises for DNS strategies. Due to the complexity of the IC constraints and their derivatives, I show the results by looking at the set of biases for which the beneficial congestion takes place. Below I describe the conditions under which increasing the number of senders revealing information about θ_1 induces i to reveal more information.

Proposition 1.4. *Let $\mathbf{b} = \{\mathbf{b}^i, \mathbf{b}^j\}$ and $\tilde{\mathbf{b}} = \{\mathbf{b}^i, \tilde{\mathbf{b}}^j\}$ be two collections of bias vectors, such that $\mathbf{b}^j \neq \tilde{\mathbf{b}}^j$. Let $k_1, k_2 \in \{0, 1\}$ be the number other truthful signals the receiver has from other senders in the equilibrium under $\mathbf{b}$, and $\tilde{k}_1 = k_1 + 1$ and $\tilde{k}_2 = k_2$ be those for the equilibrium of the game under $\tilde{\mathbf{b}}$. For any $1/2 < w < 1$, if*

$$\frac{(k_2+3)}{(k_1+3)} < \min\left\{\frac{(k_1+4)}{(k_2+3)}\,;\ \frac{4\,w\,(1-w)}{[w^2 + (1-w)^2]}\right\}$$

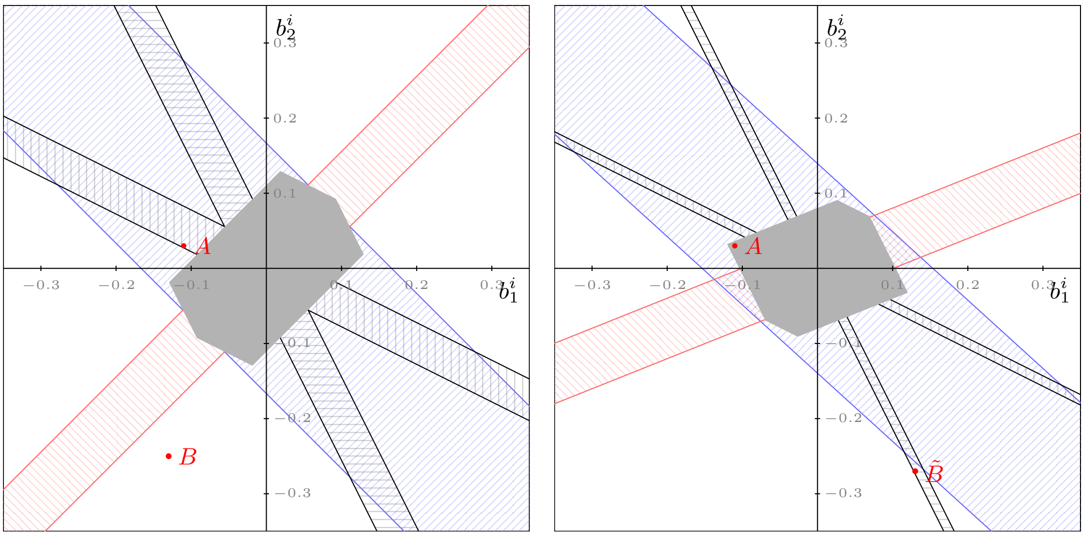
Then, there exist $\mathbf{b}^i \in \Re^2$ such that i's equilibrium message strategies are:

- *Revealing both signals when they coincide and nothing otherwise, when preferences are given by* $\mathbf{b}$*; and*

- *Revealing both signals, when preferences are given by* $\tilde{\mathbf{b}}$.

Figure 1.6 presents Proposition 1.4 with a simple example. There are two senders, A and B, each characterized by his bias vector. The figure represents A's IC constraints. I show how sender A's incentives change for different biases of sender B. The different biases will lead to different equilibrium message strategies for B, which change sender A incentives for communication. In the graph on the left, B's equilibrium message strategy is babbling, while A fully reveals when signals coincide and play babbling strategy otherwise.[23] Note that A's bias on the first dimension is relatively large, so if he observes $\mathbf{S}^A = (1, 0)$ he prefers to announce $\mathbf{m}^A = \{(0, 1)\}$—that is why full revelation cannot be an equilibrium in this case.

Figure 1.6: Effect of increasing the receiver's equilibrium information under Proposition 1.4 on A's incentives



(a) $k_1 = 0,\ k_2 = 0$ (b) $k_1 = 1,\ k_2 = 0$

Note: Corr$(\delta_1, \delta_2) = 0.8$. Both k_1 and k_2 are A's equilibrium conjectures.

Sender A's incentives change when B's preferences are as shown in $\tilde{B}$ in the right panel. Because $\tilde{B}$ is expected to reveal his information about θ_1 on path, A's influence from revealing both signals is smaller in y_1 relative to y_2. As a consequence, A is now willing to fully reveal if he observes $\mathbf{S}^A = (1, 0)$ and his equilibrium message strategy is full revelation.[24] Thus, increasing the receiver's precision on y_2 is now more important for A than the cost associated to y_1.

In summary, as the receiver has more information about θ_1 on path, sender A's influence on y_1 decreases. Given A has strong preferences over y_1, the reduced influence improves his communication incentives to

[23] Formally, $\mathbf{m}^A_{(a)} = \{\{(0,0)\}; \{(1,1)\}; \{(0,1); (1,0)\}\}$ and $\mathbf{m}^B = \{\{(0,0); (1,1); (0,1); (1,0)\}\}$.

[24] Numerically, preferences for A are $b_1^A = -0.11$ and $b_2^A = -0.03$.

the point he transmits more information than in the original situation. Note that the beneficial congestion is also possible because neither b_1^A nor b_2^A are too large in magnitude. Both informational interdependence and imperfectly informed senders are crucial features for this mechanism to arise. The mechanism can have important implications for the conformation of executive cabinets or legislative committees, which have been among the most studied applications of cheap talk models (see Austen-Smith, 1990; Dewan and Hortala-Vallve, 2011; Dewan et al., 2015).

1.6 Concluding Remarks

I studied cheap talk communication between two senders and a receiver who decides on two issues. Information relevant for each issue also influences the other. Such informational interdependence affects senders' incentives because it aggregates decision-specific biases, but communication also depends on the information structure. When senders are specialized on different issues, the aggregate conflict of interest is the only determinant of communication. When senders observe noisy signals on both issues, on the other hand, the possibility of ambiguous information also affects communication. Incentives to reveal information that influences decisions against the sender's bias become weaker when he observes other pieces of information that would have the opposite influence if revealed.

I showed that the aggregation of decision-specific biases can lead to informational spillovers. Negative spillovers arise when the multidimensional decision problem reduces the amount of information transmitted in equilibrium as compared to separate unidimensional problems—similar to Levy and Razin (2007). Positive spillovers arise because the aggregate conflict of interest of the multidimensional problem is smaller than the biases associated to each decision. Such spillovers can play a key role in the organization of governmental departments (White and Dunleavy, 2010) and negotiation teams (Trager, 2011, 2017) among others.

I also showed that information congestion can result in more information transmitted in equilibrium. In a context of imperfectly informed senders, the more information the receiver gets in equilibrium, the less influential each sender is. Deviations that may arise in the presence of favourable information that is not revealed in equilibrium are therefore less profitable.

Appendix

A1 Proofs

Senders as Specialists

Proof of Lemma 1.1. First, note that $\frac{\partial^2 U^i}{\partial y_d^2} < 0$ and $\frac{\partial^2 U^i}{\partial y_d \partial \theta_r} > 0$ because preferences are quadratic losses and additively non-separable. This guarantees that equilibrium message strategies involve finite partitions of the state space characterized by an integer $N(b)$|see Lemma 1 in Crawford and Sobel (1982). For a given equilibrium $N(b)$, let $a_0^i, a_1^i, ..., a_N^i$ denote the boundaries of this partitions.

I now derive the arbitrage condition for a typical 'boundary type'. Let assume sender 1 observes the realization of θ_1 and sender 2 that of θ_2. Also, let $\nu^i = E(\theta_i|m^i)$ denote the posterior induced on θ_i after message m^i was sent by sender i, for $i = \{1, 2\}\}$. Communication incentives for a given realization of θ_i are then given by:

$$\begin{aligned} E\left[U^i|\theta_i, \nu_i^{i*}\right] &= -E\left[(y_1 - w_{11}\theta_1 - w_{12}\theta_2 - b_1^i)^2 + (y_2 - w_{21}\theta_1 - w_{22}\theta_2 - b_2^i)^2|\theta_i, m^{i*}\right] \\ &= -E\Big[[w_{11}(\nu_1^i - \theta_1) + w_{12}(\nu_2^i - \theta_2)]^2 - 2b_1^i[w_{11}(\nu_1^i - \theta_1) + w_{12}(\nu_2^i - \theta_2)] + (b_1^i)^2 + \\ &\quad + [w_{21}(\nu_1^i - \theta_1) + w_{22}(\nu_2^i - \theta_2)]^2 - 2b_2^i[w_{21}(\nu_1^{i*} - \theta_1) + w_{12}(\nu_2^{i*} - \theta_2)] + (b_2^i)^2|\theta_i, m^{i*}\Big] \end{aligned}$$

Notice that the independence of θ_1 and θ_2 implies that sender i's messages cannot affect the receiver's beliefs on θ_j, i.e. $E\left[\nu_j^{i*}|\theta_i, m^{i*}\right] = E\left[\theta_j|\theta_i, m^{i*}\right] = E(\theta_j)$. Then, re-arranging the squared terms involving $(\nu_1^i - \theta_1)$ and $(\nu_2^i - \theta_2)$, sender i's IC constraint becomes:

$$\begin{aligned} E\left[U^i|\theta_i, \nu_i^{i*}\right] = &- E\Big[(\nu_i^{i*} - \theta_i)^2(w_{1i}^2 + w_{2i}^2) - 2(\nu_i^{i*} - \theta_i)[b_1^i w_{1i} + b_2^i w_{2i}]|\theta_i, m^{i*}\Big] \\ &- E\big[(E(\theta_j) - \theta_j)^2\big](w_{1j}^2 + w_{2j}^2) + (b_1^i)^2 + (b_2^i)^2 \end{aligned} \tag{A1.1}$$

Because of continuity of the state space, for any equilibrium partition there exist boundary types that would be indifferent between announcing two messages. More precisely, denote by a_n^i a generic boundary type for $n = \{0, 1, ..., N\}$. Then, by definition of boundary type:

$$E[U^i(m_n^i) - U^i(m_{n+1}^i)|a_n^i] = 0$$

Now solving the above expression with (A1.1) and noting that:

$$\nu_n^i = \frac{a_{n-1}^i + a_n^i}{2} \qquad \text{and} \qquad \nu_{n+1}^i = \frac{a_{n+1}^i + a_n^i}{2}$$

I get:

$$(a_{n-1}^i - a_{n+1}^i)\left[(w_{1i}^2 + w_{2i}^2)(a_{n+1}^i - 2a_n^i + a_{n-1}^i) - 4(b_1^i w_{1i} + b_2^i w_{2i})\right] = 0$$

Which holds if and only if:

$$a^i_{n+1} = 2a^i_n - a^i_{n-1} + 4\frac{(b^i_1 w_{1i} + b^i_2 w_{2i})}{(w^2_{1i} + w^2_{2i})}$$

The same Arbitrage condition as the uniform-quadratic case in Crawford and Sobel (1982), with the conflict of interest between sender i and the receiver defined by:

$$B^i \equiv \frac{(b^i_1 w_{1i} + b^i_2 w_{2i})}{(w^2_{1i} + w^2_{2i})}$$

□

Senders observe noisy signals about both states

Suppose that the decision-maker holds k^*_r signals about one of the states, for $r = \{1, 2\}$. Let ℓ^*_r denote the number of such signals that equal 1; then the conditional pdf is:

$$f(\ell^*_r|\theta_r, k^*_r) = \frac{k^*_r!}{\ell^*_r!(k^*_r - \ell^*_r)!}\theta_r^{\ell^*_r}(1-\theta_r)^{k^*_r - \ell^*_r}$$

And her posterior is:

$$h(\theta_r|\ell^*_r, k^*_r) = \frac{(k^*_r + 1)!}{\ell^*_r!(k^*_r - \ell^*_r)!}\theta_r^{\ell^*_r}(1-\theta_r)^{k^*_r - \ell^*_r}$$

As a result:

$$E(\theta_r|\ell^*_r, k^*_r) = \frac{(\ell^*_r + 1)}{(k^*_r + 2)} \qquad \text{Var}(\theta_r|\ell^*_r, k^*_r) = \frac{(\ell^*_r + 1)(k^*_r - \ell^*_r + 1)}{(k^*_r + 2)^2(k^*_r + 3)}$$

For $r = \{1, 2\}$.

A generic IC constraint. Recall that $\mathbf{S}^i$ is the vector of signals actually received by sender i and $\mathbf{m}^*$ is the vector of equilibrium message strategies for all senders. Now, let $\mathbf{m}^{i*}$ denote sender i's pure message strategy in equilibrium, and $\hat{\mathbf{m}}^i$ the deviation under consideration. In addition, denote by $y_d(\mathbf{m}^i, \mathbf{m}^{\text{-}i})$ the action the receiver would take in dimension $d = \{1, 2\}$, when she receives $\mathbf{m}^{\text{-}i}$ from players other than i, and i is influential under both $\mathbf{m}^{i*}$ and $\hat{\mathbf{m}}^i$. Given that i takes other senders' equilibrium strategies as given, and in order to simplify notation let $y_d(\mathbf{m}^{i*}(\mathbf{S}^i), \mathbf{m}^{\text{-}i}) = y_d(\mathbf{m}^{i*})$ and $y_d(\hat{\mathbf{m}}^i(\mathbf{S}^i), \mathbf{m}^{\text{-}i}) = y_d(\hat{\mathbf{m}}^i)$.

For $\delta_d = w_{d1}\theta_1 + w_{d2}\theta_2$, then sender i reports $\mathbf{m}^{i*}$ instead of $\hat{\mathbf{m}}^i$ if and only if:

$$-\int_0^1\int_0^1 \Bigg[\Big[\big(y_1(\mathbf{m}^{i*} - \delta_1 - b^i_1)\big)^2 + \big(y_2(\mathbf{m}^{i*} - \delta_2 - b^i_2)\big)^2\Big] - \\ - \Big[\big(y_1(\hat{\mathbf{m}}^i) - \delta_1 - b^i_1\big)^2 + \big(y_2(\hat{\mathbf{m}}^i) - \delta_2 - b^i_2\big)^2\Big]\Bigg] f(\theta_1, \mathbf{m}^{\text{-}i}|\mathbf{S}^i)\, f(\theta_2, \mathbf{m}^{\text{-}i}|\mathbf{S}^i)\, d\theta_1\, d\theta_2 \geq 0$$

Using the identity $a^2 - b^2 = (a+b)(a-b)$, and by noting that $f(\theta_1, \mathbf{m}^{-i}|\mathbf{S}^i) = f(\theta_1|\mathbf{m}^{-i}, S_1^i)\, P(\mathbf{m}^{-i}|S_1^i)$ and that $f(\theta_2, \mathbf{m}^{-i}|\mathbf{S}^i) = f(\theta_2|\mathbf{m}^{-i}, S_2^i)\, P(\mathbf{m}^{-i}|S_2^i)$ I get:

$$
\begin{aligned}
-\int_0^1\int_0^1 \Bigg[& \big[\big(y_1(\mathbf{m}^{i*}) + y_1(\hat{\mathbf{m}}^i)\big) - 2(\delta_1 + b_1^i)\big]\,\big(y_1(\mathbf{m}^{i*}) - y_1(\hat{\mathbf{m}}^i)\big) + \\
& + \big[\big(y_2(\mathbf{m}^{i*}) + y_2(\hat{\mathbf{m}}^i)\big) - 2(\delta_2 + b_2^i)\big]\,\big(y_2(\mathbf{m}^{i*}) - y_2(\hat{\mathbf{m}}^i)\big)\Bigg] \\
& f(\theta_1|\mathbf{m}^{-i}, S_1)\, f(\theta_2|\mathbf{m}^{-i}, S_2)\, P(\mathbf{m}^{-i}|S_1^i)\, P(\mathbf{m}^{-i}|S_2^i)\, d\theta_1\, d\theta_2 \geq 0
\end{aligned}
$$

Note that in equilibrium:

$$
y_1(\mathbf{m}^i, \mathbf{m}^{-i}) = E(\delta_1|\mathbf{m}^i, \mathbf{m}^{-i}) \qquad y_2(\mathbf{m}^i, \mathbf{m}^{-i}) = E(\delta_2|\mathbf{m}^i, \mathbf{m}^{-i})
$$

Now let denote:

$$
\begin{aligned}
\Delta(\delta_1) &= E(\delta_1|\mathbf{m}^{i*}, \mathbf{m}^{-i}) - E(\delta_1|\hat{\mathbf{m}}^i, \mathbf{m}^{-i}) \\
\Delta(\delta_2) &= E(\delta_2|\mathbf{m}^{i*}, \mathbf{m}^{-i}) - E(\delta_2|\hat{\mathbf{m}}^i, \mathbf{m}^{-i})
\end{aligned}
$$

I then get the first expression of the IC constraint:

$$
\begin{aligned}
-\int_0^1\int_0^1 \Bigg[& \Big[\frac{E(\delta_1|\mathbf{m}^{i*}, \mathbf{m}^{-i}) + E(\delta_1|\hat{\mathbf{m}}^i, \mathbf{m}^{-i})}{2} - \delta_1 - b_1^i\Big]\big[\Delta(\delta_1)\big] + \\
& + \Big[\frac{E(\delta_2|\mathbf{m}^{i*}, \mathbf{m}^{-i}) + E(\delta_2|\hat{\mathbf{m}}^i, \mathbf{m}^{-i})}{2} - \delta_2 - b_2^i\Big]\big[\Delta(\delta_2)\big]\Bigg] \\
& f(\theta_1|\mathbf{m}^{-i}, S_1^i)\, f(\theta_2|\mathbf{m}^{-i}, S_2^i)\, P(\mathbf{m}^{-i}|S_1^i)\, P(\mathbf{m}^{-i}|S_2^i)\, d\theta_1\, d\theta_2 \geq 0
\end{aligned} \tag{A1.2}
$$

Given that the equilibrium message strategies for players other than i, $\mathbf{m}^{-i}$, are independent of i's actual signal realizations, the expressions $P(\mathbf{m}^{-i}|S_1^i)$ and $P(\mathbf{m}^{-i}|S_2^i)$ can be taken out the double-integral. Then, noting that:

$$
\begin{aligned}
\int_0^1\int_0^1 \delta_1\, f(\theta_1|S_1, \mathbf{m}^{-i})\, f(\theta_2|S_2, \mathbf{m}^{-i})\, P(\mathbf{m}^{-i}|S_1)\, P(\mathbf{m}^{-i}|S_2)\, d\theta_1\, d\theta_2 &= E\left(\delta_1|S_1, S_2, \mathbf{m}^{-i}\right) \\
&= E\left(w_{11}\,\theta_1 + w_{12}\,\theta_2\big|S_1, S_2, \mathbf{m}^{-i}\right) \\
&= w_{11}E(\theta_1|S_1, \mathbf{m}^{-i}) + w_{12}E(\theta_2|S_2, \mathbf{m}^{-i})
\end{aligned}
$$

and similarly for δ_2. Note that $y_d(\mathbf{m}^i, \mathbf{m}^{-i}) = w_{d1}E(\theta_1|\mathbf{m}^i, \mathbf{m}^{-i}) + w_{d2}E(\theta_2|\mathbf{m}^i, \mathbf{m}^{-i})$ for any $\mathbf{m}^i =$

$\{\mathbf{m}^{i*}, \hat{\mathbf{m}}^i\}$. Let define the receiver's updated beliefs with respect to δ_d from i's perspective as:

$$\nu_d^{i*} = E(\delta_d|\mathbf{m}^{i*}) \qquad \hat{\nu}_d^i = E(\delta_d|\hat{\mathbf{m}}^i) \qquad \nu_d^i = E(\delta_d|\mathbf{S}^i)$$

And let $\nu_{1r}^{i*} = \nu_{2r}^{i*} = E(\theta_r|\mathbf{m}^{i*})$, $\hat{\nu}_{1r}^i = \hat{\nu}_{2r}^i = E(\theta_r|\hat{\mathbf{m}}^i)$, and $\nu_{1r}^i = \nu_{2r}^i = E(\theta_r|S_r^i)$ the posteriors associated to θ_r, respectively. Then, equation (A1.2) becomes:

$$-\left[\left[(\nu_1^{i*} + \hat{\nu}_1^i) - 2(\nu_1^i - b_1^i)\right]\Delta(\delta_1) + \left[(\nu_2^{i*} + \hat{\nu}_2^i) - 2(\nu_2^i - b_2^i)\right]\Delta(\delta_2)\right] P(\mathbf{m}^{-i}|S_1)\, P(\mathbf{m}^{-i}|S_2) \geq 0 \tag{A1.3}$$

The following lemma shows the expressions for the expected influence of each message when the sender is believed with respect to a single signal.

Lemma A1.1. *Suppose sender i is expected to truthfully reveal his information about $S_r^i = \{S_1^i, S_2^i\}$, and let $k_r = \{k_1, k_2\}$ be the other truthful messages associated to the same state that the receiver expects to receive on the equilibrium path. Then the receiver's updated beliefs about $\theta_r = \{\theta_1, \theta_2\}$ is given by.*

$$E\left(\theta_r|S_r^i = 0, \mathbf{m}^{-i}\right) = \frac{(k_r+2)}{2(k_r+3)} \qquad E\left[E\left(\ell_r, \theta_r|S_r^i = 1, \mathbf{m}^{-i}\right)\right] = \frac{(k_r+4)}{2(k_r+3)}$$

Proof.

From equation (A1.2) it is true that:

$$E\left(\delta_d|m^i = \{S_1^i, S_2^i\}, \mathbf{m}^{-i}\right) = w_{d1}E\left(\theta_1|S_1^i, \mathbf{m}^{-i}\right) + w_{d2}E\left(\theta_2|S_2^i, \mathbf{m}^{-i}\right)$$

Working out the expectation for each state conditional on the signal realization gives the following (for $\theta_r = \{\theta_1, \theta_2\}$, and $S_r^i = 0$):

$$E\left(\theta_r|S_r^i = 0, \mathbf{m}^{-i}\right) = \frac{(\ell_r+1)}{(k_r+3)}$$

By the Law of Iterated Expectations, recalling that $f(\ell_r|\theta_r, k_r) = h(\theta_r|\ell_r, k_r)/(k_r+1)$, I get:

$$\begin{aligned} E\left(\theta_r|S_r^i = 0, \mathbf{m}^{-i}\right) &= E\left[E\left(\theta_r|\ell_r, S_r^i = 0, \mathbf{m}^{-i}\right)\right] \\ &= \frac{1}{(k_r+3)} \sum_{\ell_r=0}^{k_r} \frac{(\ell_r+1)}{(k_r+1)} \\ &= \frac{(k_r+2)}{2(k_r+3)} \end{aligned}$$

And now the expectation conditional on $S_r^i = 1$:

$$E\left[E\left(\ell_r, \theta_r | S_r^i = 1, \mathbf{m}^{\text{-}i}\right)\right] = \frac{1}{(k_r+3)} \sum_{\ell_r=0}^{k_r} \frac{(\ell_r+2)}{(k_r+1)}$$
$$= \frac{(k_r+4)}{2(k_r+3)}$$

□

Proof of Lemma 1.3 .

Consider the interim equilibrium in which i is believed to be telling the truth with respect to one signal. The equivalent of equation (A1.3) in terms of the posteriors on states if given by:

$$\sum_{y_d=\{y_1,y_2\}} -\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i) + w_{d2}(\nu_{d2}^{i*} - \hat{\nu}_{d2}^i)\right]\left[w_{d1}(\nu_{d1}^{i*} + \hat{\nu}_{d1}^i - 2\nu_{d1}^i) + w_{d2}(\nu_{d2}^{i*} + \hat{\nu}_{d2}^i - 2\nu_{d2}^i) - 2b_d^i\right] \geq 0 \tag{A1.4}$$

Consider the case in which i reveals S_1^i only. This implies that $\nu_{11}^{i*} = \nu_{21}^{i*} = \nu_1^i$, and $\nu_{12}^{i*} = \nu_{22}^{i*} = 1/2$. The above IC constraint then becomes:

$$\sum_{y_d=\{y_1,y_2\}} -\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i)\right]\left[-w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i) - 2w_{d2}\left(\nu_{d2}^i - \frac{1}{2}\right) - 2b_d^i\right] \geq 0$$

Now I analyse for each possible pair of signal realizations.

1. $\mathbf{S^i} = \{(0,0)\}$, then according to Lemma A1.3:

$$\nu_{d1}^i = \nu_{d1}^{i*} = \frac{(k_1+2)}{2(k_1+3)}; \qquad \hat{\nu}_{d1}^i = \frac{(k_1+4)}{2(k_1+3)}; \text{ and } \qquad \nu_{d2}^i = \nu_{d1}^i$$

   And the IC constraint (A1.4) becomes:

$$\sum_{y_d=\{y_1,y_2\}} -\left[w_{d1}\frac{(-1)}{(k_1+3)}\right]\left[-\frac{w_{d1}}{2}\frac{(-1)}{(k_1+3)} - \frac{w_{d2}}{2}\frac{(-1)}{(k_2+3)} - b_d^i\right] \geq 0$$

   Operating the sum I get:

$$w_{11}b_1^i + w_{12}b_2^i \leq \frac{(w_{11}^2 + w_{12}^2)}{2(k_1+3)} + \frac{(w_{11}w_{12} + w_{21}w_{22})}{2(k_2+3)}$$

Multiplying both sides by $(w_{11}^2 + w_{12}^2)^{-1}$ and defining $C_1 = \frac{(w_{11}w_{12}+w_{21}w_{22})}{(w_{11}^2+w_{12}^2)}$ I get:

$$B_1^i \leq \frac{1}{2}\left[\frac{1}{(k_1+3)} + \frac{C_1}{(k_2+3)}\right]$$

2. $\mathbf{S^i} = \{(1,1)\}$, then according to Lemma A1.3:

$$\nu_{d1}^i = \nu_{d1}^{i*} = \frac{(k_1+4)}{2(k_1+3)}; \qquad \hat{\nu}_{d1}^i = \frac{(k_1+2)}{2(k_1+3)}; \text{ and } \qquad \nu_{d2}^i = \nu_{d1}^i$$

Following the same steps as above it can be shown to lead to:

$$-B_1^i \leq \frac{1}{2}\left[\frac{1}{(k_1+3)} + \frac{C_1}{(k_2+3)}\right]$$

3. $\mathbf{S^i} = \{(1,0)\}$, then according to Lemma A1.3:

$$\nu_{d1}^i = \nu_{d1}^{i*} = \frac{(k_1+4)}{2(k_1+3)} \qquad \hat{\nu}_{d1}^i = \frac{(k_1+2)}{2(k_1+3)} \text{ and } \qquad \nu_{d2}^i = \frac{(k_2+2)}{2(k_2+3)}$$

The IC constraint becomes:

$$\sum_{y_d=\{y_1,y_2\}} -\left[w_{d1}\frac{(1)}{(k_1+3)}\right]\left[\frac{w_{d1}}{2}\frac{(1)}{(k_1+3)} + \frac{w_{d2}}{2}\frac{(-1)}{(k_2+3)} - b_d^i\right] \geq 0$$

Which, following the same steps leads to

$$-B_1^i \leq \frac{1}{2}\left[\frac{1}{(k_1+3)} - \frac{C_1}{(k_2+3)}\right]$$

4. $\mathbf{S^i} = \{(0,1)\}$, then according to Lemma A1.3:

$$\nu_{d1}^i = \nu_{d1}^{i*} = \frac{(k_1+2)}{2(k_1+3)} \qquad \hat{\nu}_{d1}^i = \frac{(k_1+4)}{2(k_1+3)} \text{ and } \qquad \nu_{d2}^i = \frac{(k_2+4)}{2(k_2+3)}$$

Which, following the same steps leads to

$$B_1^i \leq \frac{1}{2}\left[\frac{1}{(k_1+3)} - \frac{C_1}{(k_2+3)}\right]$$

Note that the absolute values for left-hand sides of the above IC constraints are equal; hence, the expression with the minimum value on the right-hand side constitutes a necessary and sufficient condition for truthful revelation of S_1^i. The proof for truthful revelation of S_2^i is equivalent. □

PROOF OF LEMMA 1.4.

Here I derive the IC constraint for truthful revelation of both signals, against the deviation of lying on both of them simultaneously. In other words, if i is believed to reveal both signals on path and he observes $\mathbf{S}^i = (0,0)$ he must not find profitable to announce $\mathbf{m}^i = (1,1)$. Note that, when i reveals all his information, then $\nu^{i*}_{dr} = \nu^i_{dr}$ for both decisions and states —$y_d = \{y_1, y_2\}$ and $\theta_r = \{\theta_1, \theta_2\}$. Then, equation (A1.3) reduces to:

$$-\left[\left[-\frac{\Delta(\delta_1)}{2} - b^i_1\right]\Delta(\delta_1) + \left[-\frac{\Delta(\delta_2)}{2} - b^i_2\right]\Delta(\delta_2)\right] P(\mathbf{m}^{-i}|S_1)\, P(\mathbf{m}^{-i}|S_2) \geq 0$$

Let $\beta^i_r = w_{1r}b^i_1 + w_{2r}b^i_2$ ($\beta^i_r = B^i_r\,(w^2_{1r} + w^2_{2r})$), then:

(a). For $\mathbf{S}^i = (0,0)$ not announcing $\mathbf{m}^i = (1,1)$:

$$\frac{\beta^i_1}{(k_1+3)} + \frac{\beta^i_2}{(k_2+3)} \leq \frac{1}{2}\left[\frac{(w^2_{11}+w^2_{21})}{(k_1+3)^2} + \frac{(w^2_{12}+w^2_{22})}{(k_2+3)^2} + \frac{2[w_{11}w_{12}+w_{21}w_{22}]}{(k_1+3)(k_2+3)}\right]$$

(b). For $\mathbf{S}^i = (0,1)$ not announcing $\mathbf{m}^i = (1,0)$:

$$\frac{\beta^i_1}{(k_1+3)} - \frac{\beta^i_2}{(k_2+3)} \leq \frac{1}{2}\left[\frac{(w^2_{11}+w^2_{21})}{(k_1+3)^2} + \frac{(w^2_{12}+w^2_{22})}{(k_2+3)^2} - \frac{2[w_{11}w_{12}+w_{21}w_{22}]}{(k_1+3)(k_2+3)}\right]$$

No deviation for $\mathbf{S}^i = (1,1)$ requires a condition similar to (a) with the signs on the left-hand side changed, while no deviation for $\mathbf{S}^i = (1,0)$ requires the same as (b) with the corresponding change in signs. □

PROOF OF LEMMA 1.5.

When sender i's equilibrium message strategy includes full revelation of some signals realization and babbling for others, any information transmission must be incentive compatible against the deviation to the babbling message (and vice-versa). Then, by Lemma A1.1 the expected marginal influence against the deviation to babbling, for each possible combination of signals realizations is given by:

$$\Delta(\delta_d)_{(0,0)} = -\frac{w_{d1}}{2(k_1+3)} - \frac{w_{d2}}{2(k_2+3)} \qquad \Delta(\delta_d)_{(1,1)} = \frac{w_{d1}}{2(k_1+3)} + \frac{w_{d2}}{2(k_2+3)}$$
$$\Delta(\delta_d)_{(0,1)} = -\frac{w_{d1}}{2(k_1+3)} + \frac{w_{d2}}{2(k_2+3)} \qquad \Delta(\delta_d)_{(1,0)} = \frac{w_{d1}}{2(k_1+3)} - \frac{w_{d2}}{2(k_2+3)}$$

Plugging each of these expression into (A1.3) and working each of them out as previously:

(a). For $\mathbf{S}^i = (0,0)$:

$$\frac{\beta^i_1}{(k_1+3)} + \frac{\beta^i_2}{(k_2+3)} \leq \frac{1}{4}\left[\frac{(w^2_{11}+w^2_{21})}{(k_1+3)^2} + \frac{(w^2_{12}+w^2_{22})}{(k_2+3)^2} + \frac{2[w_{11}w_{12}+w_{21}w_{22}]}{(k_1+3)(k_2+3)}\right]$$

(b). For $\mathbf{S}^i = (0,1)$:

$$\frac{\beta_1^i}{(k_1+3)} - \frac{\beta_2^i}{(k_2+3)} \leq \frac{1}{4}\left[\frac{(w_{11}^2+w_{21}^2)}{(k_1+3)^2} + \frac{(w_{12}^2+w_{22}^2)}{(k_2+3)^2} - \frac{2[w_{11}w_{12}+w_{21}w_{22}]}{(k_1+3)(k_2+3)}\right]$$

And similarly for $\mathbf{S}^i = (1,1)$ —part (a)— and $\mathbf{S}^i = (1,0)$ —part (b). Each of the above corresponds to the incentives for the *influential type* to play his equilibrium strategy. Now, for the *babbling types* in each case, the expected influence of no announcing the influential type will be the opposite to that of the corresponding influential type; that is:

$$\Delta(\delta_d)_{\text{babbling}} = -\Delta(\delta_d)_{(S_1^i, S_2^i)}$$

Which can be proved to lead to the same IC above. Finally, pairing the corresponding IC constraints in each case leads to the corresponding conditions. □

Characterization of the Most Informative Equilibrium

Proposition A1.1. *The strategy profile $(\mathbf{y}^*, \mathbf{m}^*)$ constitutes the receiver-optimal equilibrium when for every possible message strategy from sender i to the decision-maker, then i:*

1. ***Reveals both signals** if and only if condition (1.2) holds for both signals, and (1.4) hold. The message strategy in this case is given by:*
   $\mathbf{m}^i = \Big\{\{(0,0)\};\{(0,1)\};\{(1,0)\};\{(1,1)\}\Big\}.$
2. ***Reveals S_r only:***
   (a) if (1.1) holds for S_r^i but not for $S_{\tilde{r}}^i$
   (b) if (1.1) and (1.5) hold together, then:
   $$\frac{[(w_{11})^2+(w_{21})^2]}{(k_1+2)(k_1+3)} > \frac{[(w_{12})^2+(w_{22})^2]}{(k_2+2)(k_2+3)} \tag{A1.5}$$
3. ***Reveals both signals when is of type $(0,0)$ or $(1,1)$ and no information otherwise:***
   $\mathbf{m}^i = \Big\{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\Big\}$
   (a) if condition (1.5) holds but (1.1) does not; or
   (b) if (1.5) and (1.1) hold, and condition (A1.5) does not.
4. ***Reveals both signals when is of type $(0,1)$ and $(1,0)$ and no information otherwise** if and only if condition (1.6) holds and (1.1) does not. The associated message strategy is:*
   $\mathbf{m}^i = \Big\{\{(0,0);(1,1)\};\{(0,1)\};\{(1,0)\}\Big\}.$

5. ***Reveals no information*** *(babbling strategy) if and only if none of the previous applies. That is:* $\mathbf{m}^i = \Big\{\{(0,0);(1,1);(0,1);(1,0)\}\Big\}$.

PROOF OF PROPOSITION A1.1.

This proof consists on two steps: in the first I construct the equilibrium for each of the message strategies in Proposition 1.2, while the second shows that each message strategy constitutes the receiver-optimal equilibrium for the set of preferences it applies.

Equilibrium Construction

Part 1. Sender i revealing both signals truthfully constitutes the *Fully Separating* equilibrium. Thus, for b_1^i and b_2^i satisfying (1.2) for both signals, (1.3), and (1.4), then the following system of beliefs:

$$\mu^*\big((0,0)|m^i=\{(0,0)\}\big)=1 \qquad \mu^*\big((1,0)|m^i=\{(1,0)\}\big)=1$$
$$\mu^*\big((0,1)|m^i=\{(0,1)\}\big)=1 \qquad \mu^*\big((1,1)|m^i=\{(1,1)\}\big)=1$$

And the following equilibrium actions:

$$y_d^*\left(m^i=\{(0,0)\},\mathbf{m}^{\text{-}i}\right)=w_{d1}\frac{(\ell_1+1)}{(k_1+3)}+w_{d2}\frac{(\ell_2+1)}{(k_2+3)} \qquad y_d^*\left(m^i=\{(1,0)\},\mathbf{m}^{\text{-}i}\right)=w_{d1}\frac{(\ell_1+2)}{(k_1+3)}+w_{d2}\frac{(\ell_2+1)}{(k_2+3)}$$
$$y_d^*\left(m^i=\{(0,1)\},\mathbf{m}^{\text{-}i}\right)=w_{d1}\frac{(\ell_1+1)}{(k_1+3)}+w_{d2}\frac{(\ell_2+2)}{(k_2+3)} \qquad y_d^*\left(m^i=\{(1,1)\},\mathbf{m}^{\text{-}i}\right)=w_{d1}\frac{(\ell_1+2)}{(k_1+3)}+w_{d2}\frac{(\ell_2+2)}{(k_2+3)}$$

For $d=\{1,2\}$.

Are consistent with sender i revealing both signals truthfully, that is $\mathbf{m}^{i*}=\{(0,0);(0,1);(1,0);(1,1)\}$

Part 2. For sender i's biases that satisfy (1.1) for S_1^i only, consider the following equilibrium beliefs for the receiver:

$$\mu^*\big((0,0)|\mathbf{m}^i=(0,0)\big)=\mu^*\big((0,1)|\mathbf{m}^i=(0,0)\big)=\frac{1}{2} \qquad \mu^*\big((0,1)|\mathbf{m}^i=(0,1)\big)=\mu^*\big((0,0)|\mathbf{m}^i=(0,1)\big)=\frac{1}{2}$$
$$\mu^*\big((1,0)|\mathbf{m}^i=(1,0)\big)=\mu^*\big((1,1)|\mathbf{m}^i=(1,0)\big)=\frac{1}{2} \qquad \mu^*\big((1,1)|\mathbf{m}^i=(1,1)\big)=\mu^*\big((1,0)|\mathbf{m}^i=(1,1)\big)=\frac{1}{2}$$

The above beliefs mean that upon hearing any message the receiver is certain that i reveals the realization of S_1^i truthfully but not that for S_2^i. The receiver's optimal actions being the following:

$$y_d^*\left(m^i=(0,0),\mathbf{m}^{\text{-}i}\right)=y^*\left(m^i=(0,1),\mathbf{m}^{\text{-}i}\right)=w_{d1}\frac{(\ell_1+1)}{(k_1+3)}+w_{d2}\frac{(\ell_2+1)}{(k_2+2)}$$
$$y_d^*\left(m^i=(1,0),\mathbf{m}^{\text{-}i}\right)=y^*\left(m^i=(1,1),\mathbf{m}^{\text{-}i}\right)=w_{d1}\frac{(\ell_1+2)}{(k_1+3)}+w_{d2}\frac{(\ell_2+1)}{(k_2+2)}$$

For $d=\{1,2\}$.

When $\mathbf{b}^i$ satisfies (1.1) o but not (1.3) the above system of beliefs is consistent with i optimally revealing the realization of S_1^i only, that is $\mathbf{m}^{i*}=\big\{\{(0,0),(0,1)\};\{(1,0),(1,1)\}\big\}$, and i being influential trough S_1^i

only.

Part 3. The case of i revealing the realization of S_2^i only is equivalent to the previous and, thus, omitted.

Part 4. Note that when (1.5) but not (1.1), the only incentives to lie for sender i happens between types $\mathbf{S}^i = (0,1)$ and $\mathbf{S}^i = (1,0)$. In this equilibrium types $\mathbf{S}^i = (0,1)$ and $\mathbf{S}^i = (1,0)$ play the corresponding babbling strategy, while the other types are revealing truthfully both signals.

Let consider the following equilibrium beliefs for the receiver upon having seen sender i's message $\mathbf{m}^i$, given that i's bias satisfy condition (1.5) but not (1.1).

$$\mu^*\left((0,0)|\mathbf{m}^i = (0,0)\right) = 1 \qquad \mu^*\left((1,1)|\mathbf{m}^i = (1,1)\right) = 1$$
$$\mu^*\left((0,1)|\mathbf{m}^i = (0,1)\right) = \mu^*\left((1,0)|\mathbf{m}^i = (0,1)\right) = \frac{1}{2} \qquad \mu^*\left((1,0)|\mathbf{m}^i = (1,0)\right) = \mu^*\left((0,1)|\mathbf{m}^i = (1,0)\right) = \frac{1}{2}$$

And the optimal actions by the receiver:

$$\begin{aligned}
y_d^*\left(m^i = (0,0), \mathbf{m}^{-i}\right) &= w_{d1}\frac{(\ell_1+1)}{(k_1+3)} + w_{d2}\frac{(\ell_2+1)}{(k_2+3)}\\
y_d^*\left(m^i = (1,1), \mathbf{m}^{-i}\right) &= w_{d1}\frac{(\ell_1+2)}{(k_1+3)} + w_{d2}\frac{(\ell_2+2)}{(k_2+3)}\\
y_d^*\left(m^i = (1,0), \mathbf{m}^{-i}\right) &= y_d^*\left(m^i = (0,1), \mathbf{m}^{-i}\right) = w_{d1}\frac{(\ell_1+1)}{(k_1+2)} + w_{d2}\frac{(\ell_2+1)}{(k_2+2)}
\end{aligned}$$

For $d = \{1,2\}$. The above means that upon hearing either $\mathbf{m}^i = (0,0)$ or $\mathbf{m}^i = (1,1)$ the receiver updates her estimation of both θ_1 and θ_2, but she does not update any of them otherwise. From the sender's perspective, types $\mathbf{S}^i = (0,1)$ and $\mathbf{S}^i = (1,0)$ should prefer an equilibrium babbling strategy (mixing between these two messages) rather than announcing any of the influential messages.

The lack of incentives for types $\mathbf{S}^i = (0,0)$ and $\mathbf{S}^i = (1,1)$ to pool to each other given both of them are influential is given by condition (1.3), which is implied by (1.5).

Now, incentives to reveal both signals given types $\mathbf{S}^i = (0,1)$ and $\mathbf{S}^i = (1,0)$ are babbling are given by Lemma 1.5. It is worth noting that condition (1.5) guarantees type $\mathbf{S}^i = (0,0)$ as well as $\mathbf{S}^i = (1,1)$ has no incentives to play the babbling strategy when he is believed about both signals if announces truthfully. At the same time, these conditions guarantee types $\mathbf{S}^i = (0,1)$ and $\mathbf{S}^i = (1,0)$ prefers to play the babbling strategy available rather than announcing any of the influential messages.

Part 5. In this equilibrium types $\mathbf{S}^i = (0,0)$ and $\mathbf{S}^i = (1,1)$ send the corresponding babbling messages, while the other types are revealing truthfully both signals. The proof is similar to that in Part 4, adapting the equilibrium beliefs to the messages and types that are playing each strategy.

Part 6. The babbling equilibrium is always part of the available equilibria in any cheap talk game. In Part 6 of the following section I show that for the set of preferences that do not satisfy any of the conditions above, the unique equilibrium available is the babbling.

Most Informative Equilibrium

Part 1. Trivially, the sender is fully revealing his information.

Part 2. Any strategy that is more informative than revealing only one signal has at least one type fully revealing in equilibrium (and the others revealing partially). Consider the following message strategies that would improve upon revealing S_1^i only:

- $\tilde{\mathbf{m}}^i = \{\{(0,0)\};\{(0,1)\};\{(1,0);(1,1)\}\}$
- $\hat{\mathbf{m}}^i = \{\{(0,0);(0,1)\};\{(1,0)\};\{(1,1)\}\}$

(a) Condition (1.1) holds for S_1^i only. For $\tilde{\mathbf{m}}^i$ to be an equilibrium, types $\mathbf{S}^i = (0,0)$ and $\mathbf{S}^i = (0,1)$ must have incentives to separate from each other, which requires condition (1.1) to hold for S_2^i, leading to a contradiction. The same applies to $\hat{\mathbf{m}}^i$ for separation between types $\mathbf{S}^i = (1,0)$ and $\mathbf{S}^i = (1,1)$.

(b) Condition (1.1) hold for both signals. The set of preferences for which both $\tilde{\mathbf{m}}^i$ and $\hat{\mathbf{m}}^i$ can be sustained in equilibrium are the same, leading to multiple equilibria. Moreover, both $\tilde{\mathbf{m}}^i$ and $\hat{\mathbf{m}}^i$ improve the receiver's ex-ante payoff[25] in exactly the same way|i.e. by obtaining information about S_2^i when sender is one of the fully revealing types, which occurs with probability $1/2$; thus, receiver ex-ante utility fails to select among these equilibrium message strategies. Finally, the implementation of any $\tilde{\mathbf{m}}^i$ or $\hat{\mathbf{m}}^i$ depends upon the pooling types playing mixed strategies and, thus, is restricted to all possible message strategies being played with positive probability *on path*. In other words, there are no out-of-equilibrium strategies/beliefs that sustain any of these strategies|i.e. neologism-proofness (Farrell, 1993) fails.

Part 3. Any strategy that is more informative than revealing only S_2^i must be of the form:

- $\tilde{\mathbf{m}}^i = \{\{(0,0)\};\{(1,0)\};\{(0,1);(1,1)\}\}$
- $\hat{\mathbf{m}}^i = \{\{(0,0);(1,0)\};\{(0,1)\};\{(1,1)\}\}$

Then, the same arguments as in Part 2.(a) and 2.(b) apply.

Part 4. Given the equilibrium message strategy $\mathbf{m}^i = \{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\}$ then a more informative message strategy would necessarily involve at least one more type revealing both signals, since revelation of one signal is not available due to pooling types not sharing any single realization —i.e. $(0,1)$ and $(1,0)$. As a consequence, the only message strategy that is more informative than $\mathbf{m}^i$ is the fully separating one, for which condition (1.4) must hold; a contradiction.

[25] With respect to revealing S_1^i only.

Part 5. Given the equilibrium message strategy $\mathbf{m}^i = \{\{(0,0);(1,1)\};\{(0,1)\};\{(1,0)\}\}$ then a more informative message strategy would involve at least one more type revealing both signals, since revelation of one signal is not available due to pooling types not sharing any single realization —i.e. $(0,0)$ and $(1,1)$. As a consequence, the only message strategy that is more informative than $\mathbf{m}^i$ is the fully separating one, for which condition (1.4) must hold; a contradiction.

Part 6. The equilibrium message strategy is given by $\mathbf{m}^i = \{(0,0);(0,1);(1,0);(1,1)\}$.

Let first consider the partition in which a single type fully separates. Lemma 1.5 implies that for types $(0,0)$ and $(1,1)$ (individually) the corresponding condition leads to the equilibrium in Part 4, which rules out this possibility. The same argument applies to the cases in which either types $(0,1)$ or $(1,0)$, the receiver-optimal equilibrium for the set of preferences satisfying the corresponding condition would involve the following message strategy $\mathbf{m}^i = \{\{(0,0);(0,1)\};\{(1,0)\};\{(1,1)\}\}$.

Secondly, consider the cases in which one type with coincident signals and another with non-coincident signals separate and the others play babbling. By Lemma 1.5 whenever type $(0,0)$ separates, type $(1,1)$ has incentives to do so and the deviation is profitable for the receiver; while type $(0,1)$ has incentives to separate if and only if $(1,0)$ has.

Finally, I must rule out the message strategy in which only one realization of a given signal is revealed —i.e. the equilibrium message strategies being either $\mathbf{m}^i = \{\{(0,0);(0,1)\};\{(1,0);(1,1)\}\}$ or $\mathbf{m}^i = \{\{(0,0);(1,0)\};\{(0,1);(1,1)\}\}$, with only one message being influential in each. The argument is simple: if, for instance, sender i were willing to reveal only $S_1^i = 1$; then the receiver would infer $S_1^i = 0$ when i plays the babbling strategy, which implies the equilibrium in Part 1. □

PROOF OF PROPOSITION 1.3 AND LEMMA 1.6

Both Proposition 1.3 and Lemma 1.6 are corollaries of the following result.

Proposition A1.2. *Let $1/2 \leq w \leq 1$. How informational interdependence affects incentives for communication depend on two effects: the aggregate conflict of interest and the credibility loss; as follows.*

Incentives to reveal one signal only:

$$\frac{\partial IC_{(1.1)}}{\partial w} = -\frac{1}{(k_{\tilde{r}}+3)}\frac{\partial C_r}{\partial w} - 2\frac{\partial |B_r^i|}{\partial w} \tag{A1.6}$$

Incentives to fully reveal $(S_1^i, S_2^i) = \{(0,0);(1,1)\}$:

$$\frac{\partial IC_{(1.3)}}{\partial w} = \frac{2}{(k_1+3)(k_2+3)}\left[\frac{\partial C_r}{\partial w} \pm (k_2+3)\frac{\partial B_1^i}{\partial w} \pm (k_1+3)\frac{\partial B_2^i}{\partial w}\right] \tag{A1.7}$$

Incentives to fully reveal $(S_1^i, S_2^i) = \{(0,1);(1,0)\}$:

$$\frac{\partial IC_{(1.4)}}{\partial w} = \frac{2}{(k_1+3)(k_2+3)}\left[-\frac{\partial C_r}{\partial w} \pm (k_2+3)\frac{\partial B_1^i}{\partial w} \mp (k_1+3)\frac{\partial B_2^i}{\partial w}\right] \tag{A1.8}$$

for $\theta_{\tilde{r}} \neq \theta_r$

Proof. Let $w_{11} = w_{22} = w$; then, note that:

$$\begin{aligned}\frac{\partial B_1^i}{\partial w} &= \frac{b_1^i - b_2^i}{w^2 + (1-w)^2} - \frac{[2w - 2(1-w)][wb_1^i + (1-w)b_2^i]}{[w^2 + (1-w)^2]^2} \\ &= \frac{b_1^i - b_2^i}{w^2 + (1-w)^2} - \frac{2(2w-1)}{w^2 + (1-w)^2} B_1^i\end{aligned}$$

By the rule of derivative of absolute value functions, $\frac{\partial |y(x)|}{\partial x} = \frac{\partial y(x)}{\partial x}\frac{y(x)}{|y(x)|}$, and noting that $|B_1^i| = \frac{|wb_1^i + (1-w)b_2^i|}{w^2+(1-w)^2}$, the above expression yields:

$$\frac{\partial |B_1^i|}{\partial w} = \text{sign}(B_1^i) \times \frac{(b_1^i - b_2^i)}{[w^2 + (1-w)^2]} + |B_1^i| \times \frac{2(1-2w)}{[w^2 + (1-w)^2]}$$

For higher interdependence to reduce the aggregate conflict of interest, I need $\frac{\partial |B_1^i|}{\partial w} > 0$.[26] The second term of the above equation is clearly negative; so, for the derivate to be positive a necessary condition is that the first term is positive as well. I then need that $\text{sign}(B_1^i) = \text{sign}(b_1^i - b_2^i)$, which depends on the signs of the decision-specific biases as follows.

1. if $\text{sign}(b_1^i) = \text{sign}(b_2^i)$, then $\text{sign}(B_1^i) = \text{sign}(b_1^i)$ and the necessary condition above depends on $\text{sign}(b_1^i - b_2^i)$. It is straightforward to check that the necessary condition is satisfied when $|b_1^i| > |b_2^i|$.
2. if $\text{sign}(b_1^i) \neq \text{sign}(b_2^i)$, then $\text{sign}(b_1^i - b_2^i) = \text{sign}(b_1^i)$ and the necessary condition above depends on $\text{sign}(B_1^i)$. Again, it is straightforward to notice that $\text{sign}(B_1^i) = \text{sign}(b_1^i)$ requires that $w|b_1^i > (1-w)b_2^i$.

Now, let proceed to compute the credibility loss, given by $\frac{\partial C_r}{\partial w}$. Note that $w_{11} = w_{22}$ implies that $C_1 = C_2 = \frac{2w(1-w)}{1-2w(1-w)}$; for which, after some algebra I get:

$$\frac{\partial C_r}{\partial w} = \frac{2(1-2w)}{[1-2w(1-w)]^2} \leq 0$$

□

Proof of Proposition 1.4. For $k_1' > k_1$, the statement in Proposition 1.4 is equivalent to preferences

[26]Recall that highe w means less informational interdependence.

that satisfy the following conditions:

$$|\beta_1^i| \le \frac{w^2 + (1-w)^2}{2\,(k_1' + 3)} \tag{A1.9}$$

$$|\beta_2^i| \le \frac{w^2 + (1-w)^2}{2\,(k_2 + 3)} \tag{A1.10}$$

$$\left|\frac{\beta_1^i}{(k_1'+3)} - \frac{\beta_2^i}{(k_2+3)}\right| \le \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right] \tag{A1.11}$$

$$\left|\frac{\beta_1^i}{(k_1+3)} - \frac{\beta_2^i}{(k_2+3)}\right| > \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right] \tag{A1.12}$$

The proof proceeds as follow: I first find β_1^i and β_2^i for which both (A1.11) and (A1.12) hold, and then derive the conditions under which they satisfy (A1.9) and (A1.10). First note that LHS of both (A1.11) and (A1.12) depend on the signs of β_1^i and β_2^i:

1. If sign$[\beta_1^i] \neq$ sign$[\beta_2^i]$, then

$$\left|\frac{\beta_1^i}{(k_1'+3)} - \frac{\beta_2^i}{(k_2+3)}\right| = \frac{|\beta_1^i|}{(k_1'+3)} + \frac{|\beta_2^i|}{(k_2+3)}$$

2. If sign$[\beta_1^i] =$ sign$[\beta_2^i]$, then

$$\left|\frac{\beta_1^i}{(k_1'+3)} - \frac{\beta_2^i}{(k_2+3)}\right| = \left|\frac{|\beta_1^i|}{(k_1'+3)} - \frac{|\beta_2^i|}{(k_2+3)}\right|$$

The signs of β_1^i and β_2^i are determined by both the sign and the relative magnitude of b_1^i and b_2^i. In the first of the above, biases go in opposite directions and are relatively similar in magnitude according to the interdependence: $\frac{(1-w)}{w} < \frac{|b_1^i|}{|b_2^i|} < \frac{w}{(1-w)}$. For sign$[\beta_1^i] =$ sign$[\beta_2^i]$ arises either when sign$[b_1^i] =$ sign$[b_2^i]$, or sign$[b_1^i] \neq$ sign$[b_2^i]$ but one of them being sufficiently larger in magnitude. For the sake of simplicity, assume the increase in the receiver's information on θ_1 is minimal —i.e. $k_1' = k_1 + 1$.

Case 1: sign$[\beta_1^i] \neq$ sign$[\beta_2^i]$. Then, (A1.11) becomes

$$\frac{|\beta_1^i|}{(k_1'+3)} + \frac{|\beta_2^i|}{(k_2+3)} \le \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right]$$

And (A1.12):

$$\frac{|\beta_1^i|}{(k_1+3)} + \frac{|\beta_2^i|}{(k_2+3)} > \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right]$$

The biases for which increasing information in θ_1 improves communication must satisfy:

$$\frac{(k_1+3)}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2}+\frac{w^2+(1-w)^2}{(k_2+3)^2}-\frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right]-\frac{|\beta_2^i|\,(k_1+3)}{(k_2+3)}<|\beta_1^i|\leq$$
$$\leq\frac{(k_1'+3)}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2}+\frac{w^2+(1-w)^2}{(k_2+3)^2}-\frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right]-\frac{|\beta_2^i|\,(k_1'+3)}{(k_2+3)}$$

and

$$\frac{(k_2+3)}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2}+\frac{w^2+(1-w)^2}{(k_2+3)^2}-\frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right]-\frac{|\beta_1^i|\,(k_2+3)}{(k_1+3)}<|\beta_2^i|\leq$$
$$\leq\frac{(k_2+3)}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2}+\frac{w^2+(1-w)^2}{(k_2+3)^2}-\frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right]-\frac{|\beta_1^i|\,(k_2+3)}{(k_1'+3)}$$

Then, the necessary conditions for each of the above to hold are, respectively:

$$|\beta_2^i|\leq\frac{[w^2+(1-w)^2]}{2\,(k_2+3)}-\frac{[w^2+(1-w)^2]\,(k_2+3)}{2\,(k_1+3)(k_1+4)} \tag{A1.13}$$

and

$$|\beta_1^i|>\frac{[w^2+(1-w)^2]}{2}\left[\frac{1}{(k_1+3)}+\frac{1}{(k_1+4)}\right]-\frac{2\,w(1-w)}{(k_2+3)} \tag{A1.14}$$

Note that condition (A1.13) implies condition (A1.10). Then, for it to hold it must be that the RHS is greater than zero, which occurs if:

$$(k_1+3)(k_1+4)>(k_2+3)^2$$

Now, condition (A1.14) holds together with (A1.9) only if the value of $|\beta_1^i|$ is in between two magnitudes: RHS(A1.14)$<|\beta_1^i|\leq$ RHS(A1.9); which holds if:

$$\frac{(k_2+3)}{(k_1+3)}<\frac{4\,w\,(1-w)}{[w^2+(1-w)^2]}$$

It is worth noting that both conditions above require k_1 to be sufficiently large with respect to k_2, which has the interpretation of *saturation*. After the increase in k_1 takes place, sender i is willing to reveal all his information because he will move y_1 very little and y_2 (where preferences are aligned) very much.

Case 2: sign$[\beta_1^i]$ = **sign**$[\beta_2^i]$. I consider two cases. First, the conflict of interest in the first dimension is so large that it outweighs that of the second in terms of incentives (even in the case in which they go in opposite directions across dimensions). Secondly, I analyse the case in which the bias in the second dimension outweighs that of the first.

a. $\frac{|\beta_1^i|}{(k_1'+3)}>\frac{|\beta_2^i|}{(k_2+3)}$

Then, (A1.11) becomes

$$\frac{|\beta_1^i|}{(k_1'+3)} - \frac{|\beta_2^i|}{(k_2+3)} \leq \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right]$$

And (A1.12) becomes

$$\frac{|\beta_1^i|}{(k_1+3)} - \frac{|\beta_2^i|}{(k_2+3)} > \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right]$$

Getting restrictions over $|\beta_1^i|$ and $|\beta_2^i|$ the necessary conditions are:

$$|\beta_2^i| > \frac{[w^2+(1-w)^2]}{2\,(k_2+3)} - \frac{[w^2+(1-w)^2]\,(k_2+3)}{2\,(k_1+3)(k_1+4)} \tag{A1.15}$$

and

$$|\beta_1^i| > \frac{[w^2+(1-w)^2]}{2}\left[\frac{1}{(k_1+3)} + \frac{1}{(k_1'+3)}\right] - \frac{4\,w(1-w)}{(k_2+3)} \tag{A1.16}$$

In this case both (A1.9) and (A1.10) will be upper bounds for $|\beta_1^i|$ and $|\beta_2^i|$, such that (A1.15) holds if:

$$(k_1+3)(k_1+4) > (k_2+3)^2$$

While (A1.16) holds if:

$$\frac{(k_2+3)}{(k_1+3)} < \frac{4\,w\,(1-w)}{[w^2+(1-w)^2]}$$

Note that the conditions are the same as the previous case. The differences are on the restrictions over b_1^i and b_2^i: sign$[\beta_1^i]$ = sign$[\beta_2^i]$ implies that when sign$[b_1^i] \neq$ sign$[b_2^i]$ then $(1-w)|b_1^i| > g|b_2^i|$; while in the previous case both conflict of interest had to be of similar magnitude.[27]

In summary, when biases go in opposite directions across dimensions, "beneficial congestion" occur when the magnitude of the conflict in the first dimension is sufficiently large with respect to that on the second dimension.

b. $\frac{|\beta_1^i|}{(k_1+3)} < \frac{|\beta_2^i|}{(k_2+3)}$

Then, (A1.11) becomes

$$\frac{|\beta_2^i|}{(k_2+3)} - \frac{|\beta_1^i|}{(k_1'+3)} \leq \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right]$$

[27] More precisely, when sign$[\beta_1^i] \neq$ sign$[\beta_2^i]$, $(1-w)|b_2^i| < |b_1^i| < w|b_2^i|$.

And (A1.12) becomes

$$\frac{|\beta_2^i|}{(k_2+3)} - \frac{|\beta_1^i|}{(k_1+3)} > \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right]$$

Getting restrictions over $|\beta_1^i|$ and $|\beta_2^i|$ the necessary conditions are:

$$|\beta_2^i| < \frac{[w^2+(1-w)^2]}{2\,(k_2+3)} - \frac{[w^2+(1-w)^2]\,(k_2+3)}{2\,(k_1+3)(k_1+4)} \tag{A1.17}$$

and

$$|\beta_1^i| < \frac{2\,w(1-w)}{(k_2+3)} - \frac{[w^2+(1-w)^2]}{2}\left[\frac{1}{(k_1+3)} + \frac{1}{(k_1+4)}\right] \tag{A1.18}$$

Given that (A1.17) implies (A1.10), it is only necessary to check when its RHS is greater than zero. As before, this happens for:

$$(k_1+3)(k_1+4) > (k_2+3)^2$$

Similarly, (A1.18) is greater than zero when:

$$(k_2+3)\left[\frac{1}{(k_1+3)} + \frac{1}{(k_1+4)}\right] < \frac{4\,w\,(1-w)}{[w^2+(1-w)^2]}$$

Note that the last condition is more restrictive than those of the previous cases. Hence, it is a sufficient condition for the existence of positive congestion effects for a large range of biases.

Moreover, it can be noted that when $w=1$ the above condition cannot be satisfied.[28] □

B1 Equilibrium Selection

Proof. The ex-ante expected utility for a generic player in equilibrium $(\mathbf{y},\mathbf{m})$ is given by:

$$\begin{aligned} E\left[U^i(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right] &= -E\left[(y_1-\delta_1-b_1^i)^2\right] - E\left[(y_2-\delta_2-b_2^i)^2\right] \\ &= -\sum_{d=\{1,2\}} E\left[(y_d-\delta_d)^2 - 2(y_d-\delta_d)b_d^i + \left(b_d^i\right)^2;\mathbf{m}\right] \end{aligned}$$

[28] I am leaving aside the case of $\frac{|\beta_1^i|}{(k_1'+3)} < \frac{|\beta_2^i|}{(k_2+3)} < \frac{|\beta_1^i|}{(k_1+3)}$, since the conditions above guarantee the beneficial effect of increasing the receiver's information for some range of biases.

Which, by definitions of y_d andδ_d yield:

$$E\left[U^i(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right] = -\sum_{d=\{1,2\}}\left(b_d^i\right)^2 + E\left[\Big(w_{d1}\big(E(\theta_1|\mathbf{m})-\theta_1\big) + w_{d2}\big(E(\theta_2|\mathbf{m})-\theta_2\big)\Big)^2\right] + E\left[E(\delta_d|\mathbf{m})-\delta_d\right]b_d^i$$

With some rearranging and given the last term equals zero, I have:

$$\begin{aligned} E\left[U^i(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right] = -\left[\left(b_1^i\right)^2+\left(b_2^i\right)^2\right] &- \left[(w_{11})^2+(w_{21})^2\right]E\left[\big(E(\theta_1|\mathbf{m})-\theta_1\big)^2;\mathbf{m}\right] \\ &- \left[(w_{12})^2+(w_{22})^2\right]E\left[\big(E(\theta_2|\mathbf{m})-\theta_2\big)^2;\mathbf{m}\right] \end{aligned} \tag{B1.19}$$

Now, following Galeotti et al. (2013) and Dewan and Squintani (2015), the expectation of the squared deviation for each state is given by:

$$\begin{aligned} E\left[\big(E(\theta_r|\mathbf{m})-\theta_r\big)^2;\mathbf{m}\right] &= \int_0^1 \sum_{\ell_r=0}^{k_r+1}\Big(E(\theta_r|\mathbf{m})-\theta_r\Big)^2 f(\ell_r|k_r,\theta_r)\,d\theta_r \\ &= \int_0^1 \sum_{\ell_r=0}^{k_r+1}\Big(E(\theta_r|\mathbf{m})-\theta_r\Big)^2 \frac{h(\theta_r|\ell_r,k_r)}{(k_r+2)}\,d\theta_r \\ &= \frac{1}{(k_r+2)}\sum_{\ell_r=0}^{k_r+1}\int_0^1 \Big(E(\theta_r|\mathbf{m})-\theta_r\Big)^2 h(\theta_r|\ell_r,k_r)\,d\theta_r \\ &= \frac{1}{(k_r+2)}\sum_{\ell_r=0}^{k_r+1}\mathrm{Var}(\theta_r|\ell_r,k_r) \end{aligned}$$

Plugging the above into B1.19 gives the expression for player i's ex-ante expected utility:

$$E\left[U^i(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right] = -\left[\left(b_1^i\right)^2+\left(b_2^i\right)^2\right] - \frac{\left[(w_{11})^2+(w_{21})^2\right]}{6\,(k_1+2)} - \frac{\left[(w_{12})^2+(w_{22})^2\right]}{6\,(k_2+2)}$$

In order to analyse the ex-ante optimality of each equilibrium in Proposition 1.2.6, I "break" $\mathbf{m}$ into $\mathbf{m}^i$ and $\mathbf{m}^{\text{-}i}$. Then, denoting by $\tilde{k}_r$ the equilibrium number of truthful messages for senders other than i, the expected variance of each possible message strategy for i become:

$$E\left[U_R\left(\mathbf{y},\mathbf{m}^i=\{\{(0,0);(0,1)\};\{(1,0);(1,1)\}\},\mathbf{m}^{\text{-}i}\right)\right] = -\frac{\left[(w_{11})^2+(w_{21})^2\right]}{6\,(\tilde{k}_1+3)} - \frac{\left[(w_{12})^2+(w_{22})^2\right]}{6\,(\tilde{k}_2+2)} \tag{B1.20}$$

$$E\left[U_R\left(\mathbf{y},\mathbf{m}^i=\{\{(0,0);(1,0)\};\{(0,1);(1,1)\}\},\mathbf{m}^{\text{-}i}\right)\right] = -\frac{\left[(w_{11})^2+(w_{21})^2\right]}{6\,(\tilde{k}_1+2)} - \frac{\left[(w_{12})^2+(w_{22})^2\right]}{6\,(\tilde{k}_2+3)} \tag{B1.21}$$

$$E\left[U_R\left(\mathbf{y},\mathbf{m}^i=\{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\},\mathbf{m}^{\text{-}i}\right)\right]=-\frac{1}{2}\left[\frac{[(w_{11})^2+(w_{21})^2]}{6\,(\tilde{k}_1+3)}+\frac{[(w_{12})^2+(w_{22})^2]}{6\,(\tilde{k}_2+3)}\right]$$
$$-\frac{1}{2}\left[\frac{[(w_{11})^2+(w_{21})^2]}{6\,(\tilde{k}_1+2)}+\frac{[(w_{12})^2+(w_{22})^2]}{6\,(\tilde{k}_2+2)}\right] \tag{B1.22}$$

Where the fact that in (B1.20) sender i reveals S_1 only can be seen in the different numerators of its first and second term, and the same applies to (B1.21). Now, when i reveals $(0,0)$ and $(1,1)$ only (Part 4.c) the receiver's ex-ante expected utility weights the probability of i being one of these types, and the complementary probability of being $(0,1)$ or $(1,0)$ and not receiving any information. The following step consist in finding the conditions under which each of the above expressions are ex-ante optimal for the receiver, given the equilibrium strategies of the other senders.

1. Sender i reveals S_1 only:

   (a) (B1.20) $\geq$ (B1.21):

   $$-\frac{[(w_{11})^2+(w_{21})^2]}{6\,(\tilde{k}_1+3)}-\frac{[(w_{12})^2+(w_{22})^2]}{6\,(\tilde{k}_2+2)}+\frac{[(w_{11})^2+(w_{21})^2]}{6\,(\tilde{k}_1+2)}+\frac{[(w_{12})^2+(w_{22})^2]}{6\,(\tilde{k}_2+3)}\geq 0$$

   Which leads to:

   $$\frac{[(w_{11})^2+(w_{21})^2]}{(\tilde{k}_1+2)(\tilde{k}_1+3)}\geq\frac{[(w_{12})^2+(w_{22})^2]}{(\tilde{k}_2+2)(\tilde{k}_2+3)} \tag{B1.23}$$

   (b) (B1.20) $\geq$ (B1.22):

   $$\frac{[(w_{11})^2+(w_{21})^2]}{2}\left[\frac{1}{(\tilde{k}_1+2)}-\frac{1}{(\tilde{k}_1+3)}\right]-\frac{[(w_{12})^2+(w_{22})^2]}{2}\left[\frac{1}{(\tilde{k}_2+2)}-\frac{1}{(\tilde{k}_2+3)}\right]\geq 0$$

   Which is straightforward to note that leads to (B1.23).

2. Sender i reveals S_2 only: requires that (B1.21) $\geq$ (B1.20) and (B1.21) $\geq$ (B1.22); which following the above algebra happen if and only if

   $$\frac{[(w_{11})^2+(w_{21})^2]}{(\tilde{k}_1+2)(\tilde{k}_1+3)}\leq\frac{[(w_{12})^2+(w_{22})^2]}{(\tilde{k}_2+2)(\tilde{k}_2+3)} \tag{B1.24}$$

3. Sender i reveals $(0,0)$ and $(1,1)$ only: requires that (B1.22) $\geq$ (B1.20) and (B1.22) $\geq$ (B1.21); which happen if and only if,

   $$\frac{[(w_{11})^2+(w_{21})^2]}{(\tilde{k}_1+2)(\tilde{k}_1+3)}\geq\frac{[(w_{12})^2+(w_{22})^2]}{(\tilde{k}_2+2)(\tilde{k}_2+3)}$$

and

$$\frac{[(w_{11})^2 + (w_{21})^2]}{(\tilde{k}_1 + 2)(\tilde{k}_1 + 3)} \leq \frac{[(w_{12})^2 + (w_{22})^2]}{(\tilde{k}_2 + 2)(\tilde{k}_2 + 3)}$$

Both equations above are compatible if and only if:

$$\frac{[(w_{11})^2 + (w_{21})^2]}{(\tilde{k}_1 + 2)(\tilde{k}_1 + 3)} = \frac{[(w_{12})^2 + (w_{22})^2]}{(\tilde{k}_2 + 2)(\tilde{k}_2 + 3)} \tag{B1.25}$$

The above equation tells us that the receiver's ex-ante utility is maximal when i reveals $(0,0)$ and $(1,1)$ only, when she has similar amount of truthful messages for each state in the equilibrium being played. In addition, equation (B1.25) implies that for i revealing S_1 only to be the optimal ex-ante equilibrium for the receiver (B1.23) must hold with inequality, and the same applies for S_2 and (B1.24). □

Observation 1.1. *The RHS of* (1.4) *is positive if and only if:*

$$[w_{11}(k_2 + 3) - w_{12}(k_1 + 3)]^2 + [w_{22}(k_1 + 3) - w_{21}(k_2 + 3)]^2 > 0$$

Since $\frac{w_{11}}{(k_1+3)}$ represents sender i's influence on y_1 through revealing S_1, then $w_{11}(k_2+3)-w_{12}(k_1+3) = 0$ implies that the influence on y_1 through S_1 perfectly outweighs that of S_2. Consequently, RHS of (1.4) equal to zero implies sender i has no influence on any decision when he reveals both signals (and he is either type $(0,1)$ or $(1,0)$), which again depends on the number of other sender revealing truthfully their signals. In section 1.4 I show this plays an important role on the effect of changes in the number of senders other than i revealing truthfully in equilibrium.

Chapter 2

Authority in Complex Organizations

2.1 Introduction

A principal in charge of many decisions needs to aggregate soft information from agents. The amount of information agents transmit depends on both their preferences over decisions and how the information they have affects these decisions (Battaglini, 2002; Levy and Razin, 2007; Ambrus and Takahashi, 2008). When decisions are independent, communication takes place in each dimension separately and depends on the degree of preference misalignment between each sender and the decision-maker. In some cases, the principal can delegate authority over a decision to an agent and reduce the conflict of interests with other agents, thus improving the transmission of information in that dimension (Gilligan and Krehbiel, 1987; Krishna and Morgan, 2001a; Dessein, 2002). When information most relevant for one decision also affect other decisions, on the other hand, communication depends on how this interdependence aggregates decision-specific preferences. In such cases, delegating authority over a decision will affect communication in other dimensions as well. This paper studies the allocation of authority under informational interdependence when communication is strategic.

Consider information aggregation in Multinational Corporations (MNC). The ability to generate and exploit innovations in such organizations is dispersed among subunits () but requires that each subunit forges close relationships with local partners, which creates conflicts with organizational goals (Ghoshal and Bartlett, 1990; Ghoshal and Westney, 1993; Andersson and Forsgren, 2000; Andersson et al., 2007; Ecker et al., 2013). Preference misalignment can affect the transmission of information that is valuable across the organization. To govern this complexity, the MNC needs a rather flexible organizational structure or, as Ghoshal and Westney (1993) put it, '*the management process must be able to change from product to product, from country to country, and even from decision to decision*' (1993, p. 5).

If decisions are independent, headquarters delegate control over a decision to a subsidiary if the information received from business partners and other subsidiaries compensates for the preference misalignment (Austen-Smith, 1990; Dewan and Squintani, 2015).[1] If there is informational interdependence, on the other hand, delegating control over one decision may affect incentives to transmit information relevant for other

[1] See also Gilligan and Krehbiel (1987); Aghion and Tirole (1997); Dessein (2002).

decisions. These incentives depend on how the interdependence aggregates conflicts of interest between senders and receivers. In other words, the headquarters' incentives to delegate authority depend on whether the interdependence benefits or harms information transmission. Along these lines, the international business literature found evidence that a subsidiary's autonomy over product decisions is positively correlated with the degree to which the knowledge created spills over to sister units (Andersson et al., 2007).

In this paper I construct a theoretical model of authority under informational interdependence. The headquarters of a MNC must decide on the design of two products and needs information about profitable innovations over two different attributes. There are n susidiaries, each represented by its manager. A manager has access to information about innovations that are locally profitable, and his preferences over the design of each product are common knowledge. Information can be transmitted trought costless, non-veriable (cheap talk) messages. Before any communication takes place, the headquarters allocates the right to decide over each product among all members of the organization: she can either delegate or retain authority over any decision.[2] The degree of preference misalignment (conflict of interest) between each manager and decision-maker determines the amount of information the former transmits to the latter.

Delegating authority over a decision can lead to informational gains, which in this context are of two types. Direct gains refer to the additional information a manager receives in equilibrium as compared to what the headquarters would receive if she controlled that decision. This constitutes the mechanism found in the literature (Dessein, 2002; Dewan et al., 2015). Indirect gains, by contrast, refer to the additional information *the headquarters* receives when she delegates one decision and retains authority on the other. These gains occur only under informational interdependence, and require there exist managers whose preferences are misaligned with the headquarters in one dimension but aligned in the other.[3] Delegating the high-conflict decision allows the headquarters to restrict these managers' influence to the low-conflict decision.

The intuition behind indirect informational gains leads to one of the main results of the paper. Proposition 2.2 shows that when indirect gains are sufficiently large, the optimal informational structure is partial delegation; that is, the headquarters delegates high-conflict decisions and retains authority over low-conflict ones. Indeed, because of indirect informational gains, the headquarters tolerates a larger bias on the delegated decision for a fixed 'amount' of direct gains, as compared to the case of independent decisions. For a different set of preferences, however, the headquarters tolerates less bias on the delegated decision. This takes place when there are many managers whose preferences do not align with headquarters in any dimension, but still the *aggregate conflict of interest* is small; i.e. decision-specific (large) biases counteract each other due to the interdependence. The optimal organizational structure when such preferences are prevalent is centralization. The mechanism behind this result is similar to 'mutual discipline' in public communication with multiple audiences (Farrell and Gibbons, 1989; Goltsman and Pavlov, 2011), and 'persuasive cheap talk' in multidimensional communication (Chakraborty and Harbaugh, 2010).

[2]In other words, the headquarters has limited commitment power: she can commit to delegate decision rights but cannot contract on decisions; which defines her problem as one of organizational design (Aghion and Tirole, 1997; Dessein, 2002) rather than mechanism design (Holmström, 1977).

[3]The presence of high conflict of interest in one dimension kills a manager's credibility under centralization, which led Levy and Razin to call this situation 'negative informational spillovers'.

Informational interdependence also affects incentives to acquire information. In the second part of the paper, I extend the basic framework allowing managers to decide about what information to observe once decision rights have been allocated. Managers have access to imperfect information about each type of innovation and must incur in a cost to observe it. The investment is worth making only if the expected utility gains from a given piece of information compensate its costs. However, a manager's expected gains from truthful communication are decreasing in the amount of information the decision-maker receives on the equilibrium path.[4] As a consequence, there exists an upper bound on the amount of information any decision-maker receives in equilibrium.

I show that the upper bound on information aggregation is weakly lower when the headquarters delegate at least one decision. In other words, a subsidiary aggregates a suboptimal amount of information (in expectation) when it controls only a subset of all decisions affected by that information: some senders are expected to underinvest in information because they fail to internalize the informational interdependence. A similar mechanism leads to unbalanced investment in information under delegation; that is, managers acquire information that is more relevant for the decision they influence. Finally, I show that the headquarters may prefer to restrict a manager's access to information he is not expected to reveal in equilibrium. By doing so, she reduces the set of deviations from truthful communication and communication is incentive compatible for a larger set of preferences.

2.1.1 Related Literature

The paper contributes to the literatures on cheap talk and organizational design. In multidimensional cheap talk, the receiver can extract all the information from perfectly informed senders by restricting the influence of each of them to the dimension of common interest (Battaglini, 2002). As long as the number of decisions do not exceed that of senders, the receiver can commit to ignore part of the information provided by any sender because it is provided by the others (in equilibrium). Note that, as a consequence, she does not need delegation to affect communication incentives.[5] But this argument relies on the assumption of perfectly informed senders. When this is not the case and decisions are interdependent, the receiver loses her equilibrium commitment power. Levy and Razin (2007) show this leads to communication breakdown if the conflict of interest in one dimension is sufficiently large because communication incentives depend on how information affects decisions.[6] My paper shows delegation substitutes for the receiver's ability to ignore information in Battaglini (2002) and, more generally, analyses how the allocation of authority helps to manage informational interdependence.

Strategic communication with informational interdependence has received some recent attention in the work of Deimen and Szalay (2019). Their paper focuses on an unidimensional decision problem with two payoff-relevant states of the world, such that principal and agent disagree about the state upon which the

[4] I assume signals have a fixed amount of noise and agents can acquire at most one such signal per state. As a consequence, individual incentives for communication are decreasing in the amount of information the decision-maker is expected to have in equilibrium (see Morgan and Stocken, 2008; Galeotti et al., 2013).

[5] Dewan and Hortala-Vallve (2011) apply this argument to study how a Prime Minister uses her prerogatives on ministerial appointments and allocation of portfolios to limit ministers' influence to decisions of low conflict of interest.

[6] See also Chakraborty and Harbaugh (2007); Ambrus and Takahashi (2008).

decision has to be calibrated. Hence, the conflict of interest between the two players decreases with the correlation between states. In my paper, the effect of interdependence (correlation) on communication is not monotonic because conflicts of interest are state independent (as in Crawford and Sobel, 1982). Besides, the multidimensionality of the decision problem plays a central role in shaping incentives for information acquisition under different organizational structures.

The second contribution of the paper relates to organizational design. In unidimensional decision problems, Dessein (2002) shows that the benefits from delegation are increasing in the sender's informational advantage.[7] The same intuition applies to the organization of legislative debate (Gilligan and Krehbiel, 1987; Austen-Smith, 1990; Krishna and Morgan, 2001a), policy-making cabinets (Dewan et al., 2015; Dewan and Squintani, 2015), bureaucracies (Epstein and O'Halloran, 1994; Gailmard and Patty, 2012), and multi-divisional firms (Alonso et al., 2008; Rantakari, 2008). In all these environments, however, is natural to think of multidimensional decision problems in which the relevant information is dispersed among many agents and features interdependence.

Part of organizational design literature has addressed questions similar to mine. Several papers focus on multi-divisional firms, which trade off the need for adaptation to local shocks with the need for coordination between divisions. Communication frictions may lead to inefficiencies in terms of giving up benefits from the specialization of production (Dessein and Santos, 2006) or the need for coordination through scheduled tasks instead of using communication (Dessein et al., 2016). When divisional managers' information is not verifiable, the allocation of decision rights—along with non-separability of preferences and divisional conflict of interest—shapes incentives for communication (Alonso et al., 2008, 2015; Rantakari, 2008). Differently from these papers, I focus on interdependence arising exclusively from information and not preferences. Besides matching many real-world environments,[8] this approach allows a more direct comparison with the mainstream cheap talk literature (see above) and helps study strategic acquisition of imperfect information (see Di Pei, 2015; Argenziano et al., 2016).

The allocation of decision rights also affects incentives to acquire information. Aghion and Tirole (1997) show that delegation of real authority motivates an agent to acquire information, resulting in a loss of control for the principal. But delegation may discourage information acquisition in this context, either because the agent prefers to put more effort into information that benefits him personally (Rantakari, 2012), or because he no longer has to convince a principal with divergent opinion about the best course of action (Che and Kartik, 2009). My paper shows that informational interdependence has meaningful consequences for incentives to acquire information. By allowing information acquisition about multiple states, my framework also captures different drivers of specialization.

The next section presents the baseline model with no information acquisition and the results for optimal allocation of decision-rights. In section 2.4 I integrate the allocation of decision-rights with endogenous

[7] He also shows that delegation dominates cheap talk communication for all conflicts of interest for which there is transmission of information in the latter, but this result relies on the sender being perfectly informed.

[8] In addition to aggregation of knowledge in MNC, the framework in this paper reflects the design of legislative committees to incentivize acquisition and transmission of information from legislators (Austen-Smith, 1990, 1993; Dewan et al., 2015), and the conformation of negotiation teams in international relations (see Ramsay, 2011; Trager, 2011).

information acquisition. In section 2.6 I discuss some extensions and conclude.

2.2 Baseline Model

Players and preferences. An organization consists of a principal, P, and n agents.[9] There are two decisions to be made, $\mathbf{y} \in \Re^2$, but the outcome of each of them depends on two state variables θ_1 and θ_2 (to be defined later). Optimal decisions thus require information about these states. The decision-specific uncertainty is represented by two composite states $\delta_1(\theta_1, \theta_2)$ and $\delta_2(\theta_1, \theta_2)$. Preferences for player $i = \{P, 1, \ldots, n\}$ are given by:

$$U^i(\boldsymbol{\theta}, \mathbf{y}, \mathbf{b}^i) = -\left(y_1 - \delta_1(\theta_1, \theta_2) - b_1^i\right)^2 - \left(y_1 - \delta_2(\theta_1, \theta_2) - b_2^i\right)^2$$

Where $\mathbf{b}^i \in \Re$ represents i's bias vector, which is normalized to $\mathbf{b}^P = (0, 0)$ for the principal.

Information structure. Pay-off relevant states, θ_1 and θ_2, are uniformly distributed with support in the interval $[0, 1]$, and $\theta_1 \perp \theta_2$. Information about each of these states affects both decisions, that is, there is *informational interdependence*. In the example of multinational corporations, the states can be interpreted as different technological attributes relevant for many products the firm produces. Decisions would represent the different product lines using these technologies; arguably, each technology is a salient attribute for a different product. In a policy-making example, states can be interpreted as the different goals of a policy intervention, while decisions represent the different policy instruments that address those goals; arguably, different instruments address goals with different degrees of success. The composite states δ_1 and δ_2 capture informational interdependence:

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \equiv \begin{bmatrix} w_{11}\,\theta_1 + w_{12}\,\theta_2 \\ w_{21}\,\theta_1 + w_{22}\,\theta_2 \end{bmatrix}$$

The uniform distribution of states represents the canonical example in Crawford and Sobel (1982) and has been extensively used in the cheap talk literature.Assuming independent states allows me to isolate the effect of informational interdependence through $\boldsymbol{\delta}$. The elements of the weighting matrix W are indexed by w_{dr}, for $y_d = \{y_1, y_2\}$ and $\theta_r = \{\theta_1, \theta_2\}$ (d represents decisions and r represents states). All the weights are weakly positive and I normalize them as $w_{d1} + w_{d2} = 1$. Without loss, I also take $w_{11}, w_{22} > \frac{1}{2}$, so that the index corresponding to the state also reflects which decision that state is more important for. As a consequence, the informational interdependence between decisions is linear, and captured by the ex-ante correlation between the composite states.

$$\text{Corr}(\delta_1, \delta_2) = \frac{(w_{11}w_{12} + w_{21}w_{22})}{[(w_{11}^2 + w_{21}^2)(w_{12}^2 + w_{22}^2)]}$$

Agents' signals and communication. Agents have access to noisy, non-verifiable information about both states. Each agent observes one signal associated with each state (two in total). Let $\mathbf{S}^i = (S_1^i, S_2^i) \in \mathcal{S} \equiv$

[9]As usual in this literature, I use female pronouns to refer to the principal and male pronouns for each agent.

$\{0,1\}^2$ be i's signals, and $\tilde{\mathbf{S}} \in \mathcal{S}$ be the vector of realizations. Signals are independent across players, conditionally on $\boldsymbol{\theta}$. The prior probability distribution for each signal is characterized by Pr$(\tilde{S}_1^i = 1) = \theta_1$ and Pr$(\tilde{S}_2^i = 1) = \theta_2$.[10]

Each agent sends private, non-verifiable (cheap talk) messages to decision-maker $j = \{P, 1, .., n\}$.[11] Let $\mathbf{m}_j^i \left(\mathbf{S}^i\right) \in \{0,1\}^2$ denote agent i's message to decision-maker j, in charge of $y_d = \{y_1, y_2\}$. Note that i's (pure) message strategies associated with each signal can take one of two forms (up to relabelling messages): the truthful one, $m_j^i(S_r^i) = \tilde{S}_r^i$ for all S_r^i, and the babbling one, $m_j^i(\tilde{S}_r^i = 0) = m_j^i(\tilde{S}_r^i = 1)$. Besides, the complete set of message strategies is not just based on babbling or revealing information on separate dimensions independently. An agent could, for instance, truthfully reveal both signals for some realizations and send the babbling message for others. Such message strategies arise because states are orthogonal and information about one state does not reveal information about the other. I call these strategies 'dimensional non-separable' (DNS).

Let $\mathbf{m}_j = \{..., \mathbf{m}_j^i, ...\}$ denote the matrix containing all the messages decision-maker j receives from agents (including himself if applicable). The updated expectation and variance for each state depend on the number of agents revealing the corresponding signal truthfully, $k_r(j) \leq n$, and the the number of those agents who report a 1, $\ell_r(j)$, for $\theta_r = \{\theta_1, \theta_2\}$, as follows.

$$E\left(\theta_r|\mathbf{m}_j\right) = \frac{(\ell_r(j)+1)}{(k_r(j)+2)} \qquad \text{Var}\left(\theta_r|\mathbf{m}_j\right) = \frac{(\ell_r(j)+1)(k_r(j)-\ell_r(j)+1)}{(k_r(j)+2)^2(k_r(j)+3)}$$

Allocation of decision rights. Decision-rights are allocated before each agent learns his information. Formally, the principal decides on a set of assignments that grants decision making authority over the set of decisions. The assignment grants complete jurisdiction over the delegated decision, such that authority over each decision is granted to a unique individual. Decision makers are also granted the possibility of private communication with each agent. I assume decision-makers cannot communicate the information transmitted to them by other agents. Different allocations of decision-rights lead to different organizational structures. I group these structures into three categories: under *centralization* the principal decides on both issues; under *full delegation* the principal grants authority to two different agents, each of them assigned to a different decision; under *partial delegation* the principal retains authority over one issue and delegates the other to an agent.

At this point two clarifications are necessary. First, I am not considering the case of delegation of both decisions to a single agent. Incentives for communication in such a case use the same measure of conflict of interest as under centralization. Hence, interdependence plays no role beyond the basic trade-off between informational gains and loss of control. On the contrary, the two forms of delegation considered in the paper involve different measures of conflict of interest. The second clarification relates to the distinction between

[10] A similar information structure, for unidimensional problems with one state variable, has been used in Austen-Smith and Riker (1987); Morgan and Stocken (2008); Galeotti et al. (2013) among others.

[11] When an agent controls a decision, I assume he cannot reveal any information provided by other agents at the communication stage. An in-depth analysis of informational hierarchies can be found in Migrow (2017), while discussions of communication of imperfect information in the form of binary signals can be found in Förster (2020) and Chapter 3 of this dissertation.

delegation of decision authority and decentralization on the access to information. In my framework, authority can be centralized or decentralized, the latter is called ‘delegation’ throughout the paper. Information, on the contrary, is always decentralized because it is dispersed among agents.

Equilibrium concept The equilibrium concept is pure-strategies Perfect Bayesian Equilibria (equilibrium, henceforth). A full characterization including mixed strategies is cumbersome and does not provide much insight beyond the pure-strategies case. Because communication is cheap talk, there typically exist multiple equilibria. I select among equilibrium message strategies on the basis of the decision-maker’s ex-ante expected utility.[12]

In the following section I analyse optimal organizational design when agents observe one signal associated with each state.

2.3 Organizational Design and Information Transmission

In this section I characterize the role of informational interdependence in organizational design. I first describe incentives for communication under each organizational structure (the formal analysis is left to the appendix). I then analyse the optimal organizational structure. The figure below outlines the timing of the game.

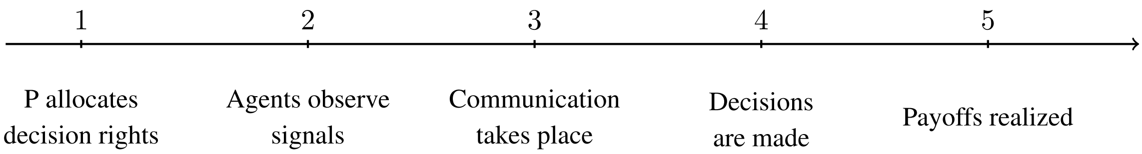


Figure 2.1: Timing of the Organizational Design Game

Before describing the incentives for communication, I introduce some notation. Let $k_r^*(j) \equiv k_r(\mathbf{m}_j^*)$ and $\ell_r^*(j)$ denote the number of truthful messages and ‘ones’ decision-maker j receives in equilibrium, respectively. Also, let $k_r(j)$ be an agent’s conjecture about the number of agents other than him who reveal information about state θ_r to decision-maker j (on path).[13] To keep track of who decides what, let $j', j'' \in \{P, 1, ..., n\}$ be the decision-makers for y_1 and y_2, respectively (the number of apostrophes coincides with the index on the decision).

An equilibrium of the game defined by the allocation of authority is characterized by the triple $\left(\mathbf{y}^*, \mathbf{m}_{j'}^*, \mathbf{m}_{j''}^*\right)$, representing the vector of decisions and the vectors of message strategies to j' and j'', respectively. Optimal actions satisfy:

$$y_1^* = w_{11}E(\theta_1|\mathbf{m}_{j'}^*) + w_{12}E(\theta_2|\mathbf{m}_{j'}^*) + b_1' \qquad y_2^* = w_{21}E(\theta_1|\mathbf{m}_{j''}^*) + w_{22}E(\theta_2|\mathbf{m}_{j''}^*) + b_2''$$

[12] See Crawford and Sobel (1982); Farrell (1993); Chen et al. (2008). Also, given the focus on pure strategies, it can be shown that this criterion satisfies neologism proofness (Farrell, 1993).

[13] Note that i’s conjecture will be correct in equilibrium, and whenever his message strategy involves revealing the corresponding signal then $k_r^*(j) = k_r(j) + 1$, otherwise $k_r^*(j) = k_r(j)$.

Where b_1' represents the bias of decision-maker j' with respect to y_1, and similarly for b_2'', j'', and y_2. Note that centralization means $j' = j'' = P$, such that the biases are equal to zero. From the principal's perspective, delegation of decision-rights has two payoff-relevant consequences. On the one hand, it implies a biased agent decides on behalf of the principal, resulting in a biased decision. On the other hand, individual incentives for communication depend on the conflict of interest between the agent and each decision-maker. Different organizational structures (and decision-makers) then result in different communication incentives. Agent i's optimal message strategy to decision-maker j solves:

$$\mathbf{m}_j^{i*}(\mathbf{S}^i, \mathbf{b}^i, b_1', b_2'') = \arg\max_{\mathbf{m}_j^i} \left\{ E\Big[-\left(y_1\left(m_{j'}^i, \mathbf{m}_{j'}^{-i}\right) - \delta_1 - b_1^i\right)^2 - \left(y_2\left(m_{j''}^i, \mathbf{m}_{j''}^{-i}\right) - \delta_2 - b_2^i\right)^2 \Big| \mathbf{S}^i \Big] \right\}$$

To simplify notation, let $k_r^{\mathrm{C}} \equiv \{k_r(j) |\, j' = j'' = P\}$ denote the number of truthful messages about $\theta_r = \{\theta_1, \theta_2\}$ the principal receives under centralization; let $k_r^{\mathrm{P}1} \equiv \{k_r(P) |\, j' = P\}$ be the number of messages received when she decides on y_1 only, and $k_r^{\mathrm{P}2} \equiv \{k_r(P) |\, j'' = P\}$ when she decides on y_2 only. For when P does not decide at all, let $k_r' \equiv k_r^*(j')$ and $k_r'' \equiv k_r^*(j'')$ refer to the number of truthful messages for decision-makers of y_1 and y_2, respectively, while keeping $k_r^j \equiv k_r^*(j)$ for a generic decision-maker.

Note that the principal's expected utility from different allocations of decision rights depends on the amount of information the different decision-makers are expected to receive on the equilibrium path. I first analyse incentives for communication for a typical agent under different organizational structures, then I study the role of informational interdependence in the principal-optimal structure.

2.3.1 Incentives for Communication

Delegation. I first describe agent i's incentives to reveal information to decision-maker j in charge of y_d. Because communication between i and j is private, incentives depend on the conflict of interest between them, represented by $|b_d^i - b_d^j|$. But since i is imperfectly informed, communication also depends on how many other agents are expected to be truthful to j on the equilibrium path. If j receives information from many agents other than i, he conjectures the decision will be 'too close' to j's ideal, and far away from his. These two determinants constitute the traditional mechanism determining communication of imperfect information via cheap talk (Austen-Smith and Riker, 1987; Morgan and Stocken, 2008; Galeotti et al., 2013). In my framework, agents observe signals about two independent states, which introduces a third determinant.

Informational interdependence implies that information about the two states affect each decision. When i observes signals $\mathbf{S}^i = \{(0, 1)\}$, for instance, his influence depends on which signal he is expected to reveal to j. Because the conflict of interest between them is unidimensional, one of these signals always moves the decision towards his bias. Therefore, i has higher incentives to follow the favourable signal, for all possible message strategies. These deviation incentives lead to a *credibility loss*, which depends on i's conjecture about the influence of the favourable signal, given the equlibrium behaviour of the other agents. As a consequence, incentives for communication now depend on how much information *about both states* other agents are expected to reveal on path.

Lemma A2.1 and Proposition A2.1 in Appendix A2 characterize the equilibrium communication between agent i and decision-maker j; here I describe the intuitions. When i is expected to reveal one signal on path, his incentives depend on the conflict of interest with j, on how many agents reveal the same information, and how many of them reveal the other signal (due to the credibility loss). When i is expected to reveal both signals on path, his influence on the decision depends on whether the signals realizations coincide or not. If they coincide, $\mathbf{S}^i = \{(0,0),(1,1)\}$, the signals' influences reinforce each other; i's expected marginal utility from revealing such signals will be larger than revealing each of them individually (conditional on k_1^j and k_2^j). Agent i would have, in principle, higher incentives to reveal both signals than revealing either of them individually. But signals may not coincide, $\mathbf{S}^i = \{(0,1),(1,0)\}$, in which case their influences counteract each other and i has incentives to follow the most favourable of them. An implication of this is that incentive compatiblity constraints for revelation of one signal and full revelation coincide, meaning that these constraints hold for the same set of bias vectors. As a result, the equilibrium under delegation involves only two message strategies: full revelation, $\mathbf{m}_j^{i*} = \{\{(0,0)\};\{(1,1)\};\{(1,0)\};\{(0,1)\}\}$, and babbling, $\mathbf{m}_j^{i*} = \{\{(0,0);(1,1);(1,0);(0,1)\}\}$.[14]

Centralization. In Chapter 1 I showed that interdependence means that information transmitted to the principal under centralization affects both decisions. But an agent's influence on decisions depend on the type and amount of information he transmits. To see this, note that revealing S_1^i has a larger influence on y_1 and, thus, the bias on the first dimension weighs more heavily in determining i's incentives. If he reveals both signals, on the contrary, the overall influence is more balanced and so are the weights of b_1^i and b_2^i on the IC constraints. This leads to different measures of conflict of interest depending on the information revealed by different message strategies.

The possibility of different measures of conflict of interest results in two main differences with respect to delegation. First, revealing information about one state only constitutes the equilibrium message strategy for a non-empty set of bias vectors. The relevant measure of conflict of interest for revealing S_1^i is given by $\beta_1 = b_1^i w_{11} + b_2^i w_{21}$, while that for revealing S_2^i is $\beta_2 = b_1^i w_{12} + b_2^i w_{22}$ (Proposition A1.1). The second difference with delegation relates to the effects of ambiguous information, i.e. $\mathbf{S}^i = \{(0,1),(1,0)\}$. Under centralization, revealing both signals has a balanced overall influence on decisions. Note that truthful revelation of $\mathbf{S}^i = \{(0,1)\}$ or $\mathbf{S}^i = \{(1,0)\}$ move decisions in opposite directions. If, for instance, i credibly announces $\mathbf{m}_P^i = \{(0,1)\}$, he would influence y_1 towards 0 and y_2 towards 1. If i's biases have the same sign, such a message leads to utility gains in one dimension and losses in the other. For some set of bias vectors i's equilibrium message strategy consists of revealing ambiguous signals and sending non-influential messages otherwise. As Proposition A1.1 shows, the principal-optimal equilibrium features the following message strategies: full revelation, revelation of information about one state, full revelation of some signal realization and nothing otherwise, and babbling. I now focus on the analysis of the optimal organizational structure.

[14]One could think that i may want to fully reveal signals when they coincide and announce the corresponding babbling message when they do not—i.e. $\mathbf{m}_j^i = \{\{(0,0)\};\{(1,1)\};\{(1,0);(0,1)\}\}$. In the Appendix I show the set of bias vectors for which this is incentive compatible is a strict subset of that for which full revelation is.

2.3.2 Optimal Organizational Structure.

In this subsection I characterize the optimal organizational structure and present the main result of the communication game: the optimal allocation of decision-rights depends on the existence and the extent of informational spillovers.

The profile of preferences, $\mathbf{B} = \{\mathbf{b}^1, ..., \mathbf{b}^n\}$, determines the amount of equilibrium information for each possible organizational structure. By allocating decision rights the principal affects agents' equlibrium message strategies; effectively, she chooses among the different equilibria induced by $\mathbf{B}$. Let $\mathrm{Var}(\theta_r|\mathbf{m}_j)$ denote the ex-ante expected variance associated to state $\theta_r = \{\theta_1, \theta_2\}$ in the equilibrium in which decision-maker $j = \{P, 1, ..., n\}$ receives messages characterized by $\mathbf{m}_j$.[15] The definition below characterizes the principal's ex-ante expected utility, for two generic decision-makers j' and j''.

Definition 2.1 (principal's ex-ante expected utility)**.** *Consider an equilibrium $(\mathbf{y}, \mathbf{m})$ in which $j', j'' = \{P, 1, ..., n\}$ decide on y_1 and y_2, respectively. Denote by $\mathbf{m}_j = \{\mathbf{m}_{j'}, \mathbf{m}_{j''}\}$ the equilibrium messages for j' and j'' under delegation, for $j', j'' = \{P, 1, 2, ...\}$. Then, the principal ex-ante expected utility is given by:*

$$\begin{aligned}\hat{U}^{\mathrm{P}}(\mathbf{B}, \mathbf{m}_{j'}, \mathbf{m}_{j''}) = &- \left[(b_1')^2 + (w_{11})^2\, \mathit{Var}(\theta_1|\mathbf{m}_{j'}) + (w_{12})^2\, \mathit{Var}(\theta_2|\mathbf{m}_{j'})\right] \\ &- \left[(b_2'')^2 + (w_{21})^2\, \mathit{Var}(\theta_1|\mathbf{m}_{j''}) + (w_{22})^2\, \mathit{Var}(\theta_2|\mathbf{m}_{j''})\right] \qquad (2.1)\end{aligned}$$

Equation (2.1) is derived in Appendix A2. The first term in square brackets represents the principal's ex-ante expected utility associated with delegation of y_1 to decision-maker j'. Because his decision will be biased, the principal's utility is decreasing in b_1'. But her utility also depends on her expectations about the decision-maker's posterior beliefs in equilibrium. The principal thus delegate authority to a player whose preferences on the associated decision are sufficiently close to other agents' preferences. This is the trade-off in the literature: informational gains must compensate for the loss of control.

But informational interdependence can lead to a different source of informational gains for the principal. Because delegation breaks the interdependence, there may be agents willing to reveal information under delegation but not under centralization. This happens when, for instance, agents' biases are very large in one dimension and small in the other. If the principal delegates the high-conflict decision and retains authority over the low-conflict decision, these agents will reveal information to her. I call these informational gains *indirect*, since they do not arise because the new decision-maker aggregates more information, but is a by-product of delegation. Indeed, indirect informational gains arise because some agents are affected by *negative informational spillovers*—under centralization, a high-conflict dimension impedes communication in a dimension of low conflict of interest (Levy and Razin, 2007).[16] The proposition below defines both types of informational gains arising in this game, and characterizes the necessary conditions for each of them.

Proposition 2.1. *Consider the equilibrium under centralization, characterized by $\mathbf{m}^*_{\mathrm{C}}$; and the equilibrium*

[15] Formally, $\mathrm{Var}(\theta_r|\mathbf{m}_j) \equiv E\left[\left(E(\theta_r|\mathbf{m}_j) - \theta_r\right)^2; \mathbf{m}_j\right] = \frac{1}{6(k_r^j+2)}$.

[16] Levy and Razin describe negative informational spillovers as the case when i's bias with respect to y_1 is so large that he would not reveal any information under centralization, even though $|b_2^i|$ is very small.

in which the principal delegates y_1 to agent j' and retains authority on y_2, characterized by $\mathbf{m}^*_{j'}$ and $\mathbf{m}^*_{\mathrm{P}2}$. Utility gains from delegation consist of:

- ***Direct Informational Gains** if:*

$$DIG_{j'}(y_1) \equiv (w_{11})^2\big[Var(\theta_1|\mathbf{m}^*_{\mathrm{C}}) - Var(\theta_1|\mathbf{m}^*_{j'})\big] + (w_{12})^2\big[Var(\theta_2|\mathbf{m}^*_{\mathrm{C}}) - Var(\theta_2|\mathbf{m}^*_{j'})\big] \geq (b'_1)^2$$

Moreover, such direct informational gains require that there exists an agent i whose preferences satisfy:

$$|b^i_1 - b'_1| < \left|b^i_1 + b^i_2\,\frac{w_{21}}{w_{11}}\right| \tag{2.2}$$

- ***Indirect Informational Gains** if:*

$$IIG(y_2) \equiv (w_{21})^2\big[Var(\theta_1|\mathbf{m}^*_{\mathrm{C}}) - Var(\theta_1|\mathbf{m}^*_{\mathrm{P}2})\big] + (w_{22})^2\big[Var(\theta_2|\mathbf{m}^*_{\mathrm{C}}) - Var(\theta_2|\mathbf{m}^*_{\mathrm{P}2})\big] \geq 0$$

Moreover, such indirect informational gains require that there exists an agent i whose preferences satisfy:

$$|b^i_2| < \left|b^i_1\,\frac{w_{12}}{w_{22}} + b^i_2\right| \tag{2.3}$$

Moreover, whenever the optimal allocation of decision rights involves delegation of y_1 to j', then either $DIG_{j'}(y_1) > b'_1$, or $IIG(y_2) > 0$, or both.

Proof. All proofs can be found in Appendix A2 □

Direct informational gains arise when more agents transmit information to the new decision-maker under delegation than to the principal under centralization. Such gains arise only if j' has more central preferences on y_1; that is, if there is at least one agent who is willing to reveal information to j' but not to the principal under centralization. Equation (2.2) reflects this condition: it shows that the relevant conflict of interest between i and j' (left-hand side) is lower than that between i and the principal under centralization (right-hand side).

Indirect informational gains arise when more agents transmit information to the principal upon delegation of one decision, as compared to centralization. For this to take place there must be at least one agent who reveals more information to the principal when she decides on y_2 only. Equation (2.3) shows that the conflict of interest with the principal under partial delegation must be lower than the aggregate conflict of interest. In this case, the bias on the first dimension is so large that communication under centralization is less informative than when the principal only decides on y_2. The presence of *negative informational spillovers* is then a necessary condition for indirect informational gains.

The optimal allocation of decision rights is fully characterized in Proposition A2.2. Full delegation is optimal when there are two different agents with central preferences and no informational spillovers associated with retaining authority. The allocation of decision rights depends then on the different communication equilibria induced by the profile of preferences. The previous discussion made clear that the argument reduces to whether the direct and/or indirect informational gains are sufficiently large, where sufficiency involves the loss of control from delegation.

2.3.3 Organizational Design with Informational Spillovers

I now present the main result of this section: how the presence of informational spillovers leads to specific organizational structures. In the previous section I showed that the presence of negative informational spillovers is a necessary condition for indirect gains, which in turn leads to partial delegation. But informational spillovers are not limited to being negative in this context. Positive spillovers occur when i's decision-specific biases are both large, but the aggregate conflict of interest (associated with the information he reveals in equilibrium) is small. When i is expected to reveal information about θ_1 only, for instance, his incentives are maximal when $b_1^i = -\frac{w_{12}}{w_{11}} b_2^i$. Intuitively, if there were many agents whose preferences are affected by positive spillovers, the principal would prefer centralization to any other organizational structure. The proposition below shows how informational spillovers affect the allocation of authority.

Proposition 2.2. *Let the triple $(\mathbf{k}^{\mathrm{C}}, \mathbf{k}', \mathbf{k}'')$ characterize the optimal organizational structure under the profile of biases $\mathbf{B}^n = (\mathbf{b}^1, ..., \mathbf{b}^n)$. Suppose that $k_1' = k_2'$, $k_1'' = k_2''$, and $k_1^{\mathrm{C}} = k_2^{\mathrm{C}}$. Now, consider the associated game consisting of $n + m$ agents, with the profile of biases for the first n being $\mathbf{B}^n$, such that $\mathbf{B}^{n+m} = (\mathbf{b}^1, ..., \mathbf{b}^n, \mathbf{b}^{n+1}, ..., \mathbf{b}^{n+m})$. For a sufficiently large $\mathfrak{b} \in \Re_+$, it is true that:*

1. *For $\mathbf{b}^{n+1} = ... = \mathbf{b}^{n+m} = (\mathfrak{b}, 0)$; then, there exists a sufficiently large m such that the optimal organizational structure in the game with $n + m$ agents is partial delegation of y_1 only.*
2. *For $\mathbf{b}^{n+1} = ... = \mathbf{b}^{\frac{2n+m+1}{2}} = (-\mathfrak{b}, \mathfrak{b})$ and $\mathbf{b}^{\frac{2n+m+1}{2}} = ... = \mathbf{b}^{n+m} = (\mathfrak{b}, \mathfrak{b})$; then, there exists a sufficiently large m such that the optimal organizational structure in the game with $n + m$ agents is centralization.*

If negative spillovers are sufficiently large with respect to one dimension, the principal's optimal organizational structure is partial delegation. If positive spillovers are sufficiently large, the principal's optimal organizational structure is centralization. Informational spillovers in Proposition 2.2 are captured by the preferences of the 'additional' agents, such that m reflects the intensity of these spillovers.

Negative spillovers lead to partial delegation because the principal finds optimal to delegate the controversial decision in order to induce the additional agents to reveal the information they have. Positive spillovers, on the other hand, lead to centralization because the additional agents are willing to play dimensional non-separable strategies under centralization. Agents whose decision-specific biases have different signs fully reveal their signals when $\mathbf{S}^i = \{(0,0); (1,1)\}$ and announce the babbling message for the other possible realizations; while agents whose biases have the same sign fully reveal their signals when $\mathbf{S}^i = \{(0,1); (1,0)\}$ and announce the babbling message otherwise. In both cases, the additional information the principal expects to receive from the m agents brings her a higher expected utility than the optimal allocation under the original profile of preferences.

Because the choice among organizational structure depends on the trade-off between informational gains and loss of control, in the following subsection I study it in more depth.

2.3.4 Informational Gains and Loss of Control

Proposition 2.3 below characterizes the relationship between informational gains and loss of control under informational interdependence, when agents are imperfectly informed.

Proposition 2.3 (Maximum Admissible Loss of Control)**.** *Let* $\mathbf{b}^{\text{D}} = (b'_1, b''_2)$ *be the biases of decision-makers for* y_1 *and* y_2*, respectively. Then, the maximum* $\mathbf{b}^{\text{D}}$ *for which the principal is willing to delegate at least one decision is given by:*

$$||\mathbf{b}^{\text{D}}|| \equiv \left[\sum_{y_d}\left[w_{d1}^2\big(Var(\theta_1|\mathbf{m}_{\text{C}}) - Var(\theta_1|\mathbf{m}_j)\big) + w_{d2}^2\big(Var(\theta_2|\mathbf{m}_{\text{C}}) - Var(\theta_2|\mathbf{m}_j)\big)\right]\right]^{\frac{1}{2}}$$

There is a positive relationship between informational gains from delegation and loss of control. The expression $||\mathbf{b}^{\text{D}}||$ represents the relevant measure for the loss of control: how far are both decisions from the principal's (state-dependent) ideal. Under partial delegation, one of the components of $||\mathbf{b}^{\text{D}}||$ is zero, which marks the maximum bias the principal tolerates in a single decision.

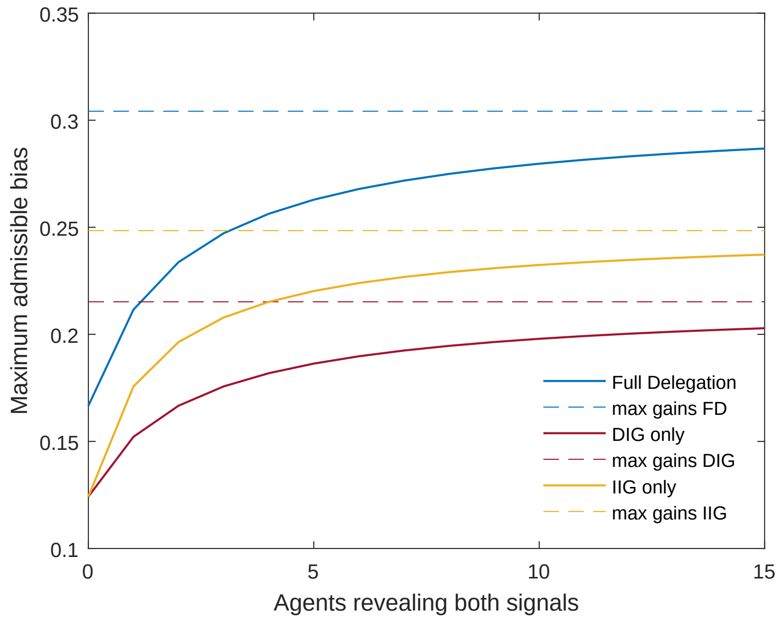
Figure 2.2 illustrates the relationship between informational gains and loss of control. It shows the maximum deviation the principal tolerates from her ideal decision—measured by $||\mathbf{b}^{\text{D}}||$—as a function of the number of agents revealing both signals in three cases.[17] First, when she delegates both decisions to different agents (in blue). Minimum informational gains in this case mean each decision-maker decides with his own signals, but no other agent reveals any additional information. The concavity of the curve represents the decreasing marginal utility of additional information, which comes from quadratic preferences and the updating process.[18] Maximum informational gains are represented with the dashed line, showing the maximum bias the principal tolerates when she expects each decision-maker *to become perfectly informed in equilibrium*. Second, the red lines show the relationship for direct informational gains only, assuming the principal retains authority on one decision and does not receive any information from agents. Minimum informational gains thus mean the agent decides with his own information and the principal decides with no information. Hence, the maximum bias the principal tolerates is lower than the full delegation case, which holds for all possible informational gains. The same intuition applies for the maximum bias the principal tolerates (dashed line). Finally, the yellow lines show the relationship when informational gains are indirect—partial delegation results in more information being received by the principal. For the delegated decision, the agent observes his own signals, which explains in part why the principal tolerates a higher bias than for direct informational gains. Because of the quadratic preferences, this additional information has a high marginal return for the principal.

The intuitions just described assume constant information under centralization ($k_1^{\text{C}} = k_2^{\text{C}}$). I now show

[17] To make the comparison clear, I assume $k_1^{\text{C}} = k_2^{\text{C}} = 0$.

[18] The marginal influence of an additional signal on the associated posterior belief is decreasing (see Morgan and Stocken, 2008; Galeotti et al., 2013).

Figure 2.2: Maximum bias the principal tolerates as a function of informational gains



Note: $w_{11} = w_{22} = \frac{2}{3}$, $k_1^{\mathrm{C}} = k_2^{\mathrm{C}} = 0$

how the information the principal expects to receive under centralization affects the loss of control she is willing to tolerate. In summary, the principal's marginal expected utility from an additional signal is decreasing in the amount of information she expects to receive under centralization.

Corollary 2.1. *The 'marginal value' of a signal for the principal is decreasing in the amount of information she receives under centralization. Let* $\Delta Var(\theta_r|k_r^{\mathrm{C}}) = Var(\theta_r|\mathbf{m}_{\mathrm{C}}) - Var(\theta_r|\mathbf{m}_j, k_r^j = k_r^{\mathrm{C}} + 1)$.

$$\frac{\partial \Delta Var(\theta_r|k_r^{\mathrm{C}})}{\partial k_r^{\mathrm{C}}} = \frac{1}{6}\left[\frac{1}{(k_r^{\mathrm{C}}+3)^2} - \frac{1}{(k_r^{\mathrm{C}}+2)^2}\right] < 0 \tag{2.4}$$

Equation (2.4) means that an additional signal represents a smaller informational gain when the principal expects to be well informed under centralization. The maximum bias she is willing to tolerate thus decreases as the profile of biases allows more agents to truthfully reveal information under centralization. The result extends Corollary 1 in Dessein (2002) to the case of imperfectly informed senders.[19]

[19]Dessein finds that incentives to delegate depend on the magnitude of the residual variance. His paper features a single, perfectly informed sender who advises the principal on a one decision problem. He analyses incentives for delegation as a function of the ex-ante variance associated with the state, against the babbling equilibrium. The residual variance here depends on the amount of information received by the principal under centralization.

2.4 Endogenous Information Acquisition

In this section I analyse agents' incentives to acquire information before the communication stage, but after decision-rights have been allocated. I first present the extended model. Next, I derive the two incentive compatibility constraints involved in information acquisition decisions and show how costs impose restrictions on informational gains from delegation. I then characterize the equilibrium strategies for a generic agent, showing the cases in which he decides to specialize. Finally, I analyse how interdependence affects incentives to acquire information under different organizational structures.

Baseline model with endogenous information acquisition. Agents have access to imperfect information about each state. In particular, each agent has access to one binary trial per state and decides which realizations to observe (if any).[20] Formally, let $\mathfrak{s}^i \in \big\{\{\emptyset\}, \{\tilde{S}_1\}, \{\tilde{S}_2\}, \{\tilde{S}_1, \tilde{S}_2\}\big\}$ be agent i's information acquisition decision. With this formulation, i's type is given by the realizations of both signals but he decides the extent to which he observes his type.

Definition 2.2. *The information structure for agent i in the game with endogenous information acquisition consists of the following elements: $\mathbf{S}^i = (S_1^i, S_2^i)$ are the signals available to him, $\tilde{\mathbf{S}}^i = (\tilde{S}_1^i, \tilde{S}_2^i)$ the realization of the corresponding signals (his type), and $\mathfrak{s}^i \in \big\{\{\emptyset\}, \{\tilde{S}_1\}, \{\tilde{S}_2\}, \{\tilde{S}_1, \tilde{S}_2\}\big\}$ the information he actually decides to observe.*

The costs of different information structures are captured by the function $C(\mathfrak{s})$, which satisfies $C\left(\{\tilde{S}_1, \tilde{S}_2\}\right) > C\left(\{\tilde{S}_1\}\right) = C\left(\{\tilde{S}_2\}\right) > C\left(\emptyset\right) = 0$. The principal has no direct access to information. The preferences of agent $i = \{1, \ldots, n\}$ are given by:[21]

$$U^i(\boldsymbol{\theta}, \mathbf{x}, \mathbf{b}^i, \mathfrak{s}^i) = -\sum_{y_d = \{y_1, y_2\}} \big(y_d - \delta_d(\theta_1, \theta_2) - b_d^i\big)^2 - C(\mathfrak{s}^i)$$

Figure 2.3 shows the timing of the game. The allocation of decision rights is observed by all agents. Knowing who decides what, each agent chooses the information he will observe (if any). I assume *overt information acquisition*—individual decisions (but not information) are observable. In Section 2.6 I discuss the implications of relaxing this assumption.

The communication stage is similar to the previous section, so I keep the notation. Let i be a generic agent and j a generic decision-maker, let $j', j'' \in \{P, 1, ..., n\}$ be the decision-makers for y_1 and y_2, respectively, such that $i = \{1, ...n\}$ and $j = \{j', j''\}$. Let $k_r^j \equiv k_r^*(\mathbf{m}_j^*(\mathfrak{s}^*))$ be the number of truthful messages decision-maker j receives in equilibrium, and $k_r' \equiv k_r^*(j')$ and $k_r'' \equiv k_r^*(j'')$ refer to the number of truthful messages to decision-makers j' and j'' (respectively).

[20] In principle, agents could have the choice on how much information about each state to observe, involving information acquisition at the intensive and the extensive margins. Here, however, I focus on the extensive margin, meaning that each agent decides whether to observe at most one binary signal per state. In section 2.6, I discuss some implications of allowing agents to acquire information on the intensive margin.

[21] The principal's preferences are captured by $U^P(\boldsymbol{\theta}, \mathbf{x}) = -\big(y_1 - \delta_1(\theta_1, \theta_2)\big)^2 - \big(y_2 - \delta_2(\theta_1, \theta_2)\big)^2$.

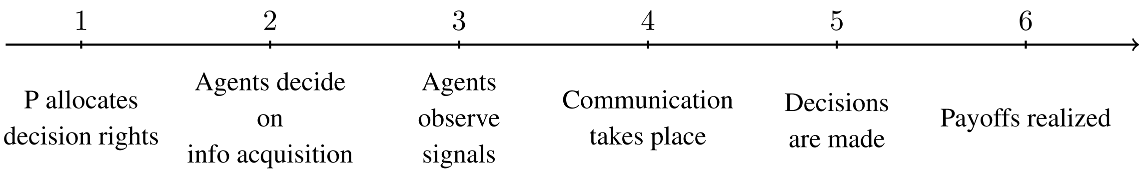


Figure 2.3: Timing of the Org. Structure / Info Acquisition game.

An equilibrium in this game is then characterized by the decision vector, $\mathbf{y}_d^*$, and collections of messagse and information acquisition strategies for each agent and decision-maker j, $\mathbf{m}_j^* = \{\ldots, \mathbf{m}_j^{i*}, \ldots\}$ and $\mathfrak{s}^* = \{\ldots, \mathfrak{s}^{i*}, \ldots\}$. The expressions for optimal actions and messages are similar to those of the previous section, noting that $k_r^*(\mathbf{m}^*(\mathfrak{s}^*))$, $y_d^*(\mathbf{m}_j^*(\mathfrak{s}^*))$, and $\mathbf{m}_j^{*i}\left(\mathfrak{s}^i, \mathbf{m}^{-i}(\mathfrak{s}^{-i})\right)$. Agent i's information acquisition strategy is given by:

$$\mathfrak{s}^{i*} = \arg\max_{\mathfrak{s}^i} \left\{ E\Big[-\left(y_1\left(\mathbf{m}_{j'}^i(\mathfrak{s}^i), \mathbf{m}_{j'}^{-i}\right) - \delta_1 - b_1^i\right)^2 - \left(y_2\left(\mathbf{m}_{j''}^i(\mathfrak{s}^i), \mathbf{m}_{j''}^{-i}\right) - \delta_2 - b_2^i\right)^2 \Big] - C(\mathfrak{s}^i) \right\}$$

The expectation is based on equilibrium beliefs. Agent i's equilibrium message strategy depends on the information he acquired in an earlier stage of the game and his conjecture about other agents' message strategies. Information acquisition are observable at the communication stage, which simplifies the beliefs space. The signals i acquired thus affect his communication strategy. When he acquires information about both states, the IC constraints for communication are the same as in the previous section. When he acquires information about one state, however, the IC constraints change significantly because his incentives to reveal information are not affected by beliefs about the other state. This kills the credibility loss and truthful communication is incentive compatible for a larger set of bias vectors. Now, let me focus on the details of these arguments.

Incentives to acquire information. For an agent to acquire a piece of information the expected utility gains must compensate its costs. First, costly information acquisition means that each agent will invest in a signal only if he expects to benefit from it. In equilibrium i only acquires signals he is willing to reveal; if he fails to reveal any piece of information (off path), no other agent will change his equilibrium message strategy.[22] The number of truthful messages does not change when i acquires a signal he does not reveal, but he still bears the costs. The lemma below formalizes this intuition—incentive compatibility at the information acquisition stage requires incentive compatibility at the communication stage.

Lemma 2.1. *Let $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$ be equilibrium strategy profiles for the principal and all agents. The equilibrium is characterized by the number of truthful messages decision-makers receive, $k_1^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$ and*

[22] Formally, i's incentives for communication depend on having acquired the signal, $\mathbf{b}^i$, and on his conjecture about k_1^j and k_2^j. Then, for i acquiring S_r^i *off-path* to change another agent h's conjecture about k_r^j, b^i should be such that he is willing to reveal that signal. In such a case, h (off-path) conjecture for k_r^j should be larger than the equilibrium value, but then i would be willing to reveal S_r^i in equilibrium and would have acquired it.

$k_2^j\left(\mathbf{m}_j^*(\mathfrak{s}^{i*})\right)$, *for* $j = \{j', j''\}$. *Then, agent* i *equilibrium information acquisition strategy,* $\mathfrak{s}^{i*}$*, satisfies:*

- $S_r \in \mathfrak{s}^{i*}$ *only if truthful revelation to* j *is incentive compatible, given* $k_r^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$*;*
- $\{S_1, S_2\} \in \mathfrak{s}^{i*}$ *only if full revelation to* j *is incentive compatible, given* $k_1^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$ *and* $k_2^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$.

The main implication of Lemma 2.1 is that the choice of organizational structure will affect agents' incentives for information acquisition, because it determines the relevant IC constraints at the communication stage. Incentives to acquire information depend on the possibility of being influential. But credibility hinges on both the conflict of interest and the number of other agents expected to reveal similar information on path. Agents thus acquire information they expect to reveal on the equilibrium path, given the allocation of decision rights. The conclusion is similar to Di Pei (2015): information structures available to agents and the cost function satisfy assumptions 1 and 2 of his paper ("Richness" and "Monotonicity"). Both seem rather natural assumptions in my framework; for a given information structure a 'coarser' alternative means investing in less signals, which will also be cheaper than the original choice.

The second element of incentive compatibility of information acquisition relates to its costs. Utility gains from revealing any piece of information are decreasing in the number of other agents revealing the same information (k_r^j). Given costs are strictly positive, there is a maximum number of agents for whom the utility gains of revealing that piece of information compensate the costs. The following lemma presents the *cost-effectiveness* condition, which captures this idea.

Lemma 2.2. *Let* k_r^j *denote* i*'s conjecture about other agents revealing* S_r *truthfully to* j.

Centralization: *acquiring signal* S_r^i *is cost-effective for* i *under centralization if:*

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^{\mathrm{C}} + 2)(k_r^{\mathrm{C}} + 3)} \tag{2.5}$$

Delegation: *acquiring signal* S_r^i *is incentive compatible for agent* i *if for at least one decision,* y_d*, with the corresponding decision-maker* j *is true that:*

$$C(S_r^i) \leq \frac{(w_{dr})^2}{6(k_r^j + 2)(k_r^j + 3)} \tag{2.6}$$

The right-hand sides in equations (2.5) and (2.6) represent the ex-ante expected utility gains from revealing one signals under centralization and delegation, respectively. An agent acquires a signal if its expected influence on decision(s) is sufficiently large. The influence of revealing a signal depends on how many other agents are expected to reveal the same information, which in turn depends on the organizational structure. Under centralization, revealing a given signal influences both decisions, which is reflected in the numerator of equation (2.5). Under delegation, the influence depends on whether i reveals the signal to one or both decision-makers. The ex-ante expected utility gains of acquiring (and revealing) a given signal are thus weakly lower under delegation.

Dimensional non-separable message strategies face a more restrictive cost-effectiveness condition because i expects to reveal information for half of the possible signal realizations. The costs of acquiring both signals must then be sufficiently low for such a strategy to be cost-effective. The latter does not hold if revealing one signal is also incentive compatible for i, in which case the most-informative equilibrium consists of acquiring and revealing information about one state.

Because the expected influence of truthful revelation is decreasing in the number of other agents revealing the same information, there exists a maximum number of agents for whom cost-effectiveness holds with respect to any signal.

Corollary 2.2. *The maximum number of agents acquiring S_r in any equilibrium under centralization is given by:*

$$K_r^{\mathrm{C}} = \left\lfloor \left[\frac{1}{4} + \frac{[(w_{1r})^2 + (w_{2r})^2]}{6\,C(S_r^i)}\right]^{1/2} - \frac{5}{2} \right\rfloor + 1 \tag{2.7}$$

And under delegation, the maximum number of agents acquiring S_r in any equilibrium is:

$$K_r^{\mathrm{D}} \in \left[\left\lfloor \left[\frac{1}{4} + \frac{(\hat{w}_{dr})^2}{6\,C(S_r)}\right]^{1/2} - \frac{5}{2} \right\rfloor + 1\,;\, K_r^{\mathrm{C}} \right] \tag{2.8}$$

Where $\hat{w}_{dr} \equiv \min\{w_{1r}, w_{2r}\}$

Expressions (2.7) and (2.8) represent the maximum number of agents, other than i, for whom investing in a signal is cost-effective under centralization (K_r^{C}) and delegation (K_r^{D}), respectively. These numbers depend on whether revealing information influences both decision or only one of them. Note that under centralization any influential agent affects both decisions; while, under delegation, the same is true only for agents revealing information to both decision-makers. Typically, however, many agents under delegation will reveal information to one decision-maker, which would make $K_r^{\mathrm{C}} > K_r^{\mathrm{D}}$.

I now discuss the equilibrium information acquisition and message strategies and how they are affected by the allocation of decision rights. I also present the analysis on specialization.

2.4.1 Specialization

Equilibrium information acquisition strategies depend on the organizational structure and the cost of information. Cost-effectiveness imposes restrictions in addition to incentive compatibility at the communication stage, which can lead agents to underinvest in information—relative to what they would be willing to reveal. In the following paragraphs, I analyse how this leads to specialization, here defined as the case in which an agent decides to acquire information about one state only.

Individual decisions on information acquisition are equivalent to choosing between message strategies given other agents' equilibrium behaviour. In particular, when agent i acquires information about one state only, his incentives for communication do not depend on information about the other state (because he does not observe any). This eliminates the credibility loss due to ambiguous information, thus enlarging the set of biases for which revealing that signal is incentive compatible. The proposition below shows the result.

Proposition 2.4. *Let* $\mathbf{k}^j = \{k_1^j, k_2^j\}$*, where* $k_r^j = k_r^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$ *be i's conjecture about other agents revealing their information about* $\theta_r = \{\theta_1, \theta_2\}$ *to decision-maker* $j = \{P, 1, ..., n\}$*. There exists a set of bias vectors,* $\mathfrak{B}_r^j = \mathfrak{B}_r^j(\mathbf{b}^j, \mathbf{k}^j)$*, such that if* $\mathbf{b}^i \in \mathfrak{B}_r^j$*, then revealing information about* θ_r *only is incentive compatible when* $\mathfrak{s}^i = \{\tilde{S}_r^i\}$*, but is not incentive compatible when* $\mathfrak{s}^i = \{\tilde{S}_1^i, \tilde{S}_2^i\}$*. Moreover, the set* $\mathfrak{B}_r^j$ *depends on the organizational structure.*

Acquiring information about one state eliminates the possibility of ambiguous information. Hence, agent i is not tempted to lie when information about the other state is more favourable: revealing information about that state is thus incentive compatible for a larger set of biases. In other words, *specialization* acts as a commitment device because the agent does not know when an unfavourable signal produces an excessive update against his preferences.

Proposition 2.4 also implies that, for a given set of biases, the principal prefers that agent i specializes, even when he has free access to information.

Corollary 2.3. *Let* $C(S_1^i) = C(S_2^i) = 0$*. If agent i's preferences satisfy:*

$$|\beta_1^i| \in \left(\frac{(w_{11})^2 + (w_{21})^2}{2}\left[\frac{1}{(k_1^{\mathrm{C}}+3)} - \frac{C_1}{(k_2^{\mathrm{C}}+3)}\right] \; ; \; \frac{(w_{11})^2 + (w_{21})^2}{2\,(k_1^{\mathrm{C}}+3)}\right]$$

Then, in the most informative equilibrium under centralization he acquires and truthfully reveals information about θ_1 *only.*

The principal prefers a less informed agent because it guarantees he will not be tempted to manipulate information. Note that the two results above hinge on the assumption that information acquisition decisions are observable. In Section 2.6, I discuss the implications of relaxing it and show that specialization still increases credibility when the cost of information is not too low. These results have implications for how firms organize subunits' access to information, since increasing the cost of some types of information may improve the quality of communication.

I now analyse the different conditions that induce an agent to specialize, in an example with two agents. Let assume $w_{11} = w_{22} = w > \frac{1}{2}$ and $C(\mathfrak{s}^i) = c \times (\#\,\mathfrak{s}^i)$, and let denote the agents by A^1 and A^2. I focus on the centralization equilibrium in which A^1 acquires information about θ_1 and A^2 acquires information about θ_2, $\mathfrak{s}^{1*} = \{\tilde{S}_1^1\}$ and $\mathfrak{s}^{2*} = \{\tilde{S}_2^2\}$. In the equilibrium under consideration, the principal is (ex-post) more informed than each of the agents since $k_1^* = 1$ and $k_2^* = 1$.[23] The Proposition below formalizes the result and panel (a) in Figure 2.4 illustrates the set of biases for which $\mathfrak{s}^{1*} = \{\tilde{S}_1^1\}$ under centralization.

[23] The paper by Alonso et al. (2015) analyses a similar situation in the form of generalist-specialist information structure, where each agent specializes in a different piece of information and fully transmits it to the principal.

Proposition 2.5 (Specialization under centralization)**.** *Suppose that there are only two agents, A^1 and A^2, and the marginal cost of each signal is linear and equal to c. There exist two cost thresholds, $\underline{c} < \bar{c}$, such that the most-informative equilibrium under centralization, $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$, consists in A^1 acquiring and revealing information on θ_1 only, and A^2 acquiring and revealing information about θ_2 only, in the following cases:*[24]

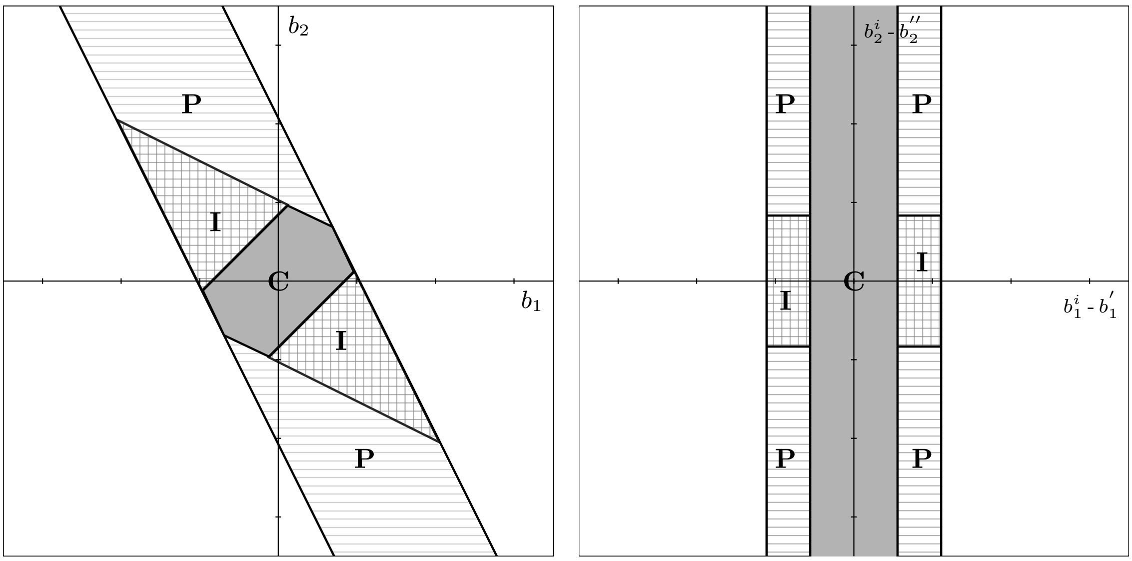
1. *For $c \leq \underline{c}$, if and only if revealing S_1^1 is IC for A^1, revealing S_2^2 is IC for A^2, and revealing both signals is not IC for any of them; or*

2. *For $\underline{c} < c \leq \bar{c}$, if and only if revealing S_1^1 is IC for A^1 and revealing S_2^2 is IC for A^2.*

Where $\underline{c} = \frac{w^2+(1-w)^2}{72}$ *and* $\bar{c} = \frac{w^2+(1-w)^2}{36}$.

When the cost of a signal is close to zero, information acquisition does not impose restrictions on communication. Agents will then acquire any information they are willing to reveal and, thus, they specialize on different signals only if these are the only IC message strategies. The stripped region in panel (a) of Figure 2.4 shows specialization *driven by preferences*.

Figure 2.4: Specialization in the two-agents model — Driven by preferences (**P**), influence (**I**), and costs (**C**)



(a) A^1 specializes on θ_1 – Centralization

(b) A^1 specializes on θ_1 – Delegation

Notes: $w = \frac{2}{3}$ and $k_1^* = k_2^* = 1$

The cross-hatched region in panel (a) represents the set of biases for which A^1 is willing to reveal information about any state if he only observes the associate signal, but this does not hold if he observes information about both states. In the equlibrium under considerantion, he expects A^2 to acquire and

[24]Formally, the equilibrium consists in $\mathfrak{s}^{1*} = \{\tilde{S}_1^1\}$ and $m^{1*} = \big\{\{(0,0),(0,1)\},\{(1,0),(1,1)\}\big\}$ for A^1, and $\mathfrak{s}^{2*} = \{\tilde{S}_2^2\}$ and $m^{2*} = \big\{\{(0,0),(1,0)\},\{(0,1),(1,1)\}\big\}$ for A^2.

reveal information about θ_2, such that the expected utility gains is larger when he specializes on θ_1. This specialization decision is *driven by expected larger influence* on the principal's beliefs, given the equilibrium strategy of the other agent.[25]

Now suppose preferences of A^1 are such that he is willing to reveal both signals under centralization. Specialization only emerges if acquiring such information is too costly. Whether he acquires information about θ_1 or θ_2 depends on what A^2 is expected to do: A^1 acquires information about the state the principal is expected to be less informed on path. The solid gray region in panel (a) illustrates the case of specialization *driven by costs*.

Specialization under delegation follows the same intuitions. I describe them using Panel (b) in Figure 2.4. Recall that information about θ_1 affects y_1 more than y_2. Therefore, A^1 specializes on θ_1 when his preferences on the first dimension are sufficiently close to j'. Indeed, whenever his preferences are close to the decision-maker of the second dimension, A^1 prefers to acquire information about θ_2. The intuitions for the different drivers of specialization—preferences, influence, and costs— are the same as for centralization, and are illustrated in the different regions of panel (b).

In the following section, I analyse the effects of different organizational structures on incentives to acquire information.

2.4.2 Information Acquisition and the Organizational Structure

The optimal organizational structure depends on both the profile of biases and the cost of information. In the previous section I showed that an agent acquires a signal if the expected utility gains from revealing it compensate its cost. If information costs are small, the principal can materialize the informational gains from delegation studied in Section 2.3. For sufficiently large costs, however, centralization is always optimal, since at the limit no agent can aquire any signal.

In this section, I study how the allocation of decision rights affects incentives to acquire information when the exact profile of biases is not known by the principal. Abstracting from the precise profile of biases circumvents the fact that information acquisition depends on how many other agents are willing to reveal each piece of information. In other words, for sufficiently small costs, there always exists a profile of biases for which delegation outperforms centralization in terms of information acquisition. This conclusion, however, stems from the analysis in section 3 more than from new insights on the effects of organizational structure on investment. From an empirical perspective, the analysis captures the idea that institutions may have persistent informational effects on outcomes. In policy-making, for instance, the allocation of authority over a set of issues to a governmental agency effectively grants policy influence to interest groups linked to it (Baumgartner and Jones, 2009). Part of that influence is about 'feeding' the agency with information likely to be more favourable to the groups' preferences. Hence, the policy becomes too responsive to these issues but insensitive to other issues it should be taking into account.

[25] An alternative equilibrium exists when both agents' bias vectors lie on cross-hatched regions. The strategies $\mathfrak{s}^{1*} = \{\tilde{S}_2^1\}$ and $\mathfrak{s}^{2*} = \{\tilde{S}_1^2\}$ can also be sustained; agents thus face a coordination problem for which there is no clear selection criterion—the principal is ex-ante (and ex-post) indifferent between any of these. Both equilibria involve specialization mainly because no agent is willing to reveal both signals.

I first analyse whether delegation affects the *expected absolute investment in information*, by focusing on the maximum number of agents for whom acquiring a signal is cost-effective. Second, I analyse whether delegation affects the *expected relative investment in information*, by assuming the bias vectors are randomly allocated within a fixed conflict of interest with the principal.

Absolute Investment in Information

When information is costly for agents, there is a limit on the informational gains the principal can obtain from delegation (Corollary 2.2). Through this channel, costs affect the ability of different organizational structures to aggregate information. The following result shows that information costs impose stronger restrictions on delegation than centralization.

Proposition 2.6. *Let κ be the maximum number of agents willing to reveal information about $\theta_r = \{\theta_1, \theta_2\}$ to both decision-makers under delegation. Then, for every $\kappa < n$, there exist costs for which the maximum number of agents willing to acquire (and reveal) information about θ_r is strictly lower under delegation than under centralization. Formally,*

$$C(S_r) \in \left(\frac{(\hat{w}_{dr})^2}{6(\kappa+2)(\kappa+3)}, \frac{(w_{1r})^2 + (w_{2r})^2}{6(\kappa+3)(\kappa+4)} \right] \;\Rightarrow\; K_r^{\mathrm{C}} > K_r^{\mathrm{D}}$$

Where $\hat{w}_{dr} = \max\{w_{1r}, w_{2r}\}$.

When information is costly, the maximum informational gains under delegation are weakly lower than under centralization. Under delegation, there can be agents who reveal information to one decision-maker only, unless there is an agent whose preferences are central in both dimensions (and these are not too far from the principal's preferences). Under centralization, on the contrary, any information transmitted affects both decisions, and agents willing to reveal any signal have larger overall influence. The expected utility gains for such agents are thus larger under centralization and there will typically be more of them willing to invest in information.

Proposition 2.6 does not mean that centralization is always optimal. For a non-empty set of cost values, there exist profiles of biases, $\mathbf{b}$, for which delegation of some sort is preferred to centralization ($K_r^{\mathrm{C}} > K_r^{\mathrm{D}}$ is not binding). But for sufficiently large costs, centralization always dominates (see Corollary A2.1). This is the case when the cost of each signal is so high that no agent acquires information under delegation.

In addition, Proposition 2.6 relates to the relationship between loss of control and informational gains in Proposition 2.3. In the previous section we learned that $||\mathbf{b}^{\mathrm{D}}||$ is increasing in the informational gains from delegation. In the present section I showed that information costs impose limits on informational gains; now I will show how $||\mathbf{b}^{\mathrm{D}}||$ is affected by costs. The maximum bias as a function of K^{D} (maximum informational gains) is given by:

$$||\mathbf{b}^{\mathrm{D}}|| = \left[\frac{[w^2 + (1-w)^2]}{6} \left[1 - \frac{2}{(K^{\mathrm{D}}+2)} \right] \right]^{\frac{1}{2}}$$

Which together with equation (2.8) in Corollary 2.2 leads to the following result.

Corollary 2.4. *The effect of information costs on $||\mathbf{b}^{\mathrm{D}}||$ is given by:*

$$\frac{\partial||\mathbf{b}^{\mathrm{D}}||}{\partial C(S_r)} < 0$$

The maximum admissible bias the principal tolerates decreases as the cost of information increases. Increasing costs reduces the number of agents willing to acquire any signal, decreasing the informational gains that can be achieved through delegation. This is relevant when information costs are large and the principal is not sure about the exact profile of biases at the moment of allocating decision rights. In such a case, she can expect the benefits from delegation to be low relative to centralization. This relationship between the information costs and distributional loss could be related to information acquisition on the intensive margin—the higher the costs, the lower the amount of information a single agent is expected to acquire and, thus, the lower his 'informational advantage' with respect to the principal. I explore this intuition in Section 2.6.

Relative Investment in Information

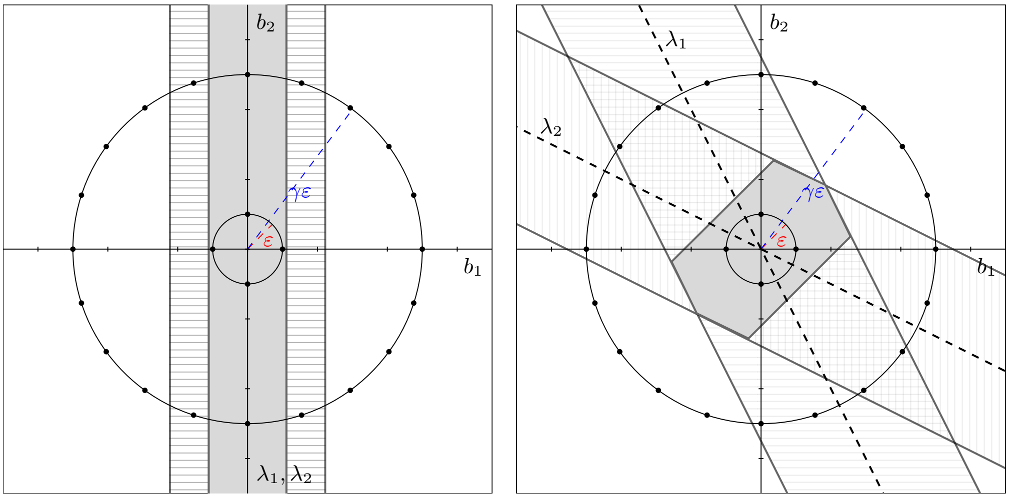
I now analyse whether the allocation of decision rights affects the relative investment in information—the amount of information about each state decision-makers are expected to receive. In order to isolate the mechanism, I make two assumptions. First, I assume the principal does not observe the exact distribution of biases when deciding on the allocation of decision rights. As in the previous analysis, this assumption allows me to abstract from the influence of the specific profile of biases. Second, I assume zero cost of information, which eliminates the influence of the costs on incentives to acquire information.

Consider the case of delegation. Agent i's incentives for communication with decision-maker j', in charge of y_1, are maximal when the conflict of interest is zero, that is $b_1^i - b_1' = 0$. More generally, any conflict of interest between i and j can be expressed as the 'horizontal distance' between $\mathbf{b}^i$ and $\mathbf{b}^j$. Communication of a given piece of information is then incentive compatible for i if this distance is small enough or, equivalently, the distance between $\mathbf{b}^i$ and the 'maximal incentives for communication' is small enough. In the Appendix I show this distance can be expressed as the projection of $\mathbf{b}^i$ onto the line that represents the maximal incentives for communication when j' decides on y_1.

Because θ_1 is more important for y_1 than θ_2, j' expects to receive more information about the former than about the latter (Lemma A2.2). In other words, under delegation, each decision-maker expects to receive proportionally more information about the state that is more important for the decision he controls. This can be seen in Panel (a) of Figure 2.5, for fixed conflict of interests ε and $\gamma\,\varepsilon$, and where the maximal incentives to reveal information about θ_1 (θ_2) are denoted by λ_1 (λ_2).

Now, consider the case of centralization. As in the case of delegation, the conflict of interest between i and the principal can be measured as the distance between $\mathbf{b}^i$ and the 'maximal incentives' for communication. Here, however, the maximal incentives for communication with the principal depend on the information i is expected to reveal. For instance, when expected to reveal information about θ_1 only, i's communication incentives are maximal when $\beta_1^i = w_{11}b_1^i + w_{12}b_2^i = 0$. Because there are different measures of 'maximal incentives', for a fixed conflict of interest, there are always agents willing to reveal information about θ_1 *and* θ_2. This is shown in panel (b) of Figure 2.5 by the two lines λ_1 and λ_2, which represent the maximal incentives to reveal information about θ_1 and θ_2, respectively. The difference between λ_1 and λ_2 implies that for any agent i who is willing to truthfully reveal S_1^i on path, there exista an agent h who is willing to reveal S_2^h on path (Lemma A2.3).

Figure 2.5: Information transmission under different organizational structures



(a) Delegation of y_2 (b) Centralization

Full Revelation S_1 only S_2 only Babbling
$(0, 0)$ and $(1, 1)$ only $(0, 1)$ and $(1, 0)$ only

Note: $w_{11} = w_{22} = \frac{2}{3}$

In Section 2.3 I showed that negative informational spillovers can provide a just foundation for partial delegation. This argument requires that the principal knows the profile of biases of informed agents, $\mathbf{B}$. When she does not observe $\mathbf{B}$, however, delegation may lead to losing control over payoff-relevant information, well beyond the distributional loss. In particular, each decision-maker under delegation is expected to receive more information on the more relevant state, becoming an *ex-post specialist*. I formalize this argument in the proposition below.

Proposition 2.7. *Let $w_{11} = w_{22} = w$. For a sufficiently large number of agents, n; for any conflict of interest $\varepsilon \in \Re_+$, such that the profile of biases, $\mathbf{B} = \{\mathbf{b}^1, ..., \mathbf{b}^n\}$, is uniformly distributed between $[0, \varepsilon]$ for agents $i = \{1, .., n\}$; for any integer κ. Then, the principal expects to receive more balanced information under centralization than when she delegates any decision to agent j; that is,*

$$|E\left[Var(\theta_1|\mathbf{m}_C^*)\right] - E\left[Var(\theta_2|\mathbf{m}_C^*)\right]| < \left|E\left[Var(\theta_1|\mathbf{m}_j^*)\right] - E\left[Var(\theta_2|\mathbf{m}_j^*)\right]\right|$$

Because of informational interdependence, the influence of revealing any signal is larger under centralization. When a principal delegates a decision, this influence decreases and, also, varies depending on the dimension in which communication is expected to take place. For instance, the decision-maker of y_1 expects to receive more information about θ_1 because expect to have a higher influence by revealing the associated signal. This leads to decision-makers being *ex-post* specialized on the issues that are more relevant for a decision. Proposition 2.7 shows this specialization in the form of higher expected precision of beliefs about the most important state for each decision.

Ex-post specialization of decision-makers under delegation is not in itself an undesired outcome. The increased precision associated with the most important state could, in principle, compensate for the lower precision of the other. Proposition 2.6, however, shows that the opposite happens: the expected absolute investment in information is (weakly) lower under delegation. These two results highlight an important limitation of delegation that goes beyond the distributional loss: the negative effects on incentives to acquire information. Hence, organizational design that aims at providing incentives to invest in information must consider the negative 'long run' informational consequences of delegation.

2.5 Covert Information Acquisition

Suppose information acquisition decisions are private information of each agent. I focus on the centralization case, restricting the analysis to pure strategies at the information acquisition and communication stages. It can be shown that focusing on equilibria in which messages do not convey information about the acquisition decision is without loss (Lemma C2.4).[26] As a result, messages sent at the communication stage do not convey information about decisions on information acquisition. Also, any deviation at the information acquisition stage results in a deviation (from truth-telling) at the communication stage (see Lemma C2.5).

In the appendix I derive the IC constraints for a typical agent i in the covert information acquisition game. There are two relevant deviations: acquiring fewer or more signals than on the equilibrium path. First, when agent i deviates by *acquiring fewer signals* than on path, he saves on information costs but he induces lower-than-optimal precision on beliefs. Suppose i is expected to acquire information about both states on path; if, instead, he decides to acquire information about θ_1 only, his message corresponding to θ_2 will induce wrong beliefs for half of its possible realizations. Because his message strategy off-path consists of announcing the most favourable of the possible realizations of S_2^i, the deviation is equivalent to lying on this signal. Incentive compatibility, hence, requires that these utility gains—lying towards his bias *plus* saving on information costs| are lower than the expected utility losses from inducing a higher expected variance.[27] Incentive compatibility constraints in the covert game are thus more restrictive than in the overt game.

The second deviation consists of *acquiring more signals* than on path. When i is expected to acquire and reveal information about θ_1, he can profitably deviate by also acquiring information about θ_2. But i cannot transmit information about θ_2 on path. Hence, the expected utility gains have to do with ambiguous information. This means he lies about θ_1 when his information is unfavourable and that about θ_2 is favourable.

[26] Also, see Lemma 3 in Argenziano et al., 2016.
[27] See equations (C2.10), (C2.11), and (C2.12).

If information costs are sufficiently low, i cannot commit himself to acquiring information about θ_1 only. The result below shows when he *can* commit.

Proposition 2.8. *Let* $(\mathfrak{s}^*, \mathbf{m}^*, \mathbf{y}^*)$ *characterize an equilibrium in the covert game under centralization, and let* $\mathbf{k}^{\mathrm{C}} = \{k_1^{\mathrm{C}}, k_2^{\mathrm{C}}\}$ *be agent i's equilibrium conjecture about other agents truthfully revealing information. Denote by* $\mathfrak{B}_r^i = \mathfrak{B}_r\left(C(S_1^i), C(S_2^i), \mathbf{k}^{\mathrm{C}}\right)$ *the set of biases for which acquiring and revealing information about θ_r is incentive compatible for agent i, where $\theta_r, \theta_{\tilde{r}} = \{\theta_1, \theta_2\}$ such that $\theta_r \neq \theta_{\tilde{r}}$.*

1. $\mathfrak{B}_r^i \neq \emptyset$ *if and only if* $\frac{(w_{1r}^2 + w_{2r}^2)}{2(k_r^{\mathrm{C}}+3)^2} - \max\left\{C(S_r^i)\,;\, \frac{(w_{1r}w_{1\tilde{r}} + w_{2r}w_{2\tilde{r}})}{2(k_r^{\mathrm{C}}+3)(k_{\tilde{r}}^{\mathrm{C}}+3)} - 2C(S_{\tilde{r}}^i)\right\} > 0$*;*
2. $\frac{\partial \mathfrak{B}^{\mathrm{i}}{}_r}{\partial C(S_r^i)} < 0$*;*
3. $\frac{\partial \mathfrak{B}_r^i}{\partial C(S_{\tilde{r}}^i)} > 0$.

Acquiring information about θ_1 in the covert game is incentive compatible for i if and only if the utility gains are sufficiently large. As in the overt game, these utility gains must compensate for the cost of acquiring the corresponding signal, $C(S_1^i)$. Unlike the overt game, however, the utility gains from increasing the precision of the principal's beliefs must compensate for the utility gains associated with having ambiguous information (rightmost term inside the curly brackets). In other words, if acquiring information about θ_2 is cheap for i, he will acquire S_2^i and lie to the principal whenever this signal favours his interests and S_1^i goes against them. Incentive compatibility, hence, requires that the cost of the signal associated with the state is low and the cost of that associated with the other state is sufficiently high.

The relationship between incentive compatibility and the costs of the different signals is shown in the last two statements of Proposition 2.8. Because of the first type of deviations—acquiring fewer signals— i's incentives to acquire and reveal information about θ_1 decrease as the cost of S_1^i increases; saving on the cost of the signal becomes more profitable for i. The second type of deviations—acquiring more signals— leads to an increase in credibility when the cost of S_2^i increases. This effect stems from the credibility loss due to ambiguous information.

2.6 Concluding remarks

Most organizations operate in complex environments: they face multi-causal problems and solutions involve many interrelated courses of action. Because actions address the causes with different degrees of success, relevant information affects many of these actions. In addition, the information is typically dispersed among the organization's members, who communicate it strategically. This paper has studied how information is acquired and aggregated under such complexity. I showed that the allocation of decision rights constitutes a key tool to govern the conflict of interests in an organization. In particular, I found a principal may want to delegate controversial decisions if that improves transmission of information on other, less controversial ones. When preferences over all decisions are extreme, centralization can 'discipline' these conflict of interests such that more information is transmitted. I have shown that complexity affects incentives to acquire information under different organizational structures. Under delegation, expected investment in information is not only lower overall but also more concentrated on issues that are salient for the corresponding decision. The analysis presented here has broad applications to the organization of policy-making bodies, advisory committees, knowledge creation in multinational corporations, and other settings where information needs to be obtained and communicated in complex environments.

Appendix

A2 Proofs and Complementary Results

Equilibrium communication under delegation

Lemma A2.1 (Incentive Compatibility of Communication on y_d.). *Consider an equilibrium $(\mathbf{y}^*, \mathbf{m}^*)$ in which the principal delegates y_d to agent j (and he does not decide on the other decision).*[28] *Then, revealing any information (either S^i_r or both signals) is incentive compatible for i if:*

$$|b^i_d - b^j_d| \leq \frac{1}{2}\left|\frac{w_{d1}}{(k^j_1+3)} - \frac{w_{d2}}{(k^j_2+3)}\right| \tag{A2.1}$$

And revealing both signals when $\tilde{\mathbf{S}}^i\{(0,0);(1,1)\}$ and announcing the babbling message for other realizations if:

$$|b^i_d - b^j_d| \leq \frac{1}{4}\left[\frac{w_{d1}}{(k^j_1+3)} + \frac{w_{d2}}{(k^j_2+3)}\right] \tag{A2.2}$$

Where $y_d = \{y_1, y_2\}$.

Proof. All proofs of results in this Appendix can be found in B2 □

The proposition below summarizes the equilibrium communication in the case of delegation.

Proposition A2.1 (Equilibrium Communication for y_d). *Let agent j be the decision-maker of y_d. In the most-informative equilibrium $(\mathbf{y}^*, \mathbf{m}^*_{j'}, \mathbf{m}^*_{j''})$ between agents i and j, i fully reveals his information if and only if condition (A2.1) hold. If the right-hand side of (A2.2) is larger than that of (A2.1), then agents with $|b^i_d - b^j_d|$ within these two values reveal both signals when $(\tilde{S}^i_1, \tilde{S}^i_2) = \{(0,0),(1,1)\}$ and send the corresponding babbling message otherwise. For other values of $\mathbf{b}^i$, i always send the babbling message consistent with his bias.*

Equilibrium communication in the case of one decision is characterized by the IC constraint in Lemma A2.1. As already discussed, full revelation dominates message strategies in which i reveals one signal because the IC constraints are the same and the decision-maker always prefer the former. The same happens with dimensional non-separable message strategies for most parameter values, so they can be overlooked in this case with little loss. Now is time to analyse communication between i and the principal when she retains authority over both decisions.

[28] By assumption, communication between decision-makers only involves own signals (not information transmitted by other agents). For an analysis on hierarchies as information intermediation see Migrow (2017).

PROOF OF EQUATION (2.1). The principal's ex-ante expected utility in the equilibrium characterized by by $\{\mathbf{y}, \mathbf{m}_{j'}, \mathbf{m}_{j''}\}$ is given by:

$$E\left[U^{\mathrm{P}}(\boldsymbol{\delta}, \mathbf{b}); \mathbf{m}\right] = -E\left[(y_1 - \delta_1)^2; \mathbf{m}_{j'}\right] - E\left[(y_2 - \delta_2)^2; \mathbf{m}_{j''}\right]$$

Which, by definitions of equilibrium y_d and δ_d yield:[29]

$$E\left[U^{\mathrm{P}}(\boldsymbol{\delta}, \mathbf{b}); \mathbf{m}\right] = -(b_1')^2 - (b_2'')^2 - \sum_{y_d=\{y_1,y_2\}} E\left[\Big(w_{d1}\big(E(\theta_1|\mathbf{m}_j) - \theta_1\big) + w_{d2}\big(E(\theta_2|\mathbf{m}_j) - \theta_2\big)\Big)^2\right]$$

With some rearrangement and given $\theta_1 \perp \theta_2$, I have:

$$E\left[U^{\mathrm{P}}(\boldsymbol{\delta}, \mathbf{b}); \mathbf{m}\right] = -\left[(b_1')^2 + (b_2'')^2\right] - \sum_{y_d=\{y_1,y_2\}} \sum_{\theta_r=\{\theta_1,\theta_2\}} \left[(w_{dr})^2 \, E\left[\big(E(\theta_r|\mathbf{m}_j) - \theta_r\big)^2; \mathbf{m}_j\right]\right] \quad \text{(A2.3)}$$

Now, the expectation of the squared deviation for each state is given by:

$$\begin{aligned}
E\left[\big(E(\theta_r|\mathbf{m}_j) - \theta_r\big)^2; \mathbf{m}_j\right] &= \int_0^1 \sum_{\ell_r=0}^{k_r^j} \Big(E(\theta_r|\mathbf{m}_j) - \theta_r\Big)^2 f(\ell_r^j|k_r^j, \theta_r)\, d\theta_r \\
&= \int_0^1 \sum_{\ell_r=0}^{k_r^j} \Big(E(\theta_r|\mathbf{m}_j) - \theta_r\Big)^2 \frac{h(\theta_r|\ell_r^j, k_r^j)}{(k_r^j+1)}\, d\theta_r \\
&= \frac{1}{(k_r^j+1)} \sum_{\ell_r^j=0}^{k_r^j} \int_0^1 \Big(E(\theta_r|\mathbf{m}_j) - \theta_r\Big)^2 h(\theta_r|\ell_r^j, k_r^j)\, d\theta_r \\
&= \frac{1}{(k_r^j+1)} \sum_{\ell_r^j=0}^{k_r^j} \mathrm{Var}(\theta_r|\ell_r^j, k_r^j) \\
&= \frac{1}{(k_r^j+1)} \sum_{\ell_r^j=0}^{k_r^j} \frac{(\ell_r^j+1)(k_r^j - \ell_r^j + 1)}{(k_r^j+2)^2(k_r^j+3)}
\end{aligned}$$

Solving the sum and plugging the above into (A2.3) yields:

$$\hat{U}^{\mathrm{P}}(\mathbf{B}, \mathbf{m}_{j'}, \mathbf{m}_{j''}) = -\left[(b_1')^2 + \frac{(w_{11})^2}{6\big(k_1'+2\big)} + \frac{(w_{12})^2}{6\big(k_2'+2\big)}\right] - \left[(b_2'')^2 + \frac{(w_{21})^2}{6\big(k_1''+2\big)} + \frac{(w_{22})^2}{6\big(k_2''+2\big)}\right]$$

[29] Note that the terms $E\left(E(\delta_d|\mathbf{m}_j) - \delta_d\right) b_d^j = 0$.

Let denote by $\text{Var}(\theta_r|\mathbf{m}_j) \equiv E\left[\left(E(\theta_r|\mathbf{m}_j) - \theta_r\right)^2; \mathbf{m}_j\right]$.

$$\begin{aligned}\hat{U}^{\text{P}}(\mathbf{B}, \mathbf{m}_{j'}, \mathbf{m}_{j''}) = &- \left[(b_1')^2 + (w_{11})^2 \text{Var}(\theta_1|\mathbf{m}_{j'}) + (w_{12})^2 \text{Var}(\theta_2|\mathbf{m}_{j'})\right] \\ &- \left[(b_2'')^2 + (w_{21})^2 \text{Var}(\theta_1|\mathbf{m}_{j''}) + (w_{22})^2 \text{Var}(\theta_2|\mathbf{m}_{j''})\right]\end{aligned}$$

□

Optimal Organizational Structure

Proposition A2.2 (Optimal Organizational Structure)**.** *Given the vector of preferences,* $\mathbf{B} = \{\mathbf{b}^1, \ldots, \mathbf{b}^n\}$, *and generic agents* i, j', *and* j''; *the organizational structure that maximizes the principal's ex-ante welfare is:*

Full Delegation. *That is, agents* j' *and* j'' *decide on* y_1 *and* y_2 *(resp.) if and only if:*

1. $DIG_{j'}(y_1) - (b_1')^2 > \max\left\{DIG_i(y_1) - (b_1^i)^2\,,\, IIG(y_1)\,,\, -\{DIG_{j''}(y_2) - (b_2'')^2\}\right\}$ *for any* $i \neq j'$; *and*
2. $DIG_{j''}(y_2) - (b_2'')^2 > \max\left\{DIG_i(y_2) - (b_2^i)^2\,,\, IIG(y_2)\,,\, -\{DIG_{j'}(y_1) - (b_1')^2\}\right\}$ *for any* $i \neq j''$.

Partial Delegation. *That is, agent* j *decides on* y_d *and the principal retains decision authority over* $y_{\tilde{d}}$; *if and only if there exist both Direct and Indirect informational gains such that:*

1. $DIG_j(y_d) - (b_d^j)^2 > \max\left\{DIG_i(y_d) - (b_d^i)^2\,,\, IIG(y_d)\,,\, -IIG(y_{\tilde{d}})\right\}$ *for any* $i \neq j$; *and*
2. $IIG(y_{\tilde{d}}) > \max\left\{DIG_i(y_{\tilde{d}}) - (b_{\tilde{d}}^i)^2\,,\, -\{DIG_j(y_d) - (b_d^j)^2\}\right\}$ *for any* $i \neq j$.

Centralization. *That is, the principal decides on both issues, if and only if there are no agent* i *and* j *such that:*

1. $DIG_j(y_d) - (b_d^j)^2 + IIG(y_{\tilde{d}}) > 0$; *nor*
2. $DIG_{j'}(y_1) - (b_1')^2 + DIG_{j''}(y_2) - (b_2'')^2 > 0$

Proof. The proof is constructive. The optimal organizational structure maximizes the principal's ex-ante expected utility. Optimality of delegation implies some informational gains, otherwise she can retain authority over both issues and decide with the information transmitted under centralization. Full Delegation is then optimal if there are two agents j' and j'' who decide on y_1 and y_2, respectively; such that the corresponding informational gains more than compensate each decision-maker's bias. These gains must be maximal among all agents, and strictly larger than if the principal retained any single decision (IIG).

Partial Delegation is optimal in either of two cases (non-exclusive). First, when direct informational gains from delegation are possible only on one decision, the principal prefers to retain authority on the other. If the DIG are sufficiently large, she may be willing to tolerate some informational losses on the retained decision; that is, receiving less information than under centralization.

The second and most interesting case is when indirect informational gains are large. From Proposition 2.2 we know that the presence of negative informational spillovers under centralization is a necessary condition. Delegating y_d thus breaks the interdependence between decisions and allows communication on the low-conflict dimension. This may hold even if there are no informational gains in the delegated decision, as long as the indirect ones are sufficiently large.

Finally, Centralization is optimal when any potential informational gain is small, such that it does not compensate the loss of control on the delegated decision(s). □

PROOF OF PROPOSITION 2.1. The characterization of the optimal organizational structure (Proposition A2.2 above) is a complement of this result. Informational gains arise when more agents reveal information to at least one decision-maker, as compared to those revealing information to the principal under centralization.

Consider direct informational gains first. For every agent who would reveal information under centralization, there must exist an agent revealing at least the same amount of information to the new decision-maker under delegation. Strict gains require that there also exist at least one agent revealing strictly more information to the new decision-maker. Let j' denote the decision-maker on y_1 and suppose the principal decides on y_2. For every other agent $h \in N$ such that β_1^h satisfies (1.1) there must exist a $i \in N$ such that b_1^i satisfies (A2.1); otherwise, k_1' will be lower than k_1^{C}.[30] At this point, there must also exist an agent such that:

$$|b_1^i w_{11} + b_2^i w_{21}| > \frac{1}{2}\left[\frac{(w_{11})^2 + (w_{21})^2}{(k_1^{\text{C}} + 3)} - \frac{w_{11}w_{12} + w_{21}w_{22}}{(k_2^{\text{C}} + 3)}\right]$$
$$|b_1^i - b_1^j| \leq \frac{1}{2}\left[\frac{w_{11}}{(k_1^j + 3)} - \frac{w_{12}}{(k_2^j + 3)}\right]$$

Multiplying the last inequality by w_{1r}, its right-hand side is strictly lower than that of the expression above. I thus get the condition that $|b_1^i - b_1^j| < |b_1^i + b_2^i \frac{w_{21}}{w_{11}}|$ for direct info gains from delegation.

Indirect informational gains mean that in equilibrium $k_2^{\text{P}2} > k_2^{\text{C}}$; the above equations must hold for y_2. An argument similar to the previous leads to: for every $h \in N$ such that β_2^h satisfies (1.1), there must exist a $i \in N$ such that b_2^i satisfies (A2.1) and, in addition, there exist exists an agent i such that:

$$|b_1^i w_{12} + b_2^i w_{22}| > \frac{1}{2}\left[\frac{(w_{12})^2 + (w_{22})^2}{(k_2^{\text{C}} + 3)} - \frac{w_{11}w_{12} + w_{21}w_{22}}{(k_1^{\text{C}} + 3)}\right]$$
$$|b_2^i| \leq \frac{1}{2}\left[\frac{w_{22}}{(k_2^j + 3)} - \frac{w_{21}}{(k_1^j + 3)}\right]$$

Again, multiplying the last inequality be w_{22} evidences that its RHS is larger than that of the first one. This reduces the necessary condition for IIG to $w_{22}|b_2^i| < |b_1^i w_{12} + b_2^i w_{22}|$. It follows that the previous is only possible when $|b_1^i|$ is sufficiently large. □

[30] Indeed, any of the agents revealing under delegation are revealing both signals.

PROOF OF PROPOSITION 2.2. Let $\mathbf{B}^n = (\mathbf{b}^1, ..., \mathbf{b}^n)$ denote a given profile of biases for n informed agents. The optimal organizational structure is characterized by the number of truthful messages decision-makers receive in equilibrium, $\mathbf{k}'$, $\mathbf{k}''$, and the number of truthful messages the principal receives under centralization, $\mathbf{k}^{\text{C}}$, as per Proposition A2.2. Suppose that $k'_1 = k'_2$, $k''_1 = k''_2$, and $k^{\text{C}}_1 = k^{\text{C}}_2$. Now consider the associated game consisting of $n+m$ agents, with the profile of biases for the first n agents being $\mathbf{B}^n$, that is $\mathbf{B}^{n+m} = (\mathbf{b}^1, ..., \mathbf{b}^n, \mathbf{b}^{n+1}, ..., \mathbf{b}^{n+m})$. Let $\mathfrak{b} \in \Re_+$ denote a large number.

Now, consider the following cases for the preferences of agents $n+1$ to $n+m$:

1. $\mathbf{b}^{n+1} = ... = \mathbf{b}^{n+m} = (\mathfrak{b}, 0)$.

   Suppose first that the optimal organizational structure under $\mathbf{B}^n$ was full delegation. Under the profile $\mathbf{B}^{n+m}$, the principal prefers to retain authority over y_2 if and only if $IIG(y_2) \geq DIG_{j''}(y_2) - (b''_2)^2$; which by Lemma 2.1 and Proposition A2.2 translates into:

$$\frac{w_{21}^2}{6}\left[\frac{1}{(k''_1+2)} - \frac{1}{(k_1^{\text{P2}}+m+2)}\right] + \frac{w_{22}^2}{6}\left[\frac{1}{(k''_2+2)} - \frac{1}{(k_2^{\text{P2}}+m+2)}\right] \geq -(b''_2)^2$$

   Then, for any $k''_1, k''_2, k_1^{\text{P2}}, k_2^{\text{P2}} \leq n$, there exists a m for which the above holds.

   Now, suppose the optimal organizational structure under $\mathbf{B}^n$ is centralization. Then the additional information she receives with partial delegation of y_1 under $\mathbf{B}^{n+m}$ must also compensate the distributional loss on the delegated decision; which means:

$$\frac{w_{21}^2}{6}\left[\frac{2}{(k_1^{\text{C}}+2)} - \frac{1}{(k_1^{\text{P2}}+m+2)}\right] + \frac{w_{22}^2}{6}\left[\frac{1}{(k_2^{\text{C}}+2)} - \frac{1}{(k_2^{\text{P2}}+m+2)}\right] \geq$$
$$\geq \frac{w_{11}^2}{6}\left[\frac{1}{(k'_1+2)} - \frac{1}{(k_1^{\text{C}}+2)}\right] + \frac{w_{12}^2}{6}\left[\frac{1}{(k'_2+2)} - \frac{1}{(k_2^{\text{C}}+2)}\right] + b'_1$$

   Note that the RHS of the expression above is non-negative because of the optimality of centralization under $\mathbf{B}^n$. A sufficient condition for optimality of partial delegation is that there exists an agent j' whose bias satisfies:[31]

$$(b'_1)^2 \leq \frac{w_{11}^2 + w_{21}^2}{6(n+2)} + \frac{w_{12}^2 + w_{22}^2}{6(n+2)} - \frac{w_{11}^2}{18} - \frac{w_{12}^2}{18}$$

   Then, there exists a finite m such that Partial Delegation (of y_1) is preferred by the principal over centralization.

2. $\mathbf{b}^{n+1} = ... = \mathbf{b}^{\frac{2n+m+1}{2}} = (-\mathfrak{b}, \mathfrak{b})$ and $\mathbf{b}^{\frac{2n+m+1}{2}} = ... = \mathbf{b}^{n+m} = (\mathfrak{b}, \mathfrak{b})$.

   Lemma 1.5 implies that, for $k_1^{\text{C}} = k_2^{\text{C}}$, senders with these preferences will have maximal incentives to

[31]The expression reflects the case in which all n agents fully reveal their signals under centralization ($k_r^{\text{C}} = n$), j' does not receive any signals from other agents in equilibrium ($k'_r = 1$), and the indirect informational gains are maximal ($m = \mathfrak{b}$).

play equilibrium DNS strategies. In particular, those in the first group satisfy:

$$\left|\frac{\beta_1^i}{(k_1^{\text{C}}+3)}+\frac{\beta_2^i}{(k_2^{\text{C}}+3)}\right|=\left|\frac{w_{11}b_1^i+w_{12}b_2^i+w_{21}b_1^i+w_{22}b_2^i}{(k^{\text{C}}+3)}\right|=\left|\frac{b_1^i+b_2^i}{(k^{\text{C}}+3)}\right|=0$$

And those in the second group:

$$\left|\frac{\beta_1^i}{(k_1^{\text{C}}+3)}-\frac{\beta_2^i}{(k_2^{\text{C}}+3)}\right|=\left|\frac{w_{11}b_1^i+w_{12}b_2^i-w_{21}b_1^i-w_{22}b_2^i}{(k^{\text{C}}+3)}\right|=\left|\frac{b_1^i-b_2^i}{(k^{\text{C}}+3)}\right|=0$$

Then, $m=2n$ is a sufficient condition for the principal to prefer centralization over any other organizational structure that was optimal with the original profile of biases.

□

PROOF OF PROPOSITION 2.3. Suppose the equilibrium organizational structure involve some form of delegation, and let $j', j'' \in \{P, 1, 2, ...\}$ be decision-makers for y_1 and y_2, respectively. The principal's ex-ante utility gain with respect to centralization is given by:

$$\begin{aligned}(b_1')^2+(b_2'')^2 \leq &\frac{(w_{11})^2}{6}\left[\frac{1}{(k_1^{\text{C}}+2)}-\frac{1}{(k_1'+2)}\right]+\frac{(w_{12})^2}{6}\left[\frac{1}{(k_2^{\text{C}}+2)}-\frac{1}{(k_2'+2)}\right]+\\ &\frac{(w_{21})^2}{6}\left[\frac{1}{(k_1^{\text{C}}+2)}-\frac{1}{(k_1''+2)}\right]+\frac{(w_{22})^2}{6}\left[\frac{1}{(k_2^{\text{C}}+2)}-\frac{1}{(k_2''+2)}\right]\end{aligned}$$

According to expression (2.1), and denoting by $\mathbf{m}_{\text{C}}$ the messages sent to the principal under centralization, the above can be expressed as:

$$\left[(b_1')^2+(b_2'')^2\right]^{\frac{1}{2}} \leq \left[\sum_{y_d}\left[w_{d1}^2\big(\text{Var}(\theta_1|\mathbf{m}_{\text{C}})-\text{Var}(\theta_1|\mathbf{m}_j)\big)+w_{d2}^2\big(\text{Var}(\theta_2|\mathbf{m}_{\text{C}})-\text{Var}(\theta_2|\mathbf{m}_j)\big)\right]\right]^{\frac{1}{2}}$$

Denote by $\hat{\mathbf{b}}^{\text{D}}$ the vector whose component satisfy the above expression with equality. Because the LHS above represents the euclidean distance of a vector with components b_1' and b_2'' to the origin, then $\hat{\mathbf{b}}^{\text{D}}$ represents the maximum conflict of interest the principal will tolerate as a function of the informational gains from delegation of the corresponding decision(s).

For the second part of the proof let assume that $k_r^j = k^{\text{C}}+1$, and denote the reduction in variance under delegation by $\Delta\text{Var}(\theta_r|\mathbf{m}_j) = \text{Var}(\theta_r|\mathbf{m}_{\text{C}}) - \text{Var}(\theta_r|\mathbf{m}_j, k_r^j = k_r^{\text{C}}+1)$, which then is given by:

$$\Delta\text{Var}(\theta_r|\mathbf{m}_j)=\frac{1}{6}\left[\frac{1}{(k_r^{\text{C}}+2)}-\frac{1}{(k_r^{\text{C}}+3)}\right]$$

Then, taking the derivative with respect to k_r^{C} gives expression (2.4). □

Proof of Lemma 2.1. The proof proceed by contradiction. I focus on the centralization case, since decentralization follows the same logic. Let $(\{\mathbf{y}^*\}, \{\mathbf{m}^*, \mathfrak{s}^*\})$ be the equilibrium strategy profiles for the receiver and all agents, respectively. Recall the equilibrium is characterized by k_1^* and k_2^*.

Acquisition of S_1. Suppose that i's equilibrium info acquisition strategy has $S_1 \in \mathfrak{s}^{i*}$ but condition (1.1) does not hold for S_1. In such a case revealing information about θ_1 is not incentive compatible for i despite he acquired information about it. Other agents base their message strategies on conjectures about k_1^*, but i is not included among agents revealing S_1 truthfully. At the information acquisition stage, i's expected payoff of $\mathfrak{s}^{i*}$ is thus given by:

$$E\left[U^i\left(\mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)), \delta, b^i\right)\right] = -E\left[\left(\mathbf{y}_1\left(m^{i*}(\mathfrak{s}^{i*}), \mathbf{m}^{\text{-}i*}\right) - \delta_1 - b_1^i\right)^2 + \left(\mathbf{y}_2\left(m^{i*}(\mathfrak{s}^{i*}), \mathbf{m}^{\text{-}i*}\right) - \delta_2 - b_2^i\right)^2\right] - C(\mathfrak{s}^{i*})$$

Now, consider the following deviation: $\hat{\mathfrak{s}}^i = \mathfrak{s}^{i*} \backslash \{S_1\}$. Note that this deviation does not affect k_1^* or k_2^*, and i's overall influence on j's decision(s) is thus unaltered—i.e. $y_d\left(m^i(\hat{\mathfrak{s}}^i), \mathbf{m}_j^{\text{-}i}\right) = y_d\left(m^i(\mathfrak{s}^{i*}), \mathbf{m}_j^{\text{-}i}\right)$. Note also that $C(\mathfrak{s}^{i*}) > C(\hat{\mathfrak{s}}^i)$, given $\#\mathfrak{s}^{i*} > \#\hat{\mathfrak{s}}^i$. Consequently,

$$E\left[U^i\left(\mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)), \delta, b^i\right)\right] - E\left[U^i\left(\mathbf{y}\left(m^i(\hat{\mathfrak{s}}^i), \mathbf{m}^{\text{-}i*}\right), \delta, b^i\right)\right] = -C(\mathfrak{s}^{i*}) + C(\hat{\mathfrak{s}}^i) < 0$$

So, $\hat{\mathfrak{s}}^i$ is a profitable deviation from $\mathfrak{s}^{i*}$. $\Rightarrow\Leftarrow$

Acquisition of both signals. The proof is similar to the previous, with i's proposed equilibrium strategy being $\mathfrak{s}^{i*} = \{S_1, S_2\}$ but conditions for Full Revelation do not hold. Then, a profitable deviation for i will be to acquire the signal he is willing to reveal on the equilibrium path (if any). $\square$

Equilibrium Information Acquisition and Cost-effectiveness condition

Before the proof of Lemma 2.2 I derive the information-acquisition IC constraint.

Observation 2.1. *Let $k_r^{j*} \equiv k_r^j\left(\mathbf{m}_j^i(\mathfrak{s}^{i*}, \mathfrak{s}^{\text{-}i}), \mathbf{m}_j^{\text{-}i}(\mathfrak{s}^{i*}, \mathfrak{s}^{\text{-}i})\right)$ and $\hat{k}_r^j \equiv k_r^j\left(\mathbf{m}_j^i(\hat{\mathfrak{s}}^i, \mathfrak{s}^{\text{-}i}), \mathbf{m}_j^{\text{-}i}(\mathfrak{s}^{i*}, \mathfrak{s}^{\text{-}i})\right)$ for $\theta_r = \{\theta_1, \theta_2\}$. Let $\mathfrak{s}^{i*}$ denote i's information acquisition strategy in an equilibrium characterized by $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$. Then, i's ex-ante expected utility from $\mathfrak{s}^{i*}$ is given by:*

$$E\left[U^i\left(\mathbf{m}^*, \mathfrak{s}^{i*}, \mathfrak{s}^{\text{-}i}, \boldsymbol{\delta}, \mathbf{b}^i\right)\right] = -\left[(b_1^i)^2 + (b_2^i)^2\right] - \sum_{y_d=\{y_1,y_2\}}\left[\frac{(w_{d1})^2}{6\left(k_1^{j*}+2\right)} + \frac{(w_{d2})^2}{6\left(k_2^{j*}+2\right)}\right]$$

Now, let $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$ be equilibrium strategy profiles. Then, $\mathfrak{s}^{i*}$ is incentive compatible for agent i if and only if, for every alternative $\hat{\mathfrak{s}}^i$:

$$\sum_{y_d=\{y_1,y_2\}}\sum_{\theta_r=\{\theta_1,\theta_2\}}\frac{(w_{dr})^2}{6}\left[\frac{1}{(\hat{k}_r^j+2)} - \frac{1}{(k_r^{j*}+2)}\right] \geq \left[C(\mathfrak{s}^{i*}) - C(\hat{\mathfrak{s}}^i)\right] \tag{A2.4}$$

Proof of Proposition 2.4. Let $S_r \in \mathfrak{s}^{i*}$ and $S_{\tilde{r}} \notin \mathfrak{s}^{i*}$ for $\theta_r \neq \theta_{\tilde{r}}$, and $k_r^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$ be i's conjecture about other agents revealing their information about θ_r to decision-maker j. Then, agent i's IC constraint for revealing S_r^i is:

- When $j = P$ decides on both issues (centralization),

$$|\beta_r^i| \leq \frac{(w_{1r})^2 + (w_{2r})^2}{2\,(k_r^{\mathrm{C}} + 3)} \tag{A2.5}$$

- When j decides on y_d only,

$$|b_d^i - b_d^j| \leq \frac{w_{dr}}{2\big(k_r^j + 3\big)} \tag{A2.6}$$

I prove the IC constraint for case of delegation, as centralization follows the same argument but requires more algebra.Suppose agent i's acquires only S_1^i; then strategy $\mathbf{m}^{i*} = \{m_{j'}^{i*}, m_{j''}^{i*}\}$ is preferred to any alternative $\hat{\mathbf{m}}$ iff (IC constraint in section B2):

$$-\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i) + w_{d2}(\nu_{d2}^{i*} - \hat{\nu}_{d2}^i)\right]\left[w_{d1}(\nu_{d1}^{i*} + \hat{\nu}_{d1}^i) + w_{d2}(\nu_{d2}^{i*} + \hat{\nu}_{d2}^i) - 2[w_{d1}\nu_{d1}^i + w_{d2}\nu_{d2}^i - (b_d^i - b_d^j)]\right] \geq 0$$

But since i has information about θ_1 only, then $E(\theta_2|S_1^i, \mathbf{m}_j^{\text{-}i}) = \nu_{d2}^i = \nu_{d2}^{i*} = \hat{\nu}_{d2}^i$. Moreover, the strategy space when i has only one signal is degenerated, such that he can only reveal it or lie. Revealing S_1^i is thus IC iff:

$$-\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i)\right]\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i) - 2(b_d^j - b_d^i)\right] \geq 0$$

Following the same steps as in section B2 it is easy to note that the above expression becomes:

$$\begin{aligned} \text{For } \tilde{S}_1^i = 0: &\qquad 2(b_d^i - b_d^j) \leq \frac{w_{d1}}{(k_1^j + 3)} \\ \text{For } \tilde{S}_1^i = 1: &\quad -2(b_d^i - b_d^j) \leq \frac{w_{d1}}{(k_1^j + 3)} \end{aligned}$$

Which together imply equation (A2.6).

Now, the vector $\mathfrak{B}_r^j(\mathbf{b}^j, \mathbf{k}^j)$ results from comparing equations (A2.1) and (A2.6). That is, assuming j decides over y_d only and $k_r^j = \{k_1^j, k_2^j\}$ are i's equilibrium conjectures about other agents revealing information to j, $\mathfrak{B}_r^{\mathrm{D}j}(\mathbf{b}^j, \mathbf{k}^j)$ can be defined as:

$$\mathfrak{B}_r^j = \left\{x : b_d^j \pm x \in \frac{1}{2} \times \left(\left|\frac{w_{d1}}{(k_1^j + 3)} - \frac{w_{d2}}{(k_2^j + 3)}\right|, \frac{w_{dr}}{(k_r^j + 3)}\right]\right\}$$

Under centralization, the vector $\mathfrak{B}_r^j(\mathbf{b}^j, \mathbf{k}^j)$ results from comparing equations (1.1) and (A2.5). Denote by k_r^{C} agent i's equilibrium conjectures about other agents revealing information about $\theta_r\{\theta_1, \theta_2\}$ to the principal under centralization, and $k^{\text{C}}_{\tilde{r}} \neq k^{\text{C}}_{r}$. Then, $\mathfrak{B}^{\text{P}}_r(\mathbf{k}^{\text{P}})$ is defined as:

$$\mathfrak{B}^{\text{P}}_r = \left\{ x : |x| \in \frac{(w_{1r})^2 + (w_{2r})^2}{2} \times \left(\left[\frac{1}{(k^{\text{C}}_r + 3)} - \frac{C_r}{(k^{\text{C}}_{\tilde{r}} + 3)} \right], 2\,(k^{\text{C}}_r + 3) \right] \right\}$$

$\square$

Proof of Lemma 2.2. I first derive the cost-effectiveness condition (2.5) and then the maximum number of agents for which acquiring a given piece of information is cost-effective —condition (2.7). In order to derive cost-effectiveness (CE), I consider each possible info acquisition strategy in equilibrium.

The number of agents revealing truthfully their signals in equilibrium, k^*_r, includes i's message strategy when he acquires (and reveals) it in equilibrium.[32] Here I need to make two clarifications. Firstly, if there were agents who acquired information on θ_1 but were not willing to reveal it when $\hat{k}_1 = k^*_1 + 1$, then only one of them changes his message strategy because when one of these agents stop revealing, then $\hat{k}_1 = k^*_1$ again. As a consequence, $\hat{k}_1 = \{k^*_1, k^*_1 + 1\}$. I take the most conservative of these approaches by making $\hat{k}_1 = k^*_1 + 1$ whenever i acquires S_1 off-path.

The second clarification relates to what happens when i acquires a signal and does not reveal it. Since other agents' message strategies will depend on conjectures about k_r, i not revealing the signal acquired off-path does not affect their equilibrium behaviour at the communication stage. In other words, Lemma 2.1 holds: i gains nothing from acquiring a signal he will not reveal.

Centralization. Let first consider the acquisition of both signals in equilibrium; that is $\mathfrak{s}^{i*} = \{S^i_1, S^i_2\}$. Expression (A2.4) for each possible alternative strategy becomes:

1) $\tilde{\mathfrak{s}}^i = \{\emptyset\}$

$$\sum_{\theta_r} \frac{(w_{1r})^2 + (w_{2r})^2}{6(k^*_r + 2)(k^*_r + 3)} \geq C(S^i_1, S^i_2)$$

When i plays DNS message strategies on path, he expects to reveal information for half of the possible realizations, such that the CE conditions becomes:

$$\frac{(w_{11})^2 + (w_{21})^2}{6(k^*_1 + 2)(k^*_1 + 3)} + \frac{(w_{12})^2 + (w_{22})^2}{6(k^*_2 + 2)(k^*_2 + 3)} \geq 2\,C(S^i_1, S^i_2) \tag{A2.7}$$

2) $\tilde{\mathfrak{s}}^i = \{S^i_r\}$

$$\frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k^*_{\tilde{r}} + 2)(k^*_{\tilde{r}} + 3)} \geq C(S^i_{\tilde{r}})$$

Now, when the equilibrium strategy consists of one signal only, $\mathfrak{s}^{i*} = \{\tilde{S}^i_r\}$, the IC constraints become:

[32] In equilibria in which i does not acquire S^i_1, k^*_1 does not count him; but in any deviation in which he does acquire it, then $\hat{k}_1 = k^*_1 + 1$.

3) $\tilde{\mathfrak{s}}^i = \{\emptyset\}$

$$\frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^* + 2)(k_r^* + 3)} \geq C(S_r^i)$$

4) $\tilde{\mathfrak{s}}^i = \{S_{\tilde{r}}^i\}$

$$\frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^* + 2)(k_r^* + 3)} \geq \frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k_{\tilde{r}}^* + 2)(k_{\tilde{r}}^* + 3)}$$

5) $\tilde{\mathfrak{s}}^i = \{S_1^i, S_2^i\}$

$$\frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k_{\tilde{r}}^* + 2)(k_{\tilde{r}}^* + 3)} < C(S_{\tilde{r}}^i)$$

Case 3) represents the necessary condition to acquire any individual signal S_r^i, since it implies case 1) (in which it holds for both signals) and case 4) (in which the agent acquires the signal that would have the highest influence). This case corresponds to equation (2.5).

Now I work out the expression for the maximum number of agents to acquire a given signal under centralization. According to the equation (2.5) as k_r increases the ex-ante expected utility of acquiring S_r decreases. So, the maximum number of agents who will acquire that signal is given by the largest k^* for which the cost-effectiveness condition hold. Re-arranging this condition I get the following polynomial:

$$-(k_r^*)^2 - 5k_r^* - \left[6 - \frac{(w_{1r})^2 + (w_{2r})^2}{6\,C(S_r^i)}\right] \geq 0$$

Then, solving for the highest positive root I get K_r^{C} in (2.7).

Delegation. As before, i is not willing to acquire signals he is not willing to reveal on-path (Lemma 2.1). But in this case there are two decision-makers and IC can refer to any of them (or both). From (A2.4) we know that acquiring S_r^i requires that i is willing to reveal it to at least one decision-maker, say:

$$C(S_r^i) \leq \frac{(w_{dr})^2}{6(k_r^j + 2)(k_r^j + 3)}$$

For at least one y_d. Consider the case of acquiring both signals, which is cost-effective in two generic cases. First, when i is willing to reveal at least one signal to a different decision-maker, the CE condition above should hold for each decision-maker. Second, when i is willing to reveal both signals to a single j the RHS of the above expression becomes larger. As a consequence, equation (2.6) is a necessary condition for investing in any individual signal.

To get the expression for the maximum number of agents to invest in S_r I need to analyse also two cases. The minimal incentives to reveal are given by the case in which all agents are willing to reveal S_r to decide on the dimension it is less important. This will define the minimum upper-bound, since the CE condition becomes

$$C(S_r^i) \leq \min_{w_{dr}} \left\{ \frac{(w_{dr})^2}{6(k_r^j+2)(k_r^j+3)} \right\}$$

Now consider the case in which all agents are willing to reveal *both signals* to *both decision-makers*. This is the maximum upper-bound. In such a case, the CE condition will be just like the centralization case; that is:

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^j+2)(k_r^j+3)}$$

Then, following the same steps as the proof of Lemma 2.2 (Appendix A2) I have the first and the second expressions in square bracket in equation (2.8), respectively. □

Agents' equilibrium strategies (endogenous information acquisition).

In this subsection I combine the results of Lemma 2.1 and Lemma 2.2 to characterize agent i's equilibrium information acquisition and message strategies. I start with the case of centralization ($j' = j'' = P$) and then proceed to the case of delegated decisions. The following result summarizes the intuitions developed in the previous discussion, presenting the equilibrium acquisition and message strategies for a typical agent under centralization.

Proposition A2.3 (Equilibrium under Centralization)**.** *In the most informative equilibrium under centralization* $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$*, agent* i *only acquires signals that are cost-effective and incentive compatible. In particular,* i*'s equilibrium strategies are given by:*

Acquiring and revealing both signals: *if and only if conditions* (2.5) *and* (A2.5) *hold for both signals, and* (1.4) *hold.*

Acquiring both signals and playing a dimensional non-separable strategy: *if condition* (A2.7) *hold for both signals and* (A2.5) *does not at all, in the following cases:*

- *Fully revealing both signals when they coincide and babbling otherwise, if condition* (1.3) *holds;*
- *Fully revealing both signals when they do not coincide and babbling otherwise, if condition* (1.4) *holds.*

Acquiring and revealing one signal only. *Agent* i *acquires and reveals* S_1^i *if* (A2.5) *and* (2.5) *hold with respect to* θ_1 *and one of the following is true:*

- *Revealing* S_2^i *is not IC —i.e.* (A2.5) *does not hold for* θ_2*; or*
- *Acquiring* S_2^i *is not CE —i.e.* (2.5) *does not hold for* θ_2*; or*
- *Acquiring* S_2^i *is CE and revealing it is IC, but revealing both signals is not IC —i.e.*(1.1) *and* (2.5) *hold for both signals, but* (1.4) *does not and* $\frac{(w_{1r})^2+(w_{2r})^2}{(k_r^*+2)(k_r^*+3)} \geq \frac{(w_{1\tilde{r}})^2+(w_{2\tilde{r}})^2}{(k_{\tilde{r}}^*+2)(k_{\tilde{r}}^*+3)}$

For $r \neq \tilde{r}$.

Acquiring no signal, *if only if any of the statements below is true:*

- *No signal is CE to acquire —i.e. condition (2.5) does not holds for any signal; and/or*

- *No signals is IC to reveal —i.e. condition (A2.5) does not hold for any signal, nor (1.4) holds.*

Proof. See Appendix B2 □

As discussed earlier, the number of agents revealing the same piece of information is an important determinant of cost-effectiveness. It could lead some agents to acquire less information than what each is willing to reveal, resulting in either specialization or non-investment. The possibility of abstaining to acquire a given signal can enhance incentives for communication (for the other signal) because it kills the effects of ambiguous information.

Dimensional non-separable message strategies can arise under centralization. As in the pure communication game, these strategies take the form of full revelation for some realizations and babbling for the rest. Because any of these involves acquiring both signals and revealing them half of the time, they arise when costs are sufficiently low and only if revealing one signal is not IC. Then, agent i typically prefers to reveal a single signal than play a DNS strategy, provided both are incentive compatible and cost-effective.[33]

Similar intuitions apply to the case of delegation. The result below characterizes equilibrium strategies for agent i and decisions y_d and $y_{\tilde{d}}$.

Proposition A2.4 (Equilibrium under Delegation)**.** *When the organizational structure involves more than one decision-maker, agent i only acquires signals that are cost-effective and for which communication is incentive compatible. In the most informative equilibrium ,$(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$, i's equilibrium strategies are:*

Acquiring and revealing both signals: *if and only if conditions (A2.1) and (2.6) hold for both signals and at least one decision-maker —and the associated decision.*

Acquiring and revealing S_1 only*, if acquiring this signal is both cost-effective and incentive compatible for agent i in the following cases:*

1. *Revealing S_2 is not IC for any decision —i.e. condition (A2.6) does not hold for S_2^i; or*

2. *Acquiring S_2 is not CE for any decision —i.e. condition (2.6) does not hold for S_2^i for any decision; or*

3. *Both S_1 and S_2 are CE and IC, but revealing both is not IC with respect to any decision-maker —i.e. conditions (A2.6) and (2.6) hold for both signals and at least one decision-maker, but (A2.1) does not hold for any of them and $\frac{(w_{dr})^2}{(k_r^*+2)(k_r^*+3)} \geq \frac{(w_{d\tilde{r}})^2}{(k_{\tilde{r}}^*+2)(k_{\tilde{r}}^*+3)}$*

For $r \neq \tilde{r}$.

Acquiring no signal *if only if any of the statements below are true:*

1. *Condition (A2.6) does not hold for any signal and any decision, nor (1.4) hold; and/or*

2. *Condition (2.6) does not holds for any signal, any decision.*

[33]For a formal discussion see Appendix B2.

Proof. See Appendix B2. $\square$

In presence of two decision-makers agents acquire a signal if it is cost-effective and incentive compatible to reveal it to at least one of them. As discussed in Proposition 2.4, the possibility of not acquiring one of the signals makes credible the strategy of revealing only one signal. When such a strategy is incentive compatible it always refers to the state the decision-maker is less informed in equilibrium. In other words, the agent will acquire the signal for which the influence is larger, which in turn implies that dimensional non-separable strategies are dominated by this strategy. As a consequence, no DNS strategy can emerge in equilibrium under delegation.

Corollary A2.1. *Let $w \equiv w_{11} = w_{22}$, $\kappa = 1$, and $\frac{\hat{w}_{dr}^2}{72} < C(S_r) \leq \frac{[(w_{1r})^2+(w_{2r})^2]}{72}$ for both θ_r. If there are no agents j' and j'' whose preferences represent a conflict of interest within $||\mathbf{b}^{\mathrm{D}}|| \leq 1-2w(1-w)$, then the principal strictly prefers centralization over any form of delegation. If there where such agents, the principal still prefers centralization (strictly) as long as there is at least one agent i who fully reveals his information —i.e. $\mathbf{b}^i$ satisfy conditions (1.1) and (1.4) with respect to both signals.*

Proof. When $\kappa = 1$, decision-makers have the stronger incentives to acquire both signals under delegation. Noting that $6(\kappa+2)(\kappa+3) = 72$, from Proposition 2.6 I get that $\frac{\hat{w}_{dr}^2}{72} < C(S_r) \leq \frac{[(w_{1r})^2+(w_{2r})^2]}{72}$ implies $K_r^{\mathrm{C}} > K_r^{\mathrm{D}} = 1$. In addition, from Proposition 2.3 we know that, for $w_{11} = w_{22} = w$ and $k_r^j = 1$:

$$||\hat{\mathbf{b}}^{\mathrm{D}}|| = \left[\frac{w^2}{6}\left(1-\frac{2}{3}\right) + \frac{(1-w)^2}{6}\left(1-\frac{2}{3}\right)\right] = 1-2w(1-w)$$

If there are no agents j' and j'' such that $||\mathbf{b}^{\mathrm{D}}|| \leq ||\hat{\mathbf{b}}^{\mathrm{D}}||$, then the distributional loss from delegation is never compensated by the maximal informational gain; as a consequence, centralization yields higher ex-ante expected utility to the principal. On the other hand, if there were agents j' and j'' such that $(b_1', b_2'') \in ||\hat{\mathbf{b}}^{\mathrm{D}}||$ the principal prefers delegation when there is no agent i whose preferences satisfy conditions (1.1) and (1.4). For if there were such an agent, he reveals both signals under centralization and, thus, delegation yields no informational gains.

Under the parameters of Corollary A2.1, costs are so high that the maximum number of truthful messages under delegation is zero. Having the chance to influence both decisions, i's acquisition of one signal is cost-effective (provided communication is IC). The result also illustrates the restrictions imposed by information costs on the optimality of different organizational structures (Proposition A2.2). For sufficiently high costs the principal always prefer to retain authority over both issues, but restrictions weaken as the costs of acquiring a signal decreases. $\square$

Proof of Proposition 2.6. Let $(\mathfrak{s}, \mathbf{m}, \mathbf{y})$ denote a generic equilibrium in which $\kappa < n$ is the maximum number of agents willing to reveal S_r to both decision-makers (suppose $\kappa > 0$). For any of such agents cost-effectiveness under delegation —condition (2.6)— is given by:

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^j+2)(k_r^j+3)}$$

But for any other agent, the CE condition is at most:

$$C(S_r^i) \le \frac{(\hat{w}_{dr})^2}{6(k_r^j+2)(k_r^j+3)}$$

Then, $C(S_r^i) \le \frac{(w_{1r})^2+(w_{2r})^2}{6(\kappa+2)(\kappa+3)}$ implies acquiring information about θ_r is CE for agents willing to reveal it to both decision-makers. But if at the same time $C(S_r^i) > \frac{(\hat{w}_{dr})^2}{6(\kappa+2)(\kappa+3)}$, agents willing to reveal S_r to at most one decision-maker do not acquire this signal because it is not CE.

On the other hand, the first of the above equations (2.7) determines the maximum number of agents for which acquiring S_r is CE under centralization. But since $C(S_r^i) \le \frac{(w_{1r})^2+(w_{2r})^2}{6(\kappa+3)(\kappa+4)}$, it should be greater or equal to $\kappa+1$. Then, $K_r^{\mathrm{D}} = \kappa < K_r^{\mathrm{C}}$ □

Relative investment in information under delegation

Consider the case of delegation. Let $\lambda_r^d \equiv \{\mathbf{z} \in \Re^2 | \ \mathbf{z}'\,\mathbf{I}_d = 0\}$ be the locus of maximal incentives to reveal about $\theta_r = \{\theta_1, \theta_2\}$ when deciding on $y_d = \{y_1, y_2\}$.[34] The locus λ_r^d captures the fact that communication depends only on the conflict of interest associated with y_d—coincides with either the vertical or the horizontal axis, for λ_r^1. and λ_r^2 (respectively). Condition (A2.6) for communication with the principal can be expressed as:

$$||\mathbf{b}^i - \mathrm{Proj}_{\lambda_r^d}(\mathbf{b}^i)|| \le \frac{w_{dr}}{2\left(k_r^{\mathrm{P}d}+3\right)}$$

Then, the lemma below shows when the principal can expect to receive more information about one state under (partial) delegation.

Lemma A2.2. *Let $w \equiv w_{11} = w_{22}$. Given $\varepsilon \in \Re_+$ and an arbitrarily large n_ε, there exists an integer κ and $i \in N_\varepsilon$ with $||\mathbf{b}^i - Proj_{\lambda_r^d}(\mathbf{b}^i)|| \le \frac{w_{dr}}{2(\kappa+3)}$ such that $||\mathbf{b}^i - Proj_{\lambda_{\tilde{r}}^d}(\mathbf{b}^i)|| > \frac{w_{d\tilde{r}}}{2(\kappa+3)}$.*

Moreover, this is true for the state associated with w_{dr} because $w_{dr} > w_{d\tilde{r}}$

Proof. Given that $\lambda_r^d = \lambda_{\tilde{r}}^d$ and $w_{dr} > w_{d\tilde{r}}$, the result holds for any $b_d^i \in \left(\frac{w_{d\tilde{r}}}{2(\kappa+3)}; \frac{w_{dr}}{2(\kappa+3)}\right]$ □

Now, consider the case of centralization. Let $\varepsilon \in \Re_+$ and $N_\varepsilon = \{1, 2, ..., n_\varepsilon\}$ be a group of agents whose preferences satisfy: for all $i \in N_\varepsilon$ then $||\mathbf{b}^i|| = \varepsilon$. Now, let $\lambda_r \equiv \left\{\mathbf{z} \in \Re^2 | \ \mathbf{z}'\,\mathbf{W}_r = 0\right\}$ be the locus with slope $-\frac{w_{1r}}{w_{2r}}$ related to θ_r. This locus represents maximal incentives to reveal information about $\theta_r = \{\theta_1, \theta_2\}$ to the principal ($\beta_r^i = 0$). The IC constraint for revealing one signal under centralization —equation (A2.5)— can be expressed as:

$$||\mathbf{b}^i - \mathrm{Proj}_{\lambda_r}(\mathbf{b}^i)|| \le \frac{\left[(w_{1r})^2 + (w_{2r})^2\right]^{\frac{1}{2}}}{2\left(k_r^{\mathrm{C}}+3\right)}$$

Where $\mathrm{Proj}_{\lambda_r}(\mathbf{b}^i)$ is the projection of i's bias vector onto the locus λ_r. Note that agents with small conflict of interest reveal both signals. The same conclusion applies to dimensional non-separable message strategies;

[34] Where $\mathbf{I}$ is the 2-by-2 identity matrix, and $\mathbf{I}_d$ is its dth column, which matches the index of the decision under consideration.

agents with the corresponding preferences reveal both signals for some realizations and reveal nothing otherwise. Hence, the principal expects to receive more information about one of the states when many agents acquire and reveal the associated signal and few agents acquire and reveal the other. The result below shows whether this is the case under centralization.

Lemma A2.3. *Let $w \equiv w_{11} = w_{22}$. Given $\varepsilon \in \Re_+$ and arbitrarily large n_ε, then for every integer κ and $i \in N_\varepsilon$ with $||\mathbf{b}^i - Proj_{\lambda_1}(\mathbf{b}^i)|| \leq \frac{\left[(w)^2+(1-w)^2\right]^{\frac{1}{2}}}{2\,(\kappa+3)}$, there exists a $j \in N_\varepsilon$ with $||\mathbf{b}^j - Proj_{\lambda_2}(\mathbf{b}^j)|| \leq \frac{\left[(1-w)^2+(w)^2\right]^{\frac{1}{2}}}{2\,(\kappa+3)}$.*

Proof. See appendix B2 □

Proof of Proposition 2.7. Note first that $\text{Var}(y_d|\mathbf{m}^*) = w_{d1}^2\text{Var}(\theta_1|\mathbf{m}^*) + w_{d2}^2\text{Var}(\theta_2|\mathbf{m})$ which, given $w_{11} = w_{22} = w$, implies that $\text{Var}(y_1|\mathbf{m}^*) - \text{Var}(y_2|\mathbf{m}^*) = w^2(1-w)^2\left[\text{Var}(\theta_1|\mathbf{m}^*) - \text{Var}(\theta_2|\mathbf{m}^*)\right] = w^2(1-w)^2\left[\frac{1}{6(\tilde{k}_1^*+2)} - \frac{1}{6(\tilde{k}_2^*+2)}\right]$.

Under centralization, the principal's expectation about her updated beliefs on each state—and, thus, the residual variance— depends on how much information she expects to receive from agents on the equilibrium path. Since biases are uniformly distributed between $[0, \varepsilon]$, Lemma A2.3 implies that for any agent revealing information about θ_1, she expects another agent with the same conflict of interest who will be willing to reveal information about θ_2. Then, in expectation, the number of agents revealing each signal coincide, that is $\tilde{k}_1^{\text{c}} = \tilde{k}^{\text{c}} = 2$, which implies that $E\left[\text{Var}(\theta_1|\mathbf{m}^*) - \text{Var}(\theta_2|\mathbf{m}^*)\right] = 0$.

Now suppose the principal delegates y_2 to agent $i = 0$. Again, given the uniform distribution of biases, by Lemma A2.2 she expects more agents willing to reveal information about θ_1 than θ_2—and the opposite pattern for $i = 0$. Then, her ex-ante expectations about equilibrium information on each state satisfy: $\tilde{k}_1^{\text{P1}} > \tilde{k}_2^{\text{P1}}$ and $\tilde{k}_1^0 < \tilde{k}_2^0$; as a consequence, $E\left[\text{Var}(\theta_1|\mathbf{m}^{\text{P1}})\right] < E\left[\text{Var}(\theta_2|\mathbf{m}^{\text{P1}})\right]$ and $E\left[\text{Var}(\theta_1|\mathbf{m}^0)\right] > E\left[\text{Var}(\theta_2|\mathbf{m}^0)\right]$. □

B2 Proofs of Complementary results

Generic IC constraints for communication

Proof. Let $j', j'' \in \{P, 1, ..., n\}$ be the decision-makers for y_1 and y_2, respectively; and $i \in \{1, ..., n\}$ be a generic sender. Let $\mathbf{m}_j^{i*}$ denote i's equilibrium message strategy with respect to j, and $\hat{\mathbf{m}}_j^i$ an alternative message strategy (deviations to be considered in each case). Then, $y_d(\mathbf{m}_j^i, \mathbf{m}_j^{-i})$ represents the action j takes when i is expected to play $\mathbf{m}_j^{i*}$ and other senders are playing $\mathbf{m}_j^{-i}$. Given i's conjectures about others' strategies are correct in equilibrium, I can simplify notation in the following way: $y_d(\mathbf{m}_j^{i*}(\mathbf{S}^i), \mathbf{m}_j^{-i}) = y_d(\mathbf{m}_j^{i*})$ and $y_d(\hat{\mathbf{m}}_j^i(\mathbf{S}^i), \mathbf{m}_j^{-i}) = y_d(\hat{\mathbf{m}}_j^i)$.

Message strategy $\mathbf{m}^{i*} = \{m_{j'}^{i*}, m_{j''}^{i*}\}$ is then incentive compatible for sender i if and only if for any alternative $\hat{\mathbf{m}}^i$:

$$-\int_0^1\int_0^1\Bigg[\Big[\left(\delta_1+b_1^i-y_1(\mathbf{m}_{j'}^{i*})\right)^2+\left(\delta_2+b_2^i-y_2(\mathbf{m}_{j''}^{i*})\right)^2\Big]-$$

$$-\Big[\left(\delta_1+b_1^i-y_1(\hat{\mathbf{m}}_{j'}^{i})\right)^2+\left(\delta_2+b_2^i-y_2(\hat{\mathbf{m}}_{j''}^{i})\right)^2\Big]\Bigg]f(\theta_1,\mathbf{m}^{\text{-}i}|\mathbf{S}^i)\,f(\theta_2,\mathbf{m}^{\text{-}i}|\mathbf{S}^i)\,d\theta_1\,d\theta_2\geq 0$$

By operating inside the square brackets with the identity $a^2-b^2=(a+b)(a-b)$, by definition of optimal decisions, $y_d^*=E(\delta_d|\mathbf{m}_j)+b_d^j$, and by denoting:

$$\Delta(\delta_1)=E(\delta_1|\mathbf{m}_{j'}^{i*},\mathbf{m}_{j'}^{\text{-}i})-E(\delta_1|\hat{\mathbf{m}}_{j'}^{i},\mathbf{m}_{j'}^{\text{-}i})\qquad\qquad \Delta(\delta_2)=E(\delta_2|\mathbf{m}_{j''}^{i*},\mathbf{m}_{j''}^{\text{-}i})-E(\delta_2|\hat{\mathbf{m}}_{j''}^{i},\mathbf{m}_{j''}^{\text{-}i})$$

I get:[35]

$$-\int_0^1\int_0^1\Bigg[\Big[\frac{E(\delta_1|\mathbf{m}_{j'}^{i*},\mathbf{m}_{j'}^{\text{-}i})+E(\delta_1|\hat{\mathbf{m}}_{j'}^{i},\mathbf{m}_{j'}^{\text{-}i})}{2}-\delta_1-(b_1^i-b_1')\Big]\Delta(\delta_1)+$$

$$+\Big[\frac{E(\delta_2|\mathbf{m}_{j''}^{i*},\mathbf{m}_{j''}^{\text{-}i})+E(\delta_2|\hat{\mathbf{m}}_{j''}^{i},\mathbf{m}_{j''}^{\text{-}i})}{2}-\delta_2-(b_2^i-b_2'')\Big]\Delta(\delta_2)\Bigg]$$

$$f(\theta_1|\mathbf{m}^{\text{-}i},S_1^i)\,f(\theta_2|\mathbf{m}^{\text{-}i},S_2^i)\,P(\mathbf{m}_{j'}^{\text{-}i}|S_1^i)\,P(\mathbf{m}_{j'}^{\text{-}i}|S_2^i)\,P(\mathbf{m}_{j''}^{\text{-}i}|S_1^i)\,P(\mathbf{m}_{j''}^{\text{-}i}|S_2^i)\,d\theta_1\,d\theta_2\geq 0$$

Given that the equilibrium message strategies for players other than i, $\mathbf{m}^{\text{-}i}$, are independent of i's actual signal realizations, the expressions $P(\mathbf{m}_j^{\text{-}i}|S_1^i)$ and $P(\mathbf{m}_j^{\text{-}i}|S_2^i)$ can be taken out the double-integral.

I denote the receiver's updated beliefs with respect to δ_d from i's perspective as:

$$\nu_d^{i*}=E(\delta_d|\mathbf{m}_j^{i*},\mathbf{m}_j^{\text{-}i})\qquad\qquad \hat{\nu}_d^{i}=E(\delta_d|\hat{\mathbf{m}}_j^{i},\mathbf{m}_j^{\text{-}i})\qquad\qquad \nu_d^{i}=E(\delta_d|\mathbf{S}_j^{i},\mathbf{m}_j^{\text{-}i})$$

Such that $\Delta(\delta_d)=\nu_d^{i*}-\hat{\nu}_d^i$. The generic IC constraint is then given by:

$$-\Bigg[\Big[(\nu_1^{i*}+\hat{\nu}_1^i)-2(\nu_1^i+b_1'-b_1^i)\Big]\,\Delta(\delta_1)+\Big[(\nu_2^{i*}+\hat{\nu}_2^i)-2(\nu_2^i+b_2''-b_2^i)\Big]\,\Delta(\delta_2)\Bigg]\,P(\mathbf{m}^{\text{-}i}|S_1)\,P(\mathbf{m}^{\text{-}i}|S_2)\geq 0 \tag{B2.8}$$

Note that under Centralization $b_1'=b_2''=0$ and we are back to the IC constraints in Chapter 1.More importantly, when i reveals one signal only ν_d^{i*} and $\hat{\nu}_d^i$ are different from ν^i. Sender i's strategies in equilibrium and in the deviation under analysis do not transmit all the information he has, and the beliefs he induces on j are different from what he believes are the optimal decisions (in the equilibrium under consideration). As I show later, this generates credibility losses for i because of the possibility of ambiguous information —i.e. signals that move decisions in opposite directions if fully revealed. □

[35]Note that $f(\theta_1,\mathbf{m}^{\text{-}i}|\mathbf{S}^i)=f(\theta_1|\mathbf{m}^{\text{-}i},S_1^i)\,P(\mathbf{m}^{\text{-}i}|S_1^i)$ and that $f(\theta_2,\mathbf{m}^{\text{-}i}|\mathbf{S}^i)=f(\theta_2|\mathbf{m}^{\text{-}i},S_2^i)\,P(\mathbf{m}^{\text{-}i}|S_2^i)$

Proof of Lemma A2.1. Consider the equilibrium $(\mathbf{y}^*, \mathbf{m}^*_{\mathbf{j}'}, \mathbf{m}^*_{\mathbf{j}''})$ in which the principal delegates y_d to agent j, who does not decide on the other decision. By the assumption on private information with each decision-maker, i's messages to j only affect y_d. The IC constraint in (B2.8) then becomes:

$$-\left[(\nu_d^{i*} + \hat{\nu}_d^i) - 2(\nu_d^i - b_d^i)\right] \Delta(\delta_d) \geq 0$$

I denote by $\nu_{dr}^{i*} = E(\theta_r|\mathbf{m}_j)$ and $\hat{\nu}_{dr}^i = E(\theta_r|\hat{\mathbf{m}}_j)$ sender i's expectations of j's posterior beliefs about θ_r in equilibrium when he plays strategies m_j^i and $\hat{m}_j$ (eqm and deviation), respectively. In addition, denote by $\nu_{dr}^i = E(\theta_r|S_r^i, \mathbf{m}_j^{-i})$ sender i's expectation of j's posterior beliefs about θ_r if j knew i's information about that state.

The IC constraint then becomes:

$$-\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i) + w_{d2}(\nu_{d2}^{i*} - \hat{\nu}_{d2}^i)\right]\left[w_{d1}(\nu_{d1}^{i*} + \hat{\nu}_{d1}^i) + w_{d2}(\nu_{d2}^{i*} + \hat{\nu}_{d2}^i) - 2[w_{d1}\nu_{d1}^i + w_{d2}\nu_{d2}^i - (b_d^i - b_d^j)]\right] \geq 0$$

When he reveals only one signal, however, the previous is true for the state associated with the signal he reveals truthfully but not for the other state. To see this recall that he is not being influential with respect to the latter, so his expectations about j's beliefs in equilibrium are different from his conjectures including his own information.[36] It is easy to check that:

$$\begin{aligned}\nu_{dr}^i = E\left(\theta_r|\tilde{S}_r^i, \mathbf{m}_j^{-i}\right) &= \frac{(\ell_r^j + 1 + \tilde{S}_r)}{(k_r^j + 3)} \\ &= \frac{\left(k_r^j + 3 - (-1)^{\tilde{S}_r^i}\right)}{2(k_r^j + 3)}\end{aligned}$$

The second equality follows from taking expectation over the realization of others' signals (by the Law of Iterated Expectations); whereas $E(\theta_r|m_j^{-i}) = \nu_{dr}^{-i} = 1/2$ —i.e. what i expects j's beliefs on θ_r are if only considers other senders' truthful messages.

Consider the equilibrium in which i reveals S_1^i only, the IC constraint becomes:

$$-\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i)\right]\left[w_{d1}(\hat{\nu}_{d1}^i - \nu_{d1}^{i*}) + 2w_{d2}(\nu_{d2}^{i*} - \nu_{d2}^i) + 2[(b_d^i - b_d^j)]\right] \geq 0$$

When i's type is given by $\tilde{\mathbf{S}}^i = (0,0)$, then $\nu_{d1}^{i*} = \nu_{d1}^i = \frac{(k_1^j+2)}{2(k_1^j+3)}$, $\hat{\nu}_{d1}^i = \frac{(k_1^j+4)}{2(k_1^j+3)}$; and $\nu_{d2}^{i*} = \hat{\nu}_{d2}^i = 1/2$, while $\nu_{d2}^i = \frac{(k_1^j+2)}{2(k_1^j+3)}$. Replacing these values on the above IC constraint I get:

$$2(b_1^i - b_1^j) \leq \frac{w_{d1}}{(k_1^j + 3)} + \frac{w_{d2}}{(k_2^j + 3)}$$

The case of $\tilde{\mathbf{S}}^i = (1,1)$ is analogous, with the LHS having negative signs. For the case of $\tilde{\mathbf{S}}^i = (0,1)$, all

[36] In other words, if i is expected to reveal S_1^i, then $\nu_{d2}^{i*} = \hat{\nu}_{d2}^i = E(\theta_s|\mathbf{m}_j^{-i})$ because i is not influential with respect to θ_2, and $\nu_{d2}^{i*} \neq \nu_{d2}^i$.

i's conjectures about j's posteriors are the same as the previous case (i reveals the same realization of the same signal), except for $\nu^i_{d2} = \frac{(k^j_1+4)}{2(k^j_1+3)}$. It can be easily checked that this leads to:

$$2(b^i_1 - b^j_1) \leq \frac{w_{d1}}{(k^j_1+3)} - \frac{w_{d2}}{(k^j_2+3)}$$

Whereas for $\tilde{\mathbf{S}}^i = (1,0)$ the LHS above has a negative sign.

Because all these IC constraints have to be satisfied in order for i to be credible in the equilibrium under consideration, and given that all of them hold for the same measure of conflict of interest between i and j, the following is a necessary and sufficient conditions for i not having incentives to lie on S^i_1:

$$|b^i_1 - b^j_1| \leq \frac{1}{2}\left[\frac{w_{d1}}{(k^j_1+3)} - \frac{w_{d2}}{(k^j_2+3)}\right]$$

Now, in equilibria in which i reveal both signals truthfully $\nu^{i*}_{dr} = \nu^i_{dr}$; that is, what he expects j's beliefs to be in equilibrium is the same as what he conjectures the optimal decisions will be according to his information. The IC constraint then becomes:

$$-\left[w_{d1}(\nu^{i*}_{d1} - \hat{\nu}^i_{d1}) + w_{d2}(\nu^{i*}_{d2} - \hat{\nu}^i_{d2})\right]\left[w_{d1}(\nu^{i*}_{d1} - \hat{\nu}^i_{d1}) + w_{d2}(\nu^{i*}_{d2} - \hat{\nu}^i_{d2}) - 2(b^i_d - b^j_d)\right] \geq 0$$

Incentive compatibility in this case means i prefers truthful revelation of both signals to any deviation, taking into account that any message he sends is believed to be truthful. So, for each type $\tilde{\mathbf{S}}^i = \{(0,0);(0,1);(1,0);(1,1)\}$ I consider deviations to announce a message different from his own. This leads to three generic deviations: lying in both signals,[37] lying on S^i_1,[38] and lying on S^i_2.[39] A substantial amount of algebra shows that the IC constraints for not lying in one signal are similar to those for revealing one signal, but without the negative term on the RHS. Incentives not to lie on both signals depend on whether the signals coincide or not, enthusiast readers can check that replacing the values for ν^{i*}_{dr} and $\hat{\nu}^i_{dr}$ for each type and deviation leads to the following IC constraints.

$$\text{For } \tilde{S}^i = \{(0,0);(1,1)\}: \qquad |b^i_1 - b^j_1| \leq \frac{1}{2}\left[\frac{w_{d1}}{(k^j_1+3)} + \frac{w_{d2}}{(k^j_2+3)}\right]$$

$$\text{For } \tilde{S}^i = \{(0,1);(1,0)\}: \qquad |b^i_1 - b^j_1| \leq \frac{1}{2}\left|\frac{w_{d1}}{(k^j_1+3)} - \frac{w_{d2}}{(k^j_2+3)}\right|$$

The last of the above is the necessary condition for full revelation given it is more restrictive (RHS is smaller). Note moreover that the expression is similar to that for revealing one signal only, which mean that whenever 'both' hold the decision-maker will prefer full revelation and, thus, will be the strategy arising in

[37] Meaning that type $(0,0)$ announces $(1,1)$ or vice-versa, and type $(0,1)$ announces $(1,0)$ or vice-versa.

[38] Meaning that type $(0,0)$ announces $(1,0)$ or vice-versa, and type $(0,1)$ announces $(1,1)$ or vice-versa.

[39] Meaning that type $(0,0)$ announces $(0,1)$ or vice-versa, and type $(1,0)$ announces $(1,1)$ or vice-versa.

equilibrium.

Finally, I analyse the existence of Dimensional Non-separable (DNS) message strategies. I consider two of them:[40] revealing both signals when they coincide and no information otherwise, $m_j^{i*} = \{\{(0,0)\},\{(1,1)\},\{(0,1),(1,0)\}\}$, and full revelation when they do not coincide and nothing otherwise, $m_j^{i*} = \{\{(0,1)\},\{(1,0)\},\{(0,0),(1,1)\}\}$. Deviation incentives between the two influential messages translate into the IC constrains derived above. Note that this rules out the DNS in which the agent fully reveals ambiguous information because the IC constraint will be similar to that for Full Revelation.

Now I show that the strategy $m_j^{i*} = \{\{(0,0)\},\{(1,1)\},\{(0,1),(1,0)\}\}$ cannot arise in equilibrium. The argument relies on the incentives of the 'non-influential types' to announce influential messages according to their bias. When $\mathbf{S}^i = (0,1)$ does not have incentives to announce $m_j^i = \{(1,1)\}$, $\nu_{1r}^{i*} = 1/2$ and $\hat{\nu}_{1r}^i = \frac{(k_r^j+4)}{2(k_r^j+3)}$ for both signals; whereas $\nu_{11}^i = \frac{(k_1^j+4)}{2(k_1^j+3)}$ and $\nu_{12}^i = \frac{(k_2^j+2)}{2(k_2^j+3)}$. Then, solving the IC constraint I get the following:

$$(b_1^i - b_1^j) \leq \frac{1}{4}\left[\frac{w_{11}}{(k_1^j+3)} - \frac{w_{12}}{(k_2^j+3)}\right]$$

Which, solving for all the relevant cases, leads to:

$$|b_1^i - b_1^j| \leq \frac{1}{4}\left|\frac{w_{11}}{(k_1^j+3)} - \frac{w_{12}}{(k_2^j+3)}\right|$$

Note that this IC constraint is more restrictive than that for Full Revelation, and the decision-maker will prefer the latter. As a consequence, no DNS strategy arises under delegation. □

Proof of Proposition A2.1. Having derived the necessary conditions for communication (Lemma A2.1), getting the receiver-optimal (R-optimal) equilibrium consist of finding the system of beliefs consistent with each message strategy (if those exist). Let denote by $\mu_j^*\left(\mathbf{S}^i | m_j^{i*}\right)$ the beliefs of decision-maker j about i's information (her type) upon receiving message m_j^{i*}.

A Fully Revealing message strategy means any of i's messages is taken at face value, that is:

$$\mu^*\left((0,0)|m_j^i = \{(0,0)\}\right) = 1 \qquad \mu^*\left((1,0)|m_j^i = \{(1,0)\}\right) = 1$$
$$\mu^*\left((0,1)|m_j^i = \{(0,1)\}\right) = 1 \qquad \mu^*\left((1,1)|m_j^i = \{(1,1)\}\right) = 1$$

From Lemma (A2.1) we know that if i's preferences satisfy condition (A2.1), then he truthfully announces his type and the beliefs above are consistent with that strategy in equilibrium. Because j also knows that, the system of beliefs defined above are implemented only if i's preferences satisfy condition (A2.1); otherwise, there is always a deviation for which the beliefs are not consistent. Now, because the IC constraints for revealing one signal are the same and given I focus on R-optimal equilibria, fully revealing dominates.

[40]In the companion paper I show that full revelation of two types and babbling for the other two are the only two DNS message strategies arising in any equilibrium. The argument is based on the equilibrium selection criterion given the similarity of IC constraints, and applies to the case of one decision as well.

For DNS, the Proof of Lemma A2.1 showed the only of such strategies emerging in equilibrium is $m_j^{i*} = \{\{(0,0)\},\{(1,1)\},\{(0,1),(1,0)\}\}$. Beliefs consistent with such a strategy are given by:

$$\mu^*\left((0,0)|m_j^i = \{(0,0)\}\right) = 1 \qquad\qquad \mu^*\left((1,1)|m_j^i = \{(1,1)\}\right) = 1$$
$$\mu^*\left((0,1)|m_j^i = \{(0,1)\}\right) = \mu^*\left((1,0)|m_j^i = \{(0,1)\}\right) = 1/2$$
$$\mu^*\left((0,1)|m_j^i = \{(1,0)\}\right) = \mu^*\left((1,0)|m_j^i = \{(1,0)\}\right) = 1/2$$

By the same argument as above, these are equilibrium beliefs if and only if i's preferences satisfy (A2.2). □

PROOF OF EQUATION (A2.7) (DNS UNDER CENTRALIZATION). For any dimensional non-separable strategy in which i fully reveals some realizations and play babbling on the others, the expected payoff in equation (A2.4) becomes.

$$\frac{[(w_{11})^2+(w_{21})^2]}{6}\left[\frac{1}{(\hat{k}_1+2)}-\frac{1}{2(k_1^*+2)}-\frac{1}{2(k_1^*+3)}\right]+$$
$$+\frac{[(w_{12})^2+(w_{22})^2]}{6}\left[\frac{1}{(\hat{k}_2+2)}-\frac{1}{2(k_2^*+2)}-\frac{1}{2(k_2^*+3)}\right]\geq\left[C(\mathfrak{s}^{i*})-C(\hat{\mathfrak{s}}^i)\right]$$

Agent i fully reveals his signals half of the time (in expectation), where k_r^* indicates the equilibrium number of agents revealing information about θ_r apart from i. Then, solving for deviations as in the previous result, I get that acquiring both signals to play a DNS message strategy is cost effective if:

$$\frac{(w_{11})^2+(w_{21})^2}{6(k_1^*+2)(k_1^*+3)}+\frac{(w_{12})^2+(w_{22})^2}{6(k_2^*+2)(k_2^*+3)}\geq 2C(S_1^i,S_2^i)$$

Now, i would prefer to acquire both signals and play DNS strategy to acquire only S_r and reveal it for sure if:

$$\frac{(w_{1\tilde{r}})^2+(w_{2\tilde{r}})^2}{6(k_{\tilde{r}}^*+2)(k_{\tilde{r}}^*+3)}-\frac{(w_{1r})^2+(w_{2r})^2}{6(k_r^*+2)(k_r^*+3)}\geq 2C(S_{\tilde{r}}^i)$$

Which is easily shown that never holds when $w_{11}=w_{22}$ and $w_{d1}+w_{d2}=1$. □

Proof of Proposition A2.3

PROOF OF PROPOSITION A2.3. We know from Lemmas 2.1 and 2.2 that any equilibrium information acquisition strategy must be CE and IC at the communication stage. Recall that the principal observes agents' choices of information, for which she knows the relevant message space for each agent. Equilibrium communication is then characterized as in Proposition A2.1 and its equivalent under centralization (Proposition A1.1).

But cost-effectiveness can impose restrictions on equilibrium communication; for instance, when i cannot afford to acquire all the information he is willing to reveal on-path. Consider the case in which i revealing both signals is IC but acquiring only one of them is CE. Revelation of each signal individually is a necessary condition for Full Revelation, so i acquires one signal if CE and reveal it in equilibrium. Now, which of those signals he actually acquires depend on the ex-ante expected utility —see case 4) in the previous proof.

Similar argument applies to any equilibria in which i is willing to reveal S_r^i only. If acquiring is CE, then he reveals that information in equilibrium; if not, he does not reveal any information —indeed, he has no message to send and the receiver observes that.

Finally, no information is transmitted when i's preferences are such that he is not willing to reveal any information, or when acquiring any signal is not cost-effective.

For the case of delegation the same argument applies, with Lemmas A2.1 and 2.1, and Corollary 2.2. $\square$

Proof of Proposition 2.5. When costs do not impose restrictions on information acquisition, incentive compatibility at the communication stage dictates agents' equilibrium strategies (expressions in Proposition A1.1). In the case of two agents and linear costs, CE does not restrict agents' communication strategies if the cost of any signal is lower than the ex-ante expected utility in the equilibrium when $k_r^* = 1$; that is:

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^*+2)(k_r^*+3)}$$
$$c \leq \frac{(w)^2 + (1-w)^2}{72}$$

Agent A^1 acquires and reveals S_1^1 in equilibrium if his preferences satisfy expression (A2.5) associated with θ_1 and with $k_1^* = 1$ (see Section B2 for the supporting system of beliefs on communication). But his preferences should not be such that full revelation is IC, otherwise the principal would be better-off under this equilibrium.[41] A similar argument applies for A^2 with respect to S_2^2.

When $\frac{(w)^2+(1-w)^2}{72} < c \leq \frac{(w)^2+(1-w)^2}{36}$ it is not CE to acquire both signals, so agents acquire the signal each of them is willing to reveal. In other words, the necessary and sufficient condition for specialization in this case is that (A2.5) holds for different signals for each agent. $\square$

Proof of Lemma A2.3. Let $w \equiv w_{11} = w_{22}$, let κ be an non-negative integer, and let $\varepsilon \in \Re_+$ with associated integer n_ε. Also, let $i \in N_\varepsilon$ be an agent whose preferences satisfy equation (A2.5) with respect to θ_r for $k_r = \kappa$. Note that $\boldsymbol{\lambda}_r = \left(1, -\frac{w_{1r}}{w_{2r}}\right)$, the associated unit vector is $\hat{\boldsymbol{\lambda}}_r = \frac{\boldsymbol{\lambda}_r}{||\boldsymbol{\lambda}_r||} = \left(\frac{w_{1r}}{(w_{1r}^2+w_{2r}^2)^{1/2}}, \frac{w_{2r}}{(w_{1r}^2+w_{2r}^2)^{1/2}}\right)$, and that $\text{Proj}_{\boldsymbol{\lambda}_r}(\mathbf{b}^i) = (\mathbf{b}^i \cdot \hat{\boldsymbol{\lambda}}_r)\boldsymbol{\lambda}_r$. Then, careful algebra leads to condition (A2.5) expressed as:

$$||\mathbf{b}^i - \text{Proj}_{\lambda_1}(\mathbf{b}^i)|| \leq \frac{\left[(w)^2 + (1-w)^2\right]^{\frac{1}{2}}}{2\,(\kappa+3)}$$

Now consider an agent j with the following preferences: $b_1^j = b_2^i$ and $b_2^j = b_1^i$. I need to show 1) $j \in N_\varepsilon$, and 2) $\mathbf{b}^j$ satisfies equation (A2.5) for θ_2. Proving the first claim is straightforward, since j's preference

[41] Note that if condition (A2.5) holds for S_2^1 (but still not those for full revelation) he will still find optimal to acquire S_1^1 only, *given A^2 is acquiring the other signal in equilibrium.*

vector is just i's with its components swapped. This says that both i and j agents have exactly the same conflict of interest with the principal.

The second part of the proof requires work out $||\mathbf{b}^j - \text{Proj}_{\lambda_2}(\mathbf{b}^j)||$, which yield:

$$
\begin{aligned}
||\mathbf{b}^j - \text{Proj}_{\lambda_2}(\mathbf{b}^j)|| &= |b_1^j\,(1-w) + b_2^j\,w|\;\left[(w)^2 + (1-w)^2\right]^{-\frac{1}{2}} \\
&= |b_2^i\,(1-w) + b_1^i\,w|\;\left[(w)^2 + (1-w)^2\right]^{-\frac{1}{2}} \\
&= ||\mathbf{b}^i - \text{Proj}_{\lambda_1}(\mathbf{b}^i)||
\end{aligned}
$$

□

C2 Covert Information Acquisition

Covert Information Acquisition Game

In the covert game the decision-maker does not observe agents' information acquisition decisions. This implies that a Perfect Bayesian Equilibrium must also specify the decision-maker's beliefs about agents' investments in information. I will focus on pure strategy equilibria at the information acquisition stage. In principle, the agent in question may try to convey information about which signals he acquired to the decision-maker by means of his cheap talk message. However, a result from Argenziano et al. (2016) allows me to restrict attention to equilibria in which agents do not signal how much information each has acquired, and this is without loss of generality. Below I present the result.

Lemma C2.4 (Argenziano et al., 2016)**.** *Any outcome supported in a Perfect Bayesian Equilibrium of the covert game in which an agent follows a pure strategy in the choice of information can be supported in a Perfect Bayesian Equilibrium in which the decision-maker's beliefs about his information acquisition decision do not vary with the agent's message.*

There would be two classes of deviations available to agents if the decision-maker's beliefs about information acquisition decisions could be affected by the choice of messages. First, an agent could acquire an off-path amount of information but still send the message corresponding to the equilibrium amount of information. Secondly, the agent could acquire an off-path amount information and send a message corresponding to an off-path information acquisition choice, which in turn may no be true. The lemma says that any equilibrium outcome under the second class of deviations can be supported as an equilibrium in which the agent cannot change the decision-maker's beliefs about his information acquisition decision.

When an agent acquires an off-path amount of information he can choose among the equilibrium messages according to his preferences. As a consequence, any deviation at the information acquisition stage implies a deviation at the communication stage. The result below summarizes this.

Lemma C2.5. *When agent i acquires fewer signals than what is expected on the equilibrium path, the messages used under the deviation are a strict subset of the equilibrium messages available. When i acquires more signals than expected on path, he uses the additional information to deviate from truth-telling for some signal realizations.*

The argument of the above lemma is straightforward. When i acquires fewer signals, he will not be able to condition his message on the information that has not been observed. As a consequence, the messages effectively used under the deviation will be fewer than those available on the equilibrium path, which implies that i is inducing beliefs that do not reflect his signal realizations. When i acquires more signals he cannot transmit that additional information on the equilibrium path (there is no way of signalling he acquired more information). Because additional information implies additional costs, i must be obtaining some utility gains with respect to equilibrium communication|by inducing beliefs according to his preferences for some signals realizations. This clearly implies that he will deviate from truth-telling when he observes the corresponding realizations.

Let $(\mathfrak{s}^*, \mathbf{m}^*(\mathfrak{s}^*), \mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)))$ be the equilibrium information acquisition decisions, message strategies, and decisions (respectively). Then, agent i's IC constraint at the information acquisition stage must consider any possible deviation $\hat{\mathfrak{s}}^i$ and the corresponding message strategy $\hat{m}^i(\hat{\mathfrak{s}}^i)$; that is,

$$E\Bigg[\int_0^1\int_0^1 -\sum_{y_d}^{\{y_1,y_2\}}\left(y_d\left(m^{i*},\mathbf{m}^{\text{-}i*}\right)-\delta_d-b_d^i\right)^2 f(\theta_1|\mathfrak{s}^{i*},\mathbf{m}^{\text{-}i*})f(\theta_2|\mathfrak{s}^{i*},\mathbf{m}^{\text{-}i*})+$$
$$+\int_0^1\int_0^1\sum_{y_d}^{\{y_1,y_2\}}\left(y_d\left(\hat{m}^{i},\mathbf{m}^{\text{-}i*}\right)-\delta_d-b_d^i\right)^2 f(\theta_1|\hat{\mathfrak{s}}^{i},\mathbf{m}^{\text{-}i*})f(\theta_2|\hat{\mathfrak{s}}^{i},\mathbf{m}^{\text{-}i*})\Bigg]\geq C(\mathfrak{s}^{i*})-C(\hat{\mathfrak{s}}^i) \quad \text{(C2.9)}$$

Then, given that deviations at the info acquisition stage do not affect the set of influential messages (Lemma C2.4) and that this deviations necessarily imply deviations at the communication stage (Lemma C2.5), the above expression can be solved by computing the expectation over all possible signals realizations and the corresponding messages on- and off-path. In particular, the utility gains from deviations will be given by the realizations in which the messages on- and off- path are different. Formally, let $\tilde{\mathbf{S}}^i$ represent the realization of the signals corresponding to agent i, independent of which of these he observes (determined by $\mathfrak{s}^i$). I can then express and compare message strategies on- and off-path as functions of i's type and the information he observes. In other words, *before deciding on information acquisition* and given the equilibrium under play, he can assess the utility gains from any info acquisition strategy and the corresponding messages he expects to send conditional on each possible pair of signal realizations. Equation C2.9 then becomes:

$$\sum_{\tilde{\mathbf{S}}\in\mathcal{S}}\Pr(\tilde{\mathbf{S}}^i)\times\int_0^1\int_0^1 -\sum_{y_d}^{\{y_1,y_2\}}\left[\left(y_d\left(m^i(\mathfrak{s}^{i*},\tilde{\mathbf{S}}^i)\right)-\delta_d-b_d^i\right)^2-\left(y_d\left(m^i(\hat{\mathfrak{s}}^{i},\tilde{\mathbf{S}}^i)\right)-\delta_d-b_d^i\right)^2\right]\times$$
$$\times f(\theta_1|\tilde{S}_1^i,\mathbf{m}^{\text{-}i*})f(\theta_2|\tilde{S}_2^i,\mathbf{m}^{\text{-}i*})\geq C(\mathfrak{s}^{i*})-C(\hat{\mathfrak{s}}^i)$$

Now, I proceed to analyse deviations from different equilibrium info acquisition strategies.

Agent i acquires both signals in equilibrium ($\mathfrak{s}^{i*}=\mathbf{S}^i$)

Let denote by $\nu_r^{i*}(\tilde{\mathbf{S}}^i)=E\left(\theta_r|m^i(\mathfrak{s}^{i*},\tilde{\mathbf{S}}^i),\mathbf{m}^{\text{-}i*}\right)$ the beliefs about $\theta_r=\{\theta_1,\theta_2\}$ induced by i under the equilibrium information acquisition strategies and the message corresponding to the realizations given by $\tilde{\mathbf{S}} \in \mathcal{S}$. Equivalently, denote by $\hat{\nu}_r^i(\tilde{\mathbf{S}}^i) = E\left(\theta_r | m^i(\hat{\mathfrak{s}}^i, \tilde{\mathbf{S}}^i), \mathbf{m}^{-i*}\right)$ be the beliefs induced under the deviation at the information acquisition stage (for the same signals realizations). Then, the IC constraint at the information acquisition stage for agent i becomes:

$$\sum_{\tilde{\mathbf{S}}^i \in \mathcal{S}} \Pr(\tilde{\mathbf{S}}^i) \Bigg[-\sum_{y_d} \left[w_{d1} \left(\nu_1^{i*}(\tilde{\mathbf{S}}^i) - \hat{\nu}_1^i(\tilde{\mathbf{S}}^i) \right) + w_{d2} \left(\nu_2^{i*}(\tilde{\mathbf{S}}^i) - \hat{\nu}_2^i(\tilde{\mathbf{S}}^i) \right) \right] \times$$
$$\left[-w_{d1} \left(\nu_1^{i*}(\tilde{\mathbf{S}}^i) - \hat{\nu}_1^i(\tilde{\mathbf{S}}^i) \right) - w_{d2} \left(\nu_2^{i*}(\tilde{\mathbf{S}}^i) - \hat{\nu}_2^i(\tilde{\mathbf{S}}^i) \right) - 2b_d^i \right] \Bigg] \geq C(\mathfrak{s}^{i*}) - C(\hat{\mathfrak{s}}^i)$$

First consider the deviation in which i only acquires information about θ_1; that is, $\hat{\mathfrak{s}}^i = \{S_1^i\}$. It is straightforward to note that this deviation *per se* does not imply any difference in induced beliefs with respect to θ_1, formally $\nu_1^{i*}(\tilde{\mathbf{S}}^i) = \hat{\nu}_1^i(\tilde{\mathbf{S}}^i)$ for all $\tilde{\mathbf{S}}^i \in \mathcal{S}$. Now, i's message associated with S_2^i does not depend on the signal's realization, but depends on $\mathbf{b}^i$ and may also depend on S_1^i.

Let consider the case in which $\hat{m}^i = \{\tilde{S}_1^i, 1\}$, i.e. i truthfully reveals his information about θ_1 and announces always a 1 for θ_2. Then, $\hat{\nu}_1^i(\tilde{\mathbf{S}}^i) = \frac{(k_2+4)}{2(k_2+3)}$ and it is different from $\nu_1^{i*}(\tilde{\mathbf{S}}^i)$ only when $\tilde{S}_2^i = \{0\}$ which, in turn, happens for $\tilde{\mathbf{S}}^i = \{(0,0);(1,0)\}$. The IC constraint in such a case is:

$$\Pr\left(\tilde{\mathbf{S}} = \{(0,0)\}\right) \left[\sum_{y_d} \left[w_{d2} \left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)} \right] \right] \left[w_{d2} \left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)} - 2b_d^i \right] \right] \right] +$$
$$\Pr\left(\tilde{\mathbf{S}} = \{(1,0)\}\right) \left[\sum_{y_d} \left[w_{d2} \left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)} \right] \right] \left[w_{d2} \left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)} - 2b_d^i \right] \right] \right] \geq C(S_2^i)$$

Given that $\Pr\left(\tilde{\mathbf{S}} = \{(0,0)\}\right) = \Pr\left(\tilde{\mathbf{S}} = \{(1,0)\}\right) = 1/4$, the IC constraint becomes.

$$\frac{1}{(k_2+3)} \left[\frac{(w_{12}^2 + w_{22}^2)}{2(k_2+3)} - \beta_2^i \right] \geq C(S_2^i)$$

That is, the expected utility gains of inducing the correct beliefs about θ_2 should be greater than the extra utility from saving in the costs of becoming informed about that state. It is easy to show that the case of $\hat{m}^i = \{\tilde{S}_1^i, 0\}$ has the sign of β_2^i reversed, for which the generic IC constraint for not acquiring signal S_r^i becomes:

$$\frac{1}{(k_r+3)} \left[\frac{(w_{1r}^2 + w_{2r}^2)}{2(k_r+3)} - |\beta_r^i| \right] \geq C(S_r^i) \tag{C2.10}$$

For the deviation involving no information acquisition, $\hat{\mathfrak{s}}^i = \{\emptyset\}$, the expression of the IC constraint will depend on the message i decides to announce at the communication stage. On the one hand, when the message is $\hat{m)}^i = \{(0,0),(1,1)\}$, the induced beliefs will coincide with the equilibrium strategy when $\tilde{\mathbf{S}}^i = (1,1)$, but also partially for other realizations. Formally $\nu_r^{i*}(1,1) = \hat{\nu}_r^i(1,1)$ for $\theta_r = \{\theta_1, \theta_2\}$, $\nu_1^{i*}(1,0) = \hat{\nu}_1^i(1,1)$, and $\nu_2^{i*}(0,1) = \hat{\nu}_r^i(1,1)$. Following the characterization of equilibrium communication under centralization, the IC constraint becomes:

$$\left[\frac{1}{(k_1+3)}\left[\frac{(w_{11}^2+w_{21}^2)}{2(k_1+3)}+\frac{(w_{11}w_{12}+w_{21}w_{22})}{(k_2+3)}-|\beta_1^i|\right]+\right.$$
$$\left.+\frac{1}{(k_2+3)}\left[\frac{(w_{12}^2+w_{22}^2)}{2(k_2+3)}+\frac{(w_{11}w_{12}+w_{21}w_{22})}{(k_1+3)}-|\beta_2^i|\right]\right]\geq 2C(S_1^i,S_2^i)+\frac{(w_{11}w_{12}+w_{21}w_{22})}{(k_1+3)(k_2+3)} \tag{C2.11}$$

Which basically is a more strict version of the IC constraint for full revelation when signals coincide (under centralization).

Similarly, when the deviation involves announcing $\hat{m)}^i=\{(0,1),(1,0)\}$ the IC constraint becomes:

$$\left[\frac{1}{(k_1+3)}\left[\frac{(w_{11}^2+w_{21}^2)}{2(k_1+3)}-\frac{(w_{11}w_{12}+w_{21}w_{22})}{(k_2+3)}-|\beta_1^i|\right]+\right.$$
$$\left.+\frac{1}{(k_2+3)}\left[\frac{(w_{12}^2+w_{22}^2)}{2(k_2+3)}-\frac{(w_{11}w_{12}+w_{21}w_{22})}{(k_1+3)}-|\beta_2^i|\right]\right]\geq 2C(S_1^i,S_2^i)+\frac{(w_{11}w_{12}+w_{21}w_{22})}{(k_1+3)(k_2+3)} \tag{C2.12}$$

Agent i acquires one signal on path ($\mathfrak{s}^{i*}=\{S_1^i\}$).

When i acquires only one signal on path, his assessment of the consequences of any deviation still conditions on each possible pair of signal realizations. As I show in this section, this becomes particularly important for deviation involving acquisition of more signals. Not is necessary to distinguish between the induced beliefs on- and off-path, and the actual information i has access to. Thus, in addition to the previously defined $\nu_r^{i*}(\tilde{\mathbf{S}}^i)$ and $\hat{\nu}_r^i(\tilde{\mathbf{S}}^i)$, I now denote by $\nu_r^i(\tilde{\mathbf{S}}^i)=E\left(\theta_r|\tilde{\mathbf{S}}^i,\mathbf{m}^{\text{-}i*}\right)$ the beliefs about θ_r that would result from the decision-maker observing the signals available to agent i (independent of his information acquisition strategy). Then, i's IC constraint at the information acquisition stage becomes:

$$\sum_{\tilde{\mathbf{S}}^i\in\mathcal{S}}\Pr(\tilde{\mathbf{S}}^i)\Bigg[-\sum_{y_d}\left[w_{d1}\left(\nu_1^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_1^i(\tilde{\mathbf{S}}^i)\right)+w_{d2}\left(\nu_2^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_2^i(\tilde{\mathbf{S}}^i)\right)\right]\times$$
$$\left[w_{d1}\left(\nu_1^{i*}(\tilde{\mathbf{S}}^i)+\hat{\nu}_1^i(\tilde{\mathbf{S}}^i)-2\nu_1^i(\tilde{\mathbf{S}}^i)\right)+w_{d2}\left(\nu_2^{i*}(\tilde{\mathbf{S}}^i)+\hat{\nu}_2^i(\tilde{\mathbf{S}}^i)-2\nu_2^i(\tilde{\mathbf{S}}^i)\right)-2b_d^i\right]\Bigg]\geq C(S_1^i)-C(\hat{\mathfrak{s}}^i)$$

When i considers the deviation of not acquiring any signals and decides to announce $\hat{m}_1^i=\{1\}$, he induces incorrect beliefs as compared to the equilibrium in two cases, namely $\tilde{\mathbf{S}}=\{(0,0);(0,1)\}$. The ex-ante expected utility losses of such strategy depends on the signal realizations, as can be noted in the expression for the IC constraint below:

$$\Pr\left(\tilde{\mathbf{S}}^i = (0,0)\right)\left[-\sum_{y_d}\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)} - \frac{(k_1+4)}{2(k_1+3)}\right)\right]\times\right.$$
$$\left.\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)} + \frac{(k_1+4)}{2(k_1+3)} - \frac{2(k_1+2)}{2(k_1+3)}\right) + w_{d2}\left(\frac{1}{2}+\frac{1}{2}-\frac{2(k_2+2)}{2(k_2+3)}\right) - 2b_d^i\right]\right]+$$
$$\Pr\left(\tilde{\mathbf{S}}^i = (0,1)\right)\left[-\sum_{y_d}\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)} - \frac{(k_1+4)}{2(k_1+3)}\right)\right]\times\right.$$
$$\left.\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)} + \frac{(k_1+4)}{2(k_1+3)} - \frac{2(k_1+2)}{2(k_1+3)}\right) + w_{d2}\left(\frac{1}{2}+\frac{1}{2}-\frac{2(k_2+4)}{2(k_2+3)}\right) - 2b_d^i\right]\right] \geq C(S_1^i)$$

Which, after some algebra gives:

$$\frac{1}{(k_1+3)}\left[\frac{(w_{11}^2+w_{21}^2)}{2(k_1+3)} - \beta_1^i\right] \geq C(S_1^i) \tag{C2.13}$$

It is straightforward to note that the generic IC constraint involves the absolute value of β_r^i.

Deviations involving the acquisition of more information have the issue that i cannot signal this to the decision-maker. This additional information will thus be used to identify situations (i.e. signal realizations) under which i will deliberately lie to the decision-maker. Such deviations are thus related to the credibility loss, because i would like to induce beliefs about the signal he is not expected to acquire on path by means of messages on the signal he is believed on path.

As analysed in the communication game, the credibility loss takes place when signals do not coincide, $\tilde{\mathbf{S}}^i = \{(0,1);(1,0)\}$, so any deviation at the communication stage will take place in one of these cases. Moreover, given that β_1^i is typically not zero, i's incentives to lie will always be in a single direction, that is either when $\tilde{\mathbf{S}}^i = (0,1)$ or when $\tilde{\mathbf{S}}^i = (1,0)$ but not in both. The IC constraint for the deviation of acquiring both signals and announcing $\hat{m}_1^i = 0$ when $\tilde{\mathbf{S}}^i = (1,0)$ will be given by:

$$-\Pr\left(\tilde{\mathbf{S}}^i = (1,0)\right)\sum_{y_d}\left[\left[w_{d1}\left(\frac{(k_1+4)}{2(k_1+3)} - \frac{(k_1+2)}{2(k_1+3)}\right)\right]\times\right.$$
$$\left.\left[w_{d1}\left(\frac{(k_1+4)}{2(k_1+3)} + \frac{(k_1+2)}{2(k_1+3)} - \frac{(k_1+4)}{(k_1+3)}\right) + w_{d2}\left(1-\frac{(k_2+2)}{(k_2+3)}\right) - 2b_d^i\right]\right] \geq C(S_2^i)$$

Which yields:

$$\left[\frac{(w_{11}^2+w_{21}^2)}{2(k_1+3)^2} - \frac{(w_{11}w_{12}+w_{21}w_{22})}{2(k_1+3)(k_2+3)} - \beta_1^i\right] \geq -2C(S_2^i) \tag{C2.14}$$

Which is equivalent to say that the cost of acquiring the second signal is too large with respect to the utility gain from deviating under ambiguous information.

Incentive compatibility then depends on $|\beta_1^i|$ being within the limits imposed by equations (A2.5), (C2.13), and (C2.14). Note that equation (C2.13) implies (A2.5), meaning that if i is willing to acquire $\tilde{S}_1^i$ instead of acquiring no signal, then he will certainly reveal it. Incentive compatibility thus is captured by equations (C2.13), and (C2.14), which lead to:

$$\frac{|\beta_1^i|}{(k^{\mathrm{C}}+3)} \leq \min\left\{\frac{(w_{11}^2+w_{21}^2)}{2(k_1^{\mathrm{C}}+3)^2} - C(S_1^i)\,;\, \frac{(w_{11}^2+w_{21}^2)}{2(k_1^{\mathrm{C}}+3)^2} - \frac{(w_{11}w_{12}+w_{21}w_{22})}{2(k_1+3)(k_2+3)} + 2C(S_2^i)\right\} \tag{C2.15}$$

Now, let define $\mathfrak{B}_1\left(C(S_1), C(S_2), \mathbf{k}^{\mathrm{C}}\right) = \{\mathbf{b} : \mathbf{b}\,\text{satisfies equation (C2.15)}\}$. Then, the LHS in equation (C2.15) is weakly positive and, thus, $\mathfrak{B}_1 \neq \emptyset$ when the RHS is strictly positive, that is,

$$\frac{(w_{11}^2+w_{21}^2)}{2(k_1^{\mathrm{C}}+3)^2} - \max\left\{C(S_1^i)\,;\, \frac{(w_{11}w_{12}+w_{21}w_{22})}{2(k_1^{\mathrm{C}}+3)(k_2^{\mathrm{C}}+3)} - 2C(S_2^i)\right\} > 0 \tag{C2.16}$$

Moreover, it is evident from the above expression that $\frac{\partial\mathfrak{B}_1}{\partial C(S_1)} < 0$ and $\frac{\partial\mathfrak{B}_1}{\partial C(S_2)} > 0$.

Chapter 3

Organizational Design and the Acquisition of Imperfect Information

3.1 Introduction

Principals who need to implement a policy and do not have enough information typically consult experts. Politicians consult advisors, headquarters consult managers before making corporate decisions, and investors consult investment bankers about the value of securities. However, effective communication is often impaired by the presence of conflicts of interest between parties: experts may want to misrepresent information to manipulate the principal's decision. This credibility problem can be solved by delegation of authority, such that all of an expert's knowledge is used for decision-making. As long as the expert is well informed, the principal typically benefits from the informational gains associated to delegation despite the decision will be biased (Dessein, 2002). But information is often costly and the expert's knowledge on the relevant matters can be limited. This paper argues that information costs play a crucial role in the expert's informational advantage under delegation, thus shaping the principal's incentives to allocate authority.

I model the interaction between a principal and a biased expert as a game with three stages. First, the principal decides whether to make the decision herself or delegate it to the expert. Secondly, the expert decides how much information he observes. The information, in turn, takes the form of noisy signals correlated with a state with known prior distribution. Finally, the expert sends a costless, non-verifiable message to the principal. Unlike similar papers in the literature, here I characterize information acquisition incentives for all biases for which communication is possible.[1] Doing so allows me to analyse information acquisition and optimal allocation of authority for intermediate biases, which leads to novel intuitions.

Under centralization, information transmission at the communication stage depends on the expert's bias *and* information. Were he perfectly informed, equilibrium communication involves a particular 'coarsening' of his information (Crawford and Sobel, 1982). When information is costly, however, the expert will typically have imperfect information. Equilibrium communication in such cases may also involve coarse messages,

[1] Argenziano et al. (2016) answer a similar question but focus on incentives to acquire information for small conflict of interests.

but there are instances in which he fully reveals his information (Fischer and Stocken, 2001; Di Pei, 2015; Förster, 2020). For small biases and large information costs, the prevalence of such communication strategies reduces the expert's informational advantage under delegation (Argenziano et al., 2016).

Incentives for delegation thus depend on how much information the expert acquires under each organizational structure. Signals have decreasing informational value but increasing costs. When in charge of the decision, the expert acquires information until the utility from the marginal reduction in variance compensates for the marginal costs. When the principal decides, however, the amount of information the expert can credibly transmit in equilibrium decreases with his bias. For intermediate biases, the use of coarse messages dilutes the informational value of each additional signal. Decisions under delegation are therefore (weakly) more informed than under centralization, but information costs will affect how large these informational gains are. Delegation is optimal only if the informational gains compensate for the loss of control.

My contribution is threefold. First, I show that centralization is optimal if the cost of information is sufficiently large—where the expert's bias determines how large costs must be. The benefits from delegation depend on how much more information is effectively used in decision-making. When the expert's bias is small, such benefits are marginal because he is willing to fully reveal large amounts of information. When the expert's bias is large, on the other hand, the informational gains that would make the principal delegate authority must compensate a larger loss of control. Therefore, the net informational benefits from delegation decrease as the unitary costs of information increase.

Secondly, I show that for intermediate biases the expert needs sufficiently many signals in order to implement an informative message strategy.[2] For the case of quadratic preferences, uniform distribution of the state, and binary signals, this restriction results in the expert acquiring more information under centralization than under delegation. Such *overinvestment* leads to optimal centralization for a range of intermediate biases, despite delegation being optimal for a non-empty set of smaller and larger biases.

Finally, when the principal does not observe how much information the expert acquires, delegation becomes (weakly) more profitable for all biases and information costs. This is due to the additional deviations available to the expert at the information acquisition stage. Acquiring fewer signals than what the principal expects in equilibrium would save the expert some costs of information. Such deviations restricts the expert's ability to become informed under centralization, while they do not affect his incentives under delegation. Therefore, informational gains from delegation in the covert game are larger than in the overt game.

3.1.1 Related Literature

The paper relates to two strands of the literature on organizational design: one dealing with endogenous information acquisition, and the other analysing optimal information quality when communication is strategic. Aghion and Tirole (1997) present one of the first analysis of authority on incentives to acquire information. In a model of project selection, a principal delegates formal authority to an agent because it encourages him to acquire information, if these increased incentives compensate for the loss of control associated to preference

[2]The arbitrage condition for the implementation of a two-partition message strategy in Crawford and Sobel (1982) holds for types closer to zero as the bias increases.

divergence. When actions are contractible, Szalay (2005) finds that the agent's incentives improve when left with extreme option because departing in the wrong direction is ex-ante more costly.

For the case of non-contractible actions and strategic communication, Dessein (2002) studies the allocation of authority when a perfectly informed expert can send cheap talk messages to the principal. He finds that full delegation dominates communication if the preference divergence between players is sufficiently small. My paper shows that this result is not robust to imperfect information. For small biases, the expert is willing to fully reveal his information to the principal.[3] Such communication incentives reduce informational gains from delegation when the cost of information is sufficiently large. The same intuition led Argenziano et al. (2016) to conclude that the principal can induce the agent to overinvest in information under centralization. Beyond the case of small biases, my paper also shows that the set of intermediate biases for which centralization is optimal increases with the costs of information.

The fact that imperfect information can lead to better communication is not new either. Fischer and Stocken (2001), for instance, find that the optimal information structure in a two-players cheap talk game consists of noisy signals the sender fully reveals in equilibrium. Such information structure divides the state space into intervals which are more even than under equilibrium communication when the sender is perfectly informed. In a more general setting, Ivanov (2010) shows that the principal prefers such information design to different forms of delegation if the conflict of interest with the expert is small. More recently, Förster (2020) shows similar results when the agent has access to binary signals.[4] Unlike these papers, my work features an expert who optimally decides how much information he acquires after he knows who controls the decision.

3.2 The Model

An organization has the opportunity to engage in a valuable project. There are potentially infinite projects but only one can be implemented. A principal, P (she), controls the critical resources of the organization, needed to initiate the project. An agent (expert), E (he), is hired to implement the project because he has access to relevant information.

Preferences. The projects available differ from each other in one dimension that can be represented by a real number $y \in Y \equiv \Re_+$. Players derive utility from each project, denoted by $u_P(y, \theta)$ for the principal and $u_E(y, \theta, b)$ for the expert, where θ is a random variable representing the payoff-relevant state and $b > 0$ is a parameter of preference dissonance between E and P. The utility of the principal reaches a unique maximum for $y = \theta$ and can be represented as:

$$u_P(y, \theta) = u_P(\theta, \theta) - \ell\left(|y - \theta|\right)$$

where $\ell''(\cdot) > 0$ and $\ell'(0) = 0$. Similarly, the utility of the expert is maximized for $y = \theta + b$. Unlike the

[3] Förster (2020) shows the expert's (equilibrium) willingness to fully reveal information is decreasing in his bias.

[4] The notion that some noise added to cheap talk communication can improve welfare is also related to mediation (Goltsman et al., 2009), and language barriers (Giovannoni and Xiong, 2019)—see also Blume et al. (2007).

principal, the expert has access to imperfect information about θ, but he must bear some costs to effectively observing that information. The information structure S_K represents the amount of information the expert decided to observe, where K (to be explained below) is associated to its precision. Acquiring more precise information is increasingly costly for the expert. The expert's preferences can be rewritten as

$$u_E(y, \theta, b) = u_E(\theta + b, \theta) - \ell\left(|y - (\theta + b)|\right) - c(S_K)$$

Information structure and timing. The state is distributed in the unit interval, $\theta \in \Theta \equiv [0, 1]$. The common prior over the state is a distribution F on Θ with continuous and strictly positive density $f(\theta)$. The expert can decide on how many binary signals associated to θ he observes. In particular, his information is represented by a vector $S_K = (s_1, s_2, ..., s_K) \in \mathcal{S}_K = \{0, 1\}^K$ for $K \in \mathcal{K} \equiv \mathbb{Z}_+$. Each $s_k \in \{0, 1\}$ is a conditionally independent realization of a Bernoulli Distribution associated with θ; that is, pr$(s_i = 1) = \theta$ and pr$(s_i = 0) = 1 - \theta$.

The cost of information structure S_K is represented by an weakly increasing and convex function $c(S_K)$, such that $c(0) = 0$. Formally, if the expert chooses to observe K signals, he incurs in the cost $c(S_K)$ and simultaneously learns the realization of all signals, $\Sigma_K = \sum_{k=1}^{K} s_k \in \{0, 1, 2, ..., K\}$. I assume that the number of signals the expert observes, K, is common knowledge at the communication stage.[5] Once informed, he sends a *cheap talk* message to the *decision-maker*, $m \in M = \Re$.

Before the expert decides on information acquisition, the principal allocates the right to decide over y. Figure 3.1 depicts the timing of the game. If the principal retains authority, the expert can use his information *indirectly* through communication with the principal. If, in the other hand, she delegates authority to the expert, he has complete discretion over y and hence uses his information *directly*. Formally, the principal decides on $a \in A = \{P, E\}$, which defines a proper subgame $\Gamma = \{\Gamma^P, \Gamma^E\}$ characterized by the player who has the right over y. A strategy for the principal is then a mapping $\alpha : A \to \Gamma$; strategies for the expert are mappings $\sigma : \mathcal{K} \to \mathcal{S}_K$ and $\mu : \mathcal{S}_K \to \Delta(M)$. Also, depending on who controls y, a decision strategy is a mapping $\psi : \Gamma \times \mathcal{K} \times M \to \Delta(Y)$.

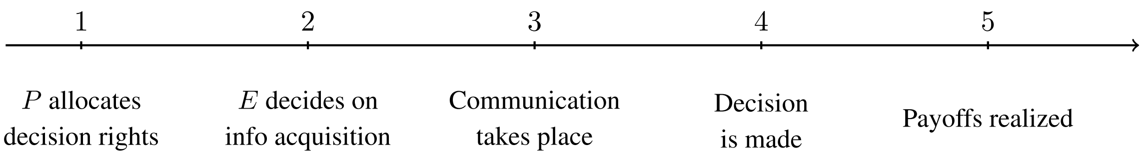


Figure 3.1: Timing of the Game

Equilibrium concept. In this game I study Perfect Bayesian Equilibria $(\alpha, \sigma, \mu, \psi) \equiv \big(\alpha, \sigma(a), \mu(S_K), \psi(\beta(m))\big)$; where $\beta(m) \to \Delta(\Theta)$ denote the posterior belief of the decision-maker upon receiving message $m \in M$ derived from the expert's message strategy μ following Bayes' rule whenever possible. As usual in the cheap

[5]Section 3.5 deals with the case of covert information acquisition.

talk literature, I assume that for any $m \in M$ such that $Pr(m|\mu) = 0$, then $\psi(m) = \psi(\mu(S_K))$ which allows us to ignore off-equilibrium messages.[6]

Let $DM = \{P, E\}$ the index for the decision-maker; then the equilibrium action profile $\psi(m)$ satisfies:

$$y^{DM} \in \arg\max_{y \in Y} \int_{\Theta} u^{DM}(y, \theta, b) f(\theta|\mu(S_K), \sigma) d\theta$$

In particular, the optimal decision for any decision-maker will depend on the updated expectation about the state: $y^{DM} = E\left[\theta|\mu(S_K), K\right] + b^{DM}$.

Given K, the communication strategy $\mu(S_K)$ satisfies:

$$m \in \arg\max_{m \in M} \int_{\Theta} u^{E}(y, \theta, b) f(\theta|\sigma) d\theta$$

And σ maximizes the expert's expected payoff under the allocation of decision rights, given equilibrium conjectures about μ and ψ:

$$K \in \arg\max_{K \in \mathcal{K}} \int_{\Theta} u^{E}(y, \theta, b) f(\theta) d\theta$$

Finally, the optimal organizational structure depends on the principal's ex-ante expected utility under centralization and delegation. Delegating the right to decide over y motivates the expert to acquire information because he can use all knows to decide; contrary to the case of centralization, where the bias determines the amount of information that can be credibly transmitted. As a consequence, having more signals does not necessarily translate into a more informed decision, since it depends on how the expert uses his information—directly when he controls y, and by means of cheap talk when the principal does.[7] Besides the potential informational gains, delegation leads to decisions that are different from the principal's ideal. This involves a trade-off between informational gains and loss of control for the principal which, in this case, is determined by how much information the expert is willing to acquire under the different organizational structures.

3.3 Equilibrium Analysis

3.3.1 Information Acquisition

Delegation. If the expert controls y, then $\mu^D(S_K) = \Sigma_K$. In this case, any additional signal improves the precision of his decision but the net marginal utility of such additional information also depends on costs.

Remark 3.1. *When the expert controls y, the following holds true:*

(i). $E\left[\ell\big(y(S_K), \theta, b\big)\right] < E\left[\ell\big(y(S_{K-1}), \theta, b\big)\right]$ *for all K;*

(ii). $E\left[\ell\big(y(S_{K-1}), \theta, b\big)\right] - E\left[\ell\big(y(S_K), \theta, b\big)\right] > E\left[\ell\big(y(S_K), \theta, b\big)\right] - E\left[\ell\big(y(S_{K+1}), \theta, b\big)\right]$

Acquiring more signals increases the precision of the expert's decision, which reduces the expected loss from decision-making for him as well as for the principal. The fact that there are no communication-related

[6]In the covert game, I have to consider deviations at the information acquisition stage as well.

[7]Förster (2020) shows that additional signals may decrease the informativeness of equilibrium communication.

distortions makes this effect is monotonic, but it is decreasing in the number of signals because of the loss function *and* the decreasing marginal value of information.[8] Together with the convexity of cost, these features make the equilibrium amount of information under centralization unique.

Lemma 3.1. *The equilibrium under delegation is characterized by a unique* $K^D = \{0, 1, 2, ...\}$ *and decision* $y^D \equiv y(S_{K^D}, b)$*:*

$$K^D \equiv \max_K \left\{ E\left[\ell\big(|E(\theta|\Sigma_{K-1}) - \theta|\big)\right] - E\left[\ell\big(|E(\theta|\Sigma_K) - \theta|\big)\right] \geq c(S_K) - c(S_{K-1}) \right\} \tag{3.1}$$
$$y^D \equiv E(\theta|\Sigma_{K^D}) + b$$

The principal's expected utility from delegation is given by $E\left[u_P\big(y, \theta, b, S_K\big)|\cdot\right] \equiv U_P(y(b, \Sigma_{K^D}), S_{K^D}) = u_P(\theta, \theta) - E\left[\ell\big(|E(\theta|\Sigma_{K^D}) - \theta + b|\big)\right]$. Note that this expression is increasing in the number of signals the expert acquires, K^D, and decreasing in his bias, b. The principal is thus willing to delegate if the loss from a biased decision is compensated (in expectation) by the *additional* information the expert acquires as compared to centralization.

Centralization. This section builds upon Förster (2020), who characterizes a sender's incentives to transmit information via cheap talk when he has access to binary signals. When the information the expert observes is imperfect, the equilibrium communication strategy can be truthful revelation even for non-zero biases (Austen-Smith and Riker, 1987; Fischer and Stocken, 2001; Morgan and Stocken, 2008; Ivanov, 2010; Galeotti et al., 2013; Argenziano et al., 2016). The lemma below characterizes the expert's incentives for communication given his bias and the number signals he observes.

Lemma 3.2 (Equilibrium communication)**.** *A message strategy* $\mu(b, S_K)$ *is an equilibrium under centralization if and only if there exist a* K *and an associated consecutive overlapping partition*[9] $\mathcal{P}_K = \{P_0, P_1, ..., P_{\kappa-1}\}$ *of* $\mathcal{S}_K$ *such that:*

(i). There exist a message associated to each element of the partition, $m_0, m_1, ..., m_{\kappa-1} \in M_K = \mathcal{S}_K$*, such that* $m_k \in \sup(\mu(b, S_K))$ *if and only if* $\Sigma_K \in P_k$ *for all* $k = 0, 1, ..., \kappa - 1$*,*

(ii). The following arbitrage conditions hold:

$$\begin{aligned} E\left[\ell\big(|E(\theta|m_{k-1}) - \theta + b|\big)|\Sigma_K = \max P_{k-1}\right] &\leq E\left[\ell\big(|E(\theta|m_k) - \theta + b|\big)|\Sigma_K = \max P_{k-1}\right] \\ E\left[\ell\big(|E(\theta|m_k) - \theta + b|\big)|\Sigma_K = \min P_k\right] &\leq E\left[\ell\big(|E(\theta|m_{k-1}) - \theta + b|\big)|\Sigma_K = \min P_k\right] \end{aligned} \tag{3.2}$$

[8] Signals have a positive but decreasing marginal value of information. Let $MSE(K) \equiv E\left[\Big(\theta - E(\theta|\Sigma_K)\Big)^2\right]$ denote the ex-ante residual variance of θ when the decision involves K signals; then, $MSE(K) - MSE(K-1) < MSE(K+1) - MSE(K)$.

[9] A *consecutive overlapping partition* of the signal space $\mathcal{S}_K$ is a $\mathcal{P}_K = \{P_0, P_1, ..., P_{\kappa-1}\}$ such that $\bigcup_{k=0}^{\kappa-1} P_k = \mathcal{S}_K$ and $\max P_{k-1} \leq \min P_k$ for all $k = 1, 2, ..., \kappa - 1$ (see Definition 3 in Förster, 2020).

There are two bias thresholds, $b^{FR}(K)$[10] and $b^{NC}(K)$,[11] such that the most informative message strategy characterized by partition $\mathcal{P}_K = \{P_0, P_1, ..., P_{\kappa-1}\}$ involves:

1. ***Full revelation** of information if $b \leq b^{FR}(K)$,*
2. ***Partial revelation** of information if $b^{FR}(K) < b < b^{NC}(K)$,*
3. ***Babbling** otherwise.*

Proof. Application of Propositions 2 and 3 in Förster (2020). □

The first part of the lemma constitutes a generalization of Crawford and Sobel (1982) to imperfect information in the form of binary signals. A message strategy is informative in equilibrium if different messages induce different actions by the decision-maker. Any of such strategies involves a partition of the signal space such that there is a unique message associated to each partition, and the expert finds incentive compatible to announce a particular message when the signals realization corresponds to the associated partition. The optimality of the expert's behaviour is captured by the incentives of the 'boundary types' in the arbitrage condition.

The second part of the lemma shows the different equilibrium message strategies. Information transmission in equilibrium depends on the conflict of interest between principal and expert, and on how much information the latter observes. When the expert observes few signals, his own posterior beliefs carry some noise. He thus faces a trade off between giving information to the principal, which result in a more accurate yet biased decision, and inducing too much residual variance that can also be costly. For small conflict of interest, his optimal decision is close enough to the principal's such that he prefers to keep the expected residual variance low.

In other words, when the expert observes few signals, his information is less precise than what he would optimally reveal to the principal through coarse messages. This intuition is captured by $b^{FR}(K) < b^{NC}(K)$ for any K, and by the fact that $b^{FR}(K)$ is decreasing in K.Indeed, for biases $b \leq b^{FR}(K)$ the equilibrium communication strategy with S_K can never be partially informative because any of such equilibria involve neglecting favourable information for some signals realizations (which, he is actually willing to fully reveal).[12]

With a partition with fewer elements the principal's best responses to the different messages are further apart from each other and, therefore, communication is less informative. Because less informative communication loosens the expert's incentive compatibility constraints, the equilibrium amount of information transmitted in the most informative equilibrium is weakly decreasing in b. In the limit, large biases prevent

[10] $b^{FR}(K) \equiv \min_{k=0,1,...,K-1} b^{FR}(k,K)$, such that $b^{FR}(k,K)$ solves $E\left[\ell\big(E(\theta|\Sigma_K = k), b^{FR}(k,K)\big)\big|\Sigma_K = k\right] = E\left[\ell\big(E(\theta|\Sigma_K = k+1), b^{FR}(k,K)\big)\big|\Sigma_K = k\right]$.

[11] $b^{NC}(K) \equiv \max_{k=0,1,...,K-1} b^{NC}(k,K)$, such that $b^{NC}(k,K)$ solves $E\left[\ell\big(E(\theta|\Sigma_K \leq k), b^{FR}(k,K)\big)\big|\Sigma_K = k\right] = E\left[\ell\big(E(\theta|\Sigma_K > k), b^{FR}(k,K)\big)\big|\Sigma_K = k\right]$.

[12] Förster (2020) shows that for any S_K, partially informative equilibria in which the expert fully reveals a subset of the available signals is not an equilibrium for $b \leq b^F R(K)$. Also, for the same biases, note that all equilibria involving fewer partitions than $\#S_K$ are dominated by full revelation.

credible information transmission. This boundary is represented by $b^{NC}(K)$, which is increasing in K because having more signals enlarges the set of posteriors about θ that can be induced in equilibrium; giving the expert more flexibility to combine them into incentive compatible messages.

The threshold $b^{FR}(K)$ represents the bias up to which the expert prefers to disclose each of the possible realizations of his K signals than to exaggerate some of them to induce higher actions.[13] Similarly, the threshold $b^{NC}(K)$ represents the bias above which the expert prefers to induce a higher action whenever the principal believes him to be revealing any information. The following two lemmas define variables associated to these thresholds, and some of they properties that will help analyse information acquisition.

Lemma 3.3. *There exists a non-negative integer $K^{FR}(b)$ defined as:*

$$K^{FR}(b) \equiv \arg\max_{K} \left\{ E\Big[\ell\big(y(\Sigma_K = k), b\big)|\Sigma_K = k\Big] \leq E\Big[\ell\big(y(\Sigma_K = k+1), b\big)|\Sigma_K = k\Big] \right\}$$

for all $k = 0, 1, ..., K-1$. Moreover, $\lim_{b \to 0} K^{FR}(b) = \infty$, $K^{FR}\left(b > b^{FR}(1)\right) = 0$, and $\frac{\partial K^{FR}(b)}{\partial b} \leq 0$.

$K^{FR}(b)$ characterizes the maximum amount of information an expert with bias b is willing to fully reveal to the principal in the most informative communication equilibrium. Note that, as the expert bias tends to zero, his incentives to be truthful to the principal involve infinitely large amount of information. Indeed, if the expert actually observes fewer than $K^{FR}(b)$ signals, he reveals them all at the communication stage, which has clear implications for the principal's incentives to delegate decision-making authority.

As the bias increases, incentives for truthful communication decrease. At some point all communication equilibria involve coarse messages. In order to do this, he needs enough signals to implement an incentive-compatible partition of the state space. The lemma below characterizes the minimum amount of information the expert needs to implement an incentive-compatible message strategy.

Lemma 3.4. *There exists a a non-negative integer $K^{NC}(b)$:*

$$K^{NC}(b) \equiv \arg\min_{K} \left\{ E\Big[\ell\big(y(\Sigma_K < k), b\big)|\Sigma_K = k\Big] \geq E\Big[\ell\big(y(\Sigma_K > k), b\big)|\Sigma_K = k\Big] \right\}$$

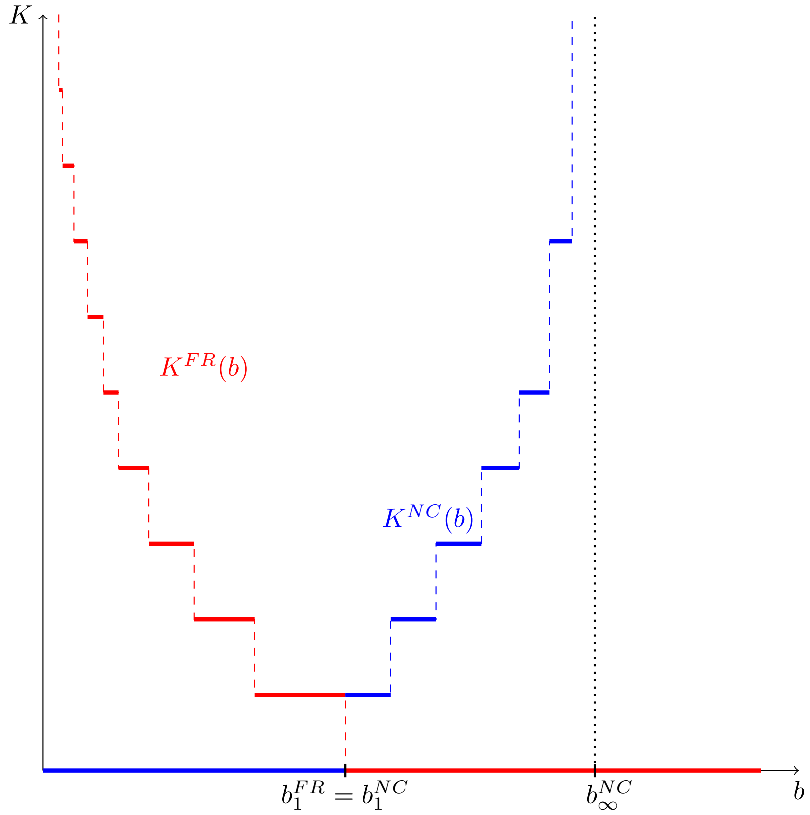
for at least one $k = 0, 1, ..., K-1$. Moreover,$K^{NC}\left(b \leq b^{FR}(K=1)\right) = 0$, and $\frac{\partial K^{NC}(b)}{\partial b} \geq 0$ for $b \in [\bar{b}, b^{NC}_{\infty}]$, where $\bar{b} > b^{FR}(K=1)$ and $b^{NC}_{\infty} \equiv \lim_{K \to \infty} b^{NC}(K)$.

When the expert's bias is so small that he is willing to fully reveal information, the minimum number of signals to implement an influential message strategy is 1. For sufficiently large biases, on the contrary, there is a single communication equilibrium which involves no information transmission—such strategy does not require any information by the expert. Hence, the restrictions on the minimum amount of signals apply to intermediate biases only.

Note that the relationship defined in Lemma 3.4 focuses on the implementation of equilibria involving only two messages ($\kappa = 2$). As the expert's bias increases, incentive compatibility of such equilibria requires

[13] For binary signals and positive bias, it is sufficient to focus on deviations involving misrepresentations of a single low signal, i.e. for $S_K = \Sigma_K$, then $\mu' = \Sigma_K + 1$.

Figure 3.2: Maximum amount of signals the expert fully reveals, $K^{FR}(b)$, and minimum amount needed to implement an informative communication equilibrium, $K^{NC}(b)$ (Lemmas 3.3 and 3.4)



the message inducing the high decision to be coarser (see Crawford and Sobel, 1982). In other words, for biases $b < b'$ such that influential communication is possible, $\kappa^*(b, \theta) \geq \kappa^*(b', \theta) \geq 2$,[14] the boundary type for $\kappa = 2$ with the former bias is larger for b than for b', i.e. $\max P_0^K(b) \geq \max P_0^K(b')$ for any K. Therefore, implementing $\mathcal{P}^K(b')$ requires a finer partition of the state space, which means acquiring more signals: $K^{NC}(b)$ is weakly increasing in b.

The relationships $K^{FR}(b)$ and $K^{NC}(b)$ characterize communication incentives when information costs are large for the expert. If the maximum amount of information he finds cost-effective to acquire is lower than $K^{FR}(b)$, the associated message strategy involves full revelation. On the contrary, an expert with large bias may find that he cannot afford to acquire $K^{NC}(b)$ signals and, hence, no information will be transmitted (or acquired) in equilibrium. Figure 3.2 shows both relationships.

I now proceed to characterize information acquisition under centralization. The expert's decision depends on the cost of a given information structure and on how much of that information he can credibly transmit to the principal at the communication stage. Because costs are weakly convex, it is worth acquiring additional

[14] $\kappa^*(b, \theta) \equiv \lim_{K \to \infty} \kappa^*(b, K)$; that is, the number of intervals in the most-informative equilibrium when an expert of bias b perfectly observes the state θ.

signals only if enough of the additional information can be transmitted through cheap talk communication. A larger bias, hence, typically means the optimal information structure involves fewer signals. The fact that information acquisition is a discrete decision implies that for sufficiently close biases, the equilibrium information acquisition decision coincide. The proposition below presents these intuitions.

Proposition 3.1. *Given $c(\cdot)$ and b, there exists two thresholds, $\underline{b}^C \leq \bar{b}^C$, such that:*

1. *for $b \leq \underline{b}^C$, then $K^C \leq \min\{K^D, K^{FR}(b)\}$ and $\mu(b, K^C) = \Sigma_{K^C}$;*
2. *for $b \geq \bar{b}^C$, then $K^C = 0$; and*
3. *for $b \in (\underline{b}^C, \bar{b}^C)$, then $K^C > \max\{K^{FR}(b), K^{NC}(b)\}$ and $\mu(b, K^C) = \{m_0, m_1, ..., m_\kappa\}$, with $\kappa < K + 1$.*

Moreover, $\underline{b}^C \geq b^{FR}(K^D)$ and $\bar{b}^C < b^{NC}_\infty$.

Equilibrium information acquisition is characterized by at least two regions. For non-zero costs K^D is always finite. Therefore, even if the expert fully reveals his information, the decision never perfectly matches the realization of the state. For sufficiently small biases, the expert fully reveals any $K^C \leq K^D$. The amount of signals he effectively acquires for $b \leq b^{FR}(K^D)$ depends on costs, but can never exceed K^D.

For sufficiently large biases, on the contrary, the expert cannot credibly transmit any information to the principal in equilibrium and, hence, acquiring information is useless. How large must the bias be for $K^C = 0$ to be optimal? This depends on information costs. A sufficient condition is that $b \geq b^{NC}_\infty$, which amounts to say that no communication can take place even if the expert has perfect information about theta. The actual $\bar{b}^C$ may be larger than $b^{NC}(K^D)$, which implies that an expert with bias $b \in (b^{NC}(K^D), \bar{b}^C)$ acquires more information than if he were in charge of the decision. As shown in section 3.4, such *overinvestment* in information arises because the expert is ex-ante better off revealing some information to the principal than leaving her uninformed. The expected marginal utility the expert derives from being able to transmit some information to the principal compensates the cost of more than K^D signals.

Equilibrium communication for intermediate biases can take the form of coarse messages, in which case $\mu(b, S_{K^C}) = \{m_0, m_1, .., m_\kappa\}$ with $\kappa < K^C + 1$. Such a possibility requires sufficiently low information costs. To see this suppose costs are such that $K^D \geq 2$ and the expert's bias greater but close to $\tilde{b} = b^{FR}(K^D)$ — $\tilde{b} = b^{FR}(K^D) + \epsilon$ for small $\epsilon > 0$. Note that with such bias, the expert would be willing to fully reveal a positive amount of signals had he observed them at the communication stage ($K^{FR}(\tilde{b}) \geq 1$). Then, among all $K \leq K^{FR}(b)$, denote by K_1 the cost effective information acquisition strategy involving full revelation at the communication stage. When $K > K^{FR}(b)$, however, the expert's communication strategy involves $\kappa < K + 1$ partitions of the state space; let K_2 denote the cost effective of such strategies. Because $K_2 > K_1$, the existence of an equilibrium involving coarse communication requires that the expected marginal utility compensates for the additional information costs. Moreover, when K_1 and K_2 are close, some of the additional signals are pooled together in the same messages and, hence, the expected utility gains (if any) are small. Relatively low information costs are then a necessary condition for $K^C = K_2$.

I now show how these intuitions shape the principal's incentives to delegate authority.

3.3.2 Decision Rights

The expert's bias determines the maximum amount of information he can credibly communicate to the principal, while information costs determines the cost-effectiveness of different information structures given the allocation of authority. Under centralization, a well-informed but biased expert typically communicates only part of his information. His his incentives to become informed are thus lower than the case in which he controls the decision. Despite this intuition is not new (Aghion and Tirole, 1997), the optimality of delegation depends on whether the *informational gains* outweigh the *loss of control* from a biased decision (Dessein, 2002; Ivanov, 2010; Argenziano et al., 2016). In the present context, the extent of the informational gains is affected by the fact that information acquisition is endogenous and costly.

Firstly, let define a notion of 'sufficient' informational gains for the case in which the principal decides with no information under centralization.

Definition 3.1. *For every $b \in \Re_+$, let*

$$K^{NI}(b) \equiv \arg\min \big\{ K : \; E\big[\ell\left(|E(\theta) - \theta|\right)\big] - E\big[\ell\left(|E(\theta|\Sigma_K) - \theta + b|\right)\big] \geq 0 \big\}$$

Moreover, there exists a bias b_∞^{NI} such that $\lim_{b \to b_\infty^{NI}} K^{NI}(b) = \infty$

The relationship $K^{NI}(b)$ determines the minimum amount of information the expert with bias b must have in order to get an *uninformed principal* to delegate authority to him. In this framework, this threshold can be interpreted in terms of babbling equilibria under centralization.[15] Next, I define a ranking of cost functions that will be useful to interpret the results on organizational structure.

Definition 3.2. *Given the expert's bias, b, a cost function $c(b,\cdot)$ leads to acquisition of more signals than another cost function $\tilde{c}(b,\cdot)$ if $K^C(b, c(b,\cdot)) \geq K^C(b, \tilde{c}(b,\cdot))$ or $K^D(c(b,\cdot)) \geq K^D(\tilde{c}(b,\cdot))$, or both. I denote such relationship as $c(b,\cdot) \preceq \tilde{c}(b,\cdot)$.*

The relationship $\preceq$ specifies a criterion to compare between cost functions, based on how much information an expert of bias b can acquire under each organizational structure. With such criterion, I now proceed to set bounds on the principal's incentives to delegate.

Proposition 3.2. *Given $c(\cdot)$ and $b \in [0, b_\infty^{NI}]$, there exist two cost thresholds, $c_1(b,\cdot) \leq c_2(b,\cdot)$, and associated information acquisition decisions, $K_1^C(b) \leq K_1^D$ and $K_2^C(b) \leq K_2^D$, such that:*

1. *Centralization is the optimal organizational structure for all $c(b,\cdot) \succeq c_1(b,\cdot)$;*

2. *Delegation is the optimal organizational structure for all $c(b,\cdot) \preceq c_2(b,\cdot)$.*

Moreover, $\lim_{b \to 0} c_2(b) = \lim_{b \to b_\infty^{NI}} c_1(b) = 0$.

[15] Equilibria in which the expert transmits no information to the principal.

Whether the principal prefers to retain or delegate authority depends on how much more information the expert acquires and uses when he controls y, as compared to the most informative equilibrium under centralization. When information is costly and the expert's bias is small, he fully reveals his information in equilibrium and, thus, the incentives to acquire information are similar to delegation. For biases sufficiently close to zero, indeed, the expert's equilibrium information acquisition under centralization and delegation coincide. The expert's communication incentives improve as the bias converges to zero, which means that the number of signals he is willing to fully reveal increases. As a consequence, the necessary informational gains that justify delegation require a larger K^D relative to $K^{FR}(b)$. In other words, as the bias converges to zero the principal will find that centralization is optimal for a larger set of costs.

For larger biases, however, the principal's expected dis-utility from biased decision becomes very large. Delegation will thus be optimal if the informational gains are sufficiently large. Note that, as the bias increases, the expert's incentives for communication given K deteriorate, which means the informational gains from any given K^D initially increase. Eventually, for sufficiently large biases, the equilibrium message strategy consists of only two different messages. From this bias onwards, the amount of information that would compensate the principal for the loss of control increases in the bias. At the limit of b_{∞}^{NI}, the expert needs to be perfectly informed for the principal to delegate authority to him. In other words, as the bias becomes large the principal will find that centralization is optimal for a larger set of costs.

In summary, Proposition 3.2 shows that the principal's incentives to delegate are non-monotonic in the bias. On the one hand, small bias means the expert is willing to acquire as much information under centralization as under delegation, and finds incentive compatible to fully reveal that information. Large biases, on the other hand, lead to a large loss of control under delegation and, thus, the amount of information that would compensate the principal can only be implemented if information is sufficiently 'cheap' for the expert. Overall, large information costs lead to centralization for a large set of biases.

3.4 An Example: The Uniform-quadratic Case

In this section I apply the results to the case of quadratic preferences and uniform distribution of the state. The uniform-quadratic case has been the workhorse formulation in the applied cheap talk literature because its closed-form solutions provide clear-cut policy implications. Players' preferences are then defined by $u_P = -(y-\theta)^2$ and $u_E = -(y-b-\theta)^2 - c(S_K)$. Information costs are given by $c(S_K) = \gamma K^{\alpha}$. I focus on the overt information acquisition game.

The state of the world is uniformly distributed in $[0, 1]$ which, together with the binary structure of each piece of information, configure a Beta-Binomial updating process in line with that of the previous chapters.

3.4.1 Information Acquisition

Delegation. It is well know in the literature that the expected utility of deciding with K signals in this context is given by the residual variance $\mathrm{V}(\theta|K) = \frac{1}{6(k+2)}$ (see Argenziano et al., 2016). Hence the expert's optimal information acquisition under delegation is given by:

$$K^D = \arg\max_{K\in\mathcal{K}} \left\{ -\frac{1}{6(K+2)} - \gamma K^\alpha \right\} \tag{3.3}$$

The principal's ex-ante expected utility is thus given by $U_P = -\frac{1}{6(K^D+2)} - b^2$.

Centralization. When the principal is in charge of the decision, the expert's communication strategy depends on how many signals he observes. After communication takes place, the principal's decision upon receiving a message associated to partition $P_i^K \in \mathcal{P}^K$ is given by:

$$y(P_i, K) = \frac{1}{|P_i|} \sum_{k\in P_i} \frac{(k+1)}{(K+2)} \tag{3.4}$$

Given the optimal decision rule, the following lemma describes the expert's incentives for communication.

Lemma 3.5 (Lemma 2'). *For the uniform-quadratic case, the expert's incentives for communication are characterized by:*

1. $K^{FR}(b) = \left\lfloor \frac{1}{2b} - 2 \right\rfloor$
2. $K^{NC}(b) = \left\lceil \frac{(8b-1)}{(1-4b)} \right\rceil$
3. $|P_{i+1}| - |P_i| \in \{4b(K+2) - 2\,,\, 4b(K+2) + 2\}$ *for all* $P_i = \{P_0, P_1, ..., P_\kappa\}$.

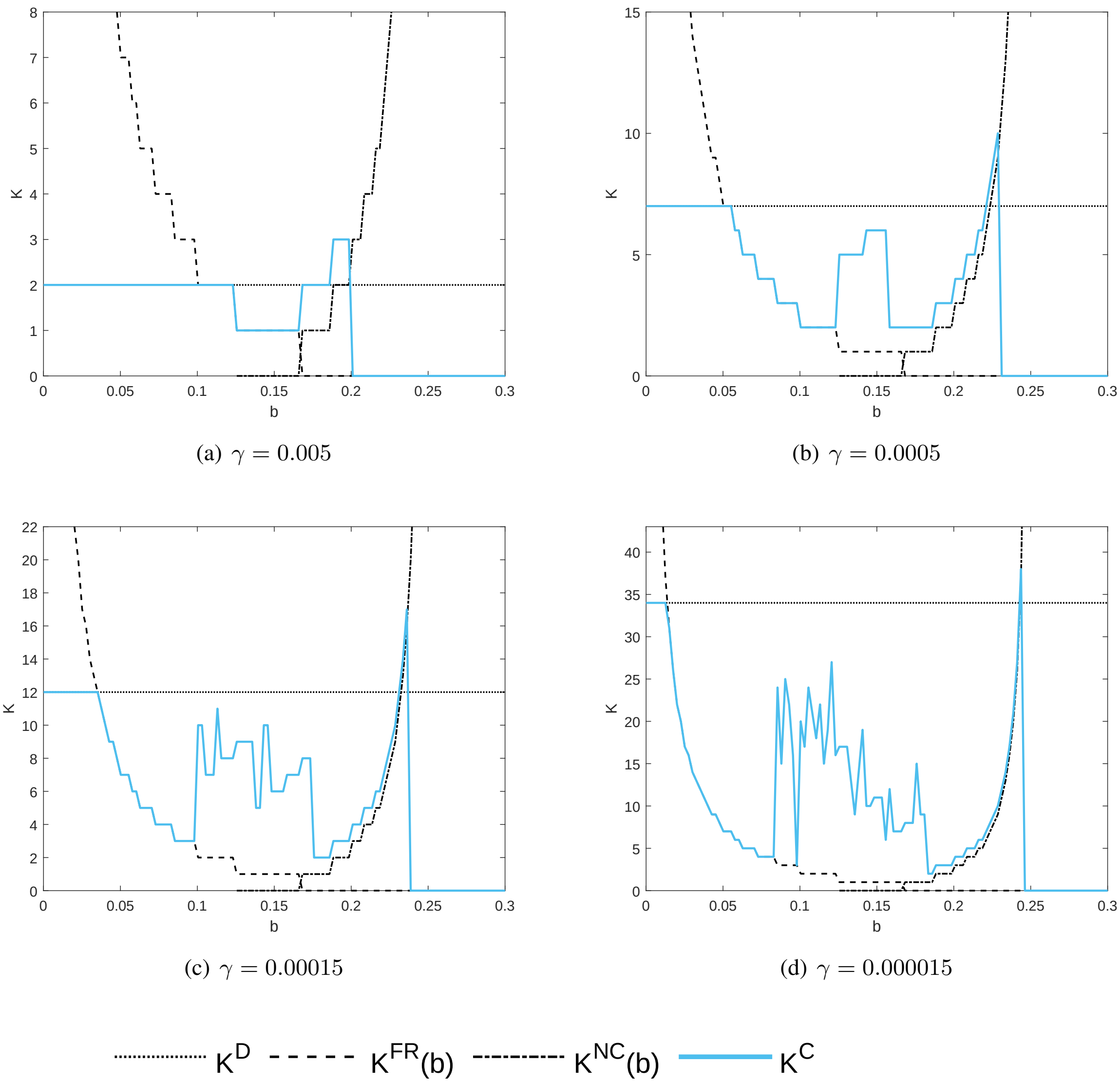
How many signals the expert acquires thus depend on the net expected utility from each possible information structure and the associated optimal communication strategy, which is characterized by partition $\mathcal{P}^K(b)$.

$$K^C(b) = \arg\max_{K\in\mathcal{K}} \left\{ -\frac{1}{(K+1)} \sum_{P_i\in\mathcal{P}^K(b)} \left[\sum_{k\in P_i} \frac{(k+1)(k+2)}{(K+2)(K+3)} - \frac{1}{|P_i|} \left(\sum_{k\in P_i} \frac{(k+1)}{(K+2)} \right)^2 \right] - b^2 - \gamma K^\alpha \right\} \tag{3.5}$$

Note that the first term of the right hand side represents (minus) the residual variance under the equilibrium communication strategy for a given number of signals K; i.e. $\mathrm{V}(\theta|S_K, \mathcal{P}^K(b)) = U_P(S_K, b)$. Given the bias, if the number of signals acquired is lower than $K^{FR}(b)$, the residual variance is $\frac{1}{6(K+2)}$ similar to the delegation case. I now proceed to analyse the equilibrium information acquisition for different cost parameters.

Figure 3.3 shows the equilibrium information acquisition strategy as a function of the expert's bias, for different values of γ. It also depicts the maximum amount of information the expert is willing to fully reveal, $K^{FR}(b)$, the minimum number of signals he needs to be able to implement an informative message strategy, $K^{NC}(b)$, and the amount of information the expert would acquire if he controls the decision, K^D.

Figure 3.3: Equilibrium information acquisition under centralization



(a) $\gamma = 0.005$

(b) $\gamma = 0.0005$

(c) $\gamma = 0.00015$

(d) $\gamma = 0.000015$

As shown in Proposition 3.1, an expert with sufficiently small bias finds optimal to acquire the same amount of information as if he had authority; also, the largest bias for which this holds decreases with the cost of information. On the contrary, when the expert's bias is large enough, his optimal decision is to acquire no information at all, and the smallest bias for which this holds is increasing in costs. Note that for non-zero information costs, $K^C(b) = 0$ is optimal for some biases for which there are informative message strategies available|but they are too costly for the expert.

For intermediate biases, information acquisition incentives are shaped by two forces: $K^{FR}(b)$ and $K^{NC}(b)$. Firstly, when the expert's bias is close to $b^{FR}(K^D)$ the amount of information he is willing to fully reveal is relatively large. Acquiring information beyond $K^{FR}(b)$ means the additional signals will not be fully revealed which, together with decreasing marginal informational value of signals makes such strategies less attractive.

As the bias increases, however, the amount of information the expert is willing to fully reveal decreases. For small enough costs the expert will be better-off implementing a message strategy involving coarse communication ($\kappa < K^C$). In such cases there exists an intermediate range of biases for which coarse communication is optimal, which can be seen in panels (b) to (d) of Figure 3.3. Indeed, the figure shows that the range of biases featuring such information acquisition and communication strategies in equilibrium is decreasing in the costs of information. For such biases, the equilibrium number of signals depends on the communication strategy that can be implemented with each K and the marginal cost of information. Panel (d) shows that, for sufficiently small costs, the amount of information acquired tends to decrease as the bias increases.

When the bias is large, the expert needs sufficiently many signals in order to implement an informative message strategy. This is captured by $K^{NC}(b)$ –meaning that the expert implements an informative message strategy if $K^C(b) > K^{NC}(b)$ and acquires no information otherwise. For large biases, the amount of information transmitted in the most informative equilibrium is small, even when the expert is perfectly informed. The expert thus finds cost-effective to acquire the minimum amount of information that allows him to implement such message strategy. Because $K^{NC}(b)$ is increasing in b, the amount of information acquired by such an expert is also increasing in the bias. Indeed, at some point $K^C(b) = K^{NC}(b) > K^D$ which means the expert is willing to acquire more signals under centralization than under delegation. This *overinvestment*, however, does not always translate in better decision-making for the principal.[16]

3.4.2 Decision Rights

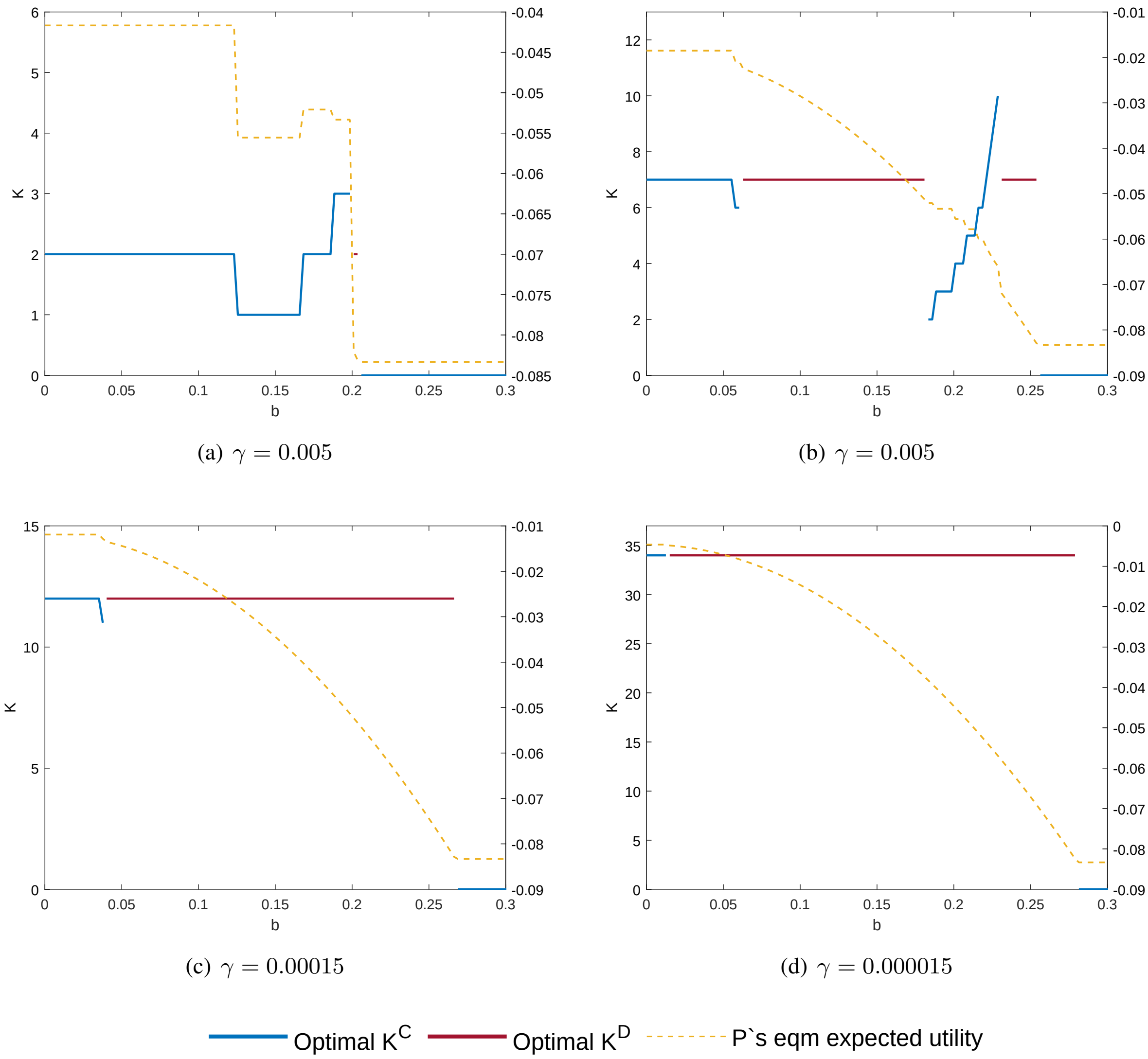
Now the question is how the expert's behaviour under centralization affects the principal's decision about who controls y. Figure 3.4 shows the expert's information investment under the optimal organizational structure as well as the principal's equilibrium ex-ante expected utility, for different values of γ.

First notice that the set of biases for which delegation is optimal (red line) is decreasing in the costs of information. Because the state is uniformly distributed, optimal delegation converges to results in Dessein (2002) as the unit costs converge to zero (panel (d)). This reflects the main result in Proposition 3.2. Further intuitions arise when information is costly. Panel (a) in Figure 3.4 illustrates the fact that centralization is optimal for (almost) all biases when cost are sufficiently large. More interestingly, though, is the fact that there are some biases for which the principal prefers to centralize despite communication is very noisy.

To see this consider the case of $b = 0.2$. An expert with such bias needs at least three signals to implement an informative message strategy but, more importantly, the same amount of information allows

[16]Unlike Argenziano et al. (2016), overinvestment in the present paper arises for large biases. Indeed, the case described in their paper does not arise here because for small biases the expert acquires at most K^D signals under centralization.

Figure 3.4: Information acquisition in the optimal organizational structure



(a) $\gamma = 0.005$ (b) $\gamma = 0.005$

(c) $\gamma = 0.00015$ (d) $\gamma = 0.000015$

him to implement a message strategy with a very small residual variance: $\mathcal{P}^{K=3}(b = 0.2) = \{\{0\}, \{1, 2, 3\}\}$. This strategy can be implemented when $\alpha = 1.5$ and $\gamma = \{0.005, 0.0005\}$. When $\gamma = 0.005$, such a strategy represents an overinvestment relative to what the expert would acquire under delegation ($K^D = 2$), which can be seen in panel (a). The rationale of why the principal prefers centralization is such a case is clear, but the expert's incentives crucially depend on his impossibility to commit ex-ante to a less generous information acquisition strategy. Indeed, the principal prefers such an expert to one with $b = 0.15$.

When the costs are lower ($\gamma = 0.005$) the expert with $b = 0.2$ still finds optimal to acquire three signals under centralization, but he would acquire many more if the had control over the decision ($K^D = 7$). The principal, however, still prefers to retain authority because the informational gains from delegation are not that large relative to the amount of information the expert is expected to transmit. Panel (c) shows that, for slightly smaller unitary costs, the associated informational gains do compensate the loss of control—delegation is optimal. To summarize, for large biases for which informative equilibria exist, the informational gains from delegation may not compensate for the loss of control when information costs are large. In such cases, the principal prefers to retain control of the decision.

The optimality of centralization for large biases and information costs crucially depend on the expert's lack of commitment at the information acquisition stage. For if he could commit to acquire no information at all, the principal prefers to give up control rights in his favour. Note that the overinvestment in information only benefits the principal for sufficiently large costs, as in panel (a). Overall, for large biases, centralization incentives are more sensitive to whether K^D compensates the principal for the loss of control.

3.5 Covert information acquisition

In this section, I analyse the game in which the principal does not observe the expert's information acquisition strategy. I focus on pure-strategy equilibrium at the information acquisition stage. When information acquisition is covert the equilibrium must specify the principal's beliefs about the expert's information acquisition strategy. The latter could, in principle, use his cheap talk messages to communicate how many signals he previously acquired. Based in Argenziano et al. (2016), I restrict attention to equilibria in which the expert cannot affect such beliefs at the communication stage, which is without loss of generality.

A pure-strategy Perfect Bayesian Equilibrium of the covert game is then characterized by the triple $\left(K_c^C, \mathcal{P}^{K_c^C}, y(\mathcal{P}^{K_c^C})\right)$; where K_c^C represents the number of signals, $\mathcal{P}^{K_c^C} = \{P_0^{K_c^C}, P_1^{K_c^C}, ..., P_\kappa^{K_c^C}\}$ is a communication partition, and $y(\mathcal{P}^{K_c^C}) = \{y_{P_i}\}_{P_i \in \mathcal{P}^{K_c^C}}$ is the principal's action profile. As in the overt game, $\mathcal{P}^{K_c^C}$ must be incentive compatible and $y(\mathcal{P}^{K_c^C})$ must be sequentially rational. The equilibrium amount of signals must maximize the expert's expected payoff given $\mathcal{P}^{K_c^C}$ and $y(\mathcal{P}^{K_c^C})$ but, as noted above, it must be immune to deviation to non-equilibrium information acquisition decisions.

$$K_c^C \in \arg\max_K \sum_{\Sigma_K=0}^{K} \left[\max_{y \in \psi(b, K_c^C)} \int_0^1 u_E\left(y, b, \theta\right) f(\theta|\Sigma_K, S_K) d\theta \right] \Pr(\Sigma_K|S_K) - c(S_K) \tag{3.6}$$

When the principal does not observe how many signals the expert acquired, the equilibrium must be immune to two deviations. First, the expert should not find profitable to acquire fewer signals than what the equilibrium specifies, despite the cost savings. This will be true when the expected utility loss associated to a larger residual variance outweighs the lower information costs. Information costs then must not be too large as compared to the gains in decision precision.

The expert could also acquire more signals than those specified in a candidate equilibrium. By doing so he can use the additional information to 'tailor' the different messages available in $\mathcal{P}^{K_c^C}$ to the different possible realizations of the information acquired off path, K'. The expert thus expects some utility gains from using the message strategy associated to the equilibrium number of signals, K_c^C, while he acquires more. For this to be true, if the principal knew the expert acquired K', then equilibrium communication would involve less information transmission than the equilibrium under K_c^C. Also, the costs of information must be sufficiently small.

I now analyse how these additional restrictions affect equilibrium information acquisition as compared to the overt game. First note that the observability of the information acquisition decision does not affect the expert's incentives under delegation. Hence, the equilibrium under delegation is characterized by Lemma 3.1.

Under centralization, the equilibrium information acquisition must be incentive compatibility in the sense of (3.6) but must also be cost effective (among all K that satisfy IC). As a consequence, the set of equilibrium information acquisition strategies available to an expert with bias b in the covert game is a subset of those available to the same expert in the overt game. The lemma below extends this intuition to show that any equilibrium in the former can be implemented as an equilibrium of the overt game in which the principal's off-path beliefs are conveniently restricted.

Lemma 3.6. *Any outcome supported in a Perfect Bayesian Equilibrium of the covert game in which the expert follows a pure strategy at the information acquisition stage can be supported in a Perfect Bayesian Equilibrium of the overt game.*

Our main result in this section uses the above lemma to guarantee the existence of cost thresholds that work as sufficient conditions for the optimality of different organizational structures, just as in the overt information acquisition game.

Proposition 3.3. *Given $c(\cdot)$ and b, there exists two cost thresholds in the covert game, $\tilde{c}_1(b,\cdot)$ and $\tilde{c}_2(b,\cdot)$, such that*

1. *Centralization is the optimal organizational structure for all $c(c,\cdot) \succeq \tilde{c}_1(b,\cdot)$;*
2. *Delegation is the optimal organizational structure for all $c(b,\cdot) \preceq \tilde{c}_2(b,\cdot)$.*

   Moreover, $\tilde{c}_1(b,\cdot) \succeq c_1(b,\cdot)$ and $\tilde{c}_2(b,\cdot) \succeq c_2(b,\cdot)$.

First note that $K_c^C = 0$ satisfies (3.6) because acquiring more information is never profitable, as there is no information transmission at the communication stage and, thus, the expert cannot make use of any additional signals acquired off path. Therefore, the set of K that satisfies (3.6) is not empty. Now, among those K within this set, the equilibrium information acquisition decision must bring the expert the highest expected utility given the optimal communication strategy $\mu(b,K)$ and the cost of that information $c(S_K)$. This constitutes the argument behind Lemma 3.6 and guarantees the existence of $\tilde{c}_1(b,\cdot)$ and $\tilde{c}_2(b,\cdot)$.

Now the question is how these thresholds compare with those of the overt game. In the appendix I show that for any two information acquisition decisions $K < K'$ and the associated message strategies characterized by $\kappa(b,K)$ and $\kappa(b,K')$, if acquiring more signals leads to more precise decisions, then cost effectiveness implies incentive compatibility (condition (3.6) holds). In other words, the IC constraints that typically bind in the covert game involve acquisition of fewer signals. As a consequence, information acquisition under centralization in the overt game involves as many signals as in the covert game. A qualification to this relates to specific cases in which more signals involve less information transmission.[17]

For large costs and small biases the informational gains from delegation are small because information acquisition decisions under both organizational structures are similar; moreover the message strategy involves full revelation. Deviations to acquire more signals than any candidate equilibrium are not profitable such that $K_c^C \leq K_o^C$. Because these (weakly) lower incentives for information acquisition, informational gains from delegation are (weakly) larger in the covert game than in overt game.

A similar argument applies to the case of small costs and bias. Suppose there exist biases for which the principal would optimally delegate were the expert perfectly informed. This is the limit case of information costs converging to zero. Increasing costs affects incentives to acquire information under centralization more than under delegation. The informational value of additional signals decreases because, for sufficiently many signals the associated message strategies consist of the same number of partitions of the state.[18] When the principal does not observe the expert's choice of signals, the additional incentive constraints hamper his information acquisition incentives, increasing the informational gains from delegation as compared to the overt game.

The same argument holds true for large biases as long as delegation is optimal when the expert is perfectly informed. Now, recall that for large biases the expert needs enough signals in order to implement the minimally-informative message strategy, $K^{NC}(b)$. Given any candidate equilibrium $K_c^C > K^{NC}(b)$, then, the expert will have incentives to acquire fewer signals rather than more. As a consequence, for large biases and costs, there will be more instances in which the expert acquire no information at all, as compared to the overt game. This further strengthen the claim that informational gains from delegation will be larger in the covert game.

3.6 Conclusion

This paper analysed the optimal allocation of decision rights in the context of strategic communication and endogenous information acquisition. A biased expert decides on how much information he acquires after a principal allocates authority over a decision. If the expert is in charge, his incentives are shaped by the marginal cost of information and by how it increases the precision of the resulting decision. From the principal's perspective, however, delegating the right to decide leads to a loss of control because the decision will be biased. Delegation is then optimal only if the loss of control is outweighed by informational gains, i.e. the expert acquires and uses more information than if the principal decides.

Under centralization, information acquisition depends on how much information the expert can credibly transmit through cheap talk messages. Incentives for communication are shaped by the conflict of interest between players *and* by the amount of information the expert decides to observe. For sufficiently small biases the equilibrium communication strategy involves full revelation of information, up to a given expected precision of the resulting decision. Such incentives to fully reveal information are decreasing in the expert's

[17] For instance, when comparing $K^{FR}(b)$ and $K^{FR}(b) + 1$.

[18] Proposition 4 in Förster (2020) shows that for sufficiently large K the most informative, cheap talk message strategy converges to the equilibrium characterized in Crawford and Sobel (1982).

bias. For sufficiently large biases, on the contrary, effective transmission of information requires that the expert has enough information. Notably, communication in the latter case involves coarsening the information the agent acquires.

I find that centralization is optimal for any conflict of interest provided the costs of information are sufficiently large. Intuitively, large costs limit the amount of information the expert acquires under delegation and thus the expected benefits for the principal. Low information costs, on the other hand, allow large investments in information under delegation. Given that the amount of information transmitted through communication is limited by the conflict of interest, sufficiently small costs leads a optimal delegation.

A more interesting set of findings stems from the application to quadratic preferences and uniform distribution of the state. Firstly, an expert with intermediate bias acquires more information under centralization. He does so because the minimum amount of information needed to implement an informative message strategy exceeds his maximum investment under delegation. Secondly, this *overinvestment* leads to optimal centralization when information costs are large. The overinvestment reduces the informational gains from delegation to the point that they no longer compensate for the loss of control. For the same costs and larger biases, however, the expert acquires no information in the equilibrium under centralization and, hence, the principal is better off delegating authority to him. These intuitions will be further investigated in future work.

Appendix

A3 Proofs and Complementary Results

Centralization

Proof of Lemma 3.3. The proof exploits the fact that $K^{FR}(b) = (b^{FR}(K))^{-1}$. By definition, $b^{FR}(K = 1)$ is the bias for which $E\left[\ell\big(E(\theta|\Sigma_1 = 0), b\big)\big|\Sigma_1 = 0\right] = E\left[\ell\big(E(\theta|\Sigma_1 = 1), b\big)\big|\Sigma_1 = 0\right]$. For $b \leq b^{FR}(K = 1)$ the left-hand side of the equality is strictly smaller because of the strict convexity of $\ell(\cdot)$, which means a looser IC constraint.

Similarly, $b^{FR}(K = 2)$ solves $E\left[\ell\big(E(\theta|\Sigma_2 = k), b\big)\big|\Sigma_2 = k\right] = E\left[\ell\big(E(\theta|\Sigma_2 = k+1), b\big)\big|\Sigma_2 = k\right]$ for $k = 0, 1$. Note that $E(\theta|\Sigma_2 = k+1) - E(\theta|\Sigma_2 = k) < E(\theta|\Sigma_1 = k+1) - E(\theta|\Sigma_1 = k)$ for all $k \leq 1$, which means that the IC constraint for full revelation of $K = 2$ is tighter than that for $K = 1$. As a consequence, $b^{FR}(K = 2) < b^{FR}(K = 1)$ which, in turn, leads to $K^{FR}(b) = 1$ for all $b \in (b^{FR}(K = 2), b^{FR}(K = 1)]$.

By the same argument, $E(\theta|\Sigma_{K+1} = k+1) - E(\theta|\Sigma_{K+1} = k) < E(\theta|\Sigma_K = k+1) - E(\theta|\Sigma_1 = K)$ for all $k \leq K$, and thus $b^{FR}(K+1) < b^{FR}(K)$.

As b goes to zero, Crawford and Sobel (1982) and Spector (2000) show that the amount of information the expert is willing to reveal in the most informative equilibrium goes to infinity; hence, $\lim_{b\to 0} K^{FR}(b) = \infty$. $\square$

Proof of Lemma 3.4. The relationship $K^{NC}(b)$ shows the minimum amount of signals the expert needs to implement the least-informative message strategy, which consists of only two partitions of the state space, $\kappa(b, K) = 2$, and thus two equilibrium messages $M_K = \{m_0, m_1\}$. Under such a strategy there are two 'boundary types', $\max P_0^K$ and $\min P_1^K$, whose incentives shape equilibrium communication and information acquisition. The proof proceeds in two steps: first, I derive $K^{NC}(b) = 1$ and $K^{NC}(b) = 2$; then, I show that $K^{NC}(b)$ is decreasing in b.

Because $b \geq 0$, I can focus on type $\max P_0^K$'s incentives to announce m_1 instead of its on-path message m_0. Let $b_1^{FR} = b^{FR}(K = 1)$ and $b_\infty^{NC} = \lim_{K\to\infty} b^{NC}(K)$ represent the largest bias for which there is truthful revelation of one signal and the largest bias for there will be some information transmission (only if the expert is perfectly informed), respectively. Similarly, let $\kappa^*(b, \theta) = \lim_{K\to\infty} \kappa^*(b, K)$ represent the number of partitions in the most-informative message strategy when the expert is perfectly informed.

Now, let $b' = b_1^{FR} + \epsilon < b_\infty^{NC}$ for sufficiently small $\epsilon > 0$. Because the loss function satisfies single crossing it is true that $E\Big[\ell\big(y(\Sigma_1 = 0), b'\big)|\Sigma_1 = 0\Big] \geq E\Big[\ell\big(y(\Sigma_1 = 1), b'\big)|\Sigma_1 = 0\Big]$.[19] But $b' < b_\infty^{NC}$ implies that $\kappa^*(b, \theta) \geq 2$ and, therefore, there exists a $K' > 1$ such that $\kappa(b, K') = 2$ can be implemented; that is, there exist $P_0^{K'}$ and $P_1^{K'}$ such that $E\Big[\ell\big(y(\Sigma_{K'} = \max P_0^{K'}), b'\big)|\Sigma_{K'} = \max P_0^{K'}\Big] \leq E\Big[\ell\big(y(P_1^{K'}), b'\big)|\Sigma_{K'} = \max P_0^{K'}\Big]$. Define $K^{NC}(b') = K' - 1 \geq 1$.

Similarly, take $\tilde{b} > b'$ such that $E\Big[\ell\big(y(\Sigma_{K'} = \max P_0^{K'}), \tilde{b}\big)|\Sigma_{K'} = \max P_0^{K'}\Big] = E\Big[\ell\big(y(P_1^{K'}), \tilde{b}\big)|\Sigma_{K'} =$

[19]By definition of $b_1^{FR} = b^{FR}(K = 1)$, $E\Big[\ell\big(y(\Sigma_{K=1} = 0), b_1^{FR}\big)|\Sigma_1 = 0\Big] = E\Big[\ell\big(y(\Sigma_{K=1} = 1), b_1^{FR}\big)|\Sigma_1 = 0\Big]$.

$\max P_0^{K'}\Big]$. Following the previous argument, define $b'' = \tilde{b} + \epsilon < b_\infty^{NC}$ such that the previous IC constraint fails to hold. Then, there must be a $K'' > K'$ and associated $P_0^{K''}$ and $P_1^{K''}$ such that $E\Big[\ell\big(y(\Sigma_{K''} = \max P_0^{K''}), b''\big)|\Sigma_{K''} = \max P_0^{K''}\Big] \leq E\Big[\ell\big(y(P_1^{K''}), b''\big)|\Sigma_{K''} = \max P_0^{K''}\Big]$. Again, $K^{NC}(b'') = K'' - 1 \geq K'$.

To see why $K'' > K'$, let first define $\tilde{\theta}_1 \equiv \lim_{K\to\infty} E[\theta|\Sigma_K = \max P_0^K] = \lim_{K\to\infty} E[\theta|\Sigma_K = \min P_1^K]$, as the boundary type for a two-message communication equilibrium when the sender is perfectly informed. Proposition 4 in Förster (2020) shows that such $\tilde{\theta}$ exists and if the expert has enough signals, such an equilibrium can be implemented.[20] Now, Lemma 2 and Corollary 1 in Crawford and Sobel (1982) show that to induce truthful communication from an expert with larger bias, the messages inducing higher actions must be less profitable. This implies that for $b < b'$, the boundary types in a two-partition message strategy satisfy $\tilde{\theta}_1^b > \tilde{\theta}_1^{b'}$, which in turn means that $\lim_{K\to\infty} E[\theta|\Sigma_K = \max P_0^K(b)] > \lim_{K\to\infty} E[\theta|\Sigma_K = \max P_0^K(b')]$. Because a higher K results in a finer partition of the state space, there exist sufficiently large $b < b_\infty^{NC}$ such that the corresponding two-partition message strategies can be implemented by truthful revelation of $\Sigma_K = 0$. Then, for $b < b'$ sufficiently close to $b < b_\infty^{NC}$ the minimum K to implement $\kappa(b', \cdot) = 2$ is never larger than the corresponding to $\kappa(b, \cdot) = 2$.

Now, the fact that the minimum amount of information needed to implement a two-partitions communication equilibrium is decreasing in the bias does not preclude that an equilibrium with more partitions can be implemented with fewer signals. Indeed, Förster (2020) shows that this is true for the uniform-quadratic case (see Example 2). In the present context, however, this can happen only for intermediate biases, since the amount of information conveyed in the most informative communication equilibrium is decreasing in the bias. For sufficiently large biases $b = b_\infty^{NC} - \epsilon$, then $\kappa^*(b, \theta) = 2$ and, therefore, $K^{NC}(b)$ is weakly increasing in b. □

PROOF OF PROPOSITION 3.1. The expert's information acquisition decision under centralization, K^C, maximizes his ex-ante expected utility given his bias and information costs; that is, for all $K' \neq K$

$$K^C = \big\{K : -E\left[\ell\big(|E(\theta|\mu(b, S_K)) - \theta - b|\big)\right] - c(S_K) \geq -E\left[\ell\big(|E(\theta|\mu(S_{K'}, b)) - \theta - b|\big)\right] - c(S_{K'})\big\} \tag{A3.1}$$

Given the equilibrium message strategy associated to K^C and b (Lemma 3.2), the resulting decision is $y^C \equiv y(\mu(b, S_{K^C})) = E(\theta|\mu(b, S_{K^C}))$.

First note that, for a given $c(\cdot)$ and sufficiently small bias, the optimal information acquisition under delegation, K^D, represents the maximum number of signals the expert acquires. Suppose, on the contrary, that $K^C(b) > K^D$ for $b < b^{FR}(K^D)$. It must then be true that $K^{FR}(b) \geq K^D$, which implies that, when the principal decides;

$$E\left[\ell\left(|E(\theta|\Sigma_{K^D}) - \theta - b|\right) - \ell\left(|E(\theta|\Sigma_{K^C}) - \theta - b|\right)\right] \geq C(S_{K^C}) - C(S_{K^D})$$

[20]Förster shows that the most-informative equilibrium with binary signals converges to that in Crawford and Sobel (1982).

But if the expert himself decides, then

$$E\left[\ell\left(|E(\theta|\Sigma_{K^D}) - \theta|\right)\right) - \ell\left(|E(\theta|\Sigma_{K^C}) - \theta|\right))] \leq C(S_{K^C}) - C(S_{K^D})$$

Which is a contradiction because it means is willing to 'use' more information when the resulting decision is biased than when it is not. Indeed, for $b = 0$ the left-hand sides of the above expressions coincide and, thus, it must be that $K^C(b = 0) = K^D$. For all $b \leq b^{FR}_{K^D}$ it is therefore true that $K^C \leq K^D$ and the acquired information is fully revealed to the principal. Moreover, because information takes the form of discrete signals, there exists a $b' \in (0, b^{FR}(K^D)]$ such that the expert will be indifferent between K^D and $K < K^D$—while for a marginal larger bias he strictly prefers the latter. The same argument applies up to $b = b^{FR}(K^D)$, beyond which the equilibrium message strategy may not involve full revelation of information.

Secondly, for sufficiently large biases no communication takes place in equilibrium; the resulting equilibrium information acquisition strategy is hence $K^C = 0$. A sufficient condition for this to be true is $b \geq b^{NC}_{\infty}$. Note that this is a sufficient condition, since an expert with bias $b = b^{NC}(K^D)$ can only implement an informative equilibrium if and only if $K^C > K^D$ and, for sufficiently low costs such an expert may find cost effective to acquire $K^C = K^D + 1$ signals. In such cases, the equilibrium communication strategy involves coarse information transmission.

Now, given the sufficient conditions for full revelation and no communication, let analyse what happens with intermediate biases. For $b \in (b^{FR}(K^D), b^{NC}_{\infty})$ the equilibrium message strategy may involve coarse communication. More precisely, for such biases there are always two candidate message strategies, one involving full revelation of signals and the other involving coarse messages. In the former case, $K^{FR}(b)$ gives the maximum amount of information the expert is willing to fully reveal. Let K_1 denote the cost effective of such strategies, that is, for all $K' \leq K^{FR}(b)$, $K_1(b)$ satisfies

$$-E\left[\ell\big(|E(\theta|\Sigma_{K_1}) - \theta - b|\big)\right] - c(S_{K_1}) \geq -E\left[\ell\big(|E(\theta|\Sigma_{K'}) - \theta - b|\big)\right] - c(S_{K'})$$

Note that $K_1(b^{FR}(K^D) + \epsilon) \leq K^C(b^{FR}(K^D))$; that is, as long as the optimal info acquisition decision involves full revelation, K^C is weakly decreasing in b.

The possibility of coarse communication arises when K^C allows the expert of bias b to implement a communication strategy involving at least two messages. Let $K_2 > K^{FR}(b)$ denote the cost effective of such strategies; that is, for all $K'' > K^{FR}(b)$ and associated $\mu(b, K'')$, K_2 and associated $\mu(b, K_2)$ satisfy:

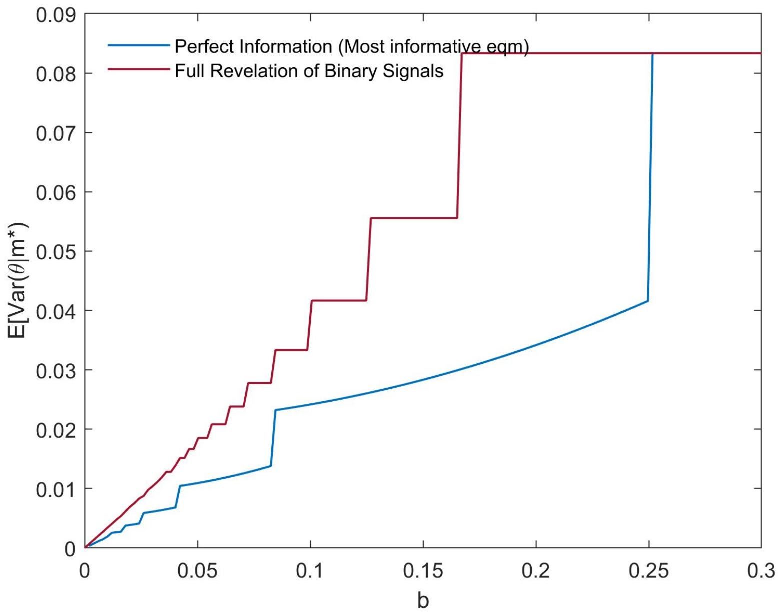
$$-E\left[\ell\big(|E(\theta|\mu(S_{K_2}, b)) - \theta|\big)\right] - c(S_{K_2}) \geq -E\left[\ell\big(|E(\theta|\mu(b, S_{K'})) - \theta|\big)\right] - c(S_{K'})$$

Proposition 4 in Förster (2020) shows that for all b there exists a sufficiently large $\tilde{K}(b) > K^{FR}(b)$ such that for all $K^C \geq \tilde{K}(b)$ the most-informative communication equilibrium implements the same number of partition as if the sender were perfectly informed. Indeed, for all $k = \{0, 1, ..., \kappa(b, K) - 1\}$, $\lim\limits_{K\to\infty} \max P^K_k = \lim\limits_{K\to\infty} \min P^K_k + 1 = \theta_k$, where θ_k represents the boundary type between partition k and

$k + 1$ of the most-informative equilibrium in Crawford and Sobel (1982). Therefore, for sufficiently small costs the expert can implement the communication strategy he would implement if perfectly informed.

I now need to show that such strategies can dominate full revelation (of fewer signals). I show this is the case when the state is uniformly distributed and preferences are quadratic losses. Figure A3.1 depicts the expected residual variance in the most informative equilibrium as a function of the expert bias. It compares the case in which the expert is perfectly informed (in blue) against the case in which he fully reveals binary signals (according to Lemma 3.2). The picture shows that coarse communication dominates full revelation of binary signals for all biases for white the latter features some information transmission. A direct implication is that, for the uniform-quadratic case, sufficiently low information costs will result in implementation of the former strategy over the latter.

Figure A3.1: Comparison between residual variances of a perfectly informed and an imperfectly informed expert who fully reveals his signals



□

Allocation of Decision Rights

Proof of Proposition 3.2. Given the costs of information and the expert's bias b, the principal delegates authority to the expert if and only if:

$$E\left[\ell\big(|E(\theta|\mu(b, S_{K^C})) - \theta|\big)\right] - E\left[\ell\big(|E(\theta|\Sigma_{K^D}) - \theta + b|\big)\right] \geq 0 \tag{A3.2}$$

Such that K^D and K^C solve (3.1) and (A3.1), respectively.

I first prove the existence of $c_1(b, \cdot)$ and $c_2(b\cdot)$. Note that information costs can be so large that $K^D = 0$ for all b; in such cases, the principal strictly prefers to retain authority over y. Decreasing $c(S_K)$ increases both K^D and K^C under the different organizational structures. As the costs go to zero, however, the expert's information acquisition under delegation converges to infinity, i.e. the ex-ante expected loss for the expert converges to zero. Proposition 3 in Dessein (2002) shows that, when the expert is perfectly informed, a principal who keeps control and communicates is on average infinite times farther away from θ than a principal who delegates control. In other words, there exists a sufficiently small bias such that the principal ex-ante prefers to delegate authority to a perfectly informed agent than to decide by herself.[21] Hence, $c_1(b, S_K)$ is cost function yielding the highest cost for each S_K for which (A3.2) holds. Similarly, for any b for which Proposition 3 in Dessein (2002) holds, $c_2(b, S_K)$ is the cost function yielding the smallest costs for each S_K for which (A3.2) fails to hold.

I now prove that $c_1(b, \cdot)$ is bounded below by the expert's information acquisition decision under centralization.

Lemma A3.1. *For all $0 \leq b \leq b^{FR}(K = 1)$, there exists a $\tilde{c}(b, \cdot)$ such that $K^D = K^C(b) \leq K^{FR}(b)$.*

Proof. First recall that for any $c(\cdot)$ and associated K^D, an expert with bias $b \leq b^{FR}(K^D)$ fully reveals his information to the principal under centralization. Also note that at $b^{FR}(K^D)$ the expert is indifferent between fully revealing K^D signals and exaggerating any low realization he could have; the marginal expected utility of acquiring K^D signals for such an expert is, therefore, small. For such costs, acquiring K^D is incentive compatible for $b = 0$; therefore, there exists a $0 < \tilde{b} \leq b^{FR}(K^D)$ such that acquiring K^D is incentive compatible for smaller biases and it is not for larger biases. $\square$

The cost $\tilde{c}(b, \cdot)$ holds for all b such that $K^{FR}(b) \geq 1$; thus, characterizes a relationship $\tilde{K}(b) \equiv K^D = K^C(b)$ which constitutes an horizontal translation of $K^{FR}(b)$. I need to show that $\lim_{b \to 0} \tilde{K}(b) = \infty$—which is easy provided that $\lim_{b \to 0} K^{FR}(b) = \infty$, and that for any non-zero cost $c(\cdot)$ and associated (finite) K^D, $\tilde{b} > 0$.[22]

The previous paragraphs showed the lower bounds for $c_1(b, \cdot)$ when the bias is sufficiently small; I now prove the corresponding bounds for large biases. For given $c(\cdot)$ and associated K^D, if the bias is sufficiently large even the maximal informational gains fail to compensate the principal for the loss of control from delegation. This threshold can be defined as

$$b^{NI}_{K^D} \equiv \big\{ b : \; E\big[\ell\left(|E(\theta) - \theta)\right]\big] - E\big[\ell\left(|E(\theta|\Sigma_{K^D}) - \theta + b|\right)\big] = 0 \big\}$$

which is increasing in b, its minimum being $b^{NI}_{K^D=0} = 0$, and its maximum, $b^{NI}_{\infty} \equiv \lim_{K^D \to \infty} b^{NI}_{K^D}$ is a finite integer.

[21] This holds for any $F(\theta)$. The argument builds upon the observation that the elements of the most-informative partition increase at a rate $4b$ and, therefore, as b tends to zero the number of partitions converge to infinity at a rate that is lower than the measure of the loss of control, b^2.

[22] Note that $K^C = K^D$ is always incentive compatible for $b = 0$.

The relationship $b^{NI}_{K^D}$ can be inverted, leading to $K^{NI}(b)$ (Definition 3.1) which represents the minimum amount of information an uninformed principal requires from an expert in order to be willing to delegate authority to him. The definition of $K^{NI}(b)$ assumes the principal is uninformed under centralization and, hence, it is binding for biases such that the most-informative communication equilibrium is the babbling on (for $K \leq K^{NI}(b)$). As a consequence, when $c(\cdot)$ and b are such that $K^D \leq \min\{K^{NC}(b), K^{NI}(b)\}$, the principal strictly prefers to retain authority over y. This show a sufficient condition on information costs that guarantees that centralization is optimal for higher costs and $b \geq b^{NC}(K = 1)$. Finally, note that $b^{NI}_\infty \leq b^{NC}_\infty$, and that:

$$\lim_{b \to b^{NI}_\infty} K^{NI}(b) = \infty$$

Example

As in the previous chapters, when $\theta \sim U[0, 1]$ and signals are i.i.d. realizations of a Bernoulli distribution, the updating process follows a Beta-binomial distribution. Given the number of signals available to the expert, K, the number of successes k is distributed according to a binomial distribution. And the posterior distribution of θ given k successes in K trials is a Beta distribution with parameters $k + 1$ and $K - k + 1$, which leads to a posterior expectation $E(\theta|k, K) = \frac{(k+1)}{(K+2)}$.

Derivation of expression (3.4) (Argenziano et al., 2016). Suppose authority is centralized. The number of signals observed by the agent is given by K, and the equilibrium communication is characterized by $\mathcal{P}^K(b)$. Upon receiving the message associated to $P_i \in \mathcal{P}^K(b)$ the principal chooses $y(P_i)$ so as to maximize $-\int_0^1 (y(P_i) - \theta)^2 f(\theta|k \in P_i^K, K)d\theta$. It is easy to see that the FOC leads to $y(P_i) = E[\theta|P_i, K]$, which can be worked out as:

$$E[\theta|P_i, K] = E\left[E[\theta|k, K]|k \in P_i\right] = \sum_{k \in P_i} E[\theta|k, K] \frac{f(k : K)}{\sum_{k \in P_i} f(k : K)} = \frac{1}{|P_i|} \sum_{k \in P_i} \frac{(k + 1)}{(K + 2)}$$

Where $|P_i|$ is the cardinality of the set P_i. Since $E(\theta|k, K) = \frac{(k+1)}{(K+2)}$ and $f(k : K) = \frac{1}{(K+1)}$. Note that, under delegation, the expert's uses all the information available to him so $\mathcal{P}^K = S_K$ and, hence, the optimal decision is given by $y = \frac{(k+1)}{(K+2)} + b$.

Derivation of expressions (3.3) and (3.5). Again, suppose authority is centralized. The expert's bias defines an optimal message strategy for each possible K, $\mathcal{P}^K(b)$ (see proof of Lemma 3.5 below). His information acquisition decision maximizes his ex-ante expected utility given by

$$E\left[-(y(P_i \in \mathcal{P}^K(b)) - \theta - b)^2\right] = E\Big[E\big[-\big(y(P_i \in \mathcal{P}^K(b)) - \theta - b\big)^2 |k \in P_i\big]|P_i \in \mathcal{P}^K(b)\Big]$$

The RHS results from the application of the Law of Iterated Expectations, and can be expressed as:

$$-\sum_{P_i\in\mathcal{P}^K(b)}\frac{\Pr(P_i)}{\sum_{P_i\in\mathcal{P}^K(b)}\Pr(P_i)}\left[\sum_{k\in P_i}\frac{\Pr(k)}{\sum_{k\in P_i}\Pr(k\in P_i)}\times\int_0^1\big(y(P_i\ni k)-\theta-b\big)^2 f(\theta;k,K)d\theta\right] \tag{A3.3}$$

Where, as before, $\frac{\Pr(k)}{\sum_{k\in P_i}\Pr(k\in P_i)}=\frac{1}{|P_i|}$ and, therefore, $\frac{\Pr(P_i)}{\sum_{P_i\in\mathcal{P}^K(b)}\Pr(P_i)}=\frac{|P_i|}{|S_K|}=\frac{|P_i|}{(K+1)}$. Note also that $E\left[-(y(P_i)-\theta-b)^2\right]=E\left[E\left[(y(P_i)-\theta)^2|P_i\in\mathcal{P}^K\right]\right]-2bE\left[E\left[(y(P_i)-\theta)|P_i\in\mathcal{P}^K\right]\right]+b^2$. Since $y(P_i)=E[\theta|P_i,K]$, the second term of the RHS equals zero and I can work out equation (A3.3) to get (3.5):

$$\begin{aligned}
&-\sum_{P_i\in\mathcal{P}^K(b)}\frac{|P_i|}{(K+1)}\left[\sum_{k\in P_i}\frac{1}{|P_i|}\times\int_0^1\big(y(P_i)-\theta\big)^2 f(\theta;k,K)d\theta+b^2\right]\\
&=-\sum_{P_i\in\mathcal{P}^K(b)}\frac{|P_i|}{(K+1)}\left[\sum_{k\in P_i}\frac{1}{|P_i|}\times\int_0^1\big(y(P_i)^2-2y(P_i)\theta+\theta^2\big)f(\theta;k,K)d\theta+b^2\right]\\
&=-\sum_{P_i\in\mathcal{P}^K(b)}\frac{|P_i|}{(K+1)}\Bigg[\left(\frac{1}{|P_i|}\sum_{k\in P_i}\frac{(k+1)}{(K+2)}\right)^2-2\left(\frac{1}{|P_i|}\sum_{k\in P_i}\frac{(k+1)}{(K+2)}\right)\sum_{k\in P_i}\frac{1}{|P_i|}\int_0^1\theta f(\theta;k,K)d\theta+\\
&\qquad\qquad+\sum_{k\in P_i}\frac{1}{|P_i|}\int_0^1\theta^2 f(\theta;k,K)d\theta+b^2\Bigg]\\
&=-\sum_{P_i\in\mathcal{P}^K(b)}\frac{|P_i|}{(K+1)}\Bigg[\left(\frac{1}{|P_i|}\sum_{k\in P_i}\frac{(k+1)}{(K+2)}\right)^2-2\sum_{k\in P_i}\left(\frac{1}{|P_i|}\sum_{k\in P_i}\frac{(k+1)}{(K+2)}\right)^2+\\
&\qquad\qquad+\sum_{k\in P_i}\frac{1}{|P_i|}\frac{(k+1)(k+2)}{(K+2)(K+3)}+b^2\Bigg]\\
&=-\sum_{P_i\in\mathcal{P}^K(b)}\frac{|P_i|}{(K+1)}\left[\sum_{k\in P_i}\frac{1}{|P_i|}\frac{(k+1)(k+2)}{(K+2)(K+3)}-\left(\frac{1}{|P_i|}\sum_{k\in P_i}\frac{(k+1)}{(K+2)}\right)^2+b^2\right]\\
&=-\frac{1}{(K+1)}\sum_{P_i\in\mathcal{P}^K(b)}\left[\sum_{k\in P_i}\frac{(k+1)(k+2)}{(K+2)(K+3)}-\frac{1}{|P_i|}\left(\sum_{k\in P_i}\frac{(k+1)}{(K+2)}\right)^2+b^2\right]
\end{aligned}$$

PROOF OF LEMMA 3.5 Condition number 3 comes from Lemma 2 in Argenziano et al. (2016), hence I refer interested readers to the corresponding proof in that paper. Now, full revelation implies that $|P_i|=1$ for all partitions $P_i = \{0, 1, ..., K\}$ and, thus, incentive compatibility requires:

$$|P_{i+1}| - |P_i| \geq 4b(K+2) - 2 \qquad\qquad |P_{i+1}| - |P_i| \leq 4b(K+2) + 2$$
$$2 \geq 4b(K+2) \qquad\qquad -2 \leq 4b(K+2)$$
$$\frac{1}{2(K+2)} \geq b \qquad\qquad -\frac{1}{2(K+2)} \leq b$$

Because I assumed $b \geq 0$, then the constraint on the right is not binding.

I now derive the equilibrium boundary types for each interval in the most informative equilibrium partition given b and K. For each interval $|P_i|$ there are two of such boundary types $\underline{k}_i = \min P_i^K$ and $\bar{k}_i = \max P_i^K$. Note that $|P_i| = \bar{k}_i - \underline{k}_i + 1$ and that $\bar{k}_i = \underline{k}_{i+1} - 1$ for all partitions. I can then rewrite $|P_{i+1}| - |P_i| = \bar{k}_{i+1} - \underline{k}_{i+1} - \bar{k}_i + \underline{k}_i = \bar{k}_{i+1} - 2\bar{k}_i + \bar{k}_{i-1}$, while $|P_i| - |P_{i-1}| = \underline{k}_{i+1} - 2\underline{k}_i + \underline{k}_{i-1}$. Incentive compatibility thus becomes

$$|P_i| - |P_{i-1}| \geq 4b(K+2) - 2$$
$$\underline{k}_{i+1} - 2\underline{k}_i + \underline{k}_{i-1} \geq 4b(K+2) - 2$$

and

$$|P_i| - |P_{i-1}| \leq 4b(K+2) + 2$$
$$\underline{k}_{i+1} - 2\underline{k}_i + \underline{k}_{i-1} \leq 4b(K+2) + 2$$

Each of which, after applying the initial condition $\underline{k}_0 = 0$, configures a second-order linear difference equations with a class of solutions parametrized by $\underline{k}_1$ as (respectively):

$$\underline{k}_i \geq i\underline{k}_1 + i(i-1)[2b(K+2) - 1] \tag{A3.4}$$
$$\underline{k}_i \leq i\underline{k}_1 + i(i-1)[2b(K+2) + 1] \tag{A3.5}$$

A necessary condition for any informative equilibrium is $\underline{k}_2 < K$. The maximum number of intervals in any informative equilibrium, $\kappa(b, K)$, is thus the largest positive integer such that $i(i-1)[2b(K+2)+1] < K + 1$,[23] which can be show to be:

$$\kappa(b, K) = \left\lceil -\frac{1}{2} + \frac{1}{2}\left[1 + \frac{4(K+1)}{(2b(K+2)+1)}\right]^{\frac{1}{2}} \right\rceil \tag{A3.6}$$

Similarly, a sufficient condition for the non-existence of an informative equilibrium, $\kappa(b, K) = 1$, is $i(i-1)[2b(K+2) - 1] > K - 1$ for $i = 2$, which translates into $b > \frac{(K+1)}{4(K+2)}$. □

[23] Recall that the cardinality of S_K is $K + 1$.

Covert Information Acquisition

DERIVATION OF EXPRESSION (3.6) The assumption of covert information acquisition only affects the expert's equilibrium strategy under centralization. Covert information acquisition implies that a Perfect Bayesian Equilibrium must specify the principal's beliefs about the expert's information acquisition decision.

I focus on pure-strategy equilibrium at the information acquisition stage. Using a result by Argenziano et al. (2016), I restrict attention to equilibria in which the expert does not use his cheap talk message to influence the principal's beliefs on K^C. The lemma below shows that this is without loss of generality.

Lemma A3.2 (Lemma 3 in Argenziano et al. (2016))**.** *Any outcome supported in a Perfect Bayesian Equilibrium of the covert game in which the expert follows a pure strategy in the choice of the number of signals can be supported in a Perfect Bayesian Equilibrium in which the principal's beliefs about the number of signals do not vary with the expert's message.*

Proof. See pp. 142 in Argenziano et al. (2016). □

If the expert could affect the principal's beliefs about the number of signals he acquires, he would face two classes of available deviations. First, he could mislead the principal by acquiring a non-equilibrium number of signals but sending a message that signals he acquired the equilibrium number. Secondly, he could acquire a non-equilibrium number of signals and send a message associated to a different, non-equilibrium number of signals to the principal.

If the expert cannot affect the principal's beliefs, on the contrary, the relevant class of deviations is the first. Hence, any equilibrium that is immune to both classes of deviations remains an equilibrium when it has to be immune to only the first of these. This results allows us to focus on equilibria in which the principal beliefs the expert acquired the equilibrium number of signals, irrespective of his message at the communication stage.

The equilibrium is characterized by the triple $\left(K_o^C, \mathcal{P}^{K_o^C}, y(\mathcal{P}^{K_o^C})\right)$. As in the overt game, $\mathcal{P}^{K_o^C}$ must be incentive compatible and $y(\mathcal{P}^{K_o^C}) = \{y_{P_i}^{K_o^C}\}_{P_i \in \mathcal{P}^{K_o^C}}$ must be sequentially rational. Let $k = \Sigma_K$ denote the number of successes for a given information acquisition decision K. The equilibrium amount of signals must, then, maximize the expert's expected payoff, that is:

$$E\left[u_E\big(y(\mathcal{P}^{K_o^C}), \theta, b\big)\right] = -\sum_{k=0}^{K_o^C} \Pr(k; K_o^C) \left[\int_0^1 \ell\big(|y_{P_i} - \theta - b|\big) f(\theta | k, K_o^C) d\theta\right]$$

Recall that $y_{P_i} = E(\theta | P_i \in \mathcal{P}^{K_o^C})$. For each equilibrium candidate K the expert must be better off, on the equilibrium path, than investing in any other amount $K' \neq K$:

$$- \sum_{k=0}^{K} \Pr(k;K) \left[\int_0^1 \ell\big(|y_{P_i}^K - \theta - b|\big) f(\theta|k,K) d\theta \right] - C(S_K) \geq$$
$$\geq - \sum_{k'=0}^{K'} \Pr(k';K') \left[\max_{y_{P_j} \in y(\mathcal{P}^K)} \int_0^1 \ell\big(|y_{P_j}^K - \theta - b|\big) f(\theta|k',K') d\theta \right] - C(S_{K'}) \tag{A3.7}$$

Note that $y_{P_j}^K$ represents the decision that maximizes the expert's utility upon observing k' successes, among those induced by $\mathcal{P}^K$. In principle, the deviations may involve fewer or more signals than K.

On the one hand, $K' < K$ means the expert saves on information costs but typically induces a larger residual variance. To see this consider the candidate equilibrium in which the expert fully reveals K. The deviation to acquiring $K' < K$ implies he must implement $K+1$ messages (actions) with $K'+1 < K+1$ available posterior beliefs about the state. To do that he must either use mixed strategies for some $k' \in K'$ or leave at least one of the $K+1$ messages unused (presumably $k = 0$ since $b \geq 0$). In both cases, the message he announces upon observing any k' induces an action that does not correspond to his posterior beliefs on the state and, thus, he is more uncertain about it being the appropriate decision than if he acquired K signals. A stronger argument for such deviations holds true when the expert's message strategy with K signals involves coarse messages because the marginal informational value of some signals is lower than if fully revealed.

On the other hand, by acquiring more than the equilibrium amount of signals, $K' > K$, the expert could better accommodate the messages on $\mathcal{P}^K$ to the additional information. Because of the additional information costs, the expected utility gains from 'gaming' the message strategy must be sufficiently large. Such a strategy will then be particularly useful if the equilibrium with K' induces fewer partitions than that under K—for instance, when $K = K^{FR}(b)$ and $\kappa(b, K') < K+1$.

Note that, given a candidate equilibrium K, the profitability of acquiring a different K' does not depend on the optimal communication strategy under K', but on the expert's expectation of implementing $\mu(b, S_K)$ with fewer or more signals. Because of this, there may be many Ks that satisfy condition (A3.7). In particular, $K_o^C = 0$ is always a candidate equilibrium, since acquiring fewer signals is not feasible and acquiring more signals cannot be used by the expert on his benefit.[24] Among all Ks that satisfy (A3.7), the expert chooses the cost effective one; that is, for all K and K'':

$$- \sum_{k=0}^{K} \Pr(k;K) \left[\int_0^1 \ell\big(|y_{P_i}^K - \theta - b|\big) f(\theta|k,K) d\theta \right] - C(S_K) \geq$$
$$\geq - \sum_{k''=0}^{K''} \Pr(k'';K'') \left[\int_0^1 \ell\big(|y_{P_j}^{K''} - \theta - b|\big) f(\theta|k'',K'') d\theta \right] - C(S_{K''}) \tag{A3.8}$$

The condition above is the same as in the overt game, which implies that condition (A3.7) reduces the set of candidate equilibria in the information acquisition stage.

[24] The equilibrium message strategy upon $K^C = 0$ is babbling.

PROOF OF **L**EMMA 3.6. Consider an equilibrium of the covert game $\mathcal{E}^o = \big(K^c(b, c(\cdot)), \mu(b, K^c, k), B^c(\cdot), \psi^c\big)$ in which the expert acquires K^c signals and follows message upon observing k successes, the principal forms beliefs $B^c : M \to \Delta\left(\{(K^c, k)|K^c, k \in \mathcal{K}, K^c \geq k\}\right)$ and follows decision strategy $\psi^c(\cdot) : B^c \to \Delta\big([0, 1]\big)$. The principal's beliefs B^c reflect the fact that in the covert game she has to form beliefs on both the number of successes k and the number of signals the expert acquires K.

Now, for the same b and $c(\cdot)$ consider an equilibrium of the overt game $\mathcal{E}^o = \big(K^o(b, c(\cdot), \mu(b, K^o, k), \beta(\cdot), \psi^o\big)$. Unlike $\mathcal{E}^c$, the principal's equilibrium belief $\beta(\cdot)$ is a mapping from the set of messages to the set of probability distributions over the number of successes $k \leq K^o$.

First note that when $K^c = K^o$, and given Lemma A3.2, then $B^c(\cdot) = \beta(\cdot)$ on the equilibrium path. Secondly, recall that, for given b and $c(\cdot)$, the equilibrium number of signals in the covert game K^c satisfies (A3.7) and (A3.8), while K^o only needs to satisfy (A3.8). Hence, if condition (A3.7) does not hold in the covert game for $K = K^o$, it means that $K^c \neq K^o$. However, I could still implement K^c in the overt game: I impose off-path restrictions of the form $\psi^o(\beta(\mu(b, K \neq K^o, k))) = E(\theta)$ for all $k \in K$. This amounts to say that the principal does not update beliefs upon the expert's message when he acquires $K \neq K^o$. $\square$

I now prove that for sufficiently large K and $K' > K$, cost effectiveness implies incentive compatibility, i.e. (A3.8) implies (A3.7).

Lemma A3.3. *For any K and $K' > K$, and b such that $\kappa(b, K) \leq \kappa(b, K')$; if K satisfies* (A3.8) *with respect to K', then it also satisfies* (A3.7).

Proof. I prove it by contradiction. Suppose condition (A3.8) holds for K with respect to K'. Because $\kappa(b, K) \leq \kappa(b, K')$, cost effectiveness means the (weakly) increased precision from acquiring more signals does not compensate the expert for the additional costs:

$$\sum_{k=0}^{K} \Pr(k; K) \left[E\Big[\ell\big(|y^K_{P_i} - \theta - b|\big)|k, K\Big]\right] - \sum_{k'=0}^{K'} \Pr(k'; K') \left[E\Big[\ell\big(|y^{K'}_{P_j} - \theta - b|\big)|k', K'\Big]\right] \leq C(S_{K'}) - C(S_K) \tag{A3.9}$$

I now suppose that condition (A3.7) is violated, that is, the expert has incentives to acquire K' when the principal expects he acquires K. The incentive compatibility constraint thus becomes:

$$\sum_{k=0}^{K} \Pr(k; K) \left[E\Big[\ell\big(|y^K_{P_i} - \theta - b|\big)|k, K\Big]\right] - \sum_{k'=0}^{K'} \Pr(k'; K') \left[E\Big[\max_{y^K_{P_j} \in y(\mathcal{P}^K)} \ell\big(|y^K_{P_j} - \theta - b|\big)|k', K'\Big]\right] > C(S_{K'}) - C(S_K$$

I now add and subtract $\sum_{k'=0}^{K'} \Pr(k'; K') \left[E\Big[\ell\big(|y^{K'}_{P_j} - \theta - b|\big)|k', K'\Big]\right]$ to the left hand side of the above expression, which gives us:

$$\left\{\sum_{k=0}^{K}\Pr(k;K)\left[E\Big[\ell\big(|y_{P_i}^{K}-\theta-b|\big)|k,K\Big]\right]-\sum_{k'=0}^{K'}\Pr(k';K')\left[E\Big[\ell\big(|y_{P_j}^{K'}-\theta-b|\big)|k',K'\Big]\right]\right\}+$$
$$+\left\{\sum_{k'=0}^{K'}\Pr(k';K')\left[E\Big[\ell\big(|y_{P_i}^{K'}-\theta-b|\big)|k',K'\Big]-E\Big[\max_{y_{P_j}^{K}\in y(\mathcal{P}^K)}\ell\big(|y_{P_j}^{K}-\theta-b|\big)|k',K'\Big]\right]\right\}>C(S_{K'})-C(S_K)$$

Note that the first term in brackets is at most equal to $C(S_{K'})-C(S_K)$lequation (A3.9). Hence, the above condition holds if the second term in brackets is larger than zero which, in turn, requires that the expert's utility is larger when he uses K' signals to mislead the principal using the message strategy associated to K than when using the message strategy that is optimal for K'. Because $K'>K$, the message strategy with $\mathcal{P}^K$ partitions of θ is available to the expert with K' signals,[25] but given $\kappa(b,K')\geq\kappa(b,K)$ the optimal message strategy for such an expert is not the same and involves at least the same number of partitions. As a consequence,

$$\sum_{k'=0}^{K'}\Pr(k';K')\left[E\Big[\ell\big(|y_{P_i}^{K'}-\theta-b|\big)|k',K'\Big]\right]\leq\sum_{k'=0}^{K'}\Pr(k';K')\left[E\Big[\max_{y_{P_j}^{K}\in y(\mathcal{P}^K)}\ell\big(|y_{P_j}^{K}-\theta-b|\big)|k',K'\Big]\right]$$

which leads to a contradiction. □

PROOF OF PROPOSITION 3.3. Note that, given an equilibrium number of signals K_c^C and Lemma 3.6, the argument in Proposition 3.2 applies to the covert game. This means that for each b there exist sufficiently large (small) information costs such that the principal prefers to centralize (delegate) control over y, which includes larger (smaller) costs. In other words, $\tilde{c}_1(b,\cdot)$ and $\tilde{c}_2(b,\cdot)$ exist for all b.

I now compare the cost thresholds of the covert versus the overt game. First, I focus on large information costs. For each b there exists a sufficiently large cost such that $K^D=K_c^C=0$ and thus the principal strictly prefers centralization. For slightly lower costs, K^D weakly increases while K_c^C increases if and only if both (A3.7) and (A3.8) relax. Note that, because costs are large, the expert will be inclined to deviate to acquiring fewer signals than in any candidate equilibrium, i.e. equation (A3.7) will tend to bind for some $K'<K$. As a result, the sufficient condition on costs for centralization to be optimal is more restrictive: $\tilde{c}_1(b,\cdot)\succeq c_1(b,\cdot)$.[26]

Secondly, I analyse the case of small information costs. In Proposition 3.2 I used results by Dessein (2002) to show that delegation dominates cheap talk communication for a non-empty set of biases and some distributions of the state when the expert is perfectly informed. This represents a sufficient condition for

[25] Possibly under mixed strategies.

[26] Note also that, because for large costs $K_o^C(b,c(\cdot))\leq\max\{K^{FR}(b),K^{NC}(b)\}$, conditions of Lemma A3.3 are satisfied; that is, cost effectiveness implies incentive compatibility and thus $K_c^C(b,c(\cdot))\leq K_c^C(b,c(\cdot))$.

delegation when information costs are zero, given that expert who controls the decision will always become perfectly informed. Now consider slightly large information costs for which the equilibrium number of signals in the overt game allows the expert to implement the same number of partitions as in Crawford and Sobel (1982). For any K and $K' > K$, conditions of Lemma A3.3 are satisfied, and the fact that the expert's optimization problem in the covert game features more restrictions than in the overt game guarantees that $K_c^C(b, c(\cdot)) \leq K_o^C(b, c(\cdot))$. As a consequence, the cost below which the principal prefers delegation to centralization is higher in the covert game than in the overt game: $\tilde{c}_2(b, \cdot) \succeq c_2(b, \cdot)$. $\square$

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