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Time is Running Out: The 2°C Target and Optimal Climate Policies

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Abstract

The quintessence of recent natural science studies is that the 2°C target can only be achieved with massive emission reductions in the next few years. The central twist of this paper is the addition of this limited time to act into a non-perpetual real options framework analysing optimal climate policy under uncertainty. The window-of-opportunity modelling setup shows that the limited time to act may spark a trend reversal in the direction of low-carbon alternatives. However, the implementation of a climate policy is evaded by high uncertainty about possible climate pathways.

Keywords: Climate policy, CO_2 scenarios, non-perpetual real options
JEL-Classification: Q51, Q54, D81

1 Introduction

The future dynamics of greenhouse gas emissions, and their implications for global climate conditions in the future, will be shaped by the way in which policy makers reach decisions while facing uncertainties in the climate projections. Under the Cancun Accord, countries recognised that deep cuts in emissions were required, so as to limit the increase in global temperature below 2°C throughout the twenty-first century, but the Accord stopped short of actually delivering a binding worldwide agreement. To reach this target, the EU strives to reduce greenhouse gas emissions by 80-95% by 2050 compared to 1990.[1] In contrast, according to the baseline scenario of the International Energy Agency (IEA) global energy demand and CO_2 emissions will more than double by 2050.[2]

The current climate debate focuses largely on current and future GHG emissions. However, climate change results from the cumulative build-up of greenhouse gases in the atmosphere over time. Greenhouse gases remain in the atmosphere and contribute to global warming long after they are emitted. Based on this insight, Meinshausen et al. (2009) identify an admissible budget for future emissions that is compatible with the 2°C target. Against the background of this so-called carbon budget approach, Rogelj et al. (2011) reanalyse a large set of published emission scenarios from integrated assessment models and conclude that global greenhouse gas emissions need to be substantially reduced in future years. The urgency to act is also confirmed by the projections in the 2011 edition of the World Energy Outlook (IEA (2011)). It is estimated that so many fossil fuelled power stations, energy intensive factories and poorly insulated buildings will be very likely erected in the next five years that the permitted emission budget is already used up. If the expansion of high-carbon infrastructure is not prevented soon, global warming will not be limited to safe levels. Hence, the limited time to act is a key aspect in every climate policy decision. Against this background, we investigate the impact of a limited timeframe on optimal climate threshold policies and their welfare values.

To this end, we develop a non-perpetual real option modelling framework in which there is limited time to act. Recent theoretical analyses of decisions under uncertainty have highlighted the effects of (partial) irreversibility in generating "real options".[3] Real options models

[1]See http://ec.europa.eu/clima/documentation/roadmap/docs/com_2011_112_en.pdf for more details.

[2]See IEA (2010). Beyond a critical temperature threshold, accelerating carbon-climate feedback processes may lead to positively skewed probability distributions for future climate change leading to non-negligible probabilities for tipping points. Needless to say, modelling tipping point phenomena in real options models would be an important priority for future research.

[3]The partial irreversibility assumption implicitly assumes that technologies to extract massive amounts of CO_2 from the atmosphere are only partly operational in the foreseeable future. Despite significant progress towards conducting test runs and in establishing legal frameworks, significant challenges to carbon capture and storage (CCS) remain. The first concerns storage. Recent estimates indicate that some 120 Gt of CO_2 must be captured and stored globally over the next four decades – an indication of the storage capacities that need to be explored, characterised, licensed, safely operated and finally closed. The status and availability of data on CO_2 storage vary significantly around the world which is potentially a major constraint to rapid, widespread CCS deployment. Furthermore, capturing and storing massive volumes of CO_2 will only become a profitable activity on a large scale if there is a globally adopted, verified and sanctioned agreement by which

provide a representation of optimal stopping problems that is convenient for many purposes. They have become increasingly popular in climate economics. While the modelling framework of these models is borrowed from financial economics, the underlying concept is the same as the discrete-time stochastic dynamic programming models familiar to economists. Policy makers reach decisions by acting with the objective of maximizing the expected value of some objective function under uncertainty. In these models the interaction of time-varying uncertainty and irreversibility leads to a range of inaction where policy makers prefer to "wait and see" rather than undertaking a costly action with uncertain consequences. The general idea behind perceiving climate policies as option rights is that implementing a climate policy can be compared, in its nature, with the purchase of a perpetual financial call option. The investor pays a premium price to get the right to buy an asset for some time at a predetermined price (exercise price) that is eventually lower than the spot market price of the asset. Analogous to this, by deciding in favour of a climate policy the decision maker pays a price that gives her the 'right' to mitigate, which reduces climate damage costs in the future. However, the introduction of a climate policy faces the following three distinctive obstacles: (1) there is uncertainty about its future payoff; (2) waiting allows policy makers to gather new information on the uncertain future; and (3) climate policies are at least partially irreversible. These characteristics are encapsulated in the concept of real options models.[4]
Aside from reflecting these features, our model captures the limited time to act by restricting the availability of the options to a certain period which is exogenously given to the policy maker. The decision maker can only take measures to meet the 2°C target before the deadline expires. Afterwards this goal moves out of reach and the economy has to bear higher climate damage costs. This set-up allows us to ask, whether such a deadline counteracts the adverse effects of uncertainty and irreversibility. Does the limited time to act framework accelerate climate policy?

The structure of the remainder of the paper is as follows. Section 2 presents a basic overview of cumulative emission trajectories that are conform to the 2°C target. In Section 3, the design of the continuous-time modelling set-up is presented. This part of the paper outlines a new way to model time-limited windows of opportunities. Subsequently, in Section 4 we illustrate the working of the model through numerical exercises and examine the sensitivity of the main results with respect to key parameters. The final section draws conclusions and presents suggestions for further research. Omitted details of some derivations are available in two technical appendices.

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governments commit to binding cuts of CO_2. See Keith et al. (2006).

[4]This strand of literature now constitutes a significant branch of the climate economics literature. A key advantage of real options models is that they share core assumptions, which makes them easily scalable to include further details sand distinctive features that are relevant to address new questions at hand. Since several surveys of the real options literature, such as Bertola (2010), Dixit & Pindyck (1994) and Stokey (2009), are available, no attempt is made to do a comprehensive survey in this paper. Golub et al. (2011) have recently provided a non-technical summary of alternative approaches modelling uncertainty in the economics of climate change.

2 Admissible Emission Trajectories for the 2°C Target

A widely supported climate policy goal is to limit global warming throughout the twenty-first century within the "comfort zone" of 1-2°C above the pre-industrial temperature. This restriction is supposed to be sufficiently low to prevent enormous climate damage, which may be also caused by passing tipping points of the Earth system. In this spirit, Article 2 of the United Nations Framework Convention on Climate Change demands stabilisation of greenhouse gas concentrations at a suitable level. However, scientific studies indicate that achieving this objective would require more than stabilisation because of the very long lifetime of carbon in the atmosphere and the response time of the temperature. As pointed out by Archer (2005), half of CO_2 emissions are removed by the natural carbon cycle within a century, but a substantial fraction will stay in the atmosphere for several millennia.[5] This is also beacuse of to positive feedback effects, which probably initiate a net release of CO_2 out of the present carbon sinks such as the terrestrial biosphere by the end of the century, see Cox et al. (2000). Therefore, the present emissions will have an irreversible effect on human timescales. Beyond this, CO_2-attributable global warming processes are rather slow. For instance, the warming of the ocean lags behind considerably, so that the full effects on temperature are not yet felt. However, the slow ocean mixing that delays the warming would be also responsible for a slow cooling. Hence, the benefits of a decrease of atmospheric carbon concentrations would be widely offset.[6]

Due to these insights, Allen et al. (2009), Meinshausen et al. (2009) and Zickfeld et al. (2009) observe that future temperature is remarkably insensitive to the shape of the emission trajectory and depends only on the cumulative total. Therefore, policy targets that are linked to the total amount of emissions are likely to be more robust to scientific uncertainty than those that refer to emission rates or concentration targets. To offer a basis for policy discussions, Meinshausen et al. (2009) provide explicit numbers that are compatible with the 2°C objective to a certain probability.[7] Having already used up a substantial part of the global carbon budget in the first 10 years of this century, the remaining cumulative total is assessed to be 750 Gt for the time period until 2050.[8] At this level, the probability of the global temperature exceeding 2°C throughout the twenty-first century is calculated to be 33 percent. Beyond this Meinshausen et al. (2009) also point out that the total proven fossil fuel reserves are large enough to move the 2°C target out of reach with a probability of 100 percent.

[5] Being based on climate models of different complexity, other studies support these findings on the whole, see for example Lenton et al. (2006), Matthews & Caldeira (2008), Mikolajewicz et al. (2007), Montenegro et al. (2007) or Tyrell et al. (2007).

[6] More information on this topic is provided by Matthews & Caldeira (2008) and Solomon et al. (2009).

[7] Zickfeld et al. (2009) present similiar computations for the period 2000-2500.

[8] Many studies examine solely the effects of CO_2 emissions. Being released to the atmosphere at rapidly growing rates, CO_2 presents itself as the most important anthropogenic climate forcing. However, mitigation of non-CO_2 forcings may grant more leeway to the admissible carbon budget. In particular, limiting the black carbon production would be cost-effective and bear fruits quickly, because this aerosol stays in the atmosphere for a relatively short life-time, see Allison et al. (2011) and Wallack & Ramanathan (2009).

What are the design choices when setting an emission reduction timetable? Figure 1 illustrates alternative global emission pathways admitting cumulative CO_2 emissions of 750 Gt during the time period 2010 - 2050.[9] The global emissions of CO_2 decreased slightly between 2008 and 2009 following the worldwide financial and economic crisis. Nevertheless, global emissions again reached record levels in 2010. Each trajectory merges an initial business as usual phase with a subsequent mitigation phase that is assumed to be delayed until 2014 (red), 2018 (orange), 2022 (green) and 2025 (blue), respectively. The up-to-date evidence implies a fast diminishing window of opportunity to stabilize CO_2 concentrations at a level of 2°C. The outcomes also illustrate that the longer the start of the mitigation phase is delayed, the steeper the subsequent reduction in emissions has to be. More precisely, even two-digit cuts in annual emissions are required once the short time slot until 2020 remains unused. This occurs due to the realistic assumption of increasing annual emissions in the business as usual scenario, so that the total carbon budget tends to be exhausted quickly. Figure 1 also highlights that we need to shut down CO_2 emissions from fossil fuels and industrial processes not later than 2050.

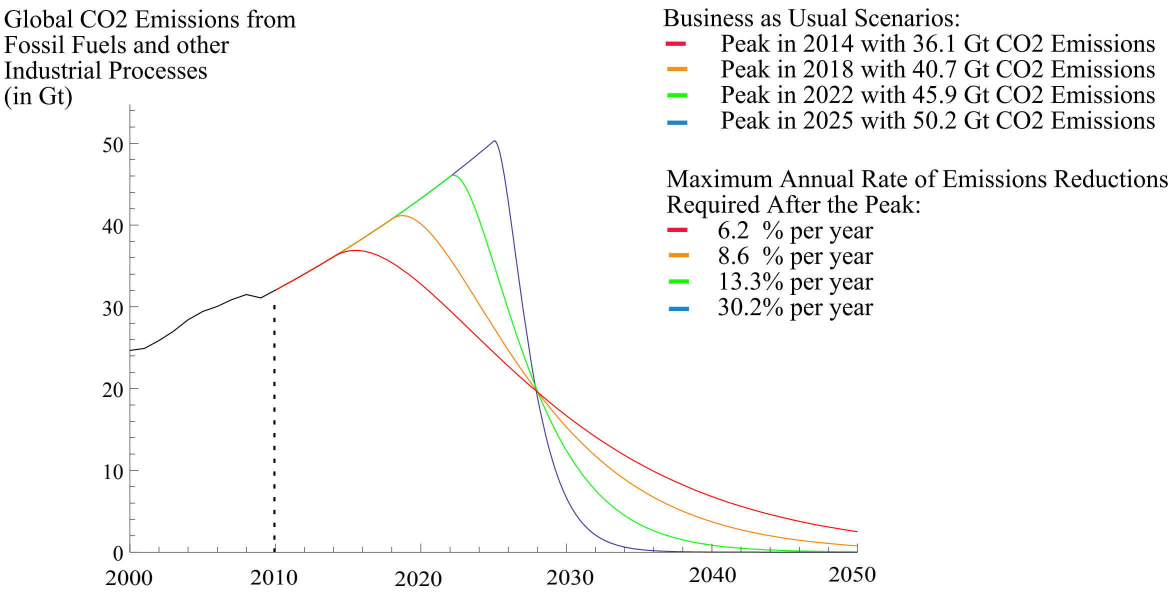

Figure 1: Alternative Carbon Dioxide Emission Pathways Complying with the 2°C Target with a Probability of 67 Percent.

Further studies also indicate that the 2 degree target requires early decreasing, rather than stabilised, CO_2 emission levels. IEA (2011) regards the atmospheric carbon dioxide level of 450 parts per million (ppm) as the threshold that is conform with the 2 °C target. The current level of 390 ppm indicates that four-fifths of the permitted budget is already locked

[9] The past emission trajectory matches observed data, which is taken from www.cerina.org/home and www.iwr.de/klima/ausstoss_welt.html. The computation of the future trajectories corresponds to the physically based equation (9) in Raupach et al. (2011), where the business-as-usual growth rate of emissions is assumed to be 3 percent.

in. Also accounting for the increasing energy demand that is triggered by global economic and population growth, IEA (2011) projects the expansion of the high-carbon infrastructure in the next five years will already mark the crossing of the 450ppm threshold. Fatih Birol, chief economist at the IEA, warns: "The door is closing. I am very worried - if we don't change direction now on how we use energy, we will end up beyond what scientists tell us is the minimum [for safety]. The door will be closed forever."[10]

Another strand of literature considers the stabilised level of radiative forcing of greenhouses gases as an anchor for climate policy targets. While most of the mitigation scenarios in the IPCC's Fourth Assessment Report base on a level of 4.5 W/m^2 in 2100, Meinshausen et al. (2006) conclude that lower numbers would be required to keep a high probability to achieve the 2°C target. Different climate models show this to be a level of 2.6 W/m^2 for the "representative concentration pathways" (RCPs), see for example Moss et al. (2008) and Moss et al. (2010). These RCPs are designed to update the SRES scenarios by explicitly accounting for information on climate policies, land-use and other socio-economic developments.[11] van Vuuren et al. (2011b) show that the RCP2.6. is technically feasible. However, emissions need to decrease significantly. Compared to the business as usual scenario, the required cumulative emission reduction over the century is assessed to be 70 percent.

Undertaking enormous reductions in such a short time frame, requires a radical redesign of the energy sector and industry. However, a swift switch to carbon-neutral alternatives will pose a vast challenge to the global economy.

How should policy makers respond to such a small window of opportunity? The answer might be less straightforward if the following reservations are considered. Firstly, the projections of climate responses and the resulting damage to the economy and human health will probably continue to involve substantial uncertainties for the next few years. Secondly, enormous emission reductions imply large sunk costs, which may not recouped before long. Thirdly, the worst effects of global warming and thus the benefits of a climate policy reducing them may not be felt for decades, whereas the costs of tackling climate change will burden the economies immediately. Hence, in spite of all warnings, policy makers are tempted to wait instead of taking action.

Despite the apparent prominence of the limited time to act issue in the climate literature, there has been very little effort to apply optimal climate policy under uncertainty to this important time limit. It is for this reason that we present a non-perpetual real options model that fits closely into the outlined facts.

[10] The statement is taken from http://www.guardian.co.uk/environment/2011/nov/09/fossil-fuel-infrastructure-climate-change?intcmp=122.

[11] For more details on RCPs see van Vuuren et al. (2011a).

3 The Window-of-Opportunity Modelling Setup

This section anchors our modelling approach in the existing real options literature. Before we begin our theoretical discussion, we believe it is helpful to characterize our use of real options models. Recent research has documented that it is more than a guideline for decision makers. More precisely, there is ample evidence that policy makers employ a "real options heuristic" [Kogut & Kulatilaka (2001)], i.e. retain the upside potential without the downside risk of fully committing up front. That means that in a situation of substantial uncertainties about the benefits of a policy, decision makers keep the options to act alive. Afraid of committing themselves to hugh expenses, they tend to wait for further information. However, as explained in section 2 the option to limit global warming to 2°C will expire at some time soon in the future. The consequential question that arises is, whether and how this affects the policy maker's decision. By incorporating the opportunity to act explicitly, the following model is set up to provide an answer.

To begin with the basis of the decision-making, it is in common economic practice to assume that a forward looking social planner strives to find the optimal timing of a climate policy by maximizing the flow of consumption over time.[12] She faces the intergenerational trade-off problem that investments into a mitigation strategy, which substantially reduces emissions, force the current economy to abstain from consumption, but avoid climate damages that would decrease future consumption potential. Moreover, bad timing will certainly lead to one of the following two irreversibility effects. On the one hand, investing too early in mitigation technologies could trigger enormous sunk costs that are not recouped for a long time. On the other hand, waiting too long may cause irreversible damage to ecological systems that are valuable to human health or the economy. However, ubiquitous uncertainties in almost every component of projections and especially in the assessment of future climate damages render a well-informed decision about the timing almost impossible. Put differently, all plans depend decisively on the unknown sensitivity of losses to climate change. Hence, particularly the uncertainties of the future climate damages and their effects are focussed on in the following, whereas any other lack of knowledge is assumed to be resolved for the sake of analytical tractability. Expressed mathematically, the policy maker solves the following objective function, which consists of the expected net present value of future consumption levels:

$$W(X, \Delta T) = E\left[(1 - w(\tau)) \int_{t=0}^{\infty} L(X_t, \Delta T_t)\, C_t e^{-rt} dt\right], \qquad (1)$$

where $E[\cdot]$ is the expectation operator and C_t is aggregate consumption over time with the initial value normalised to 1. In its simplest form, the level of consumption is assumed to

[12] In this framework the world is treated as a single entity in the interest of simplicity. The world climate policy equilibrium can be constructed as a symmetric Nash equilibrium in mitigation strategies. The equilibrium can be determined by simply looking at the single country policy which is defined ignoring the other countries' abatement policy decisions [Leahy (1993)].

be equivalent to the level of GDP. The pure rate of social time preference is expressed by r. The climate damages are measured by $L(X_t, \Delta T_t)$. This loss function is attached to the level of consumption, where ΔT_t describes scientifically estimated changes in temperature and X_t is a (positive) stochastic damage function parameter determining the sensitivity of losses to global warming. If the policy maker takes measures to limit temperature increase to some target τ, she is obliged to pay abatement costs that amount to a certain percentage $w(\tau) > 0$ of the annual GDP. In the case of no climate policy, the abatement costs $w(\tau)$ are zero.

Instead of trying to model climate impacts in any detail, we keep the problem analytically simple by assuming that damages depend only on temperature change, which is chosen as a measure of climate change. To be precise, following Pindyck (2009a,b) we assume that the damage from warming and the associated physical impacts of climate change as a fraction of GDP is implied by the exponential loss function

$$L(X_t, \Delta T_t) = e^{-X_t (\Delta T_t)^2}, \tag{2}$$

where $0 < L(X_t, \Delta T_t) \leq 1$, $\partial L / \partial (\Delta T_t) \leq 0$ and $\partial L / \partial X_t \leq 0$. This yields GDP at time t net of damage from warming in the order of $L(X_t, \Delta T_t) GDP_t$, i.e. climate-induced damages result in less GDP, and hence less consumption. Intuitively, a high value of X_t means that damages are sharply curved in ΔT_t.

Before we turn to the modelling of the uncertainty that is attached to X_t in equation (2), we briefly introduce the other component in the loss function: the temperature increase ΔT_t. For this we adopt the commonly used climate sensitivity function in Weitzman (2009a) and Pindyck (2009a,b). The single linear differential equation compresses all involved complex physical processes by capturing climate forcings and feedbacks in a simplified manner.[13] Hence, a direct link between the atmospheric greenhouse gas concentration G_t and the temperature increase ΔT_t is obtained by

$$d\Delta T_t = m_1 \left(\frac{\ln (G_t / G_p)}{\ln 2} - m_2 \Delta T_t \right) dt, \tag{3}$$

where G_p is the inherited pre-industrial baseline level of greenhouse gas, and m_1 and m_2 are positive parameters. The first term in the bracket stands for the radiative forcing induced by a doubling of the atmospheric greenhouse gases. The second term represents the net of all negative and positive feedbacks. A positive parameter for this term thus rules out a runaway greenhouse effect. The parameter m_1 describes the thermal inertia or the effective capacity to absorb heat by the earth system, which is exemplified by the oceanic heat uptake.

[13] Factors that influence the climate are categorised into forcings and feedbacks. A forcing is understood as a primary effect that changes directly the balance of incoming and outgoing energy in the earth-atmosphere system. Emissions of aerosols and greenhouse gases or changes in the solar radiation are examples. A secondary and indirect effect is described by a feedback that boosts (positive feedback) or dampens (negative feedback) a forcing. The blackbody radiation feedback exemplifies an important negative feedback, whereas, for example, the ice-albedo feedback accelerates warming by decreasing the earth's reflectivity.

Let H define the considered time horizon. In the business as usual scenario, the maximal temperture increase is assumed to double the warming after H years. This is tantamount to $\Delta T_t \to 2\Delta T_H$ for $t \to \infty$, which implies $2\Delta T_H = 1/m_2$ as the equilibrium and $m_1 m_2$ as the adjustment speed. The change in temperature increases linearly in the logarithm of greenhouse gas concentrations and thus $m_1 m_2 = \frac{\ln 2}{H}$. Cancelling terms and rearranging gives

$$d\Delta T_t = \frac{\ln(2)}{H}\left(2\Delta T_H - \Delta T_t\right) dt, \tag{4}$$

and

$$\Delta T_t = 2\Delta T_H \left(1 - e^{-\frac{\ln 2}{H} t}\right), \tag{5}$$

if the initial value ΔT_0 is set to zero. Equation (4) is an essential building block in the real options modelling setup, while equation (5) is useful for integrating the intertemporal climate change damage function.[14]

If the policy maker abates, a certain temperature target is assumed to be met after H years, i.e. $\Delta T_H \leq \tau$. In this case equations (4) and (5) are reshaped to

$$d\Delta T_t = \frac{\ln(2)}{H}\left(2\tau - \Delta T_t\right) dt, \tag{6}$$

and

$$\Delta T_t = 2\tau \left(1 - e^{-\frac{\ln 2}{H} t}\right), \tag{7}$$

respectively.

Let us now focus on the other component in equation (2), which is the sensitivity of losses to global warming. Due to the deep structural uncertainties in the natural sciences combined with uncertainty about future economic damages and the economies' ability to adapt to them, conceptualising X_t as a deterministic variable would be far-fetched. We thus need to specify a stochastic process X_t. Particularly for long timescales, uncertainty increases, because the magnitude and sequence of feedbacks to the initial forcings are hardly predictable. Hence, the process $X = (X_t)_{t \geq 0}$ is commonly presumed to follow a geometrical Brownian motion with (deterministic) drift parameter α and standard deviation σ,

$$dX_t = \alpha X_t dt + \sigma X_t dB_t, \tag{8}$$

where B is a standard Wiener process, see for example Pindyck (2000). The fluctuation of X_t over time complicates considerably the decision whether to exercise the real options of

[14] The increase in temperature is generated by some unspecified natural science climate model. Ultimately, we take ΔT_t and thus the geophysical microfoundation from climate models and impose this mathematically upon our economic model. This allows us to bypass climate and atmospheric modelling.

adopting the climate policy. Equation (8) allows one to trace the uncertainty transmission to optimal policies, as social welfare W thus evolves as

$$\begin{aligned} W(X, \Delta T) &= E\left[(1-w(\tau))\int_0^\infty e^{-X_t(\Delta T_t)^2} e^{-(r-g_0)t} dt\right] \\ &\cong E\left[[(1-w(\tau))\int_0^\infty \left(1 - X_t \Delta T_t^2 + \frac{1}{2}\left(X_t \Delta T_t^2\right)^2\right) e^{-(r-g_0)t} dt\right], \end{aligned} \tag{9}$$

with a constant consumption/GDP growth rate of g_0 and the assumption that r is greater than expected consumption growth rate g_0. Note that the exponential loss function of X_t renders an explicit analytical solution of the Ito-integral impossible. Therefore we use 2nd-order Taylor's expansions approximations in the numerical analysis below.

We need to determine optimal climate policies which imply an either-or decision, i.e. the value of action versus inaction is computed in the following. The welfare value of implementing the environmental policy now, denoted by $W^{\text{A}}(X, \Delta T; \tau) \equiv W^{\text{Action}}(X, \Delta T; \tau)$, is computed by equation (9) with $w(\tau) > 0$ and the temperature equation (7). After utilizing the relationship $E[X_t^n] = X_0^n e^{\left(n\alpha + \frac{1}{2}n(n-1)\sigma^2\right)t}$, which is derived by means of Ito's Lemma, the welfare for taking action now evolves as

$$W^{\text{A}}(X, \Delta T; \tau) = (1 - w(\tau))\left[\frac{1}{r - g_0} - 4\tau^2\gamma_1 X + 8\tau^4\gamma_2 X^2\right], \tag{10}$$

where

$$\begin{aligned} \gamma_1 &= \frac{1}{\eta_1} - \frac{2}{\eta_1 + \frac{\ln 2}{H}} + \frac{1}{\eta_1 + 2\frac{\ln 2}{H}}, \\ \gamma_2 &= \frac{1}{\eta_2} - \frac{4}{\eta_2 + \frac{\ln 2}{H}} + \frac{6}{\eta_2 + 2\frac{\ln 2}{H}} - \frac{4}{\eta_2 + 3\frac{\ln 2}{H}} + \frac{1}{\eta_2 + 4\frac{\ln 2}{H}}. \\ \eta_1 &= r - g_0 - \alpha \\ &\text{and} \\ \eta_2 &= r - g_0 - \left(2\alpha + \sigma^2\right). \end{aligned}$$

Note that it is assumed that both η_1 and η_2 are positive.[15]

Alternatively, the policy maker may want to continue to emit CO_2 emissions at the same level and therefore ΔT_t becomes ΔT_H at $t = H$, but no abatement costs are incurred, i.e. $w(\tau) = 0$. Applying the Hamilton-Jacobi-Bellman principle and Ito's Lemma to equation (9), we obtain the inaction value $W^{\text{N}}(X, \Delta T; \Delta T_H) \equiv W^{\text{No Action}}(X, \Delta T; \Delta T_H)$, which is

[15] Please see Appendix A for a derivation.

given by the partial differential equation

$$(r - g_0 - \alpha) W^{\mathrm{N}} = \left(1 - X_t \Delta T_t^2 + \frac{1}{2}\left(X_t \Delta T_t^2\right)^2\right) + \left(\frac{\ln(2)}{H}\left(2\Delta T_H - \Delta T_t\right)\right)\frac{\partial W^{\mathrm{N}}}{\partial \Delta T}$$
$$(11) \qquad + \alpha X \frac{\partial W^{\mathrm{N}}}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 W^{\mathrm{N}}}{\partial X^2} + \frac{\partial W^{\mathrm{N}}}{\partial t}.$$

The solutions to $W^{\mathrm{N}}(X, \Delta T; \Delta T_H)$ consist of two components: a particular solution and a general solution: $W^{\mathrm{N}}(X, \Delta T; \Delta T_H) = W^{\mathrm{NP}}(X, \Delta T; \Delta T_H) + W^{\mathrm{NG}}(X, \Delta T; \Delta T_H)$. Both solutions have a straightforward economic meaning. The business as usual policy is valued by the particular solution, which is derived by solving equation (9) with $w(\tau) = 0$:

$$(12) \qquad W^{\mathrm{NP}}(X, \Delta T; \Delta T_H) = \frac{1}{r - g_0} - 4\Delta T_H^2 \gamma_1 X + 8\Delta T_H^4 \gamma_2 X^2,$$

where the parameters have the same forms as in equation (10).[16] The value of the real options is obtained from the homogenous part of equation (11) being

$$(r - g_0 - \alpha) W^{\mathrm{NG}} = \left(\frac{\ln(2)}{H}\left(2\Delta T_H - \Delta T_t\right)\right)\frac{\partial W^{\mathrm{NG}}}{\partial \Delta T_t}$$
$$(13) \qquad + \alpha X \frac{\partial W^{\mathrm{NG}}}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 W^{\mathrm{NG}}}{\partial X^2} + \frac{\partial W^{\mathrm{NG}}}{\partial t}.$$

As discussed in section 2 the limited time to act implies the availability of real options in equation (13) of only a few years' time. This implies that at the end of t^* years, $0 < t^* < H$, the real options value approaches zero. This focus upon optimal policies over $(0, t^*)$ reflects the largely irreversible build-up of CO_2 in the atmosphere and clearly deviates from the infinite horizon assumption that is assumed in almost all variants of real options models. Though it is not possible to exactly pin down how many years are left for the policy maker to act before it is too late, we assume a fixed time of years left for the policy maker to pursue aggressive moves to curb emissions. What is the optimal pace of action given this critical window of opportunity?

It is the usual practice in financial derivatives that 2-factor partial differential equation (13) can be solved by 2-dimensional finite difference methods. However, we can use the method of separation of variables to reduce (13) into a one factor partial differential equation, as we know that the non-perpetual real options are related to the diffusion process X. Without the stochastic process in equation (9), the real options terms do not exist. On the contrary, the policy maker considers the process ΔT_t as an exogenous variable in the business as usual case. Furthermore, the particular solution in equation (12) implies that the solutions to equation (13) consist of the mathematical product of two different components: one for X_t and the other for ΔT_t. The discussion indicates that we can use the method of separation of variables to solve and simplify equation (13). As shown in Appendix B, equation (13) can

[16] Please see Appendix A for a derivation.

be transformed into the following one factor partial differential equation,

$$\left(r - g_0 - \alpha + 2\frac{\ln(2)}{H}\right) W^{\mathrm{NG}} = \alpha X \frac{\partial W^{\mathrm{NG}}}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 W^{\mathrm{NG}}}{\partial X^2} + \frac{\partial W^{\mathrm{NG}}}{\partial t}. \tag{14}$$

The main difference between (13) and (14) is the transition of the term $\left(\frac{\ln(2)}{H}(2\Delta T_H - \Delta T_t)\right)\frac{\partial W^{\mathrm{NG}}}{\partial \Delta T_t}$ in equation (13) into the higher effective discount rate of equation (14), increased by a factor of $2\frac{\ln(2)}{H} \cong 1.39\%$ for H =100. The meaning is straightforward, as higher changes in temperature in the future lead to an lower intertemporal value of consumption and GDP. This is equivalent to lower real options values being caused by higher effective discount rates. Equation (14) can be solved by numerical methods such as one-factor/dimensional explicit finite difference methods.[17]

Matters are made difficult when incorporating limited time to act. So far, no closed-form solutions are known for non-perpetual real options models, as in most of cases in financial derivatives pricing.[18] Therefore we seek a numerical solution instead. To this end, a simple explicit finite difference scheme is employed. The finite difference solutions of the 1-factor partial differential equation (14) may be obtained using iteration or matrix inversion techniques.[19] The value-matching condition for the optimal stopping problem for the policy maker is represented by

$$W^{\mathrm{A}}\left(\bar{X}, \Delta T; \tau\right) = W^{\mathrm{NP}}\left(\bar{X}, \Delta T; \Delta T_H\right) + W^{\mathrm{NG}}\left(\bar{X}, \Delta T; \Delta T_H\right), \tag{15}$$

where $\bar{X}$ denotes the thresholds at which the policy-maker would take action by exercising real options today, which obliges to pay the annual abatement costs $w(\tau)$ in percent of *GDP* to limit the future temperature increase to less than τ at $t = H$.[20] On the contrary, exercising the real options implies that the policy maker forgoes the option to wait and act later as more information about X_t becomes available. An algorithm for calculating equation (14) is deduced in Appendix C.

We have now laid out an applicable analytical approach that directly addresses the issue of the limited time to act. In the remainder of this paper we perform a series of calibrations of this model.

[17]For a similar algorithm in derivative pricing, see Brennan & Schwartz (1978). Using 2-dimensional finite difference methods would complicate the numerical analysis of the model without altering the basic message.

[18]See, for example, the standard textbooks Hull (2010) and Wilmott (1998).

[19]The first paper to recognize that option prices could be obtained with a finite difference solution to the partial differential equation was Schwartz (1977). The finite difference method proceeds by replacing differentials with differences and then solving over a grid of time and state variables subject to the boundary conditions. A thorough review of the state of the art in numerical finite difference techniques along with an exhaustive list of references is offered in Duffy (2006).

[20]The value-matching conditions involve the value function, while the smooth-pasting conditions concern its first-order derivatives. The smooth-pasting boundary condition will be imposed numerically via the finite difference method.

4 Numerical Simulations and Results

Formal theory is essential in enabling us to organise our knowledge about climate problems in a coherent and consistent way. However, the formal theory needs to be applied to data if it is to enhance our understanding and have relevance for practical problems. This calibration exercise will provide new insights and may thus contribute to climate policy discussions, which are certainly influenced by limited time to act. For this purpose we map the theoretical framework presented above to real-world data. Where possible, parameter values are drawn from empirical studies. However, the determination of some parameters is somewhat speculative or they are drawn from back-of-the-envelope calculations.[21] Therefore, for each parameter a sensitivity analysis over a sufficiently wide grid is performed, while keeping an eye on robustness. The unit time length corresponds to one year and annual rates are used when applicable. Our base parameters are chosen to come close to reality, which means $\alpha = 0$, $\sigma = 0.075$, $r = 0.025$, $g_0 = 0.0$, and $H = 100$. ΔT_H is assumed to be 3.4°C, which is equivalent to 4 degrees of warming since the pre-industrial level. τ is assumed to be 1.4°C by assumption which is equivalent to 2 degrees of warming compared with the pre-industrial level. In order to assess the economic costs of mitigation, Edenhofer et al. (2010) have compared the energy-environment-economy models MERGE, REMIND, POLES, TIMER and E3MG in a model comparison exercise. In order to improve model comparability, the macroeconomic drivers in the five modelling frameworks employed were harmonised to represent similar economic developments. On the other hand, different views of technology diffusion and different structural assumptions regarding the underlying economic system across the models remained. This helps to shed light on how different modelling assumptions translate into differences in mitigation costs. Despite the different structures employed in the models, four of the five models show a similar pattern in mitigation costs for achieving the first-best 400ppm CO_2 concentration pathway. After allowing for endogenous technical change and carbon capture and storage with a storage capacity of at least 120 GtC, the mitigation costs are estimated to be approximately 2 percent of worldwide GDP. These costs turned out to be of a similar order of magnitude across the models. We therefore assume that $w(\tau) = 0.02$.

As the sensitivity of losses X_t fluctuates over time, we have to pay special attention to the magnitude of the resulting climate damages. As an illustration and in order to gain an intuition Figure 1 shows the numerically simulated loss equation (2) based on the temperature equation (4) for three alternative X terms, which are assumed to be constant. The considered time period ranges from $t = 0$ to $t = 200$. Two effects must be recognized. Firstly, the minimum of $L(X_t, \Delta T_t)$ and therefore the maximum of GDP_t net of damages is obtained for the lowest value of the drift term. Secondly, as can be easily seen in the graph, L spreads out considerably during the time of undertaking no mitigation. For $t = 50$ years the damage is 3.89 percent of GDP for constant $X_t = 0.01$, 3.12 percent of GDP for $X_t = 0.008$, and

[21] Despite the increasingly detailed understanding of climate processes from a large body of research, various parameters involved remain inevitably unanswered except in retrospect.

2.35 percent of GDP for $X_t = 0.006$. After $t = 100$ years the corresponding damage is 10.92 percent of GDP for $X_t = 0.01$, 8.83 percent of GDP for $X_t = 0.008$, and 6.70 percent of GDP for $X_t = 0.006$.[22]

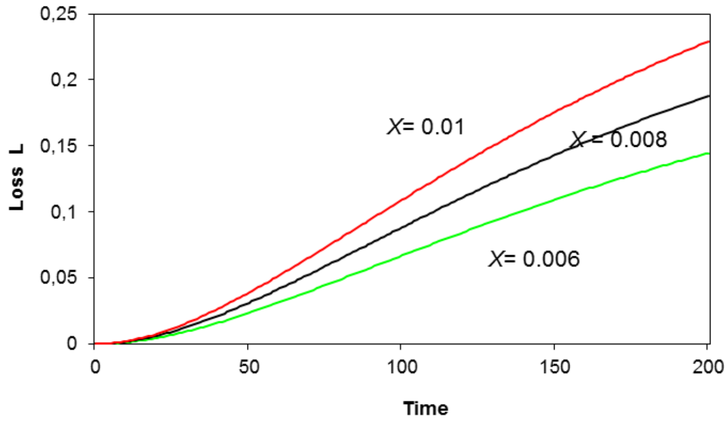

Figure 2: Simulated Loss Due To Global Warming in Percent of GDP

To simulate the full model, we have to solve equation (13) by utilizing the finite difference method. To this end, it is transformed into a one-factor partial difference equation (see Appendix B). The variable X and the parameter t need to be expressed as a network mesh of discrete points, ΔX and Δt. Afterwards, the partial differential equation can be displayed as set of finite difference equations that are numerically solvable in a backward scheme and subject to corresponding discrete-time boundary conditions (see Appendix C for more details). We use the following benchmark values for the explicit finite difference method: $X_{\max} = 0.05$, $\Delta t = 0.0001$, $\Delta X = 0.0002 \cdot \sigma$.[23]

Results for various parameters are displayed in the Figures 3 - 6. The intuitive threshold plots split the space spanned by X shocks into action and inaction areas. In the inaction area the marginal reward for pursuing CO_2 reductions is insufficient and policy makers prefer to wait. The economic explanation of the thresholds $\bar{X}$ is straightforward. The index X is part of the loss function. The smaller $\bar{X}$ is, the faster the policy response will be. For the sake of clarity, Figure 3 offers an isolated inspection of the impact of alternative time horizons upon the climate policy threshold for the baseline parameters. Broadly speaking, the results suggest that limited time to act has a significant impact upon the threshold for $t^* < 5$ years. In the case of very small t^*, rational policy makers will pursue immediate precautionary measures in the direction of low-carbon alternatives to prevent a long-term high carbon lock-in. In other words, the results elevate the urgency of climate change policies.[24]

[22]These numbers are in the range of common assumptions in the literature. In Weitzman (2009b) the damage costs are calibrated to be 9 (25) percent of GDP for 4°C (5°C) of warming and Millner et al. (2010) consider damages of 6.5 percent of GDP for 5°C of warming.

[23]The benchmark values of Δt and ΔX are chosen to make sure positive coefficient of equation (B6) and ensure convergence and stability for (B5) in Appendix C in an explicit finite difference method scheme.

[24]But a large caveat should accompany any use of that number because it assumes that the climate policies will be both efficient and effective. Unfortunately, there is a voluminous literature of government failure,

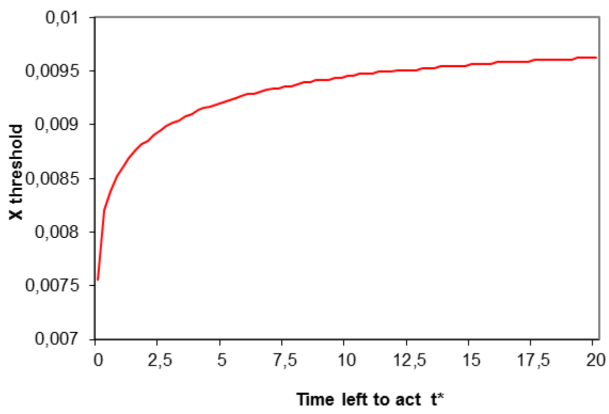

Figure 3: The Impact of Alternative Time Horizons t^* Upon the X Threshold

This result of curbing emissions aggressively contrasts the slow, incremental approach to CO_2 mitigation in reality and fits the urgency emphasized by Krugman (2010). He warns of relying on models that advocate delaying mitigation measures. For instance, the optimal policy in Nordhaus's cost-benefit model would stabilize the atmospheric carbon dioxide concentration at a level about twice its preindustrial average, which is supposed to lead to a temperature of 3°C. Decreasing emissions are not required before 2045. This strategy has only modest negative effects on global welfare according to the RICE-model.[25] However, the crucial question arises, of how trustworthy such a projection really is. One the one hand, consequences of such a warming are hardly predictable. On the other hand, looking back at historic experiences does not reveal information, as for most of the time span of human civilizations global climatic patterns have stayed within a very narrow range. Hence, it cannot be taken for granted that such a policy will not cause a dangerous climate crisis. Despite its stylised nature, the non-perpetual real option framework, however, allows one to incorporate scientific findings such as the limited time frame and thus may deliver a road map for keeping the planet safe.

How robust is this conclusion? In the following, this result is tested for its sensitivity to alternative choices of the noisiness level σ in the sensitivity of losses, the predicted temperature increase ΔT_H and the discount rate r. In particular, the assessments of the climate damage costs exhibit a broad range of uncertainty and always leads to controversies. Beyond the issue of the likely consequences of warming, it is debatable how non-market goods like human life and the intrinsic values of ecosystems are appropriately monetarised and how catastrophes that have a low probability but high impacts are included. Furthermore, the future capabilities for adapting to climate change are hardly predictable. By comparing 28 studies on marginal damages costs in different regions, Tol (2005) emphasises that the esti-

regulatory capture and the impact of rent-seeking behaviour within the policy process. Climate policy is likely to be a large source of economic rents from policy interventions. Note that this is an exploratory paper, and by no means intended to give blanket approval to any proposal for climate protection.

[25] Please see Nordhaus (2010) for more information.

mates give insights into signs, orders of magnitude and patterns of vulnerability but remain speculative. To study the effects of uncertainty in the assessments, Figure 4 illustrates the results for different values of σ. It provides an important twist in the story by revealing the adverse effects of uncertainty on the policy makers' decision. The combination of limited time to act and even moderate increases in uncertainty may make the rational policy response weaker, not stronger. The reason is that the benefits of waiting for uncertainty to dissipate overwhelm the cost of moving too slowly. Thus, rational policy makers will not necessarily behave prudently to keep nature from passing the 2°C threshold. Put differently, the high σ - small t^* constellation is a double-edged sword. For high σ the temptation to avoid tackling climate change is hard to resist although a steep near-term reduction in emissions is needed and a sound investment as indicated by climate science.

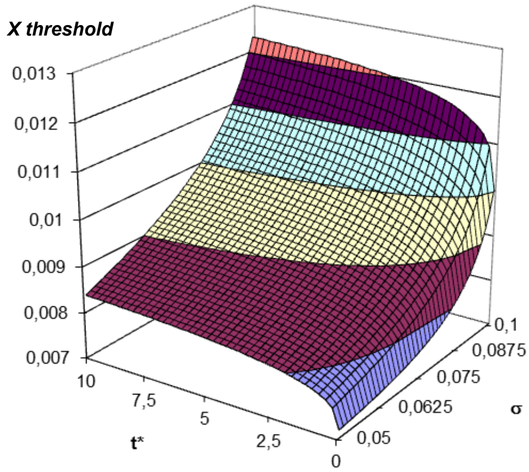

Figure 4: The Impact of t^* and σ Upon the X Threshold

Another substantial source of uncertainty is represented by the temperature increase ΔT_H. The IPCC's first assessment, published back in 1990, predicted a warming of 3°C by 2100, with no confidence bands. The second IPCC assessment, in 1995, suggested a warming of between 1°C and 3.5°C. The third, in 2001, widened the bands to project a warming of 1.4°C to 5.8°C. The fourth assessment in 2007 restrained them again, from 1.8°C to 4.0°C. At the moment it seems unlikely that scientific uncertainty will be completely resolved in the near future. Quite the reverse, Kevin Trenberth (Head, National Center for Atmospheric Research in Boulder, Colorado) recently warned in a commentary in *Nature Online* (21 January 2010) headlined "More Knowledge, Less Certainty" that "the uncertainty in AR5's predictions and projections will be much greater than in previous IPCC reports."[26] The reason for this is that "our knowledge of certain factors [responsible for global warming] does increase," he wrote, "so does our understanding of factors we previously did not account for or even recognize."[27]

[26] See http://www.nature.com/climate/2010/1002/full/climate.2010.06.html.

[27] Up-to-date climate models are trying to come to grips with a range of factors ignored or only sketchily

In other words, there is still tremendous and in some cases even increasing uncertainty in the climate projections. Figure 5 outlines the joint impact of different temperature predictions ΔT_H and the time left to act t^* to prevent the temperature from overshooting the 2°C target. For any ΔT_H the curve exhibits the same concave shape as in Figure 3. Hence, independently of the magnitude of the predicted temperature, a rational policy makers will take mitigation actions earlier for small t^*. However, the effect of varying ΔT_H is enormous and has a greater influence on the optimal policy threshold than the limited time to act. The policy threshold $\overline{X}$ doubles in size when assuming $\Delta T_H = 2.9$°C instead of $\Delta T_H = 3.9$°C and it increases even more when taking $\Delta T_H = 2.4$°C instead of our base calibration $\Delta T_H = 3.4$°C. Hence, the decision about when to implement a climate policy is radically influenced by the projection of the temperature increase. As in reality broad uncertainty ranges of the temperature dynamics exist, this simulation highlights how policy makers face hugh problems in reaching a decision in favour of a mitigation strategy.[28] What is a reasonable estimation to base climate policy decision on?

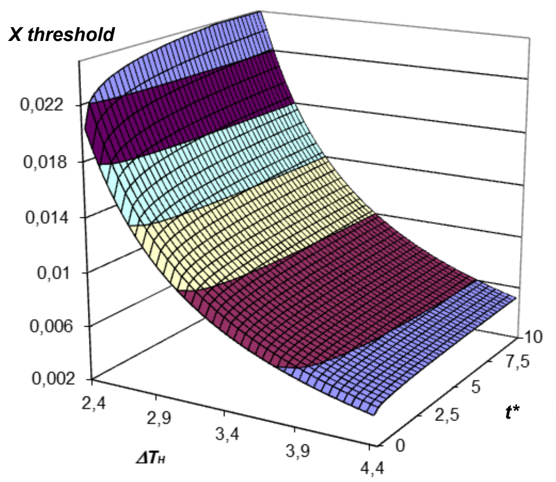

Figure 5: The Impact of ΔT_H and t^* Upon the X Threshold

Next we take a closer look at the impact of the discount rate r. To explore the sensitivity to alternative discounting assumptions, we employ a range of $0.0 < r < 0.04$. As expected, the results in Figure 6 affirm the view that higher discount rates will bolster the reasons for taking a "wait and see" attitude towards climate policy. This is due to the fact that for a larger value of r the intertemporal damage is substantially smaller. In other words, a higher discounting

dealt with in the past. One troubling aspect is the role of clouds because nobody can work out exactly whether warming will change them in a way that amplifies or moderates global warming. Another problem in understanding clouds is the role of aerosols which dramatically influence the radiation properties of clouds. It therefore comes at no surprise that the resulting error bands are extremely wide.

[28]This raises the question how much better we can expect medium run climate projections to get. Can we reduce forecast errors? How much can uncertainty go down as models improve? Although climate models have improved and societal needs push for more accurate decadal climate projections over the next 10-30 years, decadal projections are still in its infancy and the hope for useful decadal projections is far from assured [see Cane (2010)]

factor will trigger a later adoption and a lower intensity of climate policy. This highlights the importance of attaining a consensus on the discount rate before an appraisal on the optimal timing of policy implementation can be achieved. Another important conclusion from Figure 6 is that the effects of a higher discount rate trumps the effects of the limited time to act. One way to resolve this conclusion is to make it normative that future generations' welfare should figure just as highly as the welfare of the current generation.[29]

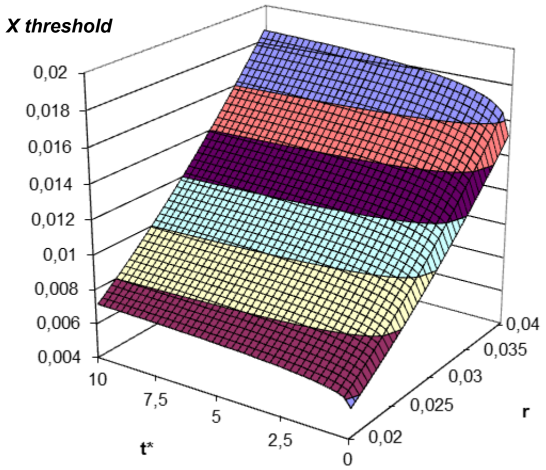

Figure 6: The Impact of t^* and r Upon the X Threshold

To sum up, we may conclude that the knowledge to have a limited time to act should accelerate climate policy significantly, in particular if the window of opportunity closes very soon. However, ubiquitous uncertainties in the projections of the temperature increase and the future damage costs as well as the different opinions on the discount rate affect the decision considerably. In particular, the uncertainties in the damage costs are shown to have adverse effects. Despite the urgency to take action, this kind of uncertainty delays the implementation of a climate policy.

5 Conclusions

Recent scientific studies by Meinshausen et al. (2009) and Rogelj et al. (2011) indicate that global greenhouse gas emissions need to be substantially reduced in upcoming years, in order to limit global warming to 2°C. This motivated us to investigate the climate policy implications of a time-limited window of opportunity. To this end, we have developed a conceptual

[29]To put a positive spin on it, one can point out that the Ramsey equation $r_t = \rho + \xi g_t$ can be employed as the rationale behind a low discount rate. r_t represents the consumption discount rate, ρ the time preference rate, ξ the coefficient of relative risk aversion in the CRRA specification, and g_t describes the consumption growth rate over $(0, t^*)$. The interest rate is then contingent upon the expected future growth rate of consumption and lower rates can be justified by making the argument that the resources needed to adapt to global warming will reduce the return on capital and the steady state growth rate. See Stokey (1998) for a corresponding endogenous growth model.

non-perpetual real options framework. The lesson to be pointed out is that, although this is a stylized representation of the real world, real options models provide a disciplined way of thinking about climate and its interaction with policy. Not least, they allow thinking afresh about the critical window of opportunity. Modelling of limited time to act is a comparatively uncharted area of climate research. In spite of its concrete climate policy relevance, nobody has explored the optimal policy response through the lens of real options models yet. Therefore this paper will not only be of interest to specialists in real options theory but also to an audience of climate scientists and policy makers. A unifying message from our paper could be stated as follows: Policy makers have to take steps to cut emissions now, so that a radical, hasty and extremely costly shift towards carbon-neutral alternatives is not necessarily required. Although a global shift in energy- and carbon-intense investment patterns is required to prevent a long-term high carbon lock-in, the policy makers will probably not take drastic action in the near future. As shown by this paper, ubiquitous uncertainties in the projections of the temperature increase and the future damage costs as well as the different opinions how to discount the future consumption flows affect the decision considerably. In particular, the uncertainties in the damage costs are shown to have adverse effects. Despite the urgency of taking action, this kind of uncertainty may lead to a range of inaction, where the policy makers prefer to postpone emission reductions. Instead of saying "there is not much time left" we unfortunately may have to note: "time runs out". That, in a nutshell, is the dilemma of climate change.

Appendices

A Derivation of Equation (10) and Equation (12)

By applying Ito's Lemma to the logarithm of X_t in equation (8) we obtain $\forall\, t \geq 0$:

$$X_t = X_0 e^{(\alpha - \frac{1}{2}\sigma^2)t + \sigma B_t}. \tag{A.1}$$

After raising equation (A.1) to the power of n, the application of the expectation value yields

$$E[X_t^n] = X_0^n e^{n(\alpha - \frac{1}{2}\sigma^2)t} E[e^{n\sigma B_t}] = X_0^n e^{(n\alpha - \frac{1}{2}n\sigma^2)t} e^{\frac{1}{2}n^2\sigma^2 t} = X_0^n e^{\left(n\alpha + \frac{1}{2}n(n-1)\sigma^2\right)t}. \tag{A.2}$$

This relationship is utilized to compute equation (9) for a climate policy:

(A.3)

$$
\begin{aligned}
&W^{\mathrm{A}}(X, \Delta T; \tau) \\
&= E\left[(1-w(\tau)) \int_0^\infty \left(1 - X_t \Delta T_t^2 + \frac{1}{2}\left(X_t \Delta T_t^2\right)^2\right) e^{-(r-g_0)t} dt\right] \\
&= E\left[(1-w(\tau)) \int_0^\infty \left(1 - X_t 4\tau^2 \left(1 - e^{-\frac{\ln 2}{H}t}\right)^2 + \frac{1}{2} X_t^2 16\tau^4 \left(1 - e^{-\frac{\ln 2}{H}t}\right)^4\right) e^{-(r-g_0)t} dt\right] \\
&= (1-w(\tau)) \int_0^\infty \left(1 - 4E[X_t]\tau^2 \left(1 - e^{-\frac{\ln 2}{H}t}\right)^2 + 8E[X_t^2]\tau^4 \left(1 - e^{-\frac{\ln 2}{H}t}\right)^4\right) e^{-(r-g_0)t} dt \\
&= (1-w(\tau)) \int_0^\infty \left(1 - 4X_0 e^{\alpha t}\tau^2 \left(1 - e^{-\frac{\ln 2}{H}t}\right)^2 + 8X_0^2 e^{\left(2\alpha+\sigma^2\right)t}\tau^4 \left(1 - e^{-\frac{\ln 2}{H}t}\right)^4\right) e^{-(r-g_0)t} dt.
\end{aligned}
$$

The second equality holds as the conduct of a climate policy is assumed to put the temperature equation (7) into effect. The third equality is obtained by applying Fubini's theorem before rearranging and taking advantage of the monotonicity of the expectation value and the last equality holds due to equation (A.2). By expanding the terms

$$
\left(1 - e^{-\frac{\ln 2}{H}t}\right)^2 = 1 - 2e^{-\frac{\ln 2}{H}t} + e^{-2\frac{\ln 2}{H}t} \tag{A.4}
$$

and

$$
\left(1 - e^{-\frac{\ln 2}{H}t}\right)^4 = 1 - 4e^{-\frac{\ln 2}{H}t} + 6e^{-2\frac{\ln 2}{H}t} - 4e^{-3\frac{\ln 2}{H}t} + e^{-4\frac{\ln 2}{H}t}, \tag{A.5}
$$

we obtain after integrating

$$
\begin{aligned}
W^{\mathrm{A}}(X, \Delta T; \tau) = (1 - w(\tau)) \Bigg[& \frac{1}{r-g_0} - 4\tau^2 X_0 \left(\frac{1}{\eta_1} - \frac{2}{\eta_1 + \frac{\ln 2}{H}} + \frac{1}{\eta_1 + 2\frac{\ln 2}{H}}\right) \\
& + 8\tau^4 X_0 \left(\frac{1}{\eta_2} - \frac{4}{\eta_2 + \frac{\ln 2}{H}} + \frac{6}{\eta_2 + 2\frac{\ln 2}{H}} - \frac{4}{\eta_2 + 3\frac{\ln 2}{H}} + \frac{1}{\eta_2 + 4\frac{\ln 2}{H}}\right)\Bigg],
\end{aligned} \tag{A.6}
$$

where

$$
\eta_1 = r - g_0 - \alpha
$$

and

$$
\eta_2 = r - g_0 - \left(2\alpha + \sigma^2\right),
$$

which is the same as equation (10).

Please note that the welfare value of the business as usual policy W^{NP} evolves in an analogue way. Hence, its solution is the same but with $w(\tau) = 0$, which gives equation (12).

B Derivation of Equation (14)

As the real options are mainly related to the diffusion process X and the process ΔT in the case of inaction is external to the policy maker, we can naturally hint that the solution to WNG has the form,

$$W^{\text{NG}} = f(\Delta T)\, Y(X,t)\,. \tag{B.1}$$

Substituting (B.1) back to equation (13) yields

$$\begin{aligned}(r-g_0-\alpha) f(\Delta T)\, Y(X,t) = {} & \frac{\ln(2)}{H}(2\Delta T_H-\Delta T)\, Y(X,t)\,\frac{df(\Delta T)}{d\Delta T} \\ & + \alpha X f(\Delta T)\frac{\partial Y(X,t)}{\partial X} + \frac{1}{2}\sigma^2 X^2 f(\Delta T)\frac{\partial^2 Y(X,t)}{\partial X^2} \\ & + f(\Delta T)\frac{\partial Y(X,t)}{\partial t}.\end{aligned} \tag{B.2}$$

Dividing both sides by $f(\Delta T)$, we get

$$\begin{aligned}(r-g_0-\alpha)\, Y(X,t) = {} & \frac{\ln(2)}{H}(2\Delta T_H-\Delta T)\frac{Y(X,t)}{f(\Delta T)}\frac{df(\Delta T)}{d\Delta T} \\ & + \alpha X\frac{\partial Y(X,t)}{\partial X} + \frac{1}{2}\sigma^2 X^2\frac{\partial^2 Y(X,t)}{\partial X^2} + \frac{\partial Y(X,t)}{\partial t}.\end{aligned} \tag{B.3}$$

To make partial differential equation (B.3) solvable by the separation of variables, $\frac{\ln(2)}{H}(2\Delta T_H-\Delta T)\frac{1}{f(\Delta T)}\frac{df(\Delta T)}{d\Delta T}$ has to be a constant linear term. This implies that the solutions of $f(\Delta T)$ take the form

$$f(\Delta T) = (2\Delta T_H-\Delta T)^2 \tag{B.4}$$

and

$$\frac{\ln(2)}{H}(2\Delta T_H-\Delta T)\frac{Y(X,t)}{f(\Delta T)}\frac{df(\Delta T)}{d\Delta T} = -2\frac{\ln(2)}{H}Y(X,t)\,. \tag{B.5}$$

Equation (B.5) ensures the separation of equations and yields the following new partial differential equation for $Y(X,t)$ by substituting (B.5) back to (B.3),

$$\left(r-g_0-\alpha+2\frac{\ln(2)}{H}\right)Y(X,t) = \alpha X\frac{\partial Y(X,t)}{\partial X} + \frac{1}{2}\sigma^2 X^2\frac{\partial^2 Y(X,t)}{\partial X^2} + \frac{\partial Y(X,t)}{\partial t}. \tag{B.6}$$

Therefore, we obtain the solution

$$W^{\text{NG}} = (2\Delta T_H-\Delta T)^2\, Y(X,t)\,, \tag{B.7}$$

where $Y(X,t)$ follows equation (B.6). The results are similar to Chen et al. (2011) apart from the fact that equation (B.6) has the term $\partial Y/\partial t$ due to the "limited time to act" real options. Equation (B.6) can be solved by numerical methods such as finite difference methods. Combining equations (B.6) and (B.7), we then obtain the desired 1-factor partial differential equation for non-perpetual real options,

$$\text{(B.8)} \qquad \left(r - g_0 - \alpha + 2\frac{\ln(2)}{H}\right) W^{\mathrm{NG}} = \alpha X \frac{\partial W^{\mathrm{NG}}}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 W^{\mathrm{NG}}}{\partial X^2} + \frac{\partial W^{\mathrm{NG}}}{\partial t},$$

which gives equation (14).

C Explicit Finite Difference Method Scheme for Equation (14)

For real options with maturity t^*, the boundary conditions are

$$\text{(C.1)} \qquad W^{\mathrm{NG}}(t, X_t = 0, \Delta T_t) = 0$$

and

$$\text{(C.2)} \qquad \begin{aligned} &\lim_{x\to\infty} W^{\mathrm{NG}}(t, X_t = x, \Delta T_t) \\ &= \max\left[\lim_{x\to\infty}\left(W^{\mathrm{A}}(t, X_t = x, \Delta T_t; \tau) - W^{\mathrm{NP}}(t, X_t = x, \Delta T_t; \Delta T_H)\right), 0\right], \end{aligned}$$

where $W^{\mathrm{A}}(t, X, \Delta T; \tau)$ and $W^{\mathrm{NP}}(t, X, \Delta T; \Delta T_H)$ are from equations (10) and (12), respectively. The terminal condition is

$$\text{(C.3)} \qquad W^{\mathrm{NG}}(t = t^*, X_{t^*}, \Delta T_{t^*}) = 0,$$

which is used as the starting points as the explicit finite difference method is backwards computing from $t = t^*$ to $t = 0$. The condition of

$$\text{(C.4)} \qquad W^{\mathrm{NG}}(t, X_t, \Delta T_t) = \max\left[W^{\mathrm{A}}(t, X_t, \Delta T_t; \tau) - W^{\mathrm{NP}}(t, X_t, \Delta T_t; \Delta T_H), 0\right]$$

is checked for every t since it is a free-boundary condition for real options in a sense that real options can be exercised at any time. Accordingly, equation (14) for real options W^{NG} can be approximated by a function that is defined on a following two-dimensional grid, i.e. $W^{\mathrm{NG}}(i\Delta t, j\Delta X) \equiv v_{i,j}$. For the explicit approximation, the partial derivatives are approximated by

$$\text{(C.5)} \qquad \frac{\partial W^{\mathrm{NG}}}{\partial X} = \frac{v_{i+1,j+1} - v_{i+1,j-1}}{2\Delta X},$$

$$\text{(C.6)} \qquad \frac{\partial^2 W^{\mathrm{NG}}}{\partial X^2} = \frac{v_{i+1,j+1} + v_{i+1,j-1} - 2v_{i+1,j}}{\Delta X^2},$$

$$\text{(C.7)} \qquad \frac{\partial W^{\mathrm{NG}}}{\partial t} = \frac{v_{i+1,j} - v_{i,j}}{\Delta t}.$$

Substituting the above equations back into equation (14) yields

$$\begin{aligned}\left(r - g_0 - \alpha + 2\frac{\ln(2)}{H}\right) v_{i,j} =& \alpha j \Delta X \left(\frac{v_{i+1,j+1} - v_{i+1,j-1}}{2\Delta X}\right) \\ &+ \frac{1}{2}\sigma^2 j^2 \Delta X^2 \left(\frac{v_{i+1,j+1} + v_{i+1,j-1} - 2v_{i+1,j}}{\Delta X^2}\right) + \left(\frac{v_{i+1,j} - v_{i,j}}{\Delta t}\right).\end{aligned} \tag{C.8}$$

Finally, rearranging and simplifying further allows us to obtain

$$v_{i,j} = a_j^* v_{i+1,j-1} + b_j^* v_{i+1,j} + c_j^* v_{i+1,j+1}, \tag{C.9}$$

where

$$a_j^* = \frac{1}{1 + \left(r - g_0 - \alpha + 2\frac{\ln(2)}{H}\right)\Delta t}\left(-\frac{1}{2}\alpha j \Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right), \tag{C.10}$$

$$b_j^* = \frac{1}{1 + \left(r - g_0 - \alpha + 2\frac{\ln(2)}{H}\right)\Delta t}\left(1 - \sigma^2 j^2 \Delta t\right), \tag{C.11}$$

$$c_j^* = \frac{1}{1 + \left(r - g_0 - \alpha + 2\frac{\ln(2)}{H}\right)\Delta t}\left(\frac{1}{2}\alpha j \Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right), \tag{C.12}$$

Analogue to equation (A.3), $W^{\mathrm{A}}(t, X_t, \Delta T_t; \tau)$ and $W^{\mathrm{NP}}(t, X_t, \Delta T_t; \Delta T_H)$ can be expressed by the following equations,

(C.13)

$$\begin{aligned}&W^{\mathrm{A}}(X_t, \Delta T_t; \tau) = (1 - w(\tau)) \int_0^\infty e^{-(r-g_0)t} \times \\ &E\left[\left(1 - 4X_t\tau^2\left(1 - e^{-\frac{\ln 2}{H}(t+j\Delta t)}\right)^2 + \frac{1}{2}\left(4X_t\tau^2\left(1 - e^{-\frac{\ln 2}{H}(t+j\Delta t)}\right)^2\right)^2\right)\right] e^{-(r-g_0)t} dt\end{aligned}$$

and

$$\begin{aligned}W^{\mathrm{NP}}(X_t, \Delta T_t; \Delta T_H) = &\int_0^\infty e^{-(r-g_0)t} \times \\ &E\left[\left(1 - 4X_t\Delta T_H^2\left(1 - e^{-\frac{\ln 2}{H}(t+j\Delta t)}\right)^2 + \frac{1}{2}\left(4X_t\Delta T_H^2\left(1 - e^{-\frac{\ln 2}{H}(t+j\Delta t)}\right)^2\right)^2\right)\right] dt,\end{aligned} \tag{C.14}$$

where the term $(t + j\Delta t)$ reflects the temperature at time $= (t + j\Delta t)$ when computing the payoffs for real options. Solving equations (C.14) and (C.13) is very time-consuming since we need to compute the integrals at each time step backwards. Note that equation (5) shows the early temperature increase is not great for small t. Furthermore, as we compute the values of $W^{\mathrm{NP}}\left(X_t, \Delta T_t; \Delta T_H\right)$ and $W^{\mathrm{A}}\left(X_t, \Delta T_t; \tau\right)$ backwards at each step of time from $t = t^*$ to $t = 0$, $(t + j\Delta t)$ approaching $(t = 0)$ at for the final values of real options, which means that at $t = 0$, (C.14) and (C.13) become

$$W^{A}\left(X_t, \Delta T_t; \Delta T_H\right) \cong (1 - w(\tau)) \left[\frac{1}{r - g_0} - 4\Delta\tau^2\gamma_1 X + 8\Delta\tau^4\gamma_2 X^2\right], \tag{C.15}$$

$$W^{\mathrm{NP}}\left(X_t, \Delta T_t; \tau\right) \cong \frac{1}{r - g_0} - 4\Delta T_H^2\gamma_1 X + 8\Delta T_H^4\gamma_2 X^2, \tag{C.16}$$

which are the same as in equations (10) and (12). Numerical testing shows that using (C.15) and (C.16), time invariant results, for the time from $t = T$ to $t = 0$ gives almost the same numerical results as using (C.14) and (C.13). The threshold for $\bar{X}_t$ at time $t = 0$ is then obtained from the above algorithm by checking numerically the points where equation (15) holds.

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