Probability-based Optimisation for a Multi-MHz IPT System with Variable Coupling

Juan M. Arteaga, George Kkelis, Samer Aldhaher, David C. Yates and Paul D. Mitcheson

Department of Electrical and Electronic Engineering

Imperial College London

j.arteaga-saenz15@imperial.ac.uk

Abstract—This paper presents the analysis and design of a dynamic inductive power transfer (IPT) system, in which coupling is treated as a stochastic variable and is therefore modelled as a probability distribution. The purpose of this formulation is to optimise the tuning of the inverter and the rectifier to the coupling value that achieves the highest charging energy-efficiency when operating at a broad range of coupling. The analysis is supported by a case study in which two rectifier designs, using the hybrid Class E topology, are tuned at different coupling values in order to verify which version achieves the highest charging efficiency. The load in the experiments is a wirelessly powered drone without a battery hovering randomly over the charging pad, and the range of motion is set by a nylon string tether. The experiments show lower energy consumption when the rectifier is tuned to present the optimal load of the link at the coupling value with the highest probability, as opposed to the first, which was designed to present the optimal load of the link at minimum coupling.

Fig. 1. Photograph of the wirelessly powered drone. A video can be seen at https://youtu.be/GyOoOeNrbfM

I. INTRODUCTION

Wireless power transfer (WPT) systems present the possibility of powering electrical loads when it is impossible or inconvenient to use a wired connection. Powering devices in motion is one of the most promising capabilities of WPT, and specifically IPT, since this technology grants a significant increase in motion independence to the device when it is charging and unplugged. Additionally, IPT allows dismissing power connectors at the link between the source and the device, which is beneficial because power connectors are known to be a significant problem in cost and reliability, especially when the device being charged is not stock-still.

The technical challenges of wirelessly feeding mobile devices using IPT, entails coping with changes in the relative position of the transmitting source and receiving-end. There are two options to address this issue: with coil design, as done for example in [1]–[3], where the magnetic flux of the link is shaped with the purpose of minimising the variability in coupling as the coils move, or by designing the circuits to minimise changes in end-to-end efficiency and output power with changes in coupling. The latter option was investigated for multi-MHz IPT systems in our previous work, [4], where the inherent regulation properties of resonant converters was exploited to achieve high efficiency for a broad range in coupling: 88-75 % dc-dc efficiency at 11-6 % coupling for a system operating at 6.78 MHz with no power throughput control.
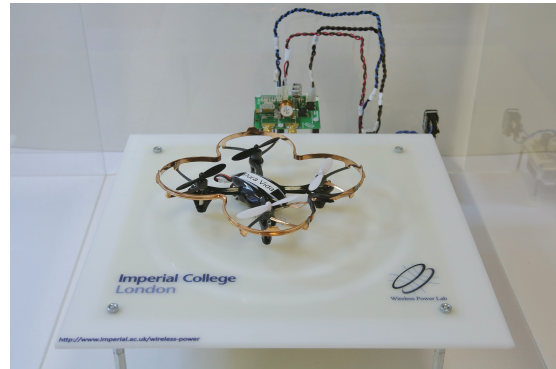
Multi-MHz IPT systems have also been utilised in applications that require regulated output power notwithstanding variable coupling. The case study of [5] presented a drone without a battery, for the first time hovering over the charging pad while receiving all the required power from a 13.56 MHz IPT source. A second version of that application was presented in [6], which tolerated a larger variance in coupling and power demand by integrating a synchronous Class E rectifier in the receiving-end. The system presented in [6] (Fig. 1) is capable of operating from the critical point of maximum power at minimum coupling, up to the point of maximum coupling and minimum power, which is when the drone is sitting on top of the charging pad and the propellers are not spinning. This broad range of operation was achieved by solely exploiting advantageous properties of Class E rectifiers for IPT [7] since no control loop in the transmitting-end was included to regulate the intensity of the magnetic field generated by the transmitting coil.

This paper proposes a design methodology to optimise the end-to-end efficiency of a multi-MHz IPT system with regulated output power and operating at highly variable coupling. Coupling is first delimited considering the spatial freedom of the receiving-end coil in two dimensions, distance and linear misalignment, and within this range, coupling is then characterised as a probability-distribution, which accounts for the dynamic behaviour of the coils obtained from statistical analysis. This distribution of coupling in time is used to find the value of coupling associated with maximum energy

throughput, to therefore tune the circuits accordingly.

the experimental work presented here shows an IPT system capable of powering a small drone without a battery for a wide range of motion. Initially, the receiving-end circuit, comprised by the rectifier and a dc-dc converter, was designed for maximum link efficiency at minimum coupling, and later redesigned taking into account the coupling distribution in time. This distribution was found by monitoring the alignment and relative distance of the coils for one minute using two synchronised video cameras. Two different hybrid Class E rectifiers capable of operating at the entire range of coupling and power were designed, built and compared.

II. DESIGN CONSIDERATIONS FOR IPT SYSTEMS WITH VARIABLE COUPLING

The coupling factor (k) of a two-coil IPT system is defined as the mutual inductance (M_{ps}) divided by the square root of the product of the self-inductance of the IPT coils (L_p and L_s):

$$k = \frac{M_{ps}}{\sqrt{L_p L_s}}. \qquad (1)$$

As the coils move, relative to one another, the mutual inductance, and henceforth k, are subject to changes resulting in large output power variations (if the system has no power-throughput control), and typically in significant degradation of the efficiency [8].

A. Power Regulation at Variable Coupling

Power throughput of a multi-MHz IPT system can be controlled in various ways. An alternative that avoids affecting the resonant frequency of the circuits at both ends, is to either adjust the amplitude of the current in the transmitting coil or change the load resistance as investigated in [4] (if parallel tuned, the resonant frequency of the receiving-end slightly changes with load). Otherwise, the system can be designed with a feature that allows diverting the operating frequency of the inverter from the resonant frequency of the IPT transmitter and receiver, and therefore detune the resonant circuits to diminish the system's power throughput [7], [9]. These techniques, however, can account for significant losses when operating away from resonance due to excessive currents circulating in the resonators, and possible non-optimal operation of the resonant power converters.

In this work we propose an IPT system with a regulated output voltage by integrating a dc-dc converter as the final power conversion stage. The system diagram is shown in Fig. 2. The only control loop of this system is that of the dc-dc converter that regulates the output voltage. Therefore, if the IPT link is modelled as the equivalent circuit shown in Fig. 3, the effect of output voltage regulation by the dc-dc converter is represented in the input resistance of the rectifier (R_{ac}). The effects of voltage regulation can also account for changes in the input reactance of the rectifier, producing a variation in the value of C_s in Fig. 3, which could detune the receiving-end circuit. The purpose of the controlled block is
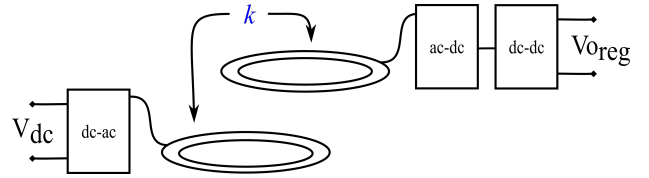

Fig. 2. Diagram of the designed dynamic IPT system.
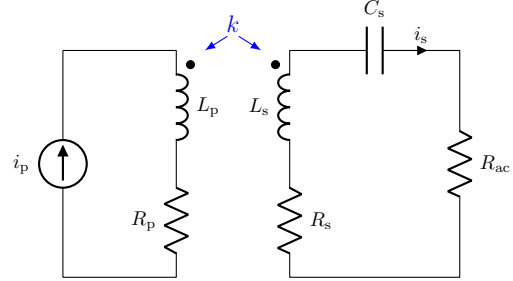

Fig. 3. Equivalent circuit model of an IPT system with a series-tuned secondary.

to compensate for changes either in the load or in the induced electromotive force (emf) at the receiving coil.

The variations in k and R_{ac} affect the efficiency of the IPT link. It is well known that for a given IPT link, there is a value of R_{ac} that achieves the maximum link efficiency ($R_{ac_{opt}}$) for a given k [10]. When the value of R_{ac} differs from its optimal value, the losses in the equivalent-series-resistance of the coils (R_p and R_s in Fig. 3) increase and therefore degrade the end-to-end efficiency of the system. Designing for $R_{ac_{opt}}$ is not always practical, especially if k changes with time ($R_{ac_{opt}}$ is a function of k), because R_{ac} is one of the parameters that dictate how much power is delivered to the load. The system that we propose varies R_{ac} as k changes not to achieve the optimal load but to regulate the output power. Therefore, the system only operates with an optimal value of R_{ac} at a certain k and output power, and interestingly some combinations of rectifier and inverter topologies, as the one presented here, allow designing such condition at a chosen combination of k and P_o.

B. The Coil Driver: A Load-Independent Class EF Inverter

The load-independent Class EF inverter [11] uses a tuning methodology that achieves zero voltage switching (ZVS) and constant amplitude output current for the entire load range. These characteristics can be exploited in multi-MHz IPT systems operating at variable k because the load independent feature implies that the inverter can operate efficiently independent of k or the amount of power required by the load, as long as it is within the range of output power of the inverter.

The output current amplitude of the inverter is proportional to the input voltage and independent of the load value. Therefore, the amplitude of the current in the transmitting coil can be determined by setting the input voltage. This could also serve as an alternative method for power regulation in multi-
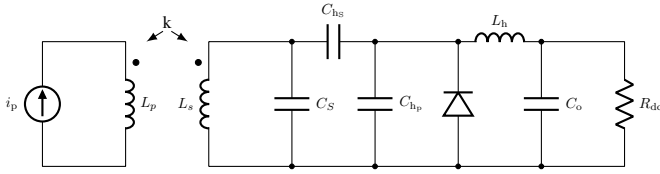
Fig. 4. Hybrid Class E rectifier schematic.

MHz IPT systems, as good results at different input voltages have been found for the same design using this topology [12].

C. The Rectifier: A Non-Synchronous Hybrid Class E

The receiving-end of the system consists of a hybrid Class E rectifier [7] and a dc-dc converter to regulate the output voltage. The hybrid Class E topology is shown in Fig. 4.

Class E rectifiers in IPT systems can be designed at a resonant frequency different than the frequency of operation ($A_r \neq 1$), given the possibility of compensating the residual reactance of the rectifier in the resonant tank of the receiving coil. This feature, investigated in [7], grants various degrees of freedom in the design, regarding among other variables, the input impedance of the rectifier and its deviation with dc-load. These rectifiers can therefore be tuned to achieve a certain input impedance for a combination of coupling and output power by solely designing the passive components of the topology. The hybrid Class E rectifier grants yet another degree of freedom, which is used in this design to better delimit the output voltage range against coupling and therefore avoid surpassing the maximum or minimum voltage at the input of the dc-dc converter.

The combination of the Class E rectifier and the dc-dc converter allows designing the maximum efficiency point of the system for a particular combination of k and P_o by selecting the reflected impedance of the receiving-end to be completely resistive (i.e. the secondary side being perfectly tuned at the frequency of operation) and at the same time matching the input resistance of the rectifier to the optimal load at that particular combination of k and P_o.

III. USING A PROBABILITY DISTRIBUTION TO OPTIMISE FOR VARIABLE COUPLING

Applying probability theory to optimise a dynamic IPT system for variable k was first proposed in [4]. The formulation is based on describing the output power of an IPT system as the product of the power consumed by the resistance reflected by the secondary side on the primary side (R_{eq}) and the receiving-end efficiency (η_r):

$$P_o = i_{Prms}^2 R_{eq} \eta_r. \tag{2}$$

The reflected resistance is a function of k and R_{ac}, and its exact formulation depends on the resonating method used at both ends of the IPT system [10].

A. Calculating Transferred Energy

The energy delivered to the load in a time interval, t_0 to t_f, can be derived from (2) as:

$$E_o\Big|_{t_0}^{t_f} = \int_{t_0}^{t_f} P_o(k, R_{ac}, i_{Prms})\, \mathrm{d}t, \tag{3}$$

re (3) as:

$$E_o\Big|_{t_0}^{t_f} = (t_f - t_0)\int_{k_{min}}^{k_{max}} P_o(k)\, f(k)\, \mathrm{d}k, \tag{4}$$

where $f(k)$ is the probability-distribution of k in the defined time interval. The energy-efficiency can therefore be calculated as:

$$\eta_{avg} = \frac{E_o}{E_i} = \frac{\displaystyle\int_{k_{min}}^{k_{max}} P_o(k)\, f(k)\, \mathrm{d}k}{\displaystyle\int_{k_{min}}^{k_{max}} P_i(k)\, f(k)\, \mathrm{d}k}, \tag{5}$$

where $P_i(k)$ describes the input power of the IPT system as a function of k. It should be noted that this model is useful when the output power is constant or is defined as a function or a distribution of k.

B. Obtaining a Probability Distribution From Observation

Analysing the distribution of k in time is advantageous because it allows characterising the efficiency of a system considering its spatial freedom. It also allows finding the highest energy-throughput point (maximum in $P_o(k)f(k)$) and therefore optimise the circuits at both ends to this value of k. The range of motion (k_{min} to k_{max}) can also be selected when analysing an IPT system with variable k since the end-to-end efficiency of the system can be calculated as a function of k and then weighted against $P_o(k)$.

The probability distribution of k can be obtained with analysis (e.g. a vehicle moving over a charging pad at constant velocity) or as a probability-mass-function by sampling the relative position of the coils throughout the charging time.

IV. CASE STUDY: OPTIMISING ENERGY-EFFICIENCY OF A WIRELESSLY POWERED DRONE

The drone shown in Fig. 1 can operate at any position within its range of motion, set by an $7.5\,\mathrm{cm}$ nylon string tether connected between the bottom of the drone and the centre of the transmitting coil. The drone's energy source is a $13.56\,\mathrm{MHz}$ IPT system comprised of a load-independent Class EF inverter at the transmitting-end, a hybrid Class E rectifier at the receiving-end and a dc-dc converter (LMZ14203 by Texas Instruments) to regulate the output voltage. The battery of the drone was removed, and therefore the energy storage system in the drone consists of solely two $100\,\mu\mathrm{H}$ ceramic capacitor at the output of the dc-dc converter. The average power consumption of the drone at full power was measured at $13\,\mathrm{W}$ with a feeding voltage of $4.3\,\mathrm{V}$.

The case study we here present investigates optimising the tuning of a resonant rectifier using the probability distribution of k, obtained by measuring the relative position of the coils over time.

A. Initial Design of the Rectifier for Variable Coupling

The initial challenges for designing the rectifier are: being able operate efficiently at a highly variable coupling factor (5 to 20 %) and a broad range in power (from maximum to zero power at any coupling), without leaving the operational range of input voltage of the dc-dc converter that follows the rectifier. In addition, the reflected impedance to the transmitting-end cannot have a significant reactance because that would detune the inverter.

These specifications were achieved by selecting, from the possible designs proposed in [7], a hybrid Class E rectifier with a resonant frequency lower than the frequency of operation. The value for A_r chosen was 0.3, which might vary slightly depending on the available commercial values for the inductance L_h.

The rectifier was designed so that the highest link efficiency was met at minimum coupling (5 %) and maximum power, considering as an assumption that the drone would mostly operate at the furthest distance from the charging pad.

B. Obtaining the Probability Distribution of a Wirelessly Powered Drone from Observation

Once the initial design is built and operating, statistical data can be gathered on the relative position of the coils, and then characterise the dynamic behaviour of the system to formulate coupling as a probability distribution.

The relative position of the coils was monitored for one minute (3600 samples) using two video cameras, one at the top and one at the side of the IPT system under test; this to measure the linear misalignment and separation of the coils. One synchronised frame from both cameras is shown in Fig. 5.

The coupling factor at each position within the range of motion of the system was calculated from simulation, and several points including the maximum and minimum coupling were verified experimentally using a Keysight E4990A impedance analyser. The distribution of k was obtained by recording the position of the drone and performing a probability mass function transformation from linear misalignment and distance to coupling.

The probability distribution is shown as a histogram in Fig. 6. The histogram was constructed with equal size bins of 0.5 % coupling, which for this case study was determined to be the minimum step change that affects the optimum rectifier design using commercial off-the-shelf components.

C. Optimised Design of the Rectifier

The histogram in Fig. 6 points out that the highest probability of coupling occurs between 8 and 10 % coupling. Therefore, the initial design that assumes that the most probable value of k was 5 %, operates with an input resistance of the rectifier different than the optimal load of the IPT link most of the time. Furthermore, most of the energy dissipated by the load in the one minute experiment was transferred wirelessly in suboptimal tuning of the IPT link.
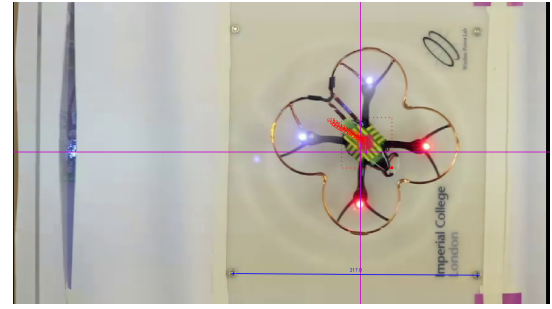
(a) Bird's eye view.

(b) Lateral view.

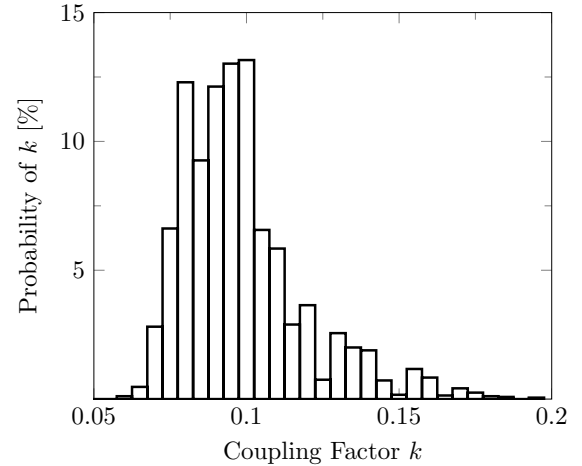Fig. 5. Capture of a key frame for position tracking of the drone.

Fig. 6. Histogram of coupling obtained from 3600 samples in one minute.

An improved version of the first rectifier, which also includes the same capabilities as the first, was built with the values of the passives as shown in Table I, in order to tune the input resistance of the rectifier to meet the optimal load at the value of coupling with the highest probability, as opposed to the first design, which was tuned to achieve the optimal load at minimum coupling.

The two designs were compared by measuring the energy consumption at the dc source of the transmitting-end. The results are shown in Table II.

TABLE I
DESIGN PARAMETERS AND COMPONENTS VALUES OF THE RECTIFIERS

Component	Design 1	Design 2	Description
k_{opt} (%)	5	10	
A_{r}	0.3	0.3	
L_{s} (nH)	360	360	
C_{p} (pF)	62	54	Vishay QUAD HIFREQ
C_{hs} (pF)	343	420	Vishay QUAD HIFREQ
C_{hp} (pF)	$16+C_{\mathrm{oss}}$	$343+C_{\mathrm{oss}}$	Vishay QUAD HIFREQ
L_{h} (μH)	3.3	2.2	Coilcraft XAL40xx Series
C_{o} (μF)	20	20	AVX X7R
D		2x Cree C3D1P7060Q	

TABLE II
AVERAGE POWER CONSUMPTION AT THE SOURCE WITH THE DIFFERENT
RECTIFIER DESIGNS

Inverter input voltage	No load	Initial Design	Optimised Design
74 V	6.6 W	26.67 W	21.78 W

V. CONCLUSIONS

This paper proposes a methodology to improve the end-to-end efficiency of an IPT system with variable coupling, by modelling the dynamic behaviour of the link as a probability distribution. Experimental results show a significant decrease in the energy consumption at the source of the system when the rectifier is tuned to achieve the highest link efficiency at the most probable value of coupling, obtained from statistical analysis.

ACKNOWLEDGEMENT

REFERENCES

[1] J. M. Miller, O. C. Onar, C. White, S. Campbell, C. Coomer, L. Seiber, R. Sepe, and A. Steyerl, "Demonstrating dynamic wireless charging of an electric vehicle: The benefit of electrochemical capacitor smoothing," *IEEE Power Electronics Magazine*, vol. 1, no. 1, pp. 12–24, March 2014.

[2] A. Zaheer, M. Neath, H. Z. Z. Beh, and G. A. Covic, "A dynamic ev charging system for slow moving traffic applications," *IEEE Trans. Transport. Electrific.*, vol. PP, no. 99, pp. 1–1, 2016.

[3] A. Pacini, A. Costanzo, S. Aldhaher, and P. D. Mitcheson, "Load- and position-independent moving MHz WPT system based on GaN-distributed current sources," *IEEE Trans. Microw. Theory Tech.*, vol. 65, no. 12, pp. 5367–5376, Dec 2017.

[4] J. M. Arteaga, S. Aldhaher, G. Kkelis, D. C. Yates, and P. D. Mitcheson, "Multi-MHz IPT systems for variable coupling," *IEEE Trans. Power Electron.*, vol. PP, no. 99, pp. 1–1, 2017.

[5] S. Aldhaher, P. D. Mitcheson, J. M. Arteaga, G. Kkelis, and D. C. Yates, "Light-weight wireless power transfer for mid-air charging of drones," in *11th European Conf. on Antennas and Propagation (EUCAP)*, March 2017, pp. 1–5.

[6] G. Kkelis, S. Aldhaher, J. M. Arteaga, D. C. Yates, and P. D. Mitcheson, "Hybrid class-e synchronous rectifier for wireless powering of quad-copters," in *IEEE Wireless Power Transfer Conf. (WPTC)*, May 2017, pp. 1–4.

[7] G. Kkelis, D. C. Yates, and P. D. Mitcheson, "Class-E half-wave zero dv/dt rectifiers for inductive power transfer," *IEEE Trans. Power Electron.*, vol. PP, no. 99, pp. 1–1, 2017.

[8] M. Pinuela, D. C. Yates, S. Lucyszyn, and P. D. Mitcheson, "Maximizing DC-to-load efficiency for inductive power transfer," *IEEE Trans. Power Electron.*, vol. 28, no. 5, pp. 2437–2447, May 2013.

[9] S. Aldhaher, P. C. K. Luk, and J. F. Whidborne, "Tuning Class E inverters applied in inductive links using saturable reactors," *IEEE Trans. on Power Electron.*, vol. 29, no. 6, pp. 2969–2978, June 2014.

[10] K. Van Schuylenbergh and R. Puers, *Inductive powering: Basic theory and application to biomedical systems*. Springer Science & Business Media, 2009.

[11] S. Aldhaher, D. C. Yates, and P. D. Mitcheson, "Load-independent class E/EF inverters and rectifiers for MHz-switching applications," *IEEE Trans. on Power Electron.*, pp. 1–1, 2018.

[12] J. M. Arteaga, S. Aldhaher, G. Kkelis, D. C. Yates, and P. D. Mitcheson, "Design of a 13.56 MHz IPT system optimised for dynamic wireless charging environments," in *2016 IEEE 2nd Annual Southern Power Electronics Conference (SPEC)*, Dec 2016, pp. 1–6.