

Rational homotopy type of the component of inclusion in the nthspace of continuous mappings from $Gr(k, n)$ to $Gr(k, n + 1)$

Otieno P.A, J.B Gatsinzi and Onyango-Otieno V

A complex manifold can be embedded in some complex projective space $CP(N)$, in particular, the Grassmann manifold $Gr(n, k)$ of k dimensional subspaces in C^n can be embedded in some complex projective space $CP(N)$. Moreover $G(k, n) \hookrightarrow G(k, n + 1)$. For $k = 1$, we get a one dimensional vector space which is the complex projective plane and is an embedding of $CP(n)$ in $CP(n + 1)$. The Grassmanian admits a CW structure and any CW structure on a space provides a filtration relative to the empty space. To a simply connected topological space, Sullivan associates a commutative differential graded algebra $(\wedge V, d)$ which encodes the rational homotopy type of X . This is called a Sullivan model of X . Given that $H^*(CP(n), Q)$ is the truncated polynomial algebra $\wedge x/(x^{n+1})$, one gets a Sullivan model of the form $(\wedge(x, y), d)$ where $|x| = 2$, $|y| = 2n + 1$ and $dx = 0$, $dy = x^{n+1}$. For $k \geq 1$, one might use the homeomorphism $G(k, n) = U(n)/(U(k) \times U(n - k))$ to find a Sullivan model. In this paper, we use a Sullivan model of the inclusion $Gr(k, n) \hookrightarrow Gr(k, n + 1)$ to compute the rational homotopy type of the component of the inclusion in the space of mappings from $Gr(k, n)$ to $Gr(k, n + 1)$.

Key words: Grassmann manifold, Commutative differential graded algebra, Sullivan model