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Development of a Dynamic Water Management Policy for Texas

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Texas A&M University

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DEVELOPMENT OF A DYNAMIC WATER MANAGEMENT POLICY FOR TEXAS

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PREFACE

The purpose of this investigation was to develop techniques to assist water planners in the optimum implementation of their plans. Specifically, techniques useful for continual evaluation of water plans and for scheduling the sequence and timing of needed project additions were sought.

Considerable effort has been directed toward developing water plans for meeting long range needs for water both in Texas and around the United States. These long range water plans have consisted of fixed systems of development projects deemed necessary at some future point in time - usually 50 years. Plans such as these are considered flexible guides to serve as water development goals. However, water planners also need implementation plans which specify the sequence and timing of construction for specific projects which are a part of the long range water plan. Previous researchers have studied the application of operations research and optimization techniques in the planning of fixed systems to meet water needs at specific times in the future. This investigation seeks the development of optimization techniques which can be used by planners and developing implementation plans.

In this research, techniques were studied to determine those which best met the research objectives. A stochastic programming formulation for obtaining an operating policy for single, multi-purpose reservoirs based on the continuity equation, stochastic inflow and demand, and chance constraints was developed. The chance constraints were converted to an equivalent linear programming problem. This formulation was then extended to a linked system of multi-purpose reservoirs. Both linear and quadratic objective functions were used with the equivalent linear constraint set.

The problem facing water resource planners during implementation of water plans was then addressed. The objective used in this problem was to select reservoir storage capacities, schedule a time for construction, and establish an operating policy such that the total cost of the linked reservoir system is minimized. In solving this problem which is in fact a mixed integer-continuous linear programming problem, an analyst defines the feasible reservoir segments in storage capacity for each time period in which expansion is possible. The resulting problem size and general structure lend themselves well to the use of a specialized decomposition technique. Use of this decomposition technique permits the problem to be separated into a linear programming problem and an integer programming problem. This approach makes the problem more computationally tractible.

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CHAPTER I

INTRODUCTION

Water resource planners face the problem of developing and evaluating alternative water resource systems at different points in time. The evaluation ultimately leads to an investment decision. The problem for the analyst is to investigate how the relevant variables affect the investment decision. The analyst is also faced with many types of uncertainties, two of which are economic uncertainty and hydrologic uncertainty.

Economic uncertainty refers to the fact that the water resource analysts often do not know the relevant benefit and loss functions necessary to evaluate the performance of various system designs and operating policies. Since water resource projects are durable, their time horizon can extend into the distant future. The benefit function will probably shift over time and it is usually impossible to determine exactly how benefit functions may effect future planning.

The dynamic variations of the benefit functions lead to questions concerning the time phasing of investment planning in water resource development. As demand grows at a steady rate over time, capacity expansion models must be used to determine an optimal investment program. Since investment programs are usually legislated, capacity expansion can only occur at finite points in time. With capacity fixed for certain time frames and demand growing almost continuously the optimal operating procedure of a reservoir system is quite dynamic.

A methodology is proposed for the analysis of time phasing of reservoir system operation with capacity expansion. The objective is to select reservoir capacities, construction timing and to establish an operating policy such that the total cost associated with a system of linked reservoirs is minimized. The formulation of this procedure is a mixed integer-continuous linear programming problem. The analyst defines the feasible reservoir segments and capacity for each time period where expanion is possible. Due to the size of the resulting problem and its general structure Benders' decomposition technique (Lasdon, 55) is applied.

Benders' method allows for the problem to be separated into a pure linear program and an almost pure integer program. The integer program is necessary due to capacity segments being added at fixed time intervals. Once the capacity segment is added a new reservoir system must be solved.

Each reservoir system has associated with its operating policy hydrologic uncertainty. The hydrologic uncertainties dealt with are the stochastic nature of streamflow and demand. The stochastic variables must be considered in order to develop models of reservoirs and reservoir systems. Since the performance of a reservoir or system of reservoirs cannot be predicted with absolute certainty chance-constraints are developed which will allow the analyst to predict with some degree of certainty a set of operating rules.

Two types of chance-constraints are presented. The first constraint is for the probability of the storage level in a reservoir not to exceed the maximum capacity of the reservoir. The second constraint is that the storage in the reservoir must exceed the minimum pool level by an allowable probability. Both constraints are based on the current ending storage or inventory level. The chance-constraints are converted to an equivalent linear deterministic set of constraints by a material balance equation. The linear constraints are used along with maximum and minimum downstream release constraints to form a linear system of time related release and pumping variables.

A continuity or material balance equation is proposed which relates the current ending storage volume to the previous time periods storage volume. The ending storage volume also considers input of random inflow, possible pumping and upstream reservoir release flow. The release or loss of inventory is due to demand, downstream release, and pumping to other reservoirs. The material balance equation allows the analyst to investigate many types of objective functions associated with a reservoir or system of reservoirs.

Bicriteria objective functions associated with the stochastic reservoir models are considered. This analysis includes additional variations to the cost function associated with certain decision variables. Application of postoptimal objective function analysis is applied to both linear and quadratic cost functions. From the resulting set of optimal solutions the analyst can select the solution which best meets his budget requirements. He is also able to eliminate the variables which will produce little or no return for additional expenditures.

As hydrologic uncertainty or the stochastic nature of streamflows is of essential consideration for developing models of water resource systems, a model is developed from the Cypress Creek Basin. An example problem is formulated based on a set of stochastic constraints.

Two types of constraints are considered for this model. The first is the uncertainty associated with meeting a downstream demand. A scheduled release is placed on each reservoir with flow into the downstream reservoir. The downstream reservoir must meet a fixed demand a certain percentage of the time. Associated with the scheduled release for each reservoir are stochastic variables which represent the actual flow supplying water to the downstream reservoir.

The second constraint imposed on this model is that the maximum capacity of the downstream reservoir must not be violated by more than a specified percent. The constraint set is converted to a linear deterministic equivalent set. This results in a linear constraint set which allows for different types of objective functions to be appended. Two techniques are investigated based on a linear objective function. The two techniques are parametric programming and linear programming and contraction mapping.

A quadratic objective function is then formulated for the linear constraint set. The objective function presented represents the cost deviation from the target releases. Two techniques similar to the linear routines are presented. The techniques are parametric quadratic programming and quadratic programming and contraction mapping.

CHAPTER II

A REVIEW OF THE APPLICABLE LITERATURE

Water resource planners have attempted to adapt techniques such as simulation, linear programming, dynamic programming, stochastic linear and dynamic programming, network analysis, queuing theory, inventory theory and combinations of these techniques to the planning of water resources systems. This review will consider past efforts in each of these areas and indicate further research needed.

Simulation

Pioneering Work

Pioneering work by Morrice and Allen (78) and Huffschmidt and Fiering (43) of the Harvard Water Program have made simulation the most nearly operational technique. The remaining research using simulation involves principally how to make the most effective use of the technique in an actual planning situation. One of the major disadvantages of simulation still remains in the large amount of computer time necessary to obtain a solution to the more complex models. In addition, even with systematic sampling of the response surface, it is very difficult to make statements concerning the optimality of the simulation results. Blanchard (10) has shown that linear programming models could be used to reduce the number and range of variables to be used in a simulation model. The concept of combining optimization techniques with simulation appears to have merit for further research.

Texas Water Development Board

The Texas Water Development Board (91) has designed, for the Texas Water Plan (90), deterministic simulation and optimization techniques. These techniques can be used by a planner to find the minimum cost physical system and operational criteria for satisfying fixed water demands with a single set of prespecified hydrologic conditions.

Report 131 (92) presents a water resource planning methodology which systematically and simultaneously relates planning variables in mathematical models to simulate and optimize over time the operation of a network of storage reservoirs, pump-canals, and river reaches in a multi-basin water resource system.

The objective function of the mathematical model in the report is formulated so as to permit optimization of a network configuration by finding a set of storage reservoirs and pump-canals that will permit a prespecified level of annual water production at least cost. The report defines the problem in such a way that the future time series of water demands can be brought to bear in the consideration of current and future alternative investments to supply the quantities demanded. Initial investment costs, operation and maintenance costs, and the possibilities of substituting investments in storage facilities at some point for costs of pumping water to another point are considered.

The procedures and methodologies presented could be further developed and refined so that they can be applied to additional water resource planning problems. More attention could have been given to defining problems where systems analysis and optimization techniques are more effectively applied in the planning and design of real systems to identify needed modeling improvements. Also needed is the incorporation of multi-level optimization and simulation procedures which would permit varying computational precision from preliminary to detailed project planning.

Mathematical Models

Linear Programming and Stochastic Design Models

Linear programming has been utilized by a number of specialists in water resources planning. Dorfman (22) considers a river basin system consisting of two possible reservoir sites, an irrigation project and a run-of-the-river power plant. Dorfman indicates that a method for piecewise linearization of the objective function is often necessary to apply linear programming. In this manner, nonlinear problems subject to linear constraints can often be efficiently solved. Although linear programming models are sometimes criticized for their abstraction of nonlinear problems, the fact remains that the simplex due to Dantzig (19) is one of the most efficient programming algorithms and should be used as a preliminary screening tool.

Another interesting application of linear programming is found in a discussion by Ramaseshan (81), which is a response to an article by Hall (35). In this paper the linear programming model takes into account monthly variation of inflow and outflow, a curvilinear cost function for canal and reservoir, and a convex benefit function given as follows:

$$B = \text{maximize} \sum_{k=1}^{3} \sum_{j=1}^{3} \sum_{i=1}^{16} C_{kji}\, q_{kji} - C_r - Q_Q$$

subject to:

discharges less than canal capacity

$$\left.\begin{aligned} \sum_{k=1}^{3} \sum_{j=1}^{3} q_{kji} &\leq Q \\ \sum_{k=1}^{2} \sum_{j=1}^{3} q_{kji} &\leq Q' \\ \sum_{j=1}^{3} q_{1ji} &\leq Q'' \end{aligned}\right\} \quad i = 1,2,\ldots,12$$

discharges have upper and lower limits

$$0 \leq q_{k1i} \leq Q_{k1i} \qquad j = 1$$

$$Q_{kji} \leq q_{kji} \leq Q'_{kji}$$

total outflow less than total inflow

$$\sum_{k=1}^{2} \sum_{j=1}^{3} \sum_{i=1}^{i=i} q_{kji} \leq \sum_{i=1}^{i=i} Q_i \qquad \begin{array}{l} i = 1,2,\ldots,11 \\ j = 1,2,3 \\ k = 1,2,3 \end{array}$$

net storage less than V

$$\left.\begin{array}{l} \sum_{i=1}^{i=i} Q_i - \sum_{k=1}^{3} \sum_{j=1}^{3} \sum_{i=1}^{i=i} q_{kji} - V \\ \\ \sum_{i=1}^{12} Q_i = \sum_{k=1}^{3} \sum_{j=1}^{3} \sum_{i=1}^{12} q_{kji} \end{array}\right\} \quad \begin{array}{l} i = 1,2,\ldots,11 \\ j = 1,2,3 \\ k = 1,2,3 \end{array}$$

where

Q,Q',Q"	-	capacity of aqueduct sections
q_{kji}	-	discharge to region k, in month i, in benefit function range j
V	-	reservoir size
C_{kji}	-	unit benefit of q_{kji}
C_r	-	cost of reservoir of volume V
Q_Q	-	cost of aqueduct section 1,2,3 of capacity Q,Q',Q"
Q_i	-	monthly (average) inflow.

For a given volume V, and assumed values of Q,Q',Q", the author suggests maximizing the net benefits by linear programming.

Values of Q,Q',Q" are then changed to increase the net benefits by some type of sampling scheme. Ramaseshan further suggests that it is possible to find the optimum V by using sensitivity analysis.

This model provides a significant improvement over the initial model given by Hall (35), however the solution procedure suggested is rather cumbersome and the model is only formulated for one reservoir and three aqueduct sections. This model and approach would be intactable if applied to a multiple reservoir and canal system.

Dorfman (22) presents a similar trial and error method to find the optimal reservoir sizes, energy and irrigation outputs for a linear programming model of a hypothetical river basin. In this model two reservoirs, an irrigation district and a power plant are assumed. Reservoir sizes are assumed, and the maximum net benefits are computed from energy and irrigation releases. Sensitivity analysis is then performed to determine whether an increase or decrease in reservoir size is required for the next trial.

Loucks (62) is responsible for a variety of linear programming models with extensions to include the inherent stochastic nature of water resources planning problems. One model considers a single reservoir with downstream users. Uncontrolled streamflow plus initial storage is available for downstream demands. The objective of the model is to determine reservoir capacity, reservoir storage and target drafts that maximize the total annual benefits less annual costs of reservoir construction and losses from deviations of storages and releases from their respective targets. Loucks derives a Markov chain for transition probabilities for summer and winter streamflow. The ergodic nature of the streamflows is then utilized to determine the steady state unconditional probability of each inflow in each season. These probabilities are used to determine the mean summer and winter inflow which are then used in the deterministic linear programming model.

Loucks also structures the same basic model as a stochastic design model assuming an operating policy of

L - the integer portion of (i+k/2)

where

L - the final reservoir volume

i - inflow

k - initial reservoir volume.

The probability of each storage level and reservoir draft is computed and added to the objective function of the basic model. Linear programming is then utilized to determine the optimal design variables for the given operating policy.

A stochastic operating policy model is next presented to determine if any improvement can be made in the operating policy by holding constant the design variables furnished by the solution of the stochastic design model. The objective function of the operating policy model conforms to the probabilistic portion of the design model objective function with the addition of one subscript. The constraints are formulated from the equations used to compute the probability of an initial storage k in period t, PS_{kt}, and the probability of a draft D_{kit} given an initial storage k, and inflow i in period t. The solution of this model specifies the steady state probabilities PD_{kit}, PS_{kt}, and P_{kiLt}, the joint probability of an initial volume k, inflow i and final volume L in period t. The joint probabilities P_{kiLt} enable one to calculate the conditional probabilities of a final volume L given an initial volume k and inflow i by the following relation:

$$\text{Prob } \{ L|k,i,t \} = \frac{P_{kiLt}}{\sum_{L} P_{kiLt}} \quad \forall \quad k,i,L,t.$$

Loucks points out that these are usually 0, 1 or pure policies in which one L is specified for a given k and i.

The design model is then resolved incorporating the revised probabilities PS_{kt} and PD_{kit} and the new operating policy in the continuity equations. If an improved design results, the operating policy model is again resolved to determine if a better operating policy exists. This two stage process is contunued until no improvement can be made. Loucks indicates that in multi-reservoir problems the policy portion of the models could become too large. He suggests that an examination of the dual variables of the continuity constraints would indicate the possibility of policy improvement.

Although the two models presented indicate possible approaches to the stochastic nature of the problem, an attempt must still be made to extend this theory to include a multi-reservoir system.

Loucks has also defined linear programming models for sequential operating policies where the design variables are assumed known and fixed. The unknowns are the reservoir releases given the initial reservoir volume, inflows, discharges and time periods. He assumes the inflow adequately represents the range of continuous variables and that the distribution of inflows will be the same in any month t regardless of the year.

The objective of this model is expressed as an expected value in the following form:

$$\text{maximize} \sum_{k} \sum_{i} \sum_{d} \sum_{t} B_{kidt} \; P_{kidt}$$

where

B_{kidt} - net benefits in period t resulting from lake levels ranging from k to k+i-d with a discharge of d.

In the event that benefits cannot be formulated, Loucks proposes the following objective:

$$\text{minimize} \sum_{k} \sum_{i} \sum_{d} \sum_{t} \{\alpha_t((k-v_t)^2) + (1-\alpha_t)((d-\delta_t)^2)\} \; P_{kidt}$$

where

v_t - initial t reservoir volume target period

δ_t - discharge target in period t

$0 \leq \alpha \leq 1$ - weighting factor expressing priority of reservoir discharges in period t.

Loucks points out that any linear or nonlinear function of k,i,d, and t can be used to measure the value of initial reservoir volume, inflow and discharge in a given period since the unknowns are the joint probabilities P_{kidt} and remain linear in the objective function. The constraints reflect the fact that these joint probabilities must satisfy certain continuity constraints. Finally, the result of the linear programming solution specifies optimal values of P_{kidt}. These values are then used to compute the conditional probabilities of discharging d given a reservoir state of k, i and t with the following expression:

$$P_t \{ d \mid k,i \} = \frac{P_{kidt}}{\Sigma_d P_{kidt}} \quad \forall \; k,i,d,t.$$

Loucks indicates that usually these probabilities will be either 0 or 1 and define an unambiguous policy. The possibility of mixed policies is alluded to but the general problem is not discussed in detail.

Loucks utilizes linear programming for comparison purposes with dynamic programming. Three models are formulated based on Markovian properties of a first order Markov process. The first model is given by the following equations:

$$\text{maximize} \quad \sum_{t}^{T} \sum_{k} \sum_{i} \sum_{d} f_t \cdot P_{kidt}$$

where

f_t - return from an initial volume k, inflow i and release d in period t

subject to:

$$\sum_{k} \sum_{i} \sum_{d} P_{kidt} = 1 \quad \forall \; t$$

$$P_{kidt} \geq 0 \quad \forall \; k,i,d,t$$

further

$$\text{if} \quad t = T \quad \text{then} \quad t + 1 = 1$$

$$k_{min} \leq K \leq k_{max}$$

$$i_{min} \leq i \leq i_{max}$$

$$\max \{d_{min}, k+i-L_{max}\} \leq d \leq \min \{d_{max}, k+i-L_{min}\}$$

$$\sum_k \sum_i \sum_d P_{kidt} P_{ij}^{(t)} = \sum_e P_{L,j,e,t+1} \qquad \forall L, j, t .$$

$$k + i - d = l$$

The final constraint specifies that the joint probability of a final volume L in period t times the probability of an inflow j in period t+l equals the joint probability of an initial volume L and inflow j in period t+l with a release of e in period t+l. The policy is then derived from the conditional probabilities given by the following calculation:

$$P\{d \mid k, i, t\} = P_{kidt} \Big/ \sum_d P_{kidt}$$

The second model formulated by Loucks is similar to the previous model with the exception that immediate past inflow is used to determine the present policy instead of present inflow. If there was an inflow h in period t-l, the following linear programming model is formulated as,

$$\text{maximize} \quad \sum_t^T \sum_h \sum_k \sum_i \sum_d P_{hi}^{(t)} f_t P_{khdt}$$

subject to:

$$\sum_k \sum_h \sum_d P_{khdt} = 1 \quad \forall\ t$$

$$\sum_k \sum_h \sum_d P_{khdt}\ P_{hi}^{(t)} = \sum_e P_{L,i,e,t+1} \quad \forall\ L,i,t$$

$$k + i - d = L$$

$$P_{khdt} \geq 0 \quad \forall\ k,h,d,t\ .$$

The discharge policy is derived in a similar manner as in the previous model using the following equation:

$$P\{\, d \mid k,h,t \,\} = P_{khdt} \Big/ \sum_d P_{khdt}$$

The third model presented by Loucks is formulated to consider the possibility that the initial reservoir volume and inflow state is transient and therefore without a release policy. The effect of discount rates is also considered in this model. Loucks defines $\lambda = 1/(1+r)$ where r is the interest rate, y as the year with $t = 1,2,\ldots,T$ periods and P_{kil} as the joint probability of a particular volume k and inflow i in period t = 1 of year y = 0. The following expressions are assumed for P_{kil}:

$$0 < P_{ki1} < 1 \qquad \forall\ k,i$$

$$\sum_k \sum_i P_{ki1} = 1$$

If an initial state probability vector whose elements satisfy the above relations is provided and $0 < \lambda \leq 1$, the following model is solved by linear programming:

$$\text{maximize} \quad \sum_{y=0}^{Y} \lambda^y \sum_t^T \sum_k \sum_i \sum_d f_t \; P_{kidt}^{(y)}$$

subject to

$$\sum_d P_{kid1}^{(0)} = P_{ki1} \qquad \forall\ k,i$$

$$\sum_k \sum_i \sum_d P_{kidt}^{(y)} P_{ij}^{(1)} = \sum_e P_{L,j,e,t+1}^{(y)} \qquad \forall\ L,j,t,y$$

where

y = the number of years to reach steady state

t = $T + 1$ in year y = $t = 1$ in year $y + 1$.

Finally, the conditional probabilities for the policy are given by

$$P\{d \mid k,i,t,y\} = P_{kidt}^{(y)} \Big/ \sum_d P_{kidt}^{(y)}$$

Although a number of other linear programming models have been formulated and may be found in a discussion paper by Manne (66), the models mentioned above represent the basic structure of most linear programming models. The majority of previous research efforts has been with single-site reservoirs. The need for more effective methods for evaluating the systems interaction is still needed in water resource planning.

A final example of linear programming points out the fact that this technique may be used to evaluate a system interaction. Sharif (88) formulated the "Backward Elimination Simplex" for the selection and ranking of reservoir sites. This model attempts to provide a means of determining how many reservoirs out of a large number of possible locations should be constructed to minimize system costs. In essence, Sharif computes an inefficiency factor for each site to reflect the amount of unused storage space. This unused storage space is then given a weighted dollar value and the most inefficient site is eliminated. The linear programming solution is then resolved. If the new problem is infeasible or gives a higher cost than the previous solution, the locations from the previous solutions are chosen. Even though the model does not reflect changes in canal and reservoir sizes, the concept appears to have merit and should be given practical consideration.

Dynamic Programming and Stochastic Design Models

Dynamic programming has been applied in a number of cases to water resources problems Buras and Hall (38,37,39) demonstrated the feasibility of dynamic programming as a solution technique for water resource studies. Meier and Beightler (71) extended the use of dynamic programming to water resource planning to include multi-stage branching systems. Meier (72) further made nonserial dynamic programming computationally feasible for water resource planning problems through the development of branch compression.

Falkson (23) extended the work of Thomas and Watermeyer (96) to include the stochastic nature of inflows utilizing Howard's (42) combination of dynamic programming and Markov processes. A discussion of every application of dynamic programming to water resource planning problems would be prohibitive, however, the close consideration of a selection of the more salient contributions is in order.

Hall (39) presented a dynamic programming model to determine the optimum aqueduct capacity for one reservoir and an aqueduct of n reaches. Hall avoided the seasonal effect on canal capacity, the capacity limitations on the reservoir, the relation between cost and capacity of the reservoir and the seasonal distribution of inflows into the reservoir. The previously discussed paper by Ramaseshan attempted to solve these deficiencies, however, the trial and error solution presented raises questions as to its practicality in a system context.

Hall (36) has also formulated a possible model to evaluate optimum aqueduct routes by dividing the problem into three subproblems. The first subproblem is the division of the general route into polyhedron surfaces in order to locate network nodes. The second subproblem involves lift and reach cost calculations in order to find the minimum link costs. Finally, dynamic programming is used to determine the most economic routes connecting any two points in the network. Both Buras (11) and Deininger (21) have suggested possible improvements in the model. Buras (11) suggests pumping directly from the lower to the upper source rather than a combination of lifts and reaches. Deininger (21) points out that Hall only considers one source and one sink, while the problem might require the transfer of water from several sources to several sinks as in a transshipment problem.

Schweig and Cole (87) used dynamic programming to determine optimal releases and transfers from two linked reservoirs with the constraint of transfers only from the larger to the smaller reservoir. The state variable was the contents of the reservoir at the beginning and end of each month.

Keckler and Larson (47) use dynamic programming to investigate four types of water resource planning problems. First, the optimum short term operation of a combined pumped hydro and irrigation storage facility for two reservoirs was solved using forward dynamic programming. In their example reservoir storage was the state variable and pumping rate the decision variable. Second, the short term operation of a multi-purpose four reservoir system with power generation, irrigation, flood control and recreation was solved using successive approximations. In this effort the decision variable was the amount released from the ith reservoir over the kth time interval with reservoir level as the state variable and each stage taken to be a time interval. Inflows were assumed constant over a day. Third, iteration in the policy space was applied to the optimal management of a single reservoir over a one year period with stochastic variation of inputs. Finally, forward dynamic programming was used to consider the optimal planning of adding additional reservoirs to a system over a 30 year period.

Loucks (62) has formulated three dynamic programming models to serve as a comparison for the three previously discussed linear programming models. The recursive optimization for the first model is given by the following equation:

$$V_{kit}^{(n)} = \max_{d} \left(f_t + \sum_{j} P_{ij}^{(t)} V_{kit} - d,j,t+1\right) \quad \forall \; k,i,t$$

where

f_t - return from initial volume k, inflow i and release d in period t.

$P_{ij}^{(t)}$ - transition probability of system states

$V_{kit}^{(n)}$ - the total expected return in n remaining periods beginning in state (k,i) in period t.

This model assumes knowledge of net inflows i during the same period in which release decisions d are made. When the release d in each period t is the same each year for a specific combination of volume and inflow, the solution has reached steady state conditions and further calculations are not necessary.

The second dynamic programming model due to Loucks (62) derives the releases d using initial volumes k and past inflows h. The recursive equations are given by the following relations:

$$n = 1 \quad V_{kht}^{(1)} = \max_{d} \left(\sum_{i} P_{hi}^{(t)} \; f_t \right)$$

$$n = 2 \quad V_{kht}^{(2)} = \max_{d} \left(\sum_{i} P_{hi}^{(t)} \left[f_t + V_{k+1-d,i,t+1}^{(1)} \right] \right)$$

$$\vdots$$

$$n = n \quad V_{kht}^{(n)} = \max_{d} \left(\sum_{i} P_{hi}^{(t)} \left[f_t + V_{k+i-d,i,t+1}^{(n-1)} \right] \right)$$

Louck's third stochastic dynamic programming model extends the first model to include a discounted return. If r is the annual discount rate, the following modification is formulated:

$$V_{kit}^{(n)} = \max_{d} \left(f_t + \lambda_t \sum_j P_{ij}^{(t)} V_{k+i-d,j,t+1}^{(n-1)}\right)$$

where

$$\lambda_t = \begin{cases} 1/(1+r) & \text{if } t = T \\ 1 & \text{otherwise.} \end{cases}$$

Sharif (88) utilized dynamic programming to determine optimal water allocations for both serial and nonserial systems with intermediate inputs. In his model the states were the inputs to and outputs from the reservoirs or stages. The problem was made mathematically tractable by a redefinition of variables. The "pseudo stage principle" developed by Meier (72) was used to convert the nonserial system into a series of equivalent serial problems.

Dynamic programming offers a number of advantages over other techniques which account for its popularity among water resources analysts. Buras (12) has summarized some of these advantages as follows:

(1) Analytically, the method always yields an absolute optimum.

(2) The technique converges to a stationary state solution for stochastic problems.

(3) Constraints given as $a_i \leq x_i \leq b_i$ speed up computations by reducing the state values.

(4) Nonlinear functions pose no theoretical difficulty.

(5) Tabular functional values are easily handled.

(6) Sensitivity information such as the effect of carrying on the process for one more stage is available.

In spite of the fact that water resources planning problems are inherently nonlinear, very little research has been done in the application of nonlinear programming as a planning tool. Young and Pisano (99) have used a gradient method in an effort to find the least cost mix of alternatives to satisfy future water demands within a region. Such alternatives as surface water, ground water, desalination, electrodialysis, and treated waste water are included in their study. The strategy, given water demands and sources, is to develop a network of possible water supplies linked by pipelines which permit the optimum sources of water to be tapped and transmitted to the area.

Queuing and Inventory Theory

Inventory and queuing theory have also been applied to water resource planning problems usually in the context of a single site-reservoirs. Avi-Itzhak and BenTuvia (1) applied inventory theory to find the combination of reservoir size and pump capacity to utilize a certain quantity of water at minimum cost. The assumptions were made that there was no serial correlation, the rate of outflow was constant, the inflow occurred only during the winter and the operation was under steady state conditions. Moran (77) has done extensive research into storage theory and the formidable mathematical problems involved. Langbein (53) and Fiering (26) have both applied queuing theory to single multi-purpose reservoirs, however, pertinent research into queuing theory applications for total water resource systems is still needed. Lloyd and Odoom (59), and Fiering (27) have applied statistical methods to various water storage problems and have done a great deal to clearify the statistical methods available to the water resource planner.

Combined Techniques

The fact that there appears to be no single technique available which solves all the problems has prompted some researchers to combine methods in an iterative format. For example, the use of linear programming and simulation has been previously mentioned. One of the more interesting combinations of techniques has been developed by Hall, Butcher and Esogbue (36). The objective of Hall's method is to find that particular operating policy which will set the firm energy and firm water commitments during a specific "critical period". The problem is formulated as a system with a "master wholesaler" and "individual producer relationship". The individual reservoir returns are optimized using dynamic programming and a set of prices furnished by the "master wholesaler". The individual reservoirs then report their outputs for each time period to the "master". With these outputs as available resources, the "master" applies linear programming to maximize the returns from water and power contracts. As part of this analysis, the "master" utilizes the dual of the linear program to furnish a new set of shadow prices to the individual reservoir operators. This procedure is continued with a new set of shadow prices and the iteration process terminates when the improvement of the solution is negligible.

Hall's method of analysis involved dynamic programming, linear programming, and a modified gradient procedure. The resulting computational time was estimated to be 20 minutes per iteration and a complete computer run was never made because of financial limitations.

CHAPTER III

STOCHASTIC METHODOLOGY FOR RESERVOIR OPERATION

Operations research methodology and systems analysis has in the last decade facilitated the water resource planner in the development of reservoir management techniques. The planner must integrate the many functions of a reservoir or system of reservoirs in order to obtain decision policies.

In a recent paper ReVelle *et al*. (83) proposed a linear decision rule for a single reservoir design and operation. The linear decision rule permitted the structure of a chance-constrained linear programming model for determining the reservoir capacity required to maintain a range of storage volumes and releases during specified time periods.

ReVelle's linear decision rule, as applied to a reservoir, has the simple form

$$x = s - b ,$$

where x is the release during a period of reservoir operation; s is the storage at the end of the previous period; and b is a decision parameter chosen to optimize some criterion function. The linear decision rule was applied in two contexts: (1) the stochastic context where the magnitudes of reservoir inputs are treated as random variables unknown in advance and (2) the deterministic context where the magnitude of each input in a sequence is specified in advance.

This chapter is an extension of the work of ReVelle for reservoir modeling. The emphasis will be mainly on stochastic systems since the deterministic case is merely a special case of the stochastic model. The linear decision rule discussed by both ReVelle and Loucks (63) is not utilized in this model. By not restricting the formulation to linear decision rules, several advantages arise. Paramount among these are the ability to include the release quantities, x_t , in the objective function, the extension to a linked multiple reservoir system is readily obtained, and the inclusion of stochastic as opposed to deterministic demands adds no conceptional difficulties. This approach is first applied to a single multi-purpose reservoir and then to the even more important case of systems of linked multi-purpose reservoirs.

Single Multi-Purpose Reservoir

In this section a single multi-purpose reservoir with chance-constraints is modeled based on a formulated continuity or material balance equation. The formulation provides decisions that specify the release during different time periods of reservoir operation. The decisions for the entire time horizon are determined by solving a linear programming problem. The linear programming problem is the deterministic equivalent of the original stochastic system. The continuity equation consists of the reservoir inventory for the previous periods random inflow, deterministic demands, and scheduled releases.

Chance-constraints for each time period are established. Chance-constrained means that the specified constraints may not be satisfied all the time, but will be satisfied at least some prespecified amount. The purpose of utilizing the chance-constrained formulation is the convenience in which the random variables can be handled in the constraints. The stochastic constraints selected involve maintaining (1) a specified maximum capacity minus a time requirement variable for upper storage space and (2) a time dependent minimum pool level.

The chance-constraints contain random inflow for each time period. The random inflows are assumed to be additive and essentially independent from one time period to the next. By making this assumption a density function for the sum of the independent random variables is obtained by convolution. The independence assumption is not necessary. It does, however, simplify the presentation of this approach and hence will be followed in further discussion. By using the convoluted random variables for each constraint, a deterministic set of equivalent linear constraints is generated.

Objective functions are then appended to this mathematical formulation for analysis of various decision policies. Both linear and quadratic objective function forms subsequently will be discussed. More general convex objective functions with linear constraints also can be handled readily (Rosen, 85, and

Goldfarb, 33). However, the size of the general problem which can be solved routinely is much smaller than the more specialized linear and quadratic forms.

Continuity Equation

The continuity equation is based on the reservoir model shown in Figure 3.1.

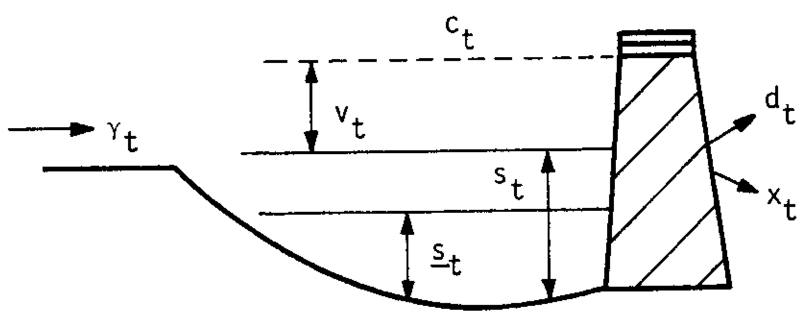

Figure 3.1: Single Multi-purpose Reservoir

The total unregulated flow γ_t enters the reservoir in time period t . The inflow is randomly distributed with a probability density function (p.d.f.) $f_t(\gamma_t)$. Therefore, the inflow in a particular period is known only with some probability. The inflow plus the storage volume s_{t-1} during the previous time period is available for downstream release x_t , and extracted demands d_t .

The current ending storage volume or inventory level s_t is then expressed as

$$s_t = e_t s_{t-1} + \gamma_t - d_t - x_t , \quad (3.1)$$

in which $-\infty < x < \infty$, $-\infty < y < \infty$ and integrated over the range $x + y \leq s$. Changes in the limits of integration are then made to the C.D.F. and

$$G(s) = \int_{-\infty}^{\infty} [\int_{-\infty}^{s-x} f(x,y)dy] dx .$$

Now, $G(s)$ is differentiated with respect to s and the p.d.f. of s is obtained

$$g(s) = \int_{-\infty}^{\infty} f(x, s-x) dx .$$

When x and y are independent, then

$$f(x,y) = f_1(x) f_2(y)$$

and the resulting p.d.f. of s , denoted by $g_c(s)$, is

$$g_c(s) = \int_{-\infty}^{\infty} f_1(x) f_2(s-x) dx .$$

For the case where it is required to obtain the convolution of three or more random variables the formulas are applied recursively until the density function of the total sum is obtained. As an example, let

$$s = x_1 + x_2 + \dots + x_n$$

where $x_1, x_2, \dots, x_n$ are independent random variables. Then to obtain the p.d.f. of s , start first by obtaining the p.d.f. of $s_2 = x_1 + x_2$. Next with regard to $s_3 = s_2 + x_3$ and continue in this manner until $s_n = s_{n-1} + x_n$.

The convolution operation also can be extended to the discrete case, that is,

$$p_c(s) = \sum_{\text{all } x} p_1(x)\, p_2(s-x) .$$

As an example of the discrete case, consider the density function of the random inflow γ_t into a reservoir over a certain time period t given by

γ_t	0	1	2
$p(\gamma_t)$	.2	.3	.5

where γ_t is the number of units of inflow (usually expressed in day-second-feet or acre-feet) into the reservoir for the time period. Assuming that this distribution is the same for each time period and that each time period is statistically independent, find the distribution of random inflow for two time periods. By the application of the convolution formula, the discrete case yields,

$$p(o) = p_1(o)\, p_2(o) = .2 \times .2 = .04$$

$$p(1) = p_1(o)\, p_2(1) + p_1(1)\, p_2(o)$$

$$= .2 \times .3 + .3 \times .2 = .12$$

$$p\ (2) = p_1\ (0)\ p_2\ (2) + p_1\ (1)\ p_2\ (1) + p_1\ (2)\ p_2\ (o)$$

$$= .2 \times .5 + .3 \times .3 + .5 \times .2 = .29$$

$$p\ (3) = p_1\ (1)\ p_2\ (2) + p_1\ (2)\ p_2\ (1)$$

$$= .3 \times .5 + .5 \times .3 = .30$$

$$p\ (4) = p_1\ (2)\ p_2\ (2) = .5 \times .5 = .25 \quad .$$

The distribution of the random inflow for the two time periods is

$\gamma_1 + \gamma_2$	0	1	2	3	4
$p(\gamma_1 + \gamma_2)$	.04	.12	.29	.30	.25

Deterministic equivalent. Chance-constraint (3.2) can now be converted to an equivalent linear constraint. For time period $t = 1$, substitute the continuity equation (3.1) into constraint (3.2) to yield

$$P\ \{\ e_1\ s_o + \gamma_1 - d_1 - x_1 \leq c_1 - v_1\ \} \geq \alpha_1\ .$$

The random variable γ_1 is taken to the right-hand side of the constraint and the inequality sign reversed to give

$$P\ \{\ c_1 - v_1 - e_1\ s_o + d_1 + x_1 \geq \gamma_1\ \} \geq \alpha_1\ .$$

Since γ_1 has a known p.d.f., the C.D.F. F_{γ_1} evaluated at the argument

$$[\ c_1 - v_1 - e_1 s_o + d_1 + x_1\ ]$$

must be greater than or equal to α_1. Thus,

$$F_{\gamma_1}[\ c_1 - v_1 - e_1 s_o + d_1 + x_1\ ] \geq \alpha_1 .$$

For specified α_1, the chance-constraint becomes

$$c_1 - v_1 - e_1 s_o + d_1 + x_1 \geq (R_1)^{\alpha_1} ,$$

where $(R_1)^{\alpha_1}$ is the value of γ_1 from the cumulative distribution F_{γ_1} such that only 100 $(1-\alpha_1)$ percent of the random values of γ_1 are greater than the argument.

For $t = 2$, constraint (3.2) is

$$P\ \{\ e_2\ s_1 + \gamma_2 - d_2 - x_2 \leq c_2 - v_2\ \} \geq \alpha_1 .$$

Substituting for s_1 from the continuity equation (3.1),

$$P\ \{\ e_2\ e_1\ s_o + e_2\ \gamma_1 - e_2\ d_1 - e_2\ x_1 + \gamma_2 - d_2 - x_2 \leq c_2 - v_2\ \} \geq \alpha_1 .$$

Grouping

$$P\ \{\ e_2\ e_1\ s_o - (e_2\ d_1 + d_2) - (e_2\ x_1 + x_2) - c_2 + v_2$$

$$\leq - (e_2\ \gamma_1 + \gamma_2)\ \} \geq \alpha_1 .$$

and reversing the inequality sign the constraint becomes

$$P\{\, c_2 - v_2 - e_2\, e_1\, s_o + (e_2\, x_1 + x_2) + e_2\, d_1 + d_2)$$

$$\geq (e_2\, \gamma_1 + \gamma_2)\,\} \geq \alpha_1 \;.$$

The C.D.F.

$$F_{e_2\gamma_1+\gamma_2}\,[\,(c_2 - v_2 - e_2\, e_1\, s_o) + (e_2\, x_1 + x_2) + (e_2\, d_1 + d_2)\,],$$

which is obtained by convoluting the p.d.f.'s of $e_2\, \gamma_1$ and γ_2, evaluated at the argument must be greater than or equal to α_1. Again for specified α_1, the chance-constraint becomes

$$(c_2 - v_2 - e_2\, e_1\, s_o) + (e_2\, x_1 + x_2) + (e_2\, d_1 + d_2)$$

$$\geq (e_2\, R_1 * R_2)^{\alpha_1} \;,$$

which is linear in x_1 and x_2.

The expression $(e_2\, R_1 * R_2)^{\alpha_1}$ represents a value on the convoluted cumulative distribution of the random variables γ_1 and γ_2 evaluated at the point α_1.

Then for the general nth time period, defining $e_{n+1} = 1$,

$$(3.5)\quad P\,\{s_o \prod_{t=1}^{n} e_t + \sum_{t=1}^{n} (\prod_{k=t+1}^{n} e_k)(\gamma_t - d_t - x_t) \leq c_n - v_n\} \geq \alpha_1 \;.$$

The sum of the random variables is taken to the right-hand side of the constraint to give

$$P\{ s_o \prod_{t=1}^{n} e_t - \sum_{t=1}^{n} [(\prod_{k=t+1}^{n} e_k)(d_t + x_t)] - c_n + v_n$$

$$\leq - \sum_{t=1}^{n} (\prod_{k=t+1}^{n} e_k) \gamma_t \} \geq \alpha_1 .$$

Rearranging,

$$P\{ c_n - v_n - s_o \prod_{t=1}^{n} e_t + \sum_{t=1}^{n} [(\prod_{k=t+1}^{n} e_k)(d_t + x_t)]$$

$$\geq \sum_{t=1}^{n} (\prod_{k=t+1}^{n} e_k) \gamma_t \} \geq \alpha_1 .$$

The C.D.F. for $\gamma_T = \sum_{t=1}^{n} (\prod_{k=t+1}^{n} e_k) \gamma_t$ evaluated at the argument is

$$F_{\gamma_t} \{ c_n - v_n - s_o \prod_{t=1}^{n} e_t + \sum_{t=1}^{n} [(\prod_{k=t+1}^{n} e_k)(d_t + x_t)] \} ,$$

which must be greater than or equal α_1 . By specifying α_1 , the chance-constraint becomes

$$(c_n - v_n - s_o \prod_{t=1}^{n} e_t) + \sum_{t=1}^{n} [(\prod_{k=t+1}^{n} e_k)(d_t + x_t)]$$

$$\geq \mathop{*}_{t=1}^{n} \{ \prod_{k=t+1}^{n} e_k \} \gamma_t$$

or

$$(3.6) \quad (c_n - v_n - s_o \prod_{t=1}^{n} e_t) + \sum_{t=1}^{n} [(\prod_{k=t+1}^{n} e_k)(d_t + x_t)] \geq (R_{*n})^{\alpha_1},$$

where $(R_{*n})^{\alpha_1}$ is the value at the α_1 point on the cumulative of the convoluted distribution.

Constraint (3.6) is the deterministic equivalent of (3.2), where the release quantities x_t are the decision variables. All other variables are state variables selected by the water resource planner.

Chance-constraint (3.3) is next converted to an equivalent linear constraint for two time periods followed by the general constraint for the nth time period constraint.

Constraint (3.3) is

$$P \{ s_t \geq \underline{s}_t \} \geq \alpha_2 ,$$

and for $t = 1$

$$P \{ s_1 \geq \underline{s}_1 \} \geq \alpha_2 .$$

Then, substitution of (3.1) into (3.3) yields

$$P \{ e_1 s_o + \gamma_1 - d_1 - x_1 \geq \underline{s}_1 \} \geq \alpha_2 .$$

Taking the random variable to the right-hand side

$$P \{ e_1 s_o - d_1 - x_1 - \underline{s}_1 \geq - \gamma_1 \} \geq \alpha_2$$

and reversing the inequality yields

$$P\{ (\underline{s}_1 - e_1 s_o) + d_1 + x_1 \leq \gamma_1 \} \geq \alpha_2 .$$

The deterministic equivalent for this equation, one minus the C.D.F., F_{γ_1} , evaluated at the argument

$$[(\underline{s}_1 - e_1 s_o) + d_1 + x_1]$$

must be greater than or equal to α_2 . Thus,

$$1 - F_{\gamma_1} [(\underline{s}_1 - e_1 s_o) + d_1 + x_1] \geq \alpha_2$$

or

$$F_{\gamma_1} [(\underline{s}_2 - e_1 s_o) + d_1 + x_1] \leq 1 - \alpha_2 .$$

For specified α_2 , the chance-constraint becomes

$$(\underline{s}_2 - e_1 s_o) + d_1 + x_1 \leq (R_1)^{1-\alpha_2} ,$$

where $(R_1)^{1-\alpha_2}$ is the value of $(1-\alpha_2)$ from the cumulative distribution F_{γ_1} .

For $t = 2$, constraint (3.3) is

$$P\{ s_2 \geq \underline{s}_2 \} \geq \alpha_2 .$$

Substitution of continuity equation (3.1) twice yields

$$P\{ - e_2 x_1 + e_2 \gamma_1 - e_2 d_1 + e_2 e_1 s_o + \gamma_2 - d_2 - x_2 \geq \underline{s}_2 \} \geq \alpha_2 .$$

Regrouping and reversing the inequality yields

$$P\{ (\underline{s}_2 - e_2 e_1 s_o) + (e_2 d_1 + d_2) + (e_2 x_1 + x_2) \leq (e_2 \gamma_1 + \gamma_2) \} \geq \alpha_2 .$$

As before, to obtain the deterministic equivalent for this equation, one minus the C.D.F. evaluated at the argument must be greater than or equal to α_2 ,

$$1 - F_{e_2\gamma_1+\gamma_2} [(\underline{s}_2 - e_2 e_1 s_o) + (e_2 d_1 + d_2) + (e_2 x_1 + x_2)] \geq \alpha_2 .$$

Letting $(e_2 R_1 * R_2)^{1-\alpha_2}$ represent the $(1-\alpha_2)$ value on the cumulative distribution $F_{e_2\gamma_1+\gamma_2}$, the deterministic constraint is

$$(\underline{s}_2 - e_2 e_1 s_o) + (e_2 d_1 + d_2) + (e_2 x_1 + x_2) \leq (e_2 R_1 * R_2)^{1-\alpha_2} .$$

The generalized formulation of equation (3.3) follows that of equation (3.2) and the convolution of the sum of the random variables ξ_n is in general:

$$\underline{s}_n - s_o \prod_{t=1}^{n} e_t + \sum_{t=1}^{n} (\prod_{k=t+1}^{n} e_k)(d_t + x_t) \leq \{ \mathop{*}_{t=1}^{n} (\prod_{k=t+1}^{n} e_k) R_t \}^{1-\alpha_2}$$

or

$$(3.7)\quad (\underline{s}_n - s_o \prod_{t=1}^{n} e_t) + \sum_{t=1}^{n} (\prod_{k=t+1}^{n} e_k)(d_t + x_t) \leq (R_{*n})^{1-\alpha_2} .$$

Constraint (3.7) is the linear deterministic equivalent of (3.3) with the release x_t $(t = 1,2,\ldots,n)$ being the decision variable necessary to insure that storage s_n exceeds the minimum pool level $\underline{s}_n$ with a probability α_2 .

Stochastic Inflows and Demands

The development for both stochastic inflows and stochastic demands is similar to that for stochastic inflows. The main difference is that the convolution of the inflows minus the demands must now be obtained.

Consider the general stochastic-constraint on reservoir capacity (3.5),

$$P \{ s_o \prod_{t=1}^{n} e_t + \sum_{t=1}^{n} (\prod_{k=t+1}^{n} e_k)(\gamma_t - d_t - x_t) \leq c_n - v_n \} \geq \alpha_1 .$$

The sum of the random variables γ_t and d_t is again taken to the right-hand side of the constraint and upon rearranging,

$$(3.8)\qquad P \{ c_n - v_n - s_o \prod_{t=1}^{n} e_t + \sum_{t=1}^{n} (\prod_{k=t+1}^{n} e_k)\, x_t$$

$$\geq \sum_{t=1}^{n} (\prod_{k=t+1}^{n} e_k)(\gamma_t - d_t) \} \geq \alpha_1 .$$

The distribution of the random variable

$$\xi_n = \sum_{t=1}^{n} \left(\prod_{k=t+1}^{n} e_k \right)(\gamma_t - d_t)$$

must now be obtained. This can be accomplished by convolution or by application of Fourier transforms (Churchill, 16, and Parzen, 80). Let F_{ξ_n} represent the distribution function (C.D.F.) of ξ_n so that equation (3.8) becomes

$$F_{\xi_n} \left[c_n - v_n - s_o \prod_{t=1}^{n} e_t + \sum_{t=1}^{n} \left(\prod_{k=t+1}^{n} e_k \right) x_t \right] \geq \alpha_1 \ .$$

Thus,

$$(3.9) \qquad c_n - v_n - s_o \prod_{t=1}^{n} e_t + \sum_{t=1}^{n} \left(\prod_{k=t+1}^{n} e_k \right) x_t \geq R^{\alpha_1}_{(\gamma * d)n} \ ,$$

where $R^{\alpha_1}_{(\gamma * d)n}$ represents the value of the random variable ξ_n for which $100\alpha_1$ percent of the area of the distribution is to the left of ξ_n, or equivalently the α_1 point on the (C.D.F.) F_{ξ_n}.

Constraint (3.9) is the deterministic equivalent of equation (3.2) when both the demands and inflows are stochastic. Similarly, equation (3.3) becomes,

$$(3.10) \qquad \underline{s}_n - s_o \prod_{t=1}^{n} e_t + \sum_{t=1}^{n} \left(\prod_{k=t+1}^{n} e_k \right) x_t \leq R^{1-\alpha_2}_{(\gamma * d)n} \ .$$

The minimum and maximum constraints on releases remain as before. Hence, for stochastic inflows and demands, the constraints on the system are equations (3.9), (3.10), and (3.4).

System of Multi-Purpose Reservoirs

The natural extension of the single multi-purpose reservoir model is to link a system of such reservoirs. The reservoirs can be linked by a series of canals or pumping facilities as well as normal flow between certain reservoirs and random inflow into all reservoirs. The formulation is structured so that each reservoir can be linked to any other reservoir.

For the development of a linked system of reservoirs, it is assumed that there are two general linkage types. These linkages consist of the normal channel flow for reservoir releases, and pipe lines or pumping canals. The model is completely general in the sense that any connecting system can be modeled. Thus, each reservoir could be connected to every other reservoir and could receive releases from any or all other reservoirs as dictated by the particular system under consideration.

For the purposes of this discussion, each reservoir in each time period is assumed to receive random unregulated inflow, regulated inflow from reservoir releases, and inflow from pumping. The reservoir level is depleted by means of scheduled releases, deterministic demands, evaporation and seepage losses, and pumping to other reservoirs. Stochastic demands can be handled by a simple extension to the model presented.

The system of multi-purpose reservoirs with chance-constraints is modeled based on material balance equations for reservoir inventory levels. The formulation provides decisions that specify the release and pumping quantities during different time periods of system operation. The release and pumping decisions for the entire planning horizon are determined by solving a linear programming problem. The linear problem is the deterministic equivalent of the original chance-constrained system.

The chance-constraints for each reservoir and each time period are identical to those developed for a single multi-purpose reservoir. The random inflows for each reservoir are assumed to be independent from one time period to the next. However, this assumption, while simplifying, is not necessary to the model development.

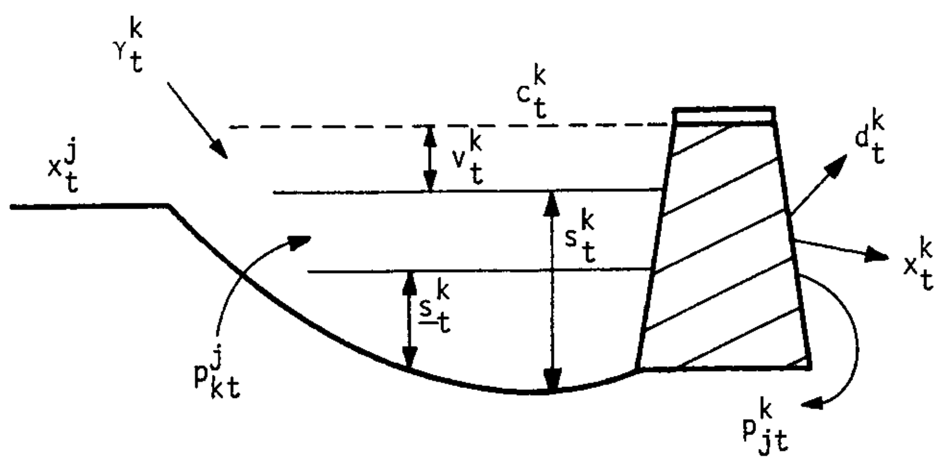

Figure 3.2: Linked Multi-purpose Reservoir

Continuity Equation

The multi-purpose linked kth reservoir is based on the model shown in Figure 3.2 and the following notation:

m - the number of reservoirs,

γ_t^k - random unregulated inflow into reservoir k in time period t,

c_t^k - reservoir k's design maximum capacity in period t,

v_t^k - conservation, flood control, or surcharge storage in the upper reservoir,

s_t^k - ending reservoir inventory level for period t,

$\underline{s}_t^k$ - minimum specified inventory level,

e_t^k - fraction of inventory remaining after evaporation and seepage losses,

x_t^k - scheduled downstream release from reservoir,

$\overline{x}_t^k$ - maximum downstream release,

$\underline{x}_t^k$ - minimum downstream release, and

d_t^k - deterministic extracted demand.

The total unregulated random inflow γ_t^k enters the kth reservoir in time period t. The inflow is randomly distributed with a p.d.f. $f_t(\gamma_t^k)$. The regulated inflow x_t^j is the release from the jth reservoir into the kth. The regulated pumped inflow p_{kt}^j

is the water pumped from reservoir j into reservoir k. The pumping and release into reservoir k can be from several reservoir, thus j can vary over all these reservoir numbers.

The releases from the kth reservoir in time period t are (1) the deterministic extracted demands d_t^k , (2) the decision variable for downstream release x_t^k , and (3) the decision variable p_{jt}^k for pumping water from reservoir k to reservoir j; again, reservoir k could pump to several different reservoirs. The releases plus the inflows and previous storage volume constitute the current inventory level s_t^k . The continuity equation for reservoir k in time period t is

$$(3.11) \quad s_t^k = e_t^k s_{t-1}^k + \gamma_t^k - d_t^k - x_t^k + \sum_{\substack{j=1 \\ j \neq k}}^{m} (I_j^k x_t^j + 0_k^j p_{kt}^j - 0_j^k p_{jt}^k) ,$$

where

$$I_j^k - \begin{cases} 1 \text{ if reservoir j releases flows into reservoir k} \\ 0 \text{ otherwise, and} \end{cases}$$

$$0_j^k - \begin{cases} 1 \text{ if reservoir k pumps to reservoir j} \\ 0 \text{ otherwise.} \end{cases}$$

Chance-Constraints with Stochastic Inflows

The chance-constraint for the probability of not exceeding the maximum capacity of the kth reservoir is

$$(3.12) \qquad P\{\, s_t^k \le c_t^k - v_t^k \,\} \ge \alpha_1^k \,, \quad k = 1,2,\ldots,m, \text{ and}$$

$$t = 1,2,\ldots,T,$$

where c_t^k is the design maximum capacity of the kth reservoir, v_t^k is the upper storage space required in time period t of the kth reservoir, and α_1^k are specified constants between zero and one.

The probability α_2^k at the end of time period t for storage s_t^k to exceed the minimum pool level is

$$(3.13) \qquad P\{\, s_t^k \ge \underline{s}_t^k \,\} \ge \alpha_2^k \,, \quad k = 1,2,\ldots,m,$$

where $\underline{s}_t^k$ is the minimum storage that must be maintained.

The downstream release x_t^k must satisfy the minimum $\underline{x}_t^k$ and the maximum $\overline{x}_t$ reservoir release constraints, i.e.,

$$(3.14) \qquad \underline{x}_t^k \le x_t^k \le \overline{x}_t^k \quad \forall \; k,t \,.$$

The chance-constraints (3.12) and 3.13) are converted to their equivalent linear deterministic constraints in a similar manner as the chance-constraints of the single multi-purpose reservoir model. The general equivalent linear deterministic result for chance-constraint (3.12) with $t = n$ is

$$(3.15) \qquad c_n^k - v_n^k - s_o^k \prod_{t=1}^{n} e_t^k + \sum_{t=1}^{n} \{ (\prod_{L=t+1}^{n} e_L) [d_t^k + x_t^k - \sum_{\substack{j=1 \\ j \neq k}}^{m} (I_j^k x_t^j + O_k^j p_{kt}^j - O_j^k p_{jt}^k)] \} \geq R_{k,*n}^{\alpha_1} ,$$

where $R_{k,*n}^{\alpha_1}$ are the values at the α_1^k points on the convoluted distribution. Chance-constraint (3.13) becomes

$$(3.16) \qquad \underline{s}_n^k - s_o^k \prod_{t=1}^{n} e_t^k + \sum_{t=1}^{n} (\prod_{L=t+1}^{n} e_L) [d_t^k + x_t^k - \sum_{\substack{j=1 \\ j \neq k}}^{m} (I_j^k x_t^j + O_k^j p_{kt}^j - O_j^k p_{jt}^k)] \leq R_{k,*n}^{(1-\alpha_2)} ,$$

where $R_{k,*n}^{(1-\alpha_2)}$ are the values at the $(1-\alpha_2)$ points on the convoluted distribution. Equations (3.15) and 3.16) are linear in the decision variables x_t^k and p_{jt}^k. The releases and pumping units during different time periods of reservoir operation are, therefore, determined by solving a linear programming problem.

Reservoir Models and Solutions

In this section two models are presented to illustrate the chance-constraint formulation. For the first model three example problems are solved. The first two example problems are solved by linear programming. One of the problems assumes stochastic inflow and the other example both stochastic inflow and demand.

The third example problem appends a quadratic objective function to the original model, which is then solved by quadratic programming. The second model presented is based on the formulation of a system of multi-purpose reservoirs. One example problem is solved using linear programming.

Single Multi-Purpose Reservoir

Linear objective function with stochastic inflow. In the single multi-purpose reservoir example, it is assumed that the objective is to maximize the profit of releasing x_t units of water for two time periods. The releases are subject to the chance-constraints

$$P \{ s_t \le c_t - v_t \} \ge \alpha_1$$

and

$$P \{ s_t \ge \underline{s}_t \} \ge \alpha_2 \quad ,$$

from which the equivalent deterministic constraints (3.6) and (3.7) were derived.

For two time periods, constraints (3.6) and (3.7) are, respectively,

$$t = 1 \begin{cases} - s_o e_1 + d_1 + x_1 + c_1 - v_1 \ge R_1^{\alpha_1} \\ \\ - s_o e_1 + d_1 + x_1 + \underline{s}_1 \qquad \le R_1^{(1-\alpha_2)} \end{cases} .$$

and

$$t = 2 \begin{cases} -s_o\, e_1\, e_2 + e_2\, x_1 + x_2 + e_2\, d_1 + d_2 + c_2 - v_2 \\ \qquad \geq (e_2\, R_1 * R_2)^{\alpha_1} \\ \\ -\, s_o\, e_1\, e_2 + e_2\, x_1 + x_2 + e_2\, d_1 + d_2 + \underline{s}_2 \\ \qquad \leq (e_2\, R_1 * R_2)^{1-\alpha_2} \quad . \end{cases}$$

The maximum and minimum release constraints must also be satisfied:

$$t = 1 \begin{cases} x_1 \geq \underline{x}_1 \\ x_1 \leq \overline{x}_1 \end{cases}$$

$$t = 2 \begin{cases} x_2 \geq \underline{x}_2 \\ x_2 \leq \overline{x}_2 \quad . \end{cases}$$

By assuming the values in Table 3.1 for the state variables, the problem to be solved is

$$
\begin{aligned}
\text{maximize} \quad & x_0 = x_1 + x_2 \\
\text{subject to} \quad & -x_1 \le 2 \\
& x_1 \le 5 \\
& x_1 \le 7 \\
& -x_1 \le -1 \\
& -.95\,x_1 - x_2 \le 11.1 \\
& .95\,x_1 + x_2 \le 5.9 \\
& x_2 \le 8.0 \\
& -x_2 \le -3.0
\end{aligned}
$$

Application of linear programming reveals the critical values of release x_1, x_2 as 1, 3 units and a maximum profit x_0 of 4 units.

Linear objective function with stochastic inflow and demand. In the previous example problem, only stochastic inflows were assumed. The distribution of inflows was known and the input to the program was listed in Table 3.1. In this example problem, the same system is assumed. Now, however, the demands and inflows are assumed to be normally distributed with the means and variances listed in Table 3.2. The corresponding convoluted inflows and demands to be used in the problem formulation are also listed. The minimum reservoir level for time period two was adjusted to a unit value. This adjustment was necessary, since the previous problem with these inflows and demands is infeasible.

Table 3.1

Stochastic Inflow Data

Variable		
Time	1	2
d_t	6.0	8.0
$c_t - v_t$	15.0	25.0
$\overline{x}_t$	7.0	8.0
$\underline{x}_t$	1.0	3.0
e_t	1.0	0.95
$\underline{s}_t$	3.0	3.0
s_o	8.0	
$R_1^{\alpha_1}$	11.0	
$R_1^{(1-\alpha_2)}$	6.0	
$(e_2 R_1 * R_2)^{\alpha_1}$		20.0
$(e_2 R_1 * R_2)^{(1-\alpha_2)}$		15.0

Table 3.2

Stochastic Inflow and Demand Data

Variable		
Time	1	2
$c_t - v_t$	15.0	25.0
$\overline{x}_t$	7.0	8.0
$\underline{x}_t$	1.0	3.0
e_t	1.0	0.95
$\underline{s}_t$	3.0	1.00
s_0	8.0	
$E\{d_t\}$	6.0	8.0
$\sigma^2\{d_t\}$	1.0	1.0
$E\{\gamma_t\}$	8.0	7.0
$\sigma^2\{\gamma_t\}$	1.0	1.0
$R_1^{\alpha_1}$	4.336	
$R_1^{(1-\alpha_2)}$	-0.336	
$(e_2 R_1 * R_2)^{\alpha_1}$		4.12
$(e_2 R_1 * R_2)^{(1-\alpha_2)}$		-2.32

The solution to this problem is again obtained by the method of linear programming. The critical values of releases in periods one and two, x_1 and x_2, are 1.3474 and 3.0, respectively, with a maximum profit x_0 of 4.347 units.

Quadratic objective function. Consider a quadratic objective function for the same system as example 1. Let the objective be:

$$\text{minimize } x_0 = x_1 + x_2 + 3\,(x_1 - 3)^2 + 5\,(x_2 - 5)^2 + 3\,x_1\,x_2 \,.$$

The solution to this problem is readily obtained by quadratic programming techniques (Frank and Wolfe, 28, and Wolfe, 97). A quadratic programming algorithm, which was developed by Schuermann (86), was used to solve the quadratic programming problem. The critical values of releases x_1 and x_2 are 1.0 and 4.6 with a minimum cost x_0 of 32.2 units.

System of Multi-Purpose Reservoirs

Linear objective function. The second model presented is a system of multi-purpose reservoirs. Figure 3.3 is the model formulated for illustration. The linked reservoir system is composed of three reservoirs, two of which have pumping capabilities. Random inflows and predetermined demands are assumed for each reservoir. Table 3.3 describes the state variables assumed for each reservoir and time period. The objective is to minimize the operating cost of the system for two time periods.

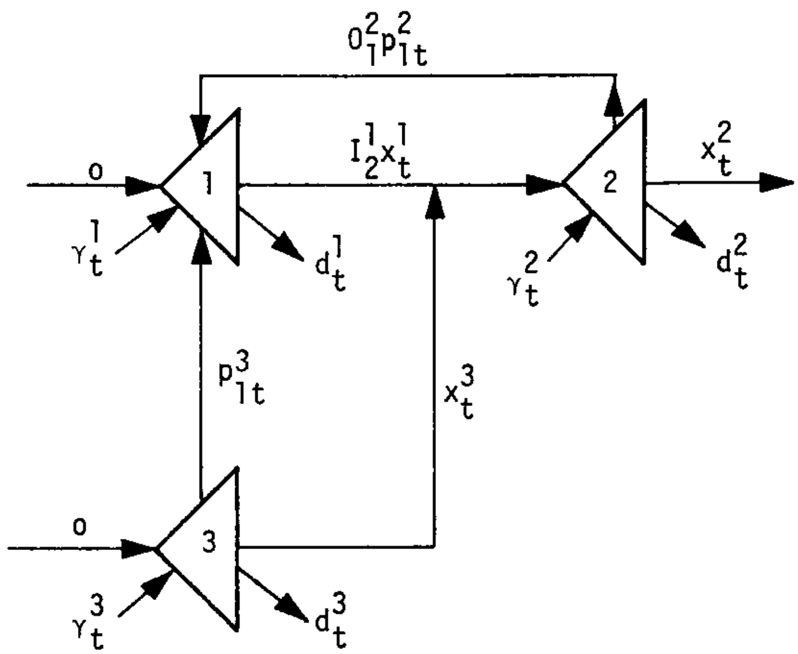

Figure 3.3: System of Connected Multi-purpose Reservoirs

The decision variables are to be determined for each time period. They are (1) the units of water released from reservoir one, two, and three and (2) the number of units of water pumped into reservoir one from reservoirs two and three. The total number of variables to be determined is the product of the number of time periods with the sum of the number of reservoirs and pumping variables.

The decision variables must satisfy the equivalent deterministic constraints (3.15) and (3.16) and the upper and lower limits on release (3.14). By taking advantage of the fact that the decision

Table 3.3

Data Used in Example Problem for Linked System of Reservoirs

Time	1			2		
Reservoir	1	2	3	1	2	3
d_t^k	6.0	5.0	10.0	8.0	7.0	7.0
$c_t^k - v_t^k$	10.0	20.0	15.0	10.0	19.0	16.0
$\bar{x}_t^k$	7.0	15.0	20.0	8.0	12.0	20.0
$\underline{x}_t^k$	1.0	2.0	1.0	3.0	3.0	1.0
e_t^k	1.0	1.0	1.0	0.95	0.97	0.98
$\underline{s}_t^k$	3.0	4.0	3.0	3.0	2.0	4.0
s_o^k	8.0	20.0	6.0			
$R_1^{\alpha_1}$	11.0	10.0	12.0			
$R_1^{(1-\alpha_2)}$	6.0	9.0	8.0			
$(e_2 R_1 * R_2)^{\alpha_1}$				20.0	15.0	20.0
$(e_2 R_1 * R_2)^{(1-\alpha_2)}$				15.0	14.0	17.0

variables are bounded from above and below, the number of constraints can be reduced considerably. Using bounded variable techniques discussed in Taha (89), the number of constraints is the product of twice the number of reservoirs multiplied by the number of time periods.

The structure of the problem can now be put in the form

$$\text{minimize} \quad Z = \underline{h}\,\underline{y}$$

$$\text{subject to} \quad (\underline{A},\underline{I})\underline{y} = \underline{b} \quad ,$$

$$\underline{\ell} \leq \underline{y} \leq \underline{u}$$

$$\underline{u} \geq \underline{\ell} \geq \underline{o} \quad ,$$

where $\underline{y}$ is the decision vector and consists of release, pumping, slack and artificial variables. The values chosen for the cost coefficient of the objective function are

$$\underline{h} = [1.0,\ -2.0,\ 0.0,\ -.75,\ .65,\ 1.0,\ -2.1,\ 0.0,\ -.80,\ .70]$$

and the linear constraints are given by specifying the matrix $\underline{A}$ and the vectors $\underline{b}$, $\underline{\ell}$, $\underline{u}$ as

$$\underline{A} = \begin{bmatrix} -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -.95 & 0 & 0 & .95 & .95 & -1 & 0 & 0 & 1 & 1 \\ .95 & 0 & 0 & -.95 & -.95 & 1 & 0 & 0 & -1 & -1 \\ .97 & -.97 & .97 & -.97 & 0 & 1 & -1 & 1 & -1 & 0 \\ -.97 & .97 & -.97 & .97 & 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & 0 & -.98 & 0 & -.98 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & .98 & 0 & .98 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\underline{b} = \begin{bmatrix} -3 \\ 5 \\ -5 \\ 20 \\ 7 \\ 1 \\ -3.9 \\ 5.9 \\ 6.45 \\ 19.55 \\ 6.92 \\ 2.08 \end{bmatrix} \qquad \underline{\ell} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \\ 3 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad \underline{u} = \begin{bmatrix} 7 \\ 15 \\ 20 \\ 10 \\ 5 \\ 8 \\ 12 \\ 20 \\ 10 \\ 5 \end{bmatrix}$$

Application of the revised simplex method in conjunction with the bounded variable technique reveals the critical values of $\underline{y}$:

$$\underline{y} = \begin{bmatrix} x_1^1 \\ x_1^2 \\ x_1^3 \\ p_{11}^2 \\ p_{11}^3 \\ x_2^1 \\ x_2^2 \\ x_3^3 \\ p_{12}^2 \\ p_{12}^3 \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ 1 \\ 4 \\ 0 \\ 8 \\ 3 \\ 1 \\ 4.85 \\ 0.1 \end{bmatrix}$$

and an optimum cost of operating the system as -16.11 or a profit of 16.11 units. The program documentation is listed in Appendix A.

Summary and Conclusion

In this Chapter an extension of the single multiple purpose stochastic constrained reservoir model was presented. The linear decision rules utilized by ReVelle et al. (83) and Loucks (63) are omitted in the model. The purpose of using linear decision rules is to disconnect the release in the nth period from the ending inventory level in period n-1. The advantage of the linear decision rule is that only the random inflow for the current period need be considered. However, the actual quantity to be released in the nth period is not known until the random inflow in periods 1 through n-1 are observed. Thus, for planning purposes where operation of the reservoir is important or when the release variables are represented in the objective function, this formulation is unsatisfactory since releases are actually random variables and not exactly determined by the reservoir planner.

The formulation proposed in the section on Single Multi-Purpose Reservoirs requires that the distributions of sums of random inflows for all time periods be obtained. This is a relatively simple task for models with a large number of time periods. Since by the central limit theorem (Parzen, 80), the distribution of the sums derived from the sampling of these parent distributions tends to become normal as the sample size increases.

By not using any form of decision rule, the constraints on upper and lower release quantities become deterministic and need not be represented by chance-constraint formulation. Also, quadratic of even general convex objective functions of the release quantities can be considered.

In the section on System of Multi-Purpose Reservoirs a mathematical model is developed for a linked system of multiple purpose reservoirs with stochastic unregulated inflows. The mathematical model is obtained as a straight forward generalization of the single reservoir model. The chance-constrained formulation for reservoir capacities and minimum inventory levels is converted to a linear system of constraints. Linear, quadratic, or even general convex objective functions can be appended to this system and the solution obtained with facility.

If linear objective functions are assumed, which could be operational or of a capacity nature, very large problems can be solved. Since the cumulative inflows will be nearly normally distributed for these problems, their formulations and solutions are a matter of course. The problem of capacity expansion dealt with in the next chapter is generally not well modeled as a continuous linear problem. Capacity expansion models are usually limited to fixed time periods and have nonlinear costs as a function of size.

CHAPTER IV

BENDERS' DECOMPOSITION APPLIED TO TIME PHASING OF CAPACITY EXPANSION MULTIPLE-RESERVOIR MODELS

An essential consideration in models of water resource planning is economic uncertainty. This uncertainty refers to the fact that water resource planners often do not know the relevant benefit and loss functions to evaluate the performance of various system designs and operating policies. Since water resource projects are durable, their time horizon can extend into the distant future. The benefit functions will probably shift over time and it is usually impossible to determine exactly how benefit functions may effect future planning. This is one major source of economic uncertainty in water resource planning. Another uncertainty is technological change which cannot be predicted perfectly, because there is economic uncertainty about future cost functions.

The dynamic variations of benefit functions posed by Marglin (67) lead to questions concerning the time phasing of investment planning in water resource development. As demand grows at an almost steady rate over time, capacity expansion models must be used to determine an optimal investment program. Since investment programs are usually legislated, capacity expansion can only occur at a finite number of points in time. With capacity fixed for certain time frames and demand growing almost continuously, the optimal operating policy for a reservoir system is quite dynamic.

In this chapter a procedure is proposed for the analysis of time phasing of reservoir system operation with capacity expansion. The formulation is based on the multiple-reservoir stochastic model developed in Chapter III. The objective is to select the reservoir (construction) sizing, timing, and to establish operating policies such that the total cost associated with the system of linked reservoirs is minimized. The formulation is a mixed integer-continuous linear programming problem. Due to the size of the resulting problem and its general structure, Benders' decomposition technique (Lasdon, 55) is applied. Benders' method allows for the problem to be separated into a pure linear program and an almost pure integer program. The computational efficiency of the solution procedure is greatly enhanced by the application of Benders' technique.

Problem Formulation

The general problem to be considered is one of selecting the set of reservoir capacities along with the system operational decisions which minimize the total reservoir system cost over a planning horizon. The system planner defines the feasible reservoir segments and sizes for each time period where expansion is feasible. For example, after the initial reservoir segment has been built, the next phase may not be feasible for five years hence. This delay could be of financial nature or due to construction lag time. Thus, the planner must select, from the various possible combinations available, the time phasing which meets a set of demands which are also varying over time. The inflow of water into the system is not known exactly in advance. However, distributions of flow can be estimated from historical data or by streamflow synthesis (Fiering, 25). The time periods associated with demands are normally much smaller than those associated with reservoir expansion periods. For instance, demands might be monthly as opposed to five year incremental opportunities for capacity expansion.

The availability of capacity in the problem formulation will be represented by binary variables (0 or 1) $y^k_{t,s}$ times the segment size R^k_s. The following notation will be used:

$$y^k_{t,s} - \begin{cases} 1 - \text{if capacity segment } s \text{ for reservoir } k \text{ is} \\ \quad\ \ \text{activated during time period } t , \\ 0 - \text{otherwise,} \end{cases}$$

R^k_s - capacity size of segment s for reservoir k,

$K^k_{t,s}$ - cost of installation and maintenance of segment from time t until the end of the planning horizon.

Thus, the available capacity for reservoir k and time period T is obtained by:

$$(4.1) \qquad c^k_T = \sum_{t\epsilon\Omega_T} \sum_{s=1}^{N_k} R^k_s \, y^k_{t,s} \quad ,$$

where N_k is the number of segments to be considered for reservoir k, and Ω_T is the set of all time periods less than or equal to T during which reservoir expansion is permitted. The cost of reservoir capacity including operation, maintenance, and replacement over the whole planning horizon of T time periods for an m reservoir system is:

$$(4.2) \qquad \sum_{k=1}^{m} \sum_{t \varepsilon \Omega_T} \sum_{s=1}^{N_k} K_{t,s}^{k} \, y_{t,s}^{k} \quad .$$

Utilizing the stochastic multi-reservoir model developed in Chapter III, operation of the system at minimum cost for a specified set of capacities is obtained by selecting the release variables x_t^k for each reservoir and each time period. If the costs associated with releases are assumed to be linear, then the total problem is a mixed integer-continuous linear programming problem. A model is now formulated for a connected system of reservoirs and will be used for the remainder of the discussion.

Connected Stochastic Multiple-Reservoir Model

Consider a connected system of multi-purpose reservoirs. The reservoirs may be connected by channels, where the release of one reservoir flows into a downstream reservoir. In addition, any network of canals or pumping links can be defined on the system. Figure 4.1 is an example of such a connected reservoir system.

Two restrictions on the reservoir ending inventory level will be considered, as well as limitations on releases and pumping. These restrictions are basic to the stochastic formulation of the system. They are: (1) the reserved capacity portion of the reservoir must not be violated more than some specified $100\alpha_1$ percent of the time, and (2) the minimum pool level cannot be violated more than $100\alpha_2$ percent of the time. These restrictions are represented mathematically as:

$$P \{ s_t^k \leq c_t^k - v_t^k \} \geq \alpha_1 \tag{4.3}$$

and

$$P \{ s_t^k \geq \underline{s}_t^k \} \geq \alpha_2 \quad . \tag{4.4}$$

The connection between ending inventory levels for successive time periods is a function of the demands, unregulated inflows, evaporation or loss, and the decisions for release and pumping. The material balance equation describing this relationship is given as:

$$s_t^k = e_t^k \, s_{t-1}^k + \gamma_t^k - d_t^k - x_t^k + \sum_{\substack{j=1 \\ j \neq k}}^{m} I_k^j \, x_t^j + \sum_{\substack{j=1 \\ j \neq k}}^{m} O_k^j \, p_{k,t}^j - \sum_{\substack{j=1 \\ j \neq k}}^{m} O_j^k \, p_{j,t}^k \quad , \tag{4.5}$$

where e_t^k is the fraction of the ending inventory quantity which is available in the next time period; the difference is the loss due to evaporation, and normal reservoir and channel losses.

The stochastic formulation utilized in this Chapter and that of ReVelle et al., differ mainly in the handling of the material balance equation (4.5). ReVelle's choice of a linear decision rule of the form

$$x_t = s_{t-1} + b_t$$

was convenient in the sense that, when substituted into his material balance equation, the current inventory level s_t became independent of s_{t-1}. However, by restricting oneself to such rules, the actual release x_t is not known until the quantity s_{t-1} is observed. Loucks (63) suggested an altered linear decision rule and obtained sufficient improvements in his results.

It is the contention that no decision rule is necessary. By working with the actual release variables and treating the stochastic inflows simultaneously, that is without disconnecting the relationship between ending reservoir inventory levels by some arbitrary decision rule, a more exacting model can be obtained. This single reservoir model has the advantage of being easily extended to multiple-reservoir systems. However, the computational effort for preparing the data for an analysis of the system is increased due to the necessity of obtaining distributions of the sums of random variables.

Equation (4.5) when repeatedly substituted into equation (4.3) yields, for some fixed time period n ,

$$P\{ s_o^k \prod_{t=1}^{n} e_t^k - \sum_{t=1}^{n} \left(\prod_{L=t+1}^{n} e_L^k \right) [d_t^k + x_t^k - \gamma_t^k + \sum_{\substack{j=1 \\ j\neq k}}^{m} (-I_k^j x_t^j - O_k^j p_{k,t}^j + O_j^k p_{j,t}^k)] \le c_t^k - v_t^k \} \ge \alpha_1 \quad ,$$

where

$$e_{n+1}^k = 1 \ .$$

Taking the random variables γ_t^k to the right-hand side and reversing the inequality sign yields:

$$P\{ c_t^k - v_t^k - s_o^k \prod_{t=1}^{n} e_t^k + \sum_{t=1}^{n} \left(\prod_{L=t+1}^{n} e_L^k \right) [d_t^k + x_t^k + \sum_{\substack{j=1 \\ j\neq k}}^{m} (-I_k^j x_t^j - O_k^j p_{k,t}^j + O_j^k p_{j,t}^k)] \ge \sum_{t=1}^{n} \left(\prod_{L=t+1}^{n} e_L^k \right) \gamma_t^k \} \ge \alpha_1 \quad .$$

The equation states that some linear function of the decision variables must be greater than or equal to the value of the sum of the random variables at least $100\alpha_1$ percent of the time. Figure 4.2 is a graphical representation of this equation. The random variable can take on any value over its range. However, the decision variables must be chosen before the random variables are observed. Thus, to insure the equation is satisfied a given percent of the time, the decision variables must be selected such that the area of the random distribution to the left of the equation value is greater than or equal to α_1 . This is equivalent to selecting the value of the random variable associated with $100\alpha_1$ percent from the cumulative distribution, denote this value by $F^{\alpha_1}_{k,*n}$, and specifying that the equation value must always be greater than or equal to this value. This procedure yields the equation:

$$(4.6) \qquad c^k_t - v^k_t - s^k_o \prod_{t=1}^{n} e^k_t + \sum_{t=1}^{n} \Big(\prod_{L=t+1}^{n} e^k_L \Big) \Big[d^k_t + x^k_t + \sum_{\substack{j=1 \\ j \neq k}}^{m} (-I^j_k \, x^j_t - O^j_k \, p^j_{k,t} + O^k_j \, p^k_{j,t}) \Big] \geq F^{\alpha_1}_{k,*n}$$

Proceeding in a similar manner, the deterministic equivalent for equation (4.4) is obtained:

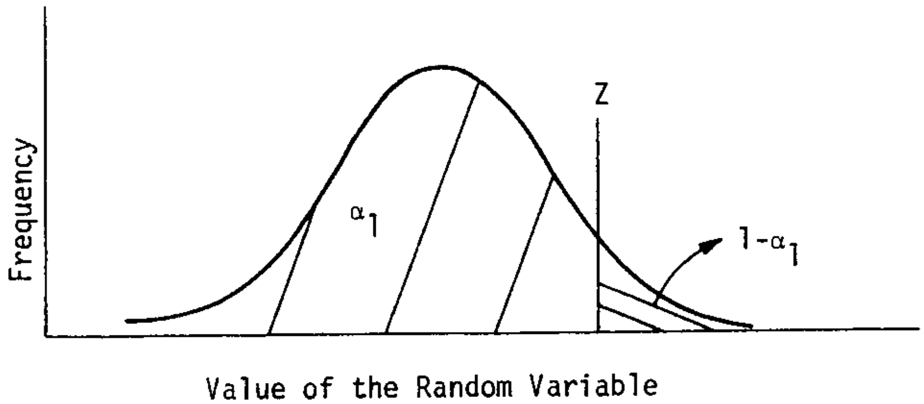

Figure 4.2: Z is the Point on the Distribution Such That $100\alpha_1$ Percent of the Values of the Random Variable Fall to the Left of Z .

$$(4.7) \qquad \underline{s}_t^k - s_o^k \prod_{t=1}^{n} e_t^k + \sum_{t=1}^{n} \left(\prod_{L=t+1}^{n} \right) \Big[d_t^k + x_t^k + \sum_{\substack{j=1 \\ j\neq k}}^{m} \left(-I_k^j \, x_t^j - O_k^j \, p_{k,t}^j + O_j^k \, p_{j,t}^k \right) \Big] \le F_{k,*n}^{(1-\alpha_2)} .$$

The flow limitation constraints for reservoir release are,

$$(4.8) \qquad \underline{x}_t^k \le x_t^k \le \bar{x}_t^k \qquad \text{for all } k, t,$$

and the restrictions on maximum pumping capacities are,

$$(4.9) \qquad 0 \le p_{j,t}^k \le \bar{p}_{j,t}^k \qquad \text{for all } k, j \text{ and } t.$$

Hence, the minimum operating cost for a fixed capacity system is obtained when equation (4.10) is minimized subject to constraints

(4.6), (4.7), (4.8), and (4.9).

$$(4.10) \qquad \sum_{t=1}^{T} \sum_{k=1}^{m} a_t^k x_t^k + \sum_{t=1}^{T} \sum_{k=1}^{m} \sum_{\substack{j=1 \\ j \neq k}}^{m} b_{j,t}^k p_{j,t}^k \quad .$$

This problem, with the assumed linear form of cost equation (4.10), can readily be solved by linear programming techniques. A bounded variables procedure is used so that all of the constraints from equations (4.8) and (4.9) can be omitted from explicit consideration in the model.

Solution Procedure

Let the capacity of each reservoir be a function of the decision variables equation (4.1), then the general capacity expansion problem for the multiple-reservoir systems is to minimize:

$$(4.11) \qquad \sum_{k=1}^{m} \sum_{t \varepsilon \Omega_T} \sum_{s=1}^{N_k} K_{t,s}^k y_{t,s}^k + \sum_{t=1}^{T} \sum_{k=1}^{m} (a_t^k x_t^k + \sum_{\substack{j=1 \\ j \neq k}}^{n} b_{j,t}^k p_{j,t}^k) \quad ,$$

subject to constraints (4.6), (4.7), (4.8), (4.9), $Y_{t,s}^k \varepsilon \{0,1\}$ and

$$(4.12) \qquad \sum_{t \varepsilon \Omega_T} y_{t,s}^k \leq 1 \qquad \text{for all } k, s.$$

Constraint set (4.12) insures that each segment for each reservoir is used at most one time.

For general reservoir systems, the number of operation and pumping variables is very large in comparison to the number of possible reservoir segments. If all variables are considered simultaneously, the resulting problem is a very large mixed integer-continuous linear program. The difficulties encountered in solving this type of problem indicates that a decomposition scheme could be used to improve the computational efficiency. The nonlinear decomposition technique of Benders (Lasdon, 55) presents an attractive solution procedure. It has been successfully applied to similar problems such as warehouse location problems by Balinski (2) and to plant location problems by Balinski and Wolfe (3) and Lasdon (55).

For simplicity of notation, the general problem of minimizing equation (4.11) subject to (4.6), (4.7), (4.8), (4.9) and (4.12) is written in matrix form as:

(4.13) minimize $\underline{ax} + \underline{by}$

subject to

$$\underline{Ax} + \underline{Fy} \leq \underline{c}$$

$$\underline{x} \in X$$

$$\underline{y} \in Y$$

where $\underline{x}$ represents the operational decision variables x_t^k and

$p^k_{j,t}$ with $\underline{a}$ as their cost coefficients, etc. Problem (4.13) lends itself immediately to Benders' problem.

Benders' decomposition. Benders' decomposition algorithm for mixed-variable programming problems assumes an initial problem (P1) form:

$$\text{(P1)} \qquad \text{minimize} \quad \underline{c}'\underline{x} + f(y)$$

$$\text{subject to} \qquad \underline{Ax} + F(y) \geq \underline{b}$$

$$\underline{x} \geq \underline{0} \qquad \underline{y} \in S \, .$$

The matrix $\underline{A}$ is mxn , $\underline{x}$ and $\underline{c}$ n vectors, $\underline{y}$ a p vector, f a scalar-valued function of $\underline{y}$, $\underline{F}$ an m vector whose components are functions of $\underline{y}$, $\underline{b}$ an m vector, and S an arbitrary subset of the p-space. The functions $f(\underline{y})$ and $\underline{F}(\underline{y})$ need not be linear and in problem (4.13) $\underline{F}(\underline{y})$ is integer. Therefore problem (P1) is a mixed integer-continuous linear problem. Problem (P1) is converted (Lasdon, 55) to the master problem:

$$\text{(MP2)} \qquad \text{minimize} \quad Z$$

$$\text{subject to} \qquad Z \geq f(\underline{y}) + [\underline{b} - \underline{F}(\underline{y})]^T \, \underline{u}_i^p \qquad i \in I_1$$

$$[\underline{b} - \underline{F}(\underline{y})]^T \, u_i^r \leq \underline{0} \qquad i \in I_2$$

$$\underline{y} \in S$$

where I_1 and I_2 are proper subsets of the integers $1,\ldots,n_p$ and $1,\ldots,n_r$, respectively. Also, $\underline{u}_i^p$, $i=1,\ldots,n_p$ and $\underline{u}_i^r$, $i=1,\ldots,n_r$ are the extreme points and extreme rays, respectively, of the polyhedron

$$P = \{ \underline{u} \mid \underline{A}'\underline{u} \leq \underline{c},\ \underline{u} \geq \underline{0} \}$$

associated with the dual constraint set:

$$\text{maximize} \quad [\underline{b} - \underline{F}(\underline{y})]^T \underline{u}$$

$$\text{subject to} \qquad \underline{A}'\underline{u} \leq \underline{c}$$

$$\underline{u} \geq \underline{0} \quad .$$

The primal of this problem is:

$$\text{minimize} \quad \underline{c}'\underline{x}$$

$$\text{subject to} \qquad \underline{Ax} \geq \underline{b} - \underline{F}(\underline{y})$$

$$\underline{x} \geq \underline{0}$$

which is obtained by fixing $\underline{y}$ in (P1). If P is empty the primal is unbounded as is the original problem (P1). If P is nonempty the dual takes on an extreme point of P or approaches $+\infty$ along an extreme ray of P . If the dual approaches $+\infty$ along an extreme ray then the primal is infeasible, however this is contrary to the assumption that $\underline{y}$ is contained in the set

$$R = \{ \underline{y} \mid [\underline{b} - \underline{F}(\underline{y})]^T \underline{u}_i^r \leq \underline{0} \ , \quad i=1,\ldots,n_r, \quad y \in S \} \quad .$$

Therefore only the extreme points $\underline{u}_i^p$, $i=1,\ldots,n_p$ of P need to be considered. Solution of the original problem (P1) is reduced to solving problem (MP2) for $(Z^0,\underline{y}^0)$ and then solving the linear program:

$$\text{minimize} \quad \underline{c}'\underline{x}$$

subject to

$$\underline{Ax} \geq \underline{b} - \underline{F}(\underline{y}^0)$$

$$\underline{x} \geq \underline{0}$$

to obtain the optimal values of $\underline{x}$.

Benders' algorithm, therefore, involves iteration between two problems. The first is the problem (MP2) in the variables $(Z,\underline{y})$ to which constraints are successively added. The second is the dual linear program (or the primal) which tests the optimality of a solution to (MP2) and, if necessary, provides a new constraint.

The following steps are a summary of the iterative procedure.

Step 1. Initialize problem (MP2), where a few or none of its constraints are binding.

Step 2. Solve the integer problem (MP2). If (MP2) is infeasible, so is (P1). Otherwise, either obtain a finite optimal solution $(Z^0,\underline{y}^0)$ or the information that the solution is unbounded. If the solution is unbounded set $Z^0 = -\infty$, let $\underline{y}^0$ be an arbitrary element of S and procede to Step 3.

Step 3. Solve the dual linear program (or the primal, if it is feasible). If the dual is infeasible then the original problem (P1) has an unbounded solution. If the dual is unbounded, go to Step 6.

Step 4. If the optimal objective value obtained in Step 3 is equal to $Z^0 - f(\underline{y}^0)$, the solution $(Z^0, \underline{y}^0)$ solves (MP2). If $\underline{x}^0$ solves the dual then $(\underline{x}^0, \underline{y}^0)$ solves (P1).

Step 5. If the optimality test in Step 3 is not passed and the dual has a finite optimal solution, which is the extreme point $\underline{u}^0$, then

$$Z^0 < f(\underline{y}^0) + [\underline{b} - \underline{F}(y^0)]^T \underline{u}^0$$

and the current solution to (MP2) does not satisfy the constraint,

$$Z \geq [\underline{b} - \underline{F}(\underline{y})]^T \underline{u}^0 + f(\underline{y})$$

which is added to (MP2) and return to Step 2.

Step 6. When the dual has an unbounded solution, the simplex method locates an extreme ray $\underline{v}^0$ and extreme point $\underline{u}^0$ such that the dual objective approaches $+\infty$ along the half-line

$$\underline{u} = \underline{u}^0 + \lambda \underline{v}^0 \quad , \quad \lambda \geq 0 \quad .$$

The inequality

$$[\underline{b} - \underline{F}(\underline{y}^0)]^T \ \underline{v}^0 > \underline{0}$$

is satisfied, so $\underline{y}^0$ does not satisfy the constraint

$$[\underline{b} - \underline{F}(\underline{y})]^T \ \underline{v}^0 \leq \underline{0} \quad .$$

This constraint is then added to (MP2). If, in addition

$$Z^0 < f(\underline{y}^0) + [\underline{b} - \underline{F}(\underline{y}^0)]^T \ \underline{u}^0$$

for the extreme point $\underline{u}^0$ the constraint

$$Z \geq [\underline{b} - \underline{F}(y)]^T \ \underline{u}^0 + f(\underline{y})$$

is added to (MP2) and return to Step 2.

The procedure will terminate in a finite number of iterations, either with the information that (P1) is infeasible or unbounded, or with an optimal solution to (P1).

Decomposition results. The result of the decomposition is the conversion of the problem to an iteration procedure between two problems. The master problem is to solve for the continuous variable Z and binary variables $\underline{y}$ from

(4.14) minimize Z

subject to $Z \geq \underline{by} + [\underline{c} - \underline{F}(\underline{y})]^T \underline{u}_i^p \qquad \forall\ i$

$[\underline{c} - \underline{F}(\underline{y})]^T \underline{u}_i^r \geq \underline{0} \qquad \forall\ i$

and $\sum_{t \in \Omega_T} y_{t,s}^k \leq 1 \qquad \forall\ k,s$.

Where $\underline{u}_i^p$ and $\underline{u}_i^r$ are the extreme points and extremal rays, respectively, of the linear problem, for fixed $\underline{y}$,

(4.15) minimize $\underline{ax}$

subject to $\underline{Ax} \leq \underline{c} - \underline{Fy}$

$\underline{x} \in \overline{\underline{X}}$

Since the number of extreme points and extremal rays can be quite large even for small linear programs, relaxation is applied to problem (4.14). Thus, only a small number of constraints are utilized at one time. For fixed $\underline{y}$'s, the linear problem (4.15) is solved ending with an extreme point $\underline{u}_i^p$ if the problem is feasible or with an extremal ray $\underline{u}_i^r$ if it is not feasible. If the associated constraint for problem (4.14) is satisfied, then the optimal solution to the original problem has been obtained. Otherwise, a new constraint is appended to problem (4.14). Problem (4.14) is resolved and the procedure repeated. Finite convergence for this procedure is guaranteed since there are only a finite number of rays and extreme points for problem (4.15).

The main difficulty of the capacity expansion reservoir system under consideration is one of obtaining a feasible system. With reservoir capacity limitations and a network of connections between reservoirs, the isolation of the reservoir whose capacity should be increased, and by how much, is not straightforward. In the analysis of a reservoir with infeasible capacity, decisions must be made concerning the effects of increasing the capacity of other reservoirs in the system, on the reservoir under consideration. These decisions are reflected in the problem by the development of an extremal ray. The method of generating the extremal rays is the critical factor in the solution of this problem. Rays which do not restrict capacities to nonoptimal levels must consider the interplay between reservoirs. That is, consideration of water that can be moved to other reservoirs in the system to minimize operating cost, meet demands, and constraint requirements.

The ray generation procedure utilized is to first isolate the most violated reservoir constraint in the linear programming subproblem. Once this constraint is determined, the inflows from all other reservoirs are analyzed. For all reservoirs which contribute nonzero inflows into the specified reservoir, the possibility of increasing these reservoir capacities as well as the isolated reservoir should be considered. Estimates are made of reduced contribution for increased capacities in these reservoirs. Thus, a constraint is added to the integer master problem which takes into consideration expansions in other reservoirs along with the infeasible reservoir. This extremal ray or additional constraint insures that the previously obtained infeasibility will be decreased, but not necessarily removed.

The integer master problem (4.14) is then solved yielding a new expansion policy. This policy is tested for operational feasibility by the linear programming subproblem (4.15). This procedure is repeated until the optimal policy has been determined.

Example Problem

Consider a three-reservoir two time period capacity expansion problem. The system of connected reservoirs is displayed in Figure 4.1. The data relative to each reservoir for this problem is contained in Table 4.1. Each reservoir has three possible expansion segment sizes for each time period. These segments may be utilized in any order. The cost of capacity expansion in each of the possible time periods varies. For this example the possible capacity expansion periods coincide with the reservoir systems operating periods. The capacity expansion data by time period is listed in Table 4.2.

The technique is started by defining an initial feasible reservoir capacities scheme. The scheme chosen for this example is to incorporate in period one reservoir segments one and two for reservoir one, segments two and three for reservoir two, and segment one of reservoir three. In the initial capacities scheme the

Table 4.1

Reservoir Data Used in Example Analysis

Parameters	Reservoir One TIME 1	Reservoir One TIME 2	Reservoir Two TIME 1	Reservoir Two TIME 2	Reservoir Three TIME 1	Reservoir Three TIME 2
Demands	6	8	5	7	10	7
Pumping Capacity	0	0	10	10	5	5
Pumping Profit	0	0	-.75	-.80	0.65	0.70
Max. Inflows	11	30	10	15	12	20
Min. Inflows	6	25	9	14	8	17
Fixed Capacity	0	0	0	1	1	0
Min. Inventory	3	3	4	2	3	4
Max. Release	7	8	15	12	20	20
Min. Release	1	3	2	3	1	1
Release Profit	1	1	-2.0	-2.1	0	0
Evaporation	1.0	.95	1.0	.97	1.0	.98
Starting Inv.	8		20		6	

Table 4.2

Capacity Expansion Data Used in Example Analysis

Parameters	Reservoir One TIME 1	Reservoir One TIME 2	Reservoir Two TIME 1	Reservoir Two TIME 2	Reservoir Three TIME 1	Reservoir Three TIME 2
Seg. 1 Capacity	5	5	10	10	10	10
Seg. 2 Capacity	5	5	10	10	6	6
Seg. 3 Capacity	15	15	10	10	5	5
Seg. 1 Cost	52	56	102	111	102	111
Seg. 2 Cost	52	56	102	111	62	67
Seg. 3 Cost	252	56	52	56	52	56

the only expansion to period two is for segment three of reservoir one. Utilizing the initial feasible capacities, the operating cost is 14.39 units. Since the linear programming solution is feasible, an extreme point is generated for the integer master program problem (4.14). However, no constraints (extremal rays) for the integer problem have been generated to insure continued feasibility of the reservoir system operation. Thus, the optimal integer solution is not to use any reservoir capacity segments. These results are displayed in iterations 0 and 1 in Table 4.3. Table 4.3 lists the linear and integer program results by iteration for the complete solution procedure.

Given that no reservoir expansion should be made, the resulting reservoir system operation is infeasible. The maximum violation of a constraint occurs for reservoir one in time period two with an infeasibility of 18.91 units. This result is obtained from the linear programming subproblem. Thus, an extremal ray must be generated to insure that this infeasibility is decreased. Since no releases from the other reservoirs flow into reservoir one, the only possible interplay is pumping. During period two, no pumping into the reservoir was made (problem (4.15) results). Hence, the infeasibility needs to be covered by expansion in reservoir one. The infeasibility occurred in time period two, therefore, the expansion can be made in either period one or period two. The corresponding integer program constraint is generated and the process repeated. For the solution to this example problem, the number of iterations is seven and the iteration results are displayed in Table 4.3.

Table 4.3

Iteration Results for Subproblems

ITERATION	L.P. SOLUTION	INTEGER SOLUTION
0		$Y^1_{2,3} = 1$ $Y^2_{1,2} = 1$ $Y^1_{1,1} = 1$ $Y^2_{1,3} = 1$ $Y^1_{1,2} = 1$ $Y^3_{1,1} = 1$
1	-14.39	All Y's = 0 $z = 67.37$
2	Res. 1, Time 2 Infeasible by 18.91	$Y^1_{1,2} = 1$ $Y^1_{2,3} = 1$ $z = 169.12$
3	Res. 1, Time 1 Infeasible by 11.0	$Y^1_{1,1} = 1$ $Y^1_{2,3} = 1$ $Y^1_{1,2} = 1$ $Y^2_{1,3} = 1$ $z = 246.63$

Table 4.3 (Continued)

ITERATION	L.P. SOLUTION	INTEGER SOLUTION	
4	Res. 3, Time 1 Infeasible by 6.0	$Y^1_{1,1} = 1$	$Y^1_{2,3} = 1$
		$Y^1_{1,2} = 1$	$Y^2_{1,3} = 1$
		$Y^3_{1,2} = 1$	
		$z = 308.63$	
5	Res. 1, Time 1 Infeasible by 4.0	$Y^1_{1,1} = 1$	$Y^1_{2,3} = 1$
		$Y^1_{1,2} = 1$	$Y^2_{1,1} = 1$
		$Y^2_{1,3} = 1$	$Y^3_{1,2} = 1$
		$z = 390.39$	
6	Res. 3, Time 2 Infeasible by 1.0	$Y^1_{1,1} = 1$	$Y^1_{2,3} = 1$
		$Y^1_{1,2} = 1$	$Y^2_{1,1} = 1$
		$Y^2_{1,3} = 1$	$Y^3_{1,1} = 1$
		$z = 430.39$	
7	-14.39 <u>Optimal Solution</u>	$Y^1_{1,1} = 1$	$Y^1_{2,3} = 1$
		$Y^1_{1,2} = 1$	$Y^2_{1,1} = 1$
		$Y^3_{1,1} = 1$	$Y^2_{1,3} = 1$
		$z = 430.39$	

The optimal capacity expansion scheme is to expand reservoir one with segments one and two in time period one and segment three is added in period two. For reservoir two, segments one and three are added in time period one. The expansion for the third reservoir consists of segment one in time period one. The resultant optimal capacity expansion cost is 416 units and 14.39 units for operating expense resulting in a total cost of 430.39 units. The computer coded documentations for Benders' decomposition applied to time phasing of capacity expansion multiple-reservoir models is in Appendix B.

Summary and Conclusion

In this chapter a procedure was developed for the analysis of time phasing of reservoir system development. The formulation was based on the multi-purpose stochastic reservoir model developed in Chapter III. The objective was to select the reservoir sizing, timing, and to establish operating policies such that the total cost associated with the system of linked reservoirs was minimized. The capacity expansion aspect was formulated as a mixed integer-continuous linear programming problem. The time periods for possible reservoir capacity expansion do not need to coincide with the operational time periods.

Due to the resulting problem size and its general structure, Benders' decomposition technique was applied. Benders' method allowed for the problem to be separated into a pure linear program and an almost pure integer program. Using Benders' approach, the size and computational speed for a solution to this type of problem is greatly enhanced.

CHAPTER V

LINKED RESERVOIRS WITH STOCHASTIC RELEASES SATISFYING FIXED DOWNSTREAM DEMANDS

The hydrologic uncertainty or the stochastic nature of streamflows is of essential consideration for developing models of water resource systems. The performance of a water resource system cannot, therefore, be predicted with absolute certainty. In particular, given the system design or level of inputs (such as storage capacities, sizes of hydropower plants, etc.) determine an operating policy (i.e., the reservoir release rules) which will satisfy a fixed downstream demand. This chapter is primarily concerned with a system of linked reservoirs having stochastic releases that must satisfy fixed downstream demands. The mathematical models presented are concerned with stochastic constraints associated with both linear and quadratic objective functions.

System Description

The Northeast Texas river basis system was selected to serve as a basis for model development and formulation. The Cypress Creek Basin in particular was chosen since it is situated in the Northeast corner of the State of Texas (Figure 5.1). The Cypress Creek Basin (Figure 5.2) is bounded on the North by the Sulphur River Basin and on the South by the Sabine River Basin. The basin is part of the Red River drainage system and is included in the

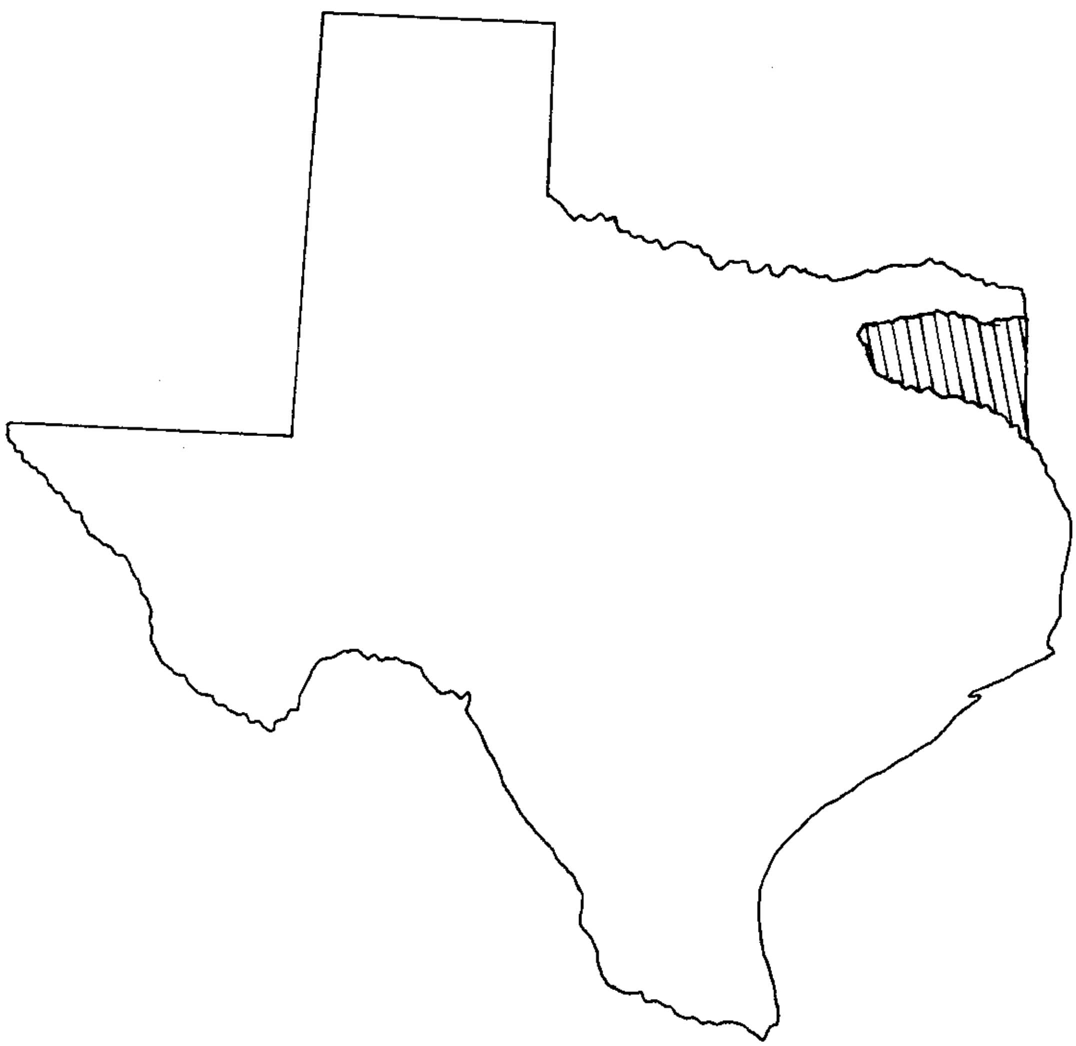

Figure 5.1: Cypress Creek Basin

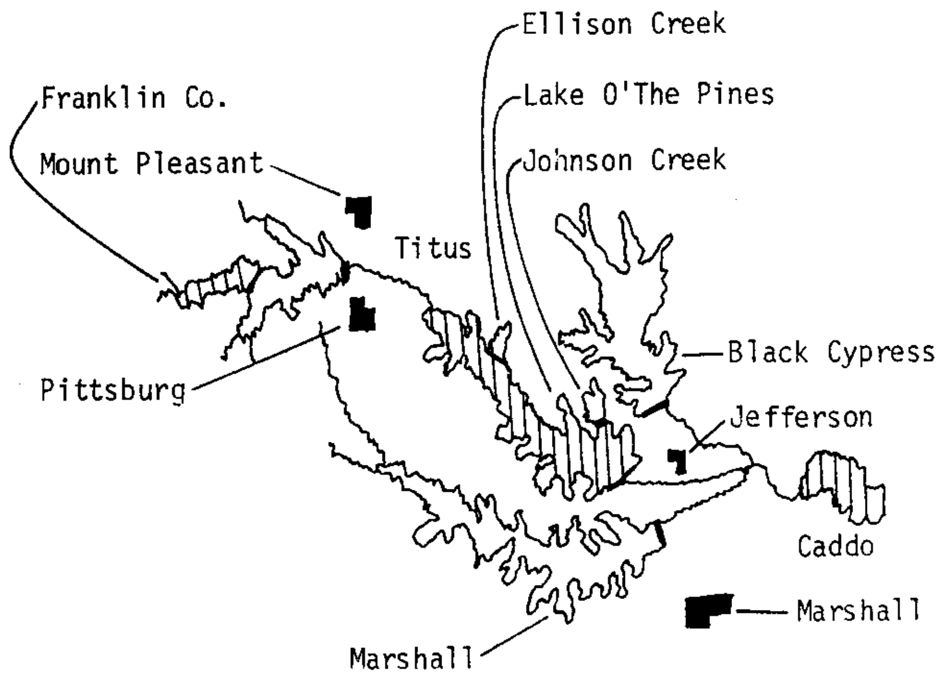

Figure 5.2: Cypress Creek Basin Enlargement

draft on the Red River. The total drainage area of the Cypress Creek Basin is approximately 2,812 square miles. Cypress Creek, which is in southeastern Hopkins County raises to an elevation of about 550 feet above sea level and flows southeasterly into Caddo Lake on the Texas-Louisiana borderline. The backwater area elevation of the streambed of Caddo Lake is approximately 168 feet.

The average annual rainfall in the basin ranges from about 48 inches at the Louisiana line to about 42 inches in the western part of the basin. The average annual runoff in the basin ranges from approximately 700 to 800 acre-feet per square mile in the western part of the basin to about 600 acre-feet in the southern part. The variations in runoff rates are due largely to physiography and geology variations within the basin.

The Cypress Creek Basin is densely forested and in 1964 less than one thousand acres was under irrigation. Scattered acreages of specialty crops and pasture lands is expected to be irrigated and to total not more than 5 thousand acres by 2020.

The chemical quality of streamflows throughout the Cypress Creek Basin is excellent. The discharge-weighted average concentrations of dissolved solids in principle streams generally ranges between 100 and 200 mg/l. The Lake O'The Pines on Cypress Creek normally contains about 100 mg/l of dissolved solids. Oil field drainage and other industrial wastes degrade the quality of Sugar, Glade, and Grays Creeks, which are tributaries of Little Cypress Creek. This problem needs to be corrected but its present effects are comparatively minor. The overall flood damage along Cypress Creek and its tributaries has been relatively minor although locally severe dammage has occurred.

There are presently three reservoirs in the Cypress Creek Basin, Lake O'The Pines, Ellison Creek, and Johnson Creek. Franklin County Reservoir is presently under construction and the existing Caddo Lake Dam is currently being replaced by a new downstream dam. This dam was designed so that it can be subsequently raised and the reservoir area enlarged. Congress has authorized the construction of projects and channel modifications to provide navigation up the Red River in Louisiana into Cypress Creek near Daingerfield, Texas.

Three major reservoirs have been proposed for construction under the Texas Water Plan (90). They are Titus County, Marshall, and Black Cypress. These reservoirs, plus existing and under-construction reservoirs, would theoretically supply all projected in-basin requirements to the year 2020. They would also develop an additional 641 thousand acre-feet of water per year.

Physical System and Model Formulation

The Cypress Creek Basin described in the previous section can be represented by a schematic diagram as shown in Figure 5.3. Since Ellison Creek and Johnson Creek reservoirs are small as compared to Lake O'The Pines they will be combined with Lake O'The Pines. Franklin County Reservoir will also be combined with Titus

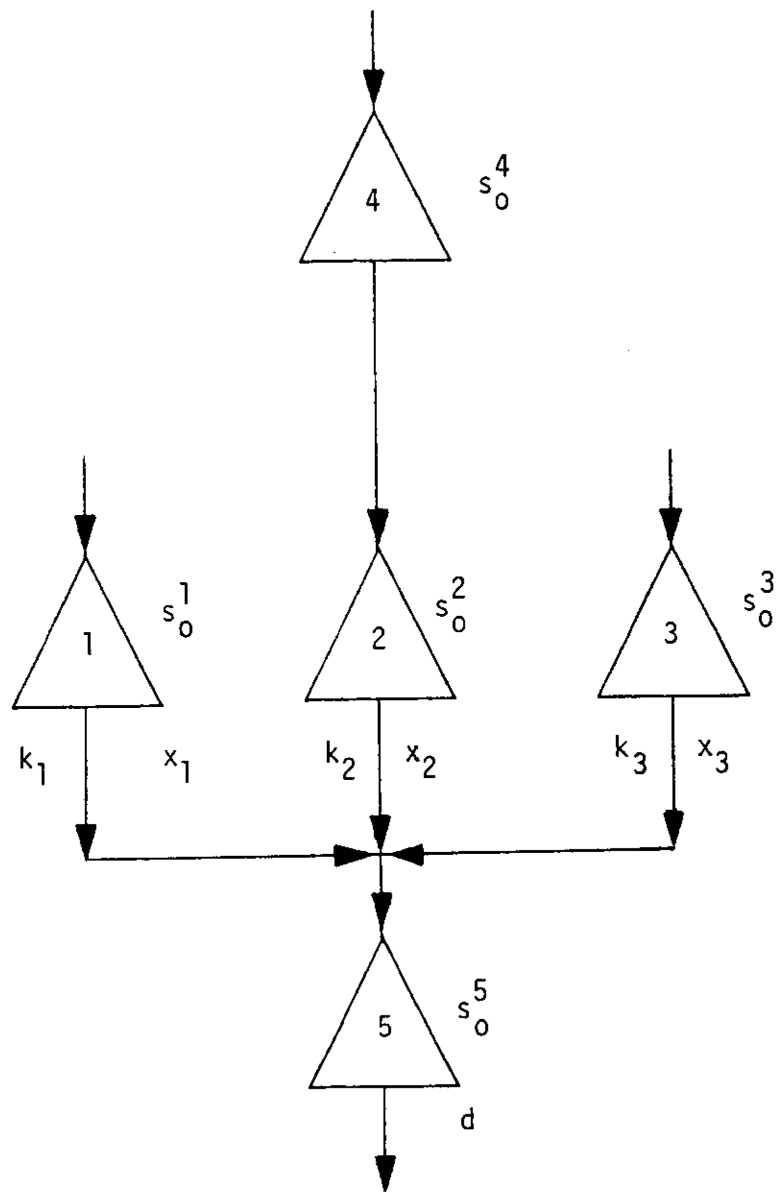

Figure 5.3: Schematic Diagram of Cypress Creek Basin

County Reservoir. The five reservoirs, two existing and three proposed, are identified as follows:

1 - Marshall

2 - Lake O'The Pines

3 - Black Cypress

4 - Titus

5 - Caddo.

The triangles correspond to existing and proposed reservoir sites. The arrows typify the river reach and the direction of streamflow.

The primary objective of this river basin system will be to meet a hypothetical navigation water demand at Caddo, Reservoir 5. Therefore, the problem is to keep the capacity of Reservoir 5 greater than or equal to a target capacity s_0^5 . The requirement to maintain the capacity is based on two stochastic constraints. Referring to Figure 5.3 the following notation is used to describe the system:

s_o^i - current level of reservoir i,

$\bar{s}_o^i$ - maximum capacity of reservoir i,

k_i - random variation of distribution of actual release per unit scheduled release $N(\mu,\sigma)$,

x_i - scheduled release at reservoir i,

d - demand or withdrawal during the period of investigation from Caddo, Reservoir 5.

Linear Objective Function Chance-Constrained

In this section two methods, developed by Curry and Wadsworth (18), for solving chance-constrained programming problems are utilized to obtain a solution to the reservoir system shown in Figure 5.3. The two solution techniques discussed are linear programming and contraction mapping, and parametric programming.

The cost function for the reservoir system is assumed to be

$$x_0 = \underline{cx}$$

where,

c_i - is the cost of meeting the target output,

n - is the number of reservoirs in the system.

The scheduled release placed on each reservoir is x_i , however only a certain percent of the water released will reach its destination. Assume that the actual flow is $k_i x_i$, $i=1,2,3$ which is the system supply available to meet the demand d at Reservoir 5. The uncertainty associated with meeting this demand is

$$(5.1) \qquad P\left\{ \sum_{i=1}^{3} k_i x_i \geq d \right\} \geq \alpha_1 .$$

It is also required that the maximum capacity $\bar{s}^5$ of Reservoir 5 must not be violated more than α_2 percent of the time, thus

(5.2) $$P \{ \sum_{i=1}^{3} k_i x_i \leq \bar{s}^5 + d - s_o^5 \} \geq \alpha_2 .$$

or

$$P \{ \sum_{i=1}^{3} k_i x_i + s_o^5 - d \leq \bar{s}^5 \} \geq \alpha_2 .$$

The scheduled releases x_i must be less than or equal to the inventory available at each reservoir

(5.3) $$0 \leq x_i \leq s_o^i \qquad i = 1,2,3,4 .$$

Also, the constraint

(5.4) $$x_4 - x_2 \leq \bar{s}^2 - s_o^2$$

will insure that the quantity of water released from Reservoir 4 for use by Reservoir 2 will not exceed Reservoir 2's capacity limit $\bar{s}^2$.

The structure of the problem is now of the form

$$\text{maximize } \underline{cx}$$

$$\text{subject to} \quad \underline{Ax} \leq \underline{b}$$

$$\underline{x} \geq \underline{0}$$

where the objective function and the right-hand side coefficients are assumed to be deterministic. Certain $a_{ij} \in \underline{A}$ are assumed normally and independently distributed, with a known mean $E \{ a_{ij} \}$

and variance σ_{ij}^2 . The procedure of chance-constrained programming (Taha, 89) is to reformulate the problem

$$\text{maximize} \quad \underline{cx}$$

$$\text{subject to} \quad P\{ \sum_{j=1}^{n} a_{ij}x_j \leq b_i \} \geq \alpha_i \qquad i = 1,2,\ldots,m$$

$$x_j \geq 0 \qquad j = 1,\ldots,n$$

to a deterministic equivalent. The name "chance-constraint" is due to the fact that each constraint

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i \ ,$$

is realized with a minimum probability of α_i , $0 \leq \alpha_i \leq 1$.

The goal is to convert the ith constraint into a legitimate linear programming constraint, so that the simplex method can be used in solving the problem. Consider the ith constraint

$$P\{ \sum_{j=1}^{n} a_{ij}x_j \leq b_i \} \geq \alpha_i \ .$$

Then, subtracting and dividing both sides of the inequality by the mean and variance respectively, the constraint becomes

$$P\left\{ \frac{\sum_{j=1}^{n} a_{ij}x_j - \sum_{j=1}^{n} E\{a_{ij}\}x_j}{(\sum_{j=1}^{n} \sigma_{ij}^2 \ x_j^2)^{1/2}} \leq \frac{b_i - \sum_{j=1}^{n} E\{a_{ij}\}x_j}{(\sum_{j=1}^{n} \sigma_{ij}^2 \ x_j^2)^{1/2}} \right\} \geq \alpha_i \ .$$

Since the a_{ij}'s are assumed to be normally distributed

$$\frac{\sum_{j=1}^{n} a_{ij}x_j - \sum_{j=1}^{n} E\{a_{ij}\}x_j}{\left(\sum_{j=1}^{n} \sigma_{ij}^2 \; x_j^2\right)^{1/2}}$$

is normally distributed with mean zero and variance one. Which means that

$$P\left\{ \sum_{j=1}^{n} a_{ij}x_j \le b_i \right\} = \Phi \left\{ \frac{b_i - \sum_{j=1}^{n} E\{a_{ij}\}x_j}{\left(\sum_{j=1}^{n} \sigma_{ij}^2 \; x_j^2\right)^{1/2}} \right\} ,$$

where Φ represents the cumulative distribution function, C.D.F., of the standard normal distribution.

Let K_{α_i} be the standard normal value such that

$$\Phi\,(K_{\alpha_i}) = \alpha_i \; .$$

Then the statement,

$$P\left\{ \sum_{j=1}^{n} a_{ij}x_j \le b_i \right\} \ge \alpha_i$$

is realized if and only if

$$\frac{b_i - \sum_{j=1}^{n} E\{a_{ij}\}x_j}{\left(\sum_{j=1}^{n} \sigma_{ij}^2 \; x_j^2\right)^{1/2}} \ge K_{\alpha_i} \quad .$$

This yields the following nonlinear constraint:

$$(5.6) \qquad \sum_{j=1}^{n} E\{a_{ij}\}\, x_j + K_{\alpha_i} \left(\sum_{j=1}^{n} \sigma_{ij}^2\, x_j^2\right)^{1/2} \le b_i \quad .$$

Now let

$$(5.7) \qquad \theta_i = K_{\alpha_i} \left(\sum_{j=1}^{n} \sigma_{ij}^2\, x_j^2\right)^{1/2} \quad ,$$

then (5.6) becomes

$$(5.8) \qquad \sum_{j=1}^{n} E\{a_{ij}\}\, x_j \le b_i - \theta_i \quad ,$$

and the equivalent problem of (5.5) is of the form

$$\text{maximize} \quad \underline{cx}$$

$$\text{subject to} \qquad \underline{Ax} \le \underline{b} - \underline{\theta}$$

$$\underline{x} \ge \underline{0}$$

<u>Linear programming and contraction mapping iterative procedure</u>. The solution to problem (5.9) is obtained by an iterative procedure of initially choosing $\underline{\theta}$, solving the associated linear subproblem with equation (5.7) relaxed, and then utilizing (5.7) to adjust the value of $\underline{\theta}$. The linear subproblem is solved by the simplex method of linear programming.

It is proved by Curry and Wadsworth (18) that the optimal solution $\underline{x}^*(\theta)$ of the parametric linear programming problem

(5.9) (with one chance-constraint) satisfies Lipschitz's condition. The proof is to consider the solution for $\theta \geq 0$. Then the vector of basic variables,

$$\underline{x}_B = \underline{B}^{-1}\,\underline{b} - \theta\underline{e}_1$$

$$\underline{x}_B = \underline{B}^{-1}\,\underline{b} - \theta\underline{B}^{-1}\,\underline{e}_1$$

$$\underline{x}_B = \underline{a} - \theta\underline{d} \qquad ,$$

is a linear function of θ . To show that $\underline{x}_B$ satisfies Lipschitz's condition, let

$$\theta^1 \leq \theta \leq \theta^2$$

be the range of θ where the set of basic variables indexed by B remains valid. Then $\underline{x}_B$ remains optimal and satisfies Lipschitz's conditions since:

$$||\underline{a} - \theta'\underline{d} - \underline{a} + \theta''\underline{d}|| = ||\underline{d}(\theta'' - \theta')||$$

$$||\underline{d}(\theta'' - \theta')|| \leq ||\underline{d}|| \cdot ||\theta'' - \theta'||$$

$$||\underline{d}|| \cdot ||\theta'' - \theta'|| \leq \beta||\theta'' - \theta'|| \quad \text{for all } \theta' , \theta'' \in [\theta^1,\theta^2].$$

With θ within its range of validity and $\theta \geq 0$, there are a finite number of critical points. The critical points are the change in the basis B . At each critical point θ^i , excluding $\theta^0 = 0$, the optimal solution is degenerate and the dual simplex procedure can be used to obtain a new optimal basis. Due to the degeneracy, the basic variables remaining in the basis will have the same value as before the basis change. The new basic variable enters at a zero level. Therefore, for each variable the parametric solution $\underline{x}_i^*(\theta)$ consists of connected linear segments each with a Lipschitz's constraint β_k . Then $\underline{x}_i^*(\theta)$ satisfies Lipschitz's condition with a constant

$$\beta = \max_k \beta_k \quad .$$

The technique was shown to converge to the optimal solution of problem (5.9) when the sequence

$$\{\theta^k\} \rightarrow \theta^*$$

where

(5.10) $$\theta^* = K_{\alpha_1} \left[\sum_{j=1}^{n} \sigma_{ij}^2 \, x_j^2 \, (\theta^*) \right]^{1/2} \quad .$$

Equation (5.7) satisfies Lipschitz's condition with constant α since the derivative exists and is bounded everywhere except at $\underline{x} = \underline{0}$. Equation (5.10) is thus the composition of two functions which are Lipschitz with a constant $\beta\alpha$. If $\beta\alpha < 1$, equation (5.10) is a contractive map and can be solved by the method of false position (Kunz, 52), that is

$$\theta^{k+1} = f(\theta^k)$$

and

$$\{\theta^{k+1}\} \rightarrow \theta^* \quad .$$

The solution is unique and the iterative procedure converges to the optimal solution of problem (5.9).

If the variance equation (5.7) is strongly contractive or if multiple chance-constraints have strongly contractive characteristics, the problem can be solved by the iterative procedure. Strongly contractive is defined to mean that the composition of the optimal solution variables and the variance constraint satisfies Lipschitz's condition, with a constant less than one.

The iterative procedure is:

Step 1. For any feasible solution $\underline{x}^0$ obtain $\underline{\theta}^1$ from equation (5.7).

Step 2. Solve a linear programming problem (5.9) for $\underline{\theta}^k$ fixed. The solution is $\underline{x}^k (\underline{\theta}^k)$.

Step 3. Compute $\underline{\theta}^{k+1}$ from (5.7) using the new $\underline{x}^k$ obtained in Step 2. Test $|\underline{\theta}^{k+1} - \underline{\theta}^k|$ to be less than a specified tolerance, if so, stop, otherwise return to Step 2.

Iterative procedure example problem. Assume that the probability of satisfying the demand at Reservoir 5 is to be maintained at least 95% of the time. Then equation (5.1)

$$P\{\sum_{i=1}^{n} k_i\ x_i \geq d\} \geq 95\%$$

is converted to the deterministic equivalent (5.8)

$$-\sum_{i=1}^{n} E\{k_i\}\ x_i \leq -d - \theta_1$$

where

$$\theta_1 = K_{\alpha_1}\ (\sum_{i=1}^{n} \sigma_i^2\ x_i^2)^{1/2}$$

and

$$K_{\alpha_1} = 1.645\ .$$

Likewise, to insure that the maximum capacity $\bar{s}^5$ is not violated by more than 95%, equation (5.2) is

$$P\{\sum_{i=1}^{3} k_i\ x_i \leq \bar{s}^5 + d - s_o^5\} \geq 95\%$$

and the deterministic equivalent

$$\sum_{i=1}^{3} E\{k_i\}\ x_i \leq \bar{s}^5 + d - s_o^5 - \theta_2$$

where θ_2 for this example is equal to θ_1 . Table 5.1 represents the data used and the resulting deterministic equivalent problem is:

(5.11) minimize $x_0 = x_1 + 2x_2 + 3x_3 + 4x_4$

subject to

$$
\begin{aligned}
x_1 \qquad\qquad\qquad &\le 5 \\
x_2 \qquad -x_4 &\le 1 \\
x_3 \qquad &\le 3 \\
-x_2 \qquad +x_4 &\le 2 \\
x_1 + x_2 + x_3 \qquad &\le 11 - \theta_1 \\
x_1 + x_2 + x_3 \qquad &\ge 6 + \theta_2
\end{aligned}
$$

$$\theta_1 = \theta_2 = 1.645 \left[.05x_1^2 + .05x_2^2 + .05x_3^2 \right]^{1/2}$$

$$x_1, x_2, x_3, x_4 \ge 0 \quad .$$

The solution to this problem is obtained in eight iterations to yield an optimal cost of 13.051 units and scheduled releases $\underline{x}^*$ of (5, 1, 2.017, 0). The stopping criterion used is a test on the change in the objective function value to be less than 0.0001. The solution results are listed in Table 5.2.

Parametric linear programming. Parametric linear programming is used to investigate the behavior of an optimal solution as a result of predetermined linear variations in the parameters of a problem. The linear programming problem before parameterization is defined as

$$\text{maximize} \quad x_0 = \underline{c}\underline{x}$$

subject to $$(\underline{A},\underline{I})\underline{x} = \underline{b}_0$$

$$\underline{x} \geq \underline{0} \quad .$$

The following four types of linear variations can be made:

1. variations in $\underline{c}$,
2. variations in $\underline{b}_0$,
3. variations in the nonbasic vector $\underline{b}_j$,
4. simultaneous variations in $\underline{c}$ and $\underline{b}_0$.

Let θ define the parameter of variation. Then the linear function necessary to change the requirements vector ${}^0\underline{b}_0$ is

$${}^{\theta}\underline{b} = {}^0\underline{b}_0 + \theta\underline{e}$$

where $\underline{e}$ is a specified, but arbitrary, vector and θ is a non-negative scalar. For the problem (5.9), it is desired to find the largest value θ for which the optimal basis vector $\underline{x}^*(\theta)$ yields a feasible solution.

The first step is to solve problem (5.9) for $\theta = 0$. If ${}^0\underline{x}_B$ is the corresponding optimal solution, then

$${}^0\underline{x}_B = {}^0\underline{B}^{-1}\, {}^0\underline{b}_0 \geq \underline{0} \quad .$$

The variations in the ${}^0\underline{b}_0$ vector can only effect the feasibility of the problem. Therefore only the feasibility of the problem need be investigated.

Table 5.1

Iterative Procedure Example Data

Reservoir i	1	2	3	4	5
c_i	1	2	3	4	0
s_i^o	5	1	3	2	7
$\bar{s}_i$	0	3	0	0	12
$E\{k_i\}$	1	1	1	1	0
σ_i^2	.05	.05	.05	0	0

Table 5.2

Iterative Procedure Results

Iteration	x_0	x_1	x_2	x_3	x_4
1	7	5	1	0	0
2	12.627	5	1	1.876	0
3	12.995	5	1	1.998	0
4	12.044	5	1	2.014	0
5	13.050	5	1	2.017	0
6	13.051	5	1	2.017	0
7	13.051	5	1	2.017	0
8	13.051	5	1	2.017	0

In order to determine the critical values of θ, the procedure is initiated by using the solution ${}^{0}\underline{x}_B$ obtained at $\theta = 0$. Next let α and β be two consecutive critical values of θ, $(\alpha \leq \beta)$, where it is assumed that the basic solution at $\theta = \alpha$ is known and given by ${}^{\alpha}\underline{x}_B$. The next critical value $\theta = \beta$ is determined in the following manner.

The basic solution ${}^{\alpha}\underline{x}_B$ will remain feasible for some range of $\theta \geq \alpha$ as long as

$$ {}^{\alpha}\underline{B}^{-1}\ {}^{\theta}\underline{b}_0 \geq \underline{0} $$

is satisfied. Which can be expanded to

$$ {}^{\alpha}\underline{B}^{-1}\ {}^{\theta}\underline{b}_0 = {}^{\alpha}\underline{B}^{-1}\ ({}^{0}\underline{b}_0 + \alpha\underline{e}) + (\theta - \alpha)\ {}^{\alpha}\underline{B}^{-1}\ \underline{e} $$

or

$$ {}^{\alpha}\underline{B}^{-1}\ {}^{\theta}\underline{b}_0 = {}^{\alpha}\underline{B}^{-1}\ {}^{\alpha}\underline{b}_0 + (\theta - \alpha)\ {}^{\alpha}\underline{B}^{-1}\ \underline{e}\ . $$

Now let $({}^{\alpha}\underline{B}^{-1}\ {}^{\alpha}\underline{b}_0)_i$ and $({}^{\alpha}\underline{B}^{-1}\ \underline{e})_i$ be the ith element of ${}^{\alpha}\underline{B}^{-1}\ {}^{\alpha}\underline{b}_0$ and ${}^{\alpha}\underline{B}^{-1}\ \underline{e}$ respectively. Since

$$ {}^{\alpha}\underline{B}^{-1}\ {}^{\alpha}\underline{b}_0 \geq \underline{0}\ , $$

it follows from the condition

$$ {}^{\alpha}\underline{B}^{-1}\ {}^{\theta}\underline{b}_0 \geq \underline{0} $$

that

1. If $({}^{\alpha}\underline{B}^{-1}\ \underline{e})_i \geq 0$, for all i , then ${}^{\alpha}\underline{x}_B$ remains feasible for all $\theta \geq \alpha$.

2. If $({}^{\alpha}\underline{B}^{-1}\ \underline{e})_i < 0$, for at least one i , there exists a critical value, $\theta = \beta$, where

$$\beta = \alpha + \min_i \left[\frac{-({}^{\alpha}\underline{B}^{-1}\ \underline{b}_0)_i}{({}^{\alpha}\underline{B}^{-1}\ \underline{e})_i} \;\middle|\; ({}^{\alpha}\underline{B}^{-1}\ \underline{e})_i < 0 \right] .$$

Therefore, for $\theta > \beta$, ${}^{\alpha}\underline{x}_B$ will no longer be feasible. At $\theta = \beta$, an alternative basic solution, ${}^{\beta}\underline{x}_B$, can be obtained by using the dual simplex method. As the variable corresponding to β is the first to go infeasible, it is selected as the leaving variable $\theta = \beta$.

The procedure is repeated on ${}^{\beta}\underline{x}_B$ to obtain a new critical value of θ , that is, the range of θ over which ${}^{\beta}\underline{x}_B$ remains feasible. When the condition occurs that no feasible solutions exist for θ the procedure is terminated. The resulting solution set of θ's and ${}^{\theta}\underline{x}_B$ are next used to find the smallest value of θ that satisfies equation (5.7). The solution to (5.7) is then the optimal solution to the problem.

For multiple chance-constraints the changes in the requirement vector (the right-hand side) are not linear functions of θ . This makes it quite difficult to obtain the parametric solution. Therefore, only multiple chance-constraints having the same variance equation can be solved by this technique.

Parametric programming example. The example problem solved is identical to problem (5.11) solved by the iterative procedure. Application of the parametric programming method reveals the critical values of θ as 0, 2.5. The solution, x_0 (θ) and $\underline{x}$ (θ) is in Table 5.3.

Table 5.3

Parametric Linear Results

Range on θ	x_0	x_1	x_2	x_3	x_4
0.0	7	5	1	0	0
2.5	14.5	5	1	2.5	0

From Table 5.3 the critical value for θ that satisfies equation (5.7) is 2.01706. The resulting optimal cost x_0^* (θ) is 13.051 units. The scheduled releases $\underline{x}^*$ (θ) are (5, 1, 2.017, 0).

Quadratic Objective Function Chance-Constrained

In this section two methods, developed by Curry and Rice (17), for solving chance-constrained problems with quadratic objective functions are used to solve the reservoir system shown in Figure 5.3. The two solution techniques discussed are, quadratic programming and contraction mapping iterative procedure, and parametric quadratic programming.

The name quadratic programming refers to the problem of maximizing or minimizing a quadratic objective function subject to linear constraints, and non-negative restrictions on the decision variables. The quadratic programming problem differs from the linear programming problem in the respect that objective function also includes x_j^2 and $x_j x_k$ $(j \neq k)$ terms. The general quadratic problem is to find $x_1, x_2, \ldots, x_n$ so as to

$$\text{minimize} \quad \left\{ \sum_{j=1}^{n} c_j x_j - 1/2 \sum_{j=1}^{n} \sum_{k=1}^{n} q_{jk} x_j x_k \right\} ,$$

subject to
$$\sum_{j=1}^{n} a_{ij} x_i \leq b_i \ , \quad \text{for} \quad i=1,2,\ldots,m,$$

and
$$x_i \geq 0 \ , \quad \text{for} \quad j=1,2,\ldots,n,$$

where the q_{jk} are given constants such that $q_{jk} = q_{kj}$.

The cost function assumed for the reservoir system in Figure 5.3 is

$$\text{minimize} \quad \sum_{i=1}^{4} c_i \ (L_i - x_i)^2$$

The constants L_i are the scheduled releases, the c_i the cost of deviating from the schedule, and x_i the variable scheduled release from the ith, i=1,2,3,4, reservoir. The objective function represents the cost or profit deviation from the target releases. The constraints are (5.1), (5.2), (5.3), and (5.4). The chance-constraints are converted to their deterministic equivalent (5.8) and the problem is formulated as (5.9) only with a quadratic objective function. The quadratic problem to be solved with chance-constraints is

$$(5.12) \qquad \text{minimize} \quad x_0 = \sum_{i=1}^{4} c_i \, (L_i - x_i)^2$$

subject to

$$\underline{Ax} \leq \underline{b} - \theta \underline{e}$$

$$\underline{x} \geq \underline{0}$$

with

$$(5.13) \qquad \theta_i = K_{\alpha_i} \left(\sum_{j=1}^{n} \sigma_{ij}^2 \, x_j^2 \right)^{1/2} \quad .$$

Quadratic programming and contraction mapping iterative procedure. The solution to problem (5.12) is obtained by an iterative procedure of initially choosing $\underline{x}$, solving the associated quadratic subproblem with (5.13) fixed. Once a solution to (5.12) is obtained, by quadratic programming, the value of $\underline{\theta}$ is adjusted with the new $\underline{x}$ value and the problem resolved. The iterative procedure is the same as discussed for linear programming and contraction mapping.

Quadratic procedure example problem. Assume the probability of satisfying the demand at Reservoir 5 is maintained by 95%. Then equation (5.1):

$$P \left\{ \sum_{i=1}^{3} k_i x_i \geq d \right\} \geq 95\%$$

is converted to the deterministic equivalent (5.8)

$$- \sum_{i=1}^{3} E\{k_i\}\, x_i \leq -d - \theta_1 \quad ,$$

where

$$\theta_1 = K_{\alpha_i} \left(\sum_{i=1}^{n} \sigma_i^2\, x_i^2 \right)^{1/2}$$

and $K_{\alpha_i} = 1.645$. Likewise, to insure that the maximum capacity $\bar{s}_5$ is not violated by more than 95%, equation (5.2) is

$$P\left\{ \sum_{i=1}^{3} k_i x_i \leq \bar{s}_5 + d - s_5^0 \right\} \geq 95\%$$

and the deterministic equivalent

$$\sum_{i=1}^{3} E\{k_i\}\, x_i \leq \bar{s}_5 + d - s_5^0 - \theta_2$$

where θ_2 for this example is equal to θ_1. Table 5.4 represents the data used and the resulting deterministic equivalent problem is

$$\text{minimize} \quad x_0 = .5(1 - x_1)^2 + 1.0(1 - x_2)^2 + 1.5(1 - x_3)^2 + 2.0(1 - x_4)^2$$

subject to

$$x_1 \leq 5$$

(5.14)

$$x_2 - x_4 \leq 1$$

$$
\begin{array}{lllll}
 & & x_3 & & \leq 3 \\
 & & & x_4 & \leq 2 \\
 & -x_2 & & +x_4 & \leq 2 \\
x_1 & +x_2 & +x_3 & & \leq 11 - \theta_1 \\
x_1 & +x_2 & +x_3 & & \geq 6 + \theta_2
\end{array}
$$

$$\theta_1 = \theta_2 = 1.645(.05x_1^2 + .05x_2^2 + .05x_3^2)^{1/2}$$

$$x_1, x_2, x_3, x_4 \geq 0 \quad .$$

Table 5.4

Quadratic Iterative Procedure Example Data

Reservoir i	1	2	3	4	5
c_i	0.5	1.0	1.5	2.0	
L_i	1.0	1.0	1.0	1.0	
s_i^0	5.0	1.0	3.0	2.0	7.0
$\overline{s}_i$		3.0			12.0
$E\{k_i\}$	1.0	1.0	1.0	1.0	
σ_i^2	0.05	0.05	0.05		

The solution to this problem is obtained in seven iterations to yield an optimum cost of 6.1331 units and scheduled releases $\underline{x}^*$ of (3.6997, 2.1166, 1.8999, 1.1166). The solution results are listed in Table 5.5

Table 5.5

Quadratic Iterative Procedure Results

Iteration	x_1	x_2	x_3	x_4	θ_1	θ_2	x_0
1	2.6364	1.8182	1.5455	1.0000	0.0000	0.0000	-7.5455
2	3.4275	2.0713	1.8092	1.0713	1.3080	1.3080	5.0864
3	3.6332	2.1055	1.8777	1.1055	1.6164	1.6164	5.8669
4	3.6835	2.1139	1.8945	1.1139	1.6920	1.6920	6.0677
5	3.6959	2.1160	1.8986	1.1160	1.7106	1.7106	6.1177
6	3.6990	2.1165	1.8997	1.1165	1.7152	1.7152	6.1301
7	3.6997	2.1166	1.8999	1.1166	1.7163	1.7163	6.1331

Parametric quadratic programming. Parametric quadratic programming is a variation of parametric linear programming. The objective function of the linear program is replaced by a quadratic function. The optimal solution is obtained as a result of predetermined linear variations in the requirements vector. The quadratic programming problem before parameterization is defined in the previous section. Let θ define the parameter of variation. Then the linear function necessary to change the requirements vector $\underline{b}_0$ is

$$\underline{b} = \underline{b}_0 + \theta \underline{e}$$

where $\underline{e}$ is a specified, but arbitrary vector.

For the example presented in this section, parametric quadratic programming is used to solve the problem of the type (5.12) and (5.13). The particular application dealt with is the change (5.13) of the requirements vector of equation (5.12). The parametric change θ results in progressively raising the value of the objective function. The smallest value of θ which satisfies equation (5.13), with $\underline{x}$ a function of θ, results in an optimal solution to (5.9) (Curry and Rice, 17).

Parametric quadratic programming example. The example problem solved is identical to problem (5.14) solved by the iterative quadratic procedure. Application of the parametric quadratic programming method reveals the critical values of θ as 0.0, 0.6667, 2.5, 3.6667, 5.1667, 5.3571. The solution $x_0(\theta)$ and $\underline{x}(\theta)$ is in Table 5.6. From Table 5.6 the critical value for θ that satisfies equation (5.13) is 1.717. The resulting optimal cost $x_0^*(\theta)$ is 6.134 units. The scheduled releases $\underline{x}^*(\theta)$ are (3.7, 2.117, 1.9, 1.117).

Table 5.6

Parametric Quadratic Results

Range on θ	x_1	x_2	x_3	x_4
0.0	2.636	1.812	1.5	1.0
0.667	3.0	2.0	1.667	1.0
2.5	4.222	2.204	2.074	1.204
3.667	5.0	2.333	2.188	1.333
5.167	5.0	2.833	3.0	1.833
5.357	5.0	3.0	3.0	2.0

Summary and Conclusion

In this chapter the stochastic nature of streamflow was considered for the development of a water resource system. In particular a system was designed from an existing river basin. The basin served for model formulation and design. This system was designed to meet a demand placed on a particular reservoir. The flow into that reservoir was not known with absolute certainty. This uncertainty lead to a set of stochastic constraints on the reservoir inventory level. The stochastic constraints were then converted to their equivalent deterministic constraints.

Two types of objective function forms were appended to the linear constraint set, linear and quadratic. The resulting linear problem was solved by two solution techniques. The first technique was that of linear programming and contraction mapping. The second linear procedure was that of parametric linear programming. Both techniques lead to identical results based on the problem formulated.

The parametric approach was to obtain the parametric solution $\underline{x}^*(\theta)$ for problem (5.9) and then solve equation (5.7) for θ. For noncontractive problems, a unique solution was not guaranteed. For multiple chance-constraints the iterative procedure was found to be superior to the parametric procedure when the variance equations (5.7) are strongly contractive. The parametric procedure, however, can be used when the variance equations are identical.

The quadratic objective function with chance-constraints was solved by two techniques. The first technique is similar to the linear programming contraction mapping technique. It essentially uses the same logic with the linear simplex code replaced by a quadratic code developed by Schuermann (86). The second quadratic technique called parametric quadratic programming was also presented. The parametric changes were restricted to changes in the requirements vector. Like the linear parametric technique it can only be used on problems where the variance equations are identical. However, the parametric quadratic approach is applicable to any single chance-constrained quadratic problem.

CHAPTER VI

BICRITERIA OBJECTIVE FUNCTIONS FOR STOCHASTIC RESERVOIR MODELS

Water resource planners face the problem of developing alternative water resource systems at various points in time. This evaluation leads to an investment decision. The problem is to investigate how the relevant variables affect that decision.

A planner should allocate his planning resources to studying those variables which have the greatest relative impact on the planning decisions, assuming that there will be a positive payoff from such a study. For some variables there will be little or no reduction in return for expenditure of substantial additional funds. For others there may be relatively large returns for small expenditures. Even if the latter is the case, if the impact on the total system is small, because of the much larger relative importance of planning variables whose returns cannot be changed by additional expenditures, there may be no justification for further study.

This chapter will apply parametric variations in the objective function of linear and quadratic programming problems to investigate the importance of the return associated with the planning variables for chance-constrained connected multi-purpose reservoir systems. This approach of mathematical programming is referred to as bicriteria mathematical programming. Bicriteria programs often arise in the application of mathematical programming when there are incommensurate objectives to be extremized.

Once an optimum solution to a reservoir or system of reservoirs is attained, the analyst may find it desirable to study the effects of discrete changes in the cost coefficients in order to observe the change in the current optimal solution and the decision variables. One way to accomplish this is to solve the problem for each change desired. This, however, may be computationally inefficient. To improve efficiency, use can be made of the properties of the simplex solution procedure for linear programming. By using parametric linear programming it is possible to reduce the additional computations considerably.

Linear Objective Function

Parametric linear programming. In this section the method of parametric linear programming (Taha, 89) will be used to investigate the postoptimal solution of the reservoir model presented in the section System of Multi-Purpose Reservoirs in Chapter III. Parametric linear programming is used to investigate the behavior of an optimal solution as a result of predetermined linear variations in the parameters of a problem. The linear programming problem before parameterization is defined as

(6.1) maximize $x_0 = \underline{c}\underline{x}$

subject to

$$(\underline{A},\underline{I})\underline{x} = \underline{b}_0$$

$$\underline{x} \geq \underline{0} \quad .$$

The variation in $\underline{c}$ is the linear variation under investigation. Let θ define the parameter of variation. Then the linear function necessary to change the objective function parameters is

$$^{\theta}\underline{c} = {}^{0}\underline{c} + \theta\underline{e}$$

or

$$^{\theta}\underline{c} = ({}^{0}c_1, \ldots, {}^{0}c_{m+n}) + \theta(e_1, \ldots, e_{m+n}) \quad ,$$

where the parameter θ is assumed to be nonnegative. Problem (6.1) is first solved with $\theta = 0$ by the revised simplex method to obtain an optimal solution. Next the effect of the predetermined changes, $\underline{e}$, in the objective function, $\underline{c}$, are determined by postoptimality analysis (Taha, 89, Chapter 9). The resulting parametric changes in θ give the analyst the complete range of solutions that result from his choice of cost variations.

<u>Parametric programming example</u>. The linked reservoir system under consideration is illustrated in Figure 3.3 (p. 58). It is composed of three reservoirs, two of which have pumping capabilities. Random inflows and predetermined demands are assumed for each reservoir. Table 3.3 (p. 59) in Chapter III describes the state

variables assumed for each reservoir and time period. The objective is to minimize the operating cost of the system for the two time periods and then to investigate parametric changes in the objective function. The parametric changes will affect the optimality of the operating policy. It will also affect the decision variables associated with changes in the objective function.

The decision variables to be determined for each time period are: the units of water released from reservoir one, two, and three; and the number of units of water pumped into reservoir one from reservoir two and three. The total number of variables to be determined is the product of the number of time periods with the sum of the number of reservoirs and pumping variables. The decision variables must satisfy the equivalent deterministic constraints (3.15), (3.16) and the upper and lower limits on release (3.14) discussed in Chapter III.

In order to solve this problem by the revised simplex procedure, slack and artificial variables must be appended to the appropriate inequalities. This increases the size of the problem to twenty-eight constraints and forty-eight variables. Ten of the original variables are of concern.

The structure of the problem is of the form:

$$\text{minimize} \quad z = (\underline{h} + \theta \underline{e})\underline{y}$$

subject to $$(\underline{A},\underline{I})\underline{y} = \underline{b}$$

$$\underline{\ell} \leq \underline{y} \leq \underline{u}$$

where $$\underline{u} \geq \underline{\ell} \geq \underline{0} \quad .$$

The values chosen for the cost coefficient of the objective function are:

$$\underline{h} = [1.0, -2.0, 0.0, -.75, .65, 1.0, -2.1, 0.0, -.80, .70] \quad .$$

In the cost coefficient vector h_2 and h_7 correspond to water released, x_1^2 and x_2^2, from the reservoir system (Figure 3.3) in time period one and two, respectively. This means that if these coefficients are parametrically changed the resulting optimal cost might be lowered. Or stated another way, a loss in profit might result in a net return. The parametric objective function vector selected to investigate these changes is then,

$$\underline{e} = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0] \quad .$$

The linear constraints are given by the matrix $\underline{A}$ and the vectors $\underline{b}$, $\underline{\ell}$, $\underline{u}$ as

$$
\underline{A} = \begin{bmatrix}
-1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-.95 & 0 & 0 & .95 & .95 & -1 & 0 & 0 & 1 & 1 \\
.95 & 0 & 0 & -.95 & -.95 & 1 & 0 & 0 & -1 & -1 \\
.97 & -.97 & .97 & -.97 & 0 & 1 & -1 & 1 & -1 & 0 \\
-.97 & .97 & -.97 & .97 & 0 & -1 & 1 & -1 & 1 & 0 \\
0 & 0 & -.98 & 0 & -.98 & 0 & 0 & -1 & 0 & -1 \\
0 & 0 & .98 & 0 & .98 & 0 & 0 & 1 & 0 & 1
\end{bmatrix}
$$

$$
\underline{b} = \begin{bmatrix} -3 \\ 5 \\ -5 \\ 20 \\ 7 \\ 1 \\ -3.9 \\ 5.9 \\ 6.45 \\ 19.55 \\ 6.92 \\ 2.08 \end{bmatrix}, \quad \underline{\ell} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \\ 3 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \underline{u} = \begin{bmatrix} 7 \\ 15 \\ 20 \\ 10 \\ 5 \\ 8 \\ 12 \\ 20 \\ 10 \\ 5 \end{bmatrix} .
$$

Application of the revised simplex method in conjunction with parametric linear variations in the cost vector reveals the following critical values of θ and $\underline{y}$:

$$\begin{bmatrix} \theta \end{bmatrix} \quad \begin{bmatrix} 0 \end{bmatrix} \quad \begin{bmatrix} 2.0 \end{bmatrix} \quad \begin{bmatrix} 2.1 \end{bmatrix}$$

$$\underline{y} = \begin{bmatrix} x_1^1 \\ x_1^2 \\ x_1^3 \\ p_{11}^2 \\ p_{11}^3 \\ x_2^1 \\ x_2^2 \\ x_2^3 \\ p_{12}^2 \\ p_{12}^3 \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ 1 \\ 4 \\ 0 \\ 8 \\ 3 \\ 1 \\ 4.85 \\ 1 \end{bmatrix} \begin{bmatrix} 7 \\ 15 \\ 1 \\ 4 \\ 0 \\ 8 \\ 3 \\ 1 \\ 4.85 \\ 1 \end{bmatrix} \begin{bmatrix} 7 \\ 15 \\ 1 \\ 4 \\ 0 \\ 8 \\ 12 \\ 1 \\ 4.85 \\ 1 \end{bmatrix} .$$

The optimal cost of operating the system is for $\theta = 0$ a cost of -16.11 units or a profit of 16.11 units. When $0 < \theta \leq 2$, there is a cost of 7.88 units which is a corresponding loss in profit. For $2 < \theta \leq 2.1$, the cost becomes 9.68 units.

It is interesting to notice that when $\theta = 2$ the release in time period one, x_1^2 , changed from 9 to 15 units. This means that it will cost 8.23 units to release an additional 6 units of water.

Also, when $\theta = 2.1$ the release in time period two, x_2^2 , changes from 3 to 12 units. Now, however, the 9 units of release only cost 1.80 units.

Linear and Quadratic Programming

Parametric quadratic programming. In this section a method of quadratic programming, developed in a paper by Wolfe (97), will be used to find a solution to the reservoir model presented in Chapter V. In Chapter V a quadratic objective function with chance-constraints was solved parametrically for changes in the requirements vector. In this section the quadratic objective function will be parametrically changed. The parametric change will be due to the addition of a linear objective function. The objective function can be thought of as bicriteria, having both a linear and quadratic function to be extremized. The quadratic function is first optimized and then parametric changes associated with the linear function are optimized.

Parametric quadratic programming example. The cost function assumed for the reservoir system in Chapter V, Figure 5.3 (p. 97) is

$$\text{minimize} \quad \sum_{i=1}^{4} c_i(L_i - x_i)^2 + \theta \sum_{i=1}^{4} x_i \quad .$$

The constants L_i are the scheduled releases, the c_i the cost of deviating from the schedule, and x_i the variable scheduled release from the ith reservoir. The objective function represents the cost deviation from the target releases plus an additional linear cost. The additional linear cost might represent a cost associated with release for recreation, irrigation or water quality and quantity control. The analyst can then observe the additional cost associated with a parametric change θ of adding additional purposes for the scheduled release, x_i, of water.

The parametric quadratic problem solved is

$$\text{(6.2)} \qquad \text{minimize} \quad x_0 = \sum_{i=1}^{4} c_i (L_i - x_i)^2 + \theta \sum_{i=1}^{4} x_i$$

subject to

$$\underline{Ax} = \underline{b}$$

$$\underline{x} \geq \underline{0} \quad .$$

The value chosen for the cost coefficients of the objective function are,

$$\underline{c} = (.5,\ 1.0,\ 1.5,\ 2.0)$$

$$\underline{L} = (1.0,\ 1.0,\ 1.0,\ 1.0) \quad ,$$

and the linear constraints are given by specifying the matrix $\underline{A}$ and the vector $\underline{b}$ as:

$$\underline{A} = \begin{bmatrix} -1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix} , \quad \underline{b} = \begin{bmatrix} -6 \\ 11 \\ 5 \\ 1 \\ 3 \\ 2 \\ 2 \end{bmatrix} ,$$

Application of the simplex method for quadratic programming yields the values of θ as 0.0, .778, 2.0, 3.0, 5.0. The solution x_0 , $\underline{x}$ are presented in Table 6.1. From Table 6.1 the analyst can observe the resulting changes in the cost, x_0 for changes in the parameter θ . When $\theta = 0$ the linear function has no effect on the quadratic function, which represents the cost or profit

Table 6.1

Parametric Quadratic Objective Function Results

θ	x_0	x_1	x_2	x_3	x_4
0.0	5.519	3.556	1.259	1.185	0.259
0.778	12.691	2.778	1.778	1.444	0.778
2.000	21.000	2.000	2.000	2.000	2.000
3.000	29.000	3.000	3.000	3.000	2.000
5.000	55.000	5.000	3.000	3.000	2.000

deviation from the target releases. This result indicated the single effect of the operating cost. No consideration is given to the additional linear expenses. However, when θ is not equal to zero the linear function, which might represent a cost for water quality control, increases the total operating cost. The changes in the scheduled release variables x_i can be observed as the linear function is brought into solution. This gives the analyst the ability to make realistic weighting consideration about scheduled releases. He can observe the cost implications with regard to individual scheduled releases.

Summary and Conclusion

Reservoir systems were modeled and solved by both linear and quadratic programming to yield an optimal solution. This analysis, however, does not include all the additional variations in the cost function associated with certain decision variables. The analyst has the choice of either computing separately each additional variation or he can use postoptimal analysis to observe the variations in the decision variables and cost function. This chapter presented the application of postoptimal analysis to illustrate how the analyst can obtain a set of optimal solutions. From the set of solutions he can select the solution which best meets his budget requirements. He can then eliminate the variables which will produce little or no return for additional expenditures. He can also determine the relative importance of the decision variables whose returns cannot be changed by additional expenditures.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The preceding chapters of this text were divided into four major sections to present methodology for the systematic planning of regional water management. Chapter III presented the stochastic formulation for a single multi-purpose reservoir and its extension to systems of multi-purpose reservoirs. Chapter IV applied this formulation to the problem of time phasing of reservoir capacity expansion with reservoir operation. In Chapter V the stochastic nature of streamflow is considered based on two types of objective functions. Parametric analysis for the requirements vector is applied in each case. The final chapter, Chapter VI, applies postoptimal analysis to linear and quadratic objective functions. The parametric objective function analysis gives a set of optimal solutions and the associated decision variables. The analyst can then decide which solution best meets his requirements and budget constraints.

In Chapter III an extension of the single multi-purpose stochastic constrained reservoir model was presented. The linear decision rules utilized by ReVelle et al. (83) and Loucks (63) are omitted in the model. The purpose of using linear decision rules is to disconnect the release in the nth period from the ending inventory level in period n-1. The advantage of the linear decision rule is that only the random inflow for the current period need be considered. However, the actual quantity to be released in the nth period is not known until the random inflow in periods 1 through n-1 are observed. Thus, for planning purposes where operation of the reservoir is important or when the release variables are represented in the objective function, this formulation is unsatisfactory since releases are actually random variables and not exactly determined by the reservoir planner.

The formulation proposed in the section on Single Multi-Purpose Reservoirs requires that the distributions of sums of random inflows for all time periods be obtained. This is a relatively simple task for models with a large number of time periods. Since by the central limit theorem (Parzen, 80), the distribution of the sums derived from the sampling of these parent distributions tends to become normal as the sample size increases.

By not using any form of decision rule, the constraints on upper and lower release quantities become deterministic and need not be represented by chance-constraint formulation. Also, quadratic or even general convex objective functions of the release quantities can be considered.

In the section on System of Multi-Purpose Reservoirs a mathematical model is developed for a linked system of multi-purpose reservoirs with stochastic unregulated inflows. The mathematical model is obtained as a straightforward generalization of the single reservoir model. The chance-constrained formulation for reservoir capacities and minimum inventory levels is converted to a linear system of constraints. Linear, quadratic, or even general convex objective functions can be appended to this system and the solution obtained.

If linear objective functions are assumed, which could be operational or of a capacity nature, very large problems can be solved. Since the cumulative inflows will be nearly normally distributed for these problems, their formulation and solutions are a matter of course. The problem of capacity expansion dealt with in Chapter IV is generally not well modeled as a continuous linear problem. Capacity expansion models are usually limited to fixed time periods and have nonlinear costs as a function of reservoir size.

In Chapter IV a procedure was developed for the analysis of time phasing of reservoir system development. The formulation was based on the multi-purpose stochastic reservoir model developed in Chapter III. The objective was to select the reservoir sizing, timing, and to establish operating policies such that the total cost associated with the system of linked reservoirs was minimized. The capacity expansion aspect was formulated as a mixed integer-continuous linear programming problem. The time periods for possible reservoir capacity expansion do not need to coincide with the operational time periods.

Due to the resulting problem size and its general structure, Benders' decomposition technique was applied. Benders' method allowed for the problem to be separated into a pure linear program and an almost pure integer program. Using Benders' approach, the size and computational speed for a solution to this type of problem is greatly enhanced.

In Chapter V the stochastic nature of streamflow was considered for the development of a water resource system. In particular a system was designed from an existing river basin. The basin served for model formulation and design. This system was designed to meet a demand placed on a particular reservoir. The flow into the reservoir was not known with absolute certainty. This uncertainty led to a set of stochastic constraints on the reservoir inventory level. The stochastic constraints were then converted to their equivalent deterministic constraints.

Two types of objective function forms were appended to the linear constraint set, linear and quadratic. The resulting linear problem was solved by two solution techniques. The first technique was that of linear programming and contraction mapping. The second linear procedure was that of parametric linear programming. Both techniques led to identical results based on the problem formulated.

The parametric approach was used to obtain the parametric solution to the problem formulated and then to solve the nonlinear variance equation for the particular solution. For noncontractive problems, a unique solution was not guaranteed. For multiple chance-constraints the iterative procedure was found to be superior to the parametric procedure when the variance equations are strongly contractive. The parametric procedure, however, can be used when the variance equations are identical.

The quadratic objective function with chance-constraints was solved by two techniques. The first technique used was similar to the linear programming contraction mapping technique. It essentially used the same logic with the linear simplex code replaced by a quadratic code developed by Schuermann (86). The second quadratic technique called parametric quadratic programming was also presented. The parametric changes were restricted to changes in the requirements vector. Like the linear parametric technique it can only be used on problems where the variance equations are identical. However, the parametric quadratic approach is applicable to any single chance-constrained quadratic problem.

Reservoir systems were modeled and solved by both linear and quadratic programming to yield an optimal solution. This analysis, however, does not include all the additional variations in the cost function associated with certain decision variables. The analyst has the choice of either computing separately each additional variation or he can use postoptimal analysis to observe the variations in the decision variables and cost function. In Chapter VI application of postoptimal analysis was used to illustrate how the analyst can obtain a set of optimal solutions. From the set of solutions he can select the solution which best meets his budget requirements. He can then eliminate the variables which will produce little or no return for additional expenditures. He can also determine the relative importance of the decision variables whose returns cannot be changed by additional expenditures.

CHAPTER VIII

RECOMMENDATIONS FOR FUTURE RESEARCH

The planning methodology presented in this text provides a means of obtaining operating policies for regional water management. There are, however, several areas associated with the methodology presented which offer opportunities for profitable future research.

In Chapter III the continuity equation (3.11) incorporated the variable e_t^k , which represented the fraction of inventory remaining after evaporation and seepage losses. This variable was assumed to be deterministic. Another interesting application would evolve if e_t^k is considered as a stochastic variable. When e_t^k is a random variable and is substituted into one of the two chance-constraints (3.12) or (3.13) the resulting convoluted distribution will involve the products of random variables.

An extension to the stochastic methodology presented in Chapter III would be to develop a production program capable of obtaining an operating policy for an existing river basin system. Once a production model is operational, it could be used to make comparisons concerning the operating policies of the system using historical streamflow data versus simulated streamflow data.

Another application concerning Chapter III would be to subdivide each time period into four possible types, depending on the unregulated streamflow. The particular types of periods might be very dry, dry, wet, and very wet. From historical or simulated streamflow records the steady-state probability of each discrete flow interval or time period could be determined for each type of period. The range of flows included in each interval and the particular discrete flow associated with each interval could be selected so that the first two moments of a probability distribution of unregulated streamflows at a particular observation station in each period could be obtained. The two moments could then be used to calculate the mean flow and the variance of the flow distribution in each of the four periods. Using this information, an operating policy for the basin under consideration could be obtained.

A final application of the stochastic methodology in Chapter III, would be to establish, based on the deterministic initial reservoir storage volume s_0^k , a set of reservoir system configurations. Each reservoir system configuration in the set would be solved to obtain several different operating policies. The sensitivity of the various operating policies could then be investigated to determine the effect of the initial storage level.

In Chapter IV, Benders' decomposition technique was applied to the time phasing of capacity expansion multiple-reservoir models. An integer code was not available which would solve the problem in a reasonable length of time. It would be worthwhile to consider the incorporation of the Balas zero-one algorithm (Taha, 89, Chapter 10) into the technique presented. Egon Balas (1965) developed the algorithm to solve linear programming problems with binary (zerc or one) variables. It would appear that his algorithm might best solve the multiple-reservoir model within an acceptable time limit.

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APPENDIX A

Chapter III Program Documentation

Chapter III Program Documentation

The linear programming system which is used to solve the multi-purpose reservoir chance-constrained problem is based on the revised simplex method with bounded variables. The bounded variables procedure is a method by which constraints that have upper or lower bounds on individual variables are handled implicitly rather than explicitly in the program. Since the work involved in solving linear programs is mainly a function of the number of constraints, speed and accuracy can be improved significantly by utilizing the bounded variables procedure. This is particularly important for linked multi-purpose reservoir models since at least one-half of the constraints are bounded. A detailed discussion of the revised simplex procedure with bounded variables is given by Taha (70, Chapter 8).

The program is FORTRAN based which offers the greatest flexibility for interfacing with the remaining subroutines and for conversion to other computers. The program was developed to run under WATFIV and OS 360 FORTRAN systems on the IBM 360/65.

Program Structure

The basic structure of the linked multi-purpose computer program consists of a master program which reads the input data, develops the linear programming formulation, submits the problem to the linear programming subroutine LPSIM, and interprets the results for printout. The general problem solved by the linear program (l.p.) subroutine is:

$$\text{maximize} \quad \underline{c}\underline{y}$$

subject to $$(\underline{A},\underline{I})\underline{y} = \underline{b} \geq \underline{o}$$

$$\underline{\ell} \leq \underline{y} \leq \underline{u} \quad ,$$

where $\underline{y}$ is the decision vector and consists of release, pumping, slack and artificial variables. The variables are stored into $\underline{y}$ in the following manner (the notation is that given in Chapter III):

$$\begin{array}{lll} \underline{y} = [\overbrace{x_1^1, x_1^2, \ldots, x_1^m}^{\text{all releases}}, & \overbrace{p_{11}^2, p_{11}^3, \ldots, p_{21}^1, \ldots}^{\text{all pumping variables}}; & \text{time } 1 \\ \quad x_2^1, x_2^2, \ldots, x_2^m, & p_{12}^2, \ \ldots \ \ p_{22}^1, \ldots; & \text{time } 2 \\ \quad x_T^1, x_T^2, \ldots, x_T^m, & p_{1T}^2, \ \ldots \qquad] & \text{time } T \ . \end{array}$$

In narrative form the sequence of variables is (1) each release variable x_t^k for each reservoir for time period one, followed by all first period pumping variables; (2) the pumping variables are ordered by all pumping into reservoir one, then reservoir two, etc.; (3) this sequence is repeated for each of the T time periods; and (4) the slack and artificial variables occur next, one each for every constraint.

<u>MASTER Program Input Formats</u>

The notation which is used in FORTRAN to designate whether a variable is an integer or a floating point variable is I or F , respectively. For I type variables, the data must be read in right adjusted in the field. For floating point variables, F designation, the decimal point may be included and the data inserted anywhere within the specified field. An alphanumeric field type of A is used for title information and can be any alpha or numeric character.

CARD	FIELD	TYPE	NAME	DESCRIPTION
1	1	I5	NT	- number of time periods,
	2	I5	NR	- number of reservoirs.

The remaining cards are read in sets. One set for each reservoir. All of the pertinent data for each reservoir is included within the set. Each reservoir in the system must be given a designation number; these numbers must be sequential starting with one. A title card also is included to identify the reservoir.

SET CARD	FIELD	TYPE	NAME	DESCRIPTION
1	1	A80	ITIB	reservoir title card, can be any alphanumeric characters, a maximum of 80 columns is available.
2	1	F10	D	reservoir extracted demand for period one,
	2	F10		demand for period two,
	.			.
	.			.
	.			.
	NT	F10		demand for period NT (If more than 8 periods are to be studied, continue data on successive cards. A maximum of 8 periods per card until NT reached.).

SET CARD	FIELD	TYPE	NAME	DESCRIPTION
3	1	I5	NP	number of other reservoirs that this reservoir can pump to (If this number is zero, place a one in column 5 of the card.),
	2	I5	IO	the number of one of the reservoirs which is pumped to,
	3	I5	IO	the number of one of the reservoirs which is pumped to, . . . (Repeat reservoir numbers until all reservoirs that are pumped to from this reservoir have been included. The maximum number on each card is 9 reservoirs on the first card. This is followed successively by 10 reservoirs per card. The limit of 9 on the first card is because NP takes up one field on this card.).

(The following set cards 4 and 5 are repeated for each reservoir pumped to from this reservoir or NP times. However, if no pumping is allowed from the reservoir omit set cards 4 and 5.)

SET CARD	FIELD	TYPE	NAME	DESCRIPTION
4	1	F10	MSIO	pumping canal maximum capacity for first time period,
		. . .		
	NT	F10		pumping canal maximum capacity for time period NT.
5	1	F10	PROPUM	profit per unit pumped through canal in first time period (costs are considered as negative profit),
	. . .			
	NT	F10		profit per unit pumped through canal in period NT.

SET CARD	FIELD	TYPE	NAME	DESCRIPTION
6	1	I5	NP	number of other reservoirs for which the normal channel release flows into this reservoir (If this number is zero, place a one in column 5.),
	2	I5	NSRF	the number of one of the reservoirs which releases into this reservoir,
	. . .			. . .
	NP	I5	NSRF	the number of one of the reservoirs which releases into this reservoir (Maximum number per card is 9 on first and 10 each on successive cards. Reasons are same as that for set card 3.).
7	1	F10	RIF	maximum inflow into reservoir in period one (This is the α_1 point from distribution inflow and was designated by $(R_{*1})^{\alpha_1}$ in previous discussions.),
	2	F10		maximum inflow into reservoir in period two, $(R_{*2})^{\alpha_1}$,
	. . .			
	NT	F10		maximum inflow into reservoir in last time period.
8	1	F10	RIFL	minimum inflow into reservoir i period one, $(R_{*1})^{1-\alpha_2}$,
	. . .			
	NT	F10		minimum reservoir inflow in period NT, $(R_{*NT})^{1-\alpha_2}$.

SET CARD	FIELD	TYPE	NAME	DESCRIPTION
9	1	F10	RCAP	maximum reservoir capacity minus surcharge $(c_1^k - v_1^k)$ for time period one,
	.			
	.			
	.			
	NT	F10		maximum reservoir capacity minus surcharge $(c_{NT}^k - v_{NT}^k)$ for time period NT.
10	1	F10	SMIN	minimum reservoir pool level $\underline{s}_1^k$ for time period one,
	.			
	.			
	.			
	NT	F10		minimum reservoir pool level $\underline{s}_{NT}^k$ for time period NT.
11	1	F10	RFUL	maximum normal channel release $\bar{x}_1^k$ from reservoir in period one,
	.			
	.			
	.			
	NT	F10		maximum normal channel release $\bar{x}_{NT}^k$ from reservoir in period NT.
12	1	F10	RFLL	minimum normal channel release $\underline{x}_1^k$ from reservoir in period 1,
	.			
	.			
	.			
	NT	F10		minimum normal channel release $\underline{x}_{NT}^k$ from reservoir in period NT.
13	1	F10	PRRES	profit per unit for releasing from reservoir in period 1 (cost considered as negative profit),
	.			
	.			
	.			
	NT	F10		profit per unit for releasing from reservoir in period NT.

SET CARD	FIELD	TYPE	NAME	DESCRIPTION
14	1	F10	E	evaporation factor, e_1^k , for time period one (This is the fraction of previous period ending reservoir inventory quantity which is not lost due to evaporation or leakage.),
	. . .			
	NT	F10		evaporation factor, e_{NT}^k , for time period NT.
15	1	F10	SO	starting reservoir water quantity.

A list of the input cards for the linked multiple-reservoir example, Figure 3.3 (p. 58), discussed in Chapter III and using the data in Table 3.3 (p. 59), is given below.

```
EXAMPLE INPUT FOR MASTER
COLUMNS
1   5    1    1    2    2    3    3    4    4    5
         0    5    0    5    0    5    0    5    0
--------------------------------------------------
    2    3
 RESERVOIR ONE
6.        8.
    1    0
    1    0
11.        20.
6.         15.
 10.0       10.0
3.        3.
7.        8.
1.        3.
1.0       1.0
1.        .95
8.
 RESERVOIR TWO
5.         7.
    1    1
10.        10.
-.75       -.80
    2    1    3
10.        15.
9.         14.0
20.0        19.0
4.         2.
15.         12.
2.          3.
-2.        -2.1
1.         .97
20.
 RESERVOIR THREE
  10.0       7.0
    1    1
5.         5.
.65         .70
    1    0
   12.0     20.0
  8.0        17.0
15.0         16.0
  3.0        4.0
  20.0        20.0
  1.0        1.0
0.
   1.0        .98
  6.0
```

RESERVOIR 3		RESERVOIR THREE	
DEMANDS	10.000	7.000	
PUMP TO 1,CAP.	5.000	5.000	
PUMPING PROFIT	0.650	0.700	
RELEASE FROM	0		
INFLOWS	12.000	20.000	
L INFLOWS	8.000	17.000	
CAP.-FREEBD.	15.000	16.000	
SMIN	3.000	4.000	
FLOW LIMIT U	20.000	20.000	
FLOW LIMIT L	1.000	1.000	
RELEASE PROFIT	0.000	0.000	
EVAPORATION	1.000	C.980	
STARTING RESERVOIR QUANTITY			6.000

VAR	LOWER BOUND	UPPER BOUND
1	1.000	7.000
2	2.000	15.000
3	1.000	20.000
4	0.000	10.000
5	0.000	5.000
6	3.000	8.000
7	3.000	12.000
8	1.000	20.000
9	0.000	10.000
10	0.000	5.000

INFEASIBLE INITIAL RHS FOR CONSTRAINT 1 -2.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT 3 -5.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT 9 -4.5500

L.P. PROBLEM SIZE NUMBER OF VARIABLES 10 NUMBER OF CONSTANTS 12

TAB	1	-1.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-3.00
TAB	2	1.00	0.00	0.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	5.00
TAB	3	1.00	-1.00	1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	-5.00
TAB	4	-1.00	1.00	-1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	20.00
TAB	5	0.00	0.00	-1.00	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	7.00
TAB	6	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00
TAB	7	-0.95	0.00	0.00	0.95	0.95	-1.00	0.00	0.00	1.00	1.00	-3.90
TAB	8	0.95	0.00	0.00	-0.95	-0.95	1.00	0.00	0.00	-1.00	-1.00	5.90
TAB	9	0.97	-0.97	0.97	-0.97	0.00	1.00	-1.00	1.00	-1.00	0.00	-3.55
TAB	10	-0.97	0.97	-0.97	0.97	0.00	-1.00	1.00	-1.00	1.00	0.00	19.55
TAB	11	0.00	0.00	-0.98	0.00	-0.98	0.00	0.00	-1.00	0.00	-1.00	6.92
TAB	12	0.00	0.00	0.98	0.00	0.98	0.00	0.00	1.00	0.00	1.00	2.08

OPTIMAL SOLUTION

X(1)=	7.000	0.1000000E 01
X(2)=	9.000	-0.2000000E 01
X(3)=	1.000	0.0000000E 00
X(4)=	4.000	-0.7500000E 00
X(6)=	8.000	0.1000000E 01
X(7)=	3.000	-0.2100000E 01
X(8)=	1.000	0.0000000E 00
X(9)=	4.850	-C.8000000E 00
X(10)=	0.100	C.7000000E 00
X(13)=	2.000	0.0000000E 00
X(17)=	15.000	0.0000000E 00
X(19)=	8.000	0.0000000E 00
X(23)=	2.000	C.0000000E 00
X(27)=	0.150	0.0000000E 00
X(29)=	15.850	0.0000000E 00
X(31)=	9.000	0.0000000E 00

OBJECTIVE FUNCTION VALUE IS -0.1611002E 02

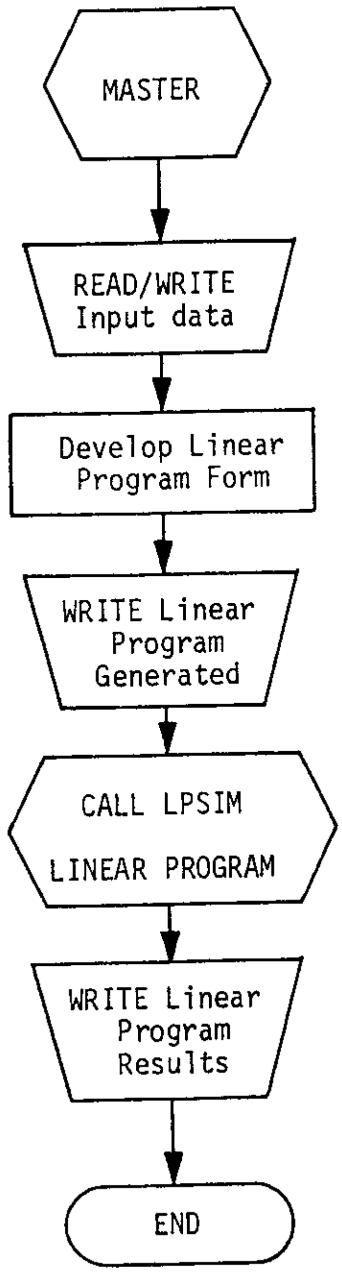
TIME PERIOD 1

RELEASE FROM RESERVOIR 1 INTO CHANNEL BED IS **7.0**
RELEASE FROM RESERVOIR 2 INTO CHANNEL BED IS **9.0**
PUMPING FROM RESERVOIR 2 TO RESERVOIR 1 IS **1.0**
RELEASE FROM RESERVOIR 3 INTO CHANNEL BED IS **4.0**
PUMPING FROM RESERVOIR 3 TO RESERVOIR 1 IS **0.0**

TIME PERIOD 2

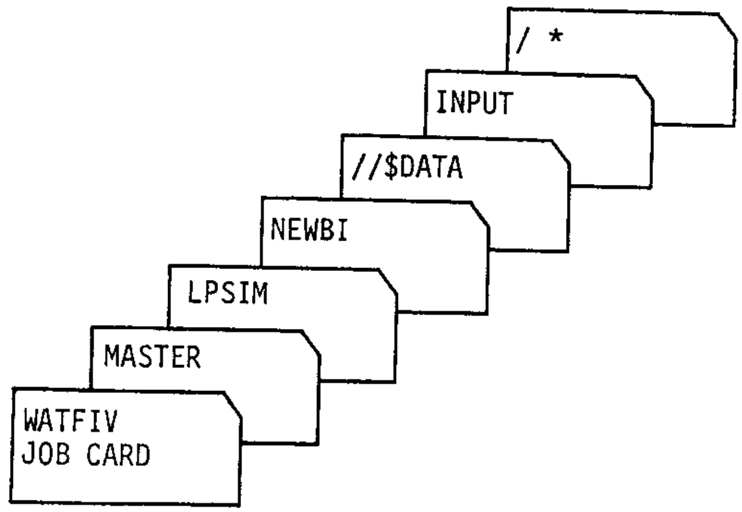
RELEASE FROM RESERVOIR 1 INTO CHANNEL BED IS **8.0**
RELEASE FROM RESERVOIR 2 INTO CHANNEL BED IS **3.0**
PUMPING FROM RESERVOIR 2 TO RESERVOIR 1 IS **1.0**
RELEASE FROM RESERVOIR 3 INTO CHANNEL BED IS **4.9**
PUMPING FROM RESERVOIR 3 TO RESERVOIR 1 IS **0.1**

Program Flowchart



Program Deck Setup and Listing

The reservoir model program is listed below. A complete input system for running the program in WATFIV is depicted as:



```
C       MASTER PROGRAM
C
        REAL MSIO
        INTEGER T
        DATA ART /-1.E9/
        COMMON NXB(25),CB(25)
        DIMENSION XUB(99),XLB(99),PA(25),P(25),C(99)
       1,XP(99),NPV(25),IO(25,25),D(25,25),RIF(20,25)
       2,RIFL(25,25),NPVSX(25),RCAP(25,25),RFUL(25,25)
       3,RFLL(25,25),SMIN(25,25),E(25,25),SO(25),NRF(25)
       4,NSRF(25,25),MSIO(25,5,25),TAB(25,99),NPTR(25)
       5,NVPTR(25,25),ITIB(20),PRRES(25,25)
       6,PROPUM(25,5,25)
3       FORMAT(8F10.3)
4       FORMAT('1')
5       WRITE(6,4)
C                            CARD 1
        READ(5,6,END=500) NT,NR
6       FORMAT(10I5)
    8   FORMAT(20A4)
        WRITE(6,10)NT,NR
10      FORMAT( 5X,'NUMBER OF TIME PERIODS= ',I5/5X,
       1  'NUMBER OF RESERVOIRS   = ' I5/)
        WRITE(6,11) (J,J=1,NT)
   11   FORMAT('-     TIME PERIOD'10(I7,3X)
       1, 10(/13X, 10I10))
        NPVV=0
        DO 40 I=1,NR
C                            SET CARD 1
        READ(5,8) ITIB
        WRITE(6,19) I, ITIB
   19   FORMAT('-    RESERVOIR' I3,10X, 20A4)
C                            SET CARD 2
        READ(5,3) (D(I,J),J=1,NT)
        WRITE(6,21)    ( D(I,J),J=1,NT)
   21   FORMAT(8X' DEMANDS' 10F10.3,10(/16X 10F10.3))
        DO 920 T=1,NT
        PROPUM(I,1,T) =0.0
  920   MSIO(I,1,T) = 0.0
C                            SET CARD 3
        READ(5,6) NP,( IO(I,J),J=1,NP)
        IF( IO(I,1) .EQ. 0) GO TO 922
C       READ PUMPING CAP. AND COSTS
        DO 921 J=1,NP
C                            SET CARD 4
        READ(5,3) ( MSIO(I,J,T),T=1,NT)
C                            SET CARD 5
  921   READ(5,3)(PROPUM(I,J,T),T=1,NT)
  922   DO 23 J=1,NP
```

```
      WRITE(6,22)    IO(I,J),  ( MSIO(I,J,T),T=1,NT)
   22 FORMAT(' PUMP TO'I3',CAP.' 10F10.3,
     1 10(/16X, 10F10.3))
   23 WRITE(6,24) ( PROPUM(I,J,T),T=1,NT)
   24 FORMAT('  PUMPING PROFIT' 10F10.3,
     1 10(/16X, 10F10.3))
      NPV(I)=NP
      IF(IO(I,1).EQ.0) NPV(I)=0
C                          SET CARD 6
      READ(5,6) NP,( NSRF(I,J),J=1,NP)
      NRF(I) = NP
      IF(  NSRF(I,1) .EQ.0) NRF(I) = 0
      WRITE(6,25)   (NSRF(I,J),J=1,NP)
25    FORMAT(4X,'RELEASE FROM' 10I10,10(16X/10I10))
C                          SET CARD 7
      READ(5,3) (RIF(I,J),J=1,NT)
      WRITE(6,28) (RIF(I,J),J=1,NT)
28    FORMAT(9X,'INFLOWS',10F10.3,10(/16X,F10.3))
C                          SET CARD 8
      READ(5,3) (RIFL(I,J),J=1,NT)
      WRITE(6,29) (RIFL(I,J),J=1,NT)
29    FORMAT(7X,'L INFLOWS',10F10.3,10(/16X,F10.3))
C                          SET CARD 9
      READ(5,3) (RCAP(I,J),J=1,NT)
      WRITE(6,30) (RCAP(I,J),J=1,NT)
30    FORMAT(4X,'CAP.-FREEBD.'10F10.3,10(/16X,F10.3))
C                          SET CARD 10
      READ (5,3) (SMIN(I,J),J=1,NT)
      WRITE(6,31)(SMIN(I,J),J=1,NT)
31    FORMAT(12X'SMIN'  10F10.3,10(/16X,10F10.3))
C                          SET CARD 11
      READ(5,3) (RFUL(I,J),J=1,NT)
      WRITE(6,32) (RFUL(I,J),J=1,NT)
32    FORMAT(4X,'FLOW LIMIT U'  10F10.3,10(/16X,10F10.3))
C                          SET CARD 12
      READ(5,3) (RFLL(I,J),J=1,NT)
      WRITE(6,34) (RFLL(I,J),J=1,NT)
34    FORMAT(4X,'FLOW LIMIT L'  10F10.3,10(/16X,10F10.3))
C
C     PROFIT FROM RELEASE BY TIME PEROID
C                          SET CARD 13
      READ(5,3) ( PRRES(I,J),J=1,NT)
      WRITE(6,35)(PRRES(I,J),J=1,NT)
   35 FORMAT(2X'RELEASE PROFIT' 10F10.3,10(/16X,10F10.3))
C                          SET CARD 14
      READ (5,3)    (E(I,J),J=1,NT)
      WRITE(6,36)   (E(I,J),J=1,NT)
36    FORMAT(5X,'EVAPORATION' ,10F10.3,10(/16X,10F10.3))
C                          SET CARD 15
```

```
      READ(5,3)     SO(I)
      WRITE(6,38)   SO(I)
38    FORMAT(5X,'STARTING RESERVOIR QUANTITY 'F10.3)
40    CONTINUE
C
C      COMPUTE THE L.P. SIZE
C
      NV=NR*NT
      NP=0
      NPVSX(1)=0
      DO 45 I=1,NR
      NP=NPV(I)+NP
      NPTR(I) = 0
45    NPVSX(I+1)=NP
C
C      SETUP INTO PUMPING VAR. ARRAY
C      NPTR(K) - NO. INFLOWS TO RES. K
C      NVPTR(K,J)-NO. OF VAR. PUMPED INTO RES. K
C
      DO 48 K=1,NR
      J= NPV(K)   .
      IF(J.LE.0) GO TO 48
      N= NPVSX(K)+ NR
      DO 47 I=1,J
C     RES. PUMPED INTO
      L= IO(K,I)
      K1= NPTR(L) + 1
      N= N+1
      NVPTR(L,K1) = N
   47 NPTR(L) = K1
   48 CONTINUE
      NV=NV+NP*NT
      NC= 2*NR*NT
      WRITE(6,57) NV,NC
57    FORMAT('1L.P. PROBLEM SIZE NUMBER OF VARIABLES '
     1,I5,5X ' NUMBER OF CONSTANTS' I5)
      NVT=2*NC+ NV
      DO  59 I=1,NVT
      C(I) = 0.0
      XLB(I) = 0.0
      XP(I) = 0.0
   59 XUB(I) = 1.0E30
      NRHS=NV+1
      NOBJ=NC+1
      DO 60 I=1,NOBJ
      DO 60 J=1,NRHS
60    TAB(I,J)=0.0
      DO 90 N=1,NT
      ICN= 2*NR*(N-1)
```

```
      DO 90 K=1,NR
      ICN=ICN+1
      PE=SO(K)
      DO 65 T=1,N
65    PE=PE*E(K,T)
C
C     START  DEVELOPMENT OF CONSTRAINT 3.15 AND 3.16
C
      TAB(ICN,NRHS)=-RIF(K,N)-PE+RCAP(K,N)
      TAB(ICN+1,NRHS)=RIFL(K,N)+PE-SMIN(K,N)
      DO 85 T=1,N
      INVR=(NP+NR)*(T-1)
      PE=1.0
      IF(T.EQ.N) GO TO 73
      K1=T+1
      DO 70 L=K1,N
70    PE=PE*E(K,L)
73    TAB(ICN,NRHS)=TAB(ICN,NRHS)+PE*D(K,T)
      TAB(ICN+1,NRHS)=TAB(ICN+1,NRHS)-PE*D(K,T)
C
C     END DEVELOPMENT OF CONSTRAINT 3.15 AND 3.16
C      RELEASE VARIABLE RES. K
C
      ISUB= INVR+K
      TAB(ICN,ISUB) = -PE
      TAB(ICN+1,ISUB   )= PE
      IF(NPV(K).EQ.0) GO TO 77
      IXV=INVR+NR+NPVSX(K)
      K1=NPV(K)
      DO 75 J=1,K1
      IXV=IXV+1
C
C      PUMPING  FROM RES. K VAR.
C
      TAB(ICN+1,IXV)=PE
75    TAB(ICN,IXV)=-PE
77    K1= NRF(K)
      IF(K1.EQ. 0) GO TO 80
      DO 79 L=1,K1
      J= NSRF(K,L)
C
C      VAR. INTO RES. K THAT ARE RELEASES FROM OTHER RES.
C
      TAB(ICN,INVR+J) = PE
79    TAB(ICN+1,INVR+J) = -PE
C
C      VAR. FOR PUMPING INTO RES. K FROM OTHER RES.
C
   80 K1= NPTR(K)
```

```
      IF( K1.EQ.0) GO TO 85
      DO 82 L=1,K1
      J= NVPTR(K,L)
      TAB( ICN, INVR+J) = PE
   82 TAB( ICN+1, INVR+J) = -PE
   85 CONTINUE
C
C      RELEASE VAR. CAP. BY TIME PEROID
C
      XUB( INVR+K) = RFUL(K,N)
      XLB( INVR+K) = RFLL(K,N)
C
C     COST FUNCTION FOR RELEASE VARIABLE
C
      C( INVR+K) = PRRES(K,N)
      ICN= ICN+1
      K1=NPV(K)
      IF(K1.EQ.0) GO TO 90
      IXV= INVR+NR+NPVSX(K)
      DO 89  L=1,K1
      IXV=IXV+1
C
C     PUMPING VAR. CAP. BY TIME PEROID
C
      XUB( IXV) = MSIO(K,L,N)
C
C     COST FUNCTION FOR PUMPING VARIABLE
C
      C( IXV) = PROPUM(K,L,N)
89    CONTINUE
   90 CONTINUE
      DO 110 I=1,NC
  110 WRITE(6,115) I,( TAB(I,J),J=1,NRHS)
  115 FORMAT(' TAB 'I3,11F6.2,9(/ 8X,11F6.2))
      IXV= NV
      NV= 2*NC+ NV
      DO 135 I=1,NC
      PA(I) = 0.0
  135 P(I) = TAB(I,NRHS)
      DO 137 I=NRHS, NV
      DO 137 J=1,NC
  137 TAB(J,I) = 0.0
      DO 140 I=1,NC,2
      IXV= IXV+ 1
      TAB(I,IXV) = 1.0
C
C      STARTING BASIS VARIABLE IN LP SUB.
C     (A,I)X = P FORM
C
```

```
      NXB(I  )=    IXV
      CB(I) = C(IXV)
      IXV= IXV+1
C
C      THIS IS THE ARTIFICIAL VARIABLE
C
      TAB(I,IXV) = -1.0
      C(IXV)=ART
      IXV= IXV+1
      TAB(I+1,IXV) = 1.0
C
C      STARTING BASIS VARIABLE IN LP SUB.
C
      NXB(I+1)=    IXV
      CB(I+1)=C(IXV)
      IXV=IXV+1
C
C     THIS IS THE ARTIFICIAL VARIABLE
C
      TAB(I+1,IXV) = -1.0
  140 C(IXV)=ART
      CALL LPSIM(NV,NC,0,0,TAB,P,PA,C,XUB,XLB,XZERO,XP)
C
C     OUTPUT LINEAR PROGRAM SOLUTION
C
      WRITE(6,300) XZERO,( XP(I),I=1,NV)
  300 FORMAT('0   SOLUTION   X0,X(I) '  1F10.4,
     1 10( /22X 5F10.4 ))
      IX= 0
      WRITE(6,4)
      DO 330 T=1,NT
      WRITE(6,320) T
  320 FORMAT('-' 14X'TIME PERIOD 'I3)
      DO 330 K=1,NR
      IX= IX+1
      WRITE(6,325) K, XP(IX)
  325 FORMAT(20X'RELEASE FROM RESERVOIR'I4,
     1 ' INTO CHANNEL BED IS' F9.1)
      NP= NPV(K)
      IF( NP.EQ.0) GO TO 330
      DO 329 J=1,NP
      IX= IX+1
      WRITE(6,327) K, IO(K,J),XP(IX)
  327 FORMAT(20X'PUMPING FROM RESERVOIR'I4,
     1 ' TO RESERVOIR'I4,' IS' F9.1)
  329 CONTINUE
  330 CONTINUE
      WRITE(6,4)
      DO 350 T=1,NT
```

```
      DO 350 K=1,NR
      IX=IX+2
      IF( XP(IX) .EQ. 0.0) GO TO 340
      WRITE(6,335) T,K,XP(IX)
  335 FORMAT(15X'TIME' I4,' RESERVOIR'I4,' CAPACITY '
     1 'CONSTRAINT VIOLATED BY'F10.1)
  340 IX= IX+2
      IF( XP(IX) .EQ. 0.0) GO TO 350
      WRITE(6,345) T,K, XP(IX)
  345 FORMAT(15X'TIME' I4,' RESERVOIR'I4,' MIN POOL '
     1 'CONSTRAINT VIOLATED BY'F10.1)
  350 CONTINUE
      GO TO 5
  500 STOP
      END
```

```
      SUBROUTINE NEWBI (NC, BI, ALPHA, IL )
      DOUBLE PRECISION BI,ALPHA,  BETA, P,DABS
      DIMENSION BI(25,25), ALPHA(25) , BETA(25)
      P = 1.0/ ALPHA(IL)
      DO  5 I=1,NC
    5 BETA(I) = BI(IL,I)
      DO 20 I=1,NC
      IF(I .EQ. IL ) GO TO 10
      DO 9 J=1,NC
      BI(I,J) = BI(I,J) - P*ALPHA(I)*BETA(J)
9     IF(DABS(BI(I,J)).LE.1.0D-09)BI(I,J)=0.0D00
      GO TO 20
   10 DO 15 J=1,NC
   15 BI(I,J) = BI(I,J)*P
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE
     1       LPSIM(NV,NC,K,L,A,P,PA,C,XUB,XLB,XZERO,XP)
C     PROGRAM TO DO REVISED SIMPLEX AND PARAMETRIC
C     RHS RANGING, NO ATTEMPT FOR PROGRAMMING
C     EFFICIENCY HAS BEEN MADE, THE PROBLEM FORM IS
C                                8-20-71   GUY CURRY
C     MAX  CX
C     S.T. (A,I)X = P GE 0, X GE 0
C
C     THE VARIABLES USED IN THIS CODE
C     FOLLOW THE STYLE OF TAHA (89).
C
C     NV - NUMBER OF VARIABLES, INCLUDES ALL SLACKS, ETC.
C     NC - NUMBER OF CONSTRAINTS
C
C     PROGRAM EXTENDED TO DO BOUNDED VARIABLES 12-8-71
C     LOWER BOUNDS ARE SUBSTITUTED OUT IN THIS MODEL
C
C     IXP(I)=0  MEANS ORIGINAL VARIABLES IN PROBLEM
C     IXP(I)=1  MEANS COMPLEMENT OF VARIABLE IN PROBLEM
C
      DATA  TOL/0.0001/
      COMMON NXB,CB
      DOUBLE PRECISION BI,ALPHA ,ZCB, ZMC
      DIMENSION A(25,99),BI(25,25),XP(99),
     1 PCTH(25,25), P(25), PA(25), C(99), CB(25), XB(25),
     2 NXB(25), ZCB(25), ALPHA(25), ZMC(99), STH(25)
     3,OBJV(25),XLB(99),XUB(99),IXP(99)
      DO 15 I=1,NV
   15 IXP(I) = 0
      DO 30 I=1,NC
      DO 25 J=1,NC
   25 BI(I,J) = 0.0
   30 BI(I,I) = 1.0
      WRITE(6,935)
  935 FORMAT('      VAR    LOWER BOUND      UPPER BOUND' )
      DO 937 I=1,NV
      IF( XUB(I).GT. 1.0E29 .AND. XLB(I).LE.0.0) GO TO 937
      WRITE(6,938) I, XLB(I),XUB(I)
      IF( XLB(I) .LE. 0.0) GO TO 937
      XUB(I) = XUB(I) -XLB(I)
      DO 936 J=1,NC
  936 P(J) = P(J) - A(J,I)*XLB(I)
  937 CONTINUE
  938 FORMAT(I5, F13.3, F16.3 )
      DO 953 I=1,NC
      IF( P(I) .GE. 0.0) GO TO 953
      WRITE(6,941) I, P(I)
  941 FORMAT('  INFEASIBLE INITIAL RHS FOR CONSTRAINT'
```

```
     1 I5,F10.4)
C
C     CONVERT TO POSITIVE RHS CONSTRAINTS
C
      P(I) = -P(I)
      DO 945 J=1,NV
  945 A(I,J) = -A(I,J)
      J= NXB(I)   +1
      NXB(I) = J
      CB(I) = C(J)
  953 XB(I) = P(I)
   39 ZMIN = 0.0
      DO 40 I=1,NC
      ZCB(I) = 0.0
      DO 40 J=1,NC
   40 ZCB(I) = ZCB(I) + CB(J) *BI(J,I)
      DO 60 I=1,NV
      ZMC(I) =  -C(I)
      DO 50 J=1,NC
   50 ZMC(I) = ZMC(I) + ZCB(J) * A(J,I)
      IF( ZMC(I) .GT. ZMIN ) GO TO 60
      ZMIN = ZMC(I)
      IMIN = I
   60 CONTINUE
C
C       IF CJ-CJ GE 0 FOR ALL J, OPT. SOL. GO TO 200
C
      IF (ZMIN .GE. -0.00100) GO TO 200
      DO 70 I=1,NC
      ALPHA(I)= 0.0
      DO 70 J=1,NC
   70 ALPHA(I) = ALPHA(I) + BI(I,J) * A(J,IMIN)
C
C      COMPUTE LEAVING VARIABLE BY MIN (XB(I)/ ALPHA(I),
C                                  FOR ALPHA(I) LT 0 )
C
      ICODE=0
      VALUE=XUB(IMIN)
      ILEAVE= 0
      DO 90 I=1,NC
      IF (ALPHA(I).LE. -TOL ) GO TO 80
      IF (ALPHA(I).LE.  TOL ) GO TO 90
C
C        POSITIVE ALPHA
C
      RATIO =XB(I)/ALPHA(I)
      IF (RATIO .GE. VALUE ) GO TO 90
      ILEAVE=I
      VALUE =RATIO
```

```
      ICODE =1
      GO TO 90
C
C        NEGATIVE ALPHA
C
   80 J=NXB(I)
      RATIO= (-XUB(J) + XB(I)) / ALPHA(I)
      IF (RATIO .GE. VALUE ) GO TO 90
      ILEAVE =I
      VALUE =RATIO
      ICODE =-1
   90 CONTINUE
      IF( VALUE .LT. 1.0E28) GO TO 91
      IF(ILEAVE) 110,110,91
   91 IF(ICODE .LE. 0 ) GO TO 125
   92 J=NXB(ILEAVE)
      CB(ILEAVE) = C(IMIN)
      NXB(ILEAVE)= IMIN
      CALL NEWBI( NC, BI , ALPHA, ILEAVE )
   97 DO 100 I=1,NC
      XB(I) = 0.0
      DO 100 J=1,NC
  100 XB(I) = XB(I) + BI(I,J) * P(J)
      GO TO 39
  110 WRITE(6,115)
  115 FORMAT('- UNBOUNDED SOLUTION ' )
      RETURN
  125 IF( ICODE.NE.0) GO TO 150
C
C     NONBASIC VARIABLE CAN NOT ENTER BECAUSE IT HAS
C     ENCOUNTERED ITS UPPER BOUND, REPLACE IT BY ITS
C     COMPLEMENT AND CONTINUE
C
      J=IMIN
      IF( IXP(J) .EQ. 0) GO TO 127
      IXP(J) = 0
      GO TO 130
  127 IXP(J) = 1
  130 DO 132 I=1,NC
      A(I,J) = -A(I,J)
  132 P(I) = P(I) + A(I,J)*XUB(J)
      C(J) = -C(J)
      GO TO 97
C
C        ALPHA IS NEGATIVE
C
  150 J=NXB(ILEAVE)
      CB(ILEAVE) = C(IMIN)
      NXB(ILEAVE)=IMIN
```

```
      CALL NEWBI (NC,BI,ALPHA,ILEAVE)
      IXP(J) =1
      DO 160 I=1,NC
      A(I,J) =-A(I,J)
  160 P(I)=P(I) +A(I,J)*XUB(J)
      C(J) = -C(J)
      GO TO 97
C
C     OPTIMAL SOLUTION HAS BEEN OBTAINED
C
  200 XZERO = 0.0
      WRITE(6,205)
  205 FORMAT('1 OPTIMAL SOLUTION'/)
      DO 206 I=1,NV
      XP(I) = XLB(I)
      IF( IXP(I) .EQ.0) GO TO 206
      C(I) = -C(I)
      XP(I) =XP(I) +XUB(I)
  206 CONTINUE
      DO 210 I=1,NC
      J= NXB(I)
      IF( IXP(J) .EQ.0) GO TO 209
      XP(J)=XP(J)-XB(I)
      GO TO 210
209   XP(J)=XP(J)+XB(I)
210   CONTINUE
      DO 212 I=1,NV
      IF( XP(I).EQ.0.0) GO TO 212
      XZERO=XZERO+C(I)* XP(I)
      WRITE(6,211)I, XP(I),C(I)
211   FORMAT(' X('I3')=' F10.3,5XE20.7)
212   CONTINUE
      WRITE(6,215) XZERO
  215 FORMAT( ' OBJECTIVE FUNCTION VALUE IS ' E20.7,/'1')
      RETURN
      END
```

APPENDIX B

Chapter IV Program Documentation

Chapter IV Program Documentation

Program Structure

The structure of the Benders computer program consists of a SUPERVISOR program, a MASTER program, an integer program, INTSUB, and a linear program, LPSIM. The SUPERVISOR program reads in the possible segment capacity expansions for each time period and the associated segment costs. The SUPERVISOR program controls the six iteration steps discussed in Chapter IV under the section Benders' Decomposition.

The MASTER program has the same input and function as discussed in Appendix A. The input to MASTER follows the capacity expansion input. The linear program LPSIM also was discussed in Appendix A. The integer subroutine, INTSUB is not an actual integer program, but was designed to simulate one. For this example problem INTSUB contains the answers that an integer program would return for each iteration.

SUPERVISOR Input Format

The notation which is used in FORTRAN to designate whether a variable is an integer or a floating point variable is I or F, respectively. For I type variables, the data must be read in right adjusted in the field. For F type variables, the decimal point may be included and the data inserted anywhere within the specified field.

```
EXAMPLE  INPUT  FOR  MASTER
COLUMNS
1    5     1     1     2     2     3     3     4     4     5
           0     5     0     5     0     5     0     5     0
---------------------------------------------------------------
    2    3
 RESERVOIR ONE
6.          8.
    1     0
    1     0
11.         30.
6.          25.
0.
3.          3.
7.          8.
1.          3.
1.0         1.0
1.          .95
8.
 RESERVOIR TWO
5.          7.
    1     1
10.         10.
-.75        -.80
    2     1     3
10.         15.
9.          14.0
0.0           1.0
4.          2.
15.          12.
2.           3.
-2.         -2.1
1.          .97
20.
 RESERVOIR THREE
  10.0        7.0
    1     1
5.          5.
.65          .70
    1     0
   12.0      20.0
  8.0         17.0
1.0            0.0
  3.0         4.0
  20.0         20.0
  1.0         1.0
0.
   1.0         .98
  6.0
```

Program Output

The program output result is explained in Chapter IV and the iteration results given in Table 4.3 (p. 88). A complete computer run for the data listed is included with the explanatory titles.

RESERVOIR SEGMENT CAPACITIES

1	5.000	5.000	15.000
2	10.000	10.000	10.000
3	10.000	6.000	5.000

COST CAP.	1	1	52.000	52.000	252.000
COST CAP.	1	2	1C2.000	102.000	52.000
COST CAP.	1	3	1C2.000	62.000	52.000
COST CAP.	2	1	56.000	56.000	56.000
COST CAP.	2	2	111.000	111.000	56.000
COST CAP.	2	3	111.000	67.000	56.000

Y(2, 1, 3)= 1.0
Y(1, 1, 1)= 1.0
Y(1, 1, 2)= 1.0
Y(1, 2, 2)= 1.0
Y(1, 2, 3)= 1.0
Y(1, 3, 1)= 1.0

NUMBER OF TIME PERIODS= 2
NUMBER OF RESERVOIRS = 3

RESERVOIR 1 RESERVOIR ONE

TIME PERIOD	1	2
DEMANDS	6.000	8.000
PUMP TO 0,CAP.	0.000	0.000
PUMPING PROFIT	0.000	0.000
RELEASE FROM	0	
INFLOWS	11.000	30.000
L INFLOWS	6.000	25.000
CAP.-FREEBD.	0.000	0.000
SMIN	3.000	3.000
FLOW LIMIT U	7.000	8.000
FLOW LIMIT L	1.000	3.000
RELEASE PROFIT	1.000	1.000
EVAPORATION	1.000	C.950

STARTING RESERVOIR QUANTITY 8.000

RESERVOIR 2 RESERVOIR TWO

TIME PERIOD	1	2
DEMANDS	5.000	7.000
PUMP TO 1,CAP.	10.000	10.000
PUMPING PROFIT	-0.750	-0.800
RELEASE FROM	1	3
INFLOWS	10.000	15.000
L INFLOWS	9.000	14.000
CAP.-FREEBD.	0.000	1.000
SMIN	4.000	2.000
FLOW LIMIT U	15.000	12.000
FLOW LIMIT L	2.000	3.000
RELEASE PROFIT	-2.000	-2.100
EVAPORATION	1.000	C.970

STARTING RESERVOIR QUANTITY 20.000

RESERVOIR 3 RESERVOIR THREE

TIME PERIOD	1	2
DEMANDS	10.000	7.000
PUMP TO 1,CAP.	5.000	5.000
PUMPING PROFIT	0.650	0.700
RELEASE FROM	0	
INFLOWS	12.000	2C.000
L INFLOWS	8.000	17.000
CAP.-FREEBD.	1.000	0.000
SMIN	3.000	4.000
FLOW LIMIT U	2C.000	20.000
FLOW LIMIT L	1.000	1.000
RELEASE PROFIT	0.000	0.000
EVAPORATION	1.000	C.980

STARTING RESERVOIR QUANTITY 6.000

L.P. PROBLEM SIZE NUMBER OF VARIABLES 10
NUMBER OF CONSTANTS 12

TAB 1	-1.00	0.00	0.00	1.00	1.00	0.00
	0.00	0.00	0.00	0.00	-13.00	
TAB 2	1.00	0.00	0.00	-1.00	-1.00	0.00
	0.00	0.00	0.00	0.00	5.00	
TAB 3	1.00	-1.00	1.00	-1.00	0.00	0.00
	0.00	0.00	0.00	0.00	-25.00	
TAB 4	-1.00	1.00	-1.00	1.00	0.00	0.00
	0.00	0.00	0.00	0.00	20.00	
TAB 5	0.00	0.00	-1.00	0.00	-1.00	0.00
	0.00	0.00	0.00	0.00	-7.00	
TAB 6	0.00	0.00	1.00	0.00	1.00	0.00
	0.00	0.00	0.00	0.00	1.00	
TAB 7	-0.95	0.00	0.00	0.95	0.95	-1.00
	0.00	0.00	1.00	1.00	-23.90	
TAB 8	0.95	0.00	0.00	-0.95	-0.95	1.00
	0.00	0.00	-1.00	-1.00	15.90	
TAB 9	0.97	-0.97	0.97	-0.97	0.00	1.00
	-1.00	1.00	-1.00	0.00	-21.55	
TAB 10	-0.97	0.97	-0.97	0.97	0.00	-1.00
	1.00	-1.00	1.00	0.00	19.55	
TAB 11	0.00	0.00	-0.98	0.00	-0.98	0.00
	0.00	-1.00	0.00	-1.00	-9.08	
TAB 12	0.00	0.00	0.98	0.00	0.98	0.00
	0.00	1.00	0.00	1.00	2.08	

REVISED SIMPLEX NUMBER OF VARIABLES IS 34
NUMBER OF CONSTRAINTS IS 12

C	1.000	-2.000	0.000	-0.750	0.650
	1.000	-2.100	0.000	-0.800	0.700

VAR	LOWER BOUND	UPPER BOUND
1	0.000	6.000
2	0.000	13.000
3	0.000	19.000
4	0.000	10.000
5	0.000	5.000
6	0.000	5.000
7	0.000	9.000
8	0.000	19.000
9	0.000	10.000
10	0.000	5.000

INFEASIBLE INITIAL RHS FOR CONSTRAINT 1 -2.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT 3 -5.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT 9 -2.5500

OPTIMAL SOLUTION

```
X(  1)=     7.000          0.1000000E 01
X(  2)=     9.000         -0.2000000E 01
X(  3)=     1.000          0.0000000E 00
X(  4)=     4.000         -0.7500000E 00
X(  6)=     8.000          0.1000000E 01
X(  7)=     3.000         -0.2100000E 01
X(  8)=     1.000          0.0000000E 00
X(  9)=     2.700         -0.8000000E 00
X( 10)=     0.100          0.7000000E 00
X( 13)=     2.000          0.0000000E 00
X( 17)=    15.000          0.0000000E 00
X( 19)=     4.000          0.0000000E 00
X( 23)=     9.150          0.0000000E 00
X( 25)=     7.850          0.0000000E 00
X( 29)=    18.000          0.0000000E 00
X( 31)=     3.000          0.0000000E 00
OBJECTIVE FUNCTION VALUE IS      -0.1438997E 02
```

ITER = 1 COST OF POLICY = 430.39 MAXIMUM ERROR =*******

	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18		
CX	45.75	56.00	45.75	56.00	233.25	56.00	81.76	103.00	81.76	103.00
CX	31.76	48.00	102.00	111.00	62.00	67.00	52.00	56.00	-67.37	
AX	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	

OPTIMAL SOLUTION 67.370

```
INFEASIBLE INITIAL RHS FOR CONSTRAINT    1   -12.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT    3   -25.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT    5    -6.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT    7   -19.9500
INFEASIBLE INITIAL RHS FOR CONSTRAINT    9   -22.5500
INFEASIBLE INITIAL RHS FOR CONSTRAINT   11    -7.1000
```

```
OPTIMAL SOLUTION

X(  1)=      6.835         0.1000000E 01
X(  2)=     15.000        -0.2000000E 01
X(  3)=      1.000         0.0000000E 00
X(  4)=     10.000        -0.7500000E 00
X(  6)=      8.000         0.1000000E 01
X(  7)=     12.000        -0.2100000E 01
X(  8)=      1.100         0.0000000E 00
X( 12)=     16.165        -0.1000000E 10
X( 13)=      8.165         0.0000000E 00
X( 16)=      7.835        -0.1000000E 10
X( 17)=      2.835         0.0000000E 00
X( 20)=      6.000        -0.1000000E 10
X( 24)=     18.907        -0.1000000E 10
X( 25)=     10.907         0.0000000E 00
X( 28)=      2.000        -0.1000000E 10
X( 32)=      7.000        -0.1000000E 10
OBJECTIVE FUNCTION VALUE IS        -0.5790654E 11
```

UR 10 0.000 0.000 0.000 0.000 0.000
-0.000 1.000 0.000 0.000 -0.000
0.000 -0.000

	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18		
CX	45.75	56.00	45.75	56.00	233.25	56.00	81.76	103.00	81.76	103.00
CX	31.76	48.00	102.00	111.00	62.00	67.00	52.00	56.00	-67.37	
AX	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	
AX	5.00	5.00	5.00	5.00	15.00	15.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18.91	

Y(1, 1, 2)= 1.0
Y(2, 1, 3)= 1.0

OPTIMAL SOLUTION 169.120

```
INFEASIBLE INITIAL RHS FOR CONSTRAINT     1    -7.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT     3   -25.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT     5    -6.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT     9   -22.5500
INFEASIBLE INITIAL RHS FOR CONSTRAINT    11    -7.1000
```

```
 OPTIMAL SOLUTION

X(   1)=       7.000                 0.1000000E 01
X(   2)=      15.000                -0.2000000E 01
X(   3)=       1.000                 0.0000000E 00
X(   4)=      10.000                -0.7500000E 00
X(   6)=       8.000                 0.1000000E 01
X(   7)=      10.910                -0.2100000E 01
X(   8)=       1.000                 0.0000000E 00
X(   9)=       1.150                -0.8000000E 00
X(  10)=       0.100                 0.7000000E 00
X(  12)=      11.000                -0.1000000E 10
X(  13)=       8.000                 0.0000000E 00
X(  16)=       8.000                -0.1000000E 10
X(  17)=       3.000                 0.0000000E 00
X(  20)=       6.000                -0.1000000E 10
X(  25)=      12.000                 0.0000000E 00
X(  28)=       2.000                -0.1000000E 10
X(  32)=       7.000                -0.1000000E 10
OBJECTIVE FUNCTION VALUE IS        -0.3399999E 11
```

```
UR     11      1.000       0.000       0.000       0.000       0.000
              -0.000      -0.000       0.000       0.000      -0.000
               0.000      -0.000
```

	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18		
CX	45.75	56.00	45.75	56.00	233.25	56.00	81.76	103.00	81.76	103.00
CX	31.76	48.00	102.00	111.00	62.00	67.00	52.00	56.00	-67.37	
AX	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	
AX	5.00	5.00	5.00	5.00	15.00	15.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18.91	
AX	5.00	0.00	5.00	0.00	15.00	0.00	0.00	0.00	0.00	0.00
AX	10.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	16.00	

```
Y( 1, 1, 1)= 1.0
Y( 1, 1, 2)= 1.0
Y( 2, 1, 3)= 1.0
Y( 1, 2, 3)= 1.0
```

UPTIMAL SOLUTION 246.630

```
INFEASIBLE INITIAL RHS FOR CONSTRAINT    1    -2.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT    3   -15.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT    5    -6.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT    9   -12.5500
INFEASIBLE INITIAL RHS FOR CONSTRAINT   11    -7.1000
```

OPTIMAL SOLUTION

```
X(  1)=     7.000        0.1000000E 01
X(  2)=    15.000       -0.2000000E 01
X(  3)=     1.000        0.0000000E 00
X(  4)=     8.000       -0.7500000E 00
X(  6)=     8.000        0.1000000E 01
X(  7)=     3.000       -0.2100000E 01
X(  8)=     1.000        0.0000000E 00
X(  9)=     3.000       -0.8000000E 00
X( 10)=     5.000        0.7000000E 00
X( 12)=     4.000       -0.1000000E 10
X( 13)=     6.000        0.0000000E 00
X( 17)=     5.000        0.0000000E 00
X( 20)=     6.000       -0.1000000E 10
X( 23)=     0.150        0.0000000E 00
X( 25)=    16.850        0.0000000E 00
X( 29)=     8.000        0.0000000E 00
X( 32)=     2.100       -0.1000000E 10
X( 34)=     4.900       -0.1000000E 10
OBJECTIVE FUNCTION VALUE IS       -0.1699999E 11
```

```
UR    12     0.000     0.000    -0.000     0.000     1.000
            -0.000     0.000     0.000    -0.000     0.000
             0.000     0.000
```

	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18		
CX	45.75	56.00	45.75	56.00	233.25	56.00	81.76	103.00	81.76	103.00
CX	31.76	48.00	102.00	111.00	62.00	67.00	52.00	56.00	-67.37	
AX	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	
AX	5.00	5.00	5.00	5.00	15.00	15.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18.91	
AX	5.00	0.00	5.00	0.00	15.00	0.00	0.00	0.00	0.00	0.00
AX	10.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	16.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	10.00	0.00	6.00	0.00	5.00	0.00	6.00	

```
Y( 1, 1, 1)= 1.0
Y( 1, 1, 2)= 1.0
Y( 2, 1, 3)= 1.0
Y( 1, 2, 3)= 1.0
Y( 1, 3, 2)= 1.0

OPTIMAL SOLUTION   308.630

INFEASIBLE INITIAL RHS FOR CONSTRAINT    1    -2.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT    3   -15.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT    9   -12.5500
INFEASIBLE INITIAL RHS FOR CONSTRAINT   11    -1.1000

OPTIMAL SOLUTION

X(  1)=      7.000          0.1000000E 01
X(  2)=     15.000         -0.2000000E 01
X(  3)=      1.000          0.0000000E 00
X(  4)=      8.000         -0.7500000E 00
X(  6)=      8.000          0.1000000E 01
X(  7)=      3.000         -0.2100000E 01
X(  8)=      1.000          0.0000000E 00
X(  9)=      3.000         -0.8000000E 00
X( 10)=      1.100          0.7000000E 00
X( 12)=      4.000         -0.1000000E 10
X( 13)=      6.000          0.0000000E 00
X( 17)=      5.000          0.0000000E 00
X( 23)=      4.050          0.0000000E 00
X( 25)=     12.950          0.0000000E 00
X( 29)=      8.000          0.0000000E 00
X( 34)=      1.000         -0.1000000E 10
OBJECTIVE FUNCTION VALUE IS       -0.4999995E 10

UR    13    1.000     0.000    -0.000     0.000     0.000
            0.000     0.000     0.000    -0.000     0.000
            0.000    -0.000
```

	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18		
CX	45.75	56.00	45.75	56.00	233.25	56.00	81.76	103.00	81.76	103.00
CX	31.76	48.00	102.00	111.00	62.00	67.00	52.00	56.00	-67.37	
AX	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	
AX	5.00	5.00	5.00	5.00	15.00	15.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18.91	
AX	5.00	0.00	5.00	0.00	15.00	0.00	0.00	0.00	0.00	0.00
AX	10.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	16.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	10.00	0.00	6.00	0.00	5.00	0.00	6.00	
AX	5.00	0.00	5.00	0.00	15.00	0.00	8.00	0.00	0.00	0.00
AX	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	16.00	

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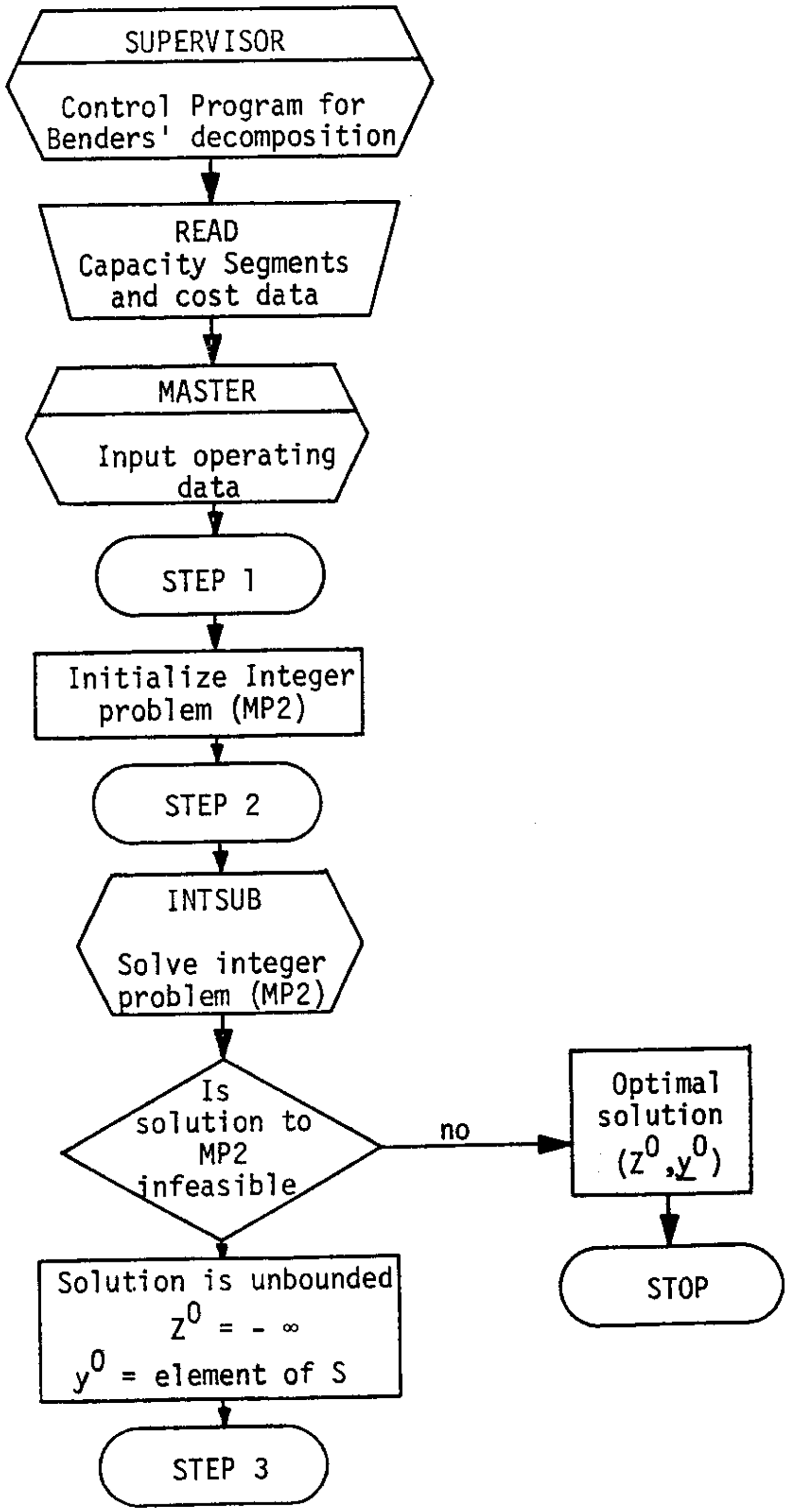
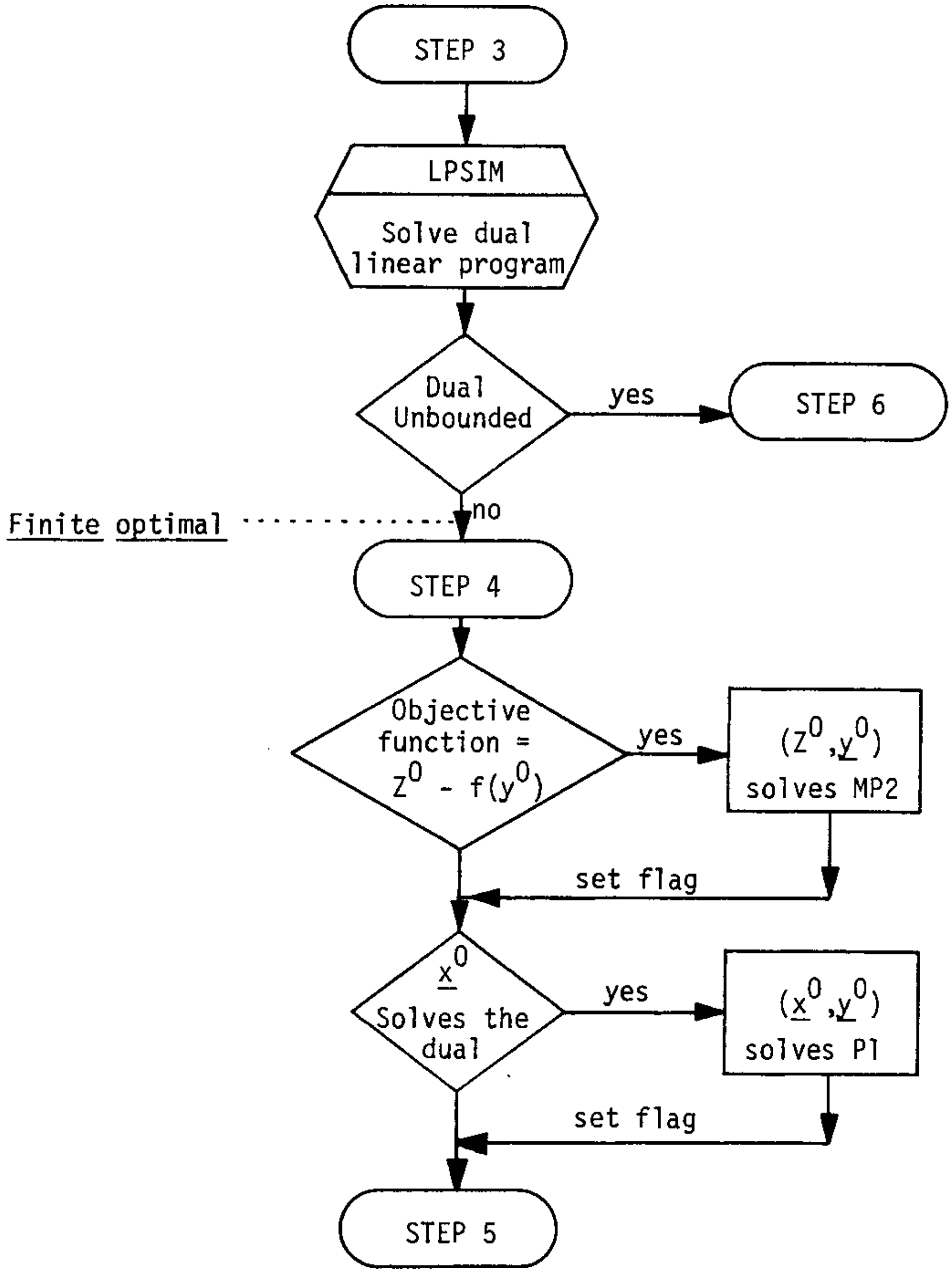
```
Y( 1, 1, 1)= 1.0
Y( 1, 1, 2)= 1.0
Y( 2, 1, 3)= 1.0
Y( 1, 2, 1)= 1.0
Y( 1, 2, 3)= 1.0
Y( 1, 3, 2)= 1.0

OPTIMAL SOLUTION   390.390


INFEASIBLE INITIAL RHS FOR CONSTRAINT    1   -2.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT    3   -5.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT    9   -2.5500
INFEASIBLE INITIAL RHS FOR CONSTRAINT   11   -1.1000


OPTIMAL SOLUTION

X(  1)=      7.000        0.1000000E 01
X(  2)=      9.000       -0.2000000E 01
X(  3)=      1.000        0.0000000E 00
X(  4)=      4.000       -0.7500000E 00
X(  6)=      8.000        0.1000000E 01
X(  7)=      3.000       -0.2100000E 01
X(  8)=      1.000        0.0000000E 00
X(  9)=      2.700       -0.8000000E 00
X( 10)=      1.100        0.7000000E 00
X( 13)=      2.000        0.0000000E 00
X( 17)=     15.000        0.0000000E 00
X( 23)=      8.150        0.0000000E 00
X( 25)=      8.850        0.0000000E 00
X( 29)=     18.000        0.0000000E 00
X( 34)=      1.000       -0.1000000E 10
OBJECTIVE FUNCTION VALUE IS       -0.9999969E 09


UR    14   -0.000    0.000   -0.000    0.000    0.000
            0.000    0.000    0.000   -0.000    0.000
            1.000   -0.000
```

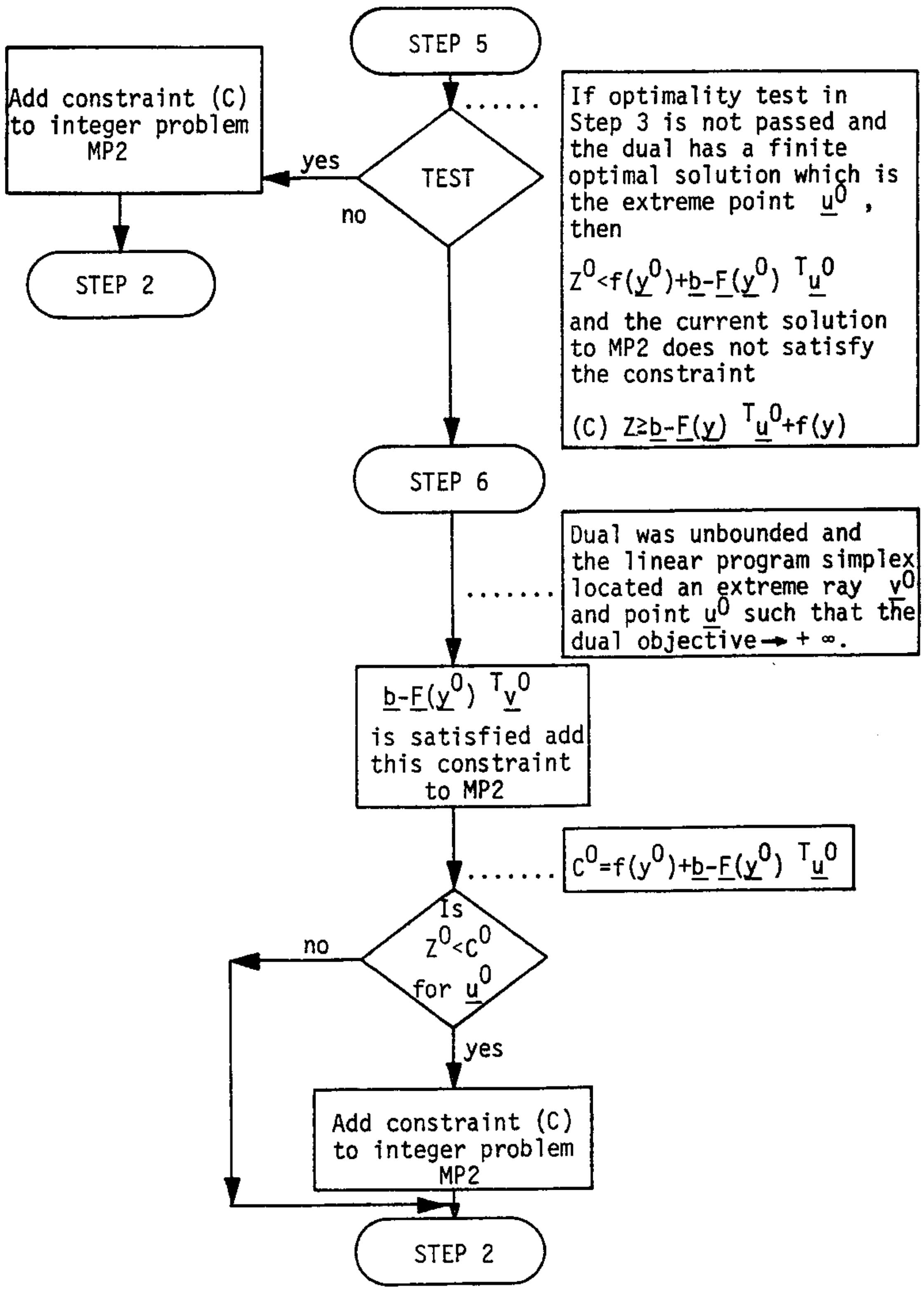
	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18		
CX	45.75	56.00	45.75	56.00	233.25	56.00	81.76	103.00	81.76	103.00
CX	31.76	48.00	102.00	111.00	62.00	67.00	52.00	56.00	-67.37	
AX	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	1.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	
AX	5.00	5.00	5.00	5.00	15.00	15.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18.91	
AX	5.00	0.00	5.00	0.00	15.00	0.00	0.00	0.00	0.00	0.00
AX	10.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	16.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	10.00	0.00	6.00	0.00	5.00	0.00	6.00	
AX	5.00	0.00	5.00	0.00	15.00	0.00	8.00	0.00	0.00	0.00
AX	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	16.00	
AX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AX	0.00	0.00	10.00	10.00	6.00	6.00	5.00	5.00	7.00	

```
Y( 1, 1, 1)= 1.0
Y( 1, 1, 2)= 1.0
Y( 2, 1, 3)= 1.0
Y( 1, 2, 1)= 1.0
Y( 1, 2, 3)= 1.0
Y( 1, 3, 1)= 1.0

OPTIMAL SOLUTION   430.390

INFEASIBLE INITIAL RHS FOR CONSTRAINT    1   -2.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT    3   -5.0000
INFEASIBLE INITIAL RHS FOR CONSTRAINT    9   -2.5500


OPTIMAL SOLUTION

X(  1)=     7.000       0.1000000E 01
X(  2)=     9.000      -0.2000000E 01
X(  3)=     1.000       0.0000000E 00
X(  4)=     4.000      -0.7500000E 00
X(  6)=     8.000       0.1000000E 01
X(  7)=     3.000      -0.2100000E 01
X(  8)=     1.000       0.0000000E 00
X(  9)=     2.700      -0.8000000E 00
X( 10)=     0.100       0.7000000E 00
X( 13)=     2.000       0.0000000E 00
X( 17)=    15.000       0.0000000E 00
X( 19)=     4.000       0.0000000E 00
X( 23)=     9.150       0.0000000E 00
X( 25)=     7.850       0.0000000E 00
X( 29)=    18.000       0.0000000E 00
X( 31)=     3.000       0.0000000E 00
OBJECTIVE FUNCTION VALUE IS       -0.1438997E 02

ITER =   7 COST OF POLICY = 430.39 MAXIMUM ERROR =    0.00


OPTIMAL SOLUTION
Y( 1, 1, 1)= 1.0
Y( 1, 1, 2)= 1.0
Y( 1, 2, 1)= 1.0
Y( 1, 2, 3)= 1.0
Y( 1, 3, 1)= 1.0
Y( 2, 1, 3)= 1.0

COST      430.4
```

Program Flowchart

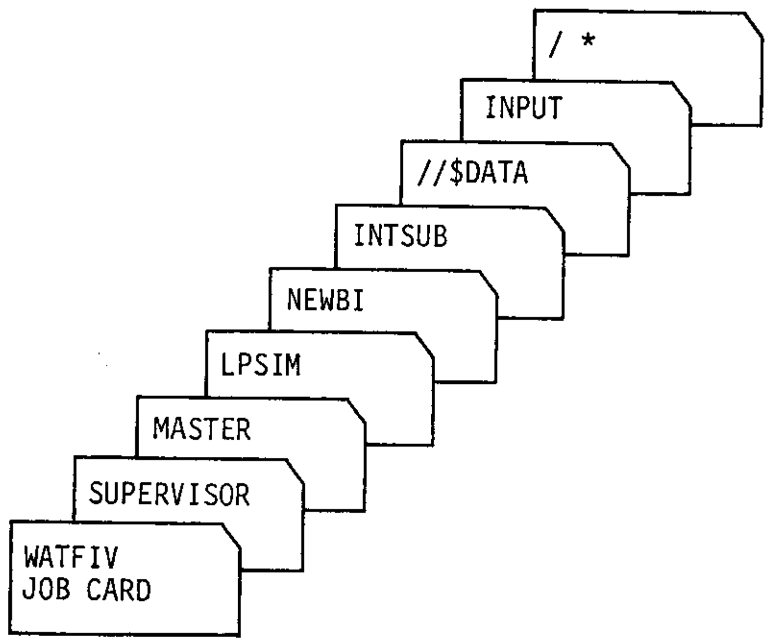



STEP 3
LPSIM
Solve dual linear program
Dual Unbounded
yes
STEP 6
no
Finite optimal
STEP 4
Objective function = $Z^0 - f(y^0)$
yes
$(Z^0, \underline{y}^0)$ solves MP2
set flag
$\underline{x}^0$ Solves the dual
yes
$(\underline{x}^0, \underline{y}^0)$ solves P1
set flag
STEP 5


STEP 5
If optimality test in Step 3 is not passed and the dual has a finite optimal solution which is the extreme point $\underline{u}^0$, then
$Z^0 < f(\underline{y}^0) + \underline{b} - \underline{F}(\underline{y}^0)\ ^T\underline{u}^0$
and the current solution to MP2 does not satisfy the constraint
(C) $Z \geq \underline{b} - \underline{F}(\underline{y})\ ^T\underline{u}^0 + f(y)$
TEST
yes
no
Add constraint (C) to integer problem MP2
STEP 2
STEP 6
Dual was unbounded and the linear program simplex located an extreme ray $\underline{v}^0$ and point $\underline{u}^0$ such that the dual objective → + ∞.
$\underline{b} - \underline{F}(\underline{y}^0)\ ^T\underline{v}^0$
is satisfied add this constraint to MP2
$C^0 = f(y^0) + \underline{b} - \underline{F}(\underline{y}^0)\ ^T\underline{u}^0$
Is $Z^0 < C^0$ for $\underline{u}^0$
no
yes
Add constraint (C) to integer problem MP2
STEP 2

Program Deck Setup and Listing

Benders' decomposition algorithm as applied to the time phasing of reservoir system operation with capacity expansion is listed below. Programs MASTER, LPSIM, and NEWBI are listed in Appendix A. A complete input system for running the program in WATFIV is depicted as:



```
C
C      SUPERVISOR
C
       INTEGER T
       DIMENSION TAB(25,99),XP(99),C(99),XLB(99)
      1,XUB(99),NXB(25),CB(25),NSRKI(25),NRESFC(25)
      2,BPR(25),CY(10,10,10),UP(25,25),PT(50)
      3,NRCS(10),JT(10),JK(10),YIN(10),JJ(10),RCS(10,10)
      4,HXO(25),HBB(25),Y(10,10,10)
       COMMON /CLINK/IO(25,25) , NPTR(25), NVPTR(25,25),NP
      1 , NRPTR(25,25) , NRF(25),  NSRF(25,25)
       COMMON /CLPSIM/ NV,NC,TAB,C,XUB,XLB,XP,NXB,CB
       COMMON /CMAINL/ NARTV, UR(25,25), NUR, NURC  ,PA(99)
      1, ASUMP(25) , ISUMC
       COMMON /CINTP/ NURH(25), NURHC(25,25),CX(100)
      1,AX(25,100),RSUM(25)
C      RCS - RESERVOIR CA?ACITY SEGMENT
C      INITY -  0 NO INITIAL Y READ
C               NOT 0  INITIAL Y READ
C      PA HAS TRUE LOWER BOUNDS STORED IN IT
       DATA TOL/ 1.0/
 10    FORMAT(16I5)
 15    FORMAT(8F10.3)
C
C      NUMBER OF RESERVOIRS AND TIME PERIODS
C
C                     CARD 1
C
 20    READ(5,10,END=500) NR,NT,INITY,MITER,
      1 (NRCS(I),I=1,NR)
       WRITE(6,22)
 22    FORMAT('1'14X'RESERVOIR SEGMENT CAPACITIES ')
       DO 25 K=1,NR
       N=NRCS(K)
C
C                     CARD 2
C
       READ(5,15) (RCS(K,J),J=1,N)
 25    WRITE(6,30) K,(RCS(K,J),J=1,N)
 30    FORMAT(10XI5,10F10.3,10(/15X,10F10.3))
       DO 35 T=1,NT
       DO 35 K=1,NR
       N=NRCS(K)
C
C      COST FOR CAPACITY
C
C                     CARD 3
C
       READ(5,15)     (CY(T,K,I),I=1,N)
```

```
      WRITE(6,32) T,K,(CY(T,K,I),I=1,N)
   32 FORMAT(5X'COST CAP.'2I5,10F10.3,10(/24X10F10.3))
      DO 35 J=1,N
   35 Y(T,K,J)=0.0
      IF(INITY.EQ.0) GO TO 60
C
C     Y VALUES REPRESENT WHICH RESERVOIR IS TO BE
C     ADDED IN TIME PERIOD SPECIFIED
C     READ INITIAL Y VALUES
C
C                       CARD 4
C
   40 READ(5,45) (JT(I),JK(I),JJ(I),YIN(I),I=1,10)
   45 FORMAT(10(3I2,F2.0))
      IF(JT(1).EQ.0) GO TO 60
      DO 50 I=1,10
      T=JT(I)
      IF(T.EQ.0) GO TO 40
      K=JK(I)
      J=JJ(I)
      WRITE(6,55) T,K,J,YIN(I)
   55 FORMAT(25X,'Y('I2','I2','I2')= ',F3.1,I5,F10.3)
   50 Y(T,K,J)=YIN(I)
      GO TO 40
C
C                             STEP 1
C
   60 CALL MASTER(BPR,NC)
C     SETUP INTEGER PROGRAM
      ZO= -1.0E30
      NADDC= 0
      DO 61 K=1,NR
   61 NURH(K) = 0
      NUR=0
      ICOL= 0
      DO 62 K=1,NR
      ICOL= ICOL+ NRCS(K)*NT
   62 NUR= NUR+ NRCS(K)
      DO 63 I=1,NUR
      DO 63 J=1,ICOL
   63 AX(I,J) = 0.0
      NUR=0
      ICOL=0
      DO 64 K=1,NR
      N= NRCS(K)
      DO 64 I=1,N
      NUR= NUR+1
      DO 65 T=1,NT
      ICOL= ICOL+ 1
```

```
   65 AX(NUR, ICOL) = 1.0
   64 RSUM(NUR) = 1.0
      NURF= NUR
C
C                             END STEP 1
C     GO TO LINEAR PROGRAM SETUP AND CALL
C
      IF( INITY.NE.0) GO TO 126
   66 IF( NADDC+NUR .EQ. 0) GO TO 250
C
C                             STEP 4
C     NO CONSTR. THEN ANY FEASIBLE Y WILL DO
C
      ITER = NADDC + NUR - NURF
      IF( ITER     .LE. MITER) GO TO 70
      WRITE(6,68) MITER
   68 FORMAT('  MAXIMUM NUMBER OF ITERATIONS 'I5)
      STOP
C
C     SETUP FOR INTEGER SUB.
C
   70 ZO= -HXO(NADDC)
      ICOL=0
      DO 80 K=1,NR
      N= NRCS(K)
      DO 80 J=1,N
      DO 79 T=1,NT
      ICOL=ICOL+1
      SUM=0.0
      DO 75 L1=T,NT
      ISUB= 1+2*(K-1)+ 2*NR*(L1-1)
   75 SUM = SUM - ABS( UP(ISUB,NADDC) )
      ZO= ZO + SUM*RCS(K,J)*Y(T,K,J)
      CX(ICOL) = SUM*RCS(K,J) + CY(T,K,J)
   79 CONTINUE
   80 CONTINUE
C
C                             END STEP 4
C
      IF( NURC.EQ.0) GO TO  91
C
C     NSRKI(K)- NUMBER OF INTEGER VARIABLE STARTING RES. K
C     VARIABLE IN INTEGER PROBLEM
C
      IVAR=1
      RSUM(NUR)=ASUMP(NUR)
      DO 81 K=1,NR
      NSRKI(K)=IVAR
   81 IVAR=NT*NRCS(K)+IVAR
```

```
C
C                          STEP 5
C
C       LOCATE MOST VIOLATED CONSTRAINT BY
C       RESERVOIR AND TIME PERIOD
C
        ICON=0
        DO 84 T=1,NT
        DO 84 K=1,NR
        ICON=ICON+1
        IF(ICON.NE.ISUMC) GO TO 83
   82   ICONT=T
        ICONR=K
        GO TO 784
   83   ICON=ICON+1
        IF(ICON.EQ.ISUMC) GO TO 82
   84   CONTINUE
C
C       NRESFC(K)-RES. K CONSTRANT FEASIBLE -0
C       CRITICAL TIME PERIOD
C       NRESFC(K)-RES. K CONSTRAINT NOT FEASIBLE -1
C       CRITICAL TIME PERIOD
C
  784   IVAR=NARTV+4*NR*(ICONT-1)+1
        DO 87 K=1,NR
        NRESFC(K)=0
        IF(XP(IVAR).NE.0.0) GO TO 85
        IF( XP(IVAR-1) .NE. 0.0) GO TO 86
   85   NRESFC(K)=1
   86   IVAR=IVAR+2
        IF(XP(IVAR).EQ.0.0) GO TO 87
        NRESFC(K) = 1
   87   IVAR=IVAR+2
C
C       ICOL IS THE TOTAL NUMBER OF INTEGER VAR.
C
        DO 88 I=1,ICOL
   88   AX(NUR,I)=0.0
C
C       GENERATE MOST VIOLATED CONSTRAINT A COEFICIENT
C
        N=NRCS(ICONR)
        IVAR=NSRKI(ICONR)
        DO 90 I=1,N
        DO 89 T=1,ICONT
        RSUM(NUR)=RCS(ICONR,I)*Y(T,ICONR,I)+RSUM(NUR)
        AX(NUR,IVAR)= RCS(ICONR,I)
   89   IVAR=IVAR+1
   90   IVAR=IVAR+NT-ICONT
```

```
      J=NPTR(ICONR)
      IF(J.EQ.0) GO TO 991
      DO 990 K1=1,J
      K= NRPTR(ICONR,K1)
C
C     K-RESERVOIR THAT PUMPS TO I CONSTRAINT
C
      NOF=(NP+NR)*(ICONT-1)+NVPTR(ICONR,K1)
      IF( XP(NOF) .LE. 0.0) GO TO 990
      SUMC=0.0
      N=NRCS(K)
      DO 895 I=1,N
      DO 895 T=1,ICONT
 895  SUMC=SUMC+ Y(T,K,I)
      ALLOC= 0.0
      IF( SUMC.EQ.0.0) GO TO 896
      ALLOC=(XUB(NOF)-XP(NOF))/SUMC
      RSUM(NUR)=RSUM(NUR)-XP(NOF) + XUB(NOF)
 896  ICOLT=NSRKI(K)
      COSTM=1.0E30
C
C     STORE AVERAGE CAPACITY UTILIZED INTO
C     ALLOCATED COEFFICIENT
C
      DO 910 I=1,N
      DO 900 T=1,ICONT
      AX(NUR,ICOLT)=ALLOC *Y(T,K,I)
      IF( T.NE. ICONT) GO TO 900
      IF(Y(T,K,I).NE.0.0) GO TO 900
      IF(CX(ICOLT).GE.COSTM) GO TO 900
      IMINC=ICOLT
      IMINS=I
      COSTM=CX(ICOLT)
 900  ICOLT=ICOLT+1
      ICOLT= ICOLT+ NT-ICONT
 910  CONTINUE
      IF(NRESFC(K).EQ.0) GO TO 990
C
C     RES. CAP. VIOLATED CHOOSE NEXT CHEAPEST
C     RES. SEQ TO ALLOC REMAINING PUMPING CAPACITY
C
      ALLOC=XP(NOF)
      IF(COSTM.GT.1.0E28) GO TO 990
      IF(ALLOC.GT.RCS(K,IMINS)) ALLOC=RCS(K,IMINS)
      AX(NUR,IMINC)=ALLOC
 990  CONTINUE
C
C     NSRF(K,J) - RES.NO. OF RES. REL. INTO K
C     NRF(K) - NO. REL TO RES. K
```

```
C      CHECK RELEASE VARIABLE
C
 991   J=NRF(ICONR)
       IF(J.EQ.0) GO TO 91
       DO 1020 K1=1,J
       K=NSRF(ICONR,K1)
C
C      K IS RES. THAT REL. TO RES. ICONR
C
       NOF=(NP+NR)* (ICONT-1)+K
       IF(XP(NOF).LE.XLB(NOF)) GO TO 1020
C
C      RELEASE EXTRA QUANTITY INTO RES. ICONR
C
       N=NRCS(K)
       SUMC=0.0
       DO 995 I=1,N
       DO 995 T=1,ICONT
 995   SUMC=SUMC+Y(T,K,I)
       ALLOC=0.0
       IF(SUMC.EQ.0.0) GO TC 1000
       ALLOC=(XUB(NOF)-XP(NOF))/SUMC
       RSUM(NUR)=RSUM(NUR)+XUB(NOF)-XP(NOF)
 1000  ICOLT=NSRKI(K)
       COSTM=1.0E30
C
C      STORE AVG. RELEASE INTO ALLOC COEF.
C
       DO 1010 I=1,N
       DO 1005 T=1,ICONT
       AX(NUR,ICOLT)=ALLOC*Y(T,K,I)
       IF( Y(T,K,I) .NE. 0.0) GO TO 1005
       IF( CX(ICOLT).GE. COSTM) GO TO 1005
       IMINC= ICOLT
       IMINS = I
       COSTM= CX(ICOLT)
 1005  ICOLT= ICOLT+ 1
 1010  ICOLT= ICOLT+ NT-ICONT
       IF( NRESFC(K) .EQ.0) GO TO 1020
C
C      PUT POSSABLE ALLOC. INTO CHEAPEST NONALLOC. RES.
C
       ALLOC = XP(NOF)- XLB(NOF)
       IF( COSTM.GT. 1.0E28) GO TO 1020
       IF( ALLOC.GT. RCS(K,IMINS)) ALLOC= RCS(K,IMINS)
       AX(NUR,IMINC) = ALLOC
 1020  CONTINUE
   91  WRITE(6,96) ( I,I=1,ICOL)
       IF(NURC.EQ.0) GO TO 92
```

```
      ZO=ZOHD
      GO TO 93
   92 ZOHD=ZO
   93 WRITE(6,94) ( CX(I),I=1,ICOL),ZO
   94 FORMAT(' CX '10F7.2 )
      IF(NUR.EQ.0) GO TO 120
      DO 95 J=1,NUR
   95 WRITE(6,97) ( AX(J,I),I=1,ICOL), RSUM(J)
   96 FORMAT(1X 10I7)
   97 FORMAT(' AX ' 10F7.2)
C
C                         END STEP 5
C
C
C                         STEP 2
C     CALL TO INTEGER SUBROUTINE
C
  120 CALL INTSUB(ZO, Y, NRCS, NT, NR)
C
C     CALL TO MASTER PROGRAM WHICH SETS UP
C     THE NEW LINEAR PROGRAM FOR L P S I M
C
      CALL MASTER (BPR,NC)
  126 DO 130 I=1,NC
  130 PT(I)= BPR(I)
      DO 145  T=1,NT
      DO 145  K=1,NR
      N=NRCS(K)
      DO 140  L=T,NT
      ISUB=1+2*(K-1)+2*NR*(L-1)
      DO 140 J=1,N
  140 PT(ISUB)=PT(ISUB)+Y(T,K,J)*RCS(K,J)
  145 CONTINUE
      NADDC=NADDC+1
C
C                         STEP 3
C
C     CALL TO LINEAR PROGRAM
C
  170 CALL LPSIM(PT,XO,UP(1,NADDC))
      HXO(NADDC) = -XO
      NADDC= NADDC-NURC
C
C     GO TO INTEGER PROGRAM SET UP AND CALL
C
      IF( NURC.NE.0) GO TO 66
C
C                         STEP 6
C
```

```
      SUM=-XO
      DO 180 T=1,NT
      DO 180 K=1,NR
      N= NRCS(K)
      DO 180 J=1,N
  180 SUM= SUM+ Y(T,K,J)*CY(T,K,J)
      AMDIF= SUM - ZO
      ITER= NADDC + NUR  - NURF
      WRITE(6,190) ITER, SUM, AMDIF
  190 FORMAT('-ITER ='I3,' COST OF POLICY ='F7.2,
     1 ' MAXIMUM ERROR =' F7.2)
C
C     SOLUTION NOT OPTIMAL, TEST IF WITHIN TOLORANCE
C     COMPUTE MAXIMUM ERROR ON OBJECTIVE FUN. VALUE
C
      IF( ABS(AMDIF) .LE. TOL) GO TO 200
      GO TO 66
C
C     STEP 2 BRANCH POINT AND END
C     OPTIMAL SOLUTION
C
  200 WRITE(6,205)
  205 FORMAT('1' 24X'OPTIMAL SOLUTION')
      IX= 0
      DO 230 T=1,NT
      DO 230 K=1,NR
      N= NRCS(K)
      IX= IX+1
      DO 230 J=1,N
      IF( Y(T,K,J) .EQ. 0.0) GO TO 230
      WRITE(6,55) T,K,J,Y(T,K,J)
  230 CONTINUE
      WRITE(6,235) SUM
  235 FORMAT('0' 24X 'COST ' F10.1,/)
      GO TO 20
C
C     NO SUBPROBLEM CONSTRAINTS , ANY VALUE FOR Y
C     WILL DO
C
  250 DO 260 K=1,NR
      N=NRCS(K)
      DO 260  J=1,N
      Y(1,K,J)=1.0
      DO 255  T=2,NT
  255 Y(T,K,J)= 0.0
  260 CONTINUE
      ZO= 1.0E30
      WRITE(6,265)
  265 FORMAT('-',15X'ARBITRARY VALUES STORED INTO Y')
```

```
      GO TO 126
  500 STOP
      END
```

```
      SUBROUTINE  INTSUB(Z,Y,NRCS,NT,NR)
C     THIS PROGRAM IS NOT AN INTEGER PROGRAM
C     IT WAS DESIGNED TO SIMULATE AN INTEGER PROGRAM
      COMMON /CINTP/ NURH(25),NURHC(25,25)
     1, CX(100),AX(25,100),RSUM(25)
      DIMENSION Y(10,10,10), NRCS(10)
      DATA ITR/0/ , MITR/6/
      ITR= ITR+1
      IF( ITR.GT. MITR) STOP
      DO 40 K=1,NR
      N= NRCS(K)
      DO 40 I=1,N
      DO 40 J=1,NT
   40 Y(J,K,I) = 0.0
      GO TO (1,2,3,4,5,6),ITR
    1 GO TO 50
    2 Y(2,1,3)  = 1.0
      Y(1,1,2)  = 1.0
      GO TO 50
    3 Y(1,1,1) = 1.0
      Y(1,1,2) = 1.0
      Y(2,1,3) = 1.0
      Y(1,2,3) = 1.0
      GO TO 50
    4 Y(1,1,1) = 1.0
      Y(1,1,2) = 1.0
      Y(2,1,3) = 1.0
      Y(1,2,3) = 1.0
      Y(1,3,2) = 1.0
      GO TO 50
    5 Y(1,1,1) = 1.0
      Y(1,1,2) = 1.0
      Y(2,1,3) = 1.0
      Y(1,2,3) = 1.0
      Y(1,2,1) = 1.0
      Y(1,3,2) = 1.0
      GO TO 50
    6 Y(1,1,1) = 1.0
      Y(1,1,2) = 1.0
      Y(2,1,3) = 1.0
      Y(1,2,3) = 1.0
      Y(1,2,1) = 1.0
      Y(1,3,1) = 1.0
   50 WRITE(6,51)
   51 FORMAT('-')
      Z=-Z
      ICOL=0
      DO 55 K=1,NR
      N= NRCS(K)
```

```
      DO 55 I=1,N
      DO 55 J=1,NT
      ICOL= ICOL+1
      IF( Y(J,K,I) .EQ. 0.0) GO TO 55
      Z= Z + Y(J,K,I) * CX(ICOL)
      WRITE(6,52) J,K,I, Y(J,K,I)
   52 FORMAT(24X'Y(' I2,',' I2',',I2')=' F4.1)
   55 CONTINUE
      WRITE(6,60) Z
   60 FORMAT('-' 20X 'OPTIMAL SOLUTION' F10.3 )
      RETURN
      END
```