A STOCHASTIC MIXED INTEGER PROGRAMMING APPROACH TO WILDFIRE MANAGEMENT SYSTEMS

A Thesis

by

WON JU LEE

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2006

Major Subject: Industrial Engineering

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Approved by:

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ABSTRACT

A Stochastic Mixed Integer Programming Approach
to Wildfire Management Systems. (August 2006)
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Chair of Advisory Committee: Dr. Lewis Ntaimo

Wildfires have become more destructive and are seriously threatening societies and our ecosystems throughout the world. Once a wildfire escapes from its initial suppression attack, it can easily develop into a destructive huge fire that can result in significant loss of lives and resources. Some human-caused wildfires may be prevented; however, most nature-caused wildfires cannot. Consequently, wildfire suppression and containment becomes fundamentally important; but suppressing and containing wildfires is costly.

Since the budget and resources for wildfire management are constrained in reality, it is imperative to make important decisions such that the total cost and damage associated with the wildfire is minimized while wildfire containment effectiveness is maximized. To achieve this objective, wildfire attack-bases should be optimally located such that any wildfire is suppressed within the effective attack range from some bases. In addition, the optimal fire-fighting resources should be deployed to the wildfire location such that it is efficiently suppressed from an economic perspective.

The two main uncertain/stochastic factors in wildfire management problems are fire occurrence frequency and fire growth characteristics. In this thesis two models for wildfire management planning are proposed. The first model is a strategic model for the optimal location of wildfire-attack bases under uncertainty in fire occurrence. The second model is a tactical model for the optimal deployment of fire-fighting resources under uncertainty in fire growth. A stochastic mixed-integer programming approach is proposed in order to take into account the uncertainty in the problem data and to allow for robust wildfire management decisions under uncertainty. For computational results, the tactical decision model is numerically experimented by two different approaches to provide the more efficient method for solving the model.

To God the Father Almighty

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CHAPTER I

INTRODUCTION

Damage caused by wildfires has been a serious problem to our society. In 2005 more than 66,000 wildfires were reported and more than 8 million acres were burned in the US alone (see Table I). Also in each year, a huge budget is allocated for wildfire suppression and containment (see Table II) [1]. It is also estimated that more than 11,000 communities adjacent to federal lands are at risk from wildfires [2].

Table I. Total fires and acres

Year	Fires	Acres	Year	Fires	Acres
2005	66,552	8,686,753	1999	93,702	5,661,976
2004	77,534	6,790,692	1998	81,043	2,329,709
2003	85,943	4,918,088	1997	89,517	3,672,616
2002	88,458	6,937,584	1996	115,025	6,701,390
2001	84,079	3,555,138	1995	130,019	2,315,730
2000	122,827	8,422,237	1994	114,049	4,724,014

It is imperative to control these catastrophic wildfires in very efficient ways because the budget and the resources are limited. One possible way to deal with this problem is to invest on the prevention effort. Prevention effort may include education, campaign, or patrol. Some human-caused wildfires may be prevented through this prevention effort. However, nature-caused wildfires cannot be prevented. Thus, it is necessary to contain the fires while they are small to minimize the associated costs and damage. Failure to contain a small fire may result in an escaped destructive huge fire. To contain the fires while they are small, it is important to deploy the fire-fighting resources in efficient ways. Consequently, it becomes very important to

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This thesis follows the style of *IEEE Transactions on Automatic Control.*

consider the strategic and tactical decisions in wildfire management.

Table II. Total suppression cost

Year	Total Suppression Cost
2004	$890,233,000
2003	$1,326,138,000
2002	$1,661,314,000
2001	$917,800,000
2000	$1,362,367,000
1999	$523,468,000

In terms of the strategic decisions, the location of the wildfire attack-bases and the allocation of the fire-fighting resources should be strategically taken into account. In terms of the tactical decision, an optimal mix of the fire-fighting resources must be determined. These strategic and tactical decisions may help to reduce the chance that the wildfire becomes a destructive huge fire. Mathematical programming such as LP may be useful to model these decisions. However, LP does not take into account the randomness or uncertainty in the problem data since fire behavior is stochastic. Thus stochastic programming approaches are needed in order to take into account the stochastic factors of the problems.

This thesis is organized as follows. In chapter II, wildfire related literatures is reviewed, and the basic concepts of the stochastic programming are introduced. In chapter III, a new stochastic mixed inter programming model for the attack-base location and resource allocation is presented. A new stochastic mixed integer programming resource allocation model for wildfire containment is presented in chapter IV. Chapter V provides solution methods for solving the proposed models. Computational results of the model proposed in chapter V are given in chapter VI. Finally, concluding remarks and future research direction are discussed in chapter VII.

CHAPTER II

LITERATURE REVIEW

Wildfires have become more destructive and are seriously threatening our ecosystems and societies all over the world. It is therefore imperative to make great efforts to reduce the devastation wildfire damage by setting up effective wildfire management plans. The scope of the wildfire management is broad. It includes wildfire containment and suppression. To utilize the limited budget and resources more efficiently, it is important to make optimal strategic decisions and tactical decisions. The strategic decisions include long-term planning for airtanker base location and resource allocation. The tactical decisions include short-term planning and scheduling for the resources with respect to actual wildfire occurrence. Mathematical programming and simulation methods are widely used in making these strategic and tactical decisions. This chapter is composed of four sections. Section A provides preliminary knowledge related to wildfire management systems. Section B provides strategic decision models of wildfire management. Section C provides tactical decision models of wildfire management, and finally section D provides a brief review of the stochastic programming ideas that will be employed to solve the proposed model later chapter.

A. Preliminaries

In terms of wildfire management, analyzing wildfire economics or estimating the probability of wildfire occurrence are important topics that give fundamental background to investigate wildfire-related problems. More fundamentally, it is also important to know how the wildfire management systems have evolved and what is needed to make the systems more effective.

One of the most important reasons for doing research on wildfire suppression is to reduce the damage to the natural resource and risk to human. [3] researched on fire risk in the wildland-urban interface (WUI). WUI is the area where houses and dense vegetation exist together. It is known that WUI covers 10% of the United States [3]. The objective of the study in [3] was to identify the risk of severe wildfire in WUI areas, and how many people and houses were statistically affected through the case of northern lower Michigan. The researchers quantified fire risk by empirical methods of fire occurrence. The results may be useful in determining the location of emergency service site or fire-fighting resource sites.

[4] conducted research on the economic analysis of the relationship between value of resource protected and suppression expenditure of wildfires. This study may play a key role in determining the economic efficiency of suppression and containment activities. [4] argue that the benefit of suppression efforts should be greater than cost of resource value changes that would have burned with no suppression effort. The authors conclude that the expenditure on suppression is worthwhile if benefit is greater than cost.

There has been much effort to evaluate and quantify wildfire risk. The National Fire Danger Rating System is the one example of this effort [5]. It is provided the methods of quantifying wildfire risk [5]. One of the significant outputs includes the fire danger map based on fire weather variables such as temperature, humidity, wind and danger variables such as burning index, fire potential index, spread component. Based on these variables, three different fire risk probabilities can be defined: (1) The probability of fire occurring, defined as the probability of a fire of size greater than 0.04ha (hectare) occurring at a given location in a given day, (2) the conditional probability of a large fire given ignition, where large fire is defined as size of more than 40ha, (3) the unconditional probability of a large fire that is defined as the product of (1) the probability of fire occurring and (2) the conditional probability of a large fire given ignition. Decisions on fire containment and suppression resource deployment may be made based on this (3) unconditional probability. These probabilities can be used to get the expected number of fires in a given region for a given time period. If a given area is divided into grids, each grid can be assigned its own probability of fire occurrence, and binary random variable (1 if fire occurs, 0 otherwise). Then the distribution of the total number of fires over entire areas is the Poisson-Binomial distribution. Thus sum of the probabilities of each grid gives the expected number of fires. Also the Poisson distribution can be employed to obtain confidence intervals.

Economists have expanded the methods of evaluating and quantifying the total economic value of wildland. Some methods of evaluating the economic value of wildland has been suggested by [6]. Wildland ecosystems can be viewed as natural capital that can produce a wide range of goods and services for mankind. Generally, timber, easily exchanged and quantified in terms of price, is considered as the most valuable goods from wildland. However there are many other outputs from wildland such as carbon storage, minerals, soil productivity, recreational use, etc. Some of them are not easily quantified as economic value. When dealing with wildfire, assessing the value of the wildland to be protected is extremely important since the solution for many kinds of wildfire suppression and containment problems depend on the economic value of the area. In this thesis, the economic value of the area is referred as value-at-risk.

Since damage caused by wildfire is composed of tangible and intangible factors, it is extremely difficult to quantify the economic value of the damages. [7] quantified the economic value of the damages caused by wildfires in Florida. They investigated seven major categories of damages, that is, pre-suppression costs, suppression costs, disaster relief expenditures, timber losses, property damage, tourism-related losses, and human health effects. This research is worthwhile because it quantifies economic impacts of wildfires systematically and empirically.

[8] proposed the theoretical framework of wildfire economics, that is the Cost plus Net Value Change (C+NVC) model. This model has been used as a principal model of evaluating wildfire economics. This model minimizes the cost of wildfire by minimizing sum of the pre-suppression cost, suppression cost, and net value change. The pre-suppression cost is the fixed cost that is spent before the fire season starts to minimize overall wildfire occurrences through education, patrol, campaign, or investment on resources or new facilities. The suppression cost is the cost that is spent during the fire season. Most of the cost is associated with fire suppression and containment operation cost. The NVC is the cost that is incurred by the damage from wildfire during the fire season. The C+NVC has become the most widely used economic theory in the text of wildfire management.

The C+NVC model was reformulated by [9] by correcting some of the assumptions. The original model treats suppression cost as a output while the corrected model treats suppression cost as a input. Also the original model assumes that suppression and pre-suppression costs are negatively correlated while the corrected model assumes that only suppression cost varies. As a result, the corrected C+NVC model provides the global minimum of the function. Figure 1 provides the corrected (C+NVC) framework. It is shown that the pre-suppression cost is fixed since it is spent before the fire season begins. Also, it can be inferred that suppression cost and Net Value Change are negatively correlated since the more suppression efforts we have, there tends to be less burned area by wildfires. The sum of the suppression cost, the pre-suppression cost, and the net value change provides the global function of the C+NVC. Since it forms a convexity, global minimum can be found by minimizing this function.

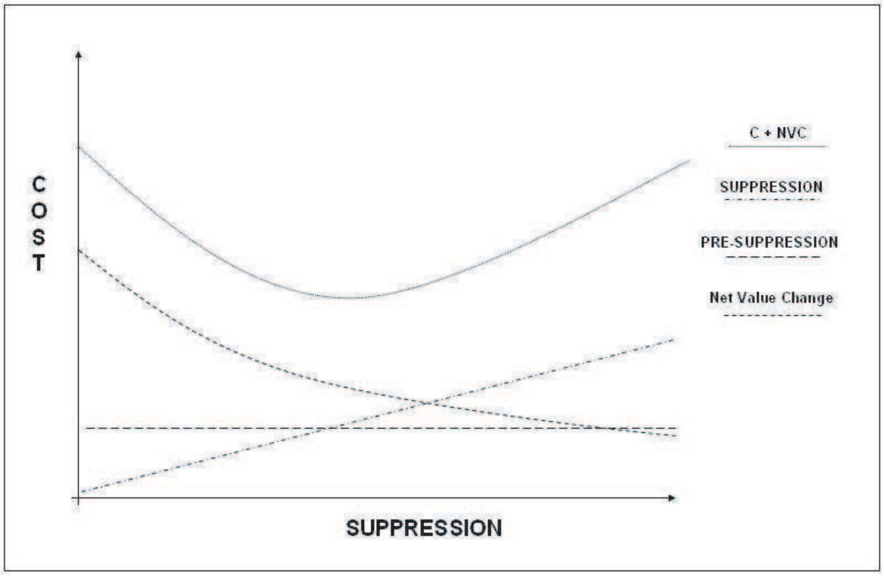

Fig. 1. Cost + NVC[9]

Before examining detailed literature on containment and suppression, it is worthwhile to review broad research on wildfire management. [10] provided good review of operational research studies in forest fire management by taking into account several topics such as prevention and fuel management, strategic and tactical detection planning, operations research techniques on fire suppression, and fire impact management policy model. In terms of operations research methods on fire suppression, they are subdivided into four topics. (1) Resource acquisition and strategic deployment involve determining what resources to acquire and where to locate them. (2) Resource mobilization involves resource re-allocation due to fluctuating fire load. (3) Initial attack dispatching deals with resources that are dispatched to the fire location in the early stages of the fire occurrence. Initial attack dispatching may be complicated since the precise fire location, size and many other fire related variables are uncertain. (4) Extended attack management involves suppression and containment effort to the fire that is escaped from initial attack and developed into a large fire.

B. Strategic Decision

Strategic decisions of wildfire management systems are the long-term plans that should be carefully considered because it may require much time and financial investment resources and may not be easily modified once it is executed. Some of the strategic decision models are conceptual, while others are more concrete. And many of the strategic models are related with the attack-base location problems. [11] provides a good review of the location problems. Some of the problems are defined as follows. Location set covering problem (LSCP) is to locate the least number of facilities that are required to cover all demand points. Maximal covering location problem (MCLP) is to seek the maximal coverage with a given number of facilities and does not necessarily require to cover all demand points. The objective of P-median problem is to optimize the average (total) distance between the demand points and the facilities, and the objective of P-center problem is to minimize the maximum distance between the demand points and the facilities.

A new fire management paradigm that takes into account broad objectives has been proposed by [12]. The authors focus on a performance-based system. Most of the previous models optimize the suppression-related problems. Instead, the proposed system is conceptual and it is composed of such elements as fire and land management plans, ecosystem and fire simulators, fire management resources, program constraints, fire response, fuels treatment, and performance evaluator. [12] recommends that the geographic information should be carefully taken into account in making the fire management plans.

A strategic decision model for location-allocation of the air-tankers is proposed in [13]. The model decides where to locate wildfire attack-bases among potential base locations. A given area is divided into a set of sub-areas in the model. The value-at-risk concept is used to quantify the value of the sub-area and to prioritize the sub-area in terms of the economic value. Also the authors take the distance from base to sub-area (demand point) into consideration in the objective function of the model since fire scale, or fire perimeter gets bigger with time. The model utilizes the historical fire data of the fire occurrence location and considers several different time periods. The model is then optimized with respect to the different time periods. Consequently, the solutions from the model may not be appropriate in making strategic decisions since each time period considered may provide different solutions of the base location.

Several different location models in locating wildfire attack-bases were analyzed in [14]. These models include covering, p-mediam, p-center, and some hybrid models. These models give various solutions that make sensitivity analysis possible. These models also utilize the historical fire occurrence data of the several different time periods as [13] did. Consequently, the solutions from the models may not be appropriate in making strategic decisions.

An optimization model that minimizes the wildfire damage by locating and deploying fire-fighting resources in critical locations was proposed in [15]. The proposed model mainly has two parts: a geographic information system (GIS) module and a mathematical programming module. Based on the GIS information, the demand area to be covered is subdivided into a number of non-uniform, and non-overlapping sub-areas. The data collected by GIS are used as input to an mathematical programming module. By taking into account the data from the GIS module, the mathematical programming module determines the optimal fire-fighting resource locations. Maximal covering location model was employed in the mathematical programming module.

As an extension of [15], an integrated framework for wildfire control is proposed in [16]. It has three major modules; a mathematical module, a simulation module and a GIS module. The mathematical module provides strategic decision solutions for long-term plans that include how many resources to need to cover the given area, and where to locate resources. The simulation module provides tactical solutions for short-term decision making. In the mathematical module, they utilize the set covering problem and the maximal covering location problem. In the set covering problem, it assumes that one vehicle assigned to each demand point can suppress all the wildfires within its coverage. The simulation module considers the resource re-allocation based on updated fire figures, thus it may not be appropriate to explain the detailed deployment plan of the optimal mix of resources for a specific fire.

A model that specifies where to locate the service site and how many resources such as emergency vehicles to be assigned at each site is provided in [17]. This emergency medical service application has much similarity to wildfire management application since the emergency location can be viewed as the wildfire location and the service site can be viewed as the site where wildfire attack resources are based. One of the significant characteristics of this model is that it takes into account the uncertainty of the demand of emergency service by employing stochastic programming approaches. Without the stochastic factors in this model, it belongs to the general class of the covering problem. It employs a probabilistic constraint approach that allows demand requests to be satisfied with a certain reliability α.

More advanced location problems through the evolution of ambulance locations and relocation models are reviewed in [18]. As previously mentioned, emergency medical service application has much similarity to wildfire management application. This paper reviews probabilistic models rather than the deterministic location models. Maximum expected covering location problem (MEXLP) and maximum availability location problem (MALP) are the probabilistic versions of the maximal covering location problem. MEXCLP maximizes the expected coverage of all demand areas. It is assumed that servers operate independently and that all servers have the same busy probability. MALP maximizes the demand that should be covered with a given reliability α. Largely MALP may be sub-divided into 2 categories; MALP I and MALP II. MALP I uses the same busy fraction for all potential location sites while MALP II relaxes the assumption that the busy fraction is identical for all sites.

C. Tactical Decision

Tactical decisions in wildfire management deal with relatively short-term decisions. These include the mix of the resources to contain a particular fire, scheduling of airtanker based on fire demands, or fire behavior simulation. The goal of these approaches is to analyze the short-term wildfire related problems and provide optimal decisions in order to minimize the damage or risk of wildfires by containing wildfires in efficient ways.

A tactical decision model that determines the optimal mix of the fire suppression resources for the particular fire to minimize the C+NVC function is proposed by [19]. This model fits well when it needs to be answered which resources to deploy and when to contain the fire with minimum cost. However, this model assumes that fire perimeter in the particular time period is deterministic. That assumption may not be applicable in reality since fire growth behavior is stochastic.

A simulation approach for the wildfire containment is provided in [20]. Most of the simulation approaches are focused on estimating a fire's capacity to spread. Instead, this model focuses on the interaction between the production of containment line and a fire's capacity to spread. It allowed a flexible choices of the fire shape, and different types of attack method by the resources. However, it may not be able to provide the optimal mix of the resources to minimize total cost within the budget and resource constraints. Rather it focuses on when to contain the fire with all available resources. By the nature of simulation, it may not be easy to find the optimal mix of resources with a small number of experiments since this simulation model only gives results with a set of given parameters in each run.

A mathematical programming model considering the daily basing rule of airtankers is proposed in [21]. After a fire load profile is obtained, it assigns airtankers in each base and the airtankers move among the bases based on the fire load. The fire load varies from period to period, and basing airtankers is dependent on the fire load data that are prepared from the long-term historic data. The objective of the model is to minimize the incurred cost of basing airtakers. Thus one airtanker is not fixed in one specific air base. One of the strengths of this model is that it actually deals with daily use of airtankers based on demand. However it does not take into account such considerations as the fire which continues more than one day, rather it assumes that all demands are satisfied in one day by the airtanker.

A model that investigates how airtanker system performance is associated with initial attack range is proposed in [22]. This model uses a simulation method to incorporate the complex systems that contain many stochastic factors. Systems that are composed of fire events and airtanker that serves fire are described as queueing systems just as they are customers and servers in the general queueing systems. It is defined that the fire arrival process as the non-stationary process since the daily fire load varies during the day. It is found that the initial attack range varies as a function of the daily fire load. As a result, they suggest that it is required to set initial attack range of individual airtanker to increase the efficiency of the suppression effort and minimize the associated cost.

A model that evaluates the performance of the initial attack of the airtankers is provided in [23]. When the probability of the fire occurrence within the initial attack zone of an airtanker base is specified, this model provides important information regarding expected flight distance between the airtanker base and a random fire location, and computes flight distance distribution. This is done by the mathematical procedure that uses data such as the coordinates of the base location, the geological distribution of fire occurrence. The model also takes the fire-start location, the distance between air-base and fire-start location as random variables. The results of the model are used to evaluate the performance of the airtankers, one of the most expensive resources in a wildfire fight.

Simulation models that simulates the wildfire spread and suppression are devised by [24], and [25]. Especially [24] employs the a discrete event system specification cell space approach. Its advantages are that it only considers active cells in computation and transmission of messages, thus it improves the efficiency of the simulation. In addition, the cells in the simulation can be dynamically created and deleted as needed. The model considers static cells that stores geographic information and dynamic cells that may have different stochastic characteristics of weather conditions. These cells decide fire spread and fire-line intensity that play key roles in the simulation. From the simulation, the cells change the status; unburned, burning, and burned. And the status tells how fast and where the fire spreads. It also includes the suppression function so that the suppressant keeps the fire in the cells from propagating further.

The National Fire Management Analysis System (NFMAS), especially the sensitivity analysis of the Initial Attack Analysis (IAA) processor is analyzed in [26]. The IAA is the fire simulator that simulates fire behavior and containment. The authors experimented the simulator with respect to different input parameters such as the fire spread rate, the production rate of suppression, the initial attack time, the fire size at detection. It is found that the fire spread rate has the most impact on the results, and the fire size has least impact to the results. Also it is found that the economic analysis of wildfire on the IAA based on the C+NVC heavily relies on the escaped fires because they are composed of much portion in NVC, and the users of the IAA processor are required to determine whether the fires are the escaped fires or not. It is suggested that this action needs to be improved since this may lead to wrong estimation of C+NVC value.

D. Stochastic Programming

From the literatures of the strategic and the tactical decisions of wildfire management problems, it is found that there is much randomness or uncertainty in wildfire occurrence and behavior. Therefore stochastic programming approaches are utilized to solve the proposed models that are introduced in later chapters. One of the most significant characteristics of the stochastic programming is that it can take into account uncertainty in the model. Good introductory reviews about stochastic programming can be found in [27], [28] and more details are provided in [29].

Since the uncertainty is inherited in fire behavior and occurrence, it is important to consider the randomness in modeling wildfire related problems. A stochastic mixed-integer programming (SMIP) deals with two generally difficult classes of problems: stochastic programs and integer programs. Therefore, by inheriting the properties of both generally difficult classes of problems, SMIP is regarded as the most challenging classes of optimization problems. In general, a stochastic program evaluates the problem by optimizing it over possible future scenarios that represent alternative outcomes of the problem data. Two-stage recourse model is widely used in solving stochastic programs. In the two-stage setting, first-stage decisions are made without full information on a random event. In the second-stage, after full information about the random event becomes available, the first-stage decisions are utilized and it takes recourse actions that may change the first-stage decisions. This procedure is repeated until the solution is converged. A general two-stage SMIP problem with recourse can be given as follows:

$$\text{SMIP1:} \quad Min\ c^{\top}x + E_{\tilde{\omega}}[f(x,\tilde{\omega},)] \tag{2.1}$$

$$\text{s.t. } Ax \ \leq \ b, \tag{2.2}$$

$$x \ \in \ X. \tag{2.3}$$

where, c is the first-stage objective function coefficient, X is the first-stage decision feasible set (it may have integrality property), and $E_{\tilde{\omega}}[.]$ is the mathematical expectation operator with respect to the random variable $\tilde{\omega}$. The function $f(x,\omega)$ denotes the second-stage recourse function under a realization ω of $\tilde{\omega}$. This evaluates the cost of recourse actions to guarantee the feasibility of the first-stage decisions x under this realization. The function $f(x,\omega)$ can be given as follows:

$$\text{SMIP2:} \ f(x,\omega) = \quad Min\ q(\omega)^{\top}y(\omega), \tag{2.4}$$

$$\text{s.t. } W(\omega)y(\omega) \leq r(\omega) - T(\omega)x, \tag{2.5}$$

$$y(\omega) \ \in \ Y. \tag{2.6}$$

where, Y is the second-stage decision feasible set (it may have integrality property), $y(\omega)$ is the second-stage decisions under a realization ω, and $W(\omega)$ and $T(\omega)$ are matrices while $r(\omega)$ is a vector, all with appropriately dimensioned sizes. In the stochastic programming literature $W(\omega)$ and $T(\omega)$ are referred to as the recourse and technology matrices, respectively. SMIP2 is referred to as a scenario subproblem. It is also generally assumed that for all (x, ω), SMIP2 is feasible. This assumption is referred to as relatively complete recourse. It is also assumed that there is a finite collection of scenarios ω denoted by Ω. If matrix W is independent of ω, problem (2.1-2.6) is said to have *fixed* recourse. Otherwise, the problem is said to have *random* recourse.

When only continuous decision variables are embedded in the two-stage recourse model, L-shaped method [30] is the most widely used algorithm to the model. This method is based on the Bender's decomposition method [31] where it is developed from the Kelley's method [32]. Because of the linearity and convexity of the L-shaped method, it performs very efficiently in large-scale problems. However, it cannot be appropriate to solve the models that have integrality property in decision variables. When the integrality is introduced in the problem, some difficulties come with this property. This includes that the problem loses its convexity property, thus it may computationally takes long time to solve the problem, and may end up with failing to get to the optimal solution within the polynomial time. One way to overcome these difficulties is to introduce a tight valid cut similar to the cutting plane methods used in solving integer programming (IP). If all the variables in the first-stage are binary, and there are only continuous variables in the second-stage, the L-shaped method may still be used to solve this problem by implementing algorithms such that integrality holds in the first-stage. If both stages have integer variables, especially binary, we may be able to use Integer L-shaped method or L^2 algorithm [29]. In this thesis, the L^2 with Benders' Cuts algorithm that takes the advantages of both L^2 algorithm and L-shaped method is utilized and implemented to solve the proposed models. The details of the algorithm are discussed in chapter V.

Equation (2.1-2.6) can also be written as one large-scale deterministic equivalent problem (DEP) in which all the scenarios are considered at once. The DEP can be given as follows:

$$\text{SMIP1:} \quad Min\ c^\top x + \sum_{\omega \in \Omega} p(\omega) q(\omega)^\top y(\omega), \tag{2.7}$$

$$\text{s.t. } Ax \ \leq \ b, \tag{2.8}$$

$$T(\omega)x + W(\omega)y(\omega) \leq r(\omega), \ \omega \in \Omega \tag{2.9}$$

$$x \ \in \ X, \ y(\omega) \ \in \ Y. \tag{2.10}$$

where $p(\omega)$ is the probability that a random event ω occurs. Since stochastic programming considers every possible scenarios in the problem, it will provide the valuable results which linear programming approach cannot provide. The DEP can be optimized by direct solvers such as CPLEX [33] since decomposition method is not applied to the approach. However this DEP may not be efficiently utilized when solving large-scale problems. This issue is discussed in chapter VI through the computational experiments.

CHAPTER III

ATTACK-BASE LOCATION AND RESOURCE ALLOCATION MODEL

In this chapter, a SMIP strategic decision model for airtanker attack-base location and resource allocation is proposed. The proposed model is extended from the deterministic model in [17]. The key questions are where to locate the attack-bases and how many resources such as airtankers to be assigned to each base to minimize the total associated costs and value-at-risk of the given area. Since fire occurrence is stochastic, the randomness of the fire occurrence should be taken into account in the model.

A. Problem Description

In terms of the strategic decision model of the wildfire management systems, locating attack-bases and allocating fire-fighting resources to the attack-bases is one of the most important decisions that should be carefully considered in long-term planning. In reality, the fire-fighting budget is generally not enough. Thus, it is extremely important to take maximum advantage of the budget by optimally locating attack-bases and allocating resources to each base. If this is not robustly planned, the wildfire suppression plan may not be working effectively. Consequently this may increase the probability of the wildfires becoming escaped fires that are usually occurred owing to the failure of initial wildfire attack. To locate the attack-bases optimally, we need to thoroughly investigate the area to be protected. This model assumes that the a set of candidate locations for the attack-bases are given and the entire area to be protected is divided by a set of sub-areas. It is also assumed that the the economic value of each sub-area, that is, the value-at-risk, and the wildfire occurrence frequency in each sub-area per the unit time are given. The unit time may be defined as the average time it takes a single fire-fighting resource to suppress a single wildfire.

The randomness is inherent in the wildfire occurrence frequency in the model. The proposed model defines a scenario as the number of fire occurrences during a specific time period. The overall year is divided into a set of several time periods, where each time period can be a month, a quarter or a fire season. This rationale comes from the fact that the wildfire occurrence in each sub-area varies in each time period. For example, it is expected that there will be more wildfires in the dry weather season than in the wet weather season. Similarly, one can expect more fires in a vacation season than in a non-vacation season. Thus, the number of the wildfire occurrences in each sub-area during the unit time differs in each scenario (each time period). The intuition of the model is that the location of the attack-bases and the allocation of the resources should be dependent on the number of wildfires in each sub-area, and the fire-fighting resources should be ready to be deployed to each sub-area based on the fire frequency during the unit time. If each sub-area is not covered by enough resources, the value-at-risk associated with the sub-area may be at risk. The objective function of the proposed model takes into account the fixed cost of locating attack-bases, the variable cost of deploying resources from the bases to the sub-areas (it also can be considered as operating cost of fire-fighting resources during the unit time), and the total value-at-risk in danger due to the failure in assigning the optimal number of resources to the sub-areas. It is worthwhile to note that the overall variable costs in the planning time horizon can be calculated by multiplying the variable cost during the unit time by the number of the unit times during the planning horizon (the planning horizon may be considered as one year in this thesis). Thus this enables the model to take into account long-term strategic planning by taking into account one-time fixed cost and overall variable cost during the planning horizon.

B. Proposed Model Formulation

In the two-stage SMIP with recourse model, it selects attack-base locations among potential attack-base locations in the first-stage, so that the fire-fighting resources can be assigned to the base location. In the second-stage, given the attack-base locations and a collection of fire occurrence scenarios Ω, the corrective (recourse) actions are made. If the required number of resources are not assigned to some sub-areas, the variable costs associated with the base location and the sub-areas are reduced, but the value-at-risk associated with the sub-areas is increased. The objective of the two-stage recourse model is to decide the optimal base location and resource allocation to minimize the total associated cost by taking into account the randomness in wildfire occurrence with respect to overall scenarios. Let i denote the index for sub-area $i \in I$ and j denote the index for potential attack-base location $j \in J$, where I and J are finite sets of sub-areas and potential attack-base locations.

1. Parameters

B_1 : budget limit of locating attack-bases

B_2 : total budget limit of fixed and variable costs

f_j : fixed cost of locating attack-base at j

M : some large constant

Pr^{ω} : probability of scenario $\omega \in \Omega$ occurring.

v_i^{ω} : value-at-risk of sub-area i under scenario $\omega \in \Omega$

c_{ij}^{ω} : variable cost of operating one resource from attack-base j to sub-area i under scenario $\omega \in \Omega$

h_i^ω : average number of wildfire occurrences during the unit time in sub-area i under scenario $\omega \in \Omega$

n : number of unit times in one year

d_{ij} : distance between sub-area i and attack-base j

d_{max} : maximum one time attack effective distance of fire-fighting resources

$N(i)$: set of potential base indices that can cover sub-area i where it can be defined as $N(i) = \{j|d_{ij} \leq d_{max}\}$

$M(j)$: set of sub area indices that can be covered by attack-base j where it can be defined as $M(j) = \{i|d_{ij} \leq d_{max}\}$

-Remarks

n can be estimated by dividing one year by unit time

Pr^ω can be estimated by dividing the total number of wildfires in an area during one year by the total number of the wildfires during the time unit (period) associated with the scenario ω. Thus it may be thought as the weight of the scenario with respect to one year.

2. Decision Variables

x_j : 1 if attack-base location j is opened, and 0 otherwise

y_{ij}^ω : integer variable that indicates the number of the resources that cover sub-area i from attack-base j under scenario $\omega \in \Omega$

z_i^ω: 1 if sub-area i is not assigned by the required number of the resources that is equivalent to h_i^ω under scenario $\omega \in \Omega$, and 0 otherwise

We can now formally state our two-stage SMIP model as follows:

$$\text{Min} \sum_{j \in J} f_j x_j + E[h(x_j, \widetilde{\omega})] \tag{3.1}$$

$$\text{s.t.} \sum_{j \in J} f_j x_j \leq B_1 \tag{3.2}$$

$$x_j \in \{0, 1\}, \ \forall j \in J \tag{3.3}$$

where for each outcome $\omega \in \Omega$,

$$h(x_j, \omega) = \text{Min} \sum_{i \in I} \sum_{j \in J} c_{ij}^{\omega} y_{ij}^{\omega} + \sum_{i \in I} v_i^{\omega} z_i^{\omega} \tag{3.4}$$

$$\text{s.t.} \sum_{i \in I} \sum_{j \in J} n * c_{ij}^{\omega} y_{ij}^{\omega} \leq B_2 - \sum_{j \in J} f_j x_j \tag{3.5}$$

$$\sum_{j \in N(i)} y_{ij}^{\omega} - h_i^{\omega} + M z_i^{\omega} \geq 0, \ \forall i \tag{3.6}$$

$$\sum_{i \in M(j)} y_{ij}^{\omega} \leq M x_j, \ \forall j \tag{3.7}$$

$$y_{ij}^{\omega} : \textit{general integer}, \ z_i^{\omega} \in \{0, 1\}, \ \forall i \in I, \ \forall j \in J \tag{3.8}$$

C. Model Description

In the model the first-stage objective function (3.1) ensures that the sum of the cost to locate the attack-bases and the expected variable cost and value-at-risk caused by wildfire occurrence is minimized. Constraint (3.2) indicates that the sum of the cost to locate the attack-bases is within the budget B_1. the constraint (3.3) enforces the binary restrictions on x_j.

The second-stage objective function (3.4) ensures that for the given opened attack-base locations, the sum of the associated variable cost and the sum of the value-at-risk associated with the sub-areas are minimized for the fire occurrence scenario $\omega \in \Omega$. The variable cost is incurred by operating one fire-fighting resource from its base to a specific sub-area and the value-at-risk associated with the sub-area is incurred if the minimum required number of resources which should be greater than or equal to h_i^ω are not provided to the associated sub-area. (3.5) requires the sum of the locating attack-bases and the sum of the variable cost based on the operation of the resources with respect to wildfire occurrence is within the total budget limit B_2. The constraint (3.6) decides whether the value-at-risk associated with each sub-area at risk or not. If it is at risk which is not a favorable case, the value-at-risk associated with the sub-area is taken into account in the second stage objective function. The constraint (3.7) indicates that if any of the resources is assigned to a specific base location, the base should be opened. The restriction (3.8) indicates that decision variables y_{ij}^ω is binary variable, z_j^ω is general integer variable by the nature of the problem.

The rationale behind (3.5) is that the budget for the attack-base location is made on long-term basis and the budget for the variable cost are made on yearly basis. Since variable cost c_{ij}^ω represents one time operating cost from attack-base j to sub-area i, to estimate the annual operating variable cost and budget, n should be multiplied to the total variable cost during the unit time. Then this gives the annual operating cost.

There are three possible cases in the constraint (3.6). Here, it is worthwhile to note that the minimum required number of fire-fighting resources in the sub-area is greater than or equal to h_i^ω. In a given fire occurrence scenario, each sub-area i is covered by the some number of resources. In first case, the decision variable y_{ij}^ω, the number of the resources that can be available for the sub-area i is greater than the required number of the resources that is equal to h_i^ω, and the decision variable z_i^ω can take on a value of either 0 or 1. In this case, the value-at-risk associated with the sub-area i is not at risk, and as an indication of this information, it is desirable that the decision variable z_i^ω takes on a value of 0 not to add value-at-risk of the sub-area

i term in the second-stage objective function. And the objective function forces z_i^ω takes on a value of 0. The second case, the decision variable y_{ij}^ω takes the same value that is equal to the value h_i^ω, and the decision variable z_i^ω can take on a value of either 0 or 1. Similar to the first case, it is desirable that z_i^ω take on a value of 0, and the same logic used in the first case is applied in the second case. Thus, z_i^ω takes on a value of 0. The third case, the decision variable y_{ij}^ω take the value that is less than h_i^ω, and the decision variable z_i^ω only takes a value of 1. Since there are not enough resources available for the sub-area i, the value-at-risk associated with the sub-area i is at risk. Thus this fact adds the value-at-risk in the objective function where its goal is to minimize the associated total cost.

The rationale of this constraint is that the third case is desirable to be avoided. However, due to the limited availability of the total number of resources forced by the budget limit, if it cannot be fully responsible for the overall area, this model recommends that the resources should be allocated to the sub-areas that have more economic value than other sub-areas.

Another important rationale of this constraint is that during the unit time as many number as h_i^ω is required to be responsible for the sub-area i. And the fire-fighting resources dedicated to the specific sub-area during the unit time cannot be deployed to the other sub-area at the same time. Thus, this model takes into account the situation where there are simultaneous fires in the area.

There will be two possible cases in the constraint (3.7). The first case is if at least one of the resources is assigned to the location j, x_j takes on a value of 1. The second case is if there is no resource assigned to the location j, x_j can take on a value of either 0 or 1. If this cases occurs, the second-stage objective function forces x_j to take on a value of 0 to minimize the objective function value. This constraint is a 'linking constraint' since it links the first and the second stages.

The proposed strategic decision model for attack-base location and resource allocation is extensions of such existing models as [13], [15], [16], [17] in several directions since it takes into account (1) the randomness in wildfire occurrence in different sub-area in different time period, (2) simultaneous fires, (3) overall operating variable cost as well as fixed cost, (4) value-at-risk which is incurred when a specific sub-area is not protected, and (5) budget limit. Also (6) it provides the solution of how many resources to be assigned to each base as well as where to locate the attack-bases. The model in [13] and [14] does not explain (1), (2), (3), (5), and (6). The model in [15] and [16] does not explain (1), (2), (3), (4), and (5). The model in [17] does not take into account (4) and (5). [17] takes into account (1) and (3), but the rationales used in [17] are different from those used in the proposed model in that the average number of the resources required to each sub-area is fixed, whereas the proposed model can be random in the proposed model, and it does not take into account the potential consequences when the required number of the resources cannot be assigned to each sub-area, whereas the proposed model in this chapter take into account the factor by introduction the value-at-risk concept and the decision variable z_i^ω. Thus incorporating the factor (1) through (6) makes this model capture much more realism than the previously proposed models. It provides valuable solution when decision makers in wildfire management want to make more realistic and systematic decisions in making long-term wildfire management plan and executing the limited budget optimally instead of making these important decisions in rule-of-thumb ways.

The proposed model can be solved by the SMIP algorithm that is described in chapter V. In this thesis, the computational experiments on this model are not conducted due to the some difficulties in acquiring the original data used in reality and extracting some important data used in the model from original data. Indeed, it is extremely difficult and takes much time to extract the realistic data such as v_i^ω, h_i^ω.

However, the model in chapter IV will be computationally experimented by the SMIP algorithm.

CHAPTER IV

RESOURCE ALLOCATION MODEL FOR WILDFIRE CONTAINMENT

In this chapter, a SMIP tactical decision model for resource allocation for wildfire containment is proposed. The proposed model is based on the deterministic model by [19]. The proposed model is also based on the C+NVC model for wildfire economics proposed [8]. The proposed model minimizes the cost of wildfire by minimizing sum of the pre-suppression cost, the suppression cost, and the net value change. The key questions are which resources to be deployed and when the wildfire to be contained by minimizing the C+NVC objective function. Since the fire growth behavior is stochastic, the randomness of the fire growth behavior should be taken into account in the model.

A. Problem Description

The proposed model provides the solution that identifies the optimal mix of the fire-fighting resources required for the wildfire containment. The basic principle of how wildfire containment works is described in [34]. If the total fire line production constructed by the fire-fighting resources is greater than the total fire perimeter, it may be concluded that the fire can be successfully contained. The proposed model adapted the basic concept of the containment. In the model, it is assumed that fire line production rate, arrival time to the fire and operating cost of the resources is *deterministic*. However, the fire growth characteristics such as fire perimeter and net value change are assumed to be *stochastic*. Another assumption is that when a fire is initially ignited, resources will be deployed to the fire time period 0. It is defined that an instance of the stochastic fire perimeter as one fire growth scenario and optimize the model over a finite collection set of the scenarios that are assumed to come from some fire simulation such as FARSITE [25] or DEVS [24]. The model considers different time periods and different type of resources. Under the budget and the resource production rate restriction, it is required to identify the fire can be whether contained or not. If it turns out to be contained, it is also required to identify the optimal mix of resources with the minimum C+NVC achieved. Since the resources cannot be fractional, it should have integrality property with the continuous fire perimeter. Thus the model has mixed integer variables.

B. Proposed Model Formulation

In the two-stage SMIP with recourse model, it selects resources and deploy them to the fire in the first-stage. In the second-stage, given resources and a collection of fire growth scenarios Ω, the corrective (recourse) actions are made on actual fire containment. Let i denote the index for fire containment resource $i \in I$ and j denote the index for time period $j \in J$, where I and J are finite sets of fire containment resources and time periods, respectively.

1. Parameters

B_1 : pre-suppression budget limit

B_2 : pre-suppression and suppression budget limit

H_j : time period counter that takes a value of j

M : some large constant

Pr^{ω} : probability of scenario $\omega \in \Omega$ occurring

PER_j^{ω} : increment in fire perimeter in period j under scenario $\omega \in \Omega$

NVC_j^{ω} : increment in net value change for the period j under scenario $\omega \in \Omega$

SP_j^{ω} : accumulated fire perimeter up to period j under scenario $\omega \in \Omega$

C_i : hourly cost of operating resource i

P_i : rental cost of resource i

PR_i : line production rate of the resource i in kilometers (km)

A_i : arrival time to the fire of resource i

2. Decision Variables

Z_i : 1 if resource i has been dispatched, 0 otherwise

Y_j^{ω} : 1 if fire is uncontained in period j under scenario $\omega \in \Omega$, 0 otherwise

D_{ij}^{ω}: 1 if containment achieved in period j using resource i under scenario $\omega \in \Omega$, 0 otherwise

L_j^{ω} : total line construction up to period j under scenario $\omega \in \Omega$

We can now formally state our two-stage SMIP model as follows:

$$\text{Min } \sum_{i \in I} P_i Z_i + E[h(Z_i, \widetilde{\omega})] \tag{4.1}$$

$$\text{s.t. } \sum_{i \in I} P_i Z_i \leq B_1 \tag{4.2}$$

$$Z_i \in \{0, 1\}, \ \forall i \in I \tag{4.3}$$

where for each outcome $\omega \in \Omega$,

$$h(Z_i, \omega) = \text{Min} \sum_{j \in J} \sum_{i \in I} C_i H_j D_{ij}^{\omega} + \sum_{j \in J} NVC_j^{\omega} Y_{j-1}^{\omega} \tag{4.4}$$

$$\text{s.t.} \sum_{i \in I} \sum_{j \in J} C_i H_j D_{ij}^{\omega} \leq B_2 - \sum_{i \in I} P_i Z_i \tag{4.5}$$

$$\sum_{j \in J} L_j^{\omega} - \sum_{j \in J} PER_j^{\omega} Y_{j-1}^{\omega} \geq 0 \tag{4.6}$$

$$\sum_{i \in I} (H_j - A_i) PR_i D_{ij}^{\omega} - L_j^{\omega} = 0, \ \forall j \tag{4.7}$$

$$SP_j^{\omega} Y_{j-1}^{\omega} - L_j^{\omega} - (M) Y_j^{\omega} \leq 0, \ \forall j \tag{4.8}$$

$$\sum_{j \in J} D_{ij}^{\omega} \leq Z_i, \ \forall i \tag{4.9}$$

$$Y_0^{\omega} = 1 \tag{4.10}$$

$$D_{ij}^{\omega}, \ Y_j^{\omega} \in \{0, 1\}, \ L_j^{\omega} \geq 0, \ \forall i \in I, \ \forall j \in J \tag{4.11}$$

C. Model Description

In the model the first-stage objective function (4.1) ensures that the sum of the pre-suppression and the expected suppression cost and net value change caused by burned area is minimized. The constraint (4.2) indicates that pre-suppression budget allowance is satisfied when resources are rented. Here the resource renting is considered as the pre-suppression cost. The restriction (4.3) indicates that decision variable Z_i will take on a value of 1 if resource $i \in I$ is rented and 0 otherwise.

The second-stage objective function (4.4) ensures that for a given mix of fire-fighting resources determined in the first-stage, the sum of the associated suppression cost and net value change is minimized for the fire growth scenario $\omega \in \Omega$. The constraint (4.5) requires the pre-suppression budget and the suppression budget is satisfied when resources are deployed. The constraint (4.6) indicates that, in a given fire behavior scenario, the total fire line production constructed by the deployed resource must exceed the total fire perimeter at some time period $j \in J$. If the dispatched resources cannot contain the fire in given time periods, the problem turns out to be infeasible. The constraint (4.7) computes total fire line production based on the resources, deployed at time period 0. Thus, decision variable L_j^ω represents total fire line construction up to and including period $j \in J$. The constraint (4.8) provides information as to whether the fire is contained or not in the time period $j \in J$. If the total fire line production constructed by the deployed resources in time period j is less than the total fire perimeter in the same time period, it is concluded that the fire is not contained yet. Then, as the indicator of the information, decision variable, Y_j^ω takes on a value of 1. Otherwise it takes on a value of either 0 or 1. If contained, the second-stage objective function forces Y_j^ω to take on a value of 0 to minimize the objective function value. This constraint is a 'linking constraint' since it links two different time periods. The constraint (4.9) ensures that in a given scenario if a particular resource $i \in I$ is used during any of the time periods, Z_i must take on a value of 1 to take into account the associated pre-suppression cost. If not used, Z_i may take a value of either 0 or 1. Then the same logic, used in the constraint (4.8), forces Z_i to take on a value of 0 to minimize objective function. The constraint (4.10) simply makes the model start by initially igniting the fire in time period 0. The restriction (4.11) indicates that the decision variables D_{ij}^ω, Y_j^ω are binary variables, and L_j^ω is a non-negative continuous variable by the nature of the problem.

CHAPTER V

SOLUTION APPROACH

In chapter III and chapter IV, the strategic and the tactical decision model are proposed. These models have binary first-stage decision variables, and mixed integer variables in the second-stage. Thus, it falls in the class of the SMIP. Due to the difficult nature of SMIP, very few algorithms have been developed for this class of problems [29]. Moreover, the proposed models have random recourse property since the second-stage objective function coefficient q or W matrix in the second-stage constraint set can have random elements. In this chapter, the solution method for the proposed models is introduced. Also, important issues in implementing the proposed algorithm are discussed.

A. L^2 with Benders' Cuts Algorithm

Both the proposed models fall in the SMIP with random recourse property. As a solution approach, both the L-shaped method and the L^2 algorithm are efficiently applied. Laporte and Louveaux derive the L^2 optimality cut for the piecewise linear approximation of the expected value function [29]. That L^2 optimality cut requires lower bound of the recourse function. Thus, LP-relaxation of the proposed model is required to get the lower bound, and L-shaped method is utilized to get the lower bound value. Tighter lower bound value is essential to make the algorithm converged faster. Once the lower bound value is obtained by the L-shaped method, the L^2 algorithm is utilized to solve the proposed models. To make the L^2 algorithm converged faster in solving large-scale problems, the Benders' cuts that are used in L-shaped method are applied in L^2 algorithm. We call the proposed algorithm 'L^2 with Benders' Cuts Algorithm'. This approach provides a very tighter initial cut as well as generates two strong valid inequalities that are the L^2 optimality cut and the Benders cut in every iteration which significantly reduce the computational time in solving large-scale SMIPs. Before stating the algorithm, some preliminaries are introduced as follows.

• Original problem is given as below.

$$\text{Min } c^\top x + q^\top y \tag{5.1}$$

$$s.t.\ Ax \geq b \tag{5.2}$$

$$Tx + Wy \geq r \tag{5.3}$$

$$x: \ binary,\ y: \ mixed\ integer \tag{5.4}$$

q, W, r may have randomness, and let $\widetilde{\omega}$ denote random, and ω denote any realization of $\widetilde{\omega}$ variable. Then it can be decomposed as below where first-stage only have deterministic data, and second-stage may have stochastic and deterministic data.

$$1st\ Stage: \quad \text{Min } c^\top x + E_{\widetilde{\omega}}[f(x, \widetilde{\omega})] \tag{5.5}$$

$$s.t.\ Ax \geq b \tag{5.6}$$

$$x: \ binary \tag{5.7}$$

where for any realization ω of $\widetilde{\omega}$ we have

$$2nd\ Stage: \qquad f(x, \omega) = \text{Min } q(\omega)^\top y \tag{5.8}$$

$$s.t.\ W(\omega) \geq r(\omega) - T(\omega)x \tag{5.9}$$

$$y: mixed\ integer \tag{5.10}$$

L-shaped Method

The L-shaped method is a efficient algorithm in solving two-stage recourse model if all the decision variables are continuous. Although the proposed models have mixed integer variables, the L-shaped method is utilized in solving the proposed models by applying LP-relaxation. This give the lower bound value L that is used in generating the L^2 optimality cut in the L^2 algorithm. Thus the following L-shaped method procedure is applied to get the lower bound L. For notational conveniences, it is assumed that there are only equality constraints.

Step[0] : Initialization

Let x^0 be given.

Set $\epsilon \geq 0,\ LB = -\infty,\ UB = \infty,\ k \leftarrow 0$

Step[1] : Solve Subproblem

Let s be scenario index, then for $s = 1,\cdots$,S

$$\begin{aligned} \text{Solve } f_s^k \ &= \ Min\ q_s^\top y \\ s.t.\ W_s y \ &= \ r_s \ - \ T_s x^k \\ y &\geq 0 \end{aligned}$$

If infeasible for some s: Generate feasibility cut

-Get dual extreme ray: μ_s^k

-Compute $\alpha_k \ = \ \mu_s^{k\top} r_s$

$\beta_k \ = \ \mu_s^{k\top} T_s$

-Go to Step[2]

Else if feasible $\forall$ s: Generate optimality cut

- Get π_s^k

- Compute $\alpha_k \ = \ \sum_s p_s \pi_s^{k\top} r_s$

$$\beta_k \ = \ \sum_S p_s {\pi_s^k}^\top T_S \qquad \text{where } p_s\text{: probability of scenario s}$$

Upper Bounding

Compute UB:

$V^k \ = \ c^\top x^k \ + \ \sum_s p_S f_s^k$

UB $= \min\{V^k, UB\}$

If UB is updated, set incumbent solution to $x^* = x^k$

Go to Step[2]

Step[2] : Add Cut to Master Problem and Solve

If some subproblem was infeasible

Add $\beta_k^\top x \ \geq \ \alpha_k$ to master problem

Else

Add $\beta_k^\top x \ + \ \eta \ \geq \ \alpha_k$

Solve the master problem to get $\{x^{k+1}, \eta^{k+1}\}$ and V^{k+1} as the master problem's objective value

Lower Bounding

LB $= \max\{V^{k+1}, LB\}$

Master Problem

$$\begin{aligned} V^{k+1} \ = \ & Min \ c^\top + \eta \\ & s.t. \ Ax = b \\ & \quad \beta_t^\top x + \eta \geq \alpha_t, \ t \in \theta_k \\ & \quad \beta_t^\top x \qquad \geq \alpha_t, \ t \in \overline{\theta_k} \\ & \qquad\qquad x \geq 0 \end{aligned}$$

Where θ_k denotes iteration index set at which an optimality cut is generated, and $\overline{\theta_k}$ denotes iteration index set at which a feasibility cut is generated

Step[3] : Termination

If $UB - LB \ \leq \ \epsilon|UB|$

Stop

ELSE

$k \leftarrow k + 1$

Return to Step[1]

L^2 with Benders' Cuts Algorithm

Once the lower bound L of the recourse function is obtained by the L-shaped method, the L^2 with Benders' Cuts algorithm is applied to solve the proposed models. The detailed steps of the algorithm are as follows.

Step[0] : Initialization

Let $\epsilon \geq 0, x^1 \in Ax \geq b, x \in \{0,1\}$, be given. Also let L be obtained by L-shaped method.

Set $v1 \leftarrow -\infty, V1 \leftarrow \infty, k \leftarrow 1$

Step[1] : Solve subproblem for all $\omega \in \Omega$

Step[1-1] : Solve LP-relaxation of the subproblem:

Apply Step[1] in L-shaped method by applying LP-relaxation to solve subproblem. (Not to apply 'Upper Bounding' part in Step[1])

Get the dual solutions of the subproblem, then create Benders cut.

Step[1-2] : Solve the mixed-integer subproblem:

Set $V_{k+1} \leftarrow min\{c^{\top}x^k + E[x^k, \widetilde{\omega}], V_k\}$ This gives upper bound

Set $F(x^k) \leftarrow E[x^k, \widetilde{\omega}]$

Step[2]: Update and solve master problem

Step[2-1] : Append Benders cut to the master problem:

If some subproblem was infeasible

Add $\beta_k^{\top} x \geq \alpha_k$ to master problem

Else

Add $\beta_k^{\top} x + \eta \geq \alpha_k$ to the master problem

Step[2-2] : Derive L^2 Cut, then append the cut to the master Problem:

Using $L,\ x^k,\ F(x^k)$, derive the L^2 optimality cut.

The cut that is a valid inequality for $E[x^k, \widetilde{\omega}]$ is defined as below.

$\eta \geq (F(x^k) - L)(\sum_{j \in S^k} x_j - \sum_{j \in \bar{S^k}} x_j - |S^k| + 1) + L,$

where $S^k = \{j | x_j^k = 1\}$

Append the cut to the master problem

Step[2-3] : Solve the master problem and get x^{k+1}:

Master Problem

$$
\begin{aligned}
v^{k+1} = \ & Min\ c^{\top} + \eta \\
& s.t.\ Ax = b \\
& \quad \beta_t^{\top} x + \eta \geq \alpha_t,\ t \in \theta_k \\
& \quad \beta_t^{\top} x \qquad \geq \alpha_t,\ t \in \overline{\theta_k} \\
& \quad \beta_t^k + \eta \geq \alpha_t, t = 1, \cdots, k \\
& \qquad x \in \{0, 1\}
\end{aligned}
$$

Where $\theta_k \equiv$ denotes iteration index set at which a Benders optimality cut is generated, $\overline{\theta_k}$ denotes iteration index set at which a Benders feasibility cut is generated and v^{k+1} denotes the optimal value of the master problem.

Let x^{k+1} be the solution of the master problem.

Step[3]: Termination

If V^{k+1} - $v^{k+1} \geq \epsilon$,

Stop

ELSE

$k \leftarrow k + 1$

Return to Step[1]

• Note that this L^2 with Benders' Cuts algorithm can be a L^2 algorithm if step[1-1] and step[2-1] are omitted from the procedures.

B. Impementation

In this section, important issues in implementing the algorithm such as use of the CPLEX callable library, data structures, standard SMPS format are discussed. The algorithm was implemented by C/C++ programming in Microsoft Visual Studio .Net 2003 environment in conjunction with CPLEX callable library [33] for solving LP and MIP problems.

The ILOG CPLEX callable library is designed to facilitate the implementation of the optimization algorithms. The library enables us to implement the algorithm, solve, modify, and interpret the results of optimization problems such as linear, mixed integer, continuous convex quadratic, and mixed integer quadratic programs. Thus the implementation of a L^2 with Benders' Cuts algorithm also requires the frequent use of the functions in the CPLEX callable library. The major routines used in the implementation are as follows.

Optimization and Result Routines : define an active problem, optimize that problem, and report the results of the optimization

Problem Modification Routines : change a problem once it has been created using CPXcreateprob()

Problem Query Routines : access information about a problem object once it has been created via CPXcreateprob()

File Reading and Writing Routines : read problems from system files

Parameter Setting and Query Routines : access and modify parameter values

Utility Routines : debug, initialize, and close the CPLEX environment

C/C++ programming data structure are used to take maximum advantage of memory allocation and computational issues. The object concept in C++ is used to inherit the methods from parent class to child class that are declared and defined in the algorithm implementation. Three objects class are used in the implementation; *LP-object-Class* (Parent Class), *Sub-problem-Class* (Child Class), and *Master-problem-Class* (Child Class). Also all the matrix data from the problem are stored as sparse matrix format for efficient memory allocation and computation.

Generally stochastic programming requires the robust algorithm implementation in order to handle the large-scale optimization problems. Therefore it is important to have the implementation that can read in standard SMPS format problem data [35]. The standard format comprises three input files: CORE file, TIME file, and STOCH file. The CORE file stores the LP/MIP problems in MPS format. The TIME file indicates the point where the second-stage variable and constraint begin in the CORE file. The STOCH file stores all the random data in the problem. There are several types of the STOCH file formats such as independent, scenario formats. In the independent type, all the random variables in the problem and their corresponding outcomes and probabilities are given in the STOCH file. In the scenario type, the random data are explicitly given in each scenario with the corresponding probabilities of outcome.

CHAPTER VI

COMPUTATIONAL EXPERIMENTS

This chapter presents a computational study of the practical application of the tactical model proposed in chapter IV. Several numerical experimental results with the L^2 with Benders' Cuts algorithm stated in chapter V are reported. The results demonstrate the use of the proposed model in providing optimal decisions on the optimal mix of the fire-fighting resources. This is with respect to different fire growth scenarios, problem parameters in terms of the time period, fire-fighting resources, budgetary constraints, and different values for the NVC. We also compare the performance of the L^2 with Benders' Cuts algorithm on the decomposed problem instances to that of the CPLEX MIP solver directly applied on the corresponding DEP instances. All the computational experiments are conducted on a 3.00 GHz Pentium D Processor with 3.5 GB of RAM. This chapter is organized as follows. In section A, numerical experiments dealing with wildfire containment decisions on an extension of the small-scale fire example given in [19] are reported. In section B, numerical results based on large-scale fire experiments conducted with the fire simulator FARSITE [25] are presented. The findings and summary of the results are discussed in section C.

A. Wildfire Containment Decisions for a Small-Scale Fire

1. Basic Data

The numerical data used in the model are based on the data used in [19]. The PER_j^ω and NVC_j^ω were randomly generated based on the data provided in [19]. Table III provides the distributions for the PER_j^ω and the formulas for NVC_j^ω. The distributions and formulas are used to generate 1 scenario. Therefore they are used

100 times to generate 100 scenarios. Different sets of instances with 1, 5, 10, 15, 30, 50, and 100 randomly generated scenarios were created. The naming convention wfcp$b_r_t_s$ is used, where wfcp stands for 'wild fire containment problem', b is the ratio of the total available budget, r is the number of resources, t is the number of time period, and s is the number of scenario. The parameters related with the fire-fighting resources are given in Table IV. It is assumed that the scenarios have the same probability of occurrence. The value of the NVC is initially fixed at $100 per hectare. To illustrate the idea of optimal wildfire containment decision-making, consider the instance wfcp_7_6_5. Note that this instance has 7 resources, 6 time period (each time period is one hour), and 5 different fire growth scenarios. The random data of the instance are given in table V and the data are generated based on the distributions and formulas shown in the table III. The data in both tables IV and V are used to solve the instance of the model. Table V provides the random parameters for the perimeter and the burned area in each time period and the different scenarios. The SP_j^ω is represented based on total fire perimeter up to time period j and the NVC_j^ω can be obtained by computing the increment of two adjacent time period from the total burned area up to time period j which is provided in Table V.

Table III. Scenario data generation for wfcp instance

	Increment in PER_j^ω	Accumulated Burned Area
Period	Distribution Used	Formula Used
1	Uniform(0.1,0.5)	$0.7*SP_1^\omega/0.3$
2	Uniform(0.5,0.9)	$5.6*SP_2^\omega/1$
3	Uniform(0.1,0.5)	$9.6*SP_3^\omega/1.3$
4	Uniform(0.3,0.7)	$15.9*SP_4^\omega/1.8$
5	Uniform(0.1,0.3)	$20.3*SP_5^\omega/2$
6	Uniform(0.1,0.3)	$24.3*SP_6^\omega/2.2$

Table IV. Fire-fighting resource characteristics

Resource	Description	A_i (hr)	C_i (\$/hr)	P_i (\$)	PR_i (km/hr)
1	Dozer	2	175	300	0.36
2	Tractor plow	2.5	150	500	0.45
3	Type I crew	0.5	125	500	0.20
4	Type II crew	1	175	600	0.25
5	Engine #1	1.5	75	400	0.09
6	Engine #2	1.5	100	900	0.10
7	Engine #3	1	124	600	0.15

Table V. Fire perimeter and burned area for 5 scenarios

Time	Scenario (perimeter)					Scenario (area)				
Period	1	2	3	4	5	1	2	3	4	5
1	0.4	0.4	0.3	0.3	0.5	1.0	1.0	0.7	0.8	1.1
2	1.3	1.3	1.2	1.1	1.2	7.3	7.2	6.6	6.3	6.9
3	1.6	1.8	1.6	1.3	1.7	11.6	13.0	11.5	9.8	12.3
4	2.2	2.2	2.0	2.0	2.0	19.7	19.3	18.1	17.4	17.7
5	2.5	2.4	2.3	2.2	2.2	25.4	24.8	22.9	22.2	22.1
6	2.8	2.7	2.4	2.4	2.4	30.5	30.0	26.7	26.4	27.0

2. Analysis of Solution of the Instance wfcp_7_6_5

The result of this instance is provided in table VI. It can be interpreted as follows. No matter which fire scenario occurs, if the fire-fighting resources, dozer and tractor plow, are deployed, the fire will be contained with the expected minimum C+NVC value of $5215. Fire containment period may differ from one scenario to another scenario. For instance, if scenario 1 occurs, the fire can be contained in period 6, while if scenario 5 occurs, the fire can be contained in period 5. For a more detailed analysis of the solutions (see [29]), it is worthwhile to review the concepts of Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS).

Table VI. Result of wfcp_7_6_5

Optimal Z*	5215	Scenario	1	2	3	4	5
Resource Deployed	Dozer, Tractor plow	Contained Period	6	6	6	5	5

Table VII. Results of 5 instances under perfect information

Scenario Index	1	2	3	4	5
Optimal Z*	4960	5860	4910	3805	4635
Fire Contained Period	4	6	5	4	5
Resource Deployment Decision					
Dozer	1	1	1	1	1
Tractor Plow		1	1		1
Type I Crew	1		1	1	
Type II Crew	1			1	
Engine #1		1			
Engine #2					
Engine #3	1				

EVPI is the difference between the objective function value of the stochastic programming solution and the average objective function value under perfect information. Because it is not certain which scenario occurs in the future, wfcp_7_6_5 considers 5 different scenarios in one instance. If the perfect information of the fire growth scenario is available for the five scenarios, it is possible to analyze the problem by considering only one scenario at a time as shown in table VII. For example, if only scenario 3 occurs, the resources, dozer, tractor plow, and type I crew, are required to achieve optimal value of Z^*=\$4910. Notice that this value is different from the optimal value obtained using the stochastic programming approach. The expected objective function value with perfect information is $0.2 * \$4960 + 0.2 * \$5860 + 0.2 *$

$\$4910 + 0.2 * \$3805 + 0.2 * \$4635 = \$4834$. Without perfect information, at best the C+NVC would be minimized with a cost of \$5215 by solving the stochastic programming formulation. Therefore, EVPI $= 5215 - 4834 = 381$. The EVPI may be thought of as the value that is worthwhile to pay for the perfect information.

VSS is the value of including the randomness in the problem. If all the random data of the problem are replaced by their mean values, there will be only one scenario available. If one instance is made based on the mean values, the optimal Z^* is found at \$4944, and the solution suggests that the resources, dozer, tractor plow and type I crew, should be deployed, and the fire is contained in period 4. The solution of this instance can be fixed as the first stage solution in the two-stage recourse problem. The recourse problem is solved to see how the mean value solution affects the second stage problems in the stochastic programming setting. The mean value solution does not explicitly account for the randomness in the problem data. In this case, the optimal Z^* is found to be \$5348. Therefore, VSS $= 5348 - 5215 = 133$. The VSS may be thought of as the value that is worthwhile to pay for using the stochastic solution rather than the mean value solution. Any stochastic programming instance can be analyzed as above. From above analysis, it can be concluded that without perfect information, the stochastic programming approach pays off. Thus the stochastic programming approach provides a robust decision making method under uncertainty for this class of problems.

3. Computational Results of wfcp Instances

This section provides the results for the large sets of scenario instances. The experiments aim at analyzing the results of the problem when the available resources are enough to contain the fire in any scenario. We also confirm the accuracy and performance of the proposed algorithm by comparing the two-stage DEP solutions to that of the corresponding instances. Table VIII provides the size of the instances for the different number of fire growth scenarios. It should be pointed out that when a scenario is added to the instance, the number of constraints and the number of variables increase. The density of the constraint matrix can be computed as follows.

$$Density = \frac{nonzeros}{variables(binary + continuous) * constraints}$$

Table IX compares the results between the CPLEX MIP solver directly applied

Table VIII. Problem size of wfcp instances

Problem Size					
Instance	Binary	Continuous	Constraint	Nonzero	Density
wfcp_7_6_1	62	7	29	189	0.094
wfcp_7_6_5	282	35	141	917	0.021
wfcp_7_6_10	557	70	281	1827	0.010
wfcp_7_6_15	832	105	421	2737	0.007
wfcp_7_6_30	1657	210	841	5467	0.003
wfcp_7_6_50	2757	350	1401	9107	0.002
wfcp_7_6_100	5507	700	2801	18207	0.001

to the corresponding DEP instances and the L^2 with Benders' Cuts decomposition approach. The iteration in the DEP refer to the number of the CPLEX iterations while the iteration in the L^2 with Benders' Cuts refers to the number of iterations conducted within the algorithm. The CPU time is given in seconds and a time limit of 300s is imposed. The Z_{IP} is the optimal objective value unless it is not mention in CPU time section. For example, the CPLEX MIP solver directly applied to the corresponding DEP instances of wfcp_7_6_100 does not provide the optimal Z_{IP} since the instance is stopped at 300s. In fact the optimal value is 0.53% away from the value indicated in the Z_{IP}. The $Z_{LP(1)}$ is the LP-relaxation value of the instances. The $Z_{LP(2)}$ is obtained by only LP-relaxing the sub-problem while the master program

Table IX. Results of wfcp instances with unconstrained budget

DEP					
Instance	Iteration	CPU Time(sec.)	Z_{IP}	$Z_{LP(1)}$	$Z_{LP(1)}$ GAP(%)
wfcp_7_6_1	26	0.11	4.96E+03	4.76E+02	90.393
wfcp_7_6_5	2545	0.36	5.22E+03	4.78E+02	90.840
wfcp_7_6_10	10596	1.42	5.15E+03	5.04E+02	90.198
wfcp_7_6_15	9556	1.5	4.59E+03	4.66E+02	89.837
wfcp_7_6_30	10854	3.11	4.31E+03	4.43E+02	89.715
wfcp_7_6_50	50270	18.64	4.03E+03	4.22E+02	89.543
wfcp_7_6_100	620181	>300(0.53%)	4.10E+03	4.15E+02	89.866
L^2 with Benders' Cuts					
Instance	Iteration	CPU Time(Sec.)	Z_{IP}	$Z_{LP(2)}$	$Z_{LP(2)}$ GAP(%)
wfcp_7_6_1	127	2.985	4.96E+03	7.11E+02	85.675
wfcp_7_6_5	128	3.703	5.22E+03	6.86E+02	86.848
wfcp_7_6_10	127	4.876	5.15E+03	7.14E+02	86.127
wfcp_7_6_15	128	5.766	4.59E+03	6.69E+02	85.400
wfcp_7_6_30	128	8.563	4.31E+03	6.46E+02	84.990
wfcp_7_6_50	127	12.43	4.03E+03	6.23E+02	84.561
wfcp_7_6_100	127	22.11	4.10E+03	6.16E+02	84.967

is solved as an IP. The $Z_{LP(1)}$ GAP(%), and the $Z_{LP(2)}$ GAP(%) are computed as follows.

$$Z_{LP(1)}GAP = \frac{Z_{IP} - Z_{LP(1)}}{Z_{IP}} * 100$$

$$Z_{LP(2)}GAP = \frac{Z_{IP} - Z_{LP(2)}}{Z_{IP}} * 100$$

Table IX deals with the case when the suppression and pre-suppression budget is not constrained in all the instances. It is observed that the CPLEX MIP solver directly applied to the corresponding DEP instances has smaller computation time for instances with smaller unmber of scenarios than the L^2 with Benders' Cuts algorithm. The CPLEX MIP solver directly applied to the corresponding DEP instances cannot solve wfcp_7_6_100 instance that has 100 scenarios within 300 seconds while the L^2 with Benders' Cuts can solve it within 23 seconds. It is also observed that $Z_{LP(1)}$ is always lower than $Z_{LP(2)}$ since the master problem is not LP-relaxed when comparing for $Z_{LP(2)}$.

Table X provides the important decisions regarding resource deployment. If the resources are deployed as they are suggested, the expected minimum C+NVC is guaranteed. It is observed that the different instances that have different number of scenarios provide different optimal mix of resources. Figure 2 illustrates how the

Table X. Optimal resource mix of wfcp instances with unconstrained budget

Solution							
Instance	Dozer	Tractor	CrewI	CrewII	EngineI	EngineII	EngineIII
wfcp_7_6_1	1		1	1			1
wfcp_7_6_5	1	1					
wfcp_7_6_10	1	1					
wfcp_7_6_15	1		1	1			
wfcp_7_6_30	1		1	1			
wfcp_7_6_50	1		1	1			
wfcp_7_6_100	1		1	1			

L^2 with Benders' Cuts algorithm is converged to the optimal objective value. This graph is based on wfcp_7_6_100 which the CPLEX MIP solver directly applied to the corresponding DEP instances cannot solve within 300s. It is observed that the overall lower bound increases as iteration increases. The $L-shaped$ upper bound and the overall lower bound meets in the beginning of the iterations. At this moment, the optimal $Z_{LP(2)}$ is found. After the optimal value is found, the L^2 upper bound is obtained and the gap between the overall lower bound and the L^2 upper bound decreases until it converges as iteration increases.

Table XI provides the results when the total available suppression and pre-suppression budget is cut in half of the maximum enough to deploy and operate all

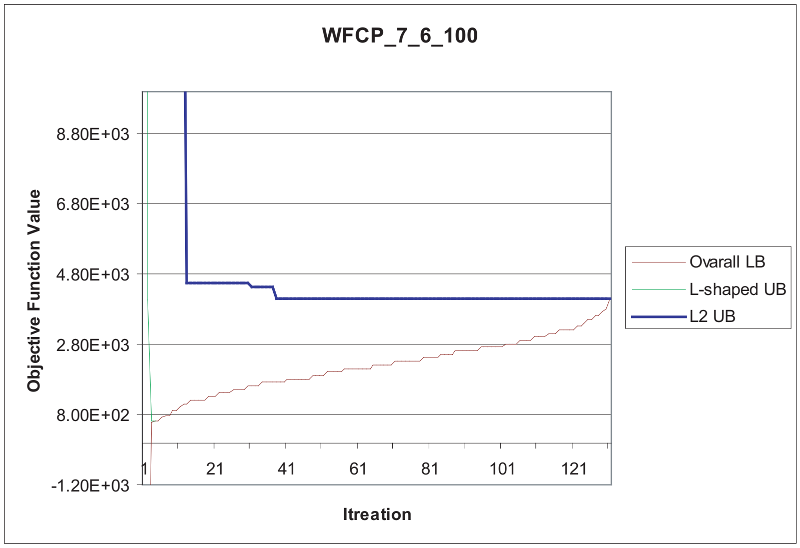

Fig. 2. Convergence of wfcp_7_6_5

the resources for all periods. When the budget is cut in half, the Z_{IP} tends to be higher than when the budget is not constrained. By limiting the budget, the pre-suppression and suppression budget is saved. However, it tends to deploy less and inexpensive resources. Consequently this will cause having higher overall NVC. In terms of the computation time, the CPLEX MIP solver directly applied to the corresponding DEP instances is not efficient enough to solve the constrained budget instances. The DEP of wfcp_7_6_100 has 7.95% of Z_{IP} optimality gap while that of unconstrained budget has 0.53%. From the first table in appendix A, it is observed that the optimal solutions of two different cases, the unconstrained and the constrained budget, are different.

Table XII provides the results when NVC per hectare is $20. When it is limited to §20, the Z_{IP} value decreases since small NVC value per hectare tends to contribute less to objective function value while the total burned area stays same. It takes relatively much time to find the optimal solution when the budget is unconstrained.

Table XI. Results of wfcp instances with constrained budget

DEP					
Instance	Iteration	CPU Time(sec.)	Z_{IP}	$Z_{LP(1)}$	$Z_{LP(1)}$ GAP(%)
wfcp_7_6_1	62	0.11	5.74E+03	4.76E+02	91.699
wfcp_7_6_5	1565	0.31	5.22E+03	4.78E+02	90.840
wfcp_7_6_10	8570	1.13	5.15E+03	5.04E+02	90.198
wfcp_7_6_15	50277	7.7	4.78E+03	4.66E+02	90.250
wfcp_7_6_30	1700689	>300(1.61%)	4.71E+03	4.43E+02	90.595
wfcp_7_6_50	801237	>300(1.94%)	4.45E+03	4.22E+02	90.521
wfcp_7_6_100	717654	>300(7.95%)	4.46E+03	4.15E+02	90.680
L^2 with Benders' Cuts					
Instance	Iteration	CPU Time(Sec.)	Z_{IP}	$Z_{LP(2)}$	$Z_{LP(2)}$ GAP(%)
wfcp_7_6_1	66	1	5.74E+03	7.11E+02	87.622
wfcp_7_6_5	67	1.39	5.22E+03	6.86E+02	86.848
wfcp_7_6_10	66	1.96	5.15E+03	7.14E+02	86.127
wfcp_7_6_15	67	2.39	4.78E+03	6.69E+02	85.993
wfcp_7_6_30	67	3.84	4.71E+03	6.46E+02	86.271
wfcp_7_6_50	67	5.7	4.45E+03	6.23E+02	86.005
wfcp_7_6_100	67	10.56	4.45E+03	6.16E+02	86.151

The inference to this phenomenon is that the small NVC value makes the problem hard to find the optimal solutions since it is sensitive to the solution. The CPLEX MIP solver directly applied to the corresponding DEP instances is less efficient than the L^2 with Benders' Cuts for all the experimented instances. The second table in appendix A provides the solution when it has \$20 NVC per hectare. From the table, it is inferred that the dozer is efficient when the budget is not restricted, and type II crew is efficient when the budget is restricted.

Table XIII provides the results when the NVC per hectare is extended to \$1000. The Z_{IP} value increases since large NVC per hectare tends to contribute much to objective function value whilte the total burned area stays same. The CPLEX MIP solver directly applied to the corresponding DEP instances may be efficient for the

Table XII. Results of wfcp instances with NVC per hectare $20

DEP					
Instance	Iteration	CPU Time(sec.)	Z_{IP}	$Z_{LP(1)}$	$Z_{LP(1)}$ GAP(%)
Uncon.Budget					
wfcp_7_5_30	2008552	>300(5.32%)	2.92E+03	3.73E+02	87.231
wfcp_7_5_50	962515	>300(7.11%)	2.88E+03	3.57E+02	87.578
wfcp_7_5_100	636277	>300(8.11%)	2.87E+03	3.52E+02	87.753
Const.Budget					
wfcp_7_5_30	16238	4.91	4.24E+05	3.73E+02	99.912
wfcp_7_5_50	19579	9.61	3.53E+05	3.57E+02	99.899
wfcp_7_5_100	557402	230	3.47E+05	3.52E+02	99.898
L^2 with Benders' Cuts					
Instance	Iteration	CPU Time(Sec.)	Z_{IP}	$Z_{LP(2)}$	$Z_{LP(2)}$ GAP(%)
Uncon.Bdget					
wfcp_7_5_30	106	9.4	2.91E+03	5.77E+02	80.210
wfcp_7_5_50	106	14.48	2.87E+03	5.58E+02	80.527
wfcp_7_5_100	106	26.95	2.86E+03	5.53E+02	80.654
Const.Budget					
wfcp_7_5_30	68	2.59	4.24E+05	5.77E+02	99.864
wfcp_7_5_50	68	3.78	3.53E+05	5.58E+02	99.842
wfcp_7_5_100	68	6.67	3.47E+05	5.53E+02	99.841

instances since the CPU time is relatively shorter than other instances. The inference to this phenomenon is that large NVC value makes the problem easier to find the optimal solution since it is not quite sensitive to the solution. The L^2 with Benders Cuts approach performs well for the instances. Table XVIII provides the optimal solution when it has $1000 NVC per hectare. Since the NVC per hectare is high, it tends to deploy more resources than when the NVC per hectare is small to minimize the NVC.

Table XIII. Results of wfcp instances with NVC per hectare $1000

DEP					
Instance	Iteration	CPU Time(sec.)	Z_{IP}	$Z_{LP(1)}$	$Z_{LP(1)}$ GAP(%)
Uncon.Budget					
wfcp_7_6_30	858	0.56	1.34E+04	1.23E+03	90.836
wfcp_7_6_50	1365	1.09	1.23E+04	1.15E+03	90.688
wfcp_7_6_100	2569	2.84	1.25E+04	1.13E+03	90.997
Const.Budget					
wfcp_7_6_30	889	0.64	1.41E+04	1.23E+03	91.275
wfcp_7_6_50	2534	1.63	1.32E+04	1.15E+03	91.272
wfcp_7_6_100	6921	6.19	1.33E+04	1.13E+03	91.524
L^2 with Benders' Cuts					
Instance	Iteration	CPU Time(Sec.)	Z_{IP}	$Z_{LP(2)}$	$Z_{LP(2)}$ GAP(%)
Uncon.Bdget					
wfcp_7_6_30	128	6.4	1.34E+04	1.43E+03	89.315
wfcp_7_6_50	128	8.81	1.23E+04	1.35E+03	89.058
wfcp_7_6_100	128	14.73	1.25E+04	1.33E+03	89.394
Const.Budget					
wfcp_7_6_30	67	2.53	1.41E+04	1.43E+03	89.828
wfcp_7_6_50	67	3.64	1.32E+04	1.35E+03	89.744
wfcp_7_6_100	67	6.23	1.33E+04	1.33E+03	90.015

4. Findings and Conclusions on wfcp Instances

From the computational results of wfcp instances, some of the important findings are observed in terms of the solution perspectives. When the pre-suppression and the suppression budget is limited, it tends to have higher Z_{IP} values. It is recommendable that the budget should not be much restricted in wildfire management. Different NVC value per hectare provides different solutions of the optimal mix of resources. It deploys as many resources as possible when the value is large, while it deploys relatively less resources when the value is small.

Since wildfire containment problem requires urgent decision-makings, the optimal

Fig. 3. FARSITE Wildfire Simulator

solution should be found in a short time. From the experiments conducted in this section, it is observed that the L^2 with Benders' Cuts algorithm performs better in larger number of scenario instances. It also is observed that the CPLEX MIP solver directly applied to the corresponding DEP instances performs bad if the budget is restricted, or NVC per hectare is small. In general, the L^2 with Benderss Cuts performs better that the CPLEX MIP solver directly applied to the corresponding DEP instances for most of the experiments conducted in this section.

B. Wildfire Containment Decisions for a Large-Scale Fire

1. Generation of Scenarios and Experimental Design

The experiments in the previous section are conducted under the relatively small-scale fire scenarios. In reality, any scale of fire may be realized. In this section, a set of large scale fire scenarios is generated from FARSITE [25]. And a set of experiments are conducted on the large-scale fire scenarios. Figure 3 shows the idea of how the simulator works. To run simulations on fire perimeter and burn area, default 'Ashley' project is utilized embedded in FARSITE. To randomly generate the large-scale fire scenarios, the changes are made in wind speed and wind direction from the default

'Ashley' project. The wind speed per hour is randomly generated from uniform(7, 13) distribution. The wind direction is randomly generated from uniform(60, 120) distribution. All the simulation runs use ignition coordinate X=166996 and Y=386878. Thirty minute unit time period is applied to the experiments in this section. Shorter unit time period provides more information on the fire growth and the burned area so that it helps to make more precise decisions. Based on the large-scale fire scenarios, different instances that have different number of scenarios are generated and experimented to see what decisions should be made under the large-scale fire scenarios. To increase the statistical accuracy of the instances, it replicates 6 times for each instance that has the same number of scenarios with different sample scenarios by sampling the scenarios from the population (100 scenarios). The naming convention wfcpu$b_r_t_s$ is used, where wfcpu stands for 'wildfire containment problem uniform', b is the ratio of total available budget, r is the number of resources, t is the number of time period, and s is the number of scenario. Letter d stands for double, t stands for triple, and q stands for quadruple in front of wfcpu. Similar to the previous section, the CPLEX MIP solver directly applied to the DEP instances and the L^2 with Benders' Cuts algorithm are compared to compare the performance of two different approaches.

2. Computational Results of wfcpu Instances

From the preliminary runs, (This is done by doubling, tripling, and quadrupling the production rate of resources) it is observed that at least tripling the production rate can contain the majority of the fire scenarios generated in this section. Thus, experiments begin with tripling the production rate of the 7 resources. This is equivalent to have 3 identical resources of each 7 resource.

Table XIV compares two different experiments conducted based on the CPLEX

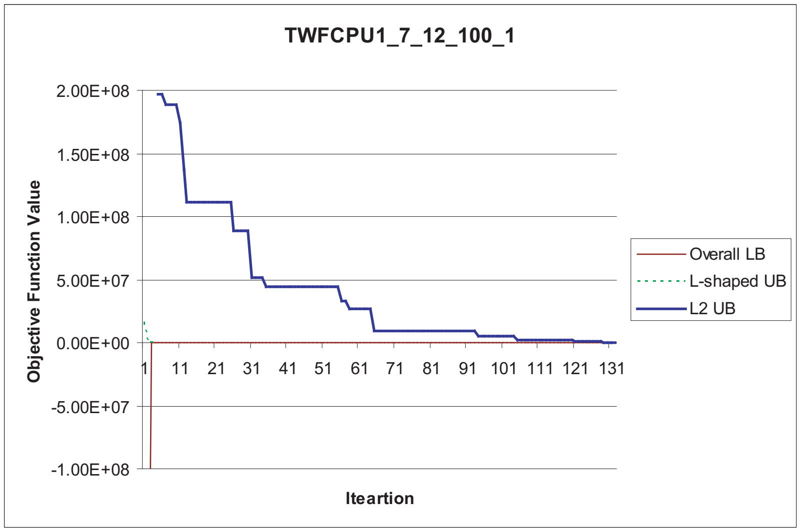

Fig. 4. Convergence of twfcpu1_7_12_100

MIP solver directly applied to the corresponding DEP instances and the L^2 with Benders' Cuts algorithm. The minimum, average, and maximum values of the CPU time, and the Z_{IP} are given in the table. The details of the results can be found in the appendix. If the pre-suppression and the suppression budget is not constrained, tripling the production rate of the 7 resources are enough to contain any fire scenario. The CPLEX MIP solver directly applied to the corresponding DEP instances seems to be efficient from the experiments. Figure 4 illustrates the convergence of the instance twfcpu1_7_12_100. It is observed that the overall lower bound increases as the iteration increases. The $L-shaped$ upper bound and the overall lower bound meets in the beginning of the iterations. At this moment, the optimal $Z_{LP(2)}$ is found. After the optimal value is found, the L^2 upper bound is obtained as iteration increases. It converges as the gap between overall lower bound and L^2 upper bound decreases.

Table XV shows the results when the pre-suppression and the suppression budget is cut in half of the maximum available budget. The optimal Z_{IP} tends to in-

Table XIV. Summary of twfcpu1 instances with unconstrained budget

DEP	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
twfcpu1_7_12_5	0	0.033	0.109	7.02E+04	7.93E+04	9.34E+04
twfcpu1_7_12_10	0.015	0.028	0.031	6.08E+04	7.27E+04	8.06E+04
twfcpu1_7_12_25	0.062	0.080	0.093	6.19E+04	6.65E+04	7.30E+04
twfcpu1_7_12_50	0.172	0.190	0.203	6.22E+04	6.61E+04	7.03E+04
twfcpu1_7_12_75	0.36	0.411	0.563	6.47E+04	6.57E+04	6.70E+04
L^2 with Benders' Cuts	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
twfcpu1_7_12_5	2.672	2.703	2.735	7.02E+04	7.93E+04	9.34E+04
twfcpu1_7_12_10	3.281	3.393	3.5	6.08E+04	7.27E+04	8.06E+04
twfcpu1_7_12_25	5.407	5.672	6.656	6.19E+04	6.65E+04	7.30E+04
twfcpu1_7_12_50	8.672	8.940	9.219	6.22E+04	6.61E+04	7.03E+04
twfcpu1_7_12_75	12.344	12.424	12.547	6.47E+04	6.57E+04	6.70E+04

crease significantly. In fact, any fire scenario cannot be contained if the budget is cut in half. Table XIX indicates that the objective function value tends to decrease when the NVC per hectare is \$20. From Table XX, it is observed that the the objective function value tends to increase when the NVC per hectare is \$1000. In both cases, changes in NVC per hectare does not affect much to the CPU time. The inference for this phenomenon comes from the concept of changing the production rate. By tripling the production rate, the optimal solution tends to be found easily since 3 identical resources of some kinds may be easily considered as the optimal solution, and it is relatively easy to be found.

Table XXI and Table XXII deal with the cases where the production rate of the fire-fighting resources is quadrupled. It is equivalent to have 4 identical resources of each kind. Table XXI provides the result when the pre-suppression and suppression budget is not constrained. Table XXII provides the result when the budget is cut in

Table XV. Summary of twfcpu0.5 instances with constrained budget

DEP	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
twfcpu0.5_7_12_5	0.015	0.039	0.125	8.80E+06	1.45E+07	2.69E+07
twfcpu0.5_7_12_10	0.047	0.070	0.094	4.44E+06	1.39E+07	2.21E+07
twfcpu0.5_7_12_25	0.187	0.258	0.281	4.59E+06	9.71E+06	1.38E+07
twfcpu0.5_7_12_50	0.5	0.805	1.235	7.07E+06	1.04E+07	1.31E+07
twfcpu0.5_7_12_75	1.578	1.656	1.766	7.67E+06	9.23E+06	1.03E+07
L^2 with Benders' Cuts	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
twfcpu0.5_7_12_5	0.828	0.865	0.89	8.80E+06	1.45E+07	2.69E+07
twfcpu0.5_7_12_10	1.156	1.169	1.187	4.44E+06	1.39E+07	2.21E+07
twfcpu0.5_7_12_25	2.031	2.050	2.078	4.59E+06	9.71E+06	1.38E+07
twfcpu0.5_7_12_50	3.469	3.497	3.547	7.07E+06	1.04E+07	1.31E+07
twfcpu0.5_7_12_75	4.859	4.891	4.953	7.67E+06	9.23E+06	1.03E+07

half. In both cases, no matter what fire growth scenario occurs, it can be contained. However, when the budget is cut in half, total C+NVC tends to increase. In terms of the efficiency between the CPLEX MIP solver directly applied to the corresponding DEP instances and the L^2 with Benders' Cuts algorithm, both approaches are efficient enough to solve all the instances. The CPU time of the L^2 with Benders' Cuts increase monotonously as the number of scenario increases while that of the CPLEX MIP solver directly applied to the corresponding DEP instances increases exponentially.

Table XXIII and Table XXIV deal with the cases where the production rate of the fire-fighting resources is doubled. Also the number of the fire-fighting resources are doubled. In other words, there are 14 different resources are available, and each of them are composed of 2 identical resources. Thus, 28 resources are prepared for the wildfire. However the decisions regarding the deployment only provide wether 2 identical resources are deployed or not for each 14 different resource. Table XXIII provides the result when the pre-suppression and the suppression budget is not constrained. Table XXIV provides the result when the budget is cut in half. For both cases, the CPLEX MIP solver directly applied to the corresponding DEP instances can provide the optimal solution if it has only small number of scenarios in the instances within 600s, while the L^2 with Benders' Cuts algorithm is not able to solve any of these instances. The optimal Z_{IP} is relatively small if the budget is not constrained. It is inferred that the number of first-stage variable affects much to the solution time of the L^2 with Benders' Cuts algorithm.

3. Findings and Conclusions of wfcpu Instances

From the computational results of wfcpu instances, several findings are observed in terms of the solution perspectives. When the pre-suppression and the suppression budget is limited, it tends to have higher Z_{IP} values. It is recommendable that the budget should not be much restricted in order to minimize the C+NVC. The optimal Z_{IP} tends to increase as the NVC per hectare increases. In terms of solution method perspectives, both the CPLEX MIP solver directly applied to the corresponding DEP instances and the L^2 with Benders' Cuts algorithm are efficient when the number of first-stage variable is limited to 7. However, it is observed that both the L^2 with Benders' Cuts algorithm cannot solve the instances if the first-stage variables are doubled.

C. Overall Findings and Conclusions

In this chapter, two different set of experiments are conducted. One set is based on small-scale fire scenarios, and the other set is based on large-scale fire scenarios.

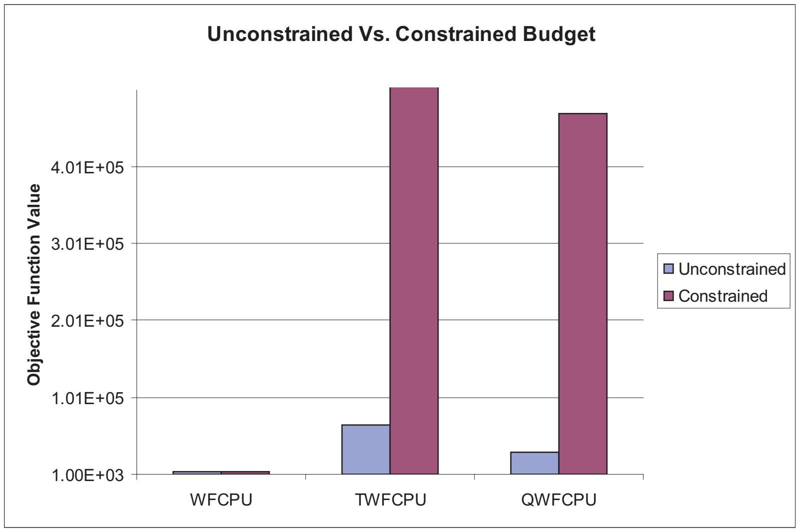

Fig. 5. Budget vs. Objective Function Value

Figure 5 and Figure 6 provide two important results about the budgetary constraint and the changes in NVC per hectare. From the graph on the left, it is inferred that the optimal Z_{IP} tends to increase if the pre-suppression and suppression budget is limited for wfcp, twfcpu and qwfcpu instances. Thus, it is recommendable that the wildfire management budget should not be much restricted in order to minimize the C+NVC. From the graph on the right, it is observed that the optimal Z_{IP} tends to increase as the NVC per hectare increases. If the value is large, it tends to deploy more resources under the same fire growth scenarios. Thus, it is recommendable to deploy more resources to minimize the NVC if the value is large. The increase in the suppression and pre-suppression cost will pay off by minimizing the NVC.

Figure 7 and Figure 8 provide the importance result about the solution time in terms of the number of scenarios when the CPLEX MIP solver directly applied to the corresponding DEP instances and the L^2 with Benders' Cuts algorithm are applied. In general, the CPU time grows as the number of scenarios grows in both cases.

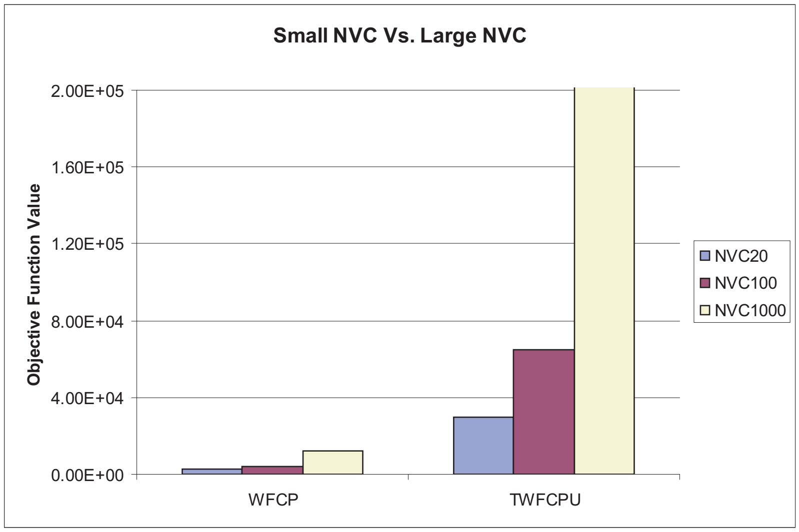

Fig. 6. NVC vs. Objective Function Value

However, the CPU time of the CPLEX MIP solver directly applied to the corresponding DEP instances grows exponentially as the number of scenarios increase, while that of the L^2 with Benders' Cuts grows monotonously. The DEP approach performs well if there are small number of scenarios in the instances while the L^2 with Benders' Cuts performs well if there are large number of scenarios in the instances. If the first-stage variables increase, both the CPLEX MIP solver directly applied to the corresponding DEP instances and the L^2 with Benders' Cuts does not efficiently perform. If a decomposition algorithm that can handle the more first-stage binary variable is developed, it will solve the realistic size of instances that have more first-stage variables and larger number of scenarios.

The wfcpu instances may not provide true optimal mix of the resource deployment decisions because the concepts of doubling, tripling, or quadrupling the production rate are embedded in the instances. To truly identify the optimal mix of the resources under more realistic fire growth scenarios, an algorithm can take into

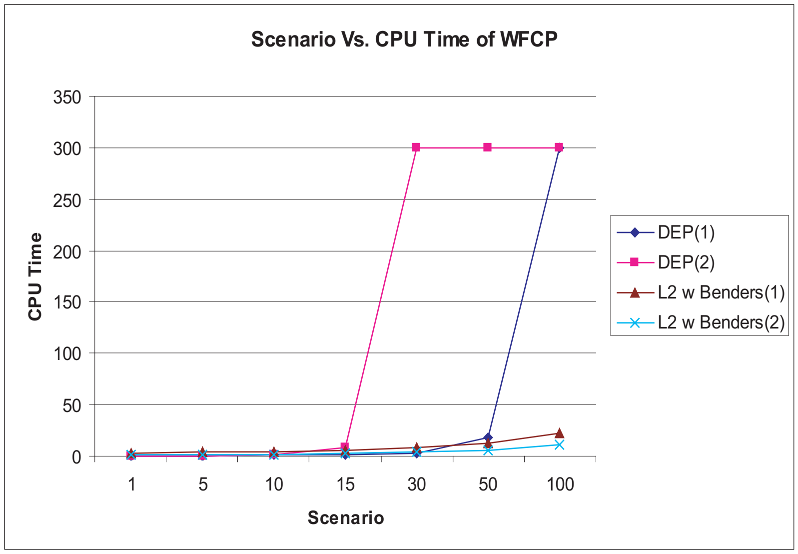

Fig. 7. DEP vs. L^2 with Benders' Cuts (Small-Scale Fire)

account more first-stage variable is required.

Earlier in this chapter, it is shown that the stochastic programming approach provides the best expected profit if the perfect information is not available. Thus the important thing to be mentioned is that the stochastic programming approach on the wildfire containment problem provides realistic decisions on the resource deployment since it takes into account all the possible scenarios that might be realized in the future to the problems.

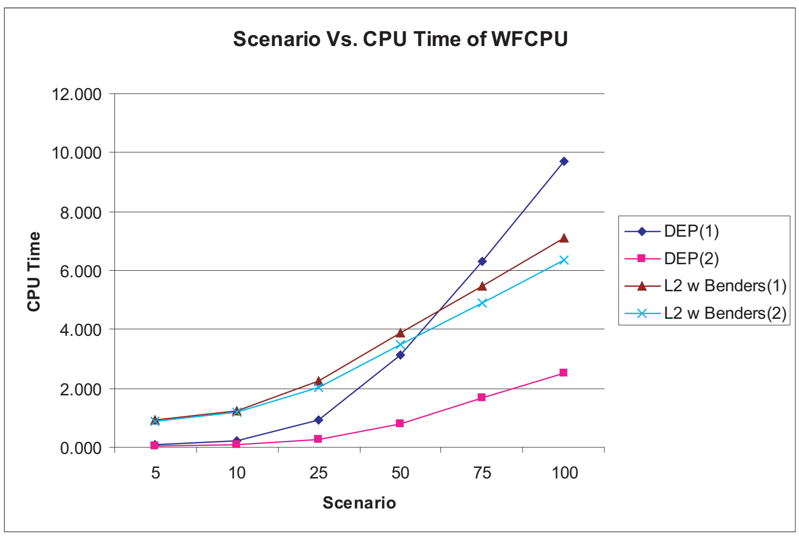

Fig. 8. DEP vs. L^2 with Benders' Cuts (Large-Scale Fire)

CHAPTER VII

CONCLUSIONS AND FUTURE WORK

A. Conclusions

This thesis proposes SMIP approaches in dealing with two different wildfire management problems. In chapter III, the strategic wildfire resource management model is proposed. This suggests that the decisions regarding the location of the attack-bases and the allocation of the fire-fighting resources should be optimally made to take advantage of the limited resources. In terms of making optimal decisions, stochastic factors such as fire occurrence should be taken into account to make the decision much more realistic. Thus a SMIP approach is utilized to take into account the random elements of the proposed model. In chapter IV, the tactical wildfire containment model is proposed. This suggests that the optimal decisions regarding the deployment of the fire-fighting resources with respect to the actual fire should be made in a much more realistic way by taking into account the randomness in the fire growth characteristic. Thus a SMIP approach is applied to this proposed model again.

In chapter V, as a solution method of the proposed models, the L^2 with Benders' Cuts algorithm is introduced. It is not an easy task to successfully implement the L^2 with Benders' Cuts algorithm. Especially the proposed models require the random recourse format and this makes the implementation of the algorithm and the experiment of the proposed model more difficult. In chapter VI, the analysis of the computational results on the wildfire containment model proposed in chapter III is provided. There are several important findings and results from the experiments and these can be summarized as follows. When the pre-suppression and the suppression budget is restricted much, the C+NVC tends to significantly increase. Thus, it is recommendable to allocate the budget as much as possible. Also if the NVC per hectare is large, it is recommendable to deploy as many fire-fighting resources as possible. In terms of the efficiency of the SMIP approach, the L^2 with Benders cuts is more efficient than the CPLEX MIP solver directly applied to the corresponding DEP instances as the number of the fire growth scenarios increase. Still there needs to have a more advanced decomposition algorithm that can take into account more number of the first-stage binary variables to solve more realistic problems.

One of the important conclusions is that the stochastic programming approach to the wildfire containment problem provides realistic decisions on the resource deployment since it takes into account all the possible scenarios that might be realized in the future. In the near future, the stochastic programming approach is expected to contribute to the wildfire management systems in more realistic ways.

B. Contributions of This Research

In terms of contributions of this research, three meaningful contributions may be considered. First, this provides the integrated framework for wildfire management systems by taking into account the realistic aspects of the strategic and tactical wildfire related problems since this provides the solution of the scopes from the attack-base location and resource allocation to the resource deployment plan for actual wildfire containment by taking into account the strategic and the tactical aspects of the systems. Second, this provides the realistic sensitivity analysis by experimenting the instances of the wildfire containment problem. If a set of information about the wildfire characteristics, and the fire-fighting resources are given, it provides the decisions about which resources to deploy to minimize C+NVC. Third, this research contributes to the theory and the application of the stochastic programming. So far very few researches have been done regarding the application of the SMIP with random recourse in the stochastic programming literatures. Successful implementation of the algorithm and application to the real-world problem make significant contributions to the theory and the application of the stochastic programming.

C. Future Work

Some Future work includes implementing a more efficient algorithm for solving realistic size instances and updating real-time fire-growth information to the model so that the real-time solution may be obtained to make robust decisions as early as possible in urgent situations. Other future work would involve conducting experiments and analysis for the strategic attack-base location and resource allocation model.

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APPENDIX A

WFCP SOLUTION AND WFCPU RESULTS SUMMARY

Table XVI. Optimal resource mix of wfcp instances with constrained budget

	Solution						
Instance	Dozer	Tractor	CrewI	CrewII	EngineI	EngineII	EngineIII
wfcp_7_6_1	1	1			1		
wfcp_7_6_5	1	1					
wfcp_7_6_10	1	1					
wfcp_7_6_15	1	1	1				
wfcp_7_6_30	1	1	1				
wfcp_7_6_50	1	1	1				
wfcp_7_6_100	1	1	1				

Table XVII. Optimal resource mix of wfcp instances with NVC per hectare $20

	Solution						
Instance	Dozer	Tractor	CrewI	CrewII	EngineI	EngineII	EngineIII
Uncon.Budget							
wfcp_7_6_30	1	1					
wfcp_7_6_50	1	1					
wfcp_7_6_100	1	1					
Const.Budget							
wfcp_7_6_30		1	1				
wfcp_7_6_50		1	1				
wfcp_7_6_100		1	1				

Table XVIII. Optimal resource mix of wfcp instances with NVC per hectare $1000

	Solution						
Instance	Dozer	Tractor	CrewI	CrewII	EngineI	EngineII	EngineIII
Uncon.Budget							
wfcp_7_6_30	1	1	1	1			1
wfcp_7_6_50	1		1	1			1
wfcp_7_6_100	1		1	1			1
Const.Budget							
wfcp_7_6_30	1	1	1	1			
wfcp_7_6_50	1	1	1	1			
wfcp_7_6_100	1	1	1	1			

Table XIX. Summary of twfcpu1 instances with NVC per hectare $20

DEP	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
twfcp20u_7_12_5	0.063	0.125	0.25	2.86E+04	3.25E+04	3.63E+04
twfcp20u_7_12_10	0.187	0.292	0.485	2.66E+04	3.11E+04	3.35E+04
twfcp20u_7_12_25	0.14	0.606	1.843	2.87E+04	2.97E+04	3.16E+04
twfcp20u_7_12_50	0.406	0.524	0.594	2.88E+04	2.99E+04	3.10E+04
twfcp20u_7_12_75	0.766	1.138	1.328	2.95E+04	2.97E+04	3.01E+04
L^2 with Benders' Cuts	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
twfcp20u_7_12_5	2.781	2.877	2.969	2.86E+04	3.25E+04	3.63E+04
twfcp20u_7_12_10	3.641	3.750	3.859	2.66E+04	3.11E+04	3.35E+04
twfcp20u_7_12_25	6.016	6.162	6.235	2.87E+04	2.97E+04	3.16E+04
twfcp20u_7_12_50	9.985	10.076	10.172	2.88E+04	2.99E+04	3.10E+04
twfcp20u_7_12_75	14.001	14.089	14.156	2.95E+04	2.97E+04	3.01E+04

Table XX. Summary of twfcpu1 instances with NVC per hectare $1000

DEP	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
twfcp1000u_7_12_5	0	0.021	0.094	5.10E+05	5.96E+05	7.26E+05
twfcp1000u_7_12_10	0.015	0.016	0.016	4.23E+05	5.35E+05	6.09E+05
twfcp1000u_7_12_25	0.031	0.044	0.047	4.35E+05	4.78E+05	5.39E+05
twfcp1000u_7_12_50	0.078	0.091	0.094	4.38E+05	4.74E+05	5.12E+05
twfcp1000u_7_12_75	0.141	0.151	0.157	4.61E+05	4.70E+05	4.82E+05
L^2 with Benders' Cuts	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
twfcp1000u_7_12_5	2.609	2.669	2.734	5.10E+05	5.96E+05	7.26E+05
twfcp1000u_7_12_10	3.25	3.966	5.672	4.23E+05	5.35E+05	6.09E+05
twfcp1000u_7_12_25	5.375	8.464	13.984	4.35E+05	4.78E+05	5.39E+05
twfcp1000u_7_12_50	8.422	11.438	21.328	4.38E+05	4.74E+05	5.12E+05
twfcp1000u_7_12_75	14.36	20.639	27	4.61E+05	4.70E+05	4.82E+05

Table XXI. Summary of qwfcpu1 instances with unconstrained budget

DEP	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
qwfcpu1_7_12_5	0.015	0.042	0.125	4.40E+04	5.08E+04	5.89E+04
qwfcpu1_7_12_10	0.062	0.104	0.172	4.30E+04	5.11E+04	5.82E+04
qwfcpu1_7_12_25	0.469	0.680	1.125	4.78E+04	5.09E+04	5.53E+04
qwfcpu1_7_12_50	0.828	1.995	2.875	4.96E+04	5.21E+04	5.33E+04
qwfcpu1_7_12_75	5.062	6.375	7.938	5.03E+04	5.16E+04	5.26E+04
L^2 with Benders' Cuts	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
qwfcpu1_7_12_5	2.906	2.948	3.047	4.40E+04	5.08E+04	5.89E+04
qwfcpu1_7_12_10	3.563	3.646	3.766	4.30E+04	5.11E+04	5.82E+04
qwfcpu1_7_12_25	5.719	5.844	5.969	4.78E+04	5.09E+04	5.53E+04
qwfcpu1_7_12_50	9.516	9.654	9.734	4.96E+04	5.21E+04	5.33E+04
qwfcpu1_7_12_75	13.375	13.516	13.719	5.03E+04	5.16E+04	5.26E+04

Table XXII. Summary of qwfcpu0.5 instances with constrained budget

DEP	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
qwfcpu0.5_7_12_5	0.062	0.099	0.203	6.56E+04	7.52E+04	8.87E+04
qwfcpu0.5_7_12_10	0.094	0.219	0.375	5.66E+04	6.92E+04	7.60E+04
qwfcpu0.5_7_12_25	0.375	0.948	1.25	5.95E+04	6.30E+04	6.90E+04
qwfcpu0.5_7_12_50	2.64	3.146	3.86	5.90E+04	6.27E+04	6.70E+04
qwfcpu0.5_7_12_75	5.938	6.313	7.328	6.11E+04	6.24E+04	6.38E+04
L^2 with Benders' Cuts	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
qwfcpu0.5_7_12_5	0.906	0.909	0.922	6.56E+04	7.52E+04	8.87E+04
qwfcpu0.5_7_12_10	1.234	1.253	1.297	5.66E+04	6.92E+04	7.60E+04
qwfcpu0.5_7_12_25	2.204	2.237	2.281	5.95E+04	6.30E+04	6.90E+04
qwfcpu0.5_7_12_50	3.844	3.870	3.906	5.90E+04	6.27E+04	6.70E+04
qwfcpu0.5_7_12_75	5.422	5.456	5.516	6.11E+04	6.24E+04	6.38E+04

Table XXIII. Summary of dwfcpu1 instances with unconstrained budget

DEP	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
dwfcpu1_14_12_5	0.047	0.214	0.297	4.38E+04	5.00E+04	5.77E+04
dwfcpu1_14_12_10	0.703	47.786	109.5	4.23E+04	5.03E+04	5.79E+04
dwfcpu1_14_12_25	600.015	600.050	600.221	4.75E+04	5.04E+04	5.50E+04
dwfcpu1_14_12_50	600.015	600.021	600.047	4.94E+04	5.17E+04	5.30E+04
dwfcpu1_14_12_75	600.015	600.026	600.032	5.01E+04	5.13E+04	5.21E+04
L^2 with Benders' Cuts	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
dwfcpu1_14_12_5	600.035	600.572	600.942	5.79E+03	5.84E+03	5.94E+03
dwfcpu1_14_12_10	600.129	600.796	601.223	5.62E+03	5.82E+03	5.98E+03
dwfcpu1_14_12_25	600.051	600.468	600.645	5.68E+03	5.71E+03	5.79E+03
dwfcpu1_14_12_50	600.114	600.512	601.051	5.68E+03	5.71E+03	5.73E+03
dwfcpu1_14_12_75	600.207	600.691	601.144	5.55E+03	5.59E+03	5.71E+03

Table XXIV. Summary of dwfcpu0.5 instances with constrained budget

DEP	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
dwfcpu0.5_14_12_5	0.234	0.419	0.812	6.54E+04	7.51E+04	8.87E+04
dwfcpu0.5_14_12_10	0.547	0.974	1.437	5.62E+04	6.90E+04	7.58E+04
dwfcpu0.5_14_12_25	4.954	45.297	123.313	5.92E+04	6.27E+04	6.87E+04
dwfcpu0.5_14_12_50	491.425	581.921	600.02	5.87E+04	6.25E+04	6.68E+04
dwfcpu0.5_14_12_75	600.019	600.022	600.035	6.08E+04	6.21E+04	6.36E+04
L^2 with Benders' Cuts	CPU Time			Z_{IP}		
	Min	Ave	Max	Min	Ave	Max
dwfcpu0.5_14_12_5	600.02	600.653	601.223	5.78E+03	5.83E+03	5.94E+03
dwfcpu0.5_14_12_10	600.238	600.730	601.222	5.62E+03	5.78E+03	5.98E+03
dwfcpu0.5_14_12_25	600.004	600.506	601.067	5.68E+03	5.71E+03	5.79E+03
dwfcpu0.5_14_12_50	600.238	600.625	600.848	5.68E+03	5.71E+03	5.73E+03
dwfcpu0.5_14_12_75	600.006	600.550	601.191	5.69E+03	5.71E+03	5.73E+03

APPENDIX B

DEP RESULT OF WFPCU INSTANCES

(1): name of the instance

(2): number of CPLEX iteration taken

(3): CPU time taken

(4): Z_{IP}

(5): Z_{LP}

(6): Z_{LP} GAP

(7): number of B&B node taken

Table XXV. Unconstrained budget (tripling resource production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
twfcpu1_7_12_5_1	112	0.109	7.75E+04	1.60E+03	97.9403	0
twfcpu1_7_12_5_2	116	0.031	7.02E+04	1.57E+03	97.7648	0
twfcpu1_7_12_5_3	134	0.032	7.04E+04	1.54E+03	97.8125	0
twfcpu1_7_12_5_4	112	0.015	7.89E+04	1.53E+03	98.0634	0
twfcpu1_7_12_5_5	101	0	8.56E+04	1.56E+03	98.1725	0
twfcpu1_7_12_5_6	78	0.015	9.34E+04	1.71E+03	98.1671	0
twfcpu1_7_12_10_1	197	0.031	8.06E+04	1.75E+03	97.8249	0
twfcpu1_7_12_10_2	199	0.031	6.55E+04	1.33E+03	97.9639	0
twfcpu1_7_12_10_3	286	0.031	6.08E+04	1.63E+03	97.321	0
twfcpu1_7_12_10_4	219	0.031	8.05E+04	1.56E+03	98.0625	0
twfcpu1_7_12_10_5	166	0.015	7.83E+04	1.56E+03	98.0077	0
twfcpu1_7_12_10_6	193	0.031	7.05E+04	1.56E+03	97.7832	0
twfcpu1_7_12_25_1	406	0.078	6.64E+04	1.50E+03	97.7458	0
twfcpu1_7_12_25_2	375	0.078	7.30E+04	1.55E+03	97.8834	0
twfcpu1_7_12_25_3	487	0.093	6.90E+04	1.64E+03	97.6156	0
twfcpu1_7_12_25_4	362	0.093	6.60E+04	1.50E+03	97.7206	0
twfcpu1_7_12_25_5	320	0.062	6.19E+04	1.54E+03	97.5153	0
twfcpu1_7_12_25_6	457	0.078	6.30E+04	1.55E+03	97.5351	0
twfcpu1_7_12_50_1	678	0.187	6.45E+04	1.59E+03	97.5346	0
twfcpu1_7_12_50_2	706	0.203	6.74E+04	1.50E+03	97.7782	0
twfcpu1_7_12_50_3	762	0.203	6.22E+04	1.55E+03	97.5139	0
twfcpu1_7_12_50_4	730	0.188	6.57E+04	1.55E+03	97.6437	0
twfcpu1_7_12_50_5	739	0.172	7.03E+04	1.52E+03	97.8375	0
twfcpu1_7_12_50_6	744	0.188	6.65E+04	1.59E+03	97.613	0
twfcpu1_7_12_75_1	1090	0.375	6.70E+04	1.57E+03	97.6544	0
twfcpu1_7_12_75_2	1135	0.563	6.48E+04	1.55E+03	97.6055	17
twfcpu1_7_12_75_3	1139	0.375	6.69E+04	1.55E+03	97.6882	0
twfcpu1_7_12_75_4	1134	0.406	6.57E+04	1.59E+03	97.5788	0
twfcpu1_7_12_75_5	1129	0.391	6.47E+04	1.56E+03	97.5913	0
twfcpu1_7_12_75_6	1144	0.36	6.49E+04	1.56E+03	97.5889	0
twfcpu1_7_12_100_1	1488	0.531	6.57E+04	1.57E+03	97.6048	0

Table XXVI. Constrained budget (tripling resource production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
twfcpu0.5_7_12_5_1	217	0.125	8.80E+06	1.60E+03	99.9819	0
twfcpu0.5_7_12_5_2	206	0.031	8.81E+06	1.57E+03	99.9822	1
twfcpu0.5_7_12_5_3	177	0.015	1.06E+07	1.54E+03	99.9855	0
twfcpu0.5_7_12_5_4	266	0.031	1.03E+07	1.53E+03	99.9852	1
twfcpu0.5_7_12_5_5	162	0.016	2.69E+07	1.56E+03	99.9942	0
twfcpu0.5_7_12_5_6	203	0.016	2.15E+07	1.71E+03	99.992	0
twfcpu0.5_7_12_10_1	494	0.078	1.41E+07	1.75E+03	99.9876	11
twfcpu0.5_7_12_10_2	367	0.078	1.46E+07	1.33E+03	99.9909	1
twfcpu0.5_7_12_10_3	514	0.094	4.44E+06	1.63E+03	99.9633	1
twfcpu0.5_7_12_10_4	457	0.062	1.86E+07	1.56E+03	99.9916	1
twfcpu0.5_7_12_10_5	318	0.047	2.21E+07	1.56E+03	99.9929	9
twfcpu0.5_7_12_10_6	442	0.062	9.73E+06	1.56E+03	99.9839	1
twfcpu0.5_7_12_25_1	1070	0.281	1.05E+07	1.50E+03	99.9857	1
twfcpu0.5_7_12_25_2	967	0.187	1.38E+07	1.55E+03	99.9888	1
twfcpu0.5_7_12_25_3	1067	0.25	9.04E+06	1.64E+03	99.9818	10
twfcpu0.5_7_12_25_4	1086	0.281	9.02E+06	1.50E+03	99.9833	10
twfcpu0.5_7_12_25_5	1095	0.266	1.13E+07	1.54E+03	99.9864	1
twfcpu0.5_7_12_25_6	1193	0.281	4.59E+06	1.55E+03	99.9662	1
twfcpu0.5_7_12_50_1	2336	0.703	1.00E+07	1.59E+03	99.9841	1
twfcpu0.5_7_12_50_2	2264	1.235	1.31E+07	1.50E+03	99.9886	102
twfcpu0.5_7_12_50_3	2315	0.75	9.13E+06	1.55E+03	99.9831	1
twfcpu0.5_7_12_50_4	2282	0.922	7.07E+06	1.55E+03	99.9781	10
twfcpu0.5_7_12_50_5	2128	0.5	1.22E+07	1.52E+03	99.9876	0
twfcpu0.5_7_12_50_6	2153	0.719	1.08E+07	1.59E+03	99.9852	1
twfcpu0.5_7_12_75_1	3237	1.594	9.84E+06	1.57E+03	99.984	1
twfcpu0.5_7_12_75_2	3233	1.672	9.38E+06	1.55E+03	99.9835	1
twfcpu0.5_7_12_75_3	3224	1.734	1.03E+07	1.55E+03	99.9851	10
twfcpu0.5_7_12_75_4	3300	1.766	9.79E+06	1.59E+03	99.9838	10
twfcpu0.5_7_12_75_5	3212	1.594	7.67E+06	1.56E+03	99.9797	1
twfcpu0.5_7_12_75_6	3133	1.578	8.34E+06	1.56E+03	99.9812	1
twfcpu0.5_7_12_100_1	4322	2.531	9.40E+06	1.57E+03	99.9833	1

Table XXVII. Fixed NVC $20 (tripling resource production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
twfcp20u1_7_12_5_1	524	0.25	3.19E+04	1.05E+03	96.6981	54
twfcp20u1_7_12_5_2	305	0.094	2.97E+04	1.03E+03	96.5369	6
twfcp20u1_7_12_5_3	291	0.094	2.86E+04	1.01E+03	96.4616	4
twfcp20u1_7_12_5_4	495	0.156	3.33E+04	1.01E+03	96.9704	15
twfcp20u1_7_12_5_5	241	0.094	3.53E+04	1.03E+03	97.0995	0
twfcp20u1_7_12_5_6	211	0.063	3.63E+04	1.11E+03	96.9527	2
twfcp20u1_7_12_10_1	772	0.187	3.33E+04	1.13E+03	96.5986	2
twfcp20u1_7_12_10_2	1358	0.328	2.93E+04	9.05E+02	96.9166	105
twfcp20u1_7_12_10_3	681	0.296	2.66E+04	1.06E+03	96.0083	19
twfcp20u1_7_12_10_4	487	0.219	3.35E+04	1.03E+03	96.9171	0
twfcp20u1_7_12_10_5	548	0.235	3.27E+04	1.04E+03	96.8293	2
twfcp20u1_7_12_10_6	1095	0.485	3.09E+04	1.03E+03	96.66	110
twfcp20u1_7_12_25_1	1579	1.063	2.89E+04	1.01E+03	96.5173	59
twfcp20u1_7_12_25_2	662	0.203	3.16E+04	1.05E+03	96.6745	0
twfcp20u1_7_12_25_3	2506	1.843	2.99E+04	1.10E+03	96.3147	290
twfcp20u1_7_12_25_4	755	0.218	2.97E+04	1.01E+03	96.5914	4
twfcp20u1_7_12_25_5	644	0.171	2.87E+04	1.05E+03	96.3518	1
twfcp20u1_7_12_25_6	717	0.14	2.91E+04	1.03E+03	96.4496	0
twfcp20u1_7_12_50_1	1298	0.594	2.94E+04	1.07E+03	96.3598	27
twfcp20u1_7_12_50_2	1200	0.484	3.02E+04	1.00E+03	96.6756	10
twfcp20u1_7_12_50_3	1284	0.563	2.88E+04	1.05E+03	96.3493	19
twfcp20u1_7_12_50_4	1261	0.5	2.98E+04	1.03E+03	96.5469	12
twfcp20u1_7_12_50_5	1173	0.406	3.10E+04	1.02E+03	96.7191	2
twfcp20u1_7_12_50_6	1252	0.594	2.99E+04	1.07E+03	96.4257	29
twfcp20u1_7_12_75_1	1894	1.266	3.01E+04	1.06E+03	96.4706	73
twfcp20u1_7_12_75_2	1949	1.328	2.95E+04	1.06E+03	96.4144	91
twfcp20u1_7_12_75_3	1802	1.016	3.01E+04	1.03E+03	96.577	28
twfcp20u1_7_12_75_4	1741	0.766	2.97E+04	1.07E+03	96.3904	11
twfcp20u1_7_12_75_5	1837	1.281	2.95E+04	1.06E+03	96.4129	69
twfcp20u1_7_12_75_6	1820	1.172	2.96E+04	1.06E+03	96.4166	50
twfcp20u1_7_12_100_1	2847	2.844	2.98E+04	1.06E+03	96.4214	192

Table XXVIII. Fixed NVC $1000 (tripling resource production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
twfcp1000u1_7_12_5_1	49	0.094	5.77E+05	7.71E+03	98.6633	0
twfcp1000u1_7_12_5_2	47	0.015	5.10E+05	7.63E+03	98.5043	0
twfcp1000u1_7_12_5_3	86	0.016	5.20E+05	7.47E+03	98.564	0
twfcp1000u1_7_12_5_4	39	0	5.92E+05	7.36E+03	98.7561	0
twfcp1000u1_7_12_5_5	55	0	6.51E+05	7.63E+03	98.8279	0
twfcp1000u1_7_12_5_6	32	0	7.26E+05	8.52E+03	98.8268	0
twfcp1000u1_7_12_10_1	105	0.016	6.08E+05	8.73E+03	98.5648	0
twfcp1000u1_7_12_10_2	105	0.015	4.67E+05	6.15E+03	98.6833	0
twfcp1000u1_7_12_10_3	127	0.015	4.23E+05	8.00E+03	98.1109	0
twfcp1000u1_7_12_10_4	106	0.016	6.09E+05	7.48E+03	98.7721	0
twfcp1000u1_7_12_10_5	45	0.016	5.87E+05	7.46E+03	98.7303	0
twfcp1000u1_7_12_10_6	110	0.015	5.14E+05	7.54E+03	98.5351	0
twfcp1000u1_7_12_25_1	124	0.047	4.76E+05	7.01E+03	98.5292	0
twfcp1000u1_7_12_25_2	128	0.047	5.39E+05	7.10E+03	98.6819	0
twfcp1000u1_7_12_25_3	268	0.047	4.98E+05	7.76E+03	98.4442	0
twfcp1000u1_7_12_25_4	139	0.031	4.74E+05	7.03E+03	98.5161	0
twfcp1000u1_7_12_25_5	112	0.047	4.35E+05	7.05E+03	98.3801	0
twfcp1000u1_7_12_25_6	227	0.047	4.44E+05	7.39E+03	98.3355	0
twfcp1000u1_7_12_50_1	248	0.094	4.60E+05	7.45E+03	98.3791	0
twfcp1000u1_7_12_50_2	252	0.094	4.86E+05	7.05E+03	98.5495	0
twfcp1000u1_7_12_50_3	275	0.094	4.38E+05	7.12E+03	98.3758	0
twfcp1000u1_7_12_50_4	281	0.094	4.69E+05	7.38E+03	98.4273	0
twfcp1000u1_7_12_50_5	237	0.078	5.12E+05	7.18E+03	98.599	0
twfcp1000u1_7_12_50_6	245	0.094	4.78E+05	7.41E+03	98.4491	0
twfcp1000u1_7_12_75_1	390	0.156	4.82E+05	7.30E+03	98.4852	0
twfcp1000u1_7_12_75_2	355	0.156	4.62E+05	7.12E+03	98.4605	0
twfcp1000u1_7_12_75_3	365	0.156	4.81E+05	7.35E+03	98.4707	0
twfcp1000u1_7_12_75_4	384	0.141	4.70E+05	7.41E+03	98.4243	0
twfcp1000u1_7_12_75_5	368	0.157	4.61E+05	7.19E+03	98.4397	0
twfcp1000u1_7_12_75_6	353	0.141	4.62E+05	7.24E+03	98.434	0
twfcp1000u1_7_12_100_1	527	0.218	4.70E+05	7.28E+03	98.4485	0

Table XXIX. Unconstrained budget (quadrupling resource production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
qwfcpu1_7_12_5_1	178	0.125	5.00E+04	1.60E+03	96.8084	0
qwfcpu1_7_12_5_2	187	0.047	4.42E+04	1.57E+03	96.4543	0
qwfcpu1_7_12_5_3	207	0.031	4.40E+04	1.54E+03	96.4974	0
qwfcpu1_7_12_5_4	155	0.015	5.03E+04	1.53E+03	96.9583	0
qwfcpu1_7_12_5_5	155	0.015	5.76E+04	1.56E+03	97.2861	0
qwfcpu1_7_12_5_6	150	0.016	5.89E+04	1.71E+03	97.0913	0
qwfcpu1_7_12_10_1	394	0.078	5.47E+04	1.75E+03	96.7991	0
qwfcpu1_7_12_10_2	391	0.063	4.39E+04	1.33E+03	96.9633	0
qwfcpu1_7_12_10_3	459	0.172	4.30E+04	1.63E+03	96.2069	0
qwfcpu1_7_12_10_4	379	0.062	5.82E+04	1.56E+03	97.3198	0
qwfcpu1_7_12_10_5	479	0.172	5.48E+04	1.56E+03	97.1539	2
qwfcpu1_7_12_10_6	413	0.078	5.20E+04	1.56E+03	96.992	2
qwfcpu1_7_12_25_1	1157	0.579	4.89E+04	1.50E+03	96.9395	13
qwfcpu1_7_12_25_2	882	0.469	5.53E+04	1.55E+03	97.2028	0
qwfcpu1_7_12_25_3	1327	0.734	5.19E+04	1.64E+03	96.8295	45
qwfcpu1_7_12_25_4	1250	0.5	5.00E+04	1.50E+03	96.9927	18
qwfcpu1_7_12_25_5	852	0.672	4.78E+04	1.54E+03	96.7828	6
qwfcpu1_7_12_25_6	1605	1.125	5.14E+04	1.55E+03	96.979	58
qwfcpu1_7_12_50_1	2826	1.906	5.20E+04	1.59E+03	96.9434	62
qwfcpu1_7_12_50_2	2322	1.938	5.33E+04	1.50E+03	97.1924	13
qwfcpu1_7_12_50_3	1963	0.828	4.96E+04	1.55E+03	96.8818	17
qwfcpu1_7_12_50_4	2676	2.875	5.11E+04	1.55E+03	96.9675	84
qwfcpu1_7_12_50_5	3175	2.719	5.31E+04	1.52E+03	97.1367	61
qwfcpu1_7_12_50_6	2229	1.703	5.32E+04	1.59E+03	97.0197	43
qwfcpu1_7_12_75_1	5407	6.906	5.19E+04	1.57E+03	96.9726	91
qwfcpu1_7_12_75_2	7989	7.781	5.15E+04	1.55E+03	96.9884	207
qwfcpu1_7_12_75_3	6354	5.468	5.19E+04	1.55E+03	97.0218	172
qwfcpu1_7_12_75_4	5661	7.938	5.26E+04	1.59E+03	96.9754	145
qwfcpu1_7_12_75_5	5447	5.062	5.03E+04	1.56E+03	96.9051	165
qwfcpu1_7_12_75_6	4926	5.094	5.13E+04	1.56E+03	96.952	180
qwfcpu1_7_12_100_1	7542	9.329	5.16E+04	1.57E+03	96.9549	279

Table XXX. Constrained budget (quadrupling resource production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
qwfcpu0.5_7_12_5_1	317	0.203	7.28E+04	1.60E+03	97.8084	5
qwfcpu0.5_7_12_5_2	286	0.062	6.56E+04	1.57E+03	97.6087	0
qwfcpu0.5_7_12_5_3	274	0.078	6.69E+04	1.54E+03	97.6977	3
qwfcpu0.5_7_12_5_4	261	0.063	7.41E+04	1.53E+03	97.9367	0
qwfcpu0.5_7_12_5_5	277	0.078	8.29E+04	1.56E+03	98.1125	0
qwfcpu0.5_7_12_5_6	315	0.11	8.87E+04	1.71E+03	98.069	3
qwfcpu0.5_7_12_10_1	548	0.094	7.59E+04	1.75E+03	97.691	0
qwfcpu0.5_7_12_10_2	695	0.297	6.18E+04	1.33E+03	97.8453	24
qwfcpu0.5_7_12_10_3	578	0.172	5.66E+04	1.63E+03	97.1238	18
qwfcpu0.5_7_12_10_4	663	0.157	7.60E+04	1.56E+03	97.9459	0
qwfcpu0.5_7_12_10_5	625	0.219	7.51E+04	1.56E+03	97.9217	44
qwfcpu0.5_7_12_10_6	1049	0.375	7.00E+04	1.56E+03	97.766	24
qwfcpu0.5_7_12_25_1	1445	1.016	6.28E+04	1.50E+03	97.6193	25
qwfcpu0.5_7_12_25_2	1602	0.922	6.90E+04	1.55E+03	97.761	55
qwfcpu0.5_7_12_25_3	1381	0.375	6.46E+04	1.64E+03	97.4551	14
qwfcpu0.5_7_12_25_4	1316	0.937	6.17E+04	1.50E+03	97.5652	41
qwfcpu0.5_7_12_25_5	1596	1.25	5.95E+04	1.54E+03	97.413	109
qwfcpu0.5_7_12_25_6	1879	1.188	6.03E+04	1.55E+03	97.4243	60
qwfcpu0.5_7_12_50_1	3003	2.64	6.11E+04	1.59E+03	97.3966	144
qwfcpu0.5_7_12_50_2	3158	3.86	6.42E+04	1.50E+03	97.6669	140
qwfcpu0.5_7_12_50_3	2924	3.031	5.90E+04	1.55E+03	97.3781	69
qwfcpu0.5_7_12_50_4	2919	2.969	6.23E+04	1.55E+03	97.515	99
qwfcpu0.5_7_12_50_5	2778	3	6.70E+04	1.52E+03	97.7299	149
qwfcpu0.5_7_12_50_6	2848	3.375	6.28E+04	1.59E+03	97.4736	87
qwfcpu0.5_7_12_75_1	6040	7.328	6.38E+04	1.57E+03	97.5388	232
qwfcpu0.5_7_12_75_2	4690	6.14	6.15E+04	1.55E+03	97.4757	123
qwfcpu0.5_7_12_75_3	5548	6.297	6.35E+04	1.55E+03	97.5649	137
qwfcpu0.5_7_12_75_4	4579	6.188	6.26E+04	1.59E+03	97.4595	175
qwfcpu0.5_7_12_75_5	4681	5.938	6.11E+04	1.56E+03	97.4492	88
qwfcpu0.5_7_12_75_6	4350	5.985	6.16E+04	1.56E+03	97.4612	104
qwfcpu0.5_7_12_100_1	7116	9.703	6.23E+04	1.57E+03	97.4753	227

Table XXXI. Unconstrained budget (doubling 14 resources production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
dwfcpu1_14_12_5_1	404	0.25	4.89E+04	1.60E+03	96.7335	96
dwfcpu1_14_12_5_2	546	0.125	4.39E+04	1.57E+03	96.4289	77
dwfcpu1_14_12_5_3	1148	0.266	4.38E+04	1.54E+03	96.483	241
dwfcpu1_14_12_5_4	1294	0.297	5.01E+04	1.53E+03	96.9492	236
dwfcpu1_14_12_5_5	1303	0.297	5.54E+04	1.56E+03	97.1776	503
dwfcpu1_14_12_5_6	214	0.047	5.77E+04	1.71E+03	97.0336	2
dwfcpu1_14_12_10_1	201471	71.328	5.44E+04	1.75E+03	96.7792	112493
dwfcpu1_14_12_10_2	13341	4.765	4.25E+04	1.33E+03	96.8629	5895
dwfcpu1_14_12_10_3	2460	0.703	4.23E+04	1.63E+03	96.1454	242
dwfcpu1_14_12_10_4	262468	109.5	5.79E+04	1.56E+03	97.3073	162076
dwfcpu1_14_12_10_5	8588	2.687	5.46E+04	1.56E+03	97.1404	2853
dwfcpu1_14_12_10_6	446649	97.734	4.99E+04	1.56E+03	96.8656	138950
dwfcpu1_14_12_25_1	698800	600.015	4.80E+04	1.50E+03	96.8818	286693
dwfcpu1_14_12_25_2	736431	600.016	5.50E+04	1.55E+03	97.19	299934
dwfcpu1_14_12_25_3	621247	600.016	5.10E+04	1.64E+03	96.775	327906
dwfcpu1_14_12_25_4	688691	600.016	4.97E+04	1.50E+03	96.9767	307792
dwfcpu1_14_12_25_5	793320	600.015	4.75E+04	1.54E+03	96.7601	307981
dwfcpu1_14_12_25_6	691527	600.221	5.11E+04	1.55E+03	96.9646	277885
dwfcpu1_14_12_50_1	312885	600.047	5.16E+04	1.59E+03	96.918	149129
dwfcpu1_14_12_50_2	447470	600.018	5.28E+04	1.50E+03	97.1651	140571
dwfcpu1_14_12_50_3	363731	600.015	4.94E+04	1.55E+03	96.8652	147533
dwfcpu1_14_12_50_4	388662	600.016	5.08E+04	1.55E+03	96.9514	148291
dwfcpu1_14_12_50_5	372493	600.016	5.26E+04	1.52E+03	97.1107	141665
dwfcpu1_14_12_50_6	615443	600.016	5.30E+04	1.59E+03	97.0043	135051
dwfcpu1_14_12_75_1	365491	600.031	5.17E+04	1.57E+03	96.9584	88401
dwfcpu1_14_12_75_2	357329	600.031	5.13E+04	1.55E+03	96.9726	89021
dwfcpu1_14_12_75_3	385869	600.016	5.16E+04	1.55E+03	97.007	86917
dwfcpu1_14_12_75_4	263159	600.032	5.21E+04	1.59E+03	96.9502	92561
dwfcpu1_14_12_75_5	282946	600.015	5.01E+04	1.56E+03	96.8893	91455
dwfcpu1_14_12_75_6	379834	600.031	5.11E+04	1.56E+03	96.9356	88261
dwfcpu1_14_12_100_1	302200	600.016	5.14E+04	1.57E+03	96.9393	64431

Table XXXII. Constrained budget (doubling 14 resources production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
dwfcpu0.5_14_12_5_1	1471	0.5	7.28E+04	1.60E+03	97.8084	41
dwfcpu0.5_14_12_5_2	1431	0.359	6.54E+04	1.57E+03	97.601	139
dwfcpu0.5_14_12_5_3	5351	0.812	6.67E+04	1.54E+03	97.6905	548
dwfcpu0.5_14_12_5_4	960	0.234	7.41E+04	1.53E+03	97.9367	47
dwfcpu0.5_14_12_5_5	974	0.297	8.27E+04	1.56E+03	98.1077	36
dwfcpu0.5_14_12_5_6	962	0.313	8.87E+04	1.71E+03	98.069	39
dwfcpu0.5_14_12_10_1	1951	0.657	7.58E+04	1.75E+03	97.6872	92
dwfcpu0.5_14_12_10_2	3073	1.046	6.16E+04	1.33E+03	97.838	153
dwfcpu0.5_14_12_10_3	4146	1.235	5.62E+04	1.63E+03	97.1031	363
dwfcpu0.5_14_12_10_4	1778	0.547	7.57E+04	1.56E+03	97.9397	85
dwfcpu0.5_14_12_10_5	2641	0.922	7.50E+04	1.56E+03	97.9192	81
dwfcpu0.5_14_12_10_6	5239	1.437	6.95E+04	1.56E+03	97.7508	665
dwfcpu0.5_14_12_25_1	12336	8.156	6.27E+04	1.50E+03	97.613	1572
dwfcpu0.5_14_12_25_2	272035	123.313	6.87E+04	1.55E+03	97.7503	68763
dwfcpu0.5_14_12_25_3	9161	4.954	6.45E+04	1.64E+03	97.4498	490
dwfcpu0.5_14_12_25_4	139023	49.579	6.15E+04	1.50E+03	97.5535	26831
dwfcpu0.5_14_12_25_5	15261	8.656	5.92E+04	1.54E+03	97.3997	2395
dwfcpu0.5_14_12_25_6	130987	77.126	6.00E+04	1.55E+03	97.411	43326
dwfcpu0.5_14_12_50_1	501100	600.019	6.08E+04	1.59E+03	97.3838	146979
dwfcpu0.5_14_12_50_2	678515	600.02	6.39E+04	1.50E+03	97.6561	143086
dwfcpu0.5_14_12_50_3	710662	600.02	5.87E+04	1.55E+03	97.3645	142632
dwfcpu0.5_14_12_50_4	548540	600.019	6.20E+04	1.55E+03	97.5032	152531
dwfcpu0.5_14_12_50_5	541274	491.425	6.68E+04	1.52E+03	97.7231	126999
dwfcpu0.5_14_12_50_6	569231	600.02	6.26E+04	1.59E+03	97.4639	157361
dwfcpu0.5_14_12_75_1	1200587	600.019	6.36E+04	1.57E+03	97.5289	75455
dwfcpu0.5_14_12_75_2	312044	600.02	6.12E+04	1.55E+03	97.4645	102464
dwfcpu0.5_14_12_75_3	389772	600.02	6.32E+04	1.55E+03	97.556	97812
dwfcpu0.5_14_12_75_4	817105	600.02	6.23E+04	1.59E+03	97.448	78728
dwfcpu0.5_14_12_75_5	1421387	600.035	6.08E+04	1.56E+03	97.4394	27938
dwfcpu0.5_14_12_75_6	1526663	600.02	6.14E+04	1.56E+03	97.4515	25264
dwfcpu0.5_14_12_100_1	1069869	600.02	6.20E+04	1.57E+03	97.465	46206

APPENDIX C

L^2 WITH BENDERS' CUTS RESULT OF WFCPU INSTANCES

(1): name of the instance

(2): number of CPLEX iteration taken

(3): CPU time taken

(4): Z_{IP}

(5): $Z_{LP(2)}$

(6): $Z_{LP(2)}$ GAP

(7): Z_{IP} GAP

Table XXXIII. Unconstrained budget (tripling resource production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
twfcpu1_7_12_5_1	132	2.687	77476.5	2282.87	97.0535	0
twfcpu1_7_12_5_2	132	2.719	70180.5	2277.48	96.7548	0
twfcpu1_7_12_5_3	132	2.735	70409	2247.25	96.8083	0
twfcpu1_7_12_5_4	132	2.672	78933	2237.08	97.1658	0
twfcpu1_7_12_5_5	132	2.687	85593.5	2277.53	97.3391	0
twfcpu1_7_12_5_6	132	2.719	93404.5	2404.27	97.426	0
twfcpu1_7_12_10_1	132	3.406	80559	2445.93	96.9638	0
twfcpu1_7_12_10_2	132	3.5	65450.7	2057.37	96.8566	0
twfcpu1_7_12_10_3	132	3.422	60817	2311.61	96.1991	0
twfcpu1_7_12_10_4	132	3.359	80521	2268.72	97.1825	0
twfcpu1_7_12_10_6	132	3.39	70509	2274.78	96.7738	0
twfcpu1_7_12_10_5	132	3.281	78301.3	2244.12	97.134	0
twfcpu1_7_12_25_1	132	5.5	66363.2	2180.49	96.7143	0
twfcpu1_7_12_25_2	132	5.407	73030.3	2188.5	97.0033	0
twfcpu1_7_12_25_3	132	6.656	68972.5	2288.56	96.6819	0
twfcpu1_7_12_25_4	132	5.531	65955.9	2188.61	96.6817	0
twfcpu1_7_12_25_5	132	5.422	61911.1	2180.37	96.4782	0
twfcpu1_7_12_25_6	132	5.516	62988.7	2237.82	96.4473	0
twfcpu1_7_12_50_1	132	8.844	64514.5	2232.95	96.5388	0
twfcpu1_7_12_50_2	132	8.672	67412.3	2182.04	96.7631	0
twfcpu1_7_12_50_3	132	8.875	62241.7	2188.56	96.4838	0
twfcpu1_7_12_50_4	132	9.219	65711.5	2232.94	96.6019	0
twfcpu1_7_12_50_5	132	9.016	70291.1	2204.29	96.8641	0
twfcpu1_7_12_50_6	132	9.015	66466.7	2228.63	96.647	0
twfcpu1_7_12_75_1	132	12.547	66992.3	2213.24	96.6963	0
twfcpu1_7_12_75_2	132	12.375	64830.9	2194.46	96.6151	0
twfcpu1_7_12_75_3	132	12.344	66854.9	2229.88	96.6646	0
twfcpu1_7_12_75_4	132	12.422	65669.2	2232.28	96.6007	0
twfcpu1_7_12_75_5	132	12.453	64679.2	2199.79	96.5989	0
twfcpu1_7_12_75_6	132	12.406	64904	2207.17	96.5993	0
twfcpu1_7_12_100_1	132	15.657	65653.4	2214.79	96.6265	0

Table XXXIV. Constrained budget (tripling resource production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
twfcpu0.5_7_12_5_1	71	0.828	8.80E+06	2282.87	99.9741	0
twfcpu0.5_7_12_5_2	71	0.859	8.81E+06	2277.48	99.9742	0
twfcpu0.5_7_12_5_3	71	0.875	1.06E+07	2247.25	99.9788	0
twfcpu0.5_7_12_5_4	71	0.875	1.03E+07	2237.08	99.9783	0
twfcpu0.5_7_12_5_5	72	0.86	2.69E+07	2277.53	99.9915	0
twfcpu0.5_7_12_5_6	72	0.89	2.15E+07	2404.27	99.9888	0
twfcpu0.5_7_12_10_1	71	1.187	1.41E+07	2445.93	99.9827	0
twfcpu0.5_7_12_10_2	71	1.172	1.46E+07	2057.37	99.9859	0
twfcpu0.5_7_12_10_3	71	1.172	4.44E+06	2311.61	99.9479	0
twfcpu0.5_7_12_10_4	72	1.172	1.86E+07	2268.72	99.9878	0
twfcpu0.5_7_12_10_6	71	1.156	9.73E+06	2274.78	99.9766	0
twfcpu0.5_7_12_10_5	72	1.157	2.21E+07	2244.12	99.9898	0
twfcpu0.5_7_12_25_1	71	2.031	1.05E+07	2180.49	99.9792	0
twfcpu0.5_7_12_25_2	71	2.032	1.38E+07	2188.5	99.9842	0
twfcpu0.5_7_12_25_3	71	2.047	9.04E+06	2288.56	99.9747	0
twfcpu0.5_7_12_25_4	71	2.062	9.02E+06	2188.61	99.9757	0
twfcpu0.5_7_12_25_5	71	2.078	1.13E+07	2180.37	99.9808	0
twfcpu0.5_7_12_25_6	71	2.047	4.59E+06	2237.82	99.9513	0
twfcpu0.5_7_12_50_1	71	3.484	1.00E+07	2232.95	99.9777	0
twfcpu0.5_7_12_50_2	71	3.469	1.31E+07	2182.04	99.9834	0
twfcpu0.5_7_12_50_3	71	3.484	9.13E+06	2188.56	99.976	0
twfcpu0.5_7_12_50_4	71	3.547	7.07E+06	2232.94	99.9684	0
twfcpu0.5_7_12_50_5	71	3.531	1.22E+07	2204.29	99.982	0
twfcpu0.5_7_12_50_6	71	3.469	1.08E+07	2228.63	99.9793	0
twfcpu0.5_7_12_75_1	71	4.906	9.84E+06	2213.24	99.9775	0
twfcpu0.5_7_12_75_2	71	4.859	9.38E+06	2194.46	99.9766	0
twfcpu0.5_7_12_75_3	71	4.86	1.03E+07	2229.88	99.9785	0
twfcpu0.5_7_12_75_4	71	4.875	9.79E+06	2232.28	99.9772	0
twfcpu0.5_7_12_75_5	71	4.953	7.67E+06	2199.79	99.9713	0
twfcpu0.5_7_12_75_6	71	4.891	8.34E+06	2207.17	99.9735	0
twfcpu0.5_7_12_100_1	71	6.359	9.40E+06	2214.79	99.9764	0

Table XXXV. Fixed NVC $20 (tripling resource production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
twfcp20u1_7_12_5_1	132	2.843	31872.8	1739.54	94.5422	0
twfcp20u1_7_12_5_2	132	2.969	29727.8	1738.31	94.1526	0
twfcp20u1_7_12_5_3	132	2.921	28637.8	1720.39	93.9926	0
twfcp20u1_7_12_5_4	132	2.781	33342.6	1718.6	94.8456	0
twfcp20u1_7_12_5_5	132	2.89	35340.7	1738.39	95.0811	0
twfcp20u1_7_12_5_6	132	2.86	36324	1799.14	95.047	0
twfcp20u1_7_12_10_1	132	3.641	33270.4	1825.41	94.5134	0
twfcp20u1_7_12_10_2	132	3.828	29342.7	1629.45	94.4468	0
twfcp20u1_7_12_10_3	132	3.719	26630.2	1745.33	93.4461	0
twfcp20u1_7_12_10_4	132	3.859	33540.2	1742.62	94.8044	0
twfcp20u1_7_12_10_6	132	3.75	30898.6	1743.79	94.3564	0
twfcp20u1_7_12_10_5	132	3.703	32671.8	1720.09	94.7352	0
twfcp20u1_7_12_25_1	132	6.187	28889.5	1690.7	94.1477	0
twfcp20u1_7_12_25_2	132	6.172	31632.9	1694.68	94.6427	0
twfcp20u1_7_12_25_3	132	6.016	29886.9	1745.4	94.16	0
twfcp20u1_7_12_25_4	132	6.235	29685.2	1697.05	94.2832	0
twfcp20u1_7_12_25_5	132	6.156	28741.8	1690.66	94.1178	0
twfcp20u1_7_12_25_6	132	6.204	29110.9	1718.76	94.0958	0
twfcp20u1_7_12_50_1	132	10.047	29380.1	1711.9	94.1733	0
twfcp20u1_7_12_50_2	132	10.046	30214.7	1688.72	94.4109	0
twfcp20u1_7_12_50_3	132	9.985	28826.5	1693.52	94.1251	0
twfcp20u1_7_12_50_4	132	10.078	29819.3	1714.26	94.2512	0
twfcp20u1_7_12_50_5	132	10.125	31002.8	1701.43	94.512	0
twfcp20u1_7_12_50_6	132	10.172	29904.9	1710.99	94.2786	0
twfcp20u1_7_12_75_1	132	14.001	30081.6	1703.6	94.3368	0
twfcp20u1_7_12_75_2	132	14.078	29499.8	1699.82	94.2378	0
twfcp20u1_7_12_75_3	132	14.063	30079.8	1713.98	94.3019	0
twfcp20u1_7_12_75_4	132	14.156	29725.8	1715.28	94.2297	0
twfcp20u1_7_12_75_5	132	14.141	29479.4	1699.33	94.2355	0
twfcp20u1_7_12_75_6	132	14.094	29604.8	1703.16	94.247	0
twfcp20u1_7_12_100_1	132	18.157	29754.6	1707.05	94.2629	0

Table XXXVI. Fixed NVC $1000 (tripling resource production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
twfcp1000u1_7_12_5_1	132	2.734	576652	8395.4	98.5441	0
twfcp1000u1_7_12_5_2	132	2.687	510442	8343.26	98.3655	0
twfcp1000u1_7_12_5_3	132	2.672	520027	8174.5	98.4281	0
twfcp1000u1_7_12_5_4	132	2.657	591825	8070.15	98.6364	0
twfcp1000u1_7_12_5_5	132	2.609	650937	8342.94	98.7183	0
twfcp1000u1_7_12_5_6	132	2.656	726213	9212.17	98.7315	0
twfcp1000u1_7_12_10_1	132	3.266	608490	9426.9	98.4508	0
twfcp1000u1_7_12_10_2	132	4.39	466824	6871.51	98.528	0
twfcp1000u1_7_12_10_3	132	3.875	423490	8682.35	97.9498	0
twfcp1000u1_7_12_10_4	132	3.344	609055	8187.41	98.6557	0
twfcp1000u1_7_12_10_6	132	5.672	514470	8248.49	98.3967	0
twfcp1000u1_7_12_10_5	132	3.25	587161	8139.52	98.6138	0
twfcp1000u1_7_12_25_1	132	7.515	476360	7690.73	98.3855	0
twfcp1000u1_7_12_25_2	132	5.375	538752	7744.05	98.5626	0
twfcp1000u1_7_12_25_3	132	13.984	498470	8399.17	98.315	0
twfcp1000u1_7_12_25_4	132	5.532	474002	7718.73	98.3716	0
twfcp1000u1_7_12_25_5	132	5.984	435065	7689.65	98.2325	0
twfcp1000u1_7_12_25_6	132	12.391	444113	8077.38	98.1812	0
twfcp1000u1_7_12_50_1	132	8.984	459776	8094.86	98.2394	0
twfcp1000u1_7_12_50_2	132	8.422	485885	7731.98	98.4087	0
twfcp1000u1_7_12_50_3	132	11.672	438163	7757.85	98.2295	0
twfcp1000u1_7_12_50_4	132	21.328	469498	8068.21	98.2815	0
twfcp1000u1_7_12_50_5	132	9.672	512285	7861.58	98.4654	0
twfcp1000u1_7_12_50_6	132	8.547	477787	8052.26	98.3147	0
twfcp1000u1_7_12_75_1	132	20.188	482237	7946.79	98.3521	0
twfcp1000u1_7_12_75_2	132	22.11	462306	7759.26	98.3216	0
twfcp1000u1_7_12_75_3	132	14.36	480575	8033.86	98.3283	0
twfcp1000u1_7_12_75_4	132	15.469	470032	8048.67	98.2876	0
twfcp1000u1_7_12_75_5	132	27	460677	7830.04	98.3003	0
twfcp1000u1_7_12_75_6	132	24.704	462020	7877.39	98.295	0
twfcp1000u1_7_12_100_1	132	28.531	469515	7926.95	98.3117	0

Table XXXVII. Unconstrained budget (quadrupling resource production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
qwfcpu1_7_12_5_1	132	2.922	49998	2582.87	94.8341	0
qwfcpu1_7_12_5_2	132	2.906	44242	2577.48	94.1741	0
qwfcpu1_7_12_5_3	132	3.047	43972	2547.25	94.2071	0
qwfcpu1_7_12_5_4	132	3	50256	2537.08	94.9517	0
qwfcpu1_7_12_5_5	132	2.906	57636	2577.52	95.5279	0
qwfcpu1_7_12_5_6	132	2.906	58860	2704.27	95.4056	0
qwfcpu1_7_12_10_1	132	3.625	54740	2745.93	94.9837	0
qwfcpu1_7_12_10_2	132	3.766	43885	2357.36	94.6283	0
qwfcpu1_7_12_10_3	132	3.594	42954	2611.6	93.92	0
qwfcpu1_7_12_10_4	132	3.656	58210	2568.71	95.5872	0
qwfcpu1_7_12_10_6	132	3.672	51962	2574.78	95.0449	0
qwfcpu1_7_12_10_5	132	3.563	54810	2544.12	95.3583	0
qwfcpu1_7_12_25_1	132	5.828	48879.2	2480.49	94.9253	0
qwfcpu1_7_12_25_2	132	5.829	55261.2	2488.5	95.4968	0
qwfcpu1_7_12_25_3	132	5.859	51870.8	2588.55	95.0096	0
qwfcpu1_7_12_25_4	132	5.969	49991.2	2488.6	95.0219	0
qwfcpu1_7_12_25_5	132	5.859	47814	2480.37	94.8125	0
qwfcpu1_7_12_25_6	132	5.719	51393.6	2537.82	95.062	0
qwfcpu1_7_12_50_1	132	9.625	52036	2532.95	95.1323	0
qwfcpu1_7_12_50_2	132	9.516	53347	2482.04	95.3474	0
qwfcpu1_7_12_50_3	132	9.734	49626	2488.55	94.9854	0
qwfcpu1_7_12_50_4	132	9.719	51059	2532.94	95.0392	0
qwfcpu1_7_12_50_5	132	9.625	53087.4	2504.29	95.2827	0
qwfcpu1_7_12_50_6	132	9.703	53235.4	2528.63	95.2501	0
qwfcpu1_7_12_75_1	132	13.485	51905.5	2513.23	95.1581	0
qwfcpu1_7_12_75_2	132	13.453	51547.4	2494.46	95.1608	0
qwfcpu1_7_12_75_3	132	13.375	51894.3	2529.88	95.1249	0
qwfcpu1_7_12_75_4	132	13.453	52567.8	2532.28	95.1828	0
qwfcpu1_7_12_75_5	132	13.61	50338.4	2499.78	95.034	0
qwfcpu1_7_12_75_6	132	13.719	51341.6	2507.17	95.1167	0
qwfcpu1_7_12_100_1	132	17.125	51641.8	2514.79	95.1303	0

Table XXXVIII. Constrained budget (quadrupling resource production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
qwfcpu0.5_7_12_5_1	71	0.906	72814	2582.87	96.4528	0
qwfcpu0.5_7_12_5_2	71	0.907	65598	2577.48	96.0708	0
qwfcpu0.5_7_12_5_3	71	0.906	66898	2547.25	96.1923	0
qwfcpu0.5_7_12_5_4	71	0.907	74088	2537.08	96.5756	0
qwfcpu0.5_7_12_5_5	71	0.922	82872	2577.52	96.8898	0
qwfcpu0.5_7_12_5_6	71	0.906	88662	2704.27	96.9499	0
qwfcpu0.5_7_12_10_1	71	1.281	75884	2745.93	96.3814	0
qwfcpu0.5_7_12_10_2	71	1.297	61849	2357.36	96.1885	0
qwfcpu0.5_7_12_10_3	71	1.235	56647	2611.6	95.3897	0
qwfcpu0.5_7_12_10_4	71	1.234	75951	2568.71	96.6179	0
qwfcpu0.5_7_12_10_6	71	1.234	69964	2574.78	96.3199	0
qwfcpu0.5_7_12_10_5	71	1.235	75058	2544.12	96.6105	0
qwfcpu0.5_7_12_25_1	71	2.219	62837.2	2480.49	96.0525	0
qwfcpu0.5_7_12_25_2	71	2.234	69039.6	2488.5	96.3956	0
qwfcpu0.5_7_12_25_3	71	2.204	64622	2588.55	95.9943	0
qwfcpu0.5_7_12_25_4	71	2.281	61746.4	2488.6	95.9696	0
qwfcpu0.5_7_12_25_5	71	2.219	59462	2480.37	95.8287	0
qwfcpu0.5_7_12_25_6	71	2.266	60279.6	2537.82	95.7899	0
qwfcpu0.5_7_12_50_1	71	3.86	61095.4	2532.95	95.8541	0
qwfcpu0.5_7_12_50_2	71	3.89	64196.6	2482.04	96.1337	0
qwfcpu0.5_7_12_50_3	71	3.844	59019.6	2488.55	95.7835	0
qwfcpu0.5_7_12_50_4	71	3.875	62308	2532.94	95.9348	0
qwfcpu0.5_7_12_50_5	71	3.906	66958.8	2504.29	96.26	0
qwfcpu0.5_7_12_50_6	71	3.844	62797.6	2528.63	95.9734	0
qwfcpu0.5_7_12_75_1	71	5.453	63844.3	2513.23	96.0635	0
qwfcpu0.5_7_12_75_2	71	5.438	61498.3	2494.46	95.9439	0
qwfcpu0.5_7_12_75_3	71	5.422	63467.9	2529.88	96.0139	0
qwfcpu0.5_7_12_75_4	71	5.437	62586.1	2532.28	95.9539	0
qwfcpu0.5_7_12_75_5	71	5.516	61075.3	2499.78	95.907	0
qwfcpu0.5_7_12_75_6	71	5.468	61637.5	2507.17	95.9324	0
qwfcpu0.5_7_12_100_1	71	7.125	62285.7	2514.79	95.9625	0

Table XXXIX. Unconstrained budget (doubling 14 resources production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
dwfcpu1_14_12_5_1	1220	600.894	5830.65	1982.87	65.9922	99.9742
dwfcpu1_14_12_5_2	1219	600.368	5825.3	1977.48	66.0535	99.972
dwfcpu1_14_12_5_3	1218	600.269	5798.28	1947.25	66.4167	99.9766
dwfcpu1_14_12_5_4	1229	600.035	5788.05	1937.08	66.533	99.9827
dwfcpu1_14_12_5_5	1229	600.942	5825.33	1977.53	66.0529	99.9867
dwfcpu1_14_12_5_6	1232	600.926	5942.5	2104.28	64.5894	99.9866
dwfcpu1_14_12_10_1	1221	600.129	5979.31	2145.93	64.1107	99.9839
dwfcpu1_14_12_10_2	1235	601.223	5622.72	1757.37	68.7452	99.9786
dwfcpu1_14_12_10_3	1221	600.739	5859.37	2011.61	65.6685	99.9541
dwfcpu1_14_12_10_4	1221	600.956	5814.9	1968.72	66.1435	99.9848
dwfcpu1_14_12_10_6	1222	601.191	5820.97	1974.78	66.0747	99.974
dwfcpu1_14_12_10_5	1219	600.535	5795.06	1944.12	66.4521	99.9868
dwfcpu1_14_12_25_1	1199	600.488	5680.5	1880.49	66.8956	99.9743
dwfcpu1_14_12_25_2	1206	600.551	5688.51	1888.5	66.8015	99.9787
dwfcpu1_14_12_25_3	1207	600.645	5788.57	1988.56	65.6468	99.9754
dwfcpu1_14_12_25_4	1204	600.645	5688.61	1888.61	66.8002	99.9704
dwfcpu1_14_12_25_5	1198	600.051	5680.38	1880.37	66.8971	99.9743
dwfcpu1_14_12_25_6	1209	600.426	5737.83	1937.82	66.2272	99.9615
dwfcpu1_14_12_50_1	1183	600.317	5732.96	1932.95	66.2835	99.9727
dwfcpu1_14_12_50_2	1179	600.254	5682.05	1882.04	66.8774	99.9778
dwfcpu1_14_12_50_3	1183	600.645	5688.56	1888.56	66.8008	99.9714
dwfcpu1_14_12_50_4	1181	601.051	5732.95	1932.94	66.2836	99.9701
dwfcpu1_14_12_50_5	1184	600.691	5704.3	1904.3	66.6165	99.9772
dwfcpu1_14_12_50_6	1183	600.114	5728.64	1928.64	66.3334	99.9734
dwfcpu1_14_12_75_1	1160	600.535	5713.24	1913.24	66.5122	99.9736
dwfcpu1_14_12_75_2	1149	600.207	5548.2	1894.47	65.8544	99.9737
dwfcpu1_14_12_75_3	1145	601.144	5581.71	1929.88	65.4249	99.9745
dwfcpu1_14_12_75_4	1147	600.879	5583.89	1932.29	65.3953	99.9737
dwfcpu1_14_12_75_5	1149	600.597	5553.95	1899.79	65.7939	99.9706
dwfcpu1_14_12_75_6	1147	600.785	5560.69	1907.17	65.7026	99.972
dwfcpu1_14_12_100_1	1120	600.176	5567.67	1914.79	65.6087	99.9735

Table XL. Constrained budget (doubling 14 resources production rate)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
dwfcpu0.5_14_12_5_1	1213	600.442	5830.65	1982.87	65.9922	99.9742
dwfcpu0.5_14_12_5_2	1210	600.394	5777.49	1977.48	65.7726	99.9722
dwfcpu0.5_14_12_5_3	1212	600.02	5798.28	1947.25	66.4167	99.9766
dwfcpu0.5_14_12_5_4	1216	601.223	5788.05	1937.08	66.533	99.9827
dwfcpu0.5_14_12_5_5	1216	600.66	5825.33	1977.53	66.0529	99.9867
dwfcpu0.5_14_12_5_6	1221	601.176	5942.5	2104.28	64.5894	99.9866
dwfcpu0.5_14_12_10_1	1211	601.222	5979.31	2145.93	64.1107	99.9839
dwfcpu0.5_14_12_10_2	1226	601.191	5622.72	1757.37	68.7452	99.9786
dwfcpu0.5_14_12_10_3	1205	600.895	5811.61	2011.61	65.3864	99.9545
dwfcpu0.5_14_12_10_4	1202	600.269	5768.72	1968.72	65.8725	99.9849
dwfcpu0.5_14_12_10_6	1202	600.566	5774.79	1974.78	65.8034	99.9742
dwfcpu0.5_14_12_10_5	1209	600.238	5744.13	1944.12	66.1546	99.9869
dwfcpu0.5_14_12_25_1	1190	600.644	5680.5	1880.49	66.8956	99.9743
dwfcpu0.5_14_12_25_2	1197	601.067	5688.51	1888.5	66.8015	99.9787
dwfcpu0.5_14_12_25_3	1194	600.004	5788.57	1988.56	65.6468	99.9754
dwfcpu0.5_14_12_25_4	1193	600.676	5688.61	1888.61	66.8002	99.9704
dwfcpu0.5_14_12_25_5	1189	600.175	5680.38	1880.37	66.8971	99.9743
dwfcpu0.5_14_12_25_6	1199	600.472	5737.83	1937.82	66.2272	99.9615
dwfcpu0.5_14_12_50_1	1172	600.504	5732.96	1932.95	66.2835	99.9727
dwfcpu0.5_14_12_50_2	1166	600.722	5682.05	1882.04	66.8774	99.9778
dwfcpu0.5_14_12_50_3	1167	600.238	5688.56	1888.56	66.8008	99.9714
dwfcpu0.5_14_12_50_4	1174	600.816	5732.95	1932.94	66.2836	99.9701
dwfcpu0.5_14_12_50_5	1163	600.848	5704.3	1904.3	66.6165	99.9772
dwfcpu0.5_14_12_50_6	1180	600.621	5728.64	1928.64	66.3334	99.9734
dwfcpu0.5_14_12_75_1	1157	600.006	5713.24	1913.24	66.5122	99.9736
dwfcpu0.5_14_12_75_2	1159	600.532	5694.47	1894.47	66.7315	99.9731
dwfcpu0.5_14_12_75_3	1160	600.503	5729.89	1929.88	66.319	99.9738
dwfcpu0.5_14_12_75_4	1158	600.551	5732.29	1932.29	66.2912	99.973
dwfcpu0.5_14_12_75_5	1158	601.191	5699.8	1899.79	66.6692	99.9699
dwfcpu0.5_14_12_75_6	1159	600.519	5707.18	1907.17	66.5829	99.9713
dwfcpu0.5_14_12_100_1	1131	600.863	5567.67	1914.79	65.6087	99.9735

APPENDIX D

100 SMALL-SCALE FIRE SCENARIOS

Table XLI. wfcp 100 scenarios

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
1	1	0.4	1.0	2	1	0.4	1.0	3	1	0.3	0.7
	2	1.3	7.3		2	1.3	7.2		2	1.2	6.6
	3	1.6	11.6		3	1.8	13.0		3	1.6	11.5
	4	2.2	19.7		4	2.2	19.3		4	2.0	18.1
	5	2.5	25.4		5	2.4	24.8		5	2.3	22.9
	6	2.8	30.5		6	2.7	30.0		6	2.4	26.7
4	1	0.3	0.8	5	1	0.5	1.1	6	1	0.4	1.0
	2	1.1	6.3		2	1.2	6.9		2	1.1	6.3
	3	1.3	9.8		3	1.7	12.3		3	1.5	10.8
	4	2.0	17.4		4	2.0	17.7		4	2.1	18.8
	5	2.2	22.2		5	2.2	22.1		5	2.3	23.4
	6	2.4	26.4		6	2.4	27.0		6	2.6	28.4
7	1	0.5	1.1	8	1	0.4	0.9	9	1	0.4	1.0
	2	1.3	7.1		2	1.2	6.8		2	1.0	5.9
	3	1.7	12.8		3	1.5	10.9		3	1.5	11.1
	4	2.1	18.2		4	2.0	18.1		4	2.2	19.1
	5	2.2	22.8		5	2.3	23.6		5	2.4	23.9
	6	2.5	27.7		6	2.5	27.5		6	2.5	27.4
10	1	0.5	1.2	11	1	0.1	0.3	12	1	0.3	0.6
	2	1.1	6.3		2	1.0	5.7		2	1.0	5.6
	3	1.5	11.1		3	1.3	9.3		3	1.3	9.5
	4	2.1	18.1		4	1.6	14.2		4	1.6	14.3
	5	2.2	22.0		5	1.8	18.0		5	1.7	17.5
	6	2.3	25.5		6	2.1	22.9		6	1.8	20.2
13	1	0.5	1.2	14	1	0.4	1.0	15	1	0.1	0.2
	2	1.3	7.1		2	0.9	5.2		2	0.9	4.8
	3	1.5	11.3		3	1.3	9.4		3	1.3	9.5
	4	2.1	18.3		4	1.9	17.0		4	1.7	15.3
	5	2.3	23.2		5	2.2	22.3		5	2.0	20.3
	6	2.5	27.8		6	2.4	26.0		6	2.3	24.9
16	1	0.5	1.1	17	1	0.4	0.8	18	1	0.3	0.6
	2	1.0	5.6		2	0.9	5.3		2	1.0	5.7
	3	1.4	10.7		3	1.2	8.7		3	1.3	9.8

Table XLI. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	4	1.7	15.5		4	1.6	14.1		4	1.8	15.9
	5	2.0	20.1		5	1.8	17.9		5	1.9	19.6
	6	2.2	24.3		6	2.1	22.7		6	2.1	22.9
19	1	0.3	0.8	20	1	0.4	0.8	21	1	0.2	0.5
	2	0.9	4.9		2	1.2	6.9		2	0.8	4.5
	3	1.2	9.2		3	1.5	11.2		3	1.1	8.3
	4	1.7	15.3		4	2.0	17.9		4	1.7	14.7
	5	1.9	19.5		5	2.2	21.8		5	1.8	18.2
	6	2.2	24.1		6	2.4	26.9		6	2.0	21.8
22	1	0.4	0.9	23	1	0.2	0.6	24	1	0.3	0.8
	2	1.0	5.8		2	1.0	5.5		2	1.1	6.4
	3	1.5	11.3		3	1.3	9.4		3	1.3	9.9
	4	2.2	19.6		4	1.6	14.0		4	1.7	15.2
	5	2.5	25.0		5	1.8	18.0		5	1.9	19.8
	6	2.6	29.3		6	2.0	22.1		6	2.2	24.2
25	1	0.3	0.6	26	1	0.5	1.2	27	1	0.3	0.7
	2	1.0	5.5		2	1.1	6.1		2	0.9	5.2
	3	1.1	8.2		3	1.3	9.2		3	1.3	9.7
	4	1.7	15.1		4	1.7	15.0		4	1.8	15.9
	5	1.9	19.1		5	2.0	19.9		5	2.1	21.0
	6	2.0	22.2		6	2.2	24.7		6	2.2	24.8
28	1	0.2	0.4	29	1	0.4	0.8	30	1	0.2	0.4
	2	1.0	5.4		2	1.1	6.1		2	0.7	3.9
	3	1.2	8.6		3	1.3	9.5		3	1.0	7.3
	4	1.8	16.2		4	1.9	17.0		4	1.5	13.4
	5	2.1	21.2		5	2.1	21.4		5	1.7	17.1
	6	2.3	24.9		6	2.3	26.0		6	1.8	20.4
31	1	0.2	0.4	32	1	0.4	0.8	33	1	0.1	0.3
	2	0.7	4.0		2	0.9	5.2		2	0.9	5.0
	3	1.0	7.5		3	1.3	9.4		3	1.0	7.4
	4	1.3	11.8		4	1.8	16.3		4	1.5	12.9
	5	1.4	14.6		5	2.0	20.4		5	1.6	16.0
	6	1.6	18.2		6	2.3	25.2		6	1.9	20.6
34	1	0.3	0.8	35	1	0.4	0.9	36	1	0.4	0.9
	2	0.9	4.9		2	1.2	7.0		2	1.2	6.6
	3	1.2	9.0		3	1.4	10.0		3	1.6	11.5
	4	1.8	15.8		4	1.7	15.0		4	2.2	19.8
	5	2.1	21.1		5	1.8	18.7		5	2.5	24.9
	6	2.2	24.6		6	1.9	21.5		6	2.7	30.0

Table XLI. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
37	1	0.5	1.1	38	1	0.4	0.9	39	1	0.2	0.5
	2	1.0	5.7		2	1.0	5.6		2	0.8	4.4
	3	1.2	8.8		3	1.1	8.4		3	1.1	8.2
	4	1.8	15.7		4	1.8	15.7		4	1.4	12.7
	5	1.9	19.6		5	1.9	19.7		5	1.7	17.4
	6	2.0	22.6		6	2.1	23.2		6	1.9	20.7
40	1	0.1	0.3	41	1	0.4	0.9	42	1	0.4	0.9
	2	0.9	5.0		2	1.1	6.0		2	1.0	5.8
	3	1.1	8.4		3	1.2	8.7		3	1.4	10.2
	4	1.6	14.2		4	1.5	13.7		4	1.9	16.3
	5	1.8	18.4		5	1.7	17.1		5	2.0	20.5
	6	2.0	22.0		6	1.8	19.9		6	2.2	24.6
43	1	0.4	0.9	44	1	0.4	0.8	45	1	0.1	0.3
	2	1.3	7.2		2	0.9	5.2		2	0.7	4.2
	3	1.7	12.6		3	1.2	8.9		3	0.9	6.5
	4	2.1	18.5		4	1.6	14.3		4	1.4	12.0
	5	2.3	22.9		5	1.8	18.6		5	1.6	16.4
	6	2.5	27.1		6	2.1	23.2		6	1.7	19.1
46	1	0.2	0.5	47	1	0.1	0.3	48	1	0.4	0.9
	2	0.9	4.8		2	0.7	3.8		2	1.1	6.0
	3	1.2	9.0		3	0.9	6.9		3	1.6	11.7
	4	1.7	15.1		4	1.5	13.6		4	2.1	18.8
	5	1.9	19.6		5	1.7	16.8		5	2.3	23.4
	6	2.2	24.6		6	1.8	20.2		6	2.5	27.2
49	1	0.2	0.4	50	1	0.2	0.4	51	1	0.2	0.4
	2	1.0	5.7		2	0.7	3.7		2	0.9	4.9
	3	1.2	8.9		3	1.1	7.9		3	1.3	10.0
	4	1.8	15.8		4	1.7	15.2		4	1.8	16.0
	5	2.0	20.7		5	1.9	19.6		5	1.9	19.6
	6	2.2	24.6		6	2.1	23.2		6	2.2	23.9
52	1	0.2	0.5	53	1	0.4	0.9	54	1	0.3	0.7
	2	0.8	4.6		2	1.0	5.6		2	0.9	5.1
	3	1.0	7.2		3	1.5	10.9		3	1.3	9.6
	4	1.5	13.3		4	1.9	16.7		4	1.8	16.0
	5	1.6	16.4		5	2.1	21.7		5	2.0	20.6
	6	1.8	19.9		6	2.3	25.7		6	2.2	23.9
55	1	0.2	0.6	56	1	0.3	0.8	57	1	0.3	0.8
	2	0.8	4.5		2	1.1	5.9		2	1.2	6.8
	3	1.2	8.8		3	1.5	11.2		3	1.5	11.0

Table XLI. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	4	1.7	14.9		4	2.0	17.8		4	2.1	18.6
	5	1.9	18.9		5	2.3	23.1		5	2.4	24.0
	6	2.1	22.7		6	2.5	28.0		6	2.5	27.4
58	1	0.3	0.7	59	1	0.2	0.6	60	1	0.2	0.4
	2	1.0	5.6		2	0.9	5.1		2	0.9	5.3
	3	1.4	10.1		3	1.0	7.6		3	1.4	10.2
	4	2.0	17.6		4	1.4	12.4		4	1.7	15.3
	5	2.2	21.9		5	1.5	15.6		5	1.9	19.4
	6	2.4	26.4		6	1.7	18.3		6	2.0	22.3
61	1	0.2	0.4	62	1	0.2	0.4	63	1	0.5	1.1
	2	0.8	4.5		2	0.9	5.1		2	1.2	6.5
	3	1.0	7.1		3	1.1	8.5		3	1.6	12.2
	4	1.6	13.7		4	1.5	13.0		4	2.1	18.7
	5	1.8	18.7		5	1.7	17.1		5	2.3	23.2
	6	2.1	23.1		6	1.9	21.4		6	2.4	27.0
64	1	0.4	0.9	65	1	0.3	0.7	66	1	0.5	1.1
	2	1.1	5.9		2	1.1	6.1		2	1.3	7.2
	3	1.4	10.3		3	1.4	10.1		3	1.8	13.2
	4	2.0	17.5		4	1.9	16.5		4	2.3	20.6
	5	2.1	21.4		5	2.2	21.9		5	2.5	25.2
	6	2.4	26.3		6	2.4	26.7		6	2.8	30.6
67	1	0.3	0.8	68	1	0.5	1.1	69	1	0.5	1.1
	2	1.0	5.4		2	1.0	5.9		2	1.0	5.7
	3	1.4	10.7		3	1.3	9.4		3	1.3	9.8
	4	1.9	17.0		4	1.8	16.1		4	1.8	16.2
	5	2.1	21.1		5	2.1	21.0		5	2.1	21.0
	6	2.3	25.1		6	2.4	26.1		6	2.3	25.5
70	1	0.5	1.1	71	1	0.3	0.6	72	1	0.4	0.9
	2	1.1	6.0		2	1.1	6.2		2	1.3	7.1
	3	1.4	10.1		3	1.4	10.0		3	1.7	12.6
	4	2.0	17.3		4	1.8	16.1		4	2.1	18.8
	5	2.1	21.5		5	2.1	21.3		5	2.4	23.9
	6	2.4	26.0		6	2.2	24.4		6	2.5	28.0
73	1	0.3	0.7	74	1	0.5	1.1	75	1	0.2	0.5
	2	1.2	6.5		2	1.1	6.4		2	1.0	5.3
	3	1.5	10.8		3	1.3	9.9		3	1.2	8.7
	4	2.1	18.6		4	1.8	15.7		4	1.7	14.8
	5	2.3	23.4		5	1.9	19.7		5	1.8	18.2
	6	2.5	28.1		6	2.2	24.0		6	1.9	21.4

Table XLI. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
76	1	0.4	0.9	77	1	0.3	0.6	78	1	0.5	1.1
	2	1.3	7.1		2	1.2	6.5		2	1.4	7.7
	3	1.5	10.8		3	1.6	11.6		3	1.6	11.9
	4	2.1	18.5		4	2.0	17.6		4	2.2	19.5
	5	2.3	22.8		5	2.2	22.5		5	2.3	23.6
	6	2.4	26.9		6	2.4	26.1		6	2.5	27.1
79	1	0.2	0.4	80	1	0.3	0.7	81	1	0.2	0.4
	2	0.7	3.9		2	1.1	6.1		2	0.7	4.1
	3	0.8	6.0		3	1.6	11.6		3	1.2	8.8
	4	1.3	11.7		4	1.9	16.5		4	1.6	13.8
	5	1.6	15.9		5	2.0	20.3		5	1.8	18.3
	6	1.7	18.5		6	2.1	23.4		6	2.0	22.0
82	1	0.2	0.4	83	1	0.1	0.3	84	1	0.5	1.2
	2	1.0	5.7		2	1.0	5.4		2	1.0	5.9
	3	1.4	10.2		3	1.4	10.6		3	1.2	8.9
	4	1.9	17.1		4	1.8	15.6		4	1.5	13.6
	5	2.2	22.3		5	1.9	19.2		5	1.7	17.4
	6	2.5	27.3		6	2.1	23.1		6	1.9	21.2
85	1	0.3	0.8	86	1	0.2	0.5	87	1	0.1	0.3
	2	1.1	6.2		2	0.8	4.7		2	1.0	5.5
	3	1.6	11.6		3	1.2	8.5		3	1.2	9.1
	4	1.9	17.1		4	1.5	13.0		4	1.6	13.8
	5	2.2	22.5		5	1.7	17.4		5	1.8	18.6
	6	2.5	27.8		6	1.9	21.4		6	2.0	21.6
88	1	0.1	0.3	89	1	0.4	1.0	90	1	0.2	0.4
	2	0.8	4.4		2	1.1	6.3		2	1.1	5.9
	3	1.2	8.5		3	1.3	9.7		3	1.2	8.8
	4	1.7	14.6		4	1.6	14.3		4	1.9	16.7
	5	1.9	19.6		5	1.8	18.3		5	2.1	20.8
	6	2.1	23.6		6	2.0	22.2		6	2.2	23.8
91	1	0.4	1.0	92	1	0.3	0.7	93	1	0.4	0.9
	2	1.1	6.0		2	1.2	6.6		2	1.1	6.1
	3	1.2	9.1		3	1.3	9.8		3	1.5	10.7
	4	1.7	15.2		4	1.8	16.3		4	1.9	16.9
	5	2.0	20.3		5	2.1	21.2		5	2.1	21.6
	6	2.1	23.6		6	2.2	24.4		6	2.3	25.6
94	1	0.1	0.2	95	1	0.4	0.9	96	1	0.2	0.5
	2	0.7	4.1		2	1.1	6.3		2	1.1	6.0
	3	1.2	9.0		3	1.3	9.9		3	1.3	9.7

Table XLI. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	4	1.5	13.6		4	1.7	14.9		4	1.9	17.2
	5	1.7	17.2		5	1.8	18.7		5	2.1	20.9
	6	2.0	21.9		6	2.1	23.6		6	2.3	25.8
97	1	0.4	1.0	98	1	0.2	0.5	99	1	0.4	0.9
	2	1.0	5.4		2	0.8	4.3		2	1.3	7.1
	3	1.4	10.1		3	1.2	9.2		3	1.4	10.6
	4	1.8	15.7		4	1.6	13.8		4	1.8	15.6
	5	1.9	19.2		5	1.7	17.7		5	1.9	19.7
	6	2.1	23.1		6	2.0	22.2		6	2.2	23.8
100	1	0.5	1.1								
	2	1.0	5.5								
	3	1.4	10.5								
	4	1.8	15.6								
	5	1.9	19.1								
	6	2.1	23.2								

APPENDIX E

100 LARGE-SCALE FIRE SCENARIOS

Table XLII. wfcpu 100 scenarios

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
1	0.5	1.1	9.3	2	0.5	1.4	12.7	3	0.5	0.9	5.9
	1	2.3	34.5		1	2.9	49.1		1	1.9	21.6
	1.5	3	58.3		1.5	3.8	87.8		1.5	3	52.3
	2	3.7	88.8		2	4.8	141.7		2	4.4	111.7
	2.5	5.2	151.7		2.5	7.3	292.7		2.5	5.4	150.3
	3	6.7	232.9		3	11.1	460.9		3	6.3	195.5
	3.5	7.7	308		3.5	11.6	551.4		3.5	7.5	271.6
	4	8.3	375.4		4	12.3	629.1		4	8.2	355.7
	4.5	10.1	481.6		4.5	13.9	727.7		4.5	12.5	625
	5	11.9	592.8		5	14.8	833.1		5	15.4	898.5
	5.5	13	656.9		5.5	16.9	990.5		5.5	15.9	935
	6	13.9	727.1		6	19.3	1170.7		6	16.4	970.7
4	0.5	1.1	8.5	5	0.5	1.2	10.9	6	0.5	0.9	5.8
	1	2.3	38.8		1	2.6	49.9		1	2	28.8
	1.5	3.7	99.6		1.5	4.1	104.1		1.5	2.8	49.4
	2	5.4	198.5		2	5.5	182.7		2	3.7	77.3
	2.5	7.1	311.5		2.5	7.3	252.7		2.5	4.3	113.9
	3	9.3	430.3		3	9.1	336.8		3	5.4	158.2
	3.5	12.5	563.4		3.5	9.4	448		3.5	7.8	317.3
	4	13.1	716.9		4	11.4	544		4	10.7	484.3
	4.5	14.3	847.3		4.5	12.1	593.5		4.5	12.8	619.9
	5	15.2	973.8		5	13.7	655.5		5	14.5	787.8
	5.5	17.1	1108.5		5.5	14.8	753.5		5.5	15	907.5
	6	19.5	1255.2		6	15.9	857.2		6	16.5	1022.2
7	0.5	0.6	3	8	0.5	0.7	4	9	0.5	0.8	4.9
	1	1.1	9		1	1.6	14.6		1	1.6	16.8
	1.5	2.4	31.1		1.5	2.5	38.6		1.5	2.6	43.3
	2	3.5	67.8		2	3.6	80.2		2	3.7	86
	2.5	5.1	128.3		2.5	4.4	104.6		2.5	5.5	178.5
	3	6.5	203.8		3	5.4	139.6		3	7.5	309.7
	3.5	7.8	267.5		3.5	6.7	206.4		3.5	9.4	414.6
	4	8.2	328.2		4	7.8	270.1		4	10.5	511.3
	4.5	10	489.7		4.5	8.7	342.6		4.5	12.4	633
	5	13.3	648.9		5	9.8	417.3		5	13.4	776.6
	5.5	14.6	734		5.5	11.2	530.7		5.5	14.9	902.5

Table XLII. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	6	15.4	823.2		6	13.3	637.5		6	16	1036.9
10	0.5	0.7	3.9	11	0.5	1.2	16.7	12	0.5	0.9	11.2
	1	1.4	14.5		1	2.6	71.1		1	1.8	50.2
	1.5	2.5	38.4		1.5	3.9	175.6		1.5	3.4	99.3
	2	3.5	73.6		2	5.1	322.7		2	5.3	173.1
	2.5	6.4	246.7		2.5	7.2	405		2.5	6.9	308.2
	3	10.7	471.7		3	10.3	488.7		3	8.6	443.4
	3.5	12.9	622.8		3.5	14.3	643.9		3.5	10.8	698.2
	4	14.1	778.8		4	17.6	819		4	12.3	1019.8
	4.5	14.3	842.9		4.5	18	924.1		4.5	13.6	1137.2
	5	14.6	900.1		5	19	1024.4		5	13.8	1232.8
	5.5	16.2	1052.6		5.5	20.6	1254		5.5	14.2	1337.2
	6	19.5	1230.6		6	22.5	1474.3		6	14.9	1442.6
13	0.5	1.1	5.5	14	0.5	0.8	8.7	15	0.5	1.5	5.4
	1	2.3	23.1		1	1.8	39		1	3.4	22.8
	1.5	3.9	76.4		1.5	2.6	91.7		1.5	5.2	41.1
	2	5.6	175.7		2	3.1	168		2	7.2	61.9
	2.5	7.3	267.1		2.5	4.3	246.1		2.5	8.8	102.2
	3	8	360.7		3	5.8	329.1		3	11.1	167
	3.5	8.4	490.3		3.5	7.4	379.6		3.5	12.9	289.2
	4	9.4	631.5		4	9.2	429.4		4	13.5	425.1
	4.5	10.8	714.4		4.5	9.8	518.8		4.5	14.1	489.4
	5	13.3	796.9		5	11.2	623.3		5	14.9	557.4
	5.5	14.1	835.2		5.5	12.8	737.1		5.5	19	707.3
	6	16	879.2		6	15.6	854.7		6	22.4	888.5
16	0.5	0.8	4.4	17	0.5	1.1	9.3	18	0.5	0.7	3.5
	1	1.7	21.9		1	2.3	38		1	1.3	12
	1.5	2.9	56		1.5	3	60.4		1.5	3.5	84.9
	2	4.1	105		2	3.7	87.2		2	6.3	234.5
	2.5	5.7	180.4		2.5	5.4	159.5		2.5	8.4	328.5
	3	7.5	269.1		3	7.2	251.6		3	9.7	459.9
	3.5	9	425.7		3.5	8.1	325.7		3.5	11.3	560.6
	4	11.5	575.6		4	8.9	390.3		4	12	658.5
	4.5	13.4	702.2		4.5	10.3	478.5		4.5	13.6	783.6
	5	14.9	847.5		5	12	564		5	15.4	913.6
	5.5	15.7	1002.7		5.5	13.2	674.1		5.5	16.7	1043.3
	6	18.4	1167		6	15.6	803.9		6	17.2	1147.1
19	0.5	1.9	24.8	20	0.5	0.6	2.9	21	0.5	0.7	3.1
	1	4.1	120.1		1	1.1	8.7		1	1.3	11.8
	1.5	5.6	205.5		1.5	1.8	21.1		1.5	2.8	50.5

Table XLII. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	2	7.1	303		2	2.7	40		2	4.6	131.2
	2.5	8.9	431.1		2.5	5	117.9		2.5	6.8	226.6
	3	11.5	552.5		3	7.3	239.9		3	8.8	324.7
	3.5	13.4	698.8		3.5	9.5	366.4		3.5	10.4	450
	4	14.4	847.9		4	12.1	514.8		4	12.4	611.4
	4.5	17.3	1109.8		4.5	12.8	603.5		4.5	14	749.9
	5	20.5	1376		5	13.8	688.7		5	16.3	894.6
	5.5	21.6	1476.8		5.5	16.6	871.5		5.5	16.9	953.9
	6	22.6	1575.9		6	19.4	1086.4		6	17	1017.7
22	0.5	0.7	3.7	23	0.5	1.4	12.8	24	0.5	1.1	9.3
	1	1.6	17.5		1	2.9	49.1		1	2.3	35.8
	1.5	3.1	57.1		1.5	3.8	94.1		1.5	3.3	71.5
	2	4.9	125.2		2	5	157.4		2	4.3	124
	2.5	7.1	230.3		2.5	6.9	264.7		2.5	6.6	283.3
	3	8.6	342		3	9.1	393.2		3	10.5	464.9
	3.5	9.7	445		3.5	10.8	505.2		3.5	11.9	524.2
	4	12	558.9		4	12.5	631.9		4	12.1	585.5
	4.5	13.6	728.1		4.5	14.6	826		4.5	12.3	686.5
	5	16	906.6		5	16.2	989.8		5	14	793.9
	5.5	17.5	1077.1		5.5	17.7	1060.7		5.5	14.7	920.8
	6	19.9	1251.9		6	18.7	1134.9		6	15.9	1055.8
25	0.5	1.1	9	26	0.5	2.1	30.2	27	0.5	0.9	5.8
	1	2.3	38.8		1	4.6	142.9		1	2	28.5
	1.5	4.3	101.8		1.5	7.4	299.2		1.5	4.9	172.9
	2	6.3	179.9		2	11.1	488.9		2	9.2	410.2
	2.5	8.3	337.6		2.5	11.8	575.4		2.5	10.8	577.2
	3	10.7	502		3	13.1	671.2		3	12.5	749.2
	3.5	11.7	586.4		3.5	13.5	752		3.5	14.3	852.8
	4	13.1	679.5		4	13.9	820.9		4	15.4	969
	4.5	14.3	860.7		4.5	14.6	898		4.5	16.2	1119.3
	5	16.5	1036.5		5	15.2	976.5		5	18.1	1230.3
	5.5	17.9	1119.7		5.5	17.5	1131.1		5.5	19	1314.8
	6	19.3	1201.4		6	19.7	1284.3		6	20.1	1381.5
28	0.5	1	7.5	29	0.5	0.7	3.2	30	0.5	0.7	3.3
	1	2	26.1		1	1.2	10		1	1.3	12
	1.5	3.2	68.2		1.5	2.5	41.7		1.5	3	59.3
	2	4.5	137.3		2	4.1	106.9		2	5.1	162.7
	2.5	6	220.8		2.5	5.6	194.2		2.5	6.3	242.8
	3	7.9	328.3		3	7.5	306.2		3	8.2	337.7
	3.5	9.9	452.3		3.5	9	392		3.5	9.8	419.6

Table XLII. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	4	11.9	575.3		4	10.4	466.9		4	10.5	493.9
	4.5	12.9	757.4		4.5	11.9	571.3		4.5	11.5	581.1
	5	14.9	945.4		5	14.4	697.1		5	13.4	678.8
	5.5	15.8	1047.2		5.5	15.4	821.7		5.5	14.8	823.3
	6	17.5	1153		6	15.4	945.9		6	15.6	970.9
31	0.5	0.8	4.7	32	0.5	1.4	12.8	33	0.5	0.8	4.2
	1	1.6	17.9		1	3.2	64.1		1	1.5	14.7
	1.5	3	60.5		1.5	4.5	123.1		1.5	2.6	43.6
	2	4.6	137.8		2	5.9	194.1		2	3.9	97.6
	2.5	5.4	176.3		2.5	7.2	256.7		2.5	5	158.4
	3	6.7	224.7		3	8.1	326.4		3	6.6	241.3
	3.5	8.2	338.8		3.5	8.9	402.4		3.5	8.3	327.5
	4	9.7	457.6		4	10.1	469.6		4	9.2	408.8
	4.5	10.2	521.4		4.5	12.3	553.7		4.5	12	583.9
	5	11.2	583.6		5	13	651		5	14.5	835.9
	5.5	12.7	662.1		5.5	14.9	783.3		5.5	14.7	899.4
	6	14.2	748.9		6	16.5	940.2		6	15.3	961.6
34	0.5	0.7	3.5	35	0.5	0.9	5.4	36	0.5	1.2	10.7
	1	1.4	12.6		1	1.8	24.1		1	2.8	55.2
	1.5	2.1	26.1		1.5	2.5	44.4		1.5	3.6	85.3
	2	2.8	47.3		2	3.2	69.7		2	4.4	125.1
	2.5	4.6	110.6		2.5	4.6	141		2.5	5.9	188.6
	3	6.3	213		3	6	243.2		3	7.3	263.7
	3.5	7.8	317.1		3.5	7.2	324.3		3.5	8.8	386.6
	4	9.4	423.4		4	8.3	404.4		4	10	496.7
	4.5	10.8	530.6		4.5	9.5	492.6		4.5	11.8	594.9
	5	12.9	642.9		5	10.3	583.6		5	13.7	709.4
	5.5	13.7	716.2		5.5	12	765.7		5.5	14	801
	6	14.4	797.3		6	13.5	984.7		6	14.5	889.1
37	0.5	1.1	8.9	38	0.5	0.8	4.3	39	0.5	0.9	4.8
	1	2.7	51.8		1	1.6	17.6		1	1.7	16.4
	1.5	4.7	157.8		1.5	2.8	53.8		1.5	2.2	30.3
	2	8.1	294		2	4.2	116.5		2	2.7	48.1
	2.5	8.4	404		2.5	5.8	217.7		2.5	4.4	103.2
	3	9.5	520		3	7.7	320.7		3	6.3	187.9
	3.5	11.5	614.4		3.5	9.6	403		3.5	7.4	291.6
	4	13	715.4		4	10.7	494.9		4	8.5	400.7
	4.5	13.3	858.6		4.5	12.6	666.7		4.5	9.5	447.9
	5	14.9	981		5	14.8	853.2		5	10.3	490.3
	5.5	17.1	1078.2		5.5	15.7	978.4		5.5	11.8	581.8

Table XLII. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	6	18.6	1188.7		6	17.2	1099.3		6	13.9	691.1
40	0.5	0.9	5.7	41	0.5	0.8	4	42	0.5	1	6.9
	1	1.8	19.2		1	1.5	14.5		1	2.1	27.8
	1.5	3.4	67.7		1.5	2.6	44.1		1.5	2.8	53.6
	2	5.4	176.1		2	3.9	96.5		2	3.6	89.6
	2.5	6.9	266		2.5	5.6	193.4		2.5	5.3	177.6
	3	8.7	365.4		3	7.7	320.1		3	7.4	299.3
	3.5	10.3	456.2		3.5	10.7	466.5		3.5	9.1	399.1
	4	11.1	542.6		4	13.3	639.4		4	10.6	499
	4.5	13.2	638		4.5	13.4	762.7		4.5	13.8	672
	5	14.2	739.7		5	13.9	874.6		5	15.3	882.9
	5.5	14.3	838		5.5	14.6	951.8		5.5	16.9	1014
	6	15.8	938		6	16.1	1029.1		6	18.6	1155.1
43	0.5	0.9	5.1	44	0.5	1.5	11.1	45	0.5	0.8	4
	1	1.8	23.5		1	2.9	41.5		1	1.5	14.3
	1.5	2.7	49.3		1.5	4.3	124.9		1.5	2.3	31.1
	2	3.7	85.5		2	6.2	255.8		2	2.9	50.9
	2.5	4.8	146.9		2.5	7.7	310.7		2.5	4	80.6
	3	6.3	228.2		3	8.3	363.6		3	4.7	120
	3.5	8.5	344.7		3.5	10.6	493.1		3.5	5.6	159
	4	10.5	506.9		4	13.1	662.4		4	6.6	198.3
	4.5	12	637		4.5	14.2	807.6		4.5	7.1	238.2
	5	13.1	766.9		5	15.1	927.5		5	8	276.8
	5.5	14	867.5		5.5	16.4	1009.7		5.5	9.2	348.5
	6	15.5	967.9		6	17.7	1091.9		6	10.3	429.5
46	0.5	0.8	4	47	0.5	0.7	3.2	48	0.5	0.8	4
	1	1.5	14.3		1	1.2	9.2		1	1.5	14.3
	1.5	2.3	31.1		1.5	2	25		1.5	2.9	57.6
	2	2.9	50.9		2	2.8	47.3		2	4.8	143.9
	2.5	4	80.6		2.5	4.4	108.5		2.5	5.9	220.7
	3	4.7	120		3	6.1	210.2		3	7.8	311.6
	3.5	5.6	159		3.5	7.4	282.4		3.5	9.5	405.9
	4	6.6	198.3		4	8.8	353.3		4	10.7	489
	4.5	7.1	238.2		4.5	9.3	444.4		4.5	12.4	631.6
	5	8	276.8		5	10.5	531		5	14.5	822.6
	5.5	9.2	348.5		5.5	11.7	588.1		5.5	15.9	1012.3
	6	10.3	429.5		6	12.8	650.7		6	19.4	1218.3
49	0.5	1	6.4	50	0.5	0.8	3.9	51	0.5	1.1	8.2
	1	2.1	31.2		1	1.5	14.2		1	2.5	44.4
	1.5	3.1	65.3		1.5	2.5	40		1.5	4.3	134.9

Table XLII. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	2	4.1	113.9		2	3.9	86.7		2	6.4	287.4
	2.5	5.2	176.5		2.5	5.1	128.7		2.5	7.9	418.6
	3	6.6	258.8		3	6	174.7		3	9.5	574.5
	3.5	8.5	371.9		3.5	7.2	245.5		3.5	10.6	687.4
	4	10.3	474.3		4	7.7	320.3		4	11.8	807.3
	4.5	12.1	585.3		4.5	8.5	356.4		4.5	13.3	1031.9
	5	14.1	717.6		5	9.2	389.5		5	15.3	1210.9
	5.5	14.4	895.3		5.5	10.1	480.4		5.5	16.5	1316.2
	6	17.2	1079.5		6	11.9	580.6		6	17.7	1405.1
52	0.5	0.6	2.5	53	0.5	0.7	3.4	54	0.5	1.1	8.1
	1	1.2	10		1	1.4	13.2		1	2.4	43
	1.5	2	27.9		1.5	2.5	44.9		1.5	3.9	112
	2	3	61.3		2	3.9	107.4		2	5.5	218
	2.5	4.1	117.1		2.5	5.1	169.4		2.5	6.6	284
	3	5.5	201.3		3	5.9	235.5		3	7.4	350.1
	3.5	6.6	297.6		3.5	7.4	363.8		3.5	8.5	456.2
	4	7.7	394		4	9	520.4		4	9.9	582
	4.5	8.8	443.7		4.5	10.4	642.6		4.5	11.1	694.2
	5	9.5	488.7		5	12	762.6		5	12.1	810.5
	5.5	10.7	592.9		5.5	13	832.9		5.5	13.1	879.7
	6	11.5	712.9		6	14	900.4		6	14.1	945.8
55	0.5	1.2	9.9	56	0.5	1.1	7.6	57	0.5	1.1	7.6
	1	2.7	51.4		1	2.3	37		1	2.3	36.9
	1.5	3.8	99.3		1.5	3.7	93.7		1.5	3.4	78.6
	2	5.1	165.1		2	5.1	176.3		2	4.5	134.8
	2.5	6	216.8		2.5	6.2	249.6		2.5	5.7	203.3
	3	6.5	274.6		3	7.2	321.1		3	6.5	265.5
	3.5	7.5	380.3		3.5	7.8	367.1		3.5	7.8	382.4
	4	8.9	502.4		4	8.7	415.5		4	9.2	521.7
	4.5	10.4	662.6		4.5	9.5	469.2		4.5	10.7	649.2
	5	11.8	843.9		5	10.2	517.6		5	12.4	775.5
	5.5	13.5	1033.3		5.5	11.2	638.4		5.5	14.2	910.4
	6	16.2	1229.4		6	12.3	777.4		6	15.7	1031.6
58	0.5	0.6	2.5	59	0.5	1.8	23.3	60	0.5	0.6	2.7
	1	1.2	10.3		1	4.1	118.2		1	1.6	15.6
	1.5	2.3	39.6		1.5	5.6	221.3		1.5	3.2	69.5
	2	3.8	106.9		2	7.3	358.5		2	4.9	173.8
	2.5	5.3	189.6		2.5	8.4	473.3		2.5	6.1	245.6
	3	6.5	282.4		3	9.7	588.5		3	7.1	327
	3.5	7.4	353.2		3.5	10.6	710.4		3.5	7.8	395

Table XLII. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	4	8.7	434.6		4	11.9	844.3		4	8.6	462.7
	4.5	9.7	519.3		4.5	13.5	1043.2		4.5	10.2	609.9
	5	10.7	605.1		5	15.8	1245.9		5	12.2	774
	5.5	12.2	734.1		5.5	17.6	1405.7		5.5	13.2	904.4
	6	13.7	872.5		6	19	1502		6	14.5	1039.9
61	0.5	0.6	2.4	62	0.5	0.6	2.9	63	0.5	0.6	2.4
	1	1.4	11.7		1	1.3	11		1	1.2	8.4
	1.5	2.9	58		1.5	2.1	34.1		1.5	3.1	67.4
	2	4.7	154		2	3.4	81.8		2	5.6	213.5
	2.5	5.9	223.3		2.5	4.4	135.3		2.5	7	319.3
	3	6.9	300.7		3	5.4	197.7		3	8.4	423
	3.5	7.5	363.2		3.5	6.7	283.3		3.5	9.4	512.3
	4	8.4	430.3		4	7.7	374.1		4	10.5	607.3
	4.5	9.8	562.1		4.5	8.9	439.7		4.5	11.7	732.1
	5	11.1	708		5	9.8	505.1		5	13	861.7
	5.5	12.7	892.9		5.5	10.6	576		5.5	14.1	999.9
	6	14.3	1094.1		6	11.4	653.6		6	16	1141.1
64	0.5	1.2	10.5	65	0.5	0.8	4	66	0.5	0.7	3.9
	1	2.8	54.6		1	1.8	23.1		1	1.5	15.8
	1.5	4.2	123.4		1.5	3.4	84.4		1.5	3.3	81.2
	2	5.7	218.4		2	5.2	195		2	5.5	223.3
	2.5	6.7	307.6		2.5	6.4	259.9		2.5	7.1	342.8
	3	8	403.8		3	7.5	330.8		3	8.4	485.4
	3.5	9	520.1		3.5	8.4	468.2		3.5	9.8	645.3
	4	10.3	648.2		4	10	625.6		4	11.6	820.3
	4.5	11.4	773		4.5	11.3	806.9		4.5	12.7	911.5
	5	12.6	898.8		5	12.8	982.8		5	13.4	990.7
	5.5	14.4	1092.4		5.5	15.9	1204.4		5.5	14.3	1095.5
	6	16.8	1299.9		6	18.7	1419.1		6	15.8	1193.5
67	0.5	0.8	4.4	68	0.5	1.1	8.1	69	0.5	1.1	8.2
	1	1.6	17.3		1	2.5	44.6		1	2.5	43.8
	1.5	2.2	33.1		1.5	3.6	93		1.5	3.8	102.1
	2	2.9	55.9		2	4.8	165.2		2	5.3	193.3
	2.5	4	98.1		2.5	6.7	322.5		2.5	6.8	323.1
	3	5.1	159.4		3	8.8	481		3	8.4	467.6
	3.5	5.7	215.4		3.5	10.1	628.2		3.5	9.7	592.8
	4	6.6	276.1		4	11.8	788.5		4	10.9	732.3
	4.5	7.6	371.3		4.5	13	900.7		4.5	13.1	964.7
	5	8.9	480.7		5	14.2	1008.2		5	16.7	1216.6
	5.5	10.3	609.1		5.5	15.5	1109.9		5.5	18.5	1344.3

Table XLII. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	6	11.9	745		6	16.9	1210.1		6	19.8	1460.5
70	0.5	0.7	3.4	71	0.5	0.6	2.9	72	0.5	1.2	10.8
	1	1.4	13.3		1	1.3	10.9		1	2.8	56.6
	1.5	2.2	32.9		1.5	3.1	68		1.5	3.8	98.1
	2	2.9	62.1		2	5.3	182.1		2	4.9	153.7
	2.5	4.2	119.2		2.5	7.1	336.8		2.5	6.1	242
	3	5.2	184.2		3	9.3	539		3	7.2	344.9
	3.5	6.2	256.4		3.5	10.8	664.9		3.5	8.6	468.4
	4	7.2	332.2		4	12.5	799.8		4	10	613.1
	4.5	8.5	432.6		4.5	15.5	1039.2		4.5	11.3	703.8
	5	9.8	542		5	18.6	1278.7		5	12.2	793.9
	5.5	12	757.9		5.5	21.3	1442.8		5.5	13.6	993.1
	6	14.3	1005.7		6	24.2	1592.2		6	16.3	1208.6
73	0.5	0.4	1.1	74	0.5	0.7	3.3	75	0.5	0.8	4.3
	1	1	6.5		1	1.4	13.6		1	1.6	17
	1.5	1.9	27.1		1.5	2	28		1.5	2.3	36.8
	2	2.9	62		2	2.8	52.6		2	3.1	68.2
	2.5	4.1	121.7		2.5	4.2	117.1		2.5	4.4	123.3
	3	5.3	198.1		3	5.5	208.7		3	5.2	176.9
	3.5	6.7	303.1		3.5	6.3	265.7		3.5	6.2	246.1
	4	8.2	410.8		4	7.1	322.4		4	7.1	315.7
	4.5	9.1	461.4		4.5	8.8	438.6		4.5	8	384.2
	5	9.6	504.4		5	10.3	569.4		5	9.3	462.6
	5.5	10.6	604.1		5.5	13.2	884.4		5.5	10.1	550.4
	6	12	726.9		6	16.5	1261.7		6	11.2	645.9
76	0.5	1.6	18.6	77	0.5	1.1	7.9	78	0.5	0.9	5.6
	1	3.6	95.2		1	2.4	40.1		1	2.1	32.1
	1.5	5.7	223.5		1.5	2.9	56.9		1.5	2.9	57.2
	2	7.5	388.9		2	3.8	82		2	3.7	89.9
	2.5	9.2	545.3		2.5	5.4	185.2		2.5	5	173.4
	3	10.7	733.5		3	7	332.4		3	6.4	289.1
	3.5	12.2	935.9		3.5	7.7	372.6		3.5	7.5	378.5
	4	14.5	1149.9		4	8.8	412.7		4	8.6	467
	4.5	16.3	1284.9		4.5	9.7	501.9		4.5	10	582.5
	5	17.6	1404		5	10.9	603.7		5	11.5	713.9
	5.5	20.3	1570.7		5.5	11.8	697.3		5.5	12.9	892.3
	6	22.6	1691.5		6	12.9	795.7		6	14.5	1083.2
79	0.5	0.8	4.2	80	0.5	0.7	3.8	81	0.5	0.6	2.6
	1	1.7	20.3		1	1.5	15		1	1.2	9
	1.5	2.8	56		1.5	2.6	49.9		1.5	2.4	40.7

Table XLII. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	2	4	113.1		2	4.2	127		2	3.9	108.9
	2.5	5	172.5		2.5	5.8	237.8		2.5	4.7	152.3
	3	5.8	227.3		3	7.3	375.3		3	6.1	208.1
	3.5	7.2	335.4		3.5	8.2	445.5		3.5	7.5	290.2
	4	8.6	465.7		4	9.1	517		4	8.4	375.7
	4.5	9.8	538.1		4.5	10.7	644.6		4.5	9.4	505.3
	5	10.7	611.8		5	12	775.3		5	10.9	646.1
	5.5	11.7	689.3		5.5	13.2	922		5.5	11.7	732.6
	6	12.4	768.5		6	14.1	1076.7		6	12.5	826.7
82	0.5	0.6	2.2	83	0.5	0.6	2.3	84	0.5	1.2	10.5
	1	1.2	8.6		1	1.2	9.5		1	2.8	54.6
	1.5	2.1	28.7		1.5	1.9	21.5		1.5	4.7	158.3
	2	3	66.3		2	2.5	39.2		2	6.7	318.9
	2.5	4.2	106.1		2.5	3.4	70.5		2.5	9.1	525
	3	4.9	151.3		3	4.3	119.5		3	10.9	794
	3.5	6.1	236.1		3.5	5.7	217		3.5	12.8	1016.2
	4	7.5	348.7		4	7.5	349.1		4	15	1247.6
	4.5	9	481.4		4.5	9.4	521		4.5	16.7	1332.8
	5	10.1	617.6		5	11.6	717.1		5	18	1404.1
	5.5	11.3	719.6		5.5	13	875		5.5	19.2	1456.2
	6	12.3	825.8		6	14.5	1036.8		6	20.5	1506.2
85	0.5	0.7	3.7	86	0.5	0.9	6.1	87	0.5	1.6	17.7
	1	1.5	16.8		1	2.1	30.6		1	3.6	91.8
	1.5	2.7	54.1		1.5	3	64.7		1.5	4.7	152.6
	2	4.2	122.5		2	4	114.9		2	5.9	225.5
	2.5	5.9	246.3		2.5	5.2	179.7		2.5	6.7	288.1
	3	7.6	403.2		3	6	253.6		3	7.9	363.9
	3.5	9.2	548.8		3.5	7.5	391.2		3.5	9.1	501.2
	4	10.9	712.7		4	9.2	550.8		4	10.7	664.1
	4.5	12.6	880.9		4.5	10.9	690.8		4.5	12.5	854.9
	5	14.5	1047.1		5	12.4	826.8		5	14.7	1044.8
	5.5	16	1143		5.5	14	1011.9		5.5	16	1108.4
	6	17.1	1235		6	16.3	1206.3		6	17.2	1163.9
88	0.5	0.7	3.4	89	0.5	0.9	6.2	90	0.5	0.6	2.9
	1	1.6	16.2		1	2	29.1		1	1.3	11
	1.5	2.4	35.9		1.5	3	62.6		1.5	3.1	68.4
	2	3	62.2		2	4.1	116.5		2	5.5	201.7
	2.5	3.8	89.6		2.5	5.8	241.7		2.5	6.4	261.6
	3	4.8	127.8		3	7.5	394.8		3	7.2	320.2
	3.5	6.3	230.5		3.5	9.3	582.8		3.5	8.1	387

Table XLII. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	4	7.6	359.9		4	11.1	802.7		4	9.2	457.5
	4.5	9	481.4		4.5	12.5	891.3		4.5	10.2	553
	5	10	605.1		5	13.3	974.8		5	11.2	655.6
	5.5	11.4	753.5		5.5	14	1048.2		5.5	12	736.2
	6	12.5	915.5		6	14.8	1117.2		6	12.8	818.9
91	0.5	0.6	2.5	92	0.5	0.6	2.4	93	0.5	1.9	23.1
	1	1.2	10		1	1.2	9.7		1	4.4	121.8
	1.5	3.1	71.7		1.5	2.5	45.9		1.5	6	236
	2	5.3	205.9		2	4.3	126.9		2	7.7	386.4
	2.5	6.3	275.5		2.5	5.7	226		2.5	8.9	520.8
	3	7.9	360.8		3	7	335		3	10.3	663.3
	3.5	9.6	501.3		3.5	7.9	402.7		3.5	11.8	849.2
	4	11.1	657.8		4	8.9	473.1		4	14.3	1033.7
	4.5	11.6	724.5		4.5	10.7	632.1		4.5	16.2	1132.8
	5	12.6	779.7		5	12.2	804.1		5	17.8	1214.8
	5.5	13.6	908.2		5.5	13.4	930.3		5.5	19.2	1361.9
	6	15	1041.5		6	14.9	1051.6		6	21.1	1516
94	0.5	0.8	4.2	95	0.5	0.7	3.3	96	0.5	0.9	5.4
	1	1.5	17		1	1.4	12.9		1	1.8	21.3
	1.5	2.5	44.1		1.5	2.7	52.2		1.5	3.1	67.9
	2	3.9	110.7		2	4.2	127.8		2	4.6	154.2
	2.5	4.9	151.9		2.5	5.1	184.9		2.5	5.9	233
	3	5.7	195.8		3	6.2	254.7		3	6.7	302.7
	3.5	6.3	246.7		3.5	7.9	361.4		3.5	8.1	422
	4	7.1	298.6		4	9.3	481.8		4	9.6	556.6
	4.5	8.7	453.4		4.5	11.3	640.8		4.5	11.1	689.6
	5	10.5	635.2		5	12.6	793.6		5	12.6	831.8
	5.5	11.6	712.4		5.5	13.6	920.1		5.5	14.5	988.5
	6	12.9	785.6		6	14.6	1038.7		6	16.3	1134.4
97	0.5	1	6.6	98	0.5	0.8	5.1	99	0.5	0.6	2.4
	1	1.9	25.9		1	2.1	30		1	1.3	10
	1.5	3.3	73		1.5	2.9	61.4		1.5	2.1	27.9
	2	4.7	147.4		2	3.8	102.5		2	2.8	55.1
	2.5	5.8	211.7		2.5	4.5	125.5		2.5	3.9	109
	3	6.6	274.8		3	5.4	159.5		3	5.3	185
	3.5	7.5	346.8		3.5	6.3	251		3.5	7	296.9
	4	8.7	425.2		4	7.5	361.1		4	8.6	419.9
	4.5	10	541.8		4.5	8.8	491.3		4.5	9.3	498.7
	5	11.1	670.4		5	10.2	633.3		5	10	569.4
	5.5	12.3	812.8		5.5	12.1	798.5		5.5	10.6	644.6

Table XLII. Continued

Scen	Hrs	Per	Area	Scen	Hrs	Per	Area	Scen	Hrs	Per	Area
	6	13.6	969.6		6	13.5	964.6		6	11.6	723
100	0.5	0.9	5.1								
	1	1.8	21.4								
	1.5	2.4	35.4								
	2	3	52.8								
	2.5	3.9	97.5								
	3	5.1	160.9								
	3.5	6.8	233.2								
	4	8.1	313.6								
	4.5	8.7	381.9								
	5	9.2	442.4								
	5.5	9.9	504.2								
	6	10.9	566								

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