ESSAYS ON ESTIMATION OF INFLATION EQUATION

A Dissertation

by

WOONG KIM

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2008

Major Subject: Economics

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Approved by:

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ABSTRACT

Essays on Estimation of Inflation Equation. (August 2008)

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Chair of Advisory Committee: Dr. Hae-shin Hwang

This dissertation improves upon the estimation of inflation equation, using the additional measures of distribution of price changes and the optimum choice of instrumental variables. The measures of dispersion and skewness of the cross-sectional distribution of price changes have been used in empirical analysis of inflation. In the first essay, we find that independent kurtosis effect can have a significant role in the approximation of inflation rate in addition to the dispersion and skewness. The kurtosis measure can improve the approximation of inflation in terms of goodness of fit. The second essay complements the first essay. It is well known that classical measures of moments are sensitive to outliers. It examines the presence of outliers in relative price changes and consider several robust alternative measures of dispersion and skewness. We find the significant relationship between inflation and robust measures of dispersion and skewness. In particular, medcouple as a measure of skewness is very useful in predicting inflation. The third essay estimates the Hybrid Phillips Curve using the optimal set of instrumental variables. Instrumental variables are usually selected from a large number of valid instruments on an ad hoc basis. It has been recognized in the literature that the estimates are sensitive to the choice of instrumental variables and to the choice of the measurement of inflation. This paper uses the L_2-boosting method that selects the best instruments from a large number of valid weakly exogenous instruments. We find that boosted instruments produce more comparable estimates of parameters across different measures of inflation and a higher joint precision of the estimates. Instruments boosted from principal components tend to give a little better results than the instruments from observed variables, but no significant difference is found between the ordinary and generalized principal components.

To my family

ACKNOWLEDGMENTS

I cannot find enough words to thank my advisor Hae-shin Hwang for his insightful guidance, enthusiasm and constant encouragement. I would also like to thank Dennis Jansen, Hwagyun Kim, and David Bessler for their valuable suggestions. I had very helpful discussions with my colleagues Deukwoo Kwon, Yanggyu Byun, Joonghoon Cho and Hyosung Yeo. I learned a lot from them and they helped me much in studying economics and writing the paper.

It is impossible to express how much I thank my dear wife, Sejung Yoon. She sacrificed herself and supported me throughout my study. Thanks to my lovely son, Andrew, for making me happy everyday. Special thanks are reserved for my parents, Jungseob Kim and Jeongi Park, and for my parents-in-law Inhak Yoon and Moonja Yoon. Without their unconditional love and support, I would not have been able to accomplish what I wanted. To them, I dedicate my dissertation.

TABLE OF CONTENTS

CHAPTER Page

LIST OF TABLES

TABLE Page

LIST OF FIGURES

FIGURE Page

FIGURE Page

CHAPTER I

INTRODUCTION

One of the stylized facts in macroeconomics is a positive relationship between inflation and relative price variability. Since Mills (1927) observed this relationship, there has been so much research on this relationship. In the past studies of Fischer (1982), Ball and Mankiw (1994), and Ball and Mankiw (1995), the key idea is that a change in the shape of the distribution can affect inflation. Fischer (1982) and Ball and Mankiw (1994) considered dispersion alone while Ball and Mankiw (1995) included skewness in addition to dispersion. However, they neglected kurtosis, which is one of the important distributional characteristics. By kurtosis, we look at the thickness and peakedness of the distribution. Therefore, I consider moments up to fourth order so as to capture the property of the distribution sufficiently. This is main motivation of for the first essay.

My key idea is to introduce kurtosis effect. Pearson's kurtosis is usually used. However, the kurtosis concept is so unclear that it is difficult to interpret since it captures both peakedness and tail heaviness as a single measure. It has been defined in many ways. Different properties of distribution can be captured by different kurtosis. Recently, Seier and Bonett (2003) introduced an alternative kurtosis measures which give more importance to the central part of the distributions so that they tend to be less correlated with skewness. Therefore, I expect that the performance of both measures can be noticeable in capturing the peakedness of the distribution. This is another motivation for the first essay.

This dissertation follows the style of *Econometrica*.

I show the importance of the independent effects of kurtosis measures by extending Ball and Mankiw's numerical analysis. I also identify the performances of alternative kurtosis measures. My questions of empirical analysis are "If we additionally consider the independent kurtosis effects that Ball and Mankiw omitted, how much can we improve the approximation of inflation in terms of the goodness of fit?" and "Which of the two kurtosis measures performs better in terms of the goodness of fit?".

The second essay complements the first essay. Ball and Mankiw analyzed the effects on the PPI inflation rate of the dispersion and skewness of the changes in prices. The dispersion and skewness are computed by the classical measurement of weighted and unweighted standard deviation and skewness of the cross sectional sample.

It is well known that classical measures of dispersion and skewness are very sensitive to the presence of outliers. This sensitivity can have a significant effect on the relationship between the skewness and inflation rate. A single positive outlier tends to significantly increase the skewness, and it will also increase the inflation rate in the same direction because the overall PPI is a weighted average of prices of individual commodities. This implies that a positive correlation between the skewness and inflation rate can be caused by outliers, particularly in a sample of small size.

The second essay examines the presence of outliers in the relative price changes and estimate unweighted and weighted robust measures of dispersion and skewness. The effects of robust measures on the inflation rate are then estimated and compared with the results based on the classical measures of dispersion and skewness.

The third essay is upon the estimation of hybrid Phillips Curve. Recent literature on the inflation dynamics focuses on two lines of research. The New Keynesian Phillips Curve (NKPC) models are based on the microeconomic foundation that introduces nominal rigidities into the forward-looking optimizing behavior. The baseline model specifies the inflation as a function of forward-looking expectations of inflation and marginal costs as the underlying driving force. Gali and Gertler (1999) extend the baseline model by introducing two types of firms: forward-looking and backward-looking firms. Their model is a hybrid model that includes past inflation and expected inflation in addition to the marginal costs as the driving force. This model has been applied in numerous empirical applications.

The model is typically estimated in a structural form or in a closed form by using the GMM. As noted in Nason and Smith (2005), estimates of NKPC parameters are sensitive to the choice of instrumental variables and to the choice of inflation data. To avoid the weak instrumental variables problem, relatively small number of instrumental variables are chosen in general on an ad hoc basis. However, since the instrumental variables are for the rational expectation of future inflation and the information set for the conditional expectation can include a large number of informational variables, it is desirable to select the best set of relatively small number of instrumental variables in a systematic way.

Another line research in inflation dynamics is the information forecasting in a data rich environment. Factor models have been used widely in the macroeconomics literature to summarize efficiently a large set of data and to use the summary statistics for a variety of purposes including forecasting. In a series of papers, Stock and Watson (1998, 2002a,b, 2005) propose to use ordinary principal components estimator of the factors, while Forni, Hallin, Lippi, and Reichlin (2000, 2003); Forni, Lippi, and Reichlin (2004); Forni, Hallin, Lippi, and Reichlin (2005) propose to use the generalized principal components estimator. Bernanke, Boivin, and Eliasz (2005) introduce the principal components estimator into the VAR model to overcome the dimensionality problem of the VAR model. The FAVAR augments the standard VAR model with a few latent factors. Bai and Ng (2007b) show that principal components of a large number of weakly exogenous variables are not only valid instruments for the endogenous regressors, but also they can be more efficient than the observed variables, if weakly exogenous instruments and the endogenous regressors share common factors. In practice, the first a few principal components, which explain the variation of indicator variables the most, are used many applications. Bai and Ng (2007a) emphasize, however, that the first a few principal components are not necessarily the best instruments for the endogenous regressors. The problem of selecting the best set of instruments still remains even when we use the principal components of weakly exogenous variables.

The third essay examines the robustness of the estimates of parameters in Gali and Gertler's hybrid model to the choice of instrumental variables. Both the structural form and closed form equations of the model are estimated by the GMM. Several sets of instruments are considered, including the set used in GG, Rudd and Whelan (2005), its subset used in Gali, Gertler, and Lopez-Salido (2001, 2005) and Rudd and Whelan (2007). Additional instrumental variable sets include a subset of observed weakly exogenous variables selected by L_2-boosting method of Buhlmann and Yu (2003), and a subset of *ordinary* and *generalized* principal components selected by the L_2-boosting method.

CHAPTER II

INFLATION AND THE DISTRIBUTION OF RELATIVE PRICE SHOCKS

A. Introduction

One of the stylized facts in macroeconomics is a positive relationship between inflation and relative price variability. Since Mills (1927) observed this relationship[1], there has been so much research on this relationship. Vinning and Elwertowski (1976), Parks (1978) and Domberger (1987) confirmed that a positive relationship holds for different periods and different countries based on their empirical findings. In more recent studies, Ball and Mankiw (1995), Debell and Lamont (1997), Peltzman (2000), Silver and Ioannidis (2001), Senda (2001), Aucremanne, Brys, Hubert, Rousseeuw, and Struyf (2002), Caraballo and Dabus (2005) and Demery and Duck (2007) investigated the empirical correlation between inflation and the moments of relative prices.

In past studies, Fischer (1981) and Fischer (1982) reviewed the previous theories that explain the inflation-relative price variability relationship [2]. According to the theories, the causality direction between inflation and relative price variability is different[3]. However, we are interested in the effects of relative price shocks as in

[1]He examined the wholesale price data over the sample period of 1890-1926 and showed relative price variability is closely related to the absolute value of inflation.

[2]He discussed three theories on the relationship between inflation and relative price variability: (i) In the presence of menu costs of changing prices, inflation can affect relative price variability. This is because inflation causes additional transaction cost so that different costs of adjusting prices in different industries result in greater relative price variability. (ii) Unexpected inflation can affect relative price variability by affecting individual prices differently. (iii) Due to the asymmetric price response, the relative price variability affect inflation.

[3]His causality tests showed no clear direction.

Friedman (1975)[4]. Thus, we focus on the theory in which causality runs from moments of relative prices to inflation. One of the theories to explain this direction is based on an asymmetric response. As an example of the asymmetric price adjustment, he considered downward price rigidity, which means prices rise more easily than they fall. He showed that if there is an asymmetric response to price shocks, changes in dispersion have an effect on inflation. The key idea is that a change in the shape of the distribution can affect inflation. Fischer (1982) considered dispersion alone. However, using his example, it can be shown that changes in other properties such as skewness and kurtosis can also affect inflation.

Ball and Mankiw (1994) also considered the asymmetric response as in Fischer (1982)[5]. They showed that if the price adjustment is asymmetric due to trend inflation, changes in dispersion can affect inflation. This is because firms consider additionally trend inflation so that more price increases are expected in the presence of positive trend inflation.

In line with this literature, Ball and Mankiw (1995) showed that changes in skewness in addition to dispersion can affect inflation under the stickiness assumption. To show this, they presented an intuitive simple model of menu costs. In their model, due to the transaction costs for changing prices, only firms with a shock larger than

[4]There are empirical evidences implying that relative price shocks can cause inflation even though they are not directly related to the monetary phenomenon. Oil shocks of the 1970s are the most obvious example. Price increases in oil-related items caused inflation and following recessions. However, as noted in Friedman (1975), relative price shocks which change firms' desired prices, logically, should not cause inflation when price adjustments are perfectly flexible. This is because price increases in particular items caused by sectoral shocks should be offset by price decreases in other items.

[5]One of the differences in both papers is the source of an asymmetric response. It is the downward price rigidity in Fischer (1982) while it is the trend inflation in Ball and Mankiw (1994). So, the asymmetric response is determined exogenously in Fischer (1982) while it is determined endogenously in Ball and Mankiw (1994).

menu cost change their prices. As a result, some firms change their actual prices and others do not. When price shocks have a symmetric distribution, positive and negative price changes are offset each other so that the net effect on inflation is zero even in the presence of menu cost. However, the distribution is asymmetric, positive and negative price changes are not offset so that the net effect is not zero. With a symmetric distribution of shocks, changes in dispersion do not affect inflation. But if the distribution is skewed, larger dispersion cause the stronger effect of skewness on inflation. That is, changes in dispersion influence inflation by means of the interaction effect between dispersion and skewness. The main idea is the same as Fischer (1982)'s in the sense that inflation can be generated by changes in the shape of the underlying distribution.

In all three papers, the key idea is that a change in the shape of the distribution can affect inflation. To capture the effect of the changes in distribution, Fischer (1982) and Ball and Mankiw (1994) considered dispersion alone while Ball and Mankiw (1995) included skewness in addition to dispersion. However, they neglected kurtosis, which is one of the important distributional characteristics. By kurtosis, we look at the thickness and peakedness of the distribution. Therefore, we consider moments up to fourth order so as to capture the property of the distribution sufficiently. This is main motivation of our study.

Since Ball and Mankiw (1995) clearly illustrated and emphasized that skewness effect is stronger than dispersion effect, both measures have been used in empirical analysis of inflation. Ball and Mankiw model was followed by considerable amount of theoretical and empirical studies[6]. In particular, it has been used to show empirical evidences for many different countries[7], implying that inflation-moments relationships are robust stylized facts even under the different price setting circumstances. Caraballo and Usabiaga (2004) extended Ball and Mankiw model by introducing kurtosis in their study of Spanish regional inflation. Based on the regression for each region, they found kurtosis measure is insignificant in most regions and concluded that kurtosis is not important in the analysis of Ball and Mankiw's framework.

However, Caraballo and Usabiaga did not notice that kurtosis can affect inflation through the interaction effect between moments like the interaction effect between dispersion and skewness as in Ball and Mankiw. We consider two distributions with the same mean,variance and skewness but different kurtosis. In the case of a symmetric distribution of shocks, changes in kurtosis do not affect inflation. But if the distribution is skewed to right, larger kurtosis cause the smaller effect of skewness on inflation. Therefore, we expect that there may be significant kurtosis interaction effects even though individual kurtosis effect can be negligible. Our interest is to capture additional properties of the distribution by using novel kurtosis interaction effect.

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[6] Debell and Lamont (1997) found the evidence that both dispersion and skewness matter at the US city level. Peltzman (2000) argued that prices tend to respond faster to a positive shock than to a negative shock, focusing on asymmetric responses. Senda (2001) studied asymmetric effects of monetary shock using Ball and Mankiw's menu cost model. Aucremanne, Brys, Hubert, Rousseeuw, and Struyf (2002) investigated the presence of outliers in the relative price changes and inflation-dispersion-skewness relationship using robust measures. Demery and Duck (2007) argued that inflation-dispersion-skewness relationship in the Ball and Mankiw model are much changed in the presence of a trend inflation.

[7] Amano and Macklem (1997) for Canada, Dopke and Pierdzioch (2003) for Germany, Nishizaki (2000) for Japan that has experienced near-zero inflation, Fielding and Mizen (2000) for EU countries, Florio (2005) for Italy, Caraballo and Dabus (2005) for Spain and Argentina, Assarsson and Riksbank (2003) for Sweden and Caraballo and Usabiaga (2004) for Spain.

What Ball and Mankiw are interested in is the impact of sectoral price shocks on inflation. Therefore, their argument has an empirical limitation because of the unobservablity of underlying price shock distribution. Alternatively, they used the characteristics of observed price changes as a proxy for unobserved price shocks. To justify using a proxy, they presented a numerical analysis which shows the linear relationship between both of them. Therefore, kurtosis measure what we are interested in can be also applied only if kurtosis of underlying price shocks and corresponding kurtosis of observed price changes are linearly related. However, Caraballo and Usabiaga neglected this essential procedure. So, it is necessary to check linear relationship between moments of underlying price shocks and corresponding moments of observed price changes by extending Ball and Mankiw's numerical analysis. If kurtosis of price changes can be used as a proxy for price shock, then we can identify both individual and interaction effect of kurtosis on inflation.

Our key idea is to introduce kurtosis effect. Pearson's kurtosis is usually used. However, the kurtosis concept is so unclear that it is difficult to interpret since it captures both peakedness and tail heaviness as a single measure. It has been defined in many ways. Different properties of distribution can be captured by different kurtosis. Compared to the other macro data, the most striking distributional features of the price changes is its peakedness. Recently, Seier and Bonett (2003) introduced an alternative kurtosis measures which give more importance to the central part of the distributions so that they tend to be less correlated with skewness. Therefore, we expect that the performance of both measures can be noticeable in capturing the peakedness of the distribution. This is another motivation for our study.

We show the importance of the independent effects of kurtosis measures by extending Ball and Mankiw's numerical analysis. We also identify the performances of alternative kurtosis measures. Our questions of empirical analysis are "If we additionally consider the independent kurtosis effects that Ball and Mankiw omitted, how much can we improve the approximation of inflation in terms of the goodness of fit?" and "Which of the two kurtosis measures performs better in terms of the goodness of fit?".

Ball and Mankiw (1995) model and Our model with independent kurtosis effect are estimated and compared for the sample period of 1947-2006. Ball and Mankiw estimated only annual data, but we estimate both annual and monthly data since we want to investigate a possible difference between annual and monthly data as Verbrugge (1999) pointed out. As expected, additional kurtosis measures have a significant effect on inflation and the alternative kurtosis measure outperforms. The improvement measured by different goodness of fit is substantial in monthly data, but is not much substantial in annual data.

The paper is organized as follows. In the next section, we show the inflation-moments relationships by extending Ball and Mankiw's numerical analysis. By comparing three models used in Fischer (1982), Ball and Mankiw (1994), and Ball and Mankiw (1995), we show whether additional properties they did not consider can also affect inflation in each model. In section 3, we introduce alternative kurtosis measures and show the usefulness of them as an ideal proxy. In section 4 and 5, the inflation equations are specified and estimated. Section 6 concludes the paper with a summary of our major findings.

B. The Relationship between Inflation and Moments of the Distribution of Relative Price Shocks

1. Models

This section shows how changes in the moments of the distribution of relative price shocks can affect inflation. We compare three models used in Fischer (1982), Ball and Mankiw (1994), and Ball and Mankiw (1995). We briefly review the firm's pricing decision rules used in three models.

In all three models, we assume that firms in each industry are subject to a common relative price shock ϵ to their desired price. In Fischer (1982), firms price adjustment is asymmetric due to the downward price rigidity. So, when realizing the sectoral shock ϵ, firms change the price by the size of ϵ if $\epsilon > 0$. But, if $\epsilon \leq 0$, they do not change the price. The critical value (0) and the magnitude of price changes can be the different value. The industry price change π_ϵ is defined as the average price changes of the firms in the industry,

$$\pi_\epsilon = \begin{matrix} 0 & \text{if } \epsilon \leq 0 \\ \epsilon & \text{if } \epsilon > 0 \end{matrix}$$

In numerical analysis, we assume a critical value (-0.15), and the magnitude of price change is equal to the size of price shock.

In Ball and Mankiw (1994), the source of the asymmetric price adjustment is trend inflation. The model assumes that there exists positive trend inflation, which firms take as given. A firm's optimal price depends on trend inflation(T) as well as a price shock (ϵ). With a heterogeneous menu cost across firms, firms are divided into two groups. For a given price shock ϵ and trend inflation T, firms with a smaller menu cost such that $c < |\epsilon + T|$ change their prices by the size of $|\epsilon + T|$. On the other hand, firms with a higher menu cost do not change their prices. This implies that the inaction range is not symmetric around zero. The proportion of firms changing prices is determined by the probability of those firms, $P(c < |\epsilon + T|)$. This probability can be measured by the cumulative distribution function of menu cost, $G(|\epsilon + T|)$. The industry price change π_ϵ is defined as

$$\pi_\epsilon = (\epsilon + T)\, G(|\epsilon + T|)$$

In numerical analysis, we assume a trend inflation of 0.025, which is a value used in Ball and Mankiw (1994).

In Ball and Mankiw (1995), firms with a menu cost lower than the absolute value of the shock $|\epsilon|$ change the price by the size of ϵ, while firms with a menu cost higher than $|\epsilon|$ do not change the price[8]. Firms have heterogeneous menu cost and the proportion of firms with a menu cost lower than $|\epsilon|$ is given by a cumulative distribution function $G(|\epsilon|)$. The industry price change π_ϵ is defined as

$$\pi_\epsilon = \epsilon G(|\epsilon|)$$

In all cases, the price shock varies across industries which is governed by a density function $f(\epsilon)$. Aggregate inflation π is then defined as a weighted average of industry price changes:

$$\pi = \int_{-\infty}^{\infty} \pi_\epsilon f(\epsilon)\, d\epsilon$$

Figure 2-1 presents the firm's price setting assumptions used in three models. The big difference is the range of inaction, in which firms do not respond to shocks.

[8] In their model, firms are assumed to have a quadratic loss function of the difference between the desired price and actual price, and they change the price if $|\epsilon|$ is greater than the square root of the menu cost. We will call the square root of menu cost simply as menu cost.

This is due to the different sticky price assumption. Under these assumptions, they showed how inflation depends on the shape of the distribution.

2. Numerical Analysis for the Theoretical Relationship

Based on these firm's price setting behaviors, we conduct a numerical analysis similar to Ball and Mankiw's analysis. To show how inflation varies with the moments of underlying price shocks, Fischer (1982), Ball and Mankiw (1994) considered dispersion alone while Ball and Mankiw (1995) included skewness in addition to dispersion. By numerical analysis, we show whether additional properties can also affect inflation in their models.

Ball and Mankiw (1995) used an exponential distribution $G\left(|\epsilon|;\alpha\right)$ for the menu cost and Azzalini (1985)'s skew normal distribution $f\left(\epsilon;\lambda\right)$ for the price shocks:

$$G\left(|\epsilon|;\alpha\right) = 1 - e^{-\alpha|\epsilon|}$$

$$f\left(\epsilon;\lambda\right) = 2\phi\left(\epsilon\right)\Phi\left(\lambda\epsilon\right)$$

where λ is the shape parameter, and $\phi\left(\epsilon\right)$ and $\Phi\left(\epsilon\right)$ are the pdf and cdf of a standard normal distribution, respectively. They used $\alpha = 7$ in the menu cost distribution and imposed a zero mean on the skew normal distribution of price shocks.

A weakness of Ball and Mankiw's numerical analysis is in their use of Azzalini's skew normal distribution for the price shocks. This distribution has a very limited range of skewness and there is a fixed linear relationship between skewness and kurtosis. As shown in Figure 2-2, the feasible set of skewness sk and kurtosis kt of the skew normal distribution is just a concave line[9] with the lowest coordinate $\{sk = 0, kt = 3\}$

[9]Figure 2-2 shows only the case of positive skewness for both the skew normal and SuN distributions.

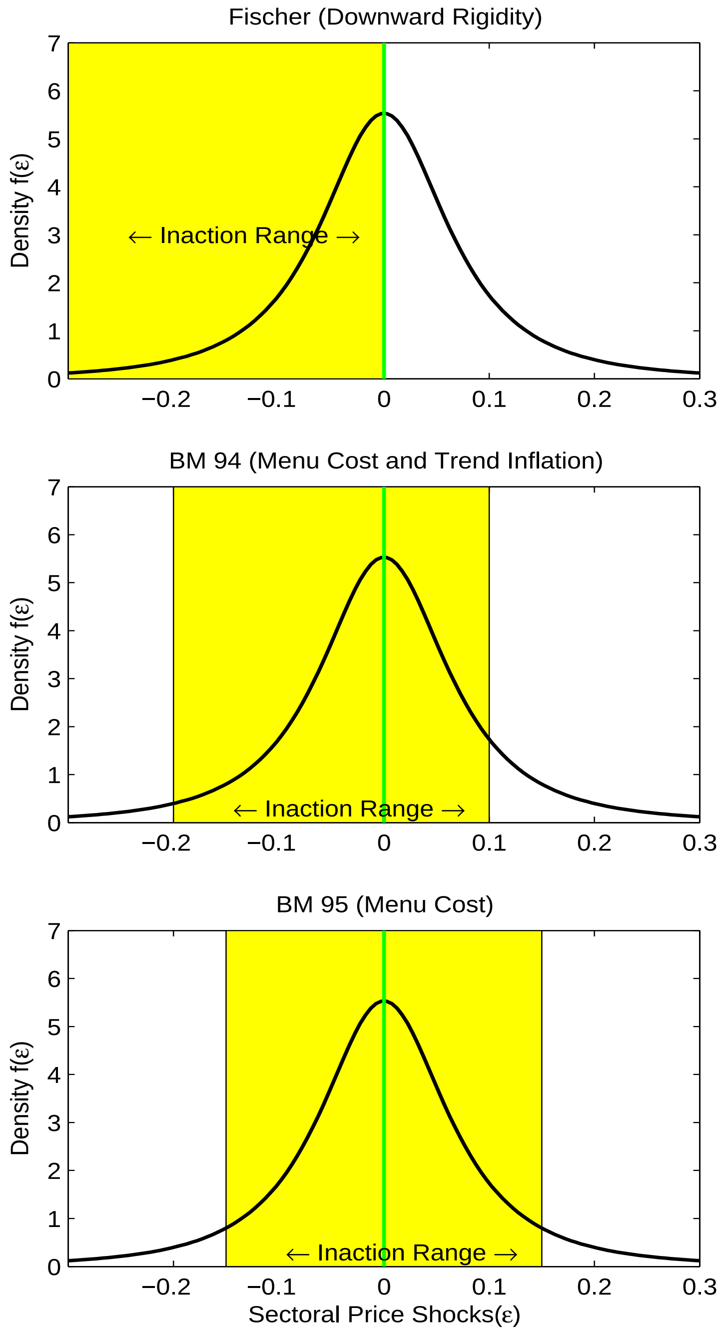


Fig. 2-1.: Theoretical Framework of Three Models: Inflation and Distribution of Price Shocks.

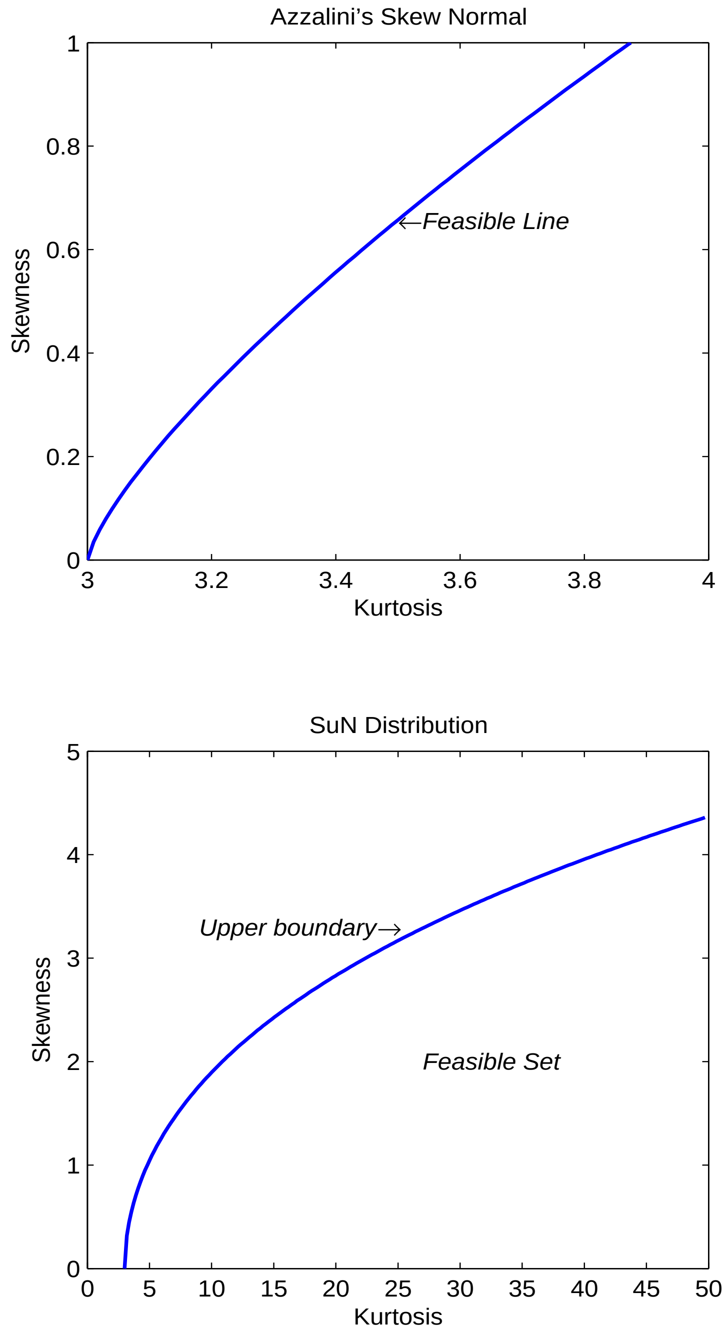
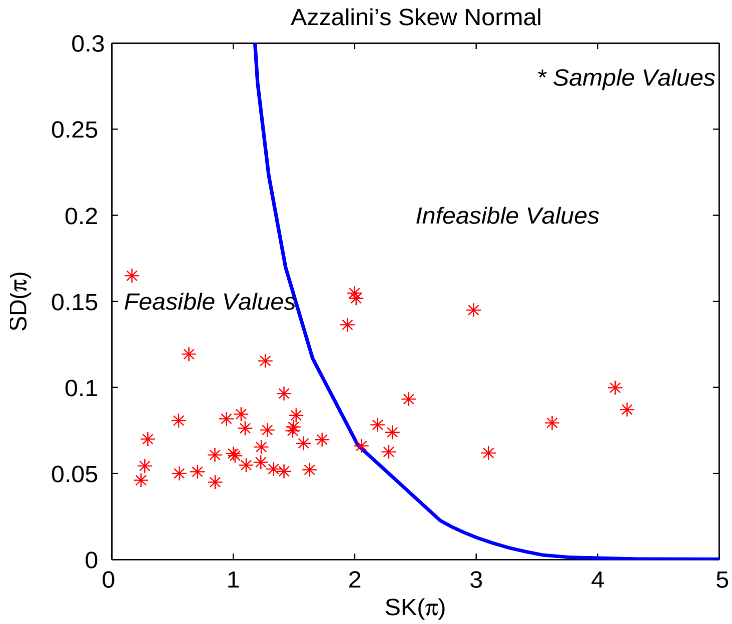


Fig. 2-2.: Feasible Sets of Skewness and Kurtosis.



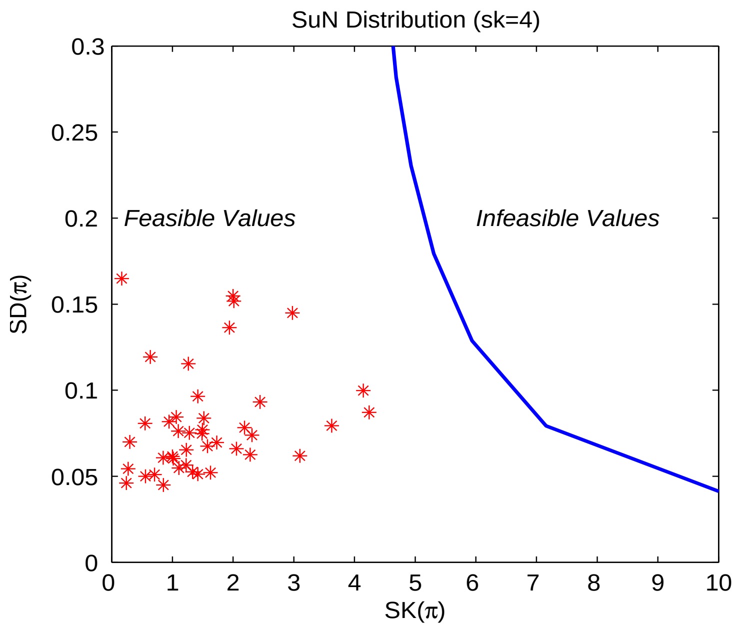


Fig. 2-3.: Feasible Set of SD and Skewness of Industry Price Changes: Skew Normal and SuN Distribution of Price Shocks (Annual).

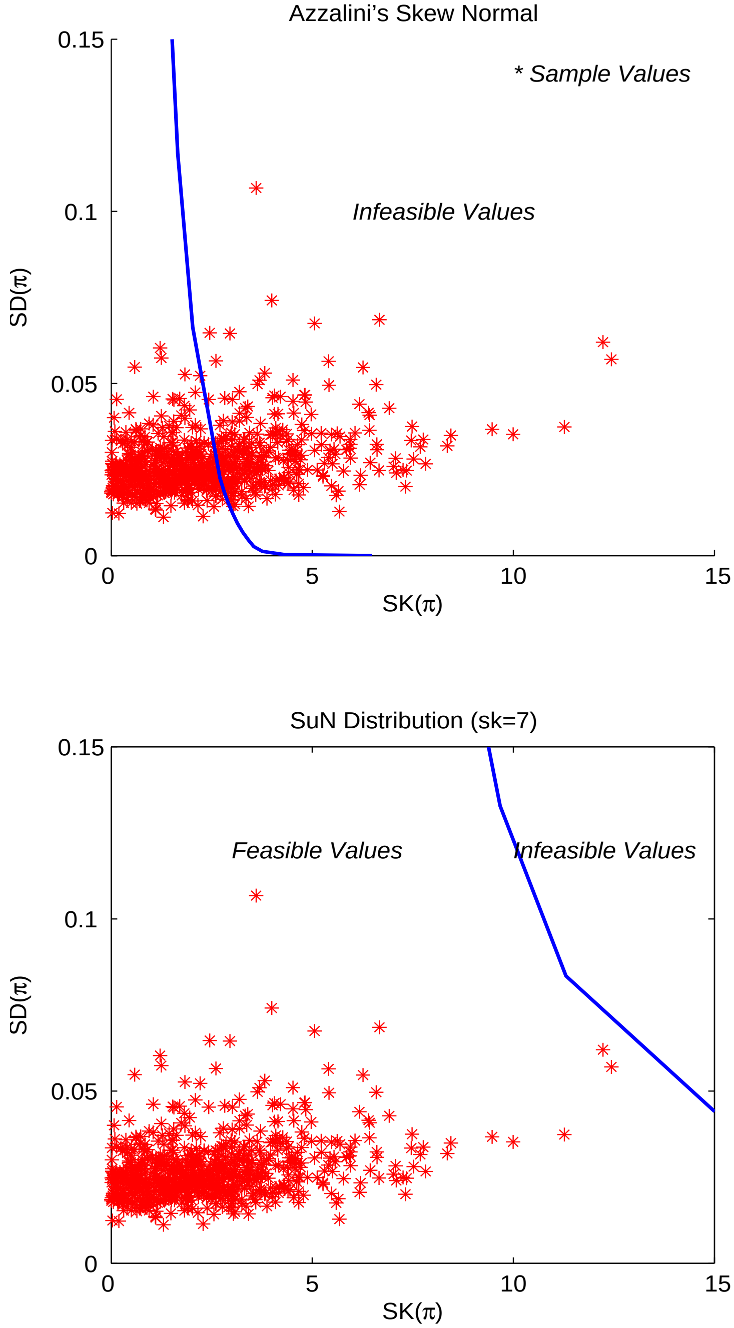


Fig. 2-4.: Feasible Set of SD and Skewness of Industry Price Changes: Skew Normal and SuN Distribution of Price Shocks (Monthly).

and the highest coordinate $\{sk = sk^*, kt = kt^*\}$ where

$$sk^* = \pm \frac{2\,(4-\pi)}{(\pi-2)^{3/2}} \approx \pm 0.9953$$

$$kt^* = \frac{3\pi^2 - 4\pi - 12}{(\pi-2)^2} \approx 3.9670$$

Because of this limited nature of skewness and kurtosis, Ball and Mankiw's numerical analysis cannot generate the range of skewness of observed industry price changes,(-4.2430,4.1443), in their sample. Figure 2-3 shows that one third of their sample has the standard deviation and skewness outside of the feasible set[10]. Furthermore, the numerical analysis of Ball and Mankiw cannot examine the effects of kurtosis of price shocks on the mean of industry price changes because the kurtosis cannot take only one value for a given skewness.

There are many alternative asymmetric leptokurtic distributions that are more flexible than Azzalini's skew normal. We consider in our numerical analysis Johnson's S_u-normal (SuN) distribution which is a hyperbolic sine transformation, $\epsilon = \sinh(X)$, of a normal random variable $X \sim N(\mu, \sigma^2)$. The density function of this SuN random variable is

$$f\left(\epsilon; \mu, \sigma^2\right) = \frac{1}{\sqrt{2\pi\sigma^2\left(\epsilon^2+1\right)}} \exp\left\{-\frac{\left(\sinh^{-1}(\epsilon)-\mu\right)^2}{2\sigma^2}\right\}$$

Figure 2-2 shows the set of feasible values of positive skewness and kurtosis of this distribution, which is the set below the upper boundary line.

Numerical analysis similar to Ball and Mankiw's analysis are conducted for the SuN distribution of price shocks. The SuN distribution has mean zero in all

[10] Sample values in Figure 2-3 show the pairs of standard deviation and absolute values of skewness in Ball and Mankiw's data set. The example of monthly data is presented in Figure 2-4.

cases. For the analysis of the relationships between the standard deviation σ_ϵ of price shocks and the moments (μ_ϵ and σ_ϵ) of industry price changes, we consider 21 evenly spaced values of σ_ϵ in the interval $[0.05, 0.25]$, the range of values considered in Ball and Mankiw analysis. These relationships are found for four values of lower skewness $sk = \{-1.0, -0.6, 0.6, 1.0\}$ and four values of low kurtosis $kt = \{5, 10, 15, 20\}$. Higher values of skewness $sk = \{-4, -2, 2, 4\}$ are paired with higher kurtosis $kt = \{45, 50, 55, 60\}$. This is necessary because the minimum feasible value of kurtosis varies with the skewness as the feasible set in Figure 2-2 indicates. The relationship between the moments of price shocks and the moments of industry price changes reveal the similar patterns regardless of the value of kurtosis. Therefore, we report the results for $kt = 10$ and $kt = 50$. Let σ_ϵ, sk_ϵ, and kt_ϵ denote the standard deviation, skewness, and kurtosis coefficient of price shocks, and let μ_π denote the mean of industry price changes (inflation), respectively.

All three papers argued that there is a positive relationship between inflation and dispersion of relative price shocks. Our first question is how changes in dispersion can affect inflation. Figure 2-5 shows the relationship between σ_ϵ and μ_π for each model. In Fischer (1982) model, μ_π rises monotonically with σ_ϵ as in the upper panels, so there is a positive relationship between μ_π and σ_ϵ. In Ball and Mankiw (1994), for a lower skewness on the left panel, σ_ϵ has a positive effect on μ_π, but for $sk = -4$, there is a weakly negative relation. The bottom panels show the result of Ball and Mankiw (1995). For a positive skewness, there is a positive relation. However, when the skewness is negative, the relationship is negative. The example presented in Figure 2-5 clearly shows this negative relationship. In this case, the problem is that μ_π depends on σ_ϵ ($\mu_\pi = a + b\sigma_\epsilon$) and the effect of σ_ϵ depends on the sk_ϵ ($b = c + dsk_\epsilon$).

These two relationships can be combined as

$$\mu_\pi = a + (c + dsk_\epsilon)\,\sigma_\epsilon$$

This means that marginal effects of dispersion $(c + dsk_\epsilon)$ depends on the sign and magnitude of skewness[11]. However, past studies including Ball and Mankiw (1995) did not clearly show that the direction of skewness can affect the marginal effect of dispersion. Vinning and Elwertowski (1976), Parks (1978),and Domberger (1987) considered the relationship between dispersion and absolute value of inflation (or squared value of inflation), just based on the empirical data. However, if we use absolute value or squared value of inflation, we cannot capture the direction of the marginal effect of dispersion. However, our numerical results clearly show that the direction of dispersion-inflation relation is determined by the sign of skewness. Thus, we can say that there a positive relationship between dispersion and the absolute value of inflation, but the direction of dispersion depends on the skewness. This finding is new in this literature.

In addition, a higher sk_ϵ raises the effects of σ_ϵ on μ_π in two Ball and Mankiw's model but it lowers the effects σ_ϵ on μ_π in Fischer model. This implies that there are substantial interaction effects between σ_ϵ and sk_ϵ in all three models. Therefore, dispersion alone is not enough to capture the effects of σ_ϵ on μ_π in Fischer (1982) and Ball and Mankiw (1994).

Our second question is how inflation varies with the changes in skewness. By this analysis, we show whether skewness is necessary in Fischer (1982) and Ball and Mankiw (1994). Figure 2-6 shows the relationship between sk_ϵ and μ_π for σ_ϵ =

[11] We can capture these interaction effect by including cross product terms in the regression since $\mu_\pi = a + (c + dsk_\epsilon)\,\sigma_\epsilon = a + c\sigma_\epsilon + dsk_\epsilon\sigma_\epsilon$.

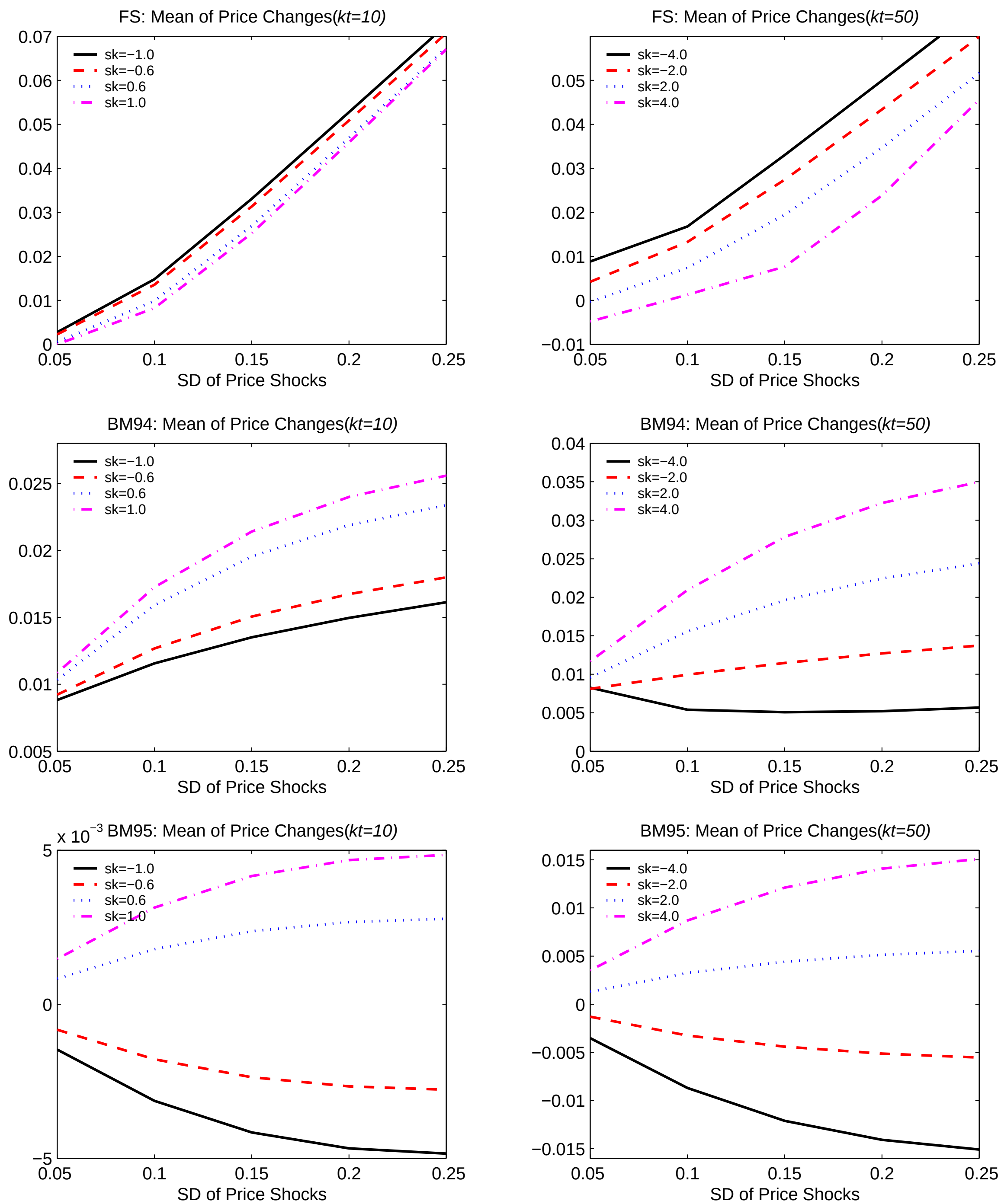


Fig. 2-5.: SD of Price Shocks and Mean of Industry Price Changes: SuN Distribution of Price Shocks.

$\{0.05, 0.10, 0.15, 0.20, 0.25\}$ over the range of sk_ϵ in $[-1, 1]$ for $kt = 10$ and over the range of sk_ϵ in $[-4, 4]$ for $kt = 50$. In two Ball and Mankiw's models, there is a monotonic relationship between sk_ϵ and μ_π. But, in Fischer model, there is a weakly negative relationship. The results for high skewness on the right panel are similar to the case of low skewness, except for that the relationship are less linear. In addition, higher σ_ϵ raises the effects of sk_ϵ on μ_π. Therefore, it is necessary to consider skewness in both Fischer (1982) and Ball and Mankiw (1994).

The third question is how inflation depends on kurtosis and whether there is a role of kurtosis to explain inflation. The relationship between the kurtosis of price shocks kt_ϵ and the mean μ_π of industry price changes are presented in Figure 2-7 for a positive skewness $sk_\epsilon = 1$ on the left panel and a negative skewness $sk_\epsilon = -1$ on the right panel. For a positive skewness,here is a nonlinear negative relationship between sk_ϵ and μ_π. But, for a negative skewness, there is a negative relationship[12] except for the case of Fischer (1982). In addition, a higher σ_ϵ raises the effects of kt_ϵ on μ_π in all cases. Also, there are substantial interaction effects between kurtosis and moments.

Numerical analyses reveal a few important results. First, the source to generate inflation-moment relationships is the change in the properties of the underlying distribution regardless of models. To capture the property of the distribution, Fischer (1982), Ball and Mankiw (1994) considered dispersion alone while Ball and Mankiw (1995) included skewness. However, kurtosis also capture the property of the underlying distribution. Second, there is a positive relationship between dispersion (kurtosis) and absolute value of inflation, but the direction and magnitude of marginal effect depends on the skewness. A positive inflation-skewness relationship depends on model assumption. A positive relationship is more intuitive. A negative inflation-skewness

[12]The example shown in Figure 2-5 shows this negative relationship.

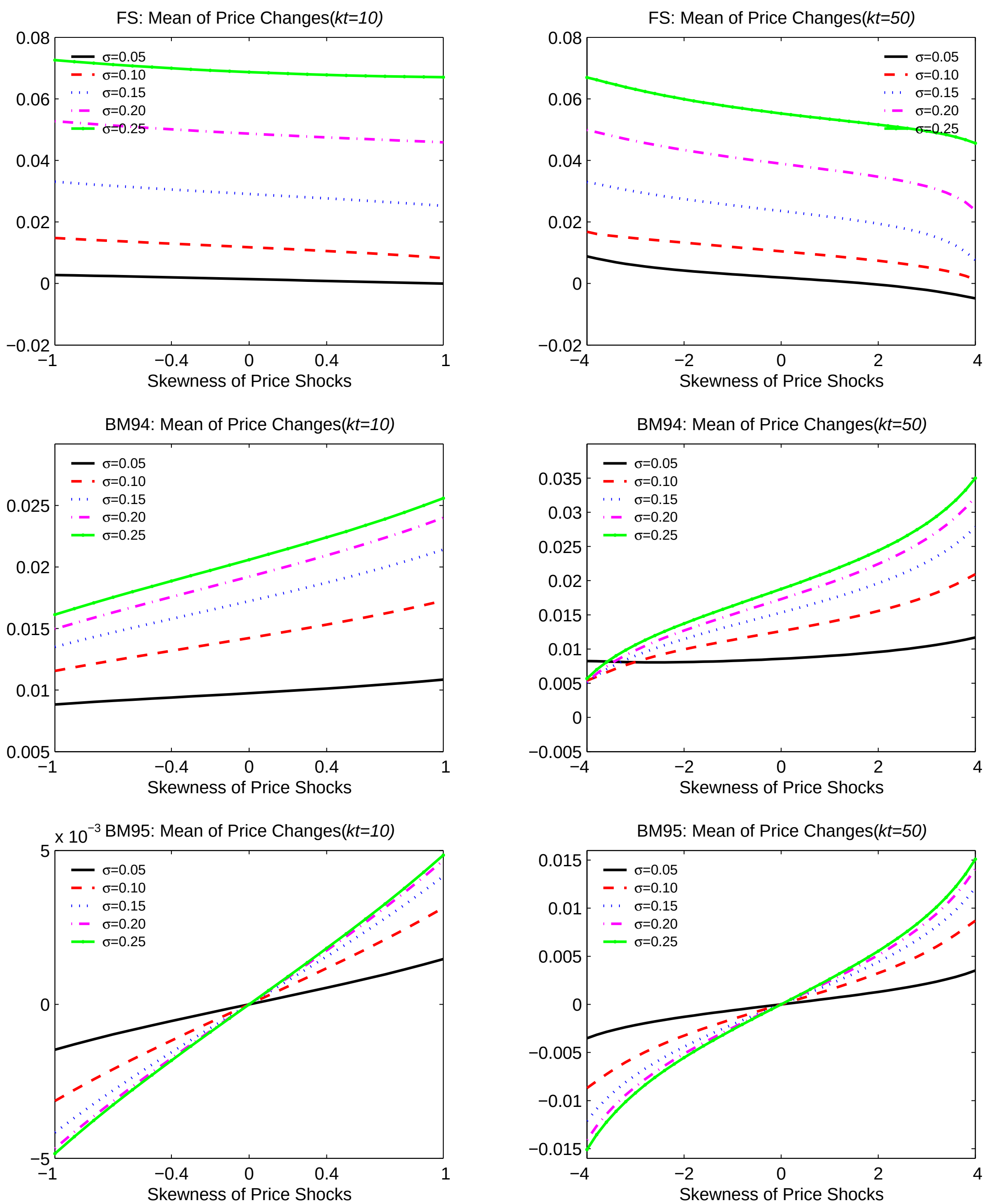


Fig. 2-6.: Skewness of Price Shocks and Mean of Industry Price Changes: SuN Distribution of Price Shocks.

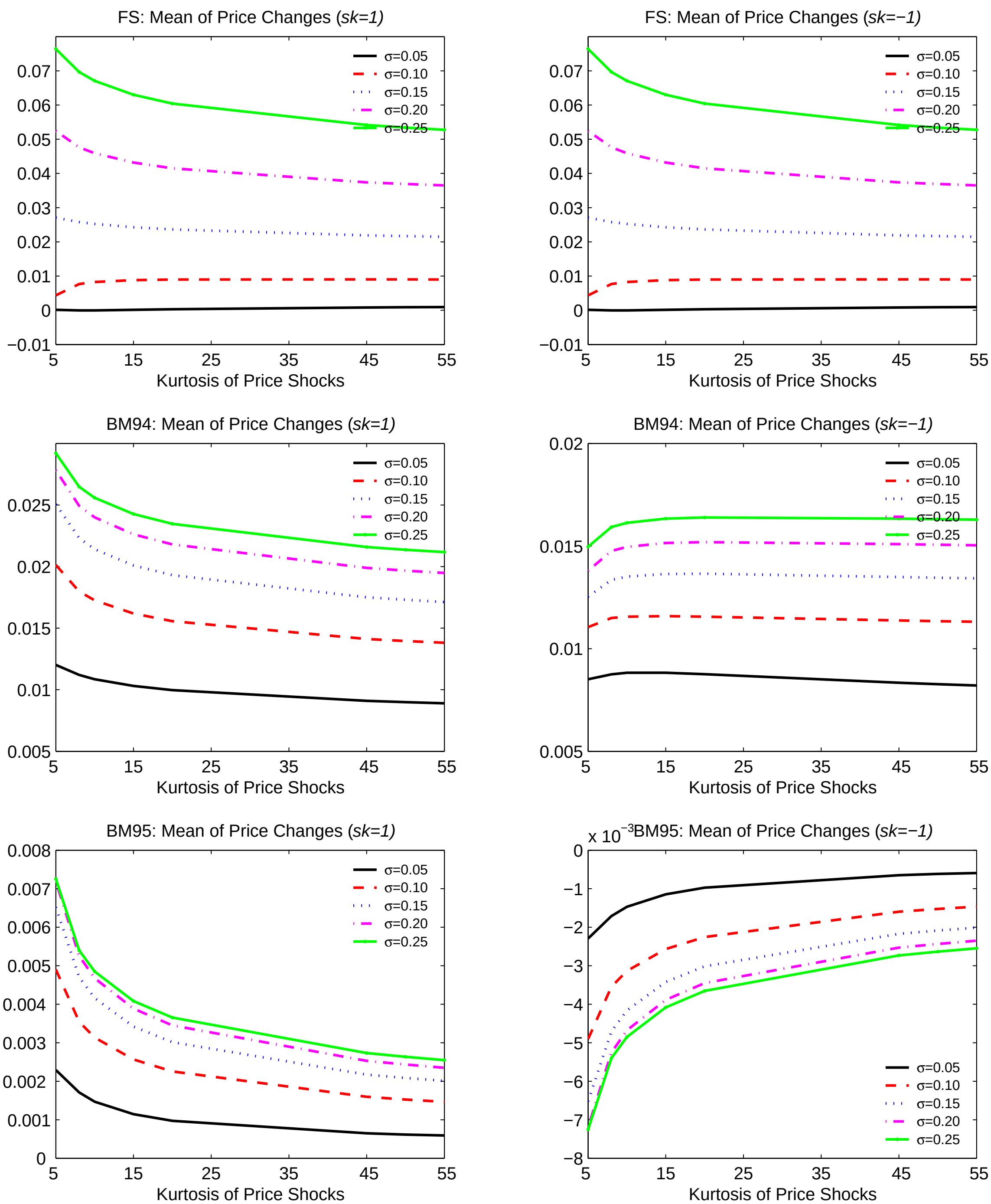


Fig. 2-7.: Kurtosis of Price Shocks and Mean of Industry Price Changes: SuN Distribution of Price Shocks.

relationship obtained from Fischer model is due to their assumption. Under the downward price rigidity, firms do not respond to the negative shock even though it is large. Thus, it is more appropriate to assume menu cost in the analysis of the effects of relative price shocks. Third, as Ball and Mankiw noted, a large standard deviation magnifies the effect of skewness on the mean, or a large skewness magnifies the effect of standard deviation on the mean. This led Ball and Mankiw to include an interaction term in the regression equation of inflation. The kurtosis also has a similar impact on the effects of standard deviation and skewness on the mean. Thus, it is desirable to include interaction terms of all three moments in the inflation regression equation.

3. Numerical Analysis for the Empirical Issues

It should be noted that the theoretical model suggests a relationship between the aggregate (mean) inflation and unobservable moments (dispersion, skewness, and kurtosis) of the distribution of underlying price shocks. Therefore, there is an empirical limitation because of the unobservablity of underlying price shock distribution. Ball and Mankiw estimated unobservable moments by the corresponding moments of observed industry price changes and used them as explanatory variables in a linear regression equation of inflation rates. This is valid procedure only if the two sets of moments are linearly related. Therefore, we present a numerical analysis to identify whether there is a linear relationship between both of them.

As shown in Figure 2-8, the relationship between σ_ϵ and σ_π are almost linear relationships in all three models. The relationship between sk_ϵ and sk_π are also almost linear except for the Fischer model of the higher kurtosis values in Figure 2-9. Also, there is a linear relationship between kt_ϵ and kt_π in Figure 2-10. The results for high kurtosis on the right panel are similar to the case of low kurtosis. Hence, the measurement errors in using σ_π, sk_π, kt_π for σ_ϵ, sk_ϵ, kt_ϵ and are minimal, and the only effect will be the magnitudes of the coefficients of these variables in the inflation regression equation.

Our numerical analyses reveal that the standard deviation, skewness and kurtosis of observed industry price changes are almost linearly related to the counterparts of underlying price shocks. Therefore, estimators of the moments of observed price changes can be used for the moments price shocks with negligible measurement errors in the linear regression analysis of the moments of price shocks on aggregate inflation.

C. Alternative Kurtosis

Our key idea is to introduce kurtosis and try to capture the additional properties of the distribution. Pearson's kurtosis is widely used. However, the kurtosis concept is so unclear that it is difficult to interpret since it captures both peakedness and tail heaviness as a single measure. It has been defined in many ways according to the focus on the properties. This imply that different properties of distribution can be captured by different kurtosis.

Recently, Seier and Bonett (2003) introduced an alternative kurtosis measures which are defined as

$$K_1(b) = E\left[ab^{-|z|}\right], \quad 2 \leq b \leq 20$$

$$K_2(b) = E\left[a\left(1 - |z|^b\right)\right], \quad 0 < b \leq 1$$

where z is the standardized variable, a is a normalizing factor to make kurtosis equal to 3 for normal distribution and parameter b is restricted to particular range. We call Seier and Bonnett's measures the 'SB kurtosis' for convenience. SB kurtosis gives more importance to the central part of the distributions while Pearson's measure, $E[z^4]$, gives more weights to the tail part of the distributions. As a result, SB

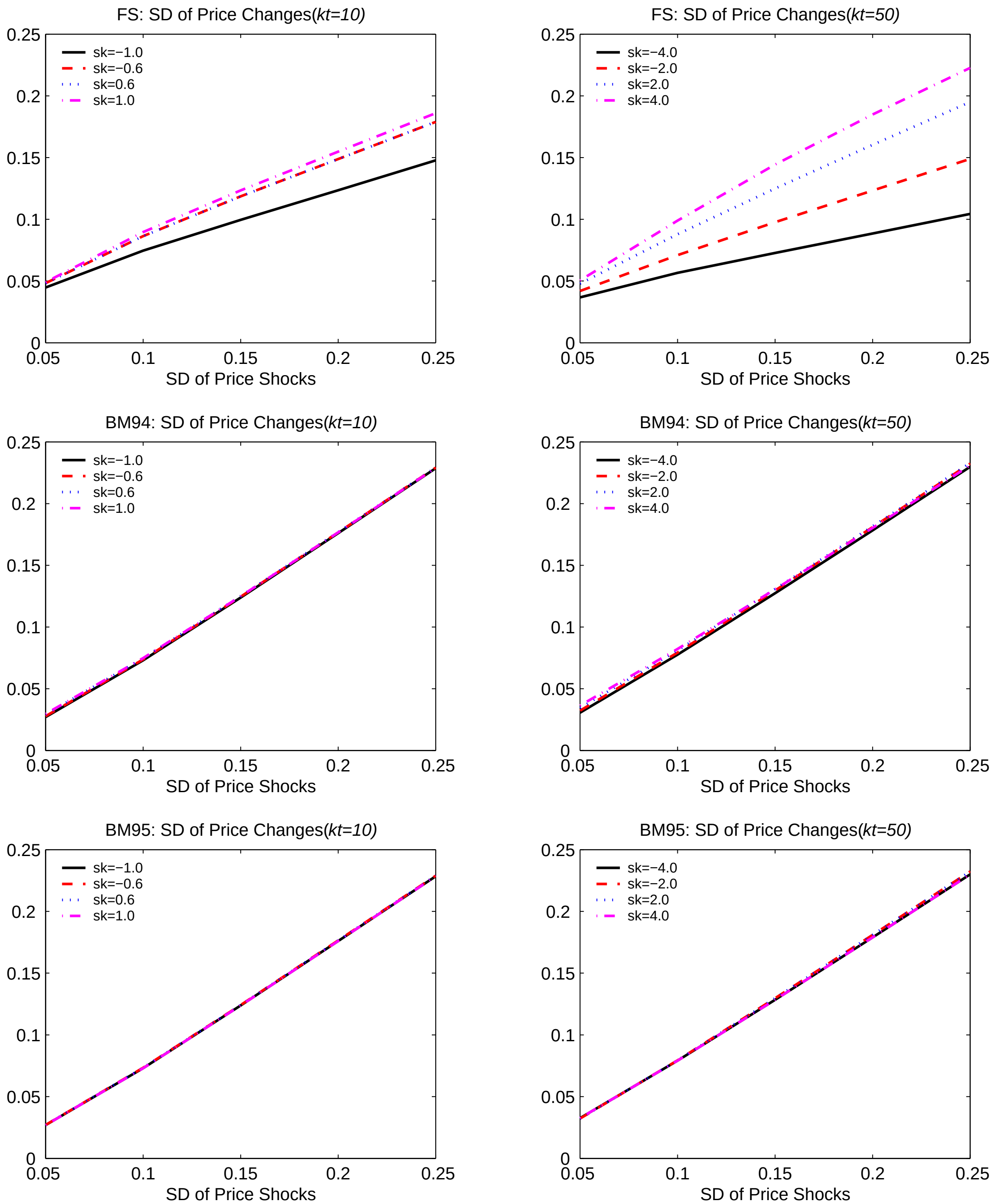


Fig. 2-8.: Linear Relationships between Price Shocks and Price Changes (SD): SuN Distribution of Price Shocks.

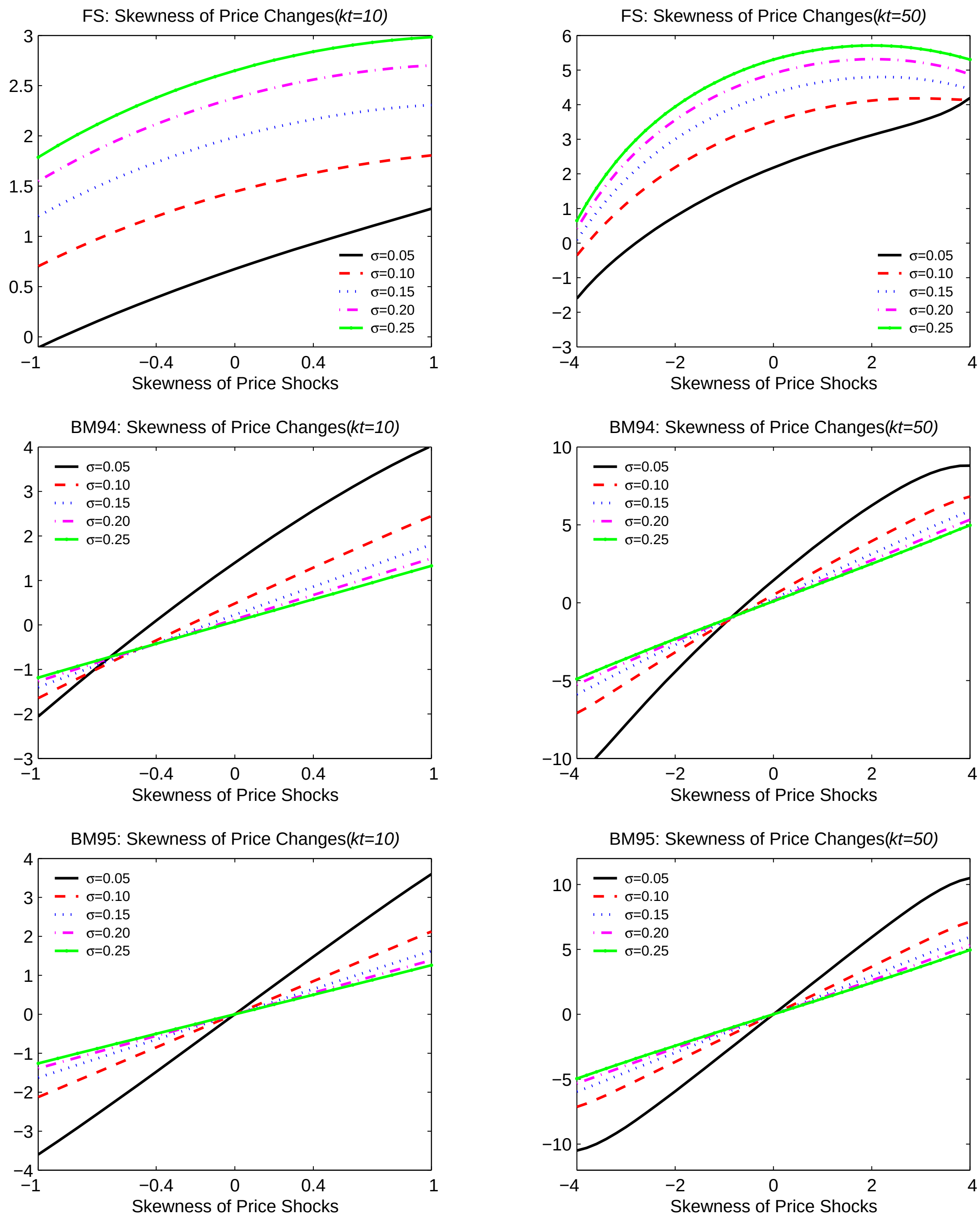


Fig. 2-9.: Linear Relationships between Price Shocks and Price Changes (Skewness): SuN Distribution of Price Shocks.

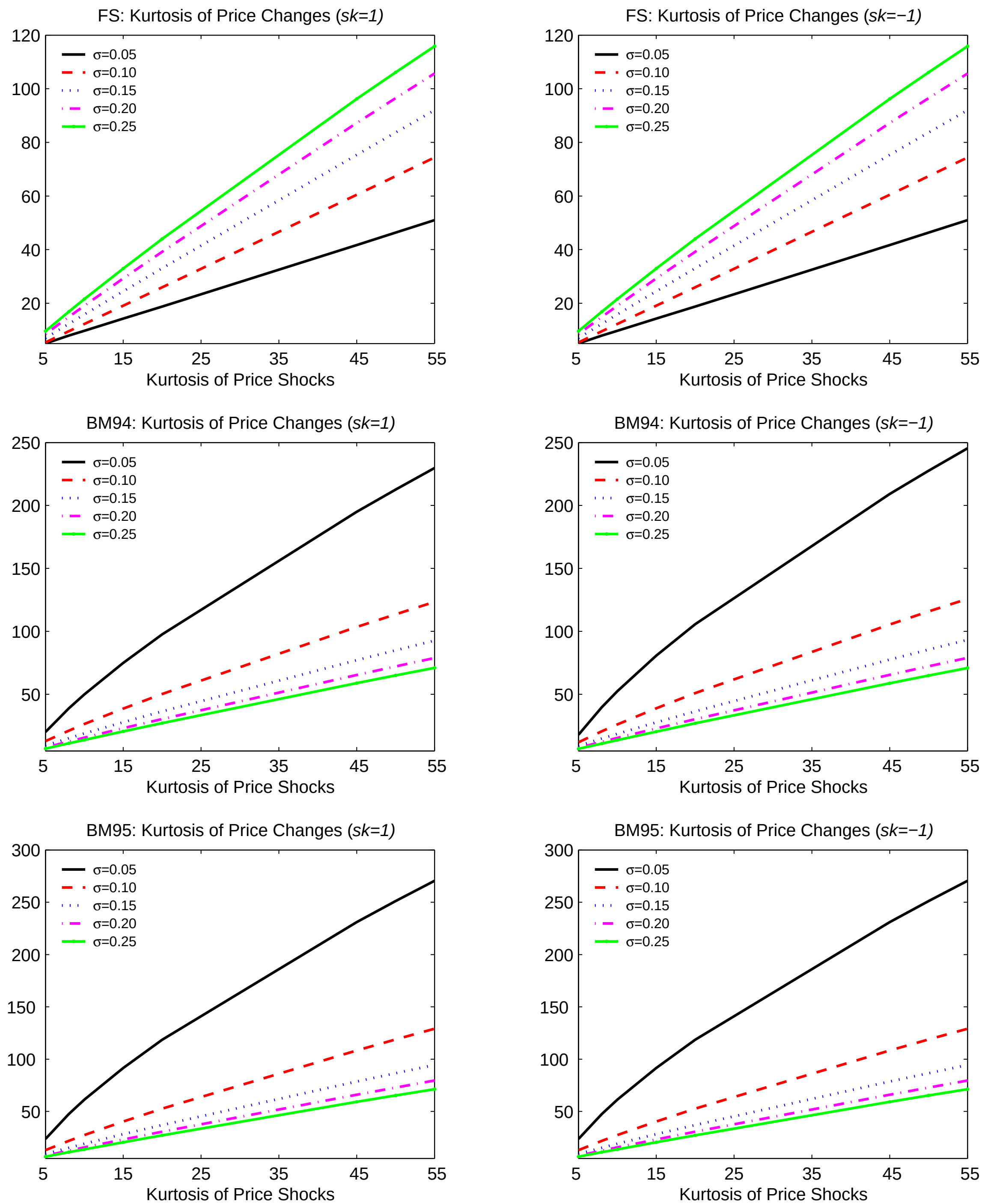


Fig. 2-10.: Linear Relationships between Price Shocks and Price Changes (Kurtosis): SuN Distribution of Price Shocks.

kurtosis is more likely to capture peakedness than tail heaviness. In particular, they argued that SB kurtosis "tends to be less correlated with skewness across a set of skewed distribution". This has something to do with our idea to capture independent kurtosis effect.

Figure 2-11 shows that two distributions with different shapes can have the same fourth moment. One is from the SuN density function and the other is from Normal Inverse Gaussian (NIG) density function, one of the popular distributions in the finance literatures. Even though two distributions have identical Pearson kurtosis, they can take quite different shapes, in particular, central part of the distribution. SB measures are likely to provide us with more information about the difference of two distributions in terms of peakedness. Therefore, we expect that the performance of both measures can be different in capturing the peakedness of the distribution.

Compared to the other macro data, the observed cross sectional price changes show strong excess kurtosis[13]. To extract the features of actual data, we estimate the density function. Figure 2-12 provides the kernel density estimation for annual and monthly data. The most striking distributional feature of the price changes is its peakedness. In particular, the density function of monthly data is more peaked than that of annual data. Thus, it is worthy of considering alternative measures which are likely to capture data properties better.

However, Ball and Mankiw's arguments are valid only if moments of underlying price shocks and corresponding moments of observed price changes are linearly related. Therefore, we compare the linear relationships of both measures. As shown in Figure 2-13 and 2-14, there is also a linear relationship between kt_ϵ and kt_π in terms of SB measures. It implies that we can use SB measures as a proxy for the unob-

[13]The ranges of kurtosis of price changes for annual and monthly data are $(4, 100)$ and $(4, 240)$, respectively.

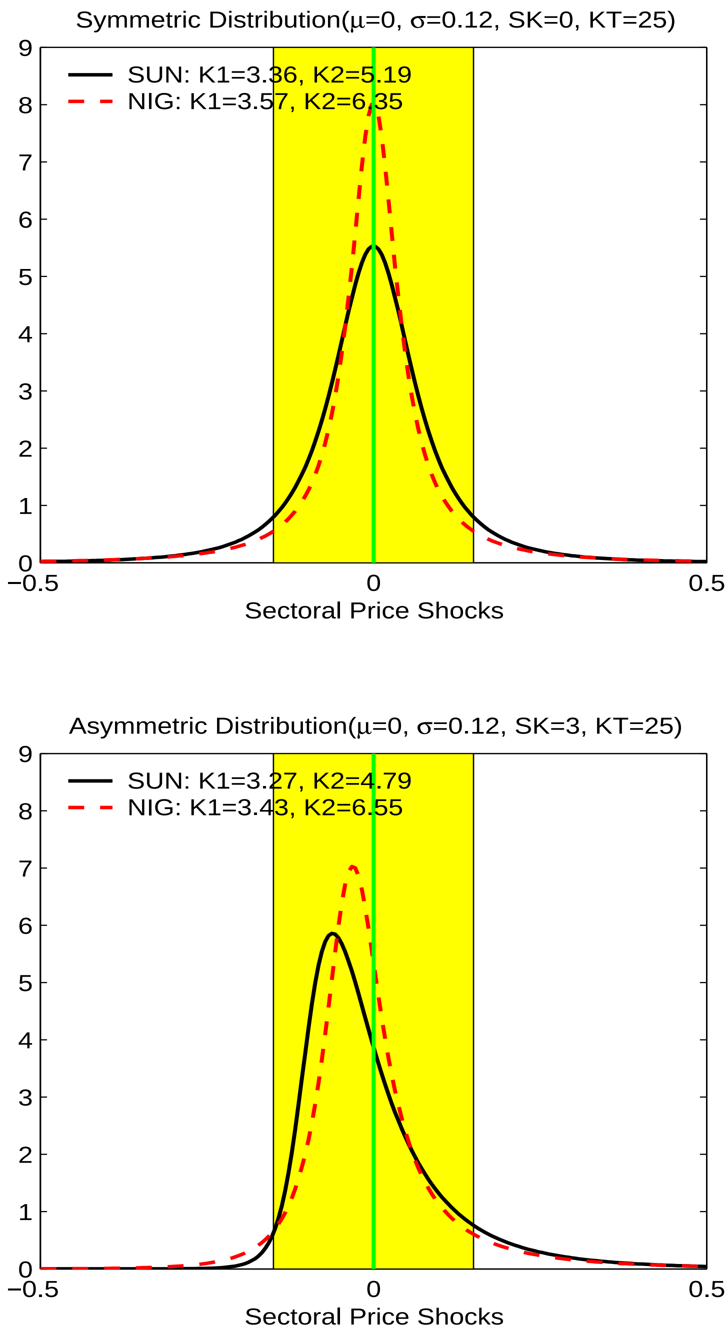


Fig. 2-11.: Same Fourth Moment and Different Shape of Distribution: SuN Distribution of Price Shocks.

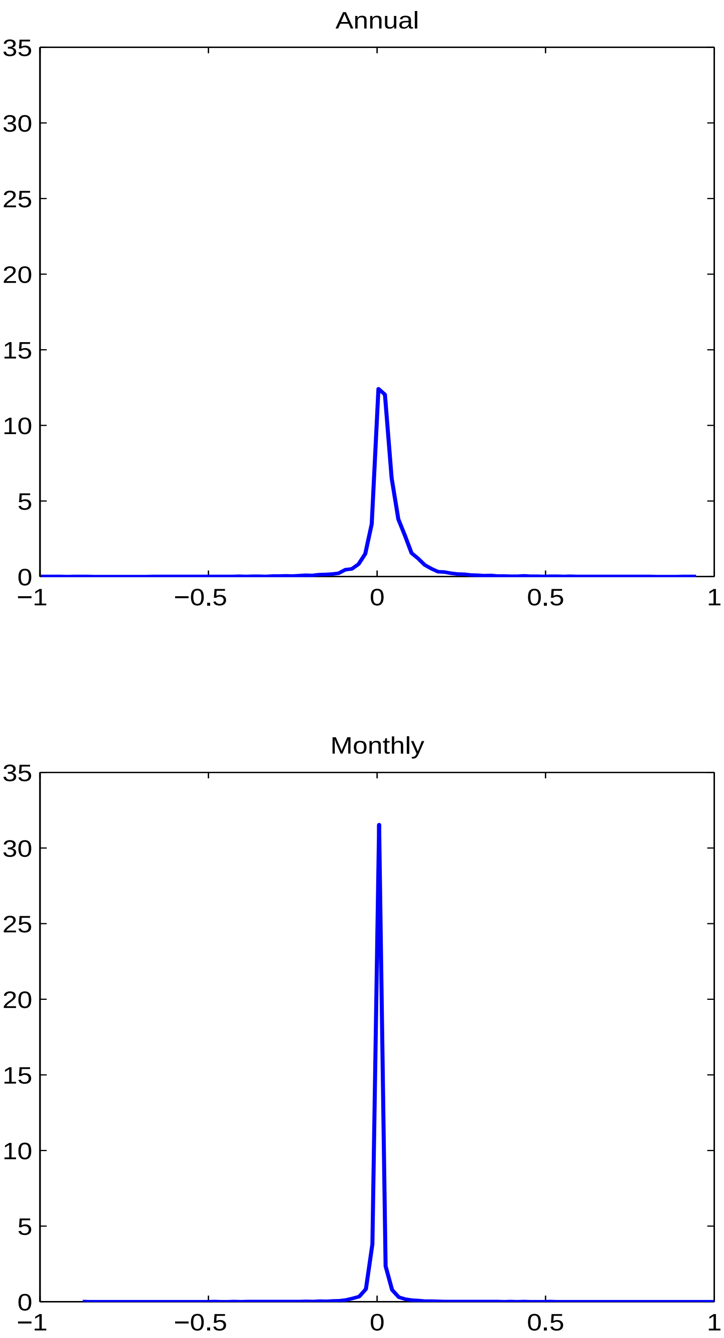


Fig. 2-12.: Kernel Density Estimation for Price Changes (1948-2006).

Notes: The estimations were carried out using Matlab Statistics Toolbox. They were based on a normal kernel function and the default bandwidth.

served price shocks in a linear regression equation of inflation. Note that Pearson's kurtosis shows fan-shaped linear relationship while SB kurtosis shows parallel linear relationship.

We conduct simple regression experiments to compare the performance of both kurtosis measures as an ideal proxy in the regression analysis. Unlike former numerical analysis, we consider a more general situation where all moments are changing at the same time. We generate the experimental price shocks by changing the four parameters of SuN distribution. These combinations of parameters are randomly chosen from the particular range, which can generate moments of price changes within the range of the actual data. We run the regression of moments of price shocks on the moments of price changes. Table 2-1 and Table 2-2 shows regression results of kurtosis of price shocks on moments of price changes by using both Pearson and SB kurtosis. Explanatory power of SB kurtosis in terms of $\overline{R}^2$ is better, particularly in the monthly experiment. Therefore, based on regression results, we expect that SB kurtosis can be a more ideal proxy for the price shocks.

D. Specification of Empirical Models

Based on numerical analysis, we found that it is desirable to include kurtosis and additional interaction terms in the inflation regression equation. In order to reflect this, we consider a single comprehensive specification that includes six major explanatory variables which turn out to be valid from our numerical analysis. It covers theoretical relationships implied by Fischer (1982), Ball and Mankiw (1994), and Ball and Mankiw (1995). Following Ball and Mankiw, the annual version of inflation equation

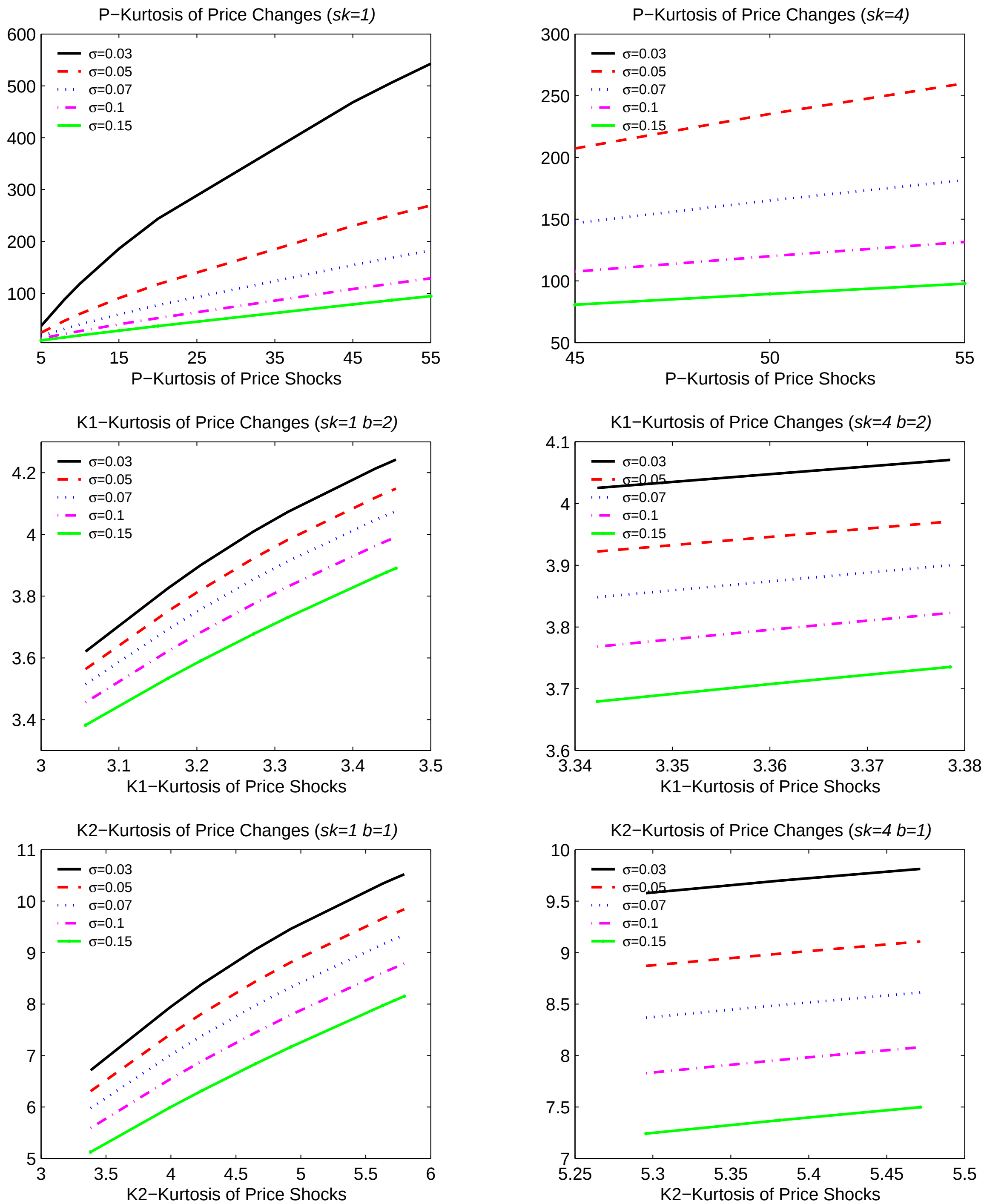


Fig. 2-13.: Comparisons of Kurtosis: Relationships between Price Shocks and Price Changes: SuN Distribution of Price Shocks.

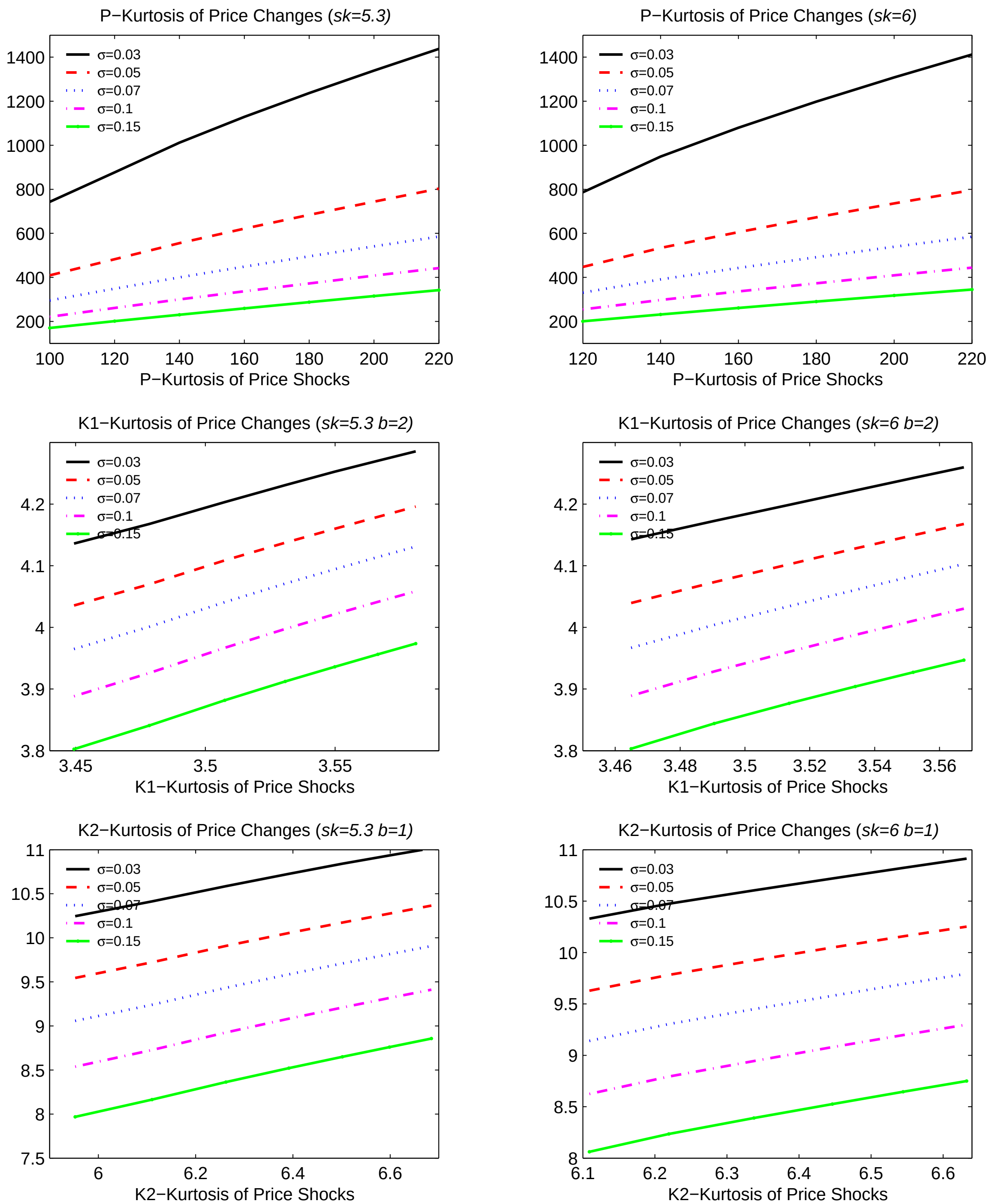


Fig. 2-14.: Comparisons of Kurtosis: Relationships between Price Shocks and Price Changes: SuN Distribution of Price Shocks.

Table 2-1.: Regression of Price Shocks on Price Changes: Annual Data.

	Pearson KT			SB KT		
	SD_ϵ	SK_ϵ	KT_ϵ	SD_ϵ	SD_ϵ	KT_ϵ
Constant	0.013	0.000	-3.874	0.005	0.000	-0.023
	(0.000)	(1.000)	(0.000)	(0.139)	(1.000)	(0.802)
SD_π	0.941	0.000	110.650	0.931	0.000	1.963
	(0.000)	(1.000)	(0.000)	(0.000)	(1.000)	(0.000)
SK_π	0.000	0.551	0.000	0.000	0.551	0.000
	(1.000)	(0.000)	(1.000)	(1.000)	(0.000)	(1.000)
KT_π	0.000	0.000	0.334	0.004	0.000	0.903
	(0.000)	(1.000)	(0.000)	(0.000)	(1.000)	(0.000)
$\overline{R}^2$	0.925	0.750	0.605	0.918	0.750	0.674

Table 2-2.: Regression of Price Shocks on Price Changes: Monthly Data.

	Pearson KT			SB KT		
	SD_ϵ	SK_ϵ	KT_ϵ	SD_ϵ	SD_ϵ	KT_ϵ
Constant	0.010	0.000	-6.398	0.042	0.000	-1.079
	(0.000)	(1.000)	(0.000)	(0.000)	(1.000)	(0.000)
SD_π	0.928	0.000	389.140	0.970	0.000	2.076
	(0.000)	(1.000)	(0.000)	(0.000)	(1.000)	(0.000)
SK_π	0.000	0.496	0.000	0.000	0.496	0.000
	(1.000)	(0.000)	(1.000)	(1.000)	(0.000)	(1.000)
KT_π	0.000	0.000	0.199	-0.009	0.000	1.208
	(0.000)	(1.000)	(0.000)	(0.000)	(1.000)	(0.000)
$\overline{R}^2$	0.862	0.748	0.539	0.867	0.748	0.801

Notes: p-values are reported in parenthesis below the estimates.

can be linearly specified as

$$\begin{aligned}\pi_t &= \alpha + \beta\pi_{t-1} + \gamma_1 SD_t + \gamma_2 SK_t + \gamma_3 \left(SD_t \cdot SK_t\right) \\ &\quad + \gamma_4 KT_t + \gamma_5 \left(SD_t \cdot KT_t\right) + \gamma_6 \left(SK_t \cdot KT_t\right) + \epsilon_t\end{aligned}$$

where π_t is the inflation rate, π_{t-1} is a lagged inflation[14] and ϵ_t is assumed to be an i.i.d disturbance term with a zero mean and a finite variance.

Most previous studies on annual data follow Ball and Mankiw's specification. However, in case of studies on monthly data, there is a wide range of specification. In particular, the selection of lags for regressors are quite different. We classify the specifications into two types of models based on the lag selections of regressors.

First type of model uses only one lagged dependent variable, following Ball and Mankiw's annual version. Assarsson and Riksbank (2003) and Caraballo and Dabus (2005) specified inflation equation as

$$\pi_t = \alpha + \beta\pi_{t-1} + \gamma_1 SD_t + \gamma_2 SK_t + \gamma_3 \left(SD_t \cdot SK_t\right) + \epsilon_t$$

Second type uses more than one lagged dependent variables. Verbrugge (2002) in his analysis of the relationship between inflation and unweighted triples U statistic specified inflation equation as

$$\pi_t = \alpha + \sum_{k=1}^{9} \beta_k \pi_{t-k} + \gamma Triple_t + \epsilon_t$$

Similarly, Caraballo and Usabiaga (2004) include two lagged dependent variables:

$$\pi_t = \alpha + \sum_{k=1}^{2} \beta_k \pi_{t-k} + \gamma_1 SD_t + \gamma_2 SK_t + \epsilon_t$$

[14]Following Ball and Mankiw, the lagged inflation is used as a proxy for the expected inflation.

Using lagged inflation is necessary to remove the autocorrelation in the residuals since inflation is observed to have considerable persistence. Aside from these two specification, there is a specification with lagged dispersion and skewness.[15] However, we cannot justify this specification because the purpose of our study is to capture the effect of the changes in the distribution over time.

We follow the second type in order to capture data property and we select the twelve lags as a counterpart of annual specification. The same lag structure is widely used in Phillips Curve literature. Therefore, the monthly version of inflation equation is linearly specified as

$$\begin{aligned}\pi_t &= \alpha + \sum_{k=1}^{12} \beta_k \pi_{t-k} + \gamma_1 SD_t + \gamma_2 SK_t + \gamma_3 (SD_t \cdot SK_t) \\ &\quad + \gamma_4 KT_t + \gamma_5 (SD_t \cdot KT_t) + \gamma_6 (SK_t \cdot KT_t) + \epsilon_t\end{aligned}$$

[15]Aucremanne, Brys, Hubert, Rousseeuw, and Struyf (2002) include lags for dispersion and skewness, not lagged inflation.

$$\pi_t = \alpha + \sum_{k=0}^{2} \gamma_{1k} SD_{t-k} + \sum_{k=0}^{2} \gamma_{2k} SK_{t-2} + \epsilon_t$$

Their results show that the coefficients for the second lags are not significant.

We consider seven different sets of regressors:

(i) $\{SD, SK\}$

(ii) $\{SD, SK, (SD \cdot SK)\}$

(iii) $\{SK, (SD \cdot SK)\}$

(iv) $\{SD, SK, (SD \cdot SK), KT\}$

(v) $\{SD, SK, (SD \cdot SK), KT, (SD \cdot KT)\}$

(vi) $\{SD, SK, (SD \cdot SK), KT, (SK \cdot KT)\}$

(vii) $\{SD, SK, (SD \cdot SK), KT, (SD \cdot KT), (SK \cdot KT)\}$

First three sets of regressors are Ball and Mankiw's model and the other four sets are our model with kurtosis measures. The individual and joint significance of regressors are evaluated by p-values. To evaluate the goodness of fit, we investigate four different criteria: i) $\overline{R}^2$, ii) Root Means Squared Errors (RMSE), iii) Akaike Information Criterion (AIC), iv) Schwartz Criterion (SC).

E. Empirical Results

1. Data

The data used in this paper is the four-digit level annual and monthly Producer Price Indices (PPI) over the available sample period of 1947-2006. This is the data set obtained from the Bureau of Labor Statistics (BLS)[16]. The price changes are defined as

$$\pi_{it} = \ln(P_{it}) - \ln(P_{it-1})$$

[16] Historical data files are directly available at following FTP site: ftp://ftp.bls.gov/pub/time.series/wp/

where P_{it} is the index for industry i at time t, following the standard definition in the literature. We use the 1997 weights that the BLS used in computing the overall PPI. The moments for the commodity price changes are computed by the classical measure of weighted standard deviation, skewness, and kurtosis of cross sectional sample. We use the published inflation rate as a mean inflation. For monthly data, we use seasonally unadjusted price changes.

The four-digit level PPI has missing observations for some industries. Ball and Mankiw used only non-missing data when they computed the moments of price changes. However, BLS computes overall inflation after estimating missing observations[17]. Therefore, it is more reasonable to consider corresponding price changes used in the computation of published inflation. By the same procedures in estimating missing item's price quote, BLS also estimates the movements of higher level missing indexes in PPI using the 'moving code' which is not published. This moving code can be the average movement of the other items within the same category. In most cases, it would be reasonable to use a higher level index to estimate the movement

[17] The Chapter 14 of the BLS Handbook of Methods gives an extensive description of its methodology: "If no price report from a participating company has been received in a particular month, the change in the price of the associated item will, in general, be estimated by averaging the price changes for the other items within the same cell (that is, for the same kind of products) for which price reports have been received". A link to this publication is http://stats.bls.gov/opub/hom/homch14_itc.htm. To figure out how BLS estimates the missing observation, we consider a following specific example.

	Item 1	Item 2	Item 3	Industry
weight	0.5	0.3	0.2	1
period 1	120	110	115	116
period 2	130	Missing	120	?

First step is to normalize the weights of two reported items: Item 1's weight is $\frac{0.5}{0.7} = 0.71$ and Item 2's weight is $\frac{0.2}{0.7} = 0.29$. Next step is to calculate the average growth rate of two items using normalized weights: $\frac{130}{120} \times 0.71 + \frac{120}{115} \times 0.29 = 1.07$. Item 2's missing index for period 2 is estimated by applying this growth rate to item 2's index for period 1: $1.07 \times 110 = 118$. Next, industry index is the weight average of all three items: $0.5 \times 130 + 0.3 \times 118 + 0.2 \times 120 = 124$.

of missing observations because a higher level index is calculated after estimating missing observations. Therefore, we estimate these missing observations using the movements of higher level indexes instead of unpublished moving code.

2. Estimation Results

Our numerical analyses show that in most cases, three models produce the similar pattern of inflation-moment relationships. However, there is a big difference in the direction of marginal effect of dispersion (kurtosis). In Ball and Mankiw (1995) model, it depends on the sign of skewness while it does not in Fischer (1982) and Ball and Mankiw (1994) models. So, we ask which theoretical implication is consistent with the feature of data. If the model assumption is right, it is likely to capture the data property well. Thus, our first empirical question is "Which of the three models is data-consistent in terms of theoretical implication?".

By numerical analysis, we find that there is role of the kurtosis and alternative kurtosis. Thus, second empirical question is "If we additionally consider the independent kurtosis effects that previous studies omitted, how much can we improve the approximation of inflation in terms of the goodness of fit?". Third question is "Which of the two kurtosis measures performs better in terms of the goodness of fit?". We will answer these three questions based on regression analysis.

To answer the first question, we examine the marginal effects of dispersion (kurtosis) with the skewness. Figure 2-15 shows the marginal effects of dispersion in annual data. In most sample periods, overtime marginal effect totally depends on the sign of skewness as in the upper panel. It is linearly related to the skewness. This relationship holds for monthly data as in Figure 2-16. In addition, the marginal effects of kurtosis also depend on the sign of skewness in Figure 2-17 and Figure 2-18. Based on these results, we can say that Ball and Mankiw (1995) model is more data-consistent. In addition, note that the direction of dispersion effect and kurtosis effect is opposite. So if we neglect the kurtosis, it can produce misleading results. Thus, this difference between dispersion and kurtosis emphasizes the role of the kurtosis.

Next, we turn to the second and third questions. To find the dominant results irrespective of the chosen sample periods and to compare the performance of models visually, we conduct the rolling regressions[18]. Figure 2-19 presents $\overline{R}^2$ of rolling regressions for annual using SB kurtosis measure. Adding kurtosis effect, in all cases, contributes greatly to the $\overline{R}^2$. Specifically, contributions of the interaction term $(SK \cdot KT)$ are greater than those of $(SD \cdot KT)$. Figure 2-20 shows the results of monthly data. Contributions to $\overline{R}^2$ by adding kurtosis effect are much substantial at the monthly frequency. In addition, interaction term $(SK \cdot KT)$ performs better in most subsamples. The overall performances of adding kurtosis measures in recent sample periods are better than those of earlier sample periods. Figure 2-21 and Figure 2-22 presents the results by using Pearson's kurtosis. They are very similar to those of SB kurtosis. Figure 2-23 and Figure 2-24 compare the performance of both measures. In most cases, the SB kurtosis performs better.

The results of rolling regression provide a few implications for the regression analysis. The additional interaction term $(SK \cdot KT)$ can help to improve the accuracy of approximation. In addition, the individual kurtosis KT and the additional interaction term $(SD \cdot KT)$ also can improve the Ball and Mankiw model even though their quantitative roles depend on sample period and data frequency. Regarding the performances of Pearson's and SB kurtosis, both measures perform almost similarly. However, SB kurtosis measure can perform slightly better than Pearson's kurtosis

[18]The rolling regression is commonly used to test the stability of coefficient between sub-samples or to identify regime changes over the sample periods.

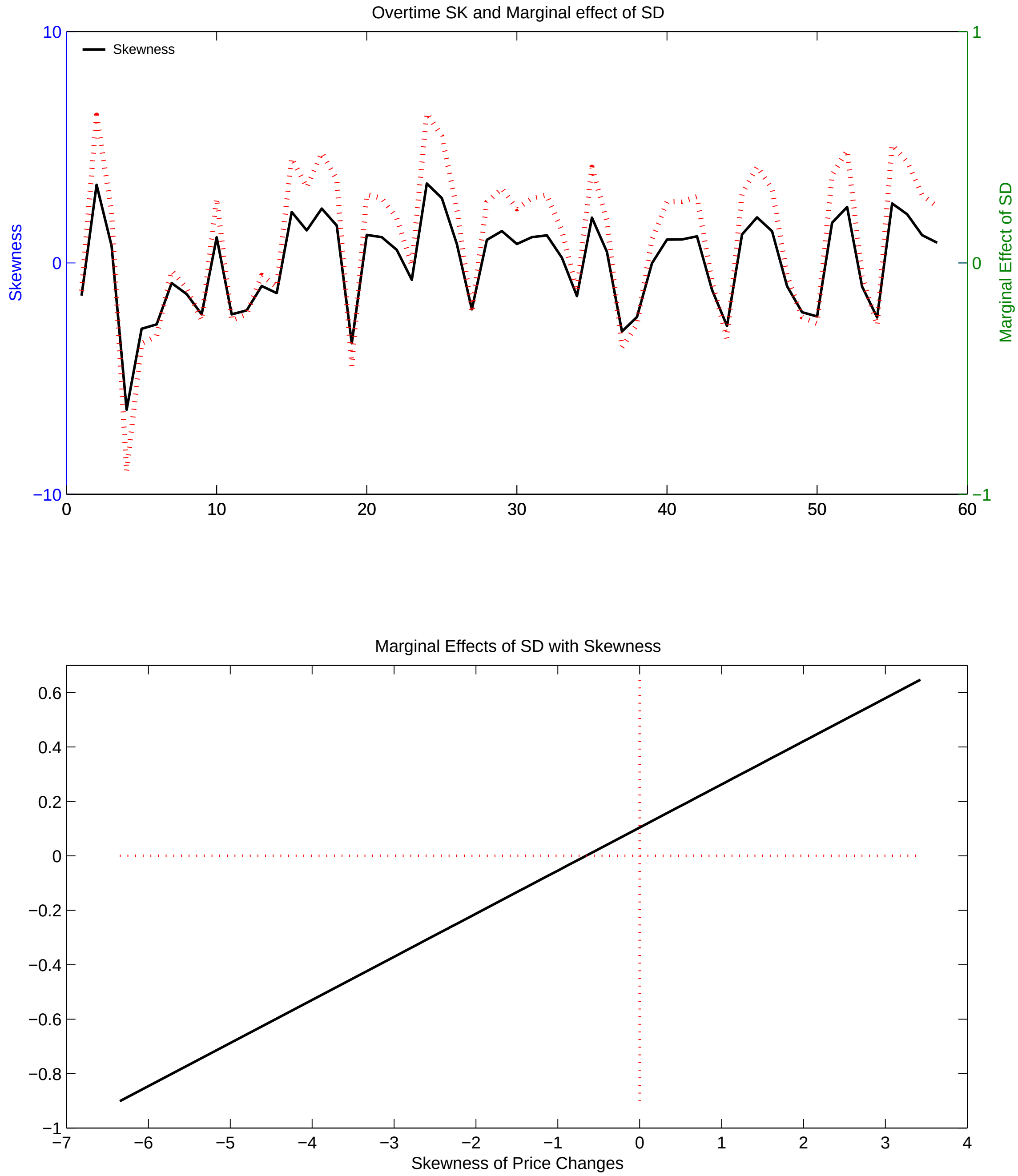


Fig. 2-15.: Marginal Effects of SD with the Skewness of Price Changes: Annual Data (1948-2006).

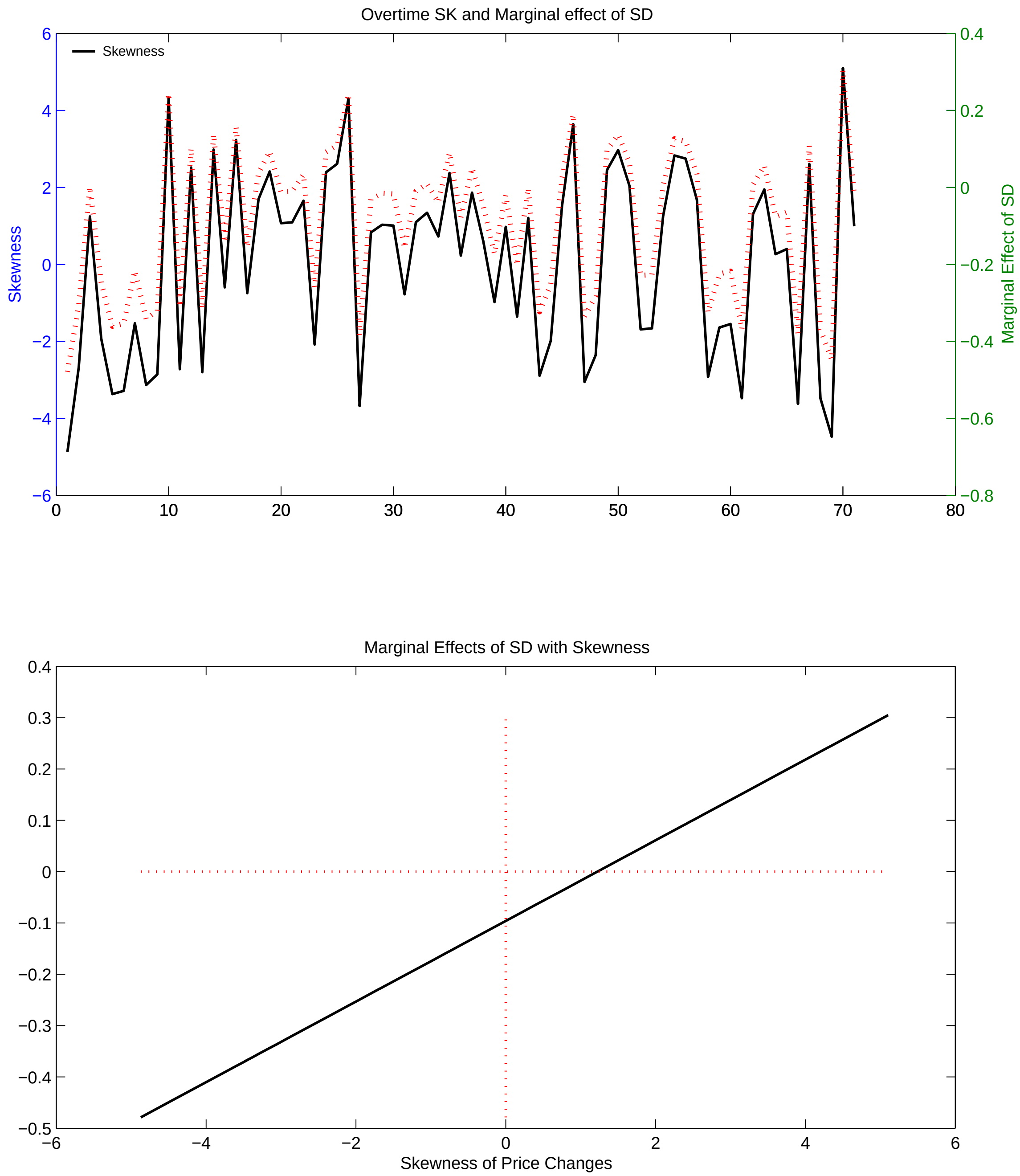
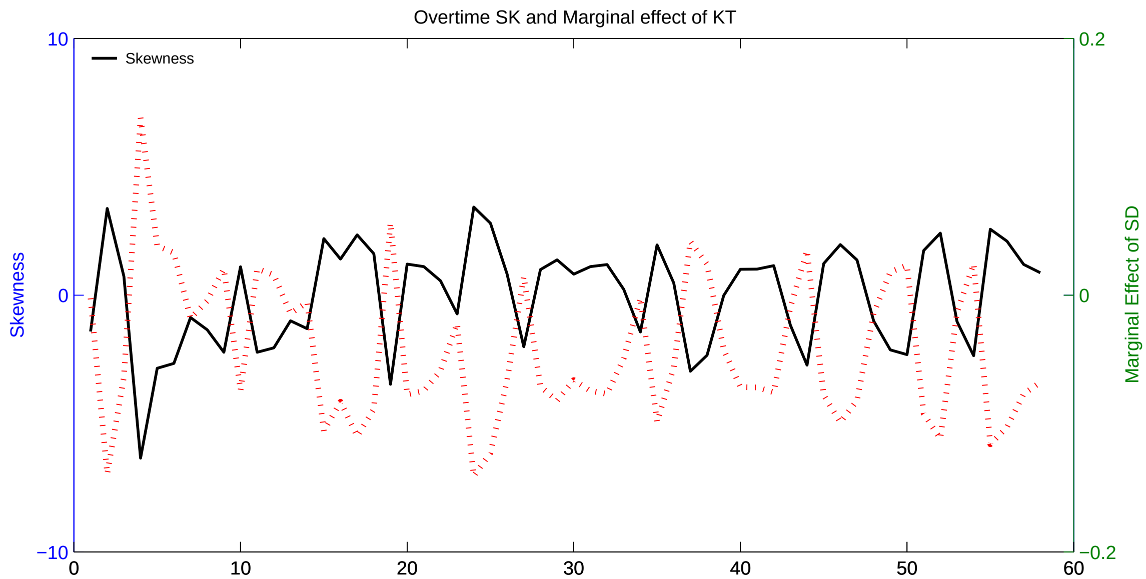


Fig. 2-16.: Marginal Effects of SD with the Skewness of Price Changes: Monthly Data (2000-2006).



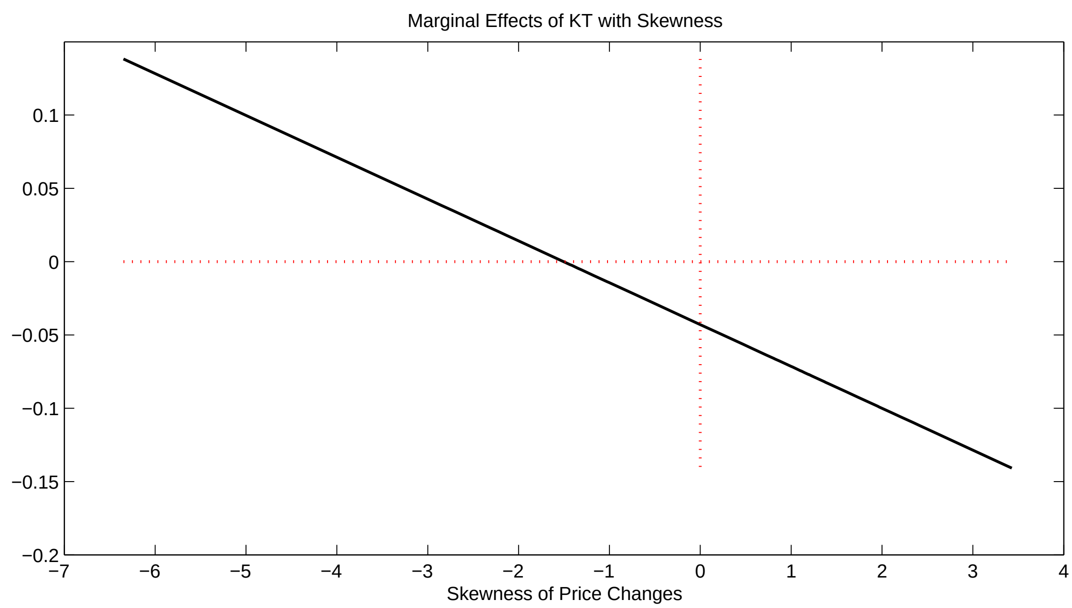


Fig. 2-17.: Marginal Effects of KT with the Skewness of Price Changes: Annual Data (1948-2006).

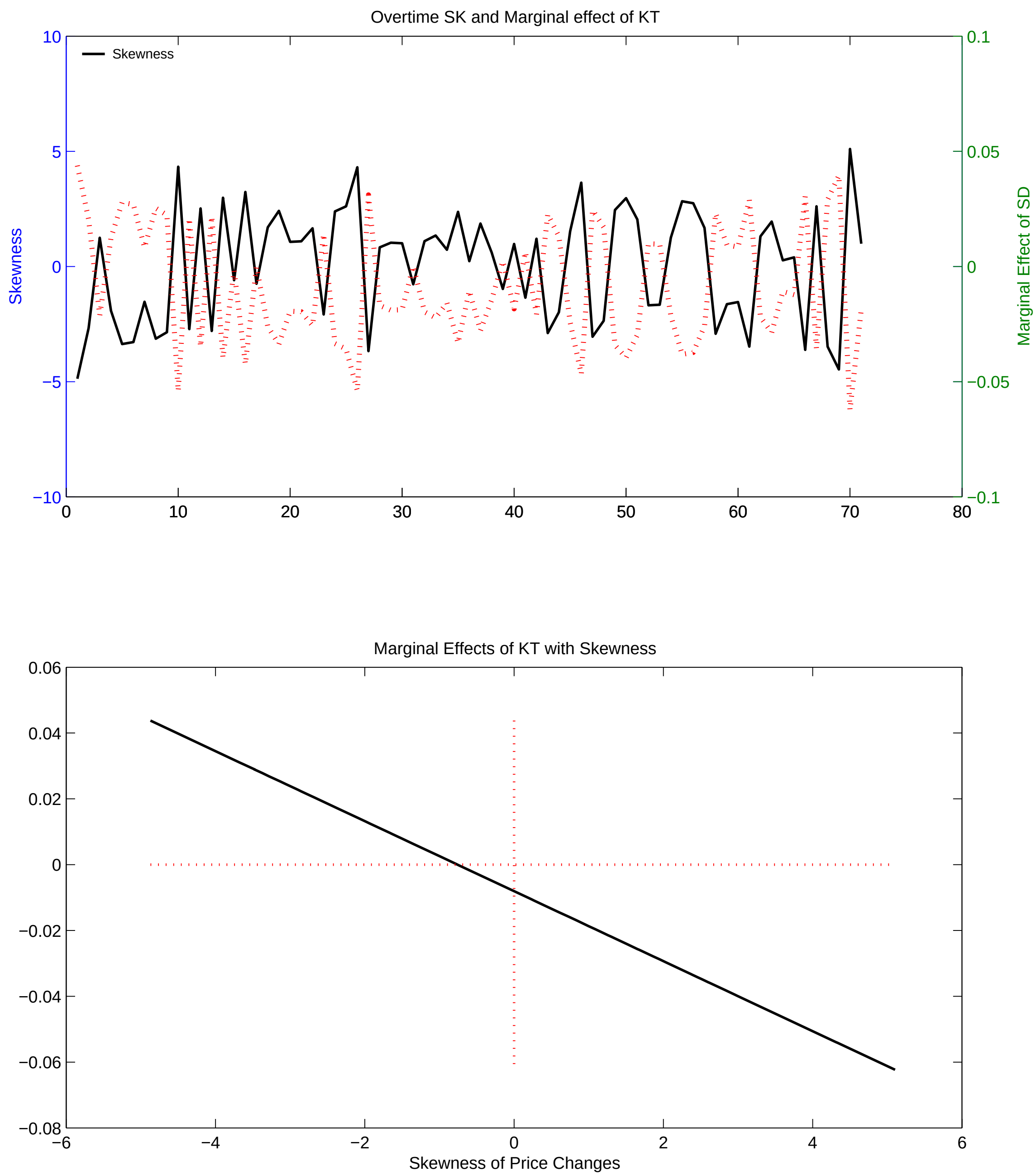


Fig. 2-18.: Marginal Effects of KT with the Skewness of Price Changes: Monthly Data (2000-2006).

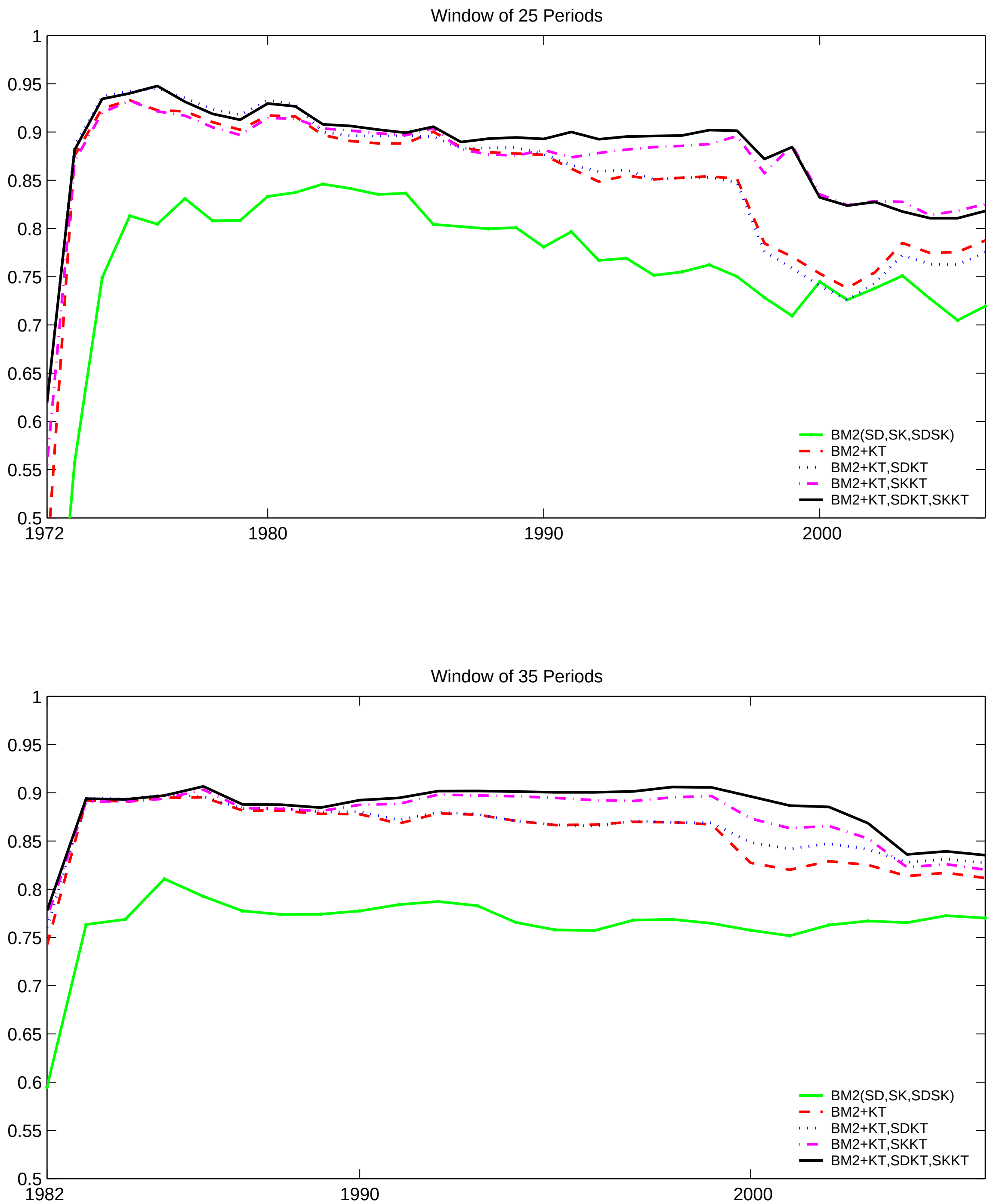


Fig. 2-19.: $\overline{R}^2$ of Rolling Regressions Using SB KT: Annual Data (1948-2006).

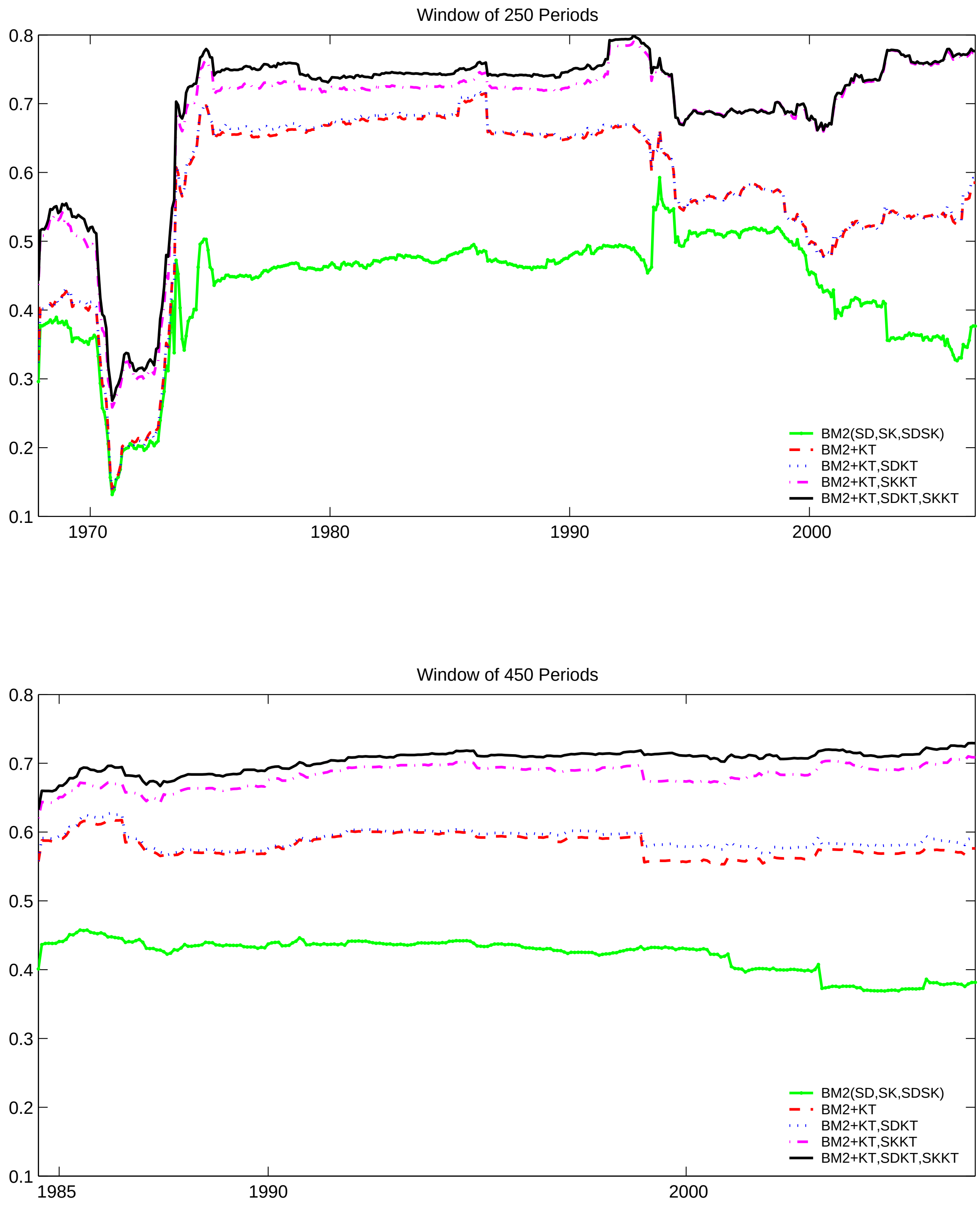
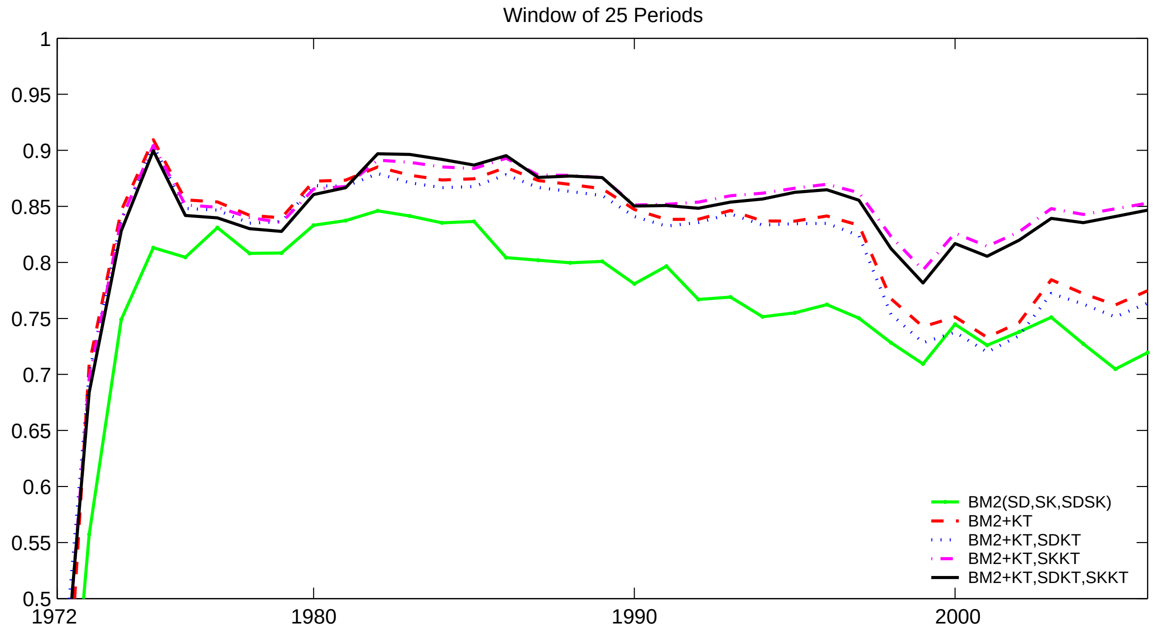


Fig. 2-20.: $\overline{R}^2$ of Rolling Regression Using SB KT: Monthly Data (1947.2-2006.12).

Notes: The seasonally unadjusted price changes are used.



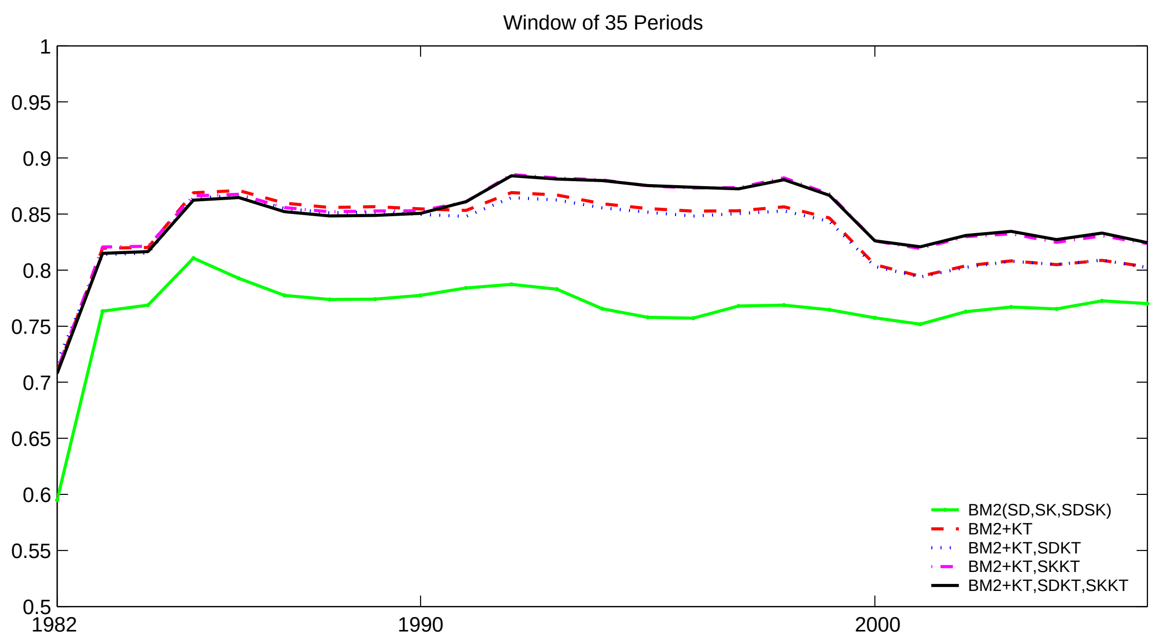
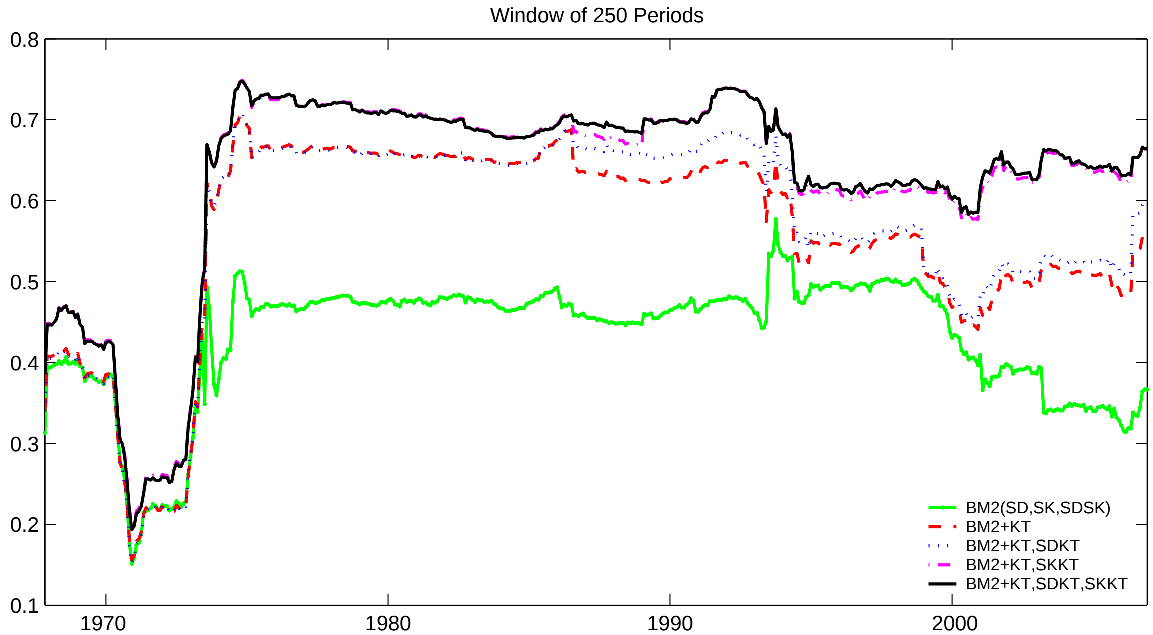


Fig. 2-21.: $\overline{R}^2$ of Rolling Regressions Using Pearson KT: Annual Data (1948-2006).



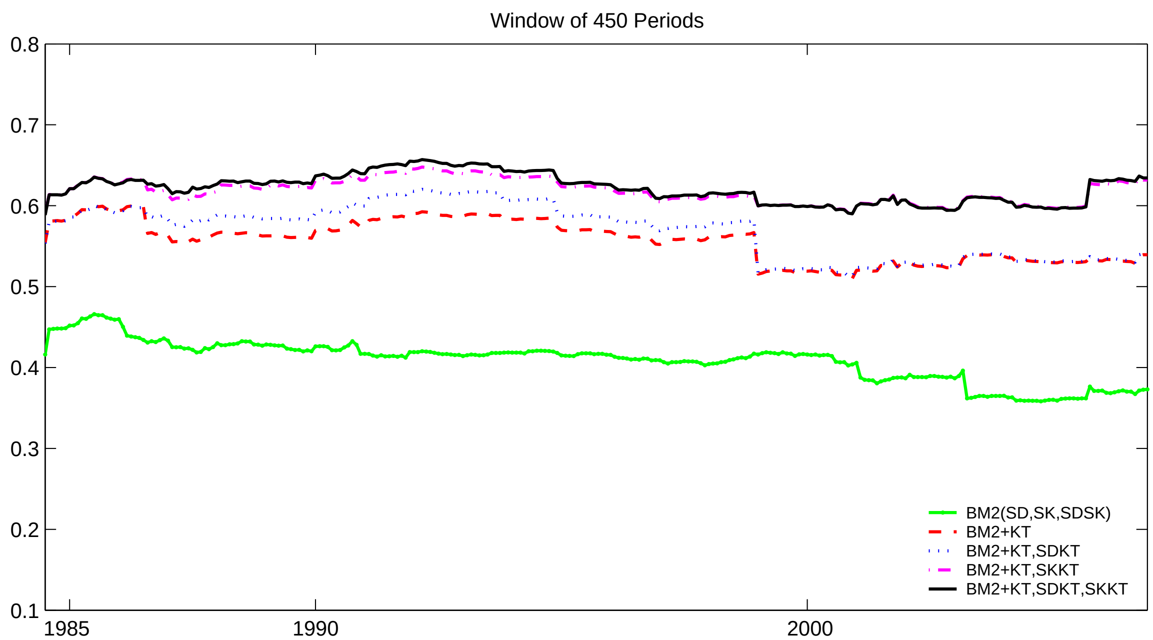
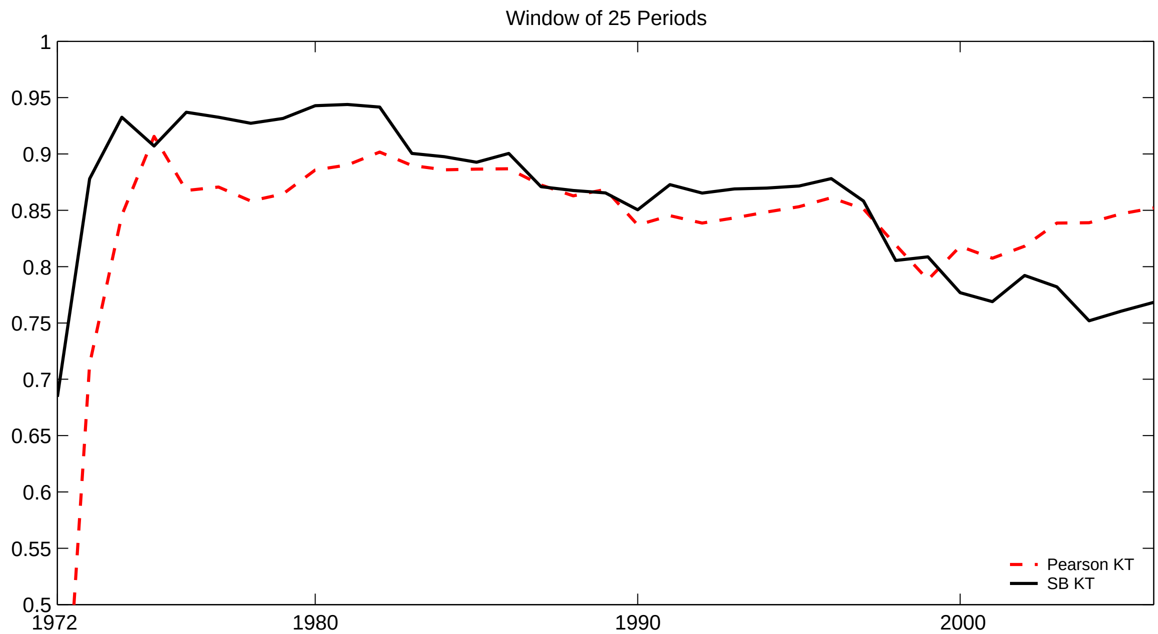


Fig. 2-22.: $\overline{R}^2$ of Rolling Regression Using Pearson KT: Monthly Data (1947.2-2006.12).

Notes: The seasonally unadjusted price changes are used.



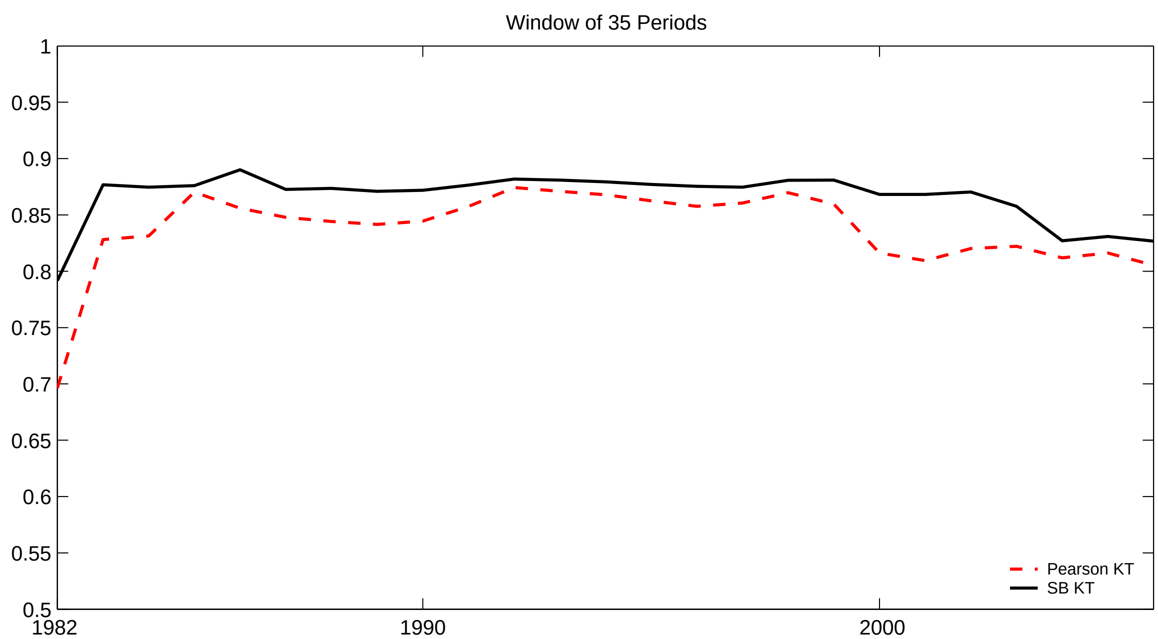


Fig. 2-23.: $\overline{R}^2$ of Rolling Regression for the Comparisons between Pearson KT and SB KT: Annual Data (1948-2006).

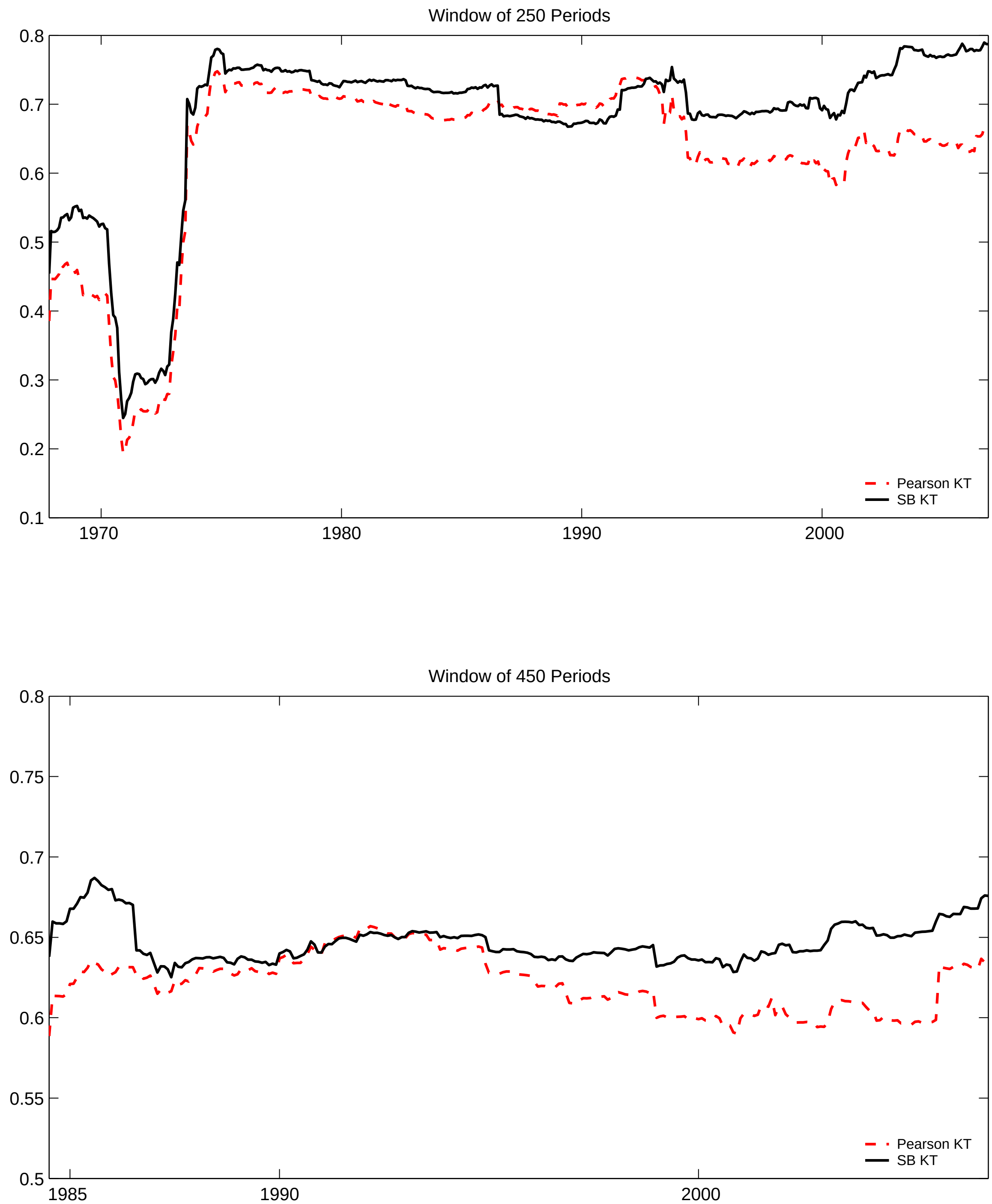


Fig. 2-24.: $\overline{R}^2$ of Rolling Regression for the Comparisons between Pearson KT and SB KT: Monthly Data (1948-2006).

measure.

Based on the rolling regression results, we suspect there may be structural changes in our sample periods. Therefore, we check parameter instability across sample periods. Because of the long spans of our data (1947-2006), it is important to ask whether there is an instability of the regression coefficients, that is, structural changes to the regression. Stability test must be taken into account since when there are structural changes, estimates of coefficients are biased and inconsistent so that they lead to a wrong conclusion. We choose our sample period based on the stability test.

The classical method, the Chow (1960) test, can be applied. However, it can be applied only when breakpoint is known so that arbitrary choice of breakpoint can be problematic because test results are very sensitive to those choices. Andrews (1993) introduced test for the case of an unknown breakpoint[19]. Following Andrews, we test Ball and Mankiw's best performed set of regressors (Constant, Lagged inflation, SD,

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[19]Consider $Y_t = X_t\beta_t + u_t$ and null hypothesis $H_0 : \beta_t = \beta$ for all t. The alternative hypothesis is that coefficients are different between two subsamples under unknown t.

$$\begin{aligned} H_1 \quad \beta_t &= \beta_1(\pi) \ \text{ for } t \le T\pi \\ &= \beta_2(\pi) \ \text{ for } t > T\pi \end{aligned}$$

where $\pi \in (0,1)$ is a trimming parameter, T is the sample size and $T\pi$ is a single breakpoint. LM test statistic for one time change occurring at change point π is defined as

$$LM(\pi) = \sum_{1}^{T\pi}\left(Y_t - X_t'\widetilde{\beta}\right)X_t'\left[\left[\sum_{1}^{T\pi}X_tX_t'\right]^{-1} + \left[\sum_{T\pi+1}^{T}X_tX_t'\right]^{-1}\right]\sum_{1}^{T\pi}\left(Y_t - X_t'\widetilde{\beta}\right)X_t/\widetilde{\sigma}^2$$

where $\widetilde{\beta} = \left[\sum_1^T X_tX_t'\right]^{-1}\sum_1^T X_tY_t$, $\quad \widetilde{\sigma}^2 = \frac{1}{T-k}\sum_1^T\left(Y_t - X_t'\widetilde{\beta}\right)^2$. Test statistics are defined as $Sup\ LM = \sup_{k\in[\pi T,(1-\pi)T]} LM_k$, exponentially weighted statistic $Exp\ LM = \ln\int\exp(F_t/2)dw(t)$ and average statistic $Ave\ LM = \int F_t dw(t)$.

SD, SD·SK). The null hypothesis of this test is that coefficients in Ball and Mankiw's model are the same throughout the entire sample period. The alternative hypothesis allows different values of coefficients across various subsample periods.

Figure 2-25 presents LM statistics over the possible breakpoints in order to illustrate testing results graphically. The 5% critical values of the Sup LM statistics are compared. We can check LM statistics exceed the Andrew's critical value for both annual and monthly data. So we can reject the null hypothesis of no structural change. Formally, Table 2-3 and Table 2-4 report three LM test statistics used in Andrews (1993), Andrews and Ploberger (1994), and p values developed by Hansen (1997). We reports only the joint statistics of all coefficients since it appears to be better to judge breakpoint based on joint statistics, rather than individual statistics. In all cases, the null hypothesis of no structural change is strongly rejected.

The LM statistics provide us with the most likely date for a structural change. We choose a single breakpoint which has the largest LM statistics for coefficient instability.[20] For annual data, it is 1957 and for monthly data, it is February 1974. To avoid the coefficient instability problem, our sample is divided into two subsamples according to these breakpoints: For annual data, $[1948, 1957]$ and $[1958, 2006]$. For monthly data, $[1947.2, 1974.2]$ and $[1974.3, 2006.12]$.

Table 2-5 reports OLS estimation results of the annual data.[21] The monthly data results are reported in Table 2-6 and Table 2-7. The tables show the comparisons between three Ball and Mankiw's models and four our models which include kurtosis

[20]We focus on test for a single breakpoint. Multiple breakpoints may be general but there are some debates on multiple breakpoints.

[21]First subsample of annual data has only 9 observations. So we will exclude it from our analysis.

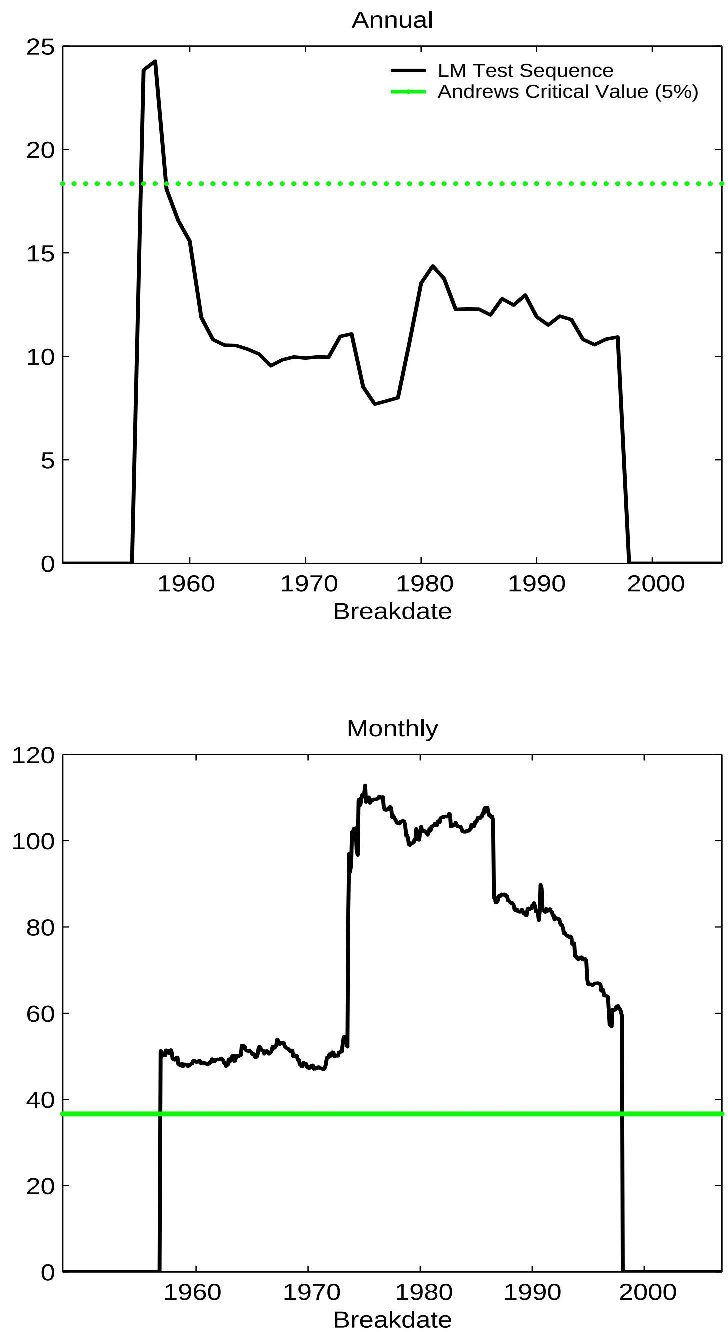

Fig. 2-25.: Testing for Structural Change of Unknown Timing: BM2 (Constant, Lagged Inflation, SD, SK, SDSK).

Table 2-3.: Testing for Structural Change of Unknown Timing: Annual Data (1948-2006).

		Critical Value			
	Test Statistic	*10%*	*5%*	*1%*	*p value*
Sup LM	24.3	16.2	18.4	22.5	0.005
Exp LM	9.1	5.2	6.1	7.9	0.003
Ave LM	12.0	7.8	9.0	11.3	0.007
Breakpoint	1957				

Table 2-4.: Testing for Structural Change of Unknown Timing: Monthly Data (1947.2-2006.12).

		Critical Value			
	Test Statistic	*10%*	*5%*	*1%*	*p value*
Sup LM	112.9	33.9	36.7	42.4	0.000
Exp LM	52.4	13.3	14.6	17.3	0.000
Ave LM	74.6	21.0	22.7	26.4	0.000
Breakpoint	1974.2				

Notes: Critical values are reported from the Andrews (1993), and Andrews and Ploberger (1994).

measures. The numbers reported in parenthesis are p-values of the coefficients. The goodness of fit of different specifications are investigated with four different criteria.

The overall estimation results are very consistent to those of our numerical analysis. All estimation results provide correct signs for the regressors except for the sign of SD in the result using monthly data. The dispersion and skewness measure have a positive effect on inflation and kurtosis measure has a negative effect on inflation. While the skewness and kurtosis measure are always significant, the significance of dispersion measure depends on data frequency. We note that when the interaction terms are included, non-interaction terms become insignificant for annual data. For example, in the first specification of Ball and Mankiw model without an interaction term, both SD and SK are significant. However, in the second specification with an interaction term, both SD and SK are not significant while the interaction term $(SD \cdot SK)$ are significant.

On the whole, the additional kurtosis measures, in particular, interaction terms can improve the Ball and Mankiw's model in terms of goodness of fit. However, the performances of adding kurtosis measures seem to depend on data frequency. For annual data, including the individual kurtosis KT or the interaction term $(SK \cdot KT)$ can improve Ball and Mankiw model, but the improvement is not substantial. However, in case of including interaction terms $(SD \cdot KT)$, the improvement is quite substantial. However, for monthly data, in case of including interaction terms $(SK \cdot KT)$, the improvement is quite substantial.

F. Conclusion

This paper shows the importance of kurtosis in the approximation of inflation, theoretically and empirically. This is because the kurtosis measure additionally captures

Table 2-5.: Regression Results: Annual Data (1958-2006).

	BM model			BM(2)+Adding KT effect			
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
Constant	-0.013	-0.007	0.008	0.119	-0.196	0.057	-0.220
	(0.090)	(0.300)	(0.028)	(0.119)	(0.263)	(0.465)	(0.198)
Lagged inflation	0.557	0.589	0.672	0.547	0.514	0.592	0.559
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
SD	0.370	0.261		0.264	5.184	0.201	4.618
	(0.001)	(0.012)		(0.010)	(0.041)	(0.049)	(0.061)
SK	0.012	-0.002	-0.004	-0.001	-0.002	0.093	0.087
	(0.000)	(0.630)	(0.302)	(0.704)	(0.612)	(0.044)	(0.052)
$SD \cdot SK$		0.185	0.222	0.170	0.111	0.168	0.116
		(0.000)	(0.000)	(0.001)	(0.051)	(0.001)	(0.037)
KT				-0.036	0.056	-0.018	0.063
				(0.098)	(0.272)	(0.427)	(0.205)
$SD \cdot KT$					-1.444		-1.294
					(0.051)		(0.072)
$SK \cdot KT$						-0.027	-0.025
						(0.041)	(0.057)
Marginal Effect SD		0.301	0.301	0.301	0.254	0.237	0.201
		(0.004)	(0.004)	(0.003)	(0.011)	(0.018)	(0.043)
Marginal Effect SK		0.011	0.011	0.010	0.010	0.011	0.010
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Marginal Effect KT					-0.043	-0.024	-0.031
					(0.039)	(0.093)	(0.043)
$\overline{R}^2$	0.761	0.816	0.793	0.824	0.835	0.837	0.846
(Ratio)				(100.9)	(102.3)	(102.5)	(103.6)
RMSE	0.019	0.017	0.018	0.016	0.016	0.016	0.015
(Ratio)				(96.8)	(92.5)	(92.1)	(88.5)
AIC	-4.882	-5.127	-4.870	-4.918	-4.930	-4.940	-4.940
(Ratio)				(100.5)	(101.4)	(101.6)	(102.4)
SC	-4.727	-4.934	-4.870	-4.918	-4.930	-4.940	-4.940
(Ratio)				(99.7)	(99.9)	(100.1)	(100.1)

Notes: p-values are reported in parenthesis below the estimates.

Table 2-6.: Regression Results: Monthly Data (1947.2-1974.2).

	BM model			BM(2)+Adding KT effect			
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
Constant	-0.004	-0.003	0.001	0.013	-0.008	0.013	-0.044
	(0.000)	(0.001)	(0.036)	(0.015)	(0.608)	(0.005)	(0.002)
SD	0.217	0.155		0.152	0.970	0.134	2.417
	(0.000)	(0.000)		(0.000)	(0.096)	(0.000)	(0.000)
SK	0.001	-0.002	-0.002	-0.002	-0.002	0.010	0.013
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$SD \cdot SK$		0.101	0.108	0.098	0.092	0.103	0.084
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
KT				-0.004	0.001	-0.004	0.011
				(0.000)	(0.727)	(0.001)	(0.004)
$SD \cdot KT$					-0.219		-0.613
					(0.159)		(0.000)
$SK \cdot KT$						-0.003	-0.004
						(0.000)	(0.000)
Marginal Effect SD		0.142	0.142	0.140	0.136	0.121	0.107
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Marginal Effect SK		0.001	0.001	0.001	0.001	0.001	0.001
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Marginal Effect KT					-0.004	-0.004	-0.004
					(0.003)	(0.002)	(0.003)
$\overline{R}^2$	0.379	0.553	0.525	0.565	0.566	0.636	0.655
(Ratio)				(102.1)	(102.3)	(115.0)	(118.4)
RMSE	0.006	0.005	0.005	0.005	0.005	0.005	0.004
(Ratio)				(98.6)	(98.2)	(89.9)	(87.4)
AIC	-7.331	-7.659	-7.599	-7.682	-7.682	-7.859	-7.909
(Ratio)				(100.3)	(100.3)	(102.6)	(103.3)
SC	-7.157	-7.473	-7.424	-7.484	-7.473	-7.649	-7.688
(Ratio)				(100.2)	(100.0)	(102.4)	(102.9)

Notes: p-values are reported in parenthesis below the estimates. Estimates of lagged inflation are not reported.

Table 2-7.: Regression Results: Monthly Data (1974.3-2006.12).

	BM model			BM(2)+Adding KT effect			
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
Constant	0.002	0.003	0.001	0.032	0.010	0.031	0.012
	(0.007)	(0.000)	(0.008)	(0.000)	(0.506)	(0.000)	(0.288)
SD	-0.038	-0.066		-0.050	0.662	-0.044	0.556
	(0.099)	(0.001)		(0.003)	(0.103)	(0.000)	(0.136)
SK	0.001	-0.001	-0.001	-0.001	-0.001	0.021	0.021
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$SD \cdot SK$		0.062	0.060	0.064	0.066	0.081	0.082
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
KT				-0.008	-0.002	-0.008	-0.003
				(0.000)	(0.630)	(0.000)	(0.384)
$SD \cdot KT$					-0.188		-0.159
					(0.097)		(0.076)
$SK \cdot KT$						-0.006	-0.006
						(0.000)	(0.000)
Marginal Effect SD		-0.065	-0.065	-0.049	-0.042	-0.043	-0.037
		(0.001)	(0.001)	(0.014)	(0.038)	(0.007)	(0.020)
Marginal Effect SK		0.001	0.001	0.001	0.001	0.001	0.001
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Marginal Effect KT					-0.008	-0.008	-0.008
					(0.000)	(0.000)	(0.000)
$\overline{R}^2$	0.386	0.526	0.514	0.551	0.554	0.722	0.724
(Ratio)				(104.8)	(105.2)	(137.2)	(137.5)
RMSE	0.005	0.005	0.005	0.005	0.005	0.004	0.004
(Ratio)				(97.2)	(96.8)	(76.4)	(76.1)
AIC	-7.462	-7.749	-7.726	-7.801	-7.803	-8.277	-8.280
(Ratio)				(100.7)	(100.7)	(106.8)	(106.9)
SC	-7.337	-7.584	-7.571	-7.625	-7.617	-8.091	-8.084
(Ratio)				(100.5)	(100.4)	(106.7)	(106.6)

Notes: p-values are reported in parenthesis below the estimates. Estimates of lagged inflation are not reported.

the effect of changes in the distribution of price shocks on inflation. Since Mills (1927), many authors have studied the relationship between inflation and moments of price changes. The source to generate these relationships is the change in the shape of the underlying distribution. To capture the shape of the distribution, earlier studies in 1970s and 1980s considered dispersion alone. Since Ball and Mankiw (1995) included skewness, both dispersion and skewness have been used. We argue that kurtosis should be considered to capture the property of the distribution sufficiently.

Empirically, we confirm the importance of kurtosis, which is consistent to our theoretical analysis. Our empirical results show that the kurtosis measure has a significant effect on inflation. In addition, we can improve the approximation of inflation in terms of the goodness of fit. Especially, the improvement is substantial in monthly data, implying dispersion and skewness are not sufficient. In this context, previous studies based on Ball and Mankiw's model have a weakness since they omitted important variables.

It is widely accepted that an inflation-dispersion relationship and an inflation-skewness relationship are one of the stylized facts in macroeconomics. We propose an inflation-kurtosis relationship as one of the stylized facts. This relationship has not been emphasized. However, it can be included in inflation-moments relationship.

CHAPTER III

INFLATION AND ROBUST MEASURES OF THE DISTRIBUTION OF PRICE CHANGES

A. Introduction

Ball and Mankiw (1995) presented an intuitive simple model of menu costs which suggests a relationship between the inflation rates and the distributional characteristics of relative price changes. In their model, firms in each industry are heterogeneous in their menu costs, but are subject to the same price shock. Faced with a shock to their desired price, firms change the price only if the price shock is large enough to make the benefit from changing the price to outweigh the menu cost. The proportion of the firms which adjust their prices and the average price thus depend on the shape of the distribution of price shocks. Ball and Mankiw focused on the dispersion and skewness of the distribution, and illustrate a positive relationship between changes in the price level and the dispersion/skewness of the distribution.

Ball and Mankiw analyzed the effects on the PPI inflation rate of the dispersion and skewness of the changes in prices of commodities that are included in the overall PPI. The dispersion and skewness are computed by the classical measurement of weighted and unweighted standard deviation and skewness of the cross sectional sample. They found significant effects of both dispersion and skewness measures.

It is well known that classical measures of dispersion and skewness are very sensitive to the presence of outliers. This sensitivity can have a significant effect on the relationship between the skewness and inflation rate. A single positive outlier tends to significantly increase the skewness, and it will also increase the inflation rate in the same direction because the overall PPI is a weighted average of prices of individual commodities. This implies that a positive correlation between the skewness and inflation rate can be caused by outliers, particularly in a sample of small size.

Verbrugge (1999) used a robust measure, triples U-statistic, in the place of the classical skewness measure in his analysis of the relationship between the unweighted median inflation and unweighted triples U-statistic which are robust to outliers. More recently, Aucremanne, Brys, Hubert, Rousseeuw, and Struyf (2002) used the interquartile range for the dispersion measure and the destandardised versions of the skewness measures of Hinkley (1975) and Groeneveld and Meeden (1984) in addition to the classical measures.

In this paper we examine the presence of outliers in the relative price changes and estimate unweighted and weighted robust measures of dispersion and skewness. The effects of robust measures on the inflation rate are then estimated and compared with the results based on the classical measures of dispersion and skewness. We find that dispersion/skewness of the distribution of price changes have a positive effect on inflation in line with Ball and Mankiw. In particular, medcouple as a measure of skewness is very useful in predicting inflation.

The paper is organized as follows. In the next section, various robust estimators of weighted dispersion and skewness are presented and several methods of outlier detection. In section 3, the inflation equations are specified and estimated. Section 4 concludes the paper with a summary of our major findings.

B. Classical and Robust Measures of Weighted Dispersion and Skewness

It is well known that classical measures of dispersion and skewness are very sensitive to the presence of outliers. We present in this section the classical and robust measures of weighted dispersion and weighted skewness, followed by a few methods of detecting outliers. Let $x = \{x_1, x_2, \cdots, x_n\}$ be the random sample of size n and $w = \{w_1, w_2, \cdots, w_n\}$ be the corresponding non-negative weights. Unless specified otherwise, we assume that the weights are normalized such that $\sum_{i=1}^{n} w_i = 1$.

1. Classical Measures

The classical measure of weighted dispersion is

$$\widehat{\sigma}^2 = \frac{1}{c}\sum_{i=1}^{n} w_i \left(x_i - \widehat{\mu}\right)^2, \quad \widehat{\mu} = \sum_{i=1}^{n} w_i x_i$$

where the constant term c takes a various forms[1]. For an unbiasedness of the estimator, $c = 1 - \sum_{i=1}^{n} w_i^2$, which is used in Aucremanne, Brys, Hubert, Rousseeuw, and Struyf (2004) and in the PyGSL program. If the location parameter is fixed, then $c = 1$. The classical measure of weighted skewness is

$$\widehat{sk} = \frac{1}{d}\sum_{i=1}^{n} w_i \left(\frac{x_i - \widehat{\mu}}{\widehat{\sigma}}\right)^3$$

where $d = 1 - 3\sum_{i=1}^{n} w_i^2 + 2\sum_{i=1}^{n} w_i^3$ for the unbiased estimator, which is used in Aucremanne, Brys, Hubert, Rousseeuw, and Struyf (2004). When the location parameter is a fixed value and is not estimated, $d = 1$. An alternative definition of the weighted skewness that is used in the SAS program[2] is

$$\widehat{sk} = \frac{1}{d}\sum_{i=1}^{n} w_i^{3/2} \left(\frac{x_i - \widehat{\mu}_w}{\widehat{\sigma}}\right)^3$$

These classical forms of dispersion and skewness measures are very sensitive to the presence of outliers.

[1] Dataplot uses $c = (n_1 - 1)/n_1$, where n_1 is the number of nonzero weights, and SAS uses $c = n$ or $c = n - 1$. Most studies uses $c = 1$.

[2] SAS program uses $d = n$ if $c = n$ and $d = (n-1)(n-2)/n$ if $c = n - 1$. Most studies uses $c = 1$.

2. Robust Measures

There are many alternative robust measures of dispersion and skewness, most of which are based on the estimates of the standard quantiles for unweighted data and on the estimates of weighted quantiles for weighted data. When the weights are nonnegative integers, the p^{th} weighted quantile of data set $x = (x_1, x_2, \cdots, x_n)$ with weight vector $w = (w_1, w_2, \cdots, w_n)$ is computed as the p^{th} quantile of the expanded data set $w \Diamond x = (w_1 \Diamond x_1, w_2 \Diamond x_2, \cdots, w_n \Diamond x_n)$, where $\Diamond$ is the duplication operator, i.e., $m \Diamond x = xx \cdots x,$, repeating x by m times. When the weights are normalized such that $\sum w_i = 1$ and $w_i \geq 0$, the p^{th} weighted quantile $Q_p^w(x)$ of x can be computed as follows. Let $\{x_{(i)}\}$ and $\{w_{(i)}\}$ denote the sorted data in ascending order and the corresponding ordered weights, respectively. Let x_k be the largest value such that $ws_{(k)} \equiv \sum_{i=1}^{k} w_{(i)} \leq p$. If $ws_{(k)} = p$, then $Q_p^w(x) = x_{(k)}$. If $ws_{(k)} < p$, then we may compute $Q_p^w(x)$ by $Q_p^w(x) = x_{(k)}$, or by a weighted average $x_{(k)}$ and $x_{(k+1)}$

$$Q_p^w(x) = \frac{x_{(k)}\left(ws_{(k+1)} - p\right) + x_{(k+1)}\left(p - ws_{(k)}\right)}{ws_{(k+1)} - ws_{(k)}}$$

The robust estimators of weighted dispersion and weighted skewness presented below will use the estimates of weighted quantiles, or the weighted order statistics[3] in the computation of L moments.

Robust Measure of Dispersion

Two most commonly used robust alternatives to the classical standard deviation are dispersion measures based on the interquartile range and the median absolute deviation (MAD). The MAD in particular is a very robust estimator. The dispersion

[3]The weighted order statistic of x with weight vector w and threshold k is defined as the k^{th} largest value of the expanded list $w \Diamond x$.

measure based on the interquartile range is defined by

$$d_{iqr} = c_{iqr}(Q_{0.75} - Q_{0.25}) = c_{iqr}IQR$$

where $c_{iqr} = 1/(2\alpha)$ and $\alpha = \Phi^{-1}(0.75) \approx 0.67449$. The normalization factor c_{iqr} is to make d_{iqr} comparable to the classical standard deviation σ when the sample is from a normal $N(\mu, \sigma^2)$[4].

The MAD is the median of the absolute distances between each data point and overall median of the data set

$$MAD(x_i) = med_i\left(|x_i - med_j(x_j)|\right)$$

where the inner median, $med_j(x_j)$, is the median of n observations and the outer median, med_i, is the median of the n absolute values of the deviations about the overall median. The dispersion measure based on the MAD is defined by

$$d_{mad} = c_{mad}MAD(x_i)$$

where the normalization factor $c_{mad} = 1/\alpha \approx 1.4826$ is to make d_{mad} comparable to σ.

The MAD statistic implicitly assumes a symmetric distribution as it measures the distance from a measure of central location (the median). Rousseeuw and Croux (1993) proposed two new statistics, S_n and Q_n, as alternatives to the MAD statistic. The S_n is defined by

$$d_{rcs} = c_{rcs}med_i(med_j(|x_i - x_j|))$$

[4]If x is distributed as a normal $N(\mu, \sigma^2)$, then $MAD(x) = \alpha\sigma$ and $IQR(x) = 2\alpha\sigma$, for unweighted MAD and IQR, where $\alpha = \Phi^{-1}(0.75)$. The relationship between σ and the weighted MAD and the weighted IQR is unknown. We will use the same normalization constant for both unweighted and weighted samples.

where the outer median, med_i, is the median of n medians of $\{|x_i - x_j|, j = 1, 2, \cdots, n\}$. The correction factor $c_{rcs} = 1.1926$ is to reduce the small sample bias in the estimation of the standard deviation. The Q_n measure of Rousseeuw and Croux is defined by

$$d_{rcq} = c_{rcq}\left\{|x_i - x_j|; i < j\right\}_{(k)}, \qquad k = \binom{h}{2}, \qquad h = [n/2] + 1$$

where $c_{rcq} = 2.2219$ and $[n/2]$ is the integer part of $n/2$. This estimator is a constant times the k^{th} order statistic of the $n(n-1)/2$ distances between data points. This estimator has a significantly better normal efficiency and it does not depend on symmetry.

Robust Measure of Skewness

One of the robust skewness measures is Hinkley's (1975) generalization of Bowley's (1920) coefficient of skewness, which is defined by

$$sk_h(p) = \frac{(Q_{1-p} - Q_{0.5}) - (Q_{0.5} - Q_p)}{(Q_{1-p} - Q_{0.5}) + (Q_{0.5} - Q_p)}, \qquad 0 < p < 1/2$$

which takes a value in the interval $[-1, 1]$. The quartile skewness with $p = 1/4$ is Bowley's measure. The quartile skewness is less sensitive to outliers than the octile skewness ($p = 1/8$), but the latter uses more information from the tails of the distribution and can be more useful in detecting asymmetry[5].

Hinkley's measure requires a choice of p and the measure may be sensitive to a particular choice. Furthermore, this measure is insensitive to the distribution in the tails outside the chosen quantiles. The skewness measure proposed by Groeneveld

[5] Aucremanne, Brys, Hubert, Rousseeuw, and Struyf (2004) used the destandardized versions of Hinkley's measure in their study of inflation rate, i.e., they used only the numerator term of the Hinkley's measure with $p = 1/4$ and $p = 1/8$.

and Meeden (1984) overcomes this problem by taking probability-weighted averages of the numerator and denominator terms in Hinkley's measure. It is defined by

$$\begin{aligned} sk_{gm} &= \frac{\int_0^{\frac{1}{2}} \left([F^{-1}(1-p) - Q_{0.5}] - [Q_{0.5} - F^{-1}(p)] \right) dp}{\int_0^{\frac{1}{2}} \left([F^{-1}(1-p) - Q_{0.5}] + [Q_{0.5} - F^{-1}(p)] \right) dp} \\ &= \frac{\mu - Q_{0.5}}{E|X - Q_{0.5}|} \end{aligned}$$

where F is the cumulative distribution function and $Q_p = F^{-1}(p)$. This measure takes a zero value for a symmetric distribution and takes a value in the interval $[-1, 1]$[6]. This estimator can be estimated by

$$sk_{gm} = \frac{\overline{x} - Q_{0.5}}{\sum_{i=1}^{n} w_i \left| x_i - Q_{0.5} \right|}, \qquad \overline{x} = \sum_{i=1}^{n} w_i x_i$$

Brys, Hubert, and Struyf (2003) introduced the medcouple (MC) as a robust measure of skewness. Let the sample be sorted in ascending order: $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$. The medcouple is defined by

$$sk_{mc} = \underset{x_{(i)} \leq Q_{0.5} \leq x_{(j)}}{med} h(x_{(i)}, x_{(j)})$$

where the kernel function is defined as

$$h(x_{(i)}, x_{(j)}) = \frac{(x_{(j)} - Q_{0.5}) - (Q_{0.5} - x_{(i)})}{(x_{(j)} - Q_{0.5}) + (Q_{0.5} - x_{(i)})}$$

for all $x_{(i)} \leq Q_{0.5} \leq x_{(j)}$. Note that, if either $x_{(i)}$ or $x_{(j)}$ coincides with the median, then $h(x_{(i)}, x_{(j)}) = 1$ for all $x_{(j)} \geq x_{(i)} = Q_{0.5}$, and $h(x_{(i)}, x_{(j)}) = -1$ for all $x_{(i)} \leq x_{(j)} = Q_{0.5}$. If there are more than one data point which coincide with the median such that $x_{(j)} = x_{(i)} = Q_{0.5}$, then the kernel function is defined as $h(x_{(i)}, x_{(j)}) = +1$

[6]Note that the denominator of sk_{gm} can be considered as a measure of dispersion. If the denominator term is replaced with the classical dispersion measure, it becomes Pearson's coefficients of skewness $sk_p = 3(\text{mean} - \text{median})/\sigma$.

if $i > j$, $h(x_{(i)}, x_{(j)}) = -1$ if $i < j$, and $h(x_{(i)}, x_{(j)}) = 0$ if $i = j$. Thus, if m number of data points coincide with the median, the kernel function takes m number of zero values and $m(m-1)/2$ number of $+1$ and -1, respectively. Since the value of the kernel function lies in the interval $(-1, 1)$ for all $x_{(i)} \leq Q_{0.5} \leq x_{(j)}$, sk_{mc} takes a value in $(-1, 1)$. Note that the kernel function is the same as Hinkley's measure of skewness except that Q_p and Q_{1-p} are replaced by order statistics $x_{(i)}$ and $x_{(j)}$.

Hosking (1990) introduced L-moments which are summary statistics for probability distributions and data samples. L-moments can characterize a wider range of distributions than the classical moments because the existence of L-moments requires the existence of only the first order moment. They are particularly useful in identifying skewed distributions and their estimators are more robust to the presence of outliers in the data. They also provide measures of location, dispersion, skewness, kurtosis, and other aspects of the shape of probability distributions or data samples.

L-moments are defined as a linear function of the expected order statistics

$$\ell_r = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} E(X_{r-k:r}), \qquad r = 1, 2, \cdots$$

where $E(X_{j:r})$ is the expectation of the j^{th} order statistic in a sample of size r drawn from the distribution of $F(x)$. These moments can also be expressed as linear functions of the weighted probability moments introduced by Greenwood, Landwehr, Matalas, and Wallis (1979)

$$\ell_r = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^{r-k-1} \binom{r-1}{k} \binom{r+k-1}{k} \beta_k, \qquad r = 1, 2, \cdots$$

where β_k is the probability weighted moment

$$\beta_k = \int x[F(x)]^k dF(x)$$

The first three L-moments can thus be written as

$$\ell_1 = \beta_0, \qquad \ell_2 = 2\beta_1 - \beta_0, \qquad \ell_3 = 6\beta_2 - 6\beta_1 + \beta_0$$

where the coefficients are those of the shifted Legendre polynomials. $\ell_1$is the sample mean, a measure of location. The second L-moment ℓ_2 is (a multiple of) Gini's mean difference statistic, a measure of the dispersion of the data values about their mean. By dividing the higher-order L-moments by the dispersion measure, we obtain L-moment ratios,

$$\tau_r = \ell_r / \ell_2, \qquad r = 3, 4, \cdots$$

These are dimensionless quantities, independent of the units of measurement of the data. Hosking shows that τ_r for $r \geq 3$ are bounded in $(-1, 1)$, and proposes to use τ_3 as a measure of skewness, which is called the L-skewness and will be denoted by sk_L.

The L-moments are estimated from the estimators b_k of the probability-weighted moments β_k,

$$b_0 = \frac{1}{n} \sum_{j=1}^{n} x_{(j)}$$
$$b_k = \frac{1}{n} \sum_{j=k+1}^{n} \frac{(j-1)(j-2)\cdots(j-k)}{(n-1)(n-2)\cdots(n-k)} x_{(j)}$$

where $x_{(j)}$ is the j^{th} order statistic of a sample sorted in ascending order.

For the weighted sample, b_k is computed by using the order statistics of the expanded data set $w \Diamond x$[7].

The triples U-statistic is proposed independently by Davis and Quade (1978) and Randles et al. (1980) to test asymmetry (skewness) around an unknown center.

[7] When weights are not integers, weights are converted to integers by multiplying a constant that is large enough to make the smallest weight to become an integer.

Verbrugge (1999) used this statistic in his analysis of the correlation between the median and skewness of the cross sectional prices. The triples U-statistic is defined as

$$sk_{tu} = \binom{n}{3}^{-1} \sum_{i<j<k} h(x_i, x_j, x_k)$$

where the kernel function is given by

$$\begin{aligned} & h(x_i, x_j, x_k) \\ = \; & \frac{1}{3}(sign\left[(x_i - x_k) + (x_j - x_k)\right] \\ & + sign\left[(x_i - x_j) + (x_k - x_j) + sign\left[(x_j - x_i) + (x_k - x_i)\right]\right]) \\ = \; & \frac{1}{3} sign\left[mean(x_i, x_j, x_k) - med(x_i, x_j, x_k)\right] \end{aligned}$$

When this statistic is used to test the asymmetry around an unknown center, the hypothesis of symmetry is rejected if the corresponding U-statistic is too large in absolute value. The second expression of the kernel function suggests that the validity of the test follows from the observation that for a sample of size three from a symmetric distribution, the sample median is equally likely to be above the sample mean as below it. We use the triples U-statistic as a measure of skewness around an unknown center. Triples U-statistic can be considered as an estimator of

$$\begin{aligned} & P(x_1 + x_2 > 2x_3) - P(x_1 + x_2 < 2x_3) \\ = \; & P\left[(x_1 - x_3) + (x_2 - x_3) > 0\right] - P\left[(x_1 - x_3) + (x_2 - x_3) < 0\right] \end{aligned}$$

3. Detection of Outliers

One of the most widely used identifiers is Tuckey's (1971, 1977) boxplot identifier which uses the first and third quartiles as reference points and determines the length of the whisker by a constant multiple of the interquartile range (IQR):

$$\textit{Boxplot identifier}: \quad [Q_{0.25} - cIQR,\, Q_{0.75} + cIQR], \quad c = 1.5$$

Vandervieren and Hubert (2004) modified Tuckey's boxplot by introducing a robust measure of skewness in the determination of whiskers

$$\textit{VH identifier}: \quad [Q_{0.25} - c_1 IQR,\, Q_{0.75} + c_3 IQR]$$
$$c_1 = 1.5e^{\alpha_1 MC},\ c_3 = 1.5e^{\alpha_3 MC}$$

where MC is the medcouple measure of skewness. $\alpha_1 = -3.5$ and $\alpha_3 = 4$ when $MC \geq 0$ and $\alpha_1 = -4$ and $\alpha_3 = 3.5$ when $MC \leq 0$.

Carling (2000) proposed the use of median $Q_{0.5}$ instead of quartiles $Q_{0.25}$ and $Q_{0.75}$ as the reference point and to use the IQR for the whisker length

$$\textit{Carling identifier}: \quad [Q_{0.5-} cIQR,\, Q_{0.5} + cIQR], \qquad c = 2 \text{ or } 3$$

This identifier is also called the *median rule.* When the dispersion estimator d_{mad} from the MAD is used instead of the IQR, it is called the Hampel identifier

$$\textit{Hampel identifier}: \quad [Q_{0.5} - cd_{mad},\, Q_{0.5} + cd_{mad}], \qquad c = 2 \text{ or } 3^{8}$$

Rousseeuw, Ruts, and Tukey (1999) proposed the bagplot which is a bivariate generalization of the boxplot, and defined the univariate fences as

$$\textit{RRT identifier}: \quad [Q_{0.5} - c(Q_{0.5} - Q_{0.25}),\, Q_{0.5} + c(Q_{0.75} - Q_{0.5})], \quad c = 3 \text{ or } 4$$

Aucremanne, Brys, Hubert, Rousseeuw, and Struyf (2004)[9] called this identifier the *asymmetric boxplot rule* and used $c = 3$, while Rousseeuw, Ruts, and Tukey (1999) used $c = 4$.

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[8]Other choices of the values of c have been used in the literature such as c=3.5 in Sabade and Walker (2002).

[9]They defined the fences of the standard boxplot rule as $[Q_{0.5} - 1.5IQR,\, Q_{0.5} + 1.5IQR]$, but this definition is not consistent with the conventional definition that is widely used in the literature.

C. Empirical Results

Data used in this study is the four-digit level annual Producer Price Indices (PPI) over the sample period of 1947-2003. This is the data set that Demery and Duck (2007) have used in their study of the effect of trend in inflation. We use the 1997 weights that the BLS used in computing the overall PPI[10].

Figure 3-1 shows the proportion of industries' PPI that are identified as outliers by various methods described in the previous section. Average proportions of outliers range from about 11% to about 12%. The Tuckey boxplot, VH boxplot and RRT identifiers show almost identical results on the average as well as over the entire sample period. The Carling and Hampel identifiers[11] give a little smaller proportion as outliers on the average, but the process of their proportions over time is similar to the process of the proportions of other identifiers.

Figure 3-2 presents the classical estimates and various robust estimates of dispersion. The biased and unbiased classical estimates of dispersion are practically identical, and their estimates are greater than robust estimates. As shown in the top panel of Figure 2, the interquartile-based dispersion d_{iqr} and the MAD-based dispersion d_{mad} are very close to each other with a correlation coefficient 0.98. The middle panel of Figure 3-2 shows that the S_n and Q_n measures of Rousseeuw and Croux (1993), denoted by d_{rcs} and d_{rcq}, respectively, are also very close to each other with a correlation coefficient 0.99. These two groups of estimators are highly correlated with the classical measure of dispersion with a correlation coefficient of about 0.7.

[10]We are grateful to Demery and Duck who generously provided the data for our study. When the PPI index is missing for some industries, the weights are normalized after excluding the industry with missing observations.

[11]We used c=3 for Hampel identifier and c=4 for RRT identifier. When smaller values of c are used, the proportion of outliers identified by each method is a little higher: 0.186 for Hampel and 0.176 for RRT.

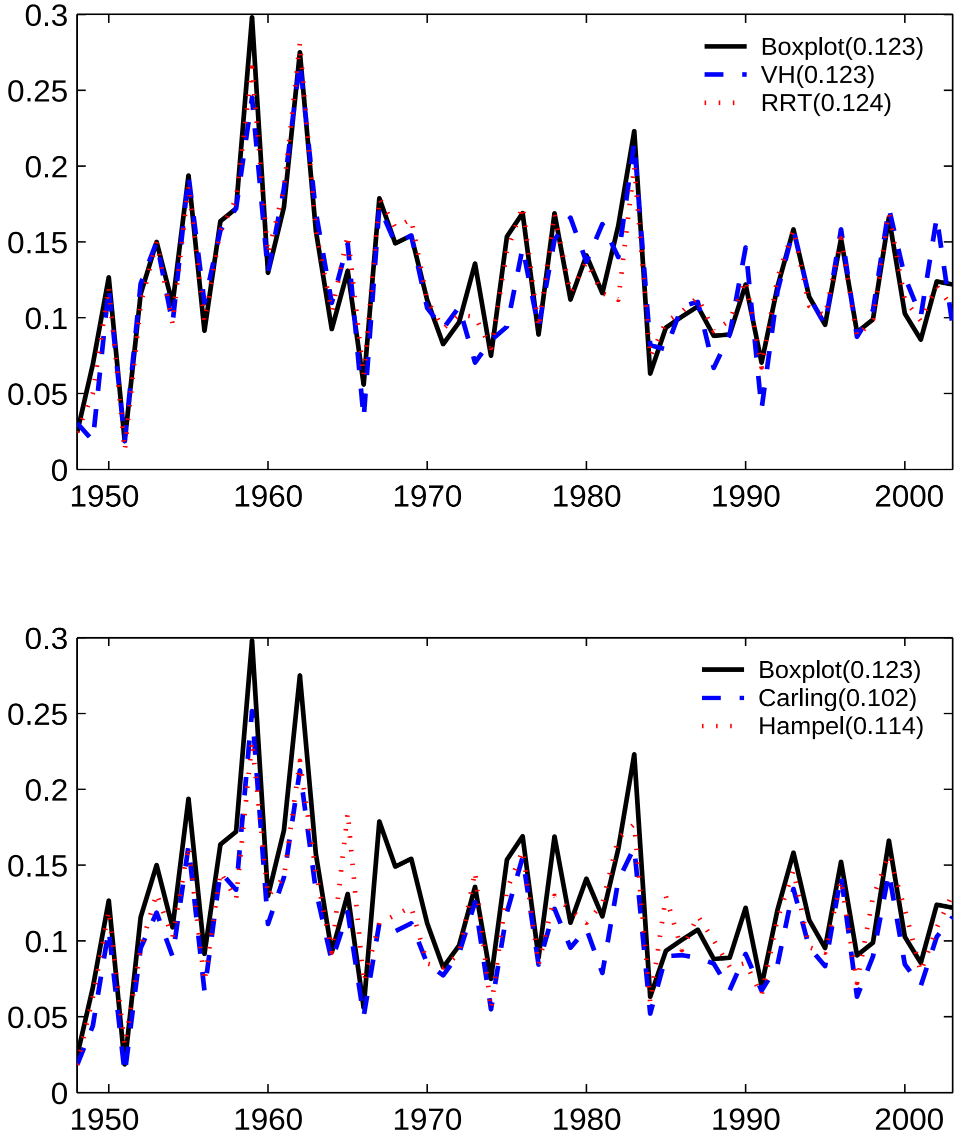

Fig. 3-1.: Proportion of Outliers.

Notes: Average Proportion in Parenthesis.

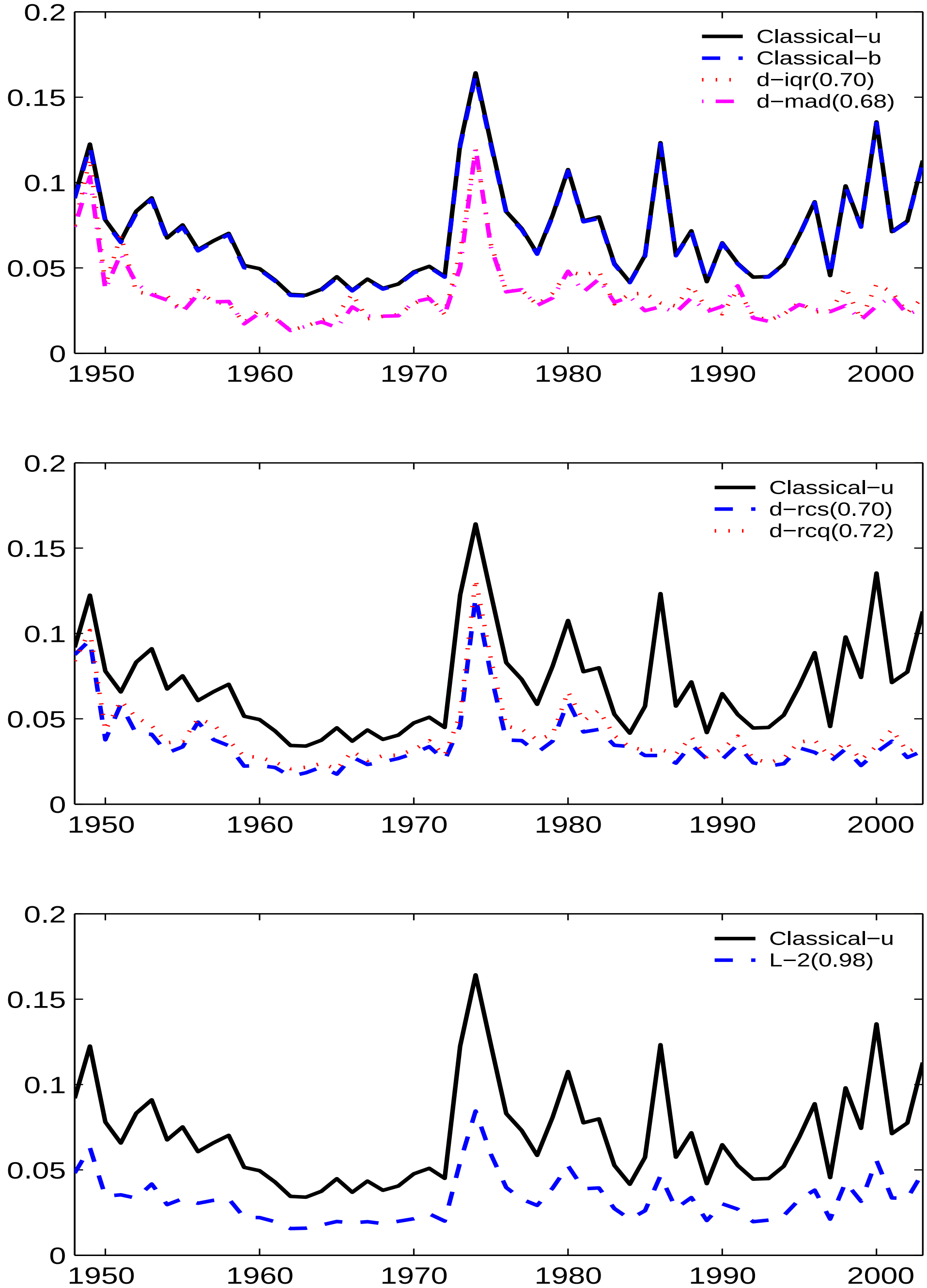

Fig. 3-2.: Alternative Dispersion Estimates.

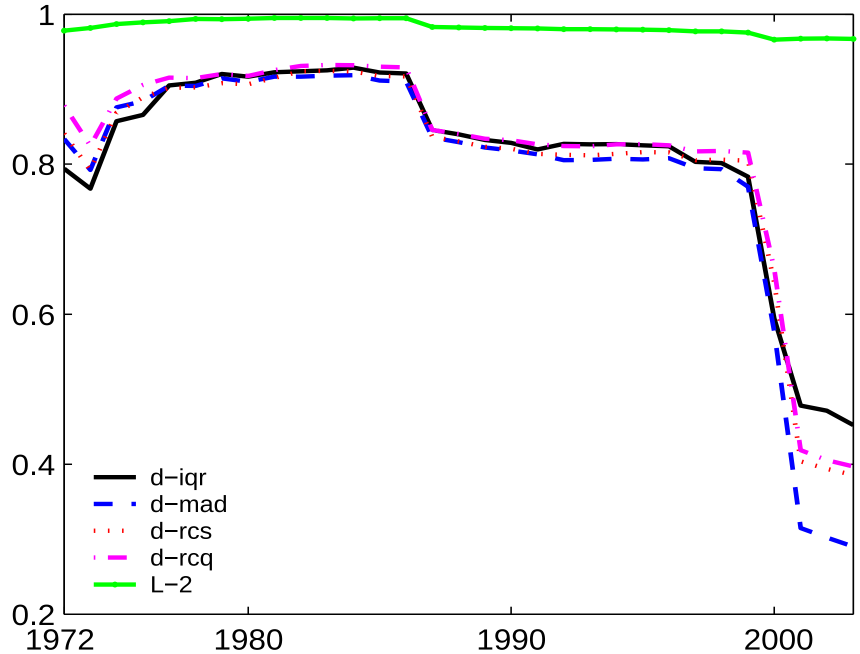

Fig. 3-3.: Correlation between Classical and Robust Estimates of Dispersion.

Notes: Rolling Window of 25 Periods.

But, Hosking's L_2 measure is even more highly correlated with the classical measure with a correlation coefficient 0.98. When L_2 is scaled up to have the same standard deviation as the classical measure, the two series become almost identical.

Although the correlations of robust estimates d_{iqr}, d_{mad}, d_{rcs} and $d_{rcq}$with the classical estimates are quite high for the entire sample period, their relationships appear different between the early sample period and later sample period. The robust estimates seem to stay flat since 1990 while the classical estimates show an upward trend. Figure 3-3 shows the change in the correlation over time, which are computed from 25 period rolling window. It clearly shows a substantial change in the correlation in 1999: there is a much greater discordance between the classical and robust estimates in more recent samples.

Table 3-1.: Correlation among Skewness Estimates.

	sk_{clu}	sk_{clb}	$sk_h(1/4)$	$sk_h(1/8)$	sk_{gm}	sk_{mc}	sk_L	sk_{tu}	sk_{wtu}
sk_{clu}	1.00								
sk_{clb}	1.00	1.00							
$sk_h(1/4)$	0.53	0.53	1.00						
$sk_h(1/8)$	0.63	0.64	0.75	1.00					
sk_{gm}	0.76	0.76	0.79	0.93	1.00				
sk_{mc}	0.63	0.64	0.90	0.89	0.93	1.00			
sk_L	0.81	0.81	0.68	0.88	0.98	0.85	1.00		
sk_{tu}	0.74	0.74	0.63	0.80	0.82	0.78	0.81	1.00	
sk_{wtu}	0.73	0.74	0.75	0.94	0.97	0.90	0.96	0.86	1.00

Robust estimators of skewness, sk_h, sk_{gm}, sk_{mc} and sk_L are bounded in an interval (-1, 1), and the triples U-statistics sk_{wtu} is bounded in an interval (-1/3, 1/3). To make them visually comparable with unbounded classical measures, all estimators are scaled to have the average standard deviation of sk_h, sk_{gm}, sk_{mc}, and sk_L. Figure 3-4 shows the estimates of skewness in three groups, where the grouping is partly based on the correlations among the estimates as reported in Table 3-1. The unbiased and biased classical measures of skewness are almost identical, and the L-skewness measure sk_L is most highly correlated with the classical measures with correlation coefficient 0.81. The robust estimators sk_{gm}, sk_L, and sk_{wtu} are almost identical.

Figure 3-5 shows the changes of relationship between the estimates of the unbiased classical measure and the estimates of robust measures of skewness over time. They are computed from 25 period rolling windows. The relationship seems to be relatively stable for the sk_{gm}, sk_L and sk_{wtu}, but sk_h and sk_{mc} show substantial decrease in their concordance with the classical measure over the period of 1974-1984. For more recent data period, the relationship between the classical and robust measure seem to converge to 0.7-0.8 of correlations.

Turning now to the regression analysis of the effects of the dispersion and skew-

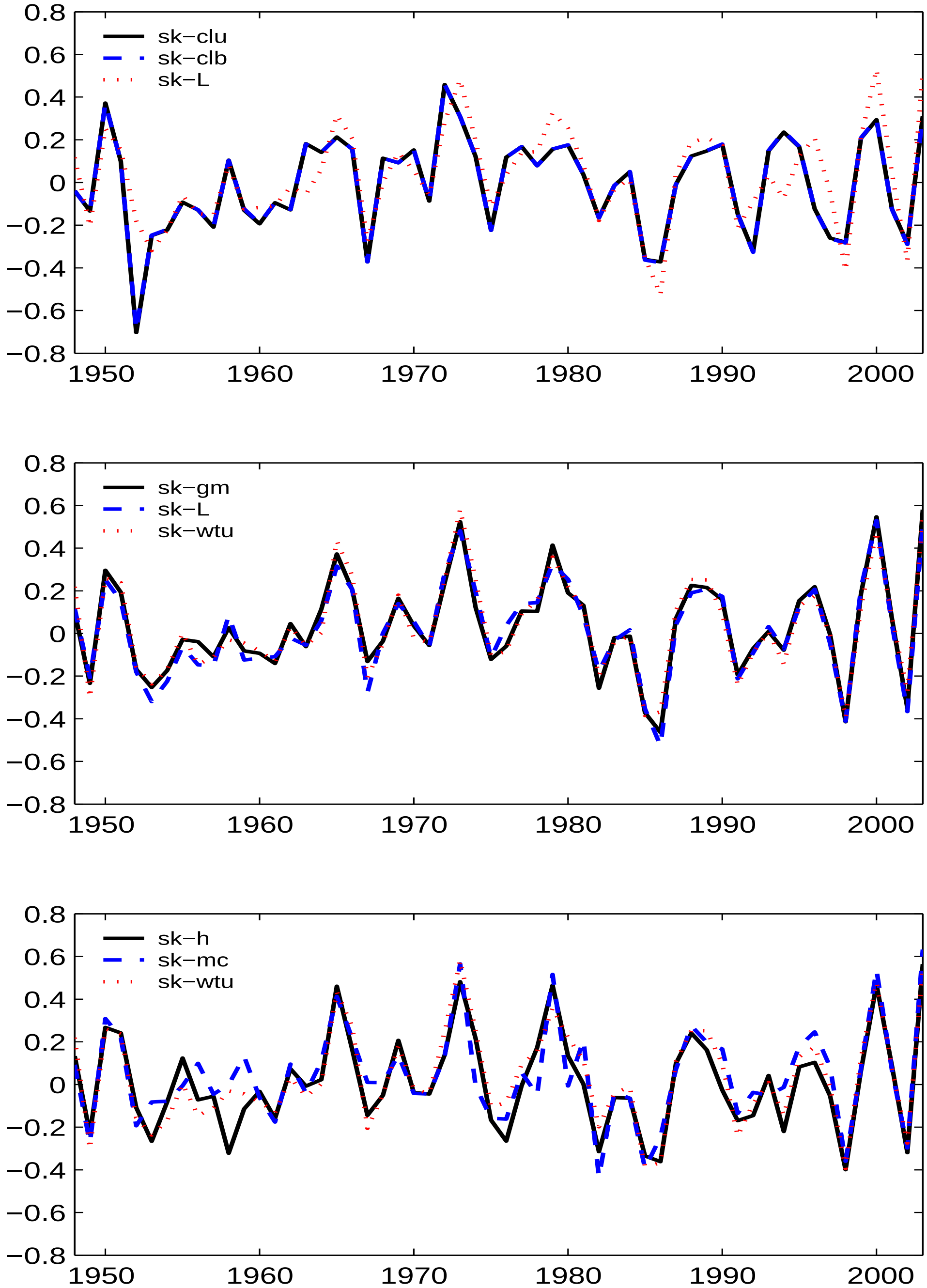

Fig. 3-4.: Alternative Estimates of Skewness.

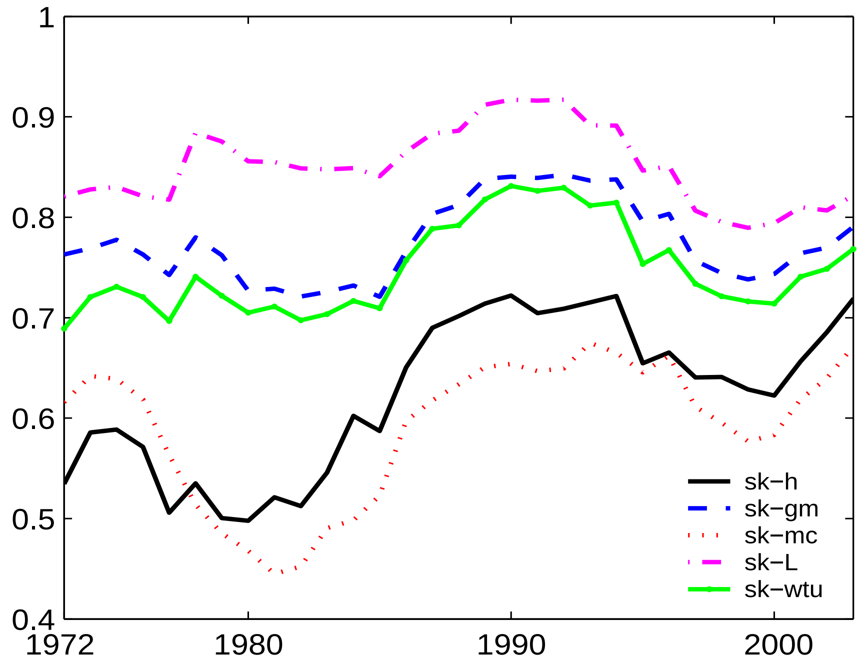

Fig. 3-5.: Correlation between Classical and Robust Estimates of Skewness.

Notes: Rolling Window of 25 Periods.

ness on the aggregate inflation rates, we estimate

$$\pi_t = \alpha + \beta\pi_{t-1} + \gamma_1 SD_t + \gamma_2 SK_t + \gamma_3 \left(SD_t \cdot SK_t\right) + \epsilon_t$$

where π_t is the inflation rate, π_{t-1} is a lagged inflation[12] , SD_t is dispersion, SK_t skewness and ϵ_t is assumed to be an i.i.d disturbance term with a zero mean and a finite variance.

Table 3-2 presents the estimation results. When classical measures are used, both dispersion and skewness have a positive effect on inflation. The effect of skewness is highly significant with a p-value close to zero. But, the effects of dispersion is not

[12]Following Ball and Mankiw, the lagged inflation is used as a proxy for the expected inflation.

Table 3-2.: Estimation Results: 1947-2003.

dispersion	skewness	Marginal Effect of SD (p-value)	Marginal Effect of SK (p-value)	$\overline{R}^2$	Ranking of $\overline{R}^2$
CLS	CLS	0.245	0.000	0.686	23
d_{iqr}	sk_h	0.203	0.000	0.796	14
	sk_{gm}	0.001	0.000	0.817	11
	sk_{mc}	0.000	0.000	0.733	21
	sk_L	0.018	0.000	0.814	12
	sk_{wtu}	0.012	0.000	0.823	10
d_{mad}	sk_h	0.351	0.000	0.806	13
	sk_{gm}	0.000	0.000	0.853	3
	sk_{mc}	0.000	0.000	0.771	19
	sk_L	0.034	0.000	0.833	7
	sk_{wtu}	0.019	0.000	0.840	6
d_{rcs}	sk_h	0.161	0.000	0.796	15
	sk_{gm}	0.000	0.000	0.863	1
	sk_{mc}	0.000	0.000	0.788	17
	sk_L	0.007	0.000	0.827	8
	sk_{wtu}	0.005	0.000	0.843	4
d_{rcq}	sk_h	0.226	0.000	0.791	16
	sk_{gm}	0.000	0.000	0.856	2
	sk_{mc}	0.000	0.000	0.771	18
	sk_L	0.013	0.000	0.824	9
	sk_{wtu}	0.011	0.000	0.841	5
$L2$	sk_h	0.410	0.000	0.692	22
	sk_{gm}	0.101	0.000	0.672	25
	sk_{mc}	0.078	0.000	0.608	26
	sk_L	0.162	0.000	0.676	24
	sk_{wtu}	0.307	0.000	0.741	20

significant with a p-value of 0.245. In the case of robust measures, both dispersion and skewness also have a positive effect on inflation irrespective of the choice of measures. The effects of skewness are highly significant in all cases. But the significance of dispersion depends on the choice of robust measures. Note that when sk_{mc} is used, the effect of dispersion becomes significant. But, when sk_h is used, the effect of dispersion becomes insignificant. It implies that the interaction effect between dispersion and skewness is greater than individual effect of dispersion. Irrespective of the choice of robust measures, the effect of skewness is positively significant. Thus, we don't find any evidence that the positive inflation-skewness relationship can be caused by outliers.

It is interesting to compare the goodness of fit according to the different choice of measures. Table also presents $\overline{R}^2$ of the regressions. When classical measures are used, it is 0.69. $\overline{R}^2$ of the regression used by robust measure are higher than $\overline{R}^2$ by classical measures, except for the cases used $L2$ as a dispersion measure. When d_{rcs} and sk_{gm} are used, the fitting of inflation is the best with a $\overline{R}^2$ of 0.86. The next best set for the fitting is (d_{rcq}, sk_{gm}), (d_{mad}, sk_{gm}) and (d_{rcs}, sk_{wtu}).

D. Conclusion

Ball and Mankiw (1995) showed a positive relationship between inflation and the dispersion/skewness of price changes. One of the issues in past studies concerns the source of the observed positive relationship. Ball and Mankiw argued that its important source is the menu cost, that is, price stickiness. However, their argument was criticized by Bryan and Cecchetti (1999). Their criticism is that the presence of outliers in price changes causes the misleading correlation between mean and the dispersion/skewness of price changes.

We showed there is a significant relationship between inflation and dispersion/skewness after considering outlier effects. Thus, the observed inflation-dispersion/skewness relationship is one of the stylized fact, rather than a spurious result caused by outliers. Consequently, our empirical results generally support the argument of Ball and Mankiw.

However, our results also partly support the criticism of Bryan and Cecchetti. We showed that using robust measures yields the higher goodness of fit in predicting inflation. In particular, medcouple as a measure of skewness is very useful. We find that adjusting outlier problems is reasonable in the study of cross-sectional distribution of price changes.

CHAPTER IV

ESTIMATION OF HYBRID PHILLIPS CURVE: OPTIMUM CHOICE OF INSTRUMENTAL VARIABLES

A. Introduction

Recent literature on the inflation dynamics focuses on two lines of research. The New Keynesian Phillips Curve (NKPC) models are based on the microeconomic foundation that introduces nominal rigidities into the forward-looking optimizing behavior of monopolistically competitive firms. The baseline model specifies the inflation as a function of forward-looking expectations of inflation and marginal costs as the underlying driving force.

Gali and Gertler (1999, GG henceforth) extend the baseline model by introducing two types of firms: forward-looking and backward-looking firms. Their model is a hybrid model that includes past inflation and expected inflation in addition to the marginal costs as the driving force. This model has been applied in numerous empirical applications. Main interests in these studies are the degree of price rigidity, relative role of forward- and backward-looking expectations, and the marginal costs as the driving force instead of more conventional measures such as output gaps.

The model is typically estimated in a structural form or in a closed form by using the GMM. As noted in Nason and Smith (2005), estimates of NKPC parameters are sensitive to the choice of instrumental variables and to the choice of inflation data. To avoid the weak instrumental variables problem, relatively small number of instrumental variables are chosen in general on an *ad hoc* basis. However, since the instrumental variables are for the rational expectation of future inflation and the information set for the conditional expectation can include a large number of informational variables, it is desirable to select the best set of relatively small number of instrumental variables in a systematic way.

Another line research in inflation dynamics is the information forecasting in a data rich environment. Factor models have been used widely in the macroeconomics literature to summarize efficiently a large set of data and to use the summary statistics for a variety of purposes including forecasting. In a series of papers, Stock and Watson (1998, 2002a,b, 2005) propose to use ordinary principal components estimator of the factors, while Forni, Hallin, Lippi, and Reichlin (2000, 2003); Forni, Lippi, and Reichlin (2004); Forni, Hallin, Lippi, and Reichlin (2005) propose to use the generalized principal components estimator. Bernanke, Boivin, and Eliasz (2005) introduce the principal components estimator into the VAR model to overcome the dimensionality problem of the VAR model. The FAVAR augments the standard VAR model with a few latent factors. Bai and Ng (2007b) show that principal components of a large number of weakly exogenous variables are not only valid instruments for the endogenous regressors, but also they can be more efficient than the observed variables, if weakly exogenous instruments and the endogenous regressors share common factors. In practice, the first a few principal components, which explain the variation of indicator variables the most, are used many applications. Bai and Ng (2007a) emphasize, however, that the first a few principal components are not necessarily the best instruments for the endogenous regressors. The problem of selecting the best set of instruments still remains even when we use the principal components of weakly exogenous variables.

This paper examines the robustness of the estimates of parameters in GG's hybrid model to the choice of instrumental variables. Both the structural form and closed form equations of the model are estimated by the GMM. Several sets of instruments are considered, including the set used in GG, Rudd and Whelan (2005, RW5 henceforth), its subset used in Gali, Gertler, and Lopez-Salido (2001, 2005, GGLS henceforth) and Rudd and Whelan (2007, RW7 henceforth). Additional instrumental variable sets include a subset of observed weakly exogenous variables selected by L_2-boosting method of Buhlmann and Yu (2003), and a subset of *ordinary* and *generalized* principal components selected by the L_2-boosting method. The L_2-boosting method is one of three methods suggested in Bai and Ng (2007a).

We find that the boosting procedure from 270 observed variables yields different sets of instruments for the GDP and NFB inflation series, and they differ from the set of instruments used in previous studies. It is interesting to note that the first instrument selected by the boosting is the NAPM (National Association of Purchasing Managers) vendor deliveries index for both inflation series, and both sets include the lagged monetary base, which is not included in GG's instrument set. Since the GDP and NFB inflation series are highly correlated and follow similar time paths, estimates of parameters and significance of hypothesis tests are expected to be similar between the two inflation series. However, GG's and GGLS's instruments give very different estimates for the fraction of backward-looking agents and p-values of some of the test statistics. Boosted instruments, on the other hand, give very comparable estimates. Furthermore, parameter estimates with boosted instruments have much higher joint precision. We can draw similar observations from the instruments boosted from the set of ordinary and generalized principal components. We do not find any significant difference in the results between the ordinary and generalized principal components, but the instruments boosted from principal components tend to perform better than the instruments boosted from observed exogenous variables.

The paper is organized as follows. In section 2, we briefly review GG's hybrid model of inflation and identify the hypotheses that the model implies. Since previous studies did not test some of the hypotheses formally, we test them based on their estimation results. Section 3 describes the L_2-boosting method, computational procedures of the ordinary and generalized principal components, and the methods of determining the number of static and dynamic factors. In section 4, estimation of parameters in GG's model and tests of hypotheses are reported and compared across different sets of instruments. Section 5 concludes the paper with a summary of findings.

B. Specification and Estimation of Phillips Curve Models

GG consider two models of inflation, a baseline model and a hybrid model. Both models are based on Calvo (1983)'s assumption that monopolistically competitive firms face some type of constraints on price adjustment. The probability that a firm may adjust its price during any given period is $(1-\theta)$ and it must keep the current price with probability θ. Each firm faces a demand of constant price elasticity. When all firms are identical ex ante, the aggregate price level p_t is given by a convex combination of p_{t-1} and the optimal reset price p_t^*

$$p_t = \theta p_{t-1} + (1-\theta)\, p_t^* \tag{1}$$

where the optimal reset price that maximizes the expected discounted profit is given by

$$p_t^* = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left(mc_{t+k}^n \right) \tag{2}$$

where mc_t^n is the nominal marginal cost, β is the subjective discount factor, and all variables are expressed as a percent deviation from their steady state values. Combining these two equations, the baseline model is derived as

$$\pi_t = p_t - p_{t-1} = \lambda_0 mc_t + \beta E_t (\pi_{t+1}) \tag{3}$$

where $\lambda_0 = (1-\theta)(1-\beta\theta)/\theta$ and mc_t is the real marginal cost. The closed form equation of (3) is derived by repeated substitution

$$\pi_t = \lambda_0 \sum_{k=0}^{\infty} \beta^k E_t (mc_{t+k}) \tag{4}$$

This closed form predicts that inflation should be determined by the expected discounted sum of future values of the real marginal cost. To make (4) empirically tractable, Rudd and Whelan (2005, RW5 henceforth) truncate the infinite sum to a finite sum plus a remainder term

$$\pi_t = \lambda_0 \sum_{k=0}^{K} \beta^k E_t (mc_{t+k}) + \beta^K E_t (\pi_{t+K+1}) \tag{5}$$

GG generalize the baseline model by introducing two types of firms: 'forward looking' firms and 'backward looking' firms. Forward-looking firms behave like the firms in the baseline model in setting their price p_t^f as in (2). Backward-looking firms set their price p_t^b to the average of newly set prices in previous period plus an adjustment for the realized inflation in previous period

$$p_t^b = \overline{p}_{t-1}^* + \pi_{t-1} = \left[\omega p_{t-1}^b + (1-\omega) p_{t-1}^f\right] + \pi_{t-1} \tag{6}$$

where ω is the fraction of backward-looking firms. Substituting these relationships into the aggregate price level $p_t = \theta p_{t-1} + (1-\theta)\overline{p}_t^*$, they derive a new hybrid model

$$\pi_t = \lambda mc_t + \gamma_f E_t (\pi_{t+1}) + \gamma_b \pi_{t-1} \tag{7}$$

where

$$\lambda = \frac{(1-\theta)(1-\omega)(1-\beta\theta)}{\phi}, \quad \gamma_f = \frac{\beta\theta}{\phi}, \quad \gamma_b = \frac{\omega}{\phi} \tag{8}$$

and $\phi = \theta + \omega\left[1 - \theta\left(1 - \beta\right)\right]$. The closed form equation of (7) is given by

$$\pi_t = \delta_1 \pi_{t-1} + \overline{\lambda} \sum_{k=0}^{\infty} {\delta_2}^{-k} E_t \left(mc_{t+k}\right) \tag{9}$$

where $\overline{\lambda} = \lambda / \left(\delta_2 \gamma_f\right)$ and

$$\delta_1 = \frac{1 - \sqrt{1 - 4\gamma_b \gamma_f}}{2\gamma_f}, \qquad \delta_2 = \frac{1 + \sqrt{1 - 4\gamma_b \gamma_f}}{2\gamma_f}$$

Empirically implementable version of (9) is specified as

$$\pi_t = \delta_1 \pi_{t-1} + \overline{\lambda} \sum_{k=0}^{\infty} \tau^k E_t \left(mc_{t+k}\right) + E_t \left[\tau^{(K+1)} \left(\pi_{t+K+1} - \delta_1 \pi_{t+K}\right)\right] \tag{10}$$

where $\tau = {\delta_2}^{-1}$. Parameters $\overline{\lambda}$, δ_1 and τ in this equation are estimated with or without the last remainder term.

RW5 argue that the estimation of the structural equation (7) can be sensitive to specification errors. If one of the instrumental variables is an omitted variable from the inflation equation, then the instrumental variable estimator of the coefficient of the expected inflation is likely biased upward. They argue that it is preferable to estimate the closed form specification of the inflation equation because it is *model consistent*, and because it is less likely to overstate the effect of forward-looking behavior even if some relevant variables are omitted from the inflation equation.

One of the major issues in the analysis of inflation dynamics is the relative importance of backward- and forward-looking behavior. GG use the relative size of γ_f and γ_b as the measure of relative importance, and draw a conclusion that forward-looking behavior is dominant because the estimate of γ_f is greater than the estimate of γ_b. Though they do not conduct a formal test, we may test GG's measure of relative importance by specifying the null and alternative hypotheses as[1]

$$H^0_{GG} : \gamma_b - \gamma_f \geq 0, \quad H^1_{GG} : \gamma_b - \gamma_f < 0 \tag{11}$$

Rudd and Whelan (2007, RW7 henceforth) criticize GG's measure of relative importance of price setting behavior. They argue that estimation of closed form (10) is preferable to the estimation of structural form (9) because of potential adverse effects of mis-specification errors in the latter. The role of forward-looking behavior is represented by $\overline{\lambda}$ or λ in (10). They argue that parameters γ_f and γ_b are "almost completely unrelated to the question... whether there is a statistically significant role for expected future labor shares." Therefore, the comparison of the estimates of γ_f and γ_b is "not useful for assessing the importance of the forward-looking component of the hybrid model." The null and alternative hypotheses[2] that RW7 prefer to test are

$$H^0_{RW} : \lambda = 0, \quad H^1_{RW} : \lambda > 0 \tag{12}$$

which are equivalent to

$$H^0_{RW} : \overline{\lambda} = 0, \quad H^1_{RW} : \overline{\lambda} > 0 \tag{13}$$

in the closed form equation (10).

In the context of GG's hybrid model with theoretical restrictions on the range of parameters ($0 < \theta < 1$, $0 < \beta \leq 1$ and $0 \leq \omega \leq 1$), this test can be written in two alternative tests. It is easy to see from the expression for λ in (8) that $\lambda = 0$ if and

[1]These hypotheses can be expressed as nonlinear hypotheses in terms of primitive parameters of GG's hybrid model as $H^0_{GG} : \omega - \beta\theta \geq 0$ and $H^1_{GG} : \omega - \beta\theta < 0$.

[2]They seem to use two-sided tests. We specify the hypotheses as a one-sided test because λ takes only non-negative values in the context of GG's model.

only if $\omega = 1$, and $\lambda > 0$ if and only if $\omega < 1$. Therefore, the null and alternative hypotheses in (12) and (13) can be written as

$$H^0_{RW} : \omega = 1, \quad H^1_{RW} : \omega < 1 \tag{14}$$

The test of RW7 is thus a test of the null hypothesis of no forward-looking agents against mere presence of forward-looking agents and it does not consider the magnitude of the effects of forward-looking agents on inflation. On the other hand, GG's test is testing not just the presence of forward-looking agents, but testing the presence of a sufficient number of forward-looking agents such that they influence the inflation dynamics more than backward-looking agents[3].

If $\beta \neq 1$, the test of RW7 is also equivalent to a test of hypotheses

$$H^0_{RW} : \gamma_f + \gamma_b = 1, \quad H^1_{RW} : \gamma_f + \gamma_b < 1 \tag{15}$$

This test is equivalent to the test in the closed form equation

$$H^0_{RW} : \tau = 1, \quad H^1_{RW} : \tau < 1 \tag{16}$$

if $\gamma_f > 1/2$, or to the test

$$H^0_{RW} : \delta_1 = 1, \quad H^1_{RW} : \delta_1 < 1 \tag{17}$$

if $\gamma_f < 1/2$.

The hypotheses in (15) further illustrates that the test of RW7 is also related to parameters γ_b and γ_f, and hence, their criticism on the use of these parameters in GG's test seems to be untenable. Note that $\gamma_f + \gamma_b = 1$ if and only if $\omega = 1$ or

[3]This interpretation is based on the fact that $\beta\theta < 1$ and the test statistic for (14) is same as the test statistic for $\omega = \beta\theta$ against $\omega < \beta\theta$. Comparison of these hypotheses with hypotheses in (15), (16) and (17) gives the stated interpretation.

$\beta = 1$. Therefore, rejection of the null hypothesis $\gamma_f + \gamma_b = 1$ cannot be interpreted as a rejection of $\omega = 1$ unless the null hypothesis $\beta = 1$ is also rejected. When the direct test of $\beta = 1$ is not feasible as in the case of estimating closed form equations, we may conclude that $\beta = 1$ if $\lambda = 0$ ($\overline{\lambda} = 0$) is rejected and $\gamma_b + \gamma_f = 1$ (or $\tau = 1$, or $\delta_1 = 1$) is not rejected.

An alternative simple way to measure the relative importance of backward- and forward-looking behavior is to consider the parameter ω itself. Since ω represents the fraction of firms of backward-looking behavior and all firms are assumed to be identical, we may use the following hypotheses to test the relative importance of two price setting behavior:

$$H_\omega^0 : \omega \geq 0.5, \quad H_\omega^1 : \omega < 0.5 \tag{18}$$

GG use the labor share in the non-farm business sector as the measure of the real marginal cost and the percentage change in the GDP deflator or the non-farm business (NFB) deflator for π_t. Both the baseline model (3) and the hybrid model (7) are estimated by the linear GMM that treats λ, γ_f and γ_b as independent parameters, and by the nonlinear GMM that takes into account the structure of λ, γ_f and γ_b as functions of primitive parameters β, θ and ω. Their set of instrumental variables includes four lags of inflation, the labor income share, the output gap, the long-short interest rate spread, wage inflation, and commodity price inflation. In subsequent papers, GGLS use a subset[4] of these instruments "in order to minimize the potential estimation bias that is known to arise in small samples when there are too many overidentifying restrictions." Data is quarterly data and cover the sample

[4]They allow for an increasing real marginal cost in this paper and their set of instruments include two lags of the real marginal cost, detrended output, wage inflation, and four (five) lags of inflation for US (Euro) data.

period 1960:1-1997:4.

RW5 augment the closed form equation (4) of the purely forward-looking model with lagged inflation rates π_{t-1} and π_{t-2}, and truncate the present value terms to 12 leads ($K = 12$) with a remainder terms as specified in (10). They use the same instruments as in GG with a slightly different sample period. GGLS5 point out that RW5's analysis is inappropriate in assessing the validity of GG's hybrid model because their closed form equation is not the closed form of GG's hybrid model. GGLS5 estimate the closed form equation (10) with 16 leads ($K = 16$) and without the remainder term[5]. In response to GGLS5's criticism, RW7 reestimate the closed form equation (10) with a remainder term and three different sets of instruments: (i) GG's set excluding all third and fourth lagged variables, which will be denoted by GG-2 set, (ii) GGLS5, and (iii) GGLS5 after dropping π_{t-3} and π_{t-4}, which will be denoted by RW set. The first four columns in Table 4-1 list the variables in each instrument set.

There are differences among these studies not only in the choice of the instrumental variables and sample periods, but also in some of the data sets. For comparability, we estimate the structural and closed form equations for the sample period 1960:I - 2003:IV and for four instrumental variables sets: GG, GGLS5, GG-2 and RW. The data set is described in more detail in the section of empirical estimation below. Estimation results are presented in Table 4-2 for the structural equation and in Table 4-3 for the closed form equation.

GG and GGLS5 report four key findings. First, the real marginal cost has a positive and statistically significant effect, and it is a proper variable as the inflation

[5]GGLS5 estimate parameters λ, δ_1 and δ_2, after substituting $\gamma_f = 0.5\,(\delta_1 + \delta_1)^{-1}$, and then compute the estimates γ_f and γ_b from the estimates of δ_1 and δ_2. RW7 estimate λ, δ_1 and τ, and then compute γ_f and γ_b.

Table 4-1.: Set of Instrumental Variables.

GG	GGLS5	GDP-L2	NFB-L2
inflation lag 1-4	✓	✓	✓
Marginal cost 1-2	✓	✓	✓
Marginal cost 3-4			
Real output 1-2	✓	lag2 only	
Real output 3-4			
Nominal Wage 1-2	✓	lag2 only	
Nominal wage 3-4			
Comm. price 1-4[a]		lag1 of two related comm. prices	lag1 of three related comm. price
Interest rate spread 1-4[b]			lag2 only
Additional instruments not in GG and GGLS5		17 real, 2 inflation, 2 monetary variables	8 real, 3 monetary variables

Notes: (a) GG use the spot market price index of all commodities. L_2-boosting selects the CPI-service and PPI-material goods for the GDP deflator, and the CPI-medical care, PPI-material goods and PPI-finished goods for the NFB deflator. (b) GG and GGLS5 define the interest rate spread as the difference between one year government bond yield and the three month treasury bill rate. The NFB deflator selects two-lagged value of interest rate spread defined by the difference between AAA corporate bond yield and the federal funds rate.

driving force. Second, the fraction of backward-looking firms (ω) is significantly different from zero and thus the pure forward-looking model is rejected. Third, the estimate of γ_b is smaller than the estimate of γ_f and hence, the forward-looking behavior is dominant[6]. Lastly, their estimates are consistent with the underlying theory of GG's hybrid model.

Table 4-2 shows a few interesting results. First, the test results of (H1) $\lambda = 0$ in the GDP inflation depend on the choice of instruments: it is rejected with GG and GGLS5, but not with GG-2 and RW. The hypothesis is accepted with large p-values

[6] GG's estimates of γ_b and γ_f subject to the restriction $\beta = 1$ sum to one as the hybrid model implies, but they report different standard deviations for the two coefficients: they should have an identical standard deviation. This is probably due to their use of delta-method in computing standard deviation of γ_f and γ_b from the estimates of primitive parameters.

Table 4-2.: Comparison of Alternative Instrumental Variables: Structural Form Equation (1960:I-2003:IV).

	GDP				NFB			
	GG	GGLS5	GG-2	RW	GG	GGLS5	GG-2	RW
ω	0.400	0.402	0.448	0.408	0.127	0.098	0.030	0.084
	(0.038)	(0.085)	(0.078)	(0.103)	(0.252)	(0.077)	(0.068)	(0.085)
θ	0.882	0.874	0.916	0.926	1.001	0.902	0.936	0.911
	(0.021)	(0.033)	(0.047)	(0.060)	(1.949)	(0.051)	(0.054)	(0.051)
β	0.950	0.948	0.980	0.992	1.000	1.005	1.009	1.005
	(0.030)	(0.038)	(0.039)	(0.034)	(0.021)	(0.030)	(0.031)	(0.034)
γ_b	0.316	0.320	0.331	0.306	0.112	0.098	0.031	0.084
	(0.022)	(0.051)	(0.038)	(0.056)	(0.046)	(0.070)	(0.068)	(0.079)
γ_f	0.663	0.659	0.662	0.690	0.888	0.906	0.977	0.920
	(0.017)	(0.025)	(0.023)	(0.024)	(0.188)	(0.030)	(0.030)	(0.031)
λ	0.009	0.010	0.004	0.003	0.000	0.008	0.004	0.007
	(0.004)	(0.006)	(0.004)	(0.005)	(0.005)	(0.010)	(0.007)	(0.009)
p-Values of Hypothesis Tests								
H1	0.006	0.033	0.213	0.282	0.500	0.207	0.307	0.218
H2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H3	0.000	0.000	0.000	0.000	0.303	0.000	0.000	0.000
H4	0.190	0.304	0.425	0.475	0.501	0.528	0.547	0.523
H5	0.050	0.089	0.302	0.406	0.508	0.565	0.607	0.560
H6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Notes: Standard errors are reported in parenthesis below the estimates. The test hypotheses are as follows.

H1: $\lambda = 0$ vs $\lambda > 0$ H2: $\gamma_b - \gamma_f \geq 0$ vs $\gamma_b - \gamma_f < 0$
H3: $\omega - \beta\theta \geq 0$ vs $\omega - \beta\theta < 0$ H4: $\gamma_b + \gamma_f = 1$ vs $\gamma_b + \gamma_f < 1$
H5: $\beta = 1$ vs $\beta < 1$ H6: $\omega = 1$ vs $\omega < 1$

Table 4-3.: Comparison of Alternative Instrumental Variables: Closed Form Equation (1960:I-2003:IV).

	GDP				NFB			
	GG	GGLS5	GG-2	RW	GG	GGLS5	GG-2	RW
$\overline{\lambda}$	0.017	0.011	0.008	0.009	0.022	0.011	0.013	0.004
	(0.006)	(0.005)	(0.005)	(0.005)	(0.010)	(0.005)	(0.010)	(0.005)
δ_1	0.798	0.747	0.781	0.757	0.756	0.611	0.737	0.489
	(0.026)	(0.049)	(0.039)	(0.049)	(0.024)	(0.061)	(0.048)	(0.094)
τ	0.673	0.925	0.824	0.913	0.631	0.913	0.773	0.972
	(0.107)	(0.040)	(0.081)	(0.048)	(0.166)	(0.042)	(0.170)	(0.027)
γ_b	0.519	0.442	0.475	0.448	0.512	0.392	0.470	0.331
	(0.032)	(0.021)	(0.027)	(0.022)	(0.046)	(0.029)	(0.050)	(0.044)
γ_f	0.438	0.547	0.501	0.540	0.427	0.586	0.492	0.659
	(0.046)	(0.024)	(0.035)	(0.025)	(0.077)	(0.034)	(0.075)	(0.044)
λ	0.011	0.007	0.005	0.005	0.015	0.007	0.008	0.003
	(0.005)	(0.003)	(0.003)	(0.003)	(0.008)	(0.003)	(0.007)	(0.003)
p-Values of Hypothesis Tests								
H1	0.008	0.016	0.053	0.034	0.033	0.019	0.119	0.195
H1($\overline{\lambda}$)	0.003	0.014	0.044	0.031	0.016	0.019	0.100	0.198
H2	0.854	0.010	0.336	0.027	0.756	0.001	0.427	0.000
H4	0.007	0.025	0.016	0.032	0.033	0.011	0.091	0.158
H4(δ_1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H4(τ)	0.001	0.030	0.016	0.035	0.014	0.021	0.092	0.152

Notes: Standard errors are reported in parenthesis below the estimates. The test hypotheses are as follows.

H1: $\lambda = 0$ vs $\lambda > 0$ H1($\overline{\lambda}$): $\overline{\lambda} = 0$ vs $\overline{\lambda} > 0$

H2: $\gamma_b - \gamma_f \geq 0$ vs $\gamma_b - \gamma_f < 0$ H4: $\gamma_b + \gamma_f = 1$ vs $\gamma_b + \gamma_f < 1$

H4(δ_1): $\delta_1 = 1$ vs $\delta_1 < 1$ H4(τ): $\tau = 1$ vs $\tau < 1$

for the NFB inflation, implying that the real marginal cost has no significant effect on the NFB inflation. Second, the estimates of ω, and thereby the estimates of γ_b, show significant differences between the GDP and NFB inflations. The estimates of ω in the NFB inflation equation are not only much smaller, but also they are not statistically different from zero while they are all significant in the GDP inflation equation. Similar results are observed with the estimates of γ_b, except that it is significant with GG instruments in the NFB inflation equation. The third conclusion of GG and GGLS5 holds for all cases in Table 4-2, and the hypothesis (H2) $\gamma_b > \gamma_f$ is strongly rejected with almost zero p-values.

Note that the null hypothesis (H5) $\beta = 1$ is not rejected at 5% level of significance, though it rejected at 10% level in the GDP inflation equation with GG and GGLS5 instruments. As discussed earlier, $\beta = 1$ implies (H4) $\gamma_b + \gamma_f = 1$. If GG's hybrid model is the true data generating mechanism, then we would expect the acceptance of H4 when $\beta = 1$ is not rejected. Table 4-2 shows that this is generally true. However, another theoretical implication that $\lambda = 0$ if and only if $\omega = 1$ does not hold: (H6) $\omega = 1$ is rejected in all cases, but $\lambda = 0$ is rejected in only the first two cases.

Turning to the estimation of the closed form equation, RW5 estimate an equation that includes π_{t-2} as an additional variable in (10) and by using GG's set of instruments and twelve leads ($K = 12$)[7]. They conclude that lagged inflation's role is far more important than can be explained by the pure forward-looking model, and that the effect of forward-looking price setting (λ) is statistically significant, but it is quantitatively unimportant. GGLS5 estimate the closed form equation (10) with 16 leads ($K = 16$) and without including the remainder term. Their instruments are different from those of RW5 and their sample period of quarterly data is 1960:1-

[7]They also estimated the equation with the output gap instead of the real marginal cost as the driving force.

1997:4. They report that the GMM estimates of λ, γ_f and γ_b are almost identical to the GMM estimates of the structural hybrid model.

RW7 report GMM estimates of the closed form parameters of NFB inflation for nine combinations of the number of lead and the set of instrumental variables[8]. The estimates of all parameters except for the estimates of $\overline{\lambda}$ and λ are highly significant in all cases. Because of statistical insignificance of $\overline{\lambda}$ and λ, RW7 conclude that there is "little empirical relationship between inflation and expectations of future values of the labor income share" and there is no empirical evidence of the presence of rational forward-looking agents[9].

Our estimates reported in Table 4-3 show that the test results of $\lambda = 0$ or $\overline{\lambda} = 0$ depend on the choice of instruments in the NFB inflation equation: the hypothesis is accepted with GG-2 and RW instruments and is rejected with GG and GGLS5 instruments. The hypothesis is rejected for the GDP inflation regardless of the choice of instruments. A similar pattern is observed in the test of (H4) $\gamma_b + \gamma_f = 1$ and its equivalent version H4(τ). Tests of H4 (δ_1) also give the same results as H4 when the estimate of γ_f is less than 1/2. The test results of (H2) $\gamma_b > \gamma_f$ also depend on the instrument set: it is rejected with GGLS5 and RW, but not rejected with GG and GG-2, for both inflation.

Results reported in Table 4-2 and 4-3 clearly illustrate the importance of the choice of instrumental variables. Different choices of instruments can lead to totally different conclusions on important issues of interest. We start the next section with a

[8]As noted earlier, GGLS5 estimate parameters λ, δ_1 and δ_2 and then compute the estimates of γ_b and γ_f from the estimates of δ_1 and δ_2, while RW7 estimate $\overline{\lambda}$, δ_1 and τ and then derive λ, γ_b and γ_f.

[9]GGLS5 and RW7 have differences not only in the estimation equation (different number of lead terms and inclusion of the remainder terms), but also in the data of real marginal costs. The two marginal cost data are highly correlated, but they lead to different conclusions in the estimates.

statistical method of selecting the optimum set of instruments from a large number of valid instruments. We then use the selected optimum instruments in the estimation of structural and closed form equations of GG's hybrid model, and compare the results with the results reviewed above.

C. Choice of Instrumental Variables

The instrumental variables for the GMM estimators of the parameters in structural or closed form inflation equation are the variables in the information set for agents' conditional expectation of inflation or marginal costs. This set can contain a large number of variables as the lagged values are valid instruments and as recent advances of information technology allow agents to have an access to data on many economic variables. It is nether practicable to include all valid instrumental variables nor desirable to include an excessive number of instrumental variables as the bias of instrumental variable estimators increases with the number of instruments.

The conventional approach to the problem is to select a few variables and their lagged values from the set of valid instrumental variables. As discussed in the previous section, GG selects lagged values of six distinct variables for instrumental variables, and GGLS1, GGLS5, RW5 and RW7 use a subset of GG's set[10]. Nason and Smith (2005) use four different combinations of lagged values of inflation and marginal cost. In a study of the effect of inflation premium in the hybrid model, Gulyas and Startz (2006) use the forward and the spot inflation premium in addition to the lagged values of inflation and the driving force variable. In the estimation of Taylor rule that involves expected inflation, the instrument set in Clarida, Gali, and Gertler (2000) includes lags of the federal funds rate, inflation, and the output gap, commodity price

[10]GG use four lags of inflation, marginal cost, detrended real output, nominal wage inflation, commodity price inflation, and interest rate spread.

inflation, M2 growth, and the spread between the long-term bond rate and the three-month Treasury Bill rate. In a VAR model of European monetary policy, Favero and Marcellino (2004) use the instrumental variables[11] that are similar to those in Clarida, Gali, and Gertler (2000) and the estimates of static principal components.

In a recent paper, Bai and Ng (2007a) address systematic procedures of selecting a subset from a large set of valid instrumental variables. They consider two large sets: one set is the panel data of observable weakly exogenous variables, and another set is the set of unobservable factors that are estimated from a dynamic factor model. We will first review the L_2 boosting method proposed by Buhlmann and Yu (2003). This is one of the three selection procedures that Bai and Ng examine in their paper. This method can be applied to both observable panel data or the set of principal component estimated from the panel data. After the selection procedure is presented, we will review the estimation methods of standard and generalized principal components from the dynamic factor model.

1. Selection of Optimal Instrumental Variables

Consider a model of interest

$$y_{1t} = \beta' Z_t + \gamma' y_{2t} + u_t, \qquad t = 1, 2, \cdots, T$$

where y_{it} are the endogenous variables and Z_t is the set of exogenous regressors included in the equation[12]. We have a set of large number of instrumental variables

[11]The set includes lagged values of the regressors, of the dependent variable, of a raw material price index, and of the real exchange rate with the US dollar.

[12]The number of endogenous regressor y_{2t} may be more than one, but we will consider the case of single y_{2t} in the following discussion. When there are more than one endogenous regressors, the procedure described below will be applied to each of them.

X_t that are weakly exogenous to the parameters. This set can include the lags of the endogenous variables, lags and functions (such as square) of other predetermined variables. It can also be a set of principal components of a panel data on weakly exogenous variables.

The conventional first stage regression of instrumental variable estimation specifies a regression of endogenous regressor y_{2t} on the included exogenous regressors Z_t and all other instrumental variables X_t. The L_2 boosting method proposed by Buhlmann and Yu (2003) for the selection of 'relevant instruments' is based on repeated first stage least squares including Z_t and one component x_{it} of X_t one at a time

$$y_{2t} = {\pi_1}' Z_t + {\pi_{i2}}' x_{it} + u_{it} \equiv {\pi_i}' W_{it} + u_{it}, \qquad i = 1, 2, \cdots, N \tag{19}$$

where N is the number of variables in X_t. The first relevant instrument x_{i^*} is the instrument that has the highest explanatory power in the least squares sense among all N instrumental variables, i.e., the regression with x_{i^*} yields the smallest sum of squared residuals. Let $\widehat{v}_1 = \widehat{u}_{i^*}$ be the residual vector of using instrument x_{i^*} in the first boosting iteration. Repeat the process with $\widehat{v}_1$ as the dependent variable, and find the second relevant instrument and the corresponding residual vector $\widehat{v}_2$ and so on. Since the search for the minimum SSR is always over the entire N instruments, a variable may be selected more than once.

Let P_j be the projection matrix defined by $W_{i^*} = (Z, x_{i^*})$ at the j-th boosting iteration. Then,

$$\widehat{v}_m = (I - P_m)\,\widehat{v}_{m-1} = \left(\prod_{j=1}^{m} (I - P_j)\right) y_2 \equiv (I - B_m)\, y_2, \qquad m = 1, 2, \cdots, M$$

where $\widehat{v}_0 = y_2$ and M is the maximum number of iterations. It is clear that $B_m y_2$ represents the estimate of the conditional mean of y_2 conditional on Z and m relevant instruments. Though B_m is not the standard projection matrix, the last expression is in the form of usual regression residuals, and B_m plays the role of standard projection matrix. It should be noted that the trace of the standard projection matrix is the number of regressors. And hence, we may use the trace of B_m as an equivalent measure of the number of regressors in the selection of the number of boosting iterations. The number of total boosting iterations is determined by the modified AIC or BIC:

$$IC(m) = \ln\left(\frac{v_m' v_m}{T}\right) + tr(B_m)\frac{A}{T}, \qquad m = 1, 2, \cdots, M$$

where $A = 2$ for the AIC, $A = \ln(T)$ for the BIC, and $tr(B_m)$ is the trace of B_m which is a measure of the 'degree of freedom.'

The procedure presented above is the case of unitary 'step length.' When the step length τ is in the interval $\tau \in (0,1)$, then $\widehat{v}_m$ is computed by

$$\widehat{v}_m = (I - \tau P_m)\,\widehat{v}_{m-1} = \prod_{j=1}^{m}(I - \tau P_j)\,y_2 \equiv (I - B_m(\tau))\,y_2$$

In their numerical analysis, Bai and Ng (2007a) use the conventional step length $\tau = 0.1$ and the maximum number of iterations equal to $M = \min\left[N^{1/3}, T^{1/3}\right]$. It should be noted that the boosting procedure can be conducted with residual matrices $\widetilde{y}_2 = (I - P_z)\,y_2$ and $\widetilde{X} = (I - P_z)\,X$, where $P_z = Z\,(Z'Z)^{-1}\,Z'$.

2. Estimation of Principal Components

Principal components have been used to reduce the dimensionality problem when panel data on a large number of variables are available. For example, in a series of papers, Stock and Watson (1998, 2002a,b, 2005, SW henceforth) consider forecasting a time series using a large number of predictors. To reduce the dimensionality problem, they model the series in terms of a relatively few number of observed variables and unobserved latent factors which are estimated by the principal components of the relevant panel data. Bernanke, Boivin, and Eliasz (2005) propose the Factor-Augmented VAR (FAVAR) model to overcome the dimensionality problem of standard VAR models. The FAVAR augments the standard VAR model with a few latent factors. Bai and Ng (2007b) consider the instrumental variable estimator when the number of available instrumental variables is large. They show that, if a large number of instruments and the endogenous regressors share common factors, the factors estimated from the panel are not only valid instruments for the endogenous regressors, but also they can be more efficient than the observed variables.

There are two types of factor models that have been used in the literature: *static* factor model and *dynamic* factor model. Let X_t be an $N \times 1$ vector of time-series observations on N economic variables with zero means[13]. X_t is a noisy measure of the underlying unobserved dynamic factors and it admits a dynamic factor representation

$$X_t = \lambda\,(L)\, f_t + u_t \tag{20}$$

where f_t is the $q \times 1$ vector of unobserved covariance stationary dynamic factors, $\lambda\,(L)$ is a matrix of lag polynomials of a finite order p, and u_t is the *idiosyncratic component*. $\lambda\,(L)$ is called the dynamic factor loadings and $\lambda\,(L)\, f_t$ is called the *common component*. X_t are noisy measures of the underlying unobserved dynamic factors. The *dynamic representation* of the dynamic factor model in (20) can also be

[13]Estimates of principal components are sensitive to the measurement units. It is therefore a common practice to standardize the data before the estimation.

written in the *static representation*

$$X_t = \begin{bmatrix} \lambda_{10} & \lambda_{11} & \cdots & \lambda_{1p} \\ & & \vdots & \\ \lambda_{n0} & \lambda_{n1} & \cdots & \lambda_{np} \end{bmatrix} \begin{bmatrix} f_t \\ f_{t-1} \\ \vdots \\ f_{t-p} \end{bmatrix} + u_t = \Lambda F_t + u_t \tag{21}$$

where λ_{ij} is a $1 \times q$ vector of factor loadings of f_{t-j}, and Λ is an $N \times r$ matrix, where $r = q\,(p+1)$. It is assumed that $E\,(u_t) = 0$ and $E\,(u_t u_t') = \Sigma_u$. The common factors and idiosyncratic components are assumed to be uncorrelated at all leads and lags, that is, $E\,(f_t u_s') = 0$ for all t and s. If Σ_u is a scalar matrix and $E\,(u_t u_s') = 0$ for all $t \neq s$, then (21) is the classical (strict) factor model. Approximate factor models relax the assumptions by allowing that u_t can be serially and cross-sectionally correlated.

Note that f_t and F_t are unique only up to premultiplication by a unitary (or orthogonal) matrix. That is, $\Lambda F_t = (\Lambda Q)\,(Q' F_t) = \Lambda^* F_t^*$ for any orthogonal matrix Q. Therefore, we cannot identify the common factors. We can estimate only the orthogonal vectors that span the linear space spanned by the common factors. We will briefly review two methods of estimating the static factor F_t: the ordinary (standard) principal component (OPC) estimator that has been used widely, and the generalized principal component (GPC) estimator proposed by Forni, Hallin, Lippi, and Reichlin (2005, FHLR henceforth).

The OPC estimator finds F_t and Λ as the solution to the nonlinear regression problem that minimizes the sum of all squared residuals

$$\min_{F_t,\Lambda} \sum_{t=1}^{T} (X_t - \Lambda F_t)'\,(X_t - \Lambda F_t) = tr\left[(X - F\Lambda')'\,(X - F\Lambda')\right] \tag{22}$$

subject to normalization $\Lambda'\Lambda = I$ and orthogonal conditions that $F'F$ is a diagonal matrix, where X is the $T \times N$ data matrix and F is a $T \times r$ matrix. The estimator of $\widehat{\Lambda}$ is the eigenvectors of $X'X$ corresponding to its r largest eigenvalues and the estimator of the static factors F is the principal component, $\widehat{F} = X\widehat{\Lambda}$. Note that the OPC estimator completely ignores the dynamics among the factors[14].

Dynamic factor models in (20) and (21) do not include any observable variables as the underlying factors of X_t. In their analysis of Factor-Augmented VAR (FAVAR) model, Bernanke, Boivin, and Eliasz (2005) include other observable variables in the static representation (21)

$$X_t = \Lambda F_t + \Psi Z_t + u_t \tag{23}$$

where Z_t in their model represents the main endogenous variables in the standard VAR model. The idea is that both Z_t and F_t represent common factors that drive the dynamics of X_t. The dynamic factor model with autoregressive idiosyncratic terms, $u_{it} = \delta_i(L)\, u_{it-1} + v_{it}$, in Stock and Watson (2005) also takes the form of (23), where $\Psi = diag\left(\delta_i(L)\right)$, $Z_t = X_{t-1}$, and the i^{th} row of Λ is specified as $\left[1 - \delta_i(L)\, L\right] \lambda_i(L)$.

Bernanke, Boivin, and Eliasz (2005) estimate the unobservable factors F_t in (23) by the principal components of X_t, ignoring the presence of observable factors Z_t and excluding Z_t from X_t[15]. Their estimator of the factors is thus an estimator of the linear space spanned by Z_t and F_t, and will be correlated with Z_t in general. Stock and Watson (2005) uses an iterative procedure: starting with an initial estimator of Ψ, F_t is estimated by the first r principal components of $X_t - \Psi Z_t$; given the estimate of F_t, $\Psi = \delta_i(L)$ and Λ are estimated by n individual regressions of X_{it} on $(F_t, X_{i,t-1}, \cdots, X_{i,t-m_i+1})$, where m_i is the order of $\delta_i(L)$. This procedure is repeated

[14]Stock and Watson (2005) suggest to augment a vector of distinct time series in X_t with its lagged values when F_t includes lags of the dynamic factors. This is referred as stacking X_t with its lags.

[15]The same procedure is used in one of the examples in an earlier version of Bai and Ng (2007b).

until convergence.

Alternatively, we can estimate F_t, Λ and Ψ in (23) as the solution to the nonlinear regression problem that minimizes the sum of all squared residuals

$$\min_{F_t,\Lambda,\Psi} \frac{1}{T}\sum_{t=1}^{T}\left(X_t-\Lambda F_t-\Psi Z_t\right)'\left(X_t-\Lambda F_t-\Psi Z_t\right) \tag{24}$$

Hwang (2006) shows that a solution for F is the principal components $F=U\Lambda$, where $U=\left[I-Z\left(Z'Z\right)^{-1}Z'\right]X$ is the matrix of regression residuals of X_t on Z_t, and Λ is the eigenvectors of $U'U$ corresponding to its r largest eigenvalues subject to $\Lambda'\Lambda=I$. Since the principal components are linear combinations of regression residuals, they are orthogonal to observed regressors Z.

While OPC's objective function takes a form of ordinary least squares, the GPC estimators are the solutions to the generalized nonlinear regression problem

$$\min_{F_t,\Lambda}\sum_{t=1}^{T}\left(X_t-\Lambda F_t\right)'\Sigma_u^{-1}\left(X_t-\Lambda F_t\right)=tr\left[\left(X-F\Lambda'\right)'\Sigma_u^{-1}\left(X-F\Lambda'\right)\right] \tag{25}$$

where Σ_u is the contemporaneous covariance matrix of the idiosyncratic component u_t. As in the case of the generalized least squares, this problem becomes the OPC problem after the transformation $\widetilde{X}_t=PX_t$, and $\widetilde{\Lambda}=P\Lambda$, where $\Sigma_u^{-1}=P'P$:

$$\min_{F_t,\Lambda}\sum_{t=1}^{T}\left(\widetilde{X}_t-\widetilde{\Lambda}F_t\right)'\left(\widetilde{X}_t-\widetilde{\Lambda}F_t\right)=tr\left[\left(\widetilde{X}-F\widetilde{\Lambda}'\right)'\left(\widetilde{X}-F\widetilde{\Lambda}'\right)\right] \tag{26}$$

The normalization constraint $\widetilde{\Lambda}'\widetilde{\Lambda}=I$ is equivalent to the restriction $\Lambda'\Sigma_u^{-1}\Lambda=I$. The estimator of F is the principal component $F=\widetilde{X}\widetilde{\Lambda}$, where the columns of $\widetilde{\Lambda}$ are the eigenvectors of $\widetilde{X}'\widetilde{X}$ corresponding to its r largest eigenvalues. The GPC can also be computed by $F=X\Lambda^*$, where Λ^* is the matrix of the generalized eigenvectors of $X'X$ in the metric of Σ_u corresponding to its r largest eigenvalues with normalization

$\Lambda^{*\prime}\Sigma_u\Lambda^* = I$[16]. When X_t is a demeaned time series, $\widehat{\Sigma}_x = X'X/T$ is an estimator of X_t, and hence λ_j^* can be computed as the generalized eigenvector of the covariance matrix $\widehat{\Sigma}_x$ of X_t in the metric of Σ_u.

To implement the GPC estimator we need an estimator of Σ_u. FHLR estimate Σ_u by the average of spectral density matrices of idiosyncratic components. The spectral density matrix of X_t at frequency ω_h is computed by using the Bartlett smoothing lag window w_k:

$$\widehat{S}_x\left(\omega_h\right) = \frac{1}{2\pi}\sum_{k=-m}^{m} w_k \widehat{\Gamma}_k e^{-ik\omega_h}, \quad w_k = 1 - \frac{|k|}{m+1}, \quad \omega_h = \frac{2\pi h}{2H}, \quad h = -H, \cdots, H$$

where $\widehat{\Gamma}_x(k)$ the estimate of k-th autocovariance of X_t. This has the spectral decomposition at each frequency

$$\begin{aligned}
\widehat{S}_x\left(\omega_h\right) &= U\left(\omega_h\right) D\left(\omega_h\right) U\left(\omega_h\right)' \\
&= U_q\left(\omega_h\right) D_q\left(\omega_h\right) U_q\left(\omega_h\right)' + U_{n-q}\left(\omega_h\right) D_{n-q}\left(\omega_h\right) U_{n-q}\left(\omega_h\right)' \\
&\equiv \widehat{S}_c\left(\omega_h\right) + \widehat{S}_u\left(\omega_h\right)
\end{aligned}$$

where $D\left(\omega_h\right)$ be the diagonal matrix with the eigenvalues of $\widehat{S}_x\left(\omega_h\right)$ on the principal diagonal in descending order, and $U\left(\omega_h\right)$ is the matrix of corresponding eigenvectors, and $U\left(\omega_h\right)'$ is the transpose of complex conjugate of $U\left(\omega_h\right)$. $U_q\left(\omega_h\right)$ and $U_{n-q}\left(\omega_h\right)$ are, respectively, the first q columns and the last $n-q$ columns of $U\left(\omega_h\right)$. $\widehat{S}_c\left(\omega_h\right)$ and $\widehat{S}_u\left(\omega_h\right)$ are the estimators of the spectral density matrix of the common components and idiosyncratic components, respectively. The covariance matrices $\widehat{\Sigma}_c$ and $\widehat{\Sigma}_u$ of the common and idiosyncratic components are computed as the average of $\widehat{S}_c\left(\omega_h\right)$ and $\widehat{S}_u\left(\omega_h\right)$ over the frequencies.

[16]The j^{th} column of λ_j^* of Λ^* is the solution to $\left(X'X - \mu_j\Sigma_u\right)\lambda_j^* = 0$ subject to the normalization restrictions $\lambda_j^{*\prime}\Sigma_u\lambda_k^* = 1$ if $j = k$ and $\lambda_j^{*\prime}\Sigma_u\lambda_k^* = 0$ if $j \neq k$.

Generalized eigenvectors Λ^* can now be computed by the eigenvectors of $\widehat{\Sigma}_x$ in the metric of $\widehat{\Sigma}_u$ subject to $\Lambda^{*\prime}\widehat{\Sigma}_u\Lambda^* = I$. FHLR compute $\widehat{\Sigma}_u$ by $\widehat{\Sigma}_u = \widehat{\Sigma}_x - \widehat{\Sigma}_c$, and compute Λ^* by the generalized eigenvectors of $\widehat{\Sigma}_c$ in the metric of a diagonal matrix $diag\left(\widehat{\Sigma}_u\right)$. They use only the diagonal elements of $\widehat{\Sigma}_u$ because this gives better results in their numerical analysis when N is large with respect to T. Note that GPC takes into account the dynamics among the factors by evaluation of the peridogram at different frequencies. Boivin and Ng (2005) point out that, if the static factor model is the true data generating process, "unnecessary estimation of the spectral density matrices could induce efficiency loss."

3. Determination of the Number of Static Factors

In a recent paper, Bai and Ng (2002) propose a few criterion functions for the determination of r. Let $\widehat{V}_k$ be the value of the objective function for the OPC divided by NT:

$$\widehat{V}_k = \frac{1}{NT}\sum_{t=1}^{T}\left(X_t - \widehat{\Lambda}_k\widehat{F}_{tk}\right)'\left(X_t - \widehat{\Lambda}_k\widehat{F}_{tk}\right)$$

where $\widehat{F}_{tk}$ is the estimate of k number of factors. Bai and Ng (2002) propose the information criterion

$$IC_{pj}\left(k\right) = \ln\left(\widehat{V}_k\right) + kg_j\left(N,T\right) \tag{27}$$

which is similar to the information criteria commonly used in the time series analysis except that current penalty function depends on both N and T. They present three penalty functions:

(28)

$$g_1 = \left(\frac{N+T}{NT}\right)\ln\left(\frac{NT}{N+T}\right), g_2 = \left(\frac{N+T}{NT}\right)\ln\left[\min\left(N,T\right)\right], g_3 = \frac{\ln\left[\min\left(N,T\right)\right]}{\min\left(N,T\right)}$$

The $IC_{p2}(k)$ criterion seems to be the most popular statistic in practice.

4. Determination of the Number of Dynamic Factors

Determination of the number of dynamic factors q is particularly important in the GPC analysis because it requires q in the computation of Σ_u. Forni, Hallin, Lippi, and Reichlin (2000) suggest a heuristic inspection of the averages of the eigenvalues of $\widehat{S}_x(\omega_h)$ over the frequencies for different number of variables. Bai and Ng (2007c) propose a more systematic way to determine q. Their method is based on the fact that, when a dynamic factor model is written in a static form such as in (21), the static factor F_t follows a autoregressive process.

Suppose that the dynamic factors follow $AR(h)$ process $f_t = B_h(L) f_{t-1} + \epsilon_t$, where ϵ_t is an *i.i.d.* innovation vector with a diagonal covariance matrix. Then, the static factors F_t can be written as an $AR(\tau)$ process $F_t = A_\tau(L) F_{t-1} + \xi_t$, where $\xi_t = R\epsilon_t$, R is a $r \times q$ matrix of rank q, and $\tau = \max(1, h-p)$. The covariance matrix of ξ_t, Σ_ξ, is a $r \times r$ matrix with a rank $q < r$. They determine q by the statistic that captures the number of nonzero eigenvalues of Σ_ξ.

Static factors are first estimated by principal components, and the number of factors is determined by using one of the information criteria in (27) and (28). Using the estimated factors $\widehat{F}_t$, Σ_ξ is estimated by the sample moments of residuals $\widehat{\Sigma}_\xi = T^{-1}\sum_{t=1}^{T}\widehat{\xi}_t\widehat{\xi}_t'$, where $\widehat{\xi}_t$ is the residual vector of the regression of $\widehat{F}_t$ on its lagged values

$$\widehat{F}_t = A_1\widehat{F}_{t-1} + \cdots + A_p\widehat{F}_{t-p} + \xi_t$$

Let the eigenvalues of $\widehat{\Sigma}_\xi$ be denoted by c_i, arranged in descending order so that the first q eigenvalues are nonzeros and the last $r-q$ eigenvalues are zeros. Define

$$D_{1,k} = \left(\frac{c_{k+1}^2}{\sum_{i=1}^{r} c_i^2} \right)^{1/2}, \quad D_{2,k} = \left(\frac{\sum_{i=k+1}^{r} c_i^2}{\sum_{i=1}^{r} c_i^2} \right)^{1/2}$$

Then, $D_{1,k} = D_{2,k} = 0$ for all $k \geq q$ because $c_k = 0$ for $k > q$. The estimator of q that Bai and Ng propose are

$$\widehat{q}_3 = \arg\min_{k} \left(D_{1,k} | D_{1,k} < m^* \right) \tag{29}$$

$$\widehat{q}_4 = \arg\min_{k} \left(D_{2,k} | D_{2,k} < m^* \right) \tag{30}$$

where $m^* = m/\min\left[T^{1/2-\delta}, N^{1/2-\delta}\right]$, $0 < \delta < 1/2$ and $0 < m < \infty$.

To implement this procedure, one needs to specify parameters m, δ, and τ. In their simulation study, Bai and Ng use $\delta = 0.1$, $m = 2, 1, 0.5$ and $\tau = 2$. Eigenvalues and eigenvectors are computed by using the singular value decomposition method.

Stock and Watson (2005) exploit a different implication of the static representation of the dynamic factor model. Substituting $F_t = A_\tau(L) F_{t-1} + \xi_t$ into (21), the model can be written as $X_t = \Lambda A_\tau(L) F_{t-1} + \eta_t$, where $\eta_t = (\Lambda R)\epsilon_t + u_t$. This is precisely the form of standard static factor model if data on η_t are available. They estimate η_t by the residuals of the regression of X_t on lagged values of $\widehat{F}_{t-1}$, and then use the information criteria in (27) and (28) to determine the number of static factors of η_t, which coincides with the number of dynamic factors q. They use $\tau = 2$ in their empirical analysis.

D. Empirical Estimation of Hybrid Phillips Curve

The data is a quarterly panel data of 138 variables over the sample period 1960:I-2003:IV. The data set includes Stock and Watson's (2005) 132 time series data and GG's six time series data that are not included in the former[17]. All data is obtained from the Global Insights Basic Economics Database (GIBED) except for the non-farm business (NFB) deflator, which is obtained from the Federal Reserve Economic Data (FRED)[18]. Following GG and Marcellino, Stock, and Watson (2003), the monthly data is aggregated to quarterly data by the quarterly averages of the monthly data. Stock and Watson (2005) transform the nonstationary series by taking the first or second differences in log or level data, while GG and GGLS5 take only the first differences of log or level data nonstationary variables[19]. Stock and Watson (2005) adjust the outliers in some of their monthly data, but we do not adjust for the outliers in our quarterly data. The inflation is measured as the the log difference in the GDP deflator (or NFB deflator) and the marginal cost is constructed as the log of labor

[17]Data covers macroeconomic variables such as industrial production, personal income, inventories, employment, payroll, new housing starts, manufacturer's new orders, stock price index, interest rate, consumer price index, the producer price index, personal consumption expenditure deflator and average hourly earnings. GG's 7 variables are GDP deflator, NFB deflator, labor income share, the interest rate spread, output gap (quadratically detrended real GDP), wage inflation (compensation per hour of nonfarm business sector), and commodity price inflation (spot market price index of all commodities). The interest rate spread appears in both GG and Stock and Watson (2005) data sets, but they are computed differently. GG compute the spread by the difference between one year government bond yield and three month treasury bill rate, while Stock and Watson define eight different spreads: for example, the difference between one year government bond yield or AAA corporate bond yield and the federal funds rate.

[18]We found that the NFB deflator data in the GIBED, in GG's study and in RW7's study are all different. RW7's data seem to be identical to FRB's data with some recent data that may have been revised. Compared to the data that Stock and Watson posted on their web site, our data on Stock and Watson's variables reflect revisions on several variables.

[19]Stock and Watson take the first difference of the commodity price inflation, but GG do not. We will follow Stock and Watson's procedure.

income share in the non-farm business sector. The ordinary and generalized principal components are estimated from 135 variables, excluding the variables that appear in the inflation equation, i.e., GDP deflator, NFB deflator and the labor income share.

1. Selection of Instruments from Observed Instrumental Variables

We first examine whether the choice of instrumental variables in GG and GGLS studies are optimal in the sense of Bai and Ng. GG use four lags of inflation, marginal cost, detrended real output, nominal wage inflation, commodity price inflation, and interest rate spread, while GGLS5 use four lags of inflation, and two lags of marginal cost, detrended real output, and nominal wage inflation. We keep four lags of inflation and two lags of marginal cost as retained instrumental variables (Z_t in equation (19), and select the best instruments for π_{t+1} from two lags of all other variables by using the L_2-boosting method[20]. The inflation rate is defined by the log difference in GDP deflator or NFB deflator.

The BIC criterion selects 25 variables for the GDP deflator and 15 variables for the NFB deflator. There are five common instrumental variables selected for both GDP and NFB deflators[21]. Table 4-1 compares the selected instrumental variables with those in GG and GGLS5. The set for the GDP deflator includes only the two-lagged values of detrended real output and nominal wage inflation. It includes one-

[20]When the GDP deflator is used for the inflation, lagged values of the NFB deflator are not included in the set of instrumental variables, and vice versa. The set of candidate instrumental variables consists of one-and two-lagged values of Stock and Watson's 132 variables, plus one-and two-lagged values of GG's 3 variables (detrended real output, nominal wage inflation and interest rate spread). The total number of variables in X_t is thus 270 variables. The step length is set to $\tau = 1$.

[21]These are one lagged values of NAPM vendor deliveries index, Monetary base-Adjusted for reserve requirement changes, and PPI-intermed Mat. Supplies & Components, and two lagged values of IP index-durable consumer goods, and average weekly hours of Prod or Nonsup workers on private nonfarm payrolls-goods-producing.

lagged values of two commodity price indices which are different from the spot market price index of all commodities that GG use. The set for the NFB deflator includes neither the detrended real output, nor nominal wage inflation. It includes one-lagged values of interest rate spread defined by the difference between AAA corporate bond yield and the federal funds rate, while GG define the spread by the difference between one year government bond yield and the three month treasury bill rate.

It is interesting to not that the first instrumental variable selected by L_2-boosting method is the lagged NAPM (National Association of Purchasing Managers) vendor deliveries index for both the GDP and NFB deflators. The second instruments that is selected is the one-lagged number of building permits in Midwest region for the GDP deflator and the one-lagged inventory to sales ratio for the NFB deflator. One-lagged value of monetary base is in the set for both GDP and NFB deflators.

We estimate the underlying model parameters (ω, θ, β) in GG's specification (7) by using the nonlinear GMM, compute the estimates of parameters $(\gamma_b, \gamma_f, \lambda)$ from the estimates of underlying model parameters, and compute the p-values of various test statistics that are discussed in section 2. Table 4-4 presents the results for GGLS5's sample period, 1960:I-1997:IV, as well as for the entire sample period, 1960:I-2003:IV. We also replicate the results in GGLS5 with their data for the first subsample, and find some differences between their results and the results reported in Table 4-4 under the heading GGLS5. The results for the GDP deflator are qualitatively similar, but there are some noticeable differences for the NFB deflator. The p-value of the test hypothesis $\lambda = 0$ for the NFB deflator is 0.140 with our data, but it is 0.019 with their data. We find that the major source of this difference is the difference in the marginal cost data[22].

The results reported in Table 4-4 with our data are similar between the two sample periods, and hence we focus our discussion on the results in the full sample period. There is a substantial difference in the estimates of the fraction of backward-looking firms ω for the GDP and NFB deflators when GGLS5's instruments are used. The estimate of ω for the GDP deflator is more than three times greater than the estimate for the NFB deflator. The difference in the estimates of ω is also reflected in the estimates of γ_b. This is rather difficult to justify as the two deflators are similar with a high correlation coefficient (0.94). The difference is much smaller when the best instruments are used: the estimate of ω for the GDP deflator is a little more than 50% greater than the estimate for the NFB deflator when the best instruments are used.

GGLS5's instruments also yield a substantial difference between the two deflators in the test of the null hypothesis $\lambda = 0$ against the alternative hypothesis $\lambda > 0$. The null hypothesis is rejected for the GDP deflator at 5% level while it is not rejected for the NFB deflator even at 20% significance level. When the L_2-boosted instruments are used, the null hypothesis is rejected very strongly for both deflators. We observe a similar results in the test of the null hypothesis $\beta = 1$ against $\beta < 1$. That is, GGLS5's instruments are used, $\beta = 1$ is easily accepted for the NFB deflator and it is rejected at 10% level of significance for the GDP deflator. Estimates with L_2-boosted instruments strongly reject $\beta = 1$.

Another significant difference between using GGLS5's instruments and L_2-boosted instruments can be seen in the test of the null hypothesis $\gamma_b + \gamma_f = 1$ against

[22]When their marginal cost is replaced with our marginal cost data, the p-value becomes 0.155. Substitution of the data of other variables do not change the p-value much from their p-value.

Table 4-4.: GMM Estimation of Phillips Curve.

	1960:I-1997:IV				1960:I-2003:IV			
	GDP		NFB		GDP		NFB	
	GGLS5	L2-OBS	GGLS5	L2-OBS	GGLS5	L2-OBS	GGLS5	L2-OBS
ω	0.410	0.527	0.146	0.412	0.402	0.511	0.098	0.337
	(0.087)	(0.035)	(0.074)	(0.042)	(0.085)	(0.040)	(0.077)	(0.049)
θ	0.873	0.849	0.893	0.881	0.874	0.857	0.902	0.886
	(0.034)	(0.015)	(0.045)	(0.033)	(0.033)	(0.017)	(0.051)	(0.031)
β	0.934	0.804	0.986	0.809	0.948	0.847	1.005	0.864
	(0.041)	(0.037)	(0.034)	(0.042)	(0.038)	(0.034)	(0.030)	(0.036)
γ_b	0.325	0.409	0.141	0.337	0.320	0.393	0.098	0.285
	(0.051)	(0.017)	(0.062)	(0.025)	(0.051)	(0.020)	(0.070)	(0.033)
γ_f	0.648	0.530	0.849	0.583	0.659	0.558	0.906	0.647
	(0.025)	(0.017)	(0.032)	(0.027)	(0.025)	(0.016)	(0.030)	(0.026)
λ	0.011	0.018	0.010	0.016	0.010	0.015	0.008	0.015
	(0.006)	(0.004)	(0.010)	(0.008)	(0.006)	(0.004)	(0.010)	(0.007)
p-Values of Hypothesis Tests								
H1	0.034	0.000	0.140	0.022	0.033	0.000	0.207	0.018
H2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H4	0.277	0.004	0.414	0.002	0.304	0.018	0.528	0.006
H5	0.056	0.000	0.341	0.000	0.089	0.000	0.565	0.000
H6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H7	0.151	0.780	0.000	0.018	0.126	0.613	0.000	0.001
GV P	5.760	0.076	8.645	1.610	3.728	0.102	9.282	1.264
GV D	1.820	0.048	12.254	0.958	1.364	0.054	17.887	1.157
J-stat	7.098	10.462	6.462	10.462	7.864	12.056	6.623	12.056
	0.419	0.999	0.487	0.999	0.345	0.996	0.469	0.996

Notes: Standard errors are reported in parenthesis below the estimates. The test hypotheses are as follows.

H1: $\lambda = 0$ vs $\lambda > 0$ H2: $\gamma_b - \gamma_f \geq 0$ vs $\gamma_b - \gamma_f < 0$
H3: $\omega - \beta\theta \geq 0$ vs $\omega - \beta\theta < 0$ H4: $\gamma_b + \gamma_f = 1$ vs $\gamma_b + \gamma_f < 1$
H5: $\beta = 1$ vs $\beta < 1$ H6: $\omega = 1$ vs $\omega < 1$
H7: $\omega \geq 0.5$ vs $\omega < 0.5$

$\gamma_b + \gamma_f < 1$. The null hypothesis is accepted with GGLS5's instruments for both deflators, but it is strongly rejected with L_2-boosted instruments. As discussed in section 2, $\gamma_b + \gamma_f = 1$ if either $\beta = 1$ or $\omega = 1$. The tests with GGLS5's instruments indicate $\beta = 1$ and $\omega < 1$. Thus, we draw a conclusion that $\gamma_b + \gamma_f = 1$ is due to $\beta = 1$. On the other hand, the tests with L_2-boosted instruments indicate that $\beta < 1$ and $\omega < 1$, which imply $\gamma_b + \gamma_f < 1$.

Table 4-4 reports the generalized variance (i.e., determinant of the covariance matrix) of the estimates of primitive parameters (ω, θ, β) and the derived parameters $(\gamma_b, \gamma_f, \lambda)$ as a measure of joint precision of the estimates. Boosted instruments give substantially smaller values of the generalized variance for both inflation measures. The J-statistics indicate that GGLS5's instruments are not rejected but the p-values of the J-statistics for the boosted instruments are much higher than that for the GGLS5's instruments.

2. Selection of Instruments from Principal Components

We use the specification in (23) as the factor model, where X_t consists of the candidate instrumental variables that is used for the L_2-boosting in the previous subsection and Z_t includes the retained variables (four lagged inflation and two lagged marginal cost). All variables are standardized before computing the principal components[23]. Ordinary principal components are computed from the residuals of the regression of X_t on Z_t by using singular-value decomposition. The principal components are thus orthogonal to the retained instrumental variables.

The number of static factors $\widehat{r}$ is determined by using Bai and Ng's (2002) in-

[23]Each lagged variable is treated as an independent variable. That is, π_{t-1} and π_{t-2} are standardized separately instead of taking the lagged values of standardized values of π_{t-1} as the standardized values of π_{t-2}.

formation criterion $IC_{p2}(k)$ as specified in (28). This procedure selects the first 10 and 7 principal components for the GDP and NFB deflators, respectively. Primitive parameters (ω, θ, β) and derived parameters $(\gamma_b, \gamma_f, \lambda)$ of the Phillips curve equation (7) are estimated by the GMM. The set of instrumental variables include the retained instrumental variables Z_t and the first $\widehat{r}$ number of principal components. The results are reported in Table 4-5 under the heading $\widehat{r}$-OPC.

As Bai and Ng (2007a) emphasize, the principal components that explain the variation in X_t the best are not necessarily the best instrumental variables for the endogenous variable π_{t+1}. Therefore, we apply L_2-boosting to the first $\widehat{r}$ number of principal components and expanded sets of principal components as the base set for the boosting. To determine the size of the expanded boosting base, we first examine the fraction of variance explained by each principal component. Figure 4-1 shows the cumulative fraction of principal components in descending order of eigenvalues. The first 10 principal components explain 59% of the variance of X_t unexplained by Z_t, and 99.8% of the variance is explained by the first 130 principal components. We thus consider the first 130 principal components as the maximum base for L_2-boosting.

Since the choice of boosting base will lead to different sets of instrumental variables and hence different estimates of parameters, we use the boosting base from the first 10 to 130 principal components, adding one additional principal component each time. Instrumental variables selected from each boosting base are used to estimate the parameters and their generalized variance. Figure 4-2 shows the effect of different boosting base on the joint precision of the estimates of primitive and derived parameters. There is a significant gain in joint precision of the estimates as the boosting base increases from a small set, but the additional gains become negligible as the base size increases beyond 60 (or at most 80) principal components for the GDP deflator and beyond 80 principal components for the NFB deflator. Estimation results with

Table 4-5.: GMM Estimation of Phillips Curve-GDP Deflator: Standardized Data for OPC (1960:I-2003:IV).

				L2 Boosting			
	GG	GGLS5	$\widehat{r}$-OPC	OBS	$\widehat{r}$-OPC	OPC80	OPC100
ω	0.400 (0.038)	0.402 (0.085)	0.365 (0.057)	0.511 (0.040)	0.397 (0.071)	0.437 (0.043)	0.450 (0.041)
θ	0.882 (0.021)	0.874 (0.033)	0.862 (0.026)	0.857 (0.017)	0.878 (0.030)	0.851 (0.020)	0.848 (0.019)
β	0.950 (0.030)	0.948 (0.038)	0.946 (0.034)	0.847 (0.034)	0.950 (0.037)	0.883 (0.026)	0.872 (0.026)
γ_b	0.316 (0.022)	0.320 (0.051)	0.302 (0.035)	0.393 (0.020)	0.316 (0.040)	0.351 (0.023)	0.360 (0.021)
γ_f	0.663 (0.017)	0.659 (0.025)	0.674 (0.021)	0.558 (0.016)	0.663 (0.022)	0.604 (0.014)	0.592 (0.014)
λ	0.009 (0.004)	0.010 (0.006)	0.013 (0.005)	0.015 (0.004)	0.010 (0.005)	0.017 (0.005)	0.017 (0.005)
p-Values of Hypothesis Tests							
H1	0.006	0.033	0.007	0.000	0.035	0.000	0.000
H2	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H3	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H4	0.190	0.304	0.222	0.018	0.286	0.029	0.019
H5	0.050	0.089	0.058	0.000	0.089	0.000	0.000
H6	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H7	0.005	0.126	0.010	0.613	0.075	0.071	0.112
GV Prim	0.156	3.728	0.878	0.102	2.107	0.121	0.097
GV Derv	0.066	1.364	0.630	0.054	0.801	0.101	0.081
# of PC			169	270	10	80	100
# of IV	18	4	10	25	5	21	23
J-stat	10.844 (0.966)	7.864 (0.345)	10.399 (0.661)	12.056 (0.996)	8.274 (0.407)	11.466 (0.985)	11.641 (0.993)

Standard errors are reported in parenthesis below the estimates. The test hypotheses are as follows.

H1: $\lambda = 0$ vs $\lambda > 0$ H2: $\gamma_b - \gamma_f \geq 0$ vs $\gamma_b - \gamma_f < 0$
H3: $\omega - \beta\theta \geq 0$ vs $\omega - \beta\theta < 0$ H4: $\gamma_b + \gamma_f = 1$ vs $\gamma_b + \gamma_f < 1$
H5: $\beta = 1$ vs $\beta < 1$ H6: $\omega = 1$ vs $\omega < 1$
H7: $\omega \geq 0.5$ vs $\omega < 0.5$

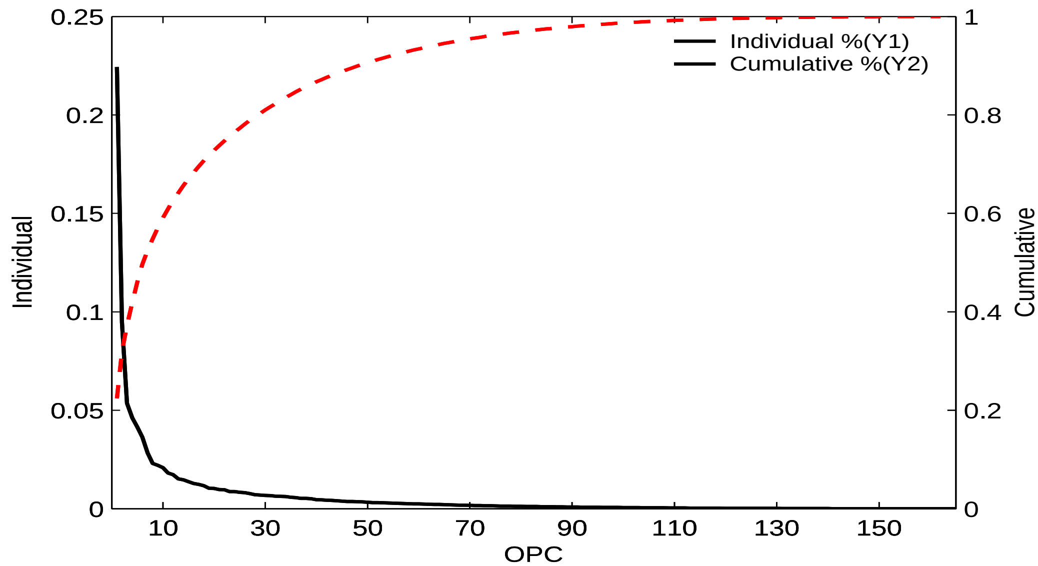

Fig. 4-1.: Percentage of Variance Explained by Ordinary Principal Components.

Notes: Standardized Data

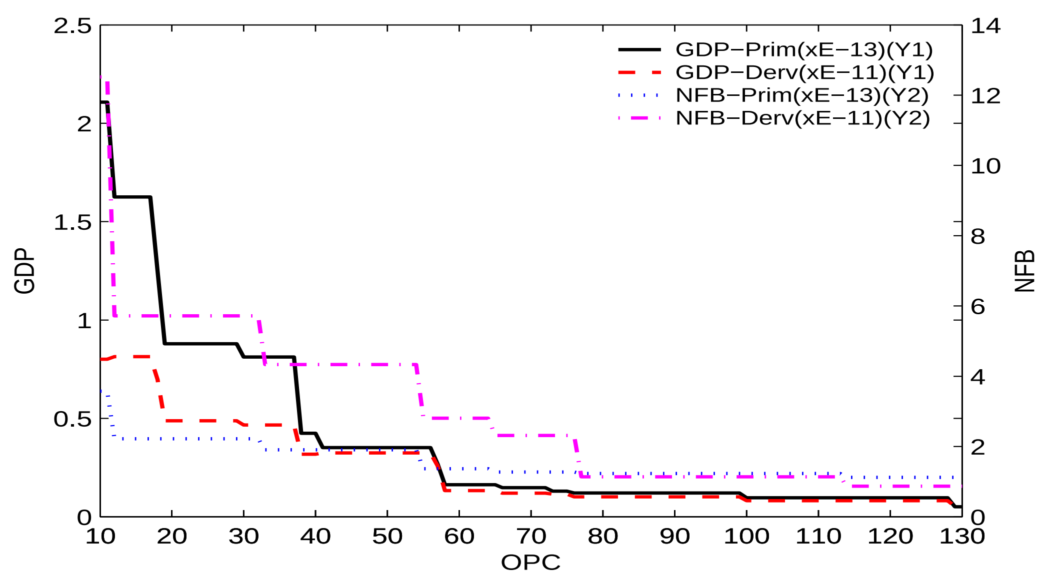

Fig. 4-2.: Generalized Variances of Primitive and Derived Parameters: OPC from Standardized Data.

the instrumental variables selected from the first 80 and 100 principal components are reported in Table 4-5.

Estimates of parameters and the p-values of hypothesis tests with the first $\widehat{r}$ number of principal components ($\widehat{r}$-OPC) as the instruments are very similar to the results of using GGLS5's instruments, except that the joint precision of parameter estimates is much higher with the former instruments than with the latter instruments. The gain in precision is not entirely due to a larger number of instruments in $\widehat{r}$-OPC, which uses 10 instruments while GGLS5 use 4 instruments, in addition to the retained instruments. This can be seen from a comparison of GGLS5 with the column $L2$-boosted $\widehat{r}$-OPC which uses 5 instruments boosted from $\widehat{r}$-OPC principal components. Results of GGLS5 and $L2$-boosted $\widehat{r}$ are extremely close to each other, but the joint precision of parameter estimates is much higher for $L2$-boosted $\widehat{r}$ than for GGLS5. The similarity of $\widehat{r}$-OPC and $L2$-boosted $\widehat{r}$ with the GGLS5 in the estimates of parameters implies that our earlier observations about the relationship between $L2$-OBS and GGLS5 also hold for the relationship between $L2$-OBS and $\widehat{r}$-OPC or $L2$-boosted $\widehat{r}$. We may conclude that using the first $\widehat{r}$ number of principal components as instruments is not as good as using instruments selected by L_2-boosting over the observed instrumental variables.

There is little difference among the estimates with the instruments boosted from 80 and 100 principal components though the number of instruments are different. The joint precision of the estimates of course increases with the number of instruments. Cursory inspection of the rows in Table 4-5 reveals no large differences in the estimated values of parameters. But, there are more variations in the estimated standard errors, and they are reflected in the differences in the p-values of hypothesis tests. The hypothesis H4: $\gamma_b + \gamma_f = 1$ is not rejected by GG, GGLS5, $\widehat{r}$-OPC and boosted $\widehat{r}$-OPC, but it is strongly rejected by other estimates with boosted instruments. Similar observations apply to the test of H5: $\beta = 1$ to a lesser degree.

A wide variations are also observed in the joint precision of the estimates. In general, the precision increases with the number of instruments. Though we are comparing estimates with different sets of instruments, this result is in line with the theory. It is more interesting to compare the joint precision based on the same number of instruments, which is shown in Table 4-6. When only four instruments are used as in GGLS5, $L2$-OPC gives a higher precision than the GGLS5 or $L2$-OBS. When 18 instruments are used as in GG, the results are mixed: $L2$-OPC gives a slightly higher precision than GG for the primitive parameters, but it is reversed for the derived parameters. One clear conclusion we can draw is that the boosting from the principal components gives a better result than boosting from the observed data. This confirms the theory of Bai and Ng (2007b).

Table 4-6.: Comparison of Generalized Variancee-GDP Deflator (Same Number of IVs).

	4 IV			18 IV		
	GGLS5	L2-OBS	L2-OPC	GG	L2-OBS	L2-OPC
GDP-Prim	3.728	4.002	2.460	0.156	0.226	0.148
GDP-Derv	1.364	2.486	1.293	0.066	0.115	0.120
NFB-Prim	9.282	4.994	2.219	1993.300	1.264[a]	1.124[b]
NFB-Derv	17.887	6.777	5.717	0.908	1.157[a]	0.871[b]

Notes: (a) These are for 15 instruments. (b) These are for 12 instruments.

Turning to the NFB deflator in Table 4-7, we find that the number of static factors and the number of boosted instruments are much smaller for the NFB deflator than for the GDP deflator. We also find a wider variation of the results across different choices of instruments. In particular, the estimate of ω ranges from 0.049 to 0.399, which propagates into a wide variation in the estimates of derived parameters and the p-

Table 4-7.: GMM Estimation of Phillips Curve-NFB Deflator: Standardized Data for OPC (1960:I-2003:IV).

				L2 Boosting			
	GG	GGLS5	$\widehat{r}$-OPC	OBS	$\widehat{r}$-OPC	OPC80	OPC114
ω	0.127	0.098	0.097	0.337	0.049	0.359	0.399
	(0.252)	(0.077)	(0.059)	(0.049)	(0.059)	(0.044)	(0.041)
θ	1.001	0.902	0.907	0.886	0.899	0.868	0.868
	(1.949)	(0.051)	(0.044)	(0.031)	(0.043)	(0.031)	(0.032)
β	1.000	1.005	0.973	0.864	0.982	0.844	0.817
	(0.021)	(0.030)	(0.026)	(0.036)	(0.025)	(0.037)	(0.038)
γ_b	0.112	0.098	0.097	0.285	0.052	0.304	0.331
	(0.046)	(0.070)	(0.055)	(0.033)	(0.060)	(0.028)	(0.025)
γ_f	0.888	0.906	0.881	0.647	0.932	0.622	0.589
	(0.188)	(0.030)	(0.026)	(0.026)	(0.027)	(0.026)	(0.025)
λ	0.000	0.008	0.010	0.015	0.012	0.019	0.019
	(0.005)	(0.010)	(0.009)	(0.007)	(0.011)	(0.009)	(0.009)
p-Values of Hypothesis Tests							
H1	0.500	0.207	0.148	0.018	0.137	0.014	0.014
H2	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H3	0.303	0.000	0.000	0.000	0.000	0.000	0.000
H4	0.501	0.528	0.315	0.006	0.373	0.003	0.002
H5	0.508	0.565	0.155	0.000	0.238	0.000	0.000
H6	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H7	0.070	0.000	0.000	0.001	0.000	0.001	0.007
GV Prim	1993.300	9.282	2.755	1.264	3.576	1.234	1.124
GV Derv	0.908	17.887	6.488	1.157	12.518	1.135	0.871
# of PC			169	270	7	80	114
# of IV	18	4	7	15	3	11	12
J-stat	9.930	6.623	9.780	12.056	7.116	9.886	10.390
	(0.980)	(0.469)	(0.460)	(0.996)	(0.310)	(0.770)	(0.795)

Notes: Standard errors are reported in parenthesis below the estimates. The test hypotheses are as follows.

H1: $\lambda = 0$ vs $\lambda > 0$ H2: $\gamma_b - \gamma_f \geq 0$ vs $\gamma_b - \gamma_f < 0$
H3: $\omega - \beta\theta \geq 0$ vs $\omega - \beta\theta < 0$ H4: $\gamma_b + \gamma_f = 1$ vs $\gamma_b + \gamma_f < 1$
H5: $\beta = 1$ vs $\beta < 1$ H6: $\omega = 1$ vs $\omega < 1$
H7: $\omega \geq 0.5$ vs $\omega < 0.5$

values of test statistics. For example, the null hypothesis $\lambda = 0$ is easily accepted with GG's and GGLS5's set of instruments, but it is strongly rejected with the instruments boosted from observed data or principal components. Similar observations can be made about the test of H4 and H5. The joint precision of the estimates of primitive parameters is quite low for the GG's and GGLS5's instrument sets. These results contrast sharply with the estimation results for the GDP inflation whose estimates are more robust to the choice of instruments. Estimates of NFB inflation is obviously more sensitive to the choice of instruments and we can use their sensitivity as a guide in choosing the proper set of instruments for both GDP and NFB inflation.

Previous studies of the GG's model that estimate both GDP and NFB inflation assume, at least implicitly, that the model can explain both inflation[24]. Under this premise, we would expect that the parameter estimates and their statistical significance will be similar as the two inflation rates show very similar paths with a high correlation coefficient (see Figure 4-3). As the estimates of some parameters are very different between the two inflation data, one way to assess the adequacy of instrumental variables is to compare the parameter estimates and the p-values of hypotheses tests between the two measures of inflation.

Estimates of ω and γ_b in GG, GGLS5, $\widehat{r}$-OPC and $L2$-boosted $\widehat{r}$ are substantially smaller for the NFB inflation than for the GDP inflation, while $L2$-OBS and $L2$-OPC give much smaller differences in the parameter estimates. Similar differences are present in the p-values of test statistics for H1 and H5. Most notable differences are observed in GG's estimates for which the p-values of H3 and H4 are quite different between the two measures of inflation.

[24]The model is based on the monopolistic competition and the driving force is the marginal cost which is measured by the marginal cost of non-farm business sector. Therefore, the model is more in line with the NFB inflation.

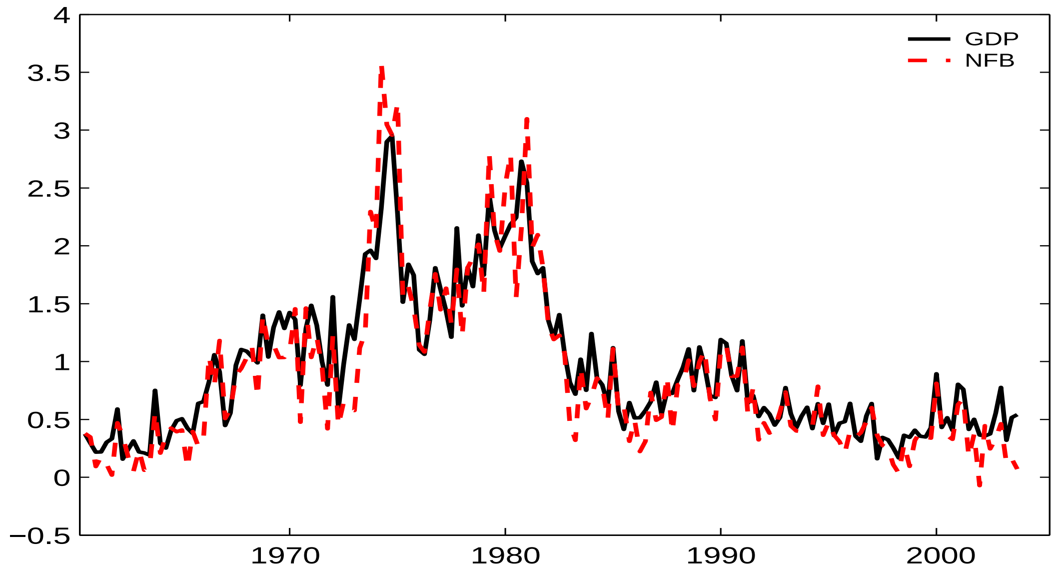

Fig. 4-3.: Comparison of Inflation Rates.

For the compatibility with the implications of GG's model, we check the consistency in the test results between the two inflation rates. As discussed in section 2, GG's model implies that H1 and H6 are equivalent hypotheses and both tests are expected to lead to the same conclusion. A similar equivalence holds for H2 and H3. The model also implies that, if H5 is not rejected (i.e., $\beta = 1$), then H4 should not be rejected. If H5 is rejected, tests of H1, H4 and H6 should give the same conclusion. We find that GG, GGLS5, $\widehat{r}$-OPC and $L2$-boosted $\widehat{r}$ give at least one conclusion that is different between GDP and NFB inflation, while $L2$-OBS and $L2$-OPC give the conclusions that are consistent ith the model in both GDP and NFB inflation. On the basis of these observations, we can conclude that $L2$-OBS and $L2$-OPC give the estimates that are more consistent between the two measures of inflation and more consistent with the implications of GG's model. The $L2$-OPC has a slight advantage over the $L2$-OBS in that the former gives a higher joint precision of parameter estimates with a smaller number of instruments, and thereby a smaller chance of the bias due to a larger number of instruments.

Finally, we estimate the generalized principal components (GPC) as described in the previous section. The number of dynamic factors is determined by using $\widehat{q}_3$ in (29) with parameters $m = 0.5$, $\delta = 0.1$ and $\tau = 2$. We find that the number of dynamic factors are 7 for the GDP inflation and 5 for the NFB inflation. The covariance matrix Σ_u of idiosyncratic terms is computed by using FHLR's method with parameter values $M = H = 3$. The generalized principal components are computed by $\widehat{F} = X\widehat{\Lambda}^*$, where $\widehat{\Lambda}^*$ is the matrix of eigenvectors of $\widehat{\Sigma}_x$ in the metric of $diag\left(\widehat{\Sigma}_u\right)$. As in the case of OPC, instrumental variables are selected from GPCs by using the L_2-boosting method with an increasing number of GPC as the boosting base. The size of the boosting base is determined by examining the generalized variance of parameter estimates.

Figure 4-4 shows the change in the joint precision of parameter estimates as the size of boosting base increases. For the GDP deflator, there are sizable reductions in the generalized variance up to 69 GPCs and then it stays quite flat. We choose 80 and 102 GPCs as the boosting base for comparability with OPC. For the NFB deflator, we select 89 and 104 GPCs for the boosting base because the former gives the same number (11) of instrumental variables as OPC80 and this number does not change until the base is extended to 104 GPCs. Table 4-8 presents the estimation results for both OPC and GPC for an easy comparison. There is practically no difference between OPC and GPC for the GDP deflator except for a little high p-value of test H4 in GPC80. The joint precision is higher whenever more instruments are used. There are a little more noticeable differences in the estimates of ω and γ_b, but the magnitudes of the differences are still very small.

Now we consider the estimation of the closed form equation (10). Principal com-

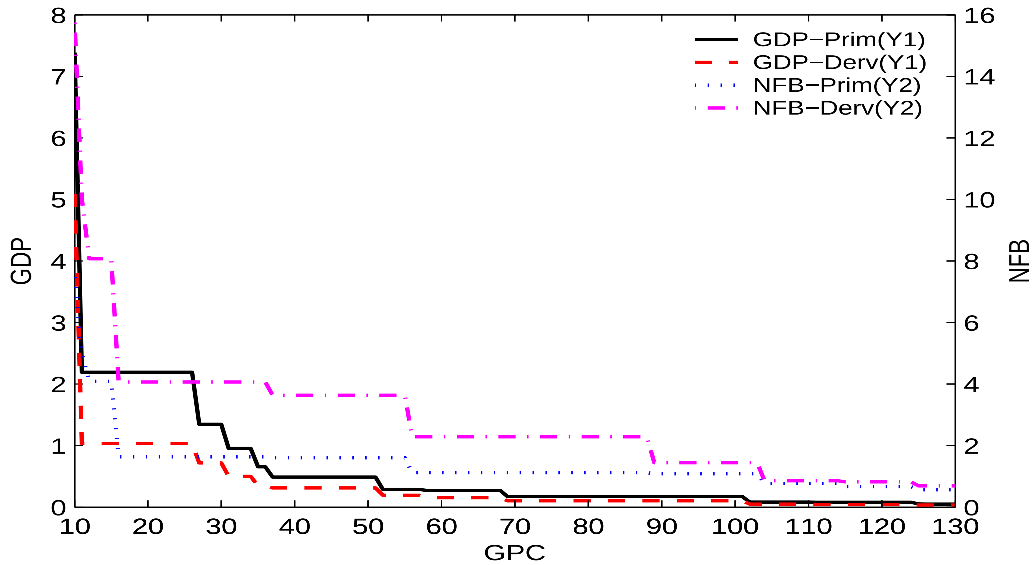

Fig. 4-4.: Generalized Variances of Primitive and Derived Parameters: GPC from Standardized Data.

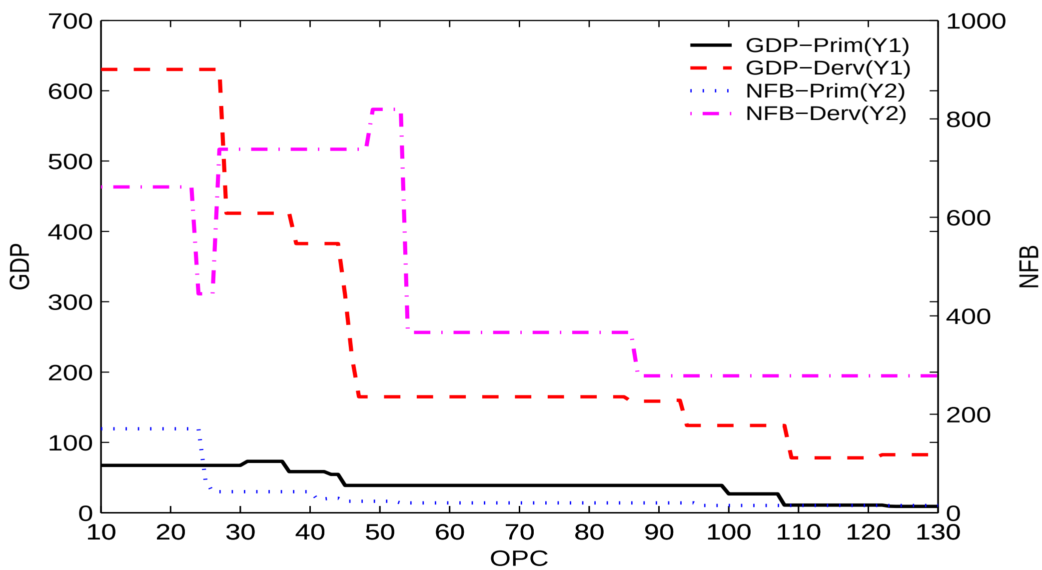

Fig. 4-5.: Generalized Variances of Primitive and Derived Parameters: Closed Form Equation.

Table 4-8.: GMM Estimation of Phillips Curve-OPC & GPC.

	GDP				NFB			
	OPC80	OPC100	GPC80	GPC102	OPC80	OPC114	GPC89	GPC104
ω	0.437	0.450	0.438	0.488	0.359	0.399	0.276	0.347
	(0.043)	(0.041)	(0.042)	(0.038)	(0.044)	(0.041)	(0.046)	(0.041)
θ	0.851	0.848	0.858	0.854	0.868	0.868	0.876	0.867
	(0.020)	(0.019)	(0.022)	(0.018)	(0.031)	(0.032)	(0.030)	(0.028)
β	0.883	0.872	0.904	0.859	0.844	0.817	0.884	0.842
	(0.026)	(0.026)	(0.029)	(0.028)	(0.037)	(0.038)	(0.032)	(0.034)
γ_b	0.351	0.360	0.348	0.380	0.304	0.331	0.245	0.297
	(0.023)	(0.021)	(0.022)	(0.019)	(0.028)	(0.025)	(0.033)	(0.028)
γ_f	0.604	0.592	0.616	0.572	0.622	0.589	0.689	0.626
	(0.014)	(0.014)	(0.016)	(0.014)	(0.026)	(0.025)	(0.024)	(0.023)
λ	0.017	0.017	0.014	0.016	0.019	0.019	0.018	0.020
	(0.005)	(0.005)	(0.005)	(0.004)	(0.009)	(0.009)	(0.008)	(0.008)
p-Values of Hypothesis Tests								
H1	0.000	0.000	0.002	0.000	0.014	0.014	0.014	0.005
H2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H4	0.029	0.019	0.060	0.009	0.003	0.002	0.015	0.001
H5	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
H6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H7	0.071	0.112	0.070	0.378	0.001	0.007	0.000	0.000
GV Prim	0.121	0.097	0.171	0.081	1.234	1.124	1.083	0.770
GV Derv	0.101	0.081	0.104	0.050	1.135	0.871	1.442	0.860
# of PC	80	100	80	102	80	114	89	104
# of IV	21	23	19	24	11	12	11	16
J-stat	11.466	11.641	11.011	12.004	9.886	10.390	10.066	11.070
	(0.985)	(0.993)	(0.975)	(0.994)	(0.770)	(0.795)	(0.757)	(0.921)

Notes: Standard errors are reported in parenthesis below the estimates. The test hypotheses are as follows.

H1: $\lambda = 0$ vs $\lambda > 0$ H2: $\gamma_b - \gamma_f \geq 0$ vs $\gamma_b - \gamma_f < 0$
H3: $\omega - \beta\theta \geq 0$ vs $\omega - \beta\theta < 0$ H4: $\gamma_b + \gamma_f = 1$ vs $\gamma_b + \gamma_f < 1$
H5: $\beta = 1$ vs $\beta < 1$ H6: $\omega = 1$ vs $\omega < 1$
H7: $\omega \geq 0.5$ vs $\omega < 0.5$

ponents are same as those used in the estimation of the structural form equation. The L_2-boosting is conducted to find the best instruments for mc_{t+1} where the retained instruments are the same as the variables that are used in the boosting for π_{t+1}. Following RW7, we estimate the closed form parameters δ_1, $\overline{\lambda}$ and τ, and derive the fundamental parameters γ_b, γ_f and λ. To determine the size of the boosting base of the principal components, we examine the generalized variance of γ_b, γ_f and λ for various base size. The results are shown in Figure 4-5. The generalized variances decline in general as the size of the base increases except for the generalized principal components in the estimation of NFB inflation. The generalized variance becomes stable at the size 110 for the OPC and 100 for the GPC in the GDP inflation equation, and at the size 110 for the OPC and 90 for the GPC in the NFB inflation equation. We use the boosted instruments from these boosting bases. The estimation results are presented in Table 4-9.

We have shown earlier in Table 4-3 that GG-2 and RW instrument sets give opposite conclusions between the GDP and NFB inflation in the test of (H1) $\lambda = 0$, and RW instrument gives opposite conclusion in the test (H4) $\gamma_b + \gamma_f = 1$. Instrument sets $\widehat{r}$-OPC and L2-OBS also give contradictory results in the test of (H2) $\gamma_b > \gamma_f$. Therefore, instrument sets that yield robust test results are GG, GGLS5 and instruments boosted from OPC and GPC. Among these four sets, latter two sets lead to a higher joint precision of the estimates without excessive number of instruments. As in the case of structural form estimation, L2-OPC instruments seem to be a little better than the L2-GPC. According to the estimates with L2-OPC, forward-looking behavior has a 'dominant' role in inflation dynamics, and the real marginal cost or the present value of future real marginal cost have a significant effect on the inflation regardless of the measurement of inflation by the GDP deflator or NFB deflator.

Table 4-9.: GMM Estimation of Phillips Curve-Closed Form Equation:Standardized Data for OPC and GPC: 1960:I-2003:IV.

	GDP				NFB			
	$\widehat{r}$-OPC	OBS	OPC	GPC	$\widehat{r}$-OPC	OBS	OPC	GPC
$\overline{\lambda}$	0.010	0.011	0.010	0.010	0.018	0.015	0.018	0.016
	(0.003)	(0.005)	(0.003)	(0.004)	(0.008)	(0.006)	(0.004)	(0.005)
δ_1	0.746	0.760	0.692	0.700	0.699	0.719	0.697	0.630
	(0.030)	(0.043)	(0.042)	(0.049)	(0.046)	(0.041)	(0.038)	(0.049)
τ	0.882	0.877	0.930	0.945	0.771	0.790	0.815	0.889
	(0.028)	(0.045)	(0.025)	(0.020)	(0.112)	(0.099)	(0.038)	(0.034)
γ_b	0.450	0.456	0.421	0.421	0.454	0.459	0.445	0.404
	(0.012)	(0.019)	(0.017)	(0.019)	(0.035)	(0.032)	(0.019)	(0.023)
γ_f	0.532	0.526	0.566	0.569	0.501	0.504	0.520	0.570
	(0.012)	(0.022)	(0.018)	(0.019)	(0.052)	(0.046)	(0.021)	(0.025)
λ	0.006	0.007	0.006	0.006	0.012	0.010	0.011	0.011
	(0.002)	(0.003)	(0.002)	(0.002)	(0.006)	(0.004)	(0.003)	(0.003)
p-Values of Hypothesis Tests								
H1	0.003	0.012	0.001	0.003	0.019	0.006	0.000	0.001
H1($\overline{\lambda}$)	0.002	0.009	0.001	0.003	0.013	0.003	0.000	0.001
H2	0.000	0.042	0.000	0.000	0.293	0.283	0.031	0.000
H4	0.001	0.008	0.004	0.005	0.025	0.015	0.000	0.001
H4(δ_1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H4(τ)	0.000	0.004	0.003	0.003	0.021	0.017	0.000	0.001
GV Prim	8.903	103.880	13.463	23.280	5315.100	471.560	70.149	330.500
GV Derv	9.406	68.708	10.777	14.612	5690.300	581.990	78.108	278.000
		270	110	100		270	110	90
	10	6	12	10	7	8	16	8
J-stat	11.980	10.515	11.704	12.535	11.508	10.383	13.688	11.628
	(0.529)	(0.310)	(0.701)	(0.484)	(0.319)	(0.496)	(0.802)	(0.392)

Notes: Standard errors are reported in parenthesis below the estimates. The test hypotheses are as follows.

H1: $\lambda = 0$ vs $\lambda > 0$ H1($\overline{\lambda}$): $\overline{\lambda} = 0$ vs $\overline{\lambda} > 0$
H2: $\gamma_b - \gamma_f \geq 0$ vs $\gamma_b - \gamma_f < 0$ H4: $\gamma_b + \gamma_f = 1$ vs $\gamma_b + \gamma_f < 1$
H4(δ_1): $\delta_1 = 1$ vs $\delta_1 < 1$ H4(τ): $\tau = 1$ vs $\tau < 1$

E. Conclusion

The NKPC equation or its hybrid version includes the rational expectation of inflation as one of the explanatory variables. Instrumental variables for the future expectation can include all variables in the current information set, but only a relatively small number of instruments are used in the estimation of the hybrid inflation equation. Instrumental variables are selected on an ad hoc basis though they are intuitively reasonable candidates for the instruments. It has been recognized that the estimates of the parameters are not robust to the choice of instruments and the effects of the choice of instruments can be substantial across different measures of inflation. Such a sensitivity is detected in recent literature.

This paper applies the L_2-boosting method of selecting the optimal set of instruments to the estimation of Gali and Gertler's hybrid inflation equation. Three sets of boosting base are used. The first boosting base is the lagged values of a large number of observed variables that Stock and Watson and many others use for inflation forecasting. The other two sets are the ordinary and generalized principal components estimators of underlying factors. Bai and Ng (2007a) show that principal components can be more efficient instrumental variables than the observed variables.

We find that the set of optimal instruments from observed boosting base is quite different from the sets used in GG, GGLS5 and RW7. There are also difference between the instrumental variable set for the GDP and for the NFB deflators. An interesting result is that the lagged monetary base is one of the optimal instruments for both inflation series, while it is not one of the instruments in previous studies of GG's inflation equation. Another interesting result is that the lagged output gap is not one of the selected instruments for the NFB deflator. We find that different sets of instruments in previous studies give substantially different estimates of parameters, the fraction of backward-looking agents in particular, and different p-values of some key test statistics, between the two inflation measures. Such differences vanish or are reduced significantly when boosted instruments are used. Furthermore, the joint precision of parameter estimates is higher when the boosted instruments are used.

Results with the boosted instruments from principal component estimates of the factors are similar to the results with the boosted instruments from observed variables. The major difference is that the former has a fewer number of instruments, and yet, it gives the better results than the latter. We find negligible differences between the ordinary principal components and the generalized principal components as the boosting base.

CHAPTER V

CONCLUSION

This dissertation consists of three essays. I try to improve the estimation of inflation equation, using the additional measures of distribution of price changes and the optimum choice of instrumental variables.

The first essay shows the importance of kurtosis in the approximation of inflation, theoretically and empirically. Since Mills (1927), many authors have studied the relationship between inflation and moments of price changes. The source to generate these relationships is the change in the shape of the underlying distribution. To capture the shape of the distribution, earlier studies in 1970s and 1980s considered dispersion alone. Since Ball and Mankiw (1995) included skewness, both dispersion and skewness have been used. We argue that kurtosis should be considered to capture the property of the distribution sufficiently. My empirical results show that the kurtosis measure has a significant effect on inflation. In addition, we can improve the approximation of inflation in terms of the goodness of fit.

The second essay is to examine the concerns about the source of the observed positive relationship between inflation and the dispersion/skewness of price changes. The concern is that the presence of outliers in price changes causes the misleading correlation between mean and the dispersion/skewness of price changes. I show there is a significant relationship between inflation and dispersion/skewness after considering outlier effects. Thus, the observed inflation-dispersion/skewness relationship is one of the stylized fact. I also show that using robust measures yields the higher goodness of fit in predicting inflation. In particular, medcouple as a measure of skewness is very useful. We find that adjusting outlier problems is reasonable in the study of cross-sectional distribution of price changes.

The third essay is to consider the GMM estimation of Hybrid Phillips Curve. It has been known that GMM estimates are sensitive to the choice of instrumental variables. Previous studies select the instrumetnal variables on an ad hoc basis from a set of reasonable predictors of inflation. This paper applies the L_2-boosting method from two boosting bases: large number of observed weakly exogenous predictors and their OPC and GPC. The instrumental variable sets used in previous studies lead to contradictory test results, depending on the measurement of inflation (GNP or NFB) and depending on estimating the structural form or closed form equation. Instrumental variable sets that are L_2-boosted from OPC and GPC give all consistent test results and joint precision of the estimates is higher.

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