QUANTIZATION, BEHAVIORS, AND ARCHETYPES: A PRACTICAL GUIDE TO MORTON FELDMAN'S *PATTERNS IN A CHROMATIC FIELD*

by

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Introduction

A doctorate in music performance is a curious thing. At once, a specialized pursuit, in keeping with the nature of any terminal degree; but at the same time, a manifest resistance to the specialization necessitated by professional engagement. Such duality of focus presents an interesting opportunity when the time comes to complete a doctoral capstone project. The project can aim to draw on the specialized nature of doctoral work to investigate a particular subject, while targeting a general audience of performers. That is exactly what I have set out to do in my project, which investigates the issues of interpretation and practicability of Morton Feldman's *Patterns in a Chromatic Field* (henceforth PCF). Rather than an investigation of specialized techniques of a difficult piece, targeted at a similarly specialized audience, my document aims to leverage the unique position of doctoral work to claw back an underperformed piece from the necessity of contemporary performance specialization. To accomplish this, my document takes a three-pronged approach. First, I define the topography of the piece through the framework of quantization.[1] Second, I use the quanta to address the nature of patterning and behavior in PCF. And finally, I use those quanta and behaviors to generate practice excerpts and techniques, based on basic and familiar pedagogical approaches, to make PCF practicable for any accomplished cellist.

Before I begin this exploration of PCF, it is important to understand the types of critical and theoretical work that have been written on the "late-period" Feldman works to which PCF belongs. My document is internally motivated by its own procedures and investigations, but there does exist a broad, if not totally specific to this piece, body of work that can provide valuable contextualization.

First is Feldman's own scholarly work, *Crippled Symmetry*. In his paper, published in the same year as PCF (1981), Feldman discusses the influences of "Near- and Middle-Eastern" rugs on his compositions of this era. Specifically, Feldman's work discusses how he tried to recreate the small imperfections in patterning, celebrated in rug-making, through rhythmic asymmetry in music. The name he gives for this feature in his music is "crippled symmetry." The focus of his paper is on the small rhythmic elements of his music (groups of a few notes or rhythmic gestures), but it also alludes to a

[1] Very briefly, and very generally, quantization is the process of either imposing or measuring the discrete states of a field or wave. The resultant quantum values are the smallest discrete elements in that system.

concept useful for the purposes of my document: patterns exist on multiple levels. A pattern can be the repetition of a particular local feature, or it can be the total sum of those repetitions, viewed from afar, like an ornate rug. The patterns of PCF discussed in my document are not those small "crippled symmetries," but the analogous rugs themselves. As my document progresses, the elements Feldman describes in his paper are reinterpreted as quanta and behaviors.

Another kind of approach to the music that Feldman created during the late-seventies and eighties looks at it from the other direction, addressing the question of how can we talk about the effects of the larger sections of a piece on the listener. In Rebecca Leydon's *Towards a Typology of Minimalist Tropes* (2002), she discusses the various approaches employed by Minimalist composers to build "musmatic" elements into topics with different global affects. Feldman is not a Minimalist composer in the vein of Steve Reich or Arvo Pärt, but the musmatic-discursive paradigm Leydon discusses is a good way of understanding how the small "crippled symmetries" Feldman was so fascinated with, when repeated, can create different affects and moods on a large scale. This larger scale interest, in part, justifies the intuitive approach to recognizing distinct patterns that I have taken in this document.

Along the lines of this larger scale, I would like to point to the work of Elizabeth Helmuth Margulis, and her book, *On Repeat,* which investigates the psychological effects of musical repetition. Margulis's work focuses on issues far beyond the scope of my document, but her simple discussion of musical repetition performing the functions of "(1) learning and level-shifting, (2) segmentation, and (3) expectation," has informed my general approach to understanding the behaviors of the quantum elements that make up each "pattern" of PCF. Her book is a wide-ranging discussion of a large number of psychological phenomena that would make for a compelling exploration of PCF. That being said, my document is more focused on the practical applications of analysis for a potential performer, rather than in how music like PCF operates at a psychological level. I revisit some of Margulis's work, and the work of the previously mentioned authors, throughout my document when appropriate. But mostly, I have relied on an approach driven by intuition to begin segmenting PCF into discrete patterns.

Finally, I want mention Fred Sherry's, *A Grand Tour of Cello Technique*. The inclusion of tone-rows alongside traditional arpeggios and scales as a source for pedagogical materials in his book, influenced my own approach to building pedagogical models for PCF. Examples of the influence of his book are found in the last chapter of my document.

Each unique section of PCF is a pattern, and a pattern is simply any stretch of the piece that employs a unique and consistent set of features or behaviors. The patterns of PCF can last for a few measures, or in some cases, 60-70. The boundaries between patterns are sometimes marked with a double bar, at other times they are indicated through a change in meter and musical texture. If two sections of the piece look rhythmically the same, but in one, the notes are drawn from one pitch set, and in the other, a different pitch set, those two sections are different patterns. The features that define patterns are explored in the first chapter. By defining a pattern as an area of self-similar features, it is possible to begin to do the work that this document makes its priority, to make an impracticable piece practicable.

The three-pronged approach discussed above maps onto the three chapters of the document. Chapter 1 discusses how each pattern can be quantized into a set of simple features. These quanta reflect almost every topographical feature that can be encountered in a pattern of PCF. This chapter is the most exhaustive in the document, and seeks to define a quantum for everything that happens in PCF. Chapter 2 uses the quantum pattern features of the first chapter to follow the behaviors, changes, and regularity of repetition over a part, or the whole, of a pattern. This chapter also begins to address the nature of patterning in PCF, in a way that owes a certain debt to Feldman's own interpretation in *Crippled Symmetry,* but that ultimately is more concerned with practical issues that PCF-like patterns create for performance. This chapter is not nearly as exhaustive as the first. Instead, it focuses on a subset of the patterns in PCF. Throughout the first two chapters, there are also occasional "performance utility" asides, which seek to orient the whole of the document, not just the last chapter, towards the goal of practicable performance. Finally, Chapter 3 synthesizes the work of the first two chapters to create simple "archetypes" and "variations" that capture the topography of PCF while eliminating the baroque notation and inscrutable polyrhythms that make this piece appear to be such a daunting undertaking. This chapter is where the pedagogical promises of the work my document undertakes come to fruition. Unlike the first two chapters, Chapter 3 does not take an exhaustive approach. Instead, the goal is a demonstration of the kind of practice that is facilitated by the concepts in the first two chapters.

My hope is that my document serves as a starting point for anyone who may be interested in undertaking Feldman's monumental piece. PCF has become dear to me over the time I spent with it, and I hope my genuine interest and appreciation come across to the reader. Ultimately, this is not an experiment in new techniques of interpretation and analysis, but rather, an attempt to apply the kinds of practice that any instrumentalist worth their salt does every time they take up a new project, to a piece that has, so far, escaped the kind of recognition it deserves.

Chapter 1: Pattern Quanta

The question Chapter 1 seeks to address is how a performer might begin to identify the smallest unique elements that make up each pattern. Broadly, the chapter focuses on the general topology of PCF patterns as a way of discovering their smallest elements. The question, in other words, is: what are the surface level features that are repeated over the course of a pattern, and what is the simplest way to define that feature? Throughout the chapter I refer to these smallest elements as "quanta," and the process of pattern feature-extraction as a kind of quantization. Stylized, the question Chapter 1 addresses can be stated thusly: how can one quantize each pattern of the piece, and along which dimensions of pitch and rhythm can the resultant quanta be defined? I have chosen to refer to these smaller elements as "quanta" because I think the analogy to the mathematical/physics concept of the smallest indivisible element is fitting for this piece. A quantum unit is the smallest possible unit of a quantized system. A quantum unit of a pattern is the smallest unit of that pattern that is repeated, what Feldman might have called a "crippled symmetry" (Feldman 1981). By isolating and defining these quanta, it becomes possible to compare patterns to each other, look for commonalities, and describe how they change over time.

Throughout the discussion of pattern quanta is a parallel discussion of the particular performative and interpretive relevance of the findings. The goal is begin to define a solution to the motivating issue of my document, as stated in the Introduction, of making a piece that is un-practicable into something that can be realistically prepared by any accomplished cellist. This parallel discussion addresses both the particular technical issues that can be solved by the insights that quantizing a pattern can bring, and how identifying certain quanta might incline a performer to make certain interpretive choices. Of course, the main discussion of this broader question is in Chapter 3, so the goal of these sections in this present chapter is just to orient the technical and analytic work towards the focus of the final chapter, so as to better facilitate the more comprehensive discussion that happens later.

This chapter is divided into four parts. The first section is a discussion of voicing within PCF. In order to begin quantizing the patterns of PCF, it is first necessary to define the boundaries between voices. These boundaries are the first way in which I begin to quantize the features of each pattern. The second section is a discussion of the pitch quanta in PCF. Here I show how there are logical and consistent pitch sets within each voice that make up the pitch-related pattern quanta. Specifically, this section explores the concepts of "pitch-element number" and pitch sets. The third section is about rhythmic quanta. This section looks at how the complex notation in PCF can be quantized in such a way that facilitates easy analysis of, and comparisons between, a pattern's rhythmic properties. Section 3 explores how tuplets and meter are used in surprising ways, and the concepts of "tuplet-element number," "meter-element number," "impulse number," impulse kinds, slurs, rhythmic simplification, and elongation. The groundwork in the three sections of Chapter 1 provide a foundation for Chapter 2, which uses the quanta of Chapter 1 to capture phenomena that become apparent when patterns are quantized, and the resultant quanta are observed over time.

1. Voicing

Before delving into the specific quanta that make up the patterns of PCF, it is necessary to detail how I have understood voicing to work in the piece. It is as necessary to have a consistent guide for how to recognize distinct voices in PCF as it is in a piece for which a traditional tonal analysis would be most appropriate. As an oversimplified example, a traditional theoretical analysis of a Schubert art song might feel most comfortable distinguishing between the voice and piano as two separate entities with their own theoretical concerns. This is done intuitively, and without much need for repeated theoretical justification. Feldman seldom grants the freedom to operate with prior assumptions, and there is no perfect parallel for Feldman's writing, but because there are often clear distinctions in what is happening in, say, the cello part, and the piano part, it is possible make generalizations about how material is divided between the cello and piano in PCF. In so doing, it becomes easier to further quantize the piece.

There are three arrangements of material between the two instruments that begin to define the simplest boundaries of the elements of PCF: unison voicing, cello + piano voicing, and solo voicing. I have chosen to call these arrangements "voicings," because "voice" implies a functional independence in a way that, for example, "cello part" does not. It is important for the pianist and cellist to know when their parts are or are not operating independently. The cello can operate as a unique voice in a larger texture with the piano, or it can be part of a general undivided "unison" texture, where there are no quantum distinctions between parts. In that situation, the cello part is simply a component of the larger overall voice created by the combination of cello and piano parts. In Section 1 there is some use of the quanta terminology that is defined in later sections, but I have tried to limit its use to instances where it is obvious how I am defining a set. Otherwise, this section can be referenced as the reader progresses through the rest of Chapter 1.

1.1. Unison Voicing

In a unison voicing, the cello, piano left hand (henceforth LH), and piano right hand (henceforth RH) operate as a single cohesive texture. This means that all the parts share a significant number of quanta, and the listener is unable to hear a meaningful distinction between the parts other than timbre. This does not mean that the cello and piano have to be in literal unison. To wit, mimicry is not a literal unison, but in the absence of individual voice deviation, can be thought of as a kind of displaced unison in the vain of a Bach two-part invention. Unison voicing is the least common arrangement in PCF, and because of that, patterns with this voicing can be quite striking. I have defined unison voicing narrowly in

Example 1. Page 63, mm. 1395–1403.

order to be able to highlight how rare these instances of synchronicity are. There are two examples, one definitive and one slightly less intuitive that help demonstrate this voicing. From here, I stylize pattern locations as "Pattern *x*—*y*," where *x* is the first measure of the pattern and *y* is the last. Additionally, all examples are from Morton Feldman's *Patterns in a Chromatic Field* unless otherwise noted.

Pattern 1395–1403 (Example 1) has several features which are typical of a unison voicing. First, there is perfect rhythmic synchronicity between the cello, and both hands of the piano (except for the LH in measure 1401). In other words, the rhythm is identical and constant between all three parts for the duration of the pattern. Because of this, the listener is likely to perceive the chord created between all the parts as one single set of pitches, rather than as a piano set of pitches distinct from a cello set of pitches. Pitch sets are one of the quanta are discussed later, but needless to say, in this example, the vertical pitch sets are most intuitively defined across all three parts. This is despite the fact that the pitch in the cello part does not always come from the set of pitches the piano is playing beneath it.

Unison voices are more often created through rhythmic synchronicity than through pitch synchronicity. Despite this, the particular unison voicing seen here is further enhanced by a brief period of pitch synchronicity. In the first measure (circled in blue), the cello notes are also contained within the set of pitches that the piano plays. The harmonic G flat and F are duplicated in the piano RH. More generally, the use of harmonics in the cello line is a pitch-related feature that enhances the sense of unison voicing over the whole pattern. The harmonics create the impression that the notes played by the cello are acoustic harmonics spun out of the notes the piano plays. Further aiding the impression in the domain of pitch, the direction of the cello notes over time is mirrored in the voicing of the piano chords. This is evident in the second measure of this passage (circled in red), where the cello notes are no longer a part of the set of pitches in the piano hands, but the half-step descent of the cello line (E to D Sharp) is mirrored in the outer notes of the piano lines (F to E on top and C to B on the bottom). Turning to a different passage, we observe a more complex example of unison voicing.

Pattern 90–100 (excerpt found in Example 2) is lacking many of the features that made the previous example an obvious unison. There is not perfect rhythmic synchronicity between the cello part and the piano hands, and the two parts do not share a pitch set. They do, however, share the pitch-class set

Example 2. Page 5, mm. 90–97.

{0,1,3,4,5,8}.[2] They also have the same rhythm, albeit reorganized: continuous eighths with two one-dot elongations and two double-dot elongations. This is the first hint at the utility of the basic descriptions offered by what is later called quanta. Because both the piano hands and the cello part share so many of the same quanta, there is nothing to distinguish them from each other. For example, the lack of rhythmic synchronicity between the cello part and the piano hands is not because the rhythmic elements that make up each part are different; they are just slightly rearranged. The complicated rhythmic relationship between the cello and the piano is like the emergent counterpoint of a two-part invention: it only exists as a result of the line being overlaid on itself. This is what leads to the cohesive nature of the pattern, and why I have labeled it as a unison. In the next section I discuss another kind of voicing arrangement, cello + piano voicing.

[2] This concept is described in great detail in Allan Forte's work, *The Structure of Atonal Music* (1973).

Example 3. Page 1, mm. 1–18.

1.2. Cello + Piano Voicing

Cello + piano voicing is the most intuitive voicing to understand, as it most closely resembles something familiar: a classical sonata. However, it is still useful to review exactly what cello + piano voicing looks like in an actual pattern of PCF. In this voicing, the distinction between the cello and piano "voices" is created by using separate, non-overlapping, pattern quanta for each instrument. Stated the other way around, each voice is distinct from the other because it has its own set of topological features, and these features may evolve in their own independent ways. Because of this independence, the cello and the piano can be thought of as their own voices in the overall texture. They both contribute something different to the pattern. To demonstrate how this voicing works in PCF I have chosen to highlight a few basic pattern quanta of a particular pattern (Pattern 1–18). I have chosen this example because the prevalent pattern quanta are obvious enough to be understood without having previously read the following sections detailing those quanta.

Pattern 1-18 (Example 3), which is the opening pattern of the piece, is an example of a pattern where cello and piano can be thought of as two voices, and the independence of these voices can by elucidated by cataloguing some basic quanta of each, and looking for commonalities. An absence of

Table 1. Basic quanta in both the Cello Voice and the Piano LH + RH.

	Cello	Piano LH + RH
Pitch Element Number	4	6
Pitch Set	{B Double Flat, A Flat, F Double Sharp, A Sharp}	{D, E Flat, F Flat}
Tuplet Number	9	{3,5}
Rhythmic Impulse Number	N/A	2
Slurs	{6,5,4,3}	N/A
Tuplet Elongations	Dotted sixteenth	N/A
Rest Elongations	N/A	{32nd,16th, 8th}
Meter	8/32	4/16

commonalities indicates two voices that should sound totally distinct, while the presence of a common element predicts voices with a greater degree of perceived similarity. To a degree, this is evident by visually comparing the cello and piano parts. Further, in the chart shown in Table 1, I have catalogued some basic quanta for each voice, to more specifically demonstrate the lack of overlap between these two voices, and thus, their independence. As can be seen in the chart, when both voices share a type of quanta, the value of that quantum is different. For example, both voices have pitch sets, but the pitches of those sets are not the same. There are also several quantum values for which there is no correlate in the other voice. For example, the cello voice has “tuplet elongations,” while the piano voice does not. As new quanta are introduced in Sections 2 and 3 of this chapter, the ways in which voices can be similar or dissimilar increase. In the next section I discuss the final of the three voicing types, solo voicing.

1.3. Solo Voicing

Solo voicing, as the name suggests, is one where all the pattern quanta are contained in either the cello part, or one of the piano hands. This arrangement is less common than the cello + piano voicing, but more common than unison voicing. There are only about four distinct patterns with this voicing in PCF, with each having two to four variations across the span of the piece. Because this voicing arrangement is so self-evident, I have not included a detailed discussion of examples and how they exhibit solo voicing. I

Example 4. Page 6, mm. 109–117.

have however included one example of a solo cello pattern in order to describe a specific and unusual pattern that has no other correlate in another voicing.

The solo cello nature of Pattern 109–117 (Example 4) is immediately apparent from the lack of any material in the piano hands. Perhaps no more needs to be said, but I have chosen to highlight this pattern because it has several unusual features that do not have correlates in any of the other patterns in the piece. First, outside of using a fairly small set of pitches, it is not really a pattern in any objective or subjective way. To use the language that is developed in the later sections of this paper, there is no apparent way to quantize this pattern. The four dotted eighth notes in the first measure seem like they might be repeated in the fourth measure, and the three quarter notes under an 8-tuplet in the second measure seem mirrored in the fifth measure, but being repeated once does not really strongly imply a pattern. That is not enough information to hear any obvious period of repetition that might help reinforce the privileged status of a particular feature.

Turning to pitch, Pattern 109–117 (Example 4) does have an obvious pitch set {G Flat, E Sharp, F Flat, A Double Flat}, and the pitch-class of that set has been used in several guises by this point in the piece {0,1,2,3}. This does make this solo voicing pattern like some of the previous patterns of the piece, but again, there is a lack of any regular repetition of those quanta that would reinforce the sense that Pattern 109–117 is actually a pattern. For the first 5 measures, the sequence of pitches is a cycle of the four-note set, where each repetition is in the same order as the first. Because this cycle happens independent of the rhythm of the voice, it creates an isorhythmic effect. However, after the fifth measure, this breaks down and the sequence of the pitches becomes random. Because this break down also happens at the same time that the two aforementioned repeated rhythms disappear (the four dotted eighths and the three quarters under an 8 tuplet), an effect of dissolution is created. Perhaps Pattern 109–117 starts with a certain kind of order, but it quickly disappears before the end (highlighted in Example 4). This is a striking departure from the affect of the previous patterns, and is far more through-composed than anything else in PCF.

1.4. Performance Utility

Explicit instruction for how to conceptualize voicing is essential for a performer. The cellist and pianist need to have an understanding of when they are playing with each other in unison, and when they need to play either a leading or supporting role. This is no less true for a piece such as PCF despite its seeming unconventionality. In addition to the purely practical component of why this information is helpful, there is also an interpretational utility to it as well. The distribution of different voicing across the piece is a component of its overall flow and pacing. Understanding this can help to create clarity in the mind of the performers for where they are going, and in turn, improve the overall performance. As a explicit example, knowing that after a pattern with multiple voices and many pattern elements there is a unison or solo pattern coming, may inspire the performers to try and heighten this discontinuity by exaggerating the inherent differences between those two ways of voicing. With voicing now defined, it is possible to begin the work of defining the quanta of the patterns of PCF, and demonstrating the values of those quanta found in the piece. The first domain that is discussed is pitch.

2. Pitch

Given the title "*Patterns in a Chromatic Field,*" it seems safe to assume that chromaticism and pitch were a focus for Feldman in creating the sonic world of PCF. In this section, I introduce the quanta of pitch-element number and pitch sets. These are two of the most salient quanta of pitch, and together, are able to describe the majority of the pitch "features" of any given pattern. The need for a distinct quantum for pitch-element number, separate from the size of the pitch set of a pattern, is made clearer as examples and analysis are provided. I also include a discussion of some of the more baroque pitch notations that Feldman frequently uses, and how I have dealt with enharmonic equivalence. Finally, I provide a brief discussion of the performance utility of the quanta detailed in this section, drawing on the same examples used to exemplify the quanta, and show specific quantum values.

2.1. Pitch-Element Number

Simply stated, the pitch-element number is the number of unique notes in each voice. Pitch-element number should be consistent over the duration of a pattern in each voice. When the variability of a voice within a pattern makes it difficult to define a narrow set of pitches, then it no longer becomes practical to discuss a voice's pitch-element number, and functionally, the pattern does not have one. For patterns that have a difficult to define pitch-element number, it may be necessary to confine the analysis of pitch-element number to a smaller rhythmic impulse, or the notes under one slur. That being said, the majority of patterns in PCF have a narrow and stable pitch-element number. This is useful, because it makes it easy to compare the pitch-element number of different patterns. By doing so, it is possible to define audible similarities based on this quantum alone, as well as to define differences that most meaningfully exist along the dimension of pitch-element number. I have chosen two examples which demonstrate how I have defined this quantum and hint at its utility in comparing various patterns to each other.

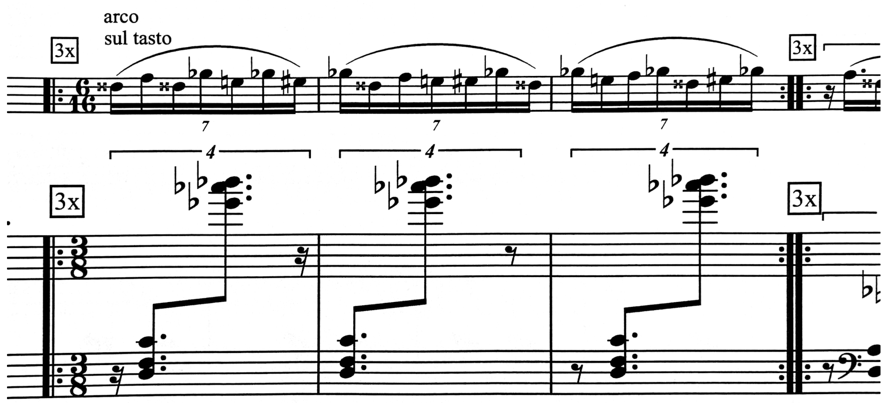
Example 5. Page 1–2, mm. 15–24.

Example 5 shows the end of one pattern and the beginning of another (Patterns 1–18 and 19–36). The junction (marked in red) is at the end of measure 18/beginning of measure 19, where the cello is tacit for a measure and the piano changes the notes of its impulses. That measure is the first measure of a new pattern (Pattern 19–36). Using these two patterns as a comparison, it is possible to demonstrate both how pitch-element number is calculated, and how it can be a useful comparative tool. The cello voice has a pitch-element number of 4 in both patterns. This is easy to see in the cello voice because it is composed of a string of four notes {B Double Flat, A Flat, F Double Sharp, A Sharp} that cycle repeatedly. The piano voice has a pitch element-number of 6 in the first pattern, {D, E Flat, F Flat, D 24va, E Flat 16va, F Flat 24va}, and 7 in the second pattern {C Sharp, D, E Flat, F Flat, E Flat 16va, C Sharp 24va, D 24va}. In these two patterns, the pitch-element number of the piano voice is fairly obvious, because each chord in the piano voices is composed of the same notes.

Pitch-class sets and pitch-element number are not the same, because pitch-element number counts the same pitch displaced by any number of octaves as a distinct element. Because of this, the piano voice of Pattern 1–18 has six pitch-elements, but a pitch-class set size of three. However, pitch-element number is similar to a pitch-class set in that it doesn't count repeated notes as new elements. This is why the cello voice in each pattern has a pitch-element number of 4 rather than 72 or 68 (the total number of notes in the entire cello voice of Patterns 1–18 and 19–36, respectively).

As for the ability of this quantum to offer comparative insights, one of the differences between these two patterns is that the piano voice of Pattern 1–18 has six pitch elements, and the piano voice of Pattern 19-36 has seven. This change in the pitch element-number from 6 to 7 in the piano voice across this pattern junction means that there is an increase in pitch density over these two patterns. Regardless of what the pitches are, it is possible to define a way in which the piano voices of these two patterns are different. It is helpful to be able to compare how one pattern might have a few sparse pitches and another might contain a large number of different pitches. Another example of the simple comparisons that can made with pitch-element number is found by comparing Patterns 1–18 and 19–36 to Pattern 1360–1362.

Pattern 1360–1362 (Example 6) is similar to Patterns 1–18 and 19–36 (shown in part in Example 5) in several ways. The cello voice in this pattern is a string of a handful of chromatic pitches, and the piano voice is a series of isolated impulses which alternate low-high. Ignoring the specifics of pitch and



Example 6. Page 61 mm. 1360–1362.

rhythm, the overall effect of all three patterns is quite similar. However, Pattern 1360–1362 is not identical to the two patterns discussed previously, for instance, in the quantum of pitch-element number. The pitch-element number of the cello voice in this pattern is 5 {D Double Sharp, F, G Flat, E, E Sharp}. Therefore, one of the ways this pattern can be defined as varied from the previous sampled patterns is that Feldman has increased the pitch-element number from 4 to 5. This is a consequence of the cello voice's pitch set being different in this later pattern than that of the two earlier patterns, which implies a notable departure from the previous two patterns. Despite this notable departure in pitch set, the pitch-element number of Pattern 1360–1362 has only increased by one, which, depending on how a listener perceives the piece, arguably might be the more "accurate" description of the change.

To further demonstrate that pitch-element number and pitch sets are not always closely related, let us examine measures 59–67 (Example 7) which are an excerpt from Pattern 37–72. This example also demonstrates how there can be both a local and global sense of pitch-element number. In this excerpt from Pattern 38–72, the cello voice and the RH voice are stable in a way that makes counting the pitch-element number a simple task, as it was in the previous two patterns (Patterns 1–18 and 19–36). The pitch set is constant, so the task is simply to count the number of distinct notes over the course of the pattern. The LH, however, introduces a degree of variability. In measures 62–63 and 65–67, the LH shoots up in register and plays unique sets of pitches (circled in red in Example 7). One way to count the pitch-element number of the LH of the piano voice would be to simply add all these unique pitches to the total pitch-

Example 7. Page 4, mm. 59–67.

element number of the piano voice. This interpretation of the pitch-element number captures a useful metric for comparison. It captures the fact that the piano voice of this pattern, despite being similar to others with shared features, contains a larger pitch-element number in the piano LH, and therefore is more varied or irregular than the piano voice of other comparable patterns (such as the one in Excerpt 3), where the LH of the piano has a smaller pitch element number. Another possible interpretation recognizes that, while the piano LH is jumping around, and increasing the total number of pitch elements, each individual cluster of notes it plays has a pitch-element number of 4. In this sense, the pitch element is actually constant across each measure, and remains at 4 for the duration of this sample. These two interpretations suggest that it is possible to define two types of pitch-element number: a local pitch-element number, or how many pitches are found in any given impulse, and a global pitch element number, or what is the total number of pitch elements a voice plays over the course of a pattern.

In conclusion, pitch-element number is a distinct pattern quantum. It captures information about the total number of unique pitches within a pattern voice. Unlike a pitch set, which may not be concerned with octave doubling, pitch-element number is concerned with all the pitches in a voice. This makes it useful for comparing different patterns, and for noticing regularity across motives and chords with different notes, but similar sizes. And because pitch-element number is unconcerned with what the pitches actually are, it is a useful tool for comparing patterns that do not share pitch sets, but are otherwise similar in how many pitches there are and how they are used.

2.2. Pitch Sets

There are two types of pitch sets that I have drawn from traditional set theory analysis as pattern quanta in PCF.[3] The first is the simple pitch set, which is just the set of pitches in a given chord, or in a single voice over time. This kind of set only reflects novel pitches, and unisons or octaves are not considered to be more than one class. It is often useful to define the quantum of pitch set for each voice in a pattern. It can, for example, show when voices share a set of pitches and when they do not. The second quantum is the pitch-class set, which reflects intervalic relationships. Any two pitches that form a major third, for example, are part of the same pitch-class set. The quantum of pitch-class set is useful for noticing more abstract similarities between voices or patterns. Pitch-class sets are a common tool in rigorous set-theoretic analysis, which is not what I have set out to do in this document; in part, because to engage in that kind of exhaustive analysis of PCF would quickly become a masochistic exercise. In my document, pitch-class sets are used as a targeted tool when comparing the relative chromaticism between two patterns. Tightly chromatic clusters are abundant in PCF, and it is valuable to recognize those pattern quanta over the piece. For the purposes of my document, I am not overly concerned with the distinctions between an {0,1,2,3} and a {0,2,3,4}. It is enough to recognize them both as highly chromatic. I have provided two examples that demonstrate how both pitch set and pitch-class set quanta are derived and their analytical utility.

The three measures of Example 8 are pulled from three different patterns, and while they all have variations in rhythm and register, the cello voices of all three share the same pitch set: {F Double Sharp, A

[3] These concepts are described in great detail in Allan Forte's work, *The Structure of Atonal Music* (1973).

Example 8. mm. 1, 38, and 211.

Flat, A sharp, B Double Flat}. Like pitch-element number, the pitch set of a pattern can be either the total set of all pitches in a voice over the duration of a pattern, or the isolated pitch set of a rhythmic impulse. Determining which one might be analytically useful is done by reflecting on the nature of the pattern. Some patterns have small sets of pitches that are cycled through repeatedly, such as in the cello voice of the measures in Example 8. In this case, it is possible to describe the pitch set that is the same both locally and over the entire length of a pattern. Other patterns might include sequences of chords with different pitch sets. In this case, it is more useful to examine the pitch sets of those isolated impulses than the total set of pitches in that voice over the length of a pattern. Another indication of what scope to examine is the presence, at some level, of a previously defined pitch set. For example, after finding the {F Double Sharp, A Flat, A sharp, B Double Flat} set in the pattern at 1–18 (Example 3), its presence in the pattern at 38–72 and at 202-216 (measures from which are sampled in Example 8), becomes conspicuous. The same pitch set in different guises in different patterns signifies an important pattern quantum in PCF.

To demonstrate the utility of pitch class sets in PCF, let us return to Pattern 90–100 (excerpt found in Example 9). This example is a perfect demonstration of how looking at pitch-class sets can reveal otherwise opaque connections. The cello part and the piano LH + RH share many features in common in the unison voicing of this pattern. The gesture and basic rhythm make the parts look quite similar. This was one of the reasons I labeled this example a unison voicing in the previous section. Upon closer examination, however, there are pitch quanta that set the two voices apart. For example, they have completely dissimilar pitch sets. The pitch set for the cello voice is {F Flat, C Sharp, B Sharp, C Flat, G

Example 9. Page 5, mm. 90–97.

Double Sharp, A Flat}, and the pitch set for the piano voice is {B, A, G Sharp, E, D Flat, C}. Despite these dissimilarities, looking at the pitch-class set for the cello part and piano hands reveals that both voices are actually composed of the same pitch-class set, {0,1,3,4,5,8}. This means that, despite their apparent differences, the two pitch sets actually contain the same or inversion-related interval content. In other words, they share the same pitch-class set hexachord. Looking at the pitch-class set of each part allows us to define a pattern quantum, which is the same between the two voices in this pattern. This shared quantum helps reinforce the sense that the pattern is a unison voicing, as it contributes to the sense that the cello and piano are related, and thus create a single cohesive texture.

In conclusion, the quanta of pitch set and pitch-class set are two ways in which it is possible to describe the pitches in PCF. Between the two quanta, it is possible to capture a large amount of information about what a pattern's pitch material looks like, and both can be used to compare and contrast different patterns. Pitch sets can compare the pitch material of voices across patterns, as in the three measures of Example 8, and pitch-class sets can be used to find less obvious connections that unify parts into a unison voicing, such as in Example 9. More generally, pitch-class sets can be used to demonstrate the prevalence of chromaticism across the whole piece.

There is one last issue of pitch that I discuss in this section on pitch quanta, and that has to do with the sometimes inscrutable notation found throughout PCF, and especially in the cello part.

2.3. Enharmonic Equivalence and Notation

Feldman's writing in PCF contains many features which appear *prima facie* to be excessively baroque or simply absurd. Within the scope of pitch, any discussion of PCF requires addressing the double sharps, double flats, C Flats, F Flats, etc. that are exceedingly common. One reaction to such notes would be to dismiss them as Feldman being eccentric, and to treat them the same as their enharmonic equivalents. However, imagining Feldman in the company of composers such as Ferneyhough and others of the New Complexity fold is not particularly convincing, especially considering the broader abstract expressionist movement which defined so much of Feldman's work and collaboration.[4] Additionally, the use of double sharps and double flats exclusively for the cello, which is not restricted to equal temperament, suggests that Feldman meant them to be acted on and interpreted.

This leads us to a set of performance-related questions: 1) Is this realistically playable? 2) Should the pitches be calculated according to Just or Pythagorean Tuning? 3) If Just Tuning is employed, what "key" or pitch center should be used? and 4) Is enharmonic equivalence allowable in the consideration of pitch set and pitch-class set, or is an F Flat never an E? These questions are most fully addressed in Chapter 3, but for now I have found it productive to consider a B Double Flat just a "weird" A. In this way, I can address similarities in pitch-class set while considering the chromatic deviations between enharmonic equivalent notes as a kind of chromatic embellishment. The one inconsistency with this approach is that, while the piano part does not have instances of double flats/sharps, there are plenty of examples of C Flats and E Sharps which are hard to rationalize given the atonality of the piece and the equal temperament of the piano. The simplest explanation is that this occurs when there is a shared pitch

[4] See Feldman (1981), or Santarelli (2013) for a discussion on the influences of abstract expressionist paining on Morton Feldman's compositions.

set with the piano voice and cello voice, and the shared notation is there to make this shared set visually apparent in the score.

2.4 Performance Utility

While Feldman's use of pitch is arguably less intimidating than his use of rhythm, it nevertheless presents its own set of challenges. The pattern quanta discussed above can help address these challenges in two ways. First, the discussion of enharmonic equivalents as chromatic embellishment opens the door to rewriting passages in a way that is easier to read, so long as the performer applies the chromatic embellishments that the original notation implies. Secondly, recognizing that there are only a few pitch sets, and an even smaller set of pitch-class sets in each instrument, greatly reduces the practicing load for the performers. If before a performer begins practicing a piece, they know that there are a few specific sets and registers that are to be used, they can specifically prepare exercises to familiarize themselves with them. In the same way that practicing a scale, thirds, sixths, and octaves in the same key and register as a challenging passage can make that passage, and all like it, easier, the pitch sets that make up PCF can be isolated and practiced to make learning its various patterns more manageable. Exploring how these sets can be turned into exercises and practiced is covered in Chapter 3.

To conclude Section 2, there are three quanta of pitch that can be used to analyze and compare the patterns of PCF: pitch-element number, pitch sets, and pitch-class sets. Each are useful in determining the pitch identity of a pattern. In Chapter 2, I explore how these quanta of pitch can be used to examine how pitches create patterns and evolve over time, and in Chapter 3, these same pitch quanta inform the exercises that I have designed to make this piece more approachable.

3. Rhythm

Despite the title, *Patterns in a Chromatic Field* —which seems to allude to a piece which explores chromaticism— I would argue that Feldman's explorations of rhythmic variation, and the effects of complex polyrhythm, are just as compelling, and in many ways, more extensive. There are several performance and interpretation challenges presented by the rhythms in PCF. The first is simply trying to decipher the complex polyrhythms that are notated. Any analytical framework aimed at aiding a performer's preparation of PCF needs to have tools that simplify its rhythmic notation. Overcoming that first challenge only leads to more, as the next step is figuring out how those rhythms fit together and how they should sound. While there are patterns in PCF where subdividing to common denominators, marking beats in the score to highlight the sequence of rhythmic events, and ample practice, might be enough to prepare for a performance, there are many more that just frankly seem impossible. The analytical framework of pattern quanta needs to aid performers in making the rhythms that are notated seem intuitive and approachable. By applying the idea of pattern quanta to the rhythmic components, I have sought to do just that. This section seeks to answer four questions: 1) How can the rhythm of voices composed of running-notes be quantized? 2) How can voices composed of isolated rhythmic impulses be quantized? 3) Is it possible to distill complicated notation into basic short-long notation? and 4) Is it possible to use the concept of elongations to reintroduce any rhythmic nuance lost in the process of quantization. The first question I tackle in this section is the quantization of running-note voices.

3.1. Running Notes

The patterns in PCF where one of the voices has a "string" of running notes are deceptively difficult rhythmically. Much of the rhythmic complexity is not located in the rhythm of the notes themselves, but in the markings around the notes, such as the tuplets and meter changes that accompany almost every running-note passage. This makes them look simple enough in the score, but actually quite challenging to put together. Additionally, Feldman does not follow notational conventions that would help show where the beats are in measures. As an example, it is not uncommon to find a string of dotted notes in a simple meter giving no indication to where the beats should actually be. In order to quantize the rhythms of running-note voices, it is necessary to subdivide their "strings" into logical and intuitive groups. There are three devises Feldman uses to accomplish this in PCF: tuplets, meter, and slurs. These three devices are the rhythm quanta for running-note voices. Each is used to create groups of notes that, much like pitch sets and pitch-element number, have consistent properties across a pattern. In other words, far from being random and arbitrary, the groups of notes that meter, tuplets, and slurs create, have consistent and predictable values.

3.1.1. Tuplet-Element Number

The tuplet-element number is the number of notes that are consistently found under a particular tuplet in a pattern. It is possible to quantize running-note voices by tuplet element number because, without exception, when Feldman introduces a tuplet in a pattern, it always groups together the same number of notes for the rest of that pattern. In fact, these numbers tend to be consistent over the entire piece. As an example, in the cello part, every running-note voice that has a 9-tuplet invariably has that tuplet over either two or four notes.

Example 10. (a) m. 415, (b) m. 505, (c) m. 1442, (d) mm. 1612–1617.

The four examples seen in Example 10 demonstrate the various ways that tuplets are used to group running notes into consistent groups. I have included two examples that include the piano voice, to show that this phenomenon is not just present in the cello voice. In Example 10 (a), the 10-tuplet is used to make room for five even 8th notes in a 9/16 bar. Therefore, the tuplet-element number of the cello voice is 5 in this pattern. In Example 10 (b), the 6-tuplet makes room for two dotted quarter notes in a 5/8 bar. Therefore the tuplet-element number of the cello voice is 2 in this pattern. In Example 10 (c) the 4-tuplet makes room for four even 32nd notes in a 3/32 bar. Therefore, the tuplet-element number of the piano RH in this pattern is 4. These previous examples all offer intuitive examples of how tuplets group consistent numbers of notes together in patterns with running notes. Example 10 (d) however is slightly less clear as a result of how it is notated, and deserves a closer look.

The tuplet-element of Example 10 (d) is the same as example 10 (c): 4. It has all of the same quanta as the previous example. The main difference between the two examples is how Example 10 (d) is barred. Why did Feldman bar the LH differently form the RH? Both hands of the piano have the same pitch set, but they differ in how that pitch set is arranged. The pitch set in the RH is regularly ordered while the LH is not. The barring of the RH is intended to show a consistent ordering of the pitch set. Perhaps Feldman notated it this way to imply that there should be an emphasis on the second 32nd note of the measure as opposed to the actual first beat. The LH is notated with a bar tying all the notes of the pattern together. This appears to accentuate the differences between the two voices in the piano. None of this changes the tuplet-element number, but it does highlight how, like meter, tuplets can operate in juxtaposition to how a measure is barred. The next quantum, meter-element number is similar to tuplet-element number.

3.1.2. Meter-Element Number

Meter-element number looks to define the same kind of thing as tuplet-element number. In patterns where a voice has running notes, meter and the number of notes in a bar can have unexpected relationships. Meter-element number is helpful in those situations in seeing past that complexity. For example, a pattern where the time signature is 5/16 might have four running-notes in it, therefore, in many cases, it can be thought of as just a measure with four notes, and a little extra room that might be used for an embellishment. Just like tuplet-element number, when Feldman establishes a relationship between the meter and the number of running-notes in a bar, that relationship will be the same for the rest of the pattern, and will likely reappear in other patterns with the same meter. Because of the similarities in how

(a) (b) (c)

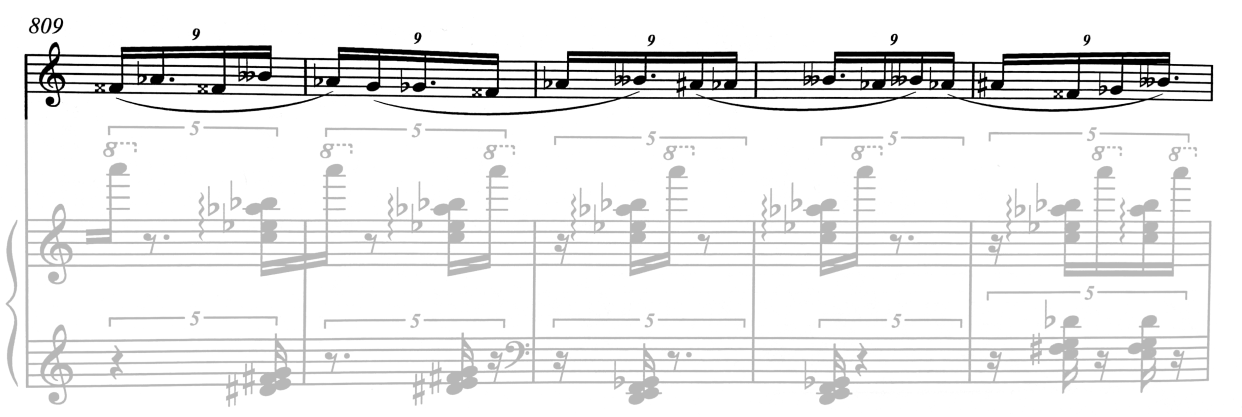
Example 11. (a) m. 307, (b) m. 1225, (c) mm. 1390–1391.

they are derived, the quanta of tuplet-element number and meter-element number are broadly comparable. This is helpful when making comparisons between patterns that look different, but sound similar, and it is also helpful for performers learning all the different patterns in PCF. If it can be shown that two patterns are actually broadly similar, then only their differences need to be tracked and practiced.

The three excerpts in Example 11 demonstrate both how meter-element number is found, and a general equivalence between meter-element number and tuplet-element number. In Example 11 (a), the cello voice is made up of running notes. There are four notes per bar in a meter of 5/16. Because there are always four notes in a measure, the meter-element number is 4. The fact that there are four notes per bar, but the meter is 5/16, creates a similar discrepancy as tuplets can in relationship to tuplet-element number. Patterns with the same meter/tuplet-element number are broadly comparable. For example, even though Example 11 (a) is not under a tuplet, to the listener, it is similar to the cello voice in the Pattern 1-18 (Example 3). In Example 11 (b), there are three double-dotted 16th notes and one double-dotted 16th rest per each 7/16 measure. Therefore, the meter-element number of the cello voice in this pattern is 4. Note that like in Example 10 (d), the way that Feldman has barred the notes in this example conflict with the bar lines, in order to hint at a privileged rhythmic grouping. In Example 11 (c), there are two measures which further demonstrate how tuplet-element number and meter-element number are analytically equivalent. Looking at the cello voice of the first measure, the tuplet-element number is 4, and in the second measure, the cello voice has a meter-element of 4. To the listener, there is very little audible difference between these two patterns. Despite the differences in notation, comparing meter-element and tuplet-element validates what is heard by the listener: the two measures are quite similar. What differences there are are created by different time signatures; this phenomenon are covered in greater detail in Section 3.3 on elongation. The final rhythm quantum defining running-note voices is slurs.

3.1.3. Slurs

Slurs are relevant to the discussion of running-note quanta in three key ways. First, slurs create intuitive groups of notes just like tuplet-element number and meter-element number. For example, the cello voice might have an uninterrupted sequence of slurs which are all six notes long. In this hypothetical, that length is a consistent and stable quantum of that pattern, which regularly divides the running notes of a voice. This is akin to the effect that meter/tuplet-element number have. Second, slur lengths might create groups of notes which are a different size than the tuplet-element number or meter-element number. When this happens, it can create isorhythmic effects. For example, the tuplet-element number of the cello voice in a pattern might be 4, but all of the uninterrupted slurs are 5 notes long. These effects are quite common throughout PCF, because many of the patterns with running notes in the cello voice are arco and slurred. In fact, this is the primary use of slurs in PCF. Finally, in some patterns, slur length varies between a defined set of several slur lengths. This variability can make slur length unlike tuplet-element number and meter-element number, which are always the same in a given pattern. However, because the slur lengths are a definable set, and the representation of that set in a pattern can be controlled by some of the quantum behaviors discussed in Chapter 2, it can still be useful to track the way slurs quantize running notes, even if it is not a static number over the course of a pattern.



Example 12. Page 36, mm. 809–813.

In the excerpt of Pattern 808–828 (excerpt found in Example 12)*,* there is a clear demonstration of how slur length and tuplet-element number can create isorhythmic effects. The tuplet-element number in this example is 4, but the slurs are all five notes long. That means that the slur will start on different notes through the measure, starting on the first note of the measure, then the second of the next, etc. until it starts on the first note of the measure again. This procession will happen at a period of fives measures. Identifying the slur length allows for both the identification of this phenomenon, and more generally, it provides another quantum with which to define this pattern. Finding a slur length of 5 creating isorhythmic effects over a tuplet-element number of 4 in this pattern hints that it might be found in others, just like all the other quanta. It also has ramifications for the practice of a performer. Understanding the set of ways that Feldman uses slurs to group notes can help the performer more intuitively practice various patterns. In this example, the cellist only needs to know that all the slurs are five notes long, and this will create an isorhythmic effect with the four tuplet-elements. This quantum can be practiced on its own, and now the cellist can be prepared for quantum when it shows up in other guises, as well.

In conclusion, the quanta that describe the rhythms of the running-note voices in PCF are all designed to create intuitive divisions of notes. Tuplet/meter-element number and slurs are ways of conceptualizing some periodicity of the string of notes. This ability is valuable for both the discussion of behaviors and regularity that are the subject of Chapter 2, and for its utility to performers wishing to learn the piece.

3.1.4. Performance Utility

The quanta that define the running note patterns have several implications for performers. The first is practical, and has to do with the process of learning the piece. Because meter and tuplets often have unexpected relationships to the notes in their measures, it can be difficult to intuit the practical beat of a measure or the pulse of a pattern. Tuplet/meter-element number and slurs are concrete and superficially apparent features that can be used to simplify a performer's sense of how beat/pulse are felt in a pattern. It is possible to capture enough about the rhythm of a measure by focusing on these quanta to begin building up a piece in practice or rehearsal. Second, these features are not just of practical use, but important to consider in matters of interpretation. Pulse and beat are not just how things fit together, but also what something sounds like. The clearer the mental representation of the pulse is for the performers, the easier it will be to get that across to the audience. By capturing audibly prevalent features of a pattern, tuplet/meter-element number and slurs can help in creating a clear mental representation.

With a framework for understanding the rhythm of running-note figures in PCF now in place, it is possible to turn to the voices made up of repeated rhythmic impulses.

3.2. Impulses

While patterns with voices that have running notes might be deceptively difficult, patterns where there are isolated rhythmic impulses are *prima facie* difficult. As a performer, all of the things that make interpreting the rhythm of the running-note patterns difficult applies to patterns with impulses, as well. Additionally, there is the complication of the irregularity created by the rests between the impulses, and the occasional variations in the length and complexity of the impulses themselves over the course of a pattern. It is possible, however, to perform the same kind of decomposition into simple quanta as it was for the rhythm of running-note passages. Like the previous rhythmic quanta, the characteristics of the impulses in a particular pattern tend to be consistent over the course of that pattern. It is possible to simply define these impulses by the number of elements in each impulse, simplifying the rhythm of these elements into short and long, and more globally, counting how many different kinds (rhythms) of impulses there are in a pattern. Just as with the quanta of tuplet-element number and meter-element number, the values of the quanta defining the rhythmic impulses in PCF are a definable set which creates opportunities for comparison and analysis of the patterns throughout the piece. The first quantum discussed is impulse-element number.

3.2.1. Impulse-Element Number

The impulse-element number is the number of notes or chords in a single rhythmic impulse. This quantum is unconcerned with the number of vertical notes. A rhythmic impulse made up of two chords would have the same impulse-element number as a rhythmic impulse made up of two single notes: 2. It is possible to easily and consistently identify the number of elements in the rhythmic impulses of a pattern, because the voices that make up a pattern are almost entirely binary: voices are either made up of running notes, or rhythmic impulses surrounded by rests. In the case of the latter, the impulse-element number(s) of a pattern are consistent across that whole pattern. The patterns of PCF use a surprisingly small set of possible rhythmic-element numbers (generally 1 or 2). By defining an impulse simply by the number of notes in it, it begins to be possible to compare impulses across patterns, and also to begin simplifying the rhythm in order to aid in performance.

Example 13. (a) m. 49, (b) mm. 721–729, (c) mm. 200–204.

The three excerpts of Example 13 demonstrate how repeated rhythmic impulses can be simplified by counting the number of notes in each impulse. The measure in Example 13 (a) gives a basic illustration of how impulse-element number is counted. In the pattern Example 13 (a) is from, the piano voice is a texture of isolated rhythmic impulses. All of the impulses are composed of one chord split between the hands, and after that, one high note in the RH. Therefore, the impulse-element number of the piano voice in this example is 2 (the chord plus the high note). The piano voice in this pattern (Pattern 37–72) is highly regular, therefore all the impulses in this pattern have an impulse-element number of 2.

In the excerpt shown in example 13 (b), the piano voice has repeated impulses separated by measures of rest throughout the pattern. The excerpt shows two of those impulses, along with a sample of the rests surrounding them. The impulses in the piano voice are made up of a sequence of four dyads in a row split between the LH and the RH. Because there are always four dyads in a row, the impulse-element number of the piano voice is 4. This is an example of a rhythmic impulse voice where the impulses are more drawn out. In the other two excerpts of Example 13, and in some of the previously sampled patterns in my document, the impulses are often short and packed tightly together. Example 13 (b) shows that this does not necessarily have to be the case in order for a pattern to have a recognizable impulse-element number.

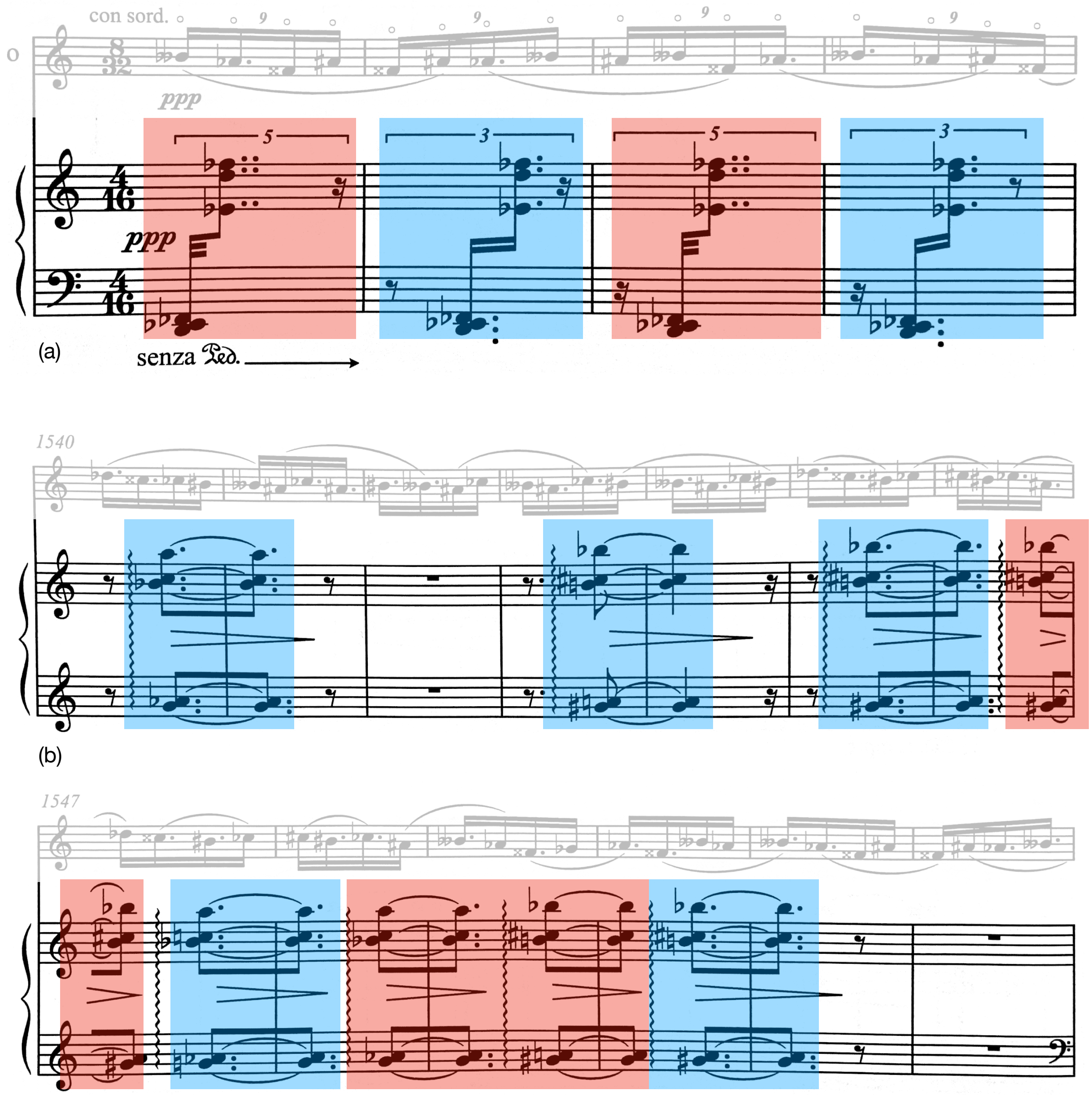
The measures shown in Example 13 (c) demonstrate an impulse-element number that is more difficult to parse. In the piano voice, there is a clear impulse with two elements in the first measure, and an impulse with four elements in the second measure (circled in red and blue respectively). One interpretation of these two different numbers would be to say that this pattern has two different impulse-element numbers: {2,4}. Another interpretation would note that in most of the previous measures, the impulses all have two elements: a chord followed by a high note. So because the four-note impulse looks like two two-note impulses back to back, it is actually just two two-note impulses back-to-back. This would mean that a possible rest length between two impulses is zero. That zero would then be a part of the set of possible rest values between impulses (rest-value sets are discussed in more detail in Section 3). Between the two, I find the latter interpretation more compelling. It reduces the number of impulses the performer needs to keep track of while not eliminating any of the complexity of the music.

With a quantum now defined that allows for the identification of impulses, it is now possible to start to define the rhythm of these impulses. Like with all the other quanta, the goal is to both extract salient features, and to define those features in a way that is easy to see and understand. For rhythmic impulses, the definition part of that process starts with a basic short-long simplification.

3.2.2. Short-Long Simplification

While impulse-element number captures the size and basic parameters of an impulse, it does not capture any information about the rhythm of the impulse itself. In other words, it does not tell us anything about the relationship of the notes in the impulse to each other. Given the baroque notation of rhythm in PCF, it would seem that defining a quantum that captures the rhythmic relationships between notes would be an impossible task. Fortunately, like the previous quanta, defining a quantum that simplifies impulses is made easier by the relatively small set of possible values that are found within PCF. As one example, most impulses have an impulse-element number between 1 and 3. As another, impulses with an impulse-element number larger than 1 can be generalized into two categories, impulses where the notes are all the same length, and impulses were the notes are different lengths. In other words, most impulses are already quite easy to describe because they are either just one note, or multiple notes of the same length. For the remaining impulses that are multiple notes of different lengths, most have an impulse-element number of 2, and these can be simplified into short-short, short-long, long-short, and long-long. While this may seem like an oversimplification, especially for the purposes of performance, in most cases, the nuances of the rhythm this leaves out are not terribly perceptible. For the instances where it is perceptible, Section 3.3 defines several elongation quanta that can be used to reintroduce some of the lost nuance back into the impulses. Even in instances where simplifying the rhythm this way can miss some of the nuance, it can still aid in the analysis of the piece by facilitating comparisons across patterns.

The two excerpts in Example 14 are rhythmic-impulse voices where the impulses are either not made of equal length notes, or where a short-long simplification helps define one-note long impulses of different lengths. In the piano voice of the pattern shown in Example 14 (a), there are two different impulses. The first, found in the first and third measures, is a 32nd note followed by a double-dotted 8th note under a 5-tuplet (highlighted in red). The second impulse, found in the second and fourth measures, is made of two consecutive dotted 16th notes under a 3-tuplet (highlighted in blue). The first impulse can be thought of as a short-long rhythm, and the second impulse can be thought of as a long-long rhythm. While this discards much of the nuance of the actual notated rhythm, it does get us closer than the previous quantum of impulse-element number, which was only concerned with the total number of elements in the impulse.



Example 14. (a) mm. 1–4, (b) mm. 1540–1552, with color-coded impulses.

Another example of the utility of this quantum is in the piano voice of the pattern shown in Example 14 (b). The impulses in this example are all one-note long (some of them are separated by a rest value of 0). These single note impulses, with one exception, come in two lengths, two 8th notes long (highlighted in red) and three 8th notes long (highlighted in blue). It is possible to simplify the one-note impulses in this pattern into short and long values. This excerpt shows how short-long simplification can be useful for comparing impulses to each other, unlike in excerpt (a), where it was used to compare the notes within the impulses.

3.2.3. Impulse-Kind Number

Now that there are quanta that define the impulses of rhythmic-impulse voices, it is possible to use them to identify and count the total number of different impulses in a pattern. The impulse-kind number is the total number of different impulses in a pattern, as defined by the short/long quanta and the impulse-element number. There are usually only a small number of different rhythmic impulses in any given pattern. The two patterns excerpted in Example 14 both have an impulse kind number of 2. In Example 14 (a), the two impulse kinds are the impulse under the 5-tuplet and the impulse under the 3-tuplet. In Example 14 (b) the two impulses kinds are the single-note impulses that are either two 8th notes long or three 8th notes long. Some voices have an impulse kind number of 1, where all the impulses are identical. This is true for all of the impulse voices excerpted in Example 13.

In conclusion, in combination with the previous two quanta, it is possible to capture a large amount of the defining characteristics of the impulses in a pattern. The quanta of impulse-element number, short-long simplification, and impulses-kind number extract impulses, define them, and count their total number. This is the first step in simplifying Feldman's baroque rhythmic notation.

3.2.4. Performance Utility

Similarly to the performance utility of the quanta that define the features of running note patterns, there is an essential utility in being able to simplify the notation around the rhythmic impulses in PCF. Perhaps most prominent is the recognition that, despite all that could be notated given the excessive use of unusual meter and tuplets, there are only a few different impulse types that are used throughout the piece. These can be isolated and practiced using various tools that are discussed in the Chapter 3. Using this set of quanta to define a set of impulses archetypes also allows the performers to discover only what is unique to each different pattern. This can accelerate the learning time associated with any given pattern by reducing the cognitive load of reading and practicing any given pattern. And just like for running notes, the clearer the mental representation of the impulses is for the performers, the easier it is to get that across to the audience. By capturing audibly prevalent features of a pattern, impulse-element number, short-long generalizing and impulse-kind number can help in creating a clear mental representation of rhythmic-impulse voices for the performers.

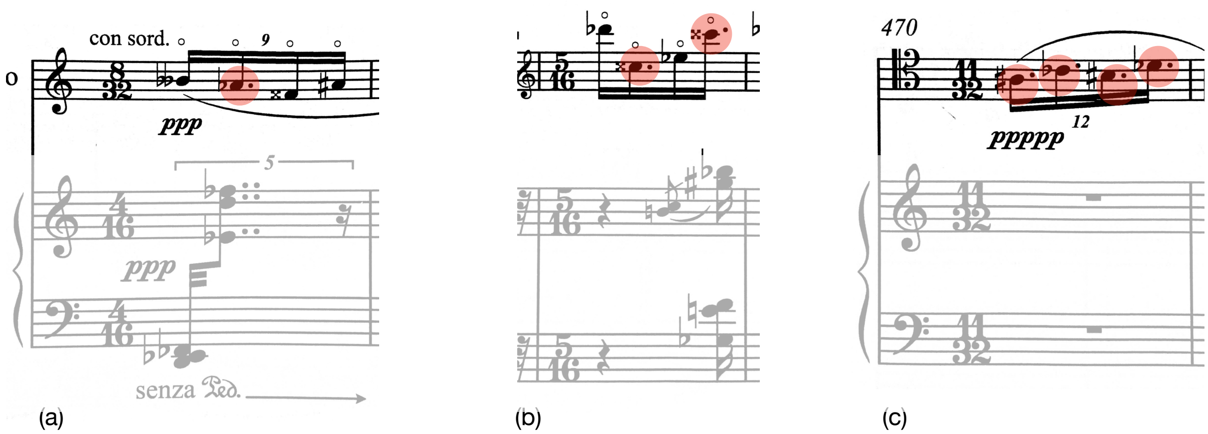
With the basic rhythms of both running-note voices and rhythmic impulse voices quantized, it is now possible to reintroduce some of the rhythmic nuance that the previous quanta left out. This is done with the quantum of elongation. In the next section I explore how this quantum applies to both types of voices, and how it captures more of the original rhythmic complexity while maintaining the conceptual simplicity achieved with the previous quanta.

3.3. Elongation

The purpose of the elongation quantum is to reintroduce the subtlety of rhythm that is not captured by the other rhythm quanta. To be a useful quantum, any method of reintroducing complexity to the simplified rhythms created in the previous section needs to be specific enough to capture the rhythmic relationships in the score, and general enough to be applicable and useful in every pattern. Up until now, the way that I have quantized the various dimensions of rhythm in PCF has been done with the intent of capturing the most basic picture of what a particular pattern's rhythmic features are. This has necessitated eliding over some of what makes a pattern unique, and sometimes has required over simplifying what is happening in the score. However, it also offers an opportunity to reintroduce the complexity of the rhythm in a way that does not rely so heavily on baroque notation. Ideally, this can be done in a way that creates the opportunity to make easy comparisons between patterns, and allows a performer to generalize a particular rhythm into known components. Section 3 examines how the quantum of elongation can be used to capture the rhythmic nuance in four domains of PCF: running notes, impulses, rests, and meter. The first elongation quanta I discuss in this section pertains to running notes.

3.3.1. Running Notes

I have already discussed how the tuplet-element number, or the meter-element number, of a voice in a pattern with running-notes can be totally removed from the tuplet or meter itself. For example, in Pattern 1–18 (Example 3) the tuplet-element number in the cello voice is 4, but the tuplet over those four notes is 9. As an example with meter-element number, see Pattern 307–314 (excerpted in Example 11 (a)), where the meter-element number of the cello voice is also 4, but the meter is 5/16. So where have the extra beats gone? Sometimes, the extra beats are evenly divided among the tuplet/meter-elements. However, more commonly, and as in Patterns 1–18 and 307–314, Feldman uses dotted rhythms to fill in the missing beats. These dots create subtle irregularities, or elongations, that can be counted and compared across patterns. As a performer, it is sometimes far more practical to slightly elongate a note, and just keep track of which tuplet-element the dot is on, rather than subdivide to the 32nd under a 9 tuplet in an 8/32 bar with a tempo of quarter=63-66 (as would be required in the pattern at measures 1-18).



Example 15. (a) m. 1, (b) m. 307, (c) m. 470, with Dot-Elongations Circled in Red.

The three excerpts in Example 15 show three different ways it is possible to conceptualize tuplet and meter as creating space for elongations within otherwise even running notes. Example 15 (a) is a measure excerpted form the opening pattern, Pattern 1-18. In this example, the cello voice of the pattern has a 9-tuplet. This 9-tuplet makes room for an additional 32nd note in the 8/32 meter. This additional space is used to add a dot to one of the 16th notes once a measure (highlighted in red). The placement of this dot changes position in each measure throughout the pattern. Therefore, ignoring the meter, this passage can be thought of simply as running 16th notes with a slight elongation on one of them per measure. This same decomposition using the quantum of elongation can be applied to measures without a tuplet.

Example 15 (b) is a measure from the cello voice with a similar effect to the cello voice in Example 15 (a). In Example 15 (b), the slight mismatch between the number of running notes and the meter allows for the same overall effect as in Example 15 (a), where there was a mismatch between the tuplet number and the tuplet-element number. In Example 15 (b), the 5/16 time signature creates space for one extra 16th note around the four 16th notes in each measure. In this pattern, that extra time is divided into two dots (highlighted in red) that are added to two of the 16th notes in each measure. Therefore, this rhythm can be thought of simply as running 16th notes with two slight elongations per measure. Like in Example 15 (a), the location of those dots changes throughout the pattern.

Sometimes dots do not contribute to any local note-to-note elongation, but are part of a scheme to elongate the whole measure. This is the case in the pattern shown in Example 15 (c). In the cello voice, the 12-tuplet makes room for a dot on every one of the four 16th notes in the measure (highlighted in red). This may be an indirect way of notating that these four notes should be a bit longer than they would be without the dots, but in Section 3.3.4 I discuss how this is actually well-motivated, and not just obscure notation for its own sake. Compared to the two previous excerpts of Example 15 (a) and (b), the pattern excerpted in Example 15 (c) has the longest overall note lengths in the cello voice of the three different patterns. This is because of the use of elongations on every note. In this way, the total voice itself is elongated compared to other similar pattern voices. The three excerpts of Example 15 are all comparable to each other because all of them have a tuplet/meter-element number of 4.

In conclusion, in running-note voices, the subtle rhythmic variations that are missed by previous quanta can be reintroduced with the quantum of elongation. Elongations are created by dots that elongate certain notes in each measure. These dots are fit into the running notes through the use of tuplets and meters that are slightly larger than needed to fit the number of notes under/in them. The quantum of elongation is more general than just dots. In the next section, I examine the way elongation can bring nuance to the voices with rhythmic impulses.

3.3.2. Impulses

Tuplets are the primary way that Feldman elongates and accentuates the characteristics of rhythmic impulses. Unlike with running notes, where the tuplets create space for the dots that elongate the notes, in patterns with isolated rhythmic impulses, tuplets themselves are the source of the elongations. The tuplet over an impulse can either subtly elongate or contract the rhythm underneath it compared to what the rhythm would be without the tuplet. The tuplet number itself acts as the quantum of elongation. As I demonstrate in the following examples, tuplet elongations can be compared across different patterns. And because there is more rhythmic variability in impulses than in running-notes, the tuplet number can also be used to compare two different impulses within a pattern.

Example 16. Page 1, mm. 1–9, with Arrows Indicating the Elongations Caused by the Tuplets.

Pattern 1-18 (excerpted in Example 16, and reproduced in its entirety in Example 3) is a perfect example of how tuplets are used to accentuate the rhythmic characteristics of impulses. In this pattern, there are two tuplets used throughout, a 3-tuplet and a 5-tuplet. The second and third measures of the pattern shows the clearest comparison with how the 3-tuplet and 5-tuplet modify the rhythm. Both measures contain a single impulse. That impulse is either short-long (first note is a 32^{nd}, second note is a double-dotted 8^{th} note), or long-long (both notes are a dotted 16^{th}). Additionally, there is either a long rest (8th), or a short rest (16th) before each impulse. The tuplet number over each measure is actually consistently coordinated with whether the elements in the measure are of the short-long or long-long variety. The 3-tuplet is matched with the longer impulse and rest, and the 5-tuplet is matched with the shorter impulse and rest. In other words, Feldman is using the tuplets to slightly exaggerate the intrinsic characteristics of the rhythm in both figures. The 3-tuplet makes 8th rest (the longer of the two rests) and dotted 16th notes (the longer of the two impulses) even longer, while the 5-tuplet makes the 16th rest and 32nd note in the short-long impulses even shorter. This exact pairing of tuplets with these notes and rests is present throughout this pattern (marked in blue and red arrows showing the way that tuplets increase and decrease the beat-space in a measure).

Voices that are composed of rhythmic impulses are not just defined by the notes that make up the impulses, but also the rests that space out those impulses. Elongation can be used to understand how changes in the rest values between impulses creates subtle nuances in the rhythm of rhythmic-impulse voices.

3.3.3. Rests

Rest elongation applies solely to patterns with isolated rhythmic impulses in one of the voices. The rests in these patterns can be thought of as a type of elongation in the empty spaces between impulses. This may seem like an anodyne statement about the role of rests in music generally, but because each pattern has a definable and consistent set of rest lengths, and these sets can be compared across different patterns, they are like any of the other pattern quanta. As an aside, it is often the case that the most basic statement about how some mechanic in music works can seem like the most profound realization when applied to Morton Feldman’s music. Perhaps that is what is so alluring to some people about this music; it elevates the most basic elements of music to something profound.

Rest elongations around impulses can be variable, and this creates interesting effects. In Pattern 1055-1062 (Example 17), the impulse-length of all the impulses in both the cello and piano voices is one

Example 17. Page 47, mm. 1055–1062.

16th note. The time between those impulses can be zero, one sixteenth, and one eighth in the piano voice; and one sixteenth, one eighth, one dotted eighth, and one quarter in the cello voice. In total, the cello has 11 impulses over the course of the pattern and the piano has 13. Given that everything else in the two voices in this pattern is very similar, it is possible to observe how the rest lengths create more length between the impulses in the cello voice, resulting in fewer total impulses compared to the piano voice. It is possible to define that difference by collecting the set of possible rest lengths and comparing them between the two voices. The fact that the set of possible rest lengths in the cello voice represents more time can be thought of as a kind of elongation of the space between the impulses compared to that same metric in the piano voice.

This more abstract kind of elongation of the entire material of a pattern is a good segue to the final dimension of the elongation quanta, meter.

3.3.4. Meter

Meter is the most abstract kind of elongation, as its effects are less local, and can be felt across patterns, even when they are located in different sections of the piece. This sense of elongation is reminiscent of the way that Elliott Carter used meter and rhythm, as it relies on the metric modulations that happen across changing time signatures.[5] Normally, the fact that Feldman prescribes one tempo for the whole piece, quarter=63-66, would mean that the overall sense of pulse would be constant from pattern to pattern, and across meter changes. However, the persistent use of tuplets in PCF means that the number of rhythmic elements in a measure often has nothing to do with the actual time signature. Because of this, the change in time signature often has an effect on the perceived tempo, rather than changing the number of perceived beats within a measure. This role reversal, where tuplets regulate the number of beats and the time signature regulates the tempo is, at first, difficult to intuit. However, this is an inevitable consequence of how rhythm has been quantized up until this point. Because I have quantized rhythms into intuitive groups of notes without regard for the time signature they are in, a disconnect

Example 18. (a) mm. 1–4, (b) mm. 470– 475.

[5] For an interesting discussion on Carter's use of metric modulation that touches on the innovations of the Cello Sonata, see Bernard (1988)

between tempo and time signature is inevitable. This relationship can most easily be demonstrated by comparing two patterns with the same tuplet/meter-element number.

There are several superficial similarities between the cello line in Pattern 1-18 (Example 18 (a)), and the cello line in Pattern 470-486 (Example 18 (b)), that facilitate a demonstration of how meter elongation works. Namely, they both have a tuplet-element number of 4, and in regards to rhythm, should actually sound very similar to each other. There are some key differences, though. In Pattern 470-486, the tuplet number is 12, and rather than using this extra space on dotted elongations of a few particular notes, every note is dotted. The result is four even notes. Because the notes are actually even, it is easy to imagine how Feldman might have simply dispensed with the tuplet and written this as a 12/32 bar. But with the prescribed tempo of quarter=63-66 applying to the whole piece, an 11/32 bar with dotted 16th notes under a 12 tuplet is actually slightly shorter than a simple 12/32 bar with no tuplet and with straight dotted 16th notes. This already represents a kind of elongation/contraction where the meter is modulating the tempo rather than the number of beats in a measure. Taking this concept even further, the identical tuplet/meter-element numbers of the two patterns in Example 18 allows us to observe how a difference in meter changes the duration of the four-note measures in relationship to each other. Specifically, the 11/32 measures are longer than the 8/32 measures. This is how meter and tuplets modulate the metric relationships between patterns, and can be thought of as a type of elongation.

Example 19. (a) mm.118–124, (b) mm. 343–353, with meter-elongation highlighted in red.

Meter can also be used to create elongations on specific pitches, and during G.P.s or rests. Pattern 118-126 (Example 19 (a)) is an example of how Feldman uses meter in order to create elongations of a

G.P. In this pattern, the time signature of the G.P. measures (highlighted in red) starts as 1/4, then in the next G.P. it is 2/4, and continues in this fashion up to 5/4. In this way, Feldman is using meter to elongate the G.P. between each 17/16 bar. Meter can similarly elongate specific pitches. In the pattern excerpted in Example 19 (b)*,* each measure with the "bend tone slightly" marking (highlighted in red) in the cello voice gets longer by one 16th note each time it occurs. Thus, the note effectively becomes longer and longer over the course of the pattern. Feldman creates the space for this note elongation by increasing the duration of the meter every time the figure reoccurs. The excerpt in Example 19 (b) also shows rest elongation after the first measure (circled in blue). Each successive period of rest in both voices is increased by one quarter. The room for this increased period of rest is created by changing the meter each measure. The ultimate effect is the same as the G.P. elongation found in Example 19 (a).

4. Performance Utility and Conclusion

The quanta that define elongation, along with the previous quanta that define the impulses, running-note figures, and pitch sets of the patterns in PCF are all an attempt to catalogue the parts that make up PCF. When I began working on PCF, the first large scale question that arose was what can be a pattern and what can't be a pattern? There is a specific identity to this piece, and even if Feldman's attitude in composing PCF was simply *a pattern is whatever I want it to be,* the set of patterns in the piece would still not be infinite. Given that there are several superficial similarities even within the first few hundred measures of the piece leads me to believe that that wasn't Feldman's attitude in the first place.

As a performer, the motivation for asking questions about what can and cannot be a pattern may be musical; what is the aesthetic world I need to recognize and recreate in my performance? It can also be deeply practical; how am I going to learn this long and complex piece before the date of the first performance? The idea of quantizing the patterns into the smallest elements that are still recognizably something from PCF is simply pulled from the already established pedagogical work all musicians do when practicing scales to help with preparation of, say, a Beethoven sonata. It helps musicians recognize consistent "patterns" in the music they perform, and it facilitates speedier learning by granting a base level of muscle memory and understanding of what is possible in any given piece. The novelty of

Feldman’s late work does not preclude this established pedagogical model, but it does necessitate the modification of some standard tools, and the creation of some new ones.

In the introduction, I mentioned Rebecca Leydon’s work discussing the musmatic-discursive paradigm, which considers the ways that small elements create larger-scale musical effects. In Chapter 2, the focus of my document, in her terms, shifts from the musmatic side of the scale to the discursive. With the quanta now defined that create the topology of the patterns in PCF —Feldman’s “crippled symmetries”— it is possible to investigate how these quanta are repeated and manipulated to create a large-scale tapestry.

Chapter 2: Quantum Behaviors

In the previous chapter, the motivating question was: what are the smallest unique parts of a pattern in PCF? In this chapter, the questions are: 1) How do quantum values change from pattern to pattern? 2) How do quantum values evolve within a pattern? and 3) How consistently are the quanta of a pattern implemented? More broadly, the question might simply be, in what ways are the patterns of PCF actually patterns, and in what ways are they not? While my intention is not to problematize the use of the word “pattern” in the title of PCF, Chapter 2 explores the limited ways that the “patterns” in the piece are actually *patterned*, and in turn, the far greater number of ways that they ultimately are not. Despite the seeming irony, or more critically, mislabeling of the piece, it is my belief that listeners would know what Feldman meant when he called the elements of the piece —large and small— “patterns.” The goal of this chapter is to capture that intuitive understanding of what a pattern is by detailing both the literal and figurative ways the quanta of PCF can be patterned.

The purpose of Chapter 2 is not to reconstruct the compositional process used by Feldman, which may or may not have included highly systematic generation of material, but to note the ways that a listener or performer might engage with what is audible in PCF. The questions of this chapter concern what is perceptible in the material, rather than being about how all this material was generated in composition. In this vein, the kinds of explicit behaviors I discuss in this chapter are all things that I believe might be attended to by the ear of a listener. As a performer, I believe that this should be one of the foremost concerns that guides a performer’s analytical study of a composition. My hope is that this level of discussion captures not what Feldman did to write the piece, but rather, what an audience might hear upon performance, and in turn, what might be necessary for a performer to acknowledge and interpret.

In this chapter I discuss three kinds of explicit behaviors: regularity, centricity, and operations. Of the three, the operations I discuss are the most akin to a knowable pattern, but regularity is the overarching idea of the chapter. The section on regularity sets the stage for the other behaviors to exist, because it describes the boundary between what might be an audible behavior for the listener, and what just sounds random. To facilitate a more detailed discussion of these behaviors, I have limited the scope of this chapter to the patterns contained within the first few hundred measures of the piece. The material of these patterns does not fully represent a cross-section of the whole piece, but does provide enough variety to drive a quasi-complete review of the kinds of behaviors that are possible. This chapter discusses the behaviors found in Patterns 1–18, 19–36, 37–72, 73–77, 78–89, 118–126, 145–181, 207–216, and 289-300. Examples are provided when necessary, but not in every discussion that includes one of these patterns. Some of them have been discussed in Chapter 1 to demonstrate quantization. It may be helpful to refer to these sections, and I note when a pattern has been discussed in some form before. As it is the overarching idea of the chapter, regularity is the first concept I discuss.

1. Regularity

Regularity is the measure of how consistent the material of a pattern is throughout its duration across the domains of the various quanta (defined in the previous chapter). Patterns are built through the repetition of elements; in the most basic sense, a pattern is built from the exact repetition of a particular sequence or idea. Feldman was not only interested in patterning as it pertains to music. His interest in patterns was also found in his interest in textiles, a domain where the very basic idea of repetition creating a pattern is intuitive and familiar.[6] But the patterns Feldman creates in PCF are hardly ever composed from perfect repetitions of the quanta that define the pattern's basic elements. This poses an interesting question: why might a listener hear a section of PCF as a Pattern (with a capitol "P") if the quanta within

[6] See not only Feldman (1981), but also Javadi and Fujieda (2020), which investigates the ways that traditional patterning in Persian and Bakhtiari rugs sometimes directly maps onto the patterning in the late works of Feldman.

that section are not repeated in a way that is systematic or predictable? In very little time, we have found ourselves back at the music-philosophical question posed just a few paragraphs ago. What can be said is that, to the degree that patterns contain a definable set of quanta, it is possible to describe several characteristics that influence the perception of the regularity of that material. How ordered or unordered are they, and how consistently does the pattern hew to the small set of quanta that are presented in its first few measures? If a pattern is not a perfect repetition of its quanta, then how close is it to that ideal? I have divided the discussion of regularity into two parts. The first discusses the regularity of the pitch quanta, and the second discusses the regularity of the rhythmic quanta.

1.1. Pitch Regularity

There are two domains in which the regularity of pitch repetition can be observed. The first is the regularity of the running-note voices within a pattern, and the second is the regularity of the pitch sets used by voices with rhythmic impulses. For the most part, I have refrained from using any kind of precise statistical modeling of regularity, and have instead opted for a more intuitive comparative kind. In other words, for any set of repeated “elements,” there is a platonic ideal “pattern,” which is an exact and regular repetition of those elements. In the quantized approaches I have taken in my document, the “elements” are the quanta described in Chapter 1. Against an ideal of regular quanta repetition, the regularity of a pattern can be defined by they ways in which it deviates from the ideal. The discussion of regularity of the pitch sets draws upon a close examination of Patterns 1–18, 19–35, and 36–72. First, I discuss running-note voices.

1.1.1. Running Note Pitch Regularity

To be able to measure the regularity of running note voices, it is first necessary to define some kind of period of repetition. How often does the listener expect to hear the same material repeated, or, how self-similar is one section of a pattern to another of the same pattern?

One option would be to look to the pitch-element number. If the pitches only occur once per repetition, then the period of that repetition is the length of the pitch-element number. In other words, the time it takes to move through the pitch set, and then start over again, defines one cycle of the pattern. In patterns where each pitch is only used once per repetition, this is a sensible metric for defining the repetitions. This is the case for Patterns 1–18 and 19–36. However, sometimes pitches are not just used once. Continuing to measure the repetition length by pitch-element number seems less intuitive in these cases, because it does not necessarily relate to what is heard by the listener. Instead, tuplet/meter-element number can be used. The measures and tuplets themselves create intuitive groups of notes, and are always the same value in patterns with running-note voices. Because this way of segmenting running-note voices is more consistent in creating intuitive groups, and it also often aligns with pitch-element grouping anyway (such as in Patterns 1–18 and 19–36), I have used tuplet/meter-element number as the pertinent property for segmentation. Now that I have defined an intuitive way to segment running-note voices, it is possible to begin assessing their regularity.

There are several questions that can be asked that pertain to the regularity of pitch in a running-note voice: 1) Are the pitches presented in the same order in each repetition? 2) Is each pitch of the set present in each repetition? 3) Are the pitches of the set evenly distributed throughout the pattern? 4) How might the comparative regularity of two patterns with the same pitch set affect the overall sound of that pattern by changing the proportions of the possible intervals that set creates? and 5) How are slurs used to enhance or complicate the cyclic nature of how a running note voice is segmented? The answers to these questions provide insight into how regularity is connected to the idea of patterning in PCF. In this section I compare Patterns 1–18 and 37–72 as a way of demonstrating how these questions can be answered.

Patterns 1–18 and 37–72 show a contrast in pitch regularity (Pattern 1–18 can be found in Example 3, and Pattern 36–72 can be found in Example 7). This is apparent because the cello voices of both patterns share multiple quanta. Both voices share the quanta of pitch set {B♭♭, A♭, Fx, A♯}, tuplet-element number (4), tuplet (9), and meter (8/32). Despite this, the cello voice of Pattern 1-18 is far more regular than that of Pattern 37–72. The most apparent difference in regularity between them is in how the pitch set is used within each tuplet. In Pattern 1-18, each measure/tuplet contains the whole pitch set, while in the pattern at measures 37–72, each measure/tuplet usually only contains a subset. This means that the cello voice in Pattern 1–18 is more regular than the cello voice at Pattern 37–72, because, while order is not preserved, each repetition defined by both pitch-element number and tuplet-element number contains the same pitches, while the cello voice of Pattern 37–72 only contains a random subset. Pattern 1-18 is much closer to a "perfect pattern," where each measure contains the same or a predictable set of pitches, and therefore, is more regular.

Despite being the more regular of the two, Pattern 1–18 is not a perfect pattern. The fact that each tuplet contains the whole set of pitches does not mean that those pitches are always presented in the same order. It is possible to continue to define ways that this random ordering is more or less regular by finding how likely any pitch is to be found at any given beat within the tuplet. Asked differently, how regularly is the pitch set distributed within the measure? For Pattern 1–18, the answer turns out to be *not very*.

Table 2. Frequency of each pitch occurring on each "beat" of the tuplet.

	1st	2nd	3rd	4th
B Double-Flat	6	4	0	8
A Flat	4	6	4	4
F Double-Sharp	4	0	12	2
A Sharp	4	8	2	4

Table 2 shows the distribution of pitches within each tuplet. As it turns out, some pitches are more likely to be found at certain positions within the tuplet than others. For example, F Double-Sharp is never found in the second beat of the tuplet, but is very likely to be found on the third beat. Of the 18 total instances of F Double-Sharp in the cello voice, 12 of them fall on the third "beat" of the tuplet. We can draw from this chart that Feldman has not regularly cycled through the pitches as he arranged the pitch set in each measure. Despite the fact that each repetition contains every pitch of the pitch set, those notes are not regularly distributed throughout the repetitions like they would be in a perfect pattern. In a perfect pattern, each note of the pitch set would be equally likely to be found on any of the "beats" of the tuplet, or it would always occur on the same "beat" of the tuplet. Either way, its location would be entirely predictable. This is not the case in Pattern 1–18.

In Pattern 37–72, this same metric is even more irregular. This is a direct consequence of the fact that this pattern does not repeat the entire pitch set in each measure. Under each tuplet, there is only ever a

Table 3. Frequency of each pitch occurring on each "beat" of the tuplet, and their respective totals in the last column.

	1st	2nd	3rd	4th	Total #
B Doube-Flat	6	17	1	16	40
A Sharp	3	1	6	3	13
A Flat	12	11	18	5	46
F Double-Sharp	14	6	10	11	41

subset of the total pitches. Table 3 shows the distribution of the pitch set within the measures of the pattern, and also includes a total count of the occurrences of each pitch over the pattern. Not only are the pitches of the set not evenly distributed within the measures; they are not equally divided within the pattern. The differences in the totals for each note show how some notes are more common than others. This was not the case in Pattern 1–18, because each note was present in every measure. Compared to both an ideal pattern, and Pattern 1–18, Pattern 37–72 is much less regular. There is a clear tendency towards A Flats (46 instances) and an avoidance of A Sharps (only 13 instances). The irregularity also gives way to something like a pitch centricity. A Flat is the most common pitch in the cello voice, and that comparative increase in frequency might be something attended to by a listener. The idea of centricities in this pattern is discussed in Section 2.1 of this chapter. The change in pitch order/frequency regularity between Patterns 1–18 and 37–72 also influences the regularity of the set of intervals formed by the pitches in each pattern.

The intervals formed from a sequence of notes can be affected by the pitch regularity discussed above. Pitch sets define a set of possible intervals between the notes that make up the voice in a pattern. While what the intervals are is constant, the prevalence of some intervals can be affected by how the notes are arranged and repeated. Specifically, it can be affected by the kinds of contrast in regularity, discussed above, between Pattern 1–18 and 37–72, where some notes are repeated more often than others. It is possible to ask: do patterns that share a pitch set share similar amounts of the possible intervals created by that set? The prevalence of some intervals over others affects the way a pattern sounds, so it is interesting to see if this quality of a pitch set is consistent or irregular across patterns.

Table 4. Interval Frequencies of Pattern 1–18.

Interval	Frequency
0	1
1	34
2	21
3	15

Table 5. Interval Frequencies of Pattern 37–72.

Interval	Frequency
0	20
1	77
2	36
3	6

Table 4 and Table 5 show the frequency of the various intervals possible in the cello voice of Patterns 1–18 and 37–72, respectively. The intervals are counted in numbers of half-steps (e.g. 3= a minor third). While there are some complications to directly comparing them, as Pattern 37–72 is longer, the fact that the longer pattern has fewer minor thirds already tells us that the intervals a pitch set creates are not constant across each use of a particular pitch set. Interestingly, it seems that Pattern 37–72 is overall shifted towards smaller intervals, implying that this pattern should sound more tightly chromatic, which is in fact what is apparent when listening to the pattern. This contrast shows that a quantum such as pitch set can be shared between two patterns, but used in such a way to produce different effects. It also demonstrates how the regularity of how a pitch set is used can affect how a voice sounds, beyond the obvious ordering of notes. It is possible to take one set of pitches, and by changing the regularity of how they are repeated, create patterns with two different affects.

The final quantum of regularity, as it pertains to running-note voices within patterns, is slurs. There are two domains in which slurs can be regular or irregular: slur lengths, and for patterns with multiple slur lengths, ordering of the lengths. The cello voice of Pattern 37–72 only has slurs that are five notes long. The slurs are consistent over the whole pattern; therefore, the slurs are highly regular. The cello voice of Pattern 1–18 does not have the same degree of regularity. There is a set of different slur lengths that are used throughout the pattern. Table 6 shows the different slur lengths that are possible in

Table 6. Slur-length Frequencies of Pattern 1–19.

Slur Length	Frequency
3	3
4	8
5	5
6	1

this pattern and their relative frequency. What this chart shows is that the distribution of slur lengths is not even; some slur lengths are more common than others. Like with the locations under the tuplet of a pitch set, a pattern where the slurs are equally likely to be any of the possible values is more regular than one where the relative frequency of the slur lengths is random.

With this set of slur lengths determined, it is possible to answer the second part of the question

Example 20. Page 1 mm. 1-9, with slur-lengths highlighted in red.

above: are the slurs ordered in any kind of regular or predictable way? Example 20 shows the first half of Pattern 1-18. At first, it appears that there is some order to the slur lengths. The first slur is six notes long, the second is five, the next two are four, and the next two are three (highlighted in red in Example 20). At first glance, it might seem that there is some kind of regular contraction of the slur lengths happening over the course of the pattern. This would be a form of regularity, because the change in length would be consistent, and the direction of the change would be predictable. However, this process does not continue for the rest of the pattern. Even in just the few last measures of the excerpt in Example 20, it is evident that this contraction does not continue. The slur lengths for the rest of the pattern are ordered in a more or less random fashion, hinting at, but not quite following, a regular process. This is a recurring theme throughout PCF. Feldman seems to like hinting at order and regular processes just enough to imply, for short periods anyway, their existence, but never enough to actually establish them as a feature of a pattern.

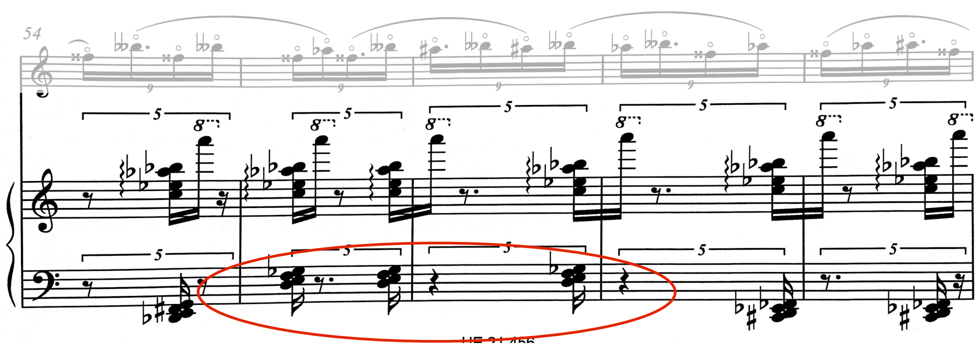
In conclusion, to restate and now answer the questions introduced at the beginning of this section: 1) Is the pitch set of the voice contained in every subdivision? As can be seen in the contrast between Patterns 1–18 and 37–72, there are examples of patterns where the pitch set is highly regular, found in every division, and highly irregular, used more or less randomly. 2) Is the pitch set of the running-note voice distributed regularly in the beat-possibility space of the tuplet/measure and pattern? As I showed in Table 2, discussing the cello voice of Pattern 1–18, even the comparatively regular cello voice of this pattern does not have an even and regular distribution of its pitch set under the tuplet/meter. 3) How do slurs affect the division of notes, and how are their lengths more or less regular? Table 6 shows how Pattern 1-18, which again, is more regular in many other ways, can have a random distribution of slur lengths. Finally, all of these properties of regularity can also have an effect on the timbre of a pattern. The respective regularity of two patterns with the same pitch set can affect the prevalence of certain intervals.

Next I discuss the pitch regularity of rhythmic impulses, which is influenced by many of the same factors that affected the pitch regularity of running-note voices.

1.1.2. Pitch Regularity of Rhythmic Impulses

Pitch regularity within rhythmic impulses is much easier to define, because the impulses themselves serve as an intuitive way to break up the material of these voices. Instead of using pitch-element number, meter/tuplet-element number, or slurs to create groups of notes to compare for consistency, the pitch sets of the individual impulses can be compared by using each impulse as its own subdivision of the voice it is in. For pitch use in rhythmic impulses, an ideal pattern would be made up of either impulses that all share the same pitch set and pitch-element number, or impulses which draw on multiple sets which are cycled or used in a predictable fashion. To demonstrate the ways in which the regularity of pitch in rhythmic impulses can be evaluated, I use examples from Patterns 1-18, 19-36, and 37-72.

Unlike in running-note voices, many rhythmic impulse voices in PCF are highly regular in the domain of pitch. The piano voice of Pattern 1–18 (excerpted in Example 20) is an example of a pattern where there is a rhythmic impulse voice that is highly regular in the domain of pitch. The piano voice uses the same pitch set and pitch element number across the whole pattern. The pitch set is {D, E-Flat, F-Flat}, and the pitch-element number is six. There is no variation in any feature of pitch across all of the rhythmic impulses in the piano voice of this pattern. In this way, the pitch use of the piano voice is highly regular.



Example 21. Page 3, mm. 54–58, with irregularity circled in red.

Pattern 37–72, however, is an example of what a rhythmic impulse voice can look like that is irregular in the domain of pitch. In the first half of this pattern, the piano voice is regular in the same ways as the piano voice in Pattern 1–18: the piano voice uses the same pitch set and pitch-element number for each of its impulses. This regularity does not persist though the whole pattern. Starting in measure 55, the LH begins varying its pitch set while keeping the same pitch-element number; see Example 21. This feature of Pattern 37–72 was discussed previously in Section 2.1 of Chapter 1, and an additional excerpt of this behavior can be found in Example 7. This variability means that the piano voice in this pattern is irregular in the domain of pitch set, but because the LH continues to only use sets of four notes at a time, it is regular in the domain of pitch-element number. There are a total of four pitch sets that the LH uses in this pattern, and there is no apparent organizing principle as to why they were chosen, or the order in which they are found, thus further increasing the perceived irregularity of this quantum.

With the factors that influence regularity of pitch, in running-note and rhythmic impulse voices defined, it is possible to discuss some of the performance utility of defining regularity.

1.1.3. Performance Utility

As is the case for all the different ways that regularity determines how quanta are used, understanding how regular the pitch of a voice is is inherently useful for the performers. It can influence everything from how much attention something needs during preparation, to how a performer might choose to practice a particular passage. It can be used to reduce the cognitive load during rehearsal and in performance. For example, as discussed above, Pattern 37–72 has some heightened irregularity in the LH of the piano. This irregularity is not found in the previous two patterns, despite the fact that they otherwise share many of the same pattern quanta. By knowing which of these patterns is or is not regular, the pianist might be able to simplify their preparation of the piece. An understanding regularity can influence how broadly the exercises created in Chapter 3 can be applied. If a quantum is highly regular, any practice of that quantum is equally beneficial for wherever it appears in a pattern. However, if a quantum is not regular, then it is difficult to build a general exercise around that quantum because its value is not consistent.

In the next section, I tackle the issue of regularity in the domain of rhythm.

1.2. Rhythmic Regularity

Rhythmic regularity is much more elusive in PCF than pitch regularity. Earlier in this chapter, I stated that it is ironic that, in a piece with “patterns” in its name, so little of the material in the piece is actually a regular pattern. This is perhaps most apparent in the domain of rhythm. Aside from a few conspicuous patterns, which are discussed in Section 3 (Operations), rhythm is rarely ever truly regular in the patterns of PCF. When it is, it is often because it is static. Despite this, it is still analytically interesting to see how quanta can be used to create regular patterns, and how the way rhythm is used in a pattern voice can hint at, but ultimately not achieve, actual regularity.

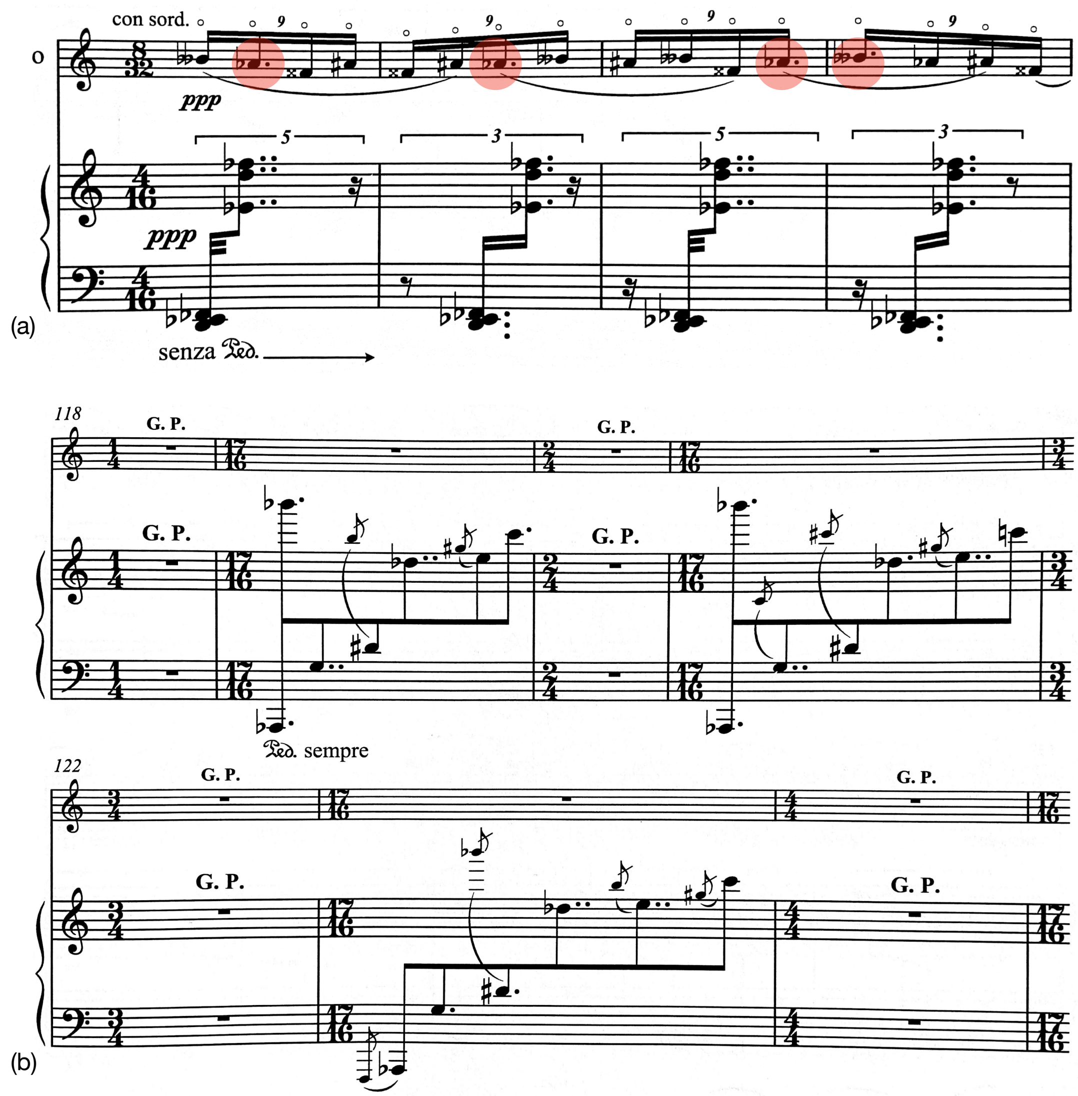
The idea of an ideal pattern, with perfectly stable repetition or evolution, is useful in this assessment, just as it is in the domain of pitch. As in the previous section, ideal patterning is the measure against which what is actually written is compared. Much like the regularity of pitch in PCF, it is helpful to discuss separately the regularity of rhythm in running-note voices and voices comprised of rhythmic impulses. Because regularity of rhythm is more elusive, and the number of rhythmic quanta is greater than that of pitch, this section draws on a larger number of excerpts from various patterns within the several-hundred-measure section that serves as the topic of this chapter.

There are several questions that pertain to the regularity of rhythm in both running-note voices and rhythmic impulse voices: 1) How regular are the elongations? 2) How regular are the interruptions in running-note voices that have them? 3) If there are multiple impulse kinds in a pattern, are they ordered in a regular way? How regular are the rests between impulses? and 4) How regular is the position of an impulse within a measure or tuplet? First, I address the questions pertaining to running-note voices.

1.2.1. Running Notes

One of the ways that the rhythm of running-note voices can be made more or less regular is in the way that elongations are used within that voice. A perfectly regular or predictable pattern in this domain would have all of the elongations at the same place in the measure, or, there would be a predictable transformation that moved their location within a measure or tuplet around. To help demonstrate the ways that elongation regularity varies, I have chosen two patterns to discuss in greater detail: Patterns 1–18 and 118–126.

The cello voice of the two examples in Example 22 both use dotted notes to create elongations. It is possible to describe the regularity of both of their elongations. The excerpt in Example 22 (a) shows the opening of the now familiar Pattern 1–18. This pattern has an elongation in the running-note figure in the cello voice. This elongation is created by the dot found on one of the 16th notes under each tuplet



Example 22. (a) mm. 1–4 with elongations circled in red, and (b) mm. 118–124.

(highlighted in red). The location of this dot within the tuplet at first appears to be regular. It starts on the second note, then is on the third, followed by the fourth, and then cycling back to the first. However, this regular cycling though the positions under the tuplet does not continue. After these first four measures, the location of the elongation becomes unpredictable. In that sense, elongation location is irregular in this pattern.

However, if elongation location was truly random, one might expect an even distribution of elongations at each spot under the tuplet. This would be a kind of global regularity, because there would be an overall even distribution of elongation locations. This is not the case in Pattern 1-18, and in fact, there is a correlation between pitch and elongation in the cello voice of this pattern. Table 7 details the number of times each note of the cello voice pitch set is elongated. It shows that there is a strong correlation between A-Flat and elongations. In other words, the elongation within any given tuplet is most likely to be found at the same place as the A-Flat under that tuplet. In this way, the use of elongations in this pattern is not actually random.

Table 7. Pattern 1–18 pitch-elongation frequencies.

Pitch	# of Elongations
B Double-Flat	2
A Flat	12
F Double-Sharp	4
A Sharp	0

The excerpt shown in Example 22 (b*)* shows how irregularity of elongation position can be used to create variation when other quanta are consistent. In this example, ignoring the grace notes for a moment, the pitch set is identical for all three 17/16 measures. The variation between the three six-note motives in these measures comes from the way that the elongations (dots) are moved around the six 8th notes. This example shows how, when there is little regularity within one quantum, Feldman is able to maintain a sense of regularity or cohesion for the whole pattern by keeping other quanta static.

Example 23. Page 9, mm. 160–165.

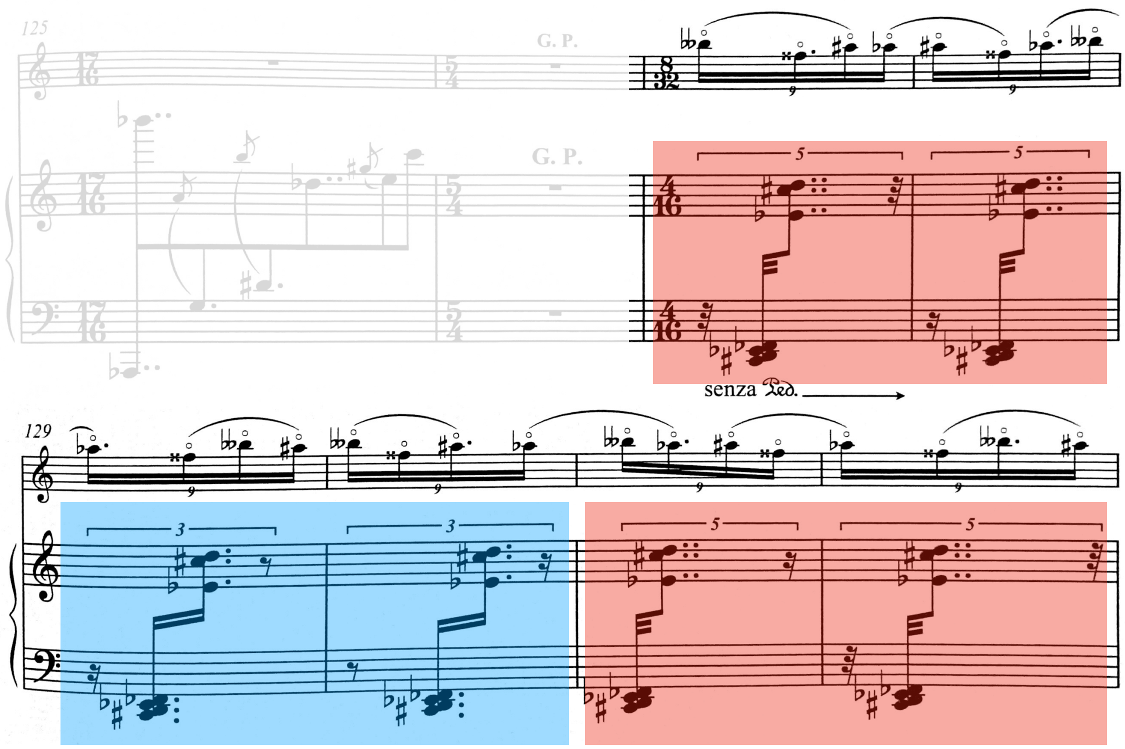
Sometimes the overall regularity of a voice is the product of irregularity of multiple quanta. The excerpt of Pattern 145–180, shown in Example 23, is an example of how rhythmic interruptions in running-note voices can be used to heighten an already irregular pattern. In the cello voice of this pattern, there are random 8th and 16th rests interrupting the running notes of the cello voice (circled in red). This pattern is otherwise like some of the previous patterns with running notes in the cello voice and rhythmic impulses in the piano voice that we have encountered up to this point. Like Pattern 37–72, the pitch-element number is 4, and the pitch set is consistent over the whole pattern. Also like Pattern 37–72, each note of the pitch set is not present under every tuplet or cycle. This already makes the cello voice of this pattern less regular than that of other patterns with running-notes such as Pattern 1–18 or Pattern 19–36. The interruptions in Pattern 145—180 are not regular, and are not even consistently present over the course of the whole pattern. The interruptions themselves are irregular, and also have the effect of adding to the overall irregularity of the cello voice in this pattern.

1.2.2. Rhythmic Impulses

The most obvious dimension in which the regularity of rhythmic impulses is more or less regular is in the variation and ordering of impulse kinds. When a pattern has an impulse kind number of 1, that pattern is totally regular in that dimension, because every impulse looks like every other impulse in the pattern. However, when an impulse kind number is greater than 1, the order of those impulses is something that can be made more or less regular.

Returning to Pattern 1--18, excerpted in Example 22 (a), we see a piano voice with an impulse kind number of 2. The two impulses are the impulse under the 3-tuplet and the impulse under the 5-tuplet. These two impulses are not ordered in any specific fashion. This already means that there is certain irregularity to the impulse kinds in this pattern. In an ideal pattern with multiple impulse kinds, those different impulses would alternate in some kind of systematic fashion. The two impulses are also not present in equal quantities. There are seven 3-tuplet measures and eleven 5-tuplet measures. This means that there is not a perfectly even distribution of impulse kinds, as would be expected if the distribution was truly random, i.e. determined by a coin flip. In this sense, there is also a global irregularity to the impulse kinds, because they are not equally represented in the pattern. The regularity of impulse kinds can also be tracked across different patterns.

The ratios of different impulse kinds are often not consistent across patterns that otherwise share pattern quanta. As an example, the piano voice of Pattern 19–35 shares all the same rhythmic impulse quanta as the piano voice of Pattern 1–18. Despite this, the order and ratios of the two impulse types is not the same between the two. In the second pattern, there are six 3-tuplet impulses and twelve 5-tuplet impulses across the whole pattern, versus the seven 3-tuplet measures and eleven 5-tuplet measures of Pattern 1–18. It is interesting that, while not totally regular, there is a preference for the 5-tuplet in both patterns. While not exactly the same, and therefore regular across patterns, they are similar, implying that the frequency of impulse kinds across similar patterns is not totally random.



Example 24. Page 7, mm 127–132.

Sometimes the ordering of the impulses in voices with an impulse kind number higher than 1 can hint at a regular ordering, but then devolve back into a more random ordering. The excerpt of Pattern 127–114 found in Example 24 shows the first six measures of the pattern. This pattern has an impulse kind number of 2 in the piano voice. The impulses are rhythmically the same as the impulses in the piano voices of Patterns 1–18 and 19–35. The first six measures of this pattern suggest there might be a regular order to the impulses, with two 5-tuplet impulses followed by two 3-tuplet impulses and then a repetition of that cycle (highlighted in red and blue). However, after the first six measures of the pattern, this regular ordering stops, and there is no longer a predictable order to the impulses. This excerpt shows how Feldman sometimes hints at a regular ordering, but then lets that implication melt away before it becomes too heavily established in the listener's ear. Interestingly, Pattern 127–114 also shows a preference for 5-tuplets over the 3-tuplets like the previous patterns with this specific set of impulse elements.

The other component of rhythmic impulse voices that can be more or less regular is the rests that separate the impulses themselves. The spaces between the impulses can be regular and all the same length, or they can be produced by a set of different rests that can occur in regular orders, or randomly. Both types are present in the patterns of PCF. The regularity of rests is similar to the regularity of impulse kinds. To demonstrate what this looks like in practice, I have chosen to examine Pattern 73–77, where both the cello and piano voices are composed of rhythmic impulses with rests separating them. In this pattern, the rests are used to create a difference in regularity between the cello and piano voices.

Example 25. Page 4, mm. 73–77.

Pattern 73–77 (Example 25) is a short pattern where both the cello and piano voices are made up of impulses. However, there is a contrast in regularity between the two. In both voices, the only variable that changes is the rest length between the intervals. The length of the impulses in the cello is always a dotted 16th note, and in the piano, it is always a 16th note with a grace note before it. The tuplet numbers in both voices stay the same throughout. In other words, rhythmically, the voices share many of the same quanta. Where they differ is in the total number of rests each voice uses, and how those rest are arranged. In the piano voice, the rest length is either a dotted 8th (or an 8th plus a 16th), or a 16th. The order of these rests in the piano is perfectly regular. The lengths alternate starting with a dotted 8th (this would be a perfect location for a short-long simplification). This behavior is regular because it is consistent and predictable. In the cello voice, there is not the same kind of regularity. The set of possible rests between the impulses in the cello voice is distributed in a seemingly random order. The regularity of rests can also influence other aspects of regularity, such as the impulse location in a meter/tuplet.

The location regularity of a repeated impulse in a tuplet/meter is the mirror-universe version of rest regularity. For example, it is possible that the rests in a rhythmic impulse voice are all the same length, and therefore are very regular, but the rest length causes the position of the impulses in the measures or under the tuplets to move around and be irregular. Ultimately, it is the rests that are influencing both rest and location regularity, but one is regular and one is irregular. Alternatively, sometimes the rest lengths are not all the same, but because the impulses are also not the same in a sympathetic way, the location of the impulses is always in the same part of the measure/tuplet. In this hypothetical, the rests are irregular themselves, but they facilitate a regularity of impulse position. Example 26 shows two excerpts that help illustrate this kind of regularity.

(a)
(b)

Example 26. (a) mm. 209–212 and, (b) mm. 295–300.

The excerpt of Pattern 207–216 shown in Example 26 (a) exemplifies how sometimes the impulses of a rhythmic-impulse-voice are not all found on the same beat of a measure. In the piano voice of this excerpt, the various rest lengths shift the two-element impulse around to unpredictable beats of the measure. Sometimes the impulse is on the first beat of the tuplet, but sometimes it is on the second eighth, or the last eighth of the tuplet. In Example 26 (b), the opposite is true. In the cello voice, the duration of the one-note impulses are correlated with a particular rest length, such that each impulse is always on the same beat of the measure. This excerpt is far more regular than the previous in this dimension, because the location of the impulses is predictable.

1.2.3. Performance Utility

If the previous chapter established its performance utility by demonstrating the ways in which patterns can be dissected into smaller parts, which could then be defined and practiced, the chapter on quantum behaviors establishes its performance utility by elucidating the larger structures these pattern quanta can construct, and the pitfalls of assuming that they result in regular, predictable patterns. For no behavior is this more apparent than with regularity. Being able to identify a regular pattern is a skill employed by musicians for all kinds of repertoire. Understanding how to decompose patterns and then see how the products of decomposition behave is just as critical for Feldman as for any other composer. Understanding how pitch and rhythmic quanta are used in more or less regular ways is just another type of information that can be used to streamline practice.

1.2.3. Conclusion

Understanding what makes a pattern in PCF is not simply about defining the set of quanta that describe a particular section of the piece. It is also about understanding the ways those quanta are used that is pattern-like. How are the quanta repeated, cycled, or changed over time? Because the answer to those questions is almost always unclear, it is necessary to have a way of thinking about repetition of quanta that allows for a degree of variability in how quanta are repeated. This is the utility of regularity. The regularity of repetition can exist on a scale, while still tying something to an ideal that is perfectly regular. It provides a practical way to talk about the patterning of things that are not actually very well-patterned. Regularity also foreshadows the other behaviors discussed in this chapter, which are sometimes emergent properties of an irregular phenomenon, and sometimes the ways that the rare "perfect" regularity is created. As an example of the former, centricity is the next behavior discussed in this chapter.

2. Centricity

Centricity is in many ways correlated to regularity. It can be created both in instances where a set of quanta are highly regular, implying a privileged position for one of the quantum values, or when the quanta are irregular, and the irregularities are used to make one pitch or rhythm more prominent than others. In the domain of pitch, centricity is usually created through irregularity of quanta besides the quantum of pitch set itself. Unlike the pitch centricity of tonal idioms, the pitch centricity that exists in PCF is not an inherent property of a pitch set, but more simply, is a product of either the irregularity in how that pitch set is used, or the irregularity of the associated rhythmic quanta. In other words, some notes are more common than others, or are emphasized in some other way that makes them stick in the ear of the listener more prominently.

In the domain of rhythm, the idea of centricity is a way of thinking about how certain impulses in patterns with an impulse kind number greater than 1 are often not evenly composed of all the possible impulse kinds. And similarly, how patterns with impulses may draw on certain rests between impulses more than others. The use of the term centricity to describe the preference of some impulse kinds over others is perhaps unusual, but because pitch and rhythmic centricities are both created the same way, I have grouped them together. To illustrate these concepts, I examine how Feldman creates a pitch centricity on A Flat in the first three patterns of the piece (Patterns 1–18, 19–36, and 37–72) , and how those same three patterns have a rhythmic "centricity" around 5-tuplet impulses.

2.1. A Flat Centricity of Measures 1-72

The pitch centricity around A Flat in the first three patterns is present in the cello voice of each pattern. The first, Pattern 1-18, uses one quantum to create this centricity: elongation. In this pattern, each tuplet/measure contains all four pitches of the pitch set. Because of this, there is no possibility to accentuate one pitch by repeating it more times than the others. While it might be possible to create an impression of prominence by always putting the same note on a strong beat, or in the same position, the notes in the cello voice of this pattern are randomly distributed throughout the beats of each tuplet. Instead, the centricity is created by the dotted elongations that are found once in each measure. Table 8 is a table showing the relative frequency of elongations on each pitch of the pitch set for the cello voice of Pattern 1-18. This table was first seen in Section 1.2.1 Chapter 2. Of the 18 total elongations in this pattern, one for each tuplet/measure, 12 of them are on an A Flat. This correlation, though not perfect, is enough to create a noticeable prominence of the A Flat. Because this pitch is often elongated, in the ear of the listener, there is an implication of some kind of centricity around A Flat.

Table 8. Pattern 1–18 Pitch-Elongation Frequencies.

Pitch	# of Elongations
B Double-Flat	2
A Flat	12
F Double-Sharp	4
A Sharp	0

The centricity around A Flat is carried through to the next two patterns. In Pattern 19–36, it is created in the same way as it is in Pattern 1–18. The whole pitch set is used each tuplet/meter, so there is no room to create a centricity by varying the occurrence of each pitch. Instead, it is created by a similar use of elongations to disproportionately elongate the A Flat. Pattern 37–72, also has the A Flat centricity; however, its pitch centricity is created in two different ways.

In Pattern 37–72, the pitches are far less regular than in the previous patterns. Each tuplet/meter does not necessarily contain each pitch of the set. This creates the opportunity to vary the relative occurrence of each pitch over the course of the pattern. In other words, some notes are repeated more often than others. The relative irregularity of Pattern 37–72, compared to the previous two, creates a novel space for the A Flat centricity to be reinforced. Table 9 shows the relative frequency of each pitch over the course of Pattern 37–72. A Flat is the most common pitch in this pattern. Repeating a particular pitch from a set more often than the others is a simplistic, but effective, way to imply a primacy or centricity to

Table 9. Frequency of each pitch occurring on each "beat" of the tuplet, and their respective totals (Pattern 37–72).

	1st	2nd	3rd	4th	Total #
B Doube-Flat	6	17	1	16	40
A Sharp	3	1	6	3	13
A Flat	12	11	18	5	46
F Double-Sharp	14	6	10	11	41

that pitch within the set. In addition, this pattern also uses elongation to more often accentuate A Flats, just like the previous two patterns. The total effect of these two features of this pattern is to create a strong sense that A Flat is a privileged note in the four-note set that these three patterns all share. The fact that this centricity is a property for all three patterns that use this pitch set, up to this point, further enhances the sense that A Flat is a central pitch.

Rhythmic centricity can be created in a similar way, as is the case with the 5-tuplet centricity of Patterns 1–18 and 19–36.

2.2. 5-Tuplet Centricity

Just as in the last example of pitch centricity in Pattern 37–72, repetition is used to create a centricity around a particular impulse kind when a rhythmic impulse voice has a kind number greater than 1. Examples of this can be found in Patterns 1–18 and 19–36. Both of these patterns share a similar piano voice with rhythmic impulses. In the piano voice of both patterns, there are either impulses under a 5-tuplet, or impulses under a 3-tuplet (for a more complete description of how the impulses work in Patterns 1–18 and 19–36, see Chapter 1 Section 3.3.2). These two impulse kinds are not present in equal quantities in either pattern. In both patterns, there is a greater number of 5-tuplet impulses than of 3-tuplet impulses. Because there is a greater number of one kind of impulse in both patterns, that specific impulse can be thought of as a centricity of those piano voices. In other words, because one is heard more often than the other, it might take on a privileged position in the ear of the listener. The 5-tuplet centricity of Patterns 1–

18 and 19–36 was also discussed in Section 1.2.2 of this chapter during the discussion of rhythmic impulse regularity.

2.3. Performance Utility

Just as with regularity, knowledge of the types of centricities discussed above can aid in a performer's preparation and performance of PCF. The discovery and description of features such as the A Flat centricity in the first three patterns of the piece are of particular interest. The running-note figure in these three patterns is especially gnarly. It goes by quickly, and there is no opportunity to check in with the pianist. It all just needs to be there. Knowing that there is a central note to this figure might help a cellist keep grounded in the otherwise disorienting string of notes. In addition to the implication of regularity discussed in the previous section, this passage begins to seem more definable, and less intimidating to learn.

3. Operations

Operations are behaviors where certain quanta are altered in a predictable or systematic way, either over the course of a pattern, or from one pattern to the next. The specific operations I discuss in this section are pitch sets repeated in retrograde, octave displacement, additive operations where rhythms become elongated over time in a systematic way, and regular alternations. Like centricity, operations are sometimes a product, or at least a correlate, of regularity. For example, an operation might be the source of a regular change, increasing the regularity of a pattern. Or, an operation might have the effect of making two similar patterns less like each other, thus decreasing the regularity of how a set of quanta are used. There are almost certainly more formal-like operations than the four I have mentioned above in PCF, but I have chosen to focus on the four that are apparent in the first several patterns of the piece, in order to show that operations exist, and to begin to elucidate how they operate on pattern quanta.

3.1. Retrograde

The first overt operation in the piece is found in the cello voice of Pattern 19–36. This pattern is very similar to Pattern 1–18. They share almost all of the same quanta. The only difference that really sets them apart as two different patterns is the pitch set change that happens in the piano voice (this junction was discussed in Section 2.1 of Chapter 1 and is shown in Example 5). This in turn means that both patterns share the same four-note pitch set for the cello voice. Not only do both cello voices share a pitch set; they are operationally related. In Pattern 19–36, the material in the cello voice is almost a perfect retrograde statement of the cello voice of Pattern 1–19. Starting in measure 20, the cello voice begins repeating the notes of the cello voice spanning measures 2–18, but in reverse. The one exception is one measure found nine measures into the second pattern. This measure is the same as the corresponding measure (measure 10 of the previous pattern), but it is stated in its non-retrograde prime order rather than in retrograde. This is an interesting "error" in the retrograde operation that deserves a closer examination.

One explanation for why this one measure is not reversed is that Feldman wanted to preserve a B Double-Flat unison at the midpoint of Pattern 19–36. In Pattern 1-18, there is a B Double-Flat unison at the junction of measures 9 and 10. This is notable because this is the midpoint of the pattern, and because there are no other instances where the same note is played twice. Together, this unison's novelty and central location in the pattern make it conspicuous to the listener. If Feldman wants to preserve this unison at the midpoint of Pattern 19–36, where the sequence is in retrograde, there is going to be a problem. While the retrograde preserves this unison, there are other differences between these two patterns that result in the unison occurring a measure later than the actual midpoint of Pattern 19–36.

Unlike Pattern 1–18, Pattern 19–36 starts with a measure of rest in the cello voice. After that measure, the retrograde string of notes begins. This means that the B Double-Flat unison is going to happen a measure later than in the previous pattern, because the whole cello voice sequence is shifted back a measure. And because the patterns are both the same length, the unison will no longer occur at the midpoint of the pattern. The measure which is kept in its prime form in Pattern 19–36 is the 9th measure of that pattern (10th measure of the Pattern 1–18), and it, and its surrounding measures, are composed in such a way that, when that one measure is flipped, it moves the B Double-Flat unison to the previous measure. This preserves the location of the B Double-Flat at the midpoint of Pattern 19–36. See Figure 1 for an annotated diagram. Red arrows show material that is copied verbatim and the blue arrows show retrograde relationships.

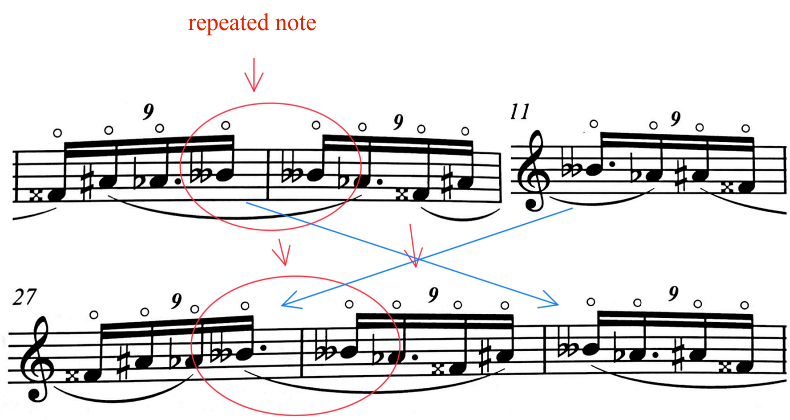
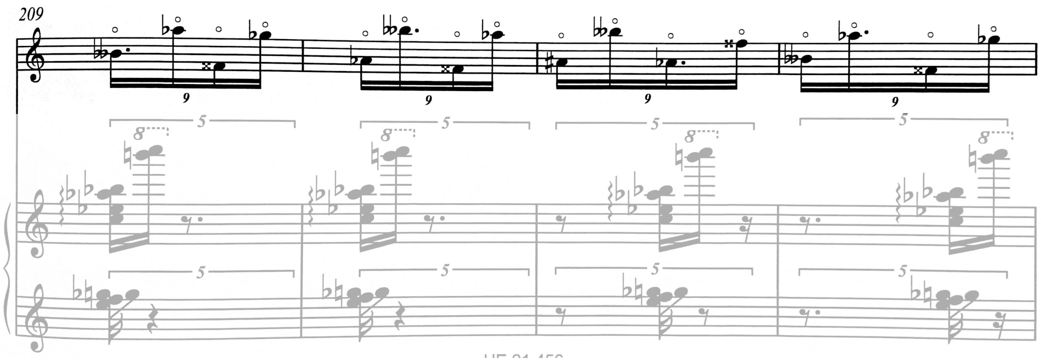


Figure 1. Diagram Showing Movement of B Double-Flat Unison.

As a performer, this is interesting for several reasons. First, it is helpful to know that new material is directly derived from something else that you have previously played. While the fact that the sequence at a pattern level is rather random might make this less audible to the audience, that this random sequence is actually related to another sequence in a way that is highly ordered is interesting to know. Second, because Feldman went out of his way to preserve this B Double-Flat unison at the midpoint of both patterns, it implies that this feature is both deliberate and important. Certainly it is something that a performer should be aware of, and perhaps, something that should be given some thought as to ways in which it might be made more obvious to the audience.

3.2. Octave Displacement

Another one of the first operations that is overtly apparent in PCF is octave displacement. Octave displacement defines the relationship between the cello voice of Patterns 1-18 and 19-36, and Patten 37-72. Pattern 37-72 uses the same pitch set in the cello voice as the two previous patterns. However, the pitch set is transposed up an octave in Pattern 37-72. Like the use of a retrograde sequence in Pattern 19-36, this particular octave displacement is happening across patterns rather than within. While the exact sequence of notes in the cello voice is not preserved, the general set used to compose that sequence is the same save for the octave displacement. This is definitely the kind of transformation that would be obvious to an audience.



Example 27. Page 11, mm. 209–212.

Another example of octave displacement of the same pitch set can be found in Pattern 202–216 (excerpted in Example 27). This pattern uses the same pitch set {B♭♭, A♭, Fx, A♯} in the cello voice as Patterns 1–18, 19–36, and 37–72, but octave displacements are used to double the total pitch-element number. The operation expands the total number of notes in a way that is predictable and apparent. This is an example of octave displacement happening at both an inter-pattern level, and an intra-pattern level. The displacement is not just an operation used to generate new material from one pattern to the next, but is a feature of what is happening within the pattern itself.

3.3. Additive

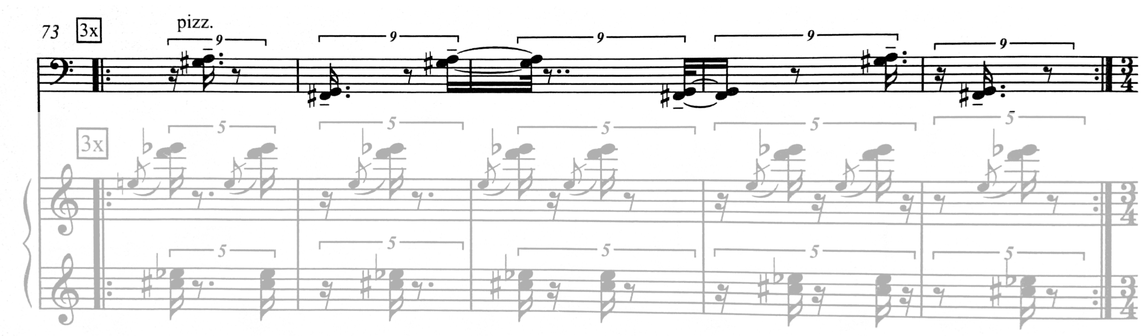
Additive operations are operations where the value of one of the quanta increases over time in a measured, predictable way. The first example of this kind of operation can be found in Pattern 78–89. In this pattern (Example 28), there is a rhythmic impulse in both voices. In the piano voice, this impulse is followed by a rest, but in the cello voice, this impulse is tied to another note (circled in red). The length of that note increases by one 16th note each time the impulse is repeated. Therefore, in this example, the quanta of impulse length grows by a predictable amount each time it is repeated. This pattern has been discussed previously because of the interesting way that Feldman uses changes in meter to create the ever-longer impulse lengths. Because meter changes are used to create this additive process, it is also possible to consider the meter itself as a quantum that can be altered through an additive operation.

Example 28. Page 5, mm. 78–89, with Additive Measures Circled in Red.

3.4. A|B|A|B Alternation

Alternation is found when there is a pattern with either multiple impulse kinds or a pitch set with a limited number of elements. In these patterns, it is often that the sequence of those elements is random or not highly regular, but sometimes, the sequence is a regular alternation of the elements. This can look like a voice alternating between high and low pitches, or a voice alternating between two different rhythmic impulses. Generally, this might be considered a measure of how regular a voice is, not an explicit operation. However, because alternation is both a common way of creating some regularity in a voice, and because a regular alternation is audible to the audience as a specific behavior, I have included it as its own operation type.

One example of this kind of operation working on pitch can be found in Pattern 73–77, (Example 29). In this pattern, the cello voice alternates between high and low pitches in a regular and predictable



Example 29. Page 4, mm. 73–77.

way. Sometimes an alternation can be implied for only a portion of a pattern before devolving into something more irregular. For example, the piano voice at the beginning of Pattern 19–36 has a rhythmic impulse kind number of 2 in the piano voice. For the first seven measures, these two impulses alternate back and forth, but after the seventh measure this pattern is broken, and the sequence of the two impulses becomes more random.

4. Conclusion

In the first chapter, the goal was to describe the smallest topographical features of patterns. This quantized approach produced data describing what patterns sounded like at a local level, but did not address what patterns did to sound *patterned* in the first place. If a pattern is a regular repetition of some element over time or space, then the patterns of PCF are not very good patterns. They are often irregular, and infrequently repeat quanta in an obvious way. And yet, PCF is composed of intuitive and apparent patterns. Even when a quantum is not repeated or altered regularly, a listener can still attend to it as a pattern, so long as it is related to something pattern-like. The concepts of regularity, centricity, and operations describe the pattern-like things that the patterns of PCF do.

Regularity is the organizing principle. It defines an ideal pattern, and then provides an assessment of how closely a pattern hews to that ideal. As discussed in this chapter, pitch and rhythm run the gamut from highly regular to totally unpredictable. But even the unpredictable behaviors of patterns suggest a space where regularity can be achieved. Centricities and operations then become the consequences of regularity, or the lack thereof. When something conspicuously patterned does arise, centricities and operations can be the frameworks for describing that behavior.

This chapter is not exhaustive, but it does demonstrate a particular utility of the quantized approach. It allows the definition of behaviors in the first place. Quantization also enables the work of the next chapter. Like behaviors, the concepts in Chapter 3 are a result of conceptualizing patterns as composed of discrete elements. The behaviors of those discrete elements discussed in this chapter help elucidate the concepts of “archetype” and “variation” that are discussed in the next chapter. These concepts, and the behaviors discussed in this chapter, begin to address the question that has been the goal of my document: how to make this impracticable piece practicable.

Chapter 3: Archetypes and Exercises

The overarching analytic question of my document has been: what can and cannot be a pattern in PCF? Chapter 1 demonstrates that it is possible to create a framework for isolating and describing the topographical features of each pattern through quantization. These pattern quanta are conceptual units, which do not describe the specific patterns of PCF, but rather, capture how a PCF-like pattern can be described. Chapter 2 demonstrates that the quanta of the first chapter can be used to elucidate how patterning works in PCF through the concepts of regularity, centricity, and operations. These concepts concern actual pattern quanta values, but not with the intention to describe what patterns are. Like the first chapter, the goal is not to circumscribe all the patterns of PCF, but to describe how a PCF-like pattern is patterned. Neither chapter has yet described in a holistic way what the actual patterns in PCF are. Again, what can and cannot be a pattern in PCF? What did Feldman actually write? That is what is discussed in Chapter 3; here, the concept of archetypes is the analytical insight that elucidates this question.

An archetype is a plain-language description of a pattern that can be broadly applied to describe a whole set of patterns within PCF. An archetype is not as specific as the set of quanta that might define a pattern, but the plain-language description of archetypes is made possible by the power that quantization has to reveal similar characteristics hidden behind complex notation. Each broad archetype contains a subset of patterns that do share a set of pattern quanta; and these subsets of highly similar patterns are variations on the overarching archetype. This taxonomy of archetype and variation is the ultimate goal of the quantum approach detailed in Chapter 1, and its payoff is how it points towards an answer to the overarching pedagogical question of my document, namely, how to make an impracticable piece practicable.

This question is the driving force behind this whole project. Archetypes and variations both have actionable insight for performance preparation. They reflect the limited set of features that exist within PCF, and provide simple descriptions of those features. Those features can be practiced outside of their original context in order to make the whole set of patterns contained within that archetype easier. The work of the previous two chapters has provided a language and framework to define pattern archetypes in PCF, and it is possible to use the generalizations the archetypes make to reduce the 1650-odd measures of the piece into a limited —and crucially, an understandable— set of like patterns. The arrival at this realization has been hinted at in the various "performance utility" sections spread throughout the previous two chapters.

In chapter 3, I not only define a set of archetypes, but demonstrate how that set of archetypes can generate exercises to make PCF easier to understand and play. Disclaimer: at the time of writing this document, I have not performed or prepared this piece. That being said, this approach is not novel, but deeply rooted in what I have learned throughout the pedagogical and theoretical work of my doctoral studies. A tremendous debt is owed is to the accrued knowledge of my teachers, and the work they inspired me to undertake. Scales, arpeggios, etudes, and exercises are all studies in archetypes. They are the archetypes of a particular canon of "classical" music. Despite the often formal language and theoretical baggage my document contains, my goal is simply to expand traditional pedagogical tools by adapting them to the archetypes of a piece, so far, outside of that canon.

The exercises in this chapter owe a debt to the pedagogical concepts of Fred Sherry's *A Grand Tour of Cello Technique*, which, more than any other technique book I know, makes it a priority to apply the features of 20th century music to familiar string pedagogy; and to my teacher and advisor at Indiana University Jacobs School of Music, Peter Stumpf, who taught me to how to construct methodical exercises from intuitively extracted features of the music at hand.

This chapter is divided into three sections. The first discusses a few examples of the archetypes in PCF. This is done using pattern excerpts to demonstrate what archetypes look like, and a few excerpts showing how the plain-language descriptions of the archetypes can group together sets of patterns that can be thought of as variations. Second, I discuss in greater detail one archetype (Archetype A), and the specific quanta that define two of its prominent variations. Finally, I use that detailed discussion of Archetype A to demonstrate how exercises can be created to aid in the preparation of the piece. Those exercises addresses the pitch and rhythm challenges this archetype presents.

1. Pattern Archetypes

In order to demonstrate the range of archetypes that exist in the piece, I have chosen two archetypes that apply to a large number of patterns in PCF. These two archetypes are not the only two that could be generated from the set of patterns in the piece; they are merely selected to show the range of patterns that can be described as part of an archetype. I am not claiming that every pattern in PCF is part of an archetype, rather, that the quantum work of the first chapter allows for the recognition of archetypes that can be applied to a significant portion of the patterns in PCF. The two archetypes I describe in this chapter are Archetype A and Archetype Z. In plain language, Archetype A describes any pattern where there running notes in the cello voice and a series of rhythmic impulses in the piano voice. Archetype Z describes any pattern where there are chords in the cello and/or the piano parts that are slow or separated enough as to not sound like a series of rhythmic impulses. The first I have stylized as Archetype A because it is very common towards the first half of the piece, and the second I have stylized as Archetype Z because it is prominent towards the end of the piece. Because there is a more in depth discussion of Archetype A later in this chapter, let us first turn to the overview of Pattern Archetype Z. Archetype A and Z can help us better understand what archetypes are, and how they describe what is written in PCF.

1.1. Archetype Z

Archetype Z is defined as a series of chords in the cello and/or piano voice(s) that are too slow or isolated to read as a series of impulses. There are around 17 patterns that fit this archetype, representing about 20% of the total measures in PCF. These are approximations, as there are some edge cases, or highly irregular patterns that I have not included in this count. Those 17 pattern can be further divided into three clear variations, or interpretations, of the main archetype. The chart shown in Table 10 provides a list of the 17 patterns of this archetype, grouped by the three main variations. Each set of patterns in the

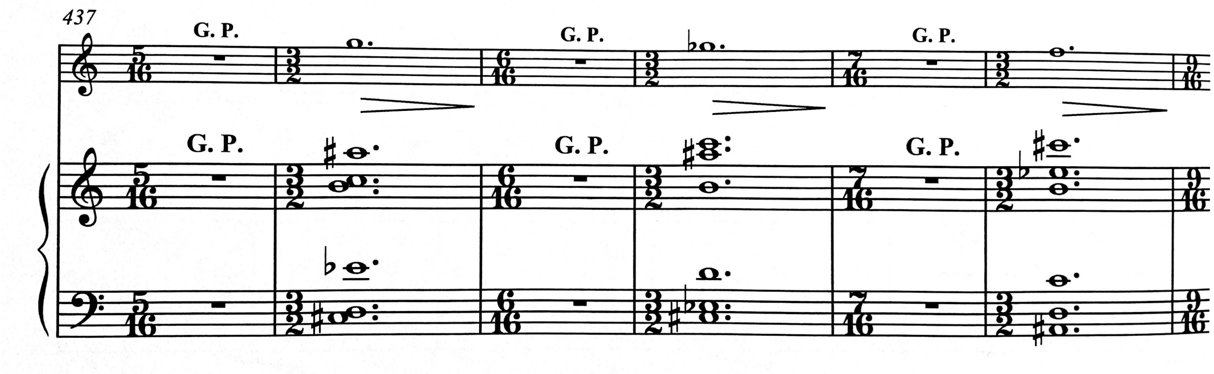
Table 10. Patterns that fit the Variations of Archetype Z.

Variation 1	Variation 2	Variation 3
271–277	433–469	554–592
280–286	1081–1094	593–612
424–430	1239–1247	622–648
1009–1017	1316–1323	829–864
1485–1608	1395–1403	865–881
	1635–1656	919–920

three variations contain a high degree of shared quantum values for at least a few salient quanta. I have chosen a pattern from each variation of Archetype Z in order to show how the plain-language definition of the archetype fits the material of PCF, and to show how the quanta of Chapter 1 can be used to describe variations.

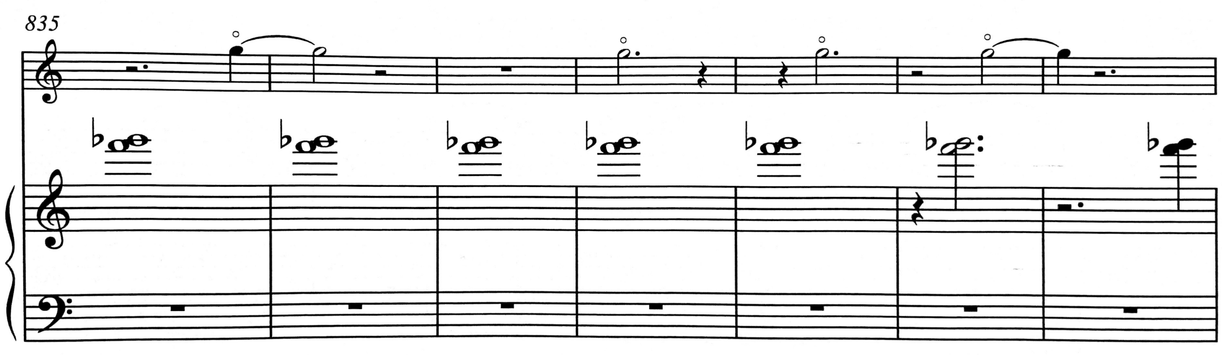
Example 30. Page 15, mm. 280–286.

Pattern 280–286 (Example 30) is one of the first patterns in the piece that fits Archetype Z, and is also an example of Variation 1 of this archetype. Its adherence to the simple description of that archetype is clear just at a glance. The pattern is a series of chords in the piano part that are evenly spaced and composed of only one impulse element each. Because of their spacing, and the regularity of that spacing, the chords do not sound like the frenetic set of impulses, but rather, they sound like a set of stand-alone events. The particular set of features that distinguish this pattern, and all the other patterns of Variation 1, are the solo voicing, the variability in the pitch set, the relative regularity of the pitch-element number, and the regularity of the impulse lengths. These particular quanta values are not necessary for a pattern of this archetype, but are what define the patterns in Variation 1.



Example 31. Page 22, mm. 437–442.

Pattern 433–469 (excerpt shown in Example 31) is the first example of Variation 2 in the piece. Despite its differences to Pattern 28–286, its adherence to the plain-language description of Archetype Z is just as clear. In this pattern, there are chords played by the cello and piano in a unison voicing. These chords are all made up of a single impulse element, which is long enough, and separated from its surroundings enough, as to not sound like a sequence of impulses, but stand-alone events themselves. Even though the particular quantum values are different from the Variation 1 example pattern, Pattern 433–469 still fits the basic description of Archetype Z. The particular quanta that describe this variation are the reduced variation in pitch set, the increase in impulse kind number (not visible in this excerpt), and the variability of the meter elongations during the G.P.s. Unlike in the first variation, there are just four different pitch sets used for the chords over the course of the whole pattern. Also unlike the first variation, the lengths of these chord impulses are much longer. It is that last quantum difference, and the use of the cello in combination with the piano to create the unison voice, that define the patterns of Variation 2.



Example 32. Page 37, mm. 835–841.

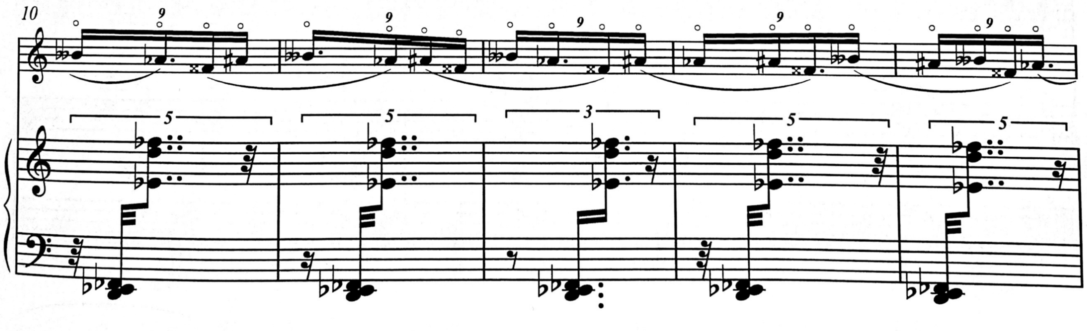
Pattern 829–864 (excerpt shown in Example 32) is an example of Variation 3 of Archetype Z. Like the previous two patterns, the length of the impulses and the length of the space between them conspire to create a pattern that fits this archetype. The texture of the pattern is placid rather than chaotic. There is no space between the piano impulses, but their whole-note length, and the instruction to hold down the pedal for the duration of the pattern, mean that the piano part sounds more like a drone than a series of connected rhythms. The features that set this variation of Archetype Z apart from the previous two are the lack of unison voicing, the total regularity of pitch material, and the irregularity of the rest elongations in the cello voice. The lack of unison voicing is made apparent by the dissimilarity of rhythm and pitch quanta between the cello and the piano parts. Unlike the first two variations of Archetype Z, the cello and piano parts are not rhythmically aligned. In addition, the cello and piano parts do not share an impulse length or rests sets. Finally, the regularity of pitch material, which is a product of the fact that every impulse uses the same pitches, is unlike the two prior patterns. Despite these differences, the description of Archetype Z fits Pattern 829–864 just as well as the previous two variation examples. The experience of listening to the three different patterns that fit the archetype is largely similar.

Before I move on to the more in-depth discussion of Archetype A, it is helpful to have a general overview of the archetype. In this next section, I discuss two variations of Archetype A in order to continue elucidating the taxonomy of archetypes and variations.

1.2 Archetype A

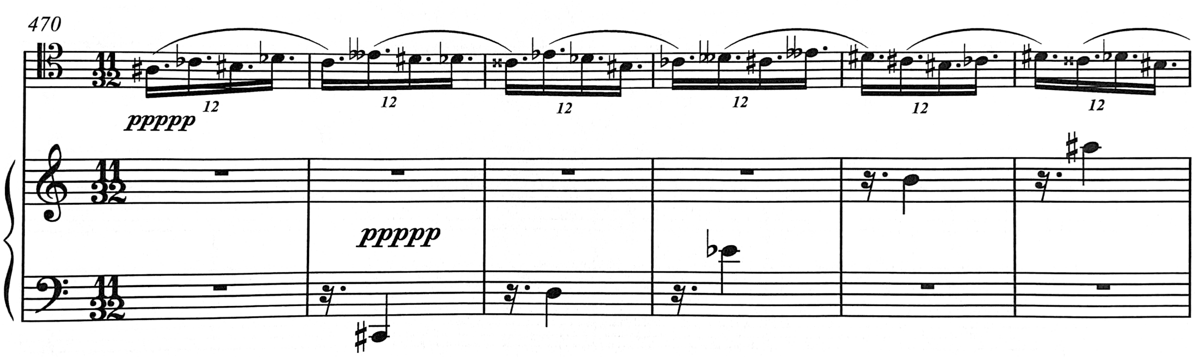
Archetype A is defined by running notes in the cello voice, accompanied by dynamic/variable rhythmic impulses in the piano voice. There are around 27 patterns in PCF that fit this description, representing about 30% of the total measures in the piece. The patterns that fit Archetype A fall into seven different variations that describe every pattern within the archetype. There is an entire section below that examines in greater depth two variations in particular, but for now, I have included a discussion of two patterns drawn from different variations on the archetype that demonstrate what the archetype looks like in practice, and to continue elucidating the concept of pattern archetypes and variations.

Pattern 1–18 (excerpted in Example 33) should be quite familiar by this point. It is the first pattern of the piece, and its various pitch and rhythmic quanta, along with its regularity and centricities,



Example 33. Page 1, mm. 10–14.

have all been discussed in previous chapters. The simple description of Archetype A (running notes in the cello voice accompanied by dynamic/variable rhythmic impulses in the piano voice), that this pattern fits into, is in part made possible by the previous work done on this pattern. The particular features that make this pattern representative of all the patterns in this variation are the pitch set and rhythm quanta used in the cello voice. All of the patterns that belong to this variation of Archetype A use the pitch set {B♭♭, A♭, Fx, A♯}, have a tuplet number of 9, and have a tuplet element number of 4, all in the cello voice. There is greater variability in the piano voice, but the impulse element number is always 2, and the pitch-



Example 34. Page 23, mm. 470–475.

element number is always regular.

Pattern 470–486 (excerpted in Example 34) looks quite different from the previous pattern. However, it still clearly fits the archetype. There are running notes in the cello voice, and the piano voice has a series of rhythmic impulses. The cello voices of this variation use entirely different rhythmic and pitch quanta than the previous variation, and yet, the quantized approach aids in the recognition that the essential nature of cello voices in this variation is very similar to cello voices of the previous. The cello voices of both variations have a tuplet element of 4 and are highly chromatic. The quanta help define the specific features that make up this pattern and its variation, and the general features that apply to all the patterns of this archetype, regardless of variant. The largest dissimilarity between the pattern in Example 33 and the pattern in Example 34 is in the piano voice. In Pattern 470–86, the values that define the rhythmic impulse quanta of the piano voice are all much smaller than in Pattern 1–18. The rhythmic-element, and pitch-element numbers are both 1 in Pattern 470–486. This dissimilarity certainly distinguishes this pattern variation. At the same time, it still complies with the basic description given by archetype A.

With a basic overview of archetypes and variations now out of the way, it is possible to move to a more specific exploration of one archetype and a few of its variations. By doing this, it is possible to pivot to an exploration of how these two concepts allow for the creation of practice methods and materials that make PCF more practicable. These methods and materials focus on the cello part, and are based in familiar string pedagogy.

2. Archetype A Variation 1 & 2 Feature Extraction and Exercise Composition

In order to show how archetypes can help group patterns in a way that can focus practice, and also how pattern quanta can be used to create sets of variations from which specific quanta can be pulled that define focused practice, I have chosen two variations of Archetype A to closely examine. I refer to the two variations I discuss in detail as Variation 1 and Variation 2. There are seven total variations that all of the patterns of Archetype A can fit into, and their patterns are scattered throughout the piece. In this section, I perform a feature extraction of these two sets of patterns using the quantum terminology developed in the first chapter. I then isolate a few salient quanta for each variation that are of particular relevance for the cellist, as they can be practiced and used to create exercises.

Table 11. Archetype A Variation 1 Pattern Quanta.

	Pattern:	Mm. 1-18	Mm. 19-36	Mm. 37-72	Mm. 127-144	Mm. 182-201	Mm. 808-828	Mm. 1519-1531	Mm. 1563-1576
Cello Voice	**Pitch-Element #**	4	4	4	4	4	5	4	4
	Pitch Set	{B♭♭, A♭, Fx, A♯}	{B♭♭, A♭, Fx, A♯}	{B♭♭, A♭, Fx, A♯}	{B♭♭, A♭, Fx, A♯}	{B♭♭, A♭, Fx, A♯}	{Fx,A♭,B♭♭,G,G♭,A♯}	{B♭♭, A♭, Fx, A♯}	{B♭♭, A♭, Fx, A♯}
	Pitch-Class Set	{0,1,2,3}	{0,1,2,3}	{0,1,2,3}	{0,1,2,3}	{0,1,2,3}	{0,1,2,3,4}	{0,1,2,3}	{0,1,2,3}
	Meter	8/32	8/32	8/32	8/32	8/32	8/32	5/16	5/16
	Meter-Element #	N/A	N/A	N/A	N/A	N/A	N/A	4	4
	Tuplet	9	9	9	9	9	9	N/A	N/A
	Tuplet Element #	4	4	4	4	4	4	N/A	N/A
	Slurs	{6,5,4,3}	{6,5,4,3}	5	3	5	5	{3,4}	{3,4}
	Elongations	One dot/tuplet	One dot/tuplet	One dot/tuplet	One dot/tuplet	One dot/tuplet	One dot/tuplet	Two dots/meter	Two dots/meter
Piano Voice	**Pitch-Element #**	6	7	RH:5, LH:4	7	RH:5, LH:4	RH:5, LH:4	5	3
	Pitch Set	{D,E♭,F♭}	{C♯,D,E♭,F♭}	RH: {C,E♭,A♭,B♭,A} LH: Var.	{C♯,D,E♭,F♭}	RH: {C,E♭,A♭,B♭,A} LH: Var.	RH: {C,E♭,A♭,B♭,A} LH: Var.	{G♯,A,B,C♯,B♭}	{C♯,E♯,D}
	Pitch Class Set	{0,1,2}	{0,1,2,3}	RH {0,1,2,4,7} LH: Var.	{0,1,2,3}	RH {0,1,2,4,7} LH: Var.	RH {0,1,2,4,7} LH: Var.	{0,1,2,3,5}	{0,1,2}
	Meter	4/16	4/16	4/16	4/16	4/16	4/16	5/16	5/16
	Tuplet	{5,3}	{5,3}	5	{5,3}	5	5	N/A	N/A
	Impulse-Element #	2	2	2	2	2	2	1	1
	Impulse-Kind #	2	2	2	2	2	2	1	1
	Elongations	Tuplet	Tuplet		Tuplet			Rests	Rests

2.1. Variation 1

Table 11 is a chart with relevant pattern quanta defined for the set of eight patterns belonging to this archetype variation. There are six features shared across all of the patterns that are crucial to understand in order to practice effectively and efficiently: pitch-element number, pitch set, meter/tuplet, tuplet/meter-element number, slurs, and elongations. With one slight exception, all of these patterns share the same pitch-element number, 4, and pitch set, {B♭♭, A♭, Fx, A♯} (set arranged in the order first presented). All of these patterns have a tuplet/meter-element number of 4. All of these patterns are slurred, with those slurs either all a fixed length 3–5 notes long, or chosen from a set of slurs 3–6 notes long. Finally, all of these patterns use dot elongations to slightly elongate one or two notes per tuplet or measure. I have collected these values in their own chart (Table 12). With this set of values, it is possible to compose a cello line that superficially resembles the cello part of any of these eight patterns, and crucially, to create exercises that target a set of features shared by a significant number of patterns. To demonstrate what that can look like, I have created several exercises using the values in Table 12 that can be used more generally to prepare for the patterns in this variation.

Table 12. Archetype A Cello Voice Quanta.

Pitch-Element #	4
Pitch Set	{B♭♭, A♭, Fx, A♯}
Tuplet-Element #	4
Slurs	{3,4,5,6}
Elongations	1 or 2 dots

Figure 2 shows the pitch set used by these patterns, recomposed to eliminate the double sharps and double flats. The first thing the cellist needs to do is decide where to play these notes. There are two places on the cello where one can use artificial harmonics to play pitches in this octave. Starting on the lowest pitch of the set, on the C-string, one can use the artificial harmonic at the fourth with the thumb on the same G as the open G-String. Alternatively, one can find the lowest pitch of the set on the G-string on the artificial harmonic at the fifth with the thumb on the C an octave above the C-string. Neither is

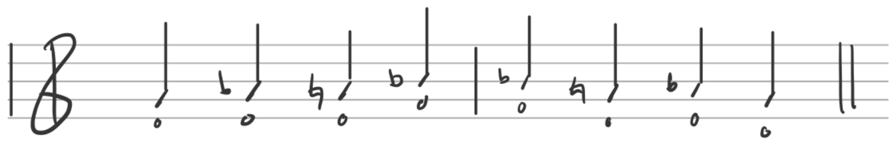


Figure 2. Pitch-Set Recomposition.

inherently better, but seeing as there is much time spent in this position, the smaller hand shape on the C-string combined with the fact that the intervals are all slightly closer in the higher position on the C-string mean that the fourth harmonic is, for many cellists, the most comfortable way to play these passages. Alternatively, if the cellist has large enough hands, the harmonic on the G-string may resonate better on their instrument.

Once the basic set has been practiced and outlined, it is possible to devise exercises to become more comfortable shifting between pitches. These can be adaptations of any shifting exercises with which the cellist is familiar. In Figure 3*,* I have provided two exercises that can be used to become more

Figure 3. Pitch-Set Exercises.

comfortable moving around these four pitches. The first is the beginning of an exercise where the first note stays the same, and the other three pitches are organized in descending order and rotated through all of the possible rotations. This can be done on any pitch and in any order. The second exercise is more like a traditional shifting exercise, where the cellist practices shifting between one of the pitches and all of the others. The goal with both is to feel totally comfortable moving around these pitches, so that it is easier just to read through the patterns with this pitch set.

The rhythm is potentially more difficult for this group of patterns. The notes go by fast enough that it can be difficult to track the elongations and changes in slur length without an amount of practice that would quickly balloon the total practice time needed for the piece, given the sheer volume of material. In order to make the rhythm easier to read without too much practice, the rhythm quanta of this archetype variation will need to be practiced separately, just like the pitch material. There are many ways to do this, but there is one way I have proposed below that highlights the ways in which the quantization in the first chapter can help the performers see past the complex notation of this pattern in particular.

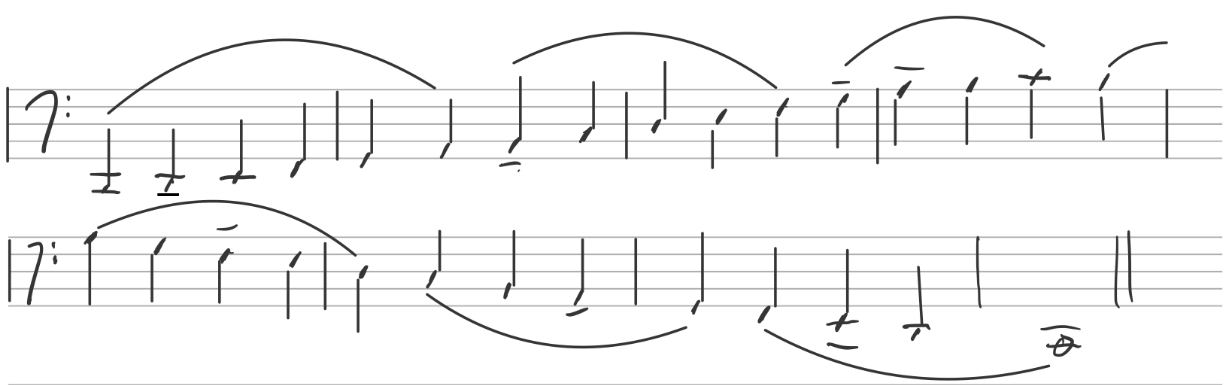


Figure 4. Pattern 1–18 Bowing and Rhythm Exercise.

Figure 4 is a two-octave C Major scale with bowings and elongations (written as tenuto marks) that are identical to those in the first eight measures of Pattern 1–18. There is no time signature, only bar lines every four notes, because the only necessary feature needed to describe the rhythmic groupings of notes in all of the patterns of Variation 1 is 4 (the tuplet/meter-element number of all of these patterns is 4). Written out this way, the rhythm of the first eight measures of Pattern 1–18 seems much more approachable. An endless number of exercises can be created this way, simplifying the tuplet/meter to just four quarter notes, and using tenuto marks to represent the subtle elongations created by the dotted rhythms in the actual patterns. Those few features can be applied to any scale or arpeggio a performer might be working on, or even to the shifting exercises described previously for the pitch set of this variation. In addition to building muscle memory, this is good practice in using the basic quanta to see past the complex notation in any of the patterns in this set.

2.2. Variation 2

Variation 2 of Archetype A, by virtue of being a part of the archetype is not significantly different from Variation 1. That means that a similar set of exercises around pitch and rhythm can be created, once the pertinent features are extracted from the set of patterns that make up this variation. Table 13 shows a

Table 13. Archetype A Variation 2 Pitch Quanta.

	Pattern:	Mm. 1180-1188	Mm. 1189-1191	Mm. 11955-1197	Mm. 1360-11362	Mm. 1366-1368
Cello Voice	Pitch-Element #	3	3	3	3	3
	Pitch Set	{E♯,G♭,Dx,F}	{E♯,G♭,Dx,F,E}	{E♯,G♭,Dx,F,E}	{E♯,G♭,Dx,F,E}	{E♯,G♭,Dx,F,E}
	Pitch-Class Set	{0,1,2}	{0,1,2}	{0,1,2}	{0,1,2}	{0,1,2}
	Meter	3/16	3/16	3/16	3/16	3/16
	Meter-Element #	N/A	N/A	N/A	N/A	N/A
	Tuplet	7	7	7	7	7
	Tuplet Element #	7	7	7	7	7
	Slurs	7	7	7	7	7
	Elongations	N/A	N/A	N/A	N/A	N/A
Piano Voice	Pitch-Element #	5	6	6	6	6
	Pitch Set	{E♯,F,G♭,B♭,G♯,A}	{D,F,C,E♭,A♭,B♭}	{A,D,B,D♭,A♭,E♭}	{D,F,C,E♭,A♭,B♭}	{A,D,B,D♭,A♭,E♭}
	Pitch Class Set	{0,1,2,4,5}	{0,1,3,5,7,10}	{0,1,2,4,6,7}	{0,1,3,5,7,10}	{0,1,2,4,6,7}
	Meter	3/16	3/16	3/16	3/16	3/16
	Tuplet	4	4	4	4	4
	Impulse-Element #	2	2	2	2	2
	Impulse-Kind #	1	1	1	1	1
	Elongation	N/A	Rests	Rests	Rests	Rests

table of all the pattern quanta for these patterns, and Table 14 shows just the salient features of the cello voice. Those features are the pitch-element number (3), the pitch set {E♯,G♭,Dx,F,E}, the tuplet-element number (7), and the slur length (7). Just as with the previous variation set, with these data, it is possible to compose a cello line that superficially resembles the cello voice of any of these five patterns, and crucially, to create exercises that target a set of features shared by a significant number of patterns.

Because this variation is similar to the previous, I focus on one problem that this feature extraction elucidates: the mismatch between the pitch-element number, and the size of the pitch set. This

Table 14. Variation 2 Cello Voice Quanta.

Pitch-Element #	3
Pitch Set	{E♯,G♭,Dx,F,E}
Tuplet-Element #	7
Slurs	7
Elongations	N/A

mismatch exists because E♯ and F are enharmonically equivalent, as are Dx and E. In Chapter 1, I stated that these unusual spellings were not just something that could be totally ignored, and that they had microtonal implications. Those implications arise when the cellist considers using a tuning system other than equal temperament, with its 100-cent half-steps. The argument for doing so was derived from the fact that Feldman frequently and consistently uses "unnecessary" spellings of notes in an atonal piece (if the cellist plays in equal temperament). The idea that Feldman was introducing complexity just for the sake of making the lives of the performers more difficult does not comport with the otherwise consequential and actionable difficulty of the rest of the piece. The suggestion that these spellings might in fact have microtonal implications is an answer to the question that arises: if Feldman wanted each "unnecessary" note spelling to mean something, what does it mean? The questions are now: 1) How should the microtonal differences between otherwise enharmonically equivalent notes be calculated? and 2) How can those differences be practiced?

The two non-equal tuning systems commonly employed by string players are just intonation and Pythagorean tuning. Just intonation is not suitable for an atonal piece such as PCF, because pitches are calculated relative to a particular tonic. This means that pitches are not fixed entities in just intonation. Pythagorean tuning on the other hand is a fixed system. A Pythagorean pitch "B" is the same in any key or context. Additionally, even if a cellist has never explicitly practiced with Pythagorean tuning, they have almost certainly used "expressive" intonation in scale practice or when performing a solo work. "Expressive" intonation, with its high leading tones and thirds is derived from Pythagorean tuning. It is also possible to find Pythagorean pitches easily on a string instrument, as they are all calculated by perfect fifth relationships. A detailed explanation of Pythagorean tuning is not the purpose of my document, and there are many resources available for performers who wish to read up on the subject.[7] The cheat-sheet version that works for the purposes of my document says that enharmonic equivalents arrived at by traveling clockwise around the circle of fifths are 24 cents higher than their equivalent, and that enharmonic equivalents arrived at by traveling counter-clockwise are 24 cents flatter than their equivalents. For the pitch set of Variation 2 of Archetype A, that means that the E♯ should be 24 cents higher than the F, and the Dx should be 24 cents higher than the E. The remaining questions are, how can this be practiced and how can microtonality be practically implemented in the patterns of PCF?

How accurately and specifically the performers adjust their intonation to match the microtonal implications of a pitch set should be influenced by the nature of the set. When there are two pitches that are enharmonically related within the same set, as there are in the pitch set of Variation 2, then it is necessary to make some provisions to treat them differently. However, when there are not two enharmonically related pitches in the same set, just one B♭♭ and one Fx such as in the set of Variation 1, perhaps it is enough just to recognize that those particular spellings squeeze the chromatic cluster of pitches of that set together, and just play the whole set with tight half-steps. That captures a degree of the chromaticism implied by how the set is spelled, without necessitating specific microtonal practice.

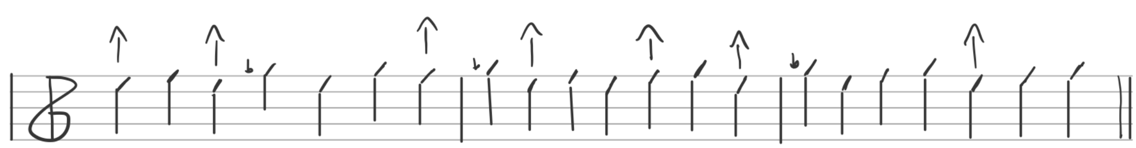


Figure 5. Pitch Recomposition.

As for the set in Variation 2, I propose an exercise such as that seen in Figure 5. In this exercise, the cellist starts out treating notes that are enharmonically equivalent as the same thing. The exercise is the cello voice from Pattern 1189–1191, recomposed using only the three unique pitches (no enharmonic equivalents). Depending on how deep down the rabbit hole the cellist wants to go, this is arguably enough to at least make it through the performance, which is an achievement of its own. However, if the cellist desires to add back in some of the chromaticism that Pythagorean tuning creates, then that can be done

[7] I recommend Hans Jørgen Jensen's Book, *Cello Mind*. There is a very lucid explanation of the 24-cent Pythagorean comma in Chapter 11.

simply by thinking of the "normal" pitch plus a slight chromatic alteration. In Figure 5*,* this is indicated by the arrows pointing up on what had before been D Double-Sharps and E Sharps. This solution, where a complicated phenomenon is broken into, in this case, two simple ideas, is another example of the application of quantization in order to make practicable something that superficially does not seem so.

3. Conclusion

There are an endless number of exercises that could be created by replicating the process I have described above. There are also an endless number of ways that a cellist could go about creating exercises around specific quanta. It would be wasteful to spend time exploring every exercise that could be created from every archetype that exists in PCF. Instead, I hope that by demonstrating just a few solutions to the practicability of PCF, the potential of the quantized approach to pattern definition has been made clear. There are many intimidating features to the score of PCF, but through quantizing the basic topography of a pattern, it is possible to translate the score into simple and intuitive parts. The way that these quanta build into patterns gets at the nature of what is a pattern in PCF. And Finally, those patterns can be grouped into archetypes and variations whose plain-language descriptions are enabled by the quantization of the first chapter.

There are many performance issues not addressed in this document that would need to be tackled by anyone interested in performing the piece. The most obvious is the work of aligning the complex rhythms of the cello and piano parts. Without being able to talk from experience, it is my hope that the work done in this document to create simple explanations for complex features would be found helpful in this task. Another issue not addressed is the extreme duration of the piece, and the kind of focus that would be required to perform the piece live. Again, my hope would be that the concepts of archetype and variation would allow a performer to consider the large scale of the piece, not with bewilderment, but a calm understanding.

It would also be interesting to see how the quanta that result from my own quantization, and the "crippled symmetries" described by Feldman, could factor into the psychological phenomenon described in Elizabeth Hellmuth Margulis's book, *Music on Repeat.* I can think of no other piece that is a better case study in how repetition of all kinds is used to create all sorts of different effects. This work could even have ramifications on performance and practice that would serve to make the piece more approachable.

Even if this document finds itself in the hands of someone with no interest in performing PCF, I hope that the processes and analytical insights it provides are still found to be interesting, and maybe even applicable to other works. 20th and 21st century pieces that have been kept out of the canon because they present a particularly novel technical difficulty deserve more than to be relegated to the auspices of contemporary performance obscurity. There is nothing wrong with specialization; in fact, without contemporary music specialists, works like PCF would almost certainly not exist. The problem arises when generalist or mainstream performers resist the expansion of the canon to which specialists are contributing. Excessive compartmentalization of the canon serves to make the musical lives of everyone involved duller, and risks perpetuating the inequities in racial, gender, and class representation endemic to the narrow canon accepted by generalist/mainstream performers. By creating new pedagogical techniques for PCF, it is my hope that at least one barrier to expanding the generalist/mainstream “canon” has been lowered.

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