FCNC, CP Violation and implications for some rare decays in a model with gauge symmetry 3-4-1

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Abstract

Models with gauge symmetry $SU(3)_c \otimes SU(4)_L \otimes U(1)_Y$ (3-4-1 models) where anomaly cancellation takes place between families (three-family models) predict the existence of new heavy neutral gauge bosons which transmit flavor changing neutral currents at tree-level. In this work, in the context of a three-family 3-4-1 model which do not contain particles with exotic electric charges, we analyze the constraints coming from neutral meson mixing on the parameters of the model. Taking into account the experimental measurements of observables related to K and B meson mixing and including CP-violating phases, we study the resulting bounds for angles and phases in the mixing matrix for the down-quark sector, as well as the implications of these bounds for the modifications of some rare decays, namely $K^+ \to \pi^+ \bar{\nu}\nu$, $K_L \to \pi^0 \nu\bar{\nu}$, $K_L \to \pi^0 l^+ l^-$ and $B_{d/s} \to \mu^+\mu^-$.

Introduction

In this work we will restrict ourselves to a particular 3-4-1 model without exotic electric charges in the fermion sector, called Model F in Refs. [1, 2]. This model predicts the existence of two heavy neutral gauge bosons Z' and Z'' which mix up with the ordinary Z boson of the SM. After the breakdown of the 3-4-1 symmetry down to $SU(3)_c \otimes U(1)_Q$, and since we have one family of quarks transforming differently from the other two under the gauge group, the new Z_3 gauge boson couples nondiagonally to ordinary quarks thus transmitting tree-level FCNC at low energies, while the Z_2 current remains flavor diagonal.

We re-examine the issue of FCNC but, instead of searching for bounds on the Z_3 mass, we will set this mass at several fixed values and we will search for information about the size of angles and phases in the V_L mixing matrix. To this purpose we will use several well measured $\Delta F = 2$ ($F = S, B$) observables, namely ΔM_K, $\Delta M_{d/s}$, ε_K, and $\sin\Phi_d$. We will also study the implications of the bounds obtained for the modifications in the rare decays $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$, $B_{d/s} \to \mu^+\mu^-$ and $K_L \to \pi^0 l^+ l^-$, where l can be an electron or a muon.

Model F

Many details of this model have been worked out in [2] and its anomaly-free fermion structure has been discussed in Ref. [1]. The model is based on the local $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ gauge symmetry, and belongs to the $b = 1,\ c = -2$ class, where b and c are parameters appearing in the expression for the electric charge

$$Q = T_{3L} + \frac{1}{\sqrt{3}} b T_{8L} + \frac{1}{\sqrt{6}} c T_{15L} + X I_4, \tag{1}$$

where $T_{iL} = \lambda_{iL}/2$, being λ_{iL} the Gell-Mann matrices for $SU(4)_L$.

Our main interest is the neutral gauge boson sector, which consists of four physical fields: the massless photon A_μ and the massive gauge bosons Z_μ, Z'_μ and Z''_μ. The Lagrangian for the neutral currents $J_\mu(\mathrm{EM})$, $J_\mu(Z)$, $J_\mu(Z')$, and $J_\mu(Z'')$ is written as

$$\begin{aligned} -\mathcal{L}^{\mathrm{NC}} = {} & eA^\mu J_\mu(\mathrm{EM}) + (g_4/C_W) Z^\mu J_\mu(Z) + g_X Z'^\mu J_\mu(Z') \\ & + g_4/(2\sqrt{2}) Z''^\mu J_\mu(Z''), \end{aligned} \tag{2}$$

The corresponding currents for the for the down quark sector are

$$J^{\mathrm{d}}_\mu(\mathrm{EM}) = -\frac{1}{3}\sum_{i=1}^{3} \bar{d}_i \gamma_\mu d_i \tag{3}$$

$$J^{\mathrm{d}}_\mu(Z) = \sum_{i=1}^{3}\left(\left(\frac{2S_W^2 - 3}{6}\right)\bar{d}_i\gamma_\mu P_L d_i + \left(\frac{S_W^2}{3}\right)\bar{d}_i\gamma_\mu P_R d_i\right), \tag{4}$$

$$J^{\mathrm{d}}_\mu(Z') = \sum_{i=1}^{3}\left(-\frac{T_W}{6}\bar{d}_i\gamma_\mu P_L d_i + \frac{T_W}{3}\bar{d}_i\gamma_\mu P_R d_i\right), \tag{5}$$

$$J^{\mathrm{d}}_\mu(Z'') = \sum_{i=1}^{2}(-\bar{d}_i\gamma_\mu P_L d_i) + \bar{d}_3\gamma_\mu P_L d_3, \tag{6}$$

$J_\mu(Z'')$ is a pure left-handed current and the neutral gauge boson Z''_μ does not mix neither with Z_μ nor with Z'_μ, but it still couples nondiagonally to ordinary fermions. As a matter of fact, its couplings to the third family of quarks are different from the ones to the first two families. Thus, at low energy, we have tree-level FCNC transmitted by Z''_μ. The neutral gauge boson Z'_μ does not transmit FCNC at low energy since it couples diagonally to ordinary fermions

Effective Lagrangian

The chiral Z_2 and Z_3 couplings, in the fermion mass eigenstate basis, read

$$B^{\psi_{L/R}}_{ij} \equiv \left(V^\psi_{L/R}\epsilon^{\psi(2)}_{L/R}V^{\psi\,\dagger}_{L/R}\right)_{ij}; \qquad K^{\psi_{L/R}}_{ij} \equiv \left(V^\psi_{L/R}\epsilon^{\psi(3)}_{L/R}V^{\psi\,\dagger}_{L/R}\right)_{ij};$$

The contribution of $\mathcal{L}_{\mathrm{eff}}$ to $\Delta S = 2$ and $\Delta B = 2$ processes is driven by the nondiagonal elements of the K^d_L matrix, that is, the coefficients $K^{(d)}_{L_{ij}}$ with $i \neq j$. The corresponding effective interaction Lagrangian for the down quark sector in the mass eigenstate basis D_i, with $D = (d\ s\ b)^T$. is

$$-\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}}\left(\frac{g_3}{g_1}\right)^2 \rho_3 \left(\overline{D}_i\gamma_\mu K^d_{L_{ij}} P_L D_j\right)\left(\overline{D}_m\gamma^\mu K^d_{L_{mn}} P_L D_n\right) \tag{7}$$

The nondiagonal elements of K^d_L read

$$K^d_{L\,ij} = 2V^d_{L\,i3}V^{d*}_{L\,j3} \tag{8}$$

whereas the ratio (g_3/g_1) can be written in terms of the Weinberg angle as

$$\left(\frac{g_3}{g_1}\right)^2 = \frac{\cos^2\theta_W}{8}. \tag{9}$$

In order to include phases, the mixing matrix V^d_L can be conveniently parametrized, generalizing the usual CKM parametrization, as a product of three rotations, and introducing a complex phase in each of them.

$\Delta F = 2$ observables

The observables receive both SM contributions arising from standard one loop diagrams, together with the new contributions from the 3-4-1-model [3], from the matrix elements M^P_{12}, that is

$$M^P_{12} = (M^P_{12})_{SM} + (M^P_{12})_{3-4-1}. \tag{10}$$

The expressions for the $\Delta F = 2$ observables are

$$\Delta m_K = 2\,\mathrm{Re}(M^K_{12});\ \Delta m_d = 2\left|M^{B_d}_{12}\right|;\ \Delta m_s = 2\left|M^{B_s}_{12}\right|; \tag{11}$$

$$\varepsilon_K = \kappa_\varepsilon \exp\left(i\frac{\pi}{4}\right)\left(\frac{\mathrm{Im}(M^K_{12})}{\sqrt{2}\Delta m_K}\right);\ \Phi_d = \arg(M^{B_d}_{12}); \tag{12}$$

The well known SM contributions to M^P_{12} are given in Refs. [4].

From the effective interaction Lagrangian in Eq. (7) the expressions for the 3-4-1 contributions can be obtained. They are

$$M^{P(3-4-1)}_{12} = \frac{8\sqrt{2}G_F}{3}\rho_3\left(\frac{g_3}{g_1}\right)^2 m_P f_P^2 \hat{B}_P \lambda_P^2, \tag{13}$$

where

$$\lambda_K = s^d_{13}s^d_{23}c^d_{13}e^{i\phi''}, \tag{14}$$

$$\lambda_{B_d} = s^d_{13}c^d_{23}c^d_{13}e^{i\phi'}, \tag{15}$$

$$\lambda_{B_s} = s^d_{23}c^d_{23}(c^d_{13})^2 e^{-i\phi_3}, \tag{16}$$

Expressions for Rare Decay Amplitudes

The decays under study are governed both by electroweak- and photon-penguins an by leptonic box diagram contributions. In the SM these contributions are described by the corresponding Inami-Lim functions $C_0(x_t)$, $D_0(x_t)$ and $B_0(x_t)$ which, in the expressions for decay amplitudes, always appear in the gauge invariant combinations $X_0(x_t)$, $Y_0(x_t)$ and $Z_0(x_t)$ [5].

For the decays $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$ and $B_{d/s} \to \mu^+\mu^-$, the contributions from new physics can be absorbed into a redefinition of the function $X(x_t)$ and $Y(x_t)$, as

$$\Delta X_{K\pi\nu\nu} = \frac{4\pi\sin^2\theta_W}{\alpha}\left(\frac{g_3}{g_1}\right)^2\rho_3\frac{V^d_{L\,23}V^{d*}_{L\,13}}{V^*_{ts}V_{td}}. \tag{17}$$

$$\Delta Y_{B\mu\mu} = -\frac{4\pi\sin^2\theta_W}{\alpha}\left(\frac{g_3}{g_1}\right)^2\rho_3\frac{V^d_{L\,33}V^{d*}_{L\,13/23}}{V^*_{tb}V_{td/ts}}. \tag{18}$$

For the decay $K_L \to \pi^0 l^+ l^-$, these new contributions will be absorbed into the matching conditions of the Wilson coefficients in the form

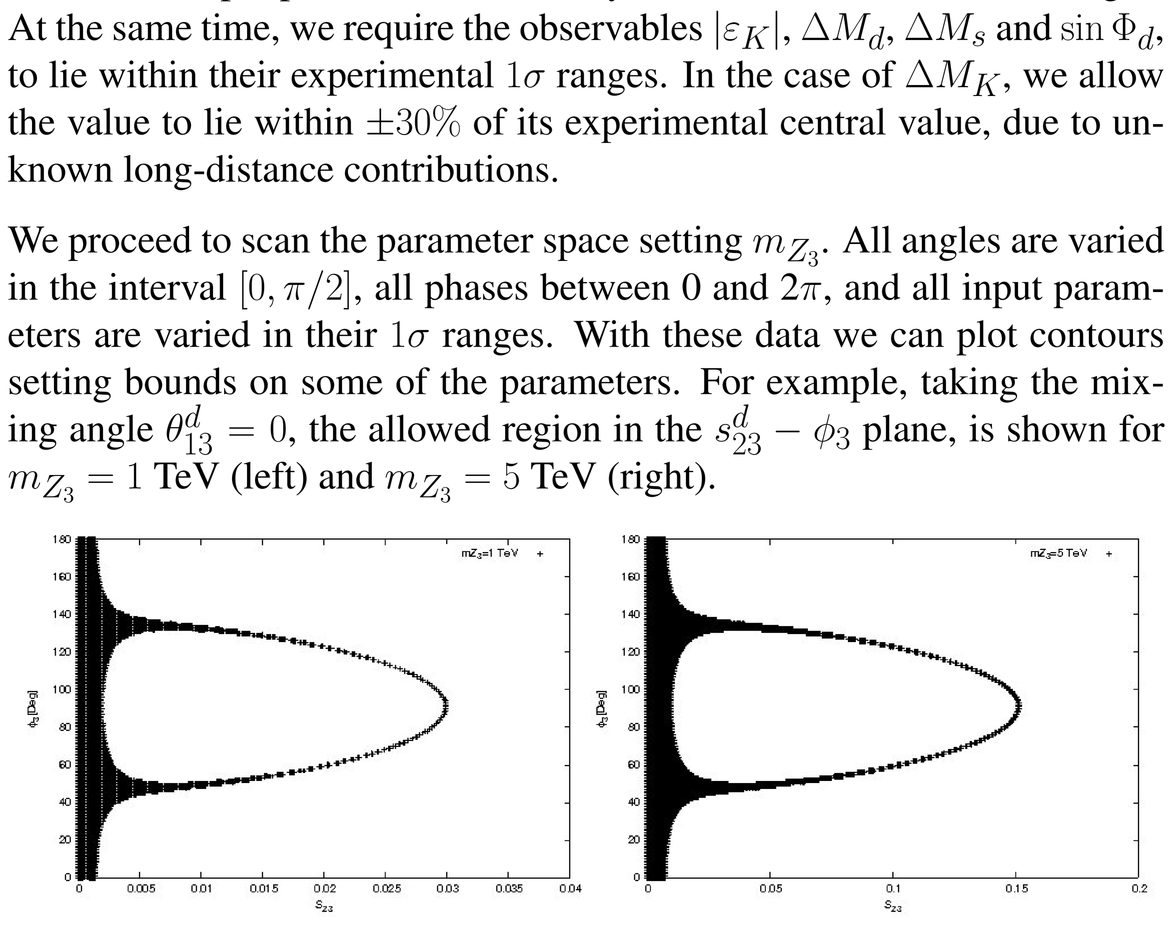
$$\Delta y_V = -2\left(\frac{g_3}{g_1}\right)^2\rho_3\frac{(V^d_{L\,23}V^{d*}_{L\,13})}{V^*_{ts}V_{td}}; \qquad \Delta y_A = 2\left(\frac{g_3}{g_1}\right)^2\rho_3\frac{(V^d_{L\,23}V^{d*}_{L\,13})}{V^*_{ts}V_{td}}. \tag{19}$$

Numerical Analysis

The 3-4-1 contributions are given in terms the mass of Z_3, the angles θ^d_{13}, θ^d_{23} and new CP-violating phases, coming from the V^d_L mixing matrix. We perform two related numerical analysis: in the first one, we fix the Z_3 mass and consider the well-measured observables ΔM_K, $\Delta M_{d/s}$, ε_K and $\sin\Phi_d$. We use them to constrain the angles and phases coming from the V^d_L mixing matrix, for selected values of m_{Z_3}. In the second analysis, we study the implications of these bounds on the rare decays mentioned previously.

We take all input parameters to be flatly distributed within their 1σ ranges. At the same time, we require the observables $|\varepsilon_K|$, ΔM_d, ΔM_s and $\sin\Phi_d$, to lie within their experimental 1σ ranges. In the case of ΔM_K, we allow the value to lie within $\pm 30\%$ of its experimental central value, due to unknown long-distance contributions.

We proceed to scan the parameter space setting m_{Z_3}. All angles are varied in the interval $[0, \pi/2]$, all phases between 0 and 2π, and all input parameters are varied in their 1σ ranges. With these data we can plot contours setting bounds on some of the parameters. For example, taking the mixing angle $\theta^d_{13} = 0$, the allowed region in the $s^d_{23} - \phi_3$ plane, is shown for $m_{Z_3} = 1$ TeV (left) and $m_{Z_3} = 5$ TeV (right).



We can estimate the order of magnitude of the upper limits for θ^d_{13} and θ^d_{23} for several values of m_{Z_3}. The results are collected in the table 1.

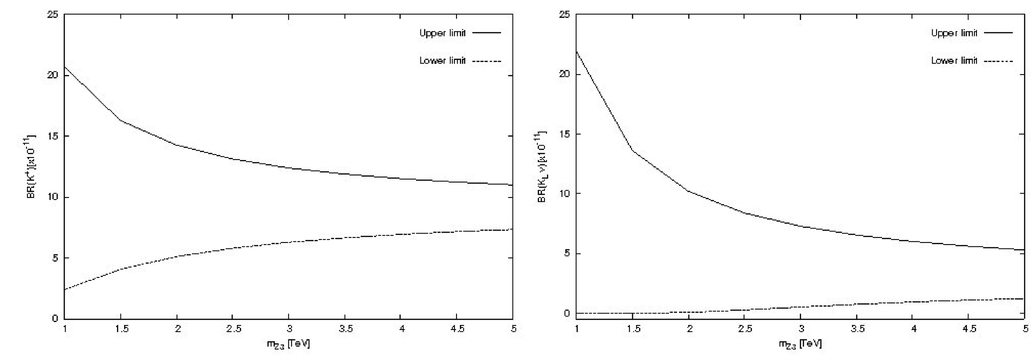
This allow us to elucidate the size of the allowed region in the parameter space and its variation with increasing values of m_{Z_3}.

Table 1: Upper limit for θ_{13} and θ_{23} for different values of m_{Z_3}.

m_{Z_3}	1 TeV	3 TeV	5 TeV	7 TeV	9 TeV
$\theta_{13,max}$	8.73×10^{-4}	2.62×10^{-3}	4.36×10^{-3}	5.24×10^{-3}	1.05×10^{-2}
$\theta_{23,max}$	1.40×10^{-2}	6.11×10^{-2}	1.52×10^{-1}	2.13×10^{-1}	9.65×10^{-1}

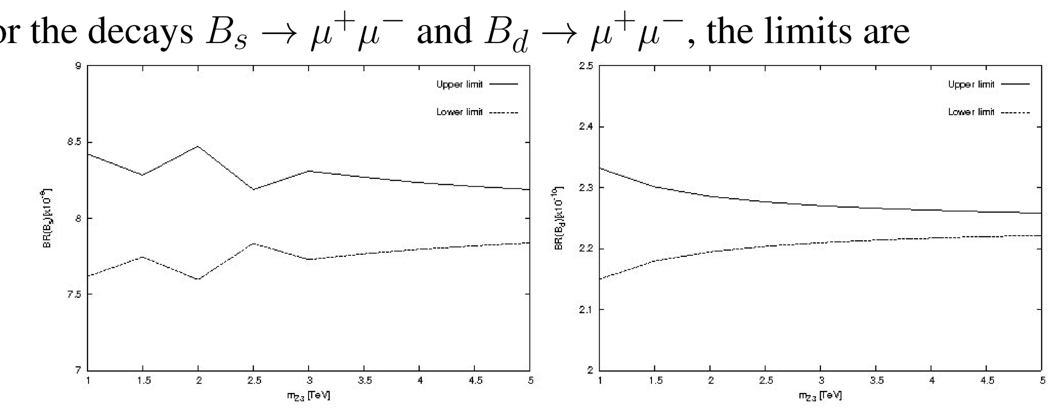
To study the implications of the bounds obtained for the modification in the rare decay amplitudes, we will move over all the effective parameter space for each selected value of m_{Z_3}, and we calculate the corresponding amplitudes for the different decays.

For the decays $K^+ \to \pi^+\bar{\nu}\nu$ and $K_L \to \pi^0\nu\bar{\nu}$, the corresponding bounds are



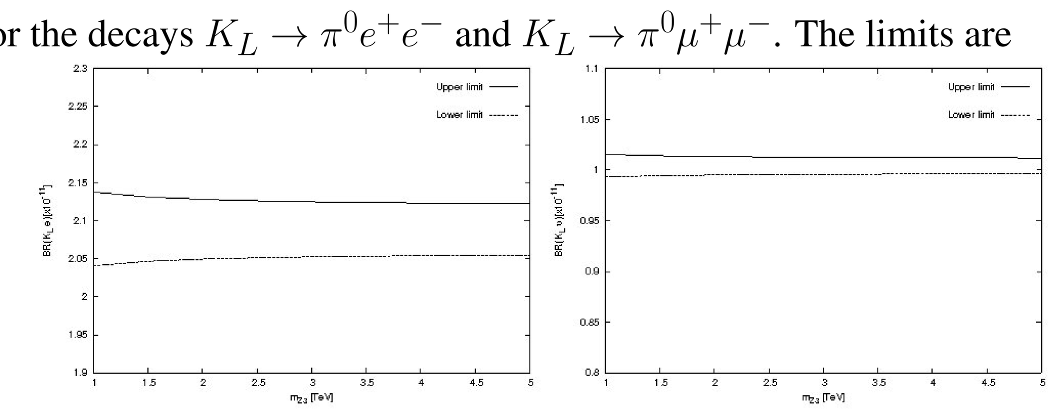
For the $K^+ \to \pi^+\bar{\nu}\nu$ decay it is seen that the experimental central value is greater than the SM prediction. The figure shows that for low values of m_{Z_3} the upper limit reach the experimental central value, but for larger m_{Z_3} both limits go closer to the SM prediction. For the decay $K_L \to \pi^0\nu\bar{\nu}$ we can see that big enhancements of the SM predictions are expected for low m_{Z_3}. The bigger enhancements are expected for m_{Z_3} lower that ~ 2.5 TeV.

For the decays $B_s \to \mu^+\mu^-$ and $B_d \to \mu^+\mu^-$, the limits are



The upper and lower bounds are higher than the SM predictions but smaller than the experimental upper limit.

For the decays $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0\mu^+\mu^-$. The limits are



We can see that the corresponding limits are lower than the SM predictions, but consistent with the upper experimental limits.

Conclusions

In this work, we have concentrated on the study of flavor mixing in the down quark sector in a three-family 3-4-1 model characterized by the values $b = 1,\ c = -2$ in the most general expression for the electric charge generator. In the analysis we have included new CP-violating phases and considered the phenomenological bounds on the model parameters arising from the experimental values of $\Delta F = 2$ observables. We have concentrated on the study of flavor mixing in the down quark sector, where $\Delta F = 2$ observables provide a set of experimental data that allows one to obtain bounds for the relevant model parameters.

We have used the experimentally measured quantities ΔM_K, $\Delta M_{d/s}$, ε_K and $\sin\Phi_d$ to constrain the size of the new mixing matrix elements for selected values of m_{Z_3}. An estimation of the upper limits for θ_{13} and θ_{23} for different values of m_{Z_3} are presented in Table 1, this allowed us to see the behavior of the parameter space for different mass of Z_3.

We have then taken these results to obtain upper and lower bounds for several very clean rare decay processes, i.e., the decays $K^+ \to \pi^+\bar{\nu}\nu$, $K_L \to \pi^0\nu\bar{\nu}$, $K_L \to \pi^0 l^+ l^-$ and $B_{d/s} \to \mu^+\mu^-$. These bounds depend on the Z_3 mass. These results present some differences from the SM predictions and can be considered as signals of the 3-4-1 model under consideration when looking at the data. From the plots it is clear that for low values of m_{Z_3} the bounds on the decays are wider than for high mass values.

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