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Electricity consumption forecasting using singular spectrum analysis

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Abstract
Singular Spectrum Analysis (SSA) is a non-parametric technique that allows the decomposition of a time series into signal and noise. Thus, it is a useful technique to trend extraction, smooth and filter a time series. The effect on performance of both Box and Jenkins' and Holt-Winters models when applied to the time series filtered by SSA is investigated in this paper. Three different methodologies are evaluated in the SSA approach: Principal Component Analysis (PCA), Cluster Analysis and Graphical Analysis of Singular Vectors. In order to illustrate and compare the methodologies, in this paper, we also present the main results of a computational experiment with the monthly residential consumption of electricity in Brazil.

Keywords: electricity consumption forecasting, singular spectrum analysis, time series, power system planning.

Previsión del consumo de electricidad mediante análisis espectral singular

Resumen
El Análisis Espectral Singular (AES) es una técnica no paramétrica que permite la descomposición de una serie de tiempo en una componente de señal y otra de ruido. De este modo, AES es una técnica útil para la extracción de la tendencia, la suavización y el filtro una serie de tiempo. En este artículo se investiga el efecto sobre el desempeño los modelos de Holt-Winters y de Box & Jenkins al ser aplicados a una serie de tiempo filtrada por AES. Tres diferentes metodologías son evaluadas con el enfoque de AES: Análisis de Componentes Principales (ACP), análisis de conglomerados y análisis gráfico de vectores singulares. Con el fin de ilustrar y comparar dichas metodologías, en este trabajo también se presentaron los principales resultados de un experimento computacional para el consumo residencial mensual de electricidad en Brasil.

Palabras clave: pronóstico del consumo de electricidad, análisis espectral singular, serie de tiempo, planeamiento del sistema eléctrico.

1. Introduction

Load forecast is a requisite to all decision-making processes in the power systems operation and planning [1]. Traditionally, the load forecasts are classified in three time periods: short-term (usually half-hourly, hourly and daily forecasts up to 1 month ahead), mid-term (1 month – 5 years ahead) and long-term (5 years onwards). The short-term load forecasting [2,3] is important for the daily operation (unit commitment). The mid-term load forecasting is essential to the maintenance scheduling, hydro resources management, schedule fuel purchases, tariff setting and energy trading [4]. The long-term load forecasting signals the need to expand the capacity of the generation and transmission systems [1].

In the Brazilian electricity market, energy trading is realized through auctions where the generators compete in order to meet the demand growth at the lowest price. The auctions procedure starts with the mid/long-term monthly demand forecasts provided by the distribution utilities and ends with the energy contracts agreed between all distributors and each generator that wins the auction [5]. The energy contracting occurs one, three or five years before supply starts with contracts lasting from five to thirty years [6]. Therefore, the mid-term electricity consumption forecasts play a fundamental role in the energy auctions. The demand for electricity can be divided into several groups: residential, commercial, industrial, rural and miscellaneous. These groups grow at different rates, thus each group must be forecast separately [1].

Traditionally, Box & Jenkins and the multiple linear regression models have been considered in mid-term electricity consumption forecasting [1,4,7]. Despite the good results obtained by these methods, efforts have been

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made to improve them [4,8, 9]. One way to improve the performance of the mid-term forecasting methods consists in filtering the time series data [4,10]. Among the available methods able to extract the signal component from a time series, the Singular Spectrum Analysis (SSA) has been successfully applied in several scientific fields [11,12].

SSA decomposes a time series into a sum of a small number of independent components interpretable as trend, oscillatory components and noise. SSA is a method for signal processing that can be used, among other applications, for example, in smoothing and filtering [12,13]. One of the advantages of SSA is its nonparametric nature, i.e., it is not necessary to know or specify a parametric model for the time series under study. A detailed description of the theoretical and practical foundations of the SSA technique can be found in [11,12].

A good example of the benefits provided by SSA filtering can be found in [14], where forecasts are provided for the industrial production in Europe. In the context of the electric power system, [15] presents the use of SSA in the monthly affluent flow forecast, essential information to the hydropower system operation and [16] presents a geometric combination approach to forecasting residential electricity consumption. In another example, [17] proposes a model-free approach for day-ahead electricity price forecasting based on SSA and [18] presents a hybrid model combining periodic autoregressive models (PAR(p)) and SSA.

In [4] the Spanish peninsular monthly electric consumption time series is split into two components: the trend and the fluctuation around it. After that, a neural network is trained to forecast each component separately. These predictions are added up to obtain an overall forecasting. The authors show that the results obtained are better than those reached when only one neural network was used to forecast the original consumption series.

This paper investigates the use of SSA in mid-term forecasting of the monthly electricity consumption for the residential class. In Brazil, the residential class is responsible for approximately 26% of the total electricity consumption and represents 85% of the consumers. This paper shows similar results related to [14], but considering three approaches in SSA before fitting the ARIMA and Holt-Winters models. Both results confirm that SSA improves the accuracy of forecasting.

The remainder of this article is organized as follows: Section 2 has a description of the SSA methods, while the traditional predictive methods are presented in Section 3. The computational experiment is presented in Section 4 and the results and discussion are reported in Section 5. Finally, in section 6 the main conclusions are drawn.

2. Singular spectrum analysis

The basic version of the SSA method has two steps: decomposition and reconstruction.

2.1. Decomposition

The decomposition step involves two stages: embedding and singular value decomposition (SVD).

Embedding is a procedure in which a time series $Y_T \in \mathbb{R}^T$ is mapped into a sequence of lagged vectors $\mathbf{X} = [X_1, \dots, X_K]_{L\times K} \in \mathbb{R}^{L\times K}$, in which $X_k = [y_K, \dots, y_{K+L-1}]' \in \mathbb{R}^L$, for all $k = 1, \dots K$, where $K = T - L + 1$ and L takes any integer value in the range $2 \le L \le T$.

The matrix $\mathbf{X}$ is known as trajectory matrix [14] and the parameter L is the window length of the trajectory matrix [11].

The trajectory matrix $\mathbf{X}$ can be expanded via singular value decomposition as (1):

$$\mathbf{X} = \textstyle\sum_{l=1}^{L} \mathbf{E}_l, \tag{1}$$

where $\mathbf{E}_l = {\lambda_l}^{1/2} U_l V_l'$, and the set $\{\lambda_l\}_{l=1}^{L}$ correspond to the eigenvalues of the positive semidefinite matrix $\mathbf{S} = \mathbf{XX}'$ taken in order of magnitude and $\{U_l\}_{l=1}^{L}$ denotes the respective eigenvectors. According to [12], $V_l = \mathbf{X}' U_l / \sqrt{\lambda_l}$.

Let d be the rank of the trajectory matrix $\mathbf{X}$ (i.e., the number of nonzero eigenvalues), then the identity described in (1) can be rewritten as:

$$\mathbf{X} = \textstyle\sum_{l=1}^{d} \mathbf{E}_l \tag{2}$$

where $d \le L$.

The collection (λ_l, U_l, V_l) is called *eigentriple* of SVD of the trajectory matrix $\mathbf{X}$. The contribution of each component in (1) can be measured by the ratio of singular values, given by $(\lambda_l)^{1/2} / \sum_{l=1}^{L} (\lambda_l)^{1/2}$ for each *l*.

2.2. Reconstruction

The reconstruction step also has two stages: grouping and diagonal averaging.

Grouping is a procedure that groups the elementary matrices into $m \le d$ disjoint groups and adding the matrices within each group. Let $I_i = \{I_{i1}, \dots, I_{ip_i}\}$ be the set of indices of the p_i elementary matrices classified in a same group i. Then the matrix corresponding to the group i is defined as: $\mathbf{X}_{I_i} = \sum_{j=1}^{p_i} \mathbf{E}_{I_{ij}}$, so the identity (2) can be rewritten as:

$$\mathbf{X} = \sum_{i=1}^{m} \mathbf{X}_{I_i}. \tag{3}$$

The contribution of the component $\mathbf{X}_{I_i}$ can be measured by the ratio of singular values given by:

$$\textstyle\sum_{j=1}^{p_i} \left(\lambda_{I_{ij}}\right)^{1/2} / \sum_{l=1}^{d} (\lambda_l)^{1/2}. \tag{4}$$

Consider the trajectory matrix $\mathbf{X}$ and assume that $L^* = \min(L, K)$ and $K^* = \max(L, K)$. Consider that $x_{l,k}^{(i)}$ is an element in the line l and column k of matrix $\mathbf{X}_{I_i}$. The element $y_t^{(i)}$ of SSA component $\left[y_t^{(i)}\right]_{1\times T}$ is computed by the *Diagonal Averaging* procedure applied to the matrix $\mathbf{X}_{I_i}$.

$$y_t^{(i)} = \begin{cases} \dfrac{\sum_{l=1}^{t} x_{l,t-l+1}^{(i)}}{t}, \; for \; 1 \leq t < L^* \\ \dfrac{\sum_{l=1}^{L^*} x_{l,t-l+1}^{(i)}}{L^*}, \; for \; L^* \leq t < K^* \\ \dfrac{\sum_{l=t-K^*+1}^{T-K^*+1} x_{l,t-l+1}^{(i)}}{T-K^*+1}, \; for \; K^* \leq t \leq T \end{cases} \quad (5)$$

Each SSA component $\left[y_t^{(i)}\right]_{1\times T}$ concentrates part of the energy of the original series $[y_t]_{1\times T}$ which can be measured by the ratio of singular values (4). According to [12], the SSA component $\left[y_t^{(i)}\right]_{1\times T}$ can be classified into three categories: trend, harmonic components (cycle and seasonality) and noise.

2.3. Separability

Separability is one of the leading concepts in SSA [20]. This property characterizes how well the different components are separated from each other. A good measure of separability is the weighted correlation (w-correlation), a function that quantifies the linear dependence between two SSA components $Y_T^{(1)}$ and $Y_T^{(2)}$:

$$\rho_{12}^{(w)} = \frac{\left(Y_T^{(1)}, Y_T^{(2)}\right)_w}{\left\|Y_T^{(1)}\right\|_w \left\|Y_T^{(2)}\right\|_w}, \quad (6)$$

Where

$$\left\|Y_T^{(i)}\right\|_w = \sqrt{\left(Y_T^{(i)}, Y_T^{(i)}\right)_w},$$
$$\left(Y_T^{(i)}, Y_T^{(j)}\right)_w = \textstyle\sum_{k=1}^{T} w_k y_k^{(i)} \, y_k^{(j)},$$
$$(i,j) = (1,2),$$

$w_k = \min\{k, L, T-k\}$ (once $L \leq T/2$).

The separability allows for a statistical check of whether two SSA components are well separated in terms of linear dependence. The matrix containing the absolute values of the w-correlations corresponding to the full decomposition can provide useful information for grouping the eigentriples [17]. If the absolute value of w-correlation is small, so the SSA components are classified as w-orthogonal (or quasi w-orthogonal) otherwise, they are said to be poorly separated. It is a useful concept in the SSA grouping stage [17].

2.4. Choice of optimal value of L

The question of the optimal value of L remains open. [19], illustrates a long discussion about the ideal value of window length assuming that this value can be fixed or variable. In several cases, the general recommendation is to choose the window length at slightly less than half the size of the series: $T/3 \leq L \leq T/2$. In [20], the suitable value of L is $median\{1, \ldots, T\}$. According to [21], the choice of L depends on several criteria including complexity of the data, the aim of the analysis and the forecasting horizon. In [21], the authors show a new bound of L for the multivariate case and study the optimum value for the number of eigenvalues N to choose. By selecting N smaller than the true number of eigenvalues, some parts of the signal(s) will be lost, and then the reconstructed series becomes less accurate. However, if one takes N greater than the value that it should be, then noise is included in the reconstructed series. Although, considerable attempts have been made and various techniques considered for selecting the optimal values of L and N in SSA, there is not enough theoretical justification for choosing these parameters. In addition, if the series has a seasonal behavior with monthly periodicity, it is advisable to choose reasonably large values of L (but smaller than $T/2$). In this paper the optimum value of L is obtained by testing values from $L = (T+1)/2$ to $L = T/3$ and performing the BDS test [22] applied to the noise series after decomposition.

3. Predictive methods

This section presents the Holt-Winters and ARIMA models; both traditionally used in mid-term consumption prediction.

3.1. Holt-Winters models

According to [23], exponential smoothing methods are based on the assumption that the data are weighted differently. Usually, recent observations contain more relevant information than older ones, so that the weighting of the data (time series) decreases exponentially as the observation becomes older. A particular case of exponential smoothing method is the multiplicative Holt-Winters method, which performs modeling dynamically (i.e. with time-varying parameters) its components: level $(a_{1,\tau})$, trend $(a_{2,\tau})$ and seasonality $(\varrho_{m(\tau)})$:

$$y_\tau = \left[a_{1,\tau} + a_{2,\tau} \times \tau\right] \times \varrho_{m(\tau)} + \epsilon_\tau, \quad (7)$$

where ϵ_τ is a stochastic error, y_τ is the observed value at time τ and $\varrho_{m(\tau)}$ is the seasonal factor in τ relative to month m. The family $\left\{\varrho_{m(\cdot)}\right\}_{m\in\Gamma}$ of seasonal factors, where Γ is the set of months of the year that satisfy the constraint $\sum_{m\in\Gamma} \varrho_{m(\cdot)} = \Lambda$, where Λ is the size of the seasonal cycle. In the process of estimating the parameters of equation (7), three hyperparameters (time invariant quantities) are used, denoted by α, β and λ which are associated, respectively, to the estimates of level, trend and seasonality and whose optimal values are in the cube $[0,1]^3$. Recently, [24] has compared the forecasts for the exponential smoothing and neural networks and removed trend and seasonality using seasonal differentiation.

3.2. Box-Jenkins models

A second order stationary stochastic process is defined as a family $\{Y_t\}_{t=1}^{T}$ of random variables whose moments (mean, variance and covariance) are time invariants for all t. Consider the sequence $\{y_t\}_{t=1}^{T}$ as a realization of $\{Y_t\}_{t=1}^{T}$. Box & Jenkins [23] proposes the following linear equation for $\{y_t\}_{t=1}^{T}$.

$$y_t = \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - \epsilon_{t-q}, \tag{8}$$

where ϵ_t is a stochastic error and y_t is the observed value in t. For seasonal time series, the following formulation is used:

$$SARIMA\ (p,d,q) \times (P,D,Q)_S. \tag{9}$$

The fitting of such SARIMA model to monthly seasonal time series is carried out in four stages; structural identification, parameters estimation, goodness of fit tests and forecasting. For details, see [23].

4. Case study

For the computational experiment, we considered the monthly time series of the residential electricity consumption in Brazil shown in Fig. 1. The time series covers the period from July 2001 to March 2013 (141 observations). In this period, the series experienced an average growth of approximately 5%. The length of the in sample period is 129 and the out of sample is 12. The computational implementation was carried out with different software: MATLAB, for the SSA approach via principal component analysis in the SVD; Caterpillar SSA [25], for detailed verification of SSA filtering via graphical analysis of singular vectors via their scatter plots and the periodogram analysis; E-Views, for the analyzes of BDS tests (independence) [26] and ARIMA models [27]; Forecast Pro for Windows for Holt-Winters modeling; R, to apply SSA using hierarchical clustering [28], and Microsoft Excel to generate graphs.

5. Results and discussions

In this study, three SSA filtering approaches were applied: principal component analysis under SVD (PCA-SVD) [28], cluster analysis integrated with PCA-SVD [29] and graphical analysis of singular vectors of SVD [30]. Each filtering approach generated a smoothed time series of

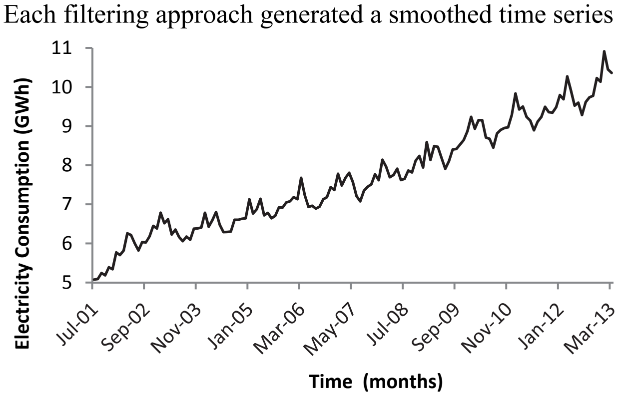


Figure 1. Residential monthly electricity consumption in Brazil, covering the period from 2001/7 to 2013/3. (Average annual growth of 5%).
Source: ANEEL - Brazilian Electricity Regulatory Agency.

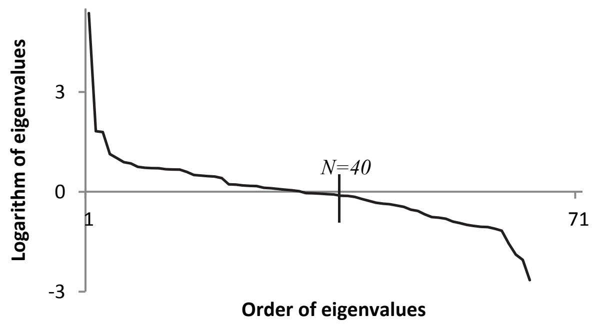


Figure 2. Optimum value of N. Beyond this value (N=40) , the logarithm of eigenvalues are negative.
Source: own elaboration.

the monthly residential electricity consumption which was modeled by two predictive methods: ARIMA and Multiplicative Holt-Winters.

5.1. *Principal component analysis under SVD (PCA-SVD)*

In the PCA-SVD approach, an optimal window length L is defined for trajectory matrix $\mathbf{X}$ and an integer N such that the SVD can be rewritten as:

$$\mathbf{X} = \{\mathbf{E}_j\}_{j=1}^{N} + \{\mathbf{E}_j\}_{j=N+1}^{L}. \tag{10}$$

Through diagonal averaging procedure, these sequences of elementary matrices in (10) generate S_T (less noisy than the original series Y_T) and R_T (the noise), respectively, such that the original time series Y_T can be written as:

$$Y_T = S_T + R_T. \tag{11}$$

The goal of this approach is to obtain a smoothed time series $\widetilde{Y}_T$ less noisy than the original time series Y_T [23].

In [31], the best values of L and N using this approach were obtained following the completion of many rounds of BDS testing for several values of these parameter. In this paper, the optimum values using the same procedure are 71 and 40 respectively. Fig. 2 shows the logarithm of the 71 eigenvalues arranged in a decreasing partial order and the point defined by the optimal value of N.

In the PCA-SVD, the first 40 eigenvectors cover the signal S_T and the last 31 remaining eigenvectors, the noise R_T. Removing R_T in (11), one obtains the filtered series $\widetilde{Y}_T$ generated by the approach PCA-SVD such that $\widetilde{Y}_T = S_T$. Table 1 shows the results of the BDS test applied to R_T.

Table 1.
The BDS test results for residual series R_T.

Approach	Dim.	BDS Statistics	*Z*-score	*p*-value.
	2	0.003570	0.802060	0.4225
	3	0.001377	0.343034	0.7316
PCA – SVD	4	0.001996	0.734919	0.4624
	5	0.001245	0.773227	0.4394
	6	0.001139	1.288628	0.1975

Source: own elaboration.

Table 2
SSA components obtained through hierarchical cluster analysis

Component	Singular Vectors
SSA 1	1,2,3
SSA 2	4-18, 21-24, 27, 28
SSA 3	19, 20, 25, 26, 29-40

Source: own elaboration.

In Table 1, one can see that the null hypothesis of independence of the BDS test is not rejected at the 5% level. So there is no empirical evidence that the time series noise R_T has any structure of temporal dependence. Based on the BDS test, one can see that the Brazilian residential electricity consumption time series can be smoothed by $\widetilde{Y}_T$.

5.2. *Cluster Analysis*

As before, in the first step, the trajectory matrix **X** is obtained from Y_T through embedding with optimum window length equal to 71. In this way, 71 singular vectors are obtained and the 31 less significant singular vectors were classified as noise, based on the BDS test using a 5% significance level, and were removed. Next, the 40 remaining singular vectors were grouped into 3 SSA components by hierarchical clustering analysis, as seen in Table 2. The implementation of this method was carried out by the general agglomerative hierarchical clustering in a R package called RSSA [32]. Initially, a matrix of dissimilarities for the L eigentriples of SVD was generated. Next, each object was assigned to its own cluster and then the algorithm proceeded iteratively, joining, at each stage, the two most similar clusters, continuing until three clusters were obtained. At each stage, Euclidean distances between clusters were recomputed using the Lance–Williams dissimilarity update formula [33].

Fig. 3 shows the plot of three SSA components obtained from the three clusters. The BDS test is applied to each SSA component in order to identify the noisy component. The aim of this approach is to obtain a less noisy time series $\hat{Y}_T$, by removing the noisy component.

According to the BDS test in Table 3, the SSA component 3 is classified as a noisy component.

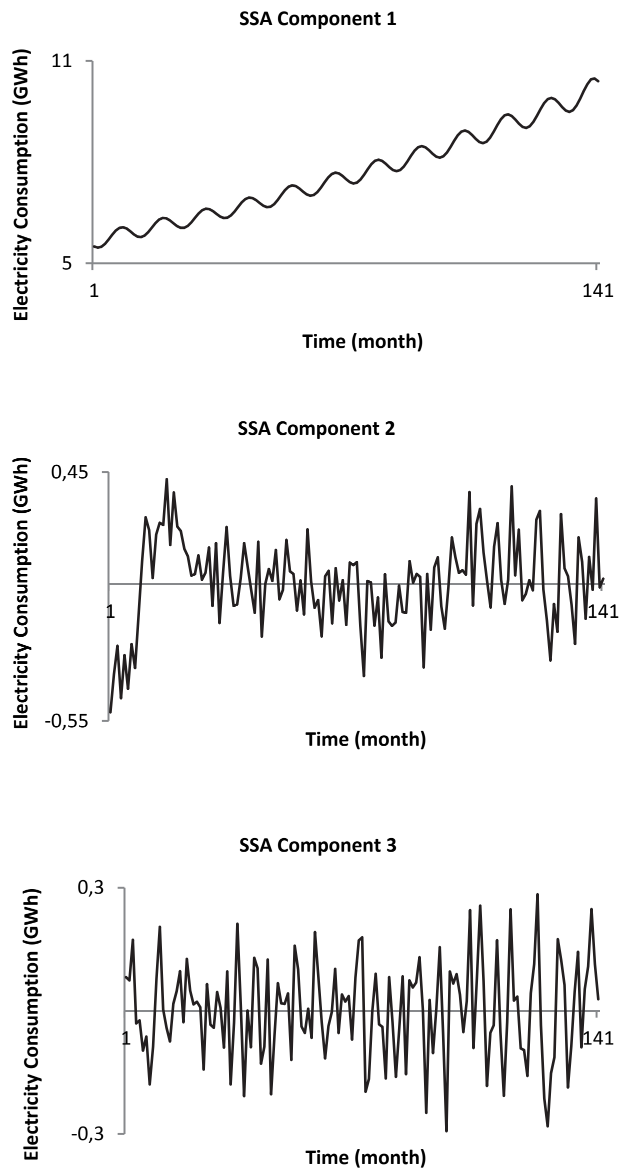


Figure 3. Three SSA components from hierarchical cluster analysis.
Source: own elaboration.

5.3. *Graphical analysis of singular vectors*

Analysis of time series coordinates on the basis defined by the singular vectors resulting from SVD identifies the components of trend, seasonality and noise present in a time series Y_T. The general problem consists in identifying and separating the oscillatory components from those that are part of the trend. According to [12], the graphical analysis of such coordinates in pairs allows us to visually identify the harmonic components of the series.

Similarly to the computational experiments above, the optimal window length was considered as $L = 71$, but with an optimal truncation $N = 50$, generating 50 singular vectors. The software used for this approach was Caterpillar SSA [25]. Through graphical analysis of pairs of singular vectors it is possible to classify them according to their behavior. Consider a pure harmonic with frequency equal to ω, phase equal to δ, amplitude equal to ξ and period $\rho = 1/\omega$ defined as a divisor of window length L and K. If the parameter ρ assumes an integer value, then ρ is classified as a harmonic period [14]. The sine and cosine functions having equal frequencies, amplitudes and phases generate a scatter plot, which displays a circular pattern [12]. Thus, the

Table 3
The BDS test results for residual SSA component 3

Approach	Dim.	BDS Statistics	*Z*-score	*p*-value.
Cluster Analysis	2	-0,006979	-1.283821	0,1992
	3	-0.003929	-0.454503	0,6495
	4	-0,003973	-0.385826	0,6996
	5	-0,001168	-0.108811	0.9134
	6	0,001095	0.105725	0.9158

Source: own elaboration.

scatter diagram shows a regular polygon with ρ vertices. For a frequency $\omega = \frac{m}{n} < 0{,}5$ with m and n integers and primes, the points are vertices of a regular polygon of n vertices [12]. Thus, the identification of components that are generated by a harmonic analysis can be performed by the pictorial analysis of the patterns determined by different pairs of components.

Fig. 4 shows the ten first singular vectors. One can see that components 1, 4, 5 and 10 correspond to the trend. It is possible to identify that components 2, 3, 6 - 9 are harmonic components.

For the other components in Fig. 4, there is no need for a deeper analysis. The first component (trend) accounts for nearly all the variability present in the time series. The domain of the trend component can be explained by the growth of the Brazilian population and the respective growth of the number of households [34]. Additionally, Brazil has experienced an improvement in income distribution, which allowed greater diffusion of home appliances, as well as the efforts to meet the universal access to electricity, expected for the year 2015. The result of classical decomposition of this series presents similar values of trend (95.83%), seasonal (2.77%) and noise (1.40%).

Fig. 5 shows three pairs of singular vectors, it is found that the singular vectors 2 and 3 are harmonic components with period equal to 12 months, while the singular vectors 8 and 9 are harmonic components with period equal to 6 months. In turn, the singular vectors 19 and 20 are harmonic components with period equal to 3 months.

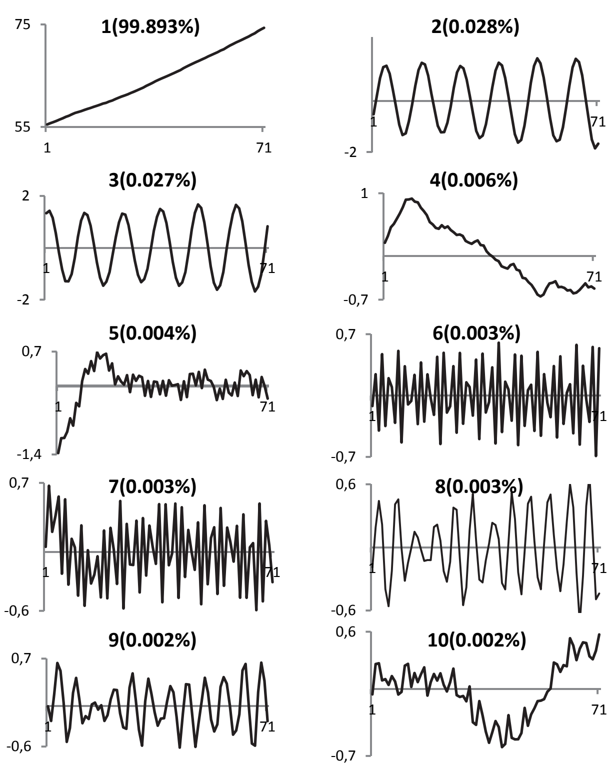


Figure 4. The 10 more significant singular vectors in SVD.
Source: own elaboration.

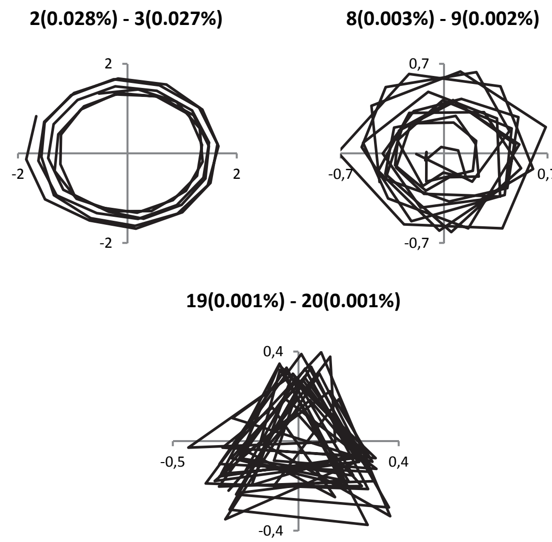


Figure. 5. Scatterplots of three pairs of singular vectors harmonics.
Source: own elaboration.

The periodogram analysis helps in the identification of a general harmonic component. The periodogram of the singular vector of each eigentriple provides information about the periodic behavior of the component and frequency (period) of the oscillations. Therefore, proper grouping can be made with the help of the periodogram analysis. For the series $Y_T = (y_1, \dots, y_T)$, the periodogram $\Pi_{Y_T}^T(\omega)$ is defined as

$$\Pi_{Y_T}^T(\omega) = \frac{1}{T}\left|\sum_{t=0}^{T-1} e^{-2\pi i\omega t} y_{t+1}\right|, \quad \omega = (-0.5, 0.5] \quad (12)$$

where $i = \sqrt{-1}$.

For the periodic components, the periodogram has sharp spikes around the component's frequency (period). Hence the visual identification is straightforward. Fig. 6 shows the respective periodogram associated to scatterplots of Fig. 5:

By exclusion, the singular vectors that are not classified as trend component or harmonic component via graphical analysis are classified as noise. After graphical analysis of the 50 singular vectors of the SVD, the classification shown in Table 4 was obtained.

Fig. 7 shows the plot of the three SSA components obtained by graphical analysis in the reconstruction phase where the elementary matrices are grouped into three groups generating the components in SVD. Note that the trend component captures a slight change in trend after 15 months and the noisy component captures the highest difference between the original time series and the smoothed times series at the 57th month.

Table 4
Grouping of singular vectors via graphical analysis.

Component	Singular Vectors
Trend	1, 4, 5, 10
Harmonic	2, 3, 6-9, 19,20
Noise	11-18, 21-50

Source: own elaboration

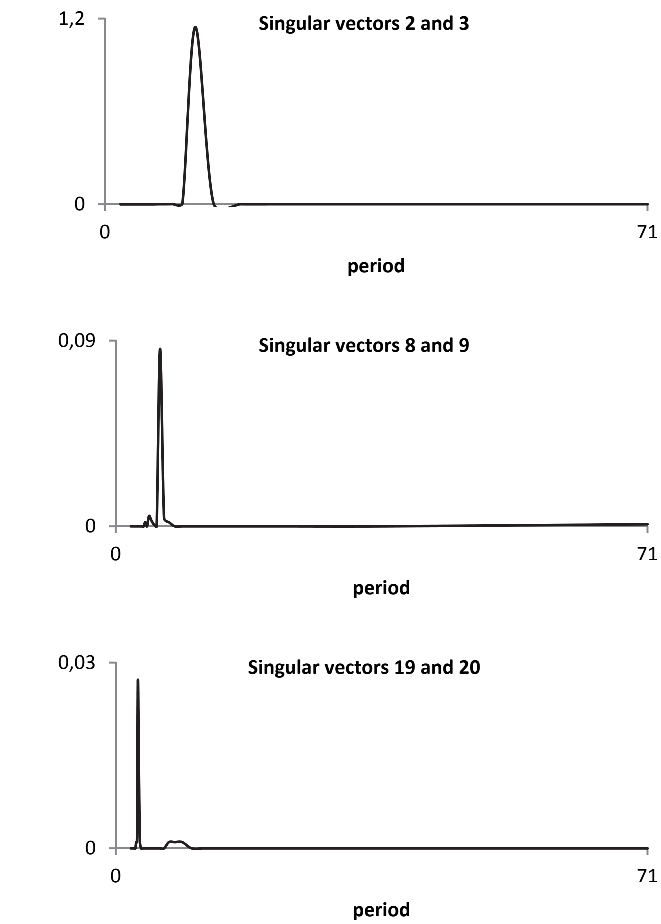
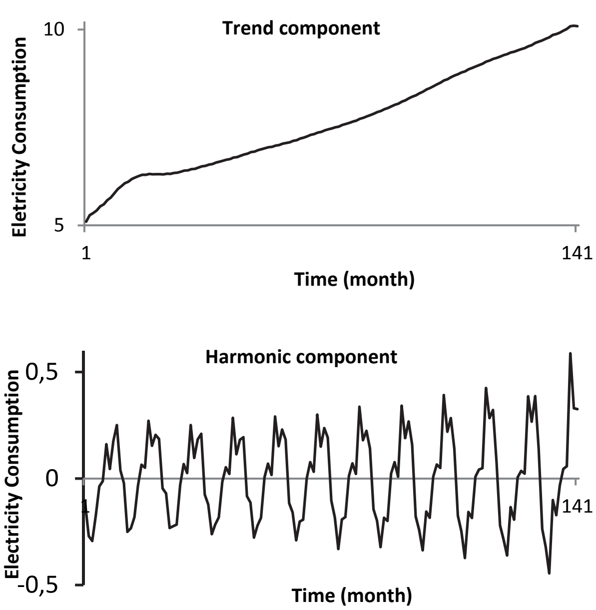


Figure 6. Periodogram of the paired singular vectors (2 and 3), (8 and 9) and (19 and 20). The periods confirm the findings in the scatter plots.
Source: own elaboration.



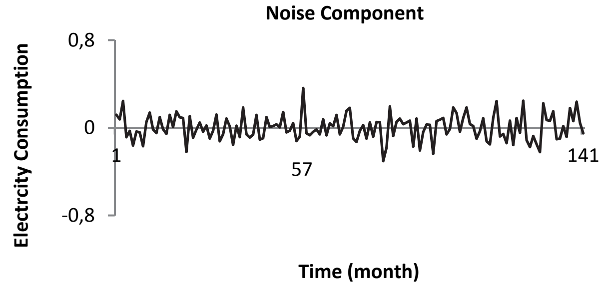


Figure 7. Plot of the three SSA components: trend, harmonic and noise via graphical analysis approach of singular vectors.
Source: own elaboration.

Table 5.
Weighted correlation between three components

Component	Trend	Harmonic	Noise
Trend	1	0	0.001
Harmonic	0	1	0,062
Noise	0.001	0,062	1

Source: own elaboration

Table 6.
The BDS test results for component noise.

Approach	Dim.	BDS Statistics	*Z*-score	*p*-value.
	2	-0,001141	-0.214661	0,8300
Graphical	3	-0.000761	-0.090085	0,9282
Analysis	4	-0,004814	-0.478405	0,6324
	5	-0,004648	-0.443188	0.6576
	6	-0.010205	-0.009020	0.3130

Source: own elaboration.

Table 5 shows the weighted correlation matrix among the three components identified by graphical analysis of the singular vectors. According to the figures presented in Table 5, the three components look well separable.

According to the BDS test in Table 6, one can see that the noisy component does not present time dependence structure. Therefore, this component can be classified as a noise.

In the PCA-SVD method (equation 11), the components aren't separated in trend, harmonic and noise, but rather in signal and noise. In this approach many noisy components are still part of the signal. In Table 2 obtained by the cluster analysis, one can see that the eigentriples 2 and 3 are added to the trend component, but the periodogram analysis (Fig. 6) shows that these eigentriples are part of a harmonic component. Similarly, the cluster analysis classifies the eigenvectors 11 – 18 as harmonic component, but they are noise components (see Table 4) obtained by graphical analysis of the singular vectors. The results above show that the graphical analysis of a singular vector is a more effective method to classify SSA eigentriples.

5.4. Forecasting models

After identifying the noisy components by the three methods under study (PCA-SVD, Cluster Analysis and Graphical Analysis of Singular Vectors), these components are extracted from the original time series resulting in smoothed time series. Thus, there are four time series to be modeled: the original and three filtered time series via SSA.

In the specification of the $SARIMA\,(p,d,q) \times (P,D,Q)_S$ model with $s = 12$, it is necessary to choose the lags $p, P, q, Q,$ the degree of differencing d and D. It is considered that the best SARIMA model minimizes the Bayesian Information Criterion (BIC) [35]. The best Holt-Winters' model (HW) also minimizes the BIC with linear trend and multiplicative seasonality. The models are shown in Table 7.

To check the predictive power of the SSA approach applied to the electricity consumption time series, Table 8 shows the in sample results of the goodness of fit statistics: R^2, mean absolute percentage error (*MAPE*), mean absolute deviation (*MAD*), and root-mean-square error (*RMSE*). One can see that the values of *MAPE, MAD,* and *RMSE* using the SSA approach are lesser than these values for the original time series.

The out of sample 1-step ahead error statistics are shown in Table 9. Note that the graphic analysis is also the best approach of SSA filtering.

Table 7.
Models fitted to the four time series.

		SARIMA $(p,d,q) \times (P,D,Q)$	HOLT-WINTERS *LEVEL*	*TREND*	*SEASONAL*
	ORIGINAL	(0,1,1) × (1,0,2)	0.339	0.014	0.248
SMOOTHED (SSA)	PCA-SVD	(0,1,1) × (1,0,2)	0.348	0.014	0.255
	CLUSTER ANALYSIS	(0,1,2) × (0,1,2)	0.474	0.013	0.499
	GRAPHICAL ANALYSIS	(0,1,1) × (0,1,3)	0.439	0.104	0.997

Source: own elaboration.

Table 8.
In sample goodness of fit statistics of the models tested

MODEL			R^2	*MAPE*	*MAD*	*RMSE*
ORIGINAL TIME SERIES		HW	0.982	0.0163	0.121	0.163
		ARIMA	0.983	0.0157	0.117	0.157
SMOOTHED (SSA)	PCA-SVD	HW	0.982	0.0163	0.121	0.160
		ARIMA	0.984	0.0152	0.114	0.152
	CLUSTER ANALYSIS	HW	0.989	0.0137	0.101	0.128
		ARIMA	0.996	0.0079	0.059	0.080
	GRAPHICAL ANALYSIS	HW	0.998	0.0058	0.040	0.055
		ARIMA	0.999	0.0028	0.020	0.025

Source: own elaboration.

Table 9.
Adherence statistics of models tested out of sample.

MODEL			*MAPE*	*MAD*	*RMSE*
ORIGINAL TIME SERIES		HW	0.014	0.141	0.182
		ARIMA	0.015	0.151	0.171
SSA	PCA-SVD	HW	0.014	0.160	0.181
		ARIMA	0.015	0.152	0.176
	CLUSTER ANALYSIS	HW	0.011	0.128	0.147
		ARIMA	0.016	0.080	0.102
	GRAPHICAL ANALYSIS	HW	0.007	0.055	0.065
		ARIMA	0.008	0.025	0.045

Source: own elaboration.

6. Conclusions

Three alternatives to remove the noisy component of a time series by SSA method were proposed in this paper. The method was applied to a real time series corresponding to monthly residential electricity consumption in Brazil. Among the three alternatives evaluated the graphical analysis of singular vector is the more effective method to remove the noisy component. In the sequence, the Holt-Winters and Box- Jenkins were applied to the original and filtered time series obtained by SSA. The in-sample and out-of-sample goodness of fit statistics (MAPE, MAD and RMSE) obtained for the eight fitted models, show that the SSA method with graphical analysis of singular vectors also provided the more accurate forecasts. This approach provides a more detailed analysis of the components, thus, the results tend to be better. In addition, the best value of L corresponds to $(T + 1)/2$ and the BDS test proved effective in identifying the noise components.

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