COMPOSITIONAL KRIGING APPLIED TO THE RESERVE ESTIMATION OF A GRANITE DEPOSIT

TÉCNICAS DE KRIGEADO COMPOSICIONAL PARA LA ESTIMACIÓN DE RESERVAS EN UN DEPÓSITO DE GRANITO

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ABSTRACT: Making an accurate estimate of quality distribution in a granite deposit is essential, both from a financial point of view, to determine the profitability of the site, and from an environmental perspective, to focus operations on the most profitable areas thereby reducing the extent of land affected by such work. Granite is extracted in blocks whose profitability and value depend on the final size of the slabs, which is an important factor in defining quality. This article uses a variant of disjunctive kriging in order to determine the quality of granite in one of the largest reserves in the world—the Porriño deposit located in northwest Spain. This method, unlike classical disjunctive kriging, considers random variables that are not necessarily binary. The advantage of using this technique compared to the classical statistical cokriging technique is that all the qualities are considered as variables with the same importance and that the sum of quality percentages in a block is one hundred percent. The validity of the method was tested in a cross-validation process. The results compared favourably with those obtained using ordinary cokriging and fuzzy kriging.

KEYWORDS: compositional kriging, cokriging, fuzzy kriging, granite, quality estimation.

RESUMEN: Realizar una estimación precisa de la distribución del granito por calidades en un yacimiento es fundamental, tanto desde el punto de vista económico, para determinar la rentabilidad del yacimiento, como ambiental, para dirigir las labores de extracción exclusivamente a las zonas más rentables, reduciendo así la extensión de los terrenos afectados por dichas labores. El granito es extraído en bloques cuya utilidad y precio dependen del tamaño final de roca que se puede extraer de los mismos, que es el factor que define la calidad. En este artículo se utiliza una variante del krigeado disyuntivo para determinar las reservas de granito por calidades en el yacimiento de Porriño, uno de los más importantes del mundo, situado en el Noroeste de España. El método utilizado, a diferencia del krigeado disyuntivo clásico, considera variables aleatorias que no son necesariamente binarias. La ventaja de utilizar esta técnica estadística frente a las técnicas clásicas de cokriging es que todas las calidades son consideradas variables de la misma importancia y que se asegura que la suma del porcentaje de las calidades en un bloque es del cien por cien. La validez del método se ha chequeado mediante un proceso de validación cruzada. La comparación con los resultados obtenidos utilizando cokrigeado ordinario y krigeado difuso ha sido favorable para el krigeado composicional.

PALABRAS CLAVE: krigeado composicional, cokriging, krigieado difuso, granito, estimación de calidad.

1. INTRODUCTION

Granite is an ornamental rock that is widely used in roofing and for interiors (flooring, worktops, etc), given its physical, chemical and aesthetic properties. It is generally quarried from opencast pits in the form of blocks that are subsequently sawn and cut into slabs of different sizes and thicknesses according to end use.

Granite deposit reserves from data collected in the field can be evaluated using the kriging methods traditionally used in the metals mining sector [1] [2] in which the variable to be estimated is continuous. The estimation method is based on traditional research methods based on geological maps, a description of fracturing in quarry fronts and vertical information provided by borehole sampling [3]. Fracturing is the parameter that ultimately defines the commercial quality of granite. Four qualities are usually defined, depending on fracturing intensity: top quality, secondary quality (both suitable for the ornamental rock market), construction quality and aggregate quality [4] [5].

Topographical and geological maps and a characterization of the structural and textural parameters of the deposit at various levels are used to define rock quality and plan exploitation methods [6]. The fact that each block of granite extracted from the quarry may have different qualities conditions the choice of which kriging technique to use. This same problem occurs with other materials. For example, Tercan and Özcelik [7] estimated the reserves in a Turkish andesite mine, from which the rock would also be extracted in blocks that could have different qualities. These authors, however, distinguished between commercially valid and other rock using indicator kriging. Recently, fuzzy kriging has been proposed as a suitable method for evaluating reserves, as it can account for the fact that a block may contain different qualities and that the definition of qualities in the field is subject to uncertainty [8]. Nonetheless, this method has the problem of having to define membership functions that adequately represent the uncertainty in determining qualities, which are assessed in the field by geologists.

Fuzzy kriging is a generalization of traditional methods of kriging in which imprecise information is typically incorporated to accompany all the sets of sample data. These generalisations can be obtained if the spatial function is considered to be a fuzzy random function, and, applying the extension principle of Zadeh [9], kriging equations are obtained that satisfy non-bias conditions and minimum prediction variance. See [10] and [11] for a discussion on fuzzy kriging fundamentals.

In this research, the problem of determining the quantities of each quality in granite blocks is tackled differently, using a kriging technique called compositional kriging.

2. MATERIALS AND METHODS

2.1 The Porriño Batholit

The reserves estimated in this research are located in Spain's most important and one of the world's most important granite deposits—the Rosa Porriño batholith situated in the province of Pontevedra (northwest Spain). Supplying technically and aesthetically high-quality ornamental rock, the licensed area measures 6.8 km^2 and a total of 39 companies operate there. A detailed description of this batholith can be found in [8]. An aerial photograph of the Porriño batholith is provided in Figure 1.
Given the size of the batholith and the textural homogeneity of the rock, this deposit is expected to be profitable for a considerable period of time (over 30 years). Nonetheless, it is clear that as greater depths are reached in the deposit, its capacity for supporting all the companies operating there will be diminished and mergers are therefore likely. It is thus important to make an accurate estimate of the reserves to enable medium-term decision making by the companies.

Figure 1. Aerial photograph of the Porriño batholith clearly showing, in the lower left part, some of the warehouses used to store and transform the granite

The rock is cut using diamond wire, which conditions the size of the primary block (10 m×10 m×10 m, i.e., 1000 m^3). The block is then further cut, using diamond wire, perforation and shape blasting, to obtain commercially sized slabs.
Depending on the degree of fracturing, qualities are assigned, with each 1000- m^3 block capable of representing different qualities. Four granite qualities are defined as follows:

- Quality 1: Rock that can be extracted in volumes that are sufficiently large to be able to obtain slabs for cutting with disk saws, in other words, rock with few fissures and yielding blocks of 6 to 12 m^3.
- Quality 2: Rock that produces blocks of less than 6 m3 but still suitable for sawing, with discontinuity spacing of over 2 m.
- Quality 3: Fractured rock that produces semi-blocks (less than 4 m^3), with discontinuity spacing of less than 2 m.
- Quality 4: Fractured rock with market value only as aggregate.

Knowing the quality of each block prior to cutting is clearly important, as it enables more realistic financial forecasting and more rational exploitation in the medium-to-long term.

2.2 Reserve evaluation

2.2.1 Data collection

For the documentation and field data-collection phase, the mining parameter of interest for defining rock quality (and therefore for estimating reserves) was the level of rock mass fracturing, given that textural features—such as grain heterogeneity and the presence of phenocrystals, weathering bands or black knots—have little bearing on the quality of a granite as homogeneous as that of the studied deposit. The fractures in the exploitable area were assessed on the basis of the following:

1) A map to scale 1:3,500 that included topographical and geological details and information on fractures. Fractures were directly observed at outcrops and represented on the map.
2) A description of seven continuous core boreholes (total perforation 304.35 m) furnishing vertical information on the non-accessible parts of the deposit.
3) A description of the fractures obtained from profiles of the areas being exploited. The fractures were characterized according to direction, dip, spacing, opening, filling and roughness, for a total of 312 diaclases and 41 faults.

Figure 2 depicts the map of granite qualities and a number of profiles obtained at outcrops (P1 to P6). Along with the borehole data, this information was the basic input to the study.

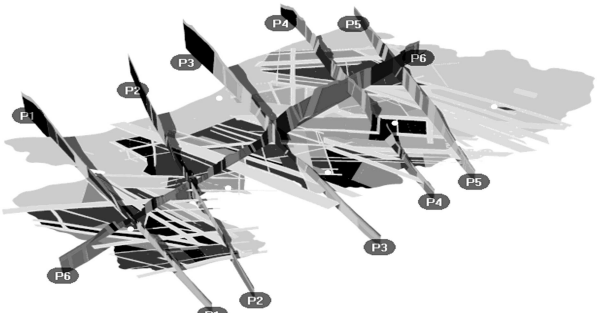

Figure 2. Map of granite qualities constructed from outcrops. Darker tones correspond to higher qualities of granite. White circles represent the location of the boreholes

2.2.2 *Compositional kriging*

Composite data are a set of non-negative vectors such that the sum of their components is a constant k. This constant is normally $k = 100$ when working with percentages, or $k = 1$ when the data is given as proportions. Denoting as $x \in D \subset \mathbf{R}^d$ the centroid of a primary block in domain D of the real space $\mathbf{R}^d$, d =1,2,3, we can define the composite random function as $\boldsymbol{Z}(x) = \left(Z_1(x), \ldots, Z_p(x)\right)$, in such a way that the one-dimensional variables $Z_j(x), j = 1, \ldots, p$ reflect the $j-$th component of the composition. Note that $Z_j(x) \geq 0$ and

$$\sum_{j=1}^{p} Z_j(x) = k \quad \forall x \in D \subset \mathbf{R}^d.$$

Given a sampling realization $\{\boldsymbol{z}(x_1), \boldsymbol{z}(x_2), \ldots, \boldsymbol{z}(x_n)\}$ for the composite random function $\boldsymbol{Z}(x)$, where each $\boldsymbol{z}(x_i) = \left(Z_1(x_i), Z_2(x_i), \ldots, Z_p(x_i)\right)$ is a composite datum, the aim is to infer the value of $\boldsymbol{Z}(x_0)$ for a new location of interest x_0.

Classical spatial statistical methods enable forecasting using kriging and the variables $Z_j(x_i), i = 1, \ldots, n$, according to $\hat{Z}_j(x_0) = \sum_{i=1}^{n} \lambda_i Z_j(x_i)$, or using cokriging and the entire set of variables $Z_j(x_i); i = 1, \ldots, n;\ j = 1, \ldots, p$, in accordance with the expression $\hat{Z}_j(x_0) = \sum_{j=1}^{p} \sum_{i=1}^{n} \lambda_{i,j} Z_j(x_i)$. However, it is a demonstrated fact that neither of these prediction methods is guaranteed to preserve the particularities of the composite data. See [12] and [13] for a more detailed explanation of these classical prediction methods.

Walvoort and de Gruijter [14] proposed a prediction method based on classical approaches. These authors included in the matrix system the constraints necessary for the predictions to take values that would be admissible in the composite random function. Other authors [15] proposed a transformation of the sample data $\boldsymbol{z}^*(x_i) = f\left(\boldsymbol{z}(x_i)\right); i = 1, 2, \ldots, n$ before applying any of the spatial prediction methods in order to obtain $\hat{\boldsymbol{z}}^*(x_0)$. If the function f has been correctly selected, admissible composite data can be obtained by inverting the transformation: $\hat{\boldsymbol{z}}(x_0) = f^{-1}\left(\hat{\boldsymbol{z}}^*(x_0)\right)$.

For this research we implemented a compositional kriging method based on the methodology developed by Tolosana-Delgado [16] and Tolosana-Delgado et al. [17]. This procedure can be viewed as a generalization of the log-normal and normal kriging techniques in $\mathbf{R}^+$.

The subset S of $\mathbf{R}^p$ formed of non-negative vectors and verifying that the sum of their components is one, can be equipped with the inner sum, external product and scalar product operations: $\{S, \oplus, \otimes, \langle \cdot, \cdot \rangle_S\}$. The space, called a Simplex, is a Euclidean $(p-1)$-dimensional space. Tolosana-Delgado [16] demonstrated that the kriging techniques can be generalized to the Euclidean simplex space and that optimal predictors can be obtained for random functions whose sample spaces are contained in a simplex. Another interesting fact is that a Euclidean space allows the selection of an orthonormal base, the calculation of the real coordinates of the elements in the simplex space with respect to this base, and the application of classical prediction methods. Changing the coordinates, the predictions can be expressed as elements in the original Euclidean space (i.e., the simplex). Furthermore, the equivalence between obtaining the predictors in the simplex space and calculating predictors based on changing the variable using an orthonormal base has been demonstrated. It can also be demonstrated that compositional kriging is the optimal predictor since it minimizes the expected Aitchinson distance between the true composition $Z(x_0)$ and its prediction $\hat{Z}(x_0)$ [16].

The procedure can be briefly summarized as follows:

a) The sample space of the composite data, contained in $S \subset R^p$, is transformed by means of a change of coordinates in a new dimension space $p-1$:

$$\boldsymbol{z}^*(x_i) = \boldsymbol{\Psi} \ln\left(\boldsymbol{z}(x_i)\right); i = 1,2,\ldots,n$$

where $\boldsymbol{\Psi}$ is the coordinate-change matrix, of dimension $(p-1)\times p$, formed of the vectors of the orthonormal base arranged in columns, $\boldsymbol{z}^*(x_i) = \left(z_1^*(x_i),\ldots,z_{p-1}^*(x_i)\right)^t$, $\ln\left(\boldsymbol{z}(x_i)\right) = \left(\ln\left(z_1(x_i)\right),\ldots,\ln\left(z_p(x_i)\right)\right)^t$, and where the superscript t means transposed.

(b) Obtained in the Euclidean space R^{p-1} using traditional cokriging techniques is the prediction $\hat{\boldsymbol{z}}^*(x_0)$.

c) The value of the prediction, $\hat{\boldsymbol{z}}(x_0) \in S$ is given by:

$$\hat{\boldsymbol{z}}(x_0) = C\left(\exp\left(\boldsymbol{\Psi}^t \hat{\boldsymbol{z}}^*(x_0)\right)\right)$$

with

$$C\left(\left(\varepsilon_1,\ldots,\varepsilon_p\right)^t\right) = \left(\varepsilon_1 \Big/ \sum_{k=1}^{p}\varepsilon_k,\ldots,\varepsilon_p \Big/ \sum_{k=1}^{p}\varepsilon_k\right)^t$$

a normalization operator. This methodology ensures admissible composite predictions.

For point (b) above, semivariograms, $\gamma_{jj}(h)$ and $(p-1)\times(p-2)/2$ cross-semivariograms, $\gamma_{jj'}(h)$, have to be calculated and fitted. The theoretical models selected to model the experimental semivariograms should verify that the variance of any linear combination of these variables is always non-negative. Put another way, it should be ensured that the prediction variance is always non-negative. To resolve this problem of model selection, the linear co-regionalization model is usually used. To ensure that the variance of any linear combination of the variables $Z_j(x)$ is always non-negative, the coefficients of the semivariograms cannot be chosen randomly but must have certain conditions verified, and this affects the process of fitting the theoretical models. See [17], [18] and [19] for a more detailed discussion on the linear co-regionalization model.

Following the structural analysis stage, in which the experimental semivariograms are suitably estimated, cokriging systems are used to estimate the random function $\boldsymbol{Z}^*(x)$

$$\hat{\boldsymbol{Z}}^*(x) = \left(\hat{Z}_1^*(x),\ldots,\hat{Z}_{p-1}^*(x)\right);\ \hat{Z}_j^*(x) = \sum_{j'=1}^{p-1}\sum_{i=1}^{n}\lambda_{i,j'}Z_{j'}^*(x_i),\ j = 1,\ldots,p-1$$

A detailed description of the compositional kriging algorithm can be found in [20].

3. RESULTS

The composite data used in this study consists of a set of n = 35,543 values such that its p = 4 components added up to one. Given an orthonormal base for a vector space of dimension p -1 = 3, $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$, the coordinate-change matrix was constructed by arranging the base elements in columns: $\boldsymbol{\Psi} = (\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$. Following a study of the principal components of the sample data, we obtained the orthogonal base that determined the main directions of variability in the observations. This base, previously normalized, constituted the orthonormal base that would give rise to the matrix $\boldsymbol{\Psi}$. As can be confirmed in [16], the choice of the orthonormal base has no great bearing on the final results. However, choosing directions close to maximum variability aims to reflect as best as possible underlying behaviour in terms of granite quality proportions. The orthonormal base was thus formed of the following vectors:

$$\boldsymbol{e}_1 = (0.4243, -0.2828, -0.1414, 0.8485)^t$$

$$\boldsymbol{e}_2 = (0.3518, 0.3518, -0.8543, 0.1506)^t$$

$$\boldsymbol{e}_3 = (-0.668, 0.7391, 0.0142, -0.0853)^t$$

Given that several components with zero value were recorded in the sample data, a positive constant was added in prior to changing the coordinates $\boldsymbol{z}^*(x_i) = \boldsymbol{\Psi}\ln\left(\boldsymbol{z}(x_i)\right); i = 1,2,\ldots,n$. It should be borne in mind that adding a constant to the data to avoid zero values might introduce some subjectivity in the results since any

error in the kriging or variance estimations is exponentially magnified.

In order to fit the linear co-regionalization model, used were $r = 2$ incorrelated variables with the following characteristics: $\gamma_1(h)$ pure nugget effect semivariogram with partial sill 1 and $\gamma_2(h)$ spherical semivariogram with range = 280 m and partial sill 1. The coefficients $\beta^r_{jj'};\ j, j' = 1,2,3;\ r = 1,2$ that complete the models were fitted using the R freeware [21].

Finally, using cokriging techniques we obtained the predictions $\hat{z}^*(x_0)$ and, after suitable transformation, the composite predictions $\hat{z}(x_0)$.

Figure 3 shows a map of the granite qualities estimated using composite kriging. The map corresponds to a height above sea level of 205 meters. It can be seen that quality 4 is the predominant.

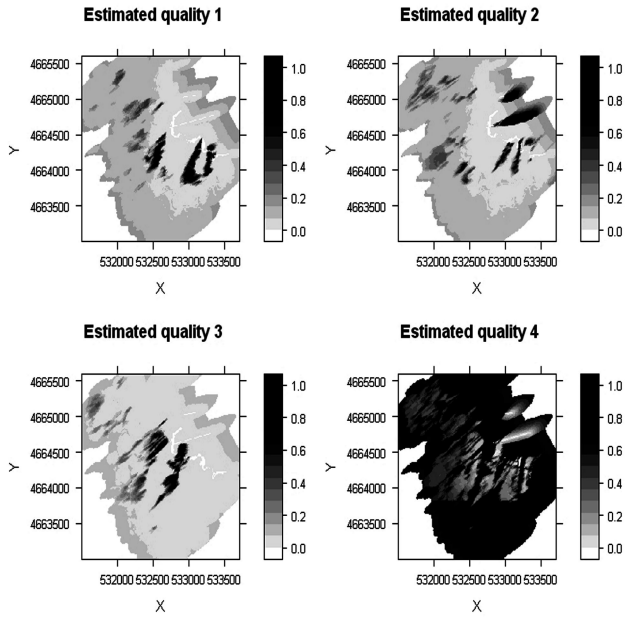

Figure 3. Maps showing the granite quality obtained using compositional kriging for a height above sea level of 205 meters. Each map corresponds to one of the qualities. Darker tones represent a higher quantity of granite for that quality

The compositional kriging described earlier was validated using a cross-validation procedure. An element was removed from the sample, $z(x_i);\ i = 1,\ldots,35,543$, and the prediction $\hat{z}(x_i)$ was calculated using the remaining data. The squared errors of the prediction were obtained for each quality proportion as $SE_j = \left(z_j(x_i) - \hat{z}_j(x_i)\right)^2;\ j = 1,\ldots,4$. These values are a good indicator of the efficacy of the prediction method. Table 1 shows descriptive coefficients calculated for the squared prediction errors and obtained by means of cross-validation. Note that $0 \le SE_j \le 1;\ j = 1,...,4$, given that the composite data reflect the proportions for the different qualities observed in each block.

These results were somewhat improved in comparison with those obtained using ordinary cokriging and adjusting the results to obtain non negative and constant sum constraints of compositional data. The mean squared errors were 0.014, 0.025, 0.021 and 0.036 for quality 1 to quality 4, respectively. The standard deviations were 0.051, 0.055, 0.052 and 0.089, respectively.

Taboada et al. [8] described a fuzzy kriging study performed using the same database. In their study, the mean squared errors obtained in a cross-validation procedure were 0.021, 0.039, 0.053 and 0.037 for quality 1 (top quality) to quality 4 (aggregate quality), respectively. The improvement in our research is evident, as the means have been reduced by values between 18.9% (quality 3) and 79.2% (quality 3).

4. CONCLUSIONES

In this research we estimated the reserves in one of the world's most important granite deposits, which sustains a large number of companies and provides employment for a significant number of people. Knowledge of the volume of reserves and distribution in terms of different quality grades is crucial to be able to implement rational exploitation over time and ensure the quarry's viability in the medium and long term. The estimation

method used in this research—compositional kriging—takes account of the fact that each block extracted from the quarry is likely to contain granite of different qualities, and, unlike other prediction methods—such as classical cokriging—ensures that the sum of the different qualities is 100% for each block. Although at first sight the method may appear to be complex, it is easily implemented in high-level language programs like R.

The results obtained indicate that the method is an improvement over other geostatistical methods, specifically, ordinary cokriging and fuzzy kriging. Any improvement in techniques to estimate qualities in granite blocks is of relevance for the companies quarrying the granite, as there are significant differences in price for the different quality grades.

Table 1. Mean squared errors and corresponding standard deviations calculated for each granite quality by means of cross-validation

	SE_1	SE_2	SE_3	SE_4
No. of values used	35,543	35,543	35,543	35,543
Mean	0.012	0.013	0.011	0.03
Std. Deviation	0.053	0.056	0.052	0.084

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