UNIVERSIDAD NACIONAL DE COLOMBIA

Hierarchical Robust Real Time Optimization with Zone Control

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Medellín, Colombia
2014

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PhD. dissertation presented as partial requirement to obtain the degree:
Doctor in Engineering

Director(a):
Ing. Jairo José Espinosa Oviedo PhD.

Research Line:
Large-Scale Systems Control
Research Group:
Grupo de Automática de la Universidad Nacional de Colombia (GAUNAL)

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2014

When one admits that nothing is certain one must, I think, also admit that some things are much more nearly certain than others.

Bertrand Russell

Acknowledgments

I am heartily thankful to my supervisor, Jairo José Espinosa Oviedo, whose encouragement, guidance and support from the initial to the final level enabled me to develop an understanding of the subject, despite his many occupations you always had a minute to give me ideas or tips.

I am grateful with the Universidad Nacional de Colombia, Facultad de Minas, the scholarship of *"estudiantes sobresalientes de posgrado"* and the Hierarchical and Distributed Model Predictive Control of Large-Scale Systems (HDMPC) project of the EU for the support during this work. I want to thank to the Grupo de Automatica de la Universidad Nacional (GAUNAL) specially to the boys of the "Línea de Matemáticas Avanzadas para el Control y los Sistemas Dinámicos", in particular to Julian Patiño, Pablo Deossa and Felipe Valencia for sharing enjoyable times both inside and outside the office. An special thanks to Cristian Portilla and Cesar Gomez for their contributions in this thesis.

Last but no least I want to thank my family, to my mother Beatriz Helena, my sisters Luisa and Laura. To Eliana thank you for being with me through thick and thin without any reproach, thank you for making me better.

Alejandro Marquez.

Resumen

El problema de control de sistemas de gran escala y en red se resuelve normalmente dividiendo el problema y aplicando técnicas locales de modelamiento y control a subsistemas más pequeños y más manejables . Dado que una partición no es natural, los subsistemas no necesariamente intercambian la información apropiada y los controladores no se comportan como deberían. La falta de cooperación hace que los controladores interactuen de manera inesperada lo cual no fue considerado en la fase de diseño. Como resultado de ello, el sistema completo puede ser muy fragil e incluso ser inestable en presencia de perturbaciones no modeladas. Actualmente, es bien sabido que con el fin de obtener un funcionamiento óptimo global de un sistema, es necesario medir, estimar y actuar en base a la información global. Hacer estas tareas es difícil e implica un serio compromiso entre la complejidad y fiabilidad, este compromiso debe ser negociado con el fin de obtener resultados reales y aplicables.

En los problemas de control de sistemas de gran escala, la eficiencia, la tratabilidad del control y el modelo son puntos claves: es necesario calcular una buena acción de control y simular el sistema de una manera precisa, teniendo en cuenta que todos los subsistemas no necesariamente tienen el mismo comportamiento dinámico, y la simulación de todo el sistema puede tener una alta carga computacional. Una partición jerárquica basada en la dinamica temporal puede ser propuesta con el fin de reducir el tamaño y la carga computacional del problema. En ese sentido y con el fin de hacer frente a estos problemas, el uso de control distribuido en el que el sistema se subdivide en varias subregiones es necesario. También habrá una distribución temporal de los controladores, lo que resulta en una estructura jerárquica en la que los controladores de nivel inferior se ocupan de la dinámica rápida en una región pequeña y en el que los controladores de nivel más alto cuidan de las dinámicas más lentas y hacen la coordinación sobre una región más grande. Así que en la configuración anterior, los sistemas de mayor funcionalidad residen en niveles más altos, mientras que en los niveles inferiores de las unidades individuales, deben garantizar funciones especificas. Además del seguimiento de referencias y objetivos económicos que se consideren explícitamente.

En esta tesis se presenta un enfoque de control jerárquico aplicado a sistemas de gran escala mediante el control predictivo basado en modelo por zonas. Este método utiliza una integración de la optimización dinámica en tiempo real (DRTO) y el control predictivo basado en modelo por zonas. En general, el control de las diferentes variables se establece en una referencia definida, pero en ocasiones, esto puede afectar la viabilidad de la solución es especial cuando el sistema está altamente interconectado. En general en los sistemas de gran escala las variables no necesariamente deben estar en una referencia específica, allí es cuando se utiliza el control por zonas, donde las variables de salida se mantienen en una zona determinada. Control por zonas es un enfoque que ayuda a encontrar una solución factible, ya que libera las variables para estar en una zona especifica y no en una referencia fija. En el enfoque de control jerárquico propuesto, en la capa superior se solucionan dos problemas: una optimización dinámica en tiempo real (DRTO) y una optimización dinámica en tiempo real robusta (RDRTO), por medio de estos problemas de optimización es posible encontrar los limites (desde el punto de vista económico) y trayectorias de referencia optimas para el coordinador (capa intermedia). El coordinador calcula las variables de entrada jerárquicas y referencias de salida que estan siempre en la zona especificada,

esta información es tomada por los controladores de los subsistemas (capa inferior) para generar las variables de entrada que se van a aplicar.

Palabras clave: Control jerárquico, control predictivo, control robusto, sistemas de gran escala.

Abstract

The control problem regarding to large, complex and networked systems is commonly solved by splitting the problem and applying local modelling and control techniques to the smallest and more manageable subsystems. Since a partition is not natural, the subsystems not necessarily exchange the appropriate information and controllers do not behave as they should. The lack of cooperation causes that the controllers interact in unexpected ways that were not considered in the design phase. As a result, the full system may have fragility and even instability in presence of unmodelled disturbances. Currently, it is well known that in order to obtain a global optimal operation of a system it is necessary to measure, to estimate, and to act based on global information. Doing these tasks is difficult and implies a very serious trade-off between complexity and reliability. This compromise must be negotiated in order to obtain real implementable results.

In the control problems in large scale systems, the tractability of the control and the models, as well as the efficiency are key issues: it is necessary to compute a good control input and to simulate the system in an accurate way, but given that all subsystems not necessarily have the same behaviour and time dynamics, and the simulation of the whole system can have a high computational cost, a temporary partition can be proposed in order to reduce the size and the computational burden of the problem. In order to deal with these issues the use of a spatially distributed control approach where the system is subdivided into several subregions is necessary. There will also be a temporal distribution of the control, resulting in a hierarchical structure where the lower-level controllers take care of the fast dynamics in a small region, and where the higher-level controllers take care of the slower dynamics and the coordination over a larger region. So in the previous configuration, systems of higher functionality reside at higher levels, while at lower levels the single units, must guarantee specific functions. In addition to tracking control, economical objectives are considered explicitly.

In this thesis a hierarchical control approach applied to large scale systems is presented using model predictive control with zone control. This approach uses an integration of dynamic real time optimization (DRTO) and model predictive control with zone control. Generally the control of different variables is set to a defined set point, but sometimes, this could affect the feasibility of the solution, when the system is highly interconnected, it is difficult to follow a given set point, but sometimes the variables do not need to be at certain set point, that is when zone control is used, where the output variables are controlled within a determined zone. Zone control is an approach that helps to find a feasible solution, because it releases the variables to be in a certain zone and not to a determined point. In the proposed hierarchical control approach, in the upper layer two problems are solved: The dynamic real time optimization (DRTO) and a Robust Dynamic Real time optimization (RDRTO), by means of these optimization problems it is possible to find the limits (economical point of view) and optimal reference trajectories for the coordinator (middle layer) that calculates hierarchical input variables and output references that are always into the designed zone, this information is taken by the subsystems controllers (lower layer) to generate the input variables are to be applied.

Keywords: Hierarchical control, Model predictive control, Robust Control, large scale systems

Contents

CHAPTER 1

Introduction

Large scale systems are frequently composed of several subsystems with special characteristics such as complex dynamics, a large number of actuators, sensors and a strong interconnection such that the local control decisions have a deep impact in the system operation. There exist many examples of large scale systems such as power valleys, transportation systems and process plants among others. The particularities of large scale systems offer several problems that must be approached for the designer of an overall control system, in order to achieve a safe, efficient and robust operation. If these features are not satisfied serious disasters and malfunctions could occur (such as the breakdowns of the power grid in North America and in Italy in 2003) [1].

In order to handle the mentioned problems and to deal with the complexity of the control task, a hierarchical control structure where the control tasks are distributed over time and space is proposed. In this approach, control systems of supervisory and strategic functionality will be located at higher decision levels (or layers), while at lower levels single control units must guarantee operational objectives. In this way, we will develop methods for designing controllers for large-scale systems based on a hierarchical control framework. There are several works in hierarchical control with MPC as the reported in [2, 3, 4], however, we propose the use of robust optimization and Model Predictive Control (MPC) with zone control, which have already proven its usefulness for control of small-scale systems, but whose performance for the control of large-scale systems has yet to be determined due to computational, coordination, and communication problems.

This chapter presents the motivation for the research addressed in this thesis. In Section 1.1 a description of the type of systems considered in this thesis (large scale systems) and the different approaches for controlling these systems are shown. In section 1.2 a discussion of the use of hierarchical control for large scale systems and a motivation on the use of model predictive control with zone control is presented. In sections 1.3 and 1.4 the academic and industrial impact of this thesis is explained. Finally, section 1.5 presents the objectives of this work and in section 1.6 the thesis outline and the main contributions of this research are shown.

1.1 Control of large scale systems

The design of controllers for complex nonlinear systems is a very hard task that rapidly becomes intractable accordingly with the system dimension [5]. If the system becomes large-scale the controller design turns to be intractable and several problems such as the lack of an effective mathematical framework upon the modeling, simulation and control become apparent [6] generating high computational requirements. This is an important problem because it does not allow real-time implementations in control. Other problem presented by large-scale systems is the possible existence of unmodeled dynamics, due to model reductions [7] or the direct decision of the model developer. The lack of representation of certain dynamics (usually faster dynamics) could lead to some effects in the stability or performance of the system.

In order to minimize computational requirements a large-scale system is decomposed into several subsystems [8]. This decomposition brings two approaches to control the system. One of them is based on total decentralization of control tasks. This approximation takes every subsystem and designs a control policy without considering the possible interactions among subsystems. With this approach an optimal control solution is not found, possibly there exists a poor controller design and finally the ability of handling input/output constraints is restricted. This approach is preferred often for reliability reasons and simplicity of design [8]. The other approach is splitted in decomposition-coordination and decomposition-cooperation methods and both are used for optimization purposes. The main problem with these methods arises from the communication and bandwidth limitations. The decomposition-coordination methods created a base for hierarchical decomposition when they were extrapolated to multilevels to deal with more complicated control problems [9].

1.2 Hierarchical control for large scale systems

Since the seventies [10], hierarchical structures have been a strategic form to provide solutions in description and control of large and more complicated problems. Current paradigm of control uses a mathematical representation of the system as an important resource to generate an optimal control policy. This mathematical representation is an important issue to achieve the goal of generating an adequate decomposition. Until 1970 there were three ways reported to hierarchize a system [10]. These ways are still being used and they are based on three heuristic principles: the ability to make abstractions from reality, ability to differentiate the complexity level in a set of objectives and the ability to make decisions.

Control of large, complex, networked systems is invariably performed by applying local modeling and control techniques to the smaller, more manageable subsystems of which they are comprised. There is little cooperation between local controllers i.e. not always the local controllers share information, often from the design phase, in spite of their possible interaction in many unexpected ways. As a result, the full system may display fragility and even instability in the face of disturbances or uncertainties. Currently, it is well known that in order to obtain global optimal operation of a system it is necessary to measure, to estimate, and to act based on global information. Doing these tasks is difficult since a very serious trade-off between complexity and reliability must be negotiated in order to obtain real implementable results. In general, centralized systems offer global optimizing strategies but they are computationally expensive or even intractable, since they need large and detailed models covering the whole system. These applications demand regular maintenance and retuning. However, such tasks are very difficult for large systems because they demand long periods of testing and tuning, due to the complex nature of the system and the strong assumption of total interaction among units. On the other hand, decentralized systems tend to demand less efforts during maintenance and model updating but at the expense of a sub-optimal operation.

For both the control and the models tractability and efficiency are key issues: It's necessary to compute a good control input and to simulate the system in an accurate way, within a given time period. This results in a trade-off between accuracy and optimality on the one hand, and computational effort on the other hand. In order to deal with these issues the use of a spatially distributed control approach where the system is subdivided into several sub-regions, each controlled by a separate controller. There will be also a temporal distribution of the control, resulting in a hierarchical structure where the lower-level controllers respond to the fast dynamics in a small region, and where the higher-level controllers take care of the slower dynamics and the coordination over a larger region. In a similar way as the control is spatially and temporally distributed, we will also use multi-level, multi-resolution models, i.e., models with various levels of spatial and temporal aggregation. So in the proposed set-up systems of higher functionality reside at higher levels, while at lower levels the single units, or local agents, must guarantee specific functions. At any level, the local agents must negotiate their outcomes and requirements with lower and higher levels. The topology of the system and of the information exchange between subsystems either at the same level or at different levels of the hierarchy can be fixed or partially modified as a function of the operating conditions while satisfying the safety and integrity requirements.

For this reason there is a need for developing controllers that explicitly account for the properties of distributed process systems, in order to design highly efficient, stable, and robust algorithms which drive distributed systems to economical optimality (to maintain the plant operation near an economic optimum) in the presence of disturbances and other external/internal changes. Furthermore, robustness requirements will also be considered for dealing with the intrinsic decentralized structure of the control system, with plant uncertainty. A further robustness issue can also be due to the impossibility for some units to guarantee the performance required by higher decision levels.

In this way, robust hierarchical controllers for complex large-scale system are developed in this thesis. The main goal is to achieve an optimal coordination of the lower layers of the hierarchical decomposition avoiding high computational requirements, reaching the level of accuracy required in the system representation and accommodating the control, modeling and simulation problems of the subsystems in the mathematical framework available. In order to achieve that, zone control concept is used in this thesis. In the next section Model Predictive Control with zone control is introduced and its potential for application to large scale systems is shown through two simulation examples.

1.2.1 Model Predictive Control with zone control

Through the optimal multivariable control strategies that exist and can be used in the problem, the one that is well studied and suitable for this type of problem is the Model Predictive Control (MPC), because is an optimal control strategy that can handle constraints. MPC can also manage the strong interaction between the states in large scale and networked systems, a particular characteristic in multivariable control that is difficult to handle. Among the different MPC approaches available in the literature, there is one of special interest. Generally the control of different variables is set to a defined set point, but sometimes, this could affect the feasibility of the solution, when the system is highly interconnected, it is difficult to follow a given set point, but sometimes the variables do not need to be at certain set point, but within a range that is when zone control is used, where the output variables are controlled to remain inside a zone. Zone control is an approach that helps to find a feasible solution, because it relaxes the variables to be in a certain zone and not in a fixed point. Often zone control is used for controlling complex systems where:

- Interactions among state, input and output variables have a high complexity.
- The manipulated variables are not enough to drive all states and/or inputs to the desired values.
- There exists high uncertainty in the system parameters and structure

Furthermore, it can be used in systems where smooth control actions are required instead of regulating the outputs and/or states to a desired value [11]. Therefore, MPC with zone control is used in cases where some constraints must be relaxed, or when a soft closed-loop operations is desired. For example, in real large scale systems where the complexity of interactions is difficult to handle (as is the case of large chemical plants), MPC with zone control becomes an effective control alternative. In addition, relaxation of the dynamic model constraints makes the MPC with zone control more robust than the traditional MPC [12]. Several approaches have been reported in the literature. In [13] a complete survey of industrial MPC applications with zone control is described. In this thesis, specifically two alternatives for implementing MPC with zone control are explored:

1. Defining upper and lower soft constraints.
2. Using the set-point approximation of soft constraints to implement the upper and lower zone boundaries.

The main drawback of these algorithms is the lack of nominal stability. In fact, these strategies were applied to a fluid catalytic cracking system in [14], but despite of the acceptable performance the closed-loop stability cannot be proved (the control system kept switching between the two alternative modes even when infinite horizon MPC was used). Addressing the stability downside of MPC with zone control, in [15] the authors proposed a control strategy where economic steady state targets were included. With this contribution, classical stability proofs were extended to the zone control strategy by considering the output set-points as additional decision variables of the control problem.

However, it is required the system to be stable in order to guarantee the conditions of the stability proofs (the authors considered a null controller as local controller). In order to extend the results of [14], in [12] the authors proposed a robust MPC with zone control. In this approach, multi-model uncertainty was assumed. In this direction the control objective of the zone control is analyzed as a set of objectives or target set in the output space. This was motivated by the fact that there is no preference about selecting one point over another inside the operating zone. In the next subsections two examples will be shown in order to motivate to use model predictive control with zone control. The first is a typical example in hierarchical control and was used by Tatjewski in [16] to illustrate different multilayer architectures in process optimization. The second one is a four-tank process, an example previously used in the development of distribute controllers [17] and by means of this the importance of zone control in large scale systems is established.

1.2.1.1 MPC with zone control for a Polymerization reactor

In order to show the performance of MPC with zone control, in this section a polymerization jacketed continuous stirred tank reactor (see Figure 1.1) is used [16]. The main idea is to compare the performance of a linear MPC with zone control versus a nonlinear MPC under disturbances. The output variable is the number average molecular weight $y = NAMW$ which will be controlled by manipulating the inlet initiator flow rate $u = F_I$. The main disturbance in the process is the unmeasured feed flow of the monomer and solvent stream $d = F$. The control objective of the process is to maintain the output of the process in a specific value (quality specifications) under unmeasured disturbances. The fundamental nonlinear process model is as follows:

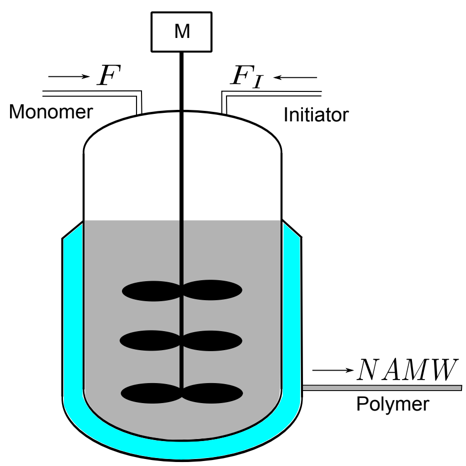


Figure 1.1: Polymerization reactor

$$\begin{aligned}
\dot{x}_1 &= 10(6 - x_1) - 2.4568x_1\sqrt{x_2} \\
\dot{x}_2 &= 80F_I - (0.10225 + 10F)x_2 \\
\dot{x}_3 &= 0.0024121x_1\sqrt{x_2} + 0.112191x_2 - 10x_3 \\
\dot{x}_4 &= 245.978x_1\sqrt{x_2} - 10x_4 \\
NAMW &= \frac{x_4}{x_3}
\end{aligned}$$

In order to find the operating point of steady-state operation we consider the disturbance $F = 2\ m^3/h$. With this the optimum operating point is,

$$\begin{aligned}
x_{ss} &= [5.6602, 0.0597, 0.0010, 34.0173]^T \\
u_{ss} &= 0.0150 \\
y_{ss} &= 3.3905 \times 10^4
\end{aligned}$$

The conventional nonlinear MPC formulation used in this example is as follows:

$$\begin{aligned}
&\min_{u_k} \sum_{j=1}^{N_p} \|(y(k+j) - y_{ss}\|_Q^2 + \sum_{j=0}^{N_p} \|u(k+j)\|_R^2 \\
&\quad + \sum_{j=0}^{N_p} \|\Delta u(k+j)\|_S^2 + \|x(k+N_p)\|_{\tilde{Q}}^2 \\
&\text{subject to:} \\
&\quad x(k+j+1) = f(x(k+j), u(k+j)) \\
&\quad y(k+j) = g(x(k+j), u(k+j)) \\
&\quad 2 \times 10^4 \leq y(k+j), \quad j = 0, \ldots, N_p \\
&\quad 0.0035 \leq u(k+j) \leq 0.033566, \quad j = 0, \ldots, N_p \\
&\quad \|u(k+j) - u(k+j-1)\| \leq \Delta u_{\max}
\end{aligned} \tag{1.1}$$

Now, the MPC optimization problem for implementing the zone control in the polymerization reactor is as follows:

$$
\begin{aligned}
&\min_{u_k, y_{ref,k}} \sum_{j=1}^{N_p} \|(y(k+j) - y_{ref,k}\|_Q^2 + \sum_{j=0}^{N_p} \|u(k+j)\|_R^2 \\
&\quad + \sum_{j=0}^{N_p} \|\Delta u(k+j)\|_S^2 + \|x(k+N_p)\|_{\tilde{Q}}^2 \\
&\text{subject to:} \\
&\quad x(k+j+1) = Ax(k+j) + Bu(k+j) \\
&\quad y(k+j) = Cx(k+j) + Du(k+j) \\
&\quad 2 \times 10^4 \leq y(k+j), \quad j = 0, \ldots, N_p \\
&\quad 0.0035 \leq u(k+j) \leq 0.033566, \quad j = 0, \ldots, N_p \\
&\quad 3.3 \times 10^4 \leq y_{ref,k} \leq 3.5 \times 10^4 \\
&\quad \|u(k+j) - u(k+j-1)\| \leq \Delta u_{\max}
\end{aligned}
\tag{1.2}
$$

where $u_k = [u^T(k), \ldots, u^T(k+N_p)]^T$ be the control sequence trajectory, with N_p the prediction horizon and $Q \in \mathbb{R}^{1\times1}$, $R \in \mathbb{R}^{1\times1}$ and $S \in \mathbb{R}^{1\times1}$ are positive definite weighting matrices. Note that in the MPC with zone control formulation the sequence of reference values $y_{ref,k}$ constitutes an additional decision variable of the optimization problem and this is subject to a constraint $3.3 \times 10^4 \leq y_{ref,k} \leq 3.5 \times 10^4$ (zone). The zone control concept is used in this thesis with the essential goal of incorporating this extra variable to the optimization problem.

Simulation results: In order to evaluate the controller performance, a change in the feed flow of the monomer (2 m^3/h to 1 m^3/h) was done at 5 h of the simulation test. It is important to note that the change in the feed flow is 100%, this is a big change for the process. Figures 1.2 and 1.3 present the simulation results. Figure 1.2 shows the number of average molecular weight ($NAMW$) for both controllers. Although the disturbances produced significant changes in the controlled variables (the overshoot is significantly large), it is important to note how the configuration with the conventional MPC (Figure 1.2 (a)) cannot reject these effects and a sustained oscillation is present, while the MPC with zone control (Figure 1.2 (b)) finds another set point inside the zone avoiding the disturbance and drives the controlled variable to the new desired value without overshoot and steady-state error. Figure 1.3 shows the behavior of the control actions: the conventional MPC (Figure 1.3 (a)) has an input variable with very fast changes, while the control action of the MPC with zone control (Figure 1.3 (b)) is very soft which could be a better operation for that kind of variables than the fast responses that could damage the actuator system.

Generally speaking, with this example we have demonstrated how the zone control gives freedom to the outputs of the systems (inside the specific zone) with the purpose of reject disturbances, while a conventional controller (in this case a MPC) tries to maintain the system in the same operating point which could drive the system to unstable operation. Thereby the zone control concept can play a key role in hierarchical control for large scale systems.

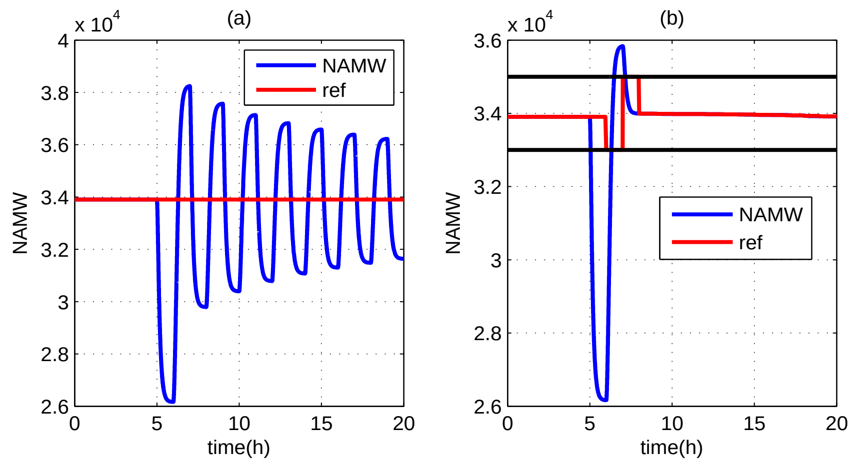


Figure 1.2: Behavior of the number average molecular weight (NAMW) of the polymerization reactor under disturbances at 5 h in F: (a) Conventional MPC, (b) MPC with zone control

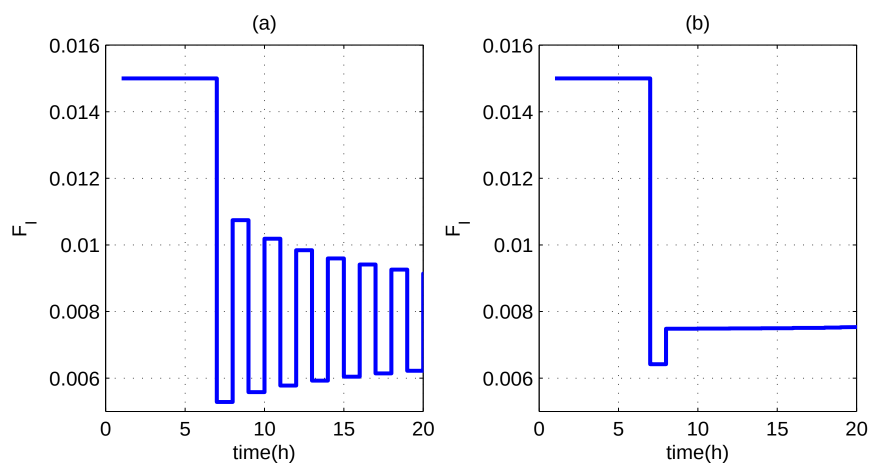


Figure 1.3: Behavior of the inlet initiator flow rate of the polymerization reactor under disturbances at 5 h in F: (a) Conventional MPC, (b) MPC with zone control

1.2.1.2 Decentralized MPC with zone control for a four-tank process

The four-tank process is a laboratory plant that has been designed to test control techniques using industrial instrumentation and control systems. The plant is a hydraulic process of four interconnected tanks inspired by the educational quadruple-tank process proposed by Johansson et al.[18]. The process constitutes a simple multivariable system with highly coupled nonlinear dynamics that can exhibit transmission zero dynamics. The control objective is to regulate the levels of the lower tanks to their desired values, by manipulating the flows feeding the tanks. Figure 1.4 shows an schematic diagram of the quadruple tank process.

In first place, a model given by (1.3) was derived [18]. In this model $h_i(t)$, A_i and a_i with $i \in \{1, 2, 3, 4\}$ refer to the level, cross section and the discharge constant of i-th tank, respectively; $q_j(t)$ and γ_j with $j \in \{a, b\}$ denote the flow and the ratio of the three-way valve of j-th pump, respectively; and g is the gravitational acceleration. The values of the parameters of the system used for the simulations are shown in Table 1.1.

$$\begin{aligned}
\frac{dh_1(t)}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1(t)} + \frac{a_3}{A_1}\sqrt{2gh_3(t)} + \frac{\gamma_a}{A_1}q_a(t) \\
\frac{dh_2(t)}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2(t)} + \frac{a_4}{A_2}\sqrt{2gh_4(t)} + \frac{\gamma_b}{A_2}q_b(t) \\
\frac{dh_3(t)}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3(t)} + \frac{1-\gamma_b}{A_3}q_b(t) \\
\frac{dh_4(t)}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4(t)} + \frac{1-\gamma_a}{A_4}q_a(t)
\end{aligned} \tag{1.3}$$

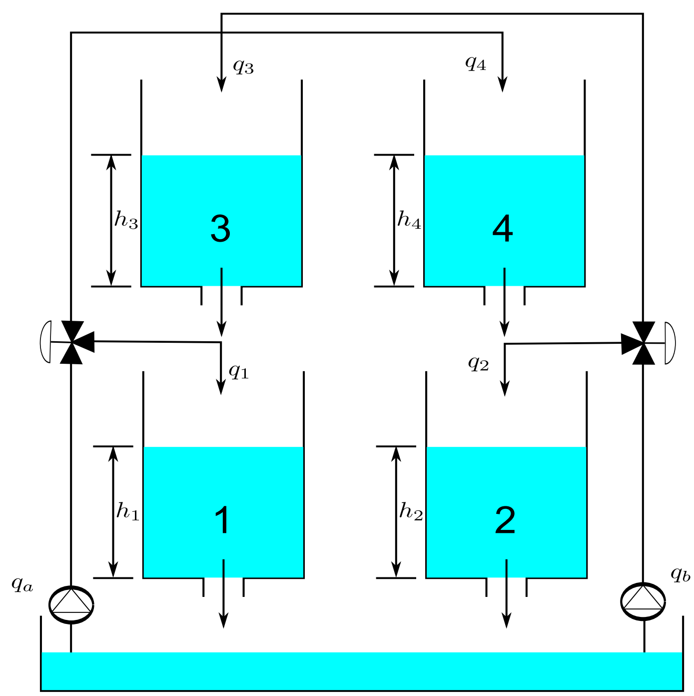


Figure 1.4: Schematic diagram of the quadruple tank system [18].

In order to show the advantages of the MPC with zone control over the typical MPC, two simulations tests are made.

- **First test:**

  The first one test is a decentralized MPC scheme. The idea is to follow the set points proposed in [17]. The mathematical formulation of each local controller in this test is as follows,

	Value	Unit	Description
h_{max}	1.36	m	Maximum level in all cases
h_{min}	0.2	m^3/h	Minimum level in all cases
q_{max}	3.26	m^3/h	Maximum flow of q_a and q_b
q_{min}	0	m^3/h	Minimum flow of q_a and q_b
A_i	0.06	m^2	Cross-section of the tanks
γ_j	0.3		Split ratio of the 3-way valve of q_a and q_b
h_1^0	0.6042	m	Linearization level of tank 1
h_2^0	0.6042	m	Linearization level of tank 2
h_3^0	0.296	m	Linearization level of tank 3
h_4^0	0.296	m	Linearization level of tank 4
q_j^0	1.63	m^3/h	Linearization flow of q_a and q_b
T_s	5	s	Sample time
a_i	1.31×10^{-4}	m^2	Discharge constant of the tanks

Table 1.1: Values of the parameters of the system used in the simulations.

$$
\begin{aligned}
\min_{u_{k_i}} J_{N_{p,i}}(x_k, u_k) = \sum_{j=1}^{N_{p,i}} \|(y_i(k+j) - y_{ref,i}\|_{Q_i}^2 + \sum_{j=0}^{N_{p,i}} \|\Delta u_i(k+j)\|_{S_i}^2 + \|x_i(k+N_p)\|_{\tilde{Q}_i}^2 \\
\text{subject to:} \\
x_i(k+j+1) = A_{ii}x_i(k+j) + B_{ii}u_i(k+j) \\
y_i(k+j) = C_i x(k+j) + D_i u_i(k+j) \\
u_{min} \leq u_i(k+j) \leq u_{max} \\
y_{min} \leq y_i(k+j) \leq y_{max} \\
\|u_i(k+j) - u_i(k+j-1)\| \leq \Delta u_i^{\max}
\end{aligned}
\tag{1.4}
$$

- **Second test:**

  The second simulation test is a decentralized MPC scheme when one of the controllers is a MPC with zone control. In this case a MPC with zone control is responsible of maintaining one output of the system in a non-constant zone i.e. in a variable zone, while the other output follows a fixed set point. The idea with the variable zone is to show that zone control can handle the variations in the additional decision variable $y_{i_{ref,k}}$, this is the case of the hierarchical structure proposed in this thesis, where the limits of the MPC with zone control are calculated by an upper layer and these are not constant in the time. Mathematical formulation of the MPC with zone control is shown below.

$$\min_{u_{k_i}, y_{i_{ref,k}}} J_{N_{p,i}}(x_k, u_k) = \sum_{j=1}^{N_{p,i}} \|(y_i(k+j) - y_{i_{ref,k}}\|^2_{Q_i} + \sum_{j=0}^{N_{p,i}} \|\Delta u_i(k+j)\|^2_{S_i}$$
$$+ \|x_i(k+N_p)\|^2_{\tilde{Q}_i}$$

subject to:

$$\begin{aligned} & x_i(k+j+1) = A_{ii}x_i(k+j) + B_{ii}u_i(k+j) \\ & y_i(k+j) = C_i x(k+j) + D_i u_i(k+j) \\ & u_{min} \leq u_i(k+j) \leq u_{max} \\ & y_{min} \leq y_i(k+j) \leq y_{max} \\ & y_{i_{ref,k}}^{min} \leq y_{i_{ref,k}} \leq y_{i_{ref,k}}^{max} \\ & \|u_i(k+j) - u_i(k+j-1)\| \leq \Delta u_i^{\max} \end{aligned} \tag{1.5}$$

As this is a motivational example, some mathematical details about the controller formulations are not given in this chapter. A complete mathematical formulation of the controllers used here can be found in chapters 3 and 4. Specific details about the four-tank process as the linear models and operating points can be found in Alvarado *et al* [17].

Simulation results: Figure 1.5 shows the results of the first test. Note that at 6000 seg the level of the first tank (Figure 1.5 (a)) can not reach the set point and the input signal of this controller (q_a) is saturated, i.e the local controller for the first tank can not drive the system to the reference, while the controller of the second tank (Figure 1.5 (b)) drives the system to the set point. Then the first tank is unable to reach the desired value because since the input of second tank is not taken into account for the first controller, becoming into a disturbance.

Now, if the level of the first tank is assumed as the most important variable in the system, it is possible to give some freedom to the second level. In this sense a MPC with zone control can be used for the second tank, with a variable zone implemented for this purpose. The main goal is to maintain the level of the first tank in a fixed reference value, while the level of the second tank remains in the specific variable zone. The Figure 1.6 shows the simulation results of the decentralized MPC with zone control. The Figure 1.6 (a) shows the level of the first tank. Note how the controller drives this variable to the fixed set points, while in Figure 1.6 (b) the level of the second tank remains in the zone thanks to the MPC with zone control. Finally, we can say that through this example it is possible to illustrate the utility of the MPC with zone control for interconnected systems. This example constitutes a motivation for using zone control in this thesis, in order to design hierarchical control systems for large scale systems.

1.3 Academic impact of the thesis

In the last decades, technologies have been developed to solve operational problems at different levels in the hierarchical control sense. However, most of them are isolated techniques, each one targeting a single problem exclusively and independently of the problems

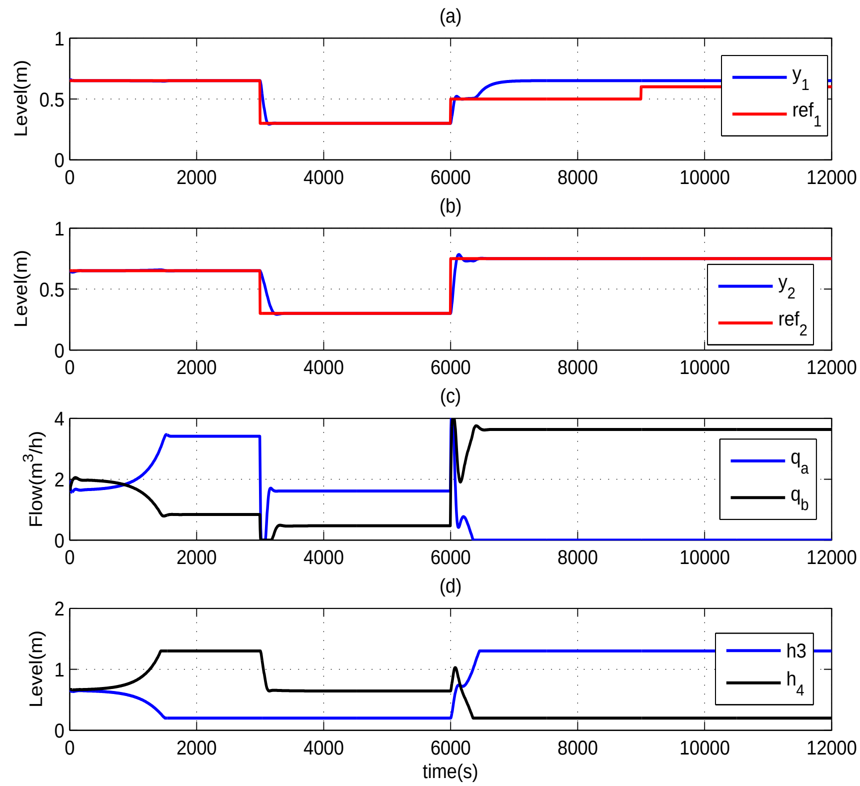


Figure 1.5: Evaluation of the control test of the decentralized MPC with zone control scheme

at other levels. In this thesis we throw light on some unsolved issues in the field of hierarchical control, by the proposition of a novel hierarchical control strategy that combines robust optimization and Model Predictive Control with zone control. A new tool is developed to find ranges of variables of interest of the large scale systems and a new algorithm for robust control is established. Finally, this algorithm is extended to the MPC with zone control and the economical case, and asymptotic stability and robust constraints satisfaction are provided.

1.4 Industrial impact of the thesis

Today industries thrive on the performance of advanced control systems. In a competition based economy, most of the developments in advanced control applications aim at better economic performance of the control system. For this reason, in addition to performing fundamental research on hierarchical control of large-scale systems, we also concentrate on applications. The scope of hierarchical control of large-scale systems is very broad and includes socially and economically relevant application fields such as process plants, power networks, road traffic networks, logistic systems, and autonomous vehicle systems. In this thesis in particular, we will consider applications on, chemical plants and hydro-power

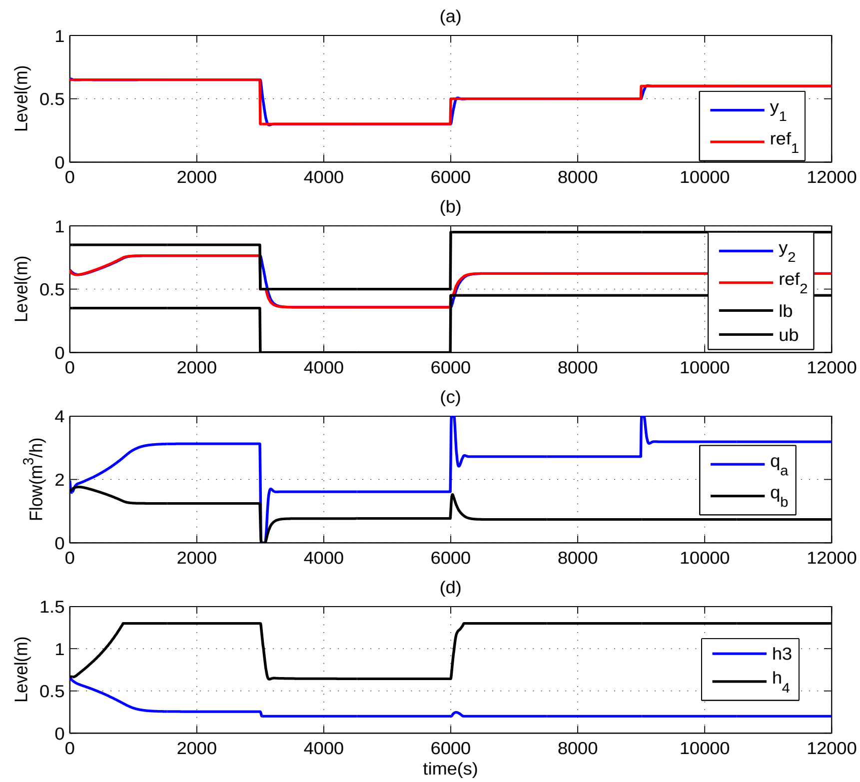


Figure 1.6: Evaluation of the control test of the decentralized MPC with zone control scheme

valley operations. It is important to mention that the method that will be developed in this thesis is generic and will thus also be applicable to a wide range of other scenarios.

1.5 Objectives

The main objectives of this dissertation are summarized in the following lines.

- To propose a methodology of hierarchical control that considers the zone control of the outputs of the systems improving the performance.
- To develop a robust optimization method that allows to generate non-conservative limits at the system output.
- To design at least one communication strategy between layers of the hierarchical structure that allows the solving of real time optimization problems applied to complex systems.

1.6 Overview of the thesis

1.6.1 Thesis outline

This thesis is organized in 7 chapters. Figure 1.7 presents an overview of them as well as the way they relate to each other.

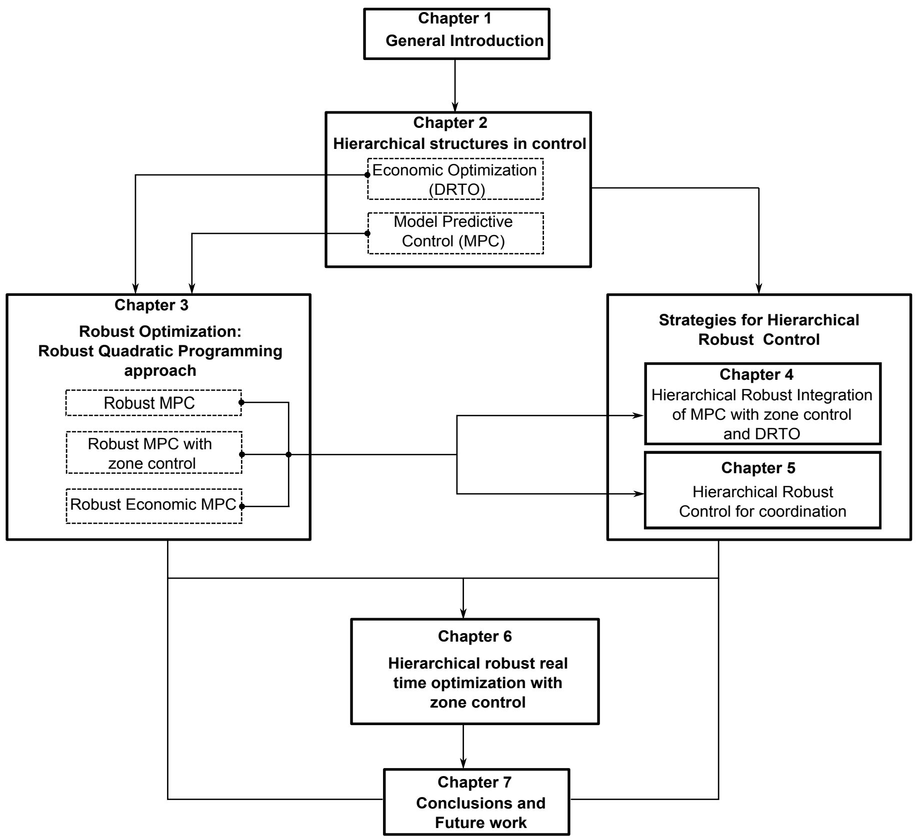


Figure 1.7: Overview and connection between the different chapters in this thesis

A brief description of each chapter is given as follows:

- **Chapter 2:** This chapter presents a literature review of hierarchical structures in control. This chapter serves as a basic for the following chapters that are based in hierarchical control.
- **Chapter 3:** This chapter presents a novel min-max MPC, min-max MPC with zone control and min-max Economic MPC. These new formulations are based in Appendix A where algorithms for Robust Quadratic Programming are presented, and the solution of the robust quadratic program is obtained by transforming the program into a Second Order Cone Program under different uncertain sets. Finally, this chapter presents one example of every robust controller in order to illustrate the advantages of every approximation

- **Chapter 4:** In this chapter a Hierarchical Robust Integration of MPC and DRTO is presented. The main idea is to combine the robust MPC approaches presented in chapter 3 and the zone control concept with the classical hierarchical structure presented in chapter 2. The main advantages of the strategy presented here are the ability of handling uncertainties, the tackling of different objectives (Economic, environmental), and the inclusion of the dynamic behavior of the system. In this way these approaches improve the structures reported in the literature.

- **Chapter 5:** In this chapter a Hierarchical Robust Control for coordination is presented. This strategy arises in problems where the upper layer of the hierarchical structure proposed in chapter 4 is not necessary. In this way in this chapter initially an example is used a motivation to use this strategy. Specifically, the example is a Hydro Power Valley(HPV), this system is a typical large scale benchmark that is used to prove control behaviour [19], because it has several characteristics which establish some control challenges.

- **Chapter 6:** In chapter 4 some strategies for hierarchical robust control using zone control were presented. However, a problem arises from these structures. The problem lies in how to calculate the limits (zone) of the MPC with zone control. Generally these limits correspond to physical limitations of the process, nervertheless, this is not always the case, and the development of a criteria to calculate these constraints is required. A reasonable criterion could be the zone where the linearization is a good approximation to the nonlinear system or physical limitations of the process, however, a better choice can arise from the economic point of view. In this sense the most remarkable contribution of this thesis is a hierarchical robust strategy where the limits of the MPC with zone control are calculated based in an economical criterion, which is presented here. This chapter ends with the application of the hierarchical methodology to a large scale chemical plant, that serves as benchmark to test the structure proposed.

- **Chapter 7:** Finally, this chapter gathers the concluding remarks and future works.

1.6.2 Contributions of this thesis

The main contributions of this dissertation can be summarized as follow.

- In appendix A a new solution of Robust Quadratic Programming (RQP) under different uncertain sets is proposed. This approach takes into account uncertainty in the quadratic, linear and constant term of the quadratic optimization problem. The main contribution of this section is the conversion of the the RQP problem into a Second Order Cone Programming (SOCP) problem. In this sense, the robust optimization problem can be transformed into a convex problem with polynomial complexity.

- In section 3.2 the result of the appendix A is extended to a new Robust Model Predictive Control formulation that inherits all convexity and complexity properties of the original RQP problem. There are two important contributions in this section, the first one is about the uncertainty representation. For the robust MPC proposed we assume uncertainty in the free and forced response; although this type of uncertainty can be expressed in typical forms reported in the robust MPC literature, in this sense the uncertainty representation presented here is fairly complete. The second and most remarkable contribution in this section is the way to find the parameters of the uncertain sets. A novel method to obtain these parameters based in mappings between the uncertainty of the free and forced response and the uncertainty in the quadratic, linear and constant term of the quadratic optimization problem is proposed. Finally, the benefits of the proposed controller are shown using an illustrative example.

- In sections 3.3 and 3.4 the results of the new robust MPC of the section 3.2 is used to propose new formulations for Robust MPC with zone control and Robust Economic MPC. As in section 3.2, a new way to find the parameters of the uncertain sets is stated. Finally, the proposed controllers are tested through two application examples.

- In Chapter 4 a Hierarchical Robust Integration of MPC and DRTO is proposed is proposed. This chapter combines the robust controllers of Chapter 3 with the hierarchical structures revised in the Chapter 2. The strategy proposed use Dynamic Real time Optimization (DRTO), robust control and a decentralized MPC scheme, the idea with this scheme is to give sufficient tools and alternatives to control a large scale system taking into account both economical and tracking objectives, and uncertainties. Finally, the strategy is tested in a chain of two reactors and flash system.

- In Chapter 5 a Hierarchical Robust Control for coordination is proposed. In this direction two hierarchical robust structures are proposed. The first one takes a robust MPC with zone control as coordinator, while in the lower layer of the hierarchical structure a decentralized scheme is employed. The second one takes a Robust Dynamic Real Time Optimization (DRTO) layer as coordinator while in the lower layer a decentralized scheme based in MPC with zone control is used. Finally, the strategy is tested in a Hydro Power Valley(HPV).

- In Chapter 6 the main contribution of this dissertation is presented. In this Chapter a hierarchical structure using robust optimization and zone control is presented. By means of robust optimization we calculate the limits of the MPC with zone control, with the idea of determining the economic limits for this controller. This strategy gives freedom from an economical point of view to the local controller of the hierarchical structure. An important aspect to highlight is that the hierarchical solution proposed here can be seen as a general hierarchical robust structure for large scale systems, because little variations over this structure can produce a classical hierarchical representation such as those shown in chapter 2. Finally, in this chapter a case of study is used to illustrate the proposed method.

CHAPTER 2

Hierarchical structures in control

2.1 Introduction

Hierarchical control has attracted the attention of the control community in the last two decades. Through these years, different architectures have been used for tackling the problem of controlling a complete process of hierarchical form. In general terms the literature shows five patterns that can be identified in hierarchical decomposition. It is possible to generate several hierarchical levels based on:

- Levels of abstraction
- Levels of complexity
- The decision-making capacity [10]
- Functional decomposition of the problem
- Temporal issues as Tatjewski proposed in 2008 [16].

In general form, the hierarchical modeling task and hierarchical controller task come together, since the controller is designed based on the way the system is represented. In this chapter a brief state of the art is presented, however, another states of the art have been published. Scatollini [20] presented a review of architectures for distributed and hierarchical Model Predictive Control, in the same direction Ochoa [21] and Tatjewski [16] show different architectures for plant wide control. In section 2.2 a classification of forms to achieve a hierarchical decomposition is showed, in this way abstraction, complexity, functional temporal and decision making layers are explained. Finally, in section 2.3 the current practices in hierarchical operations are described together with some multilayer architectures that will be used in this thesis.

2.2 Hierarchical Decomposition

2.2.1 Abstraction Levels

This approach takes as basis the human ability to create abstractions from reality. Here the system representation is divided in various "strata" where the higher strata has the higher level of abstraction. There is not implicit authority in highest layers instead there would be mutual interdependence relationships among layers. It is possible to represent and to control the system toward the thought of the designer and its ability to represent the real problem in different forms. In this category can be classified the work presented by Girard and Pappas in 2006 [5] where the three layer structure has virtual-control, virtual system representation and interpretation layer. The work presented by Abdelmoula and coworkers in 2008 [22] where a three layers hierarchical structure was proposed with modeling/estimator layer, planification layer and operation layer. All hierarchical decomposition could be inside this group since every author has different levels of abstractions, some times very similar to the reality.

There are applications to hybrid systems where different layers can deal with discrete events and other layers deal with continuous dynamics as shown by Tolani and coworkers in 2004 [23] in their work on aircraft control and by Yasuda in 2008 [24] with his work in control of manufacturing processes.

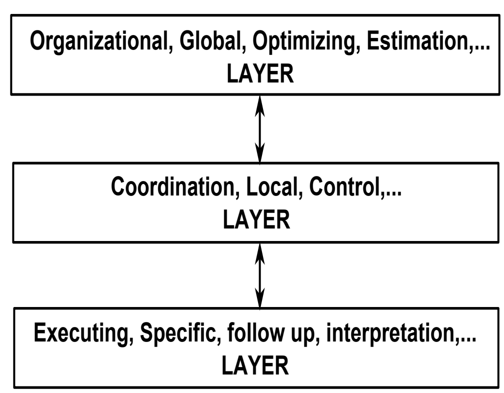


Figure 2.1: Abstraction levels decomposition

2.2.2 Complexity Levels

The hierarchical structure based on complexity levels is related with the complexity of the task developed at each level. Inside this, there are mainly optimization tasks, coordination tasks and execution levels. These tasks can be achieved at different temporal and spatial environments. There is some authority in higher levels since in order to achieve a more complex task so they could require data from the execution of "easier" tasks in lower layers. There are several forms to define the complexity of each task, based on the amount of variables, based on nonlinearity aspects, based on contribution to a global task or based on the nature of the variables (integer, complex, etc.). An interesting work was presented by Brdys and coworkers in 2008 [25] where they define a three level hierarchical structure with supervision, optimization and follow-up layers. The interesting part is the optimization layer where a multi-objective optimization is developed at three different time scales, this is a trade-off among all the dynamics of the system. This approach is presented by several authors [26], [27], [28], [29], [30], [31], [32], [33], [34], [16], [35], [36], [37] and [38].

Damba and Watanabi in [39], proposed the use of the multi-agent theory and define a hierarchical layer for every agent, the task of each agent is to limit the action field of its directly subordinated agents. This is analogous to the control by restrictions and it brings the advantage of relaxing the control requirements at lower layers.

Cheng and coworkers in 2005 [40] presented an interesting form to solve a decomposition-coordinator problem using multi agent theory, the interesting part of this work is the absence of any centralized agent, the method is totally distributed and it achieves an optimal control problem solution. This insight is aligned with the tendencies on programming: the use of totally parallel algorithms.

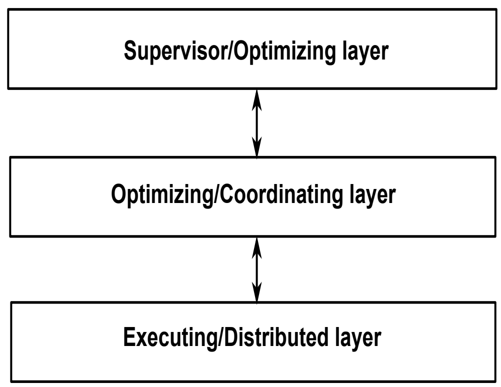


Figure 2.2: Complexity levels decomposition

2.2.3 Functional Levels

This group concerns of specific functions more than tasks. This means to decompose a system in several functional agglomerates of dynamics and to define them an internal control structure as sublayer, and the agglomerates would be coordinated by an upper layer. The functional decomposition methods are linked with the process, the optimizing and coordinating layers are not considered into this category. Tatjeski in 2008 [16] proposes this type of decomposition (Figure 2.3). The functional decomposition regularly generates a totally decentralized task at the lowest level.

Seki and Naka [41] present a methodology to obtain a Controller Group Unit (CGU) that becomes a set of functional subsystems. The features of this decomposition are a local centralization of decentralized tasks with spatial proximity and a global optimization at the coordinator layer.

Tatara and coworkers [42] uses this type of decomposition in battery of reactors in series. Something interesting is that they add a new arbitration element between the lower layer and the functional decomposition layer. This element solves possible conflicts between adjacent subsystems.

Luo and coworkers [43] propose a functional decomposition imitating the human nervous system to achieve locomotion control in a robot. Other functional decomposition is used to design a fault tolerant control system [44], and robot coordination [45].

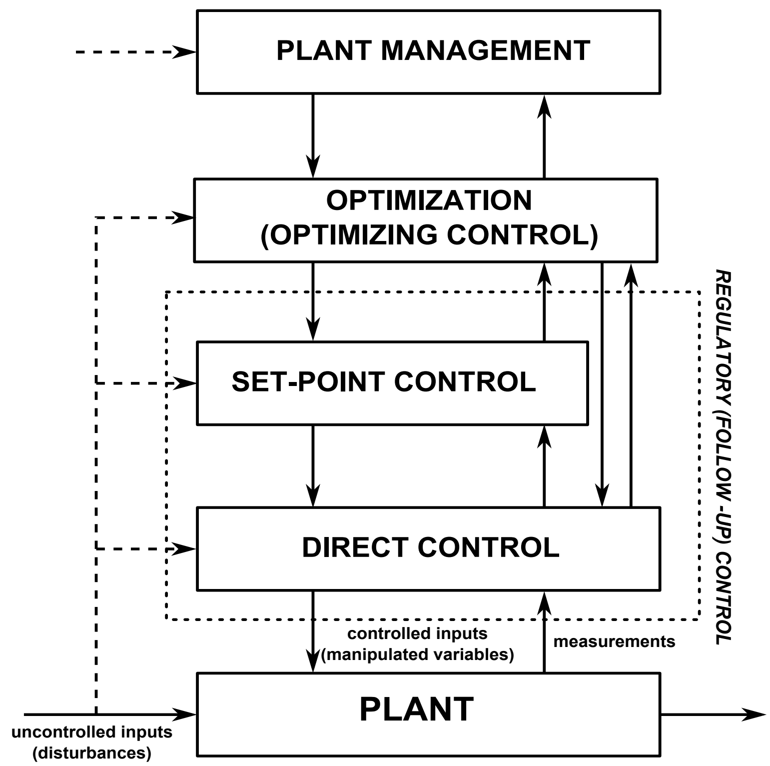


Figure 2.3: Functional multilayer control structure (taken from [16])

2.2.4 Temporal levels

Temporal decomposition is linked with the time response of each system dynamics. A large-scale system has some dynamics with time responses in the order of seconds (some actuators), some dynamics with time responses in the order of minutes (dynamics as level of a tank), some dynamics with time responses in the order of hours (as a product concentration on a chemical reactor) and so on. When the control system of a large-scale systems is designed, sometimes faster or slower dynamics are ignored in order to simplify the design. Neglecting fast dynamics most of the time has no effect on the global behaviour if these dynamics are stable. On the other hand neglecting a slow dynamics assuming interest just in local temporary events require more attention. In general a temporal decomposition brings the model capability to consider all dynamical responses working at different layers and with different models. The method is to fix slow dynamics at higher levels since these dynamics require less elaborated models. At the lower layers the fast dynamics are accommodated in order to achieve a fast response when an upper level requires it. These models will be simple because they do not require knowledge of the whole system. Regularly these decomposition generates optimization at the higher level and coordination at medium level, the lower level just needs an appropiate tuning in tracking mode to guarantee stability and good performance of the whole system [20].

There are some large-scale systems where very fast (nano or milliseconds) dynamics coexist with slow dynamics, in these cases response time and computing time are important due to the necessity of optimizing computing algorithms. Other important aspect in temporal decomposition is the definition of the horizon required for each layer [16] in order to avoid conflicts between adjacent layers.

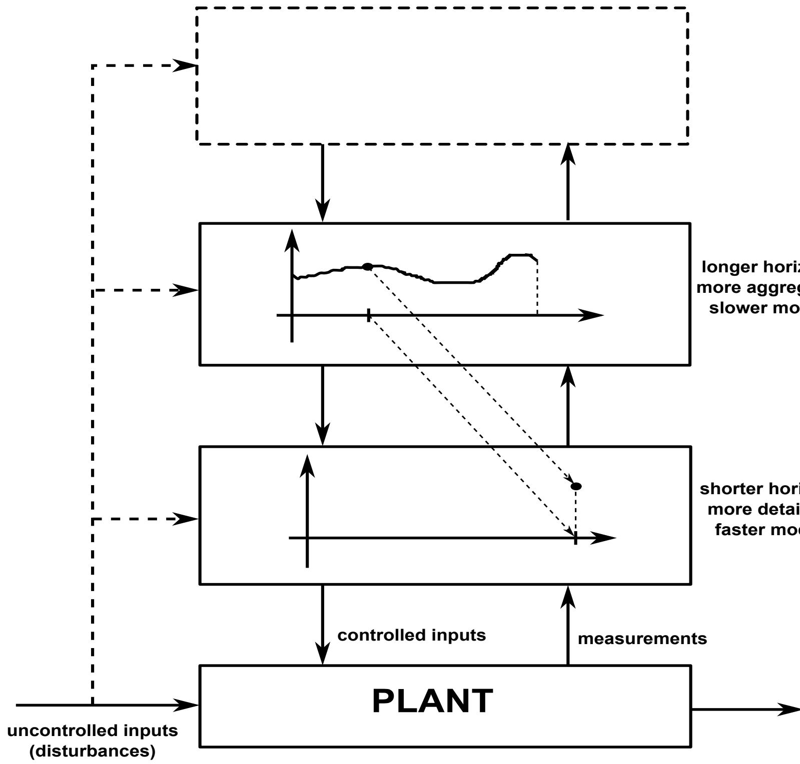


Figure 2.4: Temporal multilayer control structure (taken directly from [46])

Interesting works in this category were presented by Tatjewski in 2008 [16] where two temporal levels (fast and slow) are implanted inside a four hierarchical levels (Figure 2.4). This is very common in order to put a successful cascade structure together where the upper layer deals with slow dynamics and lower layer deals with faster dynamics. Two works in the same direction were presented by Tatjewski and coworkers in 1988 [47] and Cirre and coworker in 2009 [28] where a lower layer is concerned with the fast rejection of disturbances and the upper layer develops an optimization to govern the set point of the lower controller (typical cascade structure). Findeisen in 1978 [31] presented a structure with feed-forward actions at lower levels in order to increase the response velocity in fast dynamics and feedback actions on higher levels, this is in agreement with the actual Scattollini's statements [20]. The Brdys' work [25] was discussed in other group but it is important to recall the multi objective optimization at three time scales in order to get a trade-off among them. Baldea and Daoutidis [26] proposed stronger actions where some supervisor layers can take direct action on the system in order to achieve its temporal objective, the interesting discussion about two control actions for one control objective is not presented.

2.2.5 Decision-Making Levels

Decision-making decomposition deals with the generation of decisions based on measurements. This type of decomposition is made when there exist a set of options waiting to be used. One particular example for this decomposition is the capability to select among a set of controllers based on different conditions of the systems, this means for instance to select a PID controller when the system is far from its reference and to switch to predictive linear control when the systems is close to its reference [48]. Other applications were presented by Boston and coworkers [27] to select between a model based controller or heuristic based model depending on the amount of measurements available. Imazeki and Maeno [49] propose a structure where two control actions are computed, then a "supressor" decides which action should be applied. Another application on robotic can be found at [50]. This hierarchical decomposition does not have a defined structure due to its combination capability but it has two key layers: a decision maker and a set of available options.

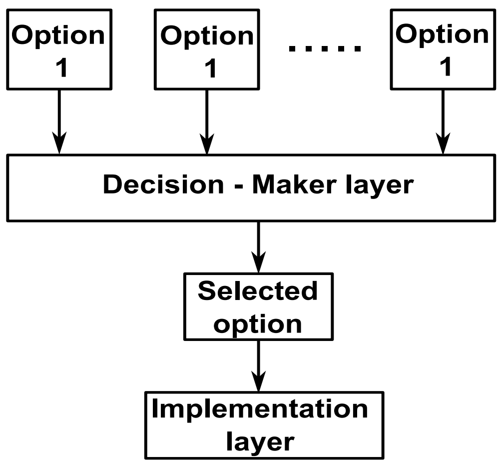


Figure 2.5: Decision-Making decomposition layer

2.3 Current Practices in Hierarchical Operation

The operation of large scale systems involves a large number of decisions which can be distributed into the called real time business decision making and automation hierarchy as shown in Figure 2.6. This hierarchical structure has been summarized in many textbooks [51, 52] and generally combine the structures explained in previous sections. The planning layer is concentrated on economic predictions and gives production objectives. This tackle aspects such as the type of products to make, which feed-stocks to buy and how much to produce and to buy, respectively. The prediction horizon in the planning layer is frequently long, typically in the scale of months or weeks. Scheduling addresses the timing of actions and events necessary to execute the chosen plan. The main objective here is the viability of the operation. The time range in the scheduling is usually in weeks or days. Additionally, the planning and scheduling also gives parameters of the cost functions (for example prices of products, raw materials, energy costs) and constraints (for example amount of raw material, production time). Nowadays, the planning and scheduling calculations are obtained using several mixed integer programming formulations [53].

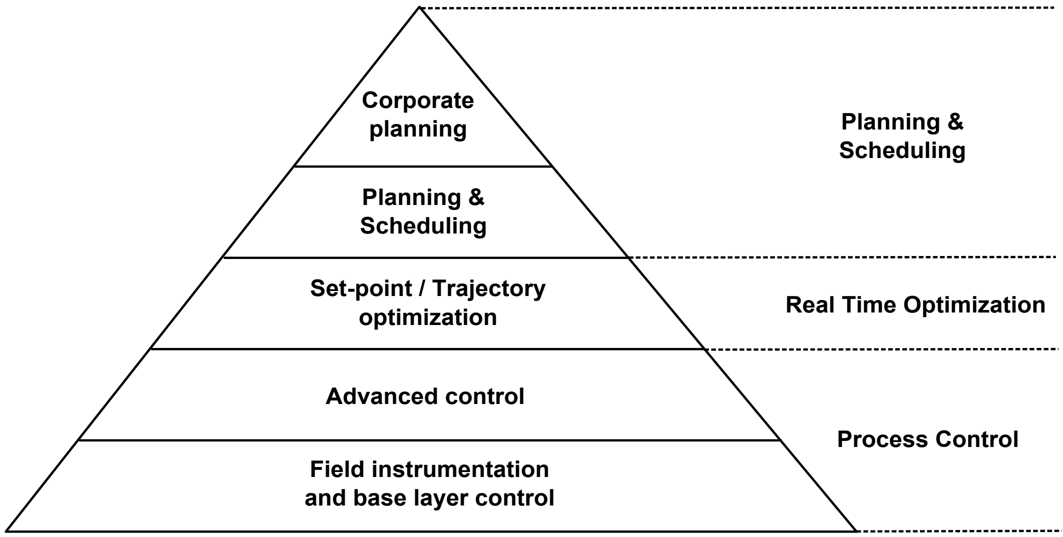


Figure 2.6: Illustration of the hierarchical operation

These business decisions are delivered to the two-layered process operation level consisting of real-time optimization (RTO) and advanced control layer. This two-layered structure is illustrated in Figure 2.7. In the Real Time Optimization (RTO) layer a business decisions and production schedules are implemented, the main idea here is to give the optimal operating point (set-point) for the lower-level advanced control. Generally, in this layer the profit is optimized based on a nonlinear steady state model of the plant and by means of real-time data reconciliation and parameter estimation an additional profit is sought. The reconciled plant data are used to compute a new set of model parameters so that the model represents the plant as accurately as possible at the current operating point [54]. Then the set-point is recalculated using the new model parameters to optimize an economic cost function while satisfying the constraints. Since the optimization is performed online, RTO provides a mechanism to react to changes and reject long term (days or hours) disturbances. It is important to highlight that the RTO layer does not manipulate the inputs of the process. The results of the optimization problem in this layer are used to give the set point to the lower layer. In this way in the hierarchical structure of Figure 2.7 the lower-level advanced controller is able to adjust the inputs to keep the process at the desired set-points at all times. These set-points are filtered by a supervisory system that usually includes the plant operators and forwarded to the advanced control layer [55]. In the last 40 years, an usual industrial practice was to use PID controllers in the lower layer. This type of controllers are able to keep the output of the system at the desired set-point and reject some disturbances. However, there are several problems with PID controllers, the major difficulties are to tune and decouple the PID controller for multi-input-multi-output (MIMO) systems. Hence, as it was mentioned in chapter 1 the optimal multivariable control strategies that exist and it is well studied in literature is the Model Predictive Control (MPC) [56]. MPC has some important features such as its capability to handle constraints and to manage the strong interaction between the states in large scale and networked systems. MPC has been widely adopted by the industrial process control community and implemented successfully in many applications [13].

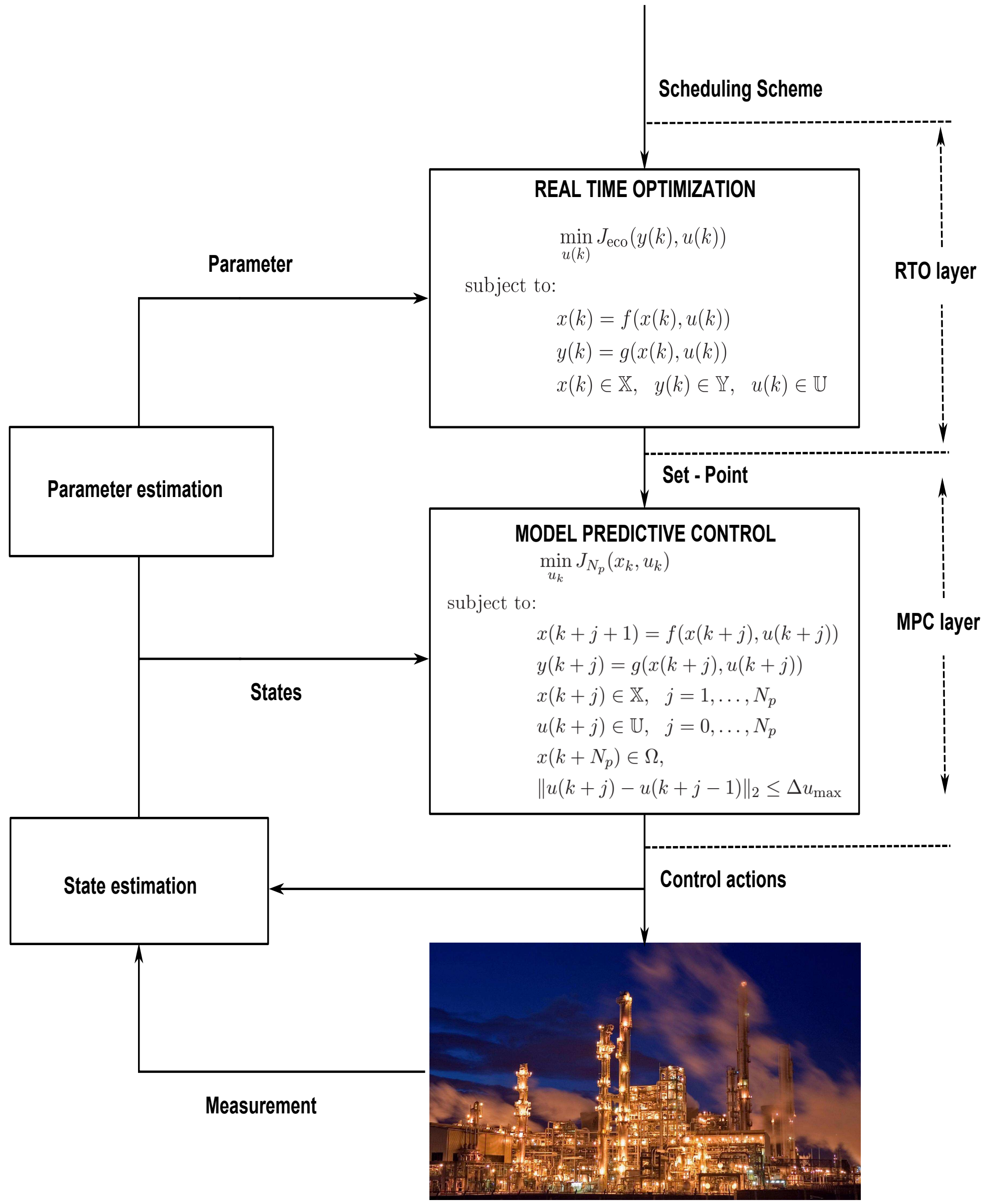


Figure 2.7: Illustration of RTO and advanced control structure

Currently, most MPC controllers still use linear models to predict the states trajectories. This implies a critical limitation in the hierarchical structure described here, since the set-points are calculated from the optimization problem based in a nonlinear model in the RTO layer and these are often inconsistent and unreachable when viewed from the dynamic layer. The reason of that are the discrepancies between the models used for steady-state optimization and dynamic regulation. Several researchers [57] have reported the problems due to the modeling inconsistency in different layers of the hierarchy. Particularly, the steady-state gains are different. Additionally, the RTO does not use all degrees of freedom in the dynamic layer which may yield suboptimal set-points. An important aspect to mention is that the layers have different time scales such that the RTO layer is delayed [58]. Moreover, if the process is operated over a wide range of conditions, a fixed model identified at a steady-state is usually not sufficient to have the predictability to cover the operating range.

2.3.1 Dynamic Real Time Optimization (DRTO)

Motivated by the previous facts, the concept of a dynamic real time optimization (DRTO) is introduced by [59]. This strategy is shown in Figure 2.8. Instead of decomposing the hierarchical structure into RTO (steady state optimization) and advanced dynamic regulation, DRTO optimizes the same economical performance of the RTO layer, but this time is over a prediction horizon (dynamic optimization) and calculates the optimal trajectories of the control actions. The DRTO has substantially the same formulation used in nonlinear model predictive control, However an economic objective is chosen to give an economically optimal operation at all times. The main idea is to translate the economical objective into process control objectives, which is the goal of a control structure synthesis first stated by Morari et al. [60]. As a result, the two-layered structure is merged into a centralized decision-making and control layer. Therefore the problems associated with the two-layered operation structure discussed in section 2.3 disappear [55].

Several researchers have contributed to refine the DRTO formulations to improve the economical performance. Zanin et al. [14] proposed a formulation and solution strategy and implemented them on a fluidized bed catalytic cracker. Adetola and Guay [61] proposed an MPC design approach that integrates RTO and MPC together. Würth et al. [62] proposed an infinite-horizon formulation for economically-oriented NMPC and Diehl et al. [63] also analyzed the nominal stability property of a general economically-oriented NMPC formulation assuming strong duality.

2.3.2 Multilayer architectures for plant wide control

This section is directly taken of the work of Ochoa et al. [21]. In this way, Ochoa et al. took the work of Findeisen et al.[64], who classified the hierarchical control into multilayer and multilevel (In the multilayer case the control of a system is split into algorithms (layers), whereas in the multilevel case control is divided into local goals and the action of each local control unit is coordinated by an additional superior unit) and propose to subdivide the multilayer (or hierarchical) architecture into: with coordination (Figure 2.9) and without coordination (Figure 2.10). Both multilayer architectures are composed by at least two different layers, i.e. an optimization (RTO) and a control layer (MPC).

2.3.2.1 Multilayer architecture with coordination

In this architecture, a coordinator is included between the RTO and the MPC layers. Usually, this coordinator drives information from both layers, and it must find the set points for each local controller close to the global solution found by the RTO layer, i.e, the responsibility of each MPC is tracking the local set points, by calculating the vector of manipulated variables for each operating unit. Details about this type of architecture

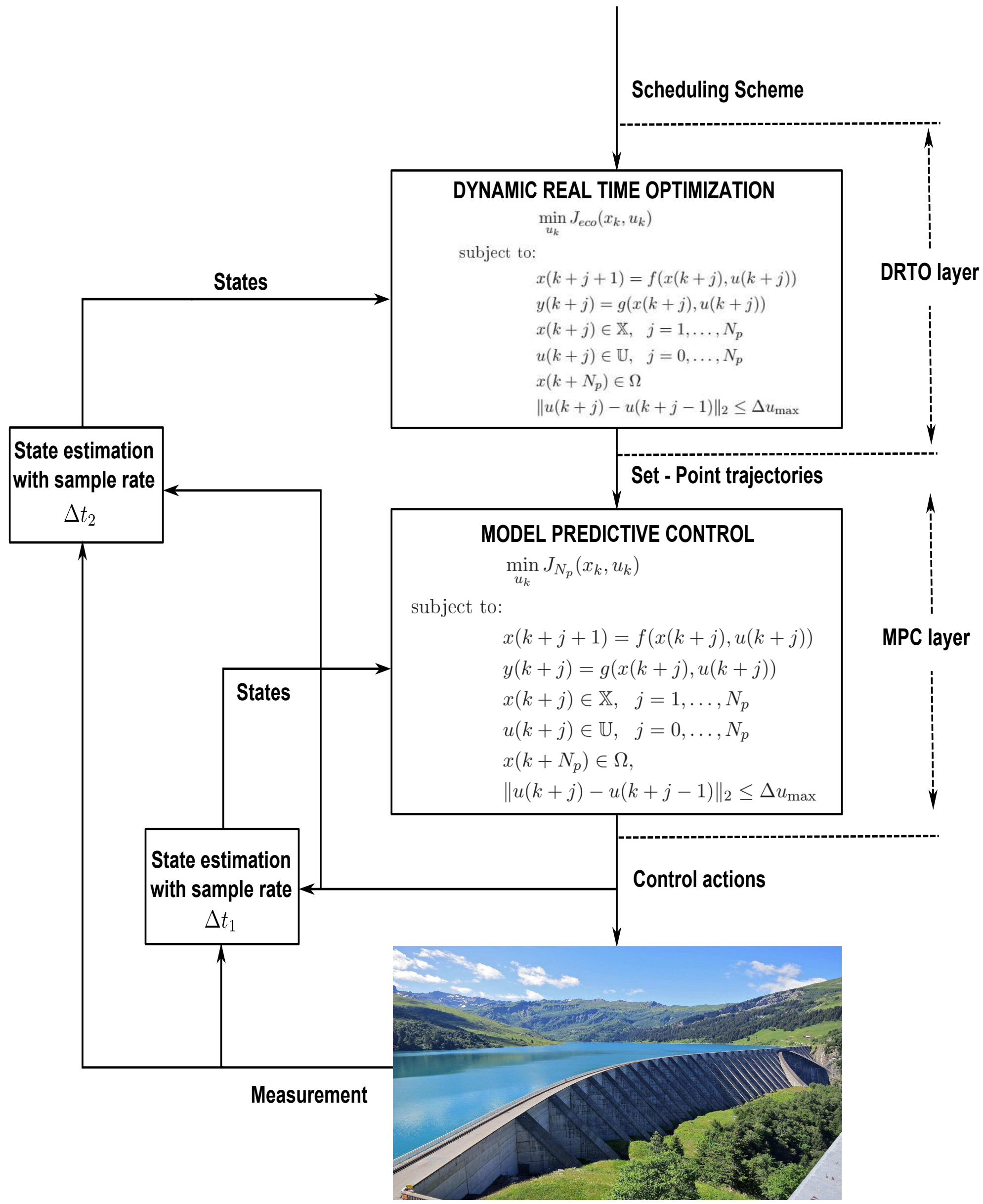


Figure 2.8: Illustration of DRTO

can be found in [65], [66], [67], [68]. An important final mention should be done about the difference between the multilayer with coordination and the distributed architectures. In some sense, both schemes include a kind of coordination, in the distributed case the coordination consists on exchanging some information between the local MPCs, whereas in the multilayer with coordination, the local MPCs are not directly communicated among them but communicated through the RTO layer.

An important aspect to highlight in this architecture is the handling of model uncertainty. Since modeling uncertainty is unavoidable in industrial problems, the multilayer architecture with coordination should handle it, however there are no important contributions in this direction, obviously MPC strategy can introduce feedback into the system, and it is well known that feedback control can provide some degree of robustness against perturbations (even if the controller is not specifically designed for this task). However, the presence of constraints and the implicit form of the control law make robustness analysis of MPC control loops a very difficult task [69]. As a result, only few approaches for analyzing robustness of nominal MPC have appeared in the literature. For this reason in the following chapters robust hierarchical approximations will be developed for large scale systems.

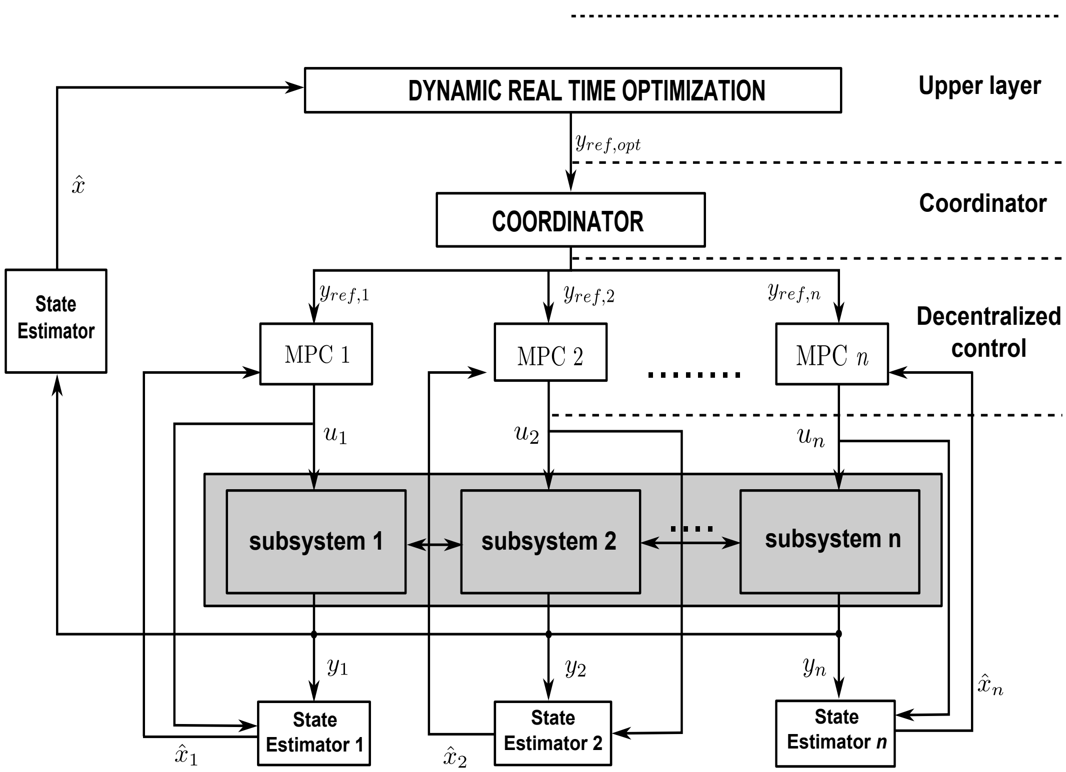


Figure 2.9: Multilayer architecture with Coordination for Plantwide Control

2.3.2.2 Multilayer architecture without coordination

When no coordination is used between the optimization and control layers, it is more suitable to replace the RTO by a DRTO layer, in order to account for the dynamic nonlinear behavior of the process. The DRTO layer is in charge of calculating the optimal set points for the process outputs, which are sent directly to the control layer. Then, the control layer calculates the set of vectors of manipulated variables for being applied in the process. Kadam et al. [70] include examples of multilayer architecture without coordination. Although this strategy seems to be attractive, the centralized nature of the controller is a very important drawback on large scale systems. For this reason the Multilayer architecture without coordination is not considered as an feasible strategy in this thesis. Finally, it is important to mention the estimation blocks shown in Figures 2.9 and 2.10. These blocks are acting as switches for recalling the optimization and control layers, when a certain condition are met. Some typical conditions for these blocks are presented in the section 4.3.1.2.

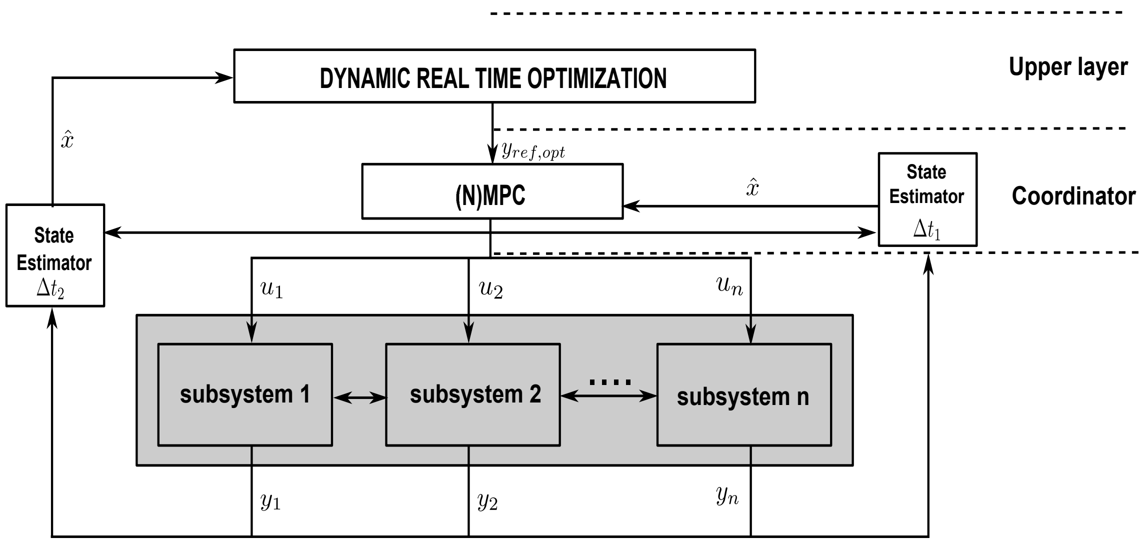


Figure 2.10: Multilayer architecture without Coordination for Plantwide Control

2.4 Summary

In this chapter a literature review of hierarchical structures in control were presented. This chapter serves as a literature review for the following chapters that are based in hierarchical control. In chapter 3 robust MPC is studied and used in chapter 4, 5 and 6 to propose hierarchical robust strategies for large scale systems based in the strategies seen in this chapter.

CHAPTER 3

Robust Model Predictive Control

3.1 Introduction

In engineering, when a problem is formulated as an optimization problem, usually the available data are not precisely known. This data uncertainty is due their random nature, measurement errors, or many other reasons. In general, the uncertainty in the data cannot be ignored; Ben-Tal and Nemirovski [71] show that a small perturbation on data can make the nominal optimal solution of the optimization problem senseless from a practical viewpoint. Therefore, there is a real need of a methodology capable of to find a robust solution that helps to minimize the effect of data uncertainty in an optimization problem. In this way, the appendix A presents some algorithms for Robust Quadratic Programming (RQP). The solution of the robust quadratic program is obtained by transforming the problem into a Second Order Cone Program [72]. In this appendix, the uncertainties include the Hessian matrix, the linear term and the constant term of the cost function under different uncertain sets and the problem is reformulated as a second order cone program (SOCP). The algorithms developed in appendix A are extended in this chapter to min-max MPC, min-max MPC with zone control and Economic MPC. The chapter is organized as follows: In section 3.2 a new algorithm for Robust MPC considering uncertainty in the free and forced responses is presented, this algorithm is based in RQP and it is extended to Robust MPC with zone control in section 3.3 and Robust Economic MPC in section 3.4.

3.2 Robust Model Predictive Control

Controlling a system with constraints in the control actions and states is one of the most important problems in control theory, but also one of the most challenging. Another important topic is robustness against uncertainties in a controlled system. For this reason solving a control problem with constraints and uncertainties can be a very hard task. The main question in robust control is how to exploit knowledge about uncertainty. Typical knowledge can be bounds on uncertain parameters in the system. Obviously, it is not possible capture all uncertainties in practice, but we can try to capture the most important and account for these. The next fact is how to incorporate the knowledge of the uncertainty in the optimization problem of the MPC, given that the calculated control law is based on an on-line optimization problem. In this way, a min-max Model Predictive Control approach for discrete time uncertain systems is proposed in this section. Min-max strategy in model predictive control (MPC) allows computing the optimal control actions where the worst-case performance with respect to the uncertainties is assumed. Unfortunately min-max formulations of predictive controllers often produce an intractable optimization problem with exponential complexity as the number of states, inputs, and outputs of the system increase. In this section a min-max algorithm for a certain type of model uncertainty is derived. The transformation of the original problem into a second-order cone program is the most remarkable feature, meaning that the min-max problem is written as a convex program. The result is an optimization problem with polynomial complexity.

3.2.1 Modelling Uncertainty

To ignore the uncertainty in control problems is generally a bad idea, because small changes in the parameters of the model could modify the behavior of the model. In this sense the uncertainty must be included and taken into account during the design process of the controller, i.e. it is necessary to modeling the uncertainty. The next sections present some of the uncertainty models that are most important in the MPC literature, including parametric and polytopic uncertainty, structured feedback uncertainty and ellipsoidal uncertainty.

3.2.1.1 Parametric and Polytopic Uncertainty

Linear Parameter-Varying Systems: Linear Parameter-Varying (LPV) systems [73] are systems of the form

$$x(k+1) = E_k + A_k(\theta)x(k) + B_k(\theta)u(k) \tag{3.1}$$

where $A(\cdot)$ and $B(\cdot)$ are matrices in terms of the parameter $\theta \in \Psi$, where Ψ is a compact set. At any time k, the parameter θ may take any value in Ψ. Thus, the system (3.1) is time-varying in general. LPV systems arise when the model parameters are not exactly known, or whose parameters may vary with time or with the system operating point.

Polytopic Systems: Another way of modeling uncertainty is the so-called polytopic uncertainty [69]. This type of uncertainty is described by the time-varying system

$$x(k+1) = E_k + A_k x(k) + B_k u(k) \tag{3.2}$$

where

$$[A_k, B_k] \in \Psi = \text{Convh}\left\{[A_1, B_1], \ldots, [A_L, B_L]\right\} \tag{3.3}$$

with Convh(·) denoting the convex hull, i.e

$$[A_k, B_k] \in \Psi = \sum_{i=1}^{L} \mu_i [A_i, B_i] \text{ with } \sum_{i=1}^{L} \mu_i = 1$$

Polytopic uncertainty models are often used when a number of "extreme" system dynamics are known, which then represent the vertices $[A_i, B_i]$ in (3.3) [74]. These extreme system dynamics can be obtained by identifying an unknown system through a sufficiently large number of measurements, or by modeling a system under different extreme operating conditions.

Relationship Between LPV and Polytopic Systems [74]: Here, there exists a direct connection between LPV and polytopic system descriptions: If the set Ψ is polytopic, and if $A(\cdot)$ and $B(\cdot)$ in (3.1) are affine functions of θ , i.e.

$$\begin{aligned} A(\theta) &= A_{\theta,0} + \sum_{i=1}^{n_A} A_{\theta,i}\theta_i \\ B(\theta) &= B_{\theta,0} + \sum_{j=1}^{n_B} B_{\theta,j}\theta_j \end{aligned} \tag{3.4}$$

then it is easy to represent a LPV system as a polytopic system. In this case, the vertex pairs $[A_i, B_i]$ in (3.3) are the mappings $[A(\theta_1), B(\theta_1)], \ldots, [A(\theta_L), B(\theta_L)]$, with θ_i being an enumeration of the L vertices of Ψ. To see this, note that under an affine mapping polytopes are again mapped into polytopes, and that the vertices of the original polytopes are mapped into the vertices of the image polytopes [75].

3.2.1.2 Regular sets

In this case the uncertainty set Ψ is represented by a regular n-dimensional region. Often, this region is an ellipsoid defined by the maximum allowable norm of the uncertainty. In this sense, it is possible to formulate the next sets $\mathbb{A}$ and $\mathbb{B}$ such that $\Psi \triangleq \mathbb{A} \times \mathbb{B}$, with

$$\begin{aligned} \mathbb{A} &= \left\{ A_0 + \sum_{l=1}^{n_A} A_l \mu_l \mid \|\mu\|_2 \leq 1 \right\} \\ \mathbb{B} &= \left\{ B_0 + \sum_{r=1}^{n_B} B_r \nu_r \mid \|\nu\|_2 \leq 1 \right\} \end{aligned} \tag{3.5}$$

where A_0 and B_0 can be assumed as the mean values of the uncertainties.

3.2.1.3 Structured Feedback Uncertainty

Another model uncertainty framework in the Robust MPC literature is the so-called structured feedback uncertainty [69]. This type of uncertainty is based on Linear Fractional Transformations (LFT), which are a well-known framework in Linear Robust Control [76]. The basic idea of the structured feedback uncertainty model is to divide the uncertain system into two components. The first component, a LTI system, includes the information that is known about the system, and the second component (a feedback loop), incorporates all of the uncertainty that appears in the model. To be specific, consider the following linear time-invariant system:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_p p(k) \\ y(k) &= Cx(k) + Du(k) \\ q(k) &= C_q x(k) + D_q u(k) \\ p(k) &= \Delta q(k) \end{aligned} \tag{3.6}$$

where extra variables are added to the system state x, the control input u, and the system output y . The feedback interconnection of this system is illustrated in Figure (3.1).

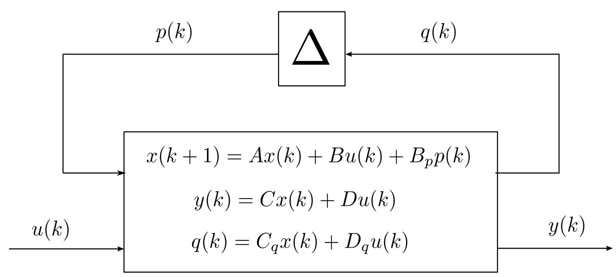


Figure 3.1: Uncertainty description by means of LFT.

There is a large number of possible causes for Structured Feedback Uncertainty. These causes include unknown parameters, unknown or neglected dynamics and nonlinearities [76]. The authors of [73] furthermore point out that the framework of structured feedback uncertainty can also be used to describe LPV systems [74].

In general terms the systems covered in this work are constrained uncertain systems of the form

$$\begin{aligned} x(k+1) &= E_k + (A_k + \delta A_k)x(k) + (B_k + \delta B_k)u(k) \\ y(k) &= F_k + C_k x(k) + D_k u(k) \\ \theta &\triangleq (\delta A_k, \delta B_k) \in \Psi \end{aligned} \tag{3.7}$$

where $\delta A_k \in \mathbb{R}^{n_x \times n_x}$, $\delta B_k \in \mathbb{R}^{n_x \times n_u}$ and $\|\theta(k)\|_2 \leq \rho$ is a bounded uncertainty. Note that the model (3.7) covers almost all models used in min-max MPC schemes. For example it is possible to transform the system (3.7) in (3.6) defining the matrices Δ, C_q, D_q and B_p

as

$$\Delta = \begin{bmatrix} \delta A & 0_{n_x \times n_u} \\ 0_{n_u \times n_x} & \delta B \end{bmatrix}, q(k) = \underbrace{\begin{bmatrix} I_{n_x} \\ 0_{n_u \times n_x} \end{bmatrix}}_{C_q} x(k) + \underbrace{\begin{bmatrix} 0_{n_x \times n_u} \\ I_{n_u} \end{bmatrix}}_{D_q} u(k), B_p = \begin{bmatrix} I_{n_x} & I_{n_u} \end{bmatrix} \tag{3.8}$$

Now, let $x^*(k)$ and $u^*(k)$ denote the state and control actions corresponding to the current operating point. The uncertainty in A and B matrices can be propagated to the free and force responses [72] by means of the prediction sequence of the output $y(k)$ as (3.9).

$$y_k = \underbrace{(\Gamma + \delta\Gamma)}_{\text{Free Response}} + \underbrace{(\Lambda + \delta\Lambda)u_k}_{\text{Forced Response}} \tag{3.9}$$

The relation among δA_k, δB_k with $\delta\Gamma$, $\delta\Lambda$ is explained in appendix C. Note that δA_k and δB_k are mapped as an uncertainty in $\delta\Gamma$ and $\delta\Lambda$, this fact is important to use RQP in min-max MPC. Our next step is to use these sets of predicted states. The prevailing approach in robust MPC, and robust control in general, is to solve min-max problems, i.e., solve problems where worst-case scenarios are accounted for. Clearly, this is a conservative approach, but it is one of the few ways we have to place the word robustness in a well-defined mathematical framework.

3.2.2 Min-max MPC

The idea behind Min-Max Model Predictive Control (also: Minimax MPC) is to optimize robust performance. That is, instead of minimizing nominal performance the controller is designed to minimize the worst-case performance achievable under any admissible uncertainty. Min-Max MPC was introduced in the seminal paper of Campo and Morari [77] and since then has become a very popular way of formulating Robust MPC problems. Most Robust MPC methods that have been developed following on the initial ideas of Campo and Morari are essentially based on the minimization of the worst-case performance. There is a vast amount of literature on min-max MPC, so let us just mention a few references that fit as reference reading to this thesis. In open-loop and closed-loop min-max MPC [78], in enumeration techniques in min-max MPC synthesis [79, 80, 81] and in LMI-based approaches to min-max MPC [73]. Now, consider a system described by a nonlinear invariant discrete time model,

$$\begin{aligned} x(k+1) &= f(x(k), u(k), \theta(k)) \\ y(k) &= g(x(k), u(k), \theta(k)) \end{aligned} \tag{3.10}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the system state, $y(k) \in \mathbb{R}^{n_y}$ is the system output, $u(k) \in \mathbb{R}^{n_u}$ is the current control vector and $\theta(k) \in \mathbb{R}^{n_d}$ are the uncertainties presents in the model. We assume that the function model $f(x(k), u(k), \theta(k))$ is continuous in x, u and θ. The system is subject to hard constraints on state $x(k) \in \mathbb{X}$, output $y(k) \in \mathbb{Y}$, input $u(k) \in \mathbb{U}$

and $\theta \in \Psi$ for all $k \geq 0$, where $\mathbb{X} \subset \mathbb{R}^{n_x}$, $\mathbb{Y} \subset \mathbb{R}^{n_y}$, $\mathbb{U} \subset \mathbb{R}^{n_u}$ and $\Psi \subset \mathbb{R}^{n_d}$ are closed sets. Then, the min-max controller design problem consists of obtaining a control law that minimizes a given performance cost index

$$\max_{\Theta_k \in \Psi^{N_p}} J_{N_p}(x_k, u_k, \Theta_k) \tag{3.11}$$

where $x_k = [x^T(k), \ldots, x^T(k+N_p)]^T$ is the state trajectories, $J_{N_p}(x_k, u_k, \Theta_k)$ defines the cost of the problem, and a sequence of control inputs $u_k = [u^T(k), \ldots, u^T(k+N_p)]^T$ is obtained assuming the uncertainty $\Theta_k = [\theta^T(k), \ldots, \theta^T(k+N_p)]^T$ maximum value. Robust nonlinear predictive control problem is written as a min-max optimization problem, in such optimization problem a nonlinear model of the system is used to forecast the state trajectories. The forecasting is done along a prediction horizon N_p. As a consequence, a sequence of control inputs u_k is obtained. Since u_k is obtained assuming the disturbances Θ_k maximum, u_k is the worst-case control action. Mathematically, this is expressed as (3.12) ([82, 83]). Subsequently different robust MPC designs have been proposed, for example those proposed in [84, 85, 86].

$$\begin{aligned}
&\min_{u_k} \max_{\Theta_k \in \Psi^{N_p}} J_{N_p}(x_k, u_k, \Theta_k) \\
&\text{subject to:} \\
&\quad x(k+j+1) = f(x(k+j), u(k+j), \theta(k+j)) \\
&\quad y(k+j) = g(x(k+j), u(k+j), \theta(k+j)) \\
&\quad x(k+j) \in \mathbb{X}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad u(k+j) \in \mathbb{U}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad x(k+N_p) \in \Omega, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad \|u(k+j) - u(k+j-1)\|_2 \leq \Delta u_{\max} \\
&\quad \theta(k+j) \in \Psi, \quad \|\theta(k+j)\|_2 \leq \rho \\
&\quad \Theta_k \in \Psi^{N_p} \triangleq \Psi \times \cdots \times \Psi
\end{aligned} \tag{3.12}$$

where Ψ^{N_p} is determined by the assumed uncertainty model, with $\|\theta(k)\|_2 \leq \rho$ a bounded uncertainty in the system dynamics. In (3.12) $J_{N_p}(x_k, u_k, \Theta_k)$ is given by:

$$J_{N_p}(x_k, u_k, \Theta_k) = \sum_{j=1}^{N_p} L(x(k+j), u(k+j)) + V(x(k+N_p))$$

where $L(\cdot)$ is the stage cost, and $V(\cdot)$ is the terminal cost. Sometimes this function penalizes the deviation of the predicted output $y(k)$ from its corresponding reference value y_{ref}. The stability of the controller (3.12) can be assured assuming that the close loop system has a Lyapunov function $V(\cdot)$, this fact will be discussed in section 3.2.4. In the next section an efficient calculation of the worst case cost will be explained based on the RQP approach.

3.2.3 Efficient calculation of the worst case cost

In this section an efficient calculation of the worst case cost presented in (3.12) will be done. The idea is to develop a method based on the RQP approach explained in section A.2, and the nominal solution of a MPC controller.

3.2.3.1 Nominal Model predictive Control

The nominal MPC problem (no uncertainty, $\|\theta(k)\|_2 = 0$) is a nonlinear programming (NLP) problem whose solution is hard to find in real time. Assuming no uncertainty, (3.12) becomes:

$$\begin{aligned}
&\min_{u_k} J_{N_p}(x_k, u_k) \\
&\text{subject to:} \\
&\quad x(k+j+1) = f(x(k+j), u(k+j)) \\
&\quad y(k+j) = g(x(k+j), u(k+j)) \\
&\quad x(k+j) \in \mathbb{X}, \quad j = 1, \ldots, N_p \\
&\quad u(k+j) \in \mathbb{U}, \quad j = 0, \ldots, N_p \\
&\quad x(k+N_p) \in \Omega, \\
&\quad \|u(k+j) - u(k+j-1)\|_2 \leq \Delta u_{\max}
\end{aligned} \tag{3.13}$$

The stability of the MPC controller (3.13) can be assured if the next hypothesis is accomplished [87]:

Hypothesis 1 (**Terminal cost and region**). *The system $x(k+1) = f(x(k), u(k))$ is such that there exist a neighborhood of the origin $\Omega \subseteq \mathbb{X}$ which is an invariant set of the system controlled by a control law $K_f : \Omega \longrightarrow \mathbb{X}$, that verify*

$$V(f(x(k+j)), K_f(x(k+j))) - V(x(k+j)) \leq -L(x(k+j), K_f(x(k+j)))$$

Then, from hypothesis 1 the following Theorem can be formulated:

Theorem 1. ***(Asymptotic stability [87])***

Let a system $x(k+1) = f(x(k), u(k))$, such as that $f(0,0) = 0$ and this is subject to the constraints $x(k+j) \in \mathbb{X}$ and $u(k+j) \in \mathbb{U}$. Then, if the hypothesis 1 is accomplished, the system controlled by the nominal MPC controller $u_{MPC} = K_f(x(k))$ is asymptotically stable.

Proof. See Mayne. et al [87] □

In the remaining of this work, it will be assumed that the system can be represented using linear models of the form,

$$
\begin{aligned}
x(k+1) &= E_k + A_k x(k) + B_k u(k) \\
y(k) &= F_k + C_k x(k) + D_k u(k)
\end{aligned} \tag{3.14}
$$

Where E_k, A_k, B_k, F_k, C_k and D_k are defined in appendix B. Assuming linear models is motivated by the fact that (3.12) recast in robust nonlinear programming (RNLP). With current computation resources is almost impossible to find the exact solution of RNLP problems in real time. In this way, in order to obtain (3.14) an algorithm that uses linear time-varying prediction models by applying a local linearization along the nominal input and state trajectory is used [88] (the local linearization method is explained in appendix B). Local linear approximation of the state equation is used to develop an optimal prediction of the future states. The output prediction is made linear with respect to the undecided control input moves assuming some smoothness in $f(\cdot)$ and $g(\cdot)$, which allowed the reduction of the nonlinear MPC optimization into a quadratic programming problem (QP)[88]. Smoothness conditions are required to guarantee the existence of the linear approximation and for guaranteeing its quality. In this way, the problem with the nonlinear MPC approach is avoided, performance analysis may be simplified, and computational efficiency is significantly improved. Let $y_k = [y^T(k), \ldots, y^T(k+N_p)]^T$ denotes the prediction sequence of the output $y(k)$. Then, the prediction model has the following form

$$
y_k = \underbrace{\Gamma}_{\text{Free Response}} + \underbrace{\Lambda u_k}_{\text{Forced Response}} \tag{3.15}
$$

where $\Gamma \in \mathbb{R}^{n_y \cdot N_p \times 1}$ and $\Lambda \in \mathbb{R}^{n_y \cdot N_p \times n_u \cdot N_p}$ are matrices stated in appendix B. Let $J_{N_p}(x_k, u_k)$ be a quadratic cost function

$$
\begin{aligned}
J_{N_p}(x_k, u_k) = &\sum_{j=1}^{N_p} \|(y(k+j) - y_{ref}\|_Q^2 + \sum_{j=0}^{N_p} \|u(k+j) - u_{ref}\|_R^2 \\
&+ \sum_{j=0}^{N_p} \|\Delta u(k+j)\|_S^2 + \|x(k+N_p)\|_{\tilde{Q}}^2
\end{aligned}
$$

where $y(k) \in \mathbb{R}^{n_y}$ and $u(k) \in \mathbb{R}^{n_u}$, $Q \in \mathbb{R}^{n_y \times n_y}$, $R \in \mathbb{R}^{n_u \times n_u}$ and $S \in \mathbb{R}^{n_u \times n_u}$ are positive definite weighting matrices. The terminal state penalty $\tilde{Q}$ is obtained by solving a Lyapunov equation, this term is added to ensure stability of the controller [89]. Using the vector notation, it is possible to rewrite the cost function in vectorial form, and replacing (3.15) into the expression for $J_{N_p}(x_k, u_k)$, a formulation of (3.13) in a quadratic program form can be written as (3.16).

$$
\begin{aligned}
&\min_{u_k} u_k^T P u_k + 2q^T u_k + r \\
&\text{subject to:} \\
&\begin{bmatrix} I \\ -I \\ \Delta_\mathrm{u} \\ \Delta_\mathrm{u} \\ \Lambda \\ -\Lambda \end{bmatrix} u_k \leq \begin{bmatrix} \tilde{u}_{\max} \\ -\tilde{u}_{\min} \\ \tilde{\Delta} u_{\max} + U_{(k-1)} \\ \tilde{\Delta} u_{\max} - U_{(k-1)} \\ \tilde{y}_{\max} - \Gamma \\ -\tilde{y}_{\min} + \Gamma \end{bmatrix}
\end{aligned}
\tag{3.16}
$$

where P, q, r, Y_{ref}, U_{ref}, $\tilde{u}_{\max}$, $\tilde{u}_{\min}$, $\tilde{y}_{\max}$, $\tilde{y}_{\min}$, $\tilde{\Delta} u_{max}$ are defined as follows:

$$
\begin{aligned}
&P = \Lambda^T \overline{Q} \Lambda + \overline{R} + \Delta_\mathrm{u}^T \overline{S} \Delta_\mathrm{u} \\
&q = (\Gamma - Y_{ref})^T \overline{Q} \Lambda - U_{(k-1)}^T \overline{S} \Delta_\mathrm{u} - U_{ref}^T \overline{R} \\
&r = (\Gamma - Y_{ref})^T \overline{Q} (\Gamma - Y_{ref}) + U_{(k-1)}^T \overline{S} U_{(k-1)} + U_{ref}^T \overline{R} U_{ref} \\
&Y_{ref} = [{y_{ref}}^T(k), \dots, {y_{ref}}^T(k+N_p)]^T \\
&U_{ref} = [{u_{ref}}^T(k), \dots, {u_{ref}}^T(k+N_p)]^T \\
&\tilde{u}_{\max} = [u_{\max}^T(k), \dots, u_{\max}^T(k+N_p)]^T \\
&\tilde{u}_{\min} = [u_{\min}^T(k), \dots, u_{\min}^T(k+N_p)]^T \\
&\tilde{y}_{\max} = [y_{\max}^T(k), \dots, y_{\max}^T(k+N_p)]^T \\
&\tilde{y}_{\min} = [y_{\min}^T(k), \dots, y_{\min}^T(k+N_p)]^T \\
&\tilde{\Delta} u_{max} = [\Delta u_{\max}^T(k), \dots, \Delta u_{\max}^T(k+N_p)]^T,
\end{aligned}
$$

Note that r is the minimum cost and cannot be changed during the optimization because it is independent of u_k. Then, the solution of (3.13) can be approximated by the solution of (3.16). In this formulation $\Delta_\mathrm{u} u_k$ denote the difference between the actual and the previous value of the control action. Hence, Δ_u is bidiagonal matrix whose elements are $-I$ and I. As mentioned before, this strategy works under the following assumptions:

1. The nonlinearities are smooth, the linear model is a good representation of the plant.
2. The control actions do not move the system far away from the region where the linearization is valid

Often, these assumptions seem to be restrictive. Then, alternatives such as robust optimization based on linear models arise. In Sections below, this approach is described.

3.2.3.2 Robust MPC problem as a RQP

In this work uncertainty in A_k and B_k is considered, however, it is possible to propagate this uncertainty in the free and force responses by means of the prediction sequence of the output $y(k)$ as (3.17)

$$y_k = \underbrace{(\Gamma + \delta\Gamma)}_{\text{Free Response}} + \underbrace{(\Lambda + \delta\Lambda)u_k}_{\text{Forced Response}} \tag{3.17}$$

Note that δA_k and δB_k are mapped as uncertainty in $\delta\Gamma$ and $\delta\Lambda$. The relation among δA_k, δB_k with $\delta\Gamma$, $\delta\Lambda$ is shown in appendix C. This section expands the nominal solution given by (3.16), and with this purpose uncertainty is added. As it was mentioned uncertainty will be restricted to the free and forced response terms $(\Gamma + \delta\Gamma)$ and $(\Lambda + \delta\Lambda)$ where $\delta\Gamma \in \Re^{n_y \cdot N_p \times 1}$ and $\delta\Lambda \in \Re^{n_y \cdot N_p \times n_u \cdot N_p}$ uncertainties. This is a modification of the work of Espinosa [90, 72], where the uncertainty was restricted to the force response. With this assumption, the original min-max MPC problem can be converted to the next RQP problem (3.18):

$$\min_{u_k} \max_{P \in \mathbb{E}, q \in \mathbb{F}, r \in \mathbb{G}} u_k^T P u_k + 2q^T u_k + r \tag{3.18}$$

where P, q and r are function of $\delta\Gamma$, $\delta\Lambda$ as follows

$$\begin{aligned} P =& \Lambda^T \overline{Q} \Lambda + \overline{R} + \Delta_{\text{u}}^T \overline{S} \Delta_{\text{u}} + \delta\Lambda^T \overline{Q} \delta\Lambda + \Lambda^T \overline{Q} \delta\Lambda + \delta\Lambda^T \overline{Q} \Lambda \\ q^T =& (\Gamma - Y_{ref})^T \overline{Q} \Lambda - U_{(k-1)}^T \overline{S} \Delta_{\text{u}} - U_{ref}^T \overline{R} + (\Gamma - Y_{ref})^T \overline{Q} \delta\Lambda + \delta\Gamma^T \overline{Q} \delta\Lambda + \delta\Gamma^T \overline{Q} \Lambda \\ r =& U_{(k-1)}^T \overline{S} U_{(k-1)} + U_{ref}^T \overline{R} U_{ref} + (\Gamma - Y_{ref})^T \overline{Q} (\Gamma - Y_{ref}) + (\Gamma - Y_{ref})^T \overline{Q} \delta\Gamma \\ &+ \delta\Gamma^T \overline{Q} \delta\Gamma + \delta\Gamma^T \overline{Q} (\Gamma - Y_{ref}) \end{aligned} \tag{3.19}$$

3.2.3.3 Robust MPC problem as a RQP with ellipsoidal uncertainty

As in appendix A.2.1 the uncertainties in P, q and r are assumed norm bounded and $\mathbb{E}$, $\mathbb{F}$ and $\mathbb{G}$ can be written as

$$\mathbb{E} = \left\{ P_0 + \sum_{i_e=1}^{m} P_{i_e} \mu_{i_e} \mid \|\mu\|_2 \leq 1 \right\}$$

$$\mathbb{F} = \left\{ q_0 + \sum_{j_e=1}^{z} q_{j_e} \nu_{j_e} \mid \|\nu\|_2 \leq 1 \right\}$$

$$\mathbb{G} = \left\{ r_0 + \sum_{l_e=1}^{n_r} r_{l_e} \xi_{l_e} \mid \|\xi\|_2 \leq 1 \right\}$$

Then, from (3.19), we have that,

$$
\begin{aligned}
&P_0 = \Lambda^T\overline{Q}\Lambda + \overline{R} + \Delta_{\mathrm{u}}^T\overline{S}\Delta_{\mathrm{u}}\\
&\sum_{i_e=1}^{m} P_{i_e}\mu_{i_e} = \delta\Lambda^T\overline{Q}\delta\Lambda + \Lambda^T\overline{Q}\delta\Lambda + \delta\Lambda^T\overline{Q}\Lambda\\
&q_0^T = (\Gamma - Y_{ref})^T\overline{Q}\Lambda - U_{(k-1)}^T\overline{S}\Delta_{\mathrm{u}} - U_{ref}^T\overline{R}\\
&\sum_{j_e=1}^{z} q_{j_e}\nu_{j_e} = (\Gamma - Y_{ref})^T\overline{Q}\delta\Lambda + \delta\Gamma^T\overline{Q}\delta\Lambda + \delta\Gamma^T\overline{Q}\Lambda\\
&r_0 = U_{(k-1)}^T\overline{S}U_{(k-1)} + U_{ref}^T\overline{R}U_{ref} + (\Gamma - Y_{ref})^T\overline{Q}(\Gamma - Y_{ref})\\
&\sum_{l_e=1}^{n_r} r_{l_e}\xi_{l_e} = (\Gamma - Y_{ref})^T\overline{Q}\delta\Gamma + \delta\Gamma^T\overline{Q}\delta\Gamma + \delta\Gamma^T\overline{Q}(\Gamma - Y_{ref})
\end{aligned}
$$

Observe that $\delta\Gamma$ and $\delta\Lambda$ uncertainties are reflected as an ellipsoidal uncertainty in P q, and r. Therefore, using the Theorem 4 stated in appendix A, the min-max MPC problem becomes the RQP problem (3.20).

$$
\begin{aligned}
&\min_{u_k, w_{i_e}, s, d, t} s + 2q_0^T u_k(k) + t + 2d + \|\overline{r}\|_2\\
&\text{subject to:}\\
&\left\|\begin{bmatrix} 2P_0^{1/2}u_k \\ s-1 \end{bmatrix}\right\|_2 \leq s+1\\
&\left\|\begin{bmatrix} 2P_{i_e}^{1/2}u_k \\ w_{i_e}-1 \end{bmatrix}\right\|_2 \leq w_{i_e}+1\\
&\|q_{j_e}u_k\|_2 \leq d\\
&\|w\|_2 \leq t\\
&0 \leq w_{i_e}\\
&0 \leq s\\
&\begin{bmatrix} I \\ -I \\ \Delta_{\mathrm{u}} \\ \Delta_{\mathrm{u}} \\ \Lambda \\ -\Lambda \end{bmatrix} u_k \leq \begin{bmatrix} \tilde{u}_{\max} \\ -\tilde{u}_{\min} \\ \tilde{\Delta u}_{\max} + \overline{u}(k-1) \\ \tilde{\Delta u}_{\max} - \overline{u}(k-1) \\ \tilde{y}_{\max} - \Gamma \\ -\tilde{y}_{\min} + \Gamma \end{bmatrix}
\end{aligned}
\tag{3.20}
$$

where $\overline{r} = [r_1, \ldots, r_{n_r}]^T$. It is worth to point out that optimization problem (3.20) is a convex optimization problem. Therefore, its solution if exists is unique. Moreover, it can be numerically computed in a finite number of steps. Then, u_k is feasible and the closed-loop system is stable.

Remark 1. *Note that in the optimization problem* (3.20) *it is required to compute the matrices* P_{i_e} *and* q_{j_e}*, as well the order of the vectors* μ *and* ν*. This can be done from the* *initial description of the uncertainty in A_k and B_k, however this is not a easy work because frequently is almost impossible to find an analytic expression for these ellipsoids. For this reason in the next paragraph we propose a method to calculate the matrices P_{i_e}, q_{j_e} and the order of the vectors μ and ν by means of algebraic relationships and without information of the initial ellipsoids.*

Computing P_{i_e} and q_{j_e}: In order to generalize the problem of computing P_{i_e}, q_{j_e} and the order of the vectors $\mu \in \mathbb{R}^m$ and $\nu \in \mathbb{R}^z$, let $\delta\Gamma \in \mathbb{R}^{p\times 1}$, $\delta\Lambda \in \mathbb{R}^{p\times n}$, $\Gamma \in \mathbb{R}^{p\times 1}$, $\Lambda \in \mathbb{R}^{p\times n}$ and $\overline{Q} \in \mathbb{R}^{p\times p}$. For computing P_{i_e} matrices, let $H \in \mathbb{R}^{n\times n}$ with $H = \delta\Lambda^T\overline{Q}\delta\Lambda + \Lambda^T\overline{Q}\delta\Lambda + \delta\Lambda^T\overline{Q}\Lambda$ a symmetric matrix. It is possible to find a general expressions for each element of the H matrix in terms of the elements of $\delta\Lambda$, Λ and $\overline{Q}$ with,

$$\delta\Lambda = \begin{bmatrix} \delta_{1,1} & \cdots & \delta_{1,n} \\ \vdots & \ddots & \vdots \\ \delta_{p,1} & \cdots & \delta_{p,n} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \lambda_{1,1} & \cdots & \lambda_{1,n} \\ \vdots & \ddots & \vdots \\ \lambda_{p,1} & \cdots & \lambda_{p,n} \end{bmatrix}, \quad \overline{Q} = \mathrm{diag}([q_1 \cdots q_p]) \tag{3.21}$$

In this case, each element of H is given by (3.22)

$$\begin{aligned} H_{ii} &= \sum_{l=1}^{p} \left[q_l \delta_{l,i}^2 + 2 q_l \lambda_{l,i}\delta_{l,i} \right] \\ H_{ij} &= \sum_{l=1}^{p} \left[q_l \delta_{l,i}\delta_{l,j} + q_l(\lambda_{l,j}\delta_{l,i} + \lambda_{l,i}\delta_{l,j}) \right], \quad \forall j > i \end{aligned} \tag{3.22}$$

Note that $H = \sum_{i_e=1}^{m} P_{i_e}\mu_{i_e}$. Hence, the elements of $\delta\Lambda$ and the different combinations of these can be associated (mapped) with each element of μ (based in (3.22)) as follows

$$\mu_{i_e} = \begin{cases} q_i\delta_{i,j} & \text{for} \quad i_e = 1, \ldots, p\cdot n \\ q_i\delta_{i,j}^2 & \text{for} \quad i_e = p\cdot n + 1, \ldots, 2p\cdot n \\ q_i\delta_{i,j}\delta_{i,\alpha} & \text{for} \quad i_e = 2p\cdot n + 1, \ldots, m \end{cases} \tag{3.23}$$

with $\alpha = \{j+1, \ldots, n\}$. Thus, the dimension of μ is $m = \frac{3}{2}(p\cdot n) + \frac{1}{2}(p\cdot n^2)$. In addition,

$$H = \sum_{i_e=1}^{m} P_{i_e}\mu_{i_e} = \begin{bmatrix} \sum_{i_e=1}^{m} P_{11,i_e}\mu_{i_e} & \cdots & \sum_{i_e=1}^{m} P_{1n,i_e}\mu_{i_e} \\ \vdots & \ddots & \vdots \\ \sum_{i_e=1}^{m} P_{1n,i_e}\mu_{i_e} & \cdots & \sum_{i_e=1}^{m} P_{nn,i_e}\mu_{i_e} \end{bmatrix} \tag{3.24}$$

where

$$\begin{aligned}\sum_{i_e=1}^{m} P_{ii,i_e}\mu_{i_e} &= H_{ii} = \sum_{l=1}^{p}\left[q_l\delta_{l,i}^2 + 2q_l\lambda_{l,i}\delta_{l,i}\right]\\ \sum_{i_e=1}^{m} P_{ij,i_e}\mu_{i_e} &= H_{ij} = \sum_{l=1}^{p}\left[q_l\delta_{l,i}\delta_{l,j} + q_l(\lambda_{l,j}\delta_{l,i} + \lambda_{l,i}\delta_{l,j})\right], \quad \forall j > i\end{aligned} \tag{3.25}$$

Now, note that it is possible to find a expression of μ_{i_e} in terms of i and j as follows

$$\mu = \begin{bmatrix}\mu_{(j-1)p+i}\\ \mu_{n\cdot p+(j-1)p+i}\\ \mu_\kappa\end{bmatrix} = \begin{bmatrix} q_i\delta_{i,j}\\ q_i\delta_{i,j}^2\\ q_i\delta_{i,j}\delta_{i,\alpha}\end{bmatrix} \tag{3.26}$$

with $\kappa = 2p\cdot n + p\cdot(j-1)(n-\frac{j}{2}) + p\cdot(\alpha - j - 1) + i$. Note that $\|\mu\|_2 \leq 1$, then a necessary condition to use this approach is:

$$\|\mu\|_2 = \left\|\begin{bmatrix}\mu_{(j-1)p+i}\\ \mu_{n\cdot p+(j-1)p+i}\\ \mu_\kappa\end{bmatrix}\right\|_2 = \left\|\begin{bmatrix} q_i\delta_{i,j}\\ q_i\delta_{i,j}^2\\ q_i\delta_{i,j}\delta_{i,\alpha}\end{bmatrix}\right\|_2 \leq 1 \tag{3.27}$$

Note that q_i regulates the previous condition, therefore an adequate election of the matrix Q can lead to accomplish the condition (3.27). Hence (3.25) can be written as,

$$\begin{aligned}\sum_{i_e=1}^{m} P_{ii,i_e}\mu_{i_e} &= H_{ii} = \sum_{l=1}^{p}\left[q_l\mu_{n\cdot p+(i-1)p+l} + 2q_l\lambda_{l,i}\mu_{(i-1)p+l}\right]\\ \sum_{i_e=1}^{m} P_{ij,i_e}\mu_{i_e} &= H_{ij} = \sum_{l=1}^{p}\left[q_l\mu_{\overline{\kappa}+l} + q_l(\lambda_{l,j}\mu_{(i-1)p+l} + \lambda_{l,i}\mu_{(j-1)p+l})\right], \quad \forall j > i\end{aligned} \tag{3.28}$$

where $\overline{\kappa} = \frac{p\cdot i}{2}(2n - i - 1) + p\cdot(n + j - 1)$. According to equation (3.28) the elements of the P_{i_e} matrices are,

$$\begin{aligned} P_{ii,i_e} &= \begin{cases} 2\lambda_{i_e-(i-1)p,i} & \text{for} \quad i_e = (i-1)p+1, \ldots, (i-1)p+p\\ 1 & \text{for} \quad i_e = p\cdot n + (i-1)p + 1, \ldots, p\cdot n + (i-1)p + p\\ 0 & \text{for} \quad \text{otherwise}\end{cases}\\ P_{ij,i_e} &= \begin{cases} \lambda_{i_e-(i-1)p,j} & \text{for} \quad i_e = (i-1)p+1, \ldots, (i-1)p+p\\ \lambda_{i_e-(j-1)p,i} & \text{for} \quad i_e = (j-1)p+1, \ldots, (j-1)p+p\\ 1 & \text{for} \quad i_e = \overline{\kappa}+1, \ldots, \overline{\kappa}+p\\ 0 & \text{for} \quad \text{otherwise}\end{cases}\end{aligned} \tag{3.29}$$

For computing the q_{j_e} matrices, let $E \in \mathbb{R}^{1\times n}$ with $E = \overline{\Gamma}^T\overline{Q}\delta\Lambda + \delta\Gamma^T\overline{Q}\delta\Lambda + \delta\Gamma^T\overline{Q}\Lambda$. It is possible to find a general expressions for each element of the E matrix in terms of the elements of $\delta\Gamma$, $\delta\Lambda$, Γ and $\overline{Q}$,

$$\delta\Gamma = \begin{bmatrix}\rho_1 & \cdots & \rho_p\end{bmatrix}^T, \quad \overline{\Gamma} = \begin{bmatrix}\sigma_1 & \cdots & \sigma_p\end{bmatrix}^T, \tag{3.30}$$

where $\overline{\Gamma} = (\Gamma - Y_{ref})$. Then each element of E can be obtained as

$$E_i = \sum_{l=1}^{p} \left[q_l\delta_{l,i}\rho_l + q_l\overline{\Gamma}_l\delta_{l,i} + q_l\lambda_{l,j}\rho_l\right] \tag{3.31}$$

Note that $E = \sum_{j_e=1}^{z} q_{j_e}\nu_{j_e}$, then each element of $\delta\Lambda$, $\delta\Gamma$ and the different combinations of these are associated (mapped) with each element of ν according to (3.31) of the next form,

$$\nu_{j_e} = \begin{cases} q_i\delta_{i,j} & \text{for} \quad j_e \;=\; 1,\ldots,p\cdot n \\ q_i\rho_i & \text{for} \quad j_e \;=\; p\cdot n+1,\ldots,p\cdot n+p \\ q_i\delta_{i,j}\rho_i & \text{for} \quad j_e \;=\; p\cdot n+p+1,\ldots,z \end{cases} \tag{3.32}$$

hence, the order of ν is $z = 2p\cdot n + p$. Following the same procedure to find P_{i_e}, the elements of the q_{j_e} are given by,

$$q_{1i,j_e} = \begin{cases} \overline{\Gamma}_{j_e-(j-1)p} & \text{for} \quad j_e = (j-1)p+1,\ldots,(j-1)p+p \\ \lambda_{j_e-p\cdot n,j} & \text{for} \quad j_e = p\cdot n+1,\ldots,p\cdot n+p \\ 1 & \text{for} \quad j_e = p\cdot n+j\cdot p+1,\ldots,p\cdot n+j\cdot p+p \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \tag{3.33}$$

Note that $\|\nu\|_2 \leq 1$, then a necessary condition to use this approach is:

$$\|\nu\|_2 = \left\| \begin{bmatrix} \nu_{(j-1)p+i} \\ \nu_{n\cdot p+i} \\ \nu_{n\cdot p+p+(j-1)p+i} \end{bmatrix} \right\|_2 = \left\| \begin{bmatrix} q_i\delta_{i,j} \\ q_i\rho_i \\ q_i\delta_{i,j}\rho_i \end{bmatrix} \right\|_2 \leq 1 \tag{3.34}$$

Again, note that q_i regulates the previous condition, therefore an adequate election of the matrix Q can lead to accomplish the condition (3.34)

3.2.3.4 Robust MPC problem as a RQP with polytopic uncertainty

In this case uncertainties in P, q and r are given by polytopes of the form

$$\begin{aligned}
\mathbb{E} &= \mathbf{Co}\{P_1, \ldots, P_m\} = \left\{ P : P = \sum_{i_p=1}^{m} P_{i_p}\mu_{i_p}, \sum_{i_p=1}^{m} \mu_{i_p} = 1, \mu_{i_p} \geq 0 \right\} \\
\mathbb{F} &= \mathbf{Co}\{q_1, \ldots, q_z\} = \left\{ q : q = \sum_{j_p=1}^{z} q_{j_p}\nu_{j_p}, \sum_{j_p=1}^{z} \nu_{j_p} = 1, \nu_{j_p} \geq 0 \right\} \\
\mathbb{G} &= \mathbf{Co}\{r_1, \ldots, r_{n_r}\} = \left\{ r : r = \sum_{l_p=1}^{n_r} r_{l_p}\xi_{l_p}, \sum_{l_p=1}^{n_r} \xi_{l_p} = 1, \xi_{l_p} \geq 0 \right\}
\end{aligned} \tag{3.35}$$

With this assumption, equation (3.16) becomes (3.36).

$$J_{N_p}(u_k) = u_k^T(P + \delta P)u_k + 2(q + \delta q)^T u_k + (r + \delta r) \tag{3.36}$$

where

$$\begin{aligned}
\delta P &= \sum_{i_p=1}^{m} P_{i_p}\mu_{i_p} = \delta\Lambda^T \overline{Q} \delta\Lambda + \Lambda^T \overline{Q} \delta\Lambda + \delta\Lambda^T \overline{Q} \Lambda \\
\delta q &= \sum_{j_p=1}^{z} q_{j_p}\nu_{j_p} = (\Gamma - Y_{ref})^T \overline{Q} \delta\Lambda + \delta\Gamma^T \overline{Q} \delta\Lambda + \delta\Gamma^T \overline{Q} \Lambda \\
\delta r &= \sum_{l_p=1}^{n_r} r_{l_p}\xi_{l_p} = (\Gamma - Y_{ref})^T \overline{Q} \delta\Gamma + \delta\Gamma^T \overline{Q} \delta\Gamma + \delta\Gamma^T \overline{Q} (\Gamma - Y_{ref})
\end{aligned} \tag{3.37}$$

Note that $\delta\Gamma$ and $\delta\Lambda$ uncertainties are reflected as an polytopic uncertainty in P q, and r. Therefore, using the result obtained in section A.2.2, the min-max MPC problem becomes the RQP problem (3.38).

$$
\begin{aligned}
&\min_{u_k, w_{i_p}, d, t} \; t + 2d + \|\overline{r}\|_2 \\
\text{subject to:}\quad & \\
&\left\| \begin{bmatrix} 2P_{i_p}^{1/2} u_k \\ w_{i_p} - 1 \end{bmatrix} \right\|_2 \leq w_{i_p} + 1 \\
&\|q_{j_p} u_k\|_2 \leq d \\
&\|w\|_2 \leq t \\
&0 \leq w_{i_p} \\
&\begin{bmatrix} I \\ -I \\ \Delta_\mathrm{u} \\ \Delta_\mathrm{u} \\ \Lambda \\ -\Lambda \end{bmatrix} u_k \leq \begin{bmatrix} \tilde{u}_{\max} \\ -\tilde{u}_{\min} \\ \tilde{\Delta} u_{\max} + \overline{u}(k-1) \\ \tilde{\Delta} u_{\max} - \overline{u}(k-1) \\ \tilde{y}_{\max} - \Gamma \\ -\tilde{y}_{\min} + \Gamma \end{bmatrix}
\end{aligned}
\tag{3.38}
$$

Remark 2. *As in section 3.2.3.3 in the optimization problem* (3.38) *it is necessary the computation of the matrices* P_{i_p} *and* q_{j_p} *also the order of the vectors* μ *and* ν*. There are two forms to do that, the first one by means of algebraic relationships between the uncertain description and equation* (3.37)*, and the second one can be done from the initial description of the uncertainty in* A_k *and* B_k*. In the next paragraphs the two forms are developed.*

Computing P_{i_p} and q_{j_p}:

First method: As it was mentioned, this method is based in algebraic relationships between the uncertain description and equation (3.37). The same methodology used in the previous section where ellipsoidal uncertainty was taken into account. In this way the result obtained is exactly the same, however a very restrictive condition is imposed. Hence, the matrices P_{i_p} and q_{j_p} are given by:

$$P_{ii,i_p} = \begin{cases} 2\lambda_{i_p-(i-1)p,i} & \text{for} \quad i_p = (i-1)p+1,\dots,(i-1)p+p \\ 1 & \text{for} \quad i_p = p\cdot n+(i-1)p+1,\dots,p\cdot n+(i-1)p+p \\ 0 & \text{for} \quad \text{otherwise} \end{cases}$$
$$P_{ij,i_p} = \begin{cases} \lambda_{i_p-(i-1)p,j} & \text{for} \quad i_p = (i-1)p+1,\dots,(i-1)p+p \\ \lambda_{i_p-(j-1)p,i} & \text{for} \quad i_p = (j-1)p+1,\dots,(j-1)p+p \\ 1 & \text{for} \quad i_p = \overline{\kappa}+1,\dots,\overline{\kappa}+p \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \tag{3.39}$$
$$q_{1i,j_p} = \begin{cases} \overline{\Gamma}_{j_p-(j-1)p} & \text{for} \quad j_p = (j-1)p+1,\dots,(j-1)p+p \\ \lambda_{j_p-p\cdot n,j} & \text{for} \quad j_p = p\cdot n+1,\dots,p\cdot n+p \\ 1 & \text{for} \quad j_p = p\cdot n+j\cdot p+1,\dots,p\cdot n+j\cdot p+p \\ 0 & \text{for} \quad \text{otherwise} \end{cases}$$

where

$$\mu_{i_p} = \begin{cases} q_i\delta_{i,j} & \text{for} \quad i_p = 1,\dots,p\cdot n \\ q_i\delta_{i,j}^2 & \text{for} \quad i_p = p\cdot n+1,\dots,2p\cdot n \\ q_i\delta_{i,j}\delta_{i,\alpha} & \text{for} \quad i_p = 2p\cdot n+1,\dots,m \end{cases}$$
$$\nu_{j_p} = \begin{cases} q_i\delta_{i,j} & \text{for} \quad j_p = 1,\dots,p\cdot n \\ q_i\rho_i & \text{for} \quad j_p = p\cdot n+1,\dots,p\cdot n+p \\ q_i\delta_{i,j}\rho_i & \text{for} \quad j_p = p\cdot n+p+1,\dots,z \end{cases} \tag{3.40}$$

with $\alpha = \{j+1,\dots,n\}$, the order of μ is $m = \frac{3}{2}(p\cdot n)+\frac{1}{2}(p\cdot n^2)$ and the dimension of ν is $z = 2p\cdot n+p$. Now, note that,

$$\mu = \begin{bmatrix} \mu_{(j-1)p+i} \\ \mu_{n\cdot p+(j-1)p+i} \\ \mu_\kappa \end{bmatrix} = \begin{bmatrix} q_i\delta_{i,j} \\ q_i\delta_{i,j}^2 \\ q_i\delta_{i,j}\delta_{i,\alpha} \end{bmatrix}$$
$$\nu = \begin{bmatrix} \nu_{(j-1)p+i} \\ \nu_{n\cdot p+i} \\ \nu_{n\cdot p+p+(j-1)p+i} \end{bmatrix} = \begin{bmatrix} q_i\delta_{i,j} \\ q_i\rho_i \\ q_i\delta_{i,j}\rho_i \end{bmatrix} \tag{3.41}$$

where $\kappa = 2p\cdot n+p\cdot(j-1)(n-\frac{j}{2})+p\cdot(\alpha-j-1)+i$. From the uncertain description $\sum_{i_p=1}^{m}\mu_{i_p} = 1$, $\mu_{i_p} \geq 0$, $\sum_{j_p=1}^{z}\nu_{j_p} = 1$ and $\nu_{j_p} \geq 0$ (an equivalent condition is $\|\mu\|_1 = 1$ and $\|\nu\|_1 = 1$), therefore a necessary condition to use this approach is,

$$\begin{aligned} \|\mu\|_1 &= \left\| \begin{bmatrix} \mu_{(j-1)p+i} \\ \mu_{n\cdot p+(j-1)p+i} \\ \mu_\kappa \end{bmatrix} \right\|_1 = \left\| \begin{bmatrix} q_i\delta_{i,j} \\ q_i\delta_{i,j}^2 \\ q_i\delta_{i,j}\delta_{i,\alpha} \end{bmatrix} \right\|_1 = 1 \\ \|\nu\|_1 &= \left\| \begin{bmatrix} \nu_{(j-1)p+i} \\ \nu_{n\cdot p+i} \\ \nu_{n\cdot p+j\cdot p+i} \end{bmatrix} \right\|_1 = \left\| \begin{bmatrix} q_i\delta_{i,j} \\ q_i\rho_i \\ q_i\delta_{i,j}\rho_i \end{bmatrix} \right\|_1 = 1 \end{aligned} \tag{3.42}$$

Again, note that q_i regulates the previous condition, therefore an adequate election of the matrix Q can lead to accomplish the condition (3.42)

Second method: This method assumes that the matrices of the polytopic uncertainty description in δP, δq and δr can be found through of the uncertainty in A_k and B_k. Let Ψ the next convex hull,

$$[A_k, B_k] \in \Psi = \sum_{i=1}^{L} \mu_i \, [A_i, B_i] \text{ with } \sum_{i=1}^{L} \mu_i = 1$$

Now, if we suppose that every pair $[A_i, B_i]$ defines the following convex hull $\overline{\Psi}$,

$$[P, q, r] \in \overline{\Psi} = \sum_{i=1}^{L} \mu_i \, [P_i, q_i, r_i] \text{ with } \sum_{i=1}^{L} \mu_i = 1$$

then, the matrices of the polytopes of (3.35) P_{i_p}, q_{j_p} and r_{l_p} can be found by means of every pair $[A_i, B_i]$ and, equations (3.15) and (3.19). It is important to mention that in this case the order of every polytope is the same, i.e $m = z = n_r = L$, with this equation (3.35) is modified as follows,

$$\begin{aligned} \mathbb{E} &= \mathbf{Co}\{P_1, \ldots, P_L\} = \left\{ P : P = \sum_{i=1}^{L} P_i \mu_i, \sum_{i=1}^{m} \mu_i = 1, \mu_i \geq 0 \right\} \\ \mathbb{F} &= \mathbf{Co}\{q_1, \ldots, q_L\} = \left\{ q : q = \sum_{i=1}^{L} q_i \nu_i, \sum_{i=1}^{z} \nu_i = 1, \nu_i \geq 0 \right\} \\ \mathbb{G} &= \mathbf{Co}\{r_1, \ldots, r_L\} = \left\{ r : r = \sum_{i=1}^{L} r_i \xi_i, \sum_{i=1}^{n_r} \xi_i = 1, \xi_i \geq 0 \right\} \end{aligned} \tag{3.43}$$

3.2.3.5 Robust MPC problem as a RQP with multi-plant uncertainty

One of the simpler ways to represent model uncertainty is to consider the multi-plant system, where we have a discrete set Ψ of plants, and the real plant is unknown, but it is assumed to be one of the members of this set. With this representation of model uncertainty, we can define the set of possible plants as $\Psi = \{\psi_1, \ldots, \psi_n\}$ where each ψ_i

corresponds to a particular plant. Also, let us assume that the true plant which lies within the set Ψ is designated as ψ_T and there is a most likely plant that also lies in Ψ and is designated as ψ_n. In addition, it is assumed that the current estimated state corresponds to the true plant. The systems covered in this section are constrained uncertain systems of the form

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i u(k) \\ y(k) &= Cx(k) + Du(k) \\ \psi_i &\triangleq (A_i, B_i) \in \Psi \end{aligned}$$

Let $y_k = [y^T(k), \ldots, y^T(k+N_p)]^T$ denotes the sequence of predicted values for the output $y(k)$. Then, the prediction model can be written as (3.44).

$$y_k = \underbrace{\Gamma_i}_{\text{Free Response}} + \underbrace{\Lambda_i u_k}_{\text{Forced Response}} \tag{3.44}$$

with $\Gamma_i = \overline{C}\Phi_i x(k)$ and $\Lambda_i = [\overline{C}H_i \quad 0] + [0 \quad \overline{D}]$, where $\overline{C} = \text{diag}([\underbrace{C, \cdots, C}_{N_p}])$, $\overline{D} = \text{diag}([\underbrace{D, \cdots, D}_{N_p}])$, and

$$\Phi_i = \begin{bmatrix} A_i \\ A_i^2 \\ \vdots \\ A_i^{N_p} \end{bmatrix}, \quad H_i = \begin{bmatrix} B_i & 0 & \cdots & 0 \\ A_i B_i & B_i & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_i^{N_p-1} B_i & A_i^{N_p-2} B_i & \cdots & B_i \end{bmatrix},$$

With this assumption, the original min-max MPC problem can be converted to the next RQP problem (3.45).

$$\min_{u_k} \max_{P_i \in \mathbb{E}, q_i \in \mathbb{F}, r_i \in \mathbb{G}} u_k^T P_i u_k + 2q_i^T u_k + r_i \tag{3.45}$$

where $\mathbb{E} = \{P_1, \ldots, P_n\}$, $\mathbb{F} = \{q_1, \ldots, q_n\}$ and $\mathbb{G} = \{r_1, \ldots, r_n\}$ and P_i, q_i and r_i are given by

$$\begin{aligned} P_i &= \Lambda_i^T \overline{Q} \Lambda_i + \overline{R} + \Delta_{\text{u}}^T \overline{S} \Delta_{\text{u}} \\ q_i^T &= (\Gamma_i - Y_{ref})^T \overline{Q} \Lambda_i - U_{(k-1)}^T \overline{S} \Delta_{\text{u}} - U_{ref}^T \overline{R} \\ r_i &= U_{(k-1)}^T \overline{S} U_{(k-1)} + U_{ref}^T \overline{R} U_{ref} + (\Gamma_i - Y_{ref})^T \overline{Q} (\Gamma_i - Y_{ref}) \end{aligned} \tag{3.46}$$

Therefore, using the result obtained in section A.2.3, the min-max MPC problem becomes the RQP problem (3.47).

$$
\begin{aligned}
&\min_{u_k,w,d,t} \; w+2t+d \\
\text{subject to:}& \\
&\left\| \begin{bmatrix} 2P_i^{1/2}u_k \\ w-1 \end{bmatrix} \right\|_2 \leq w+1 \\
&q_i u_k \leq t \\
&0 \leq w \\
&r_i \leq d \\
&\begin{bmatrix} I \\ -I \\ \Delta_{\mathrm{u}} \\ \Delta_{\mathrm{u}} \\ \Lambda \\ -\Lambda \end{bmatrix} u_k \leq \begin{bmatrix} \tilde{u}_{\max} \\ -\tilde{u}_{\min} \\ \tilde{\Delta} u_{\max} + \overline{u}(k-1) \\ \tilde{\Delta} u_{\max} - \overline{u}(k-1) \\ \tilde{y}_{\max} - \Gamma \\ -\tilde{y}_{\min} + \Gamma \end{bmatrix}
\end{aligned} \tag{3.47}
$$

3.2.4 Robust stability

Although the convexity an feasibility of the solution of (3.69),(3.38) and (3.47) guarantees system stability, is important to make an in-depth discussion about robust stability. In this case, such discussion comes from the min-max formulation of the MPC, and from the special considerations that must be taken to guarantee robust closed-loop stability. In this way, assume the existence of a compact terminal region $\Omega \subseteq \mathbb{X}$, control law $K_f : \Omega \longrightarrow \mathbb{X}$ and a class κ function $\gamma(\cdot)$ such that the following conditions holds $\forall x(k+j) \in \Omega$

$$V(f(x(k+j), K_f(x(k+j)), \theta(k+j))) - V(x(k+j)) \leq -L(x(k+j), K_f(x(k+j))) + \gamma(\rho)$$

where $L(\cdot)$ is the stage cost, and $V(\cdot)$ is the terminal cost. The previous assumption requires that for each $x(k+j) \in \Omega$, $f(x(k+j), K_f) \in \Omega$, i.e. the set Ω is control invariant under control law $u_{RMPC} = K_f(x(k+j))$. Consequently, the following Lemma arises.

Lemma 1. *If previous assumption holds and the uncertainties are modeled by* $\theta(k+j) \in \Psi$, $\|\theta\|_2 \leq \rho$, *we have that*

$$J_i^*(x(k+j)) - J_{i-1}^*(x(k+j)) \leq \gamma(\rho)$$

Proof. First, the controller will be consider in its min-max formulation, which can be expressed as the solution of the following dynamic programming problem [91]:

$$J_i^* = \min_{u \in \mathbb{U}} \left\{ \max_{\theta(k+j) \in \Psi} \{ L(x(k+j), u(k+j)) + J_{i-1}^*(f(x(k+j), u(k+j), \theta(k+j))) \} \mid \forall \theta \in \Psi \right\}$$

where $J_0^*(x(k+j)) = V(x(k+j))$. In order to simplify the notation let $x = x(k+j)$, $u = u(k+j)$ and $\theta = \theta(k+j)$. Then, in order to prove this lemma mathematical induction will be used. for $i = 1$, $\forall x \in \Omega$ we have,

$$\begin{aligned} J_1^*(x) - J_0^*(x) &= \min_{u \in \mathbb{U}} \max_{\theta \in \Psi} \{L(x,u) + V(f(x,u,\theta))\} - V(x) \\ &\leq \max_{\theta \in \Psi} \{L(x,u) + V(f(x,u,\theta))\} - V(x) \\ &\leq \gamma(\rho) \end{aligned}$$

Now we suppose that $J_i^*(x) - J_{i-1}^*(x) \leq \gamma(\rho)$, $\forall x$. Then considering the optimality of the control law $K_f(x)$ and $\forall x$ yields

$$J_{i+1}^*(x) - J_i^*(x) \leq \max_{\theta \in \Psi} \{J_i^*(f(x, K_f(x), \theta)) - J_{i-1}^*(f(x, K_f(x), \theta))\}$$

finally from hypothesis of the mathematical induction it is verified that

$$J_{i+1}^*(x) - J_i^*(x) \leq \gamma(\rho)$$

Therefore, the lemma is demonstrated.

□

The stability of the min-max controller (3.12) is established in the following theorem

Theorem 2. *Consider the system $x(k+1) = f(x(k), u(k), \theta(k))$, and suppose that uncertainties are modeled by $\theta(k+j) \in \Psi$, $\|\theta\|_2 \leq \rho$. Let $J_i^*(x(k+j)) - J_{i-1}^*(x(k+j)) \leq \gamma(\rho)$ and assuming feasibility at the initial state, then the uncertain system controlled by the min-max MPC controller $u_{RMPC} = K_f(x(k))$ is robustly stable. Furthermore, the optimal cost is a Lyapunov function.*

Proof.

$$\begin{aligned} J_{N_p}^*(x_{k+1}) - J_{N_p}^*(x_k) &= J_{N_p}^*(x_{k+1}) - \max_{\theta \in \Psi} \{L(x(k), K_f(x(k))) \\ &\quad + J_{N_p-1}^*(f(x(k), K_f(x(k)), \theta))\} \\ &\leq J_{N_p}^*(x_{k+1}) - L(x(k), K_f(x(k))) - J_{N_p-1}^*(x_{k+1}) \end{aligned}$$

considering the Lemma 1 we have

$$J_{N_p}^*(x_{k+1}) - J_{N_p-1}^*(x_{k+1}) \leq \gamma(\rho)$$

then, it is possible deduce that

$$J^*_{N_p}(x_{k+1}) - J^*_{N_p}(x_k) \leq -L(x(k), K_f(x(k))) + \gamma(\rho)$$

therefore $J^*_{N_p}(x_{k+j})$ is a Lyapunov function of the system. So the stability of the system is guaranteed by the Lemma 1.

□

3.2.5 Robust Model Predictive for a gantry crane system

In this section a Robust Model Predictive for a gantry crane (Figure 3.2) system is presented and compared with a nominal MPC under uncertainty. The pendulum dynamic system has been studied over the years and widely applied to the industry. In general, the important variables in the analysis of the crane system are angle, mass of cart, mass of object hung with cart, mass of rope and length of rope. The mathematical model for the gantry crane used here are given by:

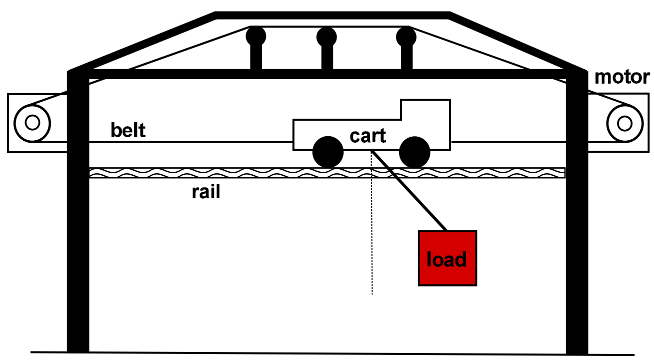


Figure 3.2: Gantry Crane.

$$\begin{aligned} ml^2\ddot{\theta} - ml\ddot{r}cos(\theta) + mglsin(\theta) = 0 \\ (M+m)\ddot{r} - ml\ddot{\theta} + ml\dot{\theta}^2\theta - U_r = 0 \end{aligned} \tag{3.48}$$

For this example, $M = 2$ Kg is the mass of the cart, $m = 1$ Kg is the mass of the load, $l = 1$ m is the length of rope between cart and mass of the load, r is the position of the cart, g is the earth gravity and U_r is the force exerted by a motor over the cart (see Figure 3.2). The objective of the controller is to regulate the angle (θ) and the position (r) of the cart. The states of the system are $x(k) = \left[\theta(k), \dot{\theta}(k), r(k), \dot{r}(k)\right]^T$, the outputs are $y(k) = [\theta(k), r(k)]^T$ and the input is $u(k) = U_r$. The linear model of the system can be found if the angle θ is supposed small, with this $cos(\theta) \approx 1$ and $sin(\theta) \approx 0$. Then, the linear model of 3.48 is given by,

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-(M+2m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-2mg}{M} & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} u \tag{3.49}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x;$$

The cost function used for the gantry crane is given by:

$$\begin{aligned} J_{N_p}(x_k, u_k) = & \sum_{j=1}^{N_p} \|(y(k+j) - y_{ref}\|_Q^2 + \sum_{j=0}^{N_p} \|u(k+j)\|_R^2 \\ & + \sum_{j=0}^{N_p} \|\Delta u(k+j)\|_S^2 + \|x(k+N_p)\|_{\tilde{Q}}^2 \end{aligned}$$

where $y_{ref} = [0, 5]^T$, $Q = 10^2 \cdot I_{2\times 2}$, $R = I_{1\times 1}$, $S = 50 \cdot I_{1\times 1}$, and $N_p = 10$. Figure 3.3 shows the responses of the nominal MPC. In order to test the nominal MPC, the system was initialized at $x_0 = [0, 0, 0]^T$, the idea is that the controller drives the system to y_{ref}. As can be seen in Figures 3.3(a) and 3.3(b) the nominal MPC controller moves the system to the set-point, however the overshot and the stabilization time are considerably high. Figures 3.3(c) and 3.3(d) show the input of the system and the cost function of the MPC controller

Now, in order to evaluate the performance of the nominal controller under uncertainties the mass m of the load is changed in 1 Kg at 50 seg. Figure 3.4 shows the simulation results. Note that in Figures 3.4(a) and 3.4(b) the controller cannot drive the system to the set point due to the uncertain in the mass of the load. Figures 3.4(c) and 3.4(d) show the input of the system and the cost function of the MPC controller which increases under uncertain scenario.

In order to control the system under uncertainties a robust MPC as the one shown in section 3.2.2 is designed. Figure 3.5 shows the simulation results of the Robust MPC. In order to test the robust controller the mass of the load was changed in 4 Kg (significantly bigger than in nominal MPC) at 15 seg. There are several facts to mention in this simulation, however, the most important facts are that the stabilization time is short versus the nominal MPC and how the uncertainty does not affect the responses. Figures 3.5(a) and 3.5(b) show the behavior of the outputs of the system. Note that the system has no overshot in the position of the crane, also the change in the mass of the load does not affect the responses. On the other hand Figures 3.5(c) and 3.5(d) show the input of the system and the cost function of the MPC controller which decreases under uncertain scenario. Finally, in general terms the gantry crane example shows the advantages of the robust MPC proposed here over a

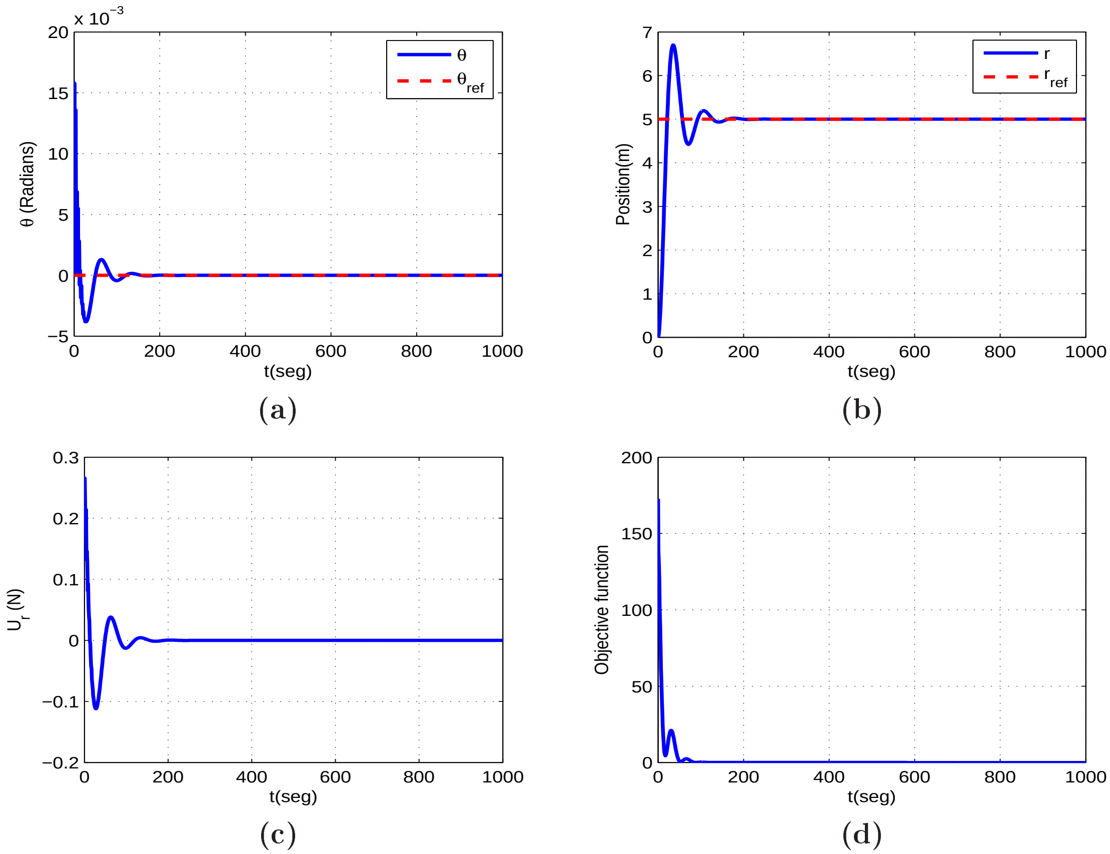


Figure 3.3: Nominal MPC response: (a) Crane angle (θ), (b) Crane Position (r), (c) Control action (U_r), (d) Objective function.

nominal MPC.

An important aspect to mention in the simulations results is the oscillation present in the objective function at the start of the simulation. This oscillation is because the angle of the crane oscillate while the car reach the set-point position, this fact can be seen in Figure 3.6 where a zoom of the crane angle and the objective function was done.

3.3 Robust Model Predictive Control with zone control

Model Predictive Control (MPC) has gained interest in the control research community, because it offers more flexibility than other optimization based control schemes. In [13] a complete review about MPC approaches is presented. One of the most interesting formulations of MPC corresponds to the zone control. In this control strategy the idea is to maintain the outputs and/or states of the controlled system inside a predefined operation zone. Then, the controlled variables are able to reach a whole set of points. In this direction, this paper proposes a robust MPC with zone control with guaranteed stability. As in the Robust MPC presented in previous section, in the proposed MPC with zone control the resulting min-max problem associated with the worst-case computation is transformed into a convex optimization problem (specifically into a second-order cone programming

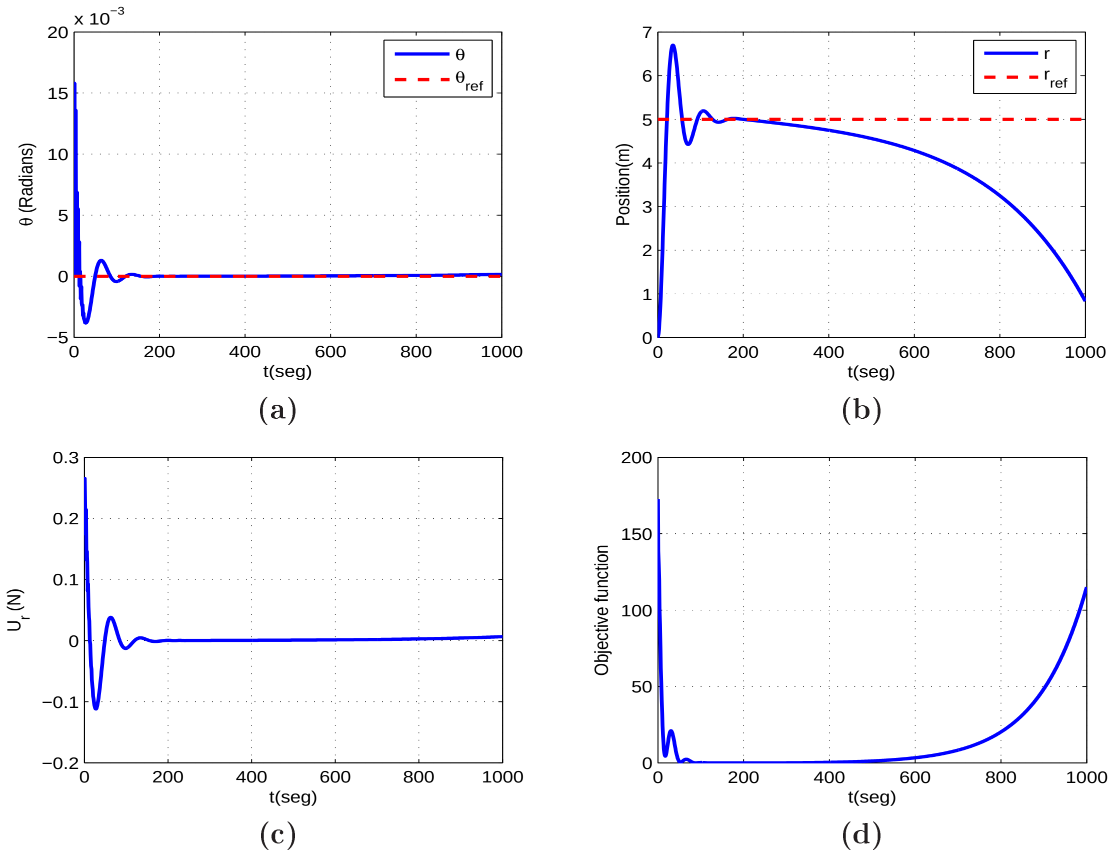


Figure 3.4: Nominal MPC response under uncertainty in mass of the load: (a)Crane angle (θ), (b) Crane Position (r), (c) Control action (U_r), (d) Objective function.

problem). Such transformation was performed by assuming norm-bounded uncertainty in the free and forced system response. Hence, the solution of the proposed robust MPC with zone control problem can be efficiently achieved in a finite number of steps using convex optimization algorithms such as interior point methods.

3.3.1 Nominal MPC with zone control (MPC-ZC)

Zone control is a control strategy often implemented in applications where a perfect tracking of the desired values for the controlled outputs is not required. Such cases appears in systems with highly correlated controlled variables, or in systems in which the manipulated variables do not allow driving independently the controlled variables to their desired values. In this optimal control strategy, both reference values for the system and a sequence of control actions are obtained by the solution of an optimization problem. With this purpose, a prediction of the system states and outputs is performed over a prediction horizon N_p. Let $x(k) \in \mathbb{R}^{n_x}$, $y(k) \in \mathbb{R}^{n_y}$, and $u(k) \in \mathbb{R}^{n_u}$ denote the system states, inputs and outputs respectively. At time step k, let $x_k = [x^T(k), \ldots, x^T(k+N_p)]^T$ and $u_k = [u^T(k), \ldots, u^T(k+N_p)]^T$ be the state trajectory and the control sequences, with N_p the prediction horizon. Let $J_{N_p}(y_{ref,k}, x_k, u_k)$ denotes the cost function, $y_{ref,k}$ being the sequence of reference values for the controlled variables. Assume the function $f(x(k), u(k))$

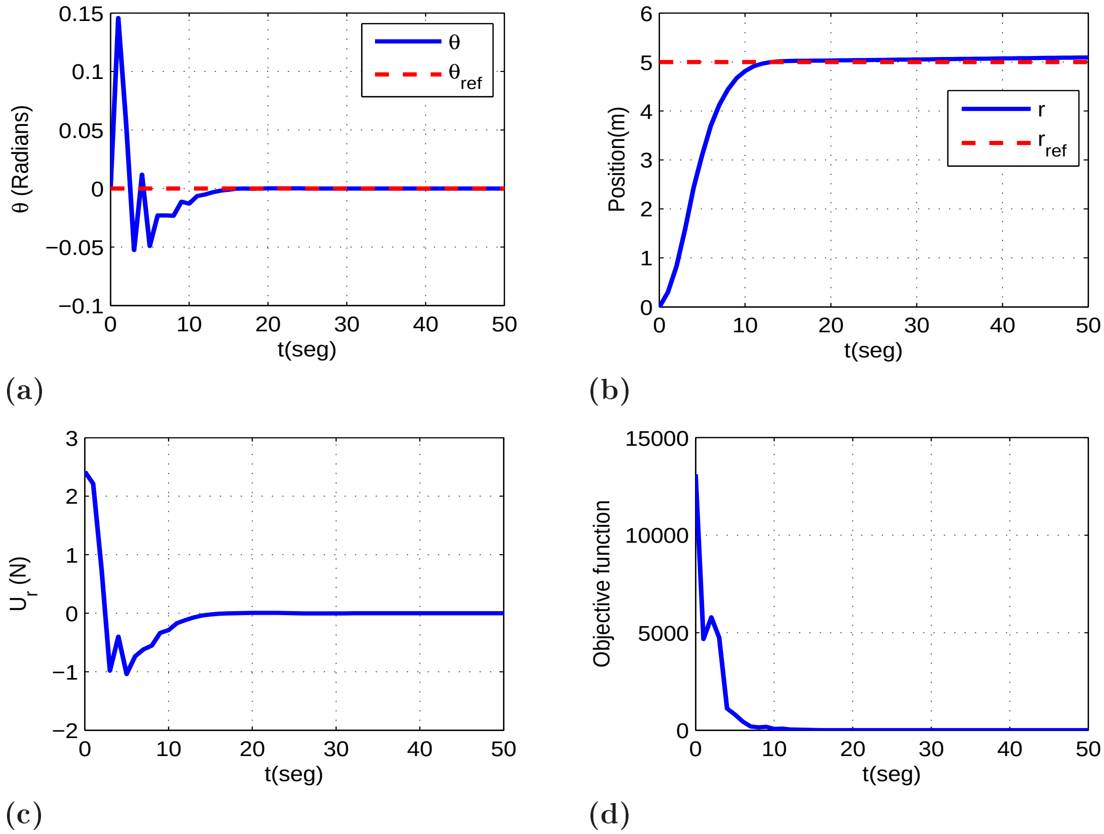


Figure 3.5: Robust MPC response under uncertainty in mass of the load: (a)Crane angle (θ), (b) Crane Position (r), (c) Control action (U_r), (d) Objective function.

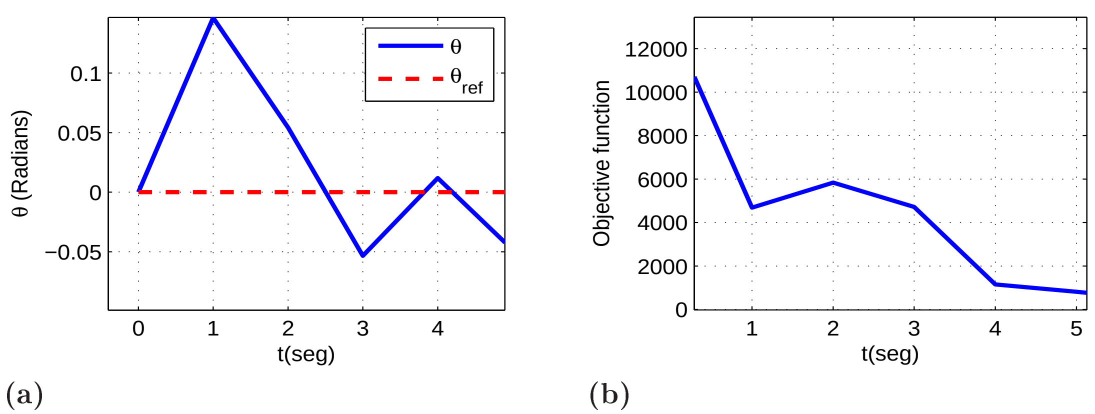


Figure 3.6: Robust MPC response under uncertainty in mass of the load: (a)Zoom in the Crane angle (θ), (b) Zoom in the objective function.

describing the state trajectories and $g(x(k), u(k))$ describing the system outputs are continuous, where $x(k) \in \mathbb{X}$, $y(k) \in \mathbb{Y}$, and $u(k) \in \mathbb{U}$ define the feasible values for the states, inputs and outputs of the system, with $\mathbb{X} \subset \mathbb{R}^{n_x}$, $\mathbb{Y} \subset \mathbb{R}^{n_y}$ and $\mathbb{U} \subset \mathbb{R}^{n_u}$ closed sets. Let $y_{ref,k} \in \mathbb{Y}_{ref}$ with $\mathbb{Y}_{ref} \subset \mathbb{R}^{n_y}$ also closed. Then, the MPC optimization problem for implementing the zone control strategy is as follows:

$$
\begin{aligned}
&\min_{u_k, y_{ref,k}} J_{N_p}(y_{ref,k}, x_k, u_k) \\
\text{subject to:}\quad & \\
& x(k+j+1) = f(x(k+j), u(k+j)) \\
& y(k+j) = g(x(k+j), u(k+j)) \\
& x(k+j) \in \mathbb{X}, \quad j = 0, \dots, N_p \\
& y(k+j) \in \mathbb{Y}, \quad j = 0, \dots, N_p \\
& u(k+j) \in \mathbb{U}, \quad j = 0, \dots, N_p \\
& x(k+N_p) \in \Omega \\
& y_{ref,k} \in \mathbb{Y}_{ref} \\
& \|u(k+j) - u(k+j-1)\|_2 \leq \Delta u_{\max}
\end{aligned}
\tag{3.50}
$$

where $J_{N_p}(x_k, u_k) = \sum_{j=1}^{N_p} L(x(k+j), u(k+j)) + V(x(k+N_p))$. The stability of the MPC controller (3.50) can be assured by means of the following theorem:

Theorem 3. ***(Asymptotic stability of the MPC with zone control)***

Let a system $x(k+1) = f(x(k), u(k))$, such as that $f(0,0) = 0$ and this is subject to the constraints $x(k+j) \in \mathbb{X}$ and $u(k+j) \in \mathbb{U}$. Then, if the hypothesis 1 is accomplished, the system controlled by the nominal MPC controller $u_{MPC-ZC} = K_f(x(k))$ is asymptotically stable.

Proof. Suppose that u_k^* and $y_{ref,k}^*$ correspond to the optimal solution of the problem (3.50) at time step k, x_k^* the optimal sequence of the states and $J_{N_p}^*(x_k)$ the cost corresponding to u_k^*. Now, consider the following variables $\tilde{u}_k = [u^*(k+1)^T, \dots, u^*(k+N_p)^T, u_f]^T$, $y_{ref,k}^*$, if $u_f \in \mathbb{U}$ is assumed, clearly $\tilde{u}(k+j) \in \mathbb{U}$ and $y_{ref,k}^* \in \mathbb{Y}_{ref}$. Since there are no discrepancies between the prediction model and the real system $x(k+1) = x^*(k+1)$, and applying $\tilde{u}_k$, the predicted state $\tilde{x}_k$ satisfies that $\tilde{x}(k+j) = x^*(k+j)$. Therefore, $\tilde{x}(k+j) \in \mathbb{X}$. Let $\tilde{J}_{N_p}(x_{k+1})$ the cost corresponding to the feasible sequence $\tilde{u}_k$. Then

$$
\begin{aligned}
\tilde{J}_{N_p}(x_{k+1}) - J_{N_p}^*(x_k) = &- L(x(k), u^*(k)) \\
&+ \{L(\tilde{x}(k+N+1), u_f) + V(\tilde{x}(k+N)) - V(x^*(k+N))\}
\end{aligned}
$$

Since $\tilde{x}(k+N) = x^*(k+N)$, then $\tilde{x}(k+N+1) = f(x^*(k+N), u_f)$, and $u_f \in \mathbb{U}$, it is possible to deduce that

$$
\begin{aligned}
\tilde{J}_{N_p}(x_{k+1}) - J_{N_p}^*(x_k) = &- L(x(k), u^*(k)) \\
&+ \{L(\tilde{x}(k+N), u_f) + V(f(x^*(k+N), u_f)) - V(x^*(k+N))\}
\end{aligned}
$$

Then from hypothesis 1 we have

$$\tilde{J}_{N_p}(x_{k+1}) - J^*_{N_p}(x_k) \leq -L(x(k), u^*(k))$$
$$J^*_{N_p}(x_{k+1}) - J^*_{N_p}(x_k) \leq \tilde{J}_{N_p}(x_{k+1}) - J^*_{N_p}(x_k) \leq -L(x(k), u^*(k))$$

therefore $J^*_{N_p}(x_{k+j})$ is a Lyapunov function of the system.

□

Now, since the optimization problem (3.50) recast in nonlinear programming, and the solution of this class of problems is not always feasible in real time and if $f(\cdot)$ and $g(\cdot)$ are assumed C^1 class functions, viz. continuous and differentiable functions, then, there exists a linear approximation of $f(\cdot)$ and $g(\cdot)$ as in (3.14). With this approximation, the nominal problem (3.50) can be transformed into a quadratic programming (QP) problem, with $y_k = [y^T(k), \ldots, y^T(k+N_p)]^T$ the sequence of predicted values for the output $y(k)$ and the prediction model written as. (3.15). Commonly, in MPC with zone control $J_{N_p}(y_{ref,k}, x_k, u_k)$ penalizes the deviation of the predicted output $y(k)$ from its corresponding reference value $y_{ref,k}$. Let $J_{N_p}(y_{ref,k}, x_k, u_k)$ be the quadratic cost function

$$\begin{aligned} J_{N_p}(y_{ref,k}, x_k, u_k) = & \sum_{j=1}^{N_p} \|(y(k+j) - y_{ref,k}\|^2_Q + \sum_{j=0}^{N_p} \|u(k+j) - u_{ref}\|^2_R \\ & + \sum_{j=0}^{N_p} \|\Delta u(k+j)\|^2_S + \|x(k+N_p)\|^2_{\tilde{Q}} \end{aligned}$$

where $Q \in \mathbb{R}^{n_y \times n_y}$, $R \in \mathbb{R}^{n_u \times n_u}$ and $S \in \mathbb{R}^{n_u \times n_u}$ are positive definite weighting matrices; u_{ref} is the reference value for the inputs, such that the pair $(y_{ref,k}, u_{ref})$ defines an optimal operating point for the system (often computed offline or by an upper layer in a hierarchical control scheme); and $\tilde{Q}$ is obtained by solving a Lyapunov equation. The terminal state penalty term $\|\tilde{x}(k+N_p)\|^2_{\tilde{Q}}$ is added for assuring the closed-loop stability of the system (hypothesis 1). Using the prediction model (3.15), the linearized version of (3.50) is given by (3.51). This version corresponds to a QP optimization problem whose solution can be efficiently obtained in real time applications.

$$\begin{aligned} & \min_{u_k, y_{ref,k}} r + 2q^T \begin{bmatrix} u_k \\ y_{ref,k} \end{bmatrix} + \begin{bmatrix} u_k^T, y_{ref,k}^T \end{bmatrix} P \begin{bmatrix} u_k \\ y_{ref,k} \end{bmatrix} \\ & \text{subject to:} \\ & \begin{bmatrix} I \\ -I \\ \Delta_\text{u} \\ \Delta_\text{u} \\ \Lambda \\ -\Lambda \end{bmatrix} u_k \leq \begin{bmatrix} \tilde{u}_{\max} \\ -\tilde{u}_{\min} \\ \tilde{\Delta} u_{\max} + \overline{u}(k-1) \\ \tilde{\Delta} u_{\max} - \overline{u}(k-1) \\ \tilde{y}_{\max} - \Gamma \\ -\tilde{y}_{\min} + \Gamma \end{bmatrix} \\ & y_{ref,\min} \leq y_{ref,k} \leq y_{ref,\max} \end{aligned} \tag{3.51}$$

In (3.51) P, q, r are defined as follows:

$$P = \begin{bmatrix} \Lambda^T\overline{Q}\Lambda + \overline{R} + \Delta_{\mathrm{u}}^T\overline{S}\Delta_{\mathrm{u}} & -\Lambda^T\overline{QI} \\ -\overline{I}^T\overline{Q}\Lambda & \overline{I}^T\overline{QI} \end{bmatrix}$$
$$q = \begin{bmatrix} \Gamma^T\overline{Q}\Lambda - U_{(k-1)}^T\overline{S}\Delta_{\mathrm{u}} - U_{ref}^T\overline{R} & -\Gamma^T\overline{QI} \end{bmatrix}$$
$$r = \Gamma^T\overline{Q}\Gamma + U_{(k-1)}^T\overline{S}U_{(k-1)} + U_{ref}^T\overline{R}U_{ref}$$

and

$$U_{ref} = [{u_{ref}}^T(k), \ldots, {u_{ref}}^T(k+N_p)]^T$$
$$\tilde{u}_{\max} = [u_{\max}^T(k), \ldots, u_{\max}^T(k+N_p)]^T$$
$$\tilde{u}_{\min} = [u_{\min}^T(k), \ldots, u_{\min}^T(k+N_p)]^T$$
$$\tilde{y}_{\max} = [y_{\max}^T(k), \ldots, y_{\max}^T(k+N_p)]^T$$
$$\tilde{y}_{\min} = [y_{\min}^T(k), \ldots, y_{\min}^T(k+N_p)]^T$$
$$\tilde{\Delta} u_{max} = [\Delta u_{\max}^T(k), \ldots, \Delta u_{\max}^T(k+N_p)]^T,$$

Hence, the MPC with zone control strategy works under the following assumptions:

1. The system nonlinearities are smooth. Then, the linear model is a good representation of the system in a neighborhood of the linearization point.

2. The control actions are such that they do not move the system outside the neighborhood where the linear model is valid.

These assumptions, in general seems to be restrictive. Then alternatives such as robust optimization based on linear models arise. In sections below, this approach is described.

3.3.2 Min-Max MPC with zone Control (RMPC-ZC)

Min-max MPC with zone control is a kind of robust MPC strategy where the worst-case scenario is formulated for a defined disturbance model. In general, a nonlinear model of the system is used to predict the trajectories of the system and its outputs. Based on these predictions a sequence of control actions u_k and reference values $y_{ref,k}$ are obtained. But in this case both u_k and $y_{ref,k}$ correspond to the best performance under the worst uncertainties. Let $\Theta_k = [\theta^T(k), \ldots, \theta^T(k+N_p)]^T$ denote the sequences of uncertainties over N_p, with $\|\theta(k)\|_2 \leq \rho$. Mathematically, min-max MPC with zone control is formulated as (3.52). Based on this formulation, several approaches have been reported in the literature as the proposed in [11, 12].

$$
\begin{aligned}
&\min_{u_k, y_{ref,k}} \max_{\Theta_k \in \Psi^{N_p}} J_{N_p}(y_{ref,k}, x_k, u_k, \Theta_k) \\
&\text{subject to:} \\
&\quad x(k+j+1) = f(x(k+j), u(k+j), \theta(k+j)) \\
&\quad y(k+j) = g(x(k+j), u(k+j), \theta(k+j)) \\
&\quad x(k+j) \in \mathbb{X}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad y(k+j) \in \mathbb{Y}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad u(k+j) \in \mathbb{U}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad x(k+N_p) \in \Omega, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad y_{ref,k} \in \mathbb{Y}_{ref}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad \|u(k+j) - u(k+j-1)\|_2 \leq \Delta u_{\max} \\
&\quad \theta(k+j) \in \Psi, \quad \|\theta(k+j)\|_2 \leq \rho \\
&\quad \Theta_k \in \Psi^{N_p} \triangleq \Psi \times \cdots \times \Psi
\end{aligned}
\tag{3.52}
$$

where Ψ^{N_p} is determined by the assumed uncertainty model. Due to the min-max MPC recast in robust nonlinear programming (RNLP), and since the current computation resources do not provide enough computational power for having an exact solution of RNLP problems in real time, linear models are used for computing equivalent solutions for (3.52). An alternative for computing these equivalent solutions consists on transforming the original optimization problem into a robust quadratic programming problem, which has efficient methods for computing its solution.

3.3.3 Robust MPC with zone Control as a RQP problem

Consider the nominal solution given by (3.51). Since this solution is based on linear models there exists a high uncertainty in the degree of representation that this model has of the system behavior. For this reason, model uncertainty will be considered and restricted to the free and forced response terms $(\Gamma + \delta\Gamma)$ and $(\Lambda + \delta\Lambda)$ where $\delta\Gamma \in \Re^{n_y \cdot N_p \times 1}$ and $\delta\Lambda \in \Re^{n_y \cdot N_p \times n_u \cdot N_p}$ are ellipsoidal uncertainties. With this assumption, equation (3.51) becomes (3.53).

$$
\min_{u_k, y_{ref,k}} \max_{P \in \mathbb{E}, q \in \mathbb{F}, r \in \mathbb{G}} r + 2q^T \begin{bmatrix} u_k \\ y_{ref,k} \end{bmatrix} + \begin{bmatrix} u_k^T, y_{ref,k}^T \end{bmatrix} P \begin{bmatrix} u_k \\ y_{ref,k} \end{bmatrix} \tag{3.53}
$$

where P, q and r are in terms of $\delta\Gamma$, $\delta\Lambda$ as follows

$$
\begin{aligned}
P &= \begin{bmatrix} \Lambda^T\overline{Q}\Lambda + \overline{R} + \Delta_{\mathrm{u}}^T\overline{S}\Delta_{\mathrm{u}} + \delta\Lambda^T\overline{Q}\delta\Lambda + \Lambda^T\overline{Q}\delta\Lambda + \delta\Lambda^T\overline{Q}\Lambda & -\Lambda^T\overline{QI} - \delta\Lambda^T\overline{QI} \\ -\overline{I}^T\overline{Q}\Lambda - \overline{I}^T\overline{Q}\delta\Lambda & \overline{I}^T\overline{QI} \end{bmatrix} \\
q^T &= \begin{bmatrix} \Gamma^T\overline{Q}\Lambda - U_{(k-1)}^T\overline{S}\Delta_{\mathrm{u}} - U_{ref}^T\overline{R} + \Gamma^T\overline{Q}\delta\Lambda + \delta\Gamma^T\overline{Q}\Lambda + \delta\Gamma^T\overline{Q}\delta\Lambda & -\Gamma^T\overline{QI} - \delta\Gamma^T\overline{QI} \end{bmatrix} \\
r &= (\Gamma + \delta\Gamma)^T\overline{Q}(\Gamma + \delta\Gamma) + U_{(k-1)}^T\overline{S}U_{(k-1)} + U_{ref}^T\overline{R}U_{ref}
\end{aligned}
$$

3.3.4 Robust MPC with zone control as a RQP problem with ellipsoidal uncertainties

As in section A.2.1 assume that uncertainties in P, q and r are norm bounded and $\mathbb{E}$, $\mathbb{F}$ and $\mathbb{G}$ can be written as

$$
\begin{aligned}
\mathbb{E} &= \left\{ P_0 + \sum_{i_e=1}^{m} P_{i_e}\mu_{i_e} \mid \|\mu\|_2 \leq 1 \right\} \\
\mathbb{F} &= \left\{ q_0 + \sum_{j_e=1}^{z} q_{j_e}\nu_{j_e} \mid \|\nu\|_2 \leq 1 \right\} \\
\mathbb{G} &= \left\{ r_0 + \sum_{l_e=1}^{n_r} r_{l_e}\xi_{l_e} \mid \|\xi\|_2 \leq 1 \right\}
\end{aligned}
$$

Then P_0, $\sum_{i_e=1}^{n} P_{i_e}\mu_{i_e}$, q_0, $\sum_{j_e=1}^{z} q_{j_e}\nu_{j_e}$, r_0 and $\sum_{l_e=1}^{n_r} r_{l_e}\xi_{l_e}$ be defined as

$$
\begin{aligned}
P_0 &= \begin{bmatrix} \Lambda^T\overline{Q}\Lambda + \overline{R} + \Delta_{\mathrm{u}}^T\overline{S}\Delta_{\mathrm{u}} & -\Lambda^T\overline{QI} \\ -\overline{I}^T\overline{Q}\Lambda & \overline{I}^T\overline{QI} \end{bmatrix} \\
\sum_{i_e=1}^{m} P_{i_e}u_{i_e} &= \begin{bmatrix} \delta\Lambda^T\overline{Q}\delta\Lambda + \Lambda^T\overline{Q}\delta\Lambda + \delta\Lambda^T\overline{Q}\Lambda & -\delta\Lambda^T\overline{QI} \\ -\overline{I}^T\overline{Q}\delta\Lambda & 0 \end{bmatrix} \\
q_0^T &= \begin{bmatrix} \Gamma^T\overline{Q}\Lambda - U_{(k-1)}^T\overline{S}\Delta_{\mathrm{u}} - U_{ref}^T\overline{R} & -\Gamma^T\overline{QI} \end{bmatrix} \\
\sum_{j_e=1}^{z} q_{j_e}v_j &= \begin{bmatrix} \Gamma^T\overline{Q}\delta\Lambda + \delta\Gamma^T\overline{Q}\Lambda + \delta\Gamma^T\overline{Q}\delta\Lambda & -\delta\Gamma^T\overline{QI} \end{bmatrix} \\
r_0 &= U_{(k-1)}^T\overline{S}U_{(k-1)} + U_{ref}^T\overline{R}U_{ref} + \Gamma^T\overline{Q}\Gamma \\
\sum_{l_e=1}^{n_r} r_{l_e}\xi_{l_e} &= \Gamma^T\overline{Q}\delta\Gamma + \delta\Gamma^T\overline{Q}\delta\Gamma + \delta\Gamma^T\overline{Q}\Gamma
\end{aligned}
\tag{3.54}
$$

Observe that $\delta\Gamma$ and $\delta\Lambda$ uncertainties are reflected as an ellipsoidal uncertainty in P, q and r. Therefore, Min-Max MPC with zone control problem becomes the RQP problem (3.55).

$$
\begin{aligned}
&\min_{u_k, y_{ref,k}, w_{i_e}, s, d, t} s + 2q_0^T \begin{bmatrix} u_k \\ y_{ref,k} \end{bmatrix} + t + 2d + \|\overline{r}\|_2 \\
\text{subject to:}& \\
&\left\| \begin{bmatrix} 2P_0^{1/2} \begin{bmatrix} u_k \\ y_{ref,k} \end{bmatrix} \\ s-1 \end{bmatrix} \right\|_2 \leq s+1 \\
&\left\| \begin{bmatrix} 2P_{i_e}^{1/2} \begin{bmatrix} u_k \\ y_{ref,k} \end{bmatrix} \\ w_{i_e}-1 \end{bmatrix} \right\|_2 \leq w_{i_e}+1 \\
&\|q_j \begin{bmatrix} u_k \\ y_{ref,k} \end{bmatrix} \|_2 \leq d \\
&\|w\|_2 \leq t \\
&0 \leq w_{i_e} \\
&0 \leq s \\
&\begin{bmatrix} I \\ -I \\ \Delta_\mathrm{u} \\ \Delta_\mathrm{u} \\ \Lambda \\ -\Lambda \end{bmatrix} u_k \leq \begin{bmatrix} \tilde{u}_{\max} \\ -\tilde{u}_{\min} \\ \tilde{\Delta u}_{\max} + \overline{u}(k-1) \\ \tilde{\Delta u}_{\max} - \overline{u}(k-1) \\ \tilde{y}_{\max} - \Gamma \\ -\tilde{y}_{\min} + \Gamma \end{bmatrix} \\
&y_{ref,\min} \leq y_{ref,k} \leq y_{ref,\max}
\end{aligned}
\tag{3.55}
$$

where $\overline{r} = [r_1, \ldots, r_{n_r}]^T$. It is worth to point out that the optimization problem (3.55) is a convex optimization problem. Therefore, its solution always exists and is unique. Moreover, it can be numerically computed in a finite number of steps.

Computing P_{i_e} and q_{j_e}: To calculate the P_{i_e} matrices, we have that,

$$
\sum_{i_e=1}^{m} P_{i_e}\mu_{i_e} = \begin{bmatrix} \sum\limits_{i_e=1}^{m} P_{1,i_e}\mu_{i_e} & \sum\limits_{i_e=1}^{m} P_{2,i_e}\mu_{i_e} \\ \sum\limits_{i_e=1}^{m} P_{3,i_e}\mu_{i_e} & \sum\limits_{i_e=1}^{m} P_{4,i_e}\mu_{i_e} \end{bmatrix} \tag{3.56}
$$

where $P_{1,i_e} \in \mathbb{R}^{n\times n}$, $P_{2,i_e} \in \mathbb{R}^{n\times N}$, $P_{3,i_e} \in \mathbb{R}^{N\times n}$ and $P_{4,i_e} \in \mathbb{R}^{N\times N}$. Then from (3.54)

$$\sum_{i_e=1}^{m} P_{1,i_e}\mu_{i_e} = \delta\Lambda^T\overline{Q}\delta\Lambda + \Lambda^T\overline{Q}\delta\Lambda + \delta\Lambda^T\overline{Q}\Lambda$$
$$\sum_{i_e=1}^{m} P_{2,i_e}\mu_{i_e} = -\delta\Lambda^T\overline{QI}$$
$$\sum_{i_e=1}^{m} P_{3,i_e}\mu_{i_e} = -\overline{I}^T\overline{Q}\delta\Lambda$$
$$\sum_{i_e=1}^{m} P_{4,i_e}\mu_{i_e} = 0$$

following the same procedure that in section 3.2.3.3, we have the next result for every P_{l,i_e} $(l = 1, 2, 3, 4)$ matrix,

$$\begin{aligned}
P_{1ii,i_e} &= \begin{cases} 2\lambda_{i_e-(i-1)p,i} & \text{for} \quad i_e = (i-1)p+1, \ldots, (i-1)p+p \\ 1 & \text{for} \quad i_e = p\cdot n + (i-1)p+1, \ldots, p\cdot n + (i-1)p+p \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \\
P_{1ij,i_e} &= \begin{cases} \lambda_{i_e-(i-1)p,j} & \text{for} \quad i_e = (i-1)p+1, \ldots, (i-1)p+p \\ \lambda_{i_e-(j-1)p,i} & \text{for} \quad i_e = (j-1)p+1, \ldots, (j-1)p+p \\ 1 & \text{for} \quad i_e = \overline{\kappa}+1, \ldots, \overline{\kappa}+p \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \\
P_{2ij,i_e} &= \begin{cases} -1 & \text{for} \quad i_e = (i-1)p+j, \ldots, (i-1)p+j+(\frac{p}{N}-1)N \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \\
P_{3ij,i_e} &= \begin{cases} -1 & \text{for} \quad i_e = (j-1)p+i, \ldots, (j-1)p+i+(\frac{p}{N}-1)N \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \\
P_{4ij,i_e} &= 0
\end{aligned} \tag{3.57}$$

where $\overline{\kappa} = \frac{p\cdot i}{2}(2n-i-1) + p\cdot(n+j-1)$, the order of μ is $m = \frac{3}{2}(p\cdot n) + \frac{1}{2}(p\cdot n^2)$ and with $\|\mu\|_2 \le 1$ as necessary condition to use this approach, namely,

$$\|\mu\|_2 = \left\| \begin{bmatrix} \mu_{(j-1)p+i} \\ \mu_{n\cdot p+(j-1)p+i} \\ \mu_{\kappa} \end{bmatrix} \right\|_2 = \left\| \begin{bmatrix} q_i\delta_{i,j} \\ q_i\delta_{i,j}^2 \\ q_i\delta_{i,j}\delta_{i,\alpha} \end{bmatrix} \right\|_2 \le 1$$

To calculate the q_{j_e} we have that,

$$\sum_{j_e=1}^{z} q_{j_e}\nu_{j_e} = \begin{bmatrix} \sum_{j_e=1}^{z} q_{1_{j_e}}\nu_{j_e} & \sum_{j_e=1}^{z} q_{2_{j_e}}\nu_{j_e} \end{bmatrix} \tag{3.58}$$

where $q_{1,j_e} \in \mathbb{R}^{1\times n}$ and $q_{2,j_e} \in \mathbb{R}^{1\times N}$. Then from (3.54)

$$\sum_{j_e=1}^{z} q_{1_{j_e}} \nu_{j_e} = \Gamma^T \overline{Q} \delta\Lambda + \delta\Gamma^T \overline{Q} \Lambda + \delta\Gamma^T \overline{Q} \delta\Lambda$$
$$\sum_{j_e=1}^{z} q_{2_{j_e}} \nu_{j_e} = -\delta\Gamma^T \overline{QI}$$

following the same procedure that in section 3.2.3.3, we have the next result for every q_{l,j_e} $(l = 1, 2)$ matrix,

$$q_{1_{i,j_e}} = \begin{cases} \Gamma_{j_e-(j-1)p} & \text{for} \quad j_e = (j-1)p+1, \ldots, (j-1)p+p \\ \lambda_{j_e-p\cdot n, j} & \text{for} \quad j_e = p \cdot n + 1, \ldots, p \cdot n + p \\ 1 & \text{for} \quad j_e = p \cdot n + j \cdot p + 1, \ldots, p \cdot n + j \cdot p + p \\ 0 & \text{for} \quad \text{otherwise} \end{cases}$$
$$q_{2_{i,j_e}} = \begin{cases} \Gamma_{j_e-(j-1)p} & \text{for} \quad j_e = (j-1)p+1, \ldots, (j-1)p+p \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \tag{3.59}$$

where the order of ν is $z = 2p \cdot n + p$. Note that $\|\nu\|_2 \leq 1$, then a necessary condition to use this approach is:

$$\|\nu\|_2 = \left\| \begin{bmatrix} \nu_{(j-1)p+i} \\ \nu_{n\cdot p+i} \\ \nu_{n\cdot p+p+(j-1)p+i} \end{bmatrix} \right\|_2 = \left\| \begin{bmatrix} q_i \delta_{i,j} \\ q_i \rho_i \\ q_i \delta_{i,j} \rho_i \end{bmatrix} \right\|_2 \leq 1$$

3.3.5 Robust Model Predictive control with zone control for a Continuous Stirred Tank Reactor

In this Section robust linear MPC with zone control (RMPC-ZC) is applied to a Continuous Stirred Tank Reactor (CSTR). The model of this process was presented in [92, 93]. The RMPC-ZC used was the same defined by (3.20). In this simulation, it was assumed that an irreversible exothermic reaction $A \rightarrow B$ takes place in the reactor. Just a liquid phase was considered. The heat produced by the reaction is removed by a cooling jacket. The dynamic behavior of this systems is described by (3.60).

$$\begin{aligned} \frac{dC_a}{dt} &= \frac{q}{v}(C_{a_0} - C_a) - k_0 C_a \exp\left(\frac{-E}{RT}\right) \\ \frac{dT}{dt} &= \frac{q}{v}(T_0 - T) + k_1 C_a \exp\left(\frac{-E}{RT}\right) \\ &\quad + k_2 \frac{q_c}{v}\left(1 - \exp\left(\frac{-k_3}{q_c}\right)\right)(T_{c_0} - T) \\ \frac{dV}{dt} &= q - k_4\sqrt{V} \end{aligned} \tag{3.60}$$

In (3.60), constants k_1, k_2, and k_3 are defined as follows:

$$k_1 = -\frac{\Delta H k_0}{\rho C_p}, \quad k_2 = \frac{\rho_c C_{pc}}{\rho C_p}, \quad k_3 = \frac{h_a}{\rho_c C_{pc}}$$

The objective of the controller is to regulate the volume V and the temperature T of the liquid inside the reactor. Such objective is carried out by manipulating the inlet flow rate q and the coolant flow q_c. The inlet temperature and concentration to the reactor T_0, C_{a_0} respectively; also the inlet temperature of the coolant flow T_{c_0} are assumed as unmeasured disturbances. Moreover, the molar concentration of A (denoted by C_a) is assumed unmeasured. Table 3.1 presents the parameters used in the simulation. In

Parameter	Value
k_0	7.2×10^{10} min^{-1}
k_4	10 L/(min $\cdot$ m$^{3/2}$)
E/R	1×10^4 K
T_0	350 K
T_{c_0}	350 K
ΔH	-2×10^5 cal/mol
C_p, C_{pc}	1 cal/(g $\cdot$ K)
ρ, ρ_c	1×10^3 g/L
h_a	7×10^5 cal/(min $\cdot$ K)
C_{a_0}	1 mol/L

Table 3.1: Values of the reactor parameters

nominal conditions the manipulated variables assume the values $q = 100$ L/min, and $q_c = 103.37$ L/min. At the same conditions $T_0 = 350$ (K), $C_{a_0} = 1$ mol/L and $T_{c_0} = 350$ K. Consequently, the CSTR model admits three steady states. For simulations purposes it was assumed the problem of controlling the CSTR around the steady state associated to the following values: $h = 100$ L, $T = 438.54$ K and $C_a = 0.1$ mol/L. The simulation was performed along 40 minutes (simulated time). During this period changes in the values of the unmeasured variables were done. Specifically, after 5 minutes a disturbance in the operating inlet concentration C_{a_0} of 35% was performed. Finally, after 20 more minutes a disturbance in the temperature of the coolant flow T_{c_0} of 10% was done.

Figures 3.7 present the simulation results. Figures 3.7 (a) and 3.7 (b) show, that the RMPC-ZC rejected the disturbances preformed during the simulation. Although disturbances produced significant changes in the controlled variables, the RMPC-ZC drove them again to their desired values. Figure 3.7 (d) presents the behaviour of the cost function along the simulation. Clearly, despite of the disturbances the cost function always converges to zero. This fact is important in order to check the stability of the proposed controller.

Also the nominal MPC with zone control results is presented, Figures 3.8 (a) and 3.8 (b) show that the MPC with zone control rejected the disturbances, even with strong perturbation, but if the variables have fast changes, the difference with the RMPC-ZC is the way it treats the variable, because RMPC-ZC seems smoother and conservative, Figures 3.7 (a) and 3.7 (b) show that variables have a slow response but the variables seem to be far from the zone limits, while Figures 3.8 (a) and 3.8 (b) show very fast variables with references that are near the zone limits. Then, both controllers can accomplish the objective of maintaining the variable into the zone. The applicability of each controller will depend on the system characteristics and the nature of the output variable: when the operation needs to be very conservative, RMPC-ZC would be an interesting option. Figures 3.7 (c) and 3.8 (c) show the behavior of the input variables, MPC with zone control has input variables with very fast changes, while RMPC-ZC has input variables that are very soft which could be a better operation for that kind of variables in presence of fast responses.

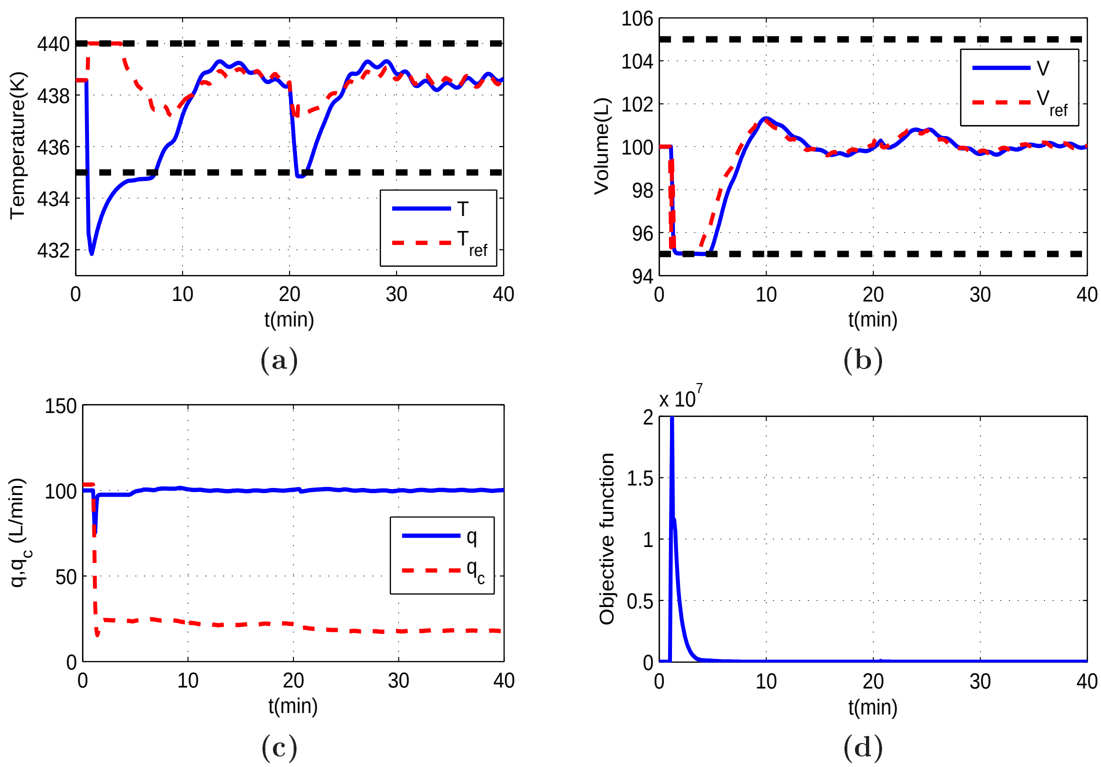
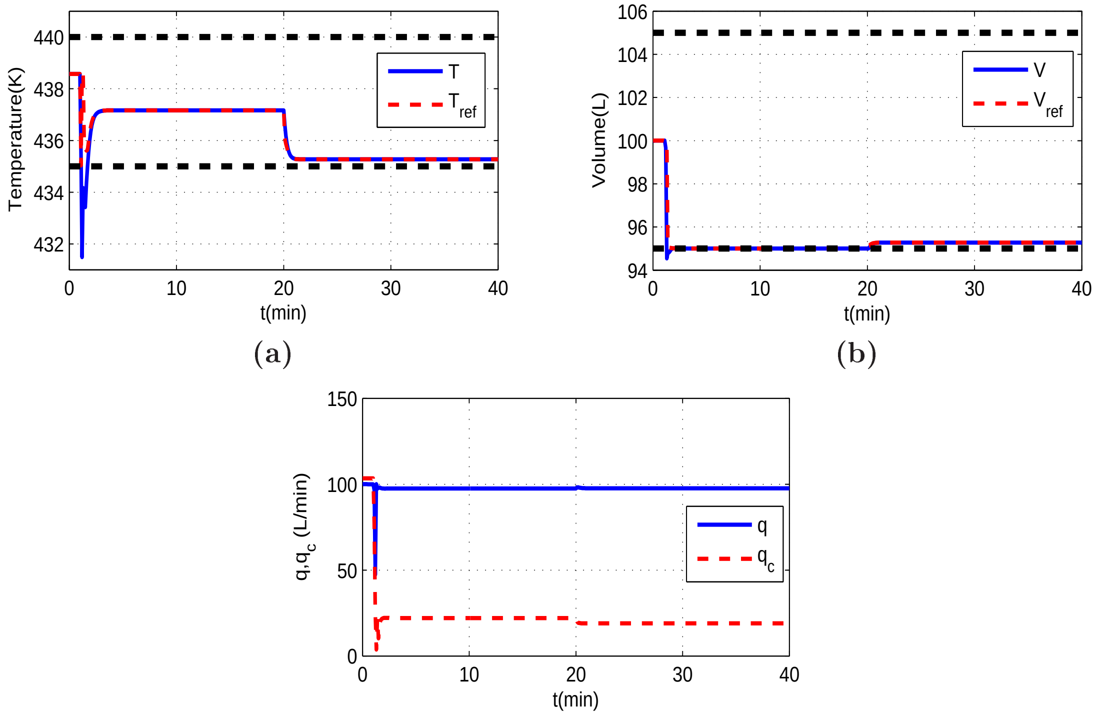


Figure 3.7: Robust MPC with zone control: (a) Temperature of CSTR, (b) Volume of CSTR, (c) Control actions of CSTR, (d) Objective function.

The results show the potential applicability of RMPC-ZC, even if the MPC with zone control shows good results. Both controllers have different behaviour, but none of them have bad performance. Application depends on the evaluated system and how uncertainty would have an important role on controllers system behaviour.

3.4 Robust Economic Model Predictive Control

Often, compliance with an economic objective is the main goal of large-scale control systems such as those found in optimal process operation. Concepts like profit maximization and operational efficiency are taken into account in order to assess the performance of the system. Very often, and especially within process control, the economic optimization considerations of a plant are usually addressed through a two-layer architecture [94]. Generally speaking, this architecture includes an upper layer, named real-time optimization



(a) (b) (c)

Figure 3.8: Nominal MPC with zone control: (a) Temperature of CSTR, (b) Volume of CSTR, (c) Control actions of CSTR

(RTO), which optimizes process operation set-points taking into account economic considerations using steady-state system models. This RTO transmits its results as setpoints to the lower layer (i.e., process control layer), and feedback control systems are employed in order to steer the process to track the set-points while rejecting disturbances. In many applications, the process control layer is implemented with model predictive control (MPC), a set of control strategies that use a model to predict the response of the system over a period of time called the prediction horizon. The positive features of MPC are flexibility, performance, robustness, and its ability to directly handle hard constraints on both inputs and states [95]. In this traditional approach, the way to distribute the information between these layers has a deep impact in the process operation. Usually, the objective function of the controller is aimed to achieve fast asymptotic tracking to setpoint changes and low output variance under disturbances, and often it is not directly linked to the economic costs of operating the plant [94].

There are an increasing number of applications where the hierarchical separation of economic analysis and control is either inefficient or inappropriate, and several approaches have been proposed in order to tackle this issue (see [94] for a brief compendium of solutions). Here, we are interested in the Economic Model Predictive Control, a term first coined in [96] for power management applications. Economic MPC employs the economic cost function used in the RTO layer directly as the tracking stage cost function [97]. Altough it is a relatively new control approach, there are several research works on applications of Economic MPC and its properties. Reports of studies on building energy systems and power management on smart grid can be found in [96, 98, 99], and for the design of sustainable policies for mitigating the effects of climate change [100]. A distributed application of economic MPC to a nonlinear chemical process network can be found in [101], and the analysis of Lyapunov stability for an economic nonlinear MPC for cyclic processes is presented in [102]. Paper [103] focuses on the development of a Lyapunov-based economic model predictive control (LEMPC) method for switched nonlinear systems, and the use of direct methods to formulate the nonlinear control problem as a large-scale NLP was proposed in [104].

In spite of the recent advances in Economic MPC, there are still many open questions. Issues such as the best design for the control system with economic goal while assuring closed-loop stability and convergence, or the quantification of the closed-loop performance of the MPC are still challenges [94]. In this sense, an economic MPC formulation with robustness characteristics is proposed and its performance under uncertainty conditions is explored. Several works have considered constrained MPC operation dealing with uncertainty and disturbances [105, 106, 107, 108, 109, 110]. However, there are few application of economic MPC in presence uncertainties such as the work of Hovgaard *et al.* [111] where a probabilistic constraints and Second Order Cone Programming (SOCP) are used together with economic MPC for power management of a refrigeration system.

3.4.1 Problem statement

Consider a system described by a nonlinear invariant discrete time model,

$$\begin{aligned} x(k+1) &= f(x(k), u(k), \theta(k)) \\ y(k) &= g(x(k), u(k), \theta(k)) \end{aligned} \tag{3.61}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the system state, $y(k) \in \mathbb{R}^{n_y}$ is the system output, $u(k) \in \mathbb{R}^{n_u}$ is the current control vector and $\theta(k) \in \mathbb{R}^{n_d}$ are the uncertainties presents in the model. We assume that the function model $f(x(k), u(k))$ is continuous. The system is subject to hard constraints on state $x(k) \in \mathbb{X}$, output $y(k) \in \mathbb{Y}$, input $u(k) \in \mathbb{U}$ and $\theta \in \Psi$ for all $k \geq 0$, where $\mathbb{X} \subset \mathbb{R}^{n_x}$, $\mathbb{Y} \subset \mathbb{R}^{n_y}$ and $\mathbb{U} \subset \mathbb{R}^{n_u}$ are closed sets. The steady state, input and output of the plant (x_s, u_s, y_s) are such that the condition $x_s = f(x_s, u_s)$ in (3.61) is accomplished. The min-max controller design problem consists of obtaining a control law that minimizes a given performance cost index

$$\max_{\Theta_k \in \Psi^{N_p}} J_{eco}(x_k, u_k, \Theta_k) \tag{3.62}$$

where $x_k = [x^T(k), \ldots, x^T(k+N_p)]^T$ is the state trajectories, $J_{eco}(x_k, u_k, \Theta_k)$ defines the stage cost and a sequence of control inputs $u_k = [u^T(k), \ldots, u^T(k+N_p)]^T$ is obtained assuming the uncertainty $\Theta_k = [\theta^T(k), \ldots, \theta^T(k+N_p)]^T$ maximum value.

3.4.2 Nominal Economic MPC (EMPC)

The Economic Model Predictive Control has substantially the same formulation used in nonlinear model predictive control, however an economic objective is chosen to give an economically optimal operation at all times. The Economic Model Predictive Control problem is defined as follows:

$$
\begin{aligned}
&\min_{u_k} J_{eco}(x_k, u_k) \\
\text{subject to:}& \\
&x(k+j+1) = f(x(k+j), u(k+j)) \\
&y(k+j) = g(x(k+j), u(k+j)) \\
&x(k+j) \in \mathbb{X}, \quad j = 1, \dots, N_p \\
&u(k+j) \in \mathbb{U}, \quad j = 0, \dots, N_p \\
&x(k+N_p) \in \Omega \\
&\|u(k+j) - u(k+j-1)\|_2 \leq \Delta u_{\max}
\end{aligned}
\tag{3.63}
$$

About the stability of the economic MPC, the typical Lyapunov arguments to prove asymptotic stability of MPC cannot be used in this case, because the optimal cost is not necessarily decreasing. In Diehl et.al. [63], asymptotic stability conditions for economic MPC are stated using Lyapunov arguments.

The objective of this section is to merge the economic MPC proposed in [63] with the new min-max MPC approach previously explained, in such way that the merged controller preserves the advantages of both formulations. Again, since the optimization problem (3.63) recast in nonlinear programming, and the solution of this class of problems is not feasible in real time and if $f(\cdot)$ and $g(\cdot)$ are assumed C^1 class functions, viz. continuous and differentiable functions, then there exists a linear approximation of $f(\cdot)$ and $g(\cdot)$ as in (3.14). With this approximation, the nominal problem (3.66) can be transformed into a quadratic programming (QP) problem, with $y_k = [y^T(k), \dots, y^T(k+N_p)]^T$ the sequence of predicted values for the output $y(k)$ and the prediction model written as. (3.15). As it was mentioned, in Economic MPC $J_{eco}(x_k, u_k)$ penalize an economic objective to provide an economically optimal operation at all times. Let $J_{eco}(x_k, u_k)$ be the next typical cost function [62]

$$
J_{\text{eco},k}(x_k, u_k) = \sum_{j=0}^{N_p} \left[\sum_{i_i=1}^{n_u} c_{u,i_i} u_{i_i}(k+j) + \sum_{i_o=1}^{n_y} c_{y,i_o} y_{i_o}(k+j) u_{i_o}(k+j) \right] \tag{3.64}
$$

where c_{u,i_i} and c_{y,i_o} are prices associated with energy and raw materials, N_p is the prediction horizon, n_y is the number of outputs and n_u is the number of inputs of the system. In order to ensure convergence, strong duality must be fulfilled [63]. In [54] this fact is used to propose a method to enforce convergence. This method includes a modification of the original stage cost function as follows:

$$\overline{J}_{\text{eco},k} = J_{\text{eco},k} + \sum_{j=0}^{N_p} \alpha(x(k+j), u(k+j))$$

This modification aims to ensure strong duality or strict dissipativity of the problem. For practical purposes, the most simple choice of $\alpha(\cdot)$ is the following tracking form,

$$\alpha(x(k+j), u(k+j)) = \|(y(k+j) - y_s\|_Q^2 + \|u(k+j) - u_s\|_R^2 + \|\Delta u(k+j)\|_S^2$$

where Q, R and S are chosen as the minimum required to achieve strict dissipativity [54]. It is important to note that adding $\alpha(\cdot)$, the economic objective is modified and the economic controller is contaminated with a tracking term, and for this reason a decrease in economic performance is expected. This decrease in performance constitutes a trade-off for stability [54]. In this way the problem stated in (3.63) is modified with a new objective function $\overline{J}_{\text{eco},k}$. Now, replacing (3.15) into the expression for $\overline{J}_{\text{eco},k}$ and making some algebraic operations it is possible to obtain a formulation of (3.63) with the modified cost in a quadratic program form like this

$$\begin{aligned} &\min_{u_k} r + 2q^T u_k + u_k^T P u_k \\ &\text{subject to:} \\ &\begin{bmatrix} I \\ -I \\ \Delta_{\text{u}} \\ \Delta_{\text{u}} \\ \Lambda \\ -\Lambda \end{bmatrix} u_k(k) \leq \begin{bmatrix} \tilde{u}_{\max} \\ -\tilde{u}_{\min} \\ \tilde{\Delta u}_{\max} + \overline{u}(k-1) \\ \tilde{\Delta u}_{\max} - \overline{u}(k-1) \\ \tilde{y}_{\max} - \Gamma \\ -\tilde{y}_{\min} + \Gamma \end{bmatrix} \end{aligned} \tag{3.65}$$

where $r = (\Gamma - Y_s)^T \overline{Q} (\Gamma - Y_s) + \overline{u}(k-1)^T \overline{S}\,\overline{u}(k-1)$. Note that r with $Y_s \in \mathbb{R}^{n_y \times N_p}$, $Y_s = [y_s^T, \ldots, y_s^T]^T$ is the minimum cost and cannot be changed during the optimization because it is independent of u_k. Then, the solution of (3.63) can be approximated by the solution of (3.65).

In (3.65) P, q, $\tilde{u}_{\max}$, $\tilde{u}_{\min}$, $\tilde{y}_{\max}$, $\tilde{y}_{\min}$, $\tilde{\Delta u}_{max}$ are defined as follows:

$$\begin{aligned} P &= \Lambda^T C_{ca} + \Lambda^T \overline{Q} \Lambda + \overline{R} + \Delta_{\text{u}}^T \overline{S} \Delta_{\text{u}} \\ q &= C_{op} + \Gamma^T C_{ca} - U_s^T \overline{R} + (\Gamma - Y_s)^T \overline{Q} \Lambda - \overline{u}(k-1)^T \overline{S} \Delta_{\text{u}} \end{aligned}$$

where C_{op} and C_{ca} are block diagonal matrices where the elements of the diagonals are $C_u = [c_{u,1}, \cdots, c_{u,n_u}]$ and $C_y = \text{diag}([c_{y,1}, \cdots, c_{y,n_y}])$ and

$$\begin{aligned}
\tilde{u}_{\max} &= [u_{\max}^T(k), \ldots, u_{\max}^T(k+N_p)]^T \\
\tilde{u}_{\min} &= [u_{\min}^T(k), \ldots, u_{\min}^T(k+N_p)]^T \\
\tilde{y}_{\max} &= [y_{\max}^T(k), \ldots, y_{\max}^T(k+N_p)]^T \\
\tilde{y}_{\min} &= [y_{\min}^T(k), \ldots, y_{\min}^T(k+N_p)]^T \\
\tilde{\Delta} u_{max} &= [\Delta u_{\max}^T(k), \ldots, \Delta u_{\max}^T(k+N_p)]^T,
\end{aligned}$$

Again, in this formulation $\Delta_{\mathrm{u}} u_k(k)$ denotes the difference between current and previous value of the control action. Hence, Δ_{u} is a bidiagonal matrix whose elements are $-I$ and I. This strategy works under the following assumptions:

1. The nonlinearities are smooth, the linear model is a good representation of the plant,
2. The control actions do not move the system far away from the region where the linearization is valid.

3.4.3 Min-Max Economic MPC

This section shows the formulation of the Robust nonlinear Economic Model Predictive Control (REMPC). REMPC problem is written as a min-max optimization problem. In such optimization problem a nonlinear model of the system is used to forecast the state trajectories. The forecasting is done along a prediction horizon N_p. Mathematically, this is expressed as (3.66). Subsequently different robust REMPC designs have been proposed, for example this one was presented in [111].

$$\begin{aligned}
&\min_{u_k} \max_{\Theta_k \in \Psi^{N_p}} J_{eco}(x_k, u_k, \Theta_k) \\
&\text{subject to:} \\
&\quad x(k+j+1) = f(x(k+j), u(k+j), \theta(k+j)) \\
&\quad y(k+j) = g(x(k+j), u(k+j), \theta(k+j)) \\
&\quad x(k+j) \in \mathbb{X}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad u(k+j) \in \mathbb{U}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad x(k+N_p) \in \Omega, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad \|u(k+j) - u(k+j-1)\|_2 \leq \Delta u_{\max} \\
&\quad \theta(k+j) \in \Psi, \quad \|\theta(k+j)\|_2 \leq \rho \\
&\quad \Theta_k \in \Psi^{N_p} \triangleq \Psi \times \cdots \times \Psi
\end{aligned} \tag{3.66}$$

where Ψ^{N_p} is determined by the assumed uncertainty model, with $\|\theta(k)\|_2 \leq \rho$ a bounded uncertainty in the system dynamics.

3.4.4 Robust Economic MPC as a RQP with ellipsoidal uncertainty

Consider the nominal solution given by (3.65). Since this solution is based on linear models there exists a high uncertainty in the degree of representation this model have of the system behavior. for this reason, model uncertainty will be considered and restricted to the free and forced response terms $(\Gamma + \delta\Gamma)$ and $(\Lambda + \delta\Lambda)$ where $\delta\Gamma \in \Re^{n_y \cdot N_p \times 1}$ and $\delta\Lambda \in \Re^{n_y \cdot N_p \times n_u \cdot N_p}$ are ellipsoidal uncertainties. With this assumption, the original min-max MPC problem can be converted to the next RQP problem (3.67):

$$\min_{u_k} \max_{P \in \mathbb{E}, q \in \mathbb{F}, r \in \mathbb{G}} u_k^T P u_k + 2q^T u_k + r \tag{3.67}$$

where P, q and r are in terms of $\delta\Gamma$, $\delta\Lambda$ as follows

$$\begin{aligned} P &= \Lambda^T C_{ca} + \Lambda^T \overline{Q} \Lambda + \overline{R} + \Delta_{\mathrm{u}}^T \overline{S} \Delta_{\mathrm{u}} + \delta\Lambda^T \overline{Q} \delta\Lambda + \Lambda^T \overline{Q} \delta\Lambda + \delta\Lambda^T \overline{Q} \Lambda + \delta\Lambda^T C_{ca} \\ q^T &= C_{op} + (\Gamma - Y_{ref})^T \overline{Q} \Lambda - U_{(k-1)}^T \overline{S} \Delta_{\mathrm{u}} - U_{ref}^T \overline{R} + (\Gamma - Y_{ref})^T \overline{Q} \delta\Lambda + \delta\Gamma^T \overline{Q} \delta\Lambda + \delta\Gamma^T \overline{Q} \Lambda \\ r &= U_{(k-1)}^T \overline{S} U_{(k-1)} + U_{ref}^T \overline{R} U_{ref} + (\Gamma - Y_{ref})^T \overline{Q} (\Gamma - Y_{ref}) + (\Gamma - Y_{ref})^T \overline{Q} \delta\Gamma \\ &\quad + \delta\Gamma^T \overline{Q} \delta\Gamma + \delta\Gamma^T \overline{Q} (\Gamma - Y_{ref}) \end{aligned} \tag{3.68}$$

Now assume that uncertainties in P, q and r are norm bounded and $\mathbb{E}$, $\mathbb{F}$ and $\mathbb{G}$ can be written as

$$\mathbb{E} = \left\{ P_0 + \sum_{i_e=1}^{m} P_{i_e} \mu_{i_e} \mid \|\mu\|_2 \leq 1 \right\}$$

$$\mathbb{F} = \left\{ q_0 + \sum_{j_e=1}^{z} q_{j_e} \nu_{j_e} \mid \|\nu\|_2 \leq 1 \right\}$$

$$\mathbb{G} = \left\{ r_0 + \sum_{l_e=1}^{n_r} r_{l_e} \xi_{l_e} \mid \|\xi\|_2 \leq 1 \right\}$$

Then, from (3.68), we have that,

$$
\begin{aligned}
&P_0 = \Lambda^T C_{ca} + \Lambda^T \overline{Q} \Lambda + \overline{R} + \Delta_{\mathrm{u}}^T \overline{S} \Delta_{\mathrm{u}} \\
&\sum_{i_e=1}^{m} P_{i_e} \mu_{i_e} = \delta\Lambda^T \overline{Q} \delta\Lambda + \Lambda^T \overline{Q} \delta\Lambda + \delta\Lambda^T \overline{Q} \Lambda + \delta\Lambda^T C_{ca} \\
&q_0^T = C_{op} + (\Gamma - Y_{ref})^T \overline{Q} \Lambda - U_{(k-1)}^T \overline{S} \Delta_{\mathrm{u}} - U_{ref}^T \overline{R} \\
&\sum_{j_e=1}^{z} q_{j_e} \nu_{j_e} = (\Gamma - Y_{ref})^T \overline{Q} \delta\Lambda + \delta\Gamma^T \overline{Q} \delta\Lambda + \delta\Gamma^T \overline{Q} \Lambda \\
&r_0 = U_{(k-1)}^T \overline{S} U_{(k-1)} + U_{ref}^T \overline{R} U_{ref} + (\Gamma - Y_{ref})^T \overline{Q} (\Gamma - Y_{ref}) \\
&\sum_{l_e=1}^{n_r} r_{l_e} \xi_{l_e} = (\Gamma - Y_{ref})^T \overline{Q} \delta\Gamma + \delta\Gamma^T \overline{Q} \delta\Gamma + \delta\Gamma^T \overline{Q} (\Gamma - Y_{ref})
\end{aligned}
$$

Observe that the uncertainties $\delta\Gamma$ and $\delta\Lambda$ uncertainties are reflected as an ellipsoidal uncertainty in P q, and r. Therefore, Min-Max MPC problem becomes the RQP problem (3.69).

$$
\begin{aligned}
&\min_{u_k, w_{i_e}, s, d, t} \; s + 2q_0^T u_k + t + 2d + \|\overline{r}\|_2 \\
&\text{subject to:} \\
&\left\| \begin{bmatrix} 2P_0^{1/2} u_k \\ s - 1 \end{bmatrix} \right\|_2 \leq s + 1 \\
&\left\| \begin{bmatrix} 2P_{i_e}^{1/2} u_k \\ w_{i_e} - 1 \end{bmatrix} \right\|_2 \leq w_{i_e} + 1 \\
&\|q_{j_e} u_k\|_2 \leq d \\
&\|w\|_2 \leq t \\
&0 \leq w_{i_e} \\
&0 \leq s \\
&\begin{bmatrix} I \\ -I \\ \Delta_{\mathrm{u}} \\ \Delta_{\mathrm{u}} \\ \Lambda \\ -\Lambda \end{bmatrix} u_k \leq \begin{bmatrix} \tilde{u}_{\max} \\ -\tilde{u}_{\min} \\ \tilde{\Delta} u_{\max} + \overline{u}(k-1) \\ \tilde{\Delta} u_{\max} - \overline{u}(k-1) \\ \tilde{y}_{\max} - \Gamma \\ -\tilde{y}_{\min} + \Gamma \end{bmatrix}
\end{aligned}
\qquad (3.69)
$$

where $\overline{r} = [r_1, \ldots, r_{n_r}]^T$. It is worth to point out that the optimization problem (3.69) is a convex optimization problem. Therefore, its solution always exists and is unique. Moreover, it can be numerically computed in a finite number of steps.

Computing P_{i_e} and q_{j_e}: This problem is the same as the one in section 3.2.3.3, the only difference is in the H matrix that has an additional term. Then, for this purpose let $H \in \mathbb{R}^{n\times n}$ with $H = \delta\Lambda^T\overline{Q}\delta\Lambda + \Lambda^T\overline{Q}\delta\Lambda + \delta\Lambda^T\overline{Q}\Lambda + \delta\Lambda^T C_{ca}$. Then each element of H is given for the next expressions

$$\begin{aligned}
H_{ii} &= \sum_{l=1}^{p}\left[q_l\delta_{l,i}^2 + (2q_l\lambda_{l,i} + C_{ca_{l,i}})\delta_{l,i}\right] \\
H_{ij} &= \sum_{l=1}^{p}\left[q_l\delta_{l,i}\delta_{l,j} + q_l(\lambda_{l,j} + C_{ca_{l,j}})\delta_{l,i} + q_l\lambda_{l,i}\delta_{l,j}\right], \quad \forall j > i
\end{aligned} \tag{3.70}$$

Following the same procedure that in section 3.2.3.3, we have the next result for P_{i_e} and q_{j_e}:

$$\begin{aligned}
P_{ii,i_e} &= \begin{cases} 2\lambda_{i_e-(i-1)p,i} + C_{ca_{i_e-(i-1)p,i}} & \text{for} \quad i_e = (i-1)p+1,\dots,(i-1)p+p \\ 1 & \text{for} \quad i_e = p\cdot n + (i-1)p+1,\dots,p\cdot n \\ & \qquad +(i-1)p+p \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \\
P_{ij,i_e} &= \begin{cases} \lambda_{i_e-(i-1)p,j} + C_{ca_{i_e-(i-1)p,j}} & \text{for} \quad i_e = (i-1)p+1,\dots,(i-1)p+p \\ \lambda_{i_e-(j-1)p,i} & \text{for} \quad i_e = (j-1)p+1,\dots,(j-1)p+p \\ 1 & \text{for} \quad i_e = \overline{\kappa}+1,\dots,\overline{\kappa}+p \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \\
q_{1i,j_e} &= \begin{cases} \overline{\Gamma}_{j_e-(j-1)p} & \text{for} \quad j_e = (j-1)p+1,\dots,(j-1)p+p \\ \lambda_{j_e-p\cdot n,j} & \text{for} \quad j_e = p\cdot n+1,\dots,p\cdot n+p \\ 1 & \text{for} \quad j_e = p\cdot n + j\cdot p+1,\dots,p\cdot n+j\cdot p+p \\ 0 & \text{for} \quad \text{otherwise} \end{cases}
\end{aligned} \tag{3.71}$$

where $\overline{\kappa} = \frac{p\cdot i}{2}(2n-i-1) + p\cdot(n+j-1)$, $\kappa = 2p\cdot n + p\cdot(j-1)(n-\frac{j}{2}) + p\cdot(\alpha-j-1)+i$, the dimension of μ is $m = \frac{3}{2}(p\cdot n) + \frac{1}{2}(p\cdot n^2)$ and the dimension of ν is $z = 2p\cdot n + p$. Note that $\|\mu\|_2 \le 1$ and $\|\nu\|_2 \le 1$, then a necessary condition to use this approach is:

$$\begin{aligned}
\|\mu\|_2 &= \left\|\begin{bmatrix} \mu_{(j-1)p+i} \\ \mu_{n\cdot p+(j-1)p+i} \\ \mu_\kappa \end{bmatrix}\right\|_2 = \left\|\begin{bmatrix} q_i\delta_{i,j} \\ q_i\delta_{i,j}^2 \\ q_i\delta_{i,j}\delta_{i,\alpha} \end{bmatrix}\right\|_2 \le 1 \\
\|\nu\|_2 &= \left\|\begin{bmatrix} \nu_{(j-1)p+i} \\ \nu_{n\cdot p+i} \\ \nu_{n\cdot p+p+(j-1)p+i} \end{bmatrix}\right\|_2 = \left\|\begin{bmatrix} q_i\delta_{i,j} \\ q_i\rho_i \\ q_i\delta_{i,j}\rho_i \end{bmatrix}\right\|_2 \le 1
\end{aligned}$$

3.4.5 Robust Economic MPC for an evaporator system

The evaporation process examined in this section is shown in Figure 3.9. This is a process which removes a volatile liquid from a non-volatile solute, thus concentrating the solution. It consists of a heat exchange vessel with a recirculating pump. The overhead vapor is condensed by the use of a process heat exchanger. The details of the mathematical model can be found in [112]. The dynamic model is given by

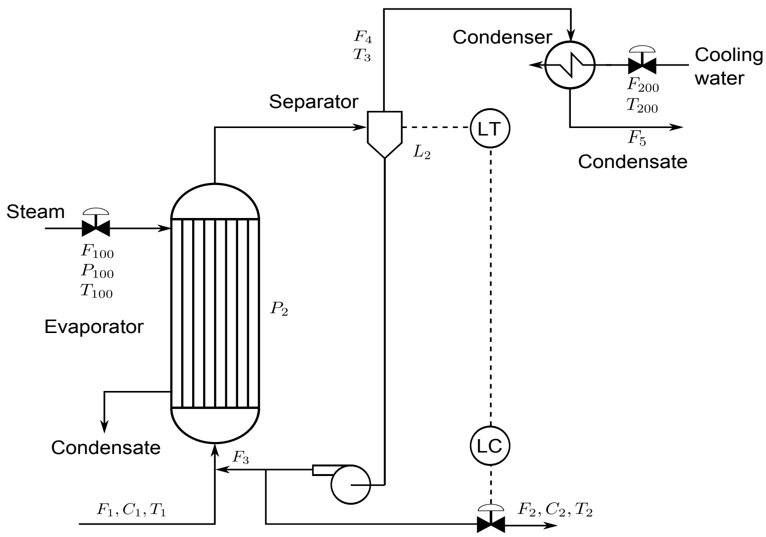


Figure 3.9: Evaporator system.

$$
\begin{aligned}
\rho A \frac{dL_2}{dt} &= F_1 - F_4 - F_2 \\
M \frac{dC_2}{dt} &= F_1 C_1 - F_2 C_2 \\
C \frac{dP_2}{dt} &= F_4 - F_5
\end{aligned}
\tag{3.72}
$$

where

$$
\begin{aligned}
F_4 =& [0.16(F_1 + F_3) \\
& \times (-0.3126 C_2 - 0.5616 P_2 + 0.1538 P_{100} + 41.54) \\
& - F_1 C_p (0.3126 C_2 + 0.5616 P_2 + 48.43 - T_1)] / \lambda_{s1}
\end{aligned}
\tag{3.73}
$$

$$
F_5 = \frac{2UA_2(0.507P_2 + 55 - T_{200}) C_p F_{200}}{\lambda_{s2}(UA_2 + 2C_p F_{200})}
$$

In the above equations ρA is the product of liquid density and the cross-sectional area of the evaporator (20 kg/m), M is the liquid holdup in the evaporator (20 kg), C is a constant (4 kg·kPa^{-1}), UA_2 is the product of the heat-transfer coefficient and the heat-transfer area in the condenser (6.84 kW·K^{-1}), C_p is the heat capacity of the cooling water (0.07 kW·kg^{-1}·min^{-1}), λ_{s1} is the latent heat of steam at saturated conditions (36.6 kW·min^{-1}·kg^{-1}) and λ_{s2} is the latent heat of evaporation of water (38.5 kW·kg^{-1}·min^{-1}). The operating cost of the evaporator consists of the cost of electricity, the cost of steam, and the cost of cooling water [113].

$$J = 1.009(F_2 + F_3) + \frac{96(F_1 + F_3)(T_{100} - T_2)}{\lambda_{s1}} + 0.6F_{200} \tag{3.74}$$

where

$$\begin{aligned} T_{100} &= 0.1538P_{100} + 90 \\ T_2 &= 0.5616P_2 + 0.3126C_2 + 48.43 \\ F_2 &= F_1 - F_4 \end{aligned} \tag{3.75}$$

The inequality constraints that define the region of feasible operation are the following

$$\begin{aligned} -C_2 + 35 &\leq 0 \\ 40 - P_2 &\leq 0 \\ P_2 - 80 &\leq 0 \\ P_{100} - 400 &\leq 0 \\ F_{200} - 400 &\leq 0 \end{aligned} \tag{3.76}$$

The outputs, inputs and disturbances of the process are

$$x = y = \begin{bmatrix} C_1 \\ P_2 \end{bmatrix}, \quad u = \begin{bmatrix} P_{200} \\ F_{200} \end{bmatrix}, \quad d = \begin{bmatrix} F_1 \\ C_1 \\ T_1 \\ T_{200} \end{bmatrix} \tag{3.77}$$

In order to find the optimum point of steady-state operation we consider the following disturbance conditions

$$F_1 = 10\text{Kg/min}, \quad C_1 = 5\%, \quad T_1 = 40°\text{C}, \quad T_{200} = 25°\text{C}$$

The objective function is optimized, subject to the equality as well as the inequality constraints. The optimum operating point is,

$$x_{ss} = \begin{bmatrix} 24.8242 \\ 50.3784 \end{bmatrix}, \quad u_{ss} = \begin{bmatrix} 193.4500 \\ 207.3300 \end{bmatrix} \tag{3.78}$$

In order to evaluate the controller performance the system was subject to disturbances in the feed flow rate F_1, the feed temperature T_1 and the coolant water inlet temperature T_{200}. The results are compared with an economic MPC using nominal models without robust considerations. Figure 3.10 show the profiles of the disturbances applied to the process. Figures 3.11 (a), 3.11 (b), 3.12 (a), and 3.12 (b) shows the state profiles and inputs as well as the corresponding instantaneous profits of the two closed loop systems (nominal and robust economic MPC). Under the mentioned disturbances the robust economic controller has a good performance in the economic sense, but compared with the nominal controller there are some important aspects to mention. For example, the response of C_2 in the robust economic MPC is slower and the drop in this variable decreases the instantaneous profit and hence the system under robust economic MPC tends to stay at a lower pressure as compared to the nominal economic MPC. The corrective control action resulting from this disturbance also increases the product composition, which in turn, increases the profit. In order to compare the economic performance of the different schemes, we take an integral index (e) based on the difference of the instantaneous profits shown in Figure 3.12 (b). Mathematically this criterion is given by:

$$e = \left(\int_0^{100} (I_{p_{REMPC}}(t) - I_{p_{EMPC}}(t))dt \right) \times 100 \tag{3.79}$$

where $I_{p_{REMPC}}$ and $I_{p_{EMPC}}$ denotes the instantaneous profit of the robust and nominal schemes. Using this index, the profit obtained from robust economic MPC is better than the nominal economic MPC in about 0.9797 %.

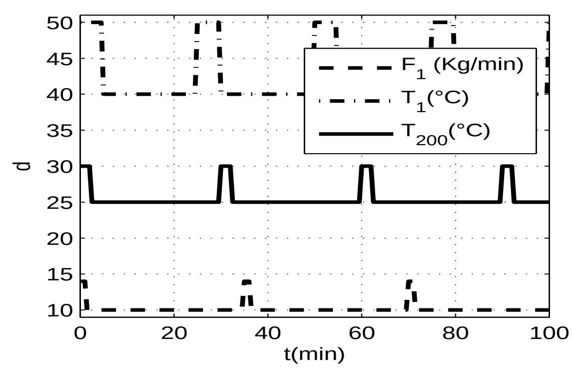
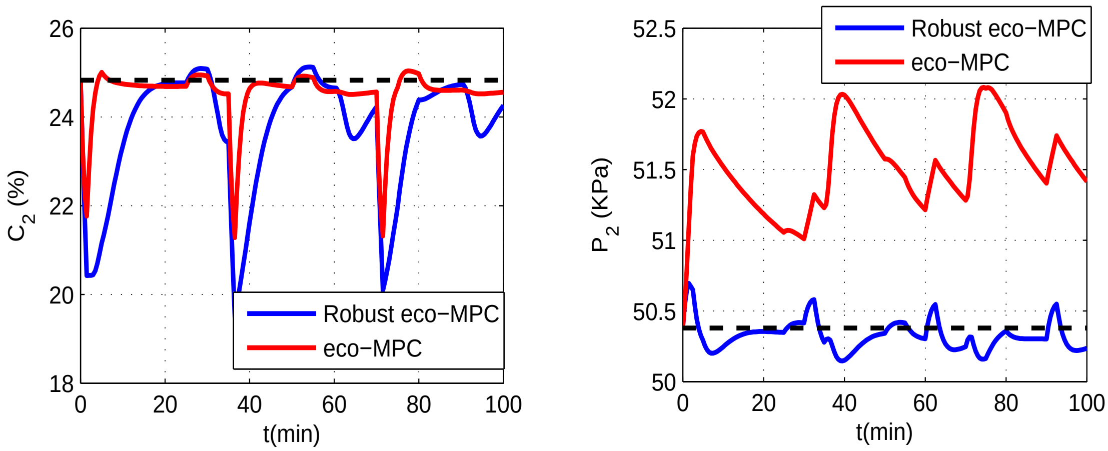
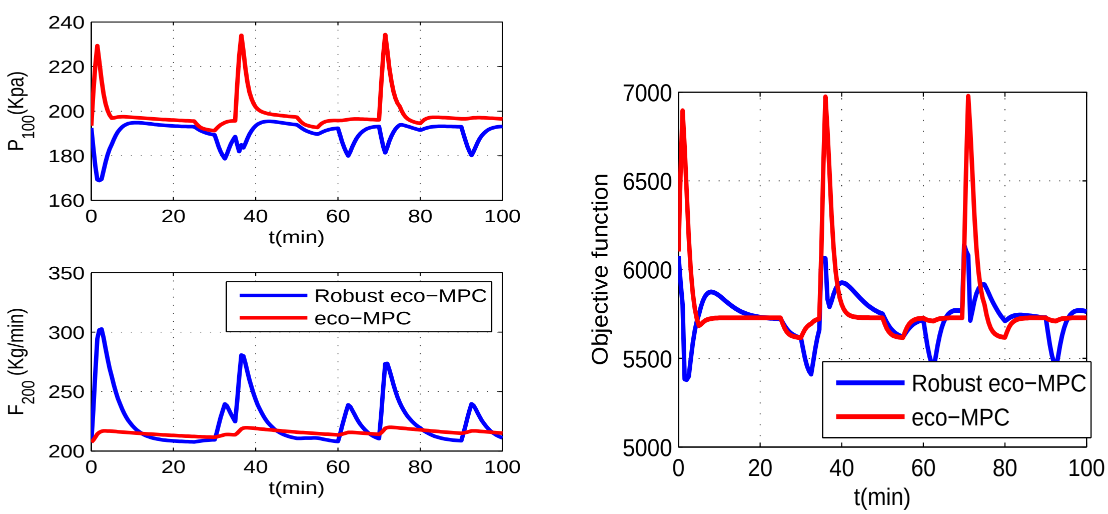


Figure 3.10: Disturbances of the process.



(a) (b)

Figure 3.11: Evaporator system: (a) Concentration C_2, (b) Pressure P_2.



(a) (b)

Figure 3.12: Evaporator system: (a) Closed-loop inputs, (b) Instantaneous profit.

3.5 Summary

This chapter presents a novel min-max MPC, min-max MPC with zone control and Economic MPC. These new formulations are based in Appendix A where algorithms for Robust Quadratic Programming are presented, and the solution of the robust quadratic program is obtained by transforming the program into a Second Order Cone Program under different uncertain sets. Finally this chapter presents one example of every robust controller in order to illustrate the advantages of every approximation. In a detailed manner,

- In section 3.2 the result of the appendix A was extended in a new Robust Model Predictive Control formulation that inherits all convexity and complexity properties of the original RQP problem. There are two important contribution in this section, the first one is about the uncertainty representation. For the robust MPC proposed we assume uncertainty in the free and forced response. This kind of uncertainty can be expressed in typical forms reported in the robust MPC literature, and in this context the uncertainty representation presented here is fairly complete. The second and most remarkable contribution in this section is the way to find the parameters of the uncertain sets. We proposed a novel method to obtain these parameters based in mappings between the uncertainty of the free and forced response and the uncertainty in the quadratic, linear and constant term of the quadratic optimization problem. Finally the benefits of the proposed controller were shown through an illustrative example.

- In sections 3.3 and 3.4 the results of the new robust MPC of the section 3.2 were used to propose new formulations for Robust MPC with zone control and Robust Economic MPC. As in the section 3.2, a new way to find the parameters of the uncertain sets is stated. Finally, the proposed controllers were tested in two application examples.

CHAPTER 4

Hierarchical Robust Integration of MPC with zone control and DRTO

4.1 Introduction

"Optimization has become a key area in control theory due to the increasing need to optimize plant operations. The optimization criterion could be to reduce operating cost or maximize profit while at the same time meeting product specifications. Consequently, the control engineer is faced with the tasks of designing a good controller, and also making sure the plant operates at an optimum. With the advent of better controllers (that guarantee good plant control), the focus has now shifted to the regulation of processes about conditions that provide maximum profitability. However, Since modeling uncertainty inevitably exists in industrial problems, our interest is on the optimization and optimal control of processes under uncertainty" Adetola et.al [61].

Nevertheless, in hierarchical control there is no tangible progress in this direction, mainly due to the lack of development of efficient algorithms for robust optimization and robust optimal control. Thereby, the design of robust hierarchical strategies remains as a challenge for the control community. In this way this chapter presents an approach to hierarchical robust control for large scale systems. The scheme proposed is based in the current practices in hierarchical operation presented in chapter 2, for this reason the strategy presented in this chapter has similar components such as a dynamic optimization and optimal control problems. This chapter is organized as follows: In section 4.2 an example that motivates the use of robust hierarchical control is presented. In this example a nominal hierarchical control as the presented in chapter 2 is studied under uncertain conditions. In section 4.3 a hierarchical robust integration of control and DRTO is shown. This strategy is basically the general robust formulation of hierarchical controllers for large scale systems. In this way, in section 4.3.1 the generalities of hierarchical robust control methodology is explained, the idea of this section is to show in general terms the methodology and the common elements of all strategies presented in this thesis. Next, every layer of the hierarchical structure with their different possible formulations are presented in sections 4.3.2, 4.3.3 and 4.3.4. Finally the strategy proposed in this chapter is implemented in the example of the section 4.2.

4.2 Motivation: Hierarchical Integration of MPC with zone control and DRTO for two reactors chain and flash system

Consider a plant with two continuous stirred-tank reactors (CSTRs) followed by a nonadiabatic flash separator, as shown in Figure 4.1. In each of the CSTRs, the desired product B is produced through the irreversible first-order reaction $A \overset{k_1}{\to} B$, k_1 being the Arrhenius constant of the reaction. An undesirable side reaction $B \overset{k_2}{\to} C$ results in the consumption of B and in the production of the unwanted side product C (here, k_2 is the Arrhenius constant of this reaction). The product stream from CSTR-2 is sent to a nonadiabatic flash separator to separate the excess of A from the product B and the side product C. In the flash separator, it is assumed that reactant A has the highest volatility and that it is the predominant component in the vapour phase. A fraction of the vapour phase is purged and the remaining stream rich in A is condensed and recycled back to CSTR-1. In order to operate the system shown in Figure 4.1, the manipulated variables of the system are the feed flow rates F_0, F_1, the cooling duties Q_r, Q_m, Q_b, and the recycle flow rate D. The measured variables are the level of liquid in the reactors H_r , H_m, H_b, the outlet mass fractions of A and B x_{Ar}, x_{Br}, x_{Am}, x_{Bm}, x_{Ab}, x_{Bb}, and the temperature of the reactors T_r, T_m, T_b. The controlled variables are the levels of the reactors and the temperatures at the reactors and at the flash separator. With the purpose of designing a controller for the system shown in Figure 4.1, a model should be derived. From the first principles of this system the evolution of the levels, concentrations, and temperatures at each device of the process can be respectively modelled as follows [114]:

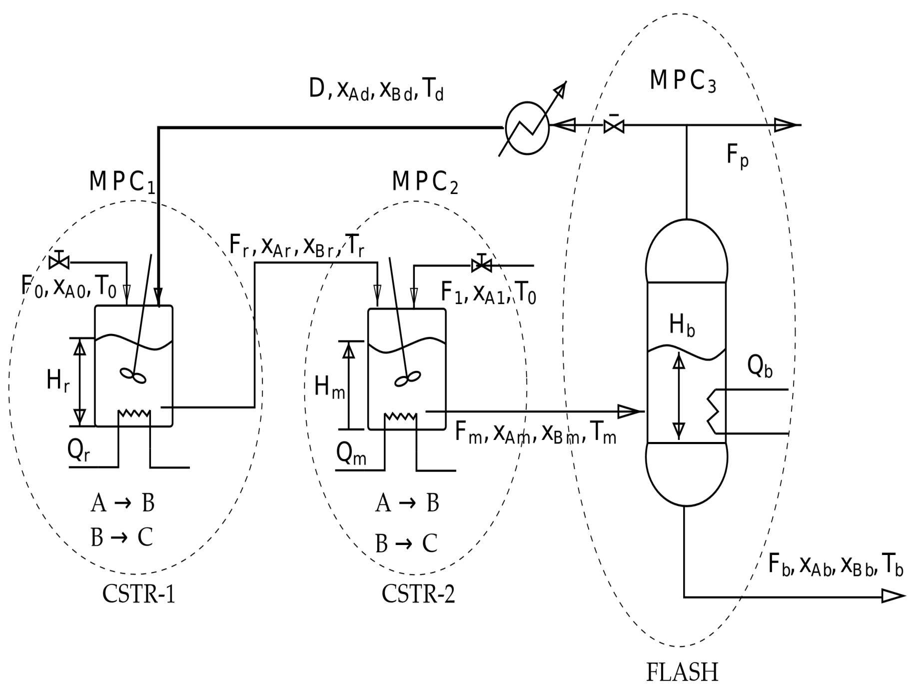


Figure 4.1: Two reactor chain followed by nonadiabatic flash [114].

- Reactor 1:

$$\begin{aligned}
\frac{dH_r}{dt} &= \frac{1}{\rho_r A_r}\left[F_0 + D - F_r\right] \\
\frac{dx_{Ar}}{dt} &= \frac{1}{\rho_r A_r H_r}\left[F_0(x_{A0} - x_{Ar}) + D(x_{Ad} - x_{Ar})\right] - k_{1r}x_{Ar} \\
\frac{dx_{Br}}{dt} &= \frac{1}{\rho_r A_r H_r}\left[F_0(x_{B0} - x_{Br}) + D(x_{Bd} - x_{Br})\right] + k_{1r}x_{Ar} - k_{2r}x_{Br} \\
\frac{dT_r}{dt} &= \frac{1}{\rho_r A_r H_r}\left[F_0(T_0 - T_r) + D(T_d - T_r)\right] - \frac{1}{C_p}\left[k_{1r}x_{Ar}\Delta H_1 + k_{2r}x_{Br}\Delta H_2\right] \\
&+ \frac{Q_r}{\rho_r A_r H_r}
\end{aligned} \tag{4.1}$$

- Reactor 2:

$$\begin{aligned}
\frac{dH_m}{dt} &= \frac{1}{\rho_m A_m}\left[F_r + F_1 - F_m\right] \\
\frac{dx_{Am}}{dt} &= \frac{1}{\rho_m A_m H_m}\left[F_r(x_{Ar} - x_{Am}) + F_1(x_{A1} - x_{Am})\right] - k_{1m}x_{Am} \\
\frac{dx_{Bm}}{dt} &= \frac{1}{\rho_m A_m H_m}\left[F_r(x_{Br} - x_{Bm}) + F_1(x_{B1} - x_{Bm})\right] + k_{1m}x_{Am} - k_{2m}x_{Bm} \\
\frac{dT_m}{dt} &= \frac{1}{\rho_m A_m H_m}\left[F_r(T_r - T_m) + F_1(T_0 - T_m)\right] - \frac{1}{C_p}\left[k_{1m}x_{Am}\Delta H_1 + k_{2m}x_{Bm}\Delta H_2\right] \\
&+ \frac{Q_m}{\rho_m A_m H_m}
\end{aligned} \tag{4.2}$$

- Nonadiabatic flash:

$$\begin{aligned}
\frac{dH_b}{dt} &= \frac{1}{\rho_b A_b}\left[F_m - F_b - D - F_p\right] \\
\frac{dx_{Ab}}{dt} &= \frac{1}{\rho_b A_b H_b}\left[F_m(x_{Am} - x_{Ab}) - (D + F_p)(x_{Ad} - x_{Ab})\right] \\
\frac{dx_{Bb}}{dt} &= \frac{1}{\rho_b A_b H_b}\left[F_m(x_{Bm} - x_{Bb}) + (D + F_p)(x_{Bd} - x_{Bmb})\right] \\
\frac{dT_b}{dt} &= \frac{1}{\rho_b A_b H_b}\left[F_b(T_m - T_b)\right] + \frac{Q_b}{\rho_b A_b H_b C_p}
\end{aligned} \tag{4.3}$$

where $F_{\rm r} = k_{\rm r}\sqrt{H_{\rm r}}$, $F_{\rm m} = k_{\rm m}\sqrt{H_{\rm m}}$, $F_{\rm b} = k_{\rm b}\sqrt{H_{\rm b}}$, $k_{\rm 1r} = k_1^* \exp^{\frac{-E_1}{RT_{\rm r}}}$, $k_{\rm 2r} = k_2^* \exp^{\frac{-E_2}{RT_{\rm r}}}$, $k_{\rm 1m} = k_1^* \exp^{\frac{-E_1}{RT_{\rm m}}}$, $k_{\rm 2m} = k_2^* \exp^{\frac{-E_2}{RT_{\rm m}}}$, $x_{\rm Cr} = 1 - x_{\rm Ar} - x_{\rm Br}$, $x_{\rm Cm} = 1 - x_{\rm Am} - x_{\rm Bm}$, $x_{\rm Cb} = 1 - x_{\rm Ab} - x_{\rm Bb}$, $\Sigma = \alpha_{\rm A}x_{\rm Ab} + \alpha_{\rm B}x_{\rm Bb} + \alpha_{\rm C}x_{\rm Cb}$, $x_{\rm Ad} = \frac{\alpha_{\rm A}x_{\rm Ab}}{\Sigma}$, $x_{\rm Bd} = \frac{\alpha_{\rm B}x_{\rm Bb}}{\Sigma}$,

$x_{\mathrm{Cd}} = \frac{\alpha_{\mathrm{C}} x_{\mathrm{Cb}}}{\Sigma}$. The values of the parameters of the system used for the simulations are shown in Tables 4.1 and 4.2 [114]. Moreover, the constraints of the system variables were: $0 \leqslant F_0 \leqslant 30$, $0 \leqslant F_1 \leqslant 30$ for the feed flow rates of the CSTRs, $0 \leqslant D \leqslant 30$ for the recycle flow rate, and $-200 \leqslant Q_{\mathrm{r}} \leqslant 200$, $-200 \leqslant Q_{\mathrm{m}} \leqslant 200$, $-200 \leqslant Q_{\mathrm{b}} \leqslant 200$ for the cooling duty of the CSTRs and the adiabatic flash separator.

	Units	Value	Description
ρ_{r} ρ_{m} ρ_{b}	[kg/m^3]	0.15	Density of the liquid
α_{A}		3.5	Volatility of x_{A}
α_{B}		1.1	Volatility of x_{B}
α_{C}		0.5	Volatility of x_{C}
k_1^*	[s^{-1}]	0.02	
k_2^*	[s^{-1}]	0.018	
A_{r}	[m^2]	0.3	Cross section area of the CSTR-1
A_{m}	[m^2]	3	Cross section area of the CSTR-2
A_{b}	[m^2]	5	Cross section area of the flash separator
T_0	[K]	313	Temperature of the manipulated flows
T_{d}	[K]	313	Temperature of the recycle flow
C_{p}	[kJ/(kgK)]	25	Heat capacity of the liquid
x_{A0}		1	Initial concentration of x_{A} at CSTR-1
x_{B0} x_{C0}		0	Initial concentration of $x_{\mathrm{b}}, x_{\mathrm{c}}$ at CSTR-1
x_{A1}		1	Initial concentration of x_{A} at CSTR-2
x_{B1} x_{C1}		0	Initial concentration of $x_{\mathrm{b}}, x_{\mathrm{c}}$ at CSTR-2
ΔH_1	[kJ/kg]	-40	Heat of reaction $A \longrightarrow B$
ΔH_2	[kJ/kg]	-50	Heat of reaction $B \longrightarrow C$
E_1/R E_2/R	[K]	150	
k_{r}	[kg/s^{-1}m$^{-1/2}$]	2.5	
k_{m}	[kg/s^{-1}m$^{-1/2}$]	2.5	
k_{b}	[kg/s^{-1}m$^{-1/2}$]	1.5	

Table 4.1: Values of the parameters used in the simulations of the two CSTRs chain followed by a nonadiabatic flash system [114]

Parameter	Units	Value	Description
$F_{0\mathrm{o}}$	[kg/s]	1.8562	Feeding flow of the CSTR-1 at the equilibrium point
$F_{1\mathrm{o}}$	[kg/s]	1.8562	Feeding flow of the CSTR-2 at the equilibrium point
D_{o}	[kg/s]	1.8562	Recycle flow of the at the equilibrium point
Q_{ro}	[kJ/s]	201.1438	Cooling duty of the CSTR-1 at the equilibrium point
Q_{mo}	[kJ/s]	201.1438	Cooling duty of the CSTR-2 at the equilibrium point
Q_{bo}	[kJ/s]	0.5565	Cooling duty of the flash separator at the equilibrium point

Table 4.2: Equilibrium point used to linearize the two CSTRs chain followed by a nonadiabatic flash model

4.2.1 Control structure

Figure 4.2 shows the control structure used in the two CSTRs chain followed by a nonadiabatic flash. In the upper layer a DRTO problem is solved with a MHE as estimator, while in the middle and lower layers a MPC with zone control and decentralized MPC scheme are employed. The outputs, inputs and states of the systems are:

$$
\begin{aligned}
y &= [H_\mathrm{r}, T_\mathrm{r}, H_\mathrm{m}, T_\mathrm{m}, H_\mathrm{b}, x_{Bb}]^T \\
u &= [F_0, F_1, Q_\mathrm{r}, Q_\mathrm{m}, Q_\mathrm{b}, D]^T \\
x &= [H_\mathrm{r}, x_{Ar}, x_{Br}, T_\mathrm{r}, H_\mathrm{m}, x_{Am}, x_{Bm}, T_\mathrm{m}, H_\mathrm{b}, x_{Ab}, x_{Bb}, T_\mathrm{b}]^T
\end{aligned}
$$

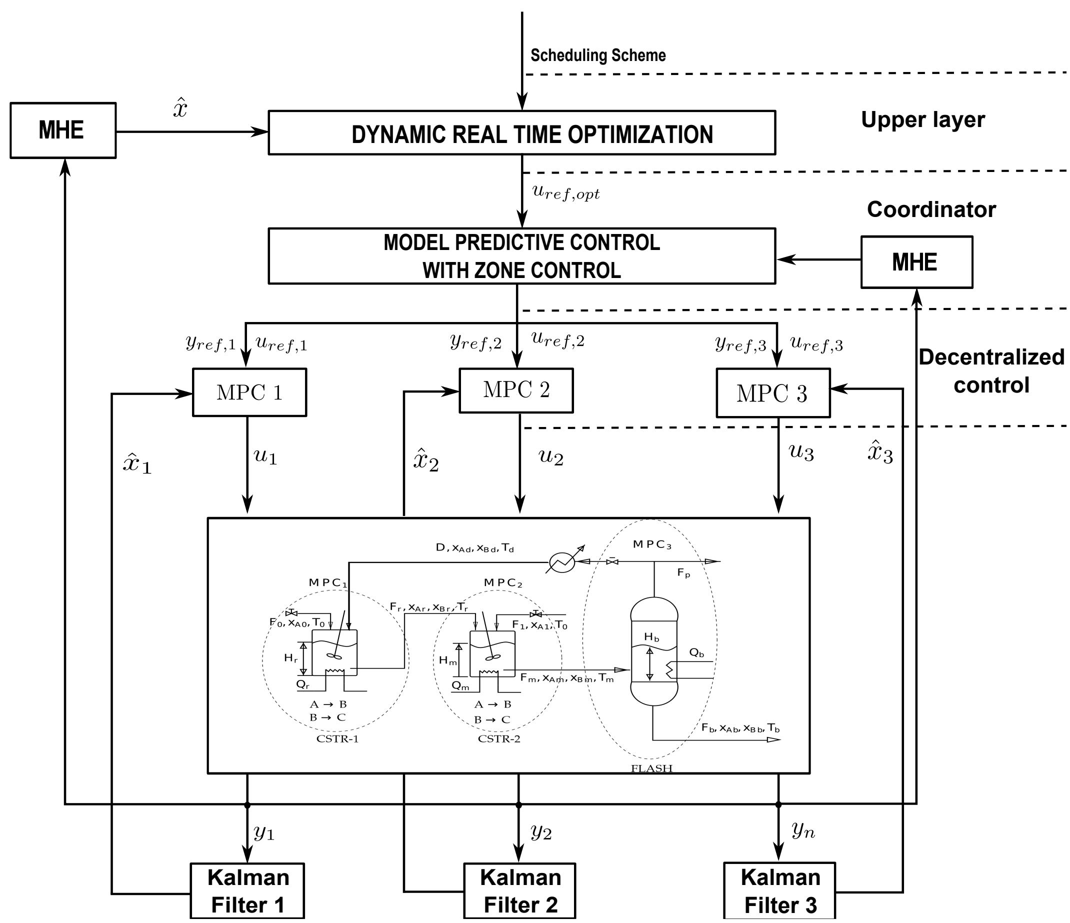


Figure 4.2: Hierarchical Integration of MPC and DRTO for two reactors chain and flash system.

The economical cost function of the system is:

$$
J_{eco} = -(2.5F_b x_{Bb} - 0.1F_0 - 0.2F_1 - 0.2(Q_r + Q_m + Q_b)) \tag{4.4}
$$

The performance of the proposed control scheme will be evaluated in a very simple scenario, the heat capacity C_p of the first and second reactor was increased a 5%. For this simulation, it was considered a sample time of the DRTO layer of $10s$ with a prediction horizon of 30 samples, while the sample time of the MPC with zone control was $1s$ with a prediction horizon of 30 samples.

4.2.2 Simulation Results and Discussion

Figures 4.3, 4.4 and 4.5 show the performance of the closed-loop system. In order to test the nominal scheme under uncertainties the heat capacity C_p of the first and second reactor was increased a 5%. In Figure 4.3 it is possible to observe how the nominal scheme cannot drive the temperature T_r to the zone and maintain the level H_r in a fixed set-point. The Figure 4.4 shows the behavior of the CSTR-2, in this Figure is important to highlight that even though all variables remain in the zone, the nominal scheme has problems to reach the set-points given by the coordinator, and clearly the inputs cannot follow the set-points. In the nonadiabatic flash we want to maintain the concentration $x_{B_b} = 0.3$ (quality specifications), note that the nominal scheme drives the concentration near of the specification, however a off-set is present. Clearly the nominal scheme cannot handle the uncertainty of the system. In this sense, a robust scheme must be used. In the following sections the a robust scheme than integrates DRTO and Robust MPC is presented.

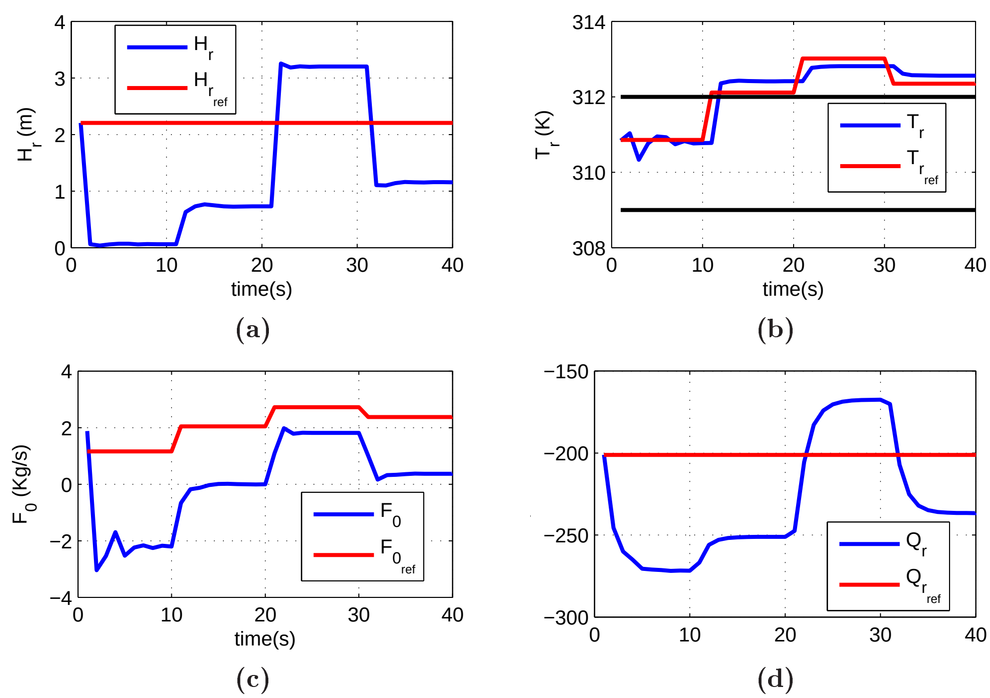


Figure 4.3: Evolution of the level and the temperature of CSTR-1, reference values, and control inputs.

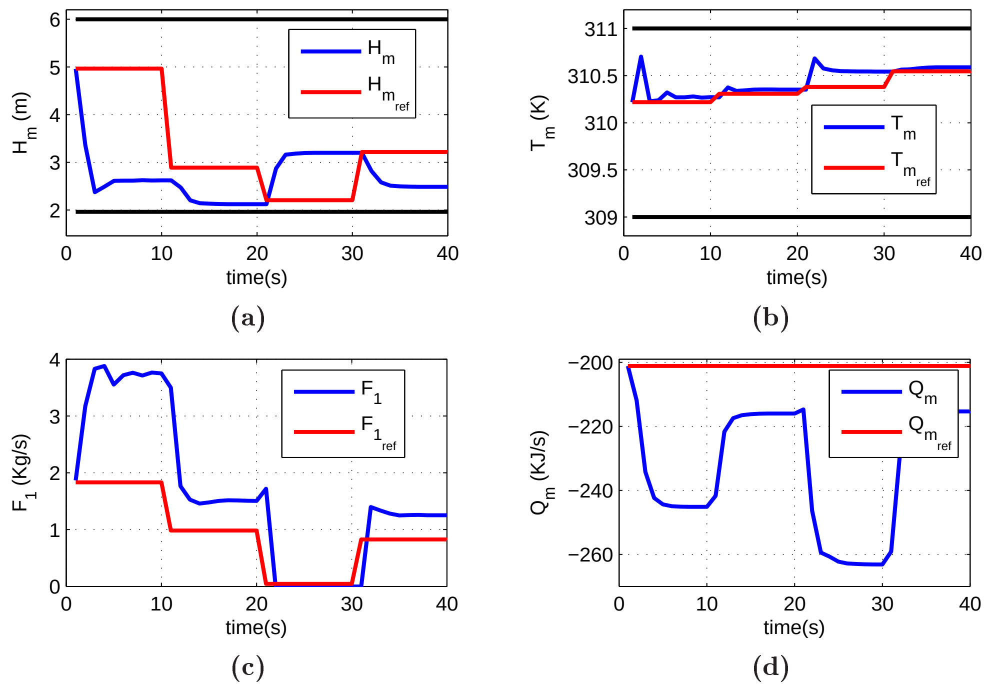


Figure 4.4: Evolution of the level and the temperature of CSTR-2, reference values, and control inputs.

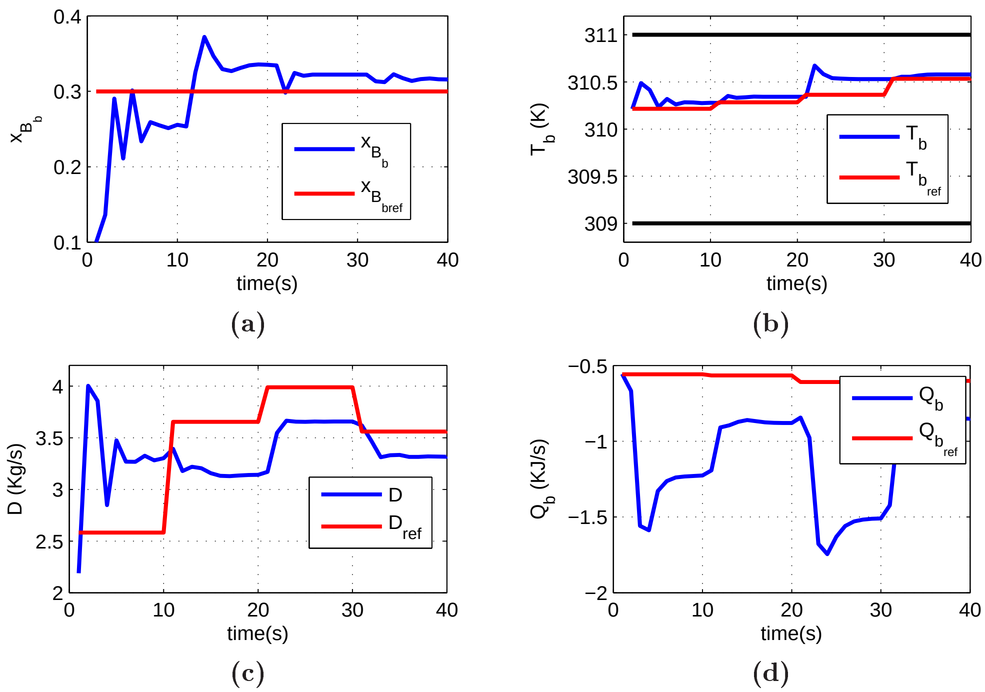


Figure 4.5: Evolution of the level and the temperature of the nonadiabatic flash, reference values, and control inputs.

4.3 Hierarchical Robust Integration of MPC with zone control and DRTO

The classic robust integration of real time optimization (RTO) and model predictive control (MPC) has been presented in several papers (see[115, 65, 116, 117, 118, 119]) as a hierarchical structure that consist of multiple tasks (local objectives) executed at different levels and time frequencies, with the objective of maximizing the profitability and while maintaining a flexible plant operation. In the upper layer of the hierarchical structure the RTO is stated, this is an optimization problem with an economic objective that uses the steady state nonlinear model. In the lower layer a robust MPC calculates, based in the RTO information, the optimal control actions to track the references given by the upper layer. However, process operation is transient due to the dynamic nature of the process, disturbances and uncertainties, and the described procedure requires that the optimal economic operating points must be reconciled with the lower layer in order to give to the MPC feasible operating points. Besides, since the MPC generally uses linear models, the discrepancies between linear models and the nonlinear nature of the process introduce more uncertainties to the control system. Robust integration addresses these problems, the procedure is described by [120] and [46] emphasizing in the integration of the economic or management level with the MPC level. This integration has some difficulties as described in [121]:

- The upper level needs to take into account the problems such as objectives and constraints of the lower levels.
- Decomposing the overall objective into local objectives at different levels is a challenging task for transient process operation.
- The operation demands a real integration of RTO and MPC in presence of disturbances and uncertainties.
- The models used in the optimization and in the control layer are not consistent.
- Since the economic optimization is only carried out when the plant reaches steady-state, the waiting time required to reach a new steady-state leads to a delay for computing a new set-point. It may thus happen that, due to uncertainties, the economically optimal operating point of the plant has shifted but the controller still tries to enforce the previously optimal operating point.

An alternative to solve the previously mentioned problems in the two-layer approach is an integration of Dynamic RTO (DRTO) and robust MPC. This section presents the following contributions to the solution of the problem:

- A formulation of an operational problem involving process economics constraints and inherent uncertainty.
- A clear separation of objectives into an economical objective and a control objective that implements the optimal solution in the presence of uncertainty.
- A new Robust MPC (RMPC) formulation. The solution transforms the original min-max problem into a second-order cone program.
- The use of a local linearization along the nominal input and state trajectory in order to avoid the nonlinear dynamic optimization in the DRTO layer.

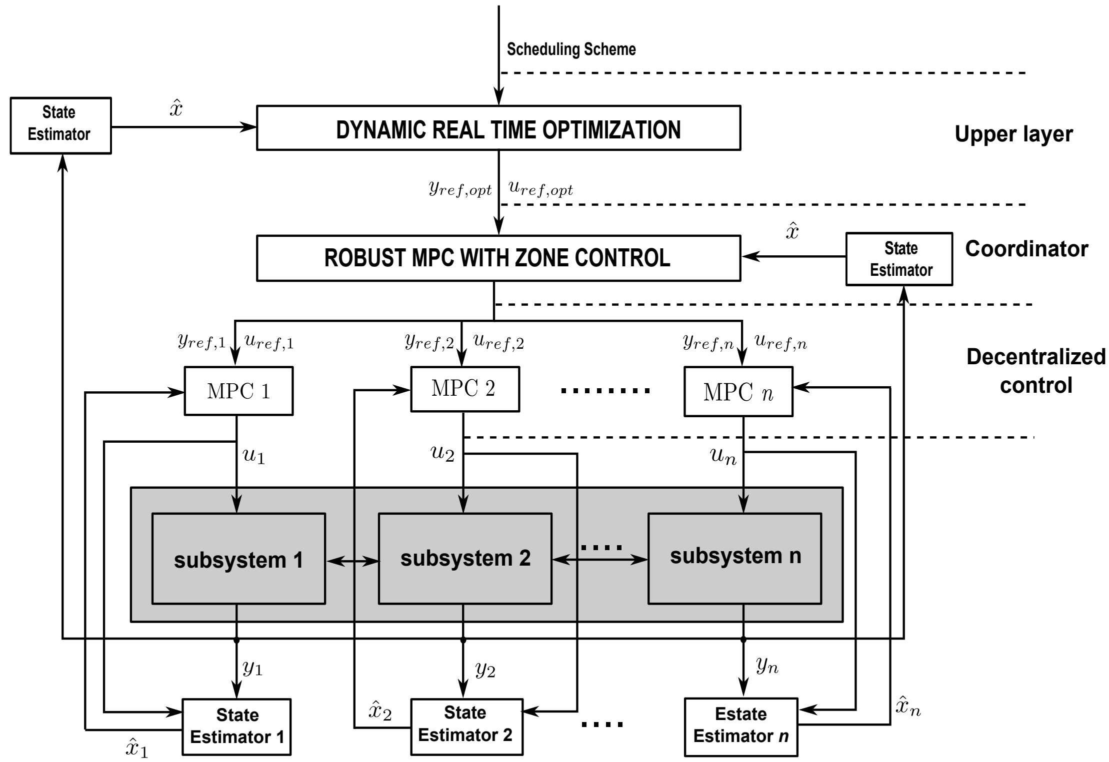


Figure 4.6: Hierarchical Robust Integration of MPC and DRTO

The structure proposed in this section is presented in Figure 4.6 and it is based on the work of Helbig et al. [122] for integration of dynamic real-time optimization (DRTO) and tracking MPC. Due to the complexity of the process (large scale systems) uncertainty is present, therefore the objective is to achieve a stable operation under process uncertainties, for this reason a robust controller is used as coordinator.

4.3.1 Generalities of the Hierarchical Robust Control Methodology

As it was mentioned before the hierarchical robust control strategies proposed here are based in the current practices in hierarchical operation presented in chapter 2. In general terms these structures are composed by three layers, however as it will be shown in chapter 5 some variations can be introduced. Figure 4.7 shows in detail the hierarchical robust structure. In the upper layer a dynamic optimization problem based in the nonlinear model of the plant is solved, the objective of this layer is to find reference trajectories for the inputs and outputs of the system based on some specific criterion (economic, environmental, etc) and send these to the lower layers. In the middle layer a robust control problem is addressed, the main idea with this layer is to handle uncertainties of the system and transform the objective given by the upper layer by means of the references trajectories in a robust control objective. The result of this layer is robust references for the lower layer where the local controllers are operating. An important aspect to highlight in the middle layer is that different types of robust control problem can be used, however in our contribution the robust MPC approaches explained in chapter 3 will be exploited. Finally, in the lower layer are local controllers, where different schemes could be used such as distributed and decentralized controllers.

In general terms the procedure to design the hierarchical robust control strategy of Figure 4.7 is composed by 2 stages.

- The first one is oriented to local control design. Two tasks can be identified in this stage. The main task is the identification of local control objectives, followed by the design of local control strategies.
- The second stage is to choose the appropriate architecture to control the system i.e multilayer with or without coordination, and in the case of a coordination scheme to chose the type of coordinator. The stage 2 can be subdivided in 4 tasks, they are the selection of the objective function of the dynamic optimization problem, choice an adequate trigger, the selection of models for dynamic optimization and tracking control problems and the identification of a set of references trajectories for the manipulated variables.

In the following sections a brief discussions about some tasks of the Hierarchical Robust Control Methodology will be made.

4.3.1.1 Selection of models for dynamic optimization and tracking control problems

- **Dynamic optimization:** The quality of a model for dynamic optimization depends basically on two factors: The first factor is the computational load of the overall optimization problem, this determines its potential for online applications. The second one is the quality of the solution of the optimization based on approximated model. In order to tackle the first factor, it is important to remember that in general for the dynamic optimization problem with economical objective a nominal model is considered. Hence, the model used for the optimization must have sufficient prediction quality and should cover a wide range of dynamics. Therefore, a fundamental process model is a natural candidate. However for large scales systems it is not always the best alternative. There are several reason for that, among them the complexity, the order of the model of the whole plant and the algorithms available to solve the dynamic optimization problem. Therefore, other alternatives must be considered to tackle these problems. In this way model reduction and linear - time variant models derived from the fundamental process model arises as an alternative. As in this thesis robust optimization is used in the hierarchical structures proposed, the previous modeling alternatives can be taken for this purpose. The second factor was studied by Forbes et.al [123] in their paper on model adequacy requirements. They talk about three manners to differentiate between alternative models. The most common used method is to compare their capability in predicting key process variables. The second method is to check the optimality conditions, which can be done if a solution of the original problem is available. Finally, it is tested whether the optimization based on the approximate model is capable of predicting the manipulated variables that coincide with the true optimal trajectories.

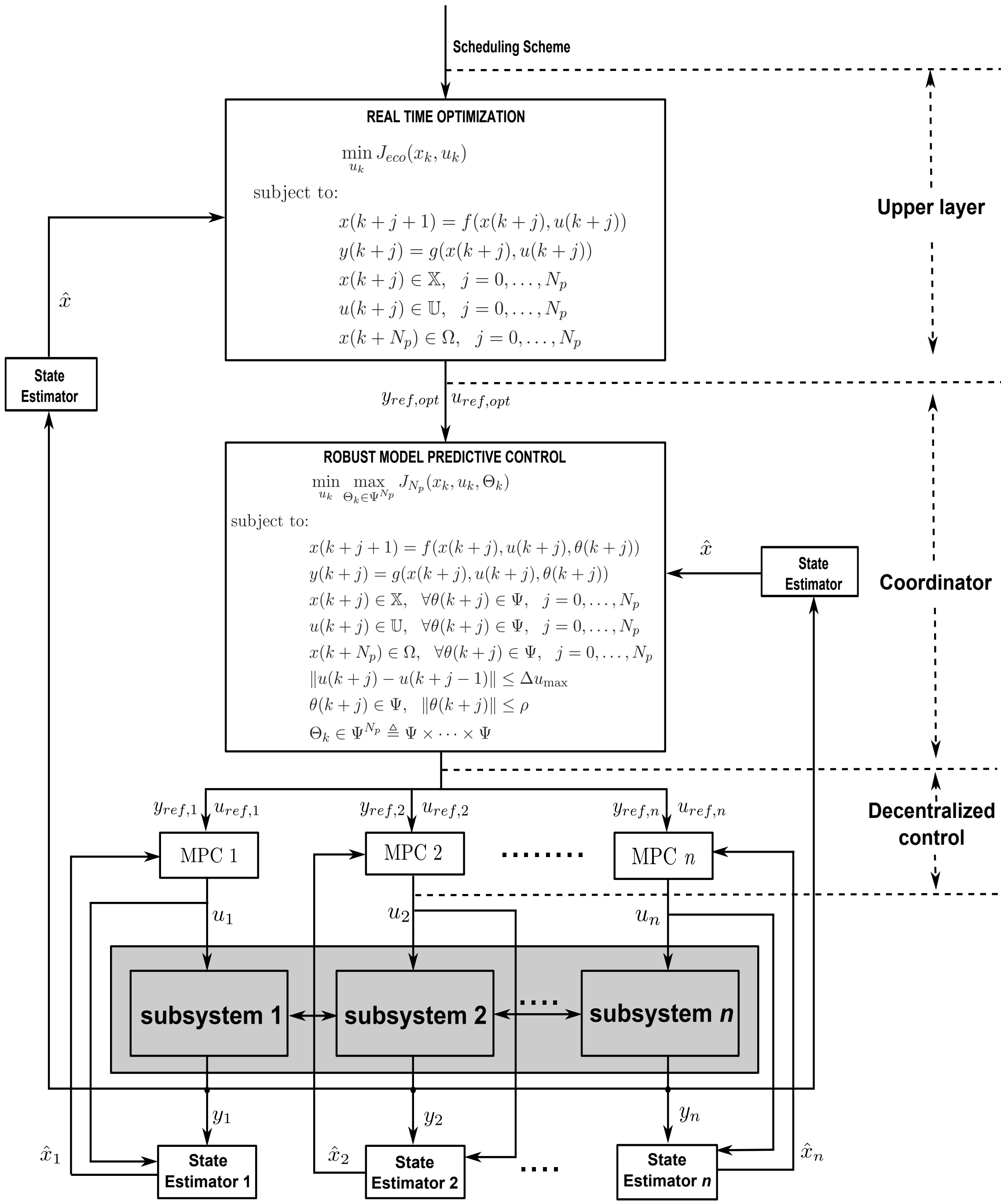


Figure 4.7: Hierarchical Robust Control structure

- **Tracking control:** As the control problem objective is to track the controlled variables, a purely linear or a linear - time variant model derived from the model used in the top layer is sufficient. Thereby, this could provide some consistency among models of optimization and control levels.

4.3.1.2 Triggers

The aim of process operation is to maximize profit. Hence, process operation is determined by economic criteria, which are incorporated in the objective function of the upper layer optimization. Moreover, the objective of process control is to achieve a stable operation under process uncertainties, which should be guaranteed by the robust controller in the lower-layer of the hierarchical structure. Thereby, typically a time-scale separation is used, as the upper layer is operating at a lower sampling rate and should only take into account the slow trend in the controlled variables. However, there are other forms to execute or trigger the different task of each level by means of the so called triggers (see Figure 4.7). The triggers fulfill an important mission in the hierarchical structure because these blocks act like switches for re-calling the optimization layer and the control layers when a certain condition is met. As can be seen in Figure 4.7, in the hierarchical structure proposed here, there are two different triggers. The first one is the trigger of the optimization layer and the second one is the trigger of the coordinator. In general there are four types of triggers described as follows [21] :

- Time criterion: in this case the problem of each layer is called periodically at predetermined frequency. This is the most used criterion in hierarchical control.
- Disturbances criterion: This criterion is based on the occurrence of disturbances, the main drawback of this criterion is to estimate the unmeasured disturbances, however to achieve that a Kalman estimator can be used.
- Profitability criterion: In this case the trigger is activated when some performance index decreases below a certain level.
- Variables deviations criterion: This case is used mainly in the tracking control layer and as the profitability criterion the trigger is activated when the deviations of the controlled variables from their optimal set points exceeds certain tolerance.

In general, the most attractive trigger is the based in disturbances because this offers several advantages such as time scale separation, state and disturbances estimation among others. For this purpose, two different state estimators are used in the proposed hierarchical structure and executed on different time-scales. A moving horizon estimator is used at the upper layer at a low sampling rate considering a window of measurements over a time horizon, while a Kalman filter is used at the lower layer employing the measurement of the last sampling time only. Below a brief description of the estimators will be done.

Kalman filter: All model predictive controllers explained in this thesis requires to know the current state of the plant for solving the optimization problem at each time instant k. However, in general, only some states are available. Therefore the use of an observer is necessary in order to estimate the state vector of the plant based in the mathematical model of the system and measuring the process input values and the process outputs. The equation of an observer is as follows,

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + Bu(k) + L_x(y(k) - \hat{y}(k)) \\ \hat{y}(k) &= C\hat{x}(k) + Du(k) \end{aligned} \tag{4.5}$$

where $\hat{x}(k)$ is the estimation of $x(k)$ and $L_x \in \mathbb{R}^{n_x \times n_u}$ is the feedback gain matrix or observer gain. When L_x is calculated by means of optimal estimation techniques, the observer is referred to as Kalman filter. Consider the following discrete-time model of the plant,

$$\begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k) + G(k)w(k) \\ y(k) &= C(k)x(k) + \nu(k) \end{aligned} \tag{4.6}$$

where $G(k) \in \mathbb{R}^{n_x \times n_w}$ is a weighting matrix, $w(k) \in \mathbb{R}^{n_w}$ is the process noise, modelled as a Gaussian white noise with zero mean and covariance matrix $\mathbf{R}_w \in \mathbb{R}^{n_w \times n_w}$, and $v(k) \in \mathbb{R}^{n_y}$ is measurement noise, modelled as a Gaussian white noise with zero mean and covariance matrix $\mathbf{R}_v \in \mathbb{R}^{n_y \times n_y}$. These covariance matrices are given by

$$\begin{aligned} \mathbf{R}_w &= \varepsilon\{w(k)w(k)^T\} \\ \mathbf{R}_v &= \varepsilon\{v(k)v(k)^T\} \end{aligned} \tag{4.7}$$

where $\varepsilon\{\cdot\}$ denotes the expected value. In addition we have that $\varepsilon\{v(k)v(k)^T\} = 0$ and it is assumed that $w(k)$ and $v(k)$ are not correlated with $x(k)$ and $y(k)$. Now, the Kalman filter is defined as the observer gain L_x such that the covariance of the estimation error $(x(k) - \hat{x}(k))$ is minimized, i.e,

$$\min_{L} \quad \frac{1}{2}\varepsilon\left\{\sum_{k=0}^{\infty}(x(k) - \hat{x}(k))^T(x(k) - \hat{x}(k))\right\} \tag{4.8}$$

The solution of this optimization problem is given by the so-called Kalman Gain,

$$L = A\overline{Q}C^T\left(C\overline{Q}C^T + \mathbf{R}_v\right)^{-1} \tag{4.9}$$

where $\overline{Q}$ is the covariance matrix of the steady-state estimation error that satisfies the Algebraic Riccati Equation,

$$\overline{Q} - A\overline{Q}A^T + A\overline{Q}C^T\left(C\overline{Q}C^T + \mathbf{R}_v\right)^{-1}C\overline{Q}A^T - G\mathbf{R}_wG^T = 0 \tag{4.10}$$

Moving Horizon Estimators (MHE): Moving Horizon Estimation (MHE) strategies were born as a dual problem of the Model Predictive Control (MPC). The basic strategy of MHE reformulates the estimation problem as a quadratic problem using a moving, fixed-size estimation window. The fixed-size window is needed to bind the computational effort to solve the problem. This is the main difference of MHE with the batch estimation problem (or full information estimator) [124, 125, 126]. Once a new measurement is available, the oldest one is discarded, using the concept of window shifting. Moreover, the main advantage of MHE in comparison with other estimation schemes (like the Kalman Filter) is the straightforward constraint addressing inside the optimization problem, and the possibility to propose the cost function. However, as MHE is a limited memory filter, stability and convergence issues arise. A review on developments on MHE procedures was published by García and Espinosa in [127].

Assume a large-scale system modeled by means of the following nonlinear difference equation:

$$\begin{aligned} x(k+1) &= f(x(k)) + g(x(k), w(k)) \\ y(k) &= h(x(k)) + \nu(k) \end{aligned} \tag{4.11}$$

where some constraints are imposed over the state variables, disturbances, and measurement noise as follows:

$$x \in \mathbb{X}, \quad w \in \mathbb{W}, \quad \text{and } \nu \in \mathbb{V} \tag{4.12}$$

with $x(k)$ and $y(k)$ the state and output at k sample respectively, $w(k)$ is the disturbance or model uncertainty, and ν is the measurement noise. Also, $f : \mathbb{R}^n \to \mathbb{R}^n$, $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ with $g(\cdot, 0) = 0$, and $h : \mathbb{R}^n \to \mathbb{R}^p$. Finally it is assumed that $\mathbb{X}$ and $\mathbb{W}$ are closed with $0 \in \mathbb{W}$.

A linear large-scale constrained system generating the measurement sequence $\{y(k)\}$ can be derived from a linearization around each operating point of (4.11) as:

$$\begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k) + G(k)w(k) \\ y(k) &= C(k)x(k) + \nu(k) \end{aligned} \tag{4.13}$$

where for simplicity $x(k) \in \mathbb{R}^n$ and $w(k) \in \mathbb{R}^w$ are the linearized state and uncertainty respectively, $\nu(k) \in \mathbb{R}^p$ is the linearized measurement noise, and $u(k) \in \mathbb{R}^m$ denotes the system input. Moreover, those variables are constrained as it is shown in (4.12). Thus, the estimation of the whole state in (4.13) can be formulated as a MHE problem as follows:

$$\begin{aligned} &\min_{z, \{w(k)\}_{t-H_e}^{t-1}} \sum_{k=t-H_e}^{t-1} \nu(k)^T R \nu(k) + w(k)^T Q w(k) + \Xi_{t-H_e}(z) \\ &\text{s.t} \quad x \in \mathbb{X}, \quad w \in \mathbb{W}, \quad \text{and } \nu \in \mathbb{V} \end{aligned} \tag{4.14}$$

with H_e the moving horizon, z and $\{w(k)\}$ are the optimization variables: the initial condition of the state in the moving horizon, and the model disturbance sequence respectively. t is the current time, and $\Xi_{t-H_e}(z)$ is the *arrival cost* [128].

4.3.1.3 Selection of a set of references trajectories

As it was mentioned before the upper layer of the hierarchical architecture provides the time variant references trajectories for control u_{ref} and outputs y_{ref}. In the case of the trajectories for control variables manipulated by the lower level control, these should contain, preferably, all of the control variables defined for the upper level dynamic optimization problem. Depending on the needs of the process operation, reference trajectories for some of the control variables are implemented as feed-forward input without update at the tracking control level. This situation usually arises when a control variable cannot be continuously changed by a controller that runs at fast sampling time. However there are situation where all the control variables cannot to have a reference trajectories, for instance when the system is not square (the number of control variables is smaller than the number of output variables) and the tracking controller can not reject disturbances due to the restricted degrees of freedom of the system. Thereby, Gonzales et.al [129] propose a way to chose the number of input variables that can drives the selected inputs to their desired optimal values. The concept of providing input reference trajectories for tracking is similar to the calculation of constant targets of controls and outputs used in MPC [14].

4.3.2 Upper layer: Dynamic Real Time Optimization (DRTO)

A dynamic optimization problem is considered for the optimization and control levels of the automation hierarchy presented in chapter 2. The dynamic optimization problem is placed in top of the automation hierarchy and addresses the overall economical objective to provide the time variant references trajectories for control u_{ref} and outputs y_{ref}. These trajectories are tracked by a lower level tracking controller. The mathematical formulation of the dynamic optimization problem is stated as follows:

$$
\begin{aligned}
\min_{u_k} \; & \sum_{j=0}^{N_p} \left[\sum_{i_i=1}^{n_u} c_{u,i_i} u_{i_i}(k+j) + \sum_{i_o=1}^{n_y} c_{y,i_o} y_{i_o}(k+j) u_{i_o}(k+j) \right] \\
& + \sum_{j=0}^{N_p} \alpha(x(k+j), u(k+j)) \\
\text{subject to:} \; & \\
& x(k+j+1) = f(x(k+j), u(k+j), d(k+j)) \\
& y(k+j) = g(x(k+j), u(k+j), d(k+j)) \\
& x(k+j) \in \mathbb{X}, \quad j = 0, \ldots, N_p \\
& u(k+j) \in \mathbb{U}, \quad j = 0, \ldots, N_p
\end{aligned}
\tag{4.15}
$$

where the input variable $u_k = [u^T(k), \ldots, u^T(k+N_p)]^T$ is the decision variable for optimization and $d_k = [d^T(k), \ldots, d^T(k+N_p)]^T)$ are the disturbances of the problem, these can be classified in: changing model parameters, disturbances and and external market conditions. This problem is commonly referred to as an open - loop optimal control problem, as no feedback from the process is explicitly considered. In general there are three different methods to solve the dynamic optimization problem 4.15 which are classified as follows:

- Dynamic programming methods [130]
- Indirect methods [131]
- Direct programming methods [132, 133]

In the case of dynamic optimization for large scale systems, the most successful method used is the direct method [134]. Another important aspect to take into account in the dynamic optimization problem is to state the objective function.

4.3.3 Coordinator: Robust Model Predictive Controwith zone control

The coordinator of the hierarchical structure is a robust MPC controller with zone control that was explained in chapter 3. Mathematically, this is expressed as (4.16).

$$
\begin{aligned}
\min_{u_k, y_{ref,k}} \max_{\Theta_k \in \Psi^{N_p}} & \sum_{j=1}^{N_p} \|(y(k+j) - y_{ref,k}\|_Q^2 + \sum_{j=0}^{N_p} \|u(k+j) - u_{ref,opt}\|_R^2 \\
& + \sum_{j=0}^{N_p} \|\Delta u(k+j)\|_S^2 + \|x(k+N_p)\|_{\tilde{Q}}^2 \\
\text{subject to:} & \\
& x(k+j+1) = A(\theta)x(k+j) + B(\theta)u(k+j) \\
& y(k+j) = Cx(k+j) + Du(k+j) \\
& x(k+j) \in \mathbb{X}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
& y(k+j) \in \mathbb{Y}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
& u(k+j) \in \mathbb{U}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
& x(k+N_p) \in \Omega, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
& y_{ref,k} \in \mathbb{Y}_{ref}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
& \|u(k+j) - u(k+j-1)\| \le \Delta u_{\max} \\
& \theta(k+j) \in \Psi, \quad \|\theta(k+j)\| \le \rho \\
& \Theta_k \in \Psi^{N_p} \triangleq \Psi \times \cdots \times \Psi
\end{aligned}
\tag{4.16}
$$

Note that robust MPC with zone control formulation (optimization problem 4.16) can be seen as a robust MPC in the case when the upper an lower limits of the zone are equal, in this sense if the hierarchical robust integration of control and DRTO consider a robust MPC with zone control as coordinator, this can be seen as a general structure for hierarchical robust control strategies.

4.3.4 Lower layer: Decentralized control scheme

Now, let the whole system be decomposed into M subsystems where $x_i(k) \in \mathbb{R}^{n_{x_i}}$, $u_i(k) \in \mathbb{R}^{n_{u_i}}$, and $y_i(k) \in \mathbb{R}^{n_{y_i}}$ are the state, inputs, and outputs of i-th subsystem and that are subject to hard constraints on state $x_i(k) \in \mathbb{X}_i$, output $y_i(k) \in \mathbb{Y}_i$ and input $u_i(k) \in \mathbb{U}_i$ for all $k \geq 0$, where $\mathbb{X}_i \subset \mathbb{R}^{n_i}$, $\mathbb{Y} \subset \mathbb{R}^{z_i}$ and $\mathbb{U} \subset \mathbb{R}^{m_i}$ are closed sets. From this definition: $x(k) = [x_1^T(k), \ldots, x_M^T(k)]^T$, $u(k) = [u_1^T(k), \ldots, u_M^T(k)]^T$, and $y(k) = [y_1^T(k), \ldots, y_M^T(k)]^T$. Let A_{ii}, B_{ii}, C_i, D_i be blocks of matrices A, B, C, D such that

$$
\begin{aligned}
x_i(k+1) &= A_{ii}x_i(k) + B_{ii}u_i(k) \\
y_i(k) &= C_i x(k) + D_i u_i(k)
\end{aligned}
$$

Then, each MPC of the lower layer can be written as (4.17).

$$
\begin{aligned}
\min_{u_{k_i}} J_{N_{p,i}}(x_k, u_k) = &\sum_{j=1}^{N_{p,i}} \|(y_i(k+j) - y_{ref,i}\|_{Q_i}^2 + \sum_{j=0}^{N_{p,i}} \|u_i(k+j) - u_i^{ref}\|_{R_i}^2 \\
&+ \sum_{j=0}^{N_{p,i}} \|\Delta u_i(k+j)\|_{S_i}^2 + \|x_i(k+N_p)\|_{\tilde{Q}_i}^2 \\
\text{subject to:}& \\
&x_i(k+j+1) = A_{ii}x_i(k+j) + B_{ii}u_i(k+j) \\
&y_i(k+j) = C_i x(k+j) + D_i u_i(k+j) \\
&u_i(k+j) = u_i^{ref} + \Delta u_i(k+j) \\
&x_i(k+j) \in \mathbb{X}_i, \quad j = 0, \ldots, N_{p,i} \\
&u_i(k+j) \in \mathbb{U}_i, \quad j = 0, \ldots, N_{p,i} \\
&x_i(k+N_p) \in \Omega_i, \quad j = 0, \ldots, N_{p,i} \\
&\|u_i(k+j) - u_i(k+j-1)\| \leq \Delta u_i^{\max}
\end{aligned}
\tag{4.17}
$$

where $y_{\text{ref}}(k) = [y_{ref,1}^T(k), \ldots, y_{ref,M}^T(k)]^T$, $\Delta u_i(k) = u_i(k) - u_i^{ref}$ for the i-th controller, and Q_i, R_i, S_i positive definite diagonal matrices. From the optimization problem (4.17), given the references for the inputs and outputs, each local controller computes the control actions to be locally applied. It is worth to point out that the optimization problem (4.16) and (4.17) have different time resolutions, i.e., optimization problem (4.17) is solved more often than optimization problem (4.16). This time-scale differentiation is required because the use of whole system model makes the upper-layer optimization problem more complex than each local optimization problem of the lower layer.

4.4 Hierarchical Robust Integration of MPC with zone control and DRTO for two reactors chain and flash system

Figures 4.8, 4.9 and 4.10 show the performance of the closed-loop system under the Hierarchical Robust Integration of MPC with zone control and DRTO scheme. In order to compare the robust with the nominal scheme, again the heat capacity C_p of the first and second reactor was increased a 5%. In Figure 4.8 it is possible to observe how all variables of the CSTR-1 remain in the specific zone under the robust scheme, while the nominal scheme (see Figure 4.3) cannot drive the temperature T_r to the zone and maintain the level H_r in a fixed set-point. In the nonadiabatic flash we want to maintain the concentration $x_{B_b} = 0.3$ (quality specifications), note that in Figure 4.10 the robust scheme can achieve this set-point, while the nominal scheme drives the concentration near of the specification, however a off-set is present (see Figure 4.5). An important feature in the behavior of the flash with the robust scheme is that this steers the inputs to the set-points, this fact leads to achieve the maximum profit for the nonadiabatic flash subsystem.

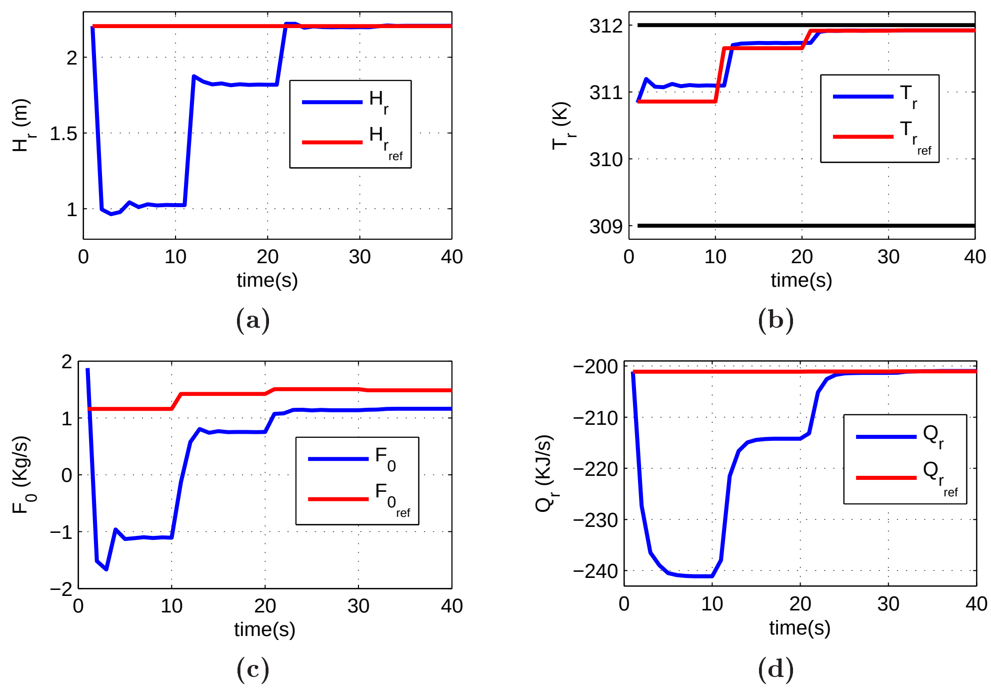


Figure 4.8: Robust scheme: Evolution of the level and the temperature of CSTR-1, reference values, and control inputs.

4.5 Summary

In this chapter a Hierarchical Robust Integration of MPC and DRTO was proposed. The main idea with this chapter was to combine the robust MPC approaches presented in chapter 3 and the zone control concept with the classical hierarchical structure presented in chapter 2. The main advantages of the strategy presented here are the ability of handling

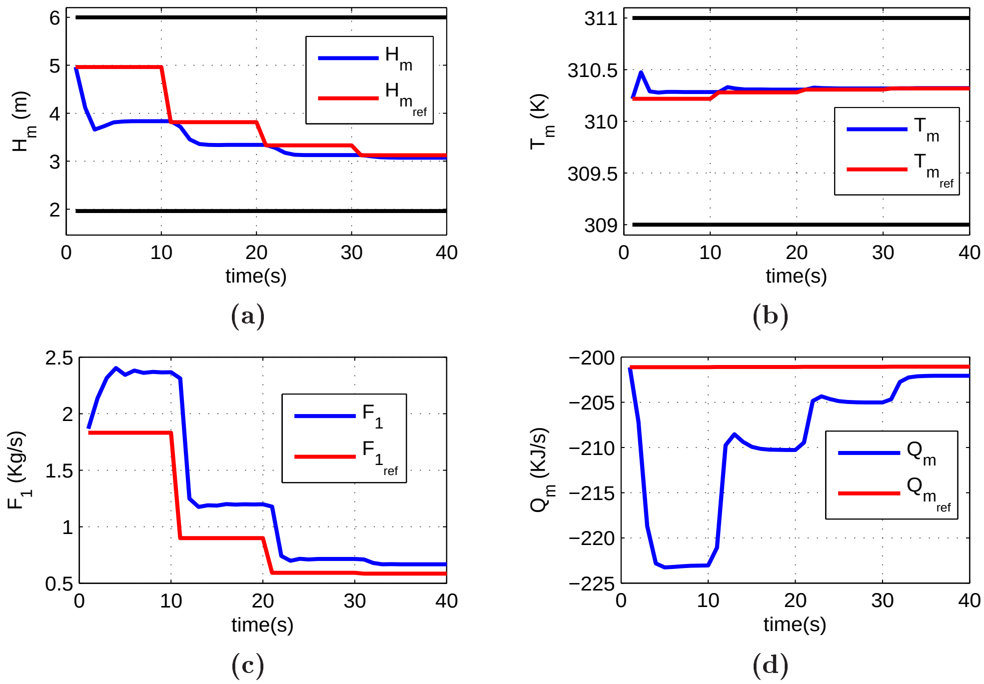


Figure 4.9: Robust scheme: Evolution of the level and the temperature of CSTR-2, reference values, and control inputs.

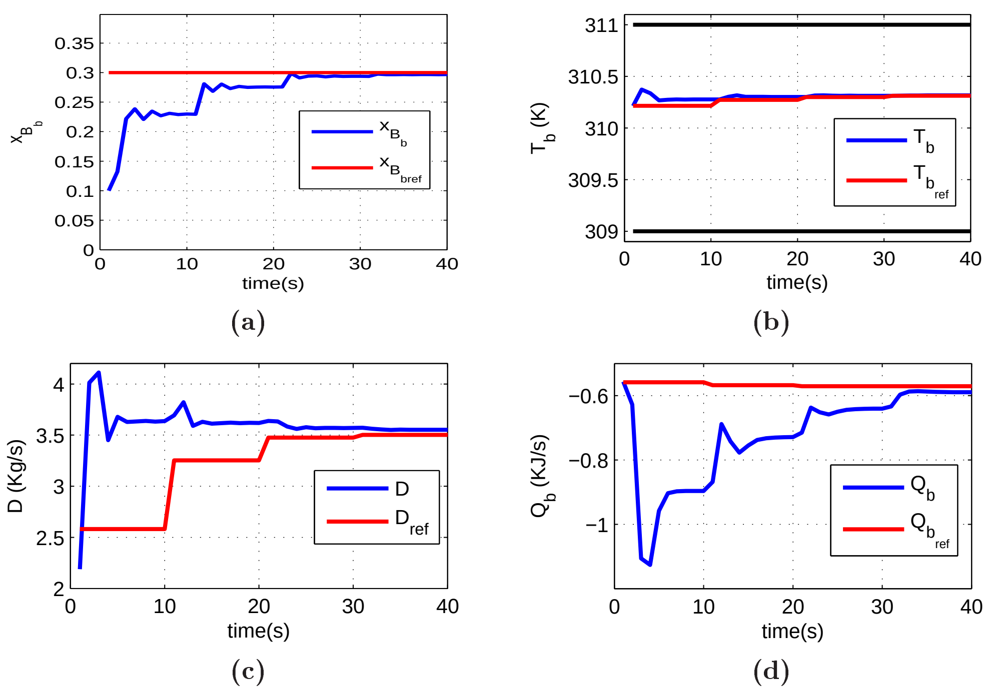


Figure 4.10: Robust scheme: Evolution of the level and the temperature of the nonadiabatic flash, reference values, and control inputs.

uncertainties, the tackling of different objectives (Economic, environmental), and the inclusion of the dynamic behavior of the system. In this way these approaches improve the structures reported in the literature.

An example (the two reactors chain and a flash system) was used in this chapter to show the advantages of the strategy proposed. This example is used in distributed MPC works such as Venkat et.al [114] and Valencia et.al [135]. There are two important contributions for this example. The first one is the operating point of the system, in the works previously mentioned the operating point was not realistic (CSTR levels in the order of kilometers). In this way we found by means of the DRTO layer realistic operating points. The second contribution was described in the simulations result, were clearly the nominal scheme was not capable to maintain the outputs of the system under the zone while the quality specifications were achieved.

In the following chapter a Hierarchical Robust Control for coordination is presented. This scheme can be seen as an special case of the hierarchical structure proposed in this chapter. However the literature and several applications lead to formulate the coordinate strategy as an specific scheme. In this sense, the following chapter use a Hydro Power Valley (HPV) as example in order to justify the need for the technique. This system is a typical large scale benchmark that is used to prove control behaviour.

CHAPTER 5

Hierarchical Robust Control for coordination

5.1 Introduction

In a very competitive economic world, production systems must find the manner to improve efficiency and productivity, as they are becoming bigger and complex. Large scale systems with networked interaction and complex behaviour are common in different engineering areas. They are characterized by the complex dynamic behaviour, which make control a very hard task. The way to avoid complexity is taking more manageable subsystems, but new challenges keeps on appearing. Currently, it is well known that in order to obtain a global optimal operation of a system, it is necessary to measure, to estimate and to act based on global information. Hierarchical control is the way to face the increasing complexity, while the large scale system is divided into various subsystems, and controlled with local controllers, a coordinator controller manages the interactions between the subsystems and local controllers.

Hierarchical Control for coordination is used in several problems in large scale systems such as Hydro Power Valleys and traffic systems. For this reason, in this thesis this type of strategy is studied considering uncertainties in the system. In this chapter two different strategies for coordination will be proposed. The first one uses robust MPC with zone control as coordinator with a decentralized scheme in the lower layer, while the second one take decisions based on economic criterion i.e the coordinator solves an economical optimization problem (Economic MPC) whereas that in the lower layer a decentralized MPC with zone control scheme is used.

In this chapter, the theoretical results are illustrated by using one widely known systems in the control field: The Hydro Power valley (HPV). Basically, the selection of this cases of study was motivated by the widely use of these systems (as large scale systems) in the test of Distribute MPC schemes, and because these systems are sufficiently illustrative to present the concepts introduced in this chapter, and interesting from the point of view of the control theory due to the coupling among their dynamics and their nonlinearities.

This chapter is organized as follows: In section 5.2 the motivation to chose the HPV benchmark as an application of the methodology proposed is stated, then, in section 5.2.1 the dynamic model of the HPV is explained, consequently, in section 5.2.2 the main challenges of this system are described. In section 5.2.3 the nominal control structure is explained, to finish with the simulations results of the nominal hierarchical structure over the HPV in section 5.2.4. Based on the results of the nominal scheme, in the section 5.3 a hierachical robust strategy for coordination is presented. Finally, in section 5.4 a Hierarchical Robust Control for coordination of a HPV is presented.

5.2 Motivation: Hierarchical Control with zone control for coordination of a Hydro Power Valley

Large scale systems problems are reported in literature [136] [137] [138], Hydro Power Valley (HPV) is a typical large scale benchmark that is used to prove control behavior [19], because it has several characteristics which establish some control challenges. In addition special control scenario is set to define a more difficult task. Nominal hierarchical control would have problems with the difficult control scenario, because it has more output variables than input variables and some output variables must follow a changing periodic reference, and not a fixed set point.

Nominal hierarchical control for coordination approach using MPC will have problems with the large scale system HPV benchmark, in order to face the challenges the HPV has. In this way, the nominal formulations should be modified in order to tackle this problems. For this reason, in this section, some mathematical formulations that helps the nominal hierarchical control to have more information about the HPV are presented.

5.2.1 Dynamic model of the HPV

This mathematical formulation was taken from [136]. Consider the HPV shown in Figure 5.1. This HPV is composed by three lakes ($L_m,\ m = 1, 2, 3$) where the water is stored, a duct (U_1) that connects two lakes, a river with six dams ($D_j,\ j = 1, \ldots, 6$), two turbines ($T_p,\ p = 1, 2$), and two turbine-pump devices (C_p). The stored water flows across ducts from one reservoir to another, or from the reservoir to power houses where the potential energy of the water is transformed into mechanical energy.

The river has a constant inflow q_{in} and a constant tributary flow $q_{tributary}$. Moreover, each dam D_j is equipped with a turbine for electric power generation. They are located in the river and divided it into six reaches (R_j), where reaches $R_1,\ R_2$ and $R_4,\ R_5$ are connected with lakes $L_1,\ L_3$ through turbines $T_1,\ T_2$ and turbine-pumps $C_1,\ C_2$ respectively. Also, lakes $L_1,\ L_2$ are connected to each other by the duct U_1. This duct is used only for transporting the water from one lake to another depending on the difference of the levels. Let us made the following assumptions about the lakes, reaches, and the river:

- The ducts are connected at the bottom of the lakes (or at the bottom of the river bed).
- The cross sections of the reaches and of the lakes are rectangular.
- The width of the reaches varies linearly along them.

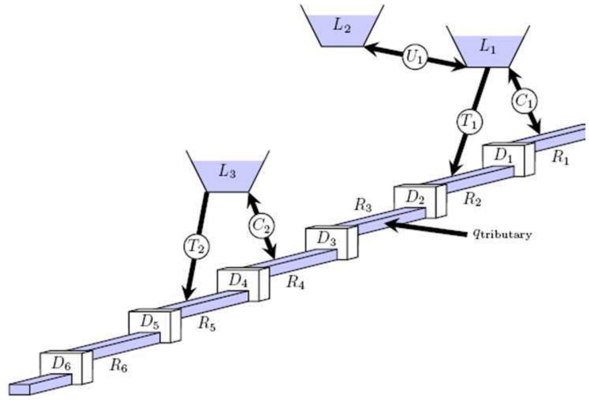


Figure 5.1: Hydro-Power Valley used as a case of study.

- The river bed slope is constant along every reach.

Based on these assumptions, the nonlinear, first-order Saint-Venant partial differential equations represent the state of the art for modeling one-dimensional river hydraulics with constant fluid density [138]. In these equations, the hydraulic state of the river is described by two variables: the water depth $h(t, z)$ and the discharge $q(t, z)$, both varying as a function of space z and time t. Thus, the dynamics of each reach is given by [136, 138, 137, 139]

$$\begin{aligned} 0 &= \frac{\partial q}{\partial z} + \frac{\partial s}{\partial t} \\ 0 &= \frac{1}{g}\frac{\partial}{\partial t}\left(\frac{q}{s}\right) + \frac{1}{2g}\frac{\partial}{\partial z}\left(\frac{q}{s}\right)^2 + \frac{\partial h}{\partial z} + I_f - I_o \end{aligned} \tag{5.1}$$

where $q = q(t, z)$, $s = s(t, z)$, $h = h(t, z)$, $I_f = I_f(t, z)$, $I_o = I_o(t, z)$, $s(t, z)$ is the wet surface, $I_f(t, z)$ is the friction slope, $I_o(t, z)$ is the river bed slope, and g is the gravitational acceleration. Since the cross sections of the reaches and lakes are assumed rectangular, the wet surface and the friction slope are given by (5.2) and (5.3) respectively

$$s(t, z) = w(z)h(t, z) \tag{5.2}$$

$$I_f(t, z) = \frac{q^2(t, z)(w(z) + 2h(t, z))^{\frac{4}{3}}}{k_{str}^2 (w(z)h(t, z))^{\frac{10}{3}}} \tag{5.3}$$

where $w(z)$ is the river width, and k_{str} is the Gauckler-Manning-Strickler coefficient. For modeling the lakes, duct, turbines, and turbine-pumps elements, (5.4)-(5.7) were used [19]:

$$\frac{\partial h(t)}{\partial t} = \frac{q_{in}(t) - q_{out}(t)}{S} \tag{5.4}$$

$$q_{U1}(t) = S_{U1}\text{sign}(H(t))\sqrt{2g|H(t)|} \tag{5.5}$$

$$p_t(t) = k_t q_t(t)\Delta h_t(t) \tag{5.6}$$

$$p_C(t) = k_C(q_C(t))q_C(t)\Delta h_C(t) \tag{5.7}$$

where sign$(\cdot)$ is the sign function, S is the surface area of the lake, S_{U1} is the section of the duct, k_t is the turbine coefficient, $q_{in}(t)$, $q_{out}(t)$, are the input and output flows of the lakes

	Inputs	Outputs
1	$u_1 = [q_{T1}, q_{C1t}, q_{C1p}]^T$	$y_1 = [P_1, h_{L1}, h_{L2}]^T$
2	$u_2 = [q_{T2}, q_{C2t}, q_{C2p}]^T$	$y_2 = [P_2, h_{L3}]^T$
3	$u_3 = q_{D1}$	$y_3 = [P_3, h_{R1}]^T$
4	$u_4 = q_{D2}$	$y_4 = [P_4, h_{R2}]^T$
5	$u_5 = q_{D3}$	$y_5 = [P_5, h_{R3}]^T$
6	$u_6 = q_{D4}$	$y_6 = [P_6, h_{R4}]^T$
7	$u_7 = q_{D5}$	$y_7 = [P_7, h_{R5}]^T$
8	$u_8 = q_{D6}$	$y_8 = [P_8, h_{R6}]^T$

Table 5.1: Input and output variables for each subsystem

respectively, $q_t(t)$ is the turbine discharge, $\Delta h_t(t)$, $\Delta h_C(t)$ are the heads of the turbine and the turbine-pump respectively, $q_{U1}(t)$ is the flow across the duct U_1, $p_t(t)$, $p_C(t)$ are the power generated by the turbines and the power generated or consumed by the turbine-pump elements respectively,

$$k_C(q_C(t)) = \begin{cases} k_{tC} \text{ if } q_C(t) \geq 0 \\ k_{pC} \text{ if } q_C(t) < 0 \end{cases}$$

is the turbine-pump coefficient, k_{tC}, k_{pC} are the gains of the turbine-pump devices in turbine or pump mode respectively, q_C is the flow in the turbine-pump elements, and $H(t) = h_{L2}(t) - h_{L1}(t) + h_{U1}$, with $h_{L1}(t)$, $h_{L2}(t)$ the levels of the lakes 1 and 2 respectively, and h_{U1} the height difference of the duct.

5.2.2 HPV challenges

Equations (5.1) to (5.7) defines HPV model, however, this model is not suitable for control purposes due to its complexity. So, a discretized version of (5.1)-(5.7) is used. Following the notation of [136], let $h_{\mathrm{L}m}(t)$ denote the level of the m-th lake. Let $q_{\mathrm{T}p}$, $q_{\mathrm{C}p}$ denote the inflow of the p-th turbine and p-th turbine-pump device respectively. For the reach R_j, let $Q_{\mathrm{R}j} = [q_{1j}(t), \ldots, q_{N_\mathrm{x}j}(t)]$ and $h_{\mathrm{R}j} = [h_{1j}(t), \ldots, h_{(N_\mathrm{x}+1)j}(t)]$ denote the vector of outflows and the vector of levels at each spatial partition, N_x being the number of spatial partitions of the reach. Then, the states vector $x(t)$ of the HPV can be defined as

$$x(t) = [h_{\mathrm{L}m}^T(t), Q_{\mathrm{R}j}^T(t), h_{\mathrm{R}j}^T(t)]^T$$

for $j = 1, 2, \ldots, 6$, $m = 1, 2, 3$. Also, let $q_{\mathrm{R}j}$ denote the outflow of the j-th turbine at the corresponding dam. From [136, 19] the HPV of Figure 5.1 can be decomposed in 8 subsystems as follows:

- Subsystem 1: lakes L_1 and L_2, turbine T_1, and turbine-pump C_1.
- Subsystem 2: lake L_3, turbine T_2, and turbine-pump C_2.
- Subsystems 3-8: reaches R_1 to R_6 coupled with dams D_1 to D_6 respectively.

In the Table 5.1 the inputs and outputs of each subsystem are defined.

Although discretization of (5.1)-(5.7) provides a suitable model for designing an MPC for the HPV, the resulting model is a non-linear model. Thus, it should be linearized in order to obtain an appropriate model for applying the hierarchical control scheme described in section 5.3. Linearizing the discrete expressions for (5.1)-(5.7) matrices A, B, C, D of the whole systems are obtained (discrete versions of (5.1)-(5.7) are in [19]). Once obtained the linear model of the HPV, some issues arise:

- Reference profile: Market requirements dictate that the HPV must follow a reference profile of power that can change each 1800 s, in this work is used a profile that is repeated for two days of operation, this profile is shown in Figure 5.10.
- Following the procedure in [19], the dimension of the whole system makes the resulting optimization complex enough to require a system decomposition and/or a model reduction for computing the zone-control of the upper layer in the proposed hierarchical zone-control scheme.
- The linear models of each subsystem of the HPV has integrating modes. Thus MPC does not guarantee system stability.

For dealing with the dimension of the linear model, a model reduction method was used for reducing the dimension of the HPV model. The reduced order model was found by balanced truncation using the Hankel norm [140]. For stabilizing the integrating modes of the linear model several approaches have been reported in the literature (see e.g., [141, 142]). Almost all of them require finding a factorization to separate the integrating modes of the HPV. Since this is not possible in all cases a state feedback control law was used. This control law has the form:

$$u_i(k) = -K_i x_i(k) + v_i(k) \tag{5.8}$$

where $v_i(k)$ is the new control action of the stabilized subsystem and $K_i \in \mathbb{R}^{n_{u_i} \times n_{x_i}}$. A necessary condition for the existence of the gain K_i is that the pair (A_i, B_i) is controllable [143]. If this condition is satisfied, the gain K_i can be computed as the solution of the equation $K_i = R_i^{-1} B_i^T P_i(k)$, where P_i matrix is given by the solution of the Ricatti equation [144]. Adding the proposed state-feedback control law for each subsystem, the linear model of the i subsystems of the HPV becomes

$$\begin{aligned} x_i(k+1) &= A_{Ki} x(k) + B_{ii} v(k) \\ y_i(k) &= C_{Ki} x(k) + D_i v(k) \end{aligned}$$

where $A_{Ki} = (A_{ii} - B_{ii} K_i)$ and $C_{Ki} = (C_i - D_i K_i)$. Thus, the model of the subsystems of the HPV are stable and the MPC can be used with guaranteed stability considering $v_i(k)$ as optimization variable.

5.2.3 Control structure

Figure 5.2 shows the control structure used in the HPV. In the next subsections every layer will be described.

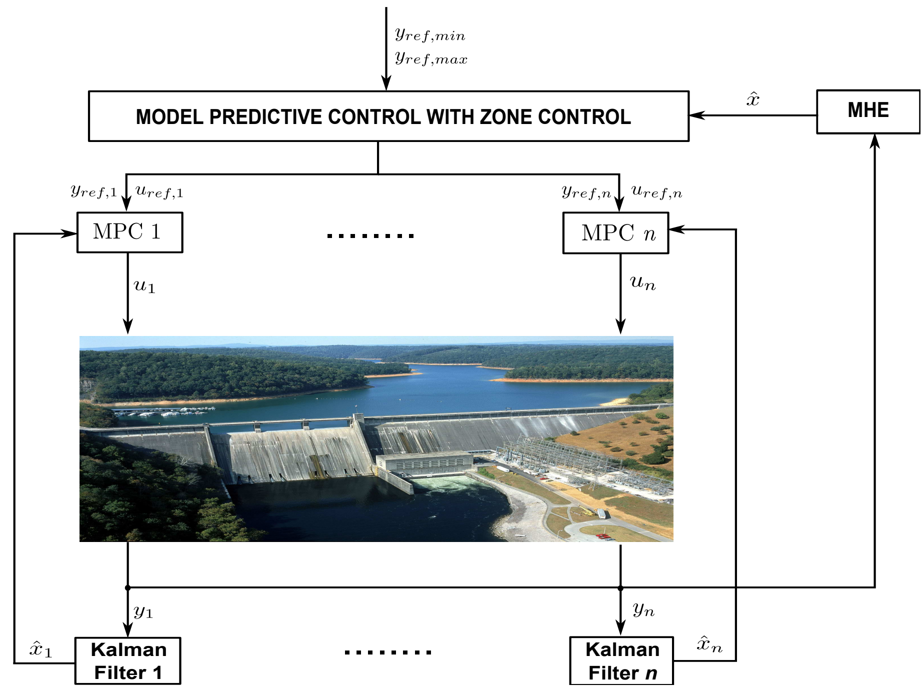


Figure 5.2: Hierarchical Robust Control with zone control for coordination for the HPV

5.2.3.1 Upper layer formulation

The upper layer of the proposed control scheme computes the reference value for the inputs and outputs of each lower-layer local controller. So, the sum of the contributions of the subsystems must be equal to the power reference established for the HPV. As it was mentioned the HPV must follow a certain reference that change every sample time, for this reason it is necessary to add the next constraint to the upper layer optimization problem:

$$C_p y = \sum_{l=1}^{8} P_l = P_{ref} \tag{5.9}$$

where C_p is a selection matrix and P_i is the power generated by the i subsystem. Therefore, the power and input references for each subsystem are given by the solution of the optimization problem (5.17).

$$\min_{u_k, y_{ref,k}} \sum_{j=1}^{N_p} \|(y(k+j) - y_{ref,k}\|_Q^2 + \sum_{j=0}^{N_p} \|u(k+j)\|_R^2 + \sum_{j=0}^{N_p} \|\Delta u(k+j)\|_S^2 + \|x(k+N_p)\|_{\tilde{Q}}^2$$

subject to:

$$\begin{aligned}
&x(k+j+1) = Ax(k+j) + Bu(k+j) \\
&y(k+j) = Cx(k+j) + Du(k+j) \\
&C_p y(k+j) = P_{ref}, \quad j = 0, \ldots, N_p \\
&x(k+j) \in \mathbb{X}, \quad j = 0, \ldots, N_p \\
&y(k+j) \in \mathbb{Y}, \quad j = 0, \ldots, N_p \\
&u(k+j) \in \mathbb{U}, \quad j = 0, \ldots, N_p \\
&x(k+N_p) \in \Omega \\
&y_{ref}^{min} \leq y_{ref,k} \leq y_{ref}^{max} \\
&\|u(k+j) - u(k+j-1)\| \leq \Delta u_{\max}, \quad j = 0, \ldots, N_p
\end{aligned} \tag{5.10}$$

where N_p is the prediction horizon, $Q \in \mathbb{R}^{n_y \times n_y}$, $R \in \mathbb{R}^{n_u \times n_u}$ and $S \in \mathbb{R}^{n_y \times n_y}$ are positive definite weighting matrices. In (5.17) all computations are done using a reduced order model. It is important to note that in the problem (5.17) the feedback control law of the each local controller is not considered. This fact leads to increase the uncertainty level in the problem, specially in the matrix A and B.

5.2.3.2 Lower layer formulation

The proposed control scheme uses MPC as local control laws. These controllers are in charge of computing the changes of the local control actions taking as a base-line the reference value provided by the coordinator. With this aim local models are used for predicting the local state trajectories. However, local models are unstable (HPV have integrating modes). As it was mentioned an state-feedback control law is used for stabilizing the local unstable modes. Thus, each local control scheme can be formulated as (5.11).

$$
\begin{aligned}
\min_{v_{k_i}} J_{N_{p,i}}(x_k, u_k) = & \sum_{j=1}^{N_{p,i}} \|(y_i(k+j) - y_{ref,i}\|^2_{Q_i} + \sum_{j=0}^{N_{p,i}} \|v_i(k+j) - v_i^{ref}\|^2_{R_i} \\
& + \sum_{j=0}^{N_{p,i}} \|\Delta v_i(k+j)\|^2_{S_i} + \|x_i(k+N_p)\|^2_{\tilde{Q}_i} \\
\text{subject to:} & \\
& x_i(k+j+1) = A_{Ki} x_i(k+j) + B_{ii} v_i(k+j) \\
& y_i(k+j) = C_{Ki} x(k+j) + D_i v_i(k+j) \\
& v_i(k+j) = v_i^{ref} + \Delta v_i(k+j) \\
& x_i(k+j) \in \mathbb{X}_i, \quad j = 0, \ldots, N_{p,i} \\
& v_i(k+j) \in \mathbb{V}_i, \quad j = 0, \ldots, N_{p,i} \\
& x_i(k+N_p) \in \Omega_i, \quad j = 0, \ldots, N_{p,i} \\
& \|v_i(k+j) - v_i(k+j-1)\| \leq \Delta v_i^{\max}
\end{aligned}
\tag{5.11}
$$

where N_{p_i} is the control horizon, $Q_i \in \mathbb{R}^{n_{y_i} \times n_{y_i}}$, $R_i \in \mathbb{R}^{n_{u_i} \times n_{u_i}}$ and $S_i \in \mathbb{R}^{n_{y_i} \times n_{y_i}}$ are positive definite weighting matrices.

In order to implement the proposed hierarchical control scheme state estimators are used to determine the unmeasured states. In the case of the HPV these state consists of the inputs flows of each segment used for the spatial discretization. For this reason a Moving Horizon Estimator (section 4.3.1.2) is used as trigger for the robust MPC with zone control, while Kalman filters are used for the local controllers.

5.2.4 Simulation Results

The control system designed for the HPV must follow a power reference, while the levels depicted earlier must be maintained into a certain zone. The power have a restrictive time dependent reference, while the levels and individual powers can move through a zone designed for the correct development of the system. In this way, for the HPV a control strategies was designed.

The nominal hierarchical control structure can manage the power reference problem in the HPV as can see in Figure 5.3. The Figure 5.4 shows the individual powers of the system. With the extra constrain the controllers can follow the total plant power reference for one day. However, the nominal scheme is able to follow the power reference for only one day (Figure 5.3), for the second day the control system cannot maintain the reference profile. The main reason for that is that the nominal scheme cannot maintain the levels under the specific zone because the nominal scheme cannot handle the uncertainty of the problem, specifically in the feedback law of the local controllers that are not included in the coordinator, i.e the state feedback of the local controllers can be seen as uncertainties in the model that use the coordinator. The Figure 5.5 shows as in the final of the day the levels are out of the zone (a big problem in order to follow the same reference for two days of operation). In Figure 5.6 the input variables are shown. Finally, Figure 5.7 shows the behavior of the MHE for some states. In this Figure it is possible to note that the MHE has a good performance for the HPV benchmark. In general, from this example it is possible conclude that the nominal hierarchical controller is not able to control the HPV, the levels of the HPV can be out of the zone leading to instability for the second day of operation.

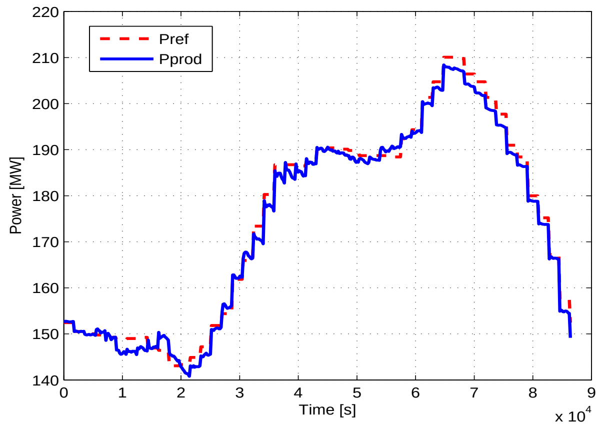


Figure 5.3: Nominal Scheme: Comparison between the power produced by the HPV with the power reference

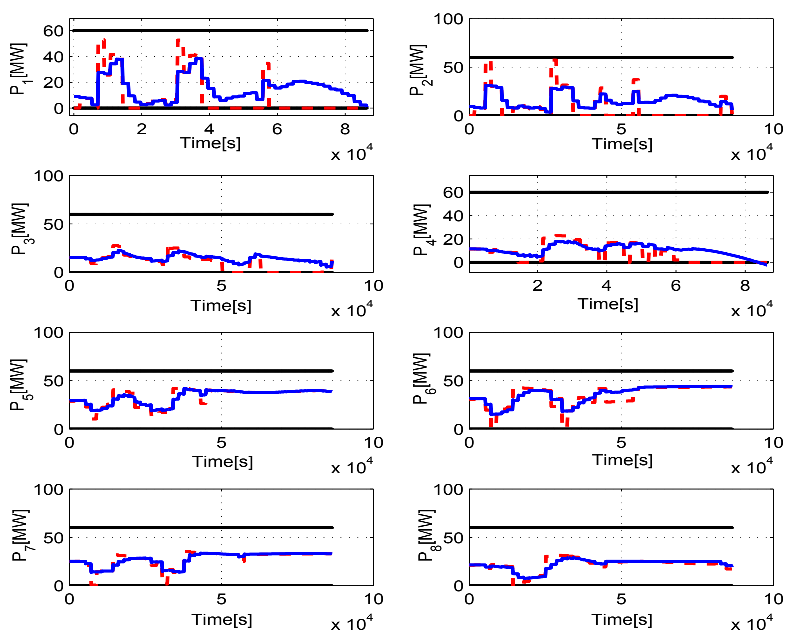


Figure 5.4: Nominal Scheme: Behavior of the individual powers of the HPV.

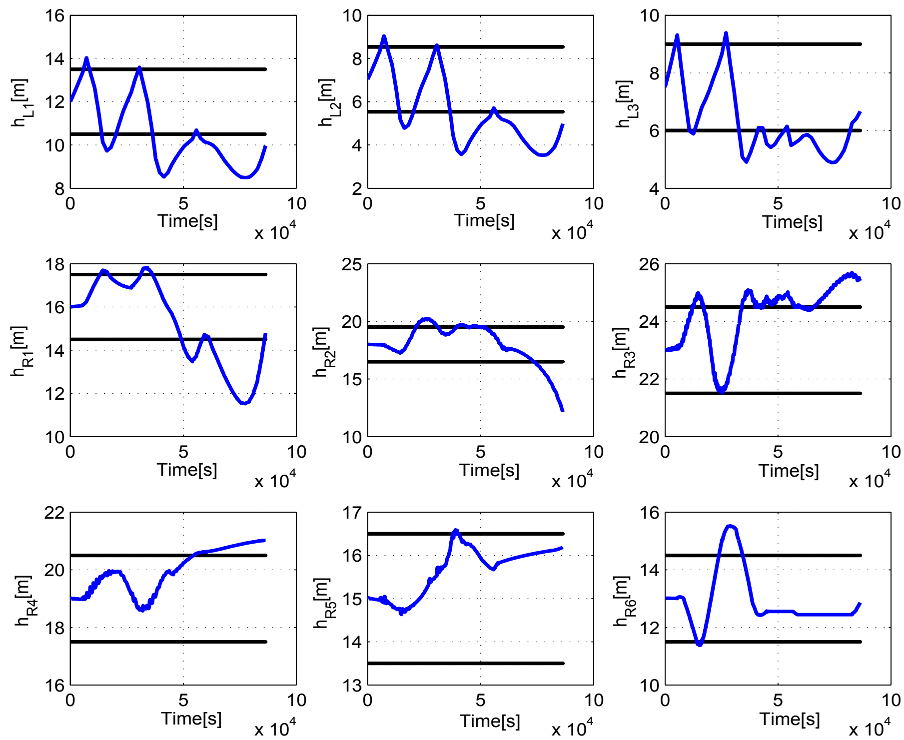


Figure 5.5: Nominal Scheme: Behavior of the levels in the lakes and the levels at the damps of the HPV.

5.3 Hierarchical Robust Control for coordination

As it was mentioned in the introduction section, the Hierarchical Control for coordination is used in many problems in large scale systems such as Hydro Power Valleys and traffic systems. For this reason, in this thesis this type of strategy is studied considering uncertainties in the system. In this section two different strategies for coordination will be proposed. The first one uses robust MPC with zone control as coordinator with a decentralized scheme in the lower layer, while the second one take decisions based on economic criterion i.e the coordinator solves an economical optimization problem (Economic MPC) whereas that in the lower layer a decentralized MPC with zone control scheme is used. It is important to mention that some aspects of the robust scheme proposed will not be detailed because it was described in the chapter 4.

5.3.1 Hierarchical Robust Control with zone control for coordination

In this section a hierarchical robust control with zone control for coordination is described. Such control scheme is illustrated in Figure 5.8. In this Figure the lower layer consists on a decentralized control scheme, while upper layer has a centralized robust controller based in a reduced order model. Since the lower layer is composed by several controllers without

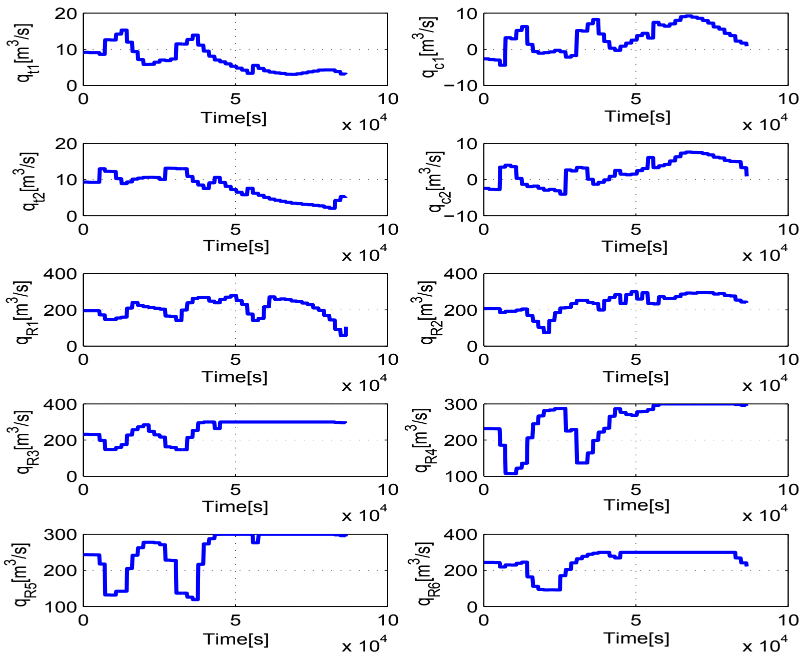


Figure 5.6: Nominal Scheme: Control actions

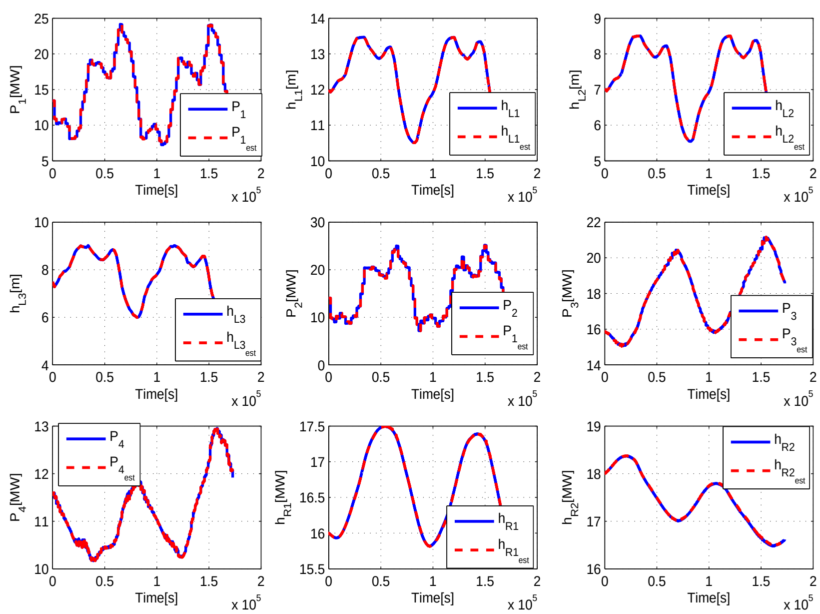


Figure 5.7: Behaviour of the MHE for some states

communication among them, the coordination layer must compute the contribution of each local controller in order to avoid conflicts. Recall that in decentralized control schemes

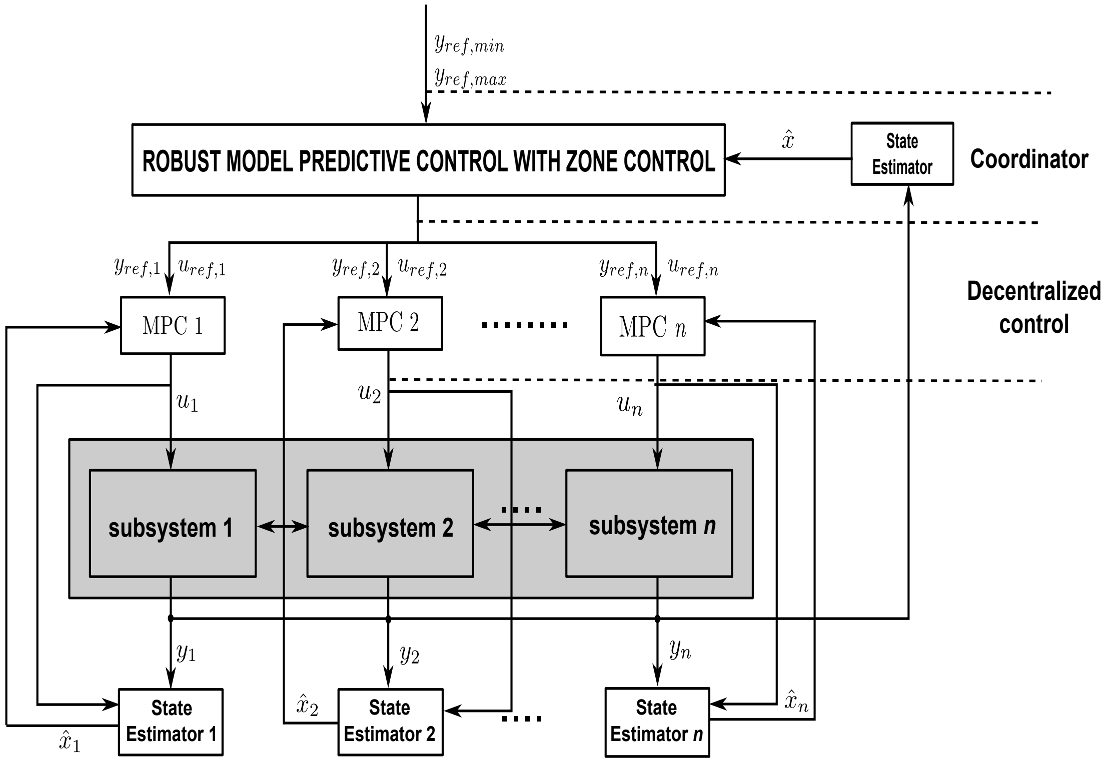


Figure 5.8: Hierarchical Robust Control with zone control for coordination

there is a huge loss of performance due to competition among local controllers. So, to avoid such competition, in the hierarchical robust control scheme of Figure 5.8, the upper layer computes robust reference values for both inputs u_{ref} and outputs y_{ref} for the whole system. In the lower layer, only local models are used to compute the control actions to be applied to the system. In both layers model predictive control (MPC) strategies are used. But, in upper layer robust zone control policy is used instead of reference tracking. In the following subsections every layer of the hierarchical structure is detailed.

5.3.1.1 Upper layer: Robust MPC with zone control

Let $x(k) \in \mathbb{R}^{n_x}$, $y(k) \in \mathbb{R}^{n_y}$, and $u(k) \in \mathbb{R}^{n_u}$ denote the system states, inputs and outputs respectively. At time step k, let $x_k = [x^T(k), \ldots, x^T(k+N_p)]^T$ and $u_k = [u^T(k), \ldots, u^T(k+N_p)]^T$ be the state trajectory and the control sequences, with N_p the prediction horizon. Let $J_{N_p}(y_{ref,k}, x_k, u_k)$ denotes the cost function, $y_{ref,k}$ being the sequence of reference values for the controlled variables. Assume the function $f(x(k), u(k))$ describing the state trajectories and $g(x(k), u(k))$ describing the system outputs are continuous, where $x(k) \in \mathbb{X}$, $y(k) \in \mathbb{Y}$, and $u(k) \in \mathbb{U}$ define the feasible values for the states, inputs and outputs of the system, with $\mathbb{X} \subset \mathbb{R}^{n_x}$, $\mathbb{Y} \subset \mathbb{R}^{n_y}$ and $\mathbb{U} \subset \mathbb{R}^{n_u}$ closed sets. Let $y_{ref,k} \in \mathbb{Y}_{ref}$ with $\mathbb{Y}_{ref} \subset \mathbb{R}^{n_y}$ also closed. Let $\Theta_k = [\theta^T(k), \ldots, \theta^T(k+N_p)]^T$ denote the sequences of uncertainties over N_p, with $\|\theta(k)\| \leq \rho$. Mathematically, min-max MPC with with zone control is formulated as (5.12).

$$\min_{u_k, y_{ref,k}} \max_{\Theta_k \in \Psi^{N_p}} \sum_{j=1}^{N_p} \|(y(k+j) - y_{ref,k}\|_Q^2 + \sum_{j=0}^{N_p} \|u(k+j)\|_R^2 + \sum_{j=0}^{N_p} \|\Delta u(k+j)\|_S^2 + \|x(k+N_p)\|_{\tilde{Q}}^2$$

subject to:

$$\begin{aligned}
&x(k+j+1) = A(\theta)x(k+j) + B(\theta)u(k+j)\\
&y(k+j) = Cx(k+j) + Du(k+j)\\
&x(k+j) \in \mathbb{X}, \quad \forall\theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p\\
&y(k+j) \in \mathbb{Y}, \quad \forall\theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p\\
&u(k+j) \in \mathbb{U}, \quad \forall\theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p\\
&x(k+N_p) \in \Omega, \quad \forall\theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p\\
&y_{ref,k} \in \mathbb{Y}_{ref}, \quad \forall\theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p\\
&\|u(k+j) - u(k+j-1)\| \leq \Delta u_{\max}\\
&\theta(k+j) \in \Psi, \quad \|\theta(k+j)\| \leq \rho\\
&\Theta_k \in \Psi^{N_p} \triangleq \Psi \times \cdots \times \Psi
\end{aligned} \tag{5.12}$$

where Ψ^{N_p} is determined by the assumed uncertainty model. Due to the min-max MPC reshaped into robust nonlinear programming (RNLP), and since the current computation resources do not provide enough computational power for having an exact solution of RNLP problems in real time, linear models are used for computing equivalent solutions for (5.12). An alternative for computing these equivalent solutions consists on transforming the original optimization problem into a robust quadratic programming problem, which has efficient methods for computing its solution.

5.3.1.2 Lower layer: Decentralized control scheme

Now, let the whole system be decomposed into M subsystems where $x_i(k) \in \mathbb{R}^{n_{x_i}}$, $u_i(k) \in \mathbb{R}^{n_{u_i}}$, and $y_i(k) \in \mathbb{R}^{n_{y_i}}$ are the state, inputs, and outputs of i-th subsystem and that are subject to hard constraints on state $x_i(k) \in \mathbb{X}_i$, output $y_i(k) \in \mathbb{Y}_i$ and input $u_i(k) \in \mathbb{U}_i$ for all $k \geq 0$, where $\mathbb{X}_i \subset \mathbb{R}^{n_i}$, $\mathbb{Y} \subset \mathbb{R}^{z_i}$ and $\mathbb{U} \subset \mathbb{R}^{m_i}$ are closed sets. From this definition: $x(k) = [x_1^T(k), \ldots, x_M^T(k)]^T$, $u(k) = [u_1^T(k), \ldots, u_M^T(k)]^T$, and $y(k) = [y_1^T(k), \ldots, y_M^T(k)]^T$. Let A_{ii}, B_{ii}, C_i, D_i be blocks of matrices A, B, C, D such that

$$\begin{aligned}
x_i(k+1) &= A_{ii}x_i(k) + B_{ii}u_i(k)\\
y_i(k) &= C_i x(k) + D_i u_i(k)
\end{aligned}$$

Then, each MPC of the lower layer can be written as (5.13).

$$
\begin{aligned}
\min_{u_{k_i}} J_{N_{p,i}}(x_k, u_k) = & \sum_{j=1}^{N_{p,i}} \|(y_i(k+j) - y_{ref,i}\|^2_{Q_i} + \sum_{j=0}^{N_{p,i}} \|u_i(k+j) - u_i^{ref}\|^2_{R_i} \\
& + \sum_{j=0}^{N_{p,i}} \|\Delta u_i(k+j)\|^2_{S_i} + \|x_i(k+N_p)\|^2_{\tilde{Q}_i}
\end{aligned}
$$

subject to:

$$
\begin{aligned}
& x_i(k+j+1) = A_{ii}x_i(k+j) + B_{ii}u_i(k+j) \\
& y_i(k+j) = C_i x(k+j) + D_i u_i(k+j) \\
& u_i(k+j) = u_i^{ref} + \Delta u_i(k+j) \\
& x_i(k+j) \in \mathbb{X}_i, \quad j = 0, \ldots, N_{p,i} \\
& u_i(k+j) \in \mathbb{U}_i, \quad j = 0, \ldots, N_{p,i} \\
& x_i(k+N_p) \in \Omega_i, \quad j = 0, \ldots, N_{p,i} \\
& \|u_i(k+j) - u_i(k+j-1)\| \leq \Delta u_i^{\max}
\end{aligned}
\tag{5.13}
$$

where $y_{\text{ref}}(k) = [y^T_{ref,1}(k), \ldots, y^T_{ref,M}(k)]^T$, $\Delta u_i(k) = u_i(k) - u_i^{ref}$ for the i-th controller, and Q_i, R_i, S_i positive definite diagonal matrices. From the optimization problem (5.13), given the references for the inputs and outputs, each local controller computes the control actions to be locally applied. It is worth to point out that the optimization problem (5.12) and (5.13) have different time resolutions, i.e., optimization problem (5.13) is solved more often than optimization problem (5.12). This time-scale differentiation is required because the use of whole system model makes the upper-layer optimization problem more complex than each local optimization problem of the lower layer.

5.3.2 Hierarchical Robust economic control for coordination

The Figure 5.9 shows the hierarchical robust economic control for coordination scheme. There are some works that propose economic MPC to control large scale systems such as the reported in [145, 146], however, the main idea in this thesis, specifically in this section is to take advantage of the MPC with zone control in decentralized schemes as it was shown in chapter 1 together with a robust operation under economic objective. In this regard, in the upper layer of the hierarchical structure a robust DRTO problem is solved. This layer sends to the lower layer robust input references. The lower layer has a decentralized scheme based on MPC with zone control. The aim of this layer is to follow the references given by the upper layer while the outputs are in an specific zone. In the following subsection the optimization problem of every layer will be explained.

5.3.2.1 Upper layer: Robust DRTO

The mathematical formulation of this layer is stated in (5.15). The objective function of this layer depends upon the specific process addressed. However, it may contain terms related to the productivity of the process, raw materials costs, energy consumption, economic

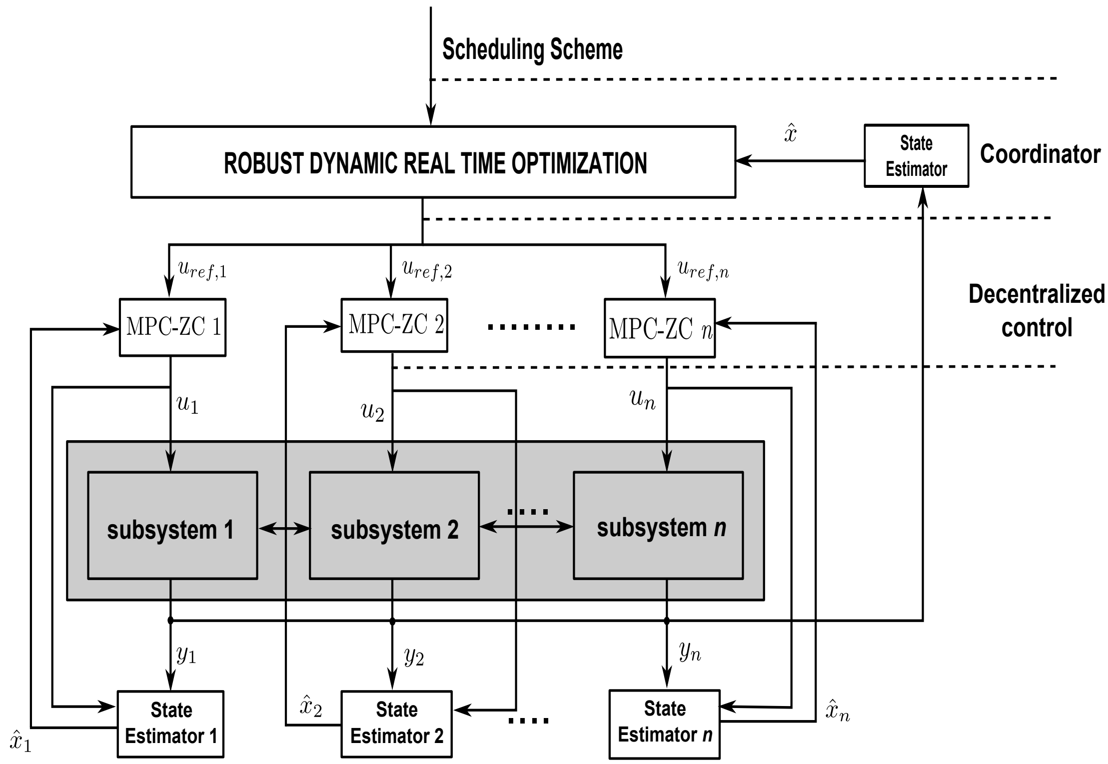


Figure 5.9: Hierarchical Robust economic control for coordination

losses, etc. So, determinate this objective function is not a trivial work. As it was stated in chapter 3 a typical economic cost function can be as follows [62]:

$$
\begin{aligned}
J_{eco}(x_k, u_k, d_k) = & \sum_{j=0}^{N_p} \left[\sum_{i_i=1}^{n_u} c_{u,i_i} u_{i_i}(k+j) + \sum_{i_o=1}^{n_y} c_{y,i_o} y_{i_o}(k+j) u_{i_o}(k+j) \right] \\
& + \sum_{j=0}^{N_p} \alpha(x(k+j), u(k+j))
\end{aligned}
\tag{5.14}
$$

where c_{u,i_i} and c_{y,i_o} are prices associated with energy and raw materials, N_p is the prediction horizon, n_y is the number of outputs, n_u is the number of inputs of the system and the term $\alpha(\cdot)$ is related to other costs.

$$
\begin{aligned}
\min_{u_k, y_{ref,k}} \max_{\Theta_k \in \Psi^{N_p}} & \sum_{j=0}^{N_p} \left[\sum_{i_i=1}^{n_u} c_{u,i_i} u_{i_i}(k+j) + \sum_{i_o=1}^{n_y} c_{y,i_o} y_{i_o}(k+j) u_{i_o}(k+j) \right] \\
& + \sum_{j=0}^{N_p} \alpha(x(k+j), u(k+j)) \\
\text{subject to:} & \\
& x(k+j+1) = A(\theta)x(k+j) + B(\theta)u(k+j) \\
& y(k+j) = Cx(k+j) + Du(k+j) \\
& x(k+j) \in \mathbb{X}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
& y(k+j) \in \mathbb{Y}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
& u(k+j) \in \mathbb{U}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
& x(k+N_p) \in \Omega, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
& y_{ref,k} \in \mathbb{Y}_{ref}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
& \|u(k+j) - u(k+j-1)\| \leq \Delta u_{\max} \\
& \theta(k+j) \in \Psi, \quad \|\theta(k+j)\| \leq \rho \\
& \Theta_k \in \Psi^{N_p} \triangleq \Psi \times \cdots \times \Psi
\end{aligned}
\tag{5.15}
$$

5.3.2.2 Lower layer: Decentralized MPC with zone control scheme

The mathematical formulation of the decentralized MPC with zone control is stated as follows:

$$
\begin{aligned}
\min_{u_{k_i}, y_{i_{ref,k}}} J_{N_{p,i}}(x_k, u_k) = & \sum_{j=1}^{N_{p,i}} \|(y_i(k+j) - y_{i_{ref,k}}\|^2_{Q_i} + \sum_{j=0}^{N_{p,i}} \|u_i(k+j) - u_i^{ref}\|^2_{R_i} \\
& + \sum_{j=0}^{N_{p,i}} \|\Delta u_i(k+j)\|^2_{S_i} + \|x_i(k+N_p)\|^2_{\tilde{Q}_i} \\
\text{subject to:} & \\
& x_i(k+j+1) = A_{ii}x_i(k+j) + B_{ii}u_i(k+j) \\
& y_i(k+j) = C_i x(k+j) + D_i u_i(k+j) \\
& u_i(k+j) = u_i^{ref} + \Delta u_i(k+j) \\
& x_i(k+j) \in \mathbb{X}_i, \quad j = 0, \ldots, N_{p,i} \\
& u_i(k+j) \in \mathbb{U}_i, \quad j = 0, \ldots, N_{p,i} \\
& x_i(k+N_p) \in \Omega_i, \quad j = 0, \ldots, N_{p,i} \\
& \|u_i(k+j) - u_i(k+j-1)\| \leq \Delta u_i^{\max}
\end{aligned}
\tag{5.16}
$$

Again here $x_i(k) \in \mathbb{R}^{n_{x_i}}$, $u_i(k) \in \mathbb{R}^{n_{u_i}}$, and $y_i(k) \in \mathbb{R}^{n_{y_i}}$ are the state, inputs, and outputs of i-th subsystem and that are subject to hard constraints on state $x_i(k) \in \mathbb{X}_i$, output $y_i(k) \in \mathbb{Y}_i$ and input $u_i(k) \in \mathbb{U}_i$ for all $k \geq 0$, where $\mathbb{X}_i \subset \mathbb{R}^{n_i}$, $\mathbb{Y} \subset \mathbb{R}^{z_i}$ and $\mathbb{U} \subset \mathbb{R}^{m_i}$ are closed sets. From this definition: $x(k) = [x_1^T(k), \ldots, x_M^T(k)]^T$, $u(k) = [u_1^T(k), \ldots, u_M^T(k)]^T$, and $y(k) = [y_1^T(k), \ldots, y_M^T(k)]^T$ and A_{ii}, B_{ii}, C_i, D_i be blocks of matrices A, B, C, D and M is the number of subsystems.

5.4 Hierarchical Robust Control for coordination of a HPV

As it was stated in section 4.3.1 the upper layer of the proposed control scheme computes the reference value for the inputs and outputs of each lower-layer local controller. Therefore, the power and input references for each subsystem are given by the solution of the robust optimization problem (5.17).

$$
\begin{aligned}
&\min_{v_k, y_{ref,k}} \max_{\Theta_k \in \Psi^{N_p}} \sum_{j=1}^{N_p} \|(y(k+j) - y_{ref,k}\|_Q^2 + \sum_{j=0}^{N_p} \|u(k+j)\|_R^2 \\
&\qquad + \sum_{j=0}^{N_p} \|\Delta u(k+j)\|_S^2 + \|x(k+N_p)\|_{\tilde{Q}}^2 \\
&\text{subject to:} \\
&\qquad x(k+j+1) = (A + \delta A_k)x(k+j) + (B + \delta B_k)u(k+j) \\
&\qquad y(k+j) = Cx(k+j) + Du(k+j), \;\; C_p y(k+j) = P_{ref} \\
&\qquad \theta(k+j) \triangleq (\delta A_k, \delta B_k) \in \Psi, \;\; \|\theta(k+j)\| \leq \rho \\
&\qquad x(k+j) \in \mathbb{X}, \;\; \forall \theta(k+j) \in \Psi, \;\; j = 0, \ldots, N_p \\
&\qquad y(k+j) \in \mathbb{Y}, \;\; \forall \theta(k+j) \in \Psi, \;\; j = 0, \ldots, N_p \\
&\qquad v(k+j) \in \mathbb{V}, \;\; \forall \theta(k+j) \in \Psi, \;\; j = 0, \ldots, N_p \\
&\qquad x(k+N_p) \in \Omega, \;\; \forall \theta(k+j) \in \Psi, \;\; j = 0, \ldots, N_p \\
&\qquad y_{ref}^{min} \leq y_{ref,k} \leq y_{ref}^{max}, \;\; \forall \theta(k+j) \in \Psi, \;\; j = 0, \ldots, N_p \\
&\qquad \|u(k+j) - u(k+j-1)\| \leq \Delta u_{\max}, \;\; \Theta_k \in \Psi^{N_p} \triangleq \Psi \times \cdots \times \Psi
\end{aligned}
\tag{5.17}
$$

where N_p is the prediction horizon, $Q \in \mathbb{R}^{n_y \times n_y}$, $R \in \mathbb{R}^{n_u \times n_u}$ and $S \in \mathbb{R}^{n_y \times n_y}$ are positive definite weighting matrices. In (5.17) all computations are done using a reduced order model. It is important to note that in the problem (5.17) the feedback control law of the each local controller is not considered. This fact leads to increase the uncertainty level in the problem, specially in the matrix A and B.

With the modification of the nominal structure, the hierarchical robust control structure is tested in two days of simulation on the HPV. Figure 5.10 shows the power generated by the HPV and the power reference for two days. In opposition to the nominal scheme the robust can manage the power reference problem in the HPV, additionally, with the extra constrain, the control system can maintain the individual powers under the zone following the reference given by the coordinator (see Figure 5.11). The Figure 5.12 shows the levels the HPV, it is important to note that they are into a designated zone under the robust scheme. In general terms, the simulations results show that the robust scheme moves the output variables to other set points while maintain the outputs in an specific range, this flexibility permits to achieve the power reference. Finally, Figure 5.13 shows the input variables.

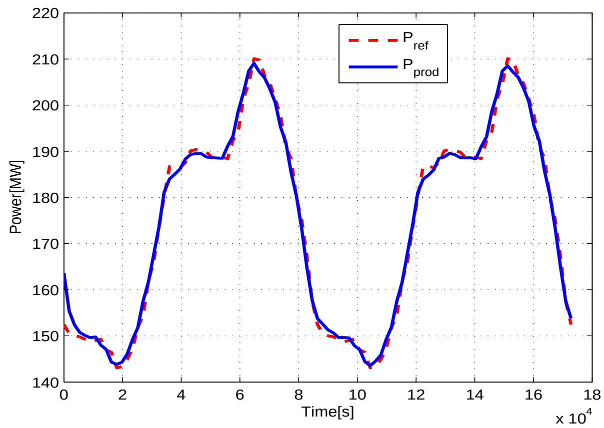


Figure 5.10: Robust Scheme: Comparison between the power produced by the HPV with the power reference

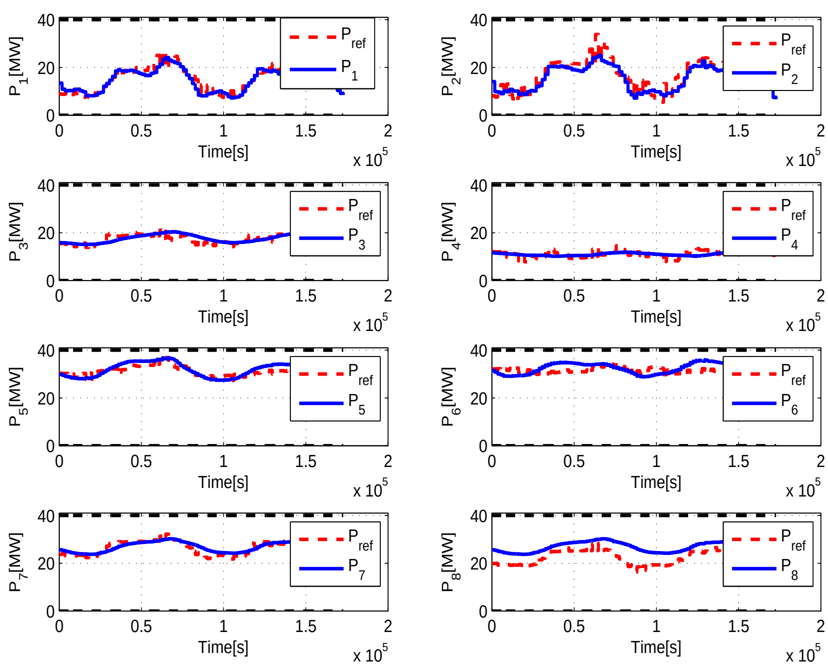


Figure 5.11: Robust Scheme: Behavior of the individual powers of the HPV.

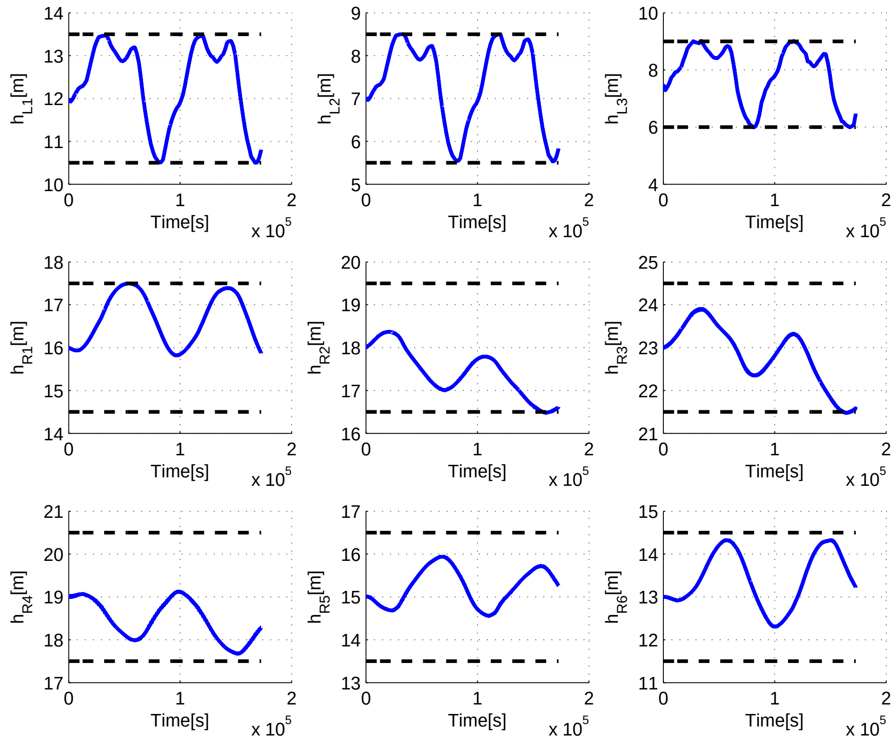


Figure 5.12: Robust Scheme: Behavior of the levels in the lakes and the levels at the damps of the HPV.

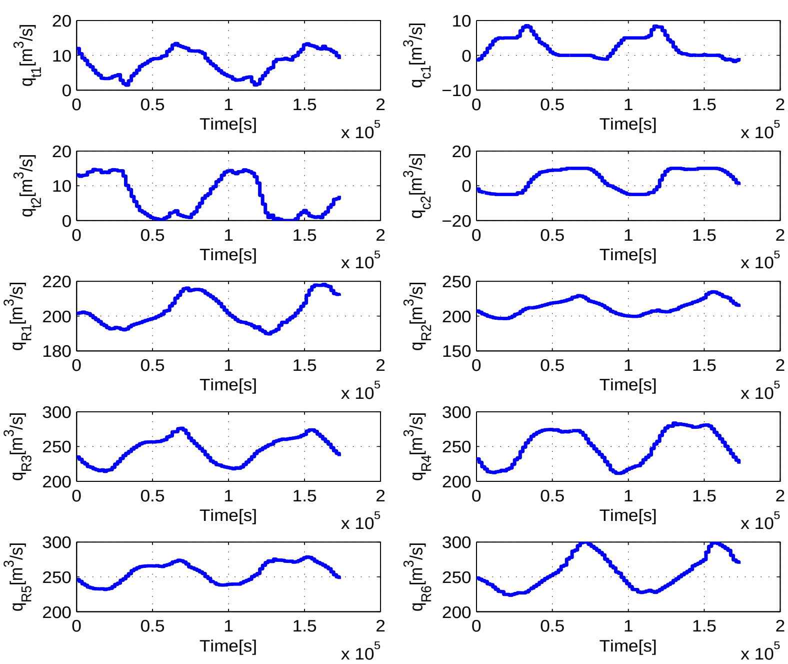


Figure 5.13: Robust Scheme: Control actions

This thesis use the HPV benchmark with a difficult control scenario to test a hierarchical robust control approach. The hierarchical robust control uses 8 MPC as local controllers in the lower layer and one Robust MPC with zone control as coordinator in the upper layer. HPV control scenario shows some challenges which could affect the performance of nominal hierarchical control, then some constraints must be added to the optimization problem. An important aspect to highlight in the HPV benchmark is that zone control helps to release output variables in order to manage the bigger amount of output variables considering the fewer input variables.

The use of Robust MPC with zone control approach and some modification to the constraints used into the controller optimization problem, help to solve an extreme complex control scenario with good performance. In general terms, the hierarchical robust structure proposed here is attractive for large-scale processes, which often lead to long computational times during optimization whereas on the other hand short sampling times are required by the process. Finally in spite of the big number of states, the linearization and the dramatic reduction of the order, the hierarchical controller has an acceptable performance.

5.5 Summary

In this chapter a Hierarchical Robust Control for coordination was presented. This strategy arises in problems where the upper layer of the hierarchical structure proposed in chapter 4 is not necessary. In this way in this chapter initially an example is used a motivation to use this strategy. Specifically, the example is a Hydro Power Valley(HPV), this system is a typical large scale benchmark that is used to prove control behaviour, because it has several characteristics which establish some control challenges. From the hierarchical robust scheme proposed, two hierarchical robust structures arise. The first one takes a robust MPC with zone control as coordinator, while in the lower layer of the hierarchical structure a decentralized scheme is employed. The second one takes a Robust Dynamic Real Time Optimization (DRTO) layer as coordinator while in the lower layer a decentralized scheme based in MPC with zone control is used. Finally, the strategy proposed was tested in the Hydro Power Valley(HPV), where the simulation results were very interesting, achieving a good performance of the scheme following a power profile during two days while the relevant variables remained in the specific zone

In the chapter 4 and 5 some strategies for hierarchical robust control were presented. The main idea was to combine the robust MPC approaches presented in chapter 3 and the zone control concept with a classical hierarchical structure presented in chapter 2. The principal advantages of the strategies presented in these chapters are the ability to hand uncertainties, to achieve different objectives (Economic, environmental), and take into account the dynamic behavior of the system. In this way these approaches improve the structures reported in the literature. However, some problems arise from the structures proposed that uses zone control approach, and the main complication lies in how to calculate the limits (zone) of the MPC with zone control. Generally these limits correspond to physical limitations of the process, nevertheless, not always this is the best choice, and it is necessary to find another way to determine these limits. A reasonable criterion can be the zone when the linearization is valid, however, an interesting criterion can arise from the economic point of view. In this sense the most remarkable contribution of this thesis is a hierarchical robust strategy where the limits of the MPC with zone control are calculated based in an economical criterion, and this strategy is presented in chapter 6.

CHAPTER 6

Hierarchical Robust Real Time Optimization with zone control

6.1 Introduction

Along this thesis the problem to control large scale systems using hierarchical control was addressed. Initially, in chapter 4 we propose some hierarchical robust strategies based in the concept of zone control and using robust optimization. This strategies were tested in chapter 5 where was demonstrated his potentiality. It is important to highlight that although there are reported some works of hierarchical control for large scale systems such as [2, 3, 4], the use of zone control and robust control (min-max in this case) have not been explored fully. In this sense, this thesis propose a general hierarchical structure to control large scale systems using robust optimization and zone control, modifying the classical structures in order to handle uncertainties such as local controllers that are not taking into account in the model of the process. However, some questions arise from the hierarchical structures proposed. The main question emerge from the zone control concept. As it was mentioned in the introductory chapter 1 and chapter 4 the limits or zone of the MPC with zone control are normally chosen based in empirical criteria or in physical constraints of the system (conservative limits). Although, theses criteria works well, there exist another attractive zones to be studied. From this point of view we propose to find this zones by means of optimization problems, i.e we want to find the limits of the MPC with zone control by means of a min-min optimization problem (best case), and a min-max optimization problem (worst case). In the light of this method, it is necessary to chose the objective function of the problems, in this way we propose an economical criterion, i.e the limits of the MPC with zone control of the hierarchical structure proposed in this chapter will be found by means of robust optimization with economic objective, of this form, the limits will be non-conservatives and economically feasible. This chapter is organized as follows: In section 6.2 the hierarchical robust real time optimization with zone control structure is detailed and tested in an illustrative example. Finally in section 6.3 the structure proposed is applied to a system composed by three distillation columns and three CSTR reactors.

6.2 Hierarchical Robust Real Time Optimization with zone control

The Figure 6.1 shows the structure proposed. In the upper layer two optimization problems are solved in order to calculate the non-conservative limits for the outputs of the system, both optimization problems with different sample time due to the time to solve the min-max optimization problem is higher than the min-min optimization problem. The upper layer is responsible of sending the limits to the MPC with zone control located in the middle layer, this controller acts as a coordinator and it is in charge of giving the set-points in a specific range to the lower layer, when a decentralized scheme is used. In the following, every part of the hierarchical structure will be explained. It is important to note that some components of the structure proposed were explained in previous chapters, in this sense certain particularities of these components are not detailed in this chapter.

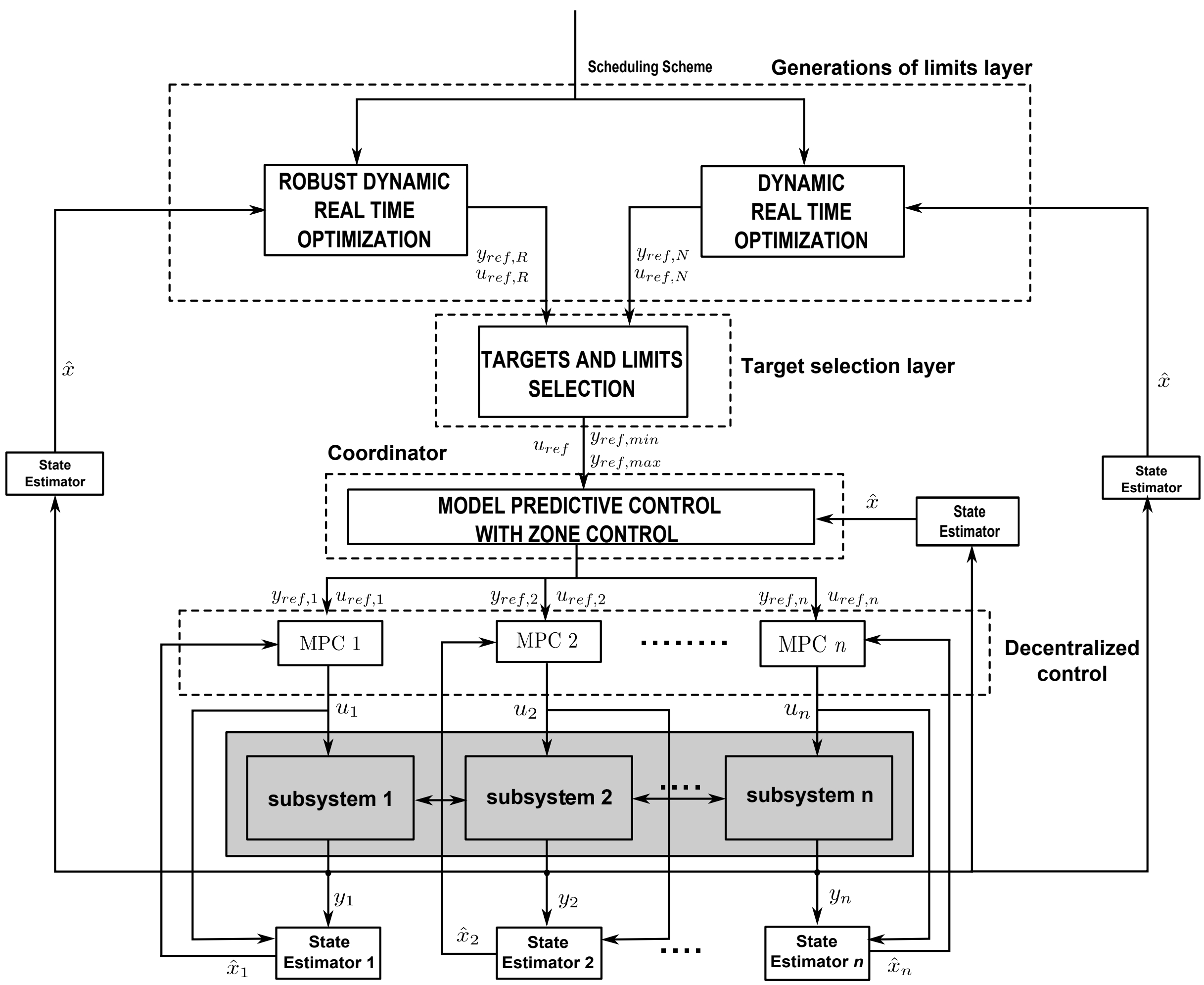


Figure 6.1: Hierarchical Robust Real Time Optimization with zone control scheme.

6.2.1 Upper layer: Generation of non-conservative limits for the outputs of the system

In the upper layer two optimization problems are solved in order to calculate the non-conservative limits for the outputs of the system. The first one problem is a robust optimization problem that considers model uncertainty (min-max optimization problem). The second one optimization problem is called the nominal problem and does not consider model uncertainty (min-min optimization problem). The main work of these two layers is to generate a couple of trajectories ($y_{ref,N}$ and $y_{ref,R}$) where the upper and the lower bound of the MPC with zone control must be selected. In the next sections the optimization problems and the targets and limits selection block will be described.

- **Min-min optimization problem**

  In this case we are considering the minimum value of the uncertainty in the optimization problem. The min-min optimization problem is described by (6.1).

$$
\begin{aligned}
&\min_{u_k} \min_{\Theta_k \in \Psi^{N_p}} J_{eco}(x_k, u_k, \Theta_k)\\
\text{subject to:}\quad &\\
&x(k+j+1) = f(x(k+j), u(k+j), \theta(k+j))\\
&y(k+j) = g(x(k+j), u(k+j), \theta(k+j))\\
&x(k+j) \in \mathbb{X}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \dots, N_p\\
&u(k+j) \in \mathbb{U}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \dots, N_p\\
&x(k+N_p) \in \Omega, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \dots, N_p\\
&\|u(k+j) - u(k+j-1)\| \leq \Delta u_{\max}\\
&\theta(k+j) \in \Psi, \quad \|\theta(k+j)\| \leq \rho\\
&\Theta_k \in \Psi^{N_p} \triangleq \Psi \times \cdots \times \Psi
\end{aligned}
\tag{6.1}
$$

An important fact in problem (6.1) is that the min-min problem can be reduced to a min problem. With this assumption and using linear time-varying prediction models (see appendix B), the problem (6.1) can be expressed as (6.2) (see chapter 3).

$$
\begin{aligned}
&\min_{u_k} r + 2q^T u_k + u_k^T P u_k\\
&\text{subject to:}\\
&\begin{bmatrix} I \\ -I \\ \Delta_{\mathrm{u}} \\ \Delta_{\mathrm{u}} \\ \Lambda \\ -\Lambda \end{bmatrix} u_k(k) \leq \begin{bmatrix} \tilde{u}_{\max} \\ -\tilde{u}_{\min} \\ \tilde{\Delta u}_{\max} + \overline{u}(k-1) \\ \tilde{\Delta u}_{\max} - \overline{u}(k-1) \\ \tilde{y}_{\max} - \Gamma \\ -\tilde{y}_{\min} + \Gamma \end{bmatrix}
\end{aligned}
\tag{6.2}
$$

In (3.65) P, q, $\tilde{u}_{\max}$, $\tilde{u}_{\min}$, $\tilde{y}_{\max}$, $\tilde{y}_{\min}$, $\tilde{\Delta u}_{max}$ are defined as follows:

$$
\begin{aligned}
P &= \Lambda^T C_{ca} + \Lambda^T \overline{Q} \Lambda + \overline{R} + \Delta_{\mathrm{u}}^T \overline{S} \Delta_{\mathrm{u}} \\
q &= C_{op} + \Gamma^T C_{ca} - U_s^T \overline{R} + (\Gamma - Y_s)^T \overline{Q} \Lambda - \overline{u}(k-1)^T \overline{S} \Delta_{\mathrm{u}} \\
r &= (\Gamma - Y_s)^T \overline{Q} (\Gamma - Y_s) + \overline{u}(k-1)^T \overline{S} \overline{u}(k-1)
\end{aligned}
$$

where C_{op} and C_{ca} are block diagonal matrices where the elements of the diagonals are $C_u = [c_{u,1}, \cdots, c_{u,n_u}]$ and $C_y = \mathrm{diag}(\left[c_{y,1}, \cdots, c_{y,n_y}\right])$.

- **Min-max optimization problem**

  This problem consider the maximum value of the model uncertainty in a optimization problem with economic objective. The robust optimization problem is as follows,

$$
\begin{aligned}
&\min_{u_k} \max_{\Theta_k \in \Psi^{N_p}} J_{eco}(x_k, u_k, \Theta_k) \\
&\text{subject to:} \\
&\quad x(k+j+1) = f(x(k+j), u(k+j), \theta(k+j)) \\
&\quad y(k+j) = g(x(k+j), u(k+j), \theta(k+j)) \\
&\quad x(k+j) \in \mathbb{X}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad u(k+j) \in \mathbb{U}, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad x(k+N_p) \in \Omega, \quad \forall \theta(k+j) \in \Psi, \quad j = 0, \ldots, N_p \\
&\quad \|u(k+j) - u(k+j-1)\| \leq \Delta u_{\max} \\
&\quad \theta(k+j) \in \Psi, \quad \|\theta(k+j)\| \leq \rho \\
&\quad \Theta_k \in \Psi^{N_p} \triangleq \Psi \times \cdots \times \Psi
\end{aligned}
\tag{6.3}
$$

Note that problem (6.3) was addressed in chapter 3, in these sense, if we consider a linear time-varying approximation of $f(\cdot)$ and $g(\cdot)$, the min-max MPC problem (6.3) becomes the SOCP problem (6.4) explained in chapter 3.

$$
\begin{aligned}
&\min_{u_k, w_{i_e}, s, d, t} \; s + 2q_0^T u_k + t + 2d + \|\overline{r}\|_2 \\
\text{subject to:} \quad & \\
&\left\| \begin{bmatrix} 2P_0^{1/2} u_k \\ s - 1 \end{bmatrix} \right\|_2 \leq s + 1 \\
&\left\| \begin{bmatrix} 2P_{i_e}^{1/2} u_k \\ w_{i_e} - 1 \end{bmatrix} \right\|_2 \leq w_{i_e} + 1 \\
&\|q_{j_e} u_k\|_2 \leq d \\
&\|w\|_2 \leq t \\
&0 \leq w_{i_e} \\
&0 \leq s \\
&\begin{bmatrix} I \\ -I \\ \Delta_{\mathrm{u}} \\ \Delta_{\mathrm{u}} \\ \Lambda \\ -\Lambda \end{bmatrix} u_k \leq \begin{bmatrix} \tilde{u}_{\max} \\ -\tilde{u}_{\min} \\ \tilde{\Delta} u_{\max} + \overline{u}(k-1) \\ \tilde{\Delta} u_{\max} - \overline{u}(k-1) \\ \tilde{y}_{\max} - \Gamma \\ -\tilde{y}_{\min} + \Gamma \end{bmatrix}
\end{aligned}
\tag{6.4}
$$

For problems (6.1) and (6.3) Ψ^{N_p} is determined by the assumed uncertainty model.

- **Targets and limits selection block:**

  This block is in charge to choose the upper and the lower bounds for the MPC of the zone control and to select the best target from the economic point of view. The main reason to add this block to the hierarchical structure is that is not possible determine the upper and the lower limits a priori from both optimization problem directly. For example we can not say that the robust problem gives the upper bound, since the worst case not necessarily generate the largest value (limit) in the output of the system. Then, this block choose the limits from $y_{ref,N}$ and $y_{ref,R}$ simply comparing which value is greater. On other hand in the classical hierarchical structures the upper layer always send targets to the bottom of the hierarchy, in this sense in the proposed structure two targets are generated $u_{ref,N}$ and $u_{ref,R}$, therefore one of these targets must be selected. The criterion to select the target in this work is very simple, the target that generates the best profit is chosen to be send to the MPC with zone control.

6.2.2 Middle layer: Generation of references in a specific range

The coordinator of the hierarchical structure is a MPC controller with zone control that was explained in chapter 3. Mathematically, this is expressed as (6.5).

$$
\begin{aligned}
&\min_{u_k, y_{ref,k}} J_{N_p}(y_{ref,k}, x_k, u_k)\\
&\text{subject to:}\\
&\quad x(k+j+1) = f(x(k+j), u(k+j))\\
&\quad y(k+j) = g(x(k+j), u(k+j))\\
&\quad x(k+j) \in \mathbb{X}, \quad j = 0, \ldots, N_p\\
&\quad y(k+j) \in \mathbb{Y}, \quad j = 0, \ldots, N_p\\
&\quad u(k+j) \in \mathbb{U}, \quad j = 0, \ldots, N_p\\
&\quad x(k+N_p) \in \Omega, \quad j = 0, \ldots, N_p\\
&\quad y_{ref,k} \in \mathbb{Y}_{ref}\\
&\quad \|u(k+j) - u(k+j-1)\| \leq \Delta u_{\max}
\end{aligned} \tag{6.5}
$$

For problems (6.1), (6.3) and (6.5), $x(k) \in \mathbb{R}^{n_x}$ is the system state, $y(k) \in \mathbb{R}^{n_y}$ is the system output, $u(k) \in \mathbb{R}^{n_u}$ is the current control vector and $\theta(k) \in \mathbb{R}^{n_d}$ are the uncertainties presents in the model. The system is subject to hard constraints on state $x(k) \in \mathbb{X}$, output $y(k) \in \mathbb{Y}$, input $u(k) \in \mathbb{U}$ and $\theta \in \Psi$ for all $k \geq 0$, where $\mathbb{X} \subset \mathbb{R}^{n_x}$, $\mathbb{Y} \subset \mathbb{R}^{n_y}$ and $\mathbb{U} \subset \mathbb{R}^{n_u}$ are closed sets.

6.2.3 Lower layer: Tracking control system

Now, let the whole system be decomposed into M subsystems where $x_i(k) \in \mathbb{R}^{n_{x_i}}$, $u_i(k) \in \mathbb{R}^{n_{u_i}}$, and $y_i(k) \in \mathbb{R}^{n_{y_i}}$ are the state, inputs, and outputs of i-th subsystem and that are subject to hard constraints on state $x_i(k) \in \mathbb{X}_i$, output $y_i(k) \in \mathbb{Y}_i$ and input $u_i(k) \in \mathbb{U}_i$ for all $k \geq 0$, where $\mathbb{X}_i \subset \mathbb{R}^{n_i}$, $\mathbb{Y} \subset \mathbb{R}^{z_i}$ and $\mathbb{U} \subset \mathbb{R}^{m_i}$ are closed sets. From this definition: $x(k) = [x_1^T(k), \ldots, x_M^T(k)]^T$, $u(k) = [u_1^T(k), \ldots, u_M^T(k)]^T$, and $y(k) = [y_1^T(k), \ldots, y_M^T(k)]^T$. Let A_{ii}, B_{ii}, C_i, D_i be blocks of matrices A, B, C, D such that

$$
\begin{aligned}
x_i(k+1) &= A_{ii} x_i(k) + B_{ii} u_i(k)\\
y_i(k) &= C_i x(k) + D_i u_i(k)
\end{aligned}
$$

Then, each MPC of the lower layer can be written as (6.6).

$$
\begin{aligned}
\min_{u_{k_i}} J_{N_{p,i}}(x_k, u_k) = & \sum_{j=1}^{N_{p,i}} \|(y_i(k+j) - y_{ref,i}\|^2_{Q_i} + \sum_{j=0}^{N_{p,i}} \|u_i(k+j) - u_i^{ref}\|^2_{R_i} \\
& + \sum_{j=0}^{N_{p,i}} \|\Delta u_i(k+j)\|^2_{S_i} + \|x_i(k+N_p)\|^2_{\tilde{Q}_i}
\end{aligned}
$$

subject to:

$$
\begin{aligned}
& x_i(k+j+1) = A_{ii} x_i(k+j) + B_{ii} u_i(k+j) \\
& y_i(k+j) = C_i x(k+j) + D_i u_i(k+j) \\
& u_i(k+j) = u_i^{ref} + \Delta u_i(k+j) \\
& x_i(k+j) \in \mathbb{X}_i, \quad j = 0, \ldots, N_{p,i} \\
& u_i(k+j) \in \mathbb{U}_i, \quad j = 0, \ldots, N_{p,i} \\
& x_i(k+N_p) \in \Omega_i, \quad j = 0, \ldots, N_{p,i} \\
& \|u_i(k+j) - u_i(k+j-1)\| \leq \Delta u_i^{\max}
\end{aligned}
\tag{6.6}
$$

where $x_i(k) \in \mathbb{R}^{n_{x_i}}$, $u_i(k) \in \mathbb{R}^{n_{u_i}}$, and $y_i(k) \in \mathbb{R}^{n_{y_i}}$ are the state, inputs, and outputs of i-th subsystem and that are subject to hard constraints on state $x_i(k) \in \mathbb{X}_i$, output $y_i(k) \in \mathbb{Y}_i$ and input $u_i(k) \in \mathbb{U}_i$ for all $k \geq 0$, where $\mathbb{X}_i \subset \mathbb{R}^{n_i}$, $\mathbb{Y} \subset \mathbb{R}^{z_i}$ and $\mathbb{U} \subset \mathbb{R}^{m_i}$ are closed sets. From this definition: $x(k) = [x_1^T(k), \ldots, x_M^T(k)]^T$, $u(k) = [u_1^T(k), \ldots, u_M^T(k)]^T$, and $y(k) = [y_1^T(k), \ldots, y_M^T(k)]^T$ and A_{ii}, B_{ii}, C_i, D_i be blocks of matrices A, B, C, D and M is the number of subsystems.

6.2.4 Illustrative example: Isothermal CSTR

This example was taken directly of [63]. Consider a single first-order, irreversible chemical reaction $A \rightarrow B$ in the isothermal CSTR shown in Figure 6.2

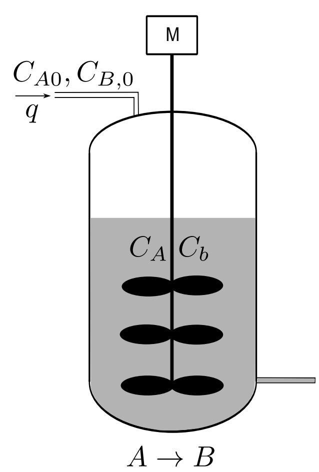


Figure 6.2: Isothermal reactor

The mathematical model of the process is:

$$\begin{aligned}\frac{dC_A}{dt} &= \frac{q}{V}(C_{A0} - C_A) - k_r C_A \\ \frac{dC_B}{dt} &= \frac{q}{V}(C_{B0} - C_B) + k_r C_A\end{aligned} \tag{6.7}$$

where k_r is the rate constant, C_A and C_B are the molar concentrations of A and B respectively, $C_{A0} = 1$ mol/L and $C_{B0} = 0$ mol/L are the feed concentrations, and q is the flow through the reactor. The volume of the reactor V is fixed at 10 L. The rate constant k_r is 1.2 L/(mol min). The available manipulated variable is the feed flow rate. Non-negativity constraints are imposed on the feed rate. An upper bound of $q_{max} = 20$ L/min is imposed on the flow rate. The economical cost function of this problem is [63],

$$J_{eco}(C_A, C_B, q) = -(2qC_B - 0.5q) \tag{6.8}$$

The optimal steady state for this cost is $C_A = C_B = 0.5$ mol/L, $q = 12$ L/min. The states of the systems are $x = [C_A, C_B]^T$, the output is $y = C_B$, the input of the system is $u = q$, while the disturbance of the system is $d = C_{A0}$.

6.2.4.1 Simulation results

Figure 6.3 show the simulations results once the methodology proposed in section 6.2 is applied. For this simulation, it was considered a sampling time of the RDRTO and DRTO layers of $10s$ with a prediction horizon of 30 samples, while the sampling time of the Robust MPC with zone control was $1s$ with a prediction horizon of 30 samples. The main reason to take this example as an illustrative example was the number of publications that report this example, specially in economic MPC approaches. In this way, we want to find limits for the output of this system when disturbances or uncertainties are presents. For this case, changes in the disturbances of the process were made as shown in Figure 6.3(c). Figure 6.3(a) shows the output and reference generated by the MPC with zone control, clearly the controller drives the output to the set-point. However, in Figure 6.3(b) the limits calculated by the upper layer of the hierarchy are presented, it is important to note that although the change in the disturbance was 50%, the limits are basically the same during the simulation time, when the change in C_{A0} is positive. This results is very interesting in the sense that not always the limits will be different in the methodology despite uncertainties and disturbances. Finally, the Figure 6.3(d) shows the input value and his reference.

6.3 Case of study: Chemical Plant

The complete chemical process is composed by three chemical reactors type CSTR (Continuous Stirred Tank Reactor) that are called R1, R2 and R3, three non reactive binary distillation columns called C1, C2 and C3 and two recycle streams called RC1 and RC3. Figure 6.4 gives a Process Flow Diagram (PFD) that helps to understand the following detailed description. It must be clear for the reader that this chemical plant case presents

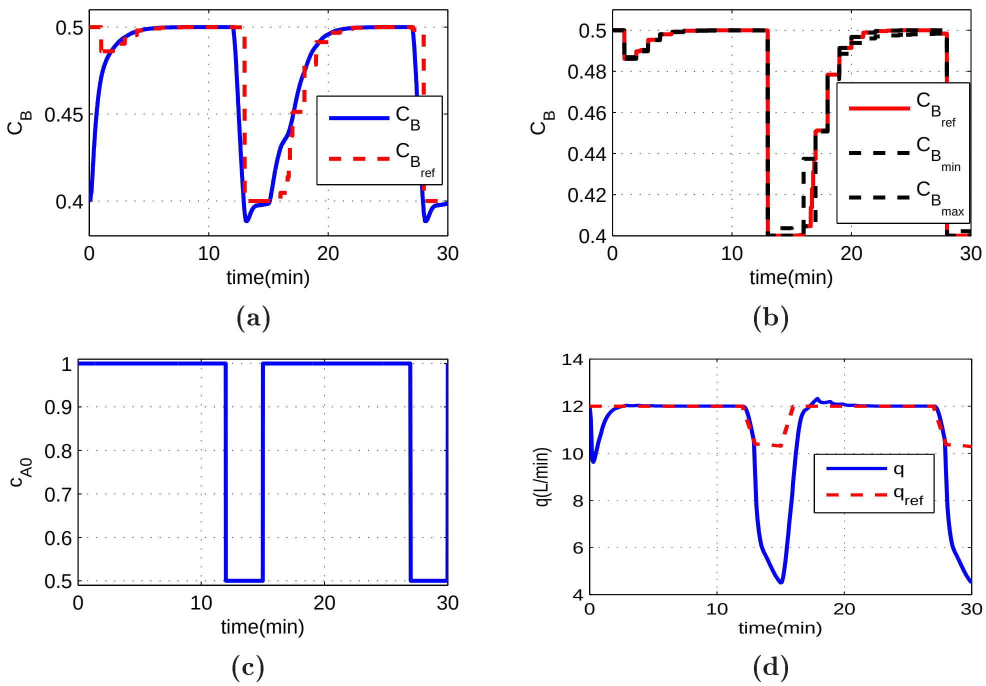


Figure 6.3: Isothermal CSTR: (a) Output of the system with the reference value. (b) Upper and lower bound generated by the top of the hierarchy. (c) Disturbances of the process. (d) Input of the system

only dynamics related with the material balance, the energy and momentum dynamics are not considered as relevant issues here. A fresh stream of A and the recycle streams RC1 and RC3 are fed to R1 where the reaction represented by (6.9) with a kinetic constant k_1 is carried out. This step produces a main intermediate product D and a byproduct C. The effluent from R1, assumed as an ideal mixture, is fed to C1 where it is separated in: top products rich in A and B called RC1, and bottom products rich in C and D. At this point the stream RC1 is fed again to R1 and the bottom stream is fed to C2. In C2, the byproduct C and the intermediate process D are separated. The top products with high concentration of C are byproducts that are removed from the process. The bottom products with high concentration of D are fed to R2. In R2 there is a fresh stream of E fed along with the bottom effluent from C2 and a new chemical reaction represented by (6.10) is carried out with a kinetic constant k_2. The products and the material without reacting from R2 pass to R3 to reach a higher conversion. Although the reaction in R2 is the same of R3, in R3 the kinetic constant is k_3 that may be different of k_2 due to effects of temperature and agitation. Then the effluent from R3 is fed to C3. In C3 the effluent from R3 is separated in bottom products with high concentration of F which is the interesting product, and top products with high concentration of B. This high B concentration stream is called RC3 and is fed again to R1.

$$A + B \rightarrow C + D \tag{6.9}$$

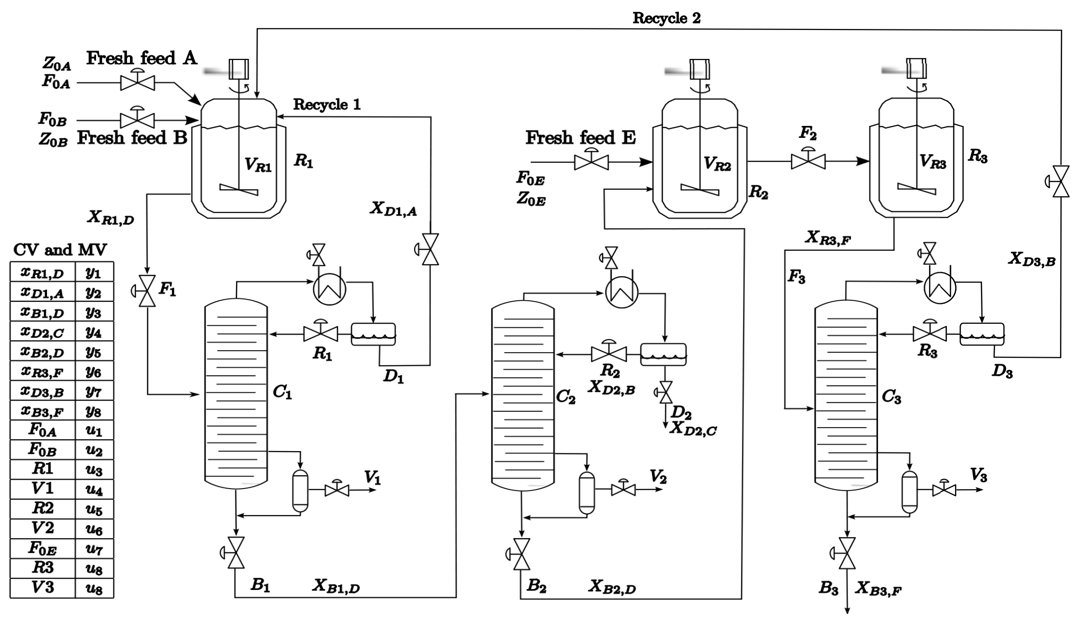


Figure 6.4: Chemical plant.

$$D + E \rightarrow F + B \tag{6.10}$$

The different mixtures are considered as ideal, there are not reactions inside the pipes, the transportation time in pipes is not considered and all the chemical reactions are considered as elemental. The process variables considered in this benchmark case will be discriminated in the following groups:

- *Controlled Variables (CV)*: There is a basic group of variables that could be used as control variables, in order to build the large amount of control strategies that could be proposed. This basic group consist of the variables depicted in Table 6.1.

$x_{R1,D}$	y_1
$x_{D1,A}$	y_2
$x_{B1,D}$	y_3
$x_{D2,C}$	y_4
$x_{B2,D}$	y_5
$x_{R3,F}$	y_6
$x_{D3,B}$	y_7
$x_{B3,F}$	y_8

Table 6.1: Controlled variables

- *Manipulated Variables (MV):* This process has 9 manipulated variables depicted in Table 6.2.

The proposed model for the chemical plant case is based on first principles, the basic material balances lead the equations describing the dynamic behaviour. This model was proposed by Scattollini [147] and it is presented in appendeix D.

F_{0A}	u_1
F_{0B}	u_2
$R1$	u_3
$V1$	u_4
$R2$	u_5
$V2$	u_6
F_{0E}	u_7
$R3$	u_8
$V3$	u_8

Table 6.2: Manipulated variables

6.3.1 Economical Aspects

6.3.1.1 Cost Function

In order to define a proper cost function for this plant, each one of the associated cost will be mentioned. As the plant is divided in six stages, each stage will have some associated operational costs in the following way.

1. Stage R1: Reaction of A + B to produce C + D, by the stoichiometry becomes clear the mole conservation in this reaction. The associated costs are:
   - cost of feeding one fresh lb-mol of A in the stage R1 (cost of A + pumping)
   - cost of feeding one fresh lb-mol of B in the stage R1 (cost of B + pumping)
   - cost of feeding one lb-mol of B recycled in the stage R1 (pumping)
2. Stage C1: Separation of A, B, C and D, in this stage is assumed that chemical reaction is not carried out and just physical separation is achieved, the associated cost is:
   - cost of feeding one lb-mol of A + B + C + D in the stage C1 (pumping)
3. Stage C2: Separation of C + D into C and D, the associated costs are:
   - cost of feeding one lb-mol of C + D in the stage C2 (pumping)
   - *possible cost of disposal of C*
4. Stage R2: Reaction of D + E to produce F + B, by the stoichiometry becomes clear the mole conservation in this reaction. The associated costs are:
   - cost of feeding one lb-mol of D in the stage R2 (pumping)
   - cost of feeding one fresh lb-mol of E in the stage R2 (cost of E + pumping)
5. Stage R3: Reaction of D + E to produce F + B, by the stoichiometry becomes clear the mole conservation in this reaction. The associated costs are:

- cost of feeding one lb-mol of D + E in the stage R3 (pumping)
- cost of feeding one lb-mol of F + B in the stage R3 (pumping)

6. Stage C3: Separation of F + B into F and B, the associated cost is:

- cost of feeding one lb-mol of F + B in the stage C3 (pumping)

All these costs must be expressed in the correct units, the units selected in this case were $[€/lb - mol_i]$ where i refers to any component. Eq. (6.11) show the units required in the function. Although the final expression is expressed in € per hours this is not relevant due to one pretend to minimize the total cost what is the same to minimize the rate of "generation of costs", due to the linearity of the cost function.

$$\left[\frac{€}{lb - mol_i}\right] * \left[\frac{lb - mol_i}{h}\right] = \left[\frac{€}{h}\right] \tag{6.11}$$

6.3.1.2 Cost Determination:

The costs mentioned above will be stated in this section. They are based on the quantity of raw material, the degree of separation(final disposal),and finally the pumping cost that is the same for all components. The value of each cost specified above is defined in Table 6.3.

ITEM	COST	UNIT
Fresh A (C_A^f)	0.5	$\left[\frac{€}{lb-mol_A}\right]$
Fresh B (C_B^f)	0.8	$\left[\frac{€}{lb-mol_B}\right]$
Fresh E (C_E^f)	0.8	$\left[\frac{€}{lb-mol_E}\right]$
Pumping (C_p)	$2.2e^{-5}\Delta h_{pump} C_{KWh}$ (*)	$\left[\frac{€}{lb-mol_i}\right]$
Disposal of C (C_d^C)	0.2	$\left[\frac{€}{lb-mol}\right]$

(*) Where Δh_{pump} is the required change in height by pumping in meters, and C_{KWh} is the cost of KWh in €/KWh

Table 6.3: Operational costs

The associated costs are multiplied by the respective stream, generating the function to be minimized, Eq. (6.12) shows the final form of the cost function.

$$f_{eco} = C_A^f F_{0A} + C_B^f F_{0B} + C_E^f F_{0E} + C_p(D_1 + D_3 + B_1 + B_3) - C_d^C D_2 x_{D2,C} - C_d^B B_3 x_{B3,F} \tag{6.12}$$

The stages and streams were defined in Figure 6.4 and the costs were specified in Table 6.3.

6.3.2 Model Reduction of the chemical plant: Hankel norm method

From a mathematical and system theoretical point of view, Hankel norm reductions are among the most fancy sort of model reduction procedures that exist today. It is one of the very few model approximation procedures that produce optimal approximate models according to some well-defined criterion. The algorithm to find the reduced order model by means of Hankel norm can be seen at [140]. For the chemical plant we achieve to reduce the original system from 203 to 20 states. This reduced order model will be used in the MPC with zone control due to well performances showed in the work of Marquez et.al [121], when a nominal hierarchical controller was designed for the chemical plant.

6.3.3 Simulation Results and Discussion

The main purpose with this case of study is to maintain the outputs of the system into a specific zone under uncertainties and disturbances in an high dimensional and interconnected system, trying to maximize the profit. Then, the simulation was carry on making some changes in parameters of the model of the system, specifically in the reactions constants k in the first and third reactor, a change of 10% of their nominal values, and in the Murphee efficiency of the third distillation column, it was changed to 90%, and a change in the inlet fresh concentration of R1 and R2 at 80 h. For this simulation, it was considered a sample time of the RDRTO and DRTO layers of 10 h with a prediction horizon of 10 samples, while the sample time of the Robust MPC with zone control was 1 h with a prediction horizon of 100 samples. Figures 6.5 and 6.6 show the simulations results. Figure 6.5 shows the outputs of the system, the limits given by the upper layer and the references generated by the MPC with zone control. Note that the limits generated in the upper layer of the hierarchy opposite to the illustrative example presented in section 6.2.4 are not the same for any output, however, there are limits or zones very close, even there are times where the limits are the same (see Figure 6.5 (a),(d),(f) and (g)), due to this the MPC with zone control generate references generally in the upper or lower limit. The chemical plant main product is the condensed F, for this reason the concentration has to be maintained in high values, then, an important aspect to highlight is about the profit and $x_{B3,F}$, note that for $x_{B3,F}$ the hierarchical controller tries to drive this output to 1. This is the maximum value expected of this output in order to maximize the profit. Finally, Figure 6.6 shows the inputs of the process, where saturation was avoided in almost all Figures.

In general, with this example we demonstrate that the Hierarchical Robust Real Time Optimization with zone control structure has the ability to control a large scale system, and the alternative of use zone control to release or give freedom to some variables is adequate. On other hand the limits found correspond to feasible limits from the economic point of view. This is in the same direction of the classical structures where in the upper layer an economical operating trajectory is found. About that, notice that it is not possible to compare the hierarchical structure proposed here with classical or even robust structures. The main reason is that we found the limits by means of an economic criterion, in this way the other solutions always will be in the zone generated by the Hierarchical Robust Real Time Optimization with zone control structure.

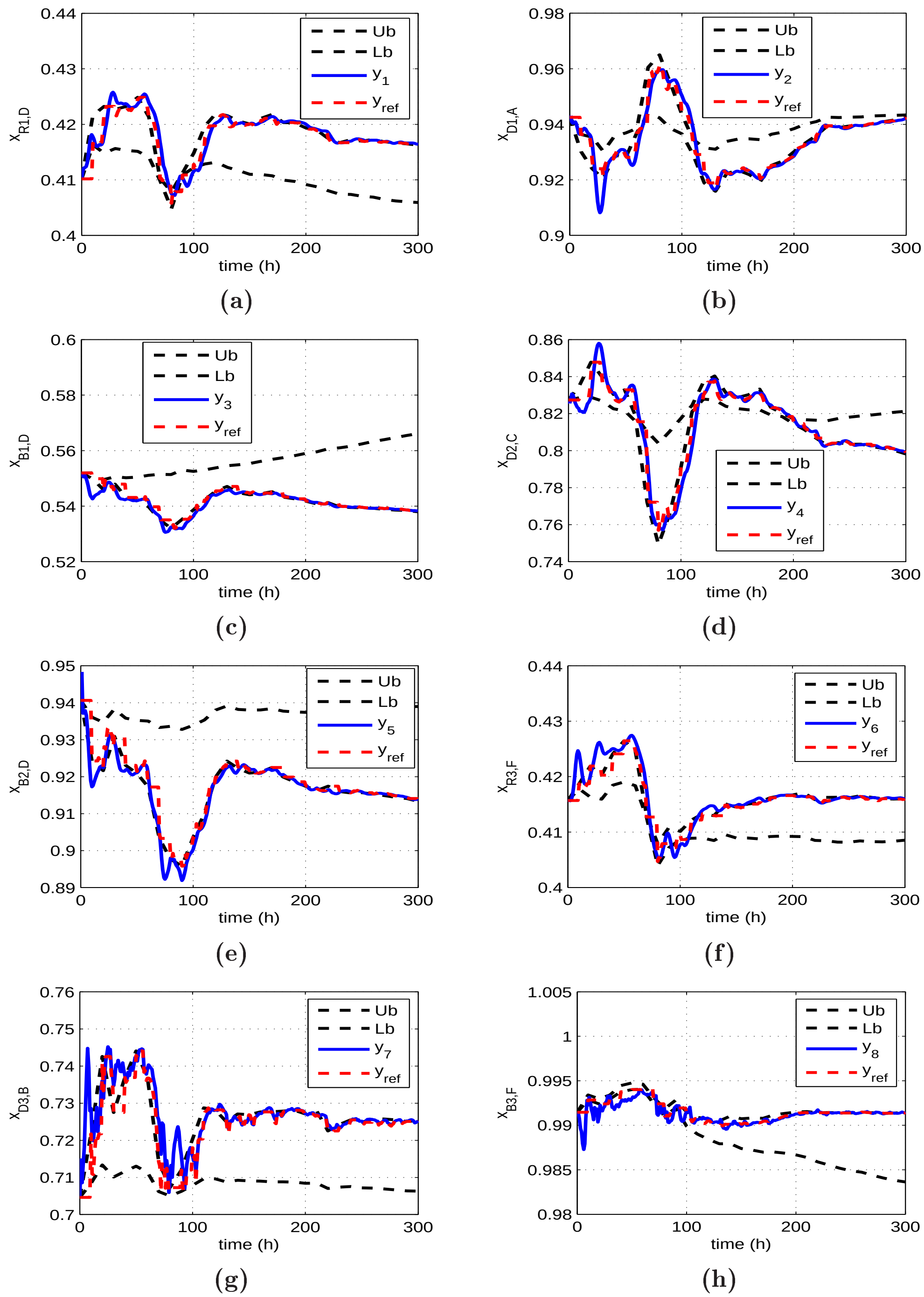


Figure 6.5: Outputs of the chemical plant

6.4 Summary

In chapter 4 some strategies for hierarchical robust control using zone control were presented. However, a problem arises from these structures. The problem lies in how to calculate the limits (zone) of the MPC with zone control. Generally these limits corre-

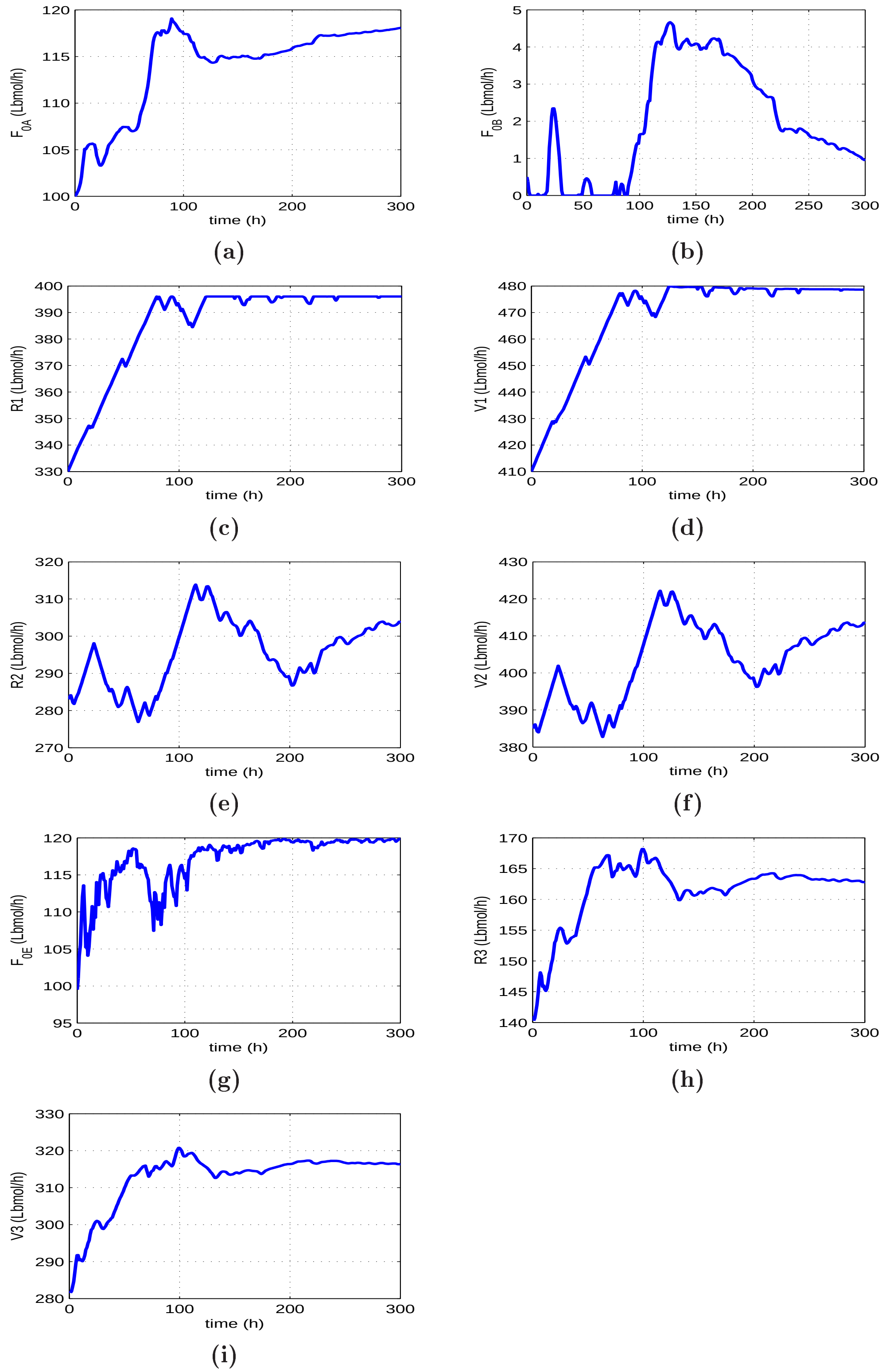


Figure 6.6: Inputs of the chemical plant

spond to physical limitations of the process, nervertheless, this is not always the case, and the development of a criteria to calculate these constraints is required. A reasonable criterion could be the zone where the linearization has good approximation to the nonlinear system or physical limitations of the process, however, a better choice can arise from the economic point of view. In this sense in this chapter a hierarchical robust strategy where the limits of the MPC with zone control are calculated based in an economical criterion was presented. An illustrative example was used in order to illustrate the strategy and to show a particular case when the limits found were the same. This chapter ends with the application of the hierarchical methodology to a large scale system, that serves as benchmark to test the structure proposed.

CHAPTER 7

Conclusions and future work

In this thesis, the problem of design hierarchical controllers for large scale interconnected systems was addressed. With this purpose, some concepts were reunited and used in order to propose a hierarchical control structure for large scale systems. The main concept was certainly zone control. This concept was introduced and used in every chapter of this thesis, demonstrating that this can be used in several applications including large scale systems. This thesis begins with a literature review about hierarchical decompositions and the current practices in hierarchical operations in chapter 2. From this literature review some research topics arose, such as fixed set points in large scale systems without cooperation between the local controllers and the robust operations in hierarchical controller. In this sense the chapter 3 was dedicated to robust controllers and robust optimization. The main contributions of chapter 3 are listed below:

- A new solution of Robust Quadratic Programming (RQP) under different uncertain sets was proposed. This approach takes into account uncertainty in the quadratic, linear and constant term of the quadratic optimization problem. The main contribution of this section is the conversion of the the RQP problem into a Second Order Cone Programming (SOCP) problem. In this sense, the robust optimization problem can be transformed in a convex problem with polynomial complexity.
- A new Robust Model Predictive Control formulation was proposed. There are two important contributions in this direction, the first one is about the uncertainty representation. For the robust MPC proposed uncertainty in the free and forced response was assumed; although this type of uncertainty can be expressed in typical forms reported in the robust MPC literature, in this sense the uncertainty representation presented here is fairly complete. The most remarkable contribution in the new Robust Model Predictive Control formulation is the way to find the parameters of the uncertain sets. we proposed a novel method to obtain these parameters based in mappings between the uncertainty of the free and forced response and the uncertainty in the quadratic, linear and constant term of the quadratic optimization problem. Finally we show the benefits of the proposed controller using an illustrative example.
- With the results of the new robust MPC, new formulations for Robust MPC with

zone control and Robust Economic MPC was proposed. Again, a new way to find the parameters of the uncertain sets is stated. Finally, we test the proposed controllers through two application examples.

In chapter 4 and 5 some strategies for hierarchical robust control were proposed. The main idea is to combine the robust MPC approaches presented in chapter 3 and the zone control concept with the classical hierarchical structure presented in chapter 2. The principal advantages of the strategies presented in these chapters are the ability of handling uncertainties, the tackling of different objectives (Economic), and the inclusion of the dynamic behavior of the system. In this way these approaches improve the structures reported in the literature. The main contributions of chapter 4 and 5 are listed below:

- The first result in chapter 4 is a general robust hierarchical strategy that uses Dynamic Real time Optimization (DRTO), robust control and a decentralized MPC scheme, the idea with this scheme is to give sufficient tools and alternatives to control a large scale system taking into account both economical and tracking objectives and uncertainties.
- The first result is the hierarchical robust control for coordination. In this direction two hierarchical robust structures are proposed, the first one takes a robust MPC with zone control as coordinator, while in the lower layer of the hierarchical structure a decentralized scheme is employed. The second one takes a Dynamic Real Time Optimization (DRTO) layer as coordinator while in the lower layer a decentralized scheme based in MPC with zone control is used.

In Chapter 4 and 5 the potential of application of the robust strategies of the Chapter 4 and 5 are shown by means of two important applications in the field of distributed systems. These examples illustrate how it is possible to handle uncertainties in distributed systems by means of robust hierarchical controllers. The example used in chapter 4 was the two reactors chain and flash system. A hierarchical robust integration of MPC and DRTO was applied to this system. There are two important contributions for this example. The first one is the operating point of the system, in previous works the operating point was not realistic (CSTR levels in the order of kilometers). In this way we found by means of the DRTO layer realistic operating points. The second contribution was described in the simulations result, were clearly the nominal scheme was not capable to maintain the outputs of the system under the zone while the quality specifications were achieved. Finally, the example used in chapter 5 was the Hydro Power Valley system. This system is used as benchmark in hierarchical and distributed MPC schemes. A hierarchical robust control with zone control for coordination was implemented in the HPV. The simulation results were very interesting, achieving a good performance of the scheme following a power profile during two days while the relevant variables remain in the specific zone.

From the strategies for hierarchical robust control using zone control presented in chapter 4 and 5 some problems arises. The problem lies in how to calculate the limits (zone) of the MPC with zone control. Generally these limits correspond to physical limitations of the process, nervertheless, this is not always the case, and the development of a criteria to calculate these constraints is required. A reasonable criterion could be the zone when the linearization has well approximation to the nonlinear system or physical limitations of the process, however, a better choice can arise from the economic point of view. In this sense the most remarkable contribution of this thesis is a hierarchical robust strategy where the limits of the MPC with zone control are calculated based in an economical criterion. This hierarchical structure was presented in chapter 6, and combines robust optimization and zone control. By means of robust optimization we calculate the limits of the MPC with zone control, with the idea of determining the economic limits for this controller. This strategy gives freedom from an economical point of view to the local controller of the hierarchical structure. An important aspect to highlight is that the hierarchical solution proposed here can be seen as a general hierarchical robust structure for large scale systems, because little variations over this structure can produce a classical hierarchical representation such as those shown in chapter 2. Finally, in chapter 6 a case of study was used to illustrate the proposed method.

7.1 Future work

From the development of this thesis several research problems have been identified, these problems are listed below:

- In the field of robust control there are basically two future work: From chapter 3, the number of constraints generated in the proposed Robust MPC method should be reduced. The second one problem is that the method proposed here needs a linealization of the original model, in this thesis a linear time-varying model is used, but this approach not always is the best election for example for batch processes. In this context the robust optimization method proposed could be extended to second order approximations of the original model.
- In the field of hierarchical control, the layer of *"Planning and Scheduling"* should be added to the robust hierarchical schemes proposed in this thesis, this layer involves discrete decisions, which needs to be optimized simultaneously with the other layers. Finally, the proposed control schemes must be tested in other large-scale systems in order to validate its applicability in real systems.

APPENDIX A

Robust Optimization: Robust Quadratic Programming approach

A.1 Formulation of Robust Optimization Problems

Optimization problems under parameter uncertainty have been a common focus of study of the mathematical programming community. In fact, it has been known that solutions to optimization problems can show a high sensitivity to uncertainties in the parameters of the problem, making the computed solution highly unfeasible, sub-optimal, or both. In this sense, approximations such as stochastic optimization assumes that the uncertainty has a probabilistic representation. This approach has its origin in the paper of Dantzig in 1955 [148]. This thesis considers Robust Optimization in a deterministic sense, i.e the model uncertainty is not stochastic. Instead, in order to shield the solution in some probabilistic sense, a solution that is optimal for any realization of the uncertainty in a given set is constructed. The motivation for this approach is twofold. First, there are many applications where the more appropriated description of parameter uncertainty is set based. Besides this, achieving a feasible degree of computational manageability is also the most important aim and motivation. This fact has influenced the theoretical trajectory of Robust Optimization, and, recently, it has been responsible for its success in a wide variety of fields. In this way, the first person to investigate explicit approaches to Robust Optimization was Soyster in 1970 [149]. This work was focused in robust linear optimization in a situation where the column vectors of the constraint matrix were constrained to belong to ellipsoidal uncertainty sets. Few years later Falk [150], following the work of Soyster published a work about inexact linear programs. After these works, the robust optimization community was relatively quiet, until the work of Ben-Tal and Nemirovski ([151, 152, 71]) and El Ghaoui et al. [153, 154] in the late 1990s. This work was performed together with advances in computing and the development of interior point methods for convex optimization.

A standard optimization problem consists typically of a given continuous objective function $f_0 : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_\theta} \rightarrow \mathbb{R}$, some compact sets $\mathbb{X} \subseteq \mathbb{R}^{n_x}$ and $\mathbb{U} \subseteq \mathbb{R}^{n_u}$ of feasible points and inequalities constraints $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_\theta} \rightarrow \mathbb{R}^m$. Here, our aim is to minimize the function f_0 over the variables which are in the sets $\mathbb{X}$ and $\mathbb{U}$. In other words, we are interested in an optimization problem of the form,

$$\begin{aligned} &\min_{x\in\mathbb{X},u\in\mathbb{U}} f_0(x,u,\theta) \\ \text{subject to:}& \\ &f(x,u,\theta) \leq 0 \\ &x\in\mathbb{X},\ u\in\mathbb{U} \end{aligned} \tag{A.1}$$

In this notation, f_0 and f depend on a parameter $\theta \in \mathbb{R}^{n_\theta}$. If we know this parameter exactly, there is nothing special about this optimization problem respect to θ. However, if that is not the case, we assume that our information about θ is that this parameter is in a given compact set $\Psi \subseteq \mathbb{R}^{n_\theta}$. In this way, it is necessary to incorporate the uncertainty θ into the optimization problem. Basically there are two forms to do this::

1. semi-infinite optimization problem: In this case the optimization problem (A.1) can be expressed as:

$$\begin{aligned} &\min_{x\in\mathbb{X},u\in\mathbb{U}} f_0(x,u,\theta) \\ \text{subject to:}& \\ &f_i(x,u,\theta) \leq 0,\ i\in\{1,...,m\},\ \forall\theta\in\Psi \\ &x\in\mathbb{X},\ u\in\mathbb{U} \end{aligned} \tag{A.2}$$

   In general, the problem (A.2) has an infinite number of constraints. This is why robust optimization problems are also called semi-infinite optimization problems, because we have on the one hand infinite constraints, but on the other hand a finite number of optimization variables.

   The most important advantages of the semi-infinite optimization are:

   (a) The natural way to extend the notation to vector or matrix valued functions f_i in combination with generalized inequalities.

   (b) In applications where the uncertainty θ is of a stochastic nature, it makes sense to constraint its own variability. This is the case where θ is a random variable with a given probability distribution.

2. Worst case: Another form to take into account the knowledge about the uncertainty θ in the optimization problem is follow the concept of robust counterpart, which has been established by Ben-Tal and Nemirovski [155, 151]. Here, the assumption is that we want to minimize the worst possible value of the function f_0, i.e., we are interested in a min-max problem of the form,

$$
\begin{aligned}
&\min_{x\in\mathbb{X},u\in\mathbb{U}} \max_{\theta\in\Psi} f_0(x,u,\theta) \\
\text{subject to:}& \\
&\max_{\theta\in\Psi} f_i(x,u,\theta) \leq 0,\ i \in \{1,...,m\} \\
&x \in \mathbb{X},\ u \in \mathbb{U}
\end{aligned}
\tag{A.3}
$$

It is important to highlight that similar to the maximization of the objective value, in the optimization problem (A.3) it is assumed that the robust counterpart is like a player that always chooses the worst possible value for the functions $f_1, \ldots, f_m$.

Due to the bi-level structure and the semi-infinite character, both formulations (A.2) and (A.3) pose challenges on the efficiency of the numerical solution [156]. Different approaches to confront these problems have been presented in a large number of articles. Methods to solve the semi-infinite optimization problem include discretization of the uncertainty set Ψ and the local reduction approach introduced by Hettich and Kortanek in 1993,[157]. In both approaches the semi-infinite optimization problem is approximated by nonlinear programming problems with a finite number of constraints. In discretization methods problem, equation (A.2) is solved on a finite grid of points $\overline{\Psi} \subseteq \Psi$ within the uncertainty set. The local reduction approach is based on the worst-case min-max formulation.

A.1.1 Existing Approaches for Robust Optimization

The last decades, robust optimization has been the center of interest of many research fields such as control, convex optimization, mathematical programming and many fields of engineering science. In general, when an optimization problem is formulated, a question about whether all parameters and inputs are known arises. In this way, many researchers are attracted by the challenges of robust optimization. In general terms there are seven approaches to the Robust Optimization problems [158] which are listed below:

- Stochastic Programming
- Classical Robust Control Theory
- Convex Robust Optimization
- Nonconvex Robust Optimization
- Classical Optimal Control Theory
- Robust Open-Loop Optimal Control
- Robust Closed-Loop Control

As a substantial part of this thesis is based on convex optimization, the idea in this chapter is to formulate an optimization problem, and then show that the problem can be shaped as some standard convex optimization problem. The main property of convex optimization problems is that it is possible to build algorithms that guarantee an optimal solution, if one exists, or verify whether there exist no solution. Additionally, it is possible to develop algorithms with polynomial complexity (interior point methods). This means that the computational effort required to find the solution grows polynomially with respect to the problem dimensions. In other words, the main objective of this chapter is to propose efficient algorithms that scale well with problem size. The introduction of convex optimization, and semidefinite programming in particular, as a standard mathematical tool has had, and still has, a profound impact on control and systems theory [159]. There is a great amount of literature on convex optimization. The book of Boyd and Vandenberghe [160] helps as an excellent introduction to both mathematical and engineering aspects of convex optimization. Another interesting source is the book of Ben-Tal and Nemirovski [161].

In the area of Convex Robust Optimization the main developments were done by Ben-Tal and Nemirovski [151],[152],[71] and independently by El-Ghaoui [153],[154]. These approaches help to transform a great number of min-max optimization problems into convex optimization problems. A typical assumption in the mentioned works is that the uncertainty set is ellipsoidal, which is in many cases the key for working out robust counterpart formulations. A common example is a linear program (LP) with uncertain data, in this case the optimization problem can be formulated as a second order cone program (SOCP). However, in the control context, polytopic uncertainty sets are also a common choice [162, 163]. The robust convex optimization field has been expanded the last years and it is still shows work in progress. However, these developments tend more and more towards approximation techniques, where the robust counterpart problem is replaced by more tractable formulations, and they also cover an increasing amount of applications [158]. For an extensive overview on robust optimization from the convex perspective, we refer to the latest book by Ben-Tal, El-Ghaoui, and Nemirovski [155].

A.2 Robust Quadratic Programming (RQP)

Robust programming is a class of optimization problems where the parameters are uncertain and belong to a defined set. In the last decade, efficient interior point methods with polynomial complexity have been developed for this type of problems. Indeed, the computational complexity for the second-order cone program (SOCP) solution (which is the method presented in this Section) is proportional to $\sqrt{l}$, with l being the number of constraints [72]. This characteristic makes these algorithms very interesting for large-scale optimization problems and open possibilities in the application of these methods not only for robust optimization but also for plant-wide optimizers . This Section is based on the work of [90] and we will focus our attention in the problem of robust quadratic programming (RQP).

Let $x \in \mathbb{R}^n$ denote the decision variables, $P \in \mathbb{R}^{n \times n}$ be a symmetric positive definite matrix, $q \in \mathbb{R}^n$ and $r \in \mathbb{R}$ be a coefficients vector and a constant term respectively. Then, the RQP problem is defined by (A.4).

$$\min_{x} \max_{P \in \mathbb{E}, q \in \mathbb{F}, r \in \mathbb{G}} \quad x^T P x + 2q^T x + r \tag{A.4}$$

Notice in (A.4) that the uncertainties are in P, q and r. Those uncertainties are respectively described by the sets $\mathbb{E}$, $\mathbb{F}$ and $\mathbb{G}$.

A.2.1 Robust Quadratic Programming with ellipsoidal uncertainties

Assume uncertainties in P, q and r are norm bounded. Then, $\mathbb{E}$, $\mathbb{F}$ and $\mathbb{G}$ can be written as

$$\begin{aligned} \mathbb{E} &= \left\{ P_0 + \sum_{i=1}^{m} P_i \mu_i \mid \|\mu\|_2 \leq 1 \right\} \\ \mathbb{F} &= \left\{ q_0 + \sum_{j=1}^{z} q_j \nu_j \mid \|\nu\|_2 \leq 1 \right\} \\ \mathbb{G} &= \left\{ r_0 + \sum_{l=1}^{n_r} r_l \xi_l \mid \|\xi\|_2 \leq 1 \right\} \end{aligned} \tag{A.5}$$

with $P_0, P_i \in \mathbb{R}^{n\times n}$, $\mu \in \mathbb{R}^m$, q_0, $q_j \in \mathbb{R}^n$, $\nu \in \mathbb{R}^z$, $r_0, r_l \in \mathbb{R}$ and $\xi \in \mathbb{R}^{n_r}$. Including the uncertainty description in (A.4), the optimization problem (A.4) can be written as an Second Order Cone Programming (SOCP) problem. Such problem is an special case of semi-definite programming (SP), but, in SOCP there exist specific efficient interior point methods for solving it (more efficient than the methods available for general SP problems). The way to transform the problem (A.4) in a SOCP problem, is stated in the following theorem:

Theorem 4. *Let $x \in \mathbb{R}^n$ be the decision variables, $P \in \mathbb{R}^{n\times n}$ be a symmetric positive definite matrix, $q \in \mathbb{R}^n$ and $r \in \mathbb{R}$ are coefficients vector and a constant term respectively of the robust optimization problem* (A.4). *Then, the problem* (A.4) *can be written as the following Second Order Cone Programming (SOCP) problem:*

$$\begin{aligned} & \min_{x,w_i,s,d,t} s + 2q_0^T x + t + 2d + \|\overline{r}\|_2 \\ & \textit{subject to:} \\ & \left\| \begin{bmatrix} 2P_0^{1/2} x \\ s-1 \end{bmatrix} \right\|_2 \leq s+1 \\ & \left\| \begin{bmatrix} 2P_i^{1/2} x \\ w_i-1 \end{bmatrix} \right\|_2 \leq w_i+1 \\ & \|q_j x\|_2 \leq d \\ & \|w\|_2 \leq t \\ & 0 \leq w \\ & 0 \leq s \end{aligned} \tag{A.6}$$

where $\overline{r} = [r_1, \ldots, r_{n_r}]^T$. Observe that in (A.6) *the search space is extended from the space of x to the spaces of s, w_i, t and d.*

Proof 1. *In order to prove the theorem, the uncertainty description* (A.5) *is replaced in* (A.4), *yielding*

$$\min_x \max_{\|\mu\|_2\leq 1, \|\nu\|_2\leq 1, \|\xi\|_2\leq 1} \quad x^T P_0 x + \sum_{i=1}^{m} x^T P_i x \mu_i + 2q_0^T x + 2\sum_{j=1}^{z} q_j^T x \nu_j + r_0 + \sum_{l=1}^{n_r} r_l \xi_l$$

The above problem is equivalent to:

$$\min_x \quad (x^T P_0 x + 2q_0^T x + r_0) + \max_{\|\mu\|_2\leq 1, \|\nu\|_2\leq 1, \|\xi\|_2\leq 1} \quad x_{p_i}\mu + 2x_{q_i}\nu + \overline{r}\xi$$

where $x_{p_i} = [x^T P_1 x, \ldots, x^T P_m x]$, $x_{q_i} = [q_1^T x, \ldots, q_z^T x]$ *and* $\overline{r} = [r_1, \ldots, r_{n_r}]^T$. *Applying the Cauchy-Schwarz inequality, we have*

$$\min_x \quad (x^T P_0 x + 2q_0^T x + r_0) + \max_{\|\mu\|_2\leq 1, \|\nu\|_2\leq 1, \|\xi\|_2\leq 1} \quad \|x_{p_i}\|\|\mu\| + 2\|x_{q_i}\|\|\nu\| + \|\overline{r}\|\|\xi\|$$

Finally the RQP problem (A.4) *can be written as an LP problem with quadratic constraints like* (A.7).

$$\begin{aligned}
&\min_{x,w_i,s,d,t} s + 2q_0^T x + t + 2d + \|\overline{r}\|_2 \\
&\textit{subject to:} \\
&\quad x^T P_0 x \leq s \\
&\quad x^T P_i x \leq w_i \\
&\quad \|q_j x\|_2 \leq d \\
&\quad \|w\|_2 \leq t
\end{aligned} \tag{A.7}$$

where $\overline{r} = [r_1, \ldots, r_{n_r}]^T$. *So, the SOCP formulation of* (A.7) *is as follows:*

$$\begin{aligned}
&\min_{x,w,s,d,t} s + 2q_0^T x + t + 2d + \|\overline{r}\|_2 \\
&\textit{subject to:} \\
&\quad \left\| \begin{bmatrix} 2P_0^{1/2} \\ s-1 \end{bmatrix} \right\|_2 \leq s+1 \\
&\quad \left\| \begin{bmatrix} 2P_i^{1/2} \\ w_i-1 \end{bmatrix} \right\|_2 \leq w_i+1 \\
&\quad \|q_j x\|_2 \leq d \\
&\quad \|w\|_2 \leq t \\
&\quad 0 \leq w_i \\
&\quad 0 \leq s
\end{aligned} \tag{A.8}$$

Observe that in (A.8) *the search space is extended from the space of x to the spaces of f, w_i, t and d*

A.2.2 Robust Quadratic Programming with polytopic uncertainties

In this case the uncertainties in P, q and r are norm bounded and $\mathbb{E}$, $\mathbb{F}$ and $\mathbb{G}$ can be written as

$$\mathbb{E} = \mathbf{Co}\{P_1, \ldots, P_m\} = \left\{ P : P = \sum_{i=1}^{m} P_i \mu_i, \sum_{i=1}^{m} \mu_i = 1, \mu_i \geq 0 \right\}$$
$$\mathbb{F} = \mathbf{Co}\{q_1, \ldots, q_z\} = \left\{ q : q = \sum_{j=1}^{z} q_j \nu_j, \sum_{j=1}^{z} \nu_j = 1, \nu_j \geq 0 \right\}$$
$$\mathbb{G} = \mathbf{Co}\{r_1, \ldots, r_{n_r}\} = \left\{ r : r = \sum_{l=1}^{n_r} r_l \xi_l, \sum_{l=1}^{n_r} \xi_l = 1, \xi_l \geq 0 \right\}$$

where the notation $\mathbf{Co}\{\cdot\}$ is used to denote the convex hull, with $P_i \in \mathbb{R}^{n \times n}$, $q_j \in \mathbb{R}^n$ and $r_l \in \mathbb{R}$. Making the same procedure of the previous section i.e. replacing the uncertainty description in (A.4), the optimization problem (A.4) considering polytopic uncertainty can be written as an Second Order Cone Programming (SOCP) problem as follows:

$$\begin{aligned} & \min_{x,w,d,t} \; t + 2d + \|\overline{r}\|_2 \\ \text{subject to:} & \\ & \left\| \begin{bmatrix} 2P_i^{1/2} x \\ w_i - 1 \end{bmatrix} \right\|_2 \leq w_i + 1 \\ & \|q_j x\|_2 \leq d \\ & \|w\|_2 \leq t \\ & 0 \leq w_i \end{aligned} \tag{A.9}$$

where $\overline{r} = [r_1, \ldots, r_{n_r}]^T$. Observe that in (A.9) the search space is extended from the space of x to the spaces of w_i, t and d.

A.2.3 Robust Quadratic Programming with finite sets uncertainties

Assume that uncertainties in P, q and r are given by

$$\mathbb{E} = \{P_1, \ldots, P_m\}$$
$$\mathbb{F} = \{q_1, \ldots, q_z\}$$
$$\mathbb{G} = \{r_1, \ldots, r_{n_r}\}$$

Again replacing the uncertainty description in (A.4), the optimization problem (A.4) can be written as,

$$\min_{x} \max_{i\in\mathsf{E},j\in\mathsf{F},l\in\mathsf{G}} \quad x^T P_i x + 2q_j^T x + r_l \tag{A.10}$$

with $\mathsf{E} = \{1,\ldots,m\}$, $\mathsf{F} = \{1,\ldots,z\}$ and $\mathsf{G} = \{1,\ldots,n_r\}$. With the previous description of the sets, the optimization problem (A.10) can be written as an Second Order Cone Programming (SOCP) problem as follows:

$$\begin{aligned}
& \min_{x,w,d,t} w + 2t + d \\
\text{subject to:} & \\
& \left\| \begin{bmatrix} 2P_i^{1/2}x \\ w-1 \end{bmatrix} \right\|_2 \leq w+1,\ i \in \mathsf{E} \\
& q_j^T x \leq t,\ j \in \mathsf{F} \\
& r_l \leq d,\ l \in \mathsf{G} \\
& 0 \leq w
\end{aligned} \tag{A.11}$$

as mentioned before, it is important to highlight that in (A.11) the search space is extended from the space of x to the spaces of w, t and d.

APPENDIX B

Local linearization

In order to implement a min-max predictive control algorithm, long-term prediction of the key states is required. Clearly, since the underlying system is nonlinear, the future states (and hence the outputs) are related to the current states and current/future inputs in a nonlinear fashion. This makes the problem of finding the optimal input sequence a complex nonlinear optimization problem. This work propose to make the relationship linear via local linearization using the approach proposed by [88].

The one-step ahead prediction of the states is defined as

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= g(x(k), u(k)) \end{aligned} \tag{B.1}$$

More generally, the multi-step prediction is

$$\begin{aligned} x(k+j) &:= f(\ldots(f(x(k), u(k))\ldots), u(k+j-1)) \\ x(k+j) &:= f_j(x(k), \{u(k+i)\}_{i=0}^{j-1}) \end{aligned}$$

Now, if $f(x(k), u(k))$ is linearized around some nominal input value $u^*(k)$ we have

$$x(k+1) = f(x(k), u^*(k)) + A_k(x(k) - x^*(k)) + B_k(u(k) - u^*(k)) \tag{B.2}$$

where

$$A_k = \frac{\partial f}{\partial x}\Big|_{x^*(k), u^*(k)}, \quad B_k = \frac{\partial f}{\partial u}\Big|_{x^*(k), u^*(k)}$$

It is possible generalize the idea to develop multi-step predictions. Note from (B.1) that $x(k+2) = f(x(k+1), u(k+1))$, where $x(k+2)$ is related in a nonlinear fashion not only to $u(k+1)$, but also to $u(k)$ appearing in expression (B.1) for $x(k+1)$. By appropriate linearization, it is possible to derive an approximation that is linear with respect to the undecided inputs $u(k)$ and $u(k+1)$. The linear relationship that approximates the local behavior can be obtained by linearizing the expression $f(x(k+1), u(k+1))$ with respect to $x^*(k+1) = f(x(k), u^*(k))$ and $u^*(k+1)$ as follows

$$\begin{aligned}
x(k+2) &= f(x^*(k+1), u^*(k+1)) + A_{k+1}(x(k+1) - x^*(k+1)) \\
&\quad + B_{k+1}(u(k+1) - u^*(k+1)), \\
x(k+2) &= f(f(x(k), u^*(k)), u^*(k+1)) + A_{k+1}(x(k+1) - f(x(k), u^*(k))) \\
&\quad + B_{k+1}(u(k+1) - u^*(k+1)), \\
x(k+2) &= f_2(x(k), u^*) + A_{k+1}(x(k+1) - f(x(k), u^*(k))) + B_{k+1}(u(k+1) - u^*(k+1))
\end{aligned} \tag{B.3}$$

where

$$A_{k+1} = \frac{\partial f}{\partial x}\Big|_{f(x(k),u^*(k)),u^*(k+1)}, \quad B_{k+1} = \frac{\partial f}{\partial u}\Big|_{f(x(k),u^*(k)),u^*(k+1)}$$

Note that u^* is a piece-wise constant input taking, for instance, values of $\{u^*(k); u^*(k+1)\}$ at the time interval $[k, k+2]$. Note from (B.2) that $x(k+1) - f(x(k), u^*(k) = A_k(x(k) - x^*(k)) + B_k(u(k) - u^*(k))$, then

$$\begin{aligned}
x(k+2) &= f_2(x(k), u^*) + A_{k+1}A_k(x(k) - x^*(k)) + A_{k+1}B_k(u(k) - u^*(k)) \\
&\quad + B_{k+1}(u(k+1) - u^*(k+1)), \\
x(k+2) &= f_2(x(k), u^*) + A_{k+1}A_k(x(k) - x^*(k)) + \begin{bmatrix} A_{k+1}B_k & B_{k+1} \end{bmatrix} \begin{bmatrix} u(k) - u^*(k) \\ u(k+1) - u^*(k+1) \end{bmatrix}
\end{aligned} \tag{B.4}$$

Carrying out the same derivations for $x(k+j)$ we obtain

$$\begin{aligned}
x(k+j) &= f_j(x(k), u^*) + \prod_{i=0}^{j-1} A_{k+i}(x(k) - x^*(k)) \\
&\quad + \begin{bmatrix} \prod_{i=1}^{j-1} A_{k+i}B_k & \prod_{i=2}^{j-1} A_{k+i}B_{k+1} & \dots & B_{k+j-1} \end{bmatrix} \begin{bmatrix} u(k) - u^*(k) \\ u(k+1) - u^*(k+1) \\ \vdots \\ u(k+j-1) - u^*(k+j-1) \end{bmatrix}
\end{aligned} \tag{B.5}$$

To reduce the computational complexity, the matrices A_{k+i} and B_{k+i} can be kept constant at the initial values of A_k and B_k throughout the prediction horizon. Hence, (B.5) simplifies to

$$\begin{aligned} x(k+j) &= f_j(x(k), u^*) + A_k^j(x(k) - x^*(k)) \\ &+ \begin{bmatrix} A_k^{j-1}B_k & A_k^{j-2}B_k & \dots & B_k \end{bmatrix} \begin{bmatrix} u(k) - u^*(k) \\ u(k+1) - u^*(k+1) \\ \vdots \\ u(k+j-1) - u^*(k+j-1) \end{bmatrix} \end{aligned} \tag{B.6}$$

In order to develop a prediction for the output that is linear with respect to the undecided input moves, we linearize $y(k+j)$ with respect to $x(k)$ and $u^*(k+j)$ and carrying out the same idea,

$$y(k+j) = g(x(k), u^*(k+j)) + C_k(x(k+j) - x^*(k)) + D_k(u(k+j) - u^*(k+j)) \tag{B.7}$$

we obtain the output prediction. Combine (B.7) with the optimal multi-step prediction equation (B.6) for to obtain the complete output prediction in a N_p horizon, let $y_k = [y^T(k), \dots, y^T(k+N_p)]^T$, then,

$$y_k = \underbrace{\Gamma}_{\text{Free Response}} + \underbrace{\Lambda \delta u_k}_{\text{Forced Response}} \tag{B.8}$$

with $\delta u_k = [\delta u^T(k), \dots, \delta u^T(k+N_p)]^T$, $\delta u(k+j) = u(k+j) - u^*(k+j)$, $\Gamma = \overline{C}\Phi$ and $\Lambda = [\overline{C}H \;\; 0] + [0 \;\; \overline{D}]$, where $\overline{C} = \text{diag}(\underbrace{[C_k, \cdots, C_k]}_{N_p})$, $\overline{D} = \text{diag}(\underbrace{[D_k, \cdots, D_k]}_{N_p})$, and

$$\Phi = \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix} x^*(k) + \begin{bmatrix} g(x(k), u^*(k)) \\ g(x(k), u^*(k+1)) \\ \vdots \\ g(x(k), u^*(k+N_p)) \end{bmatrix} + \begin{bmatrix} f_1(x(k), u^*) \\ f_2(x(k), u^*) \\ \vdots \\ f_{N_p}(x(k), u^*) \end{bmatrix} + \begin{bmatrix} A_k \\ A_k^2 \\ \vdots \\ A_k^{N_p} \end{bmatrix} (x(k) - x^*(k))$$

$$H = \begin{bmatrix} B_k & 0 & \cdots & 0 \\ A_k B_k & B_k & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_k^{N_p-1}B_k & A_k^{N_p-2}B_k & \cdots & B_k \end{bmatrix},$$

The local linearization in (B.5) makes sense only when the computed inputs $\{u(k+i)\}_{i=0}^{j-1}$ do not deviate much from u^*. For nonlinear models this can be achieved by finding a nominal trajectory $u^*(k+j)$ which is as close as possible to the optimal strategy $u_k^{\text{opt}}(k+j)$ which is the result of the MPC optimization problem. A simple but effective choice is to start with $u^*(k+j) = u_{k-1}^{\text{opt}}(k+j)$, i.e. the optimal control policy derived at the previous sample.

APPENDIX C

Uncertainty description

The systems covered in this work are constrained uncertain systems of the form

$$\begin{aligned} x(k+1) &= E_k + (A_k + \delta A_k)x(k) + (B_k + \delta B_k)u(k) \\ y(k) &= F_k + C_k x(k) + D_k u(k) \\ \theta &\triangleq (\delta A_k, \delta B_k) \in \Psi \end{aligned} \tag{C.1}$$

from appendix B we have that $E_k = f(x(k), u^*(k)) - A_k x^*(k)$ and $F_k = g(x(k), u^*(k)) - C_k x^*(k)$. In this work ellipsoidal uncertainty in A_k and B_k is considered, however, it is possible to propagate this uncertainty in the free and force responses by means of the prediction sequence of the output $y(k)$ as (C.2)

$$y_k = \underbrace{(\Gamma + \delta\Gamma)}_{\text{Free Response}} + \underbrace{(\Lambda + \delta\Lambda)u_k}_{\text{Forced Response}} \tag{C.2}$$

where $\Gamma + \delta\Gamma = \overline{C}\,(\Phi + \delta\Phi)$ and $\Lambda + \delta\Lambda = [\overline{C}\,(H + \delta H) \quad 0] + [0 \quad \overline{D}]$, then $\delta\Gamma = \overline{C}\delta\Phi$ and $\delta\Lambda = [\overline{C}\delta H \;\; 0]$. From appendix B we have that the uncertainty in A_k and B_k is propagated like this,

$$\Phi + \delta\Phi = \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix} x^*(k) + \begin{bmatrix} g(x(k), u^*(k)) \\ g(x(k), u^*(k+1)) \\ \vdots \\ g(x(k), u^*(k+N_p)) \end{bmatrix} + \begin{bmatrix} f_1(x(k), u^*) \\ f_2(x(k), u^*) \\ \vdots \\ f_{N_p}(x(k), u^*) \end{bmatrix}$$
$$+ \begin{bmatrix} (A_k + \delta A_k) \\ (A_k + \delta A_k)(A_k + \delta A_{k+1}) \\ \vdots \\ \prod_{i=0}^{N_p} (A_k + \delta A_{k+i}) \end{bmatrix} (x(k) - x^*(k))$$

$$H + \delta H =$$
$$\begin{bmatrix} (B_k + \delta B_k) & 0 & \cdots & 0 \\ (A_k + \delta A_k)(B_k + \delta B_k) & (B_k + \delta B_{k+1}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \prod_{i=0}^{N_p-1}(A_k + \delta A_{k+i})(B_k + \delta B_k) & \prod_{i=0}^{N_p-2}(A_k + \delta A_{k+i})(B_k + \delta B_{k+1}) & \cdots & (B_k + \delta B_{k+N_p}) \end{bmatrix}$$

Making some algebraic operations, $\Phi + \delta\Phi$ and $H + \delta H$ can be expressed as,

$$\Phi + \delta\Phi = \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix} x^*(k) + \begin{bmatrix} g(x(k), u^*(k)) \\ g(x(k), u^*(k+1)) \\ \vdots \\ g(x(k), u^*(k+N_p)) \end{bmatrix} + \begin{bmatrix} f_1(x(k), u^*) \\ f_2(x(k), u^*) \\ \vdots \\ f_{N_p}(x(k), u^*) \end{bmatrix} + \begin{bmatrix} A_k \\ A_k^2 \\ \vdots \\ A_k^{N_p} \end{bmatrix} (x(k) - x^*(k))$$
$$+ \begin{bmatrix} \delta A_k \\ A\delta A_{k+1} + \delta A_k(A_k + \delta A_{k+1}) \\ \vdots \\ \sum_{i=1}^{N_p} A_k^{Np-i} \delta A_{k+Np-i} \prod_{l=0}^{i-1}(A_k + \delta A_{k+Np-l}) \end{bmatrix} (x(k) - x^*(k))$$

$$H + \delta H = \begin{bmatrix} B_k & 0 & \cdots & 0 \\ A_k B_k & B_k & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_k^{N_p-1} B_k & A_k^{N_p-2} B_k & \cdots & B_k \end{bmatrix}$$
$$+ \begin{bmatrix} \delta B_k & 0 & \cdots & 0 \\ A_k \delta B_k + \delta A_k(B_k + \delta B_k) & \delta B_{k+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_k^{Np-1}\delta B_k + (B_k + \delta B_k)\eta_{N_p-1} & A_k^{Np-2}\delta B_{k+1} + (B_k + \delta B_{k+1})\eta_{N_p-2} & \cdots & (B_k + \delta B_{k+N_p}) \end{bmatrix}$$

where

$$\eta_j = \sum_{i=1}^{j} A_k^{j-i} \delta A_{k+j-i} \prod_{l=0}^{i-1} (A_k + \delta A_{k+j-l})$$

finally $\delta\Phi$ and δH are given by

$$\delta\Phi = \begin{bmatrix} \delta A_k \\ A\delta A_{k+1} + \delta A_k (A_k + \delta A_{k+1}) \\ \vdots \\ \sum_{i=1}^{N_p} A_k^{Np-i} \delta A_{k+Np-i} \prod_{l=0}^{i-1} (A_k + \delta A_{k+Np-l}) \end{bmatrix}$$

$$\delta H =$$

$$\begin{bmatrix} \delta B_k & 0 & \cdots & 0 \\ A_k \delta B_k + \delta A_k (B_k + \delta B_k) & \delta B_{k+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_k^{Np-1} \delta B_k + (B_k + \delta B_k) \eta_{N_p - 1} & A_k^{Np-2} \delta B_{k+1} + (B_k + \delta B_{k+1}) \eta_{N_p - 2} & \cdots & (B_k + \delta B_{k+N_p}) \end{bmatrix}$$

APPENDIX D

Process Models of the chemical plant

The proposed model for the chemical plant case is based on first principles, the basic material balances lead the equations describing the dynamic behaviour. This model was proposed by Scattollini [147].

D.1 Dynamic model of the reactors

Consider a chemical reactor and assume that:

- all the energy phenomena are negligible
- the hydraulic phenomena are all at the steady state
- there is perfect mixing inside the reactor;

The mass balance of the i-th component inside the reactor is then given by (D.1).

$$\frac{dc_i}{dt} = \frac{1}{V}\left[\sum_{j=1}^{n_i} c_{I\_ij}.q_{I\_j} - \sum_{j=1}^{n_0} c_{O\_ij}.q_{O\_j}\right] \pm k \prod_{r=1}^{n_r} x_r \tag{D.1}$$

where q_{I_j} is the volumetric flow rate of the j-th input, c_{I_ji} is the concentration of the i-th component in the j-th input flow rate, V is the reactor volume, c_i as the concentration inside the reactor of the i-th component, q_{O_j} is the volumetric flow rate of the j-th output, c_{O_ji} is the concentration of the i-th component in the j-th output flow rate, n_i is the number of input components, n_0 is the number of output components, n_r as the number of reacting components and k as the reaction constant.

Assuming that inside the reactor there are n components, the model will be described by a system of n differential equations besides one more equation describing the hydraulic equilibrium, that is shown by (D.2).

$$\sum_{j=1}^{n_i} q_{I\_j}(t) = \sum_{j=1}^{n_0} q_{O\_j}(t) \tag{D.2}$$

Finally, note that the dynamic model previously derived can be expressed in terms of molar fractions x_i, instead of concentrations c_i by defining(D.3).

$$x_i = \frac{c_i}{\sum_{j=1}^{n} c_j} \tag{D.3}$$

Then, with an obvious meaning of symbols, the dynamic equations can be written as (D.4) shows.

$$\frac{dx_i}{dt} = \frac{1}{V}\left[\sum_{j=1}^{n_i} x_{I\_ij}.q_{I\_j} - \sum_{j=1}^{n_0} x_{O\_ij}.q_{O\_j}\right] \pm k \prod_{r=1}^{n_r} x_r \tag{D.4}$$

and finally (D.5) shows the relation among molar fractions.

$$\sum_{j=1}^{n} x_j = 1 \tag{D.5}$$

D.2 Dynamic model of the distillation columns

The simplified model of the tray distillation column here considered assumes that it is composed by five sections:

1. Condenser
2. Enriching section
3. Feed tray
4. Stripping section
5. Reboiler

where the enriching and stripping sections can be composed by a variable number of trays.

Assume that the mixture is formed by N components and let

- x_i is the liquid molar fraction of the i-th component (i=1,...,N);
- y_i is the vapor molar fraction of the i-th component (i=1,...,N);
- α_i volatility of the i-th component (i = 1,...,N);

- α_{ij} relative volatility of the i-th component with respect to the j-th component (i; j = 1,...,N).

Straightforward computations allow to conclude that the relation among the liquid and the vapor molar fractions is given by the set of linear equations shown in (D.6).

$$\Psi Y = \Gamma \tag{D.6}$$

where,

$$\Psi = \begin{bmatrix} 1+\alpha_{1N}\frac{x_1}{x_N} & \alpha_{1N}\frac{x_1}{x_N} & ... & \alpha_{1N}\frac{x_1}{x_N} \\ \alpha_{2N}\frac{x_2}{x_N} & 1+\alpha_{2N}\frac{x_2}{x_N} & ... & \alpha_{2N}\frac{x_2}{x_N} \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ \alpha_{(N-1)N}\frac{x_{N-1}}{x_N} & \alpha_{(N-1)N}\frac{x_{N-1}}{x_N} & ... & 1+\alpha_{(N-1)N}\frac{x_{N-1}}{x_N} \end{bmatrix}$$

$$Y = [y_1, y_2, ..., y_{N-1}]^T$$

$$\Gamma = \left[\alpha_{1N}\frac{x_1}{x_N}, \alpha_{2N}\frac{x_2}{x_N}, ...\alpha_{(N-1)N}\frac{x_{N-1}}{x_N}\right]^T$$

The mathematical model of the column is derived under the fundamental assumption that the energetic phenomena are negligible, so that only mass balance equations are used. Moreover, the following simplifying hypothesis are introduced.

- The pressure inside the column is constant
- The vapor flow rate V can be directly manipulated (the reboiler has no dynamics)
- The liquid (R) and vapor (V) flow rates are constant inside the column
- The hydraulic dynamics is negligible with respect to the dynamics of the concentrations
- The vapor hold-up on the trays is negligible with respect to the liquid hold-up;
- The Murphee efficiency is constant for any (i-th) component and any (j-th) tray, as (D.7) and (D.8) show (the user can modify the efficiency in order to make more real the process).

$$E_{j_i} = \frac{y_{j_i} - y_{(j-1)_i}}{y^*_{j_i} - y_{(j-1)_i}} = 1 \tag{D.7}$$

$$y^*_{j_i} = \frac{\alpha_{iN}.x_i}{1+(\alpha_{iN}-1)x_i} \tag{D.8}$$

The mass balance for any tray and for any i-th component is:

1. Static balance of the flow rates at the condenser:

$$V = R + D \tag{D.9}$$

2. Static balance of the flow rates at the reboiler:

$$V + B = R + F \tag{D.10}$$

3. Dynamic balance at the reboiler:

$$H_{1i}\dot{x}_{1i} = -Vy_{1i} + (R+F)x_{2i} - Bx_{1i} \tag{D.11}$$

4. Dynamic balance in the stripping section:

$$H_{ji}\dot{x}_{ji} = (R+F)(x_{(j+1)i} - x_{ji}) + V(y_{(j-1)i} - y_{ji}) \tag{D.12}$$

5. Dynamic balance at the feed tray:

$$H_a\dot{x}_{ai} = Rx_{(Ne+3)i} - (R+F)x_{ai} + V(y_{(Ne+1)i} - y_{ai}) + Fx_{f_i} \tag{D.13}$$

6. Dynamic balance in the enriching section:

$$H_j\dot{x}_{ji} = R(x_{(j+1)i} - x_{ji}) + V(y_{(j-1)i} - y_{ji}) \tag{D.14}$$

7. Dynamic balance at the condenser:

$$H_{Np}\dot{x}_{Npi} = Vy_{(Np-1)i} - (R+D)x_{Npi} \tag{D.15}$$

where $x_{1i} = x_{bi}$, $x_{Npi} = x_{ti}$, $N_p = N_a + N_e + 3$ (the total number of trays, including reboiler and condenser), H_j as the liquid hold-up in the j-th tray, H_a as the liquid hold-up in the feed tray, N_a as the number of trays in the enriching section, N_e as the number of trays in the stripping section, x_{ji} as the liquid molar fraction of the i-th component in the j-th tray, y_{ji} as the vapour molar fraction of the i-th component in the j-th tray, x_{ai} as the liquid molar fraction of the i-th component in the feed tray, x_{ti} as the liquid molar fraction of the i-th component in the top product, x_{bi} as the liquid molar fraction of the i-th component in the bottom product, F as the feed flow rate, x_{f_i} as the liquid molar fraction of the i-th component in the feed flow rate and α_{iN} as the relative volatility of the i-th component with respect to the N-th component.

The complete model of a distillation column with N_p trays is then described by (D.9) to (D.15) , written for any component, besides the additional (D.6), (D.7) and (D.8).

Bibliography

[1] B. D. Schutter, "HD-MPC: Hierarchical and distributed model predictive control of large-scale systems," tech. rep., 2009-2013.

[2] A. Núñez, C. Ocampo-Martinez, B. De Schutter, F. Valencia, J. López, and J. Espinosa, "A multiobjective-based switching topology for hierarchical model predictive control applied to a hydro-power valley," in *Proceedings of the 3rd IFAC International Conference on Intelligent Control and Automation Science (ICONS 2013)*, (Chengdu, China), pp. 529–534, Sept. 2013.

[3] C. Ocampo-Martinez, D. Barcelli, V. Puig, and A. Bemporad, "Hierarchical and decentralised model predictive control of drinking water networks: Application to Barcelona case study," *IET Control Theory Applications*, vol. 6, pp. 62–71, Jan. 2012.

[4] C. Ocampo-Martinez, V. Fambrini, D. Barcelli, and V. Puig, "Model predictive control of drinking water networks: A hierarchical and decentralized approach," in *American Control Conference (ACC), 2010*, pp. 3951–3956, June 2010.

[5] A. Girard and G. Pappas, "Hierarchical control using approximate simulation relations," in *Decision and Control, 2006 45th IEEE Conference on*, pp. 264–269, Dec 2006.

[6] M. Amin, "Modeling and control of complex interactive networks [guest editorial]," *Control Systems, IEEE*, vol. 22, pp. 22–27, Feb 2002.

[7] A. M. Alimi, N. Derbel, A. Ouali, and M. B. A. Kamoun, "Hierarchical control of interconnected systems with slow and fast modes - application to the control of a doubly fed synchronous machine," in *International Conference on Systems, Man and Cybernetics. Le Touquet, France*, 1993.

[8] M. Katebi and M. Johnson, "Predictive control design for large-scale systems," *Automatica*, vol. 33, no. 3, 1997.

[9] P. P. Groumpos and A. V. Pagalos, "A two-level structural model for large scale systems," *Computers in Industry*, vol. 36, no. 1-2, pp. 147–154, 1998.

[10] M. D. Mesarovic, D. Macko, and Y. Takahara, *Theory of Hierarchical, Multilevel Systems*. New York: Academic Press, 1970.

[11] A. H. and D. Odloak, "Robust model predictive control for time delayed systems with optimizing targets and zone control," in *Robust Control, Theory and Applications* (A. Bartoszewicz, ed.), InTech, Apr. 2011.

[12] A. H. Gonzalez, J. L. Marchetti, and D. Odloak, "Robust model predictive control with zone control," *IET Control Theory & Applications*, vol. 3, pp. 121–135, Jan. 2009.

[13] S. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Engineering Practice*, vol. 11, pp. 733–764, July 2003.

[14] A. Zanin, M. Tvrzská de Gouvêa, and D. Odloak, "Integrating real-time optimization into the model predictive controller of the FCC system," *Control Engineering Practice*, vol. 10, pp. 819–831, Aug. 2002.

[15] A. González and D. Odloak, "A stable MPC with zone control," *Journal of Process Control*, vol. 19, no. 1, pp. 110 – 122, 2009.

[16] P. Tatjewski, "Advanced control and on-line process optimization in multilayer structures," *Annual Reviews in Control*, vol. 32, no. 1, pp. 71 – 85, 2008.

[17] I. Alvarado, D. Limon, D. Muñoz de la Peña, J. M. Maestre, M. A. Ridao, H. Scheu, W. Marquardt, R. R. Negenborn, B. De Schutter, F. Valencia, and J. Espinosa, "A comparative analysis of distributed MPC techniques applied to the HD-MPC four-tank benchmark," *Journal of Process Control*, vol. 21, pp. 800–815, June 2011.

[18] K. Johansson, A. Horch, O. Wijk, and A. Hansson, "Teaching multivariable control using the quadruple-tank process," in *Proceedings of the 38th IEEE Conference on Decision and Control, 1999*, vol. 1, pp. 807–812 vol.1, 1999.

[19] C. Savorgnan and D. Moritz, "Control benchmark of a hydro-power plant (draft)," tech. rep., Hierarchical and Distributed Model Predictive Control for Large-Scale Systems (HD-MPC), Katholieke Universiteit Leuven, March 23 2011.

[20] R. Scattolini, "Architectures for distributed and hierarchical model predictive control - a review," *Journal of Process Control*, vol. 19, pp. 723–731, May 2009.

[21] S. Ochoa, G. Wozny, and J.-U. Repke, "Plantwide optimizing control of a continuous bioethanol production process," *Journal of Process Control*, vol. 20, pp. 983–998, Oct. 2010.

[22] C. Abdelmoula, M. Masmoudi, and F. Chaari, "Obstacle avoidance of a mobile robot using a hierarchical control," in *Design and Technology of Integrated Systems in Nanoscale Era, 2008. DTIS 2008. 3rd International Conference on*, pp. 1–5, March 2008.

[23] D. Tolani, J. Horn, A. Ray, and J. Chen, "Hierarchical control of future generation rotorcraft," in *American Control Conference, 2004. Proceedings of the 2004*, vol. 2, pp. 1707–1712 vol.2, June 2004.

[24] G. Yasuda, "Hierarchical and distributed control of robotic manufacturing processes based on petri nets," in *Automation and Logistics, 2008. ICAL 2008. IEEE International Conference on*, pp. 221–226, Sept 2008.

[25] M. Brdys, M. Grochowski, T. Gminski, K. Konarczak, and M. Drewa, "Hierarchical predictive control of integrated wastewater treatment systems.," *Control Engineering Practice*, vol. 16, 2008.

[26] M. Baldea and P. Daoutidis, "Control of integrated process networks -a multi-time scale perspective," *Computers & Chemical Engineering*, vol. 31, no. 5-6, pp. 426 – 444, 2007.

[27] J. Boston, J. Antaki, and M. Simaan, "Hierarchical control of heart-assist devices," *Robotics Automation Magazine, IEEE*, vol. 10, pp. 54–64, Mar 2003.

[28] C. M. Cirre, M. Berenguel, L. Valenzuela, and R. Klempous, "Reference governor optimization and control of a distributed solar collector field," *European Journal of Operational Research*, vol. 193, no. 3, pp. 709 – 717, 2009.

[29] E. Delaleau and A. Stankovicc, "Flatness-based hierarchical control of the pm synchronous motor," in *American Control Conference, 2004. Proceedings of the 2004*, vol. 1, pp. 65–70 vol.1, June 2004.

[30] J. Figueiredo and J. Sa da Costa, "Two level hierarchical control strategy applied to an intelligent house," in *Intelligent Signal Processing, 2005 IEEE International Workshop on*, pp. 92–97, Sept 2005.

[31] W. Findeisen, M. Brdys, K. Malinowski, P. Tatjeski, and A. Wozniak, "On-line hierarchical control for steady-state systems," *IEEE Transactions on Automatic Control*, vol. 23, no. 2, 1978.

[32] M. Huang, "Optimal multilevel hierarchical control strategy for parallel hybrid electric vehicle," in *Vehicle Power and Propulsion Conference, 2006. VPPC '06. IEEE*, pp. 1–4, Sept 2006.

[33] S. Mohagheghi, Y. del Valle, G. Venayagamoorthy, and R. Harley, "Hierarchical control scheme for two static compensators in the brazilian 45-bus power system," in *Power Systems Conference and Exposition, 2006. PSCE '06. 2006 IEEE PES*, pp. 860–864, Oct 2006.

[34] S. Mohagheghi, G. K. Venayagamoorthy, and R. G. Harley, "Intelligent local and hierarchical control of facts devices," *IEEE PES PowerAfrica 2007 Conference and Exposition. Johannesburg, South Africa*, 2007.

[35] M. Torres-Hernandez and M. Velez-Reyes, "Hierarchical control of hybrid power systems," in *Power Electronics Congress, 2008. CIEP 2008. 11th IEEE International*, pp. 169–176, Aug 2008.

[36] X. Zhang and S. Huang, "A hierarchical control scheme for transient stability augmentation by thyristor controlled braking resistors," in *Power System Technology, 2006. PowerCon 2006. International Conference on*, pp. 1–5, Oct 2006.

[37] Y. Zhang, C. Wu, D. Xue, and W. Xiao, "Design and implementation of a hierarchical control system for wheeled mobile robot," in *Industrial Electronics and Applications, 2008. ICIEA 2008. 3rd IEEE Conference on*, pp. 331–335, June 2008.

[38] S. Zongying, X. Wenli, W. Xu, and J. Peigang, "Distributed hierarchical control system of humanoid robot THBIP-1," in *Intelligent Control and Automation, 2002. Proceedings of the 4th World Congress on*, vol. 2, pp. 1265–1269 vol.2, 2002.

[39] A. Damba and S. Watanabe, "Hierarchical control in a multiagent system," in *Innovative Computing, Information and Control, 2007. ICICIC '07. Second International Conference on*, Sept 2007.

[40] X. Cheng, Y. Zhang, L. Cao, J. Li, T. Shen, and S. Zhang, "A real-time hierarchical and distributed control scheme for reactive power optimization in multi-area power systems," in *Transmission and Distribution Conference and Exhibition: Asia and Pacific, 2005 IEEE/PES*, pp. 1–6, 2005.

[41] H. Seki and Y. Naka, "Hierarchical control structure for a reactor/separator process with recycle," in *SICE-ICASE, 2006. International Joint Conference*, pp. 4612–4617, Oct 2006.

[42] E. Tatara, A. Cinar, and F. Teymour, "Control of complex distributed systems with distributed intelligent agents," *Journal of Process Control*, vol. 17, no. 5, pp. 415 – 427, 2007.

[43] Q. Luo, B. Han, X. Mao, K. Wu, and X.Duan, "A FWN-based distributed hierarchical system for hexapod bio-robot control," in *Proceedings of the 6th World Congress on Intelligent Control and Automation*, 2006.

[44] M. Capiluppi and A. Paoli, "Hierarchical design of distributed fault tolerant control systems," in *Intelligent Control, 2005. Proceedings of the 2005 IEEE International Symposium on, Mediterrean Conference on Control and Automation*, pp. 1361–1366, June 2005.

[45] M. Campbell, R. D'Andrea, D. Schneider, A. Chaudhry, S. Waydo, J. Sullivan, J. Veverka, and A. Klochko, "Roboflag games using systems based, hierarchical control," in *American Control Conference, 2003. Proceedings of the 2003*, vol. 1, pp. 661–666 vol.1, June 2003.

[46] M. A. Brdys and P. Tatjewski, *Iterative Algorithms for Multilayer Optimizing Control*. Imperial College Press, Jan. 2005.

[47] P. Tatjewski, N. Abdullah, and P. Roberts, "Comparison of some algorithms for hierarchical steady-state optimising control of interconnected industrial processes," in *Control, 1988. CONTROL 88., International Conference on*, pp. 527–531, Apr 1988.

[48] L. P. Gutierrez Master's thesis, Universidad Nacional de Colombia, Sede Medellín.

[49] K. Imazeki and T. Maeno, "Hierarchical control method for manipulating/grasping tasks using multi-fingered robot hand," in *Intelligent Robots and Systems, 2003.*

(IROS 2003). Proceedings. 2003 IEEE/RSJ International Conference on, vol. 4, pp. 3686–3691 vol.3, Oct 2003.

[50] J. Elston and E. Frew, "Hierarchical distributed control for search and tracking by heterogeneous aerial robot networks," in *Robotics and Automation, 2008. ICRA 2008. IEEE International Conference on*, pp. 170–175, May 2008.

[51] T. E. Marlin, *Process control:designing processes and control systems for dynamic performance*. McGraw-Hill, 2000.

[52] W. L. Luyben, B. D. Tyréus, and M. L. Luyben, *Plantwide process control*. McGraw-Hill, 1999.

[53] L. T. Biegler, I. E. Grossmann, and A. W. Westerberg, "Systematic methods for chemical process design," Dec. 1997.

[54] R. Amrit, J. B. Rawlings, and D. Angeli, "Economic optimization using model predictive control with a terminal cost," *Annual Reviews in Control*, vol. 35, pp. 178–186, Dec. 2011.

[55] H. Rui, *Nonlinear Model Predictive Control and Dynamic Real Time Optimization for Large-scale Processes*. PhD thesis, Carnegie Mellon University, 2010.

[56] M. Morari and J. H. Lee, "Model predictive control: past, present and future," *Computers & Chemical Engineering*, vol. 23, pp. 667–682, May 1999.

[57] S. Engell, "Feedback control for optimal process operation," *Journal of Process Control*, vol. 17, pp. 203–219, Mar. 2007.

[58] C. R. Cutler and R. T. Perry, "Real time optimization with multivariable control is required to maximize profits," *Computers & Chemical Engineering*, vol. 7, no. 5, pp. 663–667, 1983.

[59] A. Helbig, O. Abel, and W. Marquardt, "Structural concepts for optimization based control of transient processes," in *Nonlinear Model Predictive Control* (F. Allgöwer and A. Zheng, eds.), no. 26 in Progress in Systems and Control Theory, pp. 295–311, Birkhäuser Basel, Jan. 2000.

[60] M. Morari, Y. Arkun, and G. Stephanopoulos, "Studies in the synthesis of control structures for chemical processes: Part i: Formulation of the problem. process decomposition and the classification of the control tasks. analysis of the optimizing control structures," *AIChE Journal*, vol. 26, no. 2, pp. 220–232, 1980.

[61] V. Adetola and M. Guay, "Integration of real-time optimization and model predictive control," *Journal of Process Control*, vol. 20, pp. 125–133, Feb. 2010.

[62] L. Würth, J. B. Rawlings, and W. Marquardt, "Economic dynamic real-time optimization and nonlinear model-predictive control on infinite horizons," in *In: ADCHEM 2009 (International Symposium on Advanced Control of Chemical Processes)*, pp. 225–230, 2009.

[63] M. Diehl, R. Amrit, and J. B. Rawlings, "A lyapunov function for economic optimizing model predictive control," *IEEE Transactions on Automatic Control*, vol. 56, pp. 703–707, Mar. 2011.

[64] W. Findeisen, F. N. Bailey, M. Brdys, K. Malinowski, P. Tatjewski., and A. Wozniak, "Control and coordination in hierarchical systems," Wiley, 1980.

[65] C.-M. Ying and B. Joseph, "Performance and stability analysis of lp-mpc and qp-mpc cascade control systems," *AIChE Journal*, vol. 45, no. 7, pp. 1521–1534, 1999.

[66] L. J.Z., "Challenging control problems and emerging technologies in enterprise optimization," *Control Engineering Practice*, vol. 11, no. 8, pp. 847–858, 2003.

[67] T. Tosukhowong, J. M. Lee, J. H. Lee, and J. Lu, "An introduction to a dynamic plant-wide optimization strategy for an integrated plant," *Computers & Chemical Engineering*, vol. 29, pp. 199–208, Dec. 2004.

[68] R. Cheng, J. Forbes, and W. Yip, "Price-driven coordination method for solving plant-wide MPC problems," *Journal of Process Control*, vol. 17, pp. 429–438, June 2007.

[69] A. Bemporad and M. Morari, "Robust model predictive control: A survey," in *Robustness in identification and control* (A. Garulli and A. Tesi, eds.), no. 245 in Lecture Notes in Control and Information Sciences, pp. 207–226, Springer London, Jan. 1999.

[70] J. Kadam, M. Schlegel, W. Marquardt, R. Tousain, D. van Hessem, J. van den Berg, and O. Bosgra, "A two-level strategy of integrated dynamic optimization and control of industrial processes–a case study," in *European Symposium on Computer Aided Process Engineering-12, 35th European Symposium of the Working Party on Computer Aided Process Engineering*, vol. Volume 10, pp. 511–516, Elsevier, 2002.

[71] A. Ben-Tal and A. Nemirovski, "Robust solutions of linear programming problems contaminated with uncertain data," *Mathematical Programming*, vol. 88, pp. 411–424, Sept. 2000.

[72] J. Espinosa, J. Vandewalle, and V. Wertz, *Fuzzy Logic, Identification and Predictive Control*. Springer, 2005.

[73] M. V. Kothare, V. Balakrishnan, and M. Morari, "Robust constrained model predictive control using linear matrix inequalities," *Automatica*, vol. 32, pp. 1361–1379, Oct. 1996.

[74] M. Balandat, *Constrained Robust Optimal Trajectory Tracking: Model Predictive Control Approaches*. PhD thesis, Technische Universitat Darmstadt, 2010.

[75] G. M. Ziegler, *Lectures on Polytopes*. Springer, 1995.

[76] K. Zhou, J. C. Doyle, K. Glover, *et al.*, *Robust and optimal control*, vol. 40. Prentice Hall Upper Saddle River, NJ, 1996.

[77] P. J. Campo and M. Morari, "Robust model predictive control," in *American Control Conference, 1987*, pp. 1021–1026, 1987.

[78] J. Rawlings and D. Mayne, *Model Predictive Control: Theory and Design*. Nob Hill Publishing, 2009.

[79] J. H. Lee and Z. Yu, “Worst-case formulations of model predictive control for systems with bounded parameters,” *Automatica*, vol. 33, pp. 763–781, May 1997.

[80] A. Casavola, M. Giannelli, and E. Mosca, “Min-max predictive control strategies for input-saturated polytopic uncertain systems,” *Automatica*, vol. 36, pp. 125–133, Jan. 2000.

[81] J. Rossiter and J. Schuurmans, “Robust predictive control using tight sets of predicted states,” *IEE Proceedings - Control Theory and Applications*, vol. 147, pp. 13–18, Jan. 2000.

[82] P. O. Scokaert, J. B. Rawlings, and E. S. Meadows, “Discrete-time stability with perturbations: Application to model predictive control,” *Automatica*, vol. 33, no. 3, pp. 463–470, 1997.

[83] J. a. Lee and Z. Yu, “Worst-case formulations of model predictive control for systems with bounded parameters,” *Automatica*, vol. 33, no. 5, pp. 763–781, 1997.

[84] J. Gruber, D. Ramirez, D. Limon, and T. Alamo, “Computationally efficient nonlinear Min-Max model predictive control based on volterra series models-Application to a pilot plant,” *Journal of Process Control*, vol. 23, pp. 543–560, Apr. 2013.

[85] J. Gruber, D. Ramirez, T. Alamo, and E. Camacho, “Min-Max MPC based on an upper bound of the worst case cost with guaranteed stability. application to a pilot plant,” *Journal of Process Control*, vol. 21, pp. 194–204, Jan. 2011.

[86] C. Wu, K. L. Teo, and S. Wu, “Min-max optimal control of linear systems with uncertainty and terminal state constraints,” *Automatica*, vol. 49, pp. 1809–1815, June 2013.

[87] D. Mayne, J. Rawlings, C. Rao, and P. Scokaert, “Constrained model predictive control: Stability and optimality,” *Automatica*, vol. 36, pp. 789–814, June 2000.

[88] A. Tyagunov, *High-Performance Model Predictive Control for Process Industry*. PhD thesis, Technische Universiteit Eindhoven, 2004.

[89] J. Rawlings and K. Muske, “The stability of constrained receding horizon control,” *IEEE Transactions on Automatic Control*, vol. 38, pp. 1512 –1516, Oct. 1993.

[90] J. Espinosa and J. Vandewalle, “The use of convex programming on fuzzy model based predictive control,” in *Proceedings of the 1999 IEEE International Symposium on Intelligent Control/Intelligent Systems and Semiotics, 1999*, pp. 411–416, 1999.

[91] D. Mayne, “Control of constrained dynamic systems,” *European Journal of Control*, vol. 7, no. 2-3, pp. 87–99, 2001.

[92] M. Henson and D. Seborg, “Input-output linearization of general nonlinear processes,” *AIChE Journal*, vol. 36, pp. 1753–1757, 1990.

[93] G. Lightbody and G. Irwin, "Nonlinear control structures based on embedded neural system models," *IEEE Trans. on Neural Networks*, vol. 8, pp. 553–567, 1997.

[94] J. Rawlings, D. Angeli, and C. Bates, "Fundamentals of economic model predictive control," in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*, pp. 3851–3861, 2012.

[95] M. L. Darby and M. Nikolaou, "MPC: current practice and challenges," *Control Engineering Practice*, vol. 20, no. 4, pp. 328 – 342, 2012.

[96] T. Hovgaard, K. Edlund, and J. Bagterp Jorgensen, "The potential of economic MPC for power management," in *Decision and Control (CDC), 2010 49th IEEE Conference on*, pp. 7533–7538, 2010.

[97] J. Rawlings and R. Amrit, "Optimizing process economic performance using model predictive control," in *Nonlinear Model Predictive Control* (L. Magni, D. Raimondo, and F. Allgöwer, eds.), vol. 384 of *Lecture Notes in Control and Information Sciences*, pp. 119–138, Springer Berlin Heidelberg, 2009.

[98] T. G. Hovgaard, K. Edlund, and J. B. JÃ¸rgensen, "Economic MPC for power management in the SmartGrid," in *21st European Symposium on Computer Aided Process Engineering* (M. G. E.N. Pistikopoulos and A. C. Kokossis, eds.), vol. 29 of *Computer Aided Chemical Engineering*, pp. 1839 – 1843, Elsevier, 2011.

[99] J. Ma, J. Qin, T. Salsbury, and P. Xu, "Demand reduction in building energy systems based on economic model predictive control," *Chemical Engineering Science*, vol. 67, no. 1, pp. 92 – 100, 2012.

[100] B. Chu, S. Duncan, A. Papachristodoulou, and C. Hepburn, "Using economic model predictive control to design sustainable policies for mitigating climate change," in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*, pp. 406–411, 2012.

[101] X. Chen, M. Heidarinejad, J. Liu, and P. D. Christofides, "Distributed economic MPC: application to a nonlinear chemical process network," *Journal of Process Control*, vol. 22, no. 4, pp. 689 – 699, 2012.

[102] R. Huang, E. Harinath, and L. T. Biegler, "Lyapunov stability of economically oriented nmpc for cyclic processes," *Journal of Process Control*, vol. 21, no. 4, pp. 501 – 509, 2011.

[103] M. Heidarinejad, J. Liu, and P. D. Christofides, "Economic model predictive control of switched nonlinear systems," *Systems & Control Letters*, vol. 62, no. 1, pp. 77 – 84, 2013.

[104] R. Amrit, J. B. Rawlings, and L. T. Biegler, "Optimizing process economics online using model predictive control," *Computers & Chemical Engineering*, vol. 58, no. 0, pp. 334 – 343, 2013.

[105] A. Bemporad and M. Morari, "Robust model predictive control: A survey," in *Robustness in identification and control* (A. Garulli and A. Tesi, eds.), vol. 245 of *Lecture Notes in Control and Information Sciences*, pp. 207–226, Springer London, 1999.

[106] G. Pin, D. Raimondo, L. Magni, and T. Parisini, "Robust model predictive control of nonlinear systems with bounded and state-dependent uncertainties," *Automatic Control, IEEE Transactions on*, vol. 54, no. 7, pp. 1681–1687, 2009.

[107] J. M. Grosso, J. M. Maestre, C. Ocampo-Martinez, and V. Puig, "On the assessment of tree-based and chance-constrained predictive control approaches applied to drinking water networks," in *19th IFAC World Congress*, pp. 6240–6245, 2014.

[108] J. Grosso, C. Ocampo-Martinez, and V. Puig, "A service reliability model predictive control with dynamic safety stocks and actuators health monitoring for drinking water networks," in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*, pp. 4568–4573, IEEE, 2012.

[109] J. Maestre, P. Velarde, I. Jurado, C. Ocampo-Martinez, I. Fernandez, B. Isla Tejera, and J. del Prado, "An application of chance-constrained model predictive control to inventory management in Hospitalary Pharmacy," in *2014 IEEE 53rd Annual Conference on Decision and Control (CDC)*, pp. 5901–5906, Dec. 2014.

[110] J. M. Grosso, C. Ocampo-MartÃnez, V. Puig, and B. Joseph, "Chance-constrained model predictive control for drinking water networks," *Journal of Process Control*, vol. 24, May 2014.

[111] T. Hovgaard, L. F. S. Larsen, and J. Jorgensen, "Robust economic MPC for a power management scenario with uncertainties," in *Decision and Control and European Control Conference (CDC-ECC), 2011 50th IEEE Conference on*, pp. 1515–1520, 2011.

[112] J. A. Heath, I. K. Kookos, and J. D. Perkins, "Process control structure selection based on economics," *AIChE Journal*, vol. 46, no. 10, pp. 1998–2016, 2000.

[113] F. Wang and I. Cameron, "Control studies on a model evaporation process - constrained state driving with conventional and higher relative degree systems," *Journal of Process Control*, vol. 4, pp. 59–75, May 1994.

[114] A. Venkat, J. Rawlings, and S. Wright, "Stability and optimality of distributed, linear model predictive control. part i: State feedback," *Texas-Wisconsin Modeling and Control Consortium Technical Report 3*, 2006.

[115] G. De Souza, D. Odloak, and A. C. Zanin, "Real time optimization (RTO) with model predictive control (MPC)," *Computers & Chemical Engineering*, vol. 34, pp. 1999–2006, Dec. 2010.

[116] L. A. Alvarez and D. Odloak, "Robust integration of real time optimization with linear model predictive control," *Computers & Chemical Engineering*, vol. 34, pp. 1937–1944, Dec. 2010.

[117] D. Odloak, "Robust integration of RTO and MPC," in *Computer Aided Chemical Engineering*, vol. 27, pp. 119–126, Elsevier, 2009.

[118] P. A. Martin, D. Odloak, and F. Kassab, "Robust model predictive control of a pilot plant distillation column," *Control Engineering Practice*, vol. 21, pp. 231–241, Mar. 2013.

[119] M. A. Martins, A. S. Yamashita, B. F. Santoro, and D. Odloak, "Robust model predictive control of integrating time delay processes," *Journal of Process Control*, vol. 23, pp. 917–932, Aug. 2013.

[120] J. V. Kadam and W. Marquardt, "Integration of economical optimization and control for intentionally transient process operation," in *Assessment and Future Directions of Nonlinear Model Predictive Control* (R. Findeisen, F. Allgöwer, and L. T. Biegler, eds.), vol. 358, pp. 419–434, Berlin, Heidelberg: Springer Berlin Heidelberg, 2007.

[121] A. Marquez, J. Espinosa, P. Deossa, and C. Gomez, "Hierarchical control of large scale systems: A zone control approach," in *Large Scale Complex Systems Theory and Applications*, pp. 438–443, July 2013.

[122] A. Helbig, O. Abel, and W. Marquardt, "Structural concepts for optimization based control of transient processes," in *Nonlinear Model Predictive Control* (F. Allgöwer and A. Zheng, eds.), no. 26 in Progress in Systems and Control Theory, pp. 295–311, Birkhäuser Basel, Jan. 2000.

[123] J. F. Forbes, T. E. Marlin, and J. F. MacGregor, "Model adequacy requirements for optimizing plant operations," *Computers & Chemical Engineering*, vol. 18, pp. 497–510, June 1994.

[124] K. Findeisen, *Moving Horizon State Estimation of Discrete-Time Systems*. Master's Thesis, University of Wisconsin-Madison, 1997.

[125] F. Allgöwer, T. A. J. S. Badgwell, Qin, J. B. Rawlings, and S. J. Wright, "Nonlinear predictive control and moving horizon estimation - an introductory overview," *Proceedings of the Advances in Control - Highlights of ECC '09*, pp. 391–449, 1999.

[126] C. Rao, "Moving horizon strategies for the constrained monitoring and control of nonlinear discrete-time systems," *Doctoral Dissertation, University of Wisconsin-Madison*, 2000.

[127] J. Garcia and J. Espinosa, "Moving horizon estimators for large-scale systems," *Journal of Control Engineering and Applied Informatics -CEAI-*, vol. 11, pp. 49–56, 2009.

[128] C. V. Rao, J. B. Rawlings, and J. H. Lee, "Constrained linear state estimation -a moving horizon approach," *Automatica*, no. 37, pp. 1619–1628, 2001.

[129] A. H. González, D. Odloak, and J. L. Marchetti, "Predictive control applied to heat-exchanger networks," *Chemical Engineering and Processing: Process Intensification*, vol. 45, pp. 661–671, Aug. 2006.

[130] R. Bellman, "Dynamic programming," *Princeton University Press*, 1957.

[131] A. Bryson and Y.-C. Ho, "Applied optimal control," *Taylor & Francis*, 1975.

[132] R. Sargent and G. Sullivan, "The deveopment of an efficient optimal control package," *8th IFIP Conf. Opt.Tech.Pt.2*, 1977.

[133] L. T. Biegler, "Solution of dynamic optimization problems by successive quadratic programming and orthogonal collocation," *Computers & Chemical Engineering*, vol. 8, no. 3-4, pp. 243–247, 1984.

[134] T. Binder, L. Blank, H. G. Bock, R. Bulirsch, W. Dahmen, M. Diehl, T. Kronseder, W. Marquardt, J. P. Schlöder, and O. v. Stryk, “Introduction to model based optimization of chemical processes on moving horizons,” in *Online Optimization of Large Scale Systems* (M. Grötschel, S. O. Krumke, and J. Rambau, eds.), pp. 295–339, Springer Berlin Heidelberg, Jan. 2001.

[135] F. Valencia, *Game Theory Based Distributed Model Predictive Control: An Approach to Large-Scale Systems Control*. PhD thesis, Universidad Nacional de Colombia, 2012.

[136] F. Valencia, J. Patiño, J. D. Lopez, and J. Espinosa, “Game theory based distributed model predictive control for a hydro-power valley control,” in *Proceedings of the 13th IFAC Symposium on Large Scale Complex Systems: Theory and Applications*, vol. 13, (Shanghai, China), IFAC, 2013.

[137] F. Petrone, *Model Predictive Control of a Hydro-Power Valley*. PhD thesis, Facolta di Ingegneria dell'Informazione Corso di Laurea Specialistica in Ingegneria dell'Atomazione, Politecnico di Milano, 2010.

[138] C. Setz, A. Heinrich, P. Rostalski, G. Papafotiou, and M. Morari, “Application of model predictive control to a cascade of river power plants,” in *Proceedings of the 17th IFAC World Congress*, (Seul, Korea), July 6-11 2008.

[139] Gabriela Glanzmann, Martin von Siebenthal, Tobias Geyer, George Papafotiou, and M. Morari, “Supervisory water level control for cascaded river power plants,” tech. rep., Swiss Federal Institute of Technology, Automatic Control Laboratory, CH-8092 Zurich, Switzerland, 2003.

[140] S. Weiland, “Balancing and hankel norm approximation of dynamical systems,” *Course Model Reduction*, 2006.

[141] O. Carrapiço and D. Odloak, “A stable model predictive control for integrating processes,” *Computers & Chemical Engineering*, vol. 29, pp. 1089–1099, Apr. 2005.

[142] B. F. Santoro and D. Odloak, “Closed-loop stable model predictive control of integrating systems with dead time,” *Journal of Process Control*, vol. 22, pp. 1209–1218, Aug. 2012.

[143] W.M.Wonham, “On pole assignment in multi-input controllable linear systems,” *IEEE Transactions on Automatic Control*, vol. 12, no. 6, pp. 660–665, 1967.

[144] J. Espinosa, “Control lineal de sistemas multivariables,” *Draft version*, 2003.

[145] J. Grosso, C. Ocampo-Martinez, V. Puig, D. Limon, and M. Pereira, “Economic MPC for the management of drinking water networks,” in *Control Conference (ECC), 2014 European*, pp. 790–795, June 2014.

[146] D. Limon, M. Pereira, D. Muñoz de la Peña, T. Alamo, and J. M. Grosso, “Single-layer economic model predictive control for periodic operation,” *Journal of Process Control*, vol. 24, pp. 1207–1224, Aug. 2014.

[147] R. Scattolini, "Report on literature survey and preliminary definition of the selected methods for the definition of system decomposition and hierarchical control architectures," *Internal HD-MPC project report*, 2009.

[148] G. B. Dantzig, "Linear programming under uncertainty," *Management Science*, vol. 1, pp. 197–206, Apr. 1955.

[149] A. L. Soyster, "Convex programming with set-inclusive constraints and applications to inexact linear programming," *Operations Research*, vol. 21, pp. 1154–1157, Sept. 1973.

[150] J. E. Falk, "Exact solutions of inexact linear programs," *Operations Research*, vol. 24, pp. 783–787, July 1976.

[151] A. Ben-Tal and A. Nemirovski, "Robust convex optimization," *Mathematics of Operations Research*, vol. 23, pp. 769–805, Nov. 1998.

[152] A. Ben-Tal and A. Nemirovski, "Robust solutions of uncertain linear programs," *Operations Research Letters*, vol. 25, pp. 1–13, Aug. 1999.

[153] L. El Ghaoui and H. Lebret, "Robust solutions to least-squares problems with uncertain data," *SIAM Journal on Matrix Analysis and Applications*, vol. 18, pp. 1035–1064, Oct. 1997.

[154] L. El Ghaoui, F. Oustry, and H. Lebret, "Robust solutions to uncertain semidefinite programs," *SIAM Journal on Optimization*, vol. 9, pp. 33–52, Jan. 1998.

[155] A. Ben-Tal, L. E. Ghaoui, and A. Nemirovski, *Robust Optimization*. Princeton University Press, 2009.

[156] M. Diehl, J. Gerhard, W. Marquardt, and M. Mönnigmann, "Numerical solution approaches for robust nonlinear optimal control problems," *Computers & Chemical Engineering*, vol. 32, pp. 1279–1292, June 2008.

[157] R. Hettich and K. O. Kortanek, "Semi-infinite programming: Theory, methods, and applications," *SIAM Review*, vol. 35, pp. 380–429, Sept. 1993.

[158] B. Houska, *Robust Optimization of Dynamic Systems*. PhD thesis, Katholieke Universiteit Leuven, 2011.

[159] J. Löfberg, *Minimax approaches to robust model predictive control*. PhD thesis, Linköping University, 2003.

[160] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge Univ Pr, 2004.

[161] A. Ben-Tal and A. Nemirovski, *Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications*. SIAM, 2001.

[162] A. Bartlett, C. Hollot, and H. Lin, "Root locations of an entire polytope of polynomials: It suffices to check the edges," in *American Control Conference, 1987*, pp. 1611–1616, 1987.

[163] S. Bhattacharyya, H. Chappelat, and L. Keel, *Robust Control: The Parametric Approach*. Prentice Hall, 1995.