Applying Tiab's direct synthesis technique to dilatant non-Newtonian/ Newtonian fluids

Aplicación de la técnica TDS a un yacimiento compuesto con fluidos dilatantes no newtoniano/newtoniano

Javier Andrés Martínez[1], Freddy Humberto Escobar[2], José Humberto Cantillo[3]

ABSTRACT

Non-Newtonian fluids, such as polymer solutions, have been used by the oil industry for many years as fracturing agents and drilling mud. These solutions, which normally include thickened water and jelled fluids, are injected into the formation to enhanced oil recovery by improving sweep efficiency. It is worth noting that some heavy oils behave non-Newtonianly. Non-Newtonian fluids do not have direct proportionality between applied shear stress and shear rate and viscosity varies with shear rate depending on whether the fluid is either pseudoplastic or dilatant. Viscosity decreases as shear rate increases for the former whilst the reverse takes place for dilatants. Mathematical models of conventional fluids thus fail when applied to non-Newtonian fluids. The pressure derivative curve is introduced in this descriptive work for a dilatant fluid and its pattern was observed. Tiab's direct synthesis (TDS) methodology was used as a tool for interpreting pressure transient data to estimate effective permeability, skin factors and non-Newtonian bank radius. The methodology was successfully verified by its application to synthetic examples. Also, comparing it to pseudoplastic behavior, it was found that the radial flow regime in the Newtonian zone of dilatant fluids took longer to form regarding both the flow behavior index and consistency factor.

Keywords: Dilatant fluid, consistency, viscosity, power-law, radial flow.

RESUMEN

Por muchos años, los fluidos no Newtonianos, tales como los polímeros, se han usado en la industria del petróleo como agentes de fracturamiento y en lodos de perforación. Estas soluciones las cuales normalmente contienen agua, son inyectadas en la formación para la recuperación mejorada de petróleo mediante el mejoramiento de la eficiencia de barrido. Es de resaltar que algunos crudos pesados también tienen comportamiento no Newtoniano. Los fluidos no Newtonianos no exhiben una proporcionalidad directa entre el esfuerzo de corte aplicado y la rata de corte; la viscosidad varía con la rata de corte dependiendo si el fluido es pseudoplástico o dilatante. Para los primeros, la viscosidad decrece con el incremento de la rata de corte. Para los dilatantes ocurre el caso inverso. Por ello, los modelos matemáticos de los fluidos convencionales fallan al aplicarse en fluidos No Newtonianos. En este trabajo descriptivo, se introduce la curva de derivada de presión para un fluido dilatante y se observa su comportamiento. Se usa la metodología *TDS* como herramienta para la interpretación de transientes de presión de modo que se presentan expresiones nuevas para estimar las permeabilidades efectivas, factores de daño y el radio de la zona no Newtoniana. La metodología fue satisfactoriamente verificada para su aplicación con ejemplos sintéticos. También se encontró que el flujo radial es más demorado en la zona Newtoniana de los fluidos dilatantes, comparado al caso pseudoplástico, a medida que se incrementan el índice de comportamiento de flujo y el parámetro de consistencia.

Palabras clave: fluidos dilatantes, consistencia, viscosidad, ley de potencia, flujo radial

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Introduction

Okpobiri and Ikoku (1983) presented the standard work on well test analysis for dilatant non-Newtonian fluids. They studied the behavior of both flow pattern index and fluid consistency in pressure fall-off tests and conducted reservoir characterisation by using the conventional straight-line method. The pertinent oil sector literature contains several papers on the behaviour of non-Newtonian fluids in porous media. Odeh and Yang (1979) derived a partial differential equation regarding a power-law for fluid flow through porous media; they used a power-law relating viscosity to shear rate. The power-law viscosity function was coupled to the variable viscosity diffusivity equation and a shear-rate ratio proposed by Savins (1969) to give a new partial differential equation and an approximate analytical solution. Ikoku has been the most outstanding researcher in the field of non-Newtonian power-law fluid modelling, as shown by Ikoku (1979), Ikoku and Ramey (1979a, 1979b, 1979c) and Lund and Ikoku (1981) with pseudoplastic non-Newtonian fluids.

Interpreting pressure tests for non-Newtonian fluids is differently performed to conventional Newtonian fluids. Non-Newtonian fluids have a pressure derivative curve during radial flow regime which is not horizontal but rather inclined with a negative slope for non-Newtonian dilatant fluids.

This paper analyses pressure tests and complements Tiab's di-

[1] Undergraduate level, Professional Researcher, Universidad Surcolombiana. e-mail: jandres@usco.edu.co
[2] Doctor, Oklahoma University. Professor, Universidad Surcolombiana. fescobar@usco.edu.co
[3] Master, Colorado School of Mines. Researcher, Ecopetrol S.A.-ICP. jose.cantillo@ecopetrol.com.co

rect synthesis (TDS) technique regarding a composite reservoir with dilatant non-Newtonian/Newtonian interface. The model previously developed by Lund and Ikoku (1981) was thus solved numerically to obtain pressure and pressure derivative behaviour, so new expressions for characterising the reservoir by means of the TDS technique were developed and verified with synthetic data.

Vongvuthipornchai and Raghavan (1987) used the pressure-derivative method for well test analysis of non-Newtonian fluids; however, the very first application to non-Newtonian behaviour by the TDS technique was reported by Katime-Meindl and Tiab (2001). Escobar *et al.*, (2010) conducted a numerical study to observe transient pressure behaviour when a non-Newtonian fluid is injected into a conventional reservoir containing Newtonian oil. They formulated an interpretation methodology using pressure and pressure derivative functions without type-curve matching. Escobar *et al.*, (2011) first applied the pressure derivative concept to non-Newtonian fluids in double-porosity formation, in which an extension of the TDS technique was presented. The most recent power-law non-Newtonian flow behaviour work was by Mahani *et al.*, (2011) for interpreting fall-off tests in polymer fluids using type-curve matching. Martinez *et al.*, (2011) performed a numerical experiment for studying transient pressure behaviour and develop an interpretation methodology for non-Newtonian Bingham fluids.

Mathematical model

Ikoku and Ramey (1979b) proposed a partial differential equation for non-Newtonian fluids' radial flow following a power-law relationship through porous media. Coupling the non-Newtonian Darcy's law with the continuity equation, they derived a rigorous partial differential equation. The non-linear form of the partial differential equation was solved using the Douglas-Jones predictor/corrector method for numerical solutions of non-linear partial differential equations.

A linearised approximation (Eq. 1) was also derived by Ikoku and Ramey (1979b) for analytical solutions. Linear and non-linear equation solutions were compared and found to fit very well. The errors introduced by the approximate linear equation were small and decreased as both the value of the flow behaviour index, *n*, and time increased.

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial p}{\partial r}\right)=Gr^{1-n}\frac{\partial p}{\partial t} \quad (1)$$

where:

$$G=\frac{3792.188n\phi c_t\mu_{eff}}{k_1}\left(96681.605\frac{h}{qB}\right)^{1-n} \quad (2)$$

and,

$$\mu_{eff}=\left(\frac{H}{12}\right)\left(9+\frac{3}{n}\right)^n\left(1.59344\times10^{-12}k_1\phi\right)^{(1-n)/2} \quad (3)$$

The system being considered assumed radial flow of a non-Newtonian and a slightly compressible Newtonian fluid through porous media. It was assumed that the reservoir was homogeneous and isotropic. It also had constant thickness. The reservoir was cylindrically shaped with a finite outer radius. The non-Newtonian fluid was injected through a well in the centre of the field. The fluid was considered to be non-Newtonian dilatant obeying power law behaviour. The Newtonian fluid had constant viscosity, as usually considered in well test analysis. A piston-like behaviour of the Newtonian fluid by the non-Newtonian fluid was also assumed. Figure 1 sketches the composite reservoir being considered.

Since the pressure wave was travelling through a porous medium saturated with a non-Newtonian fluid, the pressure derivative had a certain degree of inclination as the flow behaviour index increased its value. Once, the travel wave arrived in the Newtonian zone, the pressure derivative formed a plateau, as expected, after an obvious transition period. Figure 2 illustrates such a situation for different *n* values. Notice than for $n = 1$, two different plateaus were observed since two types of mobility were being dealt with. The bigger the *n*, the longer the time required to obtain horizontal radial-flow line during the Newtonian zone.

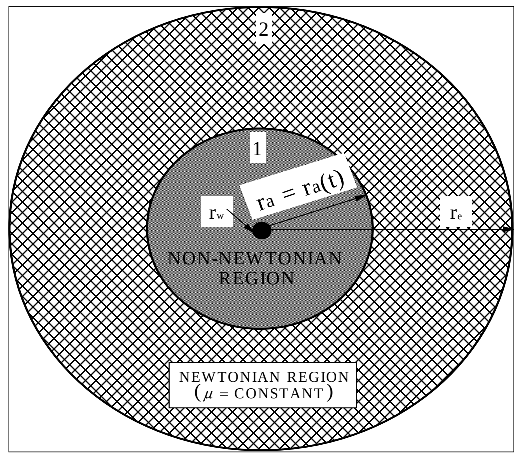

Figure 1. Composite non-Newtonian/Newtonian radial reservoir

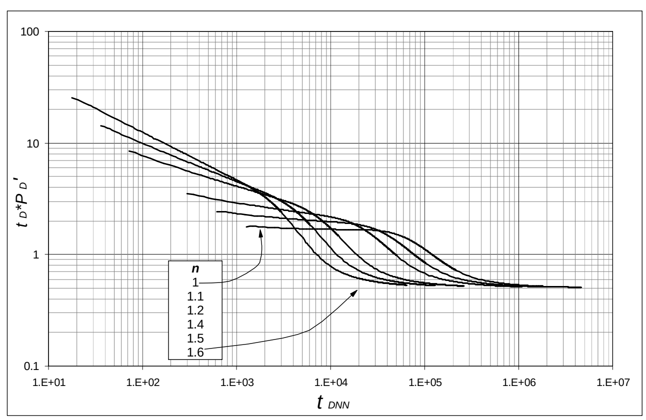

Figure 2. Newtonian dimensionless pressure derivative pattern for r_a = 120 ft

Dimensionless parameters

Dimensionless pressure, P_D, and dimensionless time, t_D, for each region were expressed as:

$$P_{DNN}=\frac{\Delta P}{141.2\left(96681.605\right)^{1-n}\left(\frac{qB}{h}\right)^n\frac{\mu_{eff}r_w^{1-n}}{k_1}} \quad (4)$$

$$t_{DNN} = \frac{t}{G r_w^{3-n}} \quad (5)$$

$$P_{DN} = \frac{k_2 h \Delta P}{141.2 q \mu_N B} \quad (6)$$

$$t_{DN} = \frac{0.0002637 k_2 t}{\phi \mu_N c_t r_w^2} \quad (7)$$

TDS formulation

The interpretation methodology followed the TDS technique philosophy introduced by Tiab (1995) which was based upon the definition of characteristic features found on the logarithmic plot of pressure and pressure derivative versus time. This meant that several specific regions on that plot were dealt with.

1. According to Katime-Meindl and Tiab (2001), the dimensionless equation for radial-flow regime derived in the non-Newtonian region without considering wellbore storage effects was:

$$\left(t_D * P_D'\right)_{rNN} = 0.00708\left(96681.605\right)^{n-1}\left(\frac{h}{qB}\right)^n \frac{k_1}{\mu_{eff}} r_w^{n-1}\left(t*\Delta P'\right)_r \quad (8)$$

The value of this derivative was:

$$\log\left[t_D * P_D'\right]_{rNN} = \alpha \log\left(t_{DNN}\right)_r + \log 0.5 \quad (9)$$

$$\left(t_D * P_D'\right)_{rNN} = 0.5 t_{DNN}^{\alpha} \quad (10)$$

Combination of Eq. 9 and 10 led to obtaining:

$$\frac{k_1}{\mu_{eff}} = \left[70.6\left(96681.605\right)^{(1-\alpha)(1-n)}\left(\frac{0.0002637 t_r}{n\phi c_t}\right)^{\alpha}\left(\frac{qB}{h}\right)^{n-\alpha(n-1)}\left(\frac{1}{\left(t*\Delta P'\right)_{r1}}\right)\right]^{\frac{1}{1-\alpha}} \quad (11)$$

Substituting Eq. 3 into Eq. 11 and solving for k_1 yielded:

$$k_1 = \left(\frac{(96681.605)^{(1-n)}\left[70.6\left(\frac{0.0002637 t_r}{n\phi c_t}\right)^{\alpha}\left(\frac{qB}{h}\right)^{n-\alpha(n-1)}\left(\frac{1}{\left(t*\Delta P'\right)_{r1}}\right)\right]^{\frac{1}{1-\alpha}}}{\left(\left(\frac{H}{12}\right)\left(9+\frac{3}{n}\right)^n\left(1.59344\times10^{-12}\phi\right)^{(1-n)/2}\right)}\right)^{\frac{2}{1+n}} \quad (12)$$

α being pressure derivative curve slope on the non-Newtonian region defined as:

$$\alpha = \frac{1-n}{3-n} \quad (13)$$

2. The governing pressure equation during non-Newtonian radial flow regime may be expressed by:

$$P_{DNNr} = 0.5\left[ln\, t_{DNN} + 0.80907 + 2s_1\right] \quad (14)$$

3. The skin factor, s_1, was obtained by dividing Eq. 14 by Eq. 9:

$$s_1 = \frac{1}{2}\left[\left[\frac{0.0002637 k_1 t_{r1}}{n\phi\mu_{eff} c_t r_w^{3-n}}\left(96681.605\frac{h}{qB}\right)^{n-1}\right]^{\alpha}\left(\frac{\Delta P}{\left(t*\Delta P\right)}\right)_{r1} - ln\left(\frac{k t_{r1}}{n\phi\mu_{eff} c_t r_w^{3-n}}\left(96681.605\frac{h}{qB}\right)^{n-1}\right) + 7.43\right] \quad (15)$$

4. The radius of the injected non-Newtonian fluid bank was calculated using the following correlation (valid for $n>1$), obtained from reading the time at which the radial flow non-Newtonian ended (t_{e_rNN}):

$$r_a = \left[\frac{G\left(0.46811 e^{0.76241n}\right)^{1/\alpha}}{t_{e\_rNN}}\right]^{1/(n-3)} \quad (16)$$

5. For the Newtonian region the permeability and skin factor were presented by Tiab (1995) as:

$$k_2 = \frac{70.6 q \mu_N B}{h(t*\Delta P')_r} \quad (17)$$

$$s_2 = \frac{1}{2}\left[\left(\frac{\Delta P}{t*\Delta P'}\right)_{r2} - \ln\left(\frac{k_2 t_{r2}}{\phi \mu_N c_t r_w^2}\right) + 7.43\right] \quad (18)$$

6. Non-Newtonian/Newtonian radial flow line intercept.

The infinite-acting dimensionless pressure derivative was given by:

$$\left(t_D * P_D'\right)_{rN} = 0.5 \quad (19)$$

Combining Eqs. 19 and 10 would result in:

$$\frac{\left(t_D * P_D'\right)_{rNN}}{t_{DNN}^{\alpha}} = \left(t_D * P_D'\right)_{rN} \quad (20)$$

Rearranging,

$$\frac{\left(t_D * P_D'\right)_{rNN}}{\left(t_D * P_D'\right)_{rN}} = t_{DNN}^{\alpha} \quad (21)$$

Replacing the dimensionless quantities and solving for k_1:

$$k_1 = \left[\left\langle\left(\frac{H}{12}\right)\left(9+\frac{3}{n}\right)^n\left(1.59344\times10^{-12}\phi\right)^{(1-n)/2}\left(96681.605\frac{h r_w}{qB}\right)^{1-n}\right\rangle^{1-1/\alpha}\frac{\mu_N^{1/\alpha}\phi c_t r_w^2 n}{0.0002637 t_{irN\_NN}}\right]^{1/2} \quad (22)$$

Table 1. Reservoir and fluid data for the synthetic examples

Parameter	Example 1	Example 2
P_R, Psi	2,500 (1.724x10^7 Pa)	2,500 (1.724x10^7 Pa)
r_e, ft	5,000 (1524 m)	10,000 (3048 m)
r_w, ft	0.3 (0.09144 m)	0.3 (0.09144 m)
h, ft	25 (7.62 m)	25 (7.62 m)
ϕ, %	20	25
k, md	100 (9.87x10^{-14} m^2)	50 (4.93x10^{-14} m^2)
q, bbl/D	250 (4.60x10^{-4} m^3/s)	100 (1.84x10^{-4} m^3/s)
B, rb/STB	1.0 (1 m^3/m^3)	1.0 (1 m^3/m^3)
c_t, 1/psi	6.89x10^{-6} (1x10^{-9} 1/Pa)	6.5x10^{-6} (9.43x10^{-10} 1/Pa)
r_a, ft	100 (30.48 m)	80 (24.384 m)
H, cp*s^{n-1}	10 (0.01 Ns^n/m^2)	5 (0.005 Ns^n/m^2)
μ_N, cp	3 (0.003 Pa.s)	10 (0.01 Pa.s)
n	1.4	1.6

where t_{iN_NN} was the time of non-Newtonian and Newtonian radial line intersection.

The above equations for permeability and skin factors were also found and verified by Escobar *et al.*, (2010) for pseudoplastic behavior.

Examples

Example 1

An injection test was simulated, using the information from Table 1. Pressure and pressure derivative data are shown in Figure 3. Permeability and skin factor in each region and the radius of non -Newtonian fluid bank had to be estimated.

Solution.

The log-log plot of pressure and pressure derivative against injection time is given in Figure 3. From that plot the following information was read:

t_{r1} = 1.0 hr	ΔP_{r1} = 3,785.19 psi	$(t^*\Delta P')_{r1}$ = 165.51 psi
t_{r2} = 190 hr	ΔP_{r2} = 4,128.33 psi	$(t^*\Delta P')_{r2}$ = 21.99 psi
t_{e_rNN} = 3.1 hr	t_{irN_NN} = 2150 hr	

First, *a* was evaluated with Eq. 13 to be -0.25. Non-Newtonian effective fluid permeability was estimated with Eq. 12, resulting in 94.16 md (9.29x10^{-14} m^2). Eq. 3 was then used to find effective viscosity, resulting in 3,098.61 cp(s/ft)$^{n-1}$ (4.984 Ns^n/m^2). Skin factor in the non-Newtonian region was then found with Eq. 15 to be -1.94.

G was estimated to be 6.13x10^{-3} hr/ft^{3-n} (147.68 s/m^{3-n}) with Eq. 2. This value was used in Eq. 16 to find the non-Newtonian fluid bank radius: 105.85 ft (32.26 m).

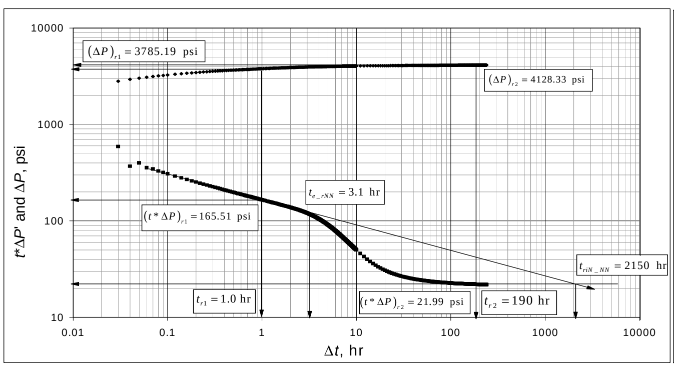

Figure 3. Pressure and pressure derivative for example 1

Eqs. 17 and 18 were used to estimate permeability and skin factor for the Newtonian zone; they were 96.32 md (9.51x10^{-14} m^2) and 65.27, respectively. Eq. 22 was used to re-estimate non-Newtonian effective fluid permeability: 109.95 md (1.09x10^{-13} m^2).

Example 2

Another injection test was simulated using the information from Table 1. Pressure and pressure derivative data are shown in Figure 4. Permeability and skin factor in each area and the radius of non-Newtonian fluid bank had to be estimated.

Solution.

The following information was read from Figure 4:

t_{r1} = 0.52 hr	ΔP_{r1} = 2,544.52 psi	$(t^*\Delta P')_{r1}$ = 105.52 psi
t_{r2} = 145 hr	ΔP_{r2} = 2,900.03 psi	$(t^*\Delta P')_{r2}$ = 59.41 psi
t_{e_rNN} = 2.6 hr	t_{irN_NN} = 1.9 hr	

As in the former example, *a* was evaluated with Eq. 13 giving a value of -0.429 and non-Newtonian effective fluid permeability was estimated with Eq. 12 to be 45.17 md (4.46x10^{-14} m^2).

Effective viscosity was calculated with Eq. 4: 31,737.06 cp(s/ft)$^{n-1}$ (64.738 Ns^n/m^2); a -1.66 skin factor was then found in the non-Newtonian region with Eq. 17.

Eq. 3 was used to find *G*: 0.01624 hr/ft^{3-n} (308.5 s/m^{3-n}); this was used in Eq. 18 to find a 80.87 ft (24.65 m) non-Newtonian fluid bank radius. Newtonian zone permeability and skin factor were estimated with Eqs. 17 and 18, respectively: 47.53 md (4.69x10^{-14} m^2) and 16.98. Non-Newtonian effective fluid permeability was verified with Eq. 22 to be 43.41 md (4.28x10^{-14} m^2).

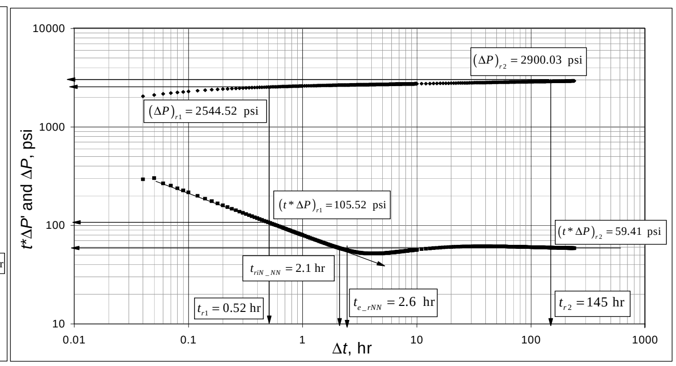

Figure. 4. Pressure and pressure derivative for example 2

Analysis of results

The simulation experiments showed that the pressure derivative pattern during the non-Newtonian zone for a dilatant-type fluid had a negative slope which increased as the flow behaviour index also increased. When n = 1 (Newtonian), the pressure derivative became a flat, horizontal line, as expected. The opposite occurred for pseudoplastic behaviour in which the pressure derivative straight-line had an increasing slope as flow decreased.

The estimation of effective permeability and skin factors matched very well with the simulated input data. It was found that the transition period, after the non-Newtonian zone, took longer to reach than for pseudoplastic behaviour and also bank zone estimation was more accurate for dilatants.

Conclusions

A new expression for estimating the non-Newtonian bank radius was developed and tested with good results; this showed that the expressions used for estimating permeability and skin factor for pseudoplastic fluids by means of the TDS technique could be used on dilatant fluids.

Compared to the pseudoplastic case, the radial flow regime in the Newtonian zone of dilatant fluids took longer to form as flow behaviour index, *n*, and consistency factor, *H*, increased.

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Nomenclature

B	Oil formation factor, rb/STB
c_t	System total compressibility, 1/psi
h	Formation thickness, ft
H	Consistency (power-law parameter), cp*s^{n-1}
k	Permeability, md
n	Flow behaviour index (power-law parameter)
p	Pressure, psi
P_R	Reservoir pressure, psi
q	Flow/injection rate, STB/D
r	Radius, ft
s	Skin factor
t	Time, hr
$t*\Delta p'$	Pressure derivative, psi

Greek symbols

α	non-Newtonian region's slope pressure derivative
Δ	Change, drop
ϕ	Porosity, fraction
μ	Viscosity, cp
μ	Effective viscosity for power-law fluids, cp*(s/ft)$^{n-1}$
λ	Mobility, md/cp

Suffixes

1	Non-Newtonian region
2	Newtonian region
a	Location of the non-Newtonian fluid front
app	Apparent
D	Dimensionless
e	External
eff	Effective
N	Newtonian
NN	Non-Newtonian
W	Wellbore

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