THE USE OF PHENOMENOLOGICAL BASED SEMI-PHYSICAL MODELS AS VIRTUAL SENSORS FOR DENSITY AND VISCOSITY OF MINERAL SLURRIES

Jenny Lorena Díaz Castañeda
Chemical Engineer

Universidad Nacional de Colombia
Facultad de Minas
Departamento de Procesos y Energía
Medellín
2017

THE USE OF PHENOMENOLOGICAL BASED SEMI-PHYSICAL MODELS AS VIRTUAL SENSORS FOR DENSITY AND VISCOSITY OF MINERAL SLURRIES

Jenny Lorena Díaz Castañeda
Chemical Engineer

Thesis work presented as a partial requirement for the degree of
Master of Engineering – Chemical Engineering

Advisor
Hernán D. Alvarez Zapata
Ch.E., M.Sc., Ph.D.

Coadvisor
Diego A. Muñoz Durango
Ch.E., M.Sc., Ph.D.

Universidad Nacional de Colombia
Facultad de Minas
Departamento de Procesos y Energía
Medellín
2017

To my parents and my lovely brother,
Jairo Díaz, Noraida Castañeda, and Jairo A. Díaz

A mis padres y mi amado hermano,
Jairo Díaz, Noraida Castañeda, y Jairo A. Díaz

Acknowledgment

Firstly, I want to thank the persons who are the most important in my life: my father, my mother and my brother. They have been my main motivation throughout my life, their support and trust have been the fuel to continue forward in this process.

I would like to express my gratitude to my advisor, Hernán Álvarez, for his permanent support, motivation and guidance for this work and his teaching about the research. He more than an academic advisor has been also a spiritual and moral mentor.

Also, I wish to thank my coadvisor Diego A. Muñoz by his ideas and disposition to cooperate. Also thanks to the professor Héctor Botero by his disposition to help me and share his knowledge in any moment during this process. To *Dynamic Processes Research Group - KALMAN* and to all members thanks by the contributions, constructive comments and discussions which contributed to developing and to improve this thesis.

To a group of unconditional persons, my cousin Angela and my friends Freiman, Alex and Melissa by their constant motivation and advice to achieve this goal. I would also like to acknowledge the families Díaz-Ortiz and Castañeda-Marín, grandfathers and grandmothers, uncles and aunts, and cousins, everyone plays an important part in my life.

Finally, I would like to extend my acknowledgments to the *Universidad Nacional de Colombia* and the company *Suministros de Colombia S.A. (SUMICOL)* for the support and accompaniment for the performance of this thesis.

Abstract

The use of phenomenological based semi-physical models as virtual sensors for density and viscosity of mineral slurries.

This thesis presents two estimation structures for on-line estimation of density and viscosity of mineral slurry flowing through a pipes and fittings assembly (PFA). The structures are developed from a phenomenological based semi-physical model (PBSM) as main component of a soft-sensor. The PBSM is developed based on the mass conservation and momentum transfer principles, in order to separate the density estimate from the viscosity, and momentum transfer, considering frictional energy losses. In this way, the model equations are obtained based on the variables directly affecting the density and viscosity of minerals slurries. The proposed model is obtained in state variables, considering the available measurements (pressure drop and volumetric flow) and the disturbances in the process. In this sense, from the theory for state observers, an observer of unknown inputs was used in the estimation structure to take into account the inherent disturbances in the process inlet. The density is estimated as a state of the model while the two options for estimating viscosity are proposed based on the availability for characterizing the rheology of the mineral slurry from laboratory tests. Finally, the performance of the proposed structures for the estimation of the density and viscosity properties of mineral slurry is tested by simulation. The conducted tests use disturbances in the mixture inlet composition and include noise in the measured variables. The results exhibit a good estimation of both slurry properties using both proposed methods.

Keywords: Soft-sensor, multiphase fluids, phenomenological based semi-physical models, loss of energy, state estimator.

Resumen

Uso de modelos semifísicos de base fenomenológica como sensores virtuales de densidad y viscosidad de pulpas minerales.

En esta tesis se presentan dos estructuras diferentes para la estimación en línea de la densidad y viscosidad de una pulpa mineral fluyendo a través de un conjunto de tuberías y accesorios. Las estructuras son desarrolladas a partir de un modelo semifísico de base fenomenológica como componente principal de un sensor virtual (*soft-sensor*). El modelo se obtiene con base en el principio de conservación de la masa y la transferencia de momento, con el fin de separar la estimación de densidad y viscosidad, y considerar las pérdidas de energía por fricción. De este modo, las ecuaciones del modelo son obtenidas en función de las variables que afectan directamente la densidad y la viscosidad de las pulpas minerales. Dicho modelo es obtenido en variables de estado y considerando las mediciones disponibles en planta (caída de presión y flujo volumétrico), además de las perturbaciones en el proceso. En este sentido, a partir de la teoría de observadores de estados, se propone un observador de entradas desconocidas de tal manera que las perturbaciones inherentes del proceso puedan ser consideradas. Con base en lo anterior, la densidad se estima como una variable de estado, mientras que para la estimación de la viscosidad se consideran dos opciones en función de la disponibilidad para caracterizar la reología de la pulpa en laboratorio. Finalmente, el desempeño de las estructuras propuestas para la estimación en línea de la densidad y viscosidad de una pulpa mineral se evalua mediante simulación. Las simulaciones fueron realizadas considerando perturbaciones en la composición de sólidos a la entrada, y ruido blanco para las variables medidas. Los resultados obtenidos presentan una buena estimación de las propiedades de la pulpa para las dos estructuras de estimación presentadas.

Palabras clave: Sensor virtual, fluidos multifásicos, modelos semifísicos de base fenomenológica, pérdidas de energía, estimadores de estado.

Contents

List of figures

List of tables

Latin letters

Symbol	Description
t	time
x, y, z	Coordinates
υ	Velocity
g	Gravitational force
D	Diameter
Q	Volumetric flow
N_i	Dimensionless number called according to sub-index i
f	Frictional factor
L	length of pipeline
K_F	The K factor - Energy losses
$\mathbf{v}$	Vector of velocity components
S	Cross section
P	Pressure
$\dot{m}$	Mass flow
W	Work
E	Energy
U	Average velocity
f	Friction factor
f_D	Darcy's Friction factor
h_L	Energy losses by friction
K	K factor for 2k method
K_1, K_∞	K factors for fittings
m	Total mass
n	Behavior index
K	Consistency factor
D_{in}	Internal diameter
$\mathbf{x}$	States vector
$\mathbf{u}$	Inputs vector
$\mathbf{y}$	Outputs vector
$\mathbf{d}$	Unknown input vector
$\mathbf{z}$	State transformation vector
$\mathbf{e}$	Estimation error
$\dot{\mathbf{e}}$	Estimation error dynamic
$\mathbf{O}$	Observability matrix
$\widehat{\mathbf{x}}$	Estimated state
$C[i, j]$	Wavelet transform coefficients

Greek letters

Symbol	Description
υ	Velocity
ρ	Density
μ	Viscosity
Δ	Gradient
τ	Tensor of momentum by molecular transport
ϵ	Roughness
ϕ	Potential energy
$\widehat{\phi}$	Potential energy per mass unit
κ	Kinetic energy
$\dot{\gamma}_{xy}$	Shear rate
τ_{xy}	Shear stress
η	Apparent viscosity
η_0	Low shear limiting viscosity
η_∞	High shear limiting viscosity
Ψ	Wavelet mother

Subscripts

Symbol	Description
in	Inlet condition
out	Outlet condition
T	Pipeline
m	Measured
um	Unmeasured
S	Solids
W	Water
V	Volumetric
P	Pulp
Re	Reynolds
1, 2	Point in the pipeline
1*B*	Point of flow division
tot	Total
ss	Stationary state

Abbreviations

Abbreviation	Description
PBSM	Phenomenological based semi-physical model
PFA	Pipe and fittings assembly
PS	Process system
DBE	Dynamic balance equation

Abbreviation	Description
BDE	Balance differential equation
LMI	Linear matrix inequalities
UIO	Unknown input observer
LPV	Linear parameter-varying
MEB	Mechanical energy balance
WT	Wavelet transform
STFT	Short-time Fourier transform
SfS	Soft sensor

CHAPTER 1
Introduction

1.1. Motivation

The pulps or slurries are the types of fluids predominant in the mineral processing industry, due to their uses in the extraction processes, particle size reduction or transport of minerals (for example pumping). The above considering that the minerals frequently are mixed with water to form a pulp or slurry before their processing [7]. Additionally, the quality of the majority of processes related to the particles separation (cyclones, hydrocyclones, jigs, etc.) strongly depends on viscosity and density of processed mineral pulps. Thus, any control system to hold efficient operation requires that the properties of the pulp are on-line measured or estimated. Hence, it is necessary to characterize pulp properties as density and viscosity. Majority of mineral pulps exhibit a non-Newtonian behavior and their properties do not depend of only one parameter or process variable. Some parameters or properties of the solid phase as concentration, size distribution, and form of solid particles, have influence on properties and rheology of the suspension. Therefore, the determination of the rheology of mineral slurries is a complex task owing to the many factors influencing pulp flow behavior [8].

Currently, there are direct and indirect methods for characterizing the rheology of the non-Newtonian fluids. The direct methods include viscometers o rheometers (coaxial cylinder viscometers, capillary viscometers, and viscometers vibrant sphere), which are used to determine off-line slurry rheology [9]. On the other hand, some indirect methods are possible using sensors by software (soft sensors), which estimate the properties of pulps using available information of other measured variables from the pulp processing or transportation. However, for the mineral processing industry [10], the implementation of soft sensors have been done based on empirical models, which do not provide information about the phenomenology of the process. Indeed, the empirical models are not generalized with other similar processes because they strongly depend on installation particularities and fluid and solid kind. The above is due to that the phenomena influencing the rheology are not considered for the construction of empirical models, thus the estimation is difficult when disturbances occur or take place changes on phases properties altering pulp rheology.

Although the importance of the rheological characterization is recognized in mineral processing literature, the majority of studies have been focused in laboratory measurements of the properties to determine the pulp rheology. Other studies are focused on developing in-line measurement devices, which operate with direct intervention of the process line. However, these devices implicate high cost of purchase, installation and maintenance [9], [11], [12], [13]. In addition, in the proposed methods in the literature, there is no a phenomenological procedure which permits to separate the estimation of density and viscosity of a mineral slurry, having to account the strong relation between these properties in the Reynolds number. This

last number is directly involved in calculates of energy losses, flow behavior and momentum transfer for the flow of a slurry through a pipeline [10], [14]. Based on the above, there is no a standard procedure for on-line measurement of the rheological variables, without intervene of the process lines and which permit to separate the estimation of density and viscosity of a pulp. On the other hand, it is known that the influence of main factors affecting the properties of mixture can be determined through phenomenological based models of the fluid flowing. These models allow to explain the dependence of slurry properties from factors and parameters governing the slurry rheology [2].

According to the previous discussion, to estimate pulp properties is necessary to construct a model that consider the phenomena involved in the flow process, also the variables and parameters influencing the system behavior. This model must quantify the interactions of different factors, which characterize the rheologycal properties of mineral slurries. Besides the features mentioned above, the structure of the model must serve as a structure for the soft sensors. This structure must allow to use available measurements of some process variables as inputs to the model to estimate the flowing slurry properties. In this sense, this research has the aim of proposing an on-line estimation for slurries flow properties using soft sensors based on a phenomenological based semi-physical model for the flow process of a mineral pulp into a pipe and fittings arrangement (PFA). This pipe and fittings assembly is responsible for transporting the pulp through the process units and provides the information about the critical variables for the tasks of design, monitoring and control of the process.

1.2. Research problem

The measurement of critical variables in industrial processes is a difficult and necessary task for control and monitoring of the process. However, some variables cannot be directly measured due to the few available devices for measuring and their high costs [15], [16]. Furthermore, some critical variables in the process have a strong dependence on parameters own of the system. Hence, these parameters must be characterized correctly to guarantee a correct estimation of the process variables or states. Uncertainty on those parameters can propagate to the estimation of unmeasured states. A case of this situation is the mineral processing industry and mineral extraction plants, where it is necessary to properly characterize the rheological properties of mineral slurries and based on these characterization, to obtain estimates of the process variables, which represent the reality of the process [10]. In this type of plants, mineral slurry is a mix of solid particles suspended in a liquid and currently with properties normally offline characterized by laboratory measurements available at long time intervals.

In many mineral processing plants, the flow of mineral slurries through the pipes can present non-Newtonian behavior due mainly to the features of solid particles in the slurry. The behavior of slurries depends on factors such as size and particles shape, solid particles distribution, particles concentration in the liquid phase, density and viscosity of the transport phase [17]. Thus, the rheological behavior of slurry must be considered for estimating its density and viscosity, which are a function of the properties of each one of the phases conforming the mixture [17]. Although both density and viscosity of mineral slurry depend on the properties of individual phases in the mixture, the dependence of density on each phase properties is different to viscosity dependence. According to literature, the density in

mineral slurries is mainly a function of solid concentration in liquid phase, while the viscosity of mixture is strongly affected by size, shape and solid particles distribution in the liquid phase [9], [14].

Until now some researches have developed on-line estimation of density and viscosity for mineral slurry as a function of properties of the solid particles [7], [12], [18]. However, these researchers are focused on the use of direct measures or characterizations of process samples in the laboratory [11], [19], [20]. The outstanding disadvantages of this methods are the high costs of installation implicated in using devices for direct measure and the high time delay for laboratory measurements respect to processing time. The last one is due to the non uniform or constant properties of process feed. In mineral processing the feed changes on time, therefore the determination of mixture properties (for this case, density and viscosity) cannot be associated in real time to changes in the feed and its properties. In addition, pressure drop and energy losses by friction are directly affected. Indeed, when it is not possible to associate the slurry properties with changes in the feed, the estimation of density and viscosity does not represent the real operation of the process.

According to the above, the research problem for this thesis is that now there is no a methodology for the on-line estimation of density and viscosity values of mineral slurry, using a phenomenological based semi-physical model (PBSM) and linking the properties of the mixture with the properties of individual phases. Additionally, there is no a procedure to separate the effects of density and viscosity on the flow regime and momentum transfer for mineral slurries flowing through the pipe. Therefore, the focus of this research is to obtain a formal methodology for on-line estimation of density and viscosity of mineral slurries flowing through pipes, based on theories and concepts valid for multiphase flow. With this, it must be possible to identify the dependence of mixture properties with properties of each phase and their interactions for representing the reality of the process. The final intention is to use estimations of mixture properties as useful tools for controlling process units that operate with mixtures of water and solids.

1.3. Objectives

The objectives that conduct this thesis are:

General objective:

Propose a methodology for the on-line estimation of density and viscosity properties of mineral slurries flowing through a pipe and fittings assembly (PFA), using soft sensors based on phenomenological based semi-physical models, which consider the phenomena of momentum transfer and the influence of the properties of each phase over mixture properties.

Specific objectives:

- Identify proposed methods from the literature for the on-line estimation of the rheological properties of multiphase fluids.

- Recognize the properties of single phases and the variables that directly affect the density and viscosity of multiphase fluids.
- Distinguish the operational principles, advantages, and limitations of each method found in the literature for on-line estimation of the multiphase fluids properties.
- Classify the variables and the single phases properties as relevant and irrelevant facts regarding mixture properties according to the degree of affectation.
- Propose a phenomenological based semi-physical model including the main variables affecting the mixture properties for mineral slurries flowing through a PFA, associated to the momentum transfer and friction energy losses.
- Develop a methodology for integrating the proposed model to a soft-sensor structure, using the variables associated with momentum transfer and energy losses as input data and correction factors.
- Evaluate the proposed methodology for the on-line estimation of the mineral slurries properties, with the simulation of a pumping process of mineral pulps in the industry of mineral processing.

1.4. Main publications

Additionally to the results obtained from the objectives, the main publications related to this work are:

Publications:

- J. L. Diaz C. and H. Alvarez, (2014). An EMSO Platform for Modeling Friction Losses in Process Plants (in Spanish). Lámpsakos - ISSN: 2145-4086. DOI: http://dx.doi.org/10.21501/issn.2145-4086, Colombia.
- D. A. Muñoz, J. L. Diaz C., Taborda S., and H. Alvarez, (2016). A phenomenological-based semiphysical model for hydrocyclones and its feasible operation region. International Journal of Mining, Materials, and Metallurgical Engineering. Accepted December 2016.
- J. L. Diaz C., D. A. Muñoz and H. Alvarez. Slurry Density and Viscosity online Estimation Using a Phenomenological Based Model as Soft-Sensor. Submitted to International Journal of Mineral Processing. February 22, 2017.

Conference papers:

- J. L. Diaz C. and F. Bolaños, (2016). Analysis of the frequency response of the pressure drop signal in the fluid transport process (in Spanish). In Latin American Conference on Automatic Control CLCA. Medellín, Colombia.

1.5. Thesis outline

The thesis is organized as follows: In chapter 2 the concepts and theory related to the transport of slurries through pipelines are exposed. In addition, this chapter provides a methodology for the formulation of phenomenological based semi-physical models. A review of main concepts for the state estimation and the state estimators commonly used for the systems with unknown inputs is presented in chapter 3. Chapter 4 explains the estimation problem for the properties of a slurry flowing through a PFA and includes the main contribution of this thesis, which is the use of phenomenological based semi-physical model to propose a structure for the construction of an unknown input observer. The application of the proposed structure to a transport process of slurries is presented in Chapter 5. Finally, in the Chapter 6 the conclusions and some future works derived from this thesis are exposed.

CHAPTER 2
Dynamic models to fluids flowing

This chapter has the intention of presenting a review of different models for the process of fluids flowing through a pipe and fittings arrangement (PFA). The variables governing the behavior of fluids, the fluid kind (compressible and incompressible fluid), and the behavior as Newtonian or non-Newtonian fluid are distinguished. Thus, a description of basic concepts related to the process modeling and some aspects concerning with the kind of models currently used are shown. A methodology to obtain phenomenological based semi-physical models (PBSM) is presented. Next, some basic concepts of the dynamic of fluids flowing into pipes and the flow regimes and the variables that influence those regimes are mentioned. Finally, the fundamental concepts about properties of the non-Newtonian fluids, the typical behaviors of flow of that fluids, and the basic models to describe them are exposed.

2.1. Process modeling

According to [21], the tasks of modeling and simulation are directly related to the concepts of systems, models and processes. The relationships between model and system are given by the modeling, while the relation between model and process correspond to model simulation. A system is a representation of the reality, an abstraction of the real process, whereas a model is a set of instructions to describe a behavior of interest in the modeled processes. The instructions set can be represented by mathematical relations, such as differential equations or any other mathematical formalisms, which are obtained from the fundamental laws governing the system [21].

The representation of a system with mathematical relations determines the structure of a model and also provide the relevant information of the system. The model structure refers to the set of relationships among parts of the model, such as variables, constants, and parameters which together form the terms of models. The incorrect use of the fundamental laws of physic and the conservation principles can generate incomplet, inconsistent or ambigous models to describe the process. An incomplete model is one not able to describe all relevant phenomena of the system. The inconsistent models present contradictions as result of the use of incompatible laws for describing the same phenomenon. Finally, a model in which a phenomenon can be described by two or more laws, each one from different approaches without knowledge about the appropriate law for the case of study, is considered an ambiguous model [21].

There are many classifications in the literature for models, however, and according to [22] the models can be classified using three approaches:

- Depending on the process knowledge implicit in the model, there are models of white, black and gray box.
- Considering the kind of information provided by the model, there are explicative, descriptive and predictive models.
- About the spatial grouping features of the model parameters, there are models with distributed parameters and lumped parameters.

The white box models or phenomenological models are obtained from theoretical knowledge (laws or principles) about the phenomena of the system and the interactions among them. Opposite to the above models, the black box models result from experimental observations applying curve fitting to find correlations. These models are commonly named empirical models and are used frequently when the predominant physical phenomenon is unknown or very complex. In the middle point between white and black box models, the gray box or semi-physical models are found. The gray box models have two subfamilies which are different according to the origin of model structure, which can be phenomenological based (white box model) or empirical based (black box model).

The gray box model with a phenomenological based structure contains one or more empirical sub-models or correlations for the unknown parameters of the model. This kind of model based on fundamental laws and conservation principles for obtaining the model structure and empirical correlations for parameters are recognized as phenomenological based semi-physical model (PBSM) [22]. The other kind of models into the gray box models has empirical structure and one o more phenomenological sub-models for the parameters. This last one are called empirical based semi-physical models and are not frequently used in engineering. The PBSM, unlike of pure phenomenological models, presents the advantage of describing part of the model parameters through correlations fitted from experimental observations. In this way, it is possible to construct new correlations without having full detail of phenomenon governing the parameter behavior. This fact simplifies the PBSM construction, allowing to obtain a parameter sub-model without phenomenological based knowledge using a simplified function (correlation).

Phenomenological based semi-physical models (PBSM) present a better approximation to qualitative and quantitative description of characteristics phenomena of the process. The PBSMs are a powerful tool in the process engineering due to the variety of process that can be described from the conservation principles, in addition to well-known correlations for process parameters, i.e., Fick, Newton, and Fourier laws. The big advantage of the PBSM is their explanatory structure, which permits to extract important information to understand and explicate the system behavior with less complex structure compared to white box models.

Due to the construction of a PBSM is not an easy task, and the construction process determines the model correctness, computed aided solution, understanding and interpretation of results, a methodology for developing PBSM is presented [23], [24].

1. Develop a verbal description and a complementary process flow diagram. These pieces of information must be clear and complete. Description and diagram are doing regarding to the real process to be modeled.

2. Propose a modeling hypothesis and set a level of detail for the model according to model object or purpose. This purpose is easily identified after elaborate the question to be answered by the model. Regarding level of detail, two main options there exist: lumped parameters or distributed parameters, both used at any size level (molecular, microscopic, mesoscopic or macroscopic).

3. Define as many process systems (PS) on the process to be modeled as required by the previously setted level of detail. A clue to PSs determination is to look for physical walls into the process, distinguishable phases or any mass characteristics marking spatial differences. Plot a block diagram relating all PSs using thin arrows for indicating mass flows and thick arrows for energy flows among PSs.

4. Apply the principle of conservation to each determined PS. It is recommended to take almost next balances: total mass balance, n component mass balances, total energy balances as thermal energy or mechanical energy balances. These equations form the Dynamic Balance Equations (DBE) set.

5 Select from DBE those equations with significant information to fulfill the model purpose as it was established in Step 2. Some DBE can be stationary equations but are included into the model structure due to their balance origin. Other DBE are redundant or are merely a numerical equality, them they must be discarded.

6. Identify variables, parameters and constants of the model. Fixed the values for all constants of the model remembering that variables values are obtained only by solving the model. Variables cannot be determined a priori. Contrary, parameters will never be the result of model solution, they need to be specified by the modeler previous to the model solution.

7. Find constitutive equations for calculating the largest number of parameters in each PS. Parameters without a constitutive equation must be identified from experimental data using auxiliary models (normally empirical ones) or the current PBSM structure converted to its stationary state provided enough experimental stationary state values.

8. Verify the degrees of freedom of the model.

9. Obtain a computational model or the solution of the mathematical model.

10. Validate the model for different operation conditions and evaluate its performance.

2.2. Modeling of fluids flowing in pipes

The dynamic of fluids regarding to momentum transfer can be described from three different levels as follows [25]:

- **Macroscopic level:** This level corresponds to a general vision about the problem. It is based on a set of balance equations describing the changes in the system due to interactions with the environment. Those are considered macroscopic balance without other levels of detail of the system. These level is the more commonly used in process engineering for representing the behavior of the fluid flowing into pipes or between pieces of equipment.

- **Microscopic level:** The aim of this level is to obtain information about variables and profiles of these into the system. The analysis is doing over a small region of the system using variation equations as Navier-Stokes to describe changes in that region. The computational fluid dynamics (CFD) is one of the common tools of this level.

- **Molecular level:** The use of this level is common when the modeler is interested on understanding the transport mechanics since molecular level. In process engineering this level has few uses except for fluid movements into porous media.

Each one of above levels is used depending on the information that is wanted to obtain from the model. Thus, the macroscopic and microscopic level are frequently used for engineering applications, whereas molecular level is normally employed in physical sciences. In addition, these levels are described in different length scales. Centimeters to meters for macroscopic, microns to centimeters for microscopic and nanometers (1 to 1000) for molecular level [25].

2.2.1. Basic concepts about fluid flowing in pipes

The flow of a fluid through a pipe is generally tackled from the macroscopic and microscopic viewpoints with the purpose of determining the changes in the system. In the case of the processes of fluids transport, the system over which it is applicable the conservation principle (mass and energy balances) is defined as a portion of the pipe. Balance equations to apply are continuity equation and momentum equation, respectively. Also, in the literature two main mechanics of momentum transport, the molecular and convective, have been identified. From a molecular viewpoint, the transport is associated to the transfer of momentum due to the action of the viscous forces, while the convective transport is related to the global movement of the fluid [2]. However, there are many concepts additional to the conservation principles necessary to understand the fluids flow in pipes, which are related to the fluids properties and the specifications of the pipe. Next, some of these concepts are explained.

2.2.1.1. Continuity equation.

According to [25] this equation describe the variation rate of the fluid density in the space respect to the time, all for a differential volume element. This equation results after obtaining a mass balance for the process system. The continuity equation can be expressed as:

$$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial}{\partial x}\rho v_x + \frac{\partial}{\partial y}\rho v_y + \frac{\partial}{\partial z}\rho v_z\right), \tag{2.1}$$

Where ρ is the density of the fluid and v_i the velocity of fluid in the spatial component i (with i equal to x, y and z). The equation 2.1 represents the rate of change of the mass into the process system considering input and output mass flows. For the case of incompressible fluids the term indicating density change is equal to zero ($\frac{\partial \rho}{\partial t} = 0$). This representation considers the transfer through all faces of the volume element.

2.2.1.2. Momentum equation.

The equation for the rate of change of momentum is obtained from a momentum balance over the system, which considers the input and output flows of momentum in addition to the external forces acting over fluid in all faces of the differential volume element [25].

$$\frac{\partial}{\partial t}\rho\boldsymbol{v} = -\left[\nabla \cdot \rho\boldsymbol{v}\boldsymbol{v}\right] - \nabla p - \left[\nabla \cdot \boldsymbol{\tau}\right] + \rho g, \tag{2.2}$$

Where ∇p is the pressure gradient and $[\nabla \cdot \boldsymbol{\tau}]$ is a vector of divergence of τ tensor representing the addition of momentum by molecular transport per unit of volume. The term $[\nabla \cdot \rho\boldsymbol{v}\boldsymbol{v}]$ is the addition rate of momentum due to the convection transport per unit of volume, and the term that corresponds to the action of external forces, such as gravity, is ρg. The detailed deduction and explication about of these mechanic energy transport are developed in [2], [25].

2.2.1.3. Flow regimes.

The introduction of the concept of flow regimen was done in 1883 by Osborn Reynolds with an experiment realized to measured the pressure drop as a function of flow rate [2]. From the obtained results, O. Reynolds proposed three kinds of flow regimen according to the flow rate in a tube. The laminar flow was considered for the stable flow in which the elements of fluid move in smooth layers relative to each other with no mixing. This occurs at low flow rates. Opposite to laminar flow regime, the flow characterized by high degree of mixing among fluids elements was named as *turbulent flow* or unstable flow and appears at higher flow rates. However, a particular case was found for the intermediate flow rates values at which it is not possible determine a tendency for the relationship between the pressure drop and the flow rate. This case is known as transitional regime because the flow is unstable but not totally mixed. This correspond to the transition of the laminar flow to turbulent flow [2], [25]. The nondimensional Reynolds number is defined according to the equation:

$$N_{Re} = \frac{D\upsilon\rho}{\mu} \tag{2.3}$$

where, ρ and μ are the density and viscosity of the fluid, whereas D and υ are the pipe inside diameter and the velocity through the pipe. The Reynolds number expresses the relationship between the inertial momentum flux in the flow direction and the viscous momentum flux in the cross direction [25]. Viscous forces result from intermolecular forces. When these forces dominate in the system, low Reynolds numbers are obtained and this is associated with the laminar flow. An opposite case occurs when the system is dominated by inertial forces, which produce high Reynolds numbers or turbulent flow [2].

Depending on features of pipes and properties of fluids, the flow regime can be identified according to the Reynolds number as: Generally, laminar flow in a pipe occurs when the value of Reynolds (N_{Re}) is less to 2000, and the turbulent flow is presented when the N_{Re} is greater than 4000. Finally, the transition region is between the laminar and turbulent region [2].

2.2.2. Energy losses by friction.

When a fluid flows through a pipe or conduit, fluid energy decreases due to energy losses as a result of the action of frictional forces. These forces correspond to the force exerted by a part of the fluid over other parts of the fluid and the forces exerted by the fluid as a whole over the solid surface of the pipe. For a fluid that flows in a pipe due to the pressure difference between the inlet and outlet points of the pipe, the frictional force is the only opposed to the flow when the pipe is in a horizontal position. For the case of pipes in a vertical position or with some inclination, the action of the gravitational force should be considered too.

For the case of flow through pipes with circular cross section, the energy lost by the frictional force can be expressed as follows [25]:

$$h_L = f\frac{L}{D}\frac{\upsilon^2}{2}, \tag{2.4}$$

Where h_L is the energy lost by friction in $\frac{m^2}{s^2}$, L and D are the length and internal diameter of the pipe or conduit in m, and f the Darcy friction factor. Remembering that friction indicates the resistance to the motion. Frictional factor is a nondimensional number representing the ratio between the wall shear stress, and the kinetic energy per unit of volume or the flux of momentum carried by the fluid. This result is known as the Fanning friction factor (f_F), however, there are other definitions such as the Darcy friction factor (f) which is equal to $4f_F$. Although the equation 2.4 is valid for calculating the energy losses in any flow regime, the friction factor is different to each regimen. There are many correlations to determine the frictional factor depending on the flow regime or Reynold number, phases of the fluid and properties of the pipe as roughness. Next, some equations for calculating the friction factor are shown.

- *Laminar regime:* For this regime, the Darcy friction factor can be expressed as [2]:

$$f = \frac{64}{N_{Re}}, \tag{2.5}$$

- *Turbulent regime:* For the turbulent regime, when the roughness of pipe becomes important, the Darcy friction factor can be expressed as [26]:

$$f = \left\{-2log\left[\frac{\epsilon/D}{3.1} - \frac{5.02}{N_{Re}}log\left(\frac{\epsilon/D}{3.71} + \frac{14.5}{N_{Re}}\right)\right]\right\}^{-2}, \tag{2.6}$$

  Where ϵ is the absolute roughness of pipe measured in m. The equation 2.6 is valid also for the transitional regime.

The above equations are used for calculating the energy losses by friction in a pipe. However, the pipeline is complemented with fittings and special elements such as elbows, contractions, expansions, valves, etc. There are different methods for estimate the energy losses in this elements. Within the most common methods are found the method of K and $2K$ and the most recent, the method of $3K$, which correlates the losses in fittings using an equivalent

length of the pipe, through a K factor [2], and using equation 2.4, the K factor is expressed as:

$$K_F = f\frac{L}{D}. \tag{2.7}$$

In the work of [27] the next expression relating the K factor with the Reynolds number and the fitting kind was presented:

$$K_F = \frac{K_1}{N_{Re}} + K_\infty\left(1 + \frac{1}{D_{in}}\right). \tag{2.8}$$

The parameters K_1 and K_∞ depends on the fitting kind and its assembly to the pipeline method (for example: welded or screwed). K values are reported in [27] for several fittings. The diameter D_{in} must be given in inches. In addition to the above correlations for friction factors calculations, there are graphics methods to estimate this factor but only to pipe, non for fittings. These graphs for pipe are based on the Reynolds number and the relative roughness, such as the Moody diagram [28].

2.2.3. About models for fluids flowing in pipes.

For the study of the problems implicating isothermal flow, the equations commonly used as an initial point for the formulation of models are the continuity equation and the momentum equation. These equations are obtained from mass and momentum balances. However, there exist other equations complementing the above equations as angular momentum balance. These equations are gotten from the general expression of momentum equation, required when angular momentum is important.

According to [25] the rate equation of mechanical energy can be deduced from a laborious rearrangement of equation 2.2 and using the equation 2.1. The final result is an equation for the kinetic energy and potential energy as a function of only mechanical terms. The above is due to this expression considers the addition of kinetic energy by convection per volume unit $\left(-\left(\nabla\cdot\frac{1}{2}\rho v^2\boldsymbol{v}\right)\right)$, the work done by the pressure of surroundings over the fluid $(\nabla\cdot p\boldsymbol{v})$, the irreversible and reversible change of kinetic energy in internal energy $(-\boldsymbol{\tau}:\nabla\boldsymbol{v}$ and $p\,(-\nabla\cdot\boldsymbol{v})$, respectively), the work of the viscous forces over the fluid $(\nabla\cdot[\boldsymbol{\tau}\cdot\boldsymbol{v}])$ and the work done by the gravity force $(\rho\,(\boldsymbol{v}\cdot g))$.

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \rho\widehat{\Phi}\right) = -\left(\nabla\cdot\left(\frac{1}{2}\rho v^2 + \rho\widehat{\Phi}\right)\boldsymbol{v}\right) - (\nabla\cdot p\boldsymbol{v}) - p\,(-\nabla\cdot\boldsymbol{v}) - (\nabla\cdot[\boldsymbol{\tau}\cdot\boldsymbol{v}]) - (-\boldsymbol{\tau}:\nabla\boldsymbol{v}), \tag{2.9}$$

Where $\widehat{\Phi}$ is the potential energy per mass unit defined as $g = -\nabla\widehat{\Phi}$, and $\boldsymbol{v}$ the velocity vector. Until now the equations 2.1, 2.2 and 2.9 were obtained from microscopic systems or system as one differential volume element. Hence, these equations are expressed in terms of partial derivatives for describing the changes of mass, momentum and mechanical energy. This kind of equations consider the interaction between the fluid and the solid surfaces through to the boundary conditions established for the partial derivatives [25]. However, the interest in this work is focused in finding equations valid for implementing

soft sensors. Thus, the macroscopic balances permit to consider the interactions and understand the relationships between the system and the surrounding in a simplified way. The macroscopic balances are the result of the integration of the variation equations through the total volume of the system, so they are expressed as ordinary differential equations.

Thus, the macroscopic balance is widely used in many engineering applications because it permits a global description of bigger systems without considering the detail of the fluid dynamics into the system. Hence, these have broad application for the tasks of monitoring and control system. Thereby, this work is developed from macroscopic approach for constructing models describing the global dynamic of the flow process, using the macroscopic balance of mass, momentum or mechanical energy. A deduction of macroscopic balances from the microscopic balances given by equations 2.1 and 2.2, for modeling fluids flowing through a horizontal pipe with constant cross section is presented below. The modeled system is shown in Figure 2.1.

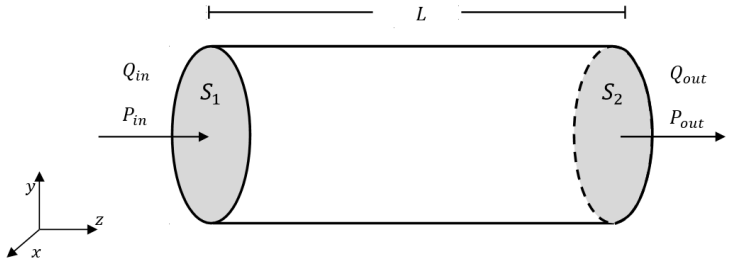

Figure 2.1. Process system to modeling the flow of a fluid through a pipe.

- *Macroscopic mass balance.*

  From the microscopic equation for continuity (Equation 2.1), considering that the flow is predominantly in the (z) axis of the pipe, the variation of density and velocity can be depreciated for the y and x components, and the change of mass per unit of volume can be expressed only as function of axial component. According to the above the equation 2.1 is reduced to:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial z}(\rho v). \tag{2.10}$$

  For a fluid approaching with an average velocity ($\langle v_1 \rangle$) through a cross section S_1, and coming out by section S_2 with an average velocity $\langle v_2 \rangle$, an integration of the equation 2.10 over total volume of the system in the Figure 2.1 produces the macroscopic balance of mass given by the following equation:

$$\frac{d}{dt} m_{tot} = \rho_1 \langle v_1 \rangle S_1 - \rho_2 \langle v_2 \rangle S_2, \tag{2.11}$$

  with m_{tot} equal to the total mass contained in the pipe between the planes 1 and 2, and the term $\dot{m}_i = \rho_i \langle v_i \rangle S_i$ is the mass flow of input or output in the plane i, with $i = 1, 2$. Thus, the equation 2.11 corresponds to the mass balance equation in unsteady state, which provides information about the change of mass as result of the input and output of mass to the system.

- *Macroscopic momentum balance.*

  From the same considerations used to develop the mass balance, and considering that the forces associates to the stress tensor τ are very low regarding to pressure forces, and the pressure does not change over the cross section in the planes 1 and 2, the change of momentum in the system can be described by the equation [25]:

$$\frac{d}{dt}\boldsymbol{P_{tot}} = \rho_1\langle {v_1}^2\rangle S_1\boldsymbol{u_1} - \rho_2\langle v_2\rangle S_2\boldsymbol{u_2} + p_1S_1\boldsymbol{u_1} - p_2S_2\boldsymbol{u_2} + \boldsymbol{F}_{s\to f} + m_{tot}g. \quad (2.12)$$

  Because the momentum is a vector quantity, in the equation 2.12 the symbols $\boldsymbol{u_1}$ and $\boldsymbol{u_2}$ represent the unit vectors in the flow direction. Thereby, this equation is the rate of change of total momentum in the system ($\boldsymbol{P_{tot}}$) as result of the input and output of momentum to the system by convection, pressure forces in the extremes of system, the gravity force acting on the fluid, and the force that the solid surface exerts over the fluid. Writing the equation 2.12 in a simplified form including the term of mass flow ($\dot{m}$), the macroscopic balance of momentum in unsteady sate is:

$$\frac{d}{dt}\boldsymbol{P_{tot}} = -\triangle\left(\frac{\langle v^2\rangle}{\langle v\rangle}\dot{m} + pS\right)\boldsymbol{u} + \boldsymbol{F}_{s\to f} + m_{tot}g. \quad (2.13)$$

  Where the symbol $\triangle$ corresponds to the difference between the inlet and output planes.

- *Macroscopic mechanical energy balance:*

  Based on equation 2.9 for describing the variation of mechanical energy, if this equation is integrated over total volume of the flow system, the macroscopic balance of mechanical energy, according to [25] can be written as:

$$\frac{d}{dt}\left(\kappa_{tot} + \Phi_{tot}\right) = -\Delta\left(\frac{1}{2}\langle v^2\rangle + \Phi + \frac{p}{\rho}\right)\dot{m} + W_m - E_c - E_v \quad (2.14)$$

  In the above equation, the first term of the right-hand side is the contribution by dissipation of kinetic and potential energy per unit of mass, in the inlet and outlet point with constant cross section. This is valid when the fluid flows in a direction parallel to the pipe walls, and the density and potential energy are constant over the cross section. In this sense, W_m correspond to the work realized over the fluid by the action of mobile parts. The terms E_c and E_v are the energy loss by compression of the fluid and the viscous loss due to the viscosity of the fluid, respectively. For incompressible fluids, the term E_c is equal to zero, while the viscous losses depend on the fluid kind. Because the last term depends on the viscosity, and according to the equations presented above to calculate energy losses, this must be calculated in according to fluid kind: Newtonian or non-Newtonian.

2.3. Slurries flow behaviors

A slurry is a mixture of two phases, generally solid particles and a carrying fluid normally water. The flow of a mixture is very different than the flow of a single-phase fluid due to the

presence of two or more phases. Multiphases presence modify the properties of the mixture and therefore, the required energy for moving the fluid is commonly higher. The change of the properties of a mixture are affected by the presence of other phases transforming the fluid behavior from Newtonian to non-Newtonian fluid. The viscosity of mixtures depends on the phase concentrations, the shear stress and the shear rate experimented by the fluid in the pipe by which it is flowing. There are two basic slurry flow kinds, homogeneous and heterogeneous flows. This classification is based on the distribution of the solid particles in the carrier fluid [1].

- **Homogeneous flow.** In this case, the particles are uniformly distributed throughout the carrier liquid. In accordance to the literature, a homogeneous mixture has typical particle sizes which are smaller than a value between $40\mu m$ to $70\mu m$, depending on the density of solids. For many mineral slurries, as increase the particles concentration the slurry becomes more viscous and develops non-Newtonian behavior [1].
- **Heterogeneous flow.** In this kind of flow, the particles are not uniformly distributed. In a horizontal plane, the particles are not uniformly mixed and present a concentration gradient in the vertical axis. This is due to the heavier particles are deposited on the bottom of the pipe while the lighter particles are flowing in suspension. The typical concentrations of these mixtures are lower to 25% [1].

The comprehension of the physical principles determining the flow of solid-liquid mixtures is based on the interactions between the phases. According to the kinds of flow, the phases can be mixed well or move in superimposed layers. Additionally, the transport of solid with a liquid is limited by other aspects such as the shape of the solids, the speed of the flow and the degree of turbulence, the interactions between the particles and pipe walls, as well as the resistance of the particles to be raised by the liquid [1].

When a slurry flows through a pipeline, the disposition of the pipeline determines the flow regime of the pulp depending on the type of flow. However, the heterogeneous flow is the most affected by this, so for the case of a horizontal pipeline it is possible to identify four regimes of flow. Figure 2.2 shows these regimes as a function of particle size and velocity of flow for a horizontal pipeline. This classification is impotant for calculating the fluid pressure drop in the pipeline [2].

Based on Figure 2.2, the heterogeneous flow dominates at low velocities and higher particle sizes, while if velocity of flow increased the flow is more homogeneous, for particles sizes not very high. For the case of a vertical disposition of the pipeline, in order to avoid a bed of solids in the pipe wall more energy is required to flow. In this way, the gradient of pressure must be higher than for a horizontal pipeline. The above is due to enough energy is required to overcome the weigh of the liquid and solids, and the resistance of these particles to flow.

2.3.1. Non-Newtonian slurries

As it was mentioned, the pressure drop is one of the variables involved in the dynamic models for the flow of a fluid through a pipeline. However, pressure drop is directly affected by the properties of the mixture, especially density and viscosity. The density of a mixture can be determined as a function of phases concentrations, while the viscosity is more affected by

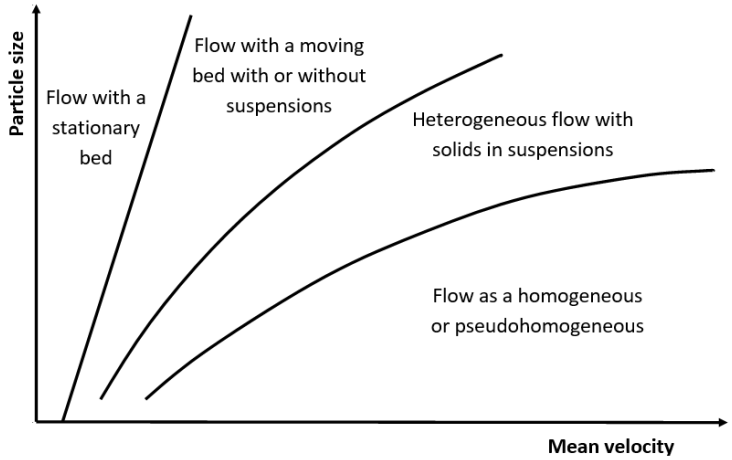

Figure 2.2. Flow regimes as a function of the particle size and flow velocity for the heterogeneous flow in the horizontal pipeline [1].

other aspects such as shape and size of particles, and the density and viscosity of each individual phase. With all these, the behavior of a slurry can be analyzed as a non-Newtonian fluid.

The Newtonian or non-Newtonian behavior of a fluid is determined based on its rheological properties. The rheology corresponds to the study of the deformation and flow behavior of materials, both fluids and mixtures of fluids with particulated solids [29]. The rheological properties are referred to the relation between the shear stress (τ_{xy}) and the shear rate ($\dot{\gamma}_{xy}$). The classical example for explicating shear stress and shear rate concepts is a fluid contained between two parallel plates, as shown in Figure 2.3. If the bottom plate is fixed and the plate in the top is free to move, the rheological properties can be classified by the way that the mobile plate responds when an external force is applied.

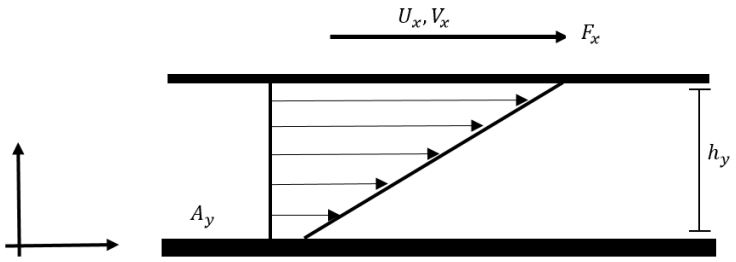

Figure 2.3. Scheme to simple shear [2].

The shear stress can be defined as the external force (F_x) per area unit of the mobile plate (A_y), while the shear rate is the displacement of the plate (U_x) respect to the distance between the two plates (h_y). In the case that the force applied is proportional to the rate of displacement, the fluid is named as a Newtonian fluid and its behavior can be represented by:

$$\tau_{xy} = \mu \dot{\gamma}_{xy}, \tag{2.15}$$

where,

$$\dot{\gamma}_{xy} = \frac{d\gamma_{xy}}{dt} = \frac{dv_x}{dy} = \frac{U_x}{h_y} \tag{2.16}$$

and μ the fluid viscosity. According to equation 2.15 if the external force is removed, the motion stops but the fluid not return to initial state. For the case in which the relationship between the shear stress and shear rate are not directly proportional when an external force is applied, the fluid is known as a non-Newtonian fluid. For this kind of fluids, the equation 2.15 continues being validated but the symbol μ is replaced by the apparent viscosity η, and the fluid behavior can be described by the equations 2.17 and 2.16.

$$\eta = \frac{\tau_{xy}}{\dot{\gamma}_{xy}}, \tag{2.17}$$

Although both kind of fluids presented different relationships for the shear stress and the shear rate, both Newtonian and non-Newtonian fluids do not have memory, i.e, they can not return to the initial state when the force is removed. These fluids are named viscous fluids, while the fluids able to return to its initial state are known as viscoelastic fluids. Because mineral slurries rarely have viscoelastic behavior, these fluids are not treated in this work. For more information about viscoelastic fluids please consult [1], [2], [9].

For characterizing the relationship between shear stress and the shear rate for non-Newtonian fluids, there exists many complex functions, which have been developed for different types of fluids. The different behaviors of the non-Newtonian fluids are shown in Figure 2.4. There the relationships between the shear stress and the shear rate for different kinds of fluids is presented. From this curves it is possible to identify the linear relation between the shear stress and shear rate of a Newtonian fluid. However, the non-Newtonian fluids present different behaviors among them. Some of the more used models to describe these behaviors are described below.

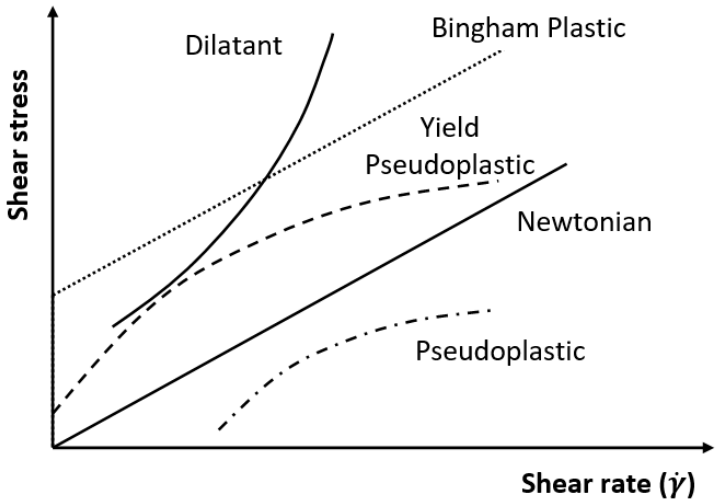

Figure 2.4. Shear stress versus shear rate for Non-Newtonian fluids [1].

2.3.2. Bingham plastics model

This type of fluid requires overcoming an initial yield stress to begin the motion. This kind of fluids has a linear behavior after that initial yield stress is exceeded. The apparent viscosity can be calculated according to:

$$\eta = \frac{\tau_0}{|\dot{\gamma}|} + \mu_\infty, \tag{2.18}$$

where τ_0 is the initial yield stress and μ_∞ is the viscosity to the higher shear rates. Therefore, for determining the behavior of these fluids it is necessary to identify the above properties. For this, it is recommended to develop experiments in the laboratory at different shear rate and shear stress. However, for the pulps, the solids concentration limits the values of yield stress and viscosity to higher values of shear rates. In the work of Thomas (1961) a correlation between the yield stress (τ_0), apparent viscosity (η), volumetric concentration of solids (C_v) and viscosity of the suspending medium (μ_0) is proposed as:

$$\tau_0 = K_1 C_v^3, \tag{2.19}$$

$$\frac{\eta}{\mu_0} = e^{K_2 C_v}, \tag{2.20}$$

with K_1 and K_2 characteristic constant associated with the particle size and shape.

2.3.3. Power law model

This model is very used for describing the behavior of non-Newtonian fluids, particularly for dilatant and pseudoplastic fluids. The power law model can be represented by the following equation:

$$\tau_{xy} = K|\dot{\gamma}|^{n-1}\dot{\gamma}, \tag{2.21}$$

and expressed in therms of the apparent viscosity,

$$\eta = K|\dot{\gamma}|^{n-1}, \tag{2.22}$$

where the terms K and n correspond to the consistency factor in $Pa \cdot s^n$ and the dimensionless behavior index, respectively. If $n = 1$, the relationship between the shear rate and shear stress is linear and this represents a Newtonian behavior. The behavior index can be positive o negative depending on the fluid kind. For the case of $n > 1$ the fluid has a dilatant behavior (or shear thickening), while that if $n < 1$ the fluid behavior is as pseudoplastic (or shear thinning) [2].

Although both fluid kinds, dilatant and pseudoplastic, require an infinitesimal shear stress to get motion, for dilatant fluid the rate of increase of shear stress with respect to the velocity gradient increases as the velocity gradient increases, whereas for a pseudoplastic fluid the rate of increase of shear stress with respect to the velocity gradient decreases as the velocity gradient increases.

For the case of the yield pseudoplastic fluids, which are characterized by the same relationship between the shear stress and the velocity gradient that of the pseudoplastic fluid, with addi-

tional condition that these fluids must overcome a considerable yield stress for getting motion, the power law model is modified as follows:

$$\tau_{xy} - \tau_0 = K\dot{\gamma}^n, \tag{2.23}$$

Where τ_0 the initial yield stress. The above is the Herschel-Buckley equation and it is accepted for describing the slurries behavior with low solids concentration. In the work of [1] there are others correlations that permit to represent the behavior of the yield pseudoplastic fluid with solids concentrations higher than 20%.

2.3.4. Complex models

Additional to above models, there are others more complex models for representing the behavior of some non-Newtonian fluids. This kind of fluids usually is related to the combination of two or more behaviors (structural fluids) presented in Figure 2.4. Some of these models are mentioned below [2], [30].

2.3.4.1. Carreau model

This model was proposed in (Carreau, 1972) and permit to describe the viscosity variation of the structural fluids using the following equation

$$\eta = \eta_\infty + \frac{\eta_0 - \eta_\infty}{(1 + (\lambda^2\dot{\gamma}^2))^p}, \tag{2.24}$$

where η_0 and η_∞ are the low shear limiting viscosity and the high shear limiting viscosity, respectively. p and λ are the shear thinning index and a time constant.

2.3.4.2. Meter model and Yasuda model

These two models were obtained as modifications of the Carreau model. These models introduce new parameters which permit a better representation of the viscosity of complex fluids.

$$\eta = \eta_\infty + \frac{\eta_0 - \eta_\infty}{(1 + \frac{\tau^2}{\sigma^2})^a} \tag{2.25}$$

$$\eta = \eta_\infty + \frac{\eta_0 - \eta_\infty}{(1 + (\lambda^2\dot{\gamma}^2)^a)^{p/a}} \tag{2.26}$$

where the parameters a and σ are the shear thinning index and a characteristic stress parameter. The disadvantage of these models is that they require a big set data covering the widest range of the shear stress values for getting a good fit of each parameter.

2.4. Summary

The concepts and models for viscosity previously presented are usually used as tools for describing the behavior of fluids flowing through pipelines or a pipes and fittings assembly (PFA). These kinds of flow are commonly used in the majority of the industrial plants. Previously viscosity models will be considered as descriptions for the proposal of a model, which can serve as the structure for the soft sensor presented in the next chapters.

CHAPTER 3
State estimation for systems with unknown inputs

A review of the most commonly used techniques for state estimation with unknown inputs are presented in this chapter. Advantages and disadvantages of these estimation techniques are exposed. The aim of this chapter is to present the tools that allows to develop the structure for estimating the properties of a mineral slurry flowing through a pipe and fittings assembly. The chapter is structured as follows. First, definitions for state estimation, state observer and virtual sensor (or soft sensor) are presented. Then, the classic observers are introduced, and based on this the improvements proposed in the literature for systems with unknown inputs are commented.

3.1. State estimators in processes

State estimation is a powerful tool used for obtaining indirect measures of state variables of a process, for which there are no available devices for direct measuring or the variable is very difficult to measure. This section begins with the basic definitions but necessaries for the understanding of the chapter. Based on the state-space theory and state estimation developed in the 1950s and 1960s it is possible to establish the next definitions [31].

- **State:** Those variables that permit a complete representation of a system are commonly named as states of the system, i.e., the variables that provide the internal condition of the system in a determined instant of time [32].
- **States estimator:** According to [33], a state estimator is a deterministic or stochastic system, dynamic or static, which based on a mathematical model and available measurements is able to provide reliable information about state variables and parameters of the process.
- **State observer:** A state observer is a kind of state estimator which is developed based on a deterministic model, normally of the dynamic kind. In Figure 3.1 the general scheme of a state observer is presented. According to the proposal of Luenberger (1971), the system S_2 corresponds to a state observer to the system S_1. If the state $\mathbf{z}$ and system model S_2 are known, the state $\mathbf{x}$ of the system S_1 can be determined from S_2 and $\mathbf{z}$.

  Indeed, the role of a state estimator is to estimate the system state based on the available measurements and a model of the process. For the nonlinear system described by the following equation:

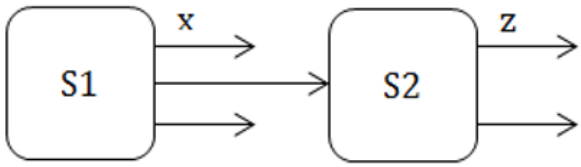

Figure 3.1. Basic scheme of a state observer [3].

$$\begin{aligned} \dot{\mathbf{x}} &= f(\mathbf{x}, \mathbf{u}), \\ \mathbf{y} &= h(\mathbf{x}). \end{aligned} \tag{3.1}$$

where $\mathbf{x}$, $\mathbf{u}$ and $\mathbf{y}$ corresponding to the vectors of state, inputs, and output, respectively, the design of a state observer consists in estimate the state vector ($\widehat{\mathbf{x}}$) in a form that the difference between the real state and estimated state is near to zero.

Thus, the estimation error

$$\mathbf{e} = \mathbf{x} - \widehat{\mathbf{x}}, \tag{3.2}$$

must satisfy the following conditions to fulfill the criteria of asymptotic convergence.

$$\widehat{\mathbf{x}}(0) = \mathbf{x}(0) \Rightarrow \widehat{\mathbf{x}}(t) = \mathbf{x}(t) \; \forall t \; > \; 0, \tag{3.3}$$

$$\| \, \widehat{\mathbf{x}}(t) - \mathbf{x}(t) \, \| \Rightarrow 0 \; \textit{when} \; t \to \infty. \tag{3.4}$$

According to the above, the general structure of a state observer for the system in 3.1 is presented as follow

$$\begin{aligned} \dot{\widehat{\mathbf{x}}} &= f(\widehat{\mathbf{x}}, \mathbf{u}) + K(\widehat{\mathbf{x}})(\mathbf{y} - \widehat{\mathbf{y}}), \\ \widehat{\mathbf{y}} &= h(\widehat{\mathbf{x}}), \end{aligned} \tag{3.5}$$

where $\widehat{\mathbf{x}} \in \Re^n$ is the estimated state vector, $\widehat{\mathbf{y}} \in \Re^m$ is the output vector and $K(\widehat{\mathbf{x}}) \in \Re^{n \times m}$ is the gain matrix of the observer. The structure presented in 3.5 corresponds to the model dynamic plus a correction term, which permits to impose to the system a desired dynamic for the estimation error. With this, the design of an observer consists on determine the gain matrix such that the conditions 3.3 and 3.4 are satisfied.

- **Soft sensor:** This is the association between one o more measurement devices (real sensors) and a state estimator (computational algorithm), which permits to do on-line measurements of unmeasured variables in a real process. This association is frequently found because for many processes there are no available devices for measuring the interest variables, or when these devices there exist, they are very expensive and have short useful life [34], [35]. In the Figures 3.2 the structure of a soft sensor is presented.

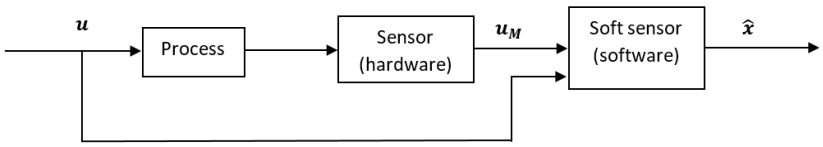

Figure 3.2. Representation scheme of a soft sensor in a process [4].

Due to different kind of models exist, and the soft sensors (SfS) are constructed from a dynamic model, these sensors can be classified based on the kind of model over which they are developed. Thus, it is possible to find SfS using phenomenological based models or using empirical models. However, the selected model limits the representation of the phenomena that describes the process and the operation region in which the SfS gives good estimation. In the case of SfS based on empirical models, the observer performance is restricted when the process is operating in a mode different from which the model was developed. For this reason, soft sensors based on phenomenological based models are preferred. With this kind of models, the SfS has no limitations in the estimation processes and the SfS designer will have a better comprehension of the system behavior.

The disadvantage of SfS constructed with a phenomenological based model is the complexity for the model deduction, especially for processes in which the phenomena governing the process is not precisely known. However, the phenomenological based semi-physical models are a good alternative for the construction of a soft sensor. These models keep the phenomenological structure for the main phenomena in the process, but using empirical correlations for describing parameters and other variables that can not be correctly described from phenomenology.

In addition to the above, there exist another kind of state estimators frequently used in engineering applications, some of these are presented below.

- State predictor: A predictor is used to obtain future values of the state based on their present values [33]. Thus, the state predictors use the observations of previous times to the time in which the estimation is done [36].
- State Smoother: This kind of estimator works with later observations to the time over which the estimation is done. These are commonly used to eliminate spurious data of signals without loss information.
- Kalman filter: It is a kind of dynamic state estimator based on a stochastic model [36]. Thus, this is an estimation algorithm for a model in the state space which permits to calculate the unmeasured state of a linear dynamic system disturbed with noisy signals. For this observer, measurements with white noise, are considered. White noise corresponds to one aleatory signal (stochastic) characterized by the fact that values in different times have not any relation. Therefore, a Kalman filter is a state estimator of a dynamic system which uses noise observations together with a process model to estimate the state through minimizing the square mean error.

  In the work of [36] the original formulation for the Kalman filter structure is proposed. According to this, for a discrete-time system described by the following equations:

$$\begin{aligned} \mathbf{x_k} &= \mathbf{A_{k-1}x_{k-1}} + \mathbf{w_{k-1}}, \\ \mathbf{z_k} &= \mathbf{Hx_k} + \upsilon_{\mathbf{k}}, \end{aligned} \tag{3.6}$$

where $\mathbf{x} \in \Re^n$ is the vector of discrete state, $\mathbf{z} \in \Re^m$ is the vector of measured outputs in the process, and $\mu \in \Re^p$ is the inputs vector. The $\mathbf{A_{k-1}}$ is the transition matrix which related the state in previous time $(k-1)$ with the actual state (k). The parameters $\mathbf{w_k} \in \Re^n$ and $\upsilon_{\mathbf{k}} \in \Re^m$, are aleatory variables which represent the noise of process phenomenon and its measurement. The noise is considered as white noise, thereby a normal probability distribution and mean equal to zero are established, as shown below:

$$\begin{aligned} p(\mathbf{w}) &\sim \ N(0, \mathbf{Q}), \\ \mathbf{E}(\mathbf{w}) &= 0, \end{aligned} \tag{3.7}$$

and

$$\begin{aligned} p(\upsilon) &\sim \ N(0, \mathbf{R}), \\ \mathbf{E}(\upsilon) &= 0. \end{aligned} \tag{3.8}$$

In the equations 3.7 and 3.8 the matrices $\mathbf{Q}$ and $\mathbf{R}$ are the covariance for the noise in the process and measurement, which change on time with each new measurement. Therefore, and with the aim of obtaining a vector of estimated state $(\widehat{\mathbf{x_k}})$, a linear function of the measurements $(\mathbf{z_i}, \ldots, \mathbf{z_k})$ that minimized the mean square error, is defined as:

$$\mathbf{E}\left[\mathbf{x_k} - \widehat{\mathbf{x_k}}\right]^T \ \mathbf{M}\left[\mathbf{x_k} - \widehat{\mathbf{x_k}}\right], \tag{3.9}$$

where $\mathbf{M}$ is a symmetric nonnegative-definite weighting matrix [37]. The solution to the estimation problem assumes that there exist a linear relation between the state and the measurement $(\mathbf{z_k} = \mathbf{Hx_k} + \upsilon_{\mathbf{k}})$, using the measurement sensitivity matrix $\mathbf{H}$, and the measurement noise $\upsilon_{\mathbf{k}}$ [36]. Thereby, the Kalman filter estimates the future state of the system applying a correction term which is proportional to a prediction factor. Due to above, the proposal of [36] to solve the estimation problem consists on two groups of equations for prediction and actualization. The first group is responsible for the state projection in the k time based on the $k-1$ time, and the intermediate actualization of the error covariance matrix $(P_k(-))$. The other group, actualization equations, is responsible for the feedback or introduction of new information from previous time for getting an improvement of the state estimation. Based on this, the mentioned equations system are:

$$\widehat{\mathbf{x_k}}(-) = \mathbf{A}_{k-1}\widehat{\mathbf{x_{k-1}}}(+), \tag{3.10}$$

$$\mathbf{P}_k(-) = \mathbf{A}_{k-1}\mathbf{P}_{k-1}(+)\mathbf{A}_{k-1}^T + \mathbf{Q}_{k-1}, \tag{3.11}$$

$$\mathbf{K}_k = \mathbf{P}_k(-)\mathbf{H}_k^T \left[\mathbf{H}_k\mathbf{P}_k(-)\mathbf{H}_k^T + \mathbf{R}_k\right]^{-1}, \tag{3.12}$$

$$\mathbf{P}_k(+) = [\mathbf{I} - \mathbf{K}_k\mathbf{H}_k]\,\mathbf{P}_k(-), \tag{3.13}$$

$$\widehat{\mathbf{x_k}}(+) = \widehat{\mathbf{x_k}}(-) + \mathbf{K}_k\left[\mathbf{z}_k - \mathbf{H}_k\widehat{\mathbf{x_k}}(-)\right], \tag{3.14}$$

The equations 3.10 and 3.11 are the prediction equations, while the equations 3.12, 3.13 and 3.14 correspond to the actualization equation. The term $\mathbf{K}_k$ is the gains matrix of observer calculated at each iteration.

The exposed above is the classic proposal of Kalman. However, at nowadays this proposal has been modified and improved by different authors. Although it was mentioned that this filter is focused in the linear systems, this can be used to the nonlinear systems. The extended Kalman filter has been developed to the nonlinear systems in discrete time. More about this and other kinds of observers can be consulted in [31] and [37].

3.2. Conventional observers for system with unknown inputs

In many engineering processes it is not possible to measure all variables of interest due to there is no available devices, or if these devices exist, they are very expensive [16]. For this case, estimation techniques to process in which several input variables are unknown have been developed. These techniques are called unknown input observers (UIO) and they have been proposed to estimate the state vector of linear or nonlinear systems based on the measurements of available inputs and outputs [38]. The state estimation for systems with unknown input disturbances have been studied from different approaches. Some works related to linear systems can be found in [39], [40], as well as for nonlinear systems in [41], [42].

Since 1970 the problem of designing observer for the system with unknown inputs has been addressed by two basic approaches. The first one assumes *a priori* information about the unmeasured inputs, such as a polynomial approximation or constant coefficient differential equation for the unknown input. The other approach considers that there is no knowledge of the inaccessible inputs [43]. Both works have as a common property that the state of the system is estimated in asymptotic way. Recent developments have introduced modifications improving the observer yield through other approaches, such as the state estimation in a finite time and robust state estimation [44].

According to [45] the design problem about UIO for nonlinear system can be classified in three categories:

- *Nonlinear state transformation-based techniques:* This name is due to the relatively large classes of systems over which it is not possible to apply any useful mathematical transformation.
- *Linearisation-based techniques:* In a similar way to the design of the extended Kalman filter, the convergence of observer only is possible under certain conditions.
- *Observers for particular classes of nonlinear system:* Into this group the UIO for polynomial and bilinear system or for Lipschitz system is included.

In the seminal work of [46] a method to design a full-order observer for linear systems with unknown inputs and the necessary conditions for the existence of the observer is presented. Based on this work many other modifications have been proposed, including the Lipschitz constraints and linear matrix inequality (LMI) for nonlinear systems [47], [48], [49]. Consider a linear time-invariant dynamical system represented in the state space by the equation 3.15, in which $\mathbf{x} \in \Re^n$, $\mathbf{u} \in \Re^m$, $\mathbf{d} \in \Re^l$ y $\mathbf{y} \in \Re^p$, are the state vector, the known inputs vector, the unknown inputs vector and outputs vector, respectively:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{D}\mathbf{d}, \\ \mathbf{y} &= \mathbf{C}\mathbf{x}, \end{aligned} \tag{3.15}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are the known constant matrices of appropriate dimensions. Assuming the rank of $\mathbf{C}$ is equal to p and the rank of $\mathbf{D}$ is equal to l and the condition of $p \geq l$ is satisfied, the full-order observer can be described as follows:

$$\begin{aligned} \dot{\mathbf{z}} &= \mathbf{N}\mathbf{z} + \mathbf{L}\mathbf{y} + \mathbf{G}\mathbf{u}, \\ \widehat{\mathbf{x}} &= \mathbf{z} - \mathbf{E}\mathbf{y}, \end{aligned} \tag{3.16}$$

where $\mathbf{z} \in \Re^n$ and $\widehat{\mathbf{x}} \in \Re^n$ are the transformation vector and the vector of estimated state. The matrices $\mathbf{N}, \mathbf{L}, \mathbf{G}$ and $\mathbf{E}$ are determined such that the estimated state $(\widehat{\mathbf{x}})$ asymptotically converge to the real state $(\mathbf{x})$. Indeed, for the system described by 3.15, the estimation error defined by 3.17 must asymptotically tend to zero regardless of the presence of unknown inputs (υ).

$$\begin{aligned} \mathbf{e} &= \widehat{\mathbf{x}} - \mathbf{x}. \\ \mathbf{e} &= \mathbf{z} - \mathbf{x} - \mathbf{E}\mathbf{y}. \end{aligned} \tag{3.17}$$

Then, the dynamic of observer estimation error is:

$$\dot{\mathbf{e}} = \mathbf{N}\mathbf{e} + (\mathbf{N}\mathbf{P} + \mathbf{L}\mathbf{P} - \mathbf{P}\mathbf{A})\,\mathbf{x} + (\mathbf{G} - \mathbf{P}\mathbf{B})\,\mathbf{u} - (\mathbf{P}\mathbf{D})\mathbf{d}, \tag{3.18}$$

with,

$$\mathbf{P} = \mathbf{I}_n + \mathbf{E}\mathbf{C}. \tag{3.19}$$

The above equations summarize the structure of an unknown inputs observer (UIO), and $\mathbf{I}_n$ is the identity matrix of order n. The observer design consists on finding the matrices of observer such that the condition of asymptomatic convergence will be satisfied.

For the case of nonlinear systems, the developments proposed in the literature have been targeted to the use of H_∞ approach [47], the linear matrix inequality [49], and the concept of sliding mode control/observer for uncertain systems [50]. A mathematical description of the nonlinear systems and their general assumptions for the existence of the unknown inputs observer are presented in [48]. Although there are many works proposed for nonlinear systems, the

majority of these impose certain conditions to the model structure in order to be implemented. For example, in some cases the model must satisfy Lipschitz constraints for developing of the observer. The above is due to such improvements have been occurred mainly based on power system and bioprocesses. Therefore, the equations of these models satisfy certain conditions of nonlinearity, which are not applicable to other processes. However, always that a nonlinear model can be linearized, the developments useful to linear systems could be used.

3.3. Common types of unknown input observers

In the above section some works for the state estimation of linear and nonlinear problems were exposed. In this section the most common kinds of observers for system with unknown inputs and the existence conditions are presented.

3.3.1. Full-order observers for linear systems with unknown inputs

According to the linear system in 3.15 and the observer structure in 3.16, to obtain an asymptotically convergence to zero of error dynamic, it is necessary to find the matrices $\mathbf{N}$, $\mathbf{G}$, $\mathbf{L}$ and $\mathbf{E}$ such that the error dynamic can be reduced to:

$$\dot{\mathbf{e}} = \mathbf{N}\mathbf{e}. \tag{3.20}$$

The above equation is satisfied if the following conditions will be used to determine the observer matrices.

$$\mathbf{P}\mathbf{D} = 0 \ \& \ (\mathbf{I_n} + \mathbf{E}\mathbf{C})\mathbf{D} = 0, \tag{3.21}$$

$$\mathbf{G} = \mathbf{P}\mathbf{B}, \tag{3.22}$$

$$\mathbf{N}\mathbf{P} + \mathbf{L}\mathbf{C} - \mathbf{P}\mathbf{A} = 0, \tag{3.23}$$

$$\mathbf{P} = \mathbf{I_n} + \mathbf{E}\mathbf{C}. \tag{3.24}$$

If conditions given by equations 3.21 to 3.24 are met, and the matrix $\mathbf{N}$ is a stable matrix, i.e., your eigenvalues have negative real part, the dynamic of the error satisfies the equation 3.20 [46]. For determining the observer gains, the equation 3.23 can be written as:

$$\mathbf{N} = \mathbf{P}\mathbf{A} - \mathbf{K}\mathbf{C}, \tag{3.25}$$

where the $\mathbf{K}$ matrix is:

$$\mathbf{K} = \mathbf{L} + \mathbf{N}\mathbf{E}. \tag{3.26}$$

If the equation 3.25 is substituted in 3.26, the expression to find the $\mathbf{L}$ matrix is:

$$\mathbf{L} = \mathbf{K}\left(\mathbf{I_n} + \mathbf{CE}\right) - \mathbf{PAE}, \tag{3.27}$$

According to the above, the matrices **E**, **P**, **G** and **L** are obtained from 3.21, 3.24, 3.22 and 3.27, while the **K** matrix is determined in such a way that $(\mathbf{PA} - \mathbf{KC})$ will be a stable matrix. However, the eigenvalues of **K** can be arbitrarily located only if $(\mathbf{PA}, \mathbf{C})$ is observable. Thus, the design of the UIO is reduced to find a matrix **E** such that equation 3.21 is satisfied, and the **K** matrix will be Hurwitz [46].

3.3.1.1. Existence and observability conditions

The following conditions of existence and observability are necessary for the design of unknown input observers. The observability of $\mathbf{PA}, \mathbf{C}$ is given by the rank of the following matrix:

$$\mathbf{O} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CPA} \\ \vdots \\ \mathbf{C}\left(\mathbf{PA}\right)^{n-1} \end{bmatrix}, \tag{3.28}$$

with n the dimension of the state vector. Therefore, if the $rank(\mathbf{O}) = n$ it is said that the pair $\mathbf{PA}, \mathbf{C}$ is observable, while that for the opposite case, the pair is not observable. For this last case, when $\mathbf{PA}, \mathbf{C}$ is not observable, the eigenvalues of **K** can not be arbitrary located and it is necessary to guarantee that $\mathbf{PA}, \mathbf{C}$ is at least detectable.

Based on the above, the condition of existence for the observer given by equation 3.16, and the condition of a detectable system are presented below.

- **Existence condition:** For the system given in 3.15, the UIO of the equation 3.16 there exist if only if the following equation is satisfied [46]:

  $$rank(\mathbf{CD}) = rank(\mathbf{D}) = l, \tag{3.29}$$

  with l the dimension of the unknown vector.

- **Condition for the arbitrary dynamic of error:** If the observability of the system is not guaranteed, it is necessary to verify that the system is at least detectable. Thus the following condition must be evaluated:

  $$rank \begin{bmatrix} sP - PA \\ C \end{bmatrix} = n, \forall s \in C. \tag{3.30}$$

3.3.2. The asymptotic observer

The asymptotic observers are a kind of observer commonly used to systems with unknown inputs. In [34] and [51] observers of this kind are implemented for process models which have in their structures balance equations related to the notion of reaction invariants.

Therefore, this kind of observer is normally used in bioporocesses and chemical processes, which implicates the transformation of material through to a chemical or biochemical reaction [34].

A general formulation of a model obtained from balances equation (mass and energy), for a reaction system in a stirred tank reactor is:

$$\dot{\mathbf{x}} = \mathbf{Y}r(\mathbf{x}) + \mathbf{F} - \mathbf{Q} - \frac{q}{V}\mathbf{x} + \mathbf{U}(T), \tag{3.31}$$

where $\mathbf{x}$ is the state vector, normally concentrations of reactants, products or biomass, and temperature. $\mathbf{Y}$ is the stoichiometric coefficients matrix, $\mathbf{Q}$ is the gaseous or liquid outflow rate vector, $\mathbf{F}$ is the feed rate vector and $\mathbf{U}(T)$ is the heat exchange term. The terms q and V represent the kinetic constant and the reactor volume. If it is consider that there are M independents reactions and M measurements, the following state partition is defined:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \end{bmatrix}, \tag{3.32}$$

with $\mathbf{x_1}$ and $\mathbf{x_2}$ the vectors corresponding to the measured and unmeasured variables, respectively. Therefore, the model proposed in 3.31 can be written as:

$$\dot{\mathbf{x}}_1 = \mathbf{Y_1}r(\mathbf{x}) + \mathbf{F_1} - \mathbf{Q_1} - \frac{q}{V}\mathbf{x_1} + \mathbf{U_1}(T), \tag{3.33}$$

$$\dot{\mathbf{x}}_2 = \mathbf{Y_2}r(\mathbf{x}) + \mathbf{F_2} - \mathbf{Q_2} - \frac{q}{V}\mathbf{x_2} + \mathbf{U_2}(T). \tag{3.34}$$

Using the following state transformation,

$$\mathbf{z} = \mathbf{x_2} - \mathbf{Y_2}{\mathbf{Y_1}}^{-1}\mathbf{x_1}, \tag{3.35}$$

and deriving respect to t, the dynamic of the transformed variable ($\mathbf{z}$) is:

$$\dot{\mathbf{z}} = -\frac{q}{V}\mathbf{z} + \mathbf{F_2} - \mathbf{Q_2} + \mathbf{U_2} - \mathbf{Y_2}{\mathbf{Y_1}}^{-1}\left(\mathbf{F_1} - \mathbf{Q_1} + \mathbf{U_1}\right). \tag{3.36}$$

Finally, the obtained equations are independent to the term $r(\mathbf{x})$ (reaction kinetics), which indicates that it is not necessary to know the expression for calculating the reaction kinetics but the matrix of stoichiometric coefficients must be known. From 3.35 and 3.36 the dynamic of the asymptotic observer is:

$$\dot{\widehat{\mathbf{z}}} = -\frac{q}{V}\widehat{\mathbf{z}} + \mathbf{F_2} - \mathbf{Q_2} + \mathbf{U_2} - \mathbf{Y_2}{\mathbf{Y_1}}^{-1}\left(\mathbf{F_1} - \mathbf{Q_1} + \mathbf{U_1}\right), \tag{3.37}$$

$$\widehat{\mathbf{x}}_2 = \widehat{\mathbf{z}} - \mathbf{Y_2}{\mathbf{Y_1}}^{-1}\mathbf{x_1}. \tag{3.38}$$

To solve previous observer is necessary to know the stoichiometric coefficients ($\mathbf{Y}$), and the flow rates $\mathbf{Q}$, $\mathbf{F}$ and $\mathbf{U}$. However, and due to it is not necessary to know the reaction kinetics

($r(\mathbf{x})$), the observer is stable and convergent if the parameter q is positive. Said it other words if there exists two constant (β and δ) [51]:

$$\beta > 0 \ \& \ \delta > 0. \tag{3.39}$$

Such that,

$$\int_t^{t+\delta} \frac{q(\tau)}{V} d\tau > \beta \ \ \forall t \geq 0. \tag{3.40}$$

The above indicates that parameter q cannot remain equal to zero by long time [34]. A detailed description of deduction and proofs to design asymptotic observer can be found in [52].

3.3.3. Nonlinear unknown input observer

In [48] a new method for designing observers for a class of system with unknown inputs and nonlinear functions satisfying the Lipschitz condition is presented. This design methodology guarantees the error system stability without requiring the rank conditions.

For a nonlinear system given by:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{D}\mathbf{d} + g(\mathbf{x}, \mathbf{u}, t), \\ \mathbf{y} &= \mathbf{C}\mathbf{x}, \end{aligned} \tag{3.41}$$

where $\mathbf{x} \in \Re^n$, $\mathbf{u} \in \Re^m$ and $\mathbf{y} \in \Re^p$ are the state, known input and output vectors, respectively. $\mathbf{A} \in \Re^{n \times n}$, $\mathbf{B} \in \Re^{n \times m}$ and $\mathbf{C} \in \Re^{p \times n}$ are known matrices, while $\mathbf{D} \in \Re^{n \times p}$ is a full rank matrix corresponding to the unknown input/disturbance distribution matrix. $\mathbf{d}$ is the vector of unknown inputs, disturbances and uncertainties such as noise. The function $g(\mathbf{x}, \mathbf{u}, t)$ represents all nonlinearity in the system.

For the successful design of the observer, the following conditions are necessaries:

- The pair $(\mathbf{C}, \mathbf{A})$ must be observable.
- The nonlinearity $g(\mathbf{x}, \mathbf{u}, t)$ must fulfills the Lipschitz condition in $\mathbf{x}$ with Lipschitz constants κ, such that:

$$\|g(\mathbf{x}, \mathbf{u}, t) - g(\widehat{\mathbf{x}}, \mathbf{u}, t)\| \leq \|\mathbf{x} - \widehat{\mathbf{x}}\|. \tag{3.42}$$

- The $\mathbf{C}$ and $\mathbf{D}$ matrices mus be satisfied, $\mathbf{CD} \neq 0$.

According to the above and following the Sedighi et al. proposal [48], a full-order observer for the system in 3.41 is:

$$\begin{aligned} \dot{\mathbf{z}} &= \mathbf{N}\mathbf{z} + \mathbf{L}\mathbf{y} + \mathbf{G}\mathbf{u} + \mathbf{H}^* g(\widehat{\mathbf{x}}, \mathbf{u}, t), \\ \widehat{\mathbf{x}} &= \mathbf{z} - \mathbf{E}\mathbf{y}, \end{aligned} \tag{3.43}$$

where $\mathbf{z} \in \Re^n$ is the state for the observer structure, and $\mathbf{N} \in \Re^{n\times n}$, $\mathbf{L} \in \Re^{n\times p}$, $\mathbf{G} \in \Re^{n\times m}$, $\mathbf{H}^* \in \Re^{n\times n}$ and $\mathbf{E} \in \Re^{n\times p}$ are the matrices of the observer. Due to the design of observers consist to find an estimated state ($\widehat{\mathbf{x}}$) which tends to the process state ($\mathbf{x}$), the dynamic of the estimation error can be described by the following equation:

$$\dot{\widehat{\mathbf{e}}} = \mathbf{Ne} + (\mathbf{HA} - \mathbf{NH} - \mathbf{LC})\mathbf{x} + (\mathbf{HB} - \mathbf{G})\mathbf{u} + \mathbf{HDv} + \mathbf{H}g(\mathbf{x}, \mathbf{u}, t) - \mathbf{H}^* g(\widehat{\mathbf{x}}, \mathbf{u}, t), \quad (3.44)$$

with,

$$\mathbf{H} = \mathbf{I_n} + \mathbf{EC}. \quad (3.45)$$

If the conditions for the system 3.41 are satisfied, the pair $(\mathbf{HA}, \mathbf{C})$ is detectable and the following conditions hold:

$$\mathbf{HD} = 0, \quad (3.46)$$

$$\mathbf{HB} - \mathbf{G} = 0, \quad (3.47)$$

$$\mathbf{HA} - \mathbf{NH} - \mathbf{LC} = 0, \quad (3.48)$$

$$\mathbf{H} = \mathbf{H}^*. \quad (3.49)$$

Then, the error dynamic can be reduced to:

$$\dot{\widehat{\mathbf{e}}} = \mathbf{Ne} + \mathbf{H}\left(g(\mathbf{x}, \mathbf{u}, t) - g(\widehat{\mathbf{x}}, \mathbf{u}, t)\right), \quad (3.50)$$

From this, the observer dynamic may be rewritten as:

$$\dot{\widehat{\mathbf{z}}} = \mathbf{Nz} + \mathbf{Ly} + \mathbf{Gu} + \mathbf{H}g(\widehat{\mathbf{x}}, \mathbf{u}, t). \quad (3.51)$$

The observer gains are selected such that $\mathbf{N}$ is Hurtwitz. In this case, assumed that P_0 is a symmetric positive definite for the algebraic Riccati equation (ARE):

$$(\mathbf{HA})^T \mathbf{P_0} + \mathbf{P_0}(\mathbf{HA}) - \mathbf{P_0}\mathbf{C}^T\mathbf{R}^{-1}\mathbf{CP_0} = -\mathbf{Q_0}, \quad (3.52)$$

with $\mathbf{Q_0} \in \Re^{n\times n}$ and $\mathbf{R} \in \Re^{p\times p}$ symmetric positive definite matrices. If $\mathbf{K} = \mathbf{P_0}\mathbf{C}^T\mathbf{R}^{-1}$, the Hurwitz condition is satisfied. By substituting $\mathbf{K}$, $\mathbf{N}$ and $\mathbf{E}$ in 3.48, the matrix $\mathbf{L}$ is obtained. However, the $\mathbf{E}$ must be determined in such a way that the given condition in 3.46 for the $\mathbf{D}$ matrix is fulfilled. This having into account that the matrices $\mathbf{E}$ and $\mathbf{D}$ are related by:

$$\mathbf{ECD} = -\mathbf{D}, \quad (3.53)$$

and, a possible solution for $\mathbf{E}$ is:

$$\mathbf{E} = -\mathbf{D}(\mathbf{CD})^+ + \Gamma(\mathbf{I} - (\mathbf{CD})(\mathbf{CD})^+), \quad (3.54)$$

where $(\mathbf{CD})^{+}$ is the pseudo-inverse of $(\mathbf{CD})$, and Γ is an arbitrary matrix.

Finally, the observer design can be summarized in the following steps [48]:

1. Define the $\mathbf{E}$ matrix.
2. Determine $\mathbf{H}$ such that the pair $(\mathbf{HA}, \mathbf{C})$ must be detectable.
3. Solve the ARE selecting the arbitrary symmetric positive defined matrices $\mathbf{Q_0}$ and $\mathbf{R}$.

3.4. Summary

Previous exposed methods are the most commonly used for designing unknown input observers. However, recently many other approaches have been proposed. Thus, in addition to the approaches developed to linear (see [46]) and nonlinear system (see [49]), the design of unknown input observers have been extended to the linear parameter-varying system (LPV). This approach is based on the description of the LPV system in a polytopic form by bounding the parameters [53]. Thus, the LPV system concerns linear dynamical system whose state-space representation depends on exogenous non-stationary parameters [54]. According to the above, the polytopic transformation is used to express the convergence conditions in terms of the linear matrix inequalities (LMI) instead of the Lyapunov theory, from which the conditions for the existence of the observer are established [53]. For more detail about this concepts it is recommended review the works of [53] and [55].

CHAPTER 4
Proposed structure for on-line estimation of density and viscosity of pulps

This chapter presents the main contribution of this thesis, which is a structure for online estimation of density and viscosity of mineral slurries (or pulps). The proposed structure is based on the soft-sensor concept, but constructed from a phenomenological based model of the process operating with available measurements from the process. This proposal have been focused in process for transporting of mineral slurries (pulps) through pipes and fittings assemblies. The process model was elaborated using the methodology proposed in [24] for developing phenomenological based semi-physical models (PBSM). In the first section of this chapter, the estimation problem of both properties (density and viscosity) is introduced and explained. Then, in the section 4.2 the description of the proposed estimation structure is presented, while in the section 4.3 the conditions of existence and convergence for the elements which conform the estimation structure are exposed.

4.1. Estimation problem of the properties of a mineral slurry

The major difficulty of estimating density and viscosity of a mineral slurry is the strong relation between these properties regarding fluid flowing. Both properties are into Reynolds number affecting as an inseparable group of variables the calculations of friction energy losses. According to the equation 4.1, the Reynolds number is a function that relates the density and viscosity of the fluid. This number is implicated in all calculations of friction factor, energy losses and pressure drop of a fluid flowing through pipelines. In addition, if the diverse behaviors presented in multiphase fluids flowing through a pipe and fittings assembly (PFA) are considered, the estimation of density and viscosity properties is a difficult task. Any estimation must be done from the typically available measurements in the units for transporting fluids, pressure drop and volumetric flow. Additonally, when a pulp is being transported, it is unusual to know with certainty the phases compositions in the mineral slurry.

$$N_{Re} = \frac{D\ v\ \rho}{\mu}. \tag{4.1}$$

Since density and viscosity depend on the concentration, shape, size and density of solid particles, the knowledge of the real concentration of solids in the mixture permits to obtain a better estimation of these properties. However, in the majority of the processes working with non-Newtonian fluids it is not possible to know this parameter. This because in many industries that used these fluids, the raw material can arbitrarily change or it is not correctly characterized. Therefore, any change in the solid concentration taking place during the fluid transport through PFA produces changes in the mixture properties, pressure and volumetric

flow measurements. Additionally, if the readings of pressure sensors are changing even when the volumetric flow remains constant, this means that the fluid properties are changing. However, from these changes on pressure readings it is not possible to attribute these changes only to one of the fluid properties, density or viscosity. Emphasising on factors influencing each one of fluid properties, it is not possible to relate the changes in readings with changes in concentrations, shape or size of solids. Therefore, to use the general mechanical energy balance, which depends on pressures and volumetric flow, the only possible to determine is the Reynolds number but ever knowing current values of density and viscosity. But if a value for Reynolds number is known, values for density and viscosity can not be individually determined because infinitum numbers of pairs of values for density and viscosity satisfy the current Reynolds number.

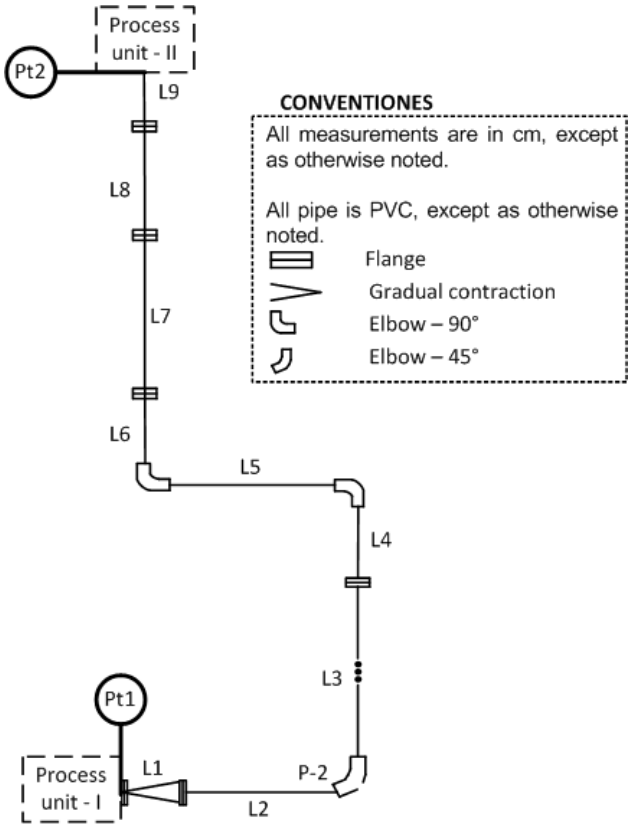

Figure 4.1. Basic scheme for the transport process of a pulp.

For the pipeline in Figure 4.1, through which a mineral slurry or pulp is transported, currently there are available sensors for the volumetric flow and the pipeline inlet and outlet pressure. By applying a mechanical energy balance between the point 1 at the inlet, and the point 2 at the outlet, the equation 4.2 is obtained. From this, if the variable P_1, P_2 and Q_P are measured, the geometric parameters A_T, L_T, z_1 and z_2 are constants, and $\frac{dQ_T}{dt}$ can be estimated from feed measurements at each sampling period of the sensor, the only unknown variables in the equation 4.2 are ρ_1, ρ_2 and $h_{f_{1-2}}$, which corresponds to fluid density and friction energy losses.

$$\frac{dQ_P}{dt} = \frac{A_T}{L_T}\left(-\frac{P_2}{\rho_2} + \frac{P_1}{\rho_1} - gz_2 + gz_1 - h_{f_{1-2}}\right). \tag{4.2}$$

If the density is assumed as constant along all the pipeline, it is possible to rewrite the density as $\rho_1 = \rho_2 = \rho_P$. The term for the energy losses considers friction in the pipe and fittings, remembering that each of these energy losses depend on Reynolds number and thus depend on fluid density and viscosity values. In addition, the equations to determine the energy losses are nonlinear functions of the Reynolds majority of cases. Equations for calculating $h_{f_{1-2}}$ are shown below:

$$h_{f_{1-2}} = (K_T + \Sigma K_F)\,\frac{1}{2}\frac{Q_P^2}{A_T^2}, \tag{4.3}$$

where K_T and K_F are the K factors for the pipe and fittings, respectively. These factors can be calculated based on the equations 4.4 and 4.5, as exposed below.

$$K_T = f_D\,\frac{D_T}{L_T}, \tag{4.4}$$

$$K_F = \frac{K_1}{N_{Re}} + K_\infty\left(1 + \frac{1}{ID_T(in)}\right), \tag{4.5}$$

where K_1 and K_∞ are constant values which are taken from Hooper work [27]. The term ID_T correspond to the pipe internal diameter always provided in inches and f_D is the Darcy friction factor. The f_D parameter can be calculated, for example, based on the Shacham formulation [26]:

$$f_D = \left\{-2\ \log\left[\frac{\epsilon/D_T}{3.1} - \frac{5.02}{N_{Re}}\log\left(\frac{\epsilon/D_T}{3.71} + \frac{14.5}{N_{Re}}\right)\right]\right\}^{-2}, \tag{4.6}$$

Finally, the relation between Reynolds number and friction losses is evident:

$$\begin{aligned} &If\ f_D = f\left(N_{Re}\right)\ and\ K_T = f\left(f_D\right) \Rightarrow K_T = f\left(N_{Re}\right), and\ K_F = f\left(N_{Re}\right) \\ &\therefore h_{f_{1-2}} = f\left(K_T, K_F\right) = f\left(N_{Re}\right). \end{aligned} \tag{4.7}$$

Based on the above it is evident that the mechanical energy balance is the only available equation, but there are two unknown parameters related to the Reynolds number. Therefore, even knowing the pressure and the volumetric flow in the pipeline, only the Reynolds number value can be determined from the mechanical energy balance. The conclusion is that to estimate and separate the density and viscosity values for the pulp flowing through to a PFA is impossible using only this equation. Finally, previous discussion allows understand the strong dependence of the effects of energy losses with the main properties involved in the flow of multiphase fluids, density and viscosity. Those variables condition the transport through a PFA and they are required to determine the material quality going to the different process units in a plant.

4.2. Description of proposed estimation structure

Considering the interest for the on-line estimation of mineral slurries properties, and due to the importance of knowing these properties during control and monitoring of the different units of the process in a plant, an estimation structure is presented below. This structure corresponds to implement a phenomenological based semi-physical model (PBSM) as a soft-sensor with a static verification of the equation of mechanical energy balance for the phenomena governing the flow process through a PFA.

4.2.1. Model structure

According to the above, and considering that the flow process through pipelines is governed by fluid to fluid and fluid to pipeline momentum transfers, it is necessary to construct a model for representing the flow behavior, which could be organized in the following way.

$$\begin{aligned} \dot{\mathbf{x}} &= f(\mathbf{x}, \mathbf{u}, \mathbf{d}), \\ \mathbf{y} &= h(\mathbf{x}), \end{aligned} \tag{4.8}$$

where $\mathbf{x} \in \Re^n$ is the process state vector, $\mathbf{y} \in \Re^m$ is the process output vector and $\mathbf{u} \in \Re^l$ and $\mathbf{d} \in \Re^p$ are the known input and unknown input vector, which depend on the available measurements in the process. As the interest of this work is focused on the transport process of pulps or mineral slurry, the system considered is a pipe and fittings assembly (PFA), taking into account that the properties with which the pulp is transported do not change into the PFA. For this reason, these properties must be correctly determined for guaranteeing the correct performance of the control systems implemented. Thus, and based on the model structure 4.8 for the case of study the state vector can be conformed by the volumetric flow (Q_P) and the contained mass in the PFA. The mass is considered because it changes as the density of the mixture according to the PFA inlet changes. Considering the pipeline volume is constant, the changes in mass are possible if the density is changing as result of changes in the concentration of solids at PFA inlet. Therefore, these changes can be related to the solid concentration in the mixture, which corresponds to the unknown input due to in the case of study the only process system is the PFA.

Based on the section 4.1 the estimation problem can be understood as the difficulty to estimate the properties of density and viscosity of a mineral slurry, because they are strongly related between them, both depend on the unknown input or process disturbances, and only one equation is available: the differential form of the mechanical energy balance. Thus, for the case of study, the density and viscosity as variables to be estimated, are a function of the solids concentration in the mixture. It must be noted that all three variables are involved in the calculation of the energy losses for the transport of pulps through a PFA. This can be summarized as the problem to estimate two parameters which are directly related and depending on an unknown input, but using only one available equation describing the transport phenomenon.

Indeed, the proposal presented below is based on the proper development of a model of the transport process, which generates an adjustable structure to implement the techniques of

state estimation. As it was mentioned at the beginning, that model will be developed using the methodology presented in [24] for obtaining phenomenological based semi-physical models (PBSM). The general idea with the model is to find other balance differential equation linking one of the unknown parameters with the conservation principle. Additionally, it is necessary that the balance differential equations of the PBSM be functions of the unknown input (or disturbance). The unknown parameter, which has not a own balance differential equation, can be related to the process states or to look for a given value used as a seed value. Thus, the model must satisfied the following:

$$\begin{aligned} \dot{\mathbf{x}}_m &= f_m(\mathbf{x}, \mathbf{u}, \mathbf{d}), \\ \dot{\mathbf{x}}_{um} &= f_{um}(\mathbf{x}, \mathbf{d}), \end{aligned} \tag{4.9}$$

where $\mathbf{x_m}$ is the vector of measured variables and $\mathbf{x_{um}}$ is the vector of unmeasured variables. The structure 4.8 is valid too when the $\mathbf{x_{um}}$ is function of the known inputs, $\mathbf{u}$. In principle, the vector of unknown inputs can be write as:

$$\mathbf{d} = \begin{bmatrix} p_1 \\ p_2 \\ d_n \end{bmatrix}, \tag{4.10}$$

where p_1 and p_2, can be the density and viscosity of a mineral pulp for the case of study, while d_n is the inherent disturbance of process, for example, the inlet solids concentration. However, one of the components of $\mathbf{d}$ must be represented as a state of one of the functions f_i in 4.9. On the other hand, for the components of $\mathbf{d}$ it is necessary to establish an equation correlating these terms with the parameter assumed as a state and the unknown input. Other alternative is to consider the resulting parameter as a known input and fix a value from the process knowledge. Obviously, that value must be recalculated when the parameter taken as state is being estimated. Thus, if the unknown parameter p_1 can be expressed as a differential equation, d_n is considered as unknown input, and p_2 is the free parameter. In this case there are two possible options:

- **Option 1.** $p_2 = g(p_1, d_n, \mathbf{x})$**:**

  For this case the model structure is given by:

$$\begin{aligned} \dot{\mathbf{x}}_m &= f_m(\mathbf{x}, \mathbf{u}, \mathbf{d}), \\ \dot{p}_1 &= f_{um}(\mathbf{x}, \mathbf{d}), \\ \mathbf{y} &= h(\mathbf{x}), \end{aligned} \tag{4.11}$$

  with f_m, f_{um} and h linear or nonlinear functions and,

$$p_2 = g(p_1, d_n, \mathbf{x}). \tag{4.12}$$

The above equations indicate that one of the parameters to be estimated must be written as a state with a balance differential equation, while the other parameter is determined based on the parameter as a state, the unknown input and the other components of the state vector, if it is necessary. The defined function $g(p_1, d_n, \mathbf{x})$ can be obtained from the experimental results or from concepts and theories from the literature. Returning to case of interest, it might be assumed that $p_1 = \rho, p_2 = \mu$ and $d_n = w_{sol}$ due to the solid fraction is a variable influencing both properties of the mixture.

- **Option 2.** p_2 **as known input:**

When one of the properties to estimate is assumed as a known input, the model structure is simplified. Thus, the vector of unknown inputs only includes the disturbances of the system. The parameter considered as the known needs to be determined with enough precision at each sampling time. Due to the above, it is necessary to find a method to calculate this parameter after the estimation problem is solved, based on an initial value for the parameter considering as known input. For this case the model structure is written as:

$$\begin{aligned} \dot{\mathbf{x}}_m &= f_m(\mathbf{x}, \mathbf{u}, \mathbf{d}), \\ \dot{p_1} &= f_{um}(\mathbf{x}, \mathbf{d}), \\ \mathbf{y} &= h(\mathbf{x}), \end{aligned} \tag{4.13}$$

with the vectors of known and unknown inputs defined as:

$$\mathbf{u} = \begin{bmatrix} p_2 \\ u_1 \\ \vdots \\ u_n \end{bmatrix}, \tag{4.14}$$

where p_2 must be known at each sampling time, and

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix}. \tag{4.15}$$

With this proposal, the consideration of $p_1 = \rho, p_2 = \mu$ can remain with the advantage of the function g can be unknown, i.e., the relation between two parameters (which represents the properties) is not required to complement the estimation structure.

4.2.2. Estimation structure

Once the phenomenological based semi-physical model is obtained, the estimation structure is constructed from this PBSM. The estimation structure are formed by a state observer for the first variable and a function to determine the second one. The proposed structure considers the available measurements in plant and the process conditions taken into account that inlet properties, such as density, viscosity and solids fraction, are unknown. This proposal

considers the model structure given in 4.11 or 4.13, with p_1 the parameter transformed as a state variable, d_n the disturbance in the process, and p_2 the parameter relating to p_1 and the state by the function $g(p_1, d_n, \mathbf{x})$. Thereby, the estimation structure must include an observer able to estimate the state variables without knowing d_n. The structure to estimate the state variables from the available measurements without the knowledge of the disturbances is presented in Figure 4.2.

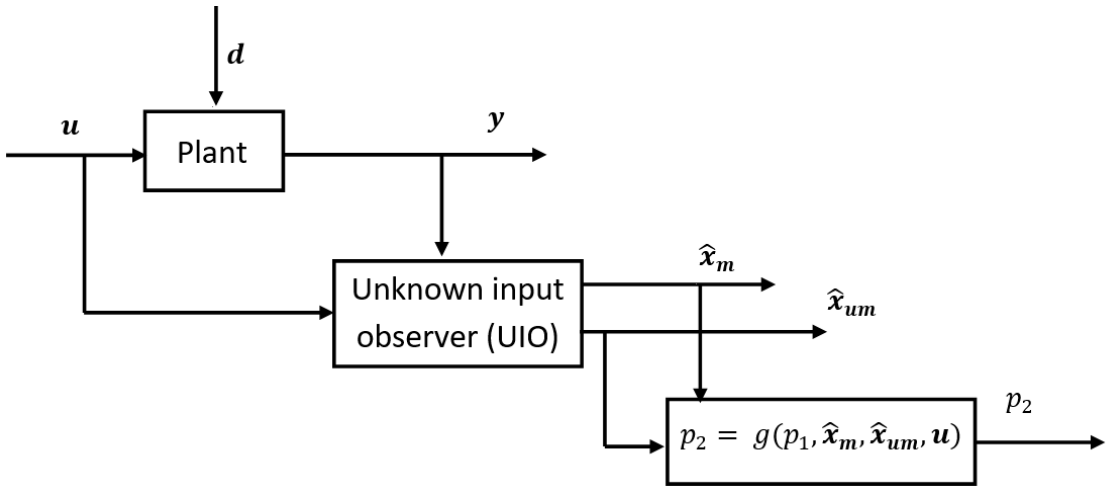

Figure 4.2. Block diagram for the estimation structure for the case in which $p_2 = g(p_1, d_n, \mathbf{x})$.

From the Figure 4.2, it is evident that the estimation of the two properties is obtained by different ways. One of these is an estimation using an unknown input observer (UIO) determining the other property through the estimation result and a function $g(\cdot)$, such that the parameter without estimation will be a function of the estimated parameter (i.e., the viscosity as a function of density). Due to the structure involves the determination of a new function for obtaining the resulting parameter, it is necessary that the function $g(\cdot)$ represents the real situation of flow in the pipeline. Taking into account the flow behavior of the fluid, depending on if the fluid is Newtonian or non-Newtonian, this function must consider the previous mentioned parameter in addition to the other parameters directly related to it. Thus, it is necessary to characterize the kind of fluid, and if it is a non-Newtonian fluid the kind of viscosity model to which it responses. Thereby, a set of steps for establishing the function $g(\cdot)$ from the experimental results and to calculate the shear rate and shear stress in a pipe is proposed here.

- **Characterization of the flow behavior.** In order to identify the flow behavior that presents the working fluid, it is necessary to perform laboratory tests at the appropriate conditions in a way that a wide range of shear stress and the shear rate can be covered. In this way, the obtained results can be extrapolated to current operating conditions in the pipeline. Besides, if the changes in the concentrations of phases in the mixture will be considered, it is recommended to repeat those laboratory tests at different solid concentrations, in such a way that the tests represent all the possible concentrations in the real process.

- **Correlation of the experimental results.** Based on the results obtained in the above steps, the idea is to determine a linear or nonlinear expression to calculate the parameter

that is no considered as a state, but as a function of the state and the main factors that influence it and which were determined in laboratory tests.

- **Establish the operation conditions in the pipeline.** Taking into account that the shear stress and the shear rate change depending on the conduit shape, the volumetric flow and other factors such as the pressure drop in the pipeline, it is necessary to fix the factors of the real process which would be related to the factors considered in the laboratory test. This is the known identification of the process variables for obtaining a function depending on the state variables too.
- **Generalization of the function $g(\cdot)$.** From the results obtained through previous steps, it is possible to obtain an expression relating the parameter considered as a state, the state variables and the known inputs with the unknown parameter. This results in an expression of like the shown in 4.12.

The procedure above mentioned can be very efficient if the pulp is correctly characterized and this always will be the processing fluid. However, if another kind of pulp or slurry is being processed, the function $g(\cdot)$ must be recalculated for guaranteeing the convergence of the estimation structure. This due to the expression for g was obtained based on another kind of mixture (i.e., when the mixture is done with another kind of solids and fluids) and the flow behavior and the selected model to represent slurry viscosity could be different. Indeed, this methodology is applicable when just one kind of the mineral slurry or pulp is processing and it is adequately characterized.

For a most general case, the second option is considered. In this option, similarly to option 1, an unknown input observer (UIO) is include into the structure, but the second part is formed by a static verification of the mechanical energy balance. This proposal is presented in Figure 4.3, in which from the estimation of the unmeasured state and the available measurements from the process, the resulting parameter (which is consider as known input) is determined by means of the static version of the mechanical energy balance.

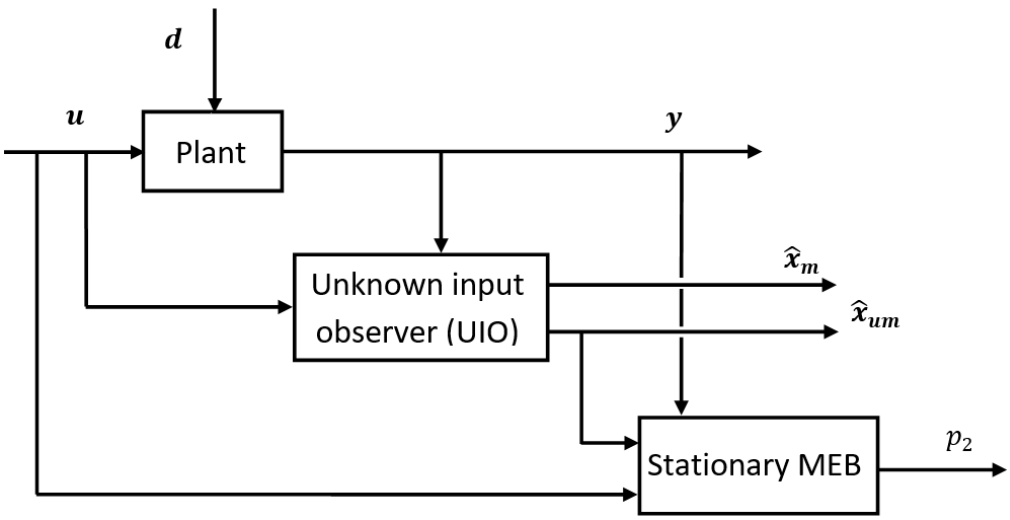

Figure 4.3. Block diagram for the estimation structure for the case in which p_2 is considered as a known input.

According to the structure presented in Figure 4.3, to calculate one of the parameters (for the case of study it corresponds to some property of the mixture), it is assumed that the working fluid in the pipeline is homogeneous in all the PFA. This assumption is stated for obtaining a system with the same number of equations and variables, because considering that the properties in the inlet are different to the outlet in the PFA, the system will have more variables that equations. Therefore, based on the estimation of the parameter transformed to state and the measurements available from the plant, a stationary state is assumed during the progress of the dynamic model for determining at each sampling time the value of the other parameter, which satisfies the stationary condition.

This approach, unlike the first structure, has not difficulties if the pulp or slurry changes, because the second part of the structure is not obtained from the experimental results for a specific pulp. With this, the structure can be applied in any case independently of the fluid being processed but provided the model structure is satisfied.

4.2.3. Unknown input observer (UIO)

According to Chapter 3, the design procedure for an unknown input observer is presented below. This proposal consist on a linear observer. Therefore, to apply this observer to a nonlinear process, it is necessary to determine the stationary state values to execute a linearization of the nonlinear model. The design was developed based on the methodology proposed in [46] where the conditions of existence and observability that the model must satisfy are stated. Worth noting that by being a linear observer, the system is presented in deviations variables, which are calculated previously as the difference between the variable and your stationary state value. Below, the procedure followed to design the unknown input observer is presented.

For a nonlinear model, as given in equations 4.16 and 4.17:

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}, \mathbf{d}), \tag{4.16}$$

$$\mathbf{y} = h(\mathbf{x}), \tag{4.17}$$

the linear version of the model can be determined based on the Jacobian matrices according to:

$$\mathbf{A} = \left[\frac{\partial f}{\partial \mathbf{x}}\right]_{ss}, \tag{4.18}$$

$$\mathbf{B} = \left[\frac{\partial f}{\partial \mathbf{u}}\right]_{ss}, \tag{4.19}$$

$$\mathbf{C} = \left[\frac{\partial h}{\partial \mathbf{x}}\right]_{ss}, \tag{4.20}$$

$$\mathbf{D} = \left[\frac{\partial f}{\partial \mathbf{d}}\right]_{ss}. \tag{4.21}$$

Next mathematical description was previously presented (see Ecs. 3.15 to 3.27). However, looking for clarity in the presentation, those equations are repeated here. Since the above matrices are evaluated in the stationary state (*ss*), the final representation of the linear model is:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{D}\mathbf{d}, \tag{4.22}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x}. \tag{4.23}$$

Using the equations 4.22 and 4.23 the design of the observer can be done. The equations 4.24 and 4.25 describe the dynamic of the observer [46].

$$\dot{\mathbf{z}} = \mathbf{N}\mathbf{z} + \mathbf{L}\mathbf{y} + \mathbf{G}\mathbf{u}, \tag{4.24}$$

$$\widehat{\mathbf{x}} = \mathbf{z} - \mathbf{E}\mathbf{y}, \tag{4.25}$$

where $\widehat{\mathbf{x}}$ and $\mathbf{z}$ are the vectors of estimated state and the transformation of state for observer design. For the vectors $\mathbf{x} \in \Re^k, \mathbf{u} \in \Re^n, \mathbf{d} \in \Re^p$ and $\mathbf{y} \in \Re^m$, the matrices $\mathbf{N}, \mathbf{L}, \mathbf{G}$ and $\mathbf{E}$ have the following dimensions: $\mathbf{N} \in \Re^{n\times n}, \mathbf{L} \in \Re^{n\times p}, \mathbf{G} \in \Re^{n\times k}$ and $\mathbf{E} \in \Re^{n\times m}$. These are constant values matrices which are selected to ensure convergence of the observer. Thus, the observer matrices are calculated based on the dynamic error, with the aim that the estimated state asymptotically converges to the real state value. According to this, the error dynamic is given by:

$$\mathbf{e} = \widehat{\mathbf{x}} - \mathbf{x} = \mathbf{z} - \mathbf{x} - \mathbf{E}\mathbf{y}. \tag{4.26}$$

If the equation for the estimated state is replaced in the above equation, the error dynamic can be expressed as:

$$\dot{\mathbf{e}} = \mathbf{N}\mathbf{e} + (\mathbf{N}\mathbf{P} + \mathbf{L}\mathbf{C} - \mathbf{P}\mathbf{A})\,\mathbf{x} + (\mathbf{G} - \mathbf{P}\mathbf{B})\,\mathbf{u} - \mathbf{P}\mathbf{D}\mathbf{d}, \tag{4.27}$$

with,

$$\mathbf{P} = \mathbf{I_n} + \mathbf{E}\mathbf{C}. \tag{4.28}$$

Based on equations 4.27 and 4.28, the expressions to calculate the matrices of the observer, which ensure the asymptotic convergence for the dynamic of the error, are as follow:

$$\mathbf{P}\mathbf{D} = 0;\ (\mathbf{I_n} + \mathbf{E}\mathbf{C})\mathbf{D} = 0, \tag{4.29}$$

$$\mathbf{G} = \mathbf{P}\mathbf{B}, \tag{4.30}$$

$$\mathbf{NP} + \mathbf{LC} - \mathbf{PA} = 0. \tag{4.31}$$

If the matrices $\mathbf{E}, \mathbf{G}$ and $\mathbf{L}$ are calculated based on the above equations, the dynamic of error is reduce to:

$$\dot{\mathbf{e}} = \mathbf{Ne}. \tag{4.32}$$

Finally, if the matrix $\mathbf{N}$ is Hurwitz (eigenvalues with real part negative), the dynamic of error is stable, and the matrix $\mathbf{N}$ can be obtained from,

$$\mathbf{N} = \mathbf{PA} - \mathbf{KC}, \tag{4.33}$$

with,

$$\mathbf{K} = \mathbf{L} + \mathbf{NE}. \tag{4.34}$$

Thus, the values of $\mathbf{K}$ (or gains) are selected to obtain a $\mathbf{K}$ matrix that satisfies the Hurwitz condition.

4.3. Convergence assessment

The convergence of the proposed structure depends on each one of its parts. Regarding the model, it must be validated using real data or experimental results before the observer will be designed. Additionally, the mentioned conditions concerning the relations among the parameters to estimate and the state expansion to find a balance differential equation for one of the parameters, must be satisfied. Also, a correct experimental part and the right characterization of the results is a necessary task if the proposal of the option 1 is implemented. In addition, it is necessary to guarantee defined, dimensionally consistent and complete model structure, i.e., with the same number of equations and variables, and the auxiliary and necessary equations for determining the internal parameters of the model. Finally, for the design of a unknown input observer it is necessary to ensure the existence and observability conditions established in the section 3.3.1.1 (equations 3.29 and 3.30).

In the Figure 4.4, a methodology for developing the proposed estimation structure is presented. This procedure has two parts. The first one is the development of the model implying the definition of the process variables and parameters based on the main phenomena governing the process and fulfilling the model goal. In addition to define these factors, model development includes obtaining the balance equations, which conform the model structure, and the constitutive equations to determine parameters values. Once the model is obtained, the best option to treat the parameter to be estimated must be selected. The second part of the procedure consists on selecting the state vectors, the known inputs, the outputs and the unknown inputs, and from this, to linearize the model for defining the observer matrices. When the observer equations are determined, it is possible to verify the conditions of existence and detectability for the observer design and to proceed with the observer implementation.

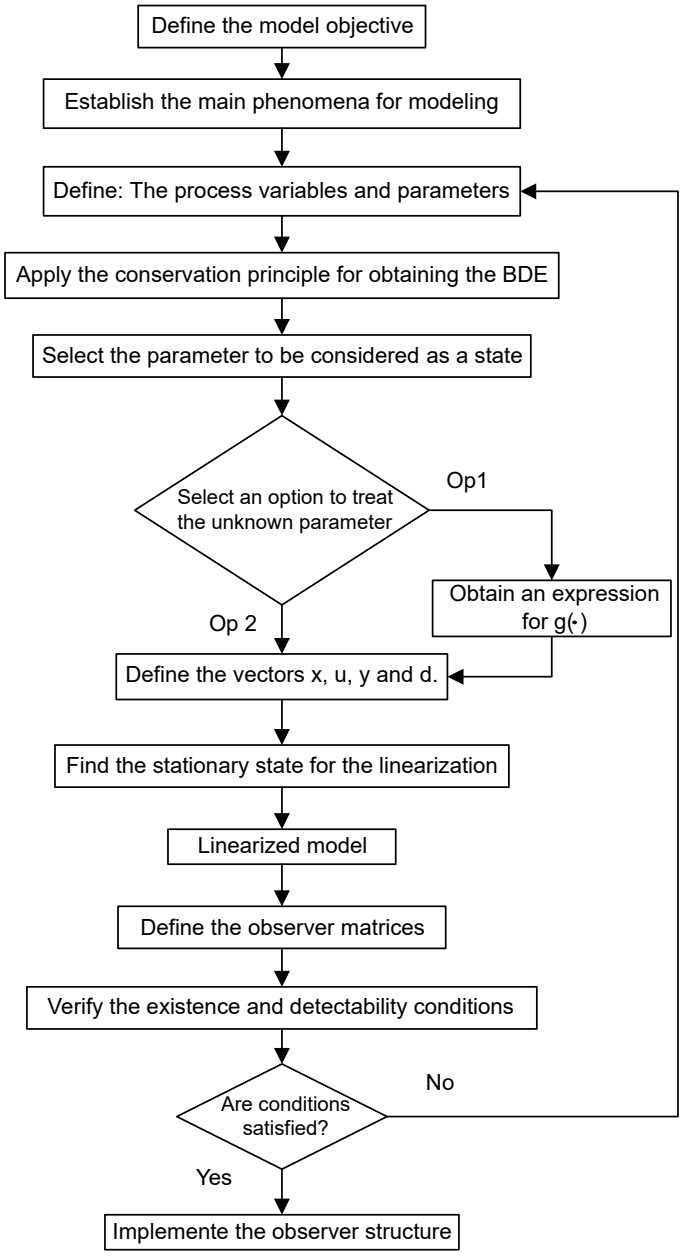

Figure 4.4. Flow diagram for the proposed estimation methodology.

4.4. Summary

In this way, the estimation structure was proposed. However, its application is conditioned to the correct deduction of the phenomenological based model, in addition to the fulfillment of the necessary conditions for the equations of the model and the relations between the parameters to estimate, as well as the relations between the parameters and the unknown inputs of the process. The general idea with these structures is to obtain the estimation of two properties, which influence the flow behavior and rheology of non-Newtonian fluids flowing through a PFA. Those two properties, density and viscosity, are strongly related to pulps or slurries flow behavior.

CHAPTER 5
Application case and results

The main objective of this chapter is to illustrate by simulation the application of the estimation structure proposed in Chapter 4, to the transport processes of pulps or slurries through a PFA. For the on-line estimation of the characteristic properties of a slurry flowing, the model of the process was deduced using the methodology presented in section 2.1. After that, the design of the unknown inputs observer from the linearization of the obtained model was carried out.

5.1. Transport of a mineral slurry

Let consider a pipe and fittings assembly (PFA) where a mineral pulp is transported from a process unit to another. To describe the behavior of a mineral pulp in a PFA is necessary to identify the measured variables and to declare the variables to be estimated. In this case, the interest is to estimate the density and viscosity of mineral pulps (mix of water and solids) from all the available measurements as inlet pressure and outlet pressure in the pipe (or pressure drop), volumetric flow, etc. A general outline of the process is shown in Figure 5.1.

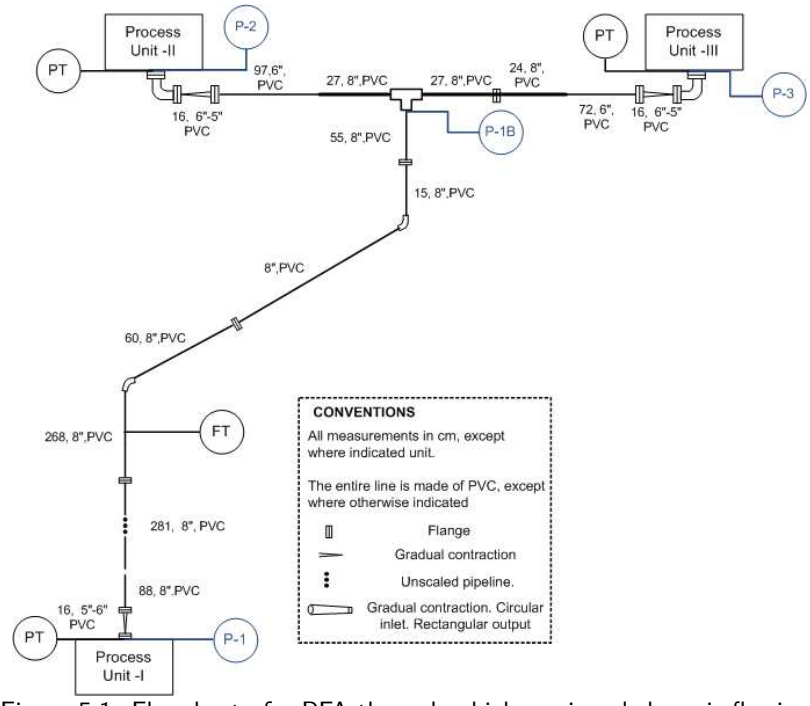

Figure 5.1. Flowsheet of a PFA through which a mineral slurry is flowing.

In the process, the mixture is flowing through a pipe and fittings assembly (PFA) with a fixed configuration. The solids in the mix are considered as spherical particles of a unique

diameter and variable concentration in the process unit I. Due to the concentration of solids in the mixture is not constant, and the density and viscosity are a function of properties of single phases and their concentration in the mixture, these properties will not be constant. Additionally, for a fluid flowing through a PFA, the pressure drop is a function of Reynolds number (N_{Re}), which is again a function of the fluid properties (density and viscosity). Therefore, changes in the feed conditions produce variations in the pressure drop. For the flow of a mineral slurry through the conduction line with known fittings and pipe sections and lengths, the fluid suffers a calculable energy loss due to the friction between the pipe walls and the fluid and the friction among different fluid portions in contact during the flow. Hence, there is an energy flow from the fluid toward the PFA wall which produces that outlet pressure be less than inlet pressure in the PFA. The energy flow toward PFA is dissipated as heat, sound, and vibration. However, these dissipation effects are unnoticeable due to the size of the pipe and the total mass being transported. Therefore, for the thermal effect the process can be considered as isothermal.

An industrial assembly as the mentioned is normally equipped with these sensors: one for volumetric flow and three for pressure. The pressure sensors are located as following: one at the outlet of processing unit I (point 1), and the other two are located at the inlet of the processing unit II (point 2) and III (point 3), respectively. In the PFA considered exists a division of flow (point 1B), but due to the segments of pipelines posterior to the division are not equal, the partition coefficient of volumetric flow must be determined by the pressure drop of each segment. Thus, with the outlet pressure of each segment and the volumetric flow at inlet of division point, the partition coefficient and the pressure at the division inlet are easily determined. This procedure is explained in more detail in the next section.

5.1.1. Model deduction

The dependence of slurries density and viscosity on the slurry solids concentration, particles size distribution (PSD), and solids density turn any on-line estimation in a very difficult task. Therefore, to obtain a correct estimation of those mixture properties, the construction of a model describing the behavior of the mineral slurry flowing must include the major factors that directly affect the properties of the mix. The model is constructed based on the most common unit of operation in plants of mineral processing, a pipe and fittings assembly (PFA) that connect two different process units. The PFA is commonly formed by a fixed quantity of fittings and line pipe sections in a known configuration, pressure sensors in the inlet and outlet of the pipe, and a flow sensor. Through the pressure sensors it is possible to obtain the pressure drop as the difference between outlet and inlet pressures. Additionally, for this research an on-line volumetric flow meter is considered currently installed in the process line. Based on the above, the process systems (PS) for modeling the process are defined as Figure 5.2 indicates.

- **Process system I (PS-I): Volume of mineral slurry contained inside the pipe and fittings assembly (PFA).**

  This process system has the same volume of the pipe. As it previously mentioned, when the fluid is in contact with the pipe, the friction between two fluid layers and among the fluid and the pipe wall produces energy losses in the fluid by mechanical energy exchange. The final result is a pressure drop between inlet and outlet points of the PFA.

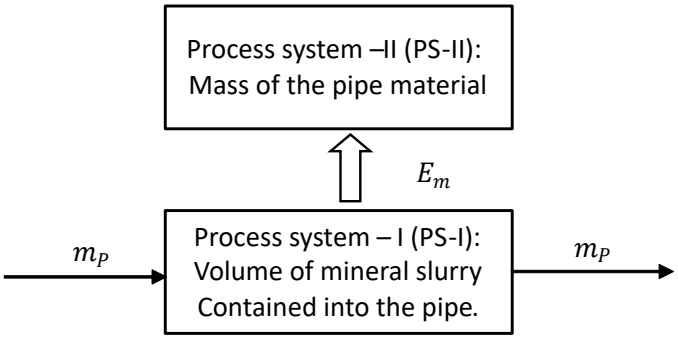

Figure 5.2. Process systems used for modeling the slurry flow through to pipe.

- **Process system II (PS-II): Mass of the PFA.**

  The SP-II contain all mass or material of the pipe sections and the fittings forming the PFA from inlet to outlet. This mass is considered as the main sump for the mechanical energy losses produced by the transport of the process fluid through the pipe and fittings assembly.

The separation of process systems presented in Figure 5.2 is made with the purpose to identify the interactions between the process fluid and the properties of the pipe, such as roughness, which are the main responsible for energy losses and the drop pressure in the pipeline. However, to guarantee a correct estimation of the mixture properties is necessary to differentiate the values of density and viscosity corresponding to the pressure drop measurement and the current volumetric flow. In the case of study of this thesis the parameters to be estimated are the density and viscosity of the mineral slurry. Remembering that one requirement for the model structure is its ability to express one of this parameters as a state. The conservation principles are applied to the process systems previous to determine what property is most feasible of being considered as a state. The balance equations for each defined process system are presented below.

Mass Balances

According to Figure 5.2 and considering that only two components form the mixture, solids and water, the notation to follow in balance equations deduction is the subscripts S for the solids and W for the liquid phase. Due to in the transport system considered there is one division of flow, the system is divided into three subsystems as it is presented in Figure 5.3. Each subsystem maintains the same two PS previously defined. The first subsystem includes the segment of PFA between the point 1 (outlet of the process unit I) and the point 1B (point just before the flow division), while the subsystems 2 and 3 correspond to each segment of PFA between the outlet of the flow division and the inlet of next process unit, i.e., point 2 and 3 respectively.

Given that the observer design only can be developed over the segment which includes the measurements of volumetric flow and the pressure drop, the subsystem 2 and 3 are only used to determine the pressure in the flow division. The possibility to evaluate the pressure in point 1B knowing P1, flow rate and head losses in the first subsystem is not used here because subsystem 1 acts as a homogenizer when the changes on solids concentration take place. One option is to state the mechanical energy balance for a system with one input and

two outputs. In this way, a most precise evaluation of flow rate distribution in the Tee will be obtained but the computational cost of solving the equations increase. In addition, after that, a new problem to find the pressure in point 1B must be solved. Therefore, it is possible to obtain the pressure drop in the selected segment for the observer design (subsystem 1). Thus, in order to determine the pressure in the point 1B the following procedure is developed, with this assumptions: 1) The Tee acting as a splitter is represented by two 90° elbows with the same diameter than the tube in each branch. 2) The pressure in the point 1B taken as input pressure to both branches is the same. 3) The head lost due to fluid pure splitting in the Tee is assumed negligible. This approach implies an error of 3% regarding the exact value found when the mentioned equation for one input and two outputs is used.

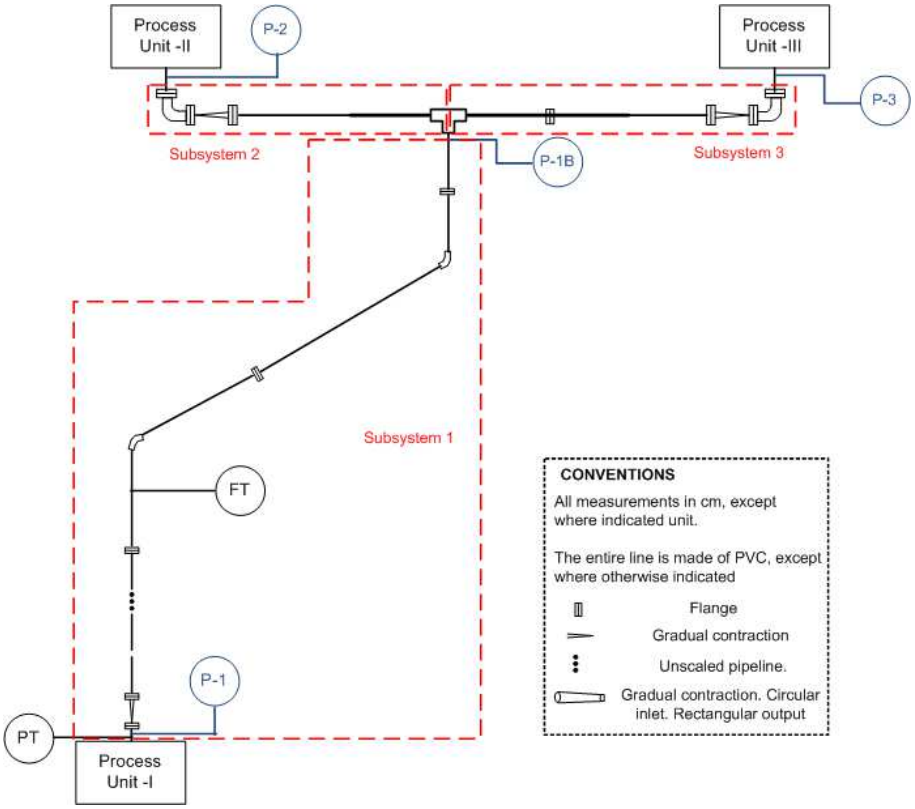

Figure 5.3. Separation in subsystem for the transport system of a mineral slurry.

If a mechanical energy balance in steady state for the subsystems 2 and 3 is done, the following expressions are obtained:

Between point 1B and 2:

$$\frac{P_2}{\rho_P} - \frac{P_{1B}}{\rho_P} + gz_2 - gz_1 + h_{f_{1B-2}} + \frac{1}{2}v_2^2 - \frac{1}{2}v_{1B,Left}^2 = 0, \tag{5.1}$$

and between point 1B and 3:

$$\frac{P_3}{\rho_P} - \frac{P_{1B}}{\rho_P} + gz_3 - gz_1 + h_{f_{1B-3}} + \frac{1}{2}v_3^2 - \frac{1}{2}v_{1B,Right}^2 = 0, \tag{5.2}$$

If the velocity of the slurry into each segment can be expressed as,

$$\upsilon_i = \frac{Q_i}{A_{T,i}}, \tag{5.3}$$

which conducts, as it is expected, to different velocities for point 1B in the mechanical energy balance for each branch, and $Q_{Sub,Main}$ is known, only it is necessary to find one expression correlating the volumetric flow in each segment. Therefore, using the continuity equation the following equality must be satisfied:

$$Q_{Sub,Main} = Q_2 + Q_3 \tag{5.4}$$

Then, if the volumetric flow in any segment can be written as a function of the known volumetric flow,

$$Q_3 = Q_{Sub,Main} - Q_2 \tag{5.5}$$

The unknown variables in the system are the volumetric flow in the subsystem 2 (Q_2) and the pressure in the point 1B (P_{1B}). Thus, as a result of replacing 5.5 in the equations 5.1 and 5.2, a system of two equations with two unknowns is obtained. From this, and using techniques for solving algebraic systems, the values of P_{1B} and Q_2 are determined, and the subsystem 1 is totally defined.

Taking into account that the subsystem 2 and 3 are used to obtain the outlet pressure for the subsystem 1, the mass and mechanical energy balances were developed only for subsystem 1. The above considering the points 1 and 1B as the flow inlet and output, respectively. The mentioned equations are shown below.

1. **Total mass balance:** The balance of total mass is stated as follows:

$$\frac{dM_P}{dt} = \dot{m}_{P,in} - \dot{m}_{P,out}, \tag{5.6}$$

where $M_P, \dot{m}_{P,in}$ y $\dot{m}_{P,out}$ are the total mass of pulp or mineral slurry, the inlet mass flow and the outlet mass flow, respectively. These terms can be expressed as:

$$M_P = \rho_P V_T, \tag{5.7}$$

$$\dot{m}_{P,in} = \rho_{P,in} Q_P, \tag{5.8}$$

$$\dot{m}_{P,out} = \rho_{P,out,} Q_P \tag{5.9}$$

with Q_P the volumetric flow of pulp, V_T the volume of the pipeline, and $\rho_{P,in}$ and $\rho_{P,out}$ the density of the mineral pulp in inlet and outlet, respectively. Differentiating the equation 5.7 for a constant volume of pipe under incompressible fluid assumption:

$$\frac{dM_P}{dt} = \frac{d\rho_P}{dt} V_T \tag{5.10}$$

Considering perfect agitation of the pulp or mixture inside the PFA: $\rho_P = \rho_{P,out}$. Thus, replacing the equations 5.8, 5.9 and 5.10 in the general balance 5.6, the expression for the total mass balance is obtained.

$$\frac{d\rho_{P,out}}{dt} = \left(\rho_{P,in} Q_P - \rho_{P,out} Q_P\right) \frac{1}{V_T}. \tag{5.11}$$

2. **Solid particles balance:** For inlet and outlet streams, the changes in the mass of the solid particles can be described by:

$$\frac{dM_S}{dt} = w_{S,in} \dot{m}_{P,in} - w_{S,out} \dot{m}_{P,out}, \tag{5.12}$$

with, $w_{S,in}$ and $w_{S,out}$ the solids concentration in the inlet and outlet of the PFA. Remembering the assumption of perfect agitation for the pulp inside the PFA, the total mass of solids is equal to:

$$M_S = w_{S,out} M_P. \tag{5.13}$$

Deriving the above equation and considering that both parts on the right-hand side are variables,

$$\frac{dM_S}{dt} = w_{S,out} \frac{dM_P}{dt} + M_P \frac{dw_{S,out}}{dt}, \tag{5.14}$$

replacing equation 5.14 in 5.12, it is obtained the mass balance for the solids in the mixture $w_{S,out}$.

$$\frac{dw_{S,out}}{dt} = \left(w_{S,in} \dot{m}_{P,in} - w_{S,out} \dot{m}_{P,out} - w_{S,out} \frac{dM_P}{dt}\right) \frac{1}{M_{Pulp}}, \tag{5.15}$$

3. **Water balance:** Having the balance equation for the solids particles, and remembering that only two components exist in the mixture, the water fraction can be obtained as follow:

$$w_{W,out} = 1 - w_{S,out}. \tag{5.16}$$

Although it is possible to obtain concentration expressions for each component in the mixture from the mass balance by components, there is an expression relating the pulp density and the mass fractions of the components forming the pulp. This equation is,

$$\rho_{P,out} = \frac{1}{\frac{w_{W,out}}{\rho_W} + \frac{w_{S,out}}{\rho_S}} \tag{5.17}$$

In this way, the equation 5.15 is discarded in the model, and the changes in the mass will be described with the equations 5.11, 5.16 y 5.17. This replacement is done because in a model is not possible to have two equations for the same variable, and by simplicity in the model the differential equation is discarded by using the algebraic equation.

From the use of the conservation principle for deducting the mass balance equation, one balance expression for the density was obtained. With this, the requirement to express one of the parameters as a state is satisfied, and the viscosity of the mineral slurry is the parameter to be considered as one known input or to be related to the function $g(\cdot)$ explained in the section 4.3, which is a function for relating the unknown parameter with the parameter taken as a state, other state variables, and the known inputs.

Mechanical Energy Balance

For the process of transport of fluids through a PFA, the changes in the kinetic energy can be described by the following equation:

$$\frac{1}{2}\frac{d(M_P v^2)}{dt} = W_f + W_g + W_{fr} + W_p, \tag{5.18}$$

where W_f, W_g, W_{fr} and W_p are the flow, gravitational, friction and machine works acting on the fluid, respectively. Each one of these terms can be represented as follows.

- **Flow work:**

$$W_f = -\left(\frac{\dot{m}_{P,out}}{\rho_{out}}P_2 - \frac{\dot{m}_{P,in}}{\rho_{in}}P_1\right). \tag{5.19}$$

- **Gravitational work:**

  For simplicity, here only the gravitational force is considered, producing the work:

$$W_g = -\left(\dot{m}_{P,out} g z_2 - \dot{m}_{P,in} g z_1\right). \tag{5.20}$$

- **Machine work:**

  In case of process system including a pump, fan, blower or compressor, one term of work for this device must be incorporated.

$$\dot{W}_p = \dot{m}_{P,i} W_{Pump}. \tag{5.21}$$

- **Friction work:**

  Due to friction losses when the fluid is flowing, it is necessary add a term for calculating the energy losses by flow.

$$W_{fr} = -\dot{m}_{P,in} h_{f_{1-2}} - \dot{m}_{P,out} h_{f_{1-2}}. \tag{5.22}$$

Replacing the equations 5.19 to 5.22 in 5.18, the dynamic equation for the mechanical energy balance is obtained:

$$\begin{aligned}\frac{1}{2}\frac{d(M_P v^2)}{dt} = &-\left(\frac{\dot{m}_{P,out}}{\rho_{out}}P_2 - \frac{\dot{m}_{P,in}}{\rho_{in}}P_1\right) \\ &- (\dot{m}_{P,out}gz_2 - \dot{m}_{P,in}gz_1) - \dot{m}_{P,in}h_{f_{1-2}} - \dot{m}_{P,out}h_{f_{1-2}} + \dot{m}_{P,i}W_{Pump}.\end{aligned} \tag{5.23}$$

Writing the derivative of he left-hand side of equation 5.23:

$$\frac{1}{2}\frac{d(M_P v^2)}{dt} = \frac{1}{2}\left(v^2\frac{dM_P}{dt} + M_P\frac{dv^2}{dt}\right). \tag{5.24}$$

If in addition, it is considered that the changes in the pressure occur significantly faster than the changes in the mass content (due only to solids concentration changes), the first term of the right-hand side of equation 5.24 is equal to zero, and applying the derivative definition on v^2 this equation can be simplified as:

$$\frac{1}{2}\frac{d(M_P v^2)}{dt} = \frac{1}{2}\left(2vM_P\frac{dv}{dt}\right). \tag{5.25}$$

Expressing the total mass of pulp as $M_P = \rho_{out}(A_T L_T)$, with A_T and L_T the pipe area and length, remembering that this total mass suffer slow changes due to solids content changes, and substituting in 5.25, the term for the change of kinetic energy is:

$$\frac{1}{2}\frac{d(M_P v^2)}{dt} = \rho_{out}(A_T L_T)v\frac{dv}{dt}. \tag{5.26}$$

Then, if the mass flow is equal to $\dot{m}_{P,out} = \rho_{out}A_T v$, the general expression for dynamic mechanical energy balance is:

$$\begin{aligned}L_T\dot{m}_{P,out}\frac{dv}{dt} = &-\left(\frac{\dot{m}_{P,out}}{\rho_{out}}P_2 - \frac{\dot{m}_{P,in}}{\rho_{in}}P_1\right) - (\dot{m}_{P,out}gz_2 - \dot{m}_{P,in}gz_1) \\ &- \dot{m}_{P,in}h_{f_{1-2}} - \dot{m}_{P,out}h_{f_{1-2}} + \dot{m}_{P,i}W_{Pump}.\end{aligned} \tag{5.27}$$

If there is no a pump, compressor or fan in the PFA, the term W_{Pump} is equal to zero. Finally, dividing the equation 5.27 by the mass flow and the total length of pipe, the equation is reduced to:

$$\frac{dv}{dt} = \frac{1}{L_T}\left(-\frac{P_2}{\rho_{out}} + \frac{\dot{m}_{P,in}}{\dot{m}_{P,out}}\frac{P_1}{\rho_{in}} - gz_2 + \frac{\dot{m}_{P,in}}{\dot{m}_{P,out}}gz_1 - \frac{\dot{m}_{P,in}}{\dot{m}_{P,out}}h_{f_{1-2}} - h_{f_{1-2}}\right). \tag{5.28}$$

In order to express the previous equation in terms of the volumetric flow, this equation must be multiplied by the pipe area (A_T):

$$\frac{dQ_P}{dt} = \frac{A_T}{L_T}\left(-\frac{P_2}{\rho_{out}} + \frac{\dot{m}_{P,in}}{\dot{m}_{P,out}}\frac{P_1}{\rho_{in}} - gz_2 + \frac{\dot{m}_{P,in}}{\dot{m}_{P,out}}gz_1 - \frac{\dot{m}_{P,in}}{\dot{m}_{P,out}}h_{f_{1-2}} - h_{f_{1-2}}\right). \quad (5.29)$$

Equations 5.29 and 5.11 represent the time variation of model states, which are used for constructing the state observer for density and viscosity of the mineral slurry.

Constitutive equations

Additionally to the balance differential equations, the model proposed is complemented with a set of algebraic equations for calculating energy losses ($h_{f_{1-2}}$) in all the pipe sections and fittings forming the PFA. These equations are expressed in a general form with the subscript i equal to *in or out*, depending on the part of the pipeline which will be treated. Thus, the calculate of energy losses in the pipeline and fittings, the K factor and the friction factor will be done using the equations 4.3 to 4.6.

Until this point, the set of balance equations and constitutive equations for the process were established. In addition, and as a consequence of these balances, the classification of the parameters as i) a state for the case of the density for which exist a differential equation and, ii) the classification of viscosity as the known input (one component of the **u** vector) or an additional equation (function $g(\cdot)$), is established. In this way, the structure of the fluid transport model is obtained. However, it is necessary to define how to determine the second parameter, which for this case corresponds to the viscosity, by the function $g(\cdot)$ or an alternative solution when the viscosity is considered as known input. Therefore, the procedure for obtaining the equation for the viscosity of the mixture is presented below. In the section 5.1.2 one procedure for determining the function $g(\cdot)$ is presented, whereas in the section 5.1.3 the alternative solution (when the viscosity is considering as a known input), such as a static verification is exposed.

5.1.2. Density and viscosity relation for a flowing slurry

According to Section 4.2 there are two possible options to address the problem of the relation between density and viscosity, for implementing the estimation structure proposed in the Chapter 4. With the density as a state and taking into account the equation 5.17, which relates the density and the components concentration in the mixture, a function $g(\cdot)$ for the viscosity must be found. This function $g(\cdot)$ relates the viscosity with the state variables and the known inputs. In order to determine the function $g(\cdot)$, the first step is characterizing the rheological behavior of the mineral slurry, which corresponds to a mixture of water and solid particles of a kaolin-type clay.

The kaolin is a kind of rock mainly conformed by kaolinite ($Al_2Si_2O_5(OH)_4$) presented as a compact or loose mass clayey with few variations of its composition. Due to the above, the following properties can be general for all varieties of kaolin: specific weight of 2.6 and hardness of 2 in the Mohs scale [5]. According to the literature, the water-kaolin mixtures exhibit non-Newtonian behaviors and its properties present strong dependence on

the concentration of solid particles. Based on the above, and to determine the kind of non-Newtonian behavior, a study of the variables affecting the rheology of the mineral slurry is required. Because the focus of this work is not the rheological characterization of the slurry, the experimental part is taken from the master thesis of Aguilera (2005). In that work, many laboratory tests were done for determining the kind of non-Newtonian behavior for different water-kaolin slurries. In addition, variations in the physical and chemical variables identified as relevant for the rheology of non-Newtonian fluids were included [5].

In the cited work, different experiments to find the apparent viscosity of a mixture of water and kaolin for different concentrations were realized. The critical volumetric fraction for mixtures without chemical additives were found. From the obtained results the non-Newtonian behavior were identified, even at low concentrations of kaolin, finding as the critical volumetric concentration a 15% of solids in volume. Indeed, it is considered that for solid concentrations higher than 15% the pulp can not be treated. Once the non-Newtonian behavior is verified, experiments to different concentrations, with gradual changes, were done for obtaining the rheograms (shear stress versus shear rate curves) and the viscosity curves. From the rheograms and according to the flow behaviors presented in the section 2.3, this kind of slurries are classified as pseudoplastic. Some of the experimental results obtained by [5] are shown in Figures 5.4 and 5.5. From these figures it is possible to observe the pseudoplastic behavior for volumetric concentrations of $w_{v,sol} = 0.05$ and $w_{v,sol} = 0.12$, respectively.

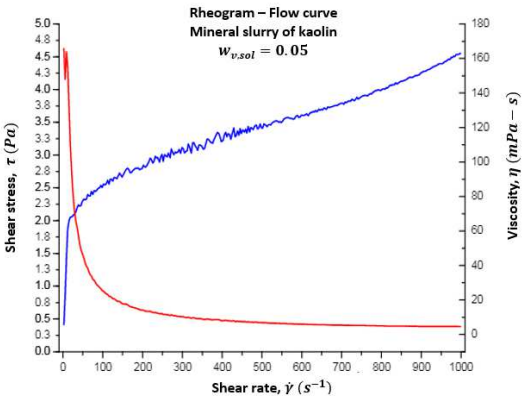

Figure 5.4. Flow curve for a mineral slurry of kaolin with solids volumetric fraction equal to 0.05 (Blue line for flow behavior and red line for viscosity) [5].

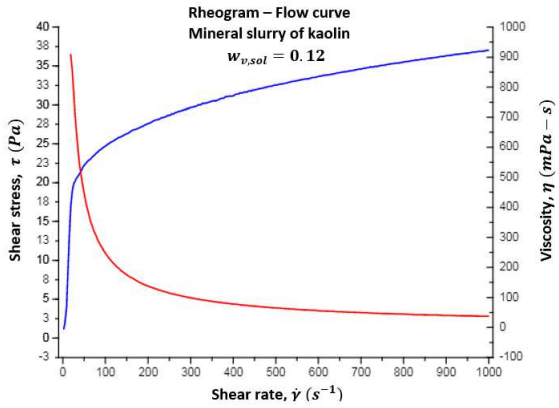

Figure 5.5. Flow curve for a mineral slurry of kaolin with solids volumetric fraction equal to 0.12 (Blue line for flow behavior and red line for viscosity) [5].

In those figures the blue line is referred to the non-Newtonian behavior of the fluid, while the red line is the variation of the slurry viscosity as a function of the shear rate. These curves were obtained using a rheometer TA Instruments Inc. CSL - 500, which permits the use of different geometries (concentric cylinder, parallel plates, etc.) under controlled shear rate and shear stress. In order to cover a big variety of scenarios, the tests were realized for a range of the shear rate between 0 to $1000s^{-1}$ and controlled temperature to $25°C$. Depending on the solids concentration in the mixture, the geometry in the rheometer was changed. For the case of low concentrations the concentric cylinder was used, whereas, for the high concentrations the parallel plates were used. Finally, in Figure 5.6 the results for all executed test are shown. According to [5], the viscosity is strongly affected by changes in the solids concentration. Indeed, the viscosity of the mixture increases as the volume fraction of solid increases, when

the shear rate and shear stress are constants.

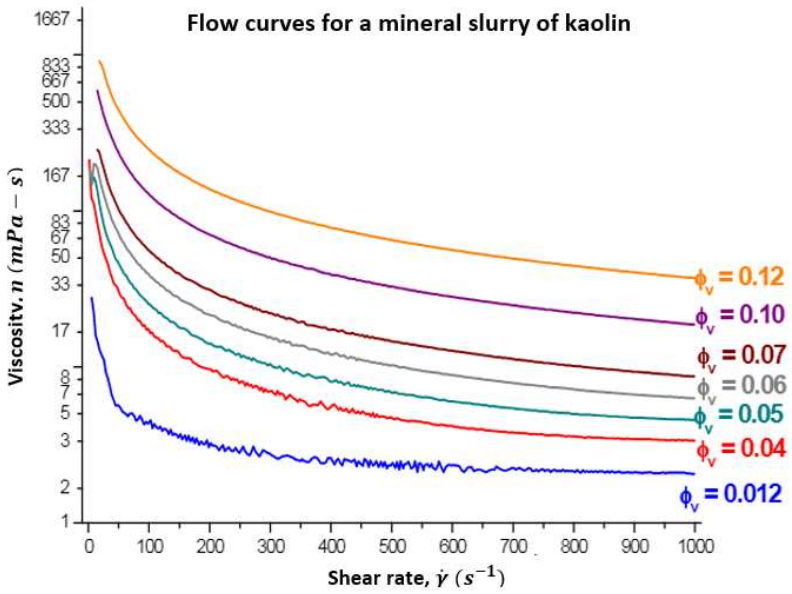

Figure 5.6. Flow curves for different concentrations of solids in mixtures of water and kaolin. [5]

Based on cited work, it is possible to establish a relation between the viscosity of the slurry and the volumetric fraction of solids. Taking into account that the power law model can represent the behavior of the pseudoplastic fluids, and using the results shown in Figure 5.6, one function for determining the parameters K and n, of the power law model, can be obtained. Therefore, remembering the power law model given by the equation 2.22, from the curves in Figure 5.6 it is possible to find the values of K and n that best fit the slurry behavior to the power law. Then, knowing the respective values of the parameters for the power law model for each considered solids volumetric fraction, expressions of the form $K = f(w_{v,sol})$ and $n = f(w_{v,sol})$ are obtained. With this, a correlation between the viscosity and the volumetric fraction is generated.

According to the flow curves for the different volumetric fractions of kaolin, the fitted values of K and n are shown in the Table 5.1. Based on this fit, the correlations between each one of parameters (K or n) and the volumetric fraction of solids are obtained. These are expressed by equations 5.30 and 5.31.

Table 5.1. Values for the parameters K and n according to the volumetric fraction of kaolin.

$w_{v,sol}$	K	n
0.012	52.30	0.486
0.040	400.0	0.300
0.050	860.4	0.243
0.060	1586	0.202
0.070	2284	0.196
0.100	5194	0.197
0.120	9626	0.207

$$K = 981072(w_{v,S})^2 - 45397(w_{v,S}) + 583.01, \tag{5.30}$$

$$n = -269.87(w_{v,S})^3 + 104.09(w_{v,S})^2 - 12.009(w_{v,S}) + 0.6188. \tag{5.31}$$

Previous two correlations for K and n were obtained as a part of the model developed in the present thesis work. By replacing equations 5.30 and 5.31 in 2.22 one function of the form $\eta = f(w_{v,sol})$ is obtained. Based on this it is possible to relate η and ρ using the equation 5.17. However, the function for determining the viscosity must be related to the operational conditions in the pipeline. For this, it is necessary to express the shear stress and the shear rate as a function of the process variables and the measured variables in the process. Thus, and continuing with the power law model as the right representation or rheological behavior of the slurry, one expression for the shear rate as a function of operational condition in the PFA is required.

In [56], the expression proposed in [57] is verified and useful for calculating the wall shear rate of non-Newtonian fluids flowing through a pipeline. This equation is defined as:

$$\dot{\gamma}_w = \frac{3}{4}\left(\frac{8\overline{u}}{D}\right) + \frac{1}{4}\left(\frac{8\overline{u}}{D}\right)\frac{d\ ln\left(\frac{8\overline{u}}{D}\right)}{d\ ln\left(\frac{D\Delta P}{4L}\right)}, \tag{5.32}$$

where the term $\frac{8\overline{u}}{D}$ corresponds to the wall shear rate for the Newtonian fluids ($\dot{\gamma}_w$), while the term $\frac{D\Delta P}{4L}$ is the wall shear stress (τ_w). For the case of fluids with rheological behavior represented by the power law model, as water-kaolin mixtures treated here, the logarithm term can be replaced with the reciprocal of the exponent of the power law model as shown in the equation 5.33 [56].

$$\frac{d\ ln\left(\frac{8\overline{u}}{D}\right)}{d\ ln\left(\frac{D\Delta P}{4L}\right)} = \frac{1}{n}. \tag{5.33}$$

Then, the wall shear rate can be determined by the following equation:

$$\dot{\gamma}_w = \frac{3n+1}{4n}\frac{8\overline{u}}{D}. \tag{5.34}$$

If the dimensions of the pipeline are known, the velocity in the line can be evaluated as:

$$\overline{u} = \frac{Q_p}{A_T}, \tag{5.35}$$

Finally, the function $g(\cdot)$ mentioned in the Chapter 4, can be established. For this, the equation 5.35 is replaced in 5.34, and the resulting expression for $\dot{\gamma}_w$ in the equation 2.22. Based on the above, the function relating slurry viscosity with slurry density and the process variables is defined as:

$$\eta = g(\rho, w_{v,sol}, Q_p) = \ K\left(\frac{3n+1}{4n}\frac{8\overline{u}}{D}\right)^{n-1}, \tag{5.36}$$

where K and n given by 5.30 and 5.31.

5.1.3. Static verification

As mentioned before, for determining one expression for density there are two options. The first of these have been explained in the above section corresponding to the correlation of experimental results. If the second option proposed for the estimation structure is selected, it is not necessary to find an expression for $g(\cdot)$. For this case, the viscosity is considered as known input and it is estimated when the state estimation is done. The procedure to obtain this estimation is through the static version of the mechanical energy balance (Bernoulli equation) given by the equation 5.29. However, the use of the static version implicates to consider some assumptions.

Using the static version of the mechanical energy balance, one condition of the steady state is considered, and this implicates that the pulp inside the pipeline is homogeneous through all the PFA. Indeed, the inlet slurry is the same as the outlet slurry, whereby in the equation 5.29 the following may be considered:

$$\rho_{in} = \rho_{out} \ \textit{and} \ \dot{m}_{in} = \dot{m}_{out}. \tag{5.37}$$

If in addition to the above, the term $\frac{dQ_P}{dt}$ in the equation 5.29 is considered as equal to zero, the static version is obtained. This consideration is done assuming the pressure changes are faster than the mass content changes. Therefore, when disturbances in the solids concentration took place at the inlet, the pressure and volumetric flow reach their new stable state faster than the mass content. In addition, from the stationary condition it is possible to reduce the system to only one unknown variable, the viscosity. Thus, at each iteration, the values of measurement and estimation are used for solving a system of one unknown variable and one equation. With this, the equation 5.29 is reduced to:

$$0 = -\frac{P_2}{\rho_P} + \frac{P_1}{\rho_P} - gz_2 + gz_1 - h_{f_{1-2}}, \tag{5.38}$$

with $h_{f_{1-2}} = f(\eta, \rho_P, Q_P)$. According to the structure presented in Figure 4.3, the evaluation of the slurry viscosity is done using a mechanical energy balance in the steady state. Thus, every time that the density estimation is available, along with pressure drop and volumetric flow measurement for the same time, through Bernoulli equation (or BEM, 5.38) the corresponding viscosity value is determined.

5.2. Design of unknown input observer (UIO)

Based on the estimation problem, the available measurements, and the unknown inlet properties (density, viscosity, and solids fraction), the observer design for the estimation structures presented in the Section 4.2 is presented.

Because the proposed model establishes differences between the properties of the inlet and outlet point in the PFA, the properties at the inlet are considered as unknown variables, remembering that the balance differential equations were obtained for the outlet point under the perfect mixture condition. However, the density and the solids concentration in the inlet can be related by equation 5.12, while the viscosity and the solids concentration are related

using the previously presented power law model. Therefore, the solids initial concentration is the only unknown input. According to the previous deduction of the model, in the Table 5.2 the definitions of the state variables ($\mathbf{x}$), the known inputs ($\mathbf{u}$), the unknown inputs ($\mathbf{d}$) and the model outputs ($\mathbf{y}$) are presented.

Table 5.2. Classification of the process variables .

State variables:	$\mathbf{x} = [\rho_{P,out}; Q_P]$
Known inputs:	$\mathbf{u} = [P_1; P_2; \mu_{P,out}]$
Unknown inputs:	$\mathbf{d} = [w_{S,in}]$
Outputs:	$\mathbf{y} = [Q_P]$

Based on the Table 5.2, the slurry viscosity is taken as known input just by the simulation, i.e. for constructing the algorithm in the software. However, the viscosity is an unknown value which is determined from equation 5.36 or 5.38, after to estimate the state variables. From the previous definitions for the process variables and the design procedure for the observer presented in the Section 4.2, the model equations can be summarized as:

$$\frac{d}{dt}\begin{bmatrix}\rho_P \\ Q_P\end{bmatrix} = \begin{bmatrix}f_1(\mathbf{x},\mathbf{u},\mathbf{d}) \\ f_2(\mathbf{x},\mathbf{u},\mathbf{d})\end{bmatrix}, \tag{5.39}$$

$$\mathbf{y} = [0,1]\ Q_P, \tag{5.40}$$

with f_1 and f_2 equal to:

$$f_1(\mathbf{x},\mathbf{u},\mathbf{d}) = (\rho_{P,in}Q_P - \rho_{P,out}Q_P)\,\frac{1}{V_T}, \tag{5.41}$$

$$f_2(\mathbf{x},\mathbf{u},\mathbf{d}) = \frac{A_T}{L_T}\left(-\frac{P_2}{\rho_{out}} + \frac{\dot{m}_{P,in}}{\dot{m}_{P,out}}\frac{P_1}{\rho_{in}} - gz_2 + \frac{\dot{m}_{P,in}}{\dot{m}_{P,out}}gz_1 - \frac{\dot{m}_{P,in}}{\dot{m}_{P,out}}h_{f_{1-2}} - h_{f_{1-2}}\right). \tag{5.42}$$

The observer design requires linearizing the model equations according to the equations presented in the Section 4.2.3 and the definition of the process variables in the Table 5.2. However, the stationary state must be determined based on the model and conditions of current operation. Thus, for the following operational conditions:

$$\begin{aligned} Q_P &= 0.0624\frac{m^3}{s}, \\ P_1 &= 487114.104Pa, \\ P_2 &= 344916.574Pa, \\ P_3 &= 347191.845Pa, \end{aligned} \tag{5.43}$$

by simulation of proposed model, the following steady state is obtained:

$$\begin{aligned} Q_P|_{ss} &= Q_P, \\ P_1|_{ss} &= P_1, \\ P_2|_{ss} &= P_2, \\ P_3|_{ss} &= P_3, \\ P_{1B}|_{ss} &= 381130.617\ Pa, \\ \rho_P|_{ss} &= 1041\ \frac{kg}{m^3}, \\ \mu_P|_{ss} &= 0.0317\ cP, \\ w_{S,out}|_{ss} &= 0.0639, \\ w_{S,in}|_{ss} &= 0.0639. \end{aligned} \tag{5.44}$$

Using the Jacobian definition and the steady state values, the matrices for the linearized model are shown below:

$$\mathbf{A} = \begin{bmatrix} -0.1614 & 0 \\ -0.0003 & -0.4465 \end{bmatrix} \tag{5.45}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -0.0314 \end{bmatrix} \tag{5.46}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{5.47}$$

$$\mathbf{D} = \begin{bmatrix} 109.6827 \\ -0.0065 \end{bmatrix} \tag{5.48}$$

From the matrices **C** and **D**, the existence condition is verified. Therefore, the matrix **CD** is given by:

$$\mathbf{CD}|_{ss} = \mathbf{C} \cdot \mathbf{D} = \begin{bmatrix} -0.0065 \end{bmatrix}, \tag{5.49}$$

then,

$$rank(\mathbf{CD}|_{ss}) = 1 \text{ and } rank(\mathbf{D}|_{ss}) = 1, \tag{5.50}$$

therefore, the existence condition is satisfied and it is possible to continue with the observer design. Before to determine the matrices of the observer, the matrix **E** must be determined as shown below:

$$\mathbf{E} = -\mathbf{D} \cdot (\mathbf{CD})^{-1} = \begin{bmatrix} 16855.55 \\ -1 \end{bmatrix}. \tag{5.51}$$

Next, the detectability of the system is verified according to the exposed in the equation 3.30 in the Chapter 4. Thus, if $s = Q_P$ (Q_P the measured variable), and the matrix **P** is computed as:

$$\mathbf{P} = \mathbf{I}_n + \mathbf{E} \cdot \mathbf{C} = \begin{bmatrix} 1 & 16856 \\ 0 & 0 \end{bmatrix}, \tag{5.52}$$

the detectability matrix of the system is given by:

$$\begin{bmatrix} s\mathbf{P} - \mathbf{PA} \\ \mathbf{C} \end{bmatrix} = \begin{bmatrix} 4.722 & 8073.194 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \tag{5.53}$$

then, the rank of detectability matrix is equal to,

$$rank \begin{bmatrix} s\mathbf{P} - \mathbf{PA} \\ \mathbf{C} \end{bmatrix} = 2. \tag{5.54}$$

From the above, the system is detectable due to the rank from 5.54 is equal to the states number in the system. Then, it is necessary to establish the gains matrix before to calculate the matrices of the observer. For this, the observer gains must be selected in such a way that the matrix $\mathbf{N}$ fulfills the Hurwitz condition.

Remembering the expression for matrix $\mathbf{N}$,

$$\mathbf{N} = \mathbf{PA} - \mathbf{KC},$$

from the above equation the matrix $\mathbf{K}$ can be expressed as:

$$\mathbf{K} = (\mathbf{PA} - \mathbf{N}) \cdot \mathbf{C}^{-1}, \tag{5.55}$$

however, based on the knowledge of the matrices $\mathbf{C}$, $\mathbf{P}$ and $\mathbf{A}$, and the products of matrices $\mathbf{PA}$ and $\mathbf{KC}$ given by:

$$\mathbf{PA} = \begin{bmatrix} -4.6595 & -7020.5 \\ 0 & 0 \end{bmatrix}, \tag{5.56}$$

$$\mathbf{KC} = \begin{bmatrix} 0 & k_1 \\ 0 & k_2 \end{bmatrix}, \tag{5.57}$$

the value of k_1 has been selected equal to the component of the first row and second column of the matrix $\mathbf{PA}$ in order to cancel the same component in the resulting matrix $\mathbf{N}$. On the other hand, the gain k_2 is a free value since the matrix $\mathbf{PA}$ does not have components in the respective position, which must be eliminated for obtaining a diagonal matrix. Thus, the values of the gains matrix ($\mathbf{K}$) were be selected such that the matrix $\mathbf{N}$ was a diagonal matrix and its eigenvalues had negative real part. Therefore, the gains of the observer were fixed as:

$$\mathbf{K} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} -7020.5 \\ 4 \end{bmatrix}. \tag{5.58}$$

Accordingly, the matrix **N** is defined as:

$$\mathbf{N} = \begin{bmatrix} -4.6595 & 0 \\ 0 & -4.0000 \end{bmatrix}, \tag{5.59}$$

Since the matrix in the equation 5.59 is diagonal, its eigenvalues are determined by the diagonal components. From this, it is possible to verify that have negative real part, and the Hurwitz condition is satisfied. Once the matrices **N** and **K** are calculated, the other matrices of the observer can be calculated from the equations 4.31 and 4.33 as follows:

$$\begin{aligned} &\mathbf{L} = \mathbf{K} \cdot (\mathbf{I} + \mathbf{CE}) - \mathbf{PAE}, \\ &\mathbf{L} = \begin{bmatrix} 71518 \\ 0 \end{bmatrix}, \end{aligned} \tag{5.60}$$

$$\begin{aligned} &\mathbf{G} = \mathbf{PB}, \\ &\mathbf{G} = \begin{bmatrix} 0.0444 & -0.0444 & -529.0263 \\ 0 & 0 & 0 \end{bmatrix}, \end{aligned} \tag{5.61}$$

Consequently, the matrices of the observer have been established to the operational conditions and the available measurements in the real process. Now, the design of the observer is finished, and the conditions of existence and detectability have been verified, in order to give validity to the estimation structure.

Then, the state estimation (slurry density and volumetric flow) is obtained based on the designed unknown input observer. Nevertheless, the slurry viscosity, initially considered as known input, must be estimated from auxiliary expressions depending on the estimation structure selected. Therefore, the structure represented by equations 5.39 and 5.40 corresponds to the UIO box in the Figures 4.2 and 4.3 for the estimation structures. The other hand, for the first estimation structure the auxiliary equation for computing the slurry viscosity is given by expression 5.36, while for the estimation structure using a static approximation the expression for the viscosity is the equation 5.38.

5.3. Simulation results: Application case

In this section, the obtained results for the estimation of density and viscosity are presented for a mineral water-kaolin slurry. Considering the application case exposed above, for two versions of the estimation structure, noise in the measurement is included. In both simulations, the process model is the same and the disturbances in the unknown input considered are among ± 10 percent of the value of the steady state, according to Figure 5.7. The behavior of the proposed model is shown in Figure 5.8.The plotted variables behaviors correspond to the PFA outlet.

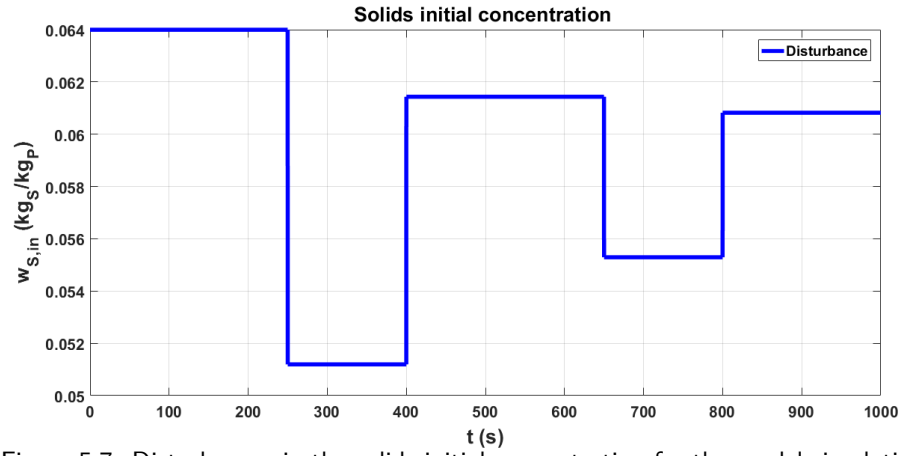

Figure 5.7. Disturbances in the solids initial concentration for the model simulation.

From Figure 5.8 it is possible to observe the response of the state variables of volumetric flow and density when disturbances in the initial concentration of solids takes place. This behavior is obtained because the pressure drop in the pipes and fittings assembly remains constant. In addition to this, the disturbances generated by the system were applied taking into account that the system has enough energy for transporting the fluid with the resulting properties of density and viscosity after disturbances appearance.

Since that the observer considered in the estimation structure is a linear one, the effect of the linearization of the model over the observer performance is evaluated. For this, the simulation results for the process under different disturbances are shown in Figure 5.9. From these, the linearized model observer has a very similar behavior to the nonlinear model observer, although the system is subject to disturbances. Notably, the linear model is valid if the operation conditions are near to the steady state. However, for this kind of process, although the system can present changes in your variables, these changes happen near to one operational condition, which normally corresponds to the steady state.

As the behavior of the linear model is valid, the estimation structure is implemented for the model developed in Section 5.1.1. For the first case, in which the power law model is used (option 1), the simulation was developed considering noise in the measurements and the function $g(\cdot)$ obtained by the explained procedure in the section 5.1.2. For this case, the estimation of the state variables (volumetric flow and density) is very near to the real behavior. Thus, as the viscosity is obtained based on the estimation of the state, the estimation error is less than 2 percent. In Figure 5.11 the simulation results are presented for different values of the free gain of the observer (k_2), i.e., the gain related to the measured state.

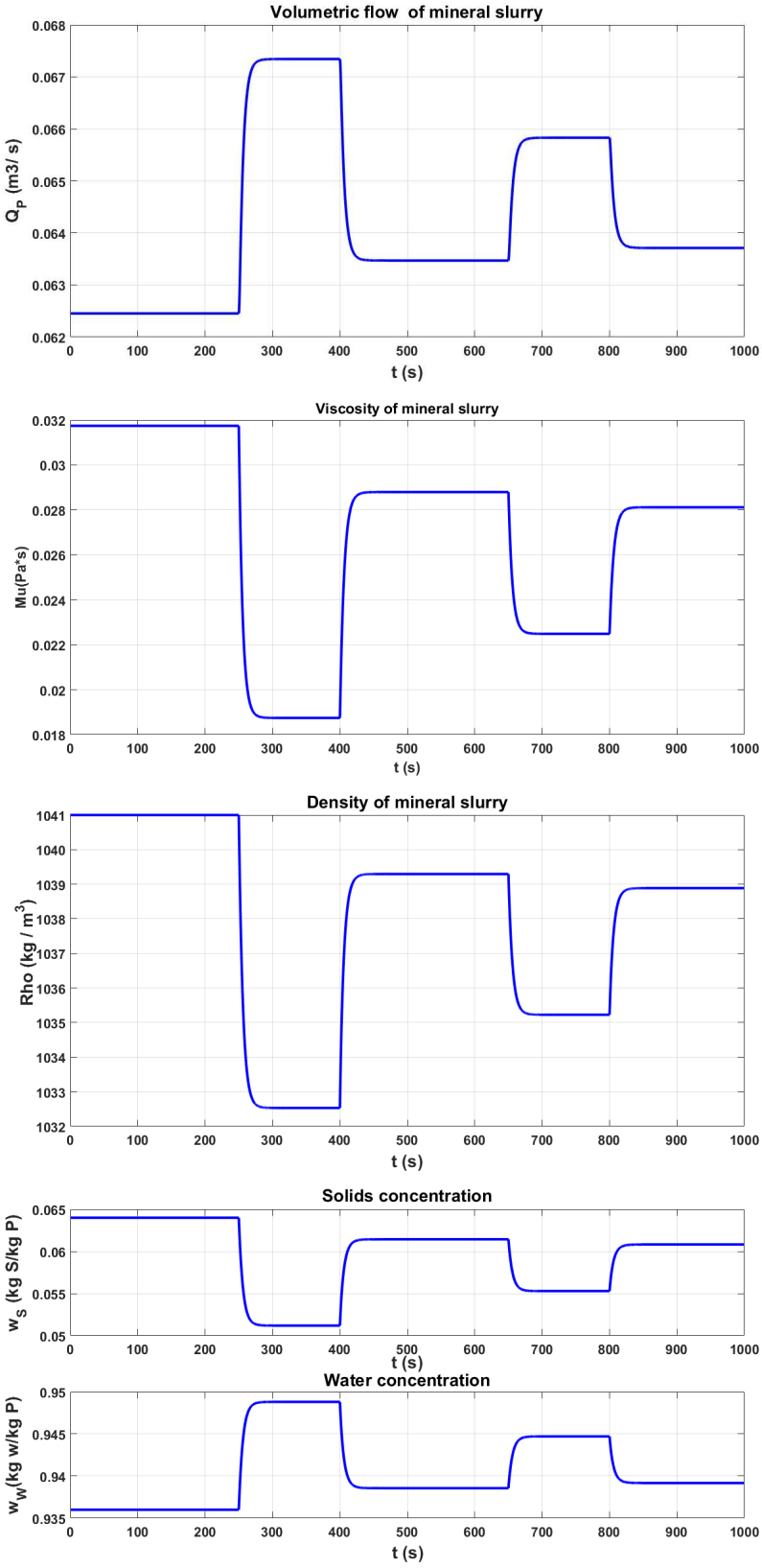

Figure 5.8. Simulation of proposed model under different disturbances for the solids concentration in the inlet.

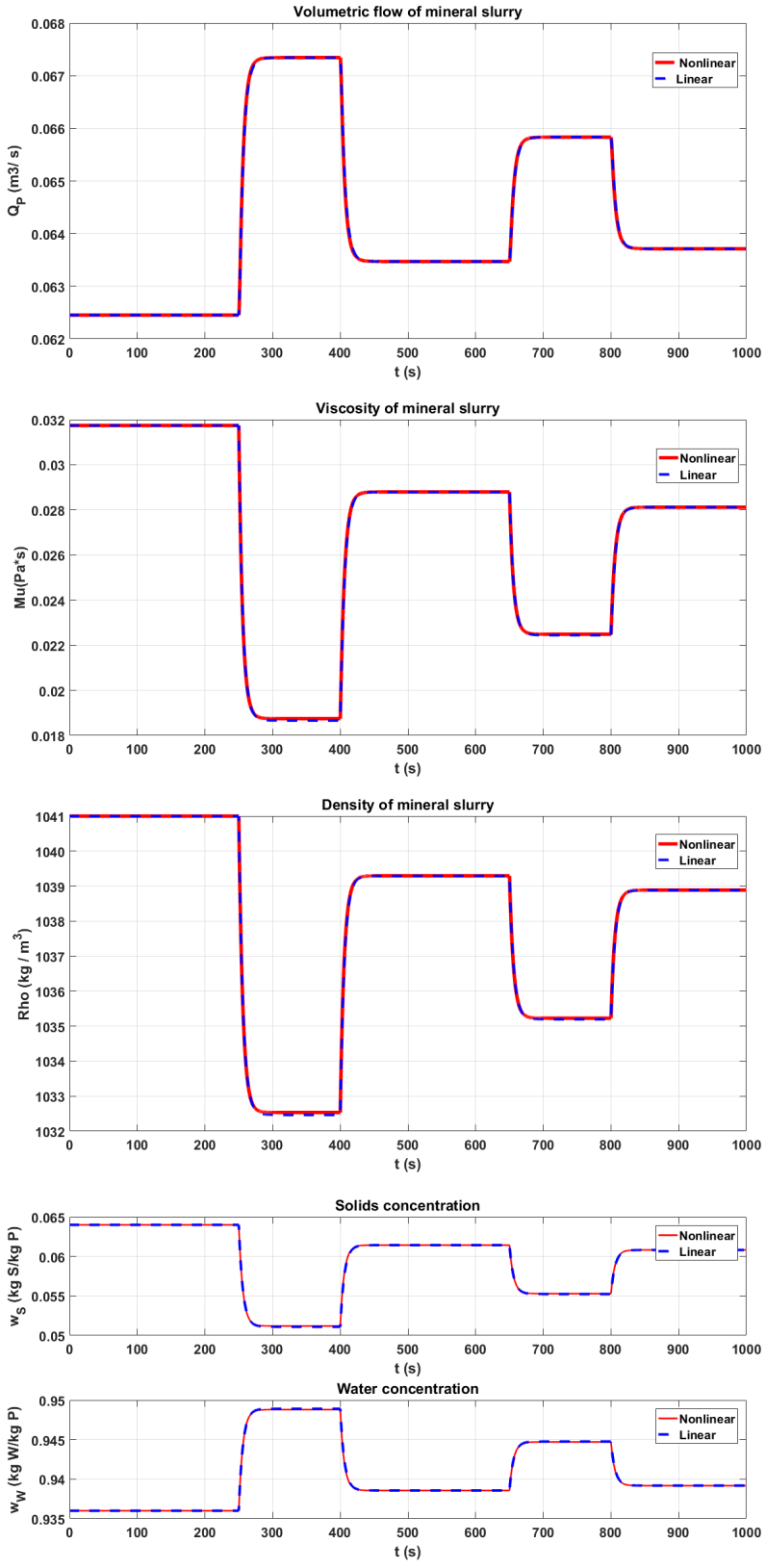

Figure 5.9. Comparison between the linear and non-linear model for the variables of volumetric flow and viscosity under different disturbances for the solids concentration in the inlet.

Based on the results presented in Figure 5.11 it is possible to appreciate that the state estimation and the viscosity do not change although the gain value for the measured variable (Q_P) is modified. The above is due to the estimation of measured value can be adjusted while for the unmeasured variable the gain is fixed by the own system of equations. For this case, the estimation could be improved if the gain for the unmeasured variable could be changed, but the existence condition and the detectability of system restrict the gain value. In Figure 5.10 the applied disturbances in the solids initial concentration are presented.

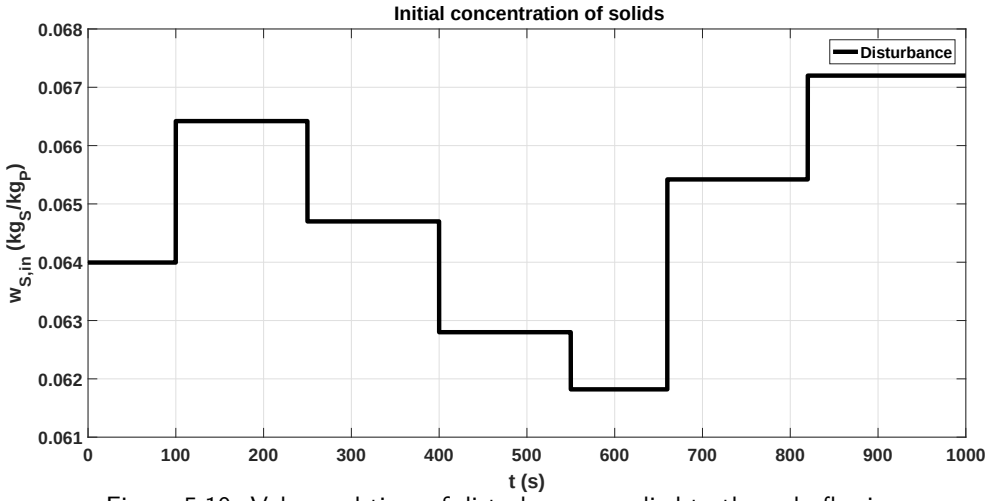

Figure 5.10. Value and time of disturbances applied to the pulp flowing.

The proposed structure was evaluated for different disturbances, among these were considered upper and lower values regarding the steady state. For this case, in which the mineral slurry is correctly characterized, the estimation of the state and the viscosity is very approximate even when the disturbance in the initial concentration of solids is higher to 0.5% for the same pressure drop. Based on the above, and having into account that the pressure measurements are considered as known inputs to the system, the variable of volumetric flow is the unique variable supporting all the changes generated by the disturbance and spreads those changes to other variables. For this reason, increasing the initial concentration of solids, the density and viscosity increase too, while the volumetric flow decreases. This behavior is due to more energy for transport the mineral slurry is required, and as the pressure drop remains constant, the transport capacity of the system for a heavier slurry is lower. On the other hand, when the initial concentration of solids is lower, the opposite case occurs and the volumetric flow increases while the density and viscosity of mineral slurry decreases.

For the case in which it is not possible to do an experimental study about flow behavior of fluid, the second option of observer must be considered. The results obtained using a static verification for the estimation of viscosity are presented in Figure 5.12. This structure was simulated for the same disturbances applied for the first option for the observer, when the experimental characterization of the rheological behavior is available. Due to the behavior of dynamic state is approximated to the steady state for each sampling time, differences between the real behavior and estimated begin to appear. This result is due to the assumptions considered for implementing the static verification, such as the steady state and the existence of a homogeneous slurry into the PFA at each sampling time.

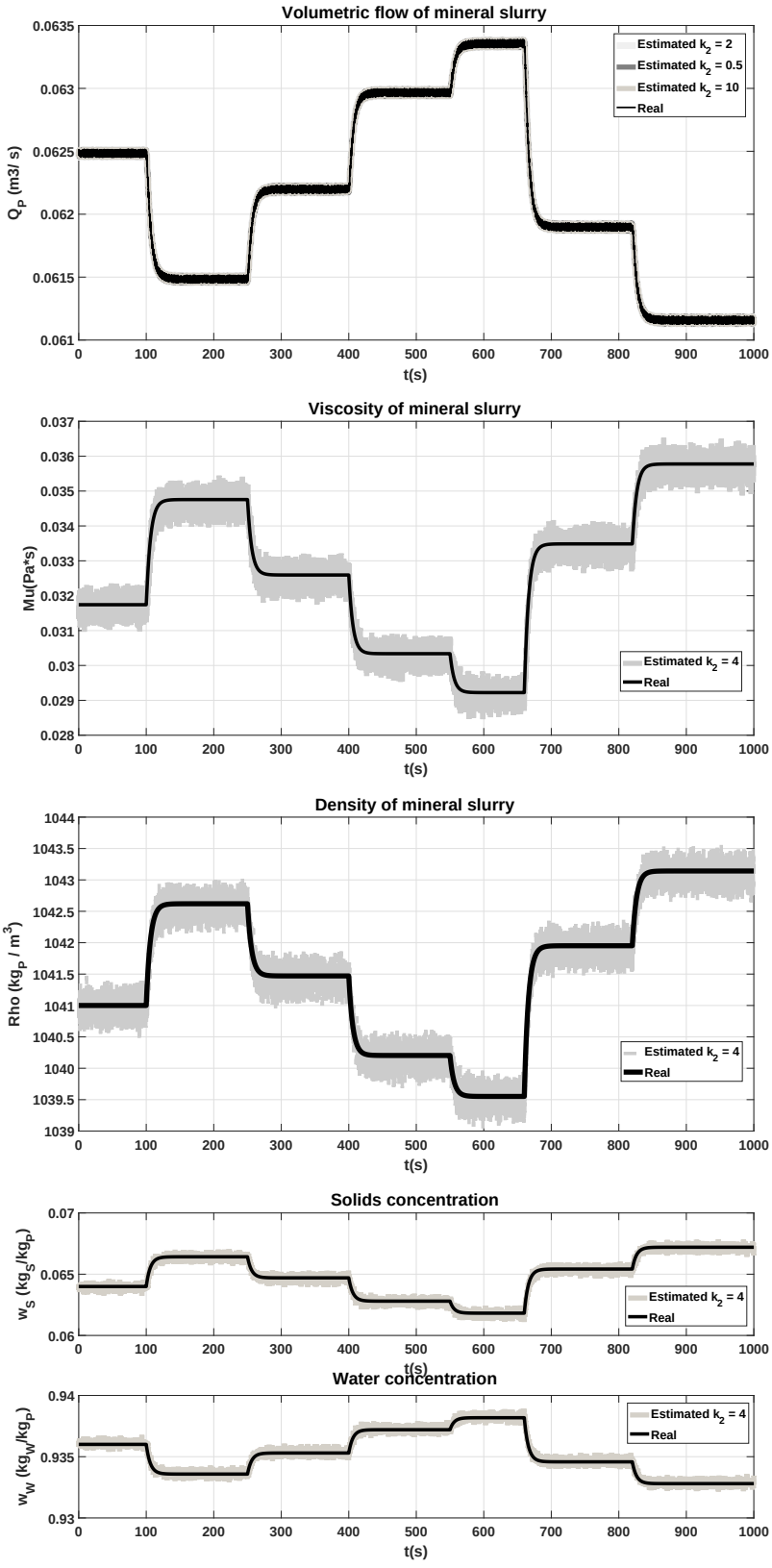

Figure 5.11. Simulation results of the process variables for the proposed estimation structure and the power law model, under different values of free gain k_2 and different disturbances for the solids concentration in the inlet.

Figure 5.12. Simulation results of the process variables for the proposed estimation structure and the static verification, under different k_2 gain values and different disturbances for the solids concentration in the inlet.

Therefore, and as result of the considerations done, the estimation of the unmeasured variable is not as approximate to the real value as in the first option. As the time passes and the system remains in the steady state, the estimation is done with better precision. However, when one disturbance takes place the estimation of the unmeasured variable loses precision, until the new steady state is reached. Although this proposal presents estimation error during the change from one steady state to another when a disturbance has happened, once the steady state is achieved the estimation is correct because the assumption of the homogeneous slurry is truth and the equations system can be simplified to a steady state mechanical energy balance. Worth noting that the estimation can be good provided the measurements of pressure drop and other available measurements are correct. In addition, the estimation has an error as results of error propagation during the dynamic state, according to the time progress.

From these results, it is possible to construct an estimation structure based on a phenomenological based model for the transport process of a mineral slurry through to a PFA, even for a mineral slurry with properties and flow behavior unknown. Hence, this is a more general option than the first one, but this generality produces a loss of precision in the estimation of the state variables and the parameters. Thereby, the loss of precision in the estimation done with the second option could be improved, if different techniques of correction are implemented, which complement the internal correction of the observer with the measured variable.

For the behavior of the system when the disturbances have taken place, the response of the system is the same in the two case. As the pressure drop for these simulations is constant, the changes in the system can be observed in the volumetric flow and the properties of the mineral slurry. Thus, when density or viscosity of the mineral slurry tend to increase, the volumetric flow tend to decrease due to with the same energy (pressure drop) it is not possible to move the same flow of a heavier slurry.

The results presented above were obtained by simulation using Matlab® academic version. The proposed algorithms were developed using a processor *Intel® Core™ i7-5500U CPU 2.40GHz and RAM of 8.0 GB*. For each simulation, the total time of simulation was of 1000*seconds* using a step size of 0.1*seconds*. Thus, the proposed algorithms form a loop with a repetitions number equal to the total time of simulation divided by the step size. The method used for solving the differential equations was the Euler method due to its didactic characteristic and easy implementation. Other equations solvers with better characteristics can be used, but this is not the scope of this thesis. Obviously, an appropriate time step size was selected for guaranteeing low error and convergence of the method. Additional, the noise in the measured variable or the system output was included using a *Random* command of Matlab®. Finally, and as a comparison for the estimation error in Table 5.3 values of the *ITAE* index for each one of the proposed structures are presented.

Table 5.3. *ITAE* index for the state estimation of the transport of a mineral slurry with noise.

Variable	Structure 1: Experimental test	Structure 2: Static verification
Q_P	334.2795	346.9432
ρ_P	162.1284	891.3047

The comparison between the two proposed structures based on the *ITAE* index shows that the first structure presents a better performance. The difference between both structures in the measured variable is due to the use of the *Random* command for the simulation of noise, with which at each simulation the value of the measured variable is always different. However, the main difference is the unmeasured variable, where the structure with an experimental develop presents a *ITAE* index lower than the structure with the static verification. Therefore, the main index for evaluating the performance of the proposed structures is the index for the unmeasured variable, so for the case of the static approximation, the index is considerably higher than the index for the first structure.

5.4. Effects of the properties of the mixture on the pressure drop

In previous section the results of two structures for the on-line estimation of the properties of density and viscosity of a mineral slurry were presented, but from this results is not possible to separate the effects of each property on the pressure drop. Thus, and due to the interest in knowing as each property of mixture affects the readings of pressure sensor, in this section a methodology using techniques of the digital signals processing is presented. Thereby, the aim of this section is to identify patterns which permit differentiate the effects of the density and viscosity on the signal of pressure drop for a mineral slurry flowing through a PFA. To know the origin of pressure drop changes can help to improve the density and viscosity estimation using previous observers proposals.

For the case of a real process, the signals to be analyzed are continuous. For the processing of the continuous signals there are different tools feasible to implement in real time. Some of these are the Wavelet transform (WT) and the short-time Fourier transform (STFT), which permit to obtain the spectral information of non-stationary signals [58]. The STFT is a technique based on the division of the signal into finite portions, over which the Fourier transform is applied for obtaining the information about the spectral components [59]. However, and due to the uncertainty principle of Heisenberg, the STFT presents problems of time-frequency resolution. For solving these problems other kinds of techniques based on the multiresolution analysis are commonly used [6].

On the other hand, the Wavelet transform is a technique based on the multiresolution analysis, in which the signal is analyzed for different frequencies with different resolutions (variable resolution). From this tool it is possible to obtain representations of the signals in the temporal and frequency domain at the same time. This advantage permits to identify the time of occurrence of certain spectral components, such as the changes of the steady state. The WT consists on a series of filter stages and decimation of the signal. Thus, with the application of the WT a filtering process in time domain using low-pass and high-pass filters is implemented. The function of these filters is to eliminate the low and high frequencies respectively. This procedure is repeated as many time as necessary, and this is known as the decomposition levels of the signal. Therefore, the result of this procedure is a group of signals analyzed to different frequency bands, which represents the same original signal.

The evaluation of the WT is done in a discrete form using the discrete Wavelet transform (DWT). In this technique a pyramidal algorithm is used to obtain a time-scale representation

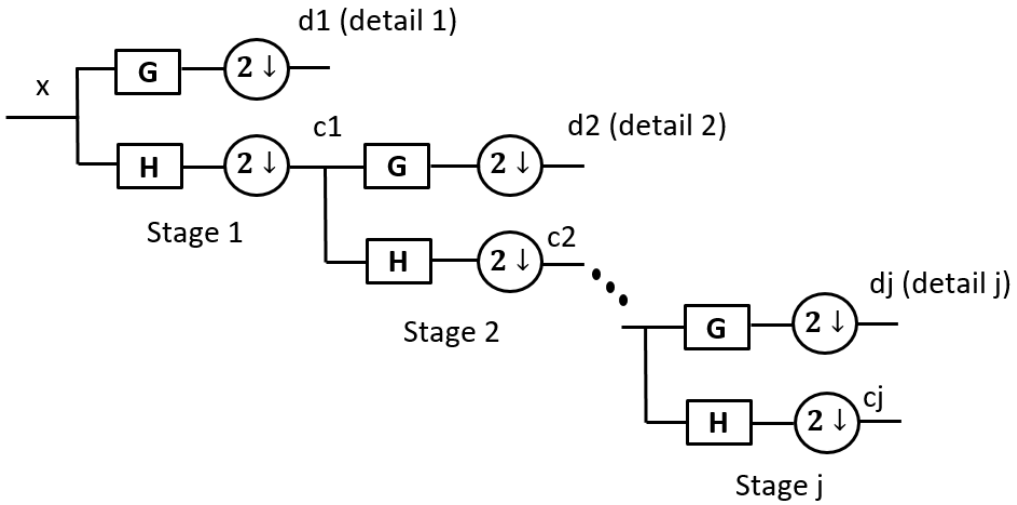

Figure 5.13. Filter bank for the decomposition steps and signal analysis. H corresponds to the low-pass and high-pass G filter to filter [6].

through successive processes of filtering of the signal. The first step is the discretization of the signal, and then the quantized signal sequence is passed through a low-pass (mid-band) filter, eliminating frequency components higher than half of the bandwidth of the original signal [58], [60]. In Figure 5.13 a representation of the decomposition process of a signal using a filters bank is presented.

According to the above, the DWT is defined as:

$$C[j,k] = \sum_{n \in Z} f[n]\Psi_{j,k}[n], \tag{5.62}$$

where $C[j,k]$ are the transform coefficients, $f[n]$ is the quantized signal and $\Psi_{j,k}[n]$ is the function of mother Wavelet transform. For this last function exist different versions in the literature, which can be consulted in [58], [60] and [61]. In the equation 5.61 the subscripts j,k are the localization in the time and frequency domain, respectively. The mother Wavelet transform can be expressed as follow:

$$\Psi_{j,k}[n] = 2^{-\frac{j}{2}}\Psi[2^{-j}n - k], \tag{5.63}$$

For the reconstruction of the signal, the equation for the synthesis of the signal, commonly named as inverse transform, is defined as:

$$f[n] = \sum_{j \in Z}\sum_{k \in Z} C[j,k]\Psi_{j,k}[n] \tag{5.64}$$

In [58], [60] and [61] it is explained in detail the deduction of the DWT and the calculating procedure through the filter bank and others alternatives different to this. Based on the above, the Wavelet transform is applied to a segment of the signal of pressure drop for the transport process of a mineral slurry, when different disturbances in density and viscosity are

applied. In Table 5.4 the values for the disturbances and simulation time are shown.

Table 5.4. Parameters of simulation - Application of Wavelet transform.

Parameter	Symbol	Value of disturbance
Simulation time	t_s	200 s
Sampling time	h	$0.1s$
Number of iterations	Num_{Ite}	2000
d_1 (ρ)	$P_{1,\rho}$	$\rho = 0.5\%\rho_0$ en $t = 20s$
d_2 (ρ)	$t_{2,\rho}$	$\rho = 0.8\%\rho_0$ en $t = 140s$
d_3 (μ)	$t_{1,\mu}$	$\mu = 0.025Pa - s$ en $t = 60s$
d_4 (μ)	$t_{2,\mu}$	$\mu = 0.045Pa - s$ en $t = 100s$

According to the plant data, in the Figure 5.14 a pressure drop signal is presented. As it was done for other simulations exposed in this work, the noise in the measured variable was considered. Thus, from Figure 5.14 it is possible to observe that each time that one disturbance occurs one change in the pressure drop takes place. Then, the idea with the Wavelet transform is to identify patterns in the frequency domain allowing to classify the origin of each disturbance that occurs in the process, based on the analysis of the signal of pressure drop. In the next section the methodology used is explained.

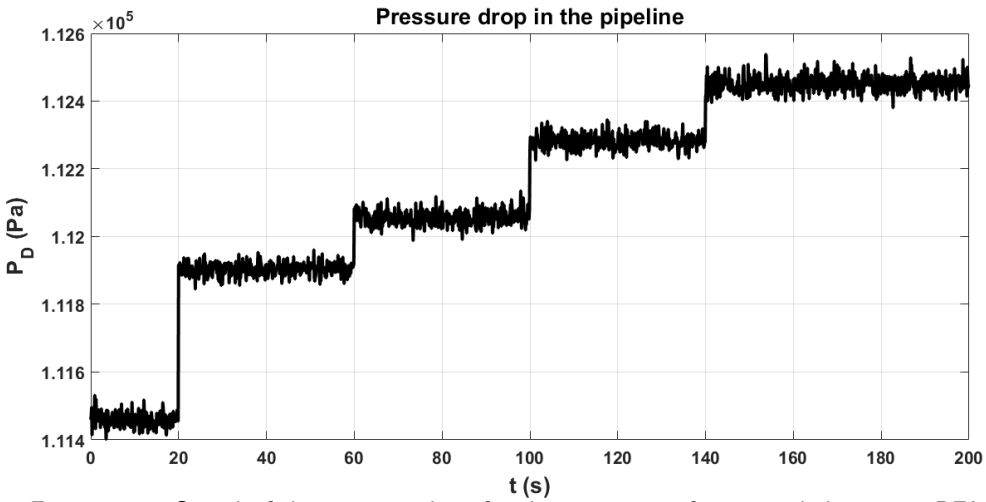

Figure 5.14. Signal of the pressure drop for the transport of a mineral slurry in a PFA.

5.4.1. Methodology for frequency response analysis

In Figure 5.15 a methodology for the analysis of signal of pressure drop is presented. Based on the measurements of plant and the disturbances affecting slurry density and viscosity at certain times, a segment of the signal for 200 seconds was selected for the analysis. From this segment and its duration, the number of iterations (Num_{Ite}) is defined based on the sampling time of 0.1 seconds. Later, the mother Wavelet transform is selected and the number of levels for the signal decomposition is fixed. These choices are done based on the aim of the analysis because with this the temporal resolution and frequency resolution are established. For this case of study, the mother Wavelet transform *Daubechies 2* was selected due to the properties

of orthogonality and asymmetry.

Then, the Wavelet transform is calculated for the segment of the signal, and from this the coefficients of the Wavelet transform are obtained. Once the coefficients are obtained, these results must be treated before graphical submission. Due to the decomposition of the signal is done to different cut-off frequencies, represented by each filter (level) in the filter bank, the length of the signal is more short than the original signal. Indeed, the signal filtered has half the length of the signal at the previous level. Therefore, a resampling process is required to transform the length of the signal to the length of the original signal. Finally, the results for the coefficients are plotted according to your respective scale.

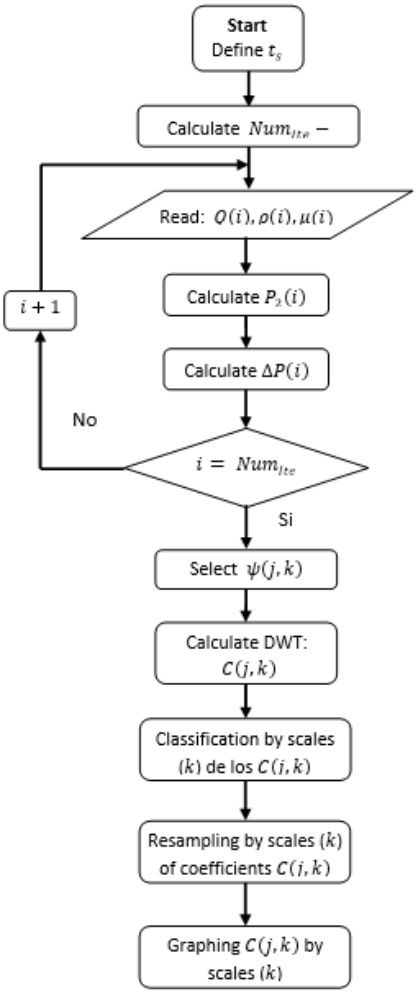

Figure 5.15. Methodology for obtaining the frequency response by DWT.

5.4.2. Results for the Wavelet transform

In order to illustrate the analysis of the frequency response for the signal of pressure drop, in Figure 5.16 the Fourier transform for the signal is presented. In addition to the above, the frequencies spectrum of the same signal filtered for a cut-off frequency of $f = 0Hz$ using a Notch filter is presented too. The above is done to eliminate the frequency associated with the steady state of the system. From these results it is possible to observe that there is

information in frequencies different to $f = 0Hz$, which can be evaluated for the identifying the origin of disturbances.

Using the exposed methodology the results obtained by simulation for the Wavelet transform are presented in Figure 5.17. For applying the transform four decomposition levels were used, resulting in four detail levels (DP_i) and one approximation or residue ($a_1(n)$). The top of Figure 5.17 is the first detail level (DP_1) and corresponds to the coefficients for the values of higher frequencies, while the following other three graphs in Figure 5.17 represent the detail levels organized in decreasingly order respect to the frequencies bands. Finally, the bottom graph in Figure 5.17 is the approximation or residue representing the coefficients for the lowest frequencies.

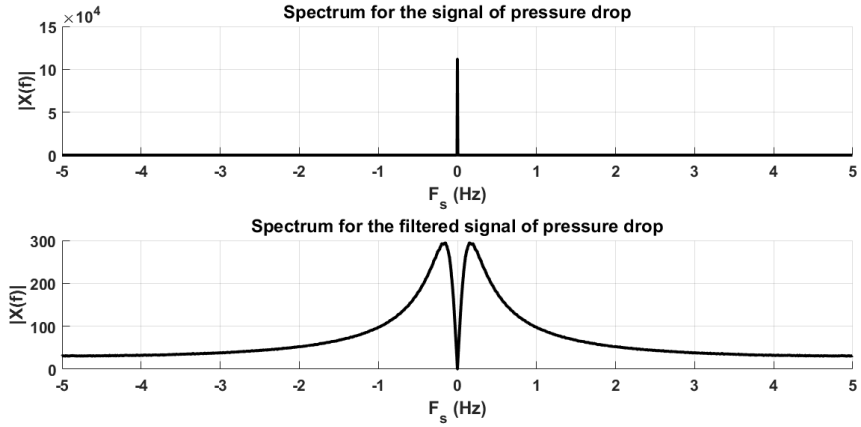

Figure 5.16. Spectrum for the pressure drop signal, and the filtered signal to the cutoff frequency $f = 0Hz$.

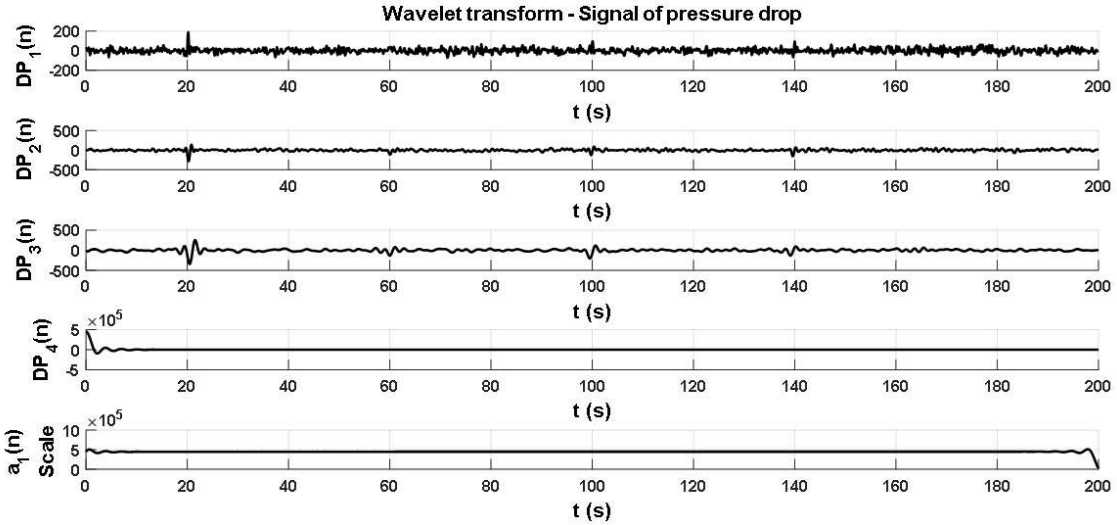

Figure 5.17. Decomposition levels using wavelet transform for signal pressure drop.

From the results for the coefficients of Wavelet transform, the fourth detail level (DP_4) and the residue ($a_1(n)$) do not have information relevant to the aim of separate the effects of density and viscosity. This last because for these it is not possible to recognize any patterns for the moments in which the disturbances occurs. However, for the second and third detail

level (DP_2, DP_3) some patterns can be distinguished. In Figures 5.18 and 5.19 each one of these levels is presented individually.

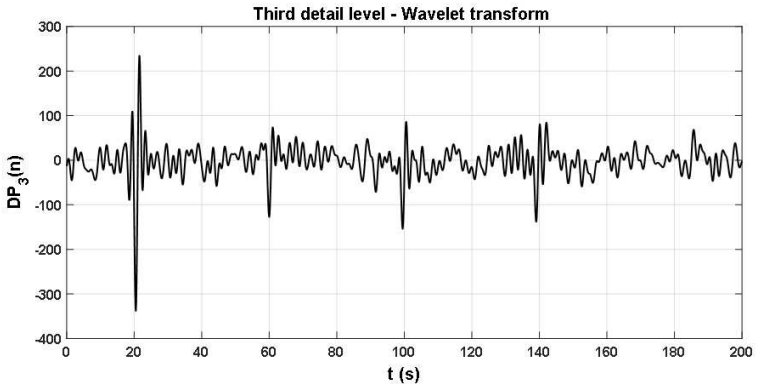

Figure 5.18. Third level of detail for the wavelet transform of the pressure drop.

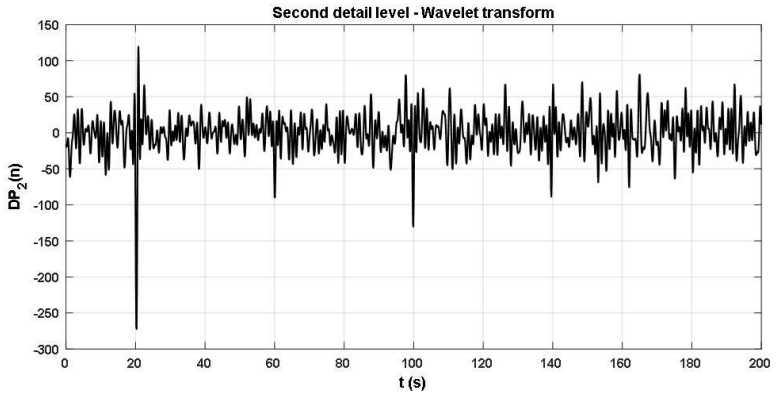

Figure 5.19. Second level of detail for the wavelet transform of the pressure drop.

In Figure 5.18 it is possible to identify significant magnitude peaks for the times in which the disturbances are applied ($t_1 = 20s, t_2 = 70s, t_3 = 100s$ y $t_4 = 140s$). Therefore, the moments of occurrence of the disturbances can be recognized with a good temporal resolution. Although in this detail level it is possible to recognize the times at which the disturbances appear, it is not possible to associate the disturbance to one of the properties. For example, in the case of the second disturbances in density and viscosity ($t_3 = 100s$, $t_4 = 140s$) the magnitude of peaks are similar even when the nature of the disturbances is different.

When the second detail level is individually analyzed, it can be appreciated that just the peaks associated with the disturbances in the density, presented magnitudes higher to -100 in the ordinate axis, whereas the disturbances in the viscosity can be confused with the rest of oscillations or peaks in the graphic. Hence, from the results in this detail level, it is possible to associate the disturbances with one property of the mineral slurry. Finally, the second and third detail level could be a useful tool for the identification of the occurrence times of the disturbances, besides they permit to characterize the disturbance origin. Indeed, the disturbances that can be identified in the frequencies band for the second level, are directly associated with disturbances in the density. Therefore, a criterion based on the thresholds according to the coefficients magnitude for the levels of the wavelet transform can be used to separate the origin of a disturbance: generated by density of by viscosity change. This

separation allows to use this information to develop additional algorithms to improve the estimation. For example, it is possible to apply only correction of estimated value over that estimated variable causing the pressure drop variation. This task is declared as future work in this thesis.

CHAPTER 6
Conclusions and future work

6.1. General conclusions

The work carried out in this thesis allows to express the following general conclusions.

- A methodology for the on-line estimation of the properties of density and viscosity of multiphase fluids, flowing through a pipeline and fittings assembly (PFA) has been developed. This proposal considers the use a phenomenological-based model as the main element for the structure of a state observer for estimating slurry properties. Thus, the structural conditions of the process model and the recommendations for selecting the state variables, known inputs, and the unknown inputs have been established. Based on this, an observer for a process with disturbances, related to the properties to be estimated, is implemented and the existence and detectability conditions for the design of the observer are given. Provided the necessary conditions are satisfied, both for the model structure and the observer design, this estimation structure allows to estimate density and viscosity of a multiphase fluid without intervening or changing the pipeline. The estimation is done with errors lower than 5%, only considering the available measurements in a plant.
- As the most significant contribution of this thesis, a phenomenological based semi-physical model which represents the flow behavior of a non-Newtonian fluid through a PFA, which can be used as the structure for a state estimator has been proposed. Although the phenomenological model for the fluids transport has been extensively studied, the structure of the dynamic mechanical energy balance used there does not fit with the mathematical transformations commonly used for the design of the unknown input observers. Besides the above, the proposed model takes into account the separability of the density and viscosity for the estimation of these properties by adding one mass balance as a function of the density, when the disturbances in the initial concentration of solids takes place. This addition completes the process model and expands the state. Resulting these model is used together with the typical measurements in this type of process, normally the volumetric flow and the pressure drop, for estimating mixture properties: density and viscosity.
- In order to comply with the general objective of this work, the direct and indirect methods, usually used for on-line estimation of the slurry properties were identified from a literary review. From these methods, was possible to identify the main variables that affect the mixture properties. These variables are the phases composition, the shape, and size of solids, and the density and viscosity of each one phase. Based on this selection of variables, and taking into account the estimation aim and the available measurements, the variables to considered in the model were selected. Thus, since only there is a measured variable, and due to the complexity that involves adding the factors of shape and

size of particles to model, the variables included in the model are the single properties of phases and the phases composition. Finally, considering the above mentioned, and the real conditions of transport process, a PBSM for the flow of a mineral slurry through a PFA was obtained, in order to use this as the structure of an observer.

- Although the proposed methodology was developed for the transport process of a specific mineral slurry, this is general for other kinds of pulp or slurries, even for the Newtonian fluids. For this, the second proposed option is a more general tool due to it works with the static verification for estimating the viscosity and experimental development is not required. The above because the second structure does not use the experimental characterization for determining equations which describe the viscosity of a fluid, but it uses the available measurements and the assumption of steady state for verifying the mechanical energy balance. In this sense the unique variable is the viscosity, which is the most affected for solids concentration in Newtonian or non-Newtonian fluid.

- In this thesis, the unknown input is a key requirement because using it is possible to expand the state and to find one balance equation for one of the properties to be estimated, the density of the mineral slurry.

- In spite of the known nonlinearity of the proposed model, especially by the terms that relate to the energy losses by friction and the correlations for determining the friction factor, a linear observer was implemented into the estimation structure. The main reason is that the design of nonlinear observers with unknown inputs require a set of restrictions in the model structure which the mechanical energy balance does not comply. Therefore the linearized model was evaluated and for simplicity, this was used for the observer design, from which the state variables density and volumetric flow are estimated. The estimation error indicates that the linearization approach to state estimation gives a good performance.

- The conditions of existence and detectability for the observer design were satisfied by the selection of the state variables, the known and unknown inputs, the output, and the equations considered in the model structure. The structure proposed is the most simple for describing this process, and the main advantage is the use of the disturbances as a tool to construct one model that can be used for designing an observer.

- If it is possible to characterize the mineral slurry for identifying the kind of fluid (Newtonian or non-Newtonian), and determining one expression for calculating the viscosity of fluid based on its behavior, the estimation can be better compared to estimate when the pulp or slurry is unknown. Hence, and in order to determine the function that represents in the best way the viscosity of the fluid, the tests to be developed experimentally must include variations of the major quantity of parameters affecting the viscosity of the fluid. Besides to include the tests relating this property with the state variables in the model.

- Based on obtained results, from an analysis on the frequency domain for the signal of pressure drop, it was possible to classify the origin of the disturbances, and propose criteria which can be implemented on-line for associate the changes in the measured variables with variations in the properties of the mixture. With this information, it is possible to think in some algorithm action to improve the variables estimation.

6.2. Future work

Based on the results obtained with this work, next aspects remain susceptible of improvement or further research.

- Due to the observer implemented is one of the linear kind, it is interesting to explore the transformation of the system using different linear parameter-varying systems (LPV). In this way it will be possible to develop different unknown inputs observers for the different steady states in the process. The general idea is that these observers can be switched depending on the steady state closer to the current values of the state variables, and from this decreases the estimation error.
- For the case of the analysis of the response on the frequency domain of the signal of pressure drop, could be convenient to realize a similar study for the signal of volumetric flow, because for the analysis done last signal remained in a constant value. In addition, and based on the plant data one analysis for the two signal changed could be a good work to complement the separation of density and viscosity effects on the signal of the process.
- The Wavelet transform was used in this thesis because its properties of multiresolution, however, other tools own of the signal digital processing can be implemented and compared with the used tool.

Bibliography

[1] Baha Abulnaga. *Slurry systems handbook*. McGraw-Hill, New York, 2002. ISBN 0071375082.

[2] Ron Darby. *Chemical engineering fluid mechanics*. Marcel Dekker, New York, 2001. ISBN 0824704444.

[3] D. Luenberger. An introduction to observers. *IEEE Transactions on Automatic Control*, 16(6):596–602, Dec 1971. ISSN 0018-9286. doi: 10.1109/TAC.1971.1099826.

[4] M. Farza, H. Hammouri, S. Othman, and K. Busawon. Nonlinear observers for parameter estimation in bioprocesses. *Chemical Engineering Science*, 52(23):4251 – 4267, 1997. ISSN 0009-2509. doi: http://dx.doi.org/10.1016/S0009-2509(97)00190-5. URL `http://www.sciencedirect.com/science/article/pii/S0009250997001905`.

[5] Gabriel F. Aguilera. Simulación de la viscosidad de suspensiones minerales mediante el uso de redes neuronales. Master's thesis, Universidad Nacional de Colombia, 2005.

[6] Julio Martínez and Rosa De Castto. *AnÃ¡lisis de la teoría de ondículas orientada a las aplicaciones en ingeniería eléctrica: Fundamentos*. E.T.S.I Industriales, Dpt. de Ingeniería Eléctrica, 2002.

[7] F.N Shi and T.J Napier-Munn. Effects of slurry rheology on industrial grinding performance. *International Journal of Mineral Processing*, 65(3):125 – 140, 2002. ISSN 0301-7516. doi: http://dx.doi.org/10.1016/S0301-7516(01)00060-6.

[8] Mingzhao He, Yanmin Wang, and Eric Forssberg. Slurry rheology in wet ultrafine grinding of industrial minerals: a review. *Powder Technology*, 147(1):94 – 112, 2004. ISSN 0032-5910. doi: http://dx.doi.org/10.1016/j.powtec.2004.09.032.

[9] R.P. Chhabra and J.F. Richardson. Chapter 2 - rheometry for non newtonian fluids. In R.P. Chhabra and J.F. Richardson, editors, *Non Newtonian Flow and Applied Rheology (Second Edition)*, pages 56 – 109. Butterworth-Heinemann, Oxford, second edition edition, 2008. ISBN 978-0-7506-8532-0. doi: http://dx.doi.org/10.1016/B978-0-7506-8532-0.00002-0.

[10] Daniel Hodouin. Methods for automatic control, observation, and optimization in mineral processing plants. *Journal of Process Control*, 21(2):211 – 225, 2011. ISSN 0959-1524. doi: http://dx.doi.org/10.1016/j.jprocont.2010.10.016. Special Issue on Automation in Mining, Minerals and Metal Processing.

[11] T.J Akroyd and Q.D Nguyen. Continuous rheometry for industrial slurries. *Experimental Thermal and Fluid Science*, 27(5):507 – 514, 2003. ISSN 0894-1777. doi: http://dx.doi.org/10.1016/S0894-1777(02)00264-9. 14th Australasian Fluid Mechanics Conference.

[12] Julia M. Rees. Towards online, continuous monitoring for rheometry of complex fluids. *Advances in Colloid and Interface Science*, 206:294 – 302, 2014. ISSN 0001-8686. doi: http://dx.doi.org/10.1016/j.cis.2013.05.006. Manuel G. Velarde.

[13] Mashuqur Rahman, Ulf Hakansson, and Johan Wiklund. In-line rheological measurements of cement grouts: Effects of water o cement ratio and hydration. *Tunnelling and Underground Space Technology*, 45:34 – 42, 2015. ISSN 0886-7798. doi: http://dx.doi.org/10.1016/j.tust.2014.09.003.

[14] A. Benretem, M. Benidir, and R. Chaib. Factors influencing slurry rheology. *World Pumps*, 2010(7):30 – 32, 2010. ISSN 0262-1762. doi: http://dx.doi.org/10.1016/S0262-1762(10)70199-1.

[15] Héctor Antonio Botero Castro and HernÁn Darío Álvarez Zapata. Una revisiÓn de los mÉtodos mÁs frecuentes para la estimaciÓn del estado en procesos quÍmicos. *DYNA*, 76:135 – 146, 06 2009. ISSN 0012-7353.

[16] J. Oliveira, J.N. Santos, and P. Seleghim Jr. Inverse measurement method for detecting bubbles in a fluidized bed reactor toward the development of an intelligent temperature sensor. *Powder Technology*, 169(3):123 – 135, 2006. ISSN 0032-5910. doi: http://dx.doi.org/10.1016/j.powtec.2006.07.021.

[17] Iván O. Gaitán. Estimación de parámetros reológicos de pulpas minerales a diferentes concentraciones de sólidos. Master's thesis, Universidad Nacional de Ingeniería - Facultad de Ingeniería Civil., 2010.

[18] Leonor Pérez-Trejo, Arturo F. Méndez Sánchez, and A. M. Paniagua Mercado. Determinación de la viscosidad de fluidos newtonianos y no newtonianos (una revisión del viscosímetro de couette). *Latin-American Journal of Physics Education*, 4(1):237 – 245, 2010. ISSN 1870-9095.

[19] Judith A Bamberger and Margaret S Greenwood. Measuring fluid and slurry density and solids concentration non-invasively. *Ultrasonics*, 42(1):563 – 567, 2004. ISSN 0041-624X. doi: http://dx.doi.org/10.1016/j.ultras.2004.01.032. Proceedings of Ultrasonics International 2003.

[20] Augustine B. Makokha and Michael H. Moys. Multivariate approach to on line prediction of in mill slurry density and ball load volume based on direct ball and slurry sensor data. *Minerals Engineering*, 26:13 – 23, 2012. ISSN 0892-6875. doi: http://dx.doi.org/10.1016/j.mineng.2011.10.009.

[21] Bernard P. Zeigler, Tag Gon Kim, and Herbert Praehofer. *Theory of Modeling and Simulation*. Academic Press, Inc., Orlando, FL, USA, 2nd edition, 2000. ISBN 0127784551.

[22] Hernán Alvarez. *Introducción al diseño simultáneo de proceso y control, La controlabilidad de estado como restricción*. EAE Editorial Académica Española, España, 1st edition, 2012.

[23] H. Álvarez, R. Lamanna, and S. Revollar. Metodología para la obtención de Modelos Semifísicos de Base Fenomenológica Aplicada a una Sulfitadora de Jugo de Caña de Azúcar. *Revista Iberoamericana de automática e información industrial*, 6(3):10–20, 2009.

[24] E. Hoyos, D. López, and H. Alvarez. A phenomenologically based material flow model for friction stir welding. *Materials & Design*, 111:321 – 330, 2016. ISSN 0264-1275. doi:

http://dx.doi.org/10.1016/j.matdes.2016.09.009. URL http://www.sciencedirect.com/science/article/pii/S0264127516311698.

[25] R Bird. *Transport phenomena*. J. Wiley, New York, 2007. ISBN 0470115394.

[26] Bernard J. Schorle, Stuart W. Churchill, and Mordechal Shacham. Comments on: An explicit equation for friction factor in pipe. *Ind. Eng. Chem. Fund*, 19(2):228–228, may 1980. doi: 10.1021/i160074a019. URL http://dx.doi.org/10.1021/i160074a019.

[27] William B. Hooper. The two k method predics head losses in pipe and fittings. *Chem Eng*, pages 96–100, August 1981.

[28] Victor Streeter. *Fluid mechanics*. McGraw Hill, Boston, 1971. ISBN 0-07-115600-3.

[29] Howard Barnes. *An introduction to rheology*. Elsevier Distributors for the U.S. and Canada, Elsevier Science Pub. Co, Amsterdam New York, 1989. ISBN 0-444-87140-3.

[30] Juan P. Hernández-Ortiz. *Boundary Integral Equations for Viscous Flows - non-Newtonian Behavior and Solid Inclusions*. PhD thesis, University of Wisconsin-Madison, 2004.

[31] Dan Simon. *Optimal state estimation : Kalman, H [infinity] and nonlinear approaches*. Wiley-Interscience, Hoboken, N.J, 2006. ISBN 978-0-471-70858-2.

[32] Vladimir Strejc. *State space theory of discrete linear control*. Wiley, Chichester England New York, 1981. ISBN 0471275948.

[33] Héctor Botero. *Formalismo para la síntesis de sensores virtuales basados en un modelo maestro de base fenomenológica*. PhD thesis, Universidad Nacional de Colombia, 2010.

[34] Denis Dochain. State and parameter estimation in chemical and biochemical processes: a tutorial. *Journal of Process Control*, 13(8):801 – 818, 2003. ISSN 0959-1524. doi: http://dx.doi.org/10.1016/S0959-1524(03)00026-X. URL http://www.sciencedirect.com/science/article/pii/S095915240300026X.

[35] Jarinah Mohd Ali, N. Ha Hoang, M.A. Hussain, and Denis Dochain. Review and classification of recent observers applied in chemical process systems. *Computers and Chemical Engineering*, 76:27 – 41, 2015. ISSN 0098-1354. doi: http://dx.doi.org/10.1016/j.compchemeng.2015.01.019. URL http://www.sciencedirect.com/science/article/pii/S0098135415000216.

[36] R. Kalman. On the general theory of control systems. *IRE Transactions on Automatic Control*, 4(3):110, 1959.

[37] Mohinder Grewal. *Kalman filtering : theory and practice using MATLAB*. Wiley, Hoboken, N.J, 2008. ISBN 978-0470173664.

[38] A. Radke and Zhiqiang Gao. A survey of state and disturbance observers for practitioners. In *2006 American Control Conference*. Institute of Electrical and Electronics Engineers (IEEE), 2006. doi: 10.1109/acc.2006.1657545. URL http://dx.doi.org/10.1109/ACC.2006.1657545.

[39] Haotian Zhang, R. Ayoub, and S. Sundaram. State estimation for linear systems with unknown inputs: Unknown input norm-observers and bibobs stability. In *2015 American Control Conference (ACC)*, pages 4186–4191, July 2015. doi: 10.1109/ACC.2015.7171986.

[40] L. Yu, G. Zheng, and F. J. Bejarano. Observer design for linear systems with commensurate delays. In *Control Conference (CCC), 2015 34th Chinese*, pages 72–76, July 2015. doi: 10.1109/ChiCC.2015.7259616.

[41] J. Tian and S. Ma. Unknown input observer design for one-sided lipschitz nonlinear continuous-time singular markovian jump systems. In *2016 12th World Congress on Intelligent Control and Automation (WCICA)*, pages 1920–1925, June 2016. doi: 10.1109/WCICA.2016.7578260.

[42] X. Liu and Z. Gao. Novel unknown input observer for fault estimation of gas turbine dynamic systems. In *2015 IEEE 13th International Conference on Industrial Informatics (INDIN)*, pages 562–567, July 2015. doi: 10.1109/INDIN.2015.7281795.

[43] Y. Guan and M. Saif. A novel approach to the design of unknown input observers. *IEEE Transactions on Automatic Control*, 36(5):632–635, May 1991. ISSN 0018-9286. doi: 10.1109/9.76372.

[44] Tobias Raff, Florian Lachner, and Frank Allgower. A finite time unknown input observer for linear systems. In *2006 14th Mediterranean Conference on Control and Automation*. Institute of Electrical and Electronics Engineers (IEEE), jun 2006. doi: 10.1109/med.2006.328795. URL `http://dx.doi.org/10.1109/MED.2006.328795`.

[45] Marcin Witczak. *Fault diagnosis and fault-tolerant control strategies for non-linear systems : analytical and soft computing approaches*. Springer, Cham New York, 2014. ISBN 978-3-319-03014-2.

[46] M. Darouach, M. Zasadzinski, and S. J. Xu. Full-order observers for linear systems with unknown inputs. *IEEE Transactions on Automatic Control*, 39(3):606–609, Mar 1994. ISSN 0018-9286. doi: 10.1109/9.280770.

[47] A. M. Pertew, H. J. Marquez, and Q. Zhao. Design of unknown input observers for lipschitz nonlinear systems. In *Proceedings of the 2005, American Control Conference, 2005.*, pages 4198–4203 vol. 6, June 2005. doi: 10.1109/ACC.2005.1470637.

[48] T. Sedighi, A.J. Koshkouei, and K.J. Burnham. Nonlinear unknown input observer design for nonlinear systems. In *UKACC International Conference on CONTROL 2010*. Institution of Engineering and Technology (IET), 2010. doi: 10.1049/ic.2010.0408. URL `http://dx.doi.org/10.1049/ic.2010.0408`.

[49] D. Koenig. Observer design for unknown input nonlinear descriptor systems via convex optimization. *IEEE Transactions on Automatic Control*, 51(6):1047–1052, June 2006. ISSN 0018-9286. doi: 10.1109/TAC.2006.876807.

[50] Jean-Pierre Barbot, Driss Boutat, and Thierry Floquet. An observation algorithm for nonlinear systems with unknown inputs. *Automatica*, 45(8):1970 – 1974, 2009. ISSN 0005-1098. doi: http://dx.doi.org/10.1016/j.automatica.2009.04.009. URL `http://www.sciencedirect.com/science/article/pii/S0005109809002106`.

[51] D. Dochain. State observers for tubular reactors with unknown kinetics. *Journal of Process Control*, 10(2-3):259–268, apr 2000. doi: 10.1016/s0959-1524(99)00020-7. URL `http://dx.doi.org/10.1016/S0959-1524(99)00020-7`.

[52] G Bastin. *On-line estimation and adaptive control of bioreactors*. Elsevier, Amsterdam New York, 1990. ISBN 978-0-444-88430-5.

[53] Dalil Ichalal and Said Mammar. On unknown input observers for LPV systems. *IEEE Transactions on Industrial Electronics*, 62(9):5870–5880, sep 2015. doi: 10.1109/tie.2015.2448055. URL http://dx.doi.org/10.1109/TIE.2015.2448055.

[54] Jeff S. Shamma. *An Overview of LPV Systems*, pages 3–26. Springer US, Boston, MA, 2012. ISBN 978-1-4614-1833-7. doi: 10.1007/978-1-4614-1833-7_1. URL http://dx.doi.org/10.1007/978-1-4614-1833-7_1.

[55] Javad Mohammadpour. *Control of linear parameter varying systems with applications*. Springer, New York, 2012. ISBN 978-1-4614-1832-0.

[56] K. Madlener, B. Frey, and H. K. Ciezki. Generalized reynolds number for non-newtonian fluids. In *Progress in Propulsion Physics*. EDP Sciences, 2009. doi: 10.1051/eucass/200901237. URL http://dx.doi.org/10.1051/eucass/200901237.

[57] Melvin Mooney. Explicit formulas for slip and fluidity. *Journal of Rheology*, 2(2):210, 1931. doi: 10.1122/1.2116364. URL http://dx.doi.org/10.1122/1.2116364.

[58] M. J. Shensa. The discrete wavelet transform: wedding the a trous and mallat algorithms. *IEEE Transactions on Signal Processing*, 40(10):2464–2482, Oct 1992. ISSN 1053-587X. doi: 10.1109/78.157290.

[59] Elkin Flórez, Salvador Cardona, and Lluisa Jordi. Selección de la ventana temporal en la transformada de fourier en tiempos cortos utilizada en el análisis de señales de vibración para determinar planos en las ruedas de un tren. *Revista Facultad de Ingeniería Universidad de Antioquia*, pages 145 – 158, 12 2009. ISSN 0120-6230.

[60] S. G. Mallat. A theory for multiresolution signal decomposition: the wavelet representation. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 11(7):674–693, Jul 1989. ISSN 0162-8828. doi: 10.1109/34.192463.

[61] Yves Meyer. *Wavelets and Operators*, volume 1. Cambridge University Press, 1993. ISBN 9780511623820. Cambridge Books Online.