FERMION MASS HIERARCHY FROM NON UNIVERSAL ABELIAN EXTENSIONS OF THE STANDARD MODEL

Master of Science Thesis

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I do not know what I may appear to the world, but to myself
I seem to have been only like a boy playing on the sea-shore,
and diverting myself in now and then finding a smoother pebble
or a prettier shell than ordinary, whilst the great ocean of truth
lay all undiscovered before me.

— Isaac Newton

Dedicated to my family and my closest friends.

ABSTRACT

The fermion mass hierarchy is addresed from the framework of an abelian extension of the Standard Model $U(1)_X$. By taking into account the cancellation of chiral anomalies, a set of $U(1)_X$ charges is presented with extended scalar and fermionic sectors. The scalar potential is shown, together with the scalar spectrum of the model which includes the respective Goldstone bosons, new physical neutral and charged scalars at TeV scale and the 125 GeV Higgs boson. Then, the mass acquisition in the fermionic sector is studied in detail. The mass matrices present an specific texture called *suppresion square texture* (SST) which suggests the mass hierarchy when they are diagonalized by algebraic and numerical methods. The model turns out to be consistent is consistent at 5σ and 3σ in the quark and lepton sectors, respectively, without unpleasant fine-tuning procedures.

Keywords: Fermion masses, fermion mass hierarchy, extended scalar sector, extended fermionic sector, beyond the Standard Model, abelian extensions.

RESUMEN

La jerarquía de masas de fermiones es abordada desde el marco de una extensión abeliana del Modelo Estándar $U(1)_X$. Teniendo en cuenta la cancelación de anomalías quirales, un conjunto de cargas de U(1) es presentada con sectores escalares y fermiónicos extendidos. Se muestra el potencial escalar junto con el espectro escalar del modelo, el cual incluye los respectivos bosones de Goldstone, escalares físicos cargados y neutros a escala de TeV y el bosón de Higgs de 125 GeV. Después, la adquisición de masas en el sector de fermiones es estudiado en detalle. Las matrices de masa presentan una textura específica llamada *textura de cuadros de supresión*, la cual sugiere la jerarquía de masas cuando son diagonalizadas por métodos tanto algebraicos como numéricos. El modelo resulta ser consistente a 5σ y 3σ en los sectores de quarks y leptones, respectivamente, sin necesidad de usar ajustes finos indeseados.

Palabras clave: Masas de fermiones, jerarquía de masas fermiónicas, sector escalar extendido, sector fermiónico extendido, más allá del Modelo Estándar, extensiones abelianas.

PUBLICATIONS

Some ideas and figures have appeared previously in the following publications:

S. F. Mantilla and R. Martinez. "A U(1) non-universal anomaly-free model with three Higgs doublets and one singlet scalar field." In: arXiv 1704.04869 (2017).
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S. F. Mantilla, R. Martinez, and F. Ochoa. "Neutrino and CP-even Higgs boson masses in a nonuniversal U(1) extension." In: Phys. Rev. D 95 (9 2017), p. 095037.
url: https://link.aps.org/doi/10.1103/PhysRevD.95.095037.
doi: 10.1103 / PhysRevD . 95 . 095037.

S. F. Mantilla, R. Martinez, F. Ochoa and C. F. Sierra. "Diphoton decay for a 750 GeV scalar boson in a $SU(6) \otimes U(1)_X$ model." In: Nuclear Physics B 911 (2016), p. 338.
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Sebastián Felipe Mantilla Serrano

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1

INTRODUCTION

Since the Antiquity there have been many models and ideas about the true nature of matter and its fundamental components. Some ideas such as the atomic theory of Ancient Greece started the road to the comprehension of the smallest pieces of matter. The law of definite proportions of Louis Proust[Pro99], the first atomic theory after John Dalton and Amadeo Avogadro, and the born of modern Chemistry gave the evidence of the discreteness of matter in chemical reactions[Dat05].

In Physics, however, the first acceptable model about the existence of atoms is due to Albert Einstein in his 1905 article about the Brownian Motion[Ein05], whose proposal was the explanation of this random motion because the collisions between water molecules and the pollen grains. Later, in 1908 Jean Perrin validated experimentally this idea and consequently, the discrete nature of matter instead of the continuous hypothesis[Per09].

Despite this successful discovery, it was the beginning of a new branch in Physics. The discovery of subatomic systems such as the electron by J. J. Thompson[F.R97], the atomic nucleus by Ernest Rutherford[Rut11] and the radioactivity by Marie and Pierre Curie[Cur04] in the early 20th century provided new natural phenomena which deserved a new theory of matter and its components. Moreover, the formulation of the new Quantum Theory at that time supplied a new framework to be employed in the search of satisfactory explanations of these discoveries.

One example of these new phenomena lies in the three kinds of radioactive decays:

- Alpha decay: A nucleus ${}^{A}_{Z}X$ transmutes into a new nucleus ${}^{A-2}_{Z-2}X$ through the emission of an α-particle ${}^{4}_{2}He$. An example of this process is the nuclear reaction present in the uranium decay chain

  $$ {}^{238}_{92}U \to {}^{234}_{90}Th + {}^{4}_{2}He. $$

- Beta decay: A nucleus ${}^{A}_{Z}X$ transmutes into a new nucleus ${}^{A-1}_{Z+1}X$ by emitting a β-particle (identified with a fast electron) and after Pauli the electronic antineutrino. The quintessential beta decay is the carbon-14 reaction

  $$ {}^{14}_{6}C \to {}^{14}_{7}N + e + \bar{\nu}_e. $$

- Gamma decay: A heavier nucleus ${}^{A}_{Z}X^*$ transmutes into a lighter ${}^{A}_{Z}X$ through the emission of a high-energy photon. An example of this process is the Technetium-99m decay

  $$ {}^{99m}_{43}Tc \to {}^{99}_{43}Tc + \gamma. $$

Each one of these processes can be identified with the three fundamental forces of Nature: the strong nuclear force, the weak nuclear force and the electromagnetic force, respectively. The electromagnetic and weak interactions are already understood under the unified Electroweak model, and the strong interaction is described by the Quantum Chromodynamics, at least in its perturbative regime. The conjunction of these schemes constitutes the Standard Model of Particle Physics (SM), the current theory of the fundamental nature of matter.

ARE THERE PHYSICS BEYOND THE STANDARD MODEL?

Nowadays, there are some non-explained phenomena by the SM. The nearest is the neutrino mass and its oscillations, but further phenomena as the dark component of the matter in the Universe, or the relation of these interactions with gravity, physical information and entropy gives new theoretical and experimental phenomena to-be-explained in the called *Physics Beyond the SM* (BSM). Some of these new schemes are:

1. Abelian extensions U(1)′: These models introduce new abelian vector bosons. The couplings of this new gauge boson to SM-fermions are determined by a new set of charges in order to forbid unwanted couplings between light fermions and scalar bosons whose VEVs are at GeV scale or above. The mass of the lightest fermions can be obtain through radiative corrections of their propagators or by Majorana masses and see-saw mechanisms in the case of neutrinos[Mar+14b; MMO17].

2. Extended scalar sectors: From the fact that there are no theoretical constraints related to the scalar spectrum, the addition of isospin singlet, doublet or triplet scalar fields is possible. The quintessential theoretical framework is the 2-Higgs-Doublet models (2HDM)[Mar+14a; Mar+15].

3. SU(3)⊗SU(3)⊗U(1) models (331): The enlargement of the weak isospin SU(2) group allows the introduction of right-handed neutrinos among other fields whose interactions with SM fermions are mediated by weak bosons $W_3^{\pm}$ and Z' heavier than the current weak bosons[CMO12; PQ14; CHMO13].

4. Left-Right symmetry: The chiral symmetry between left- and right-handed sectors can be accomplished by the introduction of the gauge group $SU(2)_R$ broken at a higher energy scale than the electroweak. The right-handed neutrino could be set at large energy scale by Majorana masses. This framework provides the preferred Fritzsch ansatz mass matrices giving an explanation to the fermion mass spectrum[Fri05; Fri78; FTY93].

5. Kaluza-Klein Theories (KK): Since the early 20th century the introduction of tiny extra dimensions in the spacetime has given an interesting framework for unifying the fundamental interactions. The KK modes could be observed at particle accelerators providing information about the size of the new space dimensions and hints of quantum gravity. Sometimes, these dimensions are suit at Planck length because the lack of observations of this extra modes[Kal21].

6. Large Extra Dimensions (LED): The Planck scale, considered as the quantum gravity scale, is so far to be testable employing the current technology. The LED

framework introduces extra dimensions with size of millimeter or micrometer increasing the dimensions of space and changing the Newton gravitational constant G. In this way, the new Planck scale could be at units or hundreds of TeV and consequently bringing closer the quantum gravity to feasible experimental corroboration[AHCG01].

7. Randall-Sundrum (RS): The hierarchy problem produced by the enormous distance between the electroweak and gravitational energy scales can be understood in the framework of branes introduced in RS models. The RS-I model proposes the universe has two branes (the electroweak scale and the Planck scale) with the bulk between them which produces the energy hierarchy. On the other hand, the RS-II model introduces only the electroweak scale[Cas+08].

There are more BSM proposals which are not reported here to make short the list. All of these schemes should satisfy the correspondence principle at low energy limit obtaining the SM as an effective theory in order to comply the current experimental constraints. Nevertheless, each one of them has its own traits which could be observed gathering more data from different laboratories and techniques. Moreover, some of these schemes can be obtained as low-energy effective theories, specially the abelian extensions of the SM, the simplest BSM framework and the most falsifiable among all of them.

The present work is devoted to present an abelian extension to the SM to address the hierarchy observed in the SM fermions masses. The part i outline the general framework for constructing abelian extensions. The chapter 2 brushes up the fundamental bases of spacetime symmetries, Yang-Mills scheme and Spontaneous Symmetry Breaking. After this, the chapter 3 reviews the most important properties of the Standard Model and comes out the general tools presented in the previous chapter. By last, to close this part the concept of fermion generation or family is studied in chapter 4 embedded in the issue of mass matrices and mixing angles. Also the charged- and neutral-current interactions involving flavor changing are reviewed.

The part ii presents the construction of an abelian extension of the Standard Model whose aim is to obtain the mass spectrum of the SM fermions. The main proposal of the work is presented in chapter 6, where the fermionic spectrum of the model is shown together with the Yukawa Lagrangians of the quark and lepton sectors and the mass matrices. The concept of *suppression square texture* is introduced, which is the cornerstone of the model. Moreover, in this chapter the mass eigenvalues and mixing angles are obtained, and the mass matrices were generated randomly by Montecarlo procedures and diagonalized numerically in order to check the actual suitability of the model to reproduce phenomenological data[PG+16].

Finally, the part iii closes the work with the chapter 7, where the results are discussed in the light of other abelian extensions.

Part I

GENERAL FRAMEWORK

This part is devoted to review basic concepts and to establish the notation before studying the new model, so this could be skipped without any problem.

The fundamental concepts related to spacetime and internal symmetries, as well as spontaneous symmetry breaking are reviewed in the chapter 2. In chapter 3, it is presented also the Standard Model of Particle Physics, the current theory about matter and fundamental interactions with experimental corroboration. Finally, the concept of particle family is introduced in chapter 4 with the mass matrices, and their corresponding rotation matrices are studied in order to understand the charged and neutral flavor-changing-currents.

2 THEORETICAL BACKGROUND

Field theories are constructed using different physical ideas with their corresponding mathematical tools. The symmetries of space and time describe the arena where fields propagate and interact. Its analysis gives suited classifications of the fields according to their mass and spin. Along with the rotations and translations described by the Poincaré group, there are three important discrete transformations: the space inversion or parity $(t, \mathbf{x}) \to (t, -\mathbf{x})$, the time reversal $(t, \mathbf{x}) \to (-t, \mathbf{x})$ and the charge conjugation. These transformations are so important for describing chiral fermions or asymmetries between matter and antimatter (CP violation). Moreover, the difference between Dirac and Majorana fermions are important ir order to study neutrino mass generation.

However, the Poincaré group does not predict any kind of interaction between the fields described by its representations in disagree with the existence of three fundamental interactions. This problem is solved by the Yang-Mills scheme (YM). It begins by proposing a global continuous symmetry in a set of fields described by a Lie group. When the continuous symmetry becomes local it is necessary to correct the derivative operator by adding a connection term which plays the role of the potential or gauge boson of the interaction whose gauge symmetry is determined by the Lie group. The electromagnetic, weak and strong interactions can be described in this scheme by the special unitary Lie groups U(1), SU(2) $\otimes$ U(1) and SU(3), respectively.

Although the successful description of electromagnetic and strong nuclear interactions in the YM scheme, the weakness of the weak interaction cannot be totally understood in this frame. Moreover, the existence of a definite energy scale given by the Fermi constant creates the necessity of another scheme. The introduction of scalar fields which develop a vacuum-expectation-value (VEV) with a smaller Lie group as its symmetry brings the spontaneous breaking of the original symmetry (SSB). An important consequence of this process is the acquisition of masses by some gauge bosons at the energy scale of the VEV. In effect, this scheme has been proven experimentally with the discoveries of the weak bosons $W^{\pm}$[Arn+83a] and Z[Arn+83b], but its final and definite corroboration was the detection of the Higgs boson[Aad+12].

Because this large set of methods and physical concepts, this chapter is devoted to do a quick review about them. The section 2.1 presents the Lie algebra associated to the Poincaré group and the set of representations with phenomenological interest. The section 2.2 brushes up the Yang-Mills scheme in constructing field theories with gauge symmetries based on Lie groups. Finally, the section 2.3 reviews a general scheme of spontaneous symmetry breaking on special unitary groups SU(N).

POINCARÉ GROUP AND LORENTZ INVARIANTS

Since the researches done by Galileo and Isaac Newton, it is known that Nature has symmetries. The homogeneity and isotropy of space described by Newton in his

Principia can be described by the Lie group composed by the semidirect product of the 3D translations $\mathbf{R}^3$ with the special orthogonal group SO(3) whose elements are the 3D rotations, known as the 3D Euclidean group or inhomogeneous SO(3) group[Gil12]

$$\mathrm{ISO}(3) = \mathbf{R}^3 \rtimes \mathrm{SO}(3). \tag{2.1}$$

On the other hand, the Galilean relativity principle proposes the invariance under 3D boosts or changes between inertial frames. Finally, there is the invariance under time translations finishing with a ten-parameter group known as the Galilean group.

The current theory of space and time is the scheme proposed by Albert Einstein, where the 3D translations is unified with the time translation as 4D translations $\mathbf{R}^{3,1}$, while 3D boosts with 3D rotations are unified in the 4D rotations with six parameters described by the Lorentz group SO(3,1). The resulting group is the Poincaré group or inhomogeneous Lorentz group

$$\mathrm{ISO}(3,1) = \mathbf{R}^{3,1} \rtimes \mathrm{SO}(3,1). \tag{2.2}$$

The Poincaré group ISO(3,1) has the associated Lie algebra $\mathfrak{iso}(3,1)$ composed by the four generators of translations P_μ and the six generators of rotations $L_{\mu\nu} = -L_{\nu\mu}$ where $\mu, \nu = 0, 1, 2, 3$. The Poincaré algebra is[Kak93]

$$\begin{aligned}
[P_\mu, P_\nu] &= 0, \\
[J_{\mu\nu}, P_\rho] &= -i\left(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu\right), \\
[J_{\mu\nu}, J_{\rho\sigma}] &= -i\left(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\sigma}J_{\mu\rho}\right)
\end{aligned} \tag{2.3}$$

where $[\eta_{\mu\nu}] = [\eta^{\mu\nu}] = \mathrm{diag}\left(+,-,-,-\right)$ is the Minkowski metric which distinguishes between time $(+)$ and space $(-)$. The explicit form of the Poincaré generators depends on the representation where the transformation acts, but there is an orbital representation which involves the spacetime coordinates and their derivative operators

$$\begin{aligned}
P_\mu &= i\partial_\mu, \\
L_{\mu\nu} &= i\left(x_\mu\partial_\nu - x_\nu\partial_\mu\right).
\end{aligned} \tag{2.4}$$

The translation generator is the same for all representations, and in the same way the orbital representation of the rotation generators. However, $J_{\mu\nu}$ can be split into its orbital part $L_{\mu\nu}$ and its spin or intrinsic part $S_{\mu\nu}$ which depends on the representation where it acts.

The Poincaré algebra can also be expressed in terms of 3D rotation scalars and vectors (3-vectors). The generators of 3D boosts and rotations are, respectively,

$$K^k = -\frac{1}{2}\epsilon^{0jk}J_{0j}, \qquad J^k = \frac{1}{2}\epsilon^{ijk}J_{ij}, \tag{2.5}$$

while the translations split them in P^0 time translation and P^i space translations. The Lie algebra turns out to be

$$\begin{aligned}
\left[J^i, J^j\right] &= +i\epsilon^{ij}{}_k J^k, & \left[K^i, K^j\right] &= -i\epsilon^{ij}{}_k J^k, \\
\left[J^i, K^j\right] &= +i\epsilon^{ij}{}_k K^k, & \left[K^i, P^j\right] &= +i\delta^{ij} P^0, \\
\left[J^i, P^j\right] &= +i\epsilon^{ij}{}_k K^k, & \left[K^i, P^0\right] &= +iP^i, \\
\left[J^i, P^0\right] &= 0, & \left[P^i, P^0\right] &= 0.
\end{aligned} \tag{2.6}$$

Poincaré representations

The different forms of $S_{\mu\nu}$ can be obtained if the Poincaré representations are determined. They can be labeled employing the Casimir operators of the Poincaré algebra: $P^2 = P_\mu P^\mu = \eta^{\mu\nu}P_\mu P_\nu$ and $W^2 = W_\mu W^\mu$ where the latter is the Pauli-Lubanski pseudovector[Ram97]

$$W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}P_\nu J_{\rho\sigma} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu S^{\rho\sigma} \tag{2.7}$$

whose orbital part vanishes because the antisymmetric symbol $\epsilon_{\mu\nu\rho\sigma}$. Their corresponding eigenvalues are

$$P^2 = m^2, \qquad W^2 = m^2 s(s+1) \tag{2.8}$$

where m is the proper mass of the field in which the Casimir operator acts and s is its spin. There are two phenomenologically interesting cases depending on the m^2 eigenvalue:

- Massive case: When a massive field is on rest its 4-momentum or translation operator takes the form[Ryd96]

$$P_\mu = (m, 0, 0, 0), \tag{2.9}$$

in order to obtain the correct eigenvalue equation $P^2 = m^2$. Hence, its Pauli-Lubanski pseudovector results with the components

$$W^0 = 0, \qquad \mathbf{W} = -m\mathbf{J}, \tag{2.10}$$

where $\mathbf{J}$ are 3-vectors which generate the little group SO(3) of 3D rotations with algebra $\mathfrak{so}(3) = \mathfrak{su}(2)$ (see the first commutator of (2.6)), so the fields with mass m can be classified by its spin ($0, \frac{1}{2}, 1$, etc.).

- Massless case: If the field is massless its 4-momentum must be[Ryd96]

$$P^\mu = (\omega, 0, 0, \omega), \tag{2.11}$$

obtaining $P^2 = 0$. ω represents the frequency (energy) of the massless field. In this conditions the Pauli-Lubanski pseudovector has the components

$$\begin{aligned} W^0 &= -\omega \mathbf{P}\cdot\mathbf{S}, \qquad & W^1 &= -\omega\left(L^1 + K^2\right), \\ W^3 &= -\omega \mathbf{P}\cdot\mathbf{S}, \qquad & W^2 &= -\omega\left(L^2 - K^1\right). \end{aligned} \tag{2.12}$$

Their commutators are

$$\begin{aligned} \left[W^1, W^2\right] &= 0, \\ \left[W^2, W^3\right] &= -i\omega W^1, \\ \left[W^3, W^1\right] &= -i\omega W^2, \end{aligned} \tag{2.13}$$

and consequently the Lie algebra of the 2D Euclidean group ISO(2) is obtained, so it is the little group for the massless representations. The suited eigenvalue is the helicity operator

$$h = \frac{\mathbf{P}\cdot\mathbf{S}}{|\mathbf{P}|}, \tag{2.14}$$

which could be ± 1. Consequently, the massless representation has only two representations labeled by positive or negative helicity.

Now, inside the massive representations there are SO(3) representations. However, they can be studied in a better way employing the homeomorphism between the Lie algebras $\mathfrak{so}(3,1) \sim \mathfrak{su}(2) \otimes \mathfrak{su}(2)$. Moreover, since $\mathfrak{su}(2) = \mathfrak{so}(3)$ the massive representations of the Lorentz group can be labeled by two numbers corresponding to their spin. The phenomenologically interesting representations are[Ram97]:

- $(\mathbf{0}, \mathbf{0})$: The null spin number in both groups leads to the interpretation of this representation as the scalar fields. In this representation $S_{\mu\nu} = 0$. Its evolution is determined by the Klein-Gordon equation

$$\left(\Box + m^2\right) \Psi = 0, \tag{2.15}$$

obtained by optimizing the Klein-Gordon action

$$S_{KG} = \int \frac{1}{2} \left(\partial_\mu \Phi \partial^\mu \Phi + m^2 \Phi^2\right) d^4x, \tag{2.16}$$

and its corresponding propagator in momentum space is[Mar02]

$$\tilde{G}_{KG}(p) = i \frac{1}{p^2 - m^2 + i\epsilon}. \tag{2.17}$$

- $(\frac{1}{2}, \mathbf{0})$: The representation carries one-half spin in the first SU(2), so that, it represents a chiral or Weyl fermion. For convention these fermions are called left-handed. Because the presence of spin, the intrinsic generators are not zero but $S_{\mu\nu} = \frac{i}{4}\left[\sigma_\mu, \sigma_\nu\right]$ where $\sigma^\mu = \left(1, \sigma^1, \sigma^2, \sigma^3\right)$ and the latter three components are the Pauli matrices.

- $(\mathbf{0}, \frac{1}{2})$: Concomitantly with the left-handed representation, the one-half spin in the second SU(2) leads to the right-handed fermions. The corresponding intrinsic rotation generators are $S_{\mu\nu} = \frac{i}{4}\left[\tilde{\sigma}_\mu, \tilde{\sigma}_\nu\right]$ where the tilde σ_μ are $\tilde{\sigma}^\mu = \left(1, -\sigma^1, -\sigma^2, -\sigma^3\right)$.

- $(\frac{1}{2}, \mathbf{0}) \oplus (\mathbf{0}, \frac{1}{2})$: The direct-sum of the two previous representations yields the Dirac or Majorana fermion $\Psi = \left(\psi_L, \psi_R\right)^T$, depending on its behavior under charge conjugation. In this representation σ^μ and $\tilde{\sigma}^\mu$ are joint together in the γ^μ matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0 \end{pmatrix}, \tag{2.18}$$

and consequently $S_{\mu\nu} = \frac{i}{4}\left[\gamma_\mu, \gamma_\nu\right]$. Its motion is dictated by the Dirac equation

$$\left(i\gamma^\mu \partial_\mu - m\right) \psi = \left(i\not{\partial} - m\right) \psi = 0, \tag{2.19}$$

obtained from the Dirac action

$$S_{Dirac} = \int \overline{\psi} \left(i\gamma^\mu \partial_\mu - m\right) \psi \, d^4x, \tag{2.20}$$

where $\overline{\psi} = \psi^\dagger \gamma^0$ is the Dirac adjoint of ψ and its corresponding propagator in momentum space is

$$\tilde{G}_{\text{Dirac}}(p) = i\frac{1}{\not{p} - m + i\epsilon} = i\frac{\not{p} + m}{p^2 - m^2 + i\epsilon}. \quad (2.21)$$

It is remarkable that chiral or Weyl fermions are massless because the mass term mixes left- and right-handed chiralities. In the case of $m = 0$ the Dirac equation and action decouples in two independent terms of ψ_L and ψ_R defined by the parity operators

$$\psi_L = \frac{1-\gamma^5}{2}\psi, \qquad \psi_R = \frac{1+\gamma^5}{2}\psi, \quad (2.22)$$

where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ is the chiral matrix, and the Dirac Lagrangian becomes in the Weyl Lagrangians

$$S_{\text{Weyl}} = \int \overline{\psi_L}\,(i\sigma^\mu \partial_\mu)\,\psi_L\, d^4x + \int \overline{\psi_R}\,(i\tilde{\sigma}^\mu \partial_\mu)\,\psi_R\, d^4x, \quad (2.23)$$

- $(\frac{1}{2}, \frac{1}{2})$: Finally, the existence of one-half spin in both SU(2) results in the vector representation which can be represented by two spin indices or one vector index $A_\mu = (\sigma_\mu)^{\alpha\dot{\alpha}} A_{\alpha\dot{\alpha}}$. Its spin generators can be expressed by

$$(S_{\mu\nu})^\rho{}_\sigma = i\left(\delta^\rho_\mu \eta_{\nu\sigma} - \delta^\rho_\nu \eta_{\mu\sigma}\right). \quad (2.24)$$

The motion of vector fields is determined by the Maxwell-Proca equation[Gre90]

$$\partial_\mu F^{\mu\nu} + m^2 A^\nu = 0, \quad (2.25)$$

obtained from the Maxwell-Proca action

$$S_{\text{MP}} = \int \bar{\psi}\left(F_{\mu\nu}F^{\mu\nu} - m^2 A_\mu A^\mu\right)\psi d^4x, \quad (2.26)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the exterior derivative of A_μ. In the next section these entities will be interpreted as the strength field tensor and the field potential, respectively. Its corresponding propagator in momentum space using the unitary gauge is

$$\tilde{G}^{\mu\nu}_{\text{MP}}(p) = i\frac{-g^{\mu\nu} + \frac{p^\mu p^\nu}{m^2}}{p^2 - m^2 + i\epsilon}. \quad (2.27)$$

YANG-MILLS FIELDS

In the same way as Galileo, Newton and Einstein proposed relativity principles which can be associated to Lie groups as the Galilean or Poincaré group, the researches done by Faraday, Maxwell and several physicists provided the electromagnetic theory. Among its different characteristics, it was the first special relativistic theory, even before the relativistic mechanics, but their main trait is its invariance under gauges of the electromagnetic scalar and vector potentials. The Maxwell equations in their covariant fashion are[Bar64]

$$\begin{aligned} \partial_\mu F^{\mu\nu} &= j^\nu \\ \partial_\mu \star F^{\mu\nu} &= 0 \end{aligned} \tag{2.28}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the strength field tensor, the exterior derivative of the electromagnetic potential A_μ, the fundamental field in the theory; $\star F_{\mu\nu}$ is the Hodge-dual of the strength tensor; and j^μ is the source of electromagnetic field, the electric current density.

When the electromagnetic potential (from now on gauge potential or gauge field) is shifted in the following way

$$A_\mu \to A_\mu - \partial_\mu \chi \tag{2.29}$$

the Maxwell equations remain unchanged. This property is called the *gauge invariance* of the electromagnetic theory, the simplest example of gauge theories.

This method could be extended to more complex fields whose gauge symmetry is described by Lie groups. Even the electromagnetic theory has its own gauge group, the unitary transformations in one dimension U(1), the simplest unitary group. For larger groups, the field is described by a multiplet, one of the representations of the Lie group. Without lose of generality the vector representation can be used for constructing the scheme. The multiplet is represented by a column vector[Mar02]

$$\Psi(x) = \begin{pmatrix} \psi^1(x) & \cdots & \psi^N(x) \end{pmatrix}^T, \tag{2.30}$$

which transforms under a gauge transformation in the following way

$$\Psi(x) \to \Psi'(x) = \mathbb{U}\Psi(x), \tag{2.31}$$

where $\mathbb{U}$ is an element of the SU(N) gauge group labeled by $N^2 - 1$ continuous parameters θ^α. It can be written using the exponential map

$$\mathbf{U}(x,\theta) = \exp\left(ig\theta^\alpha \mathbf{G}_\alpha\right) \approx \mathbf{1} - ig\theta^\alpha \mathbf{G}_\alpha + \cdots . \tag{2.32}$$

The elements $\mathbf{G}_\alpha$ are $N \times N$ matrices which span the Lie algebra $\mathfrak{su}(N)$

$$\left[\mathbf{G}_\alpha, \mathbf{G}_\beta\right] = i f^\gamma{}_{\alpha\beta} \mathbf{G}_\gamma, \tag{2.33}$$

which is determined by the structure constants $f^\gamma{}_{\alpha\beta}$.

This kind of gauge transformations does not affect the derivatives of Ψ because they are global, act in the same way at every point in spacetime. If the transformation, however, is performed locally, i.e., when the parameters depend on the spacetime coordinates $\theta^\alpha = \theta^\alpha(x)$ it is mandatory to include an affine connection $\mathbf{A}_\mu = A^\alpha_\mu \mathbf{G}_\alpha$ in order to keep the invariance of the derivative of Ψ. In this way it is obtained the covariant derivative of the gauge group[PS95]

$$D_\mu \Psi = \partial_\mu \Psi - ig\mathbf{A}_\mu \Psi = \partial_\mu \Psi - igA^\alpha_\mu \mathbf{G}_\alpha \Psi, \tag{2.34}$$

where the affine connection, also called the gauge potential, transforms in such a way that it absorbs the derivatives of the group parameters

$$A^{\alpha}_{\mu} \to A^{\alpha}_{\mu} + \frac{1}{g}\partial_{\mu}\theta^{\alpha} + f^{\alpha}{}_{\beta\gamma}A^{\beta}\theta^{\gamma}. \tag{2.35}$$

The affine connection (gauge potential) also has its associated curvature tensor or strength field tensor, which is obtained trough the self-commutator of the covariant derivative

$$\begin{aligned} [D_{\mu}, D_{\nu}] &= -ig\mathbf{F}_{\mu\nu}\Psi = -igF^{\alpha}_{\mu\nu}\mathbf{G}_{\alpha}\Psi, \\ F^{\alpha}_{\mu\nu} &= \partial_{\mu}A^{\alpha}_{\nu} - \partial_{\nu}A^{\alpha}_{\mu} + gf^{\alpha}{}_{\beta\gamma}A^{\beta}_{\mu}A^{\gamma}_{\nu}. \end{aligned} \tag{2.36}$$

This field could also be interpreted as the covariant exterior derivative of the affine connection. $F^{\alpha}_{\mu\nu}$ contains the electric-like and magnetic-like fields in analogy to electrodynamics, but in this case there are $N^2 - 1$ fields for each kind. Moreover, in order to give its own dynamics to the gauge potentials, $F^{\alpha}_{\mu\nu}$ is suited for constructing its corresponding kinetic term. In this way, the gauge field Lagrangian is

$$\mathcal{L}_{\mathrm{YM}} = \mathrm{Tr}\left(\mathbf{F}^{\mu\nu}\mathbf{F}_{\mu\nu}\right). \tag{2.37}$$

The interaction term depends on the Lorentz representation in which Ψ belongs. If Ψ is a Lorentz scalar field Φ, its coupling with the gauge fields is introduced by replacing each one of the coordinate derivatives ∂_{μ} by its gauge covariant version D_{μ} obtaining

$$\begin{aligned} \mathcal{L}_{\mathrm{KG,YM}} &= \frac{1}{2}D_{\mu}\Phi^{\dagger}D^{\mu}\Phi + \frac{m^2}{2}\Phi^2 = \frac{1}{2}\partial_{\mu}\Phi^{\dagger}\partial^{\mu}\Phi + \frac{m^2}{2}\Phi^2 \\ &+ \frac{ig}{2}\left(\Phi^{\dagger}\overleftrightarrow{\partial}^{\mu}\mathbf{G}_{\alpha}\Phi\right)A^{\alpha}_{\mu} + \frac{g^2}{2}\Phi^{\dagger}\mathbf{A}_{\mu}\mathbf{A}^{\mu}\Phi, \end{aligned} \tag{2.38}$$

where $A\overleftrightarrow{\partial}^{\mu}B = A(\partial^{\mu}B) - (\partial^{\mu}A)B$, but if Ψ is a Dirac field, the new Dirac covariant Lagrangian is

$$\mathcal{L}_{\mathrm{Dirac,YM}} = \bar{\Psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\Psi = \bar{\Psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\Psi + g\bar{\Psi}\gamma^{\mu}\mathbf{G}_{\alpha}\Psi A^{\alpha}_{\mu}. \tag{2.39}$$

In this way, the total covariant Lagrangian can be expressed in terms of the gauge kinetic term, the free KG and Dirac Lagrangians and the interaction terms $J^{\mu}_{\alpha}A^{\alpha}_{\mu}$ where J^{μ}_{α} are the gauge current densities, sources of the gauge fields. It receives contributions from scalar and fermion fields

$$J^{\mu}_{\alpha} = \frac{ig}{2}\left(\Phi^{\dagger}\overleftrightarrow{\partial}^{\mu}\mathbf{G}_{\alpha}\Phi\right) + g\bar{\Psi}\gamma^{\mu}\mathbf{G}_{\alpha}\Psi. \tag{2.40}$$

Note that although any mass term quadratic in $\mathbf{A}_{\mu}$ is forbidden because it breaks the gauge symmetry, the covariant scalar Lagrangian contains a quadratic term and so the gauge field acquires an effective mass depending on the magnitude of the scalar field. The consequences of this term when the scalar field does not vanishes in spacetime are reviewed in the next section.

SPONTANEOUS SYMMETRY BREAKING

The previous two sections were devoted for reviewing Poincaré and gauge symmetries, as well as some of their physical consequences as the spin classification of the fields and the existence of gauge fields spanning on the Lie algebra of the local internal symmetries. Nevertheless, the weakness of the weak nuclear interaction and the existence of a characteristic energy scale in the Fermi constant $\sqrt{2}G_F = (246\,\text{GeV})^{-2}$ suggest that there is an additional mechanism besides the symmetries: the symmetry breaking.

There are different examples of symmetry breaking in Physics. One of the most important is superconductivity[PS95]. Above the critical temperature T_c the electrons inside the material have the gauge symmetry U(1) of the electromagnetic interaction, but below T_c the electrons reconfigure into Cooper pairs described by a scalar field ϕ which develop a background non-zero vacuum energy $\langle\phi\rangle_0 \neq 0$ bringing the consequence of the emergence of a non-vanishing photon mass because the U(1) symmetry breaking inside the material. In effect, this scheme is adequate to describe how some gauge symmetry reduces to a smaller one.

General scheme of SSB

The general treatment with non-abelian groups is done as follows. The physical system is constituted by $\Phi(x) = [\phi^a(x)]$, a multiplet of N scalar bosons lying in the vector representation of the gauge group and the associated gauge fields A^α_μ. Their Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F^\alpha{}_{\mu\nu}F_\alpha{}^{\mu\nu} + \frac{1}{2}\left(D_\mu\Phi\right)^\dagger\left(D^\mu\Phi\right) - V(\Phi^\dagger\Phi), \tag{2.41}$$

where $D_\mu\Phi = \partial_\mu\Phi - igA^\alpha_\mu\mathbf{G}_\alpha\Phi$ and $V(\Phi^\dagger\Phi)$ is the Higgs potential[Mar02]

$$V(\Phi^\dagger\Phi) = \mu^2\Phi^\dagger\Phi + \frac{\lambda}{2}\left(\Phi^\dagger\Phi\right)^2, \tag{2.42}$$

in which $\lambda > 0$ in order to keep the potential bounded from below. These parameters determine some scalar field configurations Φ_0 at which the potential is minimized.

The optimized field configurations can be calculated by differentiating the Higgs potential respect to $\Phi^\dagger = [\phi_a]$ ($\Phi^\dagger\Phi = \phi_a\phi^a$)

$$\frac{\partial V}{\partial\phi_a} = \mu^2\phi^a + \lambda\left(\Phi^\dagger\Phi\right)\phi^a = 0. \tag{2.43}$$

This equation has two solutions depending on the sign of μ^2:

$$\begin{aligned}\left\langle\Phi^\dagger\Phi\right\rangle_0 &= 0 \qquad , \qquad \mu^2 > 0,\\ \left\langle\Phi^\dagger\Phi\right\rangle_0 &= -\frac{\mu^2}{\lambda} \qquad , \qquad \mu^2 < 0.\end{aligned} \tag{2.44}$$

The latter corresponds to a field configuration with non-vanishing VEV $\langle\Phi\rangle_0 = \Phi_0$. However, only one of these infinite configurations can be chosen, triggering the SSB mechanism. Meanwhile, the scalar field is redefined to describe field excitations over VEV

$$\Phi(x) \to \Phi(x) + \Phi_0, \tag{2.45}$$

where the new $\Phi(x)$ has no VEV and acts as small oscillations about Φ_0. Consequently, the Lagrangian becomes

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{4} F^{\alpha}{}_{\mu\nu} F_{\alpha}{}^{\mu\nu} - V(\Phi^\dagger\Phi) + \frac{1}{2}\left(\partial_\mu\Phi\right)^\dagger\left(\partial^\mu\Phi\right) \\
&+ \frac{ig}{2}\left(\Phi^\dagger \mathbf{G}_\alpha \overset{\leftrightarrow}{\partial}_\mu \Phi\right) A^{\alpha\mu} + \frac{ig}{2}\Phi_0^\dagger \mathbf{G}_\alpha A^\alpha_\mu \partial^\mu \Phi \\
&+ \frac{g^2}{2}\left(\Phi^\dagger \mathbf{G}_\alpha \mathbf{G}_\beta \Phi_0 + \Phi_0^\dagger \mathbf{G}_\alpha \mathbf{G}_\beta \Phi\right) A^\alpha_\mu A^{\beta\mu} \\
&- \frac{ig}{2}\partial_\mu \Phi^\dagger A^{\alpha\mu} \mathbf{G}_\alpha \Phi_0 + \frac{g^2}{2}\Phi_0^\dagger \mathbf{G}_\alpha \mathbf{G}_\beta \Phi_0 A^\alpha_\mu A^{\beta\mu}.
\end{aligned} \tag{2.46}$$

When the VEV is activated, the last term of the Lagrangian corresponds to an effective mass term associated to gauge fields

$$M^2_{\alpha\beta} = \frac{g^2}{2}\Phi_0^\dagger \mathbf{G}_\alpha \mathbf{G}_\beta \Phi_0, \tag{2.47}$$

which has vanishing and non-vanishing eigenvalues determining the new vector mass spectrum. If the vacuum does not remain invariant under the action of some generator, it is said that is a *broken generator*, and there will be a massless Golstone boson with an associated massive gauge boson. On the contrary, if the vacuum remains invariant, it is said that is a *non-broken generator*, and there will be a massive Higgs boson with a massless gauge boson.

- Broken generator:

$$(\mathbf{G}_\alpha)^a_b \left(\Phi_0\right)^b \neq 0 \quad \to \quad m_{\Phi^a} = 0 \quad , \quad m_{A^\alpha} \neq 0 \tag{2.48}$$

- Non-broken generator:

$$(\mathbf{G}_\alpha)^a_b \left(\Phi_0\right)^b = 0 \quad \to \quad m_{\Phi^a} \neq 0 \quad , \quad m_{A^\alpha} = 0 \tag{2.49}$$

Finally, the number of broken generators fix the number of new massive gauge fields and Goldstone bosons[Gol61; GSW62], and from the original number of generators is fixed the number of Higgs bosons and massless gauge fields which span the *remnant symmetry*.

Goldstone theorem

The last procedure associated to SSB scheme is to obtain the mass matrix for scalar fields, which is done employing the *Goldstone theorem*. It begins expanding the Higgs potential up to second order in the scalar fields[Mar02]

$$V(\Phi) = V(\Phi_0) + \frac{1}{2}\left(\frac{\partial^2 V}{\partial\phi^a \partial\phi^b}\right)_{\Phi_0} (\phi^a - \phi^a_0)(\phi^b - \phi^b_0) + \cdots \tag{2.50}$$

where the second term is interpreted as the effective mass matrix:

$$M_{ab}^2 = \left(\frac{\partial^2 V}{\partial\phi^a \partial\phi^b}\right)_{\Phi_0} \tag{2.51}$$

whose diagonal elements are not negative because prior minimizing potential.

To evaluate how each group generator acts on the VEV, the scalar fields are phase-shifted under the gauge group

$$\phi^a \to \phi^a + \delta\phi^a, \tag{2.52}$$

where

$$\delta\phi^a = \frac{\partial\phi^a}{\partial\theta^\alpha}\delta\theta^\alpha = i\delta\theta^\alpha\left((\mathbf{G}_\alpha)^a_{\ b}\,\phi^b\right), \tag{2.53}$$

which results in some variation on the potential

$$V(\Phi) \to V(\Phi) + \delta V(\Phi) \tag{2.54}$$

$$\delta V(\Phi) = \frac{\partial V}{\partial\phi^a}\delta\phi^a = i\delta\theta^\alpha \frac{\partial V}{\partial\phi^a}\left((\mathbf{G}_\alpha)^a_{\ b}\,\phi^b\right) \tag{2.55}$$

Now, differentiating δV respect to ϕ^c

$$\begin{aligned}\frac{\partial}{\partial\phi^c}\delta V = \delta\theta^\alpha \frac{\partial V}{\partial\phi^a}\frac{\partial}{\partial\phi^c}\left((\mathbf{G}_\alpha)^a_{\ b}\,\phi^b\right) \\ + \delta\theta^\alpha \frac{\partial^2 V}{\partial\phi^a\partial\phi^c}\left((\mathbf{G}_\alpha)^a_{\ b}\,\phi^b\right)\end{aligned} \tag{2.56}$$

and setting $\Phi = \Phi_0$ the optimized potential gives

$$0 = \left(\frac{\partial^2 V}{\partial\phi^a\partial\phi^c}\right)_{\Phi_0}\left((\mathbf{G}_\alpha)^a_{\ b}\,\phi^b_0\right) = M^2_{ac}\left((\mathbf{G}_\alpha)^a_{\ b}\,\phi^b_0\right) \tag{2.57}$$

This condition gives two different options. In the one hand, if the generator annihilates the VEV, its symmetry remains intact and the scalar field ϕ^b acquires mass becoming into a Higgs boson. On the other hand, if the generator is broken, the scalar field ϕ^b remains massless as a Goldstone boson and the unbroken generators span a new smaller symmetry from the remnant gauge group with the massless gauge bosons.

Fermion masses

The SSB is not only useful for explaining the gauge boson masses, but also the fermion masses. The Weyl Lagrangian

$$\mathcal{L}_{\text{Weyl}} = \overline{\psi}_L \not{\partial}\psi_L + \overline{\psi}_R \not{\partial}\psi_R, \tag{2.58}$$

describe massless fermions with definite helicity, i.e., the left- and right-handed components evolve independently. In the case of massive fermions, the Weyl Lagrangian includes the Lorentz invariant bilinear forms $\overline{\psi}_L \psi_R$ and $\overline{\psi}_R \psi_L$ such that there appear mass terms

$$\overline{\psi}_L \not{\partial} \psi_L + \overline{\psi}_R \not{\partial} \psi_R - m(\overline{\psi}_L \psi_R + \text{h.c.}). \tag{2.59}$$

In effect, the mass term couples fermions of different chiralities. However, chiral gauge theories (as SM) which describes different interactions between left- and right-handed fermions does not admit this term because it explicitly breaks the gauge symmetry.

In order to solve this drawback, instead of adding explicit mass terms, the Lagrangian contains interaction terms between fermions and scalar bosons, known as *Yukawa couplings* which are weighted by the Yukawa coupling constant h

$$\mathcal{L}_{\text{Weyl}} = \overline{\psi}_L \not{\partial} \psi_L + \overline{\psi}_R \not{\partial} \psi_R - h(\overline{\psi}_L \Phi \psi_R + \text{h.c.}). \tag{2.60}$$

This interaction term shows that the emission of a scalar boson switches the chirality of the fermion. Moreover, if the scalar field Φ acquires a non-vanishing VEV the last term will behave like an effective mass term

$$\mathcal{L}_{\text{Weyl}} = \overline{\psi}_L \not{\partial} \psi_L + \overline{\psi}_R \not{\partial} \psi_R - h\overline{\psi}_L \Phi \psi_R - m\overline{\psi}_L \psi_R + \text{h.c.}, \tag{2.61}$$

where $m = h\Phi_0$ is the mass term associated with the Dirac spinor $\Psi = (\psi_R, \psi_L)^T$. Hence, the SSB of gauge chiral symmetries also implies the acquisition of masses by the former chiral fermions.

In this way, the synthesis of spacetime symmetries described by the Poincaré group, the internal symmetries and the Yang-Mills scheme, as well as the spontaneous symmetry breaking mechanism constitute the framework for constructing theories about the microscopical nature of matter. The next chapter presents a quick review of the SM as a phenomenological application of the physical principles brushed up in the present chapter.

3

STANDARD MODEL OF PARTICLE PHYSICS

Along the development of science there were different theories about matter and its interactions, which have been improved with each one of new experimental discoveries or theoretical frameworks as the first detection of subatomic particles or the formulation of the theory of relativity or the quantum mechanics. Every one of these progresses and discoveries are condensed in the Standard Model of Particle Physics[Gla61; Sal67; Wei67] (SM), the current theory of matter. A huge number of its predictions have been corroborated experimentally, from the extremely precise prediction of the gyromagnetic factor of the electron until the detection of the Higgs boson in 2012[Aad+12].

Although there are natural phenomena which cannot be framed in the SM without important modifications or unpleasant fine-tuning of the parameters of the model, it still is the current paradigm for understanding fundamental interactions and particles. For this reason, this chapter is devoted to review the main aspects of the SM. First, the section 3.1 describes the gauge structure of the SM and proposes the fundamental features to classify the particle content. Second, the Glashow-Salam-Weinberg electroweak theory is reviewed employing the concepts of Yang-Mills fields and SSM in the section 3.2. Finally, it is presented the low-energy limit of the SM, corresponding to the Fermi model for weak interactions in the section 3.3.

SM GAUGE SYMMETRIES

The SM fundamental gauge group G_{SM} is constituted by the direct product of three special unitary groups, each one associated to a fundamental force

$$G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y. \tag{3.1}$$

The first group $SU(3)_C$ comprises the strong interaction phenomena whose gauge bosons are the eight gluons G_μ. This group gives the first benchmark to classify SM particles. Fermions which interact strongly are called *quarks* q, belonging to the vector $\mathbf{3}_C$ representation of $SU(3)_C$ and so existing in three different species or colors. On the other hand, fermions which do not interact strongly are *leptons* ℓ, belonging to the singlet $\mathbf{1}_C$ representation and so blind to gluons. Moreover, the number of quarks or leptons is a conserved quantity; there is the *baryon number* which assigns $1/3$ for each quarks (and $-1/3$ for antiquarks), and the *lepton number* which assigns $+1$ for each lepton (and -1 for antileptons).

The second group $SU(2)_L$ describes the weak isospin gauge symmetry, useful for describing $\beta-$decays and in conjunction with the last weak hypercharge group $U(1)_Y$ constitutes the *electroweak* gauge group $SU(2)_L \otimes U(1)_Y$. Its gauge bosons are $\mathbf{W}_\mu = W_\mu^\alpha T_{\alpha L}$ and B_μ, respectively, where $T_{\alpha L}$ for $\alpha = 1, 2, 3$ are the three generators of $SU(2)_L$.

The L subscript of the weak isospin group points out that $\mathbf{W}_\mu$ interact only with left-handed fermions. In this way, every left-handed fermion lies in an isospin doublet $\mathbf{2}_L$ representation of $SU(2)_L$ while right-handed fermions are singlets $\mathbf{1}_L$, ensuring the parity violation of weak interactions. On the other hand, both left- and right-handed fermions have weak hypercharges in such a way that their electromagnetic charges are obtained with the Gell-Mann - Nishijima relation

$$Q = T_{3L} + Y \tag{3.2}$$

where $T_{3L} = \text{diag}\,(1/2, -1/2)$ is the diagonal generator of the isospin group, i.e., it acts only on left-handed fermion doublets. As a consequence, the weak hypercharges of right-handed fermions are equal to their electric charges.

The last feature of a fermion which also determines its name is its electric charge. The up quark u has $+2/3$ and the down quark d has $-1/3$, while the electron has -1 and the neutrino has 0. These features are summarized in table 3.1.

	$SU(3)_C$	$SU(2)_L$	T_{3L}	Y	Q
u_L	3_C	$\mathbf{2}_L$	$+1/2$	$+1/6$	$+2/3$
d_L	3_C	$\mathbf{2}_L$	$-1/2$	$+1/6$	$-1/3$
u_R	3_C	$\mathbf{1}_L$	0	$+2/3$	$+2/3$
d_R	3_C	$\mathbf{1}_L$	0	$-1/3$	$-1/3$
ν_L	$\mathbf{1}_C$	$\mathbf{2}_L$	$+1/2$	$-1/2$	0
e_L	$\mathbf{1}_C$	$\mathbf{2}_L$	$-1/2$	$-1/2$	-1
e_R	$\mathbf{1}_C$	$\mathbf{1}_L$	0	-1	-1

Table 3.1: Representations and electroweak charges of SM-fermions.

Moreover, this set of fermions is duplicated twice, resulting with three copies of fermions with the same charges. These sets are called *families* or *generations* which will be studied in detail in chapter 4.

ELECTROWEAK THEORY: GLASHOW-SALAM-WEINBERG

The electroweak sector of SM can be described by the following Lagrangian

$$\mathcal{L}_{EW} = \mathcal{L}_{Gauge} + \mathcal{L}_{Fermion} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} \tag{3.3}$$

which has been divided in four different terms useful for their analysis. It is important to note that $\mathcal{L}_{Fermion}$ and $\mathcal{L}_{Yukawa}$ have baryon and lepton sectors because SM does not mix quarks and leptons. Each one of these Lagrangians are reviewed in the following subsections except $\mathcal{L}_{Yukawa}$ which is reviewed in the chapter 4. It is important to note that because the gauge symmetry, the normal derivative have to be replaced by the covariant derivative. For weak isospin doublets the covariant derivative is

$$D_\mu = \partial_\mu - ig\mathbf{W}_\mu - ig'YB_\mu, \tag{3.4}$$

and for singlets $D_\mu = \partial_\mu - ig'YB_\mu$. g and g' are the dimensionless coupling constants for weak isospin and hypercharge interactions, respectively.

Electroweak SSB

The origin of the electromagnetic interaction and the weakness of the weak interactions are described by the Higgs Mechanism applied to the electroweak group $SU(2)_L \otimes U(1)_Y$. This procedure is done by introducing an isospin scalar doublet hypercharged once $Y = +1$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \tag{3.5}$$

whose superscripts indicate the electric charges of the components. Its dynamics is described by the *Higgs Lagrangian*

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2} \left(D^\mu \Phi\right)^\dagger \left(D_\mu \Phi\right) - V(\Phi^\dagger \Phi), \tag{3.6}$$

where the *Higgs potential* is

$$V(\Phi^\dagger\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2. \tag{3.7}$$

The electroweak SSB follows the same procedure shown in subsection 2.3.1. Thus, evaluating the Higgs Lagrangian at the vacuum expectation value (VEV) when $\mu^2 < 0$ the gauge boson mass terms are obtained

$$\begin{aligned} \mathcal{L}_{\text{Higgs}}^{\text{VEV}} = &\frac{g^2}{2} \Phi_0^\dagger \mathbf{W}_\mu^\dagger \mathbf{W}^\mu \Phi_0 + \frac{gg'}{2} \Phi_0^\dagger \mathbf{W}_\mu^\dagger B^\mu \Phi_0 \\ &+ \frac{gg'}{2} \Phi_0^\dagger B_\mu \mathbf{W}^\mu \Phi_0 + \frac{g'^2}{2} B_\mu B^\mu \Phi_0^\dagger \Phi_0. \end{aligned} \tag{3.8}$$

On the other hand, the structure of the VEV is chosen in such a way that it would be electrically neutral, i.e., $Q\Phi_0 = (T_{3L} + Y)\,\Phi_0 = 0$. Hence, T_{1L}, T_{2L} and $T_{3L} - Y$ become broken generators and their corresponding gauge bosons acquire mass. Consequently, the Higgs field can be expressed as

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{h+v+i\eta}{\sqrt{2}} \end{pmatrix}, \tag{3.9}$$

where $v^2 = -\mu^2/\lambda$, and the expanded mass terms are

$$\begin{aligned} \mathcal{L}_{\text{Higgs}}^{\text{VEV}} = &\frac{g^2 v^2}{8} W^{+\dagger}_{\mu} W^{+\mu} + \frac{g^2 v^2}{8} W^{-\dagger}_{\mu} W^{-\mu} + \frac{g^2 v^2}{8} W_\mu^3 W^{3\mu} \\ &+ \frac{gg' v^2}{8} W_\mu^3 B^\mu + \frac{gg' v^2}{8} B_\mu W^{3\mu} + \frac{g'^2 v^2}{8} B_\mu B^\mu. \end{aligned} \tag{3.10}$$

The first two terms are the $W_\mu^\pm$ mass terms, and the other describe the mixing between B_μ and W_μ^3 which can be condensed in the following matrix

$$\frac{v^2}{8} \begin{pmatrix} B_\mu & W_\mu^3 \end{pmatrix} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} B^\mu \\ W^{3\mu} \end{pmatrix}. \tag{3.11}$$

The mass matrix is diagonalized by the following rotation on the plane (B_μ, W^3_μ)

$$\begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \tag{3.12}$$

where $t_W = g'/g$ defines the Weinberg angle. It yields the massive Z_μ and the massless A_μ identified with the photon. In this way, the gauge bosons acquire masses

$$\mathcal{L}^{\text{VEV}}_{\text{Higgs}} = m^2_W W^{+\dagger}_{\mu} W^{-\mu} + \frac{1}{2} m^2_Z Z_\mu Z^\mu + \frac{1}{2} m^2_A A_\mu A^\mu. \tag{3.13}$$

where $m_W = gv/2$, $m_Z = gv/2c_W = m_W/c_W$ and $m_A = 0$.

As a final procedure, according to the Goldstone theorem, reviewed in subsection 2.3.2, the existence of three massive and one massless gauge bosons implies the existence of three massless and one massive scalar fields. These are obtained by calculating the Hessian matrix for the minimized Higgs potential:

$$\begin{aligned} m^2_{\phi^\pm} &= \left(\frac{\partial^2 V}{\partial \phi^+ \partial \phi^-} \right)_{\Phi_0} = 0, \\ m^2_\eta &= \left(\frac{\partial^2 V}{\partial^2 \eta} \right)_{\Phi_0} = 0, \end{aligned} \tag{3.14}$$

the scalar charged bosons $\phi^\pm$ and the pseudoscalar η remains massless and get eaten by $W^\pm_\pm$ and Z_μ, respectively. On the opposite, the even scalar boson acquires mass

$$m^2_h = \left(\frac{\partial^2 V}{\partial^2 h} \right)_{\Phi_0} = -\mu^2 = \lambda v^2. \tag{3.15}$$

This is identified with the 125 GeV scalar boson discovered in 2012.

Fermion Lagrangian

The interactions between fermions and gauge bosons can be obtained from the Weyl Lagrangian for chiral femions

$$\mathcal{L}_{\text{Fermion}} = i\bar{\ell}_L \not{D} \ell_L + i\bar{e}_R \not{D} e_R + i\bar{q}_L \not{D} q_L + i\bar{u}_R \not{D} u_R + i\bar{d}_R \not{D} d_R, \tag{3.16}$$

where the minimal coupling has been applied with the covariant derivatives. By expanding the kinetic and interactions terms the Lagrangian is

$$\begin{aligned} \mathcal{L}_{\text{Fermion}} &= i\overline{q_L} \not{\partial} q_L + i\overline{u_R} \not{\partial} u_R + i\overline{d_R} \not{\partial} d_R + i\overline{\ell_L} \not{\partial} \ell_L + i\overline{e_R} \not{\partial} e_R \\ &+ g\overline{q_L} \not{\mathbf{W}} q_L + \frac{g'}{6} \overline{q_L} \not{B} q_L + \frac{2}{3} g' \overline{u_R} \not{B} u_R - \frac{g'}{3} \overline{d_R} \not{B} d_R \\ &+ g\overline{\ell_L} \not{\mathbf{W}} \ell_L - \frac{g'}{2} \overline{\ell_L} \not{B} \ell_L - g' \overline{e_R} \not{B} e_R, \end{aligned} \tag{3.17}$$

where the interaction terms of the doublets can be expressed in terms of the gauge bosons $W^{\pm}_{\mu} = (W^1_{\mu} \mp iW^2_{\mu})/\sqrt{2}$. The interaction terms for the quark sector are

$$
\begin{aligned}
&\overline{q}_L \not{\boldsymbol{W}} q_L + \frac{g'}{6}\overline{q_L}\not{B} q_L \\
&= \frac{1}{2}\overline{u_L}\left(g\not{W}^3 + \frac{g'}{3}\not{B}\right)u_L + \frac{g}{\sqrt{2}}\overline{u_L}\not{W}^+ d_L \\
&+ \frac{g}{\sqrt{2}}\overline{d_L}\not{W}^- u_L - \frac{1}{2}\overline{d_L}\left(g\not{W}^3 - \frac{g'}{3}\not{B}\right)d_L,
\end{aligned}
\tag{3.18}
$$

and for leptons

$$
\begin{aligned}
&\overline{\ell}_L \not{\boldsymbol{W}} \ell_L - \frac{g'}{2}\overline{\ell_L}\not{B}\ell_L \\
&= \frac{1}{2}\overline{\nu_L}\left(g\not{W}^3 - g'\not{B}\right)\nu_L + \frac{g}{\sqrt{2}}\overline{\nu_L}\not{W}^+ e_L \\
&+ \frac{g}{\sqrt{2}}\overline{e_L}\not{W}^- \nu_L - \frac{1}{2}\overline{e_L}\left(g\not{W}^3 + g'\not{B}\right)e_L.
\end{aligned}
\tag{3.19}
$$

Since the electroweak interaction contains the electromagnetic force, it is possible to obtain it with the rotation shown in eq. (3.12) and by defining $e = gs_W = g'c_W$, the electromagnetic coupling constant. Replacing it into the fermion Lagrangian yields

$$
\begin{aligned}
\mathcal{L}_{\text{Fermion}} &= i\overline{q_L}\not{\partial} q_L + i\overline{u_R}\not{\partial} u_R + i\overline{d_R}\not{\partial} d_R + i\overline{\ell_L}\not{\partial}\ell_L + i\overline{e_R}\not{\partial} e_R \\
&- \frac{g}{c_W}J^{\mu}_{NC,Z}Z_{\mu} - eJ^{\mu}_{NC,A}A_{\mu} - \frac{g}{\sqrt{2}}J^{\mu}_{CC,W}W^+_{\mu} - \frac{g}{\sqrt{2}}J^{\dagger\mu}_{CC,W}W^-_{\mu},
\end{aligned}
\tag{3.20}
$$

where the corresponding interaction currents are

$$
\begin{aligned}
J^{\mu}_{NC,Z} &= \left(\frac{1}{2} - 0s_W^2\right)\overline{\nu_L}\gamma^{\mu}\nu_L \\
&- \left(\frac{1}{2} - 1s_W^2\right)\overline{e_L}\gamma^{\mu}e_L - \overline{e_R}\gamma^{\mu}e_R \\
&+ \left(\frac{1}{2} + \frac{2}{3}s_W^2\right)\overline{u_L}\gamma^{\mu}u_L + \frac{2}{3}s_W^2\overline{u_R}\gamma^{\mu}u_R \\
&- \left(\frac{1}{2} - \frac{1}{3}s_W^2\right)\overline{d_L}\gamma^{\mu}d_L - \frac{1}{3}s_W^2\overline{d_R}\gamma^{\mu}d_R, \\
J^{\mu}_{NC,A} &= \frac{2}{3}\overline{u_L}\gamma^{\mu}u_L - \frac{1}{3}\overline{d_L}\gamma^{\mu}d_L - \overline{e_L}\gamma^{\mu}e_L \\
&+ \frac{2}{3}\overline{u_R}\gamma^{\mu}u_R - \frac{1}{3}\overline{d_R}\gamma^{\mu}d_R - \overline{e_R}\gamma^{\mu}e_R, \\
J^{\mu}_{CC,W} &= \overline{\nu_L}\gamma^{\mu}e_L + \overline{u_L}\gamma^{\mu}d_L.
\end{aligned}
\tag{3.21}
$$

$J^{\mu}_{NC,Z}$ and $J^{\mu}_{NC,A}$ are the *neutral currents* which conserve fermion electric charges, while $J^{\mu}_{CC,W}$ is the *charged current* where the interchange of electric charges happens due to $W^{\pm}_{\mu}$. It is important to say that the electric charges are obtained using the Gell-Mann - Nishijima relation.

Gauge Lagrangian

The dynamics of the gauge bosons is described by the YM scheme. In this way, the gauge Lagrangian is

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4}\text{Tr}\left(\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu}\right) - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \tag{3.22}$$

where $\mathbf{W}_{\mu\nu}$ and $B_{\mu\nu}$ are the strength-field tensors for weak isospin and hypercharge potentials

$$\begin{aligned} \mathbf{W}_{\mu\nu} &= \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - g\mathbf{W}_\mu \times \mathbf{W}_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \tag{3.23}$$

After the substitution of the physical gauge bosons obtained from eq. (3.12) the resulting gauge Lagrangian is

$$\begin{aligned} \mathcal{L}_{\text{Gauge}} = &-\frac{1}{2}W^{+\mu\nu}W^{-}_{\mu\nu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}Z^{\mu\nu}Z_{\mu\nu} \\ &+ \mathcal{L}_{WW} + \mathcal{L}_{AWW} + \mathcal{L}_{ZWW} + \mathcal{L}_{AZWW} \end{aligned} \tag{3.24}$$

where the new kinetic terms are

$$\begin{aligned} W^{\pm}_{\mu\nu} &= \partial_\mu W^{\pm}_\nu - \partial_\nu W^{\pm}_\mu, \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu, \end{aligned} \tag{3.25}$$

and the interactions among them are

$$\begin{aligned} \mathcal{L}_{WW} = &-\frac{g^2}{2}\left\{W^{+\mu}W^{-\mu}W^{+\nu}W^{-\nu} - W^{+\mu}W^{+\mu}W^{-\nu}W^{-\nu}\right\}, \\ \mathcal{L}_{AWW} = &\, ie\left\{\partial_\mu A_\nu \left(W^{+\mu}W^{-\nu} - W^{-\mu}W^{+\nu}\right)\right. \\ &+ A_\mu\left(W^{+}_\nu \partial^\mu W^{-\nu} - W^{-}_\nu \partial^\mu W^{+\nu}\right) \\ &\left.+ A_\mu\left(W^{+}_\nu \partial^\nu W^{-\mu} - W^{-}_\nu \partial^\nu W^{+\mu}\right)\right\}, \\ \mathcal{L}_{ZWW} = &\, igc_W\left\{\partial_\mu Z_\nu \left(W^{+\mu}W^{-\nu} - W^{-\mu}W^{+\nu}\right)\right. \\ &+ Z_\mu\left(W^{+}_\nu \partial^\mu W^{-\nu} - W^{-}_\nu \partial^\mu W^{+\nu}\right) \\ &\left.+ Z_\mu\left(W^{+}_\nu \partial^\nu W^{-\mu} - W^{-}_\nu \partial^\nu W^{+\mu}\right)\right\}, \\ \mathcal{L}_{AZWW} = &-W^{+}_\mu W^{-}_\nu\left\{g^2c_W^2 Z^\mu Z^\nu + egc_W\left(Z^\mu A^\nu + A^\mu Z^\nu\right) + e^2A^\mu A^\nu\right\} \\ &-W^{+}_\mu W^{-\mu}\left\{g^2c_W^2 Z_\nu Z^\nu + egc_W\left(Z_\nu A^\nu + A_\nu Z^\nu\right) + e^2A_\nu A^\nu\right\}. \end{aligned} \tag{3.26}$$

LOW-ENERGY LIMIT: FERMI THEORY OF β-DECAY

The weakness of the weak nuclear force may be understood from the fact that its gauge bosons $W^{\pm}_\mu$ and Z_μ had acquired masses because the electroweak VEV. Their propagators in the unitary gauge are (subsec. 2.1.1)

$$\tilde{G}^{W}_{\mu\nu}(p) = i\frac{-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_W^2}}{p^2 - m_W^2 + i\epsilon}, \qquad \tilde{G}^{Z}_{\mu\nu}(p) = i\frac{-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_Z^2}}{p^2 - m_Z^2 + i\epsilon}, \tag{3.27}$$

Boson	Spin	$\mathcal{P}$	Q	Mass (GeV)	Width (GeV)	Feature
$\phi^\pm$	0	+	±1	0	0	Goldstone
h	0	+	0	125.09 ± 0.24	< 1.7	Higgs
η	0	−	0	0	0	Goldstone
A_μ	1	−	0	$< 10^{-27}$	0	Photon $U(1)_Q$
$W^\pm_\mu$	1	−	±1	80.385 ± 0.015	2.085 ± 0.042	Weak CC
Z_μ	1	+	0	91.1876 ± 0.0021	2.4952 ± 0.0023	Weak NC
$\mathbf{G}_\mu$	1	−	0	0	0	Gluons $SU(3)_C$

Table 3.2: SM scalar and vector boson data[PG+16].

where the gauge boson masses are reported in table 3.2.

However, since there are other particles whose masses lie far below the gauge boson masses but are not the lightest masses, e.g., d quark, the associated momenta p^2 of the process are small compared to m_W^2 or m_Z^2, allowing approximate the propagators by

$$\lim_{p \ll m_W} \tilde{G}^W_{\mu\nu}(p) = i\frac{g^{\mu\nu}}{m_W^2}, \qquad \lim_{p \ll m_Z} \tilde{G}^Z_{\mu\nu}(p) = i\frac{g^{\mu\nu}}{m_Z^2}, \tag{3.28}$$

and the SM fermion Lagrangian in eq. (3.20) becomes a current-current interaction Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{Fermion}} &= i\overline{q_L}\partial\!\!\!/ q_L + i\overline{u_R}\partial\!\!\!/ u_R + i\overline{d_R}\partial\!\!\!/ d_R + i\overline{\ell_L}\partial\!\!\!/ \ell_L + i\overline{e_R}\partial\!\!\!/ e_R \\ &- eJ^\mu_{\text{NC},A}A_\mu - \frac{g^2}{8c_W^2 m_Z^2}J^{\dagger\mu}_{\text{NC},Z}J_{\text{NC},Z\mu} - \frac{g^2}{8m_W^2}J^{\dagger\mu}_{\text{CC},W}J_{\text{CC},W\mu}.\end{aligned} \tag{3.29}$$

Such interaction terms were considered in Fermi theory of β-decay whose interaction term is[GK07; Gri87]

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}}J^{\dagger\mu}_{\text{CC},W}J_{\text{CC},W\mu}. \tag{3.30}$$

which can be compared to eq. (3.29) and so finding the correspondence of Fermi constant in terms of SM parameters

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8c_W^2 m_Z^2} = \frac{g^2}{8m_W^2} = \frac{1}{2v^2}. \tag{3.31}$$

It is remarkable that the Fermi constant is proportional to the electroweak VEV, but also that G_F does not depend on g and m_W. Moreover, since $m_W = m_Z c_W$, the effective four-fermion coupling constant for NC is indeed the Fermi constant again

Consequently, for processes whose energies lies below electroweak energy scale, the effective low-energy SM fermion Lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{Fermion}} &= i\overline{q_L}\partial\!\!\!/ q_L + i\overline{u_R}\partial\!\!\!/ u_R + i\overline{d_R}\partial\!\!\!/ d_R + i\overline{\ell_L}\partial\!\!\!/ \ell_L + i\overline{e_R}\partial\!\!\!/ e_R \\ &- eJ^\mu_{\text{NC},A}A_\mu - \frac{G_F}{\sqrt{2}}J^{\dagger\mu}_{\text{NC},Z}J_{\text{NC},Z\mu} - \frac{G_F}{\sqrt{2}}J^{\dagger\mu}_{\text{CC},W}J_{\text{CC},W\mu}.\end{aligned} \tag{3.32}$$

Constant	$Q^2 = 0$	$Q^2 \approx m_Z^2$
α	$1/137.035\,999\,139(31)$	$1/128$
$G_F/(\hbar c)^2$	$1.166\,378\,7(6) \times 10^{-5}\,\text{GeV}^{-2}$	
α_S	> 1	$0.1182(12)$
s_W^2	$0.23155(5)$	$0.231\,29(5)$

Table 3.3: SM fundamental constants at low-energy and GeV scales[PG+16].

The measured value of the Fermi constant is (tab. 3.3)[PG+16]

$$G_F = 1.1663787(6) \times 10^{-5}\,\text{GeV}, \tag{3.33}$$

implying in such a way that the value of the electroweak VEV is

$$v \approx 246\,\text{GeV}. \tag{3.34}$$

Moreover, by replacing m_W and G_F in eq. (3.31) and solving for g, the electroweak isospin coupling constant. Its found value is

$$g \approx 0.66, \tag{3.35}$$

and defining the *isospin fine structure constant*

$$\alpha_g = \frac{g^2}{4\pi} \approx \frac{1}{29}, \tag{3.36}$$

the remarkable result that the weak interaction is actually stronger than the electromagnetic ones appears. This conclusion shows how the weakness of the weak interaction is due to the large masses of $W_\mu^\pm$ and Z_μ masses, but not to a small coupling constant.

It is worth mentioning that, for BSM extensions involving new gauge bosons like Z'_μ or $W_\mu^{3\pm}$ obtained from $U(1)$ or $SU(3)_L$ gauge groups, new interactions are included involving SM but also non-SM fields not discovered yet. Although these gauge bosons have not been observed since they are more massive than experimental energies achieved, their effects could be detected in a similar way than β-decays at the beginning of 20th century, and so they can be studied using Fermi-like theories involving the new SSB energy scale.

Summarizing the present chapter, the SM were briefly presented. The particle content and the different parts of the SM Lagrangian were reviewed with exception of the Yukawa Lagrangian introduced in chapter 4. The Higgs Lagrangian yields the electroweak SSB with the emergence of a non-vanishing VEV and consequently the majority of particles in the model acquire mass. The fermion Lagrangian was presented before and after SSB and the charged and neutral currents were defined. The gauge Lagrangian contains all the possible interactions among gauge bosons, and again it was presented before and after SSB. Finally, the low-energy limit was brushed up arriving at the Fermi theory. The next chapter finishes the review of general framework in part i presenting the fermion mass acquisition and the family mixing.

4 MASS MATRICES AND FAMILY MIXING

One of the best predictions of the SM, but in some way one of its greatest problems comprises the mass acquisition of chiral fermions. From the fact that the Weyl Lagrangians describe massless chiral fermions (subsec. 2.1.1) and the SM is a chiral gauge theory whose left- and right-handed fermions transform under different representations of G_{SM} in eq. (3.1), explicit fermion mass terms are forbidden and should be generated, for example, spontaneously.

This achievement was done by the SM with the electroweak SSB and Yukawa couplings among fermions and the Higgs doublet. These couplings mix both chiralities and require a very special order on its components. Since the Higgs field is described by an electroweak isospin doublet $\mathbf{2}_L$, in order to ensure $SU(2)_L$ gauge invariance it have to be contracted with the conjugate $\overline{\mathbf{2}_L}$ of the left-handed fermions $\overline{q_L}$ or $\overline{\ell_L}$. The invariance under the hypercharge gauge group $U(1)_Y$ is accomplished by adding up an isospin singlet $\mathbf{1}_L$ in such a way that $-Y_L + Y_\Phi + Y_R = 0$.

The SM fermion and scalar sectors have the suited charges for carrying out this program in the SM Yukawa Lagrangian

$$-\mathcal{L}_{\text{Yukawa}} = h^u \overline{q_L} \widetilde{\Phi} u_R + h^d \overline{q_L} \Phi d_R + h^e \overline{\ell_L} \Phi e_R + \text{h.c.}, \tag{4.1}$$

where $\widetilde{\Phi} = i\sigma_2 \Phi^*$

$$\widetilde{\Phi} = \begin{pmatrix} \frac{h+v-i\eta}{\sqrt{2}} \\ -\phi^- \end{pmatrix}, \tag{4.2}$$

and h^u, h^d and h^e are the *Yukawa coupling constants* between the Higgs scalar and the SM fermions. It is important to remark the lack of the right-handed species for neutrinos, and consequently the absence of neutrino Yukawa coupling in the SM.

The acquisition of masses is done by the electroweak VEV. By evaluating the Yukawa Lagrangian at the vacuum state it becomes

$$-\mathcal{L}^{\text{VEV}}_{\text{Yukawa}} = \frac{h^u v}{\sqrt{2}} \overline{u_L} u_R + \frac{h^d v}{\sqrt{2}} \overline{d_L} d_R + \frac{h^e v}{\sqrt{2}} \overline{e_L} e_R + \text{h.c.}, \tag{4.3}$$

where Dirac mass terms had appeared by mixing left- and right-handed chiralities. This procedure yields the acquisition of masses by three of the four SM fermions

$$m_u = \frac{h^u v}{\sqrt{2}}, \qquad m_d = \frac{h^d v}{\sqrt{2}}, \qquad m_e = \frac{h^e v}{\sqrt{2}}. \tag{4.4}$$

Moreover, if the Higgs boson h is included the remarkable conclusion that fermions couple to Higgs boson proportionally to their masses appears (h.c. terms have been summed up $f = f_L + f_R$)

$$\begin{aligned} -\mathcal{L}_{\text{Yukawa}} =& m_u \bar{u} u + m_d \bar{d} d + m_e \bar{e} e \\ &+ \frac{m_u}{v} h \bar{u} u + \frac{m_d}{v} h \bar{d} d + \frac{m_e}{v} h \bar{e} e. \end{aligned} \tag{4.5}$$

SM FAMILIES

The SM classifies the different fermions observed in Nature between quarks and leptons, but the fermionic spectrum is not composed only by one up quark, one down quark, one electron and one neutrino. Indeed, the SM proposes this spectrum is repeated thrice in Nature constituting the three *families* or *generations* of the SM which are shown in table 4.1.

First family	Second family	Third family
$q_L^1 = \begin{pmatrix} u^1 \\ d^1 \end{pmatrix}_L$	$q_L^2 = \begin{pmatrix} u^2 \\ d^2 \end{pmatrix}_L$	$q_L^3 = \begin{pmatrix} u^3 \\ d^3 \end{pmatrix}_L$
u_R^1	u_R^2	u_R^3
d_R^1	d_R^2	d_R^3
$\ell_L^e = \begin{pmatrix} \nu^e \\ e^e \end{pmatrix}_L$	$\ell_L^\mu = \begin{pmatrix} \nu^\mu \\ e^\mu \end{pmatrix}_L$	$\ell_L^\tau = \begin{pmatrix} \nu^\tau \\ e^\tau \end{pmatrix}_L$
e_R^e	e_R^μ	e_R^τ

Table 4.1: SM-fermion flavor families.

Nowadays, the main difference among families are their mass scales: masses of the first family lies at units of MeV, the second at hundreds of MeV and the third at units of GeV (table 4.2). The origin of this hierarchy and also why fermions acquire these masses are not well understood yet, but they could give some hints about the physics beyond the SM, specially neutrino oscillation data which are consistent with massive neutrinos with so light masses.

Notice the different symbols employed in both tables. Actually, the fermions listed in table 4.1 do not have definite masses because they are superpositions of the fermions listed in the table 4.2. This fact is reviewed in the next section in the context of mass and mixing matrices.

MASS MATRICES

The SM Yukawa Lagrangian shown at the beginning of this chapter in eq. (4.1) does not correspond to the observed phenomenology because the existence of three generations. Thus, the complete Yukawa Lagrangian has to contain the three types of fermions. Additionally, the right-handed counterpart of neutrinos has been included for completeness of the mass acquisition of fermions.

The three-family Yukawa Lagrangian can be expressed by

$$-\mathcal{L}_{\text{Yukawa}} = \overline{\mathbf{q}'_L}\widetilde{\Phi}H^u\mathbf{u}'_R + \overline{\mathbf{q}'_L}\Phi H^d\mathbf{d}'_R + \overline{\boldsymbol{\ell}'_L}\widetilde{\Phi}H^\nu\boldsymbol{\nu}'_R + \overline{\boldsymbol{\ell}'_L}\Phi H^e\boldsymbol{e}'_R + \text{h.c.}, \tag{4.6}$$

where the bold spinors are strings in the family space. In the flavor basis the isospin doublets are

$$\mathbf{q}'_L = \begin{pmatrix} q_L^1 \\ q_L^2 \\ q_L^3 \end{pmatrix}, \quad \boldsymbol{\ell}'_L = \begin{pmatrix} \ell_L^e \\ \ell_L^\mu \\ \ell_L^\tau \end{pmatrix}, \tag{4.7}$$

Family	Particle	Mass
1	u	$2.2^{+0.6}_{-0.4}$ MeV
	d	$4.7^{+0.5}_{-0.4}$ MeV
	e	0.511 MeV
2	c	1.27 ± 0.03 GeV
	s	96^{+8}_{-4} MeV
	μ	105.7 MeV
3	t	173.21 ± 0.71 GeV
	b	$4.18^{+0.04}_{0.03}$ GeV
	τ	1.776 GeV

Table 4.2: SM-fermion masses. The masses of the charged leptons are determined further the fourth decimal position [PG+16].

and also the isospin singlets are

$$\mathbf{u}'_R = \begin{pmatrix} u^1_R \\ u^2_R \\ u^3_R \end{pmatrix}, \quad \mathbf{d}'_R = \begin{pmatrix} d^1_R \\ d^2_R \\ d^3_R \end{pmatrix}, \quad \boldsymbol{\nu}'_R = \begin{pmatrix} \nu^e_R \\ \nu^\mu_R \\ \nu^\tau_R \end{pmatrix}, \quad \mathbf{e}'_R = \begin{pmatrix} e^e_R \\ e^\mu_R \\ e^\tau_R \end{pmatrix}. \tag{4.8}$$

In the same way, the new Yukawa couplings H^u, H^d, H^ν and H^e are matrices in the family space which connect fermions across the three families.

Again, by evaluating the Yukawa Lagrangian at the electroweak VEV the mass matrices are obtained

$$\begin{aligned} -\mathcal{L}_{\text{Yukawa}} &= \overline{\mathbf{u}'_L} M^{u\prime} \mathbf{u}'_R + \overline{\mathbf{d}'_L} M^{d\prime} \mathbf{d}'_R \\ &+ \overline{\boldsymbol{\nu}'_L} M^{\nu\prime} \boldsymbol{\nu}'_R + \overline{\mathbf{e}'_L} M^{e\prime} \mathbf{e}'_R + \text{h.c.}, \end{aligned} \tag{4.9}$$

which have to be diagonalized in order to obtain the translation, physical or mass eigenstates given by

$$\mathbf{u} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad \boldsymbol{\nu} = \begin{pmatrix} \nu^1 \\ \nu^2 \\ \nu^3 \end{pmatrix}, \quad \mathbf{e} = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}. \tag{4.10}$$

The corresponding mass matrices obtained from the electroweak SSB are

$$\begin{aligned} M^u &= \text{diag}\left(m_u, m_c, m_t\right), \qquad & M^d &= \text{diag}\left(m_d, m_s, m_b\right), \\ M^\nu &= \text{diag}\left(m_1, m_2, m_3\right), \qquad & M^e &= \text{diag}\left(m_e, m_\mu, m_\tau\right). \end{aligned} \tag{4.11}$$

Since the mass matrices are not necessary symmetric, they have to be diagonalized by a biunitary transformation in the following way

$$\begin{aligned}
\mathbf{u}'_L &= V_L^U \mathbf{u}_L, & \mathbf{u}'_R &= V_R^U \mathbf{u}_R, & M^u &= V_L^{U\dagger} M^{u\prime} V_R^U,\\
\mathbf{d}'_L &= V_L^D \mathbf{d}_L, & \mathbf{d}'_R &= V_R^D \mathbf{d}_R, & M^d &= V_L^{D\dagger} M^{d\prime} V_R^D,\\
\boldsymbol{\nu}'_L &= V_L^\nu \boldsymbol{\nu}_L, & \boldsymbol{\nu}'_R &= V_R^\nu \boldsymbol{\nu}_R, & M^\nu &= V_L^{\nu\dagger} M^{\nu\prime} V_R^\nu,\\
\boldsymbol{e}'_L &= V_L^E \boldsymbol{e}_L, & \boldsymbol{e}'_R &= V_R^E \boldsymbol{e}_R, & M^e &= V_L^{E\dagger} M^{e\prime} V_R^E.
\end{aligned} \tag{4.12}$$

The new states $\mathbf{u}$, $\mathbf{d}$, $\boldsymbol{\nu}$ and $\boldsymbol{e}$ are the physical states of the SM fermions, and consequently they have to be substituted in $\mathcal{L}_{\text{Fermion}}$ shown in eq. (3.20). The most important consequences of these replacements are studied in the next section.

FLAVOR-CHANGING-CURRENTS

The unitary transformations proposed in eq. (4.12) yield *flavor mixing*. Their off-diagonal elements induce transitions across families in such a way that the mass eigenstates turn out to be linear combinations or superpositions of the three fermion families. These transitions trigger radioactive decay chains of massive leptons or hadrons into the first and also lightest family, e.g., the charmed and strange hadrons decays. These processes are predicted when the mixing matrices (which actually have diagonalized the mass matrices) are replaced into the fermionic currents in eq. (3.21).

Neutral currents

When physical fermion states are replaced in the neutral current, it remains invariant,

$$\begin{aligned}
J^\mu_{NC,Z} = &\left(\frac{1}{2} - 0 s_W^2\right) \overline{\boldsymbol{\nu}_L} V_L^{\nu\dagger} \gamma^\mu V_L^\nu \boldsymbol{\nu}_L\\
&- \left(\frac{1}{2} - 1 s_W^2\right) \overline{\boldsymbol{e}_L} V_L^{E\dagger} \gamma^\mu V_L^E \boldsymbol{e}_L - \overline{\boldsymbol{e}_R} V_R^{E\dagger} \gamma^\mu V_R^E \boldsymbol{e}_R\\
&+ \left(\frac{1}{2} + \frac{2}{3} s_W^2\right) \overline{\mathbf{u}_L} V_L^{U\dagger} \gamma^\mu V_L^U \mathbf{u}_L + \frac{2}{3} s_W^2 \overline{\mathbf{u}_R} V_R^{U\dagger} \gamma^\mu V_R^U \mathbf{u}_R\\
&- \left(\frac{1}{2} - \frac{1}{3} s_W^2\right) \overline{\mathbf{d}_L} V_L^{D\dagger} \gamma^\mu V_L^D \mathbf{d}_L - \frac{1}{3} s_W^2 \overline{\mathbf{d}_R} V_R^{D\dagger} \gamma^\mu V_R^D \mathbf{d}_R,
\end{aligned} \tag{4.13}$$

$$\begin{aligned}
J^\mu_{NC,A} = &\frac{2}{3} \overline{\mathbf{u}_L} V_L^{U\dagger} \gamma^\mu V_L^U \mathbf{u}_L - \frac{1}{3} \overline{\mathbf{d}_L} V_L^{D\dagger} \gamma^\mu V_L^D \mathbf{d}_L - \overline{\boldsymbol{e}_L} V_L^{E\dagger} \gamma^\mu V_L^E \boldsymbol{e}_L\\
&+ \frac{2}{3} \overline{\mathbf{u}_R} V_R^{U\dagger} \gamma^\mu V_R^U \mathbf{u}_R - \frac{1}{3} \overline{\mathbf{d}_R} V_R^{D\dagger} \gamma^\mu V_R^D \mathbf{d}_R - \overline{\boldsymbol{e}_R} V_R^{E\dagger} \gamma^\mu V_R^E \boldsymbol{e}_R.
\end{aligned} \tag{4.14}$$

It is observed how mixing matrices cancel out because their unitarity $V^\dagger V = \mathbb{1}$ in both neutral currents. This result is known as the *Glashow-Iliopoulos-Maiani* (GIM) mechanism[GIM70] which ensures the absence, or at least suppression of flavor changes by emitting Z bosons or trough electromagnetic interactions called *flavor-changing-neutral-currents* (FCNC). It is worth mentioning this procedure requires the existence of left-handed doublets in each family, and at that moment it implied the prediction of c quark. Until now there are no observed events consistent with FCNC, and so any BSM scheme should predict them so suppressed to be consistent with experiments.

Charged currents

On the other hand, the charged current does change when physical fermion states are replaced in it

$$J^{\mu}_{CC,W} = \overline{\mathbf{u}_L} V_L^{U\dagger} \gamma^{\mu} V_L^{D} \mathbf{d}_L + \overline{\boldsymbol{\nu}_L} V_L^{\nu\dagger} \gamma^{\mu} V_L^{E} \boldsymbol{e}_L. \tag{4.15}$$

From the fact that mixing matrices of different flavors do not match and cancel together like the previous neutral current cases, there appears a new kind of mixing matrix which allows transitions among families only by emission of W gauge bosons, yielding flavor-changing through charged-currents. By defining these matrices as

$$V = V_L^{U\dagger} V_L^{D}, \qquad U^{\dagger} = V_L^{E\dagger} V_L^{\nu}, \tag{4.16}$$

the charged current becomes

$$J^{\mu}_{CC,W} = \overline{\mathbf{u}_L} \gamma^{\mu} V \mathbf{d}_L + \overline{\boldsymbol{\nu}_L} \gamma^{\mu} U^{\dagger} \boldsymbol{e}_L. \tag{4.17}$$

The former matrix V is called the *Cabbibo-Kobayashi-Maskawa matrix* (CKM) which describes flavor changes among quarks, while the latter U is known as the *Pontecorvo-Maki-Nakagawa-Sakata matrix* (PMNS) which does the same as CKM but among leptons.

Both matrices are parametrized as[CK84; PG+16]

$$\begin{aligned} &\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \\ &\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \end{aligned} \tag{4.18}$$

where c_{ij} and s_{ij} refer to $i-j$ family mixing, and δ modules CP asymmetries between matter-antimatter processes. In the next two sections CKM and PMNS matrices are quickly reviewed.

CKM MATRIX

Since the discovery of the pions, there have been detected a lot of different hadrons and then their decays. The non-conservation of strangeness by $\Delta S = \pm 1$, e.g., in the kaon decays, and the different rates respect to $\Delta S = 0$ decays brought the idea of flavor changing and also mixing proposed by Nicola Cabibbo[Cab63]. After the detection of c quark the mixing could be modeled by a unitary 2×2 matrix. However, the discovery of b and according to GIM mechanism, there had to exist the t quark and the mixing matrix became 3×3. The last proposal was done by Makoto Kobayashi and Toshihide Maskawa[KM73] obtaining in such a way the well-known CKM matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{4.19}$$

The CKM matrix is parametrized by the angles shown in eq. (4.18), but it is usually common to express the CKM matrix using the Wolfenstein parametrization

$$\begin{aligned} s_{12} &= \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}} \\ s_{23} &= A\lambda^2 = \lambda \left| \frac{V_{cb}}{V_{us}} \right| \\ s_{13}e^{i\delta} &= V_{ub}^* = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\overline{\rho} + i\overline{\eta})\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}\,[1 - A^2\lambda^4(\overline{\rho} + i\overline{\eta})]} \end{aligned} \tag{4.20}$$

The current absolute values of the CKM matrix are[PG+16]

$$V = \begin{pmatrix} 0.97434^{+0.00011}_{0.00012} & 0.22506 \pm 0.00050 & 0.00357 \pm 0.00015 \\ 0.22492 \pm 0.00050 & 0.97351 \pm 0.00013 & 0.0411 \pm 0.0013 \\ 0.00865^{+0.00032}_{0.00033} & 0.0403 \pm 0.0013 & 0.99915 \pm 0.00005 \end{pmatrix}. \tag{4.21}$$

They can be computed either by employing the parametrization of eq. (4.18) with the following three mixing angles and CP phase

$$\begin{aligned} \theta_{12} &= 13.04 \pm 0.05^o, & \theta_{23} &= 2.38 \pm 0.06^o, \\ \theta_{13} &= 0.201 \pm 0.011^o, & \delta &= 69 \pm 5^o. \end{aligned} \tag{4.22}$$

or by using the Wolfenstein parametrization with the following values

$$\begin{aligned} \lambda &= 0.22506 \pm 0.00050, & A &= 0.811 \pm 0.026, \\ \overline{\rho} &= 0.124^{+0.019}_{-0,018}, & \overline{\eta} &= 0.356 \pm 0.011. \end{aligned} \tag{4.23}$$

The actual matrix elements are reported on [PG+16] and the precision achieved on determining them is remarkable. It is also important to note the strong hierarchy among the mixing angles. The first and also original Cabibbo angle θ_{12} lies near 15^o, but θ_{23} and specially θ_{13} are too small such that mixing elements involving the third generation are below 10^{-1} order.

This hierarchy is fundamentally related to the large mass hierarchy in the quark sector (see table 4.2) which spans all the explored range of energies, from units of MeV to hundreds of GeV.

PMNS MATRIX

The mixing in the lepton sector was studied under a quite different light than quark sector. First, the lack of knowledge about the neutrino masses and their shocking

	NO	IO
$\Delta m^2_{21}/\text{meV}^2$	$75.0^{+1.9}_{-1.7}$	$75.0^{+1.9}_{-1.7}$
$\Delta m^2_{3\ell}/\text{meV}^2$	2524^{+39}_{40}	-2514^{+38}_{41}
$\theta_{12}/^o$	$33.56^{+0.77}_{-0.75}$	$33.56^{+0.77}_{-0.75}$
$\theta_{23}/^o$	$33.56^{+0.77}_{-0.75}$	$33.56^{+0.77}_{-0.75}$
$\theta_{13}/^o$	$8.46^{+0.15}_{-0.15}$	$8.49^{+0.15}_{-0.15}$
$\delta/^o$	261^{+51}_{-59}	277^{+40}_{-46}

Table 4.3: Three-flavor neutrino oscillation data. For NO $\ell = 1$ while for IO $\ell = 2$[Est+17].

weak interactions difficulted their study in the first half of 20th century. However, the outstanding Homestake experiment, directed by Raymond Davis[DJHH68] brought the first evidence of what would be known later as *neutrino oscillations*. An important lack of neutrinos (actually ν^e) from the sun was detected, where two thirds of predicted neutrinos arrived at the chlorine tank. This fact constitutes one of the new hints about contemporary neutrino physics.

Bruno Pontecorvo proposed that neutrino and antineutrinos could oscillate between them in a very special way that it could explain the lack of neutrinos. Notwithstanding, matter-antimatter oscillations were not observed and new proposals appeared. Ziro Maki, Masami Nakagawa and Shoichi Sakata were who proposed the PMNS matrix[MNS62]

$$U = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix}, \tag{4.24}$$

where the subscripts of each component show the mixing between charged and neutral leptons. The magnitudes of the PMNS components at 3σ are[Est+17]

$$V = \begin{pmatrix} 0.800 \to 0.844 & 0.515 \to 0.581 & 0.139 \to 0.155 \\ 0.229 \to 0.516 & 0.438 \to 0.699 & 0.614 \to 0.790 \\ 0.249 \to 0.528 & 0.462 \to 0.715 & 0.595 \to 0.776 \end{pmatrix}. \tag{4.25}$$

The principal data source for determining PMNS matrix elements are neutrino oscillation experiments. Thus, the different experiments around the world study the three main sources of neutrinos to measure each one of the mixing angles using the parametrization in eq. (4.18): solar neutrinos for θ_{12}, atmospheric neutrinos for θ_{23} and reactor/beam neutrinos for θ_{13}. In the same way, since neutrino oscillations do not give information about the individual masses of each mass eigenstate, but about squared mass differences $\Delta m^2_{ij} = m^2_i - m^2_j$, the neutrino mass hierarchy remains unknown and two schemes are considered: the *normal ordering* (NO) in which $m_1 < m_2 < m_3$ while the opposite is the *inverse ordering* (IO) where $m_3 < m_1 < m_2$. The neutrino oscillation parameters are shown in the table 4.3.

Part II

NON-UNIVERSAL U(1)′ MODEL

A new model is built starting from the claim that there exists a new nonuniversal abelian interaction $U(1)_X$ in addition to the strong, weak and electromagnetic forces. The scalar sector is extended in order to break the new symmetry. IN chapter 5, the Higgs potential es minimized, and thereafter the mass matrices and mixin angles of scalars bosons are obtained. Then, the fermion sector is introduced in chapter 6. The mass matrices are obtained from the Yukawa Lagrangians, and the mass eigenvalues, as well as the fermion mixing angles are gotten. Next, by employing Montecarlo procedures, the mass matrices were generated randomly and diagonalized numerically so as the fermion mass hierarchy can be obtained by algebraic and numerical procedures.

5 BOSONIC SECTOR

One of the most preferred extensions to the SM employs the enlargement of the scalar sector by adding new Higgs doublets (and also Higgs singlets) in order to understand some facts such as the top/bottom mass ratio or to provide the SSBs of new gauge symmetries. In particular, the abelian extensions $G_{SM} \otimes U(1)_X$ are extensively employed since it corresponds to the simplest extensions of the SM. These schemes introduce a new gauge boson called Z'_μ which should acquire mass at a higher scale than the electroweak ones, usually at TeV. Consequently, there must be a scalar field with non-zero X-quantum number such that $U(1)_X$ gets broken. The scalar sector of the model satisfies this condition by introducing three Higgs doublets with two Higgs singlets, each one of them characterized by a quantum number and a supplemental parity $\mathbb{Z}_2$ for distinguishing between doublets with the same X quantum number. The notation employed to indicate the X charge and the $\mathbb{Z}_2$ parity is $X^\pm$, and the corresponding charges of the scalar sector are shown in the table 5.1.

This chapter presents the bosonic sector of the model. First, the gauge sector of the model is studied, the masses of the gauge bosons and their mixing are obtained. Second, the Higgs potential is presented with its minimization and the resulting masses and mixing in the scalar sector.

GAUGE BOSONS AND MASSES

The gauge bosons of the model comprises the vector sector of the SM plus the additional Ξ_μ gauge boson of the abelian extension $U(1)_X$. The Gauge Lagrangian is

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4}\text{Tr}\left(\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu}\right) - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}\Xi^{\mu\nu}\Xi_{\mu\nu} \tag{5.1}$$

where $Z'_{\mu\nu}$ is the strength-field tensor of the Z'_μ gauge boson

$$\Xi_{\mu\nu} = \partial_\mu \Xi_\nu - \partial_\nu \Xi_\mu. \tag{5.2}$$

Doublets	$X^\pm$	Singlets	$X^\pm$
$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1+v_1+i\eta_1}{\sqrt{2}} \end{pmatrix}$	$+2/3^+$	$\chi = \frac{\xi_\chi + v_\chi + i\zeta_\chi}{\sqrt{2}}$	$+1/3^+$
$\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2+v_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$	$+2/3^-$	$\psi = \frac{\xi_\psi + v_\psi}{\sqrt{2}}$	0^-
$\Phi_3 = \begin{pmatrix} \phi_3^+ \\ \frac{h_3+v_3+i\eta_3}{\sqrt{2}} \end{pmatrix}$	$+1/3^+$	σ	$+1/3^-$

Table 5.1: Scalar content of the model, non-universal X quantum number and $\mathbb{Z}_2$ parity.

The gauge boson masses, on the other hand, come from the kinetic part of the Higgs Lagrangian

$$\mathcal{L}^{\text{Kin}}_{\text{Higgs}} = \frac{1}{2}\sum_{1,2,3}\left(D^{\mu}\Phi_i\right)^{\dagger}\left(D_{\mu}\Phi_i\right) + \frac{1}{2}\left(D^{\mu}\chi\right)^{*}\left(D_{\mu}\chi\right) + \frac{1}{2}\left(D^{\mu}\psi\right)\left(D_{\mu}\psi\right), \tag{5.3}$$

where de covariant derivatives are

$$D_{\mu}\Phi_i = \partial_{\mu}\Phi_i - ig\mathbf{W}_{\mu}\Phi_i - ig'YB_{\mu}\Phi_i - ig_X X_i \Xi_{\mu}\Phi_i, \tag{5.4a}$$

$$D_{\mu}\chi_i = \partial_{\mu}\chi_i - \tfrac{ig_X}{3}\Xi_{\mu}\chi_i, \qquad D_{\mu}\psi_i = \partial_{\mu}\psi_i. \tag{5.4b}$$

By evaluating the Higgs fields at their VEVs the gauge boson masses appear. The mass of the $W^{\pm}_{\mu}$ is

$$m^2_W = \frac{g^2}{4}\left(v_1^2 + v_2^2 + v_3^2\right) = \frac{g^2 v^2}{4} \tag{5.5}$$

where v is the complete electroweak VEV. In order to simplify the notation, each one of the electroweak VEVs are defined as fractions of v

$$v_1 = v\rho_1, \qquad v_2 = v\rho_2, \qquad v_3 = v\rho_3, \tag{5.6}$$

and the coefficients ρ_i satisfy the contraint

$$\rho_1^2 + \rho_2^2 + \rho_3^2 = 1. \tag{5.7}$$

This parametrization will be employed in the following sections. Regarding to the neutral gauge bosons, the mass matrix in the basis $\mathbf{W}^0_{\mu} = (B_{\mu}, W^3_{\mu}, \Xi_{\mu})$ is

$$M^2_{\mathbf{W}^0} = \begin{pmatrix} g_Y^2 v^2 & -g g_Y v^2 & \frac{2}{3} g_Y g_X v^2\left(2-\rho_3^2\right) \\ g g_Y v^2 & g^2 v^2 & \frac{2}{3} g v^2 g_X\left(2-\rho_3^2\right) \\ \frac{2}{3} g_Y g_X v^2\left(2-\rho_3^2\right) & \frac{2}{3} g v^2 g_X\left(2-\rho_3^2\right) & \frac{4}{9} g_X^2\left(\left(4-3\rho_3^2\right)v^2 + v_\chi^2\right) \end{pmatrix} \tag{5.8}$$

Its determinant is null as it is hoped because the existence of a massless gauge boson, the photon A_{μ}. In addition, there are two massive gauge bosons, the electroweak Z_{μ} at GeV scale, and the new Z'_{μ} at TeV

$$m_Z^2 \approx \frac{g^2 + g'^2}{4} v^2 = \frac{g^2 v^2}{4 c_W^2}, \tag{5.9}$$

$$m_{Z'}^2 \approx \frac{g_X^2 v_\chi^2}{9} - \frac{g_X^2 \rho_3^2 v^2}{3} + \frac{4 g_X^2 v^2}{9}. \tag{5.10}$$

The mass eigenstates $\mathbf{Z}_{\mu} = (A_{\mu}, Z_{\mu}, Z'_{\mu})$ are obained as $\mathbf{Z}_{\mu} = R_{\mathbf{W}^0}\mathbf{W}^0_{\mu}$ through the mixing matrix $R_{\mathbf{W}^0}$. In the CKM-parametrization (eq. (4.18)) its angles are

$$\tan\theta^{\mathbf{W}^0}_{12} = \frac{g'}{g}, \quad \tan\theta^{\mathbf{W}^0}_{23} = \frac{3g}{2c_W g_X}\frac{\left(\left(2-\rho_3^2\right)v^2\right)}{v_\chi^2}, \quad \tan\theta^{\mathbf{W}^0}_{13} = 0. \tag{5.11}$$

The first angle turns out to be the well-known Weinberg angle, while the second one describes the $Z_{\mu} - Z'_{\mu}$ mixing.

HIGGS POTENTIAL AND SCALAR MASSES

The scalar potential of the model is stablished according to the $U(1)_X$ charges and $\mathbb{Z}_2$ parities shown in the table 7.1. So, the most general potential invariant under the $G_{SM} \otimes U(1)_X \otimes \mathbb{Z}_2$ symmetry is

$$\begin{aligned}
V_H = &\mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_3^2 \Phi_3^\dagger \Phi_3 + \mu_\chi^2 \chi^* \chi + \mu_\psi^2 \psi^2 \\
&- \frac{f_\chi}{\sqrt{2}} \left(\Phi_3^\dagger \Phi_1 \chi + \text{h.c.} \right) - \frac{f_\psi}{\sqrt{2}} \left(\Phi_3^\dagger \Phi_2 \psi + \text{h.c.} \right) \\
&+ \lambda_{11} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \lambda_{12} \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) - \lambda'_{12} \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) \\
&+ \lambda_{22} \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_{23} \left(\Phi_2^\dagger \Phi_2 \right) \left(\Phi_3^\dagger \Phi_3 \right) - \lambda'_{23} \left(\Phi_2^\dagger \Phi_3 \right) \left(\Phi_3^\dagger \Phi_2 \right) \\
&+ \lambda_{33} \left(\Phi_3^\dagger \Phi_3 \right)^2 + \lambda_{13} \left(\Phi_1^\dagger \Phi_3 \right) \left(\Phi_3^\dagger \Phi_1 \right) - \lambda'_{13} \left(\Phi_1^\dagger \Phi_3 \right) \left(\Phi_3^\dagger \Phi_1 \right) \\
&+ \lambda_{1\chi} \left(\Phi_1^\dagger \Phi_1 \right) (\chi^* \chi) + \lambda_{2\chi} \left(\Phi_2^\dagger \Phi_2 \right) (\chi^* \chi) + \lambda_{3\chi} \left(\Phi_3^\dagger \Phi_3 \right) (\chi^* \chi) \\
&+ \lambda_{1\psi} \left(\Phi_1^\dagger \Phi_1 \right) (\psi^2) + \lambda_{2\psi} \left(\Phi_2^\dagger \Phi_2 \right) (\psi^2) + \lambda_{3\psi} \left(\Phi_3^\dagger \Phi_3 \right) (\psi^2) \\
&+ \lambda_{\chi\chi} (\chi^* \chi)^2 + \lambda_{\chi\psi} (\chi^* \chi) (\psi^2) + \lambda_{\psi\psi} (\psi^4) .
\end{aligned} \tag{5.12}$$

Minimization of the potential

The previous potential is minimized by differentiating it respect to each one of the VEVs and isolating the quadratic constants μ_i where $i = 1, 2, 3, \chi, \psi$. Thus, the following constants are obtained

$$-\mu_1^2 = \lambda_{11} v_1^2 + \Lambda_{12} v_2^2 + \Lambda_{13} v_3^2 + \frac{\lambda_{1\chi} v_\chi^2}{2} + \frac{\lambda_{1\psi} v_\psi^2}{2} - \frac{f_\chi \rho_3 v_\chi}{2\rho_1} \tag{5.13a}$$

$$-\mu_2^2 = \lambda_{22} v_2^2 + \Lambda_{23} v_3^2 + \Lambda_{12} v_1^2 + \frac{\lambda_{2\chi} v_\chi^2}{2} + \frac{\lambda_{2\psi} v_\psi^2}{2} - \frac{f_\psi \rho_3 v_\psi}{2\rho_2} \tag{5.13b}$$

$$-\mu_3^2 = \lambda_{33} v_3^2 + \Lambda_{13} v_1^2 + \Lambda_{23} v_2^2 + \frac{\lambda_{3\chi} v_\chi^2}{2} + \frac{\lambda_{3\psi} v_\psi^2}{2} - \frac{f_\chi \rho_1 v_\chi + f_\psi \rho_2 v_\psi}{2\rho_3} \tag{5.13c}$$

$$-\mu_\chi^2 = \lambda_{\chi\chi} v_\chi^2 + \frac{\lambda_{\chi\psi} v_\psi^2}{2} + \frac{v_1^2 \lambda_{1\chi}}{2} + \frac{v_2^2 \lambda_{2\chi}}{2} + \frac{v_3^2 \lambda_{3\chi}}{2} - \frac{f_\chi v_1 v_3}{2 v_\chi} \tag{5.13d}$$

$$-\mu_\psi^2 = \lambda_{\psi\psi} v_\psi^2 + \frac{\lambda_{\chi\psi} v_\chi^2}{2} + \frac{v_1^2 \lambda_{1\psi}}{2} + \frac{v_2^2 \lambda_{2\psi}}{2} + \frac{v_3^2 \lambda_{3\psi}}{2} - \frac{f_\psi v_2 v_3}{2 v_\psi} \tag{5.13e}$$

where $\Lambda_{ij} = (\lambda_{ij} - \lambda'_{ij})/2$.

Charged scalar boson masses

The mass matrix of the charged bosons is obtained by calculating the Hessian matrix respect to the charged components of the Higgs doublets. In the basis $\boldsymbol{\phi}^{\pm} = (\phi_1^{\pm}, \phi_2^{\pm}, \phi_3^{\pm})$ it turns out to be

$$M_C^2 = \frac{1}{4}\begin{pmatrix} \frac{f_\chi v_\chi \rho_3}{\rho_1} + \lambda'_{11} v^2 & -\rho_1\rho_2\lambda'_{12}v^2 & -\rho_1\rho_3\lambda'_{13}v^2 - f_\chi v_\chi \\ -\rho_1\rho_2\lambda'_{12}v^2 & \frac{f_\psi v_\psi \rho_3}{\rho_2} + \lambda'_{22}v^2 & -\rho_2\rho_3\lambda'_{23}v^2 - f_\psi v_\psi \\ -\rho_1\rho_3\lambda'_{13}v^2 - f_\chi v_\chi & -\rho_2\rho_3\lambda'_{23}v^2 - f_\psi v_\psi & \frac{\rho_1 f_\chi v_\chi + \rho_2 f_\psi v_\psi}{\rho_3} + \lambda'_{33}v^2 \end{pmatrix} \quad (5.14)$$

where $\lambda'_{11} = \lambda'_{12}\rho_2^2 + \rho_3^2\lambda'_{13}$, $\lambda'_{22} = \lambda'_{12}\rho_1^2 + \rho_3^2\lambda'_{23}$ and $\lambda'_{33} = \lambda'_{13}\rho_1^2 + \rho_2^2\lambda'_{23}$. Its determinant is null as it is hoped because the existence of $G_W^{\pm}$, the Goldstone bosons of $W_\mu^{\pm}$. Additionally there exist two physical charged bosons $H_1^{\pm}$ and $H_2^{\pm}$ which acquire mass at TeV scale with contributions at hundreds of GeV.

The masses of the physical charged bosons are (at order $\mathcal{O}(v^2)$)

$$\begin{aligned} m^2_{H^{\pm}_{1,2}} \approx\ & \frac{f_\chi\left(\rho_1^2+\rho_3^2\right)v_\chi}{8\rho_1\rho_3} + \frac{f_\psi\left(\rho_2^2+\rho_3^2\right)v_\psi}{8\rho_2\rho_3} \\ & \pm \sqrt{\frac{{f_\chi}^2(\rho_1^2+\rho_3^2)^2 v_\chi^2}{64\rho_1^2\rho_3^2} + \frac{f_\chi f_\psi(\rho_1^2\rho_2^2-\rho_3^4)v_\chi v_\psi}{32\rho_1\rho_2\rho_3^2} + \frac{{f_\psi}^2(\rho_2^2+\rho_3^2)^2 v_\psi^2}{64\rho_2^2\rho_3^2}}. \end{aligned} \quad (5.15)$$

It is straightforward to see two limit cases in this expression. The first one comprises $f_\chi = 0$ which implies an extra charged boson $h^{\pm}$ at GeV scale and other $H^{\pm}$ at TeV scale

$$m_{H_1^{\pm}} \approx \frac{\lambda'_{22}v^2}{4\left(\rho_1^2+\rho_3^2\right)}, \qquad m_{H_2^{\pm}} \approx \frac{f_\chi\left(\rho_1^2+\rho_3^2\right)v_\chi}{4\rho_1\rho_3}.$$

The opposite case is $f_\psi = 0$, and yields similar results than the previous one

$$m_{H_1^{\pm}} \approx \frac{\lambda'_{11}v^2}{4\left(\rho_1^2+\rho_3^2\right)}, \qquad m_{H_2^{\pm}} \approx \frac{f_\psi\left(\rho_2^2+\rho_3^2\right)v_\psi}{4\rho_2\rho_3}.$$

The mixing matrix R_C diagonalizes the mass matrix M_C^2 obtaining the mass eigenstates $\mathbf{H}^{\pm} = R_C\boldsymbol{\phi}^{\pm}$ which are expressed in the basis $\mathbf{H}^{\pm} = (G_W^{\pm}, H_1^{\pm}, H_2^{\pm})$. Its corresponding mixing angles in the CKM parametrization (eq. (4.18)) are

$$\tan^2\theta_{12}^C = \frac{\rho_2^2}{\rho_1^2}, \quad (5.16a)$$

$$\tan^2\theta_{23}^{odd} \approx \frac{g(f_\chi,f_\psi) - \sqrt{g(f_\chi,f_\psi)^2 - 4f_\chi f_\psi\rho_1\rho_2\rho_3^2 v_\chi v_\psi}}{g(f_\chi,f_\psi) + \sqrt{g(f_\chi,f_\psi)^2 - 4f_\chi f_\psi\rho_1\rho_2\rho_3^2 v_\chi v_\psi}}, \quad (5.16b)$$

$$\tan^2\theta_{13}^C = \frac{\rho_3}{\rho_1^2+\rho_2^2}. \quad (5.16c)$$

where $g(f_\chi,f_\psi) = f_\chi\rho_2\left(\rho_1^2+\rho_3^2\right)v_\chi + f_\psi\rho_1\left(\rho_2^2-\rho_3^2\right)v_\psi$. Similarly, the two previous limit cases give the angles

$$\begin{cases} \tan^2\theta_{12}^C = 0, & \tan^2\theta_{23}^C = 0, \quad \tan^2\theta_{13}^C = \dfrac{\rho_3}{\rho_1^2} \quad \text{if} \quad f_\psi = 0, \\ \tan^2\theta_{12}^C = \infty, & \tan^2\theta_{23}^C = 0, \quad \tan^2\theta_{13}^C = \dfrac{\rho_3}{\rho_2^2} \quad \text{if} \quad f_\chi = 0. \end{cases} \quad (5.17)$$

CP-odd boson masses

The mass matrix of the CP-odd (pseudoscalar) bosons is obtained by calculating the Hessian matrix respect to the CP-odd components of the Higgs doublets. In the basis $\boldsymbol{\eta} = (\eta_1, \eta_2, \eta_3, \zeta_\chi)$ it turns out to be

$$M^2_{\text{odd}} = \frac{1}{4}\begin{pmatrix} \frac{f_\chi v_\chi \rho_3}{\rho_1} & 0 & -f_\chi v_\chi & v f_\chi \rho_3 \\ 0 & \frac{f_\psi v_\psi \rho_3}{\rho_2} & -f_\psi v_\psi & 0 \\ -f_\chi v_\chi & -f_\psi v_\psi & \frac{f_\chi v_\chi \rho_1}{\rho_3} + \frac{f_\psi v_\psi \rho_2}{\rho_3} & -v f_\chi \rho_1 \\ v f_\chi \rho_3 & 0 & -v f_\chi \rho_1 & \frac{v^2 f_\chi \rho_1 \rho_3}{v_\chi} \end{pmatrix} \tag{5.18}$$

Its determinant is null as it is hoped because the existence of G_Z and G'_Z, the Goldstone bosons of Z_μ and Z'_μ, respectively. Additionally there exist two physical pseudoscalar bosons A_1 and A_2 which acquire mass at TeV scale with contributions at hundreds of GeV.

The masses of the physical pseudoscalar bosons are (at order $\mathcal{O}(v^2)$)

$$\begin{aligned} m^2_{A_{1,2}} &= \frac{f_\chi\left(\rho_1^2+\rho_3^2\right)v_\chi}{8\rho_1\rho_3} + \frac{f_\psi\left(\rho_2^2+\rho_3^2\right)v_\psi}{8\rho_2\rho_3} \\ &\pm \sqrt{\frac{{f_\chi}^2(\rho_1^2+\rho_3^2)^2 v_\chi^2}{64\rho_1^2\rho_3^2} + \frac{f_\chi f_\psi(\rho_1^2\rho_2^2-\rho_3^2)v_\chi v_\psi}{32\rho_1\rho_2\rho_3^2} + \frac{{f_\psi}^2(\rho_2^2+\rho_3^2)^2 v_\psi^2}{64\rho_2^2\rho_3^2}}, \end{aligned} \tag{5.19}$$

which are slightly different of $m^\pm_C$, the charged bosons masses. Thus, the two limit cases of the masses expression outlined in the previous section are similar here, with the exception that there appears an extra massless pseudoscalar boson since there are no λ'_{ij} terms in M^2_{odd}.

The mixing matrix R_{odd} diagonalizes the mass matrix M^2_{odd} obtaining the mass eigenstates $\mathbf{A} = R_{\text{odd}}\boldsymbol{\eta}$ which are expressed in the basis $\mathbf{A} = (G_Z, A_1, A_2, G'_Z)$. Moreover, the diagonalization in this case is a little more complicated because there are four bosons instead of three in comparison with the charged scalar boson sector. So, it was implemented an extended-CKM parametrization which includes mixings with a fourth component

$$\begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14} & 0 & 0 & c_{14} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{23} & s_{23} & 0 \\ 0 & -s_{23} & c_{23} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13} & 0 \\ 0 & 1 & 0 & 0 \\ -s_{13} & 0 & c_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_{12} & s_{12} & 0 & 0 \\ -s_{12} & c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13}c_{14} & c_{13}c_{14}s_{12} & c_{14}s_{13} & s_{14} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} & 0 \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} & 0 \\ -c_{12}c_{13}s_{14} & -c_{13}s_{12}s_{14} & -s_{13}s_{14} & c_{14} \end{pmatrix}. \tag{5.20}$$

Thereby, the corresponding mixing angles are

$$\tan^2\theta_{12}^{\text{odd}} = \frac{\rho_2^2}{c_{14}^2\rho_1^2}, \tag{5.21a}$$

$$\tan^2\theta_{23}^{\text{odd}} \propto \frac{g(f_\chi, f_\psi) - \sqrt{g(f_\chi, f_\psi)^2 - 4f_\chi f_\psi \rho_1\rho_2\rho_3^2 v_\chi v_\psi}}{g(f_\chi, f_\psi) + \sqrt{g(f_\chi, f_\psi)^2 - 4f_\chi f_\psi \rho_1\rho_2\rho_3^2 v_\chi v_\psi}}, \tag{5.21b}$$

$$\tan^2\theta_{13}^{\text{odd}} = \frac{\rho_3^2}{c_{14}^2\rho_1^2 + \rho_2^2 + \rho_3^2}, \tag{5.21c}$$

$$\tan^2\theta_{14}^{\text{odd}} = \frac{v\rho_1}{v_\chi}, \tag{5.21d}$$

where $g(f_\chi, f_\psi) = f_\chi\rho_2\left(\rho_1^2 + \rho_3^2\right)v_\chi + f_\psi\rho_1\left(\rho_2^2 - \rho_3^2\right)v_\psi$. Similarly, the two previous limit cases give the angles

$$\begin{cases} \tan^2\theta_{13}^{\text{odd}} = \dfrac{\rho_3}{c_{14}^2\rho_1^2}, & \tan^2\theta_{14}^{\text{odd}} = \dfrac{v\rho_1}{v_\chi}, \quad \theta_{12}^{\text{odd}} = \theta_{23}^{\text{odd}} = 0, \quad \text{if} \quad f_\psi = 0, \\ \tan^2\theta_{12}^{\text{odd}} = \infty, & \tan^2\theta_{13}^{\text{odd}} = \dfrac{\rho_3}{\rho_2^2}, \quad \theta_{23}^{\text{odd}} = \theta_{14}^{\text{odd}} = 0, \quad \text{if} \quad f_\chi = 0. \end{cases} \tag{5.22}$$

CP-even boson masses

The mass matrix of the CP-even (true scalar) bosons is obtained by calculating the Hessian matrix respect to the CP-even components of the Higgs doublets. In the basis $\mathbf{h} = (h_1, h_2, h_3, \xi_\chi, \xi_\psi)$ the CP-even mass matrix is

$$M_{\text{even}}^2 = \begin{pmatrix} \mathcal{M}_{hh} & \mathcal{M}_{h\xi} \\ \mathcal{M}_{h\xi}^T & \mathcal{M}_{\xi\xi} \end{pmatrix}, \tag{5.23}$$

where the blocks are defined as

$$\begin{aligned} \mathcal{M}_{hh} &= \begin{pmatrix} \lambda_{11}v^2{\rho_1}^2 + \frac{f_\chi v_\chi \rho_3}{4\rho_1} & \Lambda_{12}v^2\rho_1\rho_2 & \Lambda_{13}v^2\rho_1\rho_3 - \frac{f_\chi v_\chi}{4} \\ \Lambda_{12}v^2\rho_1\rho_2 & \lambda_{22}v^2{\rho_2}^2 + \frac{f_\psi v_\psi \rho_3}{4\rho_2} & \Lambda_{23}v^2\rho_2\rho_3 - \frac{f_\psi v_\psi}{4} \\ \Lambda_{13}v^2\rho_1\rho_3 - \frac{f_\chi v_\chi}{4} & \Lambda_{23}v^2\rho_2\rho_3 - \frac{f_\psi v_\psi}{4} & \lambda_{33}v^2{\rho_3}^2 + \frac{f_\chi v_\chi \rho_1 + f_\psi v_\psi \rho_2}{4\rho_3} \end{pmatrix} \\ \mathcal{M}_{h\xi} &= \begin{pmatrix} \frac{1}{2}\lambda_{1\chi}v_\chi v\rho_1 - \frac{1}{4}vf_\chi\rho_3 & \frac{1}{2}\lambda_{1\psi}v_\psi v\rho_1 \\ \frac{1}{2}\lambda_{2\chi}v_\chi v\rho_2 & \frac{1}{2}\lambda_{2\psi}vv_\psi\rho_2 - \frac{1}{4}vf_\psi\rho_3 \\ \frac{1}{2}\lambda_{3\chi}v_\chi v\rho_3 - \frac{1}{4}vf_\chi\rho_1 & \frac{1}{2}\lambda_{3\psi}vv_\psi\rho_3 - \frac{1}{4}vf_\psi\rho_2 \end{pmatrix} \\ \mathcal{M}_{\xi\xi} &= \begin{pmatrix} \lambda_{\chi\chi}v_\chi^2 + \frac{f_\chi\rho_1\rho_3 v^2}{4v_\chi} & \Lambda_{\chi\psi}v_\chi v_\psi \\ \Lambda_{\chi\psi}v_\chi v_\psi & \lambda_{\psi\psi}v_\psi^2 + \frac{f_\psi\rho_2\rho_3 v^2}{4v_\psi} \end{pmatrix}. \end{aligned} \tag{5.24}$$

The mixing matrix R_{even} which diagonalizes the mass matrix M_{even}^2 gives the mass eigenstates $\mathbf{H} = R_{\text{even}}\mathbf{h}$ which are expressed in the basis $\mathbf{H} = (h, H_1, H_2, \mathcal{H}_1, \mathcal{H}_2)$. Moreover, R_{even} splits in a see-saw rotation R_{even}^{SS} and a block-diagonal rotation R_{even}^{B} such that $R_{\text{even}} = R_{\text{even}}^{B}R_{\text{even}}^{SS}$.

Since $|\mathcal{M}_{hh}| < |\mathcal{M}_{h\xi}| < |\mathcal{M}_{\xi\xi}|$ the see-saw procedure will be implemented by following the reference [MMO17] which block-diagonalizes $\mathcal{M}_{hh}$ such that the h scalars get separated from the ξ ones. The following approximations are made on the blocks in order to avoid cumbersome expressions after rotating out the ξ scalars:

$$\mathcal{M}_{h\xi} \approx \begin{pmatrix} \frac{1}{2}\lambda_{1\chi}v_\chi v\rho_1 & \frac{1}{2}\lambda_{1\psi}v_\psi v\rho_1 \\ \frac{1}{2}\lambda_{2\chi}v_\chi v\rho_2 & \frac{1}{2}\lambda_{2\psi}v v_\psi\rho_2 \\ \frac{1}{2}\lambda_{3\chi}v_\chi v\rho_3 & \frac{1}{2}\lambda_{3\psi}v v_\psi\rho_3 \end{pmatrix}, \qquad \mathcal{M}_{\xi\xi} \approx \begin{pmatrix} \lambda_{\chi\chi}v_\chi^2 & 0 \\ 0 & \lambda_{\psi\psi}v_\psi^2 \end{pmatrix}. \tag{5.25}$$

The see-saw rotation $R_{\text{even,SS}}$ and its angle Θ_{even} are

$$R^{SS}_{\text{even}} = \begin{pmatrix} 1 & -\Theta^\dagger_{\text{even}} \\ \Theta_{\text{even}} & 1 \end{pmatrix}, \quad \Theta^\dagger_{\text{even}} = \mathcal{M}^{-1}_{\xi\xi}\mathcal{M}_{h\xi} = \begin{pmatrix} \frac{\lambda_\chi v\rho_1}{2v_\chi\lambda_{\chi\chi}} & \frac{\lambda_\psi v\rho_1}{2v_\psi\lambda_{\psi\psi}} \\ \frac{\lambda_{2\chi}v\rho_2}{2v_\chi\lambda_{\chi\chi}} & \frac{\lambda_{2\psi}v\rho_2}{2v_\psi\lambda_{\psi\psi}} \\ \frac{\lambda_{3\chi}v\rho_3}{2v_\chi\lambda_{\chi\chi}} & \frac{\lambda_{3\psi}v\rho_3}{2v_\psi\lambda_{\psi\psi}} \end{pmatrix}. \tag{5.26}$$

The block-diagonalization acts in the following way

$$R^{SS}_{\text{even}}\mathcal{M}_{hh}\left(R^{SS}_{\text{even}}\right)^T = \begin{pmatrix} M^2_{hh} & 0 \\ 0 & M^2_{\xi\xi} \end{pmatrix}. \tag{5.27}$$

where the new blocks are

$$M^2_{hh} \approx \mathcal{M}_{hh} - \mathcal{M}_{h\xi}\mathcal{M}^{-1}_{\xi\xi}\mathcal{M}^T_{h\xi}, \qquad M^2_{\xi\xi} \approx \mathcal{M}_{\xi\xi} \tag{5.28}$$

The resulting matrix M_{hh} has the same algebraic structure of $\mathcal{M}_{hh}$ with new definitions of the constants Λ_{ij}'s, where $i, j = 1, 2, 3$. The matrix turns out to be

$$\begin{aligned} M^2_{hh} &\approx \mathcal{M}_{hh} - \mathcal{M}_{h\xi}\mathcal{M}^{-1}_{\xi\xi}\mathcal{M}^T_{h\xi} \\ &= \begin{pmatrix} \widetilde{\Lambda}_{11}v^2{\rho_1}^2 + \frac{f_\chi v_\chi\rho_3}{4\rho_1} & \widetilde{\Lambda}_{12}v^2\rho_1\rho_2 & \widetilde{\Lambda}_{13}v^2\rho_1\rho_3 - \frac{f_\chi v_\chi}{4} \\ \widetilde{\Lambda}_{12}v^2\rho_1\rho_2 & \widetilde{\Lambda}_{22}v^2{\rho_2}^2 + \frac{f_\psi v_\psi\rho_3}{4\rho_2} & \widetilde{\Lambda}_{23}v^2\rho_2\rho_3 - \frac{f_\psi v_\psi}{4} \\ \widetilde{\Lambda}_{13}v^2\rho_1\rho_3 - \frac{f_\chi v_\chi}{4} & \widetilde{\Lambda}_{23}v^2\rho_2\rho_3 - \frac{f_\psi v_\psi}{4} & \widetilde{\Lambda}_{33}v^2{\rho_3}^2 + \frac{f_\chi v_\chi\rho_1 + f_\psi v_\psi\rho_2}{4\rho_3} \end{pmatrix} \end{aligned} \tag{5.29}$$

where the tilde constants are

$$\begin{aligned} \widetilde{\Lambda}_{11} &= \lambda_{11} - \frac{\lambda^2_{1\psi}}{4\lambda_{\psi\psi}} - \frac{\lambda^2_{1\chi}}{4\lambda_{\chi\chi}}, & \widetilde{\Lambda}_{12} &= \Lambda_{12} - \frac{\lambda_{1\psi}\lambda_{2\psi}}{4\lambda_{\psi\psi}} - \frac{\lambda_{1\chi}\lambda_{2\chi}}{4\lambda_{\chi\chi}}, \\ \widetilde{\Lambda}_{22} &= \lambda_{22} - \frac{\lambda^2_{2\psi}}{4\lambda_{\psi\psi}} - \frac{\lambda^2_{2\chi}}{4\lambda_{\chi\chi}}, & \widetilde{\Lambda}_{23} &= \Lambda_{23} - \frac{\lambda_{2\psi}\lambda_{3\psi}}{4\lambda_{\psi\psi}} - \frac{\lambda_{2\chi}\lambda_{3\chi}}{4\lambda_{\chi\chi}}, \\ \widetilde{\Lambda}_{33} &= \lambda_{33} - \frac{\lambda^2_{3\psi}}{4\lambda_{\psi\psi}} - \frac{\lambda^2_{3\chi}}{4\lambda_{\chi\chi}}, & \widetilde{\Lambda}_{13} &= \Lambda_{13} - \frac{\lambda_{1\psi}\lambda_{3\psi}}{4\lambda_{\psi\psi}} - \frac{\lambda_{1\chi}\lambda_{3\chi}}{4\lambda_{\chi\chi}}. \end{aligned} \tag{5.30}$$

Although the characteristic equation of M^2_{hh} is difficult to solve, the matrix suggests the same structure of M^2_C. Therefore, M^2_{hh} should have two mass eigenvalues $m^2_{H_{1,2}}$ at TeV scale and a third one m^2_h at hundreds of GeV which would be zero if the electroweak vacuum v is neglected. Indeed, the eigenvalues of

$$M^2_{hh} = \begin{pmatrix} \frac{f_\chi v_\chi\rho_3}{4\rho_1} & 0 & -\frac{f_\chi v_\chi}{4} \\ 0 & \frac{f_\psi v_\psi\rho_3}{4\rho_2} & -\frac{f_\psi v_\psi}{4} \\ -\frac{f_\chi v_\chi}{4} & -\frac{f_\psi v_\psi}{4} & \frac{f_\chi v_\chi\rho_1 + f_\psi v_\psi\rho_2}{4\rho_3} \end{pmatrix} + \mathcal{O}(v^2) \tag{5.31}$$

are the same of M_C^2. However, the non-vanishing determinant of M_{hh}^2 shows the existence of the smallest eigenvalue, which can be obtained by dividing the determinant of M_{hh}^2 by the product of the two largest eigenvalues

$$m_h \approx \frac{\mathrm{Det}[M_{hh}^2]}{m_{H_1}^2 m_{H_2}^2} = \Lambda_{hh} v^2 \tag{5.32}$$

where Λ_{hh} is the effective coupling constant of the 125 GeV Higgs boson

$$\Lambda_{hh} = \widetilde{\Lambda}_{11}\rho_1^4 + \widetilde{\Lambda}_{22}\rho_2^4 + \widetilde{\Lambda}_{33}\rho_3^4 + 2\widetilde{\Lambda}_{12}\rho_2^2\rho_1^2 + 2\widetilde{\Lambda}_{13}\rho_3^2\rho_1^2 + 2\widetilde{\Lambda}_{23}\rho_2^2\rho_3^2. \tag{5.33}$$

Regarding to R_{even}^B

$$R_{\text{even}}^B = \begin{pmatrix} R_{\text{even}}^{hh} & 0 \\ 0 & R_{\text{even}}^{\xi\xi} \end{pmatrix}, \tag{5.34}$$

R_{even}^{hh} and $R_{\text{even}}^{\xi\xi}$ diagonalize M_{hh}^2 and $M_{\xi\xi}^2$, respectively. The mixing matrix R_{even}^{hh} can be approximated to R_C because the method employed in the eigenvalue search. Thus, the corresponding mixing angles of R_{even}^{hh} are

$$\tan^2\theta_{12}^{hh} \approx \frac{\rho_2^2}{\rho_1^2}, \tag{5.35a}$$

$$\tan^2\theta_{23}^{hh} \lesssim \frac{g(f_\chi, f_\psi) - \sqrt{g^2(f_\chi, f_\psi) - 4f_\chi f_\psi \rho_1\rho_2\rho_3^2 v_\chi v_\psi}}{g(f_\chi, f_\psi) + \sqrt{g^2(f_\chi, f_\psi) - 4f_\chi f_\psi \rho_1\rho_2\rho_3^2 v_\chi v_\psi}}, \tag{5.35b}$$

$$\tan^2\theta_{13}^{hh} \approx \frac{\rho_3}{\rho_1^2 + \rho_2^2}. \tag{5.35c}$$

On the other hand, $R_{\text{even}}^{\xi\xi}$ is parametrized by the two-dimensional rotation matrix. Its angle is proportional to $\Lambda_{\chi\psi}$ and gives the largest eigenvalues of M_{even}^2,

$$m_{\mathcal{H}_1}^2 = \lambda_{\chi\chi} v_\chi^2, \qquad m_{\mathcal{H}_2}^2 = \lambda_{\psi\psi} v_\psi^2. \tag{5.36}$$

Summary of masses of the scalar and gauge sector

Boson	Spin	Mass	Boson	Spin	Mass	Boson	Spin	Mass
Gauge			SM Scalar			Non-SM Scalar		
A_μ	1	0	h	0	m_h	$H_{1,2}$	0	m_H
$W_\mu^\pm$	1	m_W	$G_W^\pm$	0	0	$H_{1,2}^\pm$	0	m_H
Z_μ	1	m_Z	G_Z	0	0	$A_{1,2}$	0	m_H
Z'_μ	1	$m_{Z'}$	$G_{Z'}$	0	0	$\mathcal{H}_{1,2}$	0	$m_{\mathcal{H}}$

Table 5.2: Summary of the bosonic mass eigenstates of the model.

6 FERMIONIC SECTOR

The set of fermions of the models is determined by three different principles: the chiral anomalies coming from the non-universal $U(1)_X$ quantum numbers, the suited mass textures and the minimal number of exotic fermions. Nevertheless, before addressing the fermionic sector, it is important to do some observations on the fermionic spectrum of the SM (see figure 6.1).

There exist four hierarchical groups: (e, u, d) at units of MeV, (s, μ) at hundreds of MeV, (c, τ, b) at units of GeV and t at hundreds of GeV (see Fig. 6.1). These groups may suggest similar mass acquisition mechanisms among them, for example, the mass acquisition of the u quark could be similar to the d quark and the electron. On the other hand, the mixing angles of the quark sector are remarkably different than the lepton sector. The CKM angles are hierarchical too, the Cabbibo angle $\theta_{12} = 13.04^o$ is one order of magnitude larger than the $\theta_{23} = 2.3^o$, which is also larger than $\theta_{13} = 0.2^o$. This behavior is not observed in lepton mixing where $\theta_{12} = 33^o$ and $\theta_{23} = 45^o$ lie at the same order of magnitude, while $\theta_{13} = 8^o$ is the only small angle.

The references [Mar+14b], [MMO17] and [MM17] present some models where the first family acquires mass through radiative corrections done by the new exotic fermions together with the scalar σ which does not have VEV. On the contrary, the model presented here does not require any kind of radiative corrections in order to get the phenomenological spectrum of fermion masses. Furthermore, this chapter presents how the model can be consistent with the aforementioned observations without unpleasant fine-tuning procedures. The mass matrices suggest the mass and CKM angle hierarchies in a natural manner, while the PMNS angles can also be obtained because of the existence of Majorana fermions in the neutral sector, which allow larger mixings among active neutrinos after performing the seesaw with the heavier neutrino species.

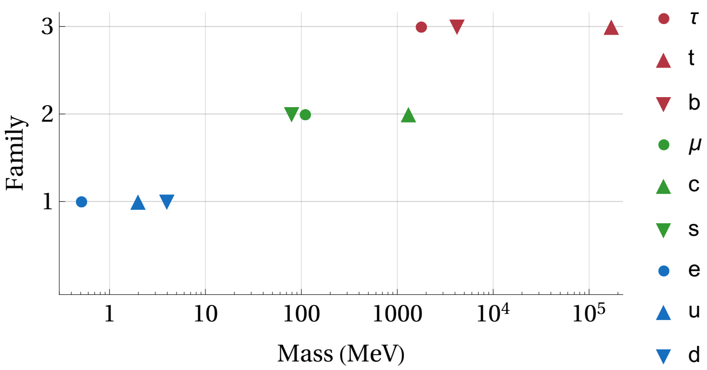

Figure 6.1: Orders of magnitude of the SM fermion masses. It is easy to realize about how the fermions get organized in four hierarchical groups.

CHIRAL ANOMALIES EQUATIONS

The most important constrains on the X-charges of the fermions are the scalar sector and the cancellation of chiral anomalies. Since the Higgs doublets and singlets are $U(1)_X$-charged (see Tab. 5.1), the Yukawa coupling constants require the left- and right-handed fermions with different $U(1)_X$ quantum numbers and the posibility to obtain chiral anomalies in the model appears. Thus, in order to cancel such anomalies from the very beginning, the non-universal X charges must vanish the following equations which comprise the different chiral anomalies in the model[Mar+14b]:

$$
\begin{aligned}
[SU(3)_C]^2\, U(1)_X \to \quad A_C &= \sum_Q X_{Q_L} - \sum_Q X_{Q_R} \\
[SU(2)_L]^2\, U(1)_X \to \quad A_L &= \sum_\ell X_{\ell_L} + 3\sum_Q X_{Q_L} \\
[U(1)_Y]^2\, U(1)_X \to \quad A_{Y^2} &= \sum_{\ell,Q} \left[Y_{\ell_L}^2 X_{\ell_L} + 3Y_{Q_L}^2 X_{Q_L}\right] - \sum_{\ell,Q} \left[Y_{\ell_R}^2 X_{L_R} + 3Y_{Q_R}^2 X_{Q_R}\right] \\
U(1)_Y\, [U(1)_X]^2 \to \quad A_Y &= \sum_{\ell,Q} \left[Y_{\ell_L} X_{\ell_L}^2 + 3Y_{Q_L} X_{Q_L}^2\right] - \sum_{\ell,Q} \left[Y_{\ell_R} X_{\ell_R}^2 + 3Y_{Q_R} X_{Q_R}^2\right] \\
[U(1)_X]^3 \to \quad A_X &= \sum_{\ell,Q} \left[X_{\ell_L}^3 + 3X_{Q_L}^3\right] - \sum_{\ell,Q} \left[X_{\ell_R}^3 + 3X_{Q_R}^3\right] \\
[\mathrm{Grav}]^2\, U(1)_X \to \quad A_G &= \sum_{\ell,Q} \left[X_{\ell_L} + 3X_{Q_L}\right] - \sum_{\ell,Q} \left[X_{\ell_R} + 3X_{Q_R}\right]
\end{aligned} \tag{6.1}
$$

These equations get cancelled by the fermionic spectrum shown in tables 6.1 and 6.2. It includes the three families of the SM, two up-like quarks $\mathcal{T}^{1,2}$, two down-like quarks $\mathcal{J}^{1,2}$ and three charged leptons $\mathcal{E}^{1,2,3}$. These fields were added to cancel chiral anomalies, but they turned out to be really important in understanding the fermion mass hierarchy. Moreover, there were also included three Majorana fermions $\mathcal{N}_R^{1,2,3}$ which do not contribute to chiral anomalies but play an important role in neutrino mass acquisition.

SUPPRESSION SQUARES TEXTURE

The majority of textures propose finite and null components of the mass matrices in order to get the suited mass eigenvalues and mixing angles. However, the finite components should be at the same order of magnitude. A new extension of this concept may be the existence of two or three orders of magnitude in the finite elements produced through the Yukawa couplings with more than one Higgs doublet whose VEVs have a vaccum hierarchy (VH), their VEVs are at different orders of magnitude. In this way, the cornerstone of the model to achieve in a natural way the fermionic mass hierarchy is the concept of *suppression squares texture* (SST), which proposes the existence of elements at two different orders of magnitude in a very special location inside the mass matrix.

The simplest example of the SST comprises two fermions f and $\mathcal{F}$ coupled by two Higgs scalars $\phi_{1,2}$ with the VH $v_1 < v_2$. The Yukawa Lagrangian is

$$-\mathcal{L}_Y = A e^{ia}\overline{f_L}\phi_1\left(s_\alpha f_R + c_\alpha \mathcal{F}_R\right) + B e^{ib}\overline{f_L}\phi_2\left(s_\beta f_R + c_\beta \mathcal{F}_R\right). \tag{6.2}$$

where the Yukawa coupling constants are parametrized in polar coordinates, i.e., the coupling constant among f_L, $\mathcal{F}_R$ and ϕ_1 is $Ae^{ia}c_\alpha$. This parametrization not only simplifies the algebra, but also helps to realize how the SST works and suggests relations among Yukawa coupling constants. The corresponding mass matrix after evaluating at the VEVs is

$$M_{\text{supp}} = \begin{pmatrix} Ae^{ia}v_1 \sin\alpha & Ae^{ia}v_1 \cos\alpha \\ Be^{ib}v_2 \sin\beta & Be^{ib}v_2 \cos\beta \end{pmatrix}. \tag{6.3}$$

The diagonalization may be done on either $MM^\dagger$ or $M^\dagger M$. Both matrices give the mass eigenvalues

$$m_f^2 \approx A^2 v_1^2 \sin^2(\alpha-\beta), \tag{6.4}$$
$$m_{\mathcal{F}}^2 \approx B^2 v_2^2 + A^2 v_1^2 \cos^2(\alpha-\beta), \tag{6.5}$$

and the mixing angles of the left- and right-handed fermions are

$$\tan\theta_L \approx \frac{Av_1}{Bv_2}\cos(\alpha-\beta)e^{i(a-b)}, \tag{6.6}$$
$$\tan\theta_R \approx \tan\beta. \tag{6.7}$$

There are some remarkable features in the expressions obtained above. The first and most important is the suppression in the first eigenvalue of the matrix through the sine of the difference between α and β. Similarly, the left-handed mixing angle is also suppressed because of the VH. On the other hand, the second eigenvalue is not supressed but enhanced by the addition of the complementary function of the first eigenvalue, and the right-handed mixing angle turns out to be the angle β in the second row of the matrix, i.e., the angle associated to the largest VEV.

The model implements extensively the SST such that the mass hierarchy can be obtained without any kind of assumption on Yukawa coupling constants. Moreover, some suppression squares involve also the exotic fermions which have been added to cancel chiral anomalies, so they play an important role in obtaining the fermionic mass hierarchy. The next sections present the mass acquisition of the fermionic sector of the model from the SST in the mass matrices, and after showing the mass eigenvalues and mixing angles, the suppression squares of each mass matrix are explained in detail.

MASS MATRICES

Before addressing the fermionic spectrum of the model, this section shows the general procedure to obtain the fermion masses and mixing angles. The fermions of each sector are described employing two bases: the flavor basis **F** and the mass basis **f**. Thus, once the Yukawa Lagrangian is evaluated at VEVs, the mass terms can be expressed as

$$-\mathcal{L}_F = \overline{\mathbf{F}_L}\mathbb{M}_F\mathbf{F}_R + \text{h.c.} \tag{6.8}$$

Since the mass matrix $\mathbb{M}_F$ is not Hermitian, it has to be diagonalized by the biunitary transformation

$$\mathbb{M}_F^{diag} = \left(\mathbb{V}_L^F\right)^{\dagger} \mathbb{M}_F \mathbb{V}_R^F, \tag{6.9}$$

and consequently the mass and flavor bases will be related via the mixing matrices $\mathbb{V}_L^F$ and $\mathbb{V}_R^F$ in the following way

$$\mathbf{F}_L = \mathbb{V}_L^F \mathbf{f}_L, \qquad \mathbf{F}_R = \mathbb{V}_R^F \mathbf{f}_R. \tag{6.10}$$

In particular, the left-handed mixing matrix can be expressed as the product of two mixing matrices

$$\mathbb{V}_L^F = \mathbb{V}_{L,SS}^F \mathbb{V}_{L,B}^F. \tag{6.11}$$

The former matrix rotates out the exotic fermions with a see-saw procedure by taking advantage of the VH. The procedure begins by splitting by blocks the whole symmetric mass matrices ($\mathbb{M}_F \mathbb{M}_F^{\dagger}$ for charged fermions and $\mathbb{M}_N$ for neutrinos)[GL01]

$$\mathbb{M}_F^{sym} = \begin{pmatrix} \mathcal{M}_{3\times 3}^{f} & \mathcal{M}_{3\times n}^{f\mathcal{F}} \\ \mathcal{M}_{n\times 3}^{\mathcal{F}f} & \mathcal{M}_{n\times n}^{\mathcal{F}} \end{pmatrix}, \tag{6.12}$$

where $\mathcal{M}^{\mathcal{F}f} = \left(\mathcal{M}^{f\mathcal{F}}\right)^T$ and n is the number of exotic fermions for each sector (2 for up- and down-like quarks, 3 for charged leptons, and 6 for neutrinos). The see-saw rotation matrix is

$$\mathbb{V}_{L,SS}^F = \begin{pmatrix} 1 & \Theta_L^{F\dagger} \\ -\Theta_L^F & 1 \end{pmatrix}, \tag{6.13}$$

where $\Theta_L^F = \left(\mathcal{M}^{\mathcal{F}}\right)^{-1} \mathcal{M}^{\mathcal{F}f}$. The resulting block-diagonalized mass matrix is

$$\left(\mathbb{V}_{L,SS}^F\right)^T \mathbb{M}_F^{sym} \mathbb{V}_{L,SS}^F = \begin{pmatrix} m_{F,SM}^{sym} & 0_{3\times n} \\ 0_{n\times 3} & M_{F,exot}^{sym} \end{pmatrix}, \tag{6.14}$$

where $m_{F,SM}^{sym}$ is the SM mass matrix given by

$$m_{F,SM}^{sym} \approx \mathcal{M}^f - \mathcal{M}^{f\mathcal{F}} \left(\mathcal{M}^{\mathcal{F}}\right)^{-1} \mathcal{M}^{\mathcal{F}f} \tag{6.15}$$

and $M_{F,exot}^{sym} \approx \mathcal{M}^{\mathcal{F}}$ is the mass matrix of the exotic species. The latter matrix in eq. (6.11), $\mathbb{V}_{L,B}^F$ describes the diagonalization of $m_{F,SM}^{sym}$ and $M_{F,exot}^{sym}$. It has the structure

$$\mathbb{V}_B^F = \begin{pmatrix} V_{SM}^F & 0_{3\times n} \\ 0_{n\times 3} & V_{exot}^F \end{pmatrix} \tag{6.16}$$

where V_{SM}^F is parametrized by

$$V_{SM}^F = R_{13}(\theta_{13}^F, \delta_{13}^F) R_{23}(\theta_{23}^F, \delta_{23}^F) R_{12}(\theta_{12}^F, \delta_{12}^F) \tag{6.17}$$

and the matrices R_{ij} are

$$R_{12}(\theta^F_{12},\delta^F_{12}) = \begin{pmatrix} c^F_{12} & s^F_{12}e^{-i\delta^F_{12}} & 0 \\ -s^F_{12}e^{i\delta^F_{12}} & c^F_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{6.18a}$$

$$R_{23}(\theta^F_{23},\delta^F_{23}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c^F_{23} & s^F_{23}e^{-i\delta^F_{23}} \\ 0 & -s^F_{23}e^{i\delta^F_{23}} & c^F_{23} \end{pmatrix}, \tag{6.18b}$$

$$R_{13}(\theta^F_{13},\delta^F_{13}) = \begin{pmatrix} c^F_{13} & 0 & s^F_{13}e^{-i\delta^F_{13}} \\ 0 & 1 & 0 \\ -s^F_{13}e^{i\delta^F_{13}} & 0 & c^F_{13} \end{pmatrix}, \tag{6.18c}$$

where $c^F_{ij} = \cos\theta^F_{ij}$ and $s^F_{ij} = \sin\theta^F_{ij}$. The angles θ^F_{ij} are specified by their tangents $t^F_{ij} = \tan\theta^F_{ij}$ which could be calculated exactly or approximately by taking advantage of VH. On the other hand, the Dirac phases δ^F_{ij} can be chosen in such a way that they correspond to the experimental measurements.

In the following subsections the mass matrices, mass eigenvalues and mixing angles (involving SM and exotic fermions) are obtained using the previous procedure by taking advantage of the VH established by the following VEVs

$$\begin{aligned} v_\chi &= 2.5\,\text{TeV}, & v_1 &= 245.7\,\text{GeV}, \\ v_\psi &= 1.0\,\text{TeV}, & v_2 &= 12.14\,\text{GeV}, \\ \mu_N &\sim 1\,\text{keV}, & v_3 &= 250\,\text{MeV}. \end{aligned} \tag{6.19}$$

where μ_N is the mass scale of the Majorana fermions $\mathcal{N}^{1,2,3}_R$. These values of VEVs are employed to do numerical explorations by Montecarlo procedures on the mass matrices in order to test their suitability to address fermion mass hierarchy and mixing angles. The results of these searches are shown after the algebraic treatment of the mass matrices.

hadronic sector

The hadronic sector of the model contains the SM fields with four exotic chiral quarks: two up-like quarks $\mathcal{T}^1, \mathcal{T}^2$ and two down-like quarks $\mathcal{J}^1, \mathcal{J}^2$. The non-universal quantum numbers and parities are shown in table 6.1, and the Lagrangians under the symmetry $U(1)_X \otimes \mathbb{Z}_2$ in the quark sector are

$$\begin{aligned} -\mathcal{L}_{Y,U} = & A_U e^{ia_U}\overline{q^1_L}\widetilde{\Phi}_3\left(s^U_\alpha u^1_R + c^U_\alpha u^3_R\right) + B_U e^{ib_U}\overline{q^1_L}\widetilde{\Phi}_2\left(s^U_\beta u^2_R + c^U_\beta \mathcal{T}^1_R\right) \\ & + C_U e^{ic_U}\overline{q^2_L}\widetilde{\Phi}_1\left(s^U_\gamma u^2_R + c^U_\gamma \mathcal{T}^1_R\right) + D_U e^{id_U}\overline{q^3_L}\widetilde{\Phi}_1\left(s^U_\delta u^1_R + c^U_\delta u^3_R\right) \\ & + E_U e^{ie_U}\overline{\mathcal{T}^1_L}\chi^*\left(s^U_\epsilon u^2_R + c^U_\epsilon \mathcal{T}^1_R\right) + F_U e^{if_U} s^U_{\zeta 2}\overline{\mathcal{T}^2_L}\chi\left(s^U_{\zeta 1} u^2_R + c^U_{\zeta 1}\mathcal{T}^1_R\right) \\ & + F_U e^{if_U} c^U_{\zeta 2}\overline{\mathcal{T}^2_L}\chi^*\mathcal{T}^2_R \end{aligned} \tag{6.20}$$

Left-handed	$X^{\pm}$	Right-handed	$X^{\pm}$
SM Quarks			
$q_L^1 = \begin{pmatrix} u^1 \\ d^1 \end{pmatrix}_L$	0^+	u_R^1	$+2/3^+$
		d_R^1	$-2/3^+$
$q_L^2 = \begin{pmatrix} u^2 \\ d^2 \end{pmatrix}_L$	$+1/3^-$	u_R^2	$+2/3^-$
		d_R^2	$-1/3^-$
$q_L^3 = \begin{pmatrix} u^3 \\ d^3 \end{pmatrix}_L$	$+1/3^+$	u_R^3	$+2/3^+$
		d_R^3	$-1/3^-$
Non-SM Quarks			
$\mathcal{T}_L^1$	$+1/3^-$	$\mathcal{T}_R^1$	$+2/3^-$
$\mathcal{T}_L^2$	$+1^-$	$\mathcal{T}_R^2$	$+4/3^-$
$\mathcal{J}_L^1$	$-1/3^+$	$\mathcal{J}_R^1$	$-2/3^+$
$\mathcal{J}_L^2$	0^+	$\mathcal{J}_R^2$	$+1/3^+$

Table 6.1: Hadronic sector of the model, non-universal X quantum number and $\mathbb{Z}_2$ parity.

$$\begin{aligned} -\mathcal{L}_{Y,D} =& A_D e^{ia_D}\overline{q_L^1}\Phi_3\left(s_\alpha^D d_R^1 + c_\alpha^D \mathcal{J}_R^1\right) + B_D e^{ib_D}\overline{q_L^2}\Phi_3\left(s_\beta^D d_R^2 + c_\beta^D d_R^3\right) \\ &+ C_D e^{ic_D}\,\overline{q_L^3}\Phi_2\left(s_\gamma^D d_R^2 + c_\gamma^D d_R^3\right) + D_D e^{id_D}\overline{\mathcal{J}_L^1}\chi\left(s_\delta^D d_R^1 + c_\delta^D \mathcal{J}_R^1\right) \\ &+ E_D e^{ie_D}\overline{\mathcal{J}_L^1}\psi\left(s_\epsilon^D d_R^2 + c_\epsilon^D d_R^3\right) + F_D e^{if_D}\overline{\mathcal{J}_L^2}\chi^*\mathcal{J}_R^2 \end{aligned} \tag{6.21}$$

Next, the Yukawa Lagrangian of the quark sector evaluated at the VEVs yield the mass matrices of the up-like and down-like quarks. Their eigenvalues, as well as their mixing angles and the results of the numerical diagonalization are shown below.

Up-like quarks

The up-like quark sector is described in the bases **U** and **u**, where the former is the flavor basis while the latter is the mass basis

$$\begin{aligned} \mathbf{U} &= (u^1, u^2, u^3, \mathcal{T}^1, \mathcal{T}^2), \\ \mathbf{u} &= (u, c, t, T^1, T^2). \end{aligned} \tag{6.22}$$

The mass term in the flavor basis turns out to be

$$-\mathcal{L}_U = \overline{\mathbf{U}_L}\mathbb{M}_U\mathbf{U}_R + \text{h.c.}, \tag{6.23}$$

where $\mathbb{M}_U$ is the up-like quarks mass matrix

$$\mathbb{M}_U = \begin{pmatrix} \mathcal{M}_U^{SM,\Phi} & \mathcal{M}_U^{Ex,\Phi} \\ \mathcal{M}_U^{SM,\chi} & \mathcal{M}_U^{Ex,\chi} \end{pmatrix} \tag{6.24}$$

and the blocks are

$$\mathcal{M}_U^{SM,\Phi} = \begin{pmatrix} A_U e^{ia_U} s_\alpha^U v_3 & B_U e^{ib_U} s_\beta^U v_2 & A_U e^{ia_U} c_\alpha^U v_3 \\ 0 & C_U e^{ic_U} s_\gamma^U v_1 & 0 \\ D_U e^{id_U} s_\delta^U v_1 & 0 & D_U e^{id_U} c_\delta^U v_1 \end{pmatrix} \tag{6.25}$$

$$\mathcal{M}_U^{Ex,\Phi} = \begin{pmatrix} B_U e^{ib_U} c_\beta^U v_2 & 0 \\ C_U e^{ic_U} c_\gamma^U v_1 & 0 \\ 0 & 0 \end{pmatrix} \tag{6.26}$$

$$\mathcal{M}_U^{SM,\chi} = \begin{pmatrix} 0 & E_U e^{ie_U} s_\epsilon^U v_\chi & 0 \\ 0 & F_U e^{if_U} s_{\zeta 1}^U s_{\zeta 2}^U v_\chi & 0 \end{pmatrix} \tag{6.27}$$

$$\mathcal{M}_U^{Ex,\chi} = \begin{pmatrix} E_U e^{ie_U} c_\epsilon^U v_\chi & 0 \\ F_U e^{if_U} c_{\zeta 1}^U s_{\zeta 2}^U v_\chi & F_U e^{if_U} c_{\zeta 2}^U \end{pmatrix} \tag{6.28}$$

Since the determinant of $\mathbb{M}_U$ is non-vanishing, the up-like quarks acquire mass. It was assumed $\zeta_2^U = 0$ to simplify algebraic expressions without spoiling the suppression mechanisms. Then, the mass eigenvalues corresponding with the SM quark masses are

$$\begin{aligned} m_u^2 &\approx A_U^2 \sin^2\left(\alpha^U - \delta^U\right) \frac{v_3^2}{2}, \\ m_c^2 &\approx B_U^2 \sin^2\left(\beta^U - \epsilon^U\right) \frac{v_2^2}{2} + C_U^2 \sin^2\left(\gamma^U - \epsilon^U\right) \frac{v_1^2}{2}, \\ m_t^2 &\approx D_U^2 \frac{v_1^2}{2} + A_U^2 \cos^2\left(\alpha^U - \gamma^U\right) \frac{v_3^2}{2}, \end{aligned} \tag{6.29}$$

while the masses of the exotic up-like quarks are

$$\begin{aligned} m_{T1}^2 &\approx E_U^2 \frac{v_\chi^2}{2} + B_U^2 \cos^2\left(\beta^U - \epsilon^U\right) \frac{v_2^2}{2} + C_U^2 \cos^2\left(\gamma^U - \epsilon^U\right) \frac{v_1^2}{2}, \\ m_{T2}^2 &\approx F_U^2 \frac{v_\chi^2}{2}. \end{aligned} \tag{6.30}$$

The corresponding left-handed rotation matrix can be expressed by

$$\mathbb{V}_L^U = \mathbb{V}_{L,SS}^U \mathbb{V}_{L,B}^U, \tag{6.31}$$

where the see-saw angle is

$$\Theta_L^{U\dagger} = \begin{pmatrix} \frac{Bv_2}{Fv_\chi} c_{\beta-\epsilon}^U e^{i(b_U - e_U)} & 0 \\ \frac{Cv_1}{Fv_\chi} c_{\gamma-\epsilon}^U e^{i(c_U - e_U)} & 0 \\ 0 & 0 \end{pmatrix} \tag{6.32}$$

while $\mathbb{V}_{L,B}^U$ diagonalizes only the SM-up quarks. Its angles are given by

$$\begin{aligned} \tan\theta_{13}^{U,L} &\approx \frac{A_U v_3}{D_u v_1} c_{\alpha-\delta}^U e^{i(a_U - d_U)} \\ \tan\theta_{23}^{U,L} &\approx \frac{A_U B_U C_U v_3 v_2}{D_U^3 v_1^2} c_{\alpha-\delta}^U s_{\beta-\epsilon}^U s_{\gamma-\epsilon}^U e^{i(a_U - b_U + c_U - d_U)} \\ \tan\theta_{12}^{U,L} &\approx \frac{B_U v_2\, s_{\beta-\epsilon}^U}{C_U v_1\, s_{\gamma-\epsilon}^U} e^{i(b_U - c_U)} \end{aligned} \tag{6.33}$$

The exotic species T^1 and T^2 got masses through v_χ at units of TeV. The SM t quark has acquired mass with v_1 without any suppression, so its mass remains at the scale of v_1, hundreds of GeV. On the contrary, the c quark have acquired mass with v_1 and v_2 but through the rectangular SST

$$\begin{pmatrix} B_U e^{ib_U} s^U_\beta v_2 & B_U e^{ib_U} c^U_\beta v_2 \\ C_U e^{ic_U} s^U_\gamma v_1 & C_U e^{ic_U} c^U_\gamma v_1 \\ E_U e^{ie_U} s^U_\epsilon v_\chi & E_U e^{ie_U} c^U_\epsilon v_\chi \end{pmatrix},$$

yielding the suppressed mass of the c quark because of T^1,

$$m_c^2 \approx B_U^2 \sin^2\left(\beta^U - \epsilon^U\right) \frac{v_2^2}{2} + C_U^2 \sin^2\left(\gamma^U - \epsilon^U\right) \frac{v_1^2}{2}$$
$$m_{T1}^2 \approx E_U^2 \frac{v_\chi^2}{2} + B_U^2 \cos^2\left(\beta^U - \epsilon^U\right) \frac{v_2^2}{2} + C_U^2 \cos^2\left(\gamma^U - \epsilon^U\right) \frac{v_1^2}{2}$$

Finally, the u quark has acquired mass through v_3 with a similar suppression but with the t quark instead of T^1,

$$\begin{pmatrix} A_U e^{ia_U} s^U_\alpha v_3 & A_U e^{ia_U} c^U_\alpha v_3 \\ D_U e^{id_U} s^U_\delta v_1 & D_U e^{id_U} c^U_\delta v_1 \end{pmatrix}.$$

Consequently, the mass of the u quark gets suppressed by t,

$$m_u^2 \approx A_U^2 \sin^2\left(\alpha^U - \delta^U\right) \frac{v_3^2}{2},$$
$$m_t^2 \approx D_U^2 \frac{v_1^2}{2} + A_U^2 \cos^2\left(\alpha^U - \gamma^U\right) \frac{v_3^2}{2}.$$

Down-like quarks

The down-like quarks are described in the bases **D** and **d**, where the former is the flavor basis while the latter is the mass basis

$$\begin{aligned} \mathbf{D} &= (d^1, d^2, d^3, \mathcal{J}^1, \mathcal{J}^2), \\ \mathbf{d} &= (d, s, b, J^1, J^2). \end{aligned} \tag{6.34}$$

The mass term in the flavor basis is

$$-\mathcal{L}_D = \overline{\mathbf{D}_L} \mathbb{M}_D \mathbf{D}_R + \text{h.c.}, \tag{6.35}$$

where $\mathbb{M}_D$ turns out to be

$$\mathbb{M}_D = \begin{pmatrix} \mathcal{M}_D^{SM,\Phi} & \mathcal{M}_D^{Ex,\Phi} \\ \mathcal{M}_D^{SM,\chi} & \mathcal{M}_D^{Ex,\chi} \end{pmatrix} \tag{6.36}$$

with the blocks given by

$$\mathcal{M}_D^{SM,\Phi} = \begin{pmatrix} A_D e^{ia_D} s^D_\alpha v_3 & 0 & 0 \\ 0 & B_D e^{ib_D} s^D_\beta v_3 & B_D e^{ib_D} c^D_\beta v_3 \\ 0 & C_D e^{ic_D} s^D_\gamma v_2 & C_D e^{ic_D} c^D_\gamma v_2 \end{pmatrix} \tag{6.37}$$

$$\mathcal{M}_D^{\text{Ex},\Phi} = \begin{pmatrix} A_D e^{i a_D} c_\alpha^D v_3 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{6.38}$$

$$\mathcal{M}_D^{\text{SM},\chi} = \begin{pmatrix} D_D e^{i d_D} s_\delta^D v_\chi & E_D e^{i e_D} s_\epsilon^D v_\psi & E_D e^{i e_D} c_\epsilon^D v_\psi \\ 0 & 0 & 0 \end{pmatrix} \tag{6.39}$$

$$\mathcal{M}_D^{\text{Ex},\chi} = \begin{pmatrix} D_D e^{i d_D} c_\delta^D v_\chi & 0 \\ 0 & F_D e^{i f_D} v_\chi \end{pmatrix} \tag{6.40}$$

Thus, the mass eigenvalues of the SM quarks are

$$\begin{aligned} m_d^2 &\approx A_D^2 \sin^2(\alpha^D - \delta^D)\frac{v_3^2}{2}, \\ m_s^2 &\approx B_D^2 \sin^2(\beta^D - \gamma^D)\frac{v_3^2}{2}, \\ m_b^2 &\approx C_D^2 \frac{v_2^2}{2} + B_D^2 \cos^2(\beta^D - \gamma^D)\frac{v_3^2}{2}, \end{aligned} \tag{6.41}$$

and the masses of the exotic species are given by

$$\begin{aligned} m_{J1}^2 &\approx D_D^2 \frac{v_\chi^2}{2} + E_D^2 \frac{v_\psi^2}{2} + A_D^2 \cos^2\left(\alpha^U - \delta^U\right)\frac{v_3^2}{2}, \\ m_{J2}^2 &\approx F_D^2 \frac{v_\chi^2}{2}. \end{aligned} \tag{6.42}$$

The corresponding left-handed rotation matrix is

$$\mathbb{V}_L^D = \mathbb{V}_{L,SS}^D \mathbb{V}_{L,B}^D, \tag{6.43}$$

where the see-saw angle which rotates out the species $J^{1,2}$ is

$$\Theta_L^{D\dagger} = \begin{pmatrix} \dfrac{A_D v_3}{D_D v_\chi} c_{\alpha-\delta}^D & 0 \\ \dfrac{B_D E_D v_3 v_\psi}{D_D^2 v_\chi^2} c_{\beta-\epsilon}^D & 0 \\ \dfrac{C_D E_D v_2 v_\psi}{D_D^2 v_\chi^2} c_{\gamma-\epsilon}^D & 0 \end{pmatrix}, \tag{6.44}$$

and the SM angles of $\mathbb{V}_{L,B}^D$ are given by

$$\begin{aligned} \tan\theta_{13}^{D,L} &\approx \frac{A_D E_D v_3 v_\psi}{C_D D_D v_2 v_\chi} c_{\alpha-\delta}^D c_{\gamma-\epsilon}^D, \\ \tan\theta_{23}^{D,L} &\approx \frac{B_D v_3}{C_D v_2} c_{\beta-\gamma}^D \\ \tan\theta_{12}^{D,L} &\approx \frac{A_D E_D v_\psi}{B_D D_D v_\chi} \frac{s_{\gamma-\epsilon}^D}{s_{\beta-\gamma}^D} c_{\alpha-\delta}^D. \end{aligned} \tag{6.45}$$

The heaviest quarks J^1 and J^2 acquired mass at TeV scale due to v_χ, while the b quark obtained its mass through v_2 at units of GeV. The s quark has acquired its mass through v_3 at hundreds of MeV with the suppression due to the b quark in the SST

$$\begin{pmatrix} B_D e^{ib_D} s^D_\beta v_3 & B_D e^{ib_D} c^D_\beta v_3 \\ C_D e^{ic_D} s^D_\gamma v_2 & C_D e^{ic_D} c^D_\gamma v_2 \end{pmatrix},$$

yielding the masses

$$m_s^2 \approx B_D^2 \sin^2(\beta^D - \gamma^D)\frac{v_3^2}{2},$$
$$m_b^2 \approx C_D^2 \frac{v_2^2}{2} + B_D^2 \cos^2(\beta^D - \gamma^D)\frac{v_3^2}{2}.$$

Similarly, the quark d got its mass through the SST with the exotic species J^1

$$\begin{pmatrix} A_D e^{ia_D} s^D_\alpha v_3 & A_D e^{ia_D} c^D_\alpha v_3 \\ D_D e^{id_D} s^D_\delta v_\chi & D_D e^{id_D} c^D_\delta v_\chi \end{pmatrix},$$

whose associated masses are

$$m_d^2 \approx A_D^2 \sin^2(\alpha^D - \delta^D)\frac{v_3^2}{2},$$
$$m_{J1}^2 \approx D_D^2 \frac{v_\chi^2}{2} + A_D^2 \cos^2\left(\alpha^U - \delta^U\right)\frac{v_3^2}{2}.$$

Numerical exploration in the quark sector

In order to test the suitability of $\mathbb{M}_U$ and $\mathbb{M}_D$ to achieve the fermion mass hierarchy, such matrices were generated with random coupling constants by Montecarlo procedures and then diagonalized numerically such that they can reproduce the phenomenological data of quark masses and CKM mixing angles. The results in reproducing such data at 5σ are presented in Fig. 6.2, showing that the model is able to generate mass and mixing angle hierarchies.

However, the absence of unpleasant fine-tunings is shown in Figs. 6.3 and 6.4. The first one shows how the angle differences $\alpha^U - \delta^U$, $\beta^U - \epsilon^U$ and $\gamma^U - \epsilon^U$ observed in the masses of eq. (6.29) get smaller as the magnitude of the moduli A_U, B_U and C_U get larger. This behavior is produced by the SST present in $\mathbb{M}_U$ which acts on the masses of the u and c quarks to supress them from hundreds to units of GeV and MeV, respectively. The second one, instead, shows similar results in the dependence of $\alpha^D - \delta^D$ on A_D of eq. (6.29), but $\beta^D - \gamma^D$ does not depend on B_D as $\alpha^D - \delta^D$. Such an anomalous behavior is produced because the mass of the s quark does not need any suppression because it actually lies at hundreds of MeV, so the SST cannot act in the same way.

The numerical exploration by Montecarlo procedures presents how the SST deals with the fermion mass hierarchy by matching the angles involved in the suppression squares as the moduli increase. This result suggests the posibility of correlations among the coupling constants in each one of the suppression squares without imposing too small couplings by hand.

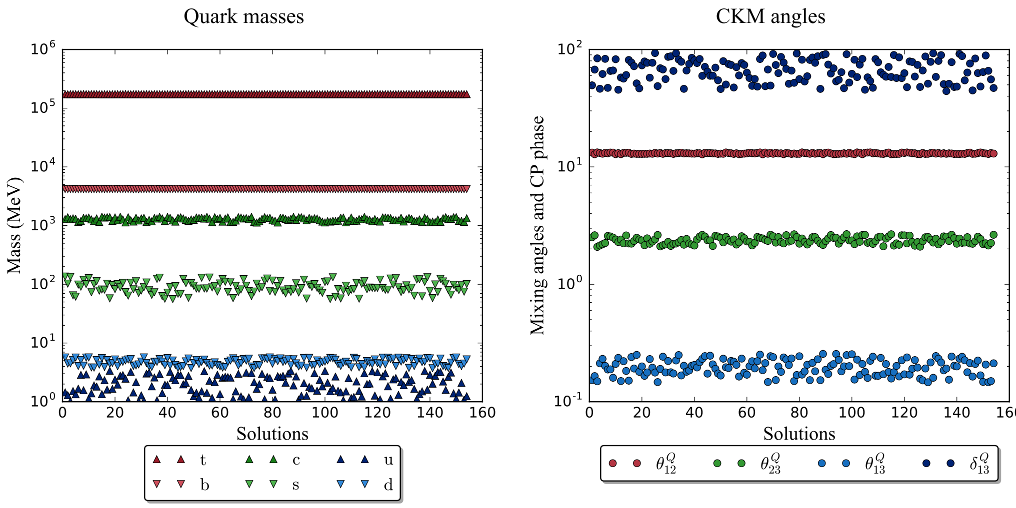

Figure 6.2: Quark masses and CKM mixing angles at 5σ obtained from random mass matrices $\mathbb{M}_{\mathrm{U}}$ and $\mathbb{M}_{\mathrm{D}}$ with the VH of eq. (6).

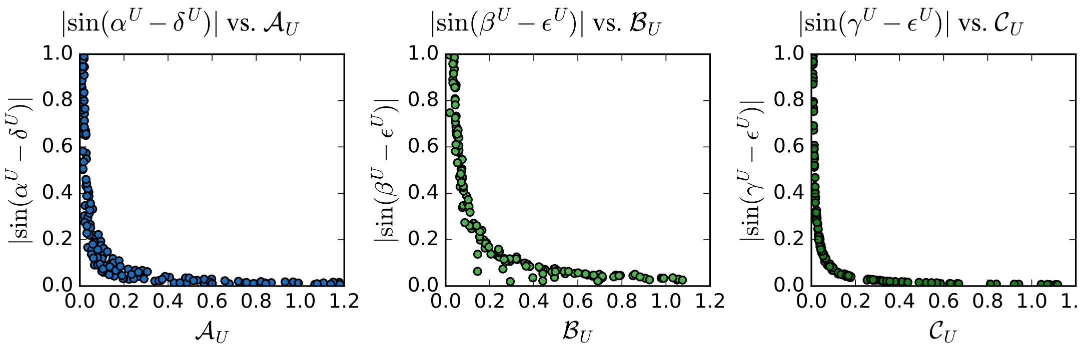

Figure 6.3: Dependeces of the angle differences $\alpha^{\mathrm{U}} - \delta^{\mathrm{U}}$, $\beta^{\mathrm{U}} - \epsilon^{\mathrm{U}}$ and $\gamma^{\mathrm{U}} - \epsilon^{\mathrm{U}}$ on the magnitude of the moduli A_{U}, B_{U} and C_{U}. The fact that the larger the modulus the smaller the angle difference shows the action of the SST on the mass eigenvalues in order to get the hierarchy.

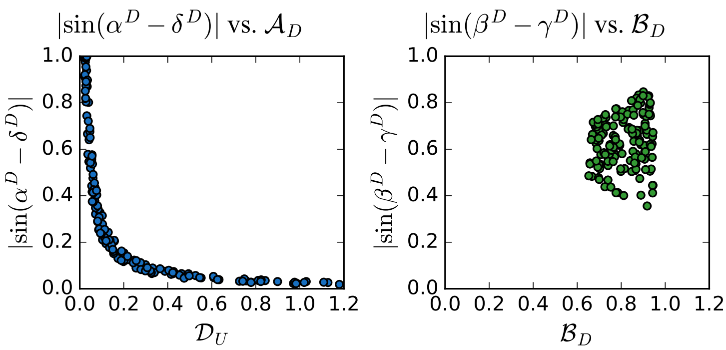

Figure 6.4: Dependences of the angle differences $\alpha^{\mathrm{D}} - \delta^{\mathrm{D}}$ and $\beta^{\mathrm{D}} - \gamma^{\mathrm{D}}$ on the magnitude of the moduli A_{D} and B_{D}. The modulus A_{D} shows the action of the SST on the d mass in order to suppress it at unts of MeV, but the anomalous behavior of B_{D} is produced by the lack of suppression in the s mass since it alredy is at hundreds of MeV.

Left-handed	$X^{\pm}$	Right-handed	$X^{\pm}$
SM Leptons + RH neutrinos			
$\ell_L^e = \begin{pmatrix} \nu^e \\ e^e \end{pmatrix}_L$	$-2/3^{+}$	ν_R^e	$+1/3^{+}$
		e_R^e	$-4/3^{+}$
$\ell_L^\mu = \begin{pmatrix} \nu^\mu \\ e^\mu \end{pmatrix}_L$	$-1/3^{-}$	ν_R^μ	0^{-}
		e_R^μ	-1^{-}
$\ell_L^\tau = \begin{pmatrix} \nu^\tau \\ e^\tau \end{pmatrix}_L$	-1^{+}	ν_R^τ	$-1/3^{-}$
		e_R^τ	$-4/3^{+}$
Non-SM Leptons			
$\mathcal{E}_L^1$	$+1^{-}$	$\mathcal{E}_R^1$	$+4/3^{-}$
$\mathcal{E}_L^2$	-1^{+}	$\mathcal{E}_R^2$	$-4/3^{+}$
$\mathcal{E}_L^3$	$+5/3^{-}$	$\mathcal{E}_R^3$	$+4/3^{-}$
Majorana Fermions		$\mathcal{N}_R^1$	0^{+}
		$\mathcal{N}_R^2$	0^{-}
		$\mathcal{N}_R^3$	0^{+}

Table 6.2: Leptonic sector of the model, non-universal X quantum number and $\mathbb{Z}_2$ parity.

LEPTONIC SECTOR

The leptonic sector of the model contains the SM fields with three exotic chiral charged leptons $\mathcal{E}^1, \mathcal{E}^2, \mathcal{E}^3$ and three Majorana fermions $\boldsymbol{\mathcal{N}}_R = (\mathcal{N}^1, \mathcal{N}^2, \mathcal{N}_R^3)$. The non-universal quantum numbers and parities are shown in table 6.2, and the Lagrangians under the symmetry $U(1)_X \otimes \mathbb{Z}_2$ in the lepton sector are

$$\begin{aligned}
-\mathcal{L}_{Y,N} = & B_{N1}\overline{\ell_L^2}\widetilde{\Phi}_2\nu_R^e + \mathbf{D}_N\overline{\nu_R^{e\,C}}\chi\boldsymbol{\mathcal{N}}_R + \\
& + A_{N1}\overline{\ell_L^1}\widetilde{\Phi}_2\nu_R^\mu + B_{N2}\overline{\ell_L^2}\widetilde{\Phi}_1\nu_R^\mu + \mathbf{E}_N'\overline{\nu_R^{\mu\,C}}\psi\boldsymbol{\mathcal{N}}_R + \\
& + A_{N2}\overline{\ell_L^1}\widetilde{\Phi}_1\nu_R^\tau + C_N\overline{\ell_L^3}\widetilde{\Phi}_3\nu_R^\tau + \mathbf{F}_N\overline{\nu_R^{\tau\,C}}\chi\boldsymbol{\mathcal{N}}_R + \\
& + \frac{\mu_{\mathcal{N}}}{2}\overline{\boldsymbol{\mathcal{N}}_R^C}G_N\boldsymbol{\mathcal{N}}_R + \text{h.c.}
\end{aligned} \tag{6.46}$$

while the Yukawa Lagrangian of the charged leptons can be expressed as

$$\begin{aligned}
-\mathcal{L}_{Y,E} = & A_E e^{ia_E}\overline{\ell_L^1}\Phi_3\left(s_{\alpha1}^E s_{\alpha2}^E e_R^e + c_{\alpha1}^E s_{\alpha2}^E e_R^\tau + c_{\alpha2}^E\mathcal{E}_R^2\right) + \\
& + C_E e^{ic_E}\overline{\ell_L^3}\Phi_1\left(s_{\gamma1}^E s_{\gamma2}^E e_R^e + c_{\gamma1}^E s_{\gamma2}^E e_R^\tau + c_{\gamma2}^E\mathcal{E}_R^2\right) \\
& + + B_E e^{ib_E}\overline{\ell_L^2}\Phi_3 e_R^\mu + D_E e^{id_E}\overline{\mathcal{E}_L^1}\chi\left(s_\delta^E\mathcal{E}_R^1 + c_\delta^E\mathcal{E}_R^3\right) + \\
& + E_E e^{ie_E}\overline{\mathcal{E}_L^2}\chi\left(s_{\epsilon1}^E s_{\epsilon2}^E e_R^e + c_{\epsilon1}^E s_{\epsilon2}^E e_R^\tau + c_{\epsilon2}^E\mathcal{E}_R^2\right) \\
& + + F_E e^{if_E}\overline{\mathcal{E}_L^2}\psi e_R^\mu + G_E e^{ig_E}\overline{\mathcal{E}_L^3}\chi\left(s_\zeta^E\mathcal{E}_R^1 + c_\zeta^E\mathcal{E}_R^3\right) + \text{h.c.}
\end{aligned} \tag{6.47}$$

Neutral leptons

Neutrinos involve Dirac and Majorana masses in their Yukawa Lagrangian. Since $\mathcal{N}_R^i$ are Majorana fermions, the bases are chiral and the mass basis describes Majorana neutrinos. The flavor and mass bases are, respectively,

$$\begin{aligned} \mathbf{N}_L &= (\nu_L^{e,\mu,\tau}, (\nu_R^{e,\mu,\tau})^C, (\mathcal{N}_R^{e,\mu,\tau})^C), \\ \mathbf{n}_L &= (\nu_L^{1,2,3}, (N_R^{1,2,3})^C, (\tilde{N}_R^{1,2,3})^C). \end{aligned} \tag{6.48}$$

The mass term expressed in the flavor basis is

$$-\mathcal{L}_N = \frac{1}{2}\overline{\mathbf{N}_L^C}\mathbb{M}_N\mathbf{N}_L, \tag{6.49}$$

where the mass matrix has the following block structure

$$\mathbb{M}_N = \begin{pmatrix} 0 & \mathcal{M}_\nu^T & 0 \\ \mathcal{M}_\nu & 0 & \mathcal{M}_\mathcal{N}^T \\ 0 & \mathcal{M}_\mathcal{N} & M_\mathcal{N} \end{pmatrix}, \tag{6.50}$$

with $\mathcal{M}_\nu$ as the Dirac mass matrix between ν_L and ν_R

$$\mathcal{M}_\nu = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & A_{N1}v_2 & A_{N2}v_1 \\ B_{N1}v_2 & B_{N2}v_1 & 0 \\ 0 & 0 & C_N v_3 \end{pmatrix}, \tag{6.51}$$

$\mathcal{M}_N$ the Dirac mass matrix between ν_R^C and $\mathcal{N}_R$

$$\mathcal{M}_N = \frac{v_\chi}{\sqrt{2}}\begin{pmatrix} D_{N1} & D_{N2} & D_{N3} \\ E_{N1} & E_{N2} & E_{N3} \\ F_{N1} & F_{N2} & F_{N3} \end{pmatrix}, \tag{6.52}$$

where $E_{N1} = \rho_\psi E'_{N1}$ with $\rho_\psi = v_\psi/v_\chi$, and $M_\mathcal{N} = \mathbb{G}_N\mu_\mathcal{N}$ is the Majorana mass of $\mathcal{N}_R$.

By employing the inverse SSM because of the VH in eq. (6.19), it is found that

$$(\mathbb{V}_{L,SS}^N)^\dagger\,\mathbb{M}_N\mathbb{V}_{L,SS}^N = \begin{pmatrix} m_\nu & 0 & 0 \\ 0 & m_N & 0 \\ 0 & 0 & m_{\tilde{N}} \end{pmatrix} \tag{6.53}$$

where the new 3×3 blocks are[CMO12; Dia+12]

$$\begin{aligned} m_\nu &= \mathcal{M}_\nu^T\left(\mathcal{M}_\mathcal{N}^T\right)^{-1} M_\mathcal{N}\left(\mathcal{M}_\mathcal{N}\right)^{-1}\mathcal{M}_\nu, \\ M_N &\approx \mathcal{M}_\mathcal{N} - M_\mathcal{N}, \quad M_{\tilde{N}} \approx \mathcal{M}_\mathcal{N} + M_\mathcal{N}. \end{aligned} \tag{6.54}$$

It was assumed $\mathcal{M}_N$ diagonal and

$$\mathbb{G}_N = \begin{pmatrix} G_{N1} & G_{N4} & 0 \\ G_{N4} & G_{N2} & 0 \\ 0 & 0 & G_{N3} \end{pmatrix} \tag{6.55}$$

so as it can yield the adequate mixing angles to fit PMNS matrix. By rejecting terms proportional to v_3 in m_ν, the neutrino ν_L^1 turns out to be massless, the masses of the other two neutrinos are

$$\begin{aligned}
m_{\nu 2}^2 &\approx \frac{B_{N2}^2 G_{N2}}{E_{N2}^2}\frac{\mu_N v_1^2}{v_\chi^2} - \frac{2A_{N1}B_{N2}^3 E_{N1} G_{N2} G_{N4}}{G_{N2}(A_{N2}^2 E_{N2}^2 G_{N1} - B_{N2}^2 D_{N1}^2 G_{N2})}\frac{\mu_N v_1 v_2}{v_\chi^2},\\
m_{\nu 3}^2 &\approx \frac{A_{N2}^2 G_{N1}}{D_{N1}^2}\frac{\mu_N v_1^2}{v_\chi^2} + \frac{2A_{N1}A_{N2}^2 B_{N2} G_{N2} G_{N1} G_{N4}}{E_{N1}(A_{N2}^2 E_{N2}^2 G_{N1} - B_{N2}^2 D_{N1}^2 G_{N2})}.\frac{\mu_N v_1 v_2}{v_\chi^2}
\end{aligned} \tag{6.56}$$

and the masses of the exotic species are

$$\begin{aligned}
N_R^1 &= D_{1N}\frac{v_\chi}{\sqrt{2}} - \frac{G_{N1}\mu_N}{2}, & \tilde{N}_R^1 &= D_{1N}\frac{v_\chi}{\sqrt{2}} + \frac{G_{N1}\mu_N}{2},\\
N_R^2 &= E_{2N}\frac{v_\chi}{\sqrt{2}} - \frac{G_{N2}\mu_N}{2}, & \tilde{N}_R^2 &= E_{2N}\frac{v_\chi}{\sqrt{2}} + \frac{G_{N2}\mu_N}{2},\\
N_R^3 &= F_{3N}\frac{v_\chi}{\sqrt{2}} - \frac{G_{N3}\mu_N}{2}, & \tilde{N}_R^3 &= F_{3N}\frac{v_\chi}{\sqrt{2}} + \frac{G_{N3}\mu_N}{2}.
\end{aligned} \tag{6.57}$$

The left-handed rotation matrix can be expressed by

$$\mathbb{V}_L^E = \mathbb{V}_{L,SS}^E \mathbb{V}_{L,B}^E, \tag{6.58}$$

where the see-saw angle is

$$\Theta_L^{N\dagger} = \begin{pmatrix}
\frac{B_{N1}G_{N_4}v_2}{D_{N1}E_{N2}} & \frac{B_{N2}G_{N_4}v_1}{D_{N1}E_{N2}} + \frac{A_{N1}G_{N_1}v_2}{D_{N1}{}^2} & \frac{A_{N2}G_{N_1}v_1}{D_{N1}{}^2}\\
\frac{B_{N1}G_{N_2}v_2}{E_{N2}{}^2} & \frac{B_{N2}G_{N_2}v_1}{E_{N2}{}^2} + \frac{A_{N1}G_{N_4}v_2}{D_{N1}E_{N2}} & \frac{A_{N2}G_{N_4}v_1}{D_{N1}E_{N2}}\\
0 & 0 & \frac{C_N G_{N_3}v_3}{F_{N3}{}^2}
\end{pmatrix} \tag{6.59}$$

and $\mathbb{V}_{L,SM}^E$, contained in the block-diagonal mixing matrix $\mathbb{V}_{L,B}^E$ after rotating out the heavy species has the angles

$$\begin{aligned}
\tan\theta_{13}^{E,L} &\approx \frac{A_{N1}B_{N1}v_2^2}{A_{N2}B_{N2}v_1^2},\\
\tan\theta_{23}^{E,L} &\approx \frac{A_{N2}B_{N1}D_{N1}E_{N2}G_{N4}}{A_{N2}^2 E_{N2}^2 G_{N1} - B_{N2}^2 D_{N1}^2 G_{N2}}\frac{v_2}{v_1}\\
\tan\theta_{12}^{E,L} &\approx \frac{B_{N1}v_2}{B_{N2}v_1}.
\end{aligned} \tag{6.60}$$

Charged leptons

The charged leptons are described in the bases **E** and **e**, where the former is the flavor basis while the latter is the mass basis

$$\begin{aligned}
\mathbf{E} &= (e^e, e^\mu, e^\tau, \mathcal{E}^1, \mathcal{E}^2),\\
\mathbf{e} &= (e, \mu, \tau, E^1, E^2).
\end{aligned} \tag{6.61}$$

The mass term obtained from the Yukawa Lagrangian is

$$-\mathcal{L}_E = \overline{\mathbf{E}_L}\mathbb{M}_E\mathbf{E}_R + \text{h.c.} \tag{6.62}$$

where $\mathbb{M}_E$ turns out to be

$$\mathbb{M}_E = \begin{pmatrix} \mathcal{M}_E^{SM,\Phi} & \mathcal{M}_E^{Ex,\Phi} \\ \mathcal{M}_E^{SM,\chi} & \mathcal{M}_E^{Ex,\chi} \end{pmatrix} \tag{6.63}$$

and the blocks are

$$\mathcal{M}_E^{SM,\Phi} = \begin{pmatrix} A_E e^{ia_E} s^E_{\alpha 1} s^E_{\alpha 2} v_3 & 0 & A_E e^{ia_E} c^E_{\alpha 1} s^E_{\alpha 2} v_3 \\ 0 & B_E e^{ib_E} v_3 & 0 \\ C_E e^{ic_E} s^E_{\gamma 1} s^E_{\gamma 2} v_1 & 0 & C_E e^{ic_E} c^E_{\gamma 1} s^E_{\gamma 2} v_1 \end{pmatrix} \tag{6.64}$$

$$\mathcal{M}_E^{Ex,\Phi} = \begin{pmatrix} 0 & A_E e^{ia_E} c^E_{\alpha 2} v_3 & 0 \\ 0 & 0 & 0 \\ 0 & C_E e^{ic_E} c^E_{\gamma 2} v_1 & 0 \end{pmatrix} \tag{6.65}$$

$$\mathcal{M}_E^{SM,\chi} = \begin{pmatrix} 0 & 0 & 0 \\ E_E e^{ie_E} s^E_{\epsilon 1} s^E_{\epsilon 2} v_\chi & F_E e^{if_E} v_\psi & E_E e^{ie_E} c^E_{\epsilon 1} s^E_{\epsilon 2} v_\chi \\ 0 & 0 & 0 \end{pmatrix} \tag{6.66}$$

$$\mathcal{M}_E^{Ex,\chi} = \begin{pmatrix} D_E e^{id_E} s^E_{\delta} v_\chi & 0 & D_E e^{id_E} c^E_{\delta} v_\chi \\ 0 & E_E e^{ie_E} c^E_{\epsilon 2} v_\chi & 0 \\ G_E e^{ig_E} s^E_{\zeta} v_\chi & 0 & G_E e^{ig_E} c^E_{\zeta} v_\chi \end{pmatrix} \tag{6.67}$$

The determinant of $\mathbb{M}_E$ is non-vanishing ensuring that all charged leptons acquire mass. In order to simplify the algebraic expressions, ϵ_1 was set equal to γ_1. Thus, the eigenvalues of the mass matrix yields the masses of the SM leptons

$$\begin{aligned} m_e^2 &\approx A_E^2 \sin^2\left(\alpha_1^E\right) \sin^2\left(\alpha_2^E - \gamma_2^E\right) \frac{v_3^2}{2}, \\ m_\mu^2 &\approx B_U^2 \frac{v_3^2}{2}, \\ m_\tau^2 &\approx C_E^2 \sin^2\left(\gamma_2^E - \epsilon_2^E\right), \frac{v_1^2}{2} \end{aligned} \tag{6.68}$$

and the masses of the new exotic charged leptons

$$\begin{aligned} m_{E1}^2 &\approx D_E^2 \frac{v_\chi^2}{2}, \\ m_{E2}^2 &\approx E_E^2 \frac{v_\chi^2}{2} + C_E^2 \cos^2\left(\gamma_2^E - \epsilon_2^E\right) \frac{v_1^2}{2}, \\ m_{E3}^2 &\approx G_E^2 \frac{v_\chi^2}{2}. \end{aligned} \tag{6.69}$$

The left-handed rotation matrix can be expressed by

$$\mathbb{V}_L^E = \mathbb{V}_{L,SS}^E \mathbb{V}_{L,B}^E, \tag{6.70}$$

where the see-saw angle is

$$\Theta_L^{E\dagger} = \begin{pmatrix} 0 & \frac{A_E E_E v_3}{E_E^2 v_\chi^2 + F_E^2 v_\psi^2} c^E_{\alpha 2-\epsilon 2} e^{i(a_E - e_E)} & 0 \\ 0 & \frac{B_E F_E v_3 v_\psi}{E_E^2 v_\chi^2 + F_E^2 v_\psi^2} e^{i(b_E - f_E)} & 0 \\ 0 & \frac{C_E E_E v_1 v_\chi}{E_E^2 v_\chi^2 + F_E^2 v_\psi^2} c^E_{\gamma 2-\epsilon 2} e^{i(c_E - e_E)} & 0 \end{pmatrix}, \tag{6.71}$$

and $\mathbb{V}^E_{L,SM}$, contained in $\mathbb{V}^E_{L,B}$ has the mixing angles

$$\begin{aligned} \tan\theta^{E,L}_{13} &\approx \frac{A_E v_3 s^E_{\alpha 2-\epsilon 2}}{C_E v_1 s^E_{\gamma 2-\epsilon 2}} e^{i(a_E - c_E)} \\ \tan\theta^{E,L}_{23} &\approx -\frac{B_E E_E F_E v_3 v_\psi v_\chi}{C_E v_1 \left(F_E^2 v_\psi^2 + E_E^2 v_\chi^2 s^E_{\gamma 2-\epsilon 2}\right)} \\ \tan\theta^{E,L}_{12} &\approx \frac{A_E v_\psi s^E_{\alpha 2-\gamma 2}}{E_E v_\chi s^E_{\gamma 2-\epsilon 2}} \frac{B_E F_E e^{i(a_E - b_E - e_E + f_E)}}{B_E^2 - A_E^2 \left(s^E_{\alpha 2} s^E_{\alpha 1-\gamma 1}\right)^2} \end{aligned} \tag{6.72}$$

The exotic charged leptons $E^{1,2,3}$ have acquired mass at TeV scale. Due to the existence of the SST (in this case extended in a rectangle)

$$\mathcal{M}_E^{SM,\Phi} = \begin{pmatrix} C_E e^{ic_E} s^E_{\gamma 1} s^E_{\gamma 2} v_1 & C_E e^{ic_E} c^E_{\epsilon 1} s^E_{\gamma 2} v_1 & C_E e^{ic_E} c^E_{\gamma 2} v_1 \\ E_E e^{ie_E} s^E_{\gamma 1} s^E_{\epsilon 2} v_\chi & E_E e^{ie_E} c^E_{\gamma 1} s^E_{\epsilon 2} v_\chi & E_E e^{ie_E} c^E_{\epsilon 1} s^E_{\epsilon 2} v_\chi \end{pmatrix}, \tag{6.73}$$

the heaviest SM lepton τ acquired a supressed mass at GeV scale through v_1 so as it does not acquire mass at hundreds of GeV, but at units of GeV. The lepton μ has acquired mass through v_3 without any suppression, so its mass remains at hundreds of MeV. Finally, the lightest lepton, e, got its mass through the largest SST involving the half of the mass matrix

$$\mathcal{M}_E^{SM,\Phi} = \begin{pmatrix} A_E e^{ia_E} s^E_{\alpha 1} s^E_{\alpha 2} v_3 & A_E e^{ia_E} c^E_{\alpha 1} s^E_{\alpha 2} v_3 & A_E e^{ia_E} c^E_{\alpha 2} v_3 \\ C_E e^{ic_E} s^E_{\gamma 1} s^E_{\gamma 2} v_1 & C_E e^{ic_E} c^E_{\gamma 1} s^E_{\gamma 2} v_1 & C_E e^{ic_E} c^E_{\gamma 2} v_1 \\ E_E e^{ie_E} s^E_{\gamma 1} s^E_{\epsilon 2} v_\chi & E_E e^{ie_E} c^E_{\gamma 1} s^E_{\epsilon 2} v_\chi & E_E e^{ie_E} c^E_{\epsilon 2} v_\chi \end{pmatrix} \tag{6.74}$$

and yielding three masses of the charged leptons,

$$\begin{aligned} m_e^2 &\approx A_E^2 \sin^2\left(\alpha_1^E\right) \sin^2\left(\alpha_2^E - \gamma_2^E\right) \frac{v_3^2}{2}, \\ m_\tau^2 &\approx C_E^2 \sin^2\left(\gamma_2^E - \epsilon_2^E\right) \frac{v_1^2}{2}, \\ m_{E2}^2 &\approx E_E^2 \frac{v_\chi^2}{2} + C_E^2 \cos^2\left(\gamma_2^E - \epsilon_2^E\right) \frac{v_1^2}{2}. \end{aligned} \tag{6.75}$$

Numerical exploration in the lepton sector

Similarly with the quark sector, the lepton mass matrices $\mathbb{M}_N$ and $\mathbb{M}_N$ were explored numerically in order to test their suitability and consistency with current neutrino

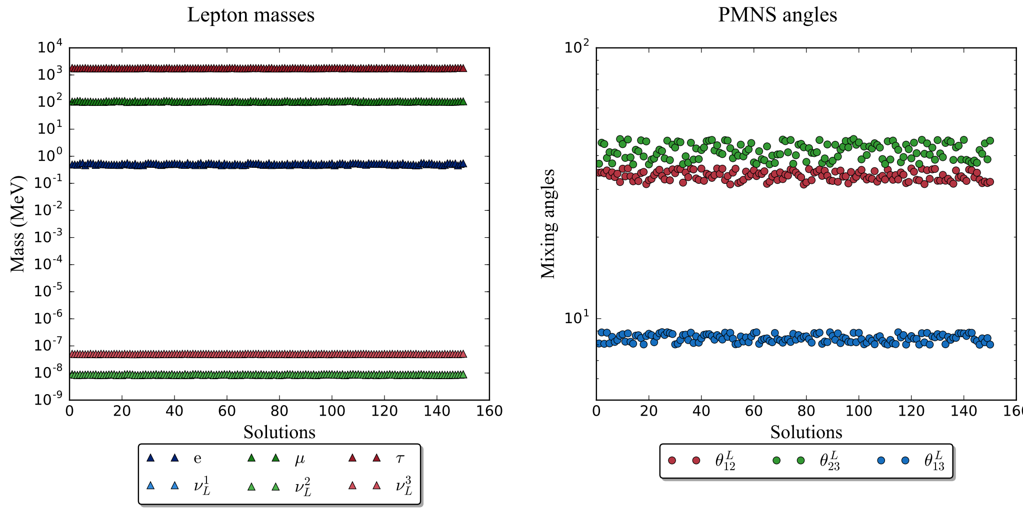

Figure 6.5: Lepton masses and PMNS mixing angles at 3σ according to ref. [GGMS14] obtained from random mass matrices $\mathbb{M}_N$ and $\mathbb{M}_E$ with the VH of eq. (6).

oscillation data[GGMS14]. The numerical results in reproducing such data at 3σ are presented in Fig. 6.5, so that the model is consistent with charged lepton masses and neutrino oscillation data. It is important to remark on the massless neutrino ν_L^1 which determines, with the squared-mass differences, the masses of the neutrinos ν_L^2 and ν_L^3.

The VH presented in eq. (6.19) fixes all the VEVs, but the Majorana mass scale μ_N was only constraint about the scale of units of keV because it only fixes the mass scale of active neutrinos $\nu_L^{1,2,3}$. Therefore, in order to determine which is the best value of μ_N, the matrix $\mathbb{M}_N$ was generated with a Montecarlo procedure with different values of μ_N, from 10^{-5} to 10^2 keV by exponential steps of $10^{+0.5}$ and ten million trials per step. Thereafter, the number of solutions consistent at 3σ with the data reported by ref. [GGMS14] were counted. Finally, such results were plotted in the Fig. 6.6 such that the maximum of solutions points to the best value of μ_N near to $10^{-1.5}$ keV.

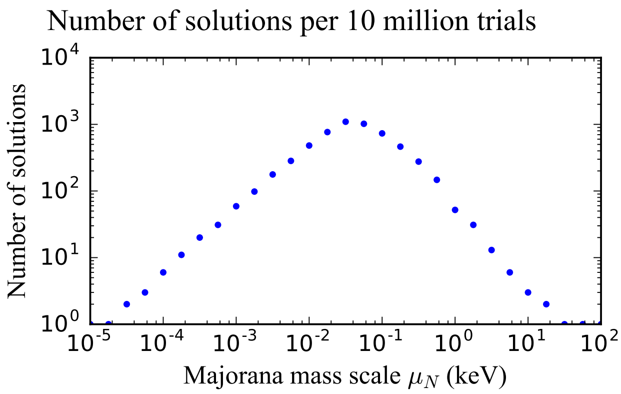

Figure 6.6: Search of the best value of μ_N for $\mathbb{M}_N$ consistent at 3σ with the ref. [GGMS14].

The neutral sector of the model shows a large variety of behaviors among the Yukawa coupling constants of the matrix $\mathbb{M}_N$ in order to reproduce neutrino oscil-

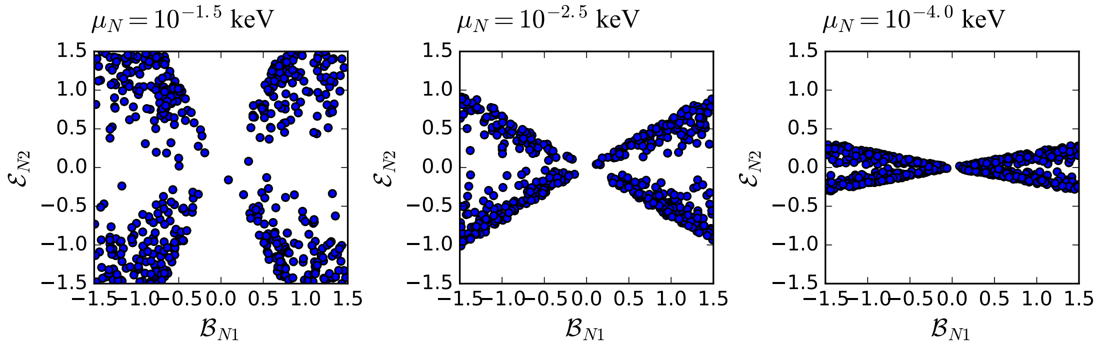

Figure 6.7: Sample of the parameter space available to reproduce neutrino oscillation data in function of the Majorana mass scale μ_N. From left to right the parameter space contracts, consistently with the Fig. 6.6.

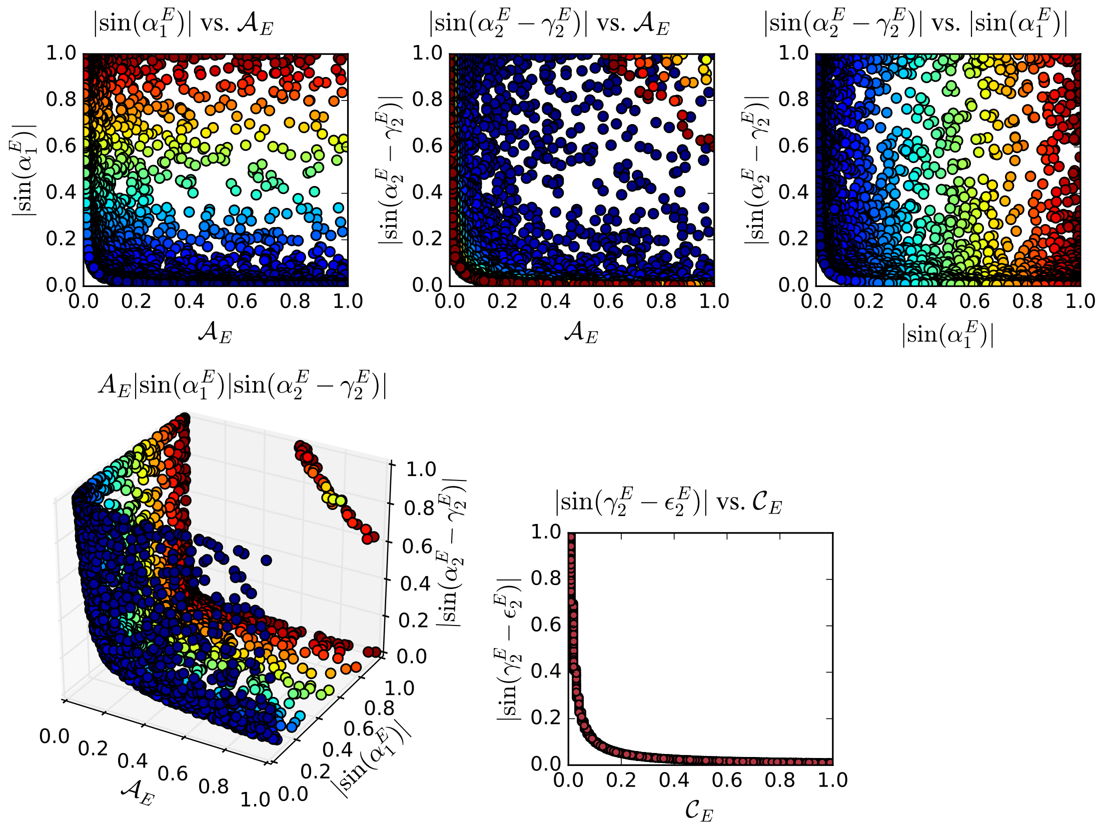

Figure 6.8: Dependences of the angles α_1^E, $\alpha_2^E - \gamma_2^E$ and $\gamma_2^E - \epsilon_2^E$ on the magnitude of the moduli A_E and C_E. The behavior of C_E is similar to C_U in the Fig. 6.3 since the τ lepton, as well as the c quark, gets supressed by the SST. However, the dependence on A_E, α_1^E and $\alpha_2^E - \gamma_2^E$ offers an extended 3D parameter space.

lation data. Specially, when the Majorana mass scale changes the parameter space expands and contracts about the value $\mu_N = 10^{-1.5}$. A sample of this observation is shown in figure 6.7, in which the plane E_{N2} vs. B_{N1} presents how the parameter space gets contracted as the Majorana scale diminishes.

On the other hand, the charged sector appears more similar to the quarks than neutrinos but new elements are introduced. First, the dependence of the angle difference $\gamma_2^E - \epsilon_2^E$ on the modulus C_E in the mass of the τ lepton is similar to the dependence given in the c quark mass. This can be interpreted as both fermions present the same suppression mechanism such that τ and c turned out to be in the same mass scale, units of GeV. Nevertheless, the mass of the electron presents a completely new behavior. Since m_e depends on A_E, α_1^E and $\alpha_2^E - \gamma_2^E$, the actual parameter space is tridimensional, and 2D scatter plots would not show the action of the SST on the electron. In fact, the numerical diagonalization reveals that m_e is suppressed by the angle difference $\alpha_2^E - \gamma_2^E$ and, at the same time, by the angle α_1^E, a new behavior not observed before in the model. Consequently, the results of the numerical exploration of the mass matrix $\mathbb{M}_E$ does show how the SST acts, consistently with the approximated algebraic results outlined above.

The algebraic expressions, as well as the Montecarlo procedures of generating random mass matrices in the quark and lepton sectors has shown the suitaility of the model in addressing the fermion mass hierarchy without unpleasant fine-tunings. Even more, the model suggests relations among different parameters, specially moduli and angle differences in the polar parametrization of the Yukawa coupling constants. Furthermore, the mass matrices in the quark sector are able to reproduce the angle hierarchy and CP-phase in the CKM mixing matrix. Reciprocally, the existence of Majorana fermions in the neutral lepton sector makes $\mathbb{M}_N$ able to deal with the large angles θ_{12}^L and θ_{23}^L of the PMNS matrix. Finally, the fermionic sector of the model has shown that an abelian extension to the SM, together with a discrete symmetry and the suited set of X-charges and new exotic fields might present a new framework to understand the fermion mass hierarchy and mixing angles.

Part III

CONCLUSIONS

7 CONCLUDING REMARKS

The SM of particle physics has been a successful framework to understand from atomic physics to high-energy phenomena. Virtually, the SM has not changed since its original formulation[Gla61; Sal67; Wei67] because of its remarkable agreement with the majority of phenomenology. Nevertheless, the existence of some unexplained facts such as fermion mass hierarchy might not be approachable from the original SM. In this way, models BSM propose new scenarios by extending the current model in so many ways, from extra dimensions to abelian extensions $U(1)_X$ in order to explain such observations. The present work is focused on the latter, abelian extensions of the SM, in which a nonuniversal set of $U(1)_X$ charges is found such that every kind of chiral anomaly gets cancelled identically. This requirement implies to extend the scalar and fermionic sectors.

The scalar sector, presented in chap. 5 of the model includes three Higgs doublets $\Phi_{1,2,3}$ and two singlets χ and ψ (see tab. 7.1). The scalar singlets introduce the scale of units of TeV, while the VEVs of doublets constitute the electroweak scale

$$v_1^2 + v_2^2 + v_3^2 = (246\,\text{GeV})^2\,. \tag{7.1}$$

The singlet χ spontaneously breaks the group $U(1)_X$ by giving mass to the gauge boson Z'_μ, while the doublets $\Phi_{1,2,3}$ perform the electroweak symmetry breaking, yelding the photon A_μ and the weak bosons $W^\pm_\mu$ and Z_μ. Concomitantly, the scalar potential gives the respective Goldstone bosons G'_Z, $G^\pm_W$ and G_Z, four charged $H^\pm_{1,2}$, two CP-odd $A_{1,2}$ and five CP-even physical bosons, $H_{1,2}$, $\mathcal{H}_{1,2}$ and h. The last boson, h, is associated with the Higgs boson of 125 GeV detected at the LHC.

An important feature of the model comprises the vacuum hierarchy (VH) among the VEVs of the scalar fields to obtain suited algebraic expressions and numerical results consistent with the fermion mass hierarchy. The numerical values are

$$\begin{aligned} v_\chi &= 2.5\,\text{TeV}, & v_1 &= 245.7\,\text{GeV},\\ v_\psi &= 1.0\,\text{TeV}, & v_2 &= 12.14\,\text{GeV},\\ \mu_N &\sim 1\,\text{keV}, & v_3 &= 250\,\text{MeV}. \end{aligned} \tag{7.2}$$

Doublets	$X^\pm$	Singlets	$X^\pm$
$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1+v_1+i\eta_1}{\sqrt{2}} \end{pmatrix}$	$+2/3^+$	$\chi = \frac{\xi_\chi + v_\chi + i\zeta_\chi}{\sqrt{2}}$	$+1/3^+$
$\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2+v_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$	$+2/3^-$	$\psi = \frac{\xi_\psi + v_\psi}{\sqrt{2}}$	0^-
$\Phi_3 = \begin{pmatrix} \phi_3^+ \\ \frac{h_3+v_3+i\eta_3}{\sqrt{2}} \end{pmatrix}$	$+1/3^+$	σ	$+1/3^-$

Table 7.1: Scalar content of the model, non-universal X quantum number and $\mathbb{Z}_2$ parity.

The fermionic sector is composed by the three families of SM fermions and new exotic species, three right-handed neutrinos $\nu_R^{e,\mu,\tau}$, two up-like quarks $\mathcal{T}^{1,2}$, two down-like quarks $\mathcal{J}^{1,2}$, three charged leptons $\mathcal{E}^{1,2,3}$ and three Majorana fermions $\mathcal{N}_R^{1,2,3}$ (see tab. 7.3). This particle content allows the cancellation of chiral anomalies, together with the posibility to obtain nonuniversal set of $U(1)_X$ charges in order to build suited Yukawa Lagrangians and mass matrices.

The exotic species get heavy because all of them couple with χ and ψ, while the SM fermions couple with the doublets in a very special way so as the fermion mass hierarchy can be achieved. The central concept to deal with it is the concept of *suppression square texture* (SST), which based on the VH, yields eigenvalues from the mass matrices that suggest the hierarchy in their algebraic expressions and numerical behavior with random Yukawa coupling constants.

Family		Mass		Mass
Quarks				
1	u	$A_U s_{\alpha U-\delta U} \frac{v_3}{\sqrt{2}}$	d	$A_D s_{\alpha D-\delta D} \frac{v_3}{\sqrt{2}}$
2	c	$C_U s_{\gamma U-\epsilon U} \frac{v_1}{\sqrt{2}}$	s	$B_D s_{\beta D-\gamma D} \frac{v_3}{\sqrt{2}}$
3	t	$\frac{D_U v_1}{\sqrt{2}}$	b	$\frac{C_D v_2}{\sqrt{2}}$
Leptons				
1	ν_L^1	0	e	$A_E s_{\alpha E-\delta E} \frac{v_3}{\sqrt{2}}$
2	ν_L^2	$\frac{B_{N2}^2 G_{N2}}{E_{N2}^2} \frac{\mu_N v_1^2}{v_\chi^2}$	μ	$\frac{B_E v_3}{\sqrt{2}}$
3	ν_L^3	$\frac{A_{N2}^2 G_{N1}}{D_{N1}^2} \frac{\mu_N v_1^2}{v_\chi^2}$	τ	$C_E s_{\gamma E-\epsilon E} \frac{v_1}{\sqrt{2}}$
Exotic Quarks				
1	T^1	$\frac{E_U v_\chi}{\sqrt{2}}$	J^1	$\frac{D_D v_\chi}{\sqrt{2}}$
2	T^2	$\frac{F_U v_\chi}{\sqrt{2}}$	J^2	$\frac{F_D v_\chi}{\sqrt{2}}$
Exotic Leptons				
1	N_R^1	$\frac{D_{1N} v_\chi}{\sqrt{2}}$	E^1	$\frac{D_E v_\chi}{\sqrt{2}}$
2	N_R^2	$\frac{E_{2N} v_\chi}{\sqrt{2}}$	E^2	$\frac{E_E v_\chi}{\sqrt{2}}$
3	N_R^3	$\frac{F_{3N} v_\chi}{\sqrt{2}}$	E^3	$\frac{F_E v_\chi}{\sqrt{2}}$

Table 7.2: Summary of fermion masses.

The chapter 6 presents in detail the fermionc mass acquisition with the different SSTs present in the mass matrices of each sector: up-like quarks, down-like quarks, neutral, and charged leptons. Regarding neutral sector, active neutrinos ν_L acquire light masses by the inverse seesaw mechanism (ISS) with ν_R and $\mathcal{N}_R$. Now, the charged sector presents different kinds of SSTs which yields suppressed masses such that the actual mass turns out to be smaller than its VEV. The masses are summarized in tab. 7.2.

The mass matrices were not only diagonalized algebraically, but also numerically. Each one of the matrices were generated with random coupling constants by Mon-

Left-handed	$X^{\pm}$	Right-handed	$X^{\pm}$
SM Quarks			
$q_L^1 = \begin{pmatrix} u^1 \\ d^1 \end{pmatrix}_L$	0^+	u_R^1	$+2/3^+$
		d_R^1	$-2/3^+$
$q_L^2 = \begin{pmatrix} u^2 \\ d^2 \end{pmatrix}_L$	$+1/3^-$	u_R^2	$+2/3^-$
		d_R^2	$-1/3^-$
$q_L^3 = \begin{pmatrix} u^3 \\ d^3 \end{pmatrix}_L$	$+1/3^+$	u_R^3	$+2/3^+$
		d_R^3	$-1/3^-$
SM Leptons + RH neutrinos			
$\ell_L^e = \begin{pmatrix} \nu^e \\ e^e \end{pmatrix}_L$	$-2/3^+$	ν_R^e	$+1/3^+$
		e_R^e	$-4/3^+$
$\ell_L^\mu = \begin{pmatrix} \nu^\mu \\ e^\mu \end{pmatrix}_L$	$-1/3^-$	ν_R^μ	0^-
		e_R^μ	-1^-
$\ell_L^\tau = \begin{pmatrix} \nu^\tau \\ e^\tau \end{pmatrix}_L$	-1^+	ν_R^τ	$-1/3^-$
		e_R^τ	$-4/3^+$
Non-SM Quarks			
$\mathcal{T}_L^1$	$+1/3^-$	$\mathcal{T}_R^1$	$+2/3^-$
$\mathcal{T}_L^2$	$+1^-$	$\mathcal{T}_R^2$	$+4/3^-$
$\mathcal{J}_L^1$	$-1/3^+$	$\mathcal{J}_R^1$	$-2/3^+$
$\mathcal{J}_L^2$	0^+	$\mathcal{J}_R^2$	$+1/3^+$
Non-SM Leptons			
$\mathcal{E}_L^1$	$+1^-$	$\mathcal{E}_R^1$	$+4/3^-$
$\mathcal{E}_L^2$	-1^+	$\mathcal{E}_R^2$	$-4/3^+$
$\mathcal{E}_L^3$	$+5/3^-$	$\mathcal{E}_R^3$	$+4/3^-$
Majorana Fermions		$\mathcal{N}_R^1$	0^+
		$\mathcal{N}_R^2$	0^-
		$\mathcal{N}_R^3$	0^+

Table 7.3: Fermionic content of the model, non-universal X quantum number and $\mathbb{Z}_2$ parity.

tecarlo procedures and diagonalized numerically in order to test the suitability of the model in achieving the fermion mass hierarchy without unpleasant fine-tuning procedures.

In both sectors, quarks and leptons, the model reproduces the phenomenological data at 5σ and 3σ, respectively. In the quark sector, the CKM mixing angles and CP-violating phase, as well as the masses were found, while in the lepton sector the masses of charged leptons and neutrino oscillation data were obtained. Such searchings reveal numerical relations among Yukawa coupling constants which can be interpreted by the results with the approximate algebraic methods in table 7.2.

BIBLIOGRAPHY

[Aad+12] Georges Aad, T Abajyan, B Abbott, J Abdallah, S Abdel Khalek, AA Abdelalim, O Abdinov, R Aben, B Abi, M Abolins, et al. "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC." In: *Physics Letters B* 716.1 (2012), pp. 1–29.

[AHCG01] Nima Arkani-Hamed, Andrew G Cohen, and Howard Georgi. "Electroweak symmetry breaking from dimensional deconstruction." In: *Physics Letters B* 513.1 (2001), pp. 232–240.

[Arn+83a] G Arnison, A Astbury, B Aubert, C Bacci, G Bauer, A Bézaguet, R Böck, TJV Bowcock, M Calvetti, T Carroll, et al. "Experimental observation of isolated large transverse energy electrons with associated missing energy at s= 540 GeV." In: *Physics Letters B* 122.1 (1983). Discovery of vector charge W boson., pp. 103–116.

[Arn+83b] G Arnison, A Astbury, B Aubert, C Bacci, G Bauer, A Bezaguet, R Böck, TJV Bowcock, M Calvetti, P Catz, et al. "Experimental observation of lepton pairs of invariant mass around 95 GeV/c2 at the CERN SPS collider." In: *Physics Letters B* 126.5 (1983), pp. 398–410.

[Bar64] Asim Orhan Barut. *Electrodynamics and classical theory of fields & particles*. Courier Corporation, 1964.

[Cab63] Nicola Cabibbo. "Unitary symmetry and leptonic decays." In: *Physical Review Letters* 10.12 (1963), p. 531.

[CHMO13] AE Carcamo Hernandez, R Martinez, and F Ochoa. "Radiative seesaw-type mechanism of quark masses in $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$." In: *Physical Review D* 87.7 (2013), p. 075009.

[Cas+08] S Casagrande, F Goertz, U Haisch, M Neubert, and T Pfoh. "Flavor physics in the Randall-Sundrum model I. Theoretical setup and electroweak precision tests." In: *Journal of High Energy Physics* 2008.10 (2008), p. 094.

[CMO12] E. Cataño, R. Martinez, and F. Ochoa. "Neutrino masses in a 331 model with right-handed neutrinos without doubly charged Higgs bosons via inverse and double seesaw mechanisms." In: *Physical Review D* 86.3 (2012), p. 073015.

[CK84] Ling-Lie Chau and Wai-Yee Keung. "Comments on the parametrization of the Kobayashi-Maskawa matrix." In: *Physical Review Letters* 53.19 (1984), p. 1802.

[Cur04] Marie Curie. *Recherches sur les substances radioactives*. Gauthier-Villars, 1904.

[Dat05] N.C. Datta. *The Story of Chemistry*. Universities Press, 2005. ISBN: 9788173715303. URL: https://books.google.com.co/books?id=IIZkAvdFJhMC.

[DJHH68] Raymond Davis Jr, Don S Harmer, and Kenneth C Hoffman. "Search for neutrinos from the sun." In: *Physical Review Letters* 20.21 (1968), p. 1205.

[Dia+12] Alex G Dias, CA de S Pires, PS Rodrigues da Silva, and Adriano Sampieri. "Simple realization of the inverse seesaw mechanism." In: *Physical Review D* 86.3 (2012), p. 035007.

[Ein05] Albert Einstein. "Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen." In: *Annalen der physik* 322.8 (1905), pp. 549–560.

[Est+17] Ivan Esteban, MC Gonzalez-Garcia, Michele Maltoni, Ivan Martinez-Soler, and Thomas Schwetz. "Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity." In: *Journal of High Energy Physics* 2017.1 (2017), p. 87.

[F.R97] J. J. Thomson M.A. F.R.S. "XL. Cathode Rays." In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 44.269 (1897), pp. 293–316. DOI: 10.1080/14786449708621070. eprint: https://doi.org/10.1080/14786449708621070. URL: https://doi.org/10.1080/14786449708621070.

[Fri78] Harald Fritzsch. "Weak-interaction mixing in the six-quark theory." In: *Physics Letters B* 73.3 (1978), pp. 317–322.

[Fri05] Harald Fritzsch. *Elementary Particles: Building blocks of matter*. World Scientific, 2005.

[FTY93] Masataka Fukugita, Morimitsu Tanimoto, and Tsutomu Yanagida. "Phenomenological lepton mass matrix." In: *Progress of Theoretical Physics* 89.1 (1993), pp. 263–267.

[Gil12] Robert Gilmore. *Lie groups, Lie algebras, and some of their applications*. Courier Corporation, 2012.

[GK07] Carlo Giunti and Chung W Kim. *Fundamentals of neutrino physics and astrophysics*. OUP Oxford, 2007.

[Gla61] Sheldon L Glashow. "Partial-symmetries of weak interactions." In: *Nuclear Physics* 22.4 (1961), pp. 579–588.

[GIM70] Sheldon L Glashow, Jean Iliopoulos, and Luciano Maiani. "Weak interactions with lepton-hadron symmetry." In: *Physical Review D* 2.7 (1970), p. 1285.

[Gol61] Jeffrey Goldstone. "Field theories with «Superconductor» solutions." In: *Il Nuovo Cimento (1955-1965)* 19.1 (1961), pp. 154–164.

[GSW62] Jeffrey Goldstone, Abdus Salam, and Steven Weinberg. "Broken symmetries." In: *Physical Review* 127.3 (1962), p. 965.

[GGMS14] MC Gonzalez-Garcia, Michele Maltoni, and Thomas Schwetz. "Updated fit to three neutrino mixing: status of leptonic CP violation." In: *arXiv preprint arXiv:1409.5439* (2014).

[Gre90] Walter Greiner. *Relativistic quantum mechanics*. Vol. 3. Springer, 1990.

[Gri87] David J Griffiths. *Introduction to Elementary Particles*. John Wiley & Sons, 1987.

[GL01] Walter Grimus and Luís Lavoura. "The seesaw mechanism at arbitrary order: disentangling the small scale from the large scale." In: *Journal of High Energy Physics* 2000.11 (2001), p. 042.

[Kak93] Michio Kaku. *Quantum field theory*. Vol. 378. Oxford University Press Oxford, 1993.

[Kal21] Theodor Kaluza. "Zum unitätsproblem der physik." In: *Sitzungsber. Preuss. Akad. Wiss. Berlin.(Math. Phys.)* 1921.966972 (1921), p. 45.

[KM73] Makoto Kobayashi and Toshihide Maskawa. "CP-violation in the renormalizable theory of weak interaction." In: *Progress of Theoretical Physics* 49.2 (1973), pp. 652–657.

[MNS62] Ziro Maki, Masami Nakagawa, and Shoichi Sakata. "Remarks on the unified model of elementary particles." In: *Progress of Theoretical Physics* 28.5 (1962), pp. 870–880.

[MM17] S. F. Mantilla and R. Martinez. "A U(1) non-universal anomaly-free model with three Higgs doublets and one singlet scalar field." In: *arXiv* 1704.04869 (2017). URL: https://arxiv.org/abs/1704.04869.

[MMO17] S. F. Mantilla, R. Martinez, and F. Ochoa. "Neutrino and CP-even Higgs boson masses in a nonuniversal $U(1)'$ extension." In: *Phys. Rev. D* 95 (9 2017), p. 095037. DOI: 10.1103/PhysRevD.95.095037. URL: https://link.aps.org/doi/10.1103/PhysRevD.95.095037.

[Mar+14a] R Martinez, J Nisperuza, F Ochoa, and JP Rubio. "Scalar dark matter with $CERN-LEP$ data and Z' search at the LHC in an $U(1)'$ model." In: *Physical Review D* 90.9 (2014), p. 095004.

[Mar+14b] R Martinez, J Nisperuza, F Ochoa, and JP Rubio. "Some phenomenological aspects of a new $U(1)'$ model." In: *Physical Review D* 89.5 (2014), p. 056008.

[Mar+15] R Martinez, J Nisperuza, F Ochoa, JP Rubio, and CF Sierra. "Scalar coupling limits and diphoton *Higgs* decay from *LHC* in an $U(1)'$ model with scalar dark matter." In: *Physical Review D* 92.3 (2015), p. 035016.

[Mar02] Roberto Martínez. *Teoria cuántica de campos*. Universidad Nacional de Colombia, Facultad de Ciencias, Departamento de Física, 2002.

[PG+16] C Patrignani, Particle Data Group, et al. "Review of particle physics." In: *Chinese physics C* 40.10 (2016), p. 100001.

[Per09] Jean Perrin. "Mouvement brownien et réalité moléculaire." In: *Annales de Chimie et de Physique*. Vol. 18. 1909, pp. 5–104.

[PS95] Michael Peskin and Dan Schroeder. *An introduction to quantum field theory*. Perseus Books Publishing, 1995.

[PQ14] Stefano Profumo and Farinaldo S Queiroz. "Constraining the Z' Mass in 331 *Models* using *Direct Dark Matter Detection*." In: *The European Physical Journal C* 74.7 (2014), p. 2960.

[Pro99] JL Proust. "Recherches sur le cuivre." In: *Annales de chimie*. Vol. 32. 1799, pp. 30–31.

[Ram97] P. Ramond. *Field Theory*. Frontiers in Physics Series. Westview Press, 1997. ISBN: 9780813346564. URL: https://books.google.com.co/books?id=Ctr9K61fY4kC.

[Rut11] Ernest Rutherford. "LXXIX. The scattering of α and β particles by matter and the structure of the atom." In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 21.125 (1911), pp. 669–688.

[Ryd96] Lewis H Ryder. *Quantum field theory*. Cambridge university press, 1996.

[Sal67] A Salam. "Elementary Particle Theory: Relativistic Groups and Analiticity, ed. N. Svartholm, Stockholm (1968) S. Weinberg." In: *Phys. Rev. Lett* 19 (1967), p. 1264.

[Wei67] Steven Weinberg. "A model of leptons." In: *Physical review letters* 19.21 (1967), p. 1264.

DECLARATION

I Sebastián Felipe Mantilla Serrano, student of the Master of Science in Physics of the Faculty of Physics at the Universidad Nacional de Colombia, aware of my responsibility of the penal law, declare and certify with my signature that my thesis entitled "Fermion mass hierarchy from non universal abelian extensions of the Standard Model " is entirely the result of my own work.

Bogotá, Colombia, 2017

Sebastián Felipe Mantilla Serrano

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