



Synchronising H_∞ Robust Distributed Controller for Multi-Robotic Manipulators^{*}

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Abstract: In this paper the problem of synchronisation of multiple robotic manipulators using H_∞ robust distributed control systems with respect to the parameter uncertainties and disturbance inputs acting on the manipulators is addressed. Robust synchronising controllers only use the information of outputs of the manipulators, and the corresponding parameters of these output-feedback controllers are designed by computing a series of linear matrix inequalities instead of solving the complex differential (e.g. Hamilton-Jacobi) (in)equalities. The proposed controller can guarantee H_∞ robust performance with respect to the external disturbance inputs and parameters uncertainties, asymptotic stability and synchronisation in the networked manipulators. Using an illustrative example we compare the results extracted in this paper to other works existing in the literature.

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Keywords: Robotic manipulators, distributed controller, linear matrix inequalities (LMIs), disturbance inputs, parameter uncertainties

1. INTRODUCTION

The concept of synchronisation has readily shown its advantages and usefulness for coordinating the behaviour of multiple agents (Sun (2016)). Networked multiple robotic manipulators face however many issues with respect to the synchronisation as documented in numerous studies, (Dai, et al. (2016)). Synthesis of the automatic systems to control these multi-robotic manipulators can rely on synchronising distributed controllers where the local information of each manipulator and the information of its neighbours are used to complete the structure of the controllers (Dai, et al. (2016)).

Using a master-slave control design methodology, a synchronising and tracking controller was presented in (Bondhus, et al. (2004)). Only the position information of the manipulators was assumed available to design the control gain. Difficulties however appeared towards scalability when having more than two manipulators (Bondhus, et al. (2004)). By means of Lagrangian models of the manipulators, the synchronisation problem of interconnected multiple manipulators was investigated in (Chung, et al. (2009)) where diffusive couplings between the agents were considered. The nominal parameters of the manipulators were assumed available in (Bondhus, et al. (2004), Chung,

et al. (2009)) where the problem of parameter uncertainties affecting synchronisation was however not considered.

Using adaptive and robust control methods, parameter uncertainties can be accommodated for in the control synthesis of interconnected robotic systems (Liu, et al. (2010) and Zhang, et al. (2017)). Relying on H_∞ robust controller enables to cope with these parameter uncertainties but also with disturbance inputs (Rigatos, et al. (2017), Miyasato (2010)). Only limited research has however been devoted to deploying H_∞ robust control methods for interconnected multiple manipulators. In (Mehrabian (2013) and Levi, et al. (2007)), the trajectory tracking problem of multiple manipulators was addressed by having Euler-Lagrange models of the agents with the solving of algebraic Riccati equations. The aforementioned works were based on H_∞ control systems which require knowledge on the dynamics of the manipulators represented by complex non-linear interconnected equations, resulting in interconnected multi-Euler-Lagrange systems. In order to tackle the problem of H_∞ distributed controller design in the works that are aforementioned, complex Hamilton-Jacobi inequalities/equalities need to be solved that are computationally difficult and sometimes intractable.

In this paper, we primarily develop the dissipative equality existing in (Williems (2007)) for the case of interconnected multi-agent systems. Secondly, we synthesise H_∞ robust distributed controllers for multi-robotic manipulators to

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significantly decrease the transient time of manipulators in the networked system. Finally, differently from the method proposed in (Mehrabian (2013)), we introduce a control design procedure using the linear matrix inequalities (LMIs) to extract the controllers’ parameters. Hence, we significantly simplify the control design procedure using LMIs compared to the Hamilton-Jacobi (in)equalities that have been introduced in (Mehrabian (2013)). Conclusively, the control design method proposed in this paper leads to a more straightforward strategy than existing methods such as in (Mehrabian (2013)). In an illustrative example we demonstrate the potential of our presented method.

This paper is organised as follows. In Section 2, the problem definition of synchronisation and trajectory tracking of a group of interconnected robotic manipulators is stated. In Section 3, we present the synthesis of H_∞ robust distributed controllers to guarantee the definitions presented in Section 2. Finally, we illustrate the reliability and effectiveness of our proposed method using a comparative example.

2. PROBLEM DEFINITION

We consider N robotic manipulators that have the same number of joints and links, and that are fully actuated with respect to rotational movement (in the joints). We furthermore suppose that the manipulators having n joints that are non-identical regarding their masses and lengths. It is worth mentioning that the angular positions and absolute velocities of each manipulator are measurable in order to synthesise the controller proposed in this paper. Without considering parameter uncertainties, the dynamic of each manipulator with nominal parameters can be formulated by Euler-Lagrange equations as follows

$$A_i(x_i)\ddot{x}_i + B_i(x_i, \dot{x}_i)\dot{x}_i + C_i(x_i) = \tau_i + \omega_i \quad (1)$$

$$i \in \{1, 2, \dots, N\},$$

in which the vector $x_i \in \mathbb{R}^n$ denotes the group of angular positions of the joints, $A_i(x_i) \in \mathbb{R}^{n \times n}$, $B_i(x_i, \dot{x}_i) \in \mathbb{R}^{n \times n}$ and $C_i(x_i)$ denote the inertia matrix, the matrix of coriolis and centripetal forces, and gravitational forces, respectively. $\tau_i \in \mathbb{R}^m$ and $\omega_i \in \mathbb{R}^w$ indicate the torques as control input at the n joints of the manipulator and the external disturbances acting on the i th manipulator, respectively.

Before proceeding further we recall a lemma from (Liu, et al. (2010)) which is used in Section 3 for the synthesis of the controller parameters. It is mentioned in (Liu, et al. (2010)) that the Euler-Lagrange system (1) has skew-symmetry property since it satisfies $\mathbf{v}^T[\frac{1}{2}\dot{A}_i(x_i) - B_i(x_i, \dot{x}_i)]\mathbf{v} = 0$ for any nonzero vector \mathbf{v} .

We suppose that the dynamical models of the multiple manipulators contain parameter uncertainties. Thus, the identified parameters are available to design controllers instead of the exact values. The system matrices in (1) are related to the exact parameter values of the manipulators whereas we will denote the identified parameters that contain parameter uncertainties as $\hat{A}_i(x_i)$, $\hat{B}_i(x_i, \dot{x}_i)$ and $\hat{C}_i(x_i)$, respectively.

In this paper, each robotic manipulator is supposed to be interconnected with its neighbours. Hence, we can model the networked robotic manipulators as a graph in such a way that manipulators are assumed as nodes and the interconnecting communication between them are considered as edges. Furthermore, each i th manipulator is controlled to asymptotically track the desired reference angular position $x_i^r(t)$ as function of time. In order to significantly improve the tracking performance towards the reference input and reduce the transient time, an auxiliary synchronisation control signal τ_{si} as torque force is added to the joints of each manipulator to attain synchronisation in the networked manipulators. The relevant control input τ_i acting on the i th manipulator can now be described as follows

$$\tau_i = \hat{A}_i + \hat{B}_i q_i + \hat{D}_i - K_i e_i + \tau_{si}, \quad (2)$$

in which q_i and \dot{q}_i are defined as $q_i = \dot{x}_i^r - \beta \bar{x}_i$ and $\dot{q}_i = \ddot{x}_i^r - \beta \dot{\bar{x}}_i$, respectively, with angular position tracking error $\bar{x}_i(t) = x_i(t) - x_i^r(t)$ and positive definite weight matrix β . The control gain K_i and β need to be chosen in such a way that the desired reference tracking can be achieved in the i th manipulator even without the synchronising controller τ_{si} . The weighted tracking error and the time derivative of the tracking error can be summarized by

$$e_i = \dot{\bar{x}}_i + \beta \bar{x}_i. \quad (3)$$

Combining (1), (2) and (3), results in a closed loop model of the i th manipulator

$$A_i \dot{e}_i = -(B_i + K_i)e_i + \bar{A}_i \dot{q}_i + \bar{B}_i q_i + \bar{C}_i + \omega_i + \tau_{si}, \quad (4)$$

in which $\bar{A}_i = \hat{A}_i - A_i$, $\bar{B}_i = \hat{B}_i - B_i$ and $\bar{C}_i = \hat{C}_i - C_i$ are defined as system identification errors. Obviously system identification errors affect the performance of tracking and synchronisation. We thus consider the function $d_i = \omega_i + \bar{A}_i \dot{q}_i + \bar{B}_i q_i + \bar{C}_i$ as general disturbance inputs including identification errors. Hence, regarding the identification errors the dynamical model of each manipulator can be rewritten as follows

$$\dot{e}_i = -A_i^{-1}(B_i + K_i)e_i + A_i^{-1}d_i + A_i^{-1}\tau_{si}. \quad (5)$$

$e_i(t)$ reflects the synchronisation state evolution within each i th manipulator. This leads us to the following definition.

Definition 1: The group of interconnected robotic manipulators (1) are synchronised if the following relation holds

$$\lim_{t \rightarrow \infty} \|e_j(t) - e_i(t)\| = 0 \quad \forall i \in \{1, \dots, N\}, \forall j \in N_i, \quad (6)$$

where N_i denotes the neighbours interconnected with the i th manipulator. $e_j - e_i$ is the synchronisation error between two manipulators.

Before proceeding further, we briefly provide the definition of so-called storage functions that will be used further in stability analysis and controller synthesis in Theorem 1 and Theorem 2.

Definition 2: A real-valued function $V : \mathbb{X} \mapsto \mathbb{R}$ defined on state space \mathbb{X} of a system with behaviour set \mathbb{B} and state $\mathbb{B} \times [0, \infty) \mapsto \mathbb{X}$ is called a Lyapunov function if

$t \mapsto V(t) = V(x(t)) = V(x(y(\cdot)), t)$ is a non-increasing function of time for every $y \in \mathbb{B}$. One can say that Lyapunov functions have an explicit upper bound imposed on their increments along system trajectories

$$V(x(y(\cdot), t_1)) - V(x(y(\cdot), t_0)) \leq 0, \forall t_1 \geq t_0 \geq 0, y \in \mathbb{B}.$$

A useful generalisation of this is given by storage functions.

3. DISTRIBUTED H_∞ ROBUST CONTROL FOR INTERCONNECTED NONLINEAR SYSTEMS

In this section, a distributed H_∞ robust controller is synthesised for the group of interconnected manipulators described by nonlinear Euler-Lagrange models.

We consider a group of N nonlinear manipulators that can be expressed, based on (6), by following state space equations

$$\begin{aligned} \dot{e}_i &= E_i(e_i, t)e_i + F_i(e_i, t)u_i + G_i(x_i, t)d_i \\ z_i &= H_{i,1}e_i + H_{i,2} \sum_{j \in N_i} (e_i - e_j) + J_i u_i, \end{aligned} \quad (7)$$

in which E_i , F_i , G_i , H_i and J_i are time varying matrices consisting of nonlinearities depending on the states of the i th manipulator. The system matrices are defined as: $E_i = -A_i^{-1}(B_i + K_i)$ and $F_i = G_i = A_i^{-1}$. In (7), $e_i \in \mathbb{R}^n$ and $e_j \in \mathbb{R}^n$ denote the measurable outputs of manipulators that are used to synthesise the control input $u_i = \tau_{si} \in \mathbb{R}^n$. $z_i \in \mathbb{R}^p$ indicates the controlled outputs with $H_{1,i}$ and $H_{2,i}$ representing the constant weight matrices associated to e_i and $e_i - e_j$ respectively.

The main issue of H_∞ controller synthesis is to reduce the effect of the disturbance input d_i on the controlled output z_i with respect to the desired disturbance attenuating constant for each i th manipulator. The synchronisation between manipulators, see (6), and their stability need to hold when synthesising the controller.

We now introduce Lemma 1 to describe the stability of multiple interconnected manipulators that follow the dynamics in (7). This lemma is used for assuring H_∞ robustness and asymptotic stability by defining the storage function, cfr. Definition 2, of the networked manipulators as

$$V_c(e_c, t) = \sum_{i=1}^N V_i(e_i, t), \quad (8)$$

in which $V_i(e_i, t)$ indicates the storage function of the i th manipulator and e_c denotes the concatenation of all synchronisation states being $e_c = [e_1^T, e_2^T, \dots, e_N^T]^T$.

Lemma 1: If there exists a stabilising control law u_i for the i th manipulator (7), such that the closed-loop system is dissipative according to the positive-definite storage function $V_i(e_i, t)$ with $V_i(0, t) = 0$ and for the disturbance attenuating gains γ_i for $i \in \{1, 2, \dots, N\}$ (see (9) and (10)), we have the following inequality for each manipulator

$$V_i(e_i(t), t) - V_i(e_i(0), 0) \leq 1/2 \int_0^t (\gamma_i^2 d_i^T d_i - z_i^T z_i) dt'.$$

Regarding (8) and the inequality defined above, the following inequality holds for the networked manipulators

$$\begin{aligned} V_c(e_c(t), t) - V_c(e_c(0), 0) &\leq \\ &1/2 \sum_{i=1}^N \int_0^t (\gamma_i^2 d_i^T d_i - z_i^T z_i) dt'. \end{aligned} \quad (9)$$

Furthermore, the following relation assures the attenuation of the vector of disturbance inputs to the vector of controlled outputs with respect to the supremum of the attenuating L_2 gains

$$\begin{aligned} \int_0^t z_c^T z_c dt' &\leq \int_0^t \gamma^2 d_c^T d_c dt' \\ \int_0^t z_i^T z_i dt' &\leq \int_0^t \gamma_i^2 d_i^T d_i dt', \quad \forall i \in \{1, 2, \dots, N\} \end{aligned} \quad (10)$$

in which $z_c = [z_1^T, z_2^T, \dots, z_N^T]^T$, $d_c = [d_1^T, d_2^T, \dots, d_N^T]^T$, and $\gamma = \sup_i \gamma_i$ for $i \in \{1, 2, \dots, N\}$.

Lemma 1 shows that the disturbance attenuation can be achieved with respect to the L_2 gains that are less or equal to γ , and furthermore the asymptotic stability can be obtained for the overall networked manipulators.

We propose the output-feedback synchronising controller as

$$u_i = \tau_{si} = K_{si} \sum_{j \in N_i} (e_j - e_i), \quad (11)$$

with gain K_{si} . The structure of controller (11) is synthesised in such a way that it can work in a distributed manner, interconnecting each manipulator with its neighbours. It is worth mentioning that each manipulator can compute its synchronising control action by the control law (11) using the information received from its neighbours, concretely via $(e_j - e_i)$ in (11). Despite the fact that the control law (11) represents a distributed controller, the desired reference trajectory $x_i^r(t)$ is also delivered to each manipulator to assure that tracking can be achieved even without synchronising controllers.

Combining (7) and (11) leads to the following closed loop dynamics of each i th manipulator

$$\begin{aligned} \dot{e}_i &= E_i e_i + Q_i \sum_{j \in N_i} (e_i - e_j) + G_i d_i \\ z_i &= H_{i,1} e_i + T_i \sum_{j \in N_i} (e_i - e_j) \end{aligned} \quad (12)$$

where $Q_i = F_i K_{si}$ and $T_i = H_{i,2} - J_i K_{si}$.

In the following theorem, we analyse the control parameters K_i and K_{si} of the i th manipulator to determine the problem of asymptotic stability and synchronisation.

Theorem 1: Suppose a network of N interconnected manipulators with state space dynamics described by (12). There exist K_{si} and the positive-definite functions $V_i(e_i, t)$ with $V_i(0, t) = 0$, for $i \in \{1, 2, \dots, N\}$ such that the following Hamilton-Jacobi inequalities hold with the constant values for $\gamma_i > 0$ and the given weight matrices of the controlled outputs z_i that are defined as $H_{i,1}$ and $H_{i,2}$ and J_i in (7).

$$\frac{\partial V_i}{\partial t} + \frac{\partial V_i}{\partial e_i} E_i e_i + \frac{1}{2\gamma_i^2} \frac{\partial V_i}{\partial e_i} G_i G_i^T \frac{\partial V_i^T}{\partial e_i^T} + \frac{1}{2} e_i^T H_{i,1}^T H_{i,1} e_i < 0 \tag{13}$$

$$\begin{aligned} & \frac{1}{2} e_i^T H_{i,1} \sum_{j \in N_i} T_i(e_i - e_j) - \sum_{j \in N_i} \frac{\partial V_i}{\partial e_i} Q_i(e_i - e_j) + \\ & \frac{1}{2} \sum_{j \in N_i} (e_i - e_j)^T T_i^T H_{i,1} e_i + \\ & \frac{1}{2} \sum_{j \in N_i} (e_i - e_j)^T T_i^T \sum_{j \in N_i} T_i(e_i - e_j) \leq 0 \end{aligned}$$

Then, the control gain K_{si} can guarantee H_∞ robustness with respect to the L_2 gains, stability of networked manipulators and synchronisation between them.

Proof: Taking the time derivative of the positive definite storage function $V_i(e_i, t)$ of the i th manipulator (12) results in

$$\begin{aligned} \dot{V}_i &= \frac{dV_i}{dt} = \frac{\partial V_i}{\partial t} + \frac{\partial V_i}{\partial e_i} E_i e_i + \\ & \frac{\partial V_i}{\partial e_i} G_i d_i + \frac{\partial V_i}{\partial e_i} Q_i \sum_{j \in N_i} (e_i - e_j). \end{aligned} \tag{14}$$

Using the method of completing the squares to (14) and regarding the inequalities described in (13), we have the following inequality with respect to \dot{V}_i

$$\begin{aligned} \dot{V}_i &\leq -\frac{1}{2} (e_i^T H_{i,1}^T \sum_{j \in N_i} T_i(e_i - e_j) + \\ & e_i^T H_{i,1}^T H_{i,1} e_i + \sum_{j \in N_i} (e_i - e_j)^T T_i^T H_{i,1} e_i \\ & \sum_{j \in N_i} (e_i - e_j)^T T_i^T \sum_{j \in N_i} T_i(e_i - e_j) + \gamma_i^2 \|d_i\|^2). \end{aligned} \tag{15}$$

Integrating both sides of (15) and considering (8) lead to the following inequality

$$\begin{aligned} V_c(e_c(t), t) - V_c(e_c(0), 0) &\leq \\ & 1/2 \sum_{i=1}^N \int_0^t (\gamma_i^2 d_i^T d_i - z_i^T z_i) dt'. \end{aligned} \tag{16}$$

According to Lemma 1, H_∞ robustness from the disturbance inputs to the controlled outputs can be obtained with respect to the attenuating L_2 gains, in which the disturbance inputs consist of external disturbance inputs and identification errors.

It is worth noting that it is effortful to directly determine the controller parameters according to the Hamilton-Jacobi inequalities defined in (13). In the next theorem, we present a straightforward method to compute the synchronising controller parameters. Instead of solving the inequalities (13), we propose a series of LMIs to synthesise the synchronising controller parameters.

Let us consider the parameter uncertainties in the inertia matrix A_i of the i th manipulator being denoted as $\delta_i \in [\delta_i^{\min}, \delta_i^{\max}] \subset \mathbb{R}^m$. We furthermore consider the following positive definite storage functions

$$V_i(e_i, t) = \frac{1}{2} e_i^T P_i(x_i, \delta_i) e_i, \quad i \in \{1, 2, \dots, N\} \tag{17}$$

with $P_i(x_i, \delta_i) \succ 0$ determined by a positive-definite symmetric matrix $H_i \in \mathbb{R}^{n \times n}$ that should be invertible and $\Delta_i(x_i, \delta_i)$ representing the polytopic parameters uncertainties: $P_i(x_i, \delta_i) = H_i \Delta_i(x_i, \delta_i)$. It is worth mentioning that the matrix $\Delta_i(x_i, \delta_i)$ consists of trigonometric nonlinear expressions which are bounded in the interval $[-1, +1]$.

In the following theorem, we present a series of LMIs that can be solved to directly synthesise the synchronising controller parameters K_{si} for each i th manipulator such that stabilisation, synchronisation and attenuation from the disturbance inputs to the controlled outputs can be achieved with respect to the attenuating L_2 gains γ_i and the given values of the weight matrices $H_{i,1}$, $H_{i,2}$ and J_i for the networked manipulators formulated in (12).

Theorem 2: If there exist the positive-definite matrix H_i and tracking controller parameters K_i independently from the synchronising controller parameters K_{si} for each i th manipulator, based on the given values of attenuating L_2 gains γ_i and the weight matrices $H_{i,1}$, $H_{i,2}$ and J_i , the controller parameters (11) are synthesised by solving the following linear matrix inequalities

$$\begin{bmatrix} -((H_i K_i)^T + H_i K_i) & H_i & H_{i,1}^T \\ * & -\gamma_i^2 I_n & O_{n \times n} \\ * & * & -I_n \end{bmatrix} \leq 0 \tag{18}$$

$$\begin{bmatrix} \phi & L_{N_i \times 1} \otimes (H_{i,2} - J_i K_{si})^T \\ * & -I_n \end{bmatrix} \leq 0 \tag{19}$$

$$H_i \Delta_i^{(v)}(x_i, \delta_i) + (H_i \Delta_i^{(v)}(x_i, \delta_i))^T > 0 \tag{20}$$

$$\begin{aligned} \phi &= I_{N_i} \otimes (H_{i,1}^T H_{i,2} + (H_{i,1}^T H_{i,2} - H_{i,1}^T J_i K_{si} - \\ & (H_{i,1} J_i K_{si})^T - H_i K_{si} - (H_i K_{si})^T), \end{aligned}$$

in which $O_{n \times n} \in \mathbb{R}^{n \times n}$, $L_{N_i \times 1} \in \mathbb{R}^{N_i \times 1}$ and $I_{N_i} \in \mathbb{R}^{N_i \times N_i}$ indicate the zero matrix, the vector of ones, and identity matrix respectively. Furthermore, $\Delta_i^{(v)}$ symbolises Δ_i computed at the polytope vertices.

Proof: Combining the positive-definite storage function (17) and the inequalities (14) leads to the following inequalities

$$\begin{aligned} e_i^T [1/2(\dot{P}_i + \dot{P}_i^T)^T + (P_i E_i + E_i^T P_i)^T + H_{i,1}^T H_{i,1} \\ 1/\gamma_i^2 P_i G_i G_i^T P_i^T] e_i \leq 0 \end{aligned} \tag{21}$$

$$\sum_{j \in N_i} e_i^T P_i Q_i(e_i - e_j) + \frac{1}{2} (\sum_{j \in N_i} (e_i - e_j)^T T_i^T e_i + \tag{22}$$

$$\begin{aligned} & \sum_{j \in N_i} (e_i - e_j)^T T_i^T \sum_{j \in N_i} T_i(e_i - e_j) + \\ & e_i^T H_{i,1}^T \sum_{j \in N_i} T_i(e_i - e_j)) \leq 0. \end{aligned}$$

Substituting the relations $E_i = -A_i^{-1}(B_i + K_i)$, $F_i = G_i = A_i^{-1}$ and (17) in (21) and considering skew-symmetry property presented in (Liu, et al. (2010)), the following inequality emerges

$$e_i^T [-(H_i K_i + K_i^T H_i^T)^T + \frac{1}{\gamma_i^2} H_i H_i^T + H_{i,1}^T H_{i,1}] e_i \leq 0, \tag{23}$$

since we consider the case that each i th manipulator is strongly interconnected with neighbours in network, we have

$$2 \sum_{i=1}^N \sum_{j \in N_i} e_i^T e_i = \sum_{i=1}^N \sum_{j \in N_i} e_j^T e_j \tag{24}$$

Using the relations $Q_i = F_i K_{si}$, $T_i = H_{i,2} - J_i K_{si}$, $P_i(x_i, \delta_i) = H_i \Delta_i(x_i, \delta_i)$ and substituting (8) and (24) in (22), (22) can be rewritten as follows

$$\sum_{i=1}^N e_{ij} [\frac{1}{2} I_{N_i} \otimes (H_{i,1}^T H_{i,2} + H_{i,2}^T H_{i,1} - H_{i,1}^T J_i K_{si} - (H_{i,1}^T J_i K_{si}^T - H_i K_{si} K_{si}^T H_i^T) + L_{N_i \times N_i} \otimes (H_{i,2} - J_i K_{si})^T (H_{i,2} - J_i K_{si}))] e_{ij} \leq 0, \tag{25}$$

where $e_{ij} = \{(e_i - e_j) \mid \forall j \in N_i\}$. Using the Schur complement for the inequalities (23) and (25), we extract the inequalities (20) and (21) presented in Theorem 2. Furthermore, as we know $P_i(x_i, \delta_i) \succ 0$ implies the inequality $P_i(x_i, \delta_i) + P_i^T(x_i, \delta_i) \succ 0$. Using the relation $P_i(x_i, \delta_i) = H_i \Delta_i(x_i, \delta_i)$, then we have the inequality $\psi_i = H_i \Delta(x_i, \delta_i) + \Delta_i(x_i, \delta_i) H_i^T \succ 0$. This inequality is nonlinear that obviously can not be solved directly, since the matrix $\Delta_i(x_i, \delta_i)$ consists of parameters uncertainties and trigonometric nonlinear expressions. Parameters uncertainties are supposed to be bounded in certain polytopes and the trigonometric nonlinearities vary in the interval $[-1, +1]$. Hence, the nonlinear matrix inequality $\psi_i \succ 0$ is bounded and can be evaluated only at the vertices of the corresponding bounds of parameters uncertainties and trigonometric nonlinearities. Thus, solving the nonlinear inequality $\psi_i \succ 0$ leads to solving a finite number of LMIs (22) presented in Theorem 2.

According to Lemma 1 and Theorem 1, we can conclude that the networked robotic manipulators (14) has the H_∞ robustness from the disturbance inputs d_i to the controlled outputs z_i with respect to the attenuating L_2 gains γ_i in such a way that the asymptotic stability and synchronisation can be achieved. It concludes the proof of Theorem 2. \square

4. ILLUSTRATIVE EXAMPLE

Using an illustrative example we demonstrate in this section the effectiveness of our proposed synchronising distributed control strategy. We consider $N = 3$ robotic manipulators consisting of $n = 2$ joints with three rigid links that are supposed to track the desired reference trajectories $x_r^*(t)$. The merit of our proposed control synthesis is shown by comparing our results with other results existing in the literature (Mehrabian (2013)).

We assume in this example that each manipulator is strongly interconnected with all manipulators, meaning that each manipulator receives information from its two neighbours. The Euler-Lagrange dynamic equation of i th manipulator according to (1) is

$$A_i(x_i) = \begin{bmatrix} (1, 1) & (1, 2) \\ (2, 1) & I_{i,2} + m_{i,2} l_{c2}^2 \end{bmatrix} \tag{26}$$

$$B_i(x_i, \dot{x}_i) = \begin{bmatrix} \widehat{(1, 1)} & \widehat{(1, 2)} \\ m_{i,2} l_1 l_{c2} \sin(x_{i,2}) \dot{x}_{i,1} & 0 \end{bmatrix}$$

$$C_i(x_i) = \begin{bmatrix} \widehat{(1)} \\ m_{i,2} l_{c2} g \cos(x_{i,1} + x_{i,2}) \end{bmatrix}$$

with

$$(1, 1) = I_{i,1} + I_{i,2} + m_{i,1} l_{c1}^2 + m_{i,2} (l_1^2 + l_{c2}^2 + 2l_1 l_{c2} \cos(x_{i,2}))$$

$$(1, 2) = I_{i,2} + m_{i,2} (l_{c2}^2 + l_1 l_{c2} \cos(x_{i,2}))$$

$$(2, 1) = I_{i,2} + m_{i,2} (l_{c2}^2 + l_1 l_{c2} \cos(x_{i,2}))$$

$$\widehat{(1, 1)} = -m_{i,2} l_1 l_{c2} \sin(x_{i,2}) \dot{x}_{i,2}, I_{i,1} = \frac{1}{12} m_{i,1} l_1^2$$

$$\widehat{(1, 2)} = -m_{i,2} l_1 l_{c2} \sin(x_{i,2}) (\dot{x}_{i,1} + \dot{x}_{i,2}), I_{i,2} = \frac{1}{12} m_{i,2} l_2^2$$

$$\widehat{(1)} = (m_{i,1} l_{c1} + m_{i,2} l_1) g \cos(x_{i,1}) + m_{i,2} l_{c2} g \cos(x_{i,1}) + x_{i,2},$$

in which $x_i = [x_{i,1}^T, x_{i,2}^T]^T$ and $\tau_i = [\tau_{i,1}^T, \tau_{i,2}^T]^T$ respectively indicate the angular positions of the i th manipulator and the control inputs applied to the joints of the i th manipulator, $i \in \{1, 2, 3\}$. $m_{i,1}$ and $m_{i,2}$ denote the masses of links and $l_1 = l_2$ indicate the length of the links. $g = 9.8$ m/s² and $l_{c,1}$ and $l_{c,2} = 0.3$ m are the centres of masses that are supposed to be equal for all three manipulators. Here, we aim to synchronise the joints of all three robotic manipulators to track the desired reference inputs $x^r(t) = [40 \sin(2\pi t) \ 20 \sin(2\pi t)]$. We assume the initial conditions of angular positions as $x_1(0) = [80^\circ \ 10^\circ]$, $x_2(0) = [100^\circ \ 8^\circ]$ and $x_3(0) = [105^\circ \ 8^\circ]$. As discussed before the parameters of manipulators models are uncertain and we suppose that the parameters uncertainties variations are bounded by values given in Table 1.

Table 1. Nominal values of parameters, identified parameters and parameters uncertainties

$m_{1,1} = m_{1,2} =$ 11	$\hat{m}_{1,1} = \hat{m}_{1,2} =$ 11.5	Bound of masses: [10 12]
$m_{2,1} = m_{2,2} =$ 8	$\hat{m}_{2,1} = \hat{m}_{2,2} =$ 8.5	Bound of masses: [7 9]
$m_{3,1} = m_{3,2} =$ 7.5	$\hat{m}_{3,1} = \hat{m}_{3,2} =$ 8	Bound of masses: [7 8.5]
$l_1 = l_2 =$ 0.45	$\hat{l}_1 = \hat{l}_2 =$ 0.48	Bound of lengths: [0.4 0.5]

Using our presented approach, the synchronising controller parameters K_{si} can be synthesised by solving the LMIs presented in Theorem 2. It is worth noting that in this example the matrix $A_i(x_i)$ contains trigonometric nonlinear terms which are bounded in interval $[-1, +1]$. Hence, $A_i(x_i)$ and (22) are bounded and can be evaluated at the polytope vertices. Given the attenuating L_2 gains as $\gamma_1 = \gamma_2 = \gamma_3 = 0.75$, the tracking controllers $K_i = 6I_{2 \times 2}$ and the weight matrices $H_{1,i} = H_{2,i} = J_i = 3I_{2 \times 2}$, and $\beta = 40I_{2 \times 2}$ for $i \in \{1, 2, 3\}$, using YALMIP tools

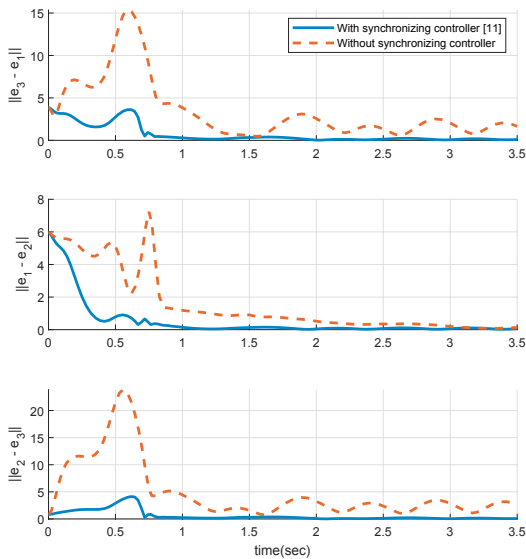


Fig. 1. Synchronisation error between three robotic manipulators, using only tracking controllers and combination of tracking and synchronising controllers.

to solve the LMIs presented in Theorem 2, the results for the positive-definite matrix H_i and the synchronising controllers parameters K_{si} are

$$K_{si} = \begin{bmatrix} 5.5 & -0.001 \\ 0.0025 & 6 \end{bmatrix}, H_i = \begin{bmatrix} 4.314 & -0.0441 \\ -0.0441 & 4.523 \end{bmatrix}.$$

Using the synchronising controller K_{si} , it can be observed from Fig. 1 that our synchronising controller leads to tracking of the desired reference trajectories with significantly less transient time compared to the case of only using the tracking controller with gain K_i . Furthermore, it is apparent from Fig. 1 that our proposed synchronising H_∞ robust distributed controller improves the performance of a networked system to attenuate the effects of disturbance inputs.

To quantitatively benchmark our method compared to approaches based on Hamilton-Jacobi inequalities (e.g. Mehrabian (2013)), we consider the mean square of synchronisation errors ($MSE_{i,j}$) for each i th manipulator, $MSE_{i,j} = \frac{1}{4} \int_0^4 \|e_i(t) - e_j(t)\|^2 dt$.

Results are also detailed in Table 2 for this illustrative example. Up to 40% improvement can be achieved when using the proposed synchronising controller. It is worth mentioning that the controller gains used in (Mehrabian (2013)) are chosen by trial and error according to the Lyapunov function analysis. Opposed to other methods as in (Mehrabian (2013)), we have proposed with Theorem 2 a way to directly synthesise the controller parameters.

5. CONCLUSION

This paper addresses the synchronisation of multiple robotic manipulators by H_∞ robust distributed controllers. The manipulators are torque controlled in their multiple joints and are subject to disturbances. Furthermore the system models describing the dynamics of the

Table 2. Comparing our results with (Mehrabian (2013))

	$MSE_{1,2}$	$MSE_{1,3}$	$MSE_{2,3}$
proposed	21.2	26.51	8.32
Hamilton-Jacobi based on (Mehrabian (2013))	35.3	44.12	16.23

multi-robotic manipulators contain parameter uncertainties. The manipulators need to synchronise and track desired reference trajectories where the effect of external disturbance inputs and identification errors are mitigated. We propose H_∞ controller synthesis method that instead of solving complex non-linear Hamilton-Jacobi inequalities, a set of LMIs are directly solved. This work is based on the proper definition for synchronised robotic manipulators and uses storage functions for analysing the asymptotic stability and synchronisation. The stability aspects and the control synthesis are each formalised in theorems. Using an illustrative example we demonstrate the effectiveness of our proposed synchronising distributed control strategy where the LMIs are solved using YALMIP. When comparing the evolution and mean square of synchronisation errors it is clear that including the synchronising controller reduces tracking errors and that time transients are reduced. Finally, the proposed approach enables to directly synthesise the controller parameters.

REFERENCES

- D. Sun, Synchronization and control of multiagent systems, vol. 41. *CRC Press*, 2016.
- G-B. Dai, and Liu Y-C. Liu, Distributed coordination and cooperation control for networked mobile manipulators, *IEEE Transactions on Industrial electronics*, Pages. 5065 -5074, 2016.
- A. K. Bondhus, K. Y. Pettersen, and H. Nijmeijer, Master slave synchronisation of robotic manipulators, *IFAC Proceeding Volumes*, 2004.
- S. J.. Chung and J. J. E. Slotine, Cooperative robot control and concurrent synchronisation of Lagrangian systems, *IEEE Transactions on Robotics*, Pages. 686-700, 2009.
- Y. C. Liu, and N. Chopra, Robust controlled synchronisation of interconnected robotic systems, *In Proceeding of The American Control Conference*, Pages:1434-1439, 2010.
- D. Zhang, and B. Wei, A review on model reference adaptive control of robotic manipulators, *Annual Reviews in Control*, Pages:188-198, 2017.
- G. Rigatos, P. Siano, and G. Raffo, A nonlinear H_∞ control method for multi-DOF robotic manipulators, *Nonlinear Dynamics*, Pages: 329-348, 2017.
- Y. Miyasato, Adaptive H_∞ formation control for Euler-Lagrange systems, *In 49th IEEE Conference on Decision and Control*, Pages: 2614-2619, 2010.
- A. R. Mehrabian, Distributed control of networked non-linear Euler-Lagrange systems, *PhD. Dissertation*, University of Concordia, 2013.
- L. Levi, N. Berman, and A. Ailon, Robust adaptive nonlinear H_∞ control for robot manipulators. *Mediterranean Conference on Control and Automation*, Pages: 1-6, 2007.
- J. C. Willems, Dissipative dynamical systems, *European Journal of Control*, Pages: 134-151, 2007.