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Effect of Beam-tunnels on Resonant Frequency of Cylindrical Reentrant Cavity

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ABSTRACT

Analytical formulations for the resonant frequency of a reentrant cavity for klystron are available in the literature only for such cavities having a single beam-tunnel. An improved analytical formulation has been proposed in this paper for the calculation of cavity gap-capacitance of reentrant cavities having single and multiple beam-tunnels and its effects on the resonant frequency are studied. The results obtained through analysis have been validated against those obtained from the 3D electromagnetic field simulations and measurements. The proposed analytical formulation provides good estimation of resonant frequency of cavity with single and multiple beam-tunnels.

Keywords: Cylindrical reentrant cavity; Equivalent circuit analysis; Klystron; Multiple- beam klystron; Rectangular reentrant cavity

1. INTRODUCTION

Klystrons are high-power amplifiers and find wide applications in military radars as well as in communication systems. Conventional Single-beam Klystrons (SBKs) and Multiple-beam Klystrons (MBKs) employ cylindrical reentrant cavities or rectangular reentrant cavities in their interaction structure[1-5]. The cylindrical reentrant cavities are analytically treated using equivalent circuit analysis where reentrant cavity could be represented by lumped parallel resonant circuit[6,7]. Carter & Liu[7] proposed an analytical formulation for the calculation of resonant frequency of the cylindrical reentrant cavity with inclusion of the effect of single beam-tunnel. Bansiwal[8], *et al.* proposed approximate analysis as well as rigorous analysis[9] for calculation of the resonant frequency of rectangular reentrant cavity with single beam-tunnel.

The analytical formulations work reasonably well for the cavities with single beam-tunnel with a wide variation in the cavity and beam-tunnel dimensions. However, when same analytical formulation is extended for the reentrant cavities with multiple beam-tunnels, analyses suffers from large deviation (~15%) in the estimation of cavity resonant frequency.

An improved analytical formulation for the calculation of cavity-gap capacitance has been proposed which is then used for calculating the resonant frequency of the reentrant cavities with single and multiple beam-tunnels. Analytical formulation has been benchmarked against the results obtained through 3D electromagnetic solver for wide range of variation in beam-tunnel radius along with variation in number of beam-tunnels. Simulations have been carried out on CST Studio 2019[10].

The proposed formulation estimates the resonant frequency of the cylindrical reentrant cavity with reasonably good accuracy. The results obtained through proposed analysis are having smaller deviation from simulation results compared with the existing analysis by Carter & Liu[7]. The proposed analysis has also been validated against the measurements and a good agreement between the results has been found.

2. ANALYSIS

A typical cylindrical reentrant cavity, used in Klystrons, having circular cylindrical ferrule and with single beam-tunnel, is shown in Fig. 1(a) and with multiple beam-tunnels is shown in Fig. 1(b). Both the cavities are operating in TM_{010} mode. Cylindrical reentrant cavities have their dimensions as outer radius (r_1), ferrule radius (r_0), height (h), drift-gap (d), beam-tunnel radius (r_b) and number of beam-tunnels (N).

Cylindrical reentrant cavities can be represented as lumped parallel resonant circuit. Representation of lumped parameters in reentrant cavity[3,6] and its equivalent lumped circuit[3,6] are shown in Figure 2(a). In the equivalent circuit analysis, magnetic field in drift-gap region is assumed to be negligible and ignored. Thus, the inductance of the cavity depends only on radial region (other than the ferrule region) of the cavity. Capacitance of the cavity is assumed as two capacitances in parallel, namely, 1) capacitance of the ferrule region (C_g) having constant electric field and 2) capacitance of the radial region (C_r) having varying electric field. The beam-tunnels are accommodated in ferrules for transporting the electron- beams through the interaction structure without interacting with each other. Hence, presence of these beam-tunnels affects only the capacitance of drift-gap region (C_g).

Carter & Liu[7] proposed an analytical formulation for C_g to account for the presence of these beam-tunnels while calculating the resonant frequency of the cylindrical reentrant cavities. The analytical formulation[7] considers electric field in

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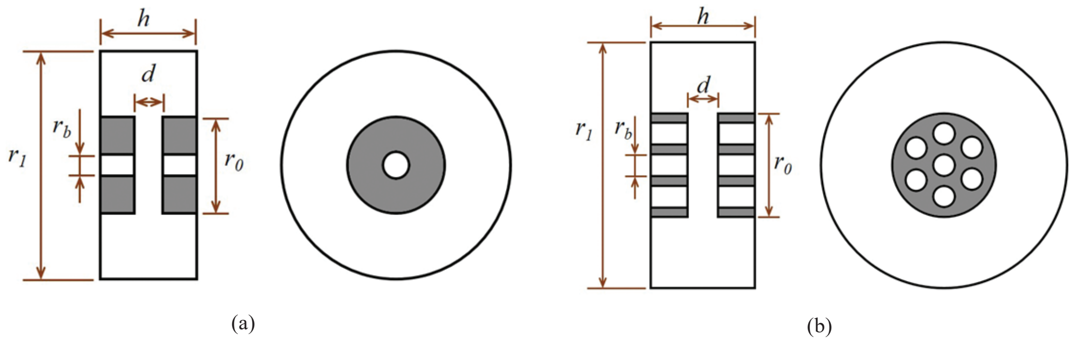


Figure 1. Schematic of typical circular cylindrical reentrant cavities with single beam-tunnel (a), with multiple beam-tunnels (b).

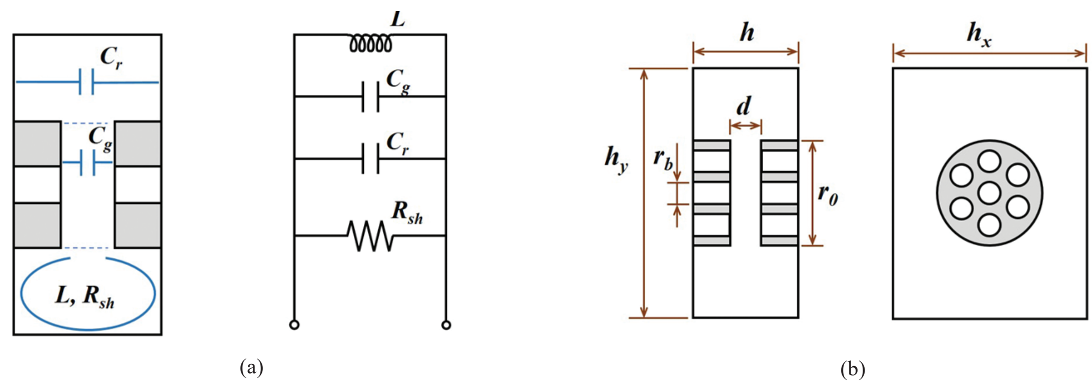


Figure 2. Representation of lumped parameters and equivalent lumped circuit of a reentrant cavity (a) and schematic of a rectangular reentrant cavity with multiple beam-tunnels (b).

the beam tunnel up to the extent of cavity side wall in all cases. Therefore, it provides good estimation of resonant frequency for cylindrical reentrant cavities having single beam-tunnel, but the deviation increases as the number of beam-tunnel as well as radius of the beam-tunnels are increased.

Here, an improved analytical formulation is proposed for calculation of cavity gap-capacitance of reentrant cavities with circular cylindrical ferrule and having multiple beam-tunnels. As per the published literaure[2], the electric field extends into the beam-tunnels and the electric field up to the extent of beam-tunnel radius only contributes to effective interaction. The drift-gap length and beam-tunnel radius are normally kept equal to have acceptable variation in modulation coefficient over gridless gaps, that is beam-tunnel area. Material property for beam-tunnels is vacuum and is the same as that for the cavity volume. Based on these assumptions. The gap capacitance (C_g) of reentrant cavities with single or multiple beam-tunnels is proposed as

$$C_g = \frac{\varepsilon_0 \pi \left(r_0^2 - N r_b^2 \right)}{d} + \frac{\varepsilon_0 N \pi r_b^2}{d + r_b} \tag{1}$$

here, ε_0 is the permittivity of free space.

The presence of beam-tunnels does not affect the cavity inductance (L) and also the capacitance C_r. The inductance (L) for the cylindrical single- and double-reentrant cavities is expressed as[6,8]

$$L = \frac{\mu_0 h}{2\pi} \ln\left(\frac{r_1}{r_0} \right) \text{, and} \tag{2}$$

the capacitance C_r for cylindrical single-reentrant cavity is expressed as

$$C_r = 4\varepsilon_0 r_0 \ln\left(\frac{e\sqrt{(r_1 - r_0)^2 + h^2}}{2d} \right) \text{and} \tag{3}$$

that for double-reentrant cavity is expressed as

$$C_r = 2\varepsilon_0 r_0 \ln\left(\frac{e\sqrt{(r_1 - r_0)^2 + 0.25h^2}}{d} \right) \tag{4}$$

here, μ_0 is the permeability of the free space.

The resonant frequency (f_0) of the cylindrical reentrant cavity is calculated using Eqns (1), (2) and (3) or (4) and is given as

$$f_0 = \frac{1}{2\pi\sqrt{L(C_g + C_r)}} \tag{5}$$

Resonant frequency of a rectangular reentrant cavity (refer Fig. 2(b)) operating in fundamental TM_{110} mode can be calculated following an available analysis[8], by geometrically transforming the cavity to an equivalent cylindrical reentrant cavity operating in TM_{010} mode. The transformed equivalent cylindrical reentrant cavity would have ferrule radius (r_0), drift-gap (d) and cavity height (h) the same as that of the rectangular reentrant cavity. The outer radius (r_1) of the transformed cylindrical reentrant cavity is obtained using an empirical relation from available analysis[8] and is given as

$$r_1 = \left(0.95 - 0.1\left(\frac{h_y}{h_x} - 1\right)\right)\sqrt{\frac{h_x h_y}{\pi}} \tag{6}$$

here, h_x is length and h_y is the width (longer side) of the transverse cross-section of rectangular reentrant cavity. Once all the dimensions of an equivalent cylindrical reentrant cavity are obtained, then L, C and f_0 of the cavity can be obtained using Eqns (1)-(5). One may also use the exact analytical relations for inductance and capacitance of a rectangular reentrant cavity as available in published literature[9,11] with improved relation for C_g as given in Eqn (1).

Beam tunnel diameter and drift-gap length of the cylindrical reentrant cavity are normalised with respect to the cavity height while comparing the results in Results and discussion section.

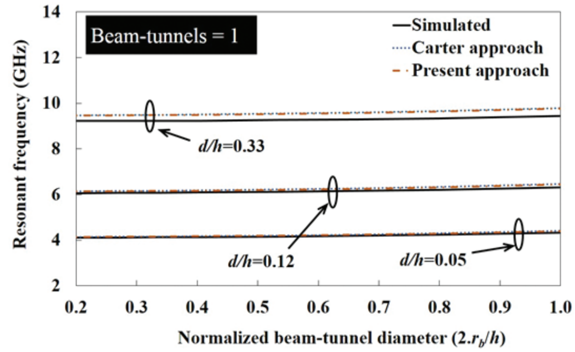
3. RESULTS AND DISCUSSION

For numerical appreciation of the proposed analytical formulation, a typical cylindrical double-reentrant cavity having resonant frequency in *C-X*-band and operating in TM_{010} mode, has been considered. The cavity resonant frequency has been simulated using 3D electromagnetic field solver CST studio[10]. Resonant frequencies of the cavity have been obtained for fixed values of cavity outer radius (r_1) = 8.5 mm, cavity height (h) = 3.0 mm and ferrule radius (r_0) = 4.0 mm; fixed discrete values of normalised drift-gap (d/h = 0.05, 0.12 and 0.33) and number of beam-tunnels ($N = 1, 7$ *and* 19), while the normalised beam-tunnel radius ($2r_b/h$) has been varied. The ranges for parameters are chosen such that most of practical klystron cavities can be brought under the analysis regime.

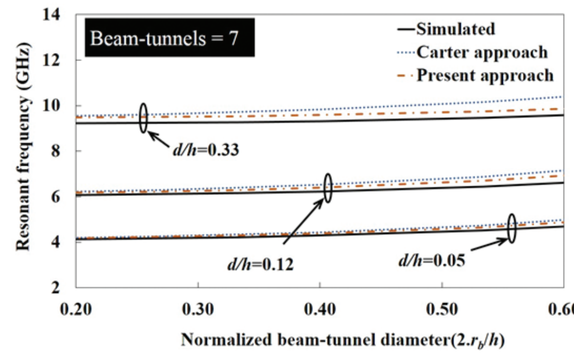
Resonant frequencies obtained through simulations, using Carter's analytical formulation and proposed improved formulation are compared in Figs. 3(a)-(c). Carter's analytical formulation provides very good estimation of resonant frequency for cavity with single beam-tunnel for most of the variation in normalised drift-gap and variation in normalised beam-tunnel diameter. The deviation in resonant frequency obtained through Carter's formulation increases with increase in number of beam-tunnels, d/h ratio and $2r_b/h$ ratio. Resonant frequency obtained through proposed analysis provides good agreement with results obtained through the simulations and provides better estimation (within 8% deviation) than the Carter's approach (~15% deviation) in multiple beam-tunnel case.

Equivalent circuit analysis assumes no magnetic field and constant electric field within the drift-gap region. However, in practical situation, magnetic field would exist within drift-gap region, though negligible, and its magnitude increases with increase in drift-gap (d). Further, the electric field also does not remain constant and varies over the drift-gap region. This two factors are the cause for an increased deviation in estimation of resonant frequency when d/h and $2r_b/h$ ratios are increased.

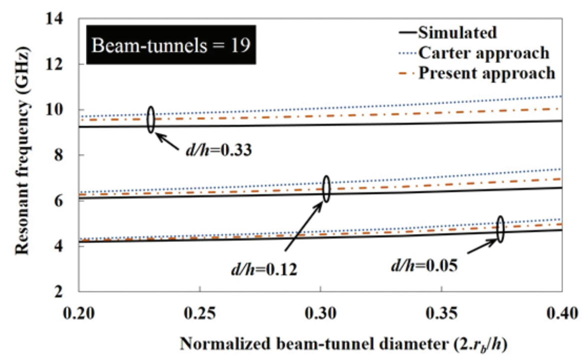
The present analysis has also been validated against the published measurement data. Two cylindrical reentrant cavities (**Cavity-1**: r_0 = 5.18 mm, r_1 = 7.0 mm, d = 0.75 mm,



(a)



(b)



(c)

Figure 3. Comparison of the resonant frequency of cylindrical reentrant cavity obtained through simulations, Carter's approach and present approach.

h = 3.95 mm, r_b = 0.55 mm and N = 1; **Cavity-2**: r_0 = 5.2 mm, r_1 = 8.55 mm, d = 0.73 mm, h = 3.96 mm, r_b = 0.55 mm and N = 1) and two rectangular reentrant cavities (**Cavity-3**: h_x = 28 mm, h_y = 42 mm, h = 11 mm, r_0 = 8.5 mm, r_b = 1.1 mm, d = 1.65 mm and N = 1; **Cavity-4**: $h_x = h_y$ = 35 mm, h = 12.5 mm, r_0 = 11.25 mm, d = 2.5 mm, r_b = 1.75 mm and N = 19) have been considered. Resonant frequency measurement was carried out through return loss measurement using network analyser and a coaxial probe with least perturbation. Results obtained through measurements are compared with those obtained through simulations, carter's formulation and proposed improved formulation in Table 1.

The proposed formulation provides a similar accuracy in estimation of resonant frequency of cavities with single beam-tunnel compared with that from analysis of Carter and Liu, whereas better accuracy for the cavity with multiple beam-tunnel. The difference between measurement and

Table 1. Comparison of the resonant frequency obtained through simulations, measurements, Carter's formulation, and proposed formulation

Reentrant cavity	Resonant frequency (GHz)			
	Simulated	Measured	Carter's formulation	Proposed formulation
Cavity-1 (Single beam-tunnel)	8.574	8.536	8.833	8.819
Cavity-2 (Single beam-tunnel)	6.645	6.604	6.687	6.676
Cavity-3 (Single beam-tunnel)	3.143	3.155	3.139	3.130
Cavity-4 (Multiple beam-tunnels)	3.316	3.337	3.816	3.506

simulated results are due to the flow of excess braze material, variation in cavity dimension after brazing and loading of cavity due to intrusion of coaxial probe while measuring the frequency.

4. CONCLUSIONS

The beam-tunnels are used for transportation of electron beam through the cavity interaction circuit of the klystron. Presence of these beam-tunnels, reduces the gap capacitance compared with beam-tunnel absent case. Carter & Liu[7] proposed analytical formulation for the calculation of resonant frequency of a cylindrical reentrant including the effect of single beam-tunnel but deviation in estimation of resonant frequency increases significantly when analysis is extended for the cavities with multiple beam-tunnels. Here, an improved analytical formation is proposed for the calculation of cavity gap-capacitance and then the resonant frequency of reentrant cavities having single and multiple beam-tunnels. Results obtained through the simulations, Carter's formulation and proposed formulation have been compared. The proposed formulation provides better estimation of resonant frequency of the reentrant cavities with multiple beam-tunnels, compared with carter's formulation. Proposed formulation has also been validated against the measurements and a good agreement has been obtained between the results. This analysis will be very helpful to the klystron designer in estimating the resonant frequency of the cavities having multiple beam-tunnels more accurately.

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