Journal of Accountancy

Volume 52 | Issue 1 Article 3

7-1931

Depreciation Based on Unit Cost

A. W. Moser

Depreciation Based on Unit Cost

By A. W. Moser

During the last ten or fifteen years American business has striven mightily to become a science. In harmony with this, the twofold purpose of accountancy has found more and more recognition and practical application. First, a record of past transactions must be furnished, so that there may be clearly established at any time the sums due a company by its debtors, how much the company owes to creditors and what is the equity of the stockholders or proprietors. These facts are periodically brought out in the usual financial statements, reaching not only the management, but also the stockholders and others. The second purpose is to provide detailed information on certain operating phases, destined for the management only, so that the latter may be in a position, by properly interpreting the figures and acting on the basis of conclusions thus reached, so to shape the organization of the business, its financial and operating policies and its sales and working methods, that the competitive parity of the enterprise will be best maintained and greatest possible profits and financial stability assured to the company. Accordingly, accuracy of thought, analysis of available data and search for the truth have become the aims of any progressive management.

While much progress, notably through the introduction of budgetary control, has been accomplished in various directions, there is nevertheless one element of cost whose manner of handling, in my opinion, has not participated to the same extent in that development. This is depreciation of wasting assets.

This is a fertile subject for discussion, and one lending itself to treatment from a number of angles. I shall, however, limit the discussion to a consideration of the subject from that viewpoint from which unit costs appear as an essential factor in determining periodical depreciation charges. This is done by what is known as the unit-cost method of depreciation. And even the sphere of discussion thus defined shall be further limited to a study of the main principles involved. At another time there may present itself an opportunity to consider also certain questions that may arise in the practical application of the method. The problem of depreciation appears to be rather poorly treated so far as unit cost is concerned in the existing literature. Many works which give it space contain theoretical misconceptions of importance. What wonder, then, that the method is almost unknown in practice?

Any method of depreciation, to be worth that name, must in the first place fulfill the one condition that, if properly carried out, the periodical charges shall reach by the end of the normal service life of the depreciating asset a total equal to the total amount to be written off, which ordinarily is the original cost less final scrap value.

Under all ordinary methods of estimating depreciation the attempt is made to accomplish still another thing, namely so to distribute the depreciation burden over the useful life of the asset that as good as possible an approximation to actual operating results may be obtained. For this purpose, much stress is often laid on knowing the actual course of depreciation, and extensive investigations to that end have been made. While these are valuable in many respects, their application in those methods does not portray a correct or interpretable picture. Take as an extreme case that of a plowshare: In the first year, assume that it is being much used, and there is little depreciation; in the second year, being left idle, it is rusting away. If the actual course of depreciation were the deciding factor, the second year would have to bear the major part of the depreciation, although the asset had not performed any service in that period. Hence it appears that charges according to depreciation's actual course are not necessarily the proper ones.

Next let us consider the analytical basis of depreciation methods, such as the straight-line method, the reducing-balance method and others. With the possible exception of the production method, they do not contain any element which relates the periodical charges they furnish to the actual course of depreciation. They fix in advance a certain sum to be regularly charged to depreciation account, without regard to actual experience in the course of the asset's usefulness. Neither do they show how and to what extent to take into consideration, at the moment of setting up the periodical charges, variations in operating intensity. Yet the fact that times of depression alternate with periods of feverish activity is a common phenomenon. Thus it will be seen that even in case of the actual run of depreciation being known there would be little gain in the way of increased accuracy of results.

It is true that wear and tear from use is normally to a larger extent the effective depreciation than was indicated in the example of the plowshare. Efforts are made frequently to determine how much of the depreciation is due to the cause just named, that is to operation, and how much to the action of time and elements, so that through adjustments some better alignment between operating intensity and temporal distribution of depreciation may be reached. This would mean, in the first place, that the adopted depreciation method could not be integrally followed through, which in itself may be of little consequence. More serious, however, is the question as to the fitness of the elements used for making the adjustments. As pointed out before, to rely on a somehow determined course of depreciation, to whatever cause this may be due, will not necessarily furnish results that are most proper and, above all, to the best interest of the business concerned over a long term.

On the other hand, basing depreciation on unit cost automatically takes care of all the incongruities that go with other methods and to cope with which these prove inadequate. More may be said, however, in favor of the unit-cost plan. This is also the one whose results, as will be seen later, are definitely related to production and cost of production, whereby important managerial information is obtained.

Because of the more accurate results obtainable, there implicitly follows also a more accurate valuation of assets and consequently of the worth of a given enterprise.

Let us now consider briefly one or two of the main objections that are usually directed at the plan of basing depreciation on unit cost, before proceeding further with the chosen thema. One of them is that the depreciation estimate in a given case is at the best based on so many unknown and unknowable elements that refinements of computation and method lose something of their value.

To this I answer with an attempt to outline a plan that comprehensively and systematically is based on quantity of output and quantity of costs, the two essential elements making for success or failure of a business. No other method does that. Also we need not be satisfied with depreciation estimates only, but can get true figures, as actual output and costs of operation may dictate. In this plan, nothing is borrowed from or based on any other plan of depreciation. It is standing on its own feet, so that it really amounts to more than a refinement of computation and method.

It will become apparent, too, that many elements entering into the plan, even if appearing as rough estimates only, will not appreciably distort the true picture. An error in estimated figures of say 100 per cent., for instance, may cause an error of less than 10 per cent. in the results, consisting in a corresponding shifting of the burden from one period to another or others. If the burden correctly allottable to a given period be 1,000, for instance, and certain estimated figures are incorrect to the extent of 100 per cent., the actual charge for that period may result in 900, for instance, while in another period the actual charge may be 1,200 instead of 1,100, the correct amount. This illustrates an extreme case, however, and yet the estimated error can hardly be considered as important. Normally, a much closer estimate of those certain figures is possible with more insignificant errors in results.

Another criticism of the method is raised on the ground of its more complicated nature, not so easily understood by the layman. This is true to a certain extent. More important than this mere circumstance in any given case, however, is the question as to whether the possible gain in results justifies a somewhat more elaborate procedure. As a general rule, one is the more inclined toward an affirmative answer to this question, when confronted with specific facts, the greater are a company's wasting assets in relation to its total assets. Thus, manufacturing concerns with expensive operating units, real-estate companies and so on will probably fall within that category. The topic also is reminiscent of the times when double-entry bookkeeping was about to supersede the single-entry system: it looked more complicated, but promised and did bring better results, and so won its way. Similar experiences have attended the introduction of cost-accounting systems.

Consequently, the question of distributing depreciation in accordance with the unit-cost method resolves itself, too, into a query as to the advantages. To this, the reader will find his own answer when reading the following. In order to prove the claims put forth, I found it unavoidable to use some mathematics; but this should not be a deterrent to giving the subject a fair hearing. For the successful application in practice fewer mathematics are needed; it is imperative, however, that the principles involved be well understood and strictly followed.

Experience teaches that despite all care the time does come when it is no longer possible or advisable to continue the use of a building, a piece of machinery or equipment, etc. The corresponding losses, as far as they can not be compensated through current repairs, are called depreciation. In order to have them in their financial effect distributed more or less evenly over the useful life of the depreciating asset, a common practice is regularly to set aside sums, called depreciation or replacement charges, and to accumulate these sums as a reserve for depreciation. The book value of an article at any time is then its cost less the value of the accumulated depreciation charges. The wearing value of an article at a time given is understood to be its book value less the salvage or scrap value, while the total wearing value is its cost less final salvage value and is equal to the sum of the yearly depreciation charges.

In dealing with the depreciation of any productive property, there is not alone the question of distributing the charges in some way over its lifetime to be considered, so that the capital invested in the property can be returned when it has outlived its usefulness —a point of even greater importance that should be kept in mind, and to which the final replacement of capital is really subordinated, is the principle of productive or competitive parity of an enterprise with respect to others of the same kind. As long as this parity is maintained, the capital invested is safe, other things being equal. If the productive parity is impaired, the investments in the property are not necessarily safe, as situations are easily conceivable and do arise in practice where, for instance, too small depreciation charges in the early years result in an undue burden for later periods, thus cutting into profits or making profitable competition entirely impossible. This consideration is the weightier, of course, the greater the proportion of wasting assets of an enterprise to its total assets.

Competitive parity on the part of an enterprise requires, in substance, that the latter be able to produce a unit of output or unit of service at any time as profitably as any rival can produce it. As far as the influence of depreciation goes, it must be avoided, that the corresponding charge be neither unduly low at one time and unduly high for another period, or vice versa; and that end is attained when the book value of a property is diminished by depreciation charges from year to year to such an extent that the unit cost of its output is at any time the same as for a new unit of property that could be found to replace the given one. This expresses the principle of the unit-cost method of depreciation. In other words, it treats the depreciation of a given service unit at any stage of its service life as the difference in worth of two units which would perform the same service at the same total cost per unit of service, one having an estimated service life equal to the total estimated service life of the unit under consideration, and the other an estimated service life equal to the estimated remaining service life of the unit. On this principle, the question of how depreciation progresses with time is translated into a function of production conditions, into which enter the two essential elements: quantity of output and quantity of costs.

The unit-cost method of depreciation has been described as the soundest plan of depreciation which, when applied with intelligence, furnishes the true measure of accrued depreciation, and it also reflects in its periodic charges the varying intensity of service to a much higher degree than any other depreciation method. In its application, however, difficulties are encountered on account of involved mathematical processes that so far have interfered with any great practical value.

Nevertheless, the fact that the method is one of the soundest in principle should be incentive enough for attempting to "put it on a better working basis." Such an attempt is made in the following paragraphs, where as successive steps I shall describe:

(a) Exact formulas with example;
(b) Simplified formulas with example;
(c) Procedure for certain adjustments;
(d) Treatment of obsolescence;
(e) Analysis of other depreciation procedures and comparison with the unit-cost method.

Mathematically, the problem is usually stated as follows:

If C = first cost of new machine;
N = number of years of useful life;
O = annual operating expenses including repairs;
Y = number of units of output per annum;
S = scrap value;
i = current interest rate;

$s_{\overline{n}|} = \frac{(1+i)^n - 1}{i}$ = amount of an immediate annuity of 1 unit a year, payable at the end of each year;

Then the average cost of a unit of service or a unit of output is

$$X = \frac{O + Ci + \frac{C}{s_{\overline{N|}}}}{Y}$$

If there is a scrap value, C should be replaced by $\frac{C-S}{s_{\overline{N|}}}$

Using lower case letters to represent the same quantities with reference to an old machine, there will result as average unit cost

$$x = \frac{o + ci + \frac{c}{s_{\overline{n|}}}}{y}$$

If now, according to the principle underlying the method, the unit cost of output must be the same for the two machines, the two equations may be equated and solved for c, the value of the old machine. The accrued depreciation on the latter would then amount to $C-c$. At this point, however, attention need be called to a few details which will make it clear that the expressions for X and x derived above do not correctly formulate the conditions of the principle in question and that proceeding on the basis indicated would possibly lead, as soon as $N>1$, to erroneous conclusions and results. This has been brought forth by J. S. Taylor in a paper entitled "Statistical study of depreciation, based on unit cost."

Making the book value of an article at the end of each year that value which would make the unit cost of production the same as for a new machine implies the existence of the following equations, where the digits 1, 2, 3 . . . r designate the successive years of service life.

$$X_1 = x_1 \text{ or } \frac{O_1 + Ci + \frac{C}{s_{\overline{N|}}}}{Y_1} = \frac{o_1 + ci + \frac{c}{s_{\overline{n|}}}}{y_1}$$

$$X_2 = x_2 \text{ or } \frac{O_2 + Ci + \frac{C}{s_{\overline{N|}}}}{Y_2} = \frac{o_2 + ci + \frac{c}{s_{\overline{n|}}}}{y_2}, \text{ etc.}$$

$$X_r = x_r \text{ or } \frac{O_r + Ci + \frac{C}{s_{\overline{N|}}}}{Y_r} = \frac{o_r + ci + \frac{c}{s_{\overline{n|}}}}{y_r}$$

If now a value c be obtained from the expressions for X and x for a machine that had been in use one year and whose first cost was C, the depreciation charges for each of the $N-1$ remaining years would amount to $\frac{c}{s_{\overline{N-1|}}}$, while the original depreciation schedule was based on an annual charge of $\frac{C}{s_{\overline{N|}}}$. Hence an adjustment will be necessary, if not by change $\frac{c}{s_{\overline{N-1|}}}=\frac{C}{s_{\overline{N|}}}$, or a deficiency in the accumulating reserve for depreciation may result. This is not all, however. The same value c should also satisfy the equations as indicated for the subsequent years. For that purpose it would be necessary that for each such period $\frac{C}{s_{\overline{N|}}}+Ci=\frac{c}{s_{\overline{N-1|}}}+ci$, and that the O's and Y's stand in some fixed relation to the o's and y's. This would be a case of rare coincidence which may indeed never happen and may be dismissed as practically non-existent. Furthermore, the unit cost having been obtained on the assumption that the annual depreciation charge would be $\frac{C}{s_{\overline{N|}}}$ and the annual interest charge Ci, a change of these quantities would mean that the average unit cost as derived for the unit new was incorrect, so that the whole basis of comparison is vitiated and the method in the form as presented is liable to lead to fundamentally wrong conclusions and to impairment of capital invested.

The concept of a property to be so depreciated that the total depreciation at a given moment is its first cost C minus c, where c has such a value as to make the unit cost of production under the existing economic conditions the same as when the property was new, clearly indicates that the periodical depreciation charges can not be fixed ones, but must vary, depending on the operating expenses, quantity of output and scrap value for each year. Only then will it be possible to distribute the charges, from the point of view of a going concern, so that each unit of output will bear its just proportion of the burden.

Using in addition to the symbols earlier indicated,

$B_r = C - D_1 - D_2 - \ldots - D_{r-1}$ = book value during r^{th} year,
$I_r = iB_r$ = interest charge for r^{th} year,
$W_r = C - S_r$ = wearing value if the article is used r years,

the unit cost for the r^{th} year is expressed by

$$X_r = \frac{O_r + I_r + D_r}{Y_r}$$

Besides operating and labor costs, O_r may be made to include such expenses as insurance, a part of the overhead, etc., according to the nature of the problem to be solved.

It will usually be desirable to have the unit cost kept constant over the life term of an asset, so that

$$X_1 = X_2 = X_3 = \ldots X, \text{ or}$$

$$X = \frac{O_1 + I_1 + D_1}{Y_1} = \frac{O_2 + I_2 + D_2}{Y_2} = \ldots \frac{O_r + I_r + D_r}{Y_r} = \frac{O_n + I_n + D_n}{Y_n} \quad \text{(a)}$$

These equations together with the one

$$D_1 + D_2 + D_3 + \ldots D_n = W_n \quad \text{(b)}$$

give a total of n equations in the n unknown quantities D_1, D_2, D_3, $\ldots D_n$. The I's are unknown, too, but functions of the D's, namely

$$I_1 = Ci$$
$$I_2 = iB_2 = i(C - D_1) = I_1 - iD_1$$
$$I_3 = iB_3 = i(C - D_1 - D_2) = I_2 - iD_2, \text{ etc.}$$
$$\cdot \; \cdot \; \cdot \; \cdot \; \cdot$$
$$I_r = I_{r-1} - iD_{r-1} \quad \text{(c)}$$

From equations (a) follows

$$XY_r = O_r + I_r + D_r, \text{ or } D_r = XY_r - O_r - I_r$$

and from this and (b)

$$D_1 = XY_1 - O_1 - Ci$$
$$D_2 = XY_2 - O_2 - I_2 = XY_2 - O_2 - i(C - D_1),$$
$$D_3 = XY_3 - O_3 - I_3 = XY_3 - O_3 - I_2 + iD_2$$
$$= (Y_3 + iY_2)X - (O_3 + iO_2) - i(1+i)(C - D_1).$$

Proceeding in this manner, expressions for I_r and D_r, correct for all values of $r > 2$, will be obtained in the following form:

$$I_r = i\{(1+i)^{r-2}(C - D_1) + O_{r-1} + (1+i)O_{r-2} + \ldots (1+i)^{r-3}O_2$$
$$-[Y_{r-1} + (1+i)Y_{r-2} + \ldots (1+i)^{r-3}Y_2]X\} \quad \text{(d)}$$

$$\begin{aligned}\text{and } D_r = & [Y_r + iY_{r-1} + i(1+i)Y_{r-2} + \ldots i(1+i)^{r-3}Y_2]X \\ & -[O_r + iO_{r-1} + i(1+i)O_{r-2} + \ldots i(1+i)^{r-3}O_2] \\ & -i(1+i)^{r-2}(C-D_1) \qquad (1)\end{aligned}$$

Thus D_r is expressed in terms of D_1 and constants.

Substituting the values for D_2, D_3, . . . D_r in equation (b) gives, after properly collecting terms,

$$\begin{aligned}W_n = & D_1 + X[(1+i)^{n-2}Y_2 + (1+i)^{n-3}Y^3 + \ldots (1+i)Y_{n-1} + Y_n] \\ & -[(1+i)^{n-2}O_2 + (1+i)^{n-3}O_3 + \ldots (1+i)O_{n-1} + O_n] \\ & -i(C-D_1)\frac{(1+i)^{n-1}-1}{i}\end{aligned}$$

Considering that $\frac{(1+i)^n - 1}{i} = s_{\overline{n}|}$ and substituting in the above the value for D_1 obtainable from relation (a), namely $D_1 = XY_1 - O_1 - Ci$, then as value for X will finally result

$$X = \frac{W_n + Cis_{\overline{n}|} + \sum_1^n (1+i)^{n-r}O_r}{\sum_1^n (1+i)^{n-r}Y_r} \qquad (2)$$

The condition being that the unit cost be the same for each year of useful life of a given asset, it is necessary to consider that life period as a whole and to determine n so that X, the unit cost, shall become a minimum for the n years. This result may be simply obtained by computing X from formula (2) separately for $n = 1$, 2, 3 . . . etc., until that state is reached where X, after having decreased for a certain number of years, begins to increase. The general procedure of applying this rule and formula (2) will be made clearer by a study of an illustrative (hypothetical) problem in the course of which it may be noted, too, how advantage can be taken of a result obtained for computing the next one in order considerably to reduce the labor involved.

Let be $C = \$10{,}000$, first cost of an equipment, $i = 5\%$, and the annual operating costs, units output and scrap values as indicated in the table below:

Year	1	2	3	4	5	6	7
O.	4000	4100	4250	4500	4700	5500	6500
Y.	1000	1000	950	1000	900	700	500
S.	7000	5500	4000	2800	1900	1200	600

The first step required is to determine the average cost if the equipment is used one year, 2 years, 3 years, etc.

If used one year only

$$X=\frac{3000+500+4000}{1000}=\frac{7500}{1000}=7.50$$

If used 2 years only

$$X=\frac{4500+500\times 2.05+4000\times 1.05+4100}{1000\times 1.05+1000}=\frac{5525+8300}{2050}=6.74$$

If used 3 years

$$X=\frac{6000+500\times 3.1525+8300\times 1.05+4250}{2050\times 1.05+950}=\frac{7576.25\times 12965}{3102.50}$$

$$=\frac{20541.25}{3102.50}=6.62$$

If used 4 years

$$X=\frac{7200+500\times 4.3101+12965\times 1.05+4500}{3102.50\times 1.05+1000}$$

$$=\frac{9355.05+18113.25}{4257.62}=\frac{27468.30}{4257.62}=6.451$$

If used 5 years

$$X=\frac{8100+500\times 5.5256+18113.25\times 1.05+4700}{4257.62\times 1.05+900}$$

$$=\frac{10862.80\times 23718.90}{5370.50}=\frac{34581.70}{5370.50}=\underline{6.439195}$$

If used 6 years

$$X=\frac{8800+500\times 6.8019+23718.90\times 1.05+5500}{5370.50\times 1.05+700}$$

$$=\frac{12200.95+30404.85}{6339.03}=\frac{42605.80}{6339.03}=6.72$$

If used 7 years

$$X=\frac{9400+500\times 8.1420+30404.85\times 1.05+6500}{6339.03\times 1.05+500}$$

$$=\frac{13471+38425.09}{7155.98}=\frac{51896.09}{7155.98}=7.25$$

With the data as given, it will be observed that $n=5$ yields the least average unit cost. The corresponding value of $X=6.439195$ will therefore be used to compute the annual depreciation charges by means of the relations

$$I_r = I_{r-1} - iD_{r-1}$$

and

$$D_r = XY_r - O_r - I_r.$$

$$\begin{aligned}
D_1 &= 6.439195 \times 1000 - 4000 - 500 &&= 1939.20\\
D_2 &= 6.439195 \times 1000 - 4100 - 403.05 &&= 1936.15\\
D_3 &= 6.439195 \times 950 - 4250 - 306.24 &&= 1561.00\\
D_4 &= 6.439195 \times 1000 - 4500 - 228.19 &&= 1711.01\\
D_5 &= 6.439195 \times 900 - 4700 - 142.64 &&= 952.64\\
&\text{Total} &&\$8100.00
\end{aligned}$$

The results may be checked by inserting the value obtained in formula

$$X = \frac{D_r + O_r + I_r}{Y_r}$$

For each year, X must and will become the same, namely the minimum 6.439195. For instance

$$X_1 = \frac{D_1 + O_1 + I_1}{Y_1} = \frac{1939.20 + 4000 + 500}{1000} = 6.4392.$$

It might be argued that in order to simplify the mathematical part of the procedure, the element of interest should not be included in the calculations, in which case as unit cost for the r^{th} year will result

$$X_r = \frac{O_r + D_r}{Y_r}$$

and as average unit cost, if the property is used n years,

$$X = \frac{W_n + \sum_1^n O_r}{\sum_1^n Y_r} \qquad (3)$$

into which merges equation (2) if there $i=O$, i.e., the unit cost is equal to the total operating costs for n years plus the wearing value, divided by the total number of units output. The depreciation charge for the r^{th} year is then given by

$$D_r = XY_r - O_r.$$

Testing formula (3) by applying it to the preceding problem requires again as first step the determination of n for which X assumes the lowest value.

If the equipment is used one year only

$$X=\frac{3000+4000}{1000}=7.00$$

If used 2 years

$$X=\frac{4500+8100}{2000}=\frac{12600}{2000}=6.30$$

If used 3 years

$$X=\frac{6000+12350}{2950}=\frac{18350}{2950}=6.22$$

If used 4 years

$$X=\frac{7200+16850}{3950}=\frac{24050}{3950}=6.0886$$

If used 5 years

$$X=\frac{8100+21550}{4850}=\frac{29650}{4850}=6.11340$$

If used 6 years

$$X=\frac{8800+27050}{5550}=\frac{35850}{5550}=6.46$$

The unit cost as thus derived shows an appreciable difference from the correct value, with the least value for $n=4$ instead of for $n=5$.

It would appear, therefore, that where greater accuracy is desired, the interest should be included. (It should be well understood that the interest only enters into the calculations as a constant and that it does not form any part of the depreciation burden itself; neither do all the items making up O.) The problem presenting itself, then, is to express the interest charges by some simple function which will give a yearly interest on the book value as near as possible to the true values. Such a function is for instance

$$I_r=I_1-(r-1)iD_1$$

which gives $D_r=XY_r-O_r-I_1+(r-1)iD_1$ (4)

$$\text{and } X=\frac{W_n+\sum_1^n O_r+nI_1+\frac{n(n-1)}{2}i(O_1+I_1)}{\sum_1^n Y_r+\frac{n(n-1)}{2}iY_1} \qquad (5)$$

or also $$I_r = I_1 - iD_1 - (r-2)\frac{W_r}{r}i, \text{ which gives}$$

$$D_r = XY_r - O_r - I_1 + iD_1 + \frac{r-2}{r}iW_r$$

and $$X = \frac{W_n + \sum_1^n O_r + ni\left(\frac{I_1}{i} + I_1 + O_1\right) - \sum_1^n \frac{r-2}{r} iW_r}{\sum_1^n Y_r + niY_1} \qquad (6)$$

Equations (5) and (6) prove, so far, to be simpler than (2) as all multiplications with an interest factor are eliminated. The results they furnish will naturally not turn out quite as accurate, although satisfactory for probably all practical purposes.

Submitting formula (5) to a test by applying it to the already known illustration, the following results are obtained:

For $n=1$

$$X = \frac{3000+4000+500}{1000} = 7.50$$

For $n=2$

$$X = \frac{4500+8100+1000+0.05\times4500}{2000+50} = \frac{13600+225}{2050} = \frac{13825}{2050} = 6.74$$

For $n=3$

$$X = \frac{6000+12350+1500+0.05\times3\times4500}{2950+3\times50} = \frac{18350+1500+675}{3100} =$$

$$\frac{20525}{3100} = 6.62$$

For $n=4$

$$X = \frac{7200+16850+2000+6\times0.05\times4500}{3950+6\times50} = \frac{26050+1350}{4250} =$$

$$\frac{27400}{4250} = 6.447$$

For $n=5$

$$X = \frac{8100+21550+2500+10\times0.05\times4500}{4850+10\times50} = \frac{32150+2250}{5350} =$$

$$\frac{34400}{5350} = 6.429906$$

For $n=6$

$$X=\frac{8800+27050+3000+15\times 0.05\times 4500}{5550+15\times 50}=\frac{38850+3375}{6300}=$$

$$\frac{42225}{6300}=6.70$$

As yearly burdens:

		Exact value
$D_1=6.4299\times 1000-4000-500$	$=1929.90$	1939.20
$D_2=6.4299\times 1000-4100-500+\ 96.50$	$=1926.40$	1936.15
$D_3=6.4299\times\ 950-4250-500+193.00$	$=1551.40$	1561.00
$D_4=6.4299\times 1000-4500-500+289.50$	$=1719.40$	1711.01
$D_5=6.4299\times\ 900-4700-500+386.00$	$=\ 972.90$	952.64
Total	8100.00	8100.00

so that

$$X_1=\frac{1929.90+4000+500}{1000}=6.4299$$

$$X_2=\frac{1926.40+4100+403.50}{1000}=6.4299$$

etc.

$$X_5=\frac{972.90+4700+114}{900}=6.4299$$

That is, $X_1=X_2=\ .\ .\ .\ X_5$, in accordance with the problem set.

(To be continued)