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## VON WRIGHTS THEORY AS THE SUBSYSTEM OF THE THEORY D

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The current interest in the research of deontic logics was stimulated by the classic paper by von Wright «Deontic Logic (1951)». The presentation purpose is to compare the formal theory by von Wright with the formal theory D described in the book «Fitting, Mendelsohn – First-Order Modal Logic (1998)» in order to show that the former is included in the latter as a subsystem.

First of all, the language L by von Wright should be considered. In this language, deontic operators are prefixed to names of acts, not to descriptions of states of affairs [3.P.2]. The word «act» is ambiguous. It may be used to refer to general characteristics of acts (e.g. smoking) or to individual acts (e.g. smoking of a particular person). In von Wright's language, deontic operators can only be prefixed to names of acts in the former sense. This approach has a significant syntactic consequence. If deontic operators are prefixed to names of acts then their iteration is impossible, since expressions containing deontic operators are not names of acts.

The performance or non-performance of a certain act (by an agent) von Wright defines as performance-value [3.P.2]. An act is defined as the performance-function of certain other acts if its performance-value depends upon the performance-values of those acts by the same agent. The performance-function concept is similar to the truth-function concept in propositional logic.

The vocabulary of the language W by von Wright contains the standard set of logical connectives that can denote both performance- and truth- functions ( $\sim, \&, \rightarrow, \equiv$ ), the set of atomic names (p, q, r, ...) and two deontic operators O (obligatory) and P (permitted).

In language by von Wright the operator P is primitive. The operator O is introduced as an abbreviation:

$$OX \text{ is } \sim P \sim X$$

Let us give the definition of the molecular complex of acts names [3.P.3].

Def.: [Molecular complex] The set of molecular complexes of acts is specified by the following rules:

1. Every atomic name is a molecular complex;
2. If A is a molecular complex, so is  $\sim A$ ;
3. If A and B are molecular complexes, and  $\circ$  is a binary connective, then  $(A \circ B)$  is a molecular complex.

Sentences of the form «PA» and «OA», where A is a molecular complex, are called P-sentences and O-sentences respectively. Let us give the definition of well-formed formula for the language W.

Def.: [Formula] The set of well-formed formulas of the language W is specified by the following rules:

1. Every P-sentence or O-sentence is a formula;
2. If A is a formula, so is  $\sim A$ ;
3. If A and B are formulas, and  $\circ$  is a binary connective,  $(A \circ B)$  is a formula.

Von Wright offers the following set of axiom schemes:

Def.: [Axioms] The axioms for W are in three categories.  
 CLASSICAL BASIS All tautologies [2.P.8].  
 SCHEMA A1 All formulas of the form  $P(pvq) \equiv Pp \vee Pq$   
 SCHEMA A2 All formulas of the form  $Pp \vee P\sim p$   
 Von Wright accepts the following rules of inference:

<b>Modus Ponens</b>	$\frac{X \quad X \rightarrow Y}{Y}$	<b>The rule of extensionality</b>	$\frac{X \equiv Y \quad OX}{OY}$
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Now let us consider the language D of the modern deontic logic [1.P.71].  
 The vocabulary of the language D contains a standard set of logical connectives ( $\sim$ ,  $\&$ ,  $\rightarrow$ ,  $\equiv$ ), a set of proposition letters ( $p$ ,  $q$ ,  $r$ , ...) and two deontic operators ( $O$ ,  $P$ ).

Let us give the definition of well-formed formula for the language D [1.P.6].  
 Def: [Formula] The set of well-formed formulas of the language D is specified by the following rules:

1. Every propositional letter is a formula;
2. If A is a formula, so is  $\sim A$ ;
3. If A and B are formulas, and  $\circ$  is a binary connective,  $(A \circ B)$  is a formula;
4. If A is a formula, so are  $OA$  and  $PA$ .

The formal theory D includes the following set of axiom schemes:

CLASSICAL BASIS All tautologies  
 SCHEMA K All formulas of the form  $O(p \rightarrow q) \rightarrow (Op \rightarrow Oq)$   
 SCHEMA D All formulas of the form  $Op \rightarrow Pp$   
 D has two rules of inference [1.P.69]:

<b>Modus Ponens</b>	$\frac{X \quad X \rightarrow Y}{Y}$	<b>Necessitation</b>	$\frac{Y}{OY}$
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Now let us compare W and D. The main difference is that in von Wright's logic, letters denote acts, while in logic D they denote statements. The set of well-formed formulas of the theory W is included in the set of well-formed formulas of the theory D. There is no converse inclusion, since in logic D the iteration of modal operators is possible and in logic W formulas in which logical connectives are used to combine deontic and non-deontic components (e.g.  $Op \rightarrow p$ ) are not accepted as well-formed. We can justify the rule of extensionality in theory D. It is easy to prove that the schema A1 is equivalent to the schema K and that the schema A2 is equivalent to the schema D. Thus, it can be argued that the set of theorems W is included in the set of theorems D. There is no converse inclusion, because firstly, modal operators can be iterated in D, secondly, the Necessity rule cannot be justified in von Wright's logic. So the system D includes the system W as a subsystem.

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