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Pentamode Metamaterials under Dynamic Loading

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***Abstract*—The field of metamaterials has grown considerably in the last few decades due to the advances in new manufacturing technologies. Metamaterials currently are of interest for a wide variety of applications including damping systems. This work is aimed to evaluate dissipative effect of pentamode metamaterials subjected to dynamic loading. The results of numerical modelling of the mechanical behavior of pentamode metamaterials from alpha titanium alloys are received and compared with available experimental data. The model of inelastic deformation and ductile damage criterion are used to describe the ductility of the unit cell of metamaterials in a wide range of strain rates, temperature and stress triaxiality. A methodology for analyzing the energy dissipation due to inelastic deformation of metamaterials at high strain rates is presented. It is shown that the values of the energy dissipation coefficient during uniaxial dynamic compression of the pentamode metamaterial are 1.5 times higher than for the bulk alloy counterpart.**

Keywords—metamaterials, alpha titanium alloys, energy dissipation, dynamic response.

I. INTRODUCTION

Devices and aerospace technical objects are often subjected to intensive dynamic impacts during operation. Development of damping materials and technologies of their manufacturing is one of the important problems for modern designs. Various composite and porous materials are widely used in modern aerospace objects.

In the works by Bragov et al, it was shown that in the case of pulsed mechanical loadings, including shock impacts, porous and frame-reinforced polymer composite materials exhibited higher dissipative properties compared to structural alloys [1–3]. Kadic et al. and Zadpoor in the review articles noted that dissipative effects played a major role in the observed non-proportional stress behavior in the loading path under cyclic loading of cellular metamaterials [4, 5]. Mohsenizadeh et al. showed that lightweight metamaterials have higher dissipative properties compared to other existing materials, such as open-cell metal foams [6].

The concept of mechanical metamaterials has been implemented in additive manufacturing technologies, which has enabled to produce materials with a complex spatial micro/nano-architecture.

Extreme metamaterials produced using honeycomb grids have an anisotropic deformation resistance. Milton and Cherkaev showed that specific combinations may be used to manufacture extreme materials [7]. Volumetric cells of extreme materials can be combined to produce materials with the required elasticity tensor. Metamaterials with two, three, four, or five small eigenvalues of the effective elastic moduli tensor components are called bimode, trimode, quadramode, and pentamode metamaterials, respectively. It was shown by Kadic and Bückmann that the experimental samples of pentamode metamaterials may exhibit the Poisson coefficient close to −1 (dilatation metamaterial) [8, 9].

The pentamodal structure of the material, proposed by Milton and Cherkaev [7, 10] and subsequently studied by Kadic [9], Bückmann and his colleagues [8, 11], consists of elements representing the connection of two truncated cones with a fixed length. Ideally, the structural elements are connected to each other in the region of their vertices and form a structure that resembles a face-centered cubic crystalline structure. The work by Mejica and Lantad [12] demonstrates the possibility of creating pentamode metamaterials with structures of other Bravais lattices.

Hedayati et al. [13] showed the possibility to produce pentamode metamaterials based on a titanium alloy by selective laser melting. The possibility of additive manufacturing of pentamode materials with a normalized mass density ρ/ρ_s from 0.24 to 4.24 ± 0.02 (ρ_s is the mass density of the material of the cell elements) from the Ti-6Al-4V alloy powders is shown. A feature of the lattice structures of pentamode materials is that the stress arising in the elements of the lattice structure is transmitted through the vertices of the cones. The parameters of the periodic structure and the geometric parameters of the elements can be set in accordance with the capabilities of modern 3D printing technologies. An increase in the diameter

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of the cones has a minimal effect on the rigidity of the structure of pentamode metamaterials, but affects the mass density. As a result, the properties of extreme pentamode metamaterials differ from the properties of most porous materials and lattice structures, for which the dependence of the elastic modulus on the mass density exhibit a power-law behavior. Due to this feature of the mechanical properties of pentamode materials, they can be used to control the dynamics of stress waves and acoustic waves [14].

Of interest is the study on dissipative properties of pentamode metamaterials subjected to pulse mechanical loadings. Pentamode metamaterials based on light alloys can be used as mechanical dampers placed between support plates.

II. Simulation of a Pentamode Metamaterial Subjected to Pulse Loading

In this work, we performed numerical studies of the mechanical response of a titanium alloy pentamode metamaterial layer with a relative mass density ρ/ρ_s = 3.15% under initial temperatures of 300 K and 900 K and a pulse loading of 100 m/s.

To study the dynamic response of pentamode metamaterials with a skeleton structure based on the alpha-titanium alloy Ti-5Al-2.5Sn (Grade 6), a numerical simulation was carried out using the finite element method. The model metamaterial volume had effective dimensions of 10 mm × 4.8 mm x 5 mm, and cell elements parameters were D = 0.4 mm, d = 0.11 mm, h = 1.6 mm.

The explicit finite element LS-DYNA solver of WB ANSYS was used to solve the boundary problem.

The model of mechanical behavior of the pentamode metamaterial includes the system of mass, momentum and energy conservation equations, kinematic relations, elastic-viscoplastic constitutive equations of the unit cell material, equations of the kinetics of the damage nucleation and growth, the fracture criterion [15–19].

The Gurson-Tvergaard condition was used as a yield criterion of the porous material [17, 19]:

$$(\sigma_{eq}{}^2/\sigma_{ys}{}^2)+2q_1 f^* \cosh(-q_2\, p/2\sigma_{ys})-1-q_3(f^*)^2=0\,, \quad (1)$$

where σ_{ys} – the yield stress, p = $\sigma_{kk}/3$ – the pressure, $\sigma_{eq}=\sqrt{(3/2)S_{ij}S_{ij}}$, $S_{ij}=\sigma_{ij}+p\,\delta_{ij}$ – components of the deviatoric stress tensor, q_1, q_2 и q_3 – model parameters, f – damage parameter of the material.

To describe the change in the stress of plastic flow in alpha phase titanium alloys with a hexagonal close-packed crystal lattice, the following ratio was used [15]:

$$\begin{aligned}\sigma_{ys} &= \sigma_{s0}+C_2[\exp(-C_3T+C_4T\ln(\dot{\varepsilon}_{eq}/\dot{\varepsilon}_{eq0})]\times \\ &\times\{C_5(\varepsilon^p_{eq})^n+C_6\}[B_1+B_2T+B_3T^2]\end{aligned}\,, \quad (2)$$

where σ_{ys} – yield stress, $\sigma_{s0,}$ C_2, C_3, C_4, C_5, C_6, n, B_1, B_2, B_3 – coefficients of constitutive equations, T – temperature, $\varepsilon^p_{eq}=\sqrt{(2/3)\varepsilon^p_{ij}\varepsilon^p_{ij}}$ – equivalent plastic strain, $\dot{\varepsilon}_{eq}$ – equivalent strain rate, $\dot{\varepsilon}_{eq0}$ = $1.0\cdot10^3$ s^{-1}.

To describe the evolution of damage (growth and nucleation of discontinuities) and fracture of the material of unit cell of pentamode metamaterials, the Needleman model was used [19].

The temperature increase in the plastic strain zone at high-speed deformation was calculated in the adiabatic approximation:

$$T=T_0+(0.9/\rho_s\, C_p)\int_0^{\varepsilon^p_{eq}}\sigma_{eq}\, d\varepsilon^p_{eq}\;, \quad (3)$$

where T_0 is the initial temperature in the material of metamaterial unit cell, ρ_s = $4.4\cdot10^{-6}$ kg/mm^3 is the mass density of the titanium alloy Ti-5Al-2.5Sn, C_p = $458+0.35\cdot T-1.929\cdot10^{-4}\cdot T^2+8.758\cdot10^{-8}\cdot T^3$ [J/kg·K]is the specific heat capacity of the titanium alloy Ti-5Al-2.5Sn.

The relation for estimation of the normalized Young's modulus E/E_s for a metamaterial with a diamond–like lattice structure was used [13, 20]:

$$E/E_s=[\sqrt{6}\;\pi(3/4)^2\;(d/2h)^4]/[1+1.5(d/2h)^2]\,, \quad (4)$$

where E_s is the Young's modulus of the unit cell elements material, d and h are the minimal diameter and length of unit cell elements of metamaterial, respectively.

The analytical formula was used for estimation of the yield strength of a metamaterial [16]:

$$\sigma_y/\sigma_{ys}=[9\pi/4\sqrt{6}](d/2h)^3\,, \quad (5)$$

where σ_y and σ_{ys} are the effective yield stress of metamaterial and the yield stress of the unit cell elements material, respectively.

The damping properties characterizing the dissipation of mechanical energy during loading of the pentamode metamaterial with the Ti-5Al-2.5Sn titanium alloy skeleton subjected to high-speed deformation were evaluated by the amount of dissipated energy and the energy dissipation coefficient [21]:

$$\Delta W(t)=W(t)-W_{\rm int}(t),\quad \lambda(t)=\int_0^t\frac{\Delta W(t)}{W(t)}dt\,, \quad (6)$$

where ΔW is the increment of dissipated energy, W is the work of loading force on displacement, W_{int} is the internal energy of the metamaterial volume, λ is the energy dissipation coefficient.

The specific energy W supplied to the model volume of the metamaterial under pulse compression was calculated by:

$$W = \frac{1}{m_0} \int_0^t F(t) u(t) dt \, , \tag{7}$$

where $u(t)$ is the displacement of the movable support of the metamaterial, $F(t)$ is the calculated resistance force in the movable support plane, $m_0 = \rho_0 V_0$ is the initial mass of the deformable metamaterial volume, ρ_0 is the initial effective mass density of the metamaterial, V_0 is the initial effective volume of the metamaterial.

The value of the specific internal energy of the volume element of metamaterial was calculated by the formula:

$$W_{\mathrm{int}}(t) = \frac{1}{m_0} \int_v \Big(\int_0^{\varepsilon_{ij}^e} \sigma_{ij} \, d\varepsilon_{ij}^e \Big) dv \, , \tag{8}$$

where W_{int} is the internal energy of deformed unit cell elements material, V is the total volume of unit cell elements, σ_{ij} is the stress tensor components, calculated in the unit cell elements material, $d\varepsilon_{ij}^e$ is increments of the elastic stain tensor components in the unit cell elements material.

The initial conditions assumed a constant initial temperature and the absence of stresses and strains in the unit cell elements. The boundary conditions were set in accordance with the loading conditions of the model volume of the metamaterial. The model volume of the pentamode metamaterial was placed between the upper fixed plane and the lower movable plane. The upper boundary of the metamaterial volume support was considered rigid and immovable. Uniaxial compression of the metamaterial volume was realized when the movable plane was displaced at a constant velocity.

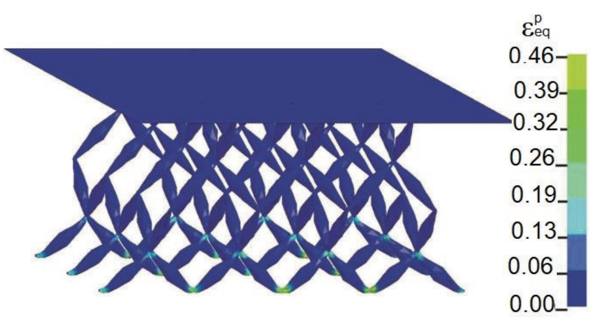
The following values of the model coefficients were used in the calculations (1)−(5): σ_{s0} = 0.25 GPa, C_2 = 0.8 GPa, C_3 = 0.0043, C_4, C_5, C_6, n = 0.25, B_1 = 1.0, B_2 = 0.000466, B_3 = 2.43 10^{-7}, q_1 = 1.0, q_2 = 1.3, q_3 = 1.0, ε_N = 0.25, F_N = 0.04, S_N = 0.1, f_F = 0.26, f_c = 0.117.

The grid convergence of the calculation results within a few percent was provided by the choice of the discretization step of the grid model.

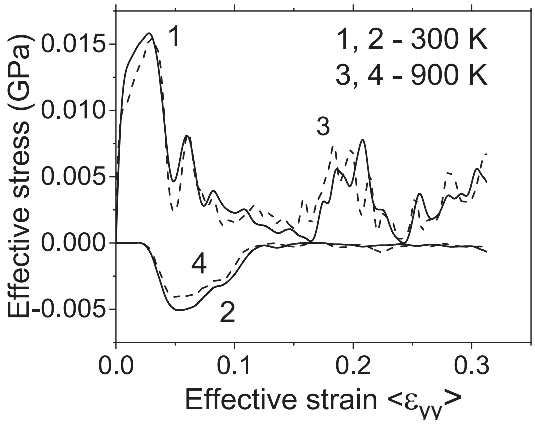
III. Results

The results of simulation indicate the localization of inelastic strains in the region of the joint of the unit cell elements. The initial normalized mass density of the model volume of the pentamode metamaterial ρ/ρ_s was 3.145 %.

Fig. 1a shows the calculated equivalent plastic strain in the unit cell elements of the metamaterial compressed by 39%.



(a)



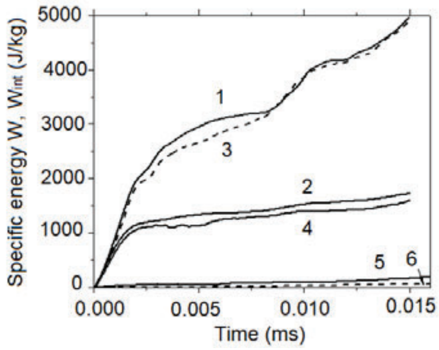
(b)

Fig. 1. Calculated equivalent plastic strain at t=0 (a); Reaction force under vertical uniaxial compression at the movable plane (curves 1 and 3) and reaction force at the fixed plane (curves 2 and 4) versus time (b).

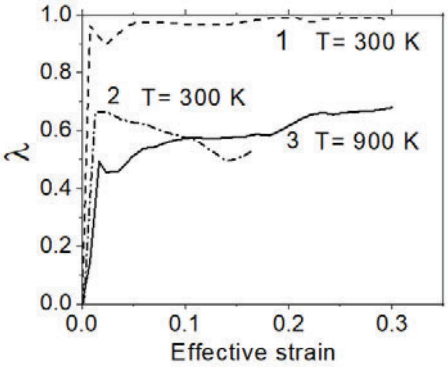
Fig. 1b shows the effective engineering stress versus engineering strain under axial compression of the metamaterial model volume. Curves 1 and 3 correspond to the force acting on the movable support surface, and curves 2 and 4 to the immovable one. Curves 1 and 2 were obtained at the initial temperature of 300 K, and curves 3 and 4 at the initial temperature of 900 K.

The calculated effective stress versus effective engineering strain diagrams are in qualitative agreement with the experimental stress-strain diagrams of titanium pentamode material created using the selective laser melting technology [8]. The delay in the arrival of a stress pulse to the fixed support surface is determined by the propagation velocity of elastic waves along the metamaterial cell elements. The initial stage of deformation was characterized by close to linear dependence of the force on the relative value of compression. This made it possible to identify it as quasi-elastic reaction and evaluate the effective values of elastic modules (Young's modulus, Poisson's ratio, and shear modulus).

Fig. 2a shows the calculated values of the specific mechanical energy W over time (curves 1 and 3) and the specific internal energy W_{int} (curves 2 and 4) over time. Curves 5 and 6 indicate the change in W and W_{int} during high-speed deformation of the bulk titanium alloy Ti-5Al-2.5 Sn at a strain rate of 100 s^{-1}. Fig. 2b shows the coefficient of energy dissipation λ versus the effective deformation <εyy> at initial temperature of 300 K (curves 1 and 2) and 900 K (curve 3).



(a)



(b)

Fig. 2. Specific internal energy of deformed unit cell elements under compression at the velocity of 100 m/s. Curves 1, 2 correspond to *W*(*t*), and curves 3,4 correspond to Wint (t)), respectively (a). The coefficient of energy dissipation λ versus effective strain <ε_{yy}> at the initial temperature of 300 K (curves 1 and 2) and 900 K (curve 3) (b).

IV. Discussion

The value of $\langle E\rangle/E_s$ obtained by simulation was $2.9\cdot10^{-5}$ at $E_s = 107$ GPa, and the value of the normalized effective yield stress σ_y/σ_{ys} was estimated as 0.00087 at $\sigma_{ys} = 0.95$ GPa and $T = 300$ K. The data obtained in the calculations are consistent with the experimental data ($\langle E\rangle/E_s = 2.74\cdot10^{-5}$ and $\sigma_y/\sigma_{ys} = 0.00024$ at $\rho/\rho_s = 3.15\%$ for pentamode material obtained by selective laser melting of a Ti-6Al-4V powder using high -energy density laser beam [8]). When comparing the data, the difference between normalization parameters from [13] and simulation was taken into account, such as the values ($E_s = 113$ GPa, $\sigma_{ys} = 0.9$ GPa, $\rho_s = 4.43$ 103 kg/m^3) of Ti-6Al-4V were slightly differ from the corresponding parameters for the Ti-5Al-2.5Sn alloy. The theoretical estimation of $\langle E\rangle/E_s$ by (4) and σ_y/σ_{ys} by (5) were of $0.8\cdot10^{-5}$ and $1.17\cdot10^{-4}$ (for the parameter $d/2h = 0.034375$ of pentamode metamaterial), respectively. The metamaterial is deformed inelastically until the stability of the elements of the cellular structure is lost, since the stresses inside the cellular elements have reached critical values. Further deformation is accompanied by wedging of unit cell fragments, which results in a temporary increase in the resistance to compression. The energy dissipation occurs as a result of the localization of plastic strain; damage and fracture of the cell elements. The triaxiality stress state parameter $\eta = -p/\sigma_{eq}$ may change significantly during inelastic deformation.

As a result of the work dissipation on the plastic deformation, the temperature rises in local zones of the cell elements. The evolution of the cell structure causes the different values of the dissipated work and energy dissipation coefficient. As the initial temperature increases from 300 K to 900 K, the predicted value of the energy dissipation coefficient λ decreases from 2 to 1.5 times with increasing compression strain of the metamaterial. The decrease in the energy dissipation coefficient is associated with the decrease in stresses acting in the unit cell elements at higher temperature. A growth in the energy dissipation coefficient λ along with compression degree shows non-linear behavior.

The λ increments are predominantly caused by the dissipation of the mechanical work due to plastic deformation in the metamaterial elements. Additional increments of the energy dissipation are associated with the nucleation and growth of damage in the material of the unit cell elements. A slight increase in the energy dissipation coefficient λ takes place due to these increments.

When bulk alloys are deformed, there are no analogues to the physical mechanisms of wedging and compaction of fragmented elements of the metamaterial cell.

The higher values of the energy dissipation coefficient for pentamode metamaterials as compared with the value for the bulk alloy counterpart are due to the significant difference between the specific energy W supplied to the volume of metamaterial and bulk alloy. Thus, pentamode metamaterials based on titanium alloys manufactured by selective laser melting are perspective materials for the development of structures of pulsed mechanical dampers.

V. Conclusion

A computational model describing mechanical response of titanium pentamode metamaterials to pulse loading has been developed.

The numerical simulation of the mechanical response of a pentamode metamaterial from the Ti-5Al-2.5Sn titanium alloy was carried out during dynamic compression at 100 m/s.

The calculated normalized Young's modulus $\langle E\rangle/E_s$ and the value of the normalized effective yield stress σ_y/σ_{ys} in good agreement with the experimental data obtained by Hedayati and coworkers [9].

A methodology for analyzing the energy dissipation due to inelastic deformation of structured materials at high strain rates was presented for metamaterials.

The values of the energy dissipation coefficient were determined for uniaxial compression of the pentamode metamaterial with the relative mass density of 3.145 % at strain rates of ~ $20.8\cdot10^3$ s^{-1} and initial temperatures of 300 K and 900 K.

The values of the energy dissipation coefficient during uniaxial dynamic compression of the pentamode metamaterial are 1.5 times higher than for the bulk alloy counterpart.

The energy dissipation coefficient under uniaxial compression decreases by a factor of 1.5–2 with an increase in the initial temperature from 300 to 900 K.

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