A taxonomy for the mereology of entangled quantum systems

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The emerging field of quantum mereology considers part-whole relations in quantum systems. Entangled quantum systems pose a peculiar problem in the field, since their total states are not reducible to that of their parts. While there exist several established proposals for modelling entangled systems, like monistic holism or relational holism, there is considerable unclarity, which further positions are available. Using the lambda operator and plural logic as formal tools, we review and develop conceivable models and evaluate their consistency and distinctness. The main result is an exhaustive taxonomy of six distinct and precise models that both provide information about the mereological features as well as about the entangled property. The taxonomy is well-suited to serve as the basis for future systematic investigations.

1. Introduction

Quantum mereology concerns the question how the quantum realm is structured into parts and wholes. The fact that quantum systems can be *entangled* (Einstein, Podolsky & Rosen 1935, Schrödinger 1935), and most of them are, seems to make the quantum realm peculiar also[1] concerning the part-whole relation because micro-reductionism fails in entangled systems (Maudlin 2006 / 1998, Hüttemann 2005): Entangled states do not supervene on intrinsic states of the parts, and consequently the entangled property of a two-particle system ("macro property") does not reduce to properties of the one-particle systems ("micro properties").[2] Typically, entanglement has been understood to imply some kind of "quantum holism", which has been spelt out in different ways.[3]

However, most of this discussion relies on a rather intuitive and imprecise notion of "part" and "whole". As a consequence, it often remains an open question whether certain positions have substantial disagreement (or in fact are equivalent), and even whether certain positions would be consistent given a clear meaning of the notions.

Only in recent years, philosophers have started to apply the resources of the more developed debate about the mereology of ordinary objects to the quantum realm (e.g. Calosi, Fano & Tarozzi 2011; Calosi

[1] It is well-known that quantum entanglement poses a spatio-temporal problem (Bell 1964, Maudlin 2011/1994), a problem for individuation (Redhead & Teller 1992; Saunders 2006) and a causal problem (Näger 2016).

[2] As usual in the philosophical debate here and in the following the attributes "micro" and "macro" do not denote the microscopic or macroscopic scale, respectively, in the physical sense, but rather discern between the level of parts and the level of the composite quantum system, which both belong to the microscopic scale in the physical sense.

[3] Authors have assumed that there are fundamental entanglement relations ("relational holism", Teller 1986), irreducible / non-separable wholes (Howard 1989, Esfeld 2001) or undivided wholes (Bohm 1993).

& Tarozzi 2014). It proceeds from a precise formal theory of parts and wholes, called "classical extensional mereology" (CEM).[4] We agree with the pioneering authors in the field of quantum mereology that using the precise resources from the classical debate is an appropriate way to better understand the mereological peculiarities of the quantum realm.

An important step in an emerging field is to get an overview of the available options. On the one hand there are models in quantum mereology that have been (at least in their rough forms) established for decades: It is well-known that entangled quantum states can be regarded to describe the irreducible intrinsic property of a (possibly undivided) whole ("monistic holism"); or they can be regarded to describe an irreducible relational property between entangled micro objects (Teller 1986; "relational holism"). On the other hand, the debate has recently become a new twist by additional proposals that regard the entangled property to describe a plural property that is collectively carried by the micro objects (Bohn 2012; Brenner 2018). If these latter models are admissible, they would extend the space of available options. But are they? Are these options reliable, distinct models? And: Are there other conceivable models that one needs to take into account?

This paper aims at making clear which models are in fact available for the quantum mereologist in entangled system. It considers both the suggested models as well as alternative, prima facie sensible ones, carefully analyses the models and tries to clarify their relations. It will turn out that some models are in fact equivalent while others are even inconsistent. The main result of this paper will be a detailed, exhaustive taxonomy of six distinct and precise models for the relation between parts and wholes in entangled systems.

On the way to our result we make use of formal methods that are not common in the debate yet. In order to characterize complex predicates we shall apply the so called "lambda operator"; the emerging formalism of plural logic will help us to characterize certain suggestions more clearly; and occasionally, we shall even combine the two. Note that this paper is not about quantum logic: Though using somewhat advanced tools of logic, the basic underlying assumption is classical first-order predicate logic.

The paper is organised in three main sections. In a first part (I) we introduce possible mereological models that describe the part-whole relations in entangled systems. A second part (II) examines how the irreducible entangled quantum state can be modelled in terms of properties or relations. Finally, we discuss which combinations of the models are viable and discuss immediate results (III).

I. Objects and mereology

2. Formal mereological models and quantum theory

Classical extensional mereology (CEM; Tarski 1929, Leonard & Goodman 1940, Simons 1987) is about objects being parts of objects. There are two questions, which a mereological model must be informative about:

(i) Which objects are there?
(ii) Which objects stand in the is-part-of relation?

[4] "Classical" in "classical extensional mereology" does not contrast with "quantum". It does not matter here what it contrasts with. As to the name, we follow Hovda (2009).

We call the conjunction of (i) and (ii) the "basic mereological question". An answer to them is called a "mereological model".

CEM is general enough to be compatible with many different interpretations of quantum theory and does not commit one to a specific reading of the theory. Nevertheless we restrict our considerations in the following to *realistic* readings of quantum states, i.e. interpretations which assume that irreducible quantum states describe real properties;[5] for interpretations of the theory that imply that entangled quantum states describe real, irreducible properties are the most debated and the most interesting ones from a mereological perspective. This is the main assumption we need to make concerning the interpretation of quantum theory, so the models we shall develop here are compatible with a wide range of interpretations such as a standard textbook interpretation, a GRW mass density theory (Ghirardi, Grassi & Benatti 1995), a propensity interpretation (Popper 1957) etc.[6] We shall now sketch CEM's fundamental notions and thereby explain, why the theory is rather general and thus well-suited for describing the quantum realm.

First, the notion of an object required by CEM is rather thin. As CEM is an extension of classical first order predicate logic, the minimal requirements for an object are determined by that theory. There are three aspects. First, CEM's standard semantics requires that an answer to question (i) indicates a nonempty domain of *well-distinguished* objects, i.e. objects capable of being elements of some set. Second, disregarding their ordering as parts and wholes, the objects are basically all on a par. Especially, considerations concerning ontological priority of parts to composites (or vice versa) do not play any role in the theory. Third, first order predicate logic assumes that it is intelligible to distinguish between objects (e.g. a proton) and the *properties that they bear* (for instance, having mass 1u). The properties are not themselves elements of the domain but are defined extensionally. Thus, the theory is not committed to any specific theory of substances. The objects may even be space-time points as in a moderate event ontology that assumes events to be instances of properties at space-time points.[7]

Being based on first order predicate logic, the semantics of classical mereology involves the usual strong concept of identity and discernibility, and especially it assumes that one is able to give names to objects.[8] It is true that in systems with quantum objects of the same kind the objects are widely assumed to not have strong identity (either they are non-individuals or only have weak identity).[9] Since

[5] By this assumption we preclude anti-realistic readings of the theory such as epistemic interpretations (which understand the wave function as a state of knowledge, e.g. Friederich 2011) or a GRW flash interpretation (according to which there are only flash events in space-time and quantum states are mere means to predict such flashes; see Bell's ontological elaboration (2004, ch. 22) of Ghirardi, Rimini & Weber's theory (1986)).
[6] While our arguments in the following are based on non-relativistic quantum mechanics, we should note that we expect them to be generalizable to quantum field theory (as the formal structure of entangled quantum states there remain unchanged).
[7] CEM is formally even compatible with certain ontologies that deny the existence of property bearers on a fundamental level (e.g. event ontologies or certain forms of ontic structural realism). Consider, for instance, a radical event ontology that assumes events to be instantiations of properties without bearer: If such events are well-individuated by the properties that constitute them, these events can be referred to as the objects of the domain. So when in the following we speak of objects bearing properties, one can understand this to include such radical ontologies as well (although this might need additional adaptions at certain points).
[8] Individual constants are clearly names, and so are variables, if provisionary ones, as the arguments of a value assignment.
[9] Lyre (2018) provides an overview of the debate. On maintaining objects that are weakly discernible cf. Saunders (2006). For an approach which models non-individuals by abandoning the usual set-theoretic semantics and works with – today not very well-explored – quasi-sets cf. French & Krause (2006).

a mereology for non-individuals (or weakly identical objects) is currently not at hand,[10] this yields the following dilemma for any quantum mereologist: Either one cannot make precise statements about parts and wholes in the quantum realm at all (since one misses an appropriate mereology) or one makes precise statements from a classical point of view (if one applies classical mereology), which, however, might be false. Fortunately, there is an elegant way out of the dilemma that becomes evident when one regards the quantum mechanical description of objects of the same kind: Quantum mechanics refers to the objects by labels (i.e. names) and then symmetrizes their properties, yielding models that can either be interpreted to involve non-individuals or objects with weak identity. The same strategy is available for mereological models: We here use names for objects and require possible models to symmetrically distribute all properties among the entangled objects. The resulting mereological models can then be understood in the one or the other way. Thereby one can separate the question about the mereological features of entangled systems from that about their individuation, and this paper focusses on the former.

The second central concept for CEM is the parthood relation, which the theory characterizes axiomatically. For convenience of presentation we choose *proper* parthood (notated by "$\ll$") as our primitive notion.[11] Accordingly, in what follows, the default meaning of "part" is "proper part". Proper parthood is characterized as asymmetric (thus irreflexive) and transitive.

For instance, a simple mereological model (for most part of this paper we shall not transcend the complexity of this example) might be that there are three distinct objects, say a, b and c, and that a is part of c and b is part of c. Stipulating that the model is complete, implies three facts. First, a and b do not overlap: They have no common parts; otherwise these would have to be recognised as objects in the model. Second, c has the special status of being the mereological sum of a and b (and thus, clearly, is a "composite object" or "whole"). And third, a and b compose c (since a and b are disjunct and c is their sum).[12]

The concept of a mereological sum (sometimes called "fusion") is a defined, not a primitive notion. In order to state its definition,[13] we need the notion of reflexive parthood (notated by "<"), which is simply proper parthood *or* identity. Moreover, we need the notion of overlap: x overlaps y (notation: x O y) if and only if there is something, which is both a part of x and of y. CEM allows for using definite descriptions of sums.[14] We can always speak of *the* sum over the condition Φ, and we use "σ" in order to notate this as $\ulcorner\sigma x\, \Phi x\urcorner$. $\ulcorner\sigma x\, \Phi x\urcorner$ is defined as an abbreviation for

$\ulcorner \iota y\, (\forall z\, (\Phi z \rightarrow z < y) \wedge \forall z\, (z < y \rightarrow \exists z'\, (\Phi z' \wedge z'\, O\, y)) \urcorner$

The definition says: "The sum over condition Φ" means "that object y such that (1) everything that fulfills condition Φ is a reflexive part of y and such that (2) every reflexive part of y overlaps something

[10] But see the efforts towards that aim, most notably Krause (2011) who proposes a mereology based on quasi-set theory.

[11] The theory can be equivalently formulated in a number of ways by choosing other notions as fundamental (and appropriately adjusting the axioms). For instance, Leonard & Goodman (1940) take the notion of overlap as fundamental.

[12] This agrees with the definition of "composition" in Inwagen (1990), which is standard today. The *definition* is independent of Inwagen's rejection of mereological universalism.

[13] The concept of a mereological sum which we shall use is type-2-fusion according to Hovda (2009). Other definitions differ in nuances; these differences, however, disappear if the chosen framework is CEM.

[14] Not every conceivable mereology does.

that fulfills condition Φ". As an abbreviation for "σx(x | x=a $\vee$ x=b)" ("the sum over the condition of being identical with a or b"), we shall use "σ(a,b)", which is read as "the sum of a and b".

3. Axioms of sum existence and metaphysics

CEM requires axiomatically that there is a sum over every nonempty condition ("axiom of universal sum existence"). A fortiori, any two (or more) objects have a sum, which may be a scattered object. Therefore, a model assuming just the existence of two distinct objects a and b is not a model of CEM. Sums are not in any way second-rate objects. So CEM is not metaphysically innocent, but encapsulates mereological universalism.

Others have weakened the axiom of sum existence by claiming that only under *certain* conditions two (or more) given objects have a sum (yielding a weaker alternative theory to CEM). In what follows, we shall allow for all mereological models that are consistent with theories that have a weaker axiom of sum existence. Some of the models we consider are even compatible with the most radical form of weakening, which is to claim that there are no composite objects at all (mereological nihilism).

4. Basic mereological models for an entangled two-particle system

Entanglement is a special kind of state for composite systems according to quantum theory. The simplest case of entanglement is an entangled spin state of a two-particle system, e.g. the spin singlet state of a two-electron system:

$$|\psi\rangle_{12} = (|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2)/\sqrt{2} \qquad (1)$$

This entangled state describes the spin property[15] of a so called "two-particle system", which is meant to be a neutral name. It is neither meant to say that there exist two classical particles, nor that there is an object corresponding to the system as a whole. It is just meant to say that when measuring at the system, one finds two localized objects ("particles") whose measured spin properties are best predicted by the noted spin state. We should mention that besides a spin state, a quantum system further has a position state as well as state independent properties like mass, charge, total spin.[16]

We shall denote the object corresponding to the two-particle system by "c" and the object corresponding to the one-particle system by "a" or by "b", respectively. Whether these objects exist is not stated by the quantum mechanical formalism, but is a matter of metaphysical interpretation. There are three basic mereological models for an entangled two-particle system:

Model I: c exists, but a and b do not (i). In this case there is no further object which could be a proper part of c, so, trivially, there are no instances of the parthood relation (ii). According to this model, c is an undivided whole. While we shall make explicit below that any model of

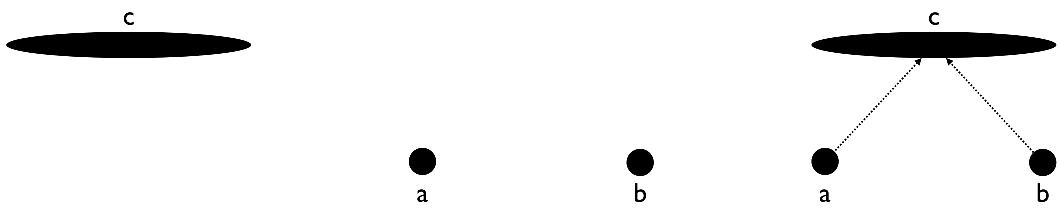
[15] According to the singlet state the total spin is zero. The ket vectors indexed by 1 or 2 indicate the spin states of particle 1 or 2, respectively, e.g. "$|\uparrow\rangle_1$" says that particle 1 has spin up. Note that due to the minus sign the complete state is in a superposition such that neither particle is in a definite spin state. More formally, the spin state spaces of the single particles are two-dimensional complex Hilbert spaces, $\mathcal{H}_1$ and $\mathcal{H}_2$, and the state space $\mathcal{H}_{12}$ of the two-particle system is the tensor product of these spaces, $\mathcal{H}_{12} = \mathcal{H}_1 \otimes \mathcal{H}_2$, which is complex four-dimensional.

[16] While position states can be entangled as well, it has become common in the debate about entanglement to focus on the mathematically much simpler cases of spin entanglement; in this paper, we follow this convention.

entangled quantum systems involves *some* kind of holism, this position is the most radical form of holism since the entangled system does not even have parts.[17]

Model II: a and b exist, but c does not (i). This model neither involves instances of the parthood relation nor (non-trivial)[18] sums (ii). The model is compatible with mereological nihilism which claims that composition does not occur.

Model III: a, b and c exist (i). In this case, a is a part of c and b is a part of c (ii). Then, according to the definition of a mereological sum, it follows that c is the mereological sum of a and b, c = σ(a,b).[19] Since a and b compose c, the model is compatible with mereological universalism as well as moderate compositionalism.



(a) Model I *(b) Model II* *(c) Model III*

Figure 1: Basic mereological models. Blobs represent objects, dotted arrows the parthood relation.

Models where only a and c, only b and c, just a or just b exist are implausible because they introduce an asymmetry concerning the existence of the one-particle systems – and there is no justification in sight for such an assumption. Hence, the three models presented here, exhaust the sensible mereological options for a two-particle quantum system.

II. Properties and reduction

5. Entanglement and the failure of micro-reduction

Entanglement is an interesting case for mereology for the very reason that synchronic micro-reductionism *fails* in entangled systems (Hüttemann 2005).

Consider again a two-particle system whose spin property is described by the singlet state (1). The state of such a two-particle system is called "entangled" if and only if it cannot be written (according to any basis) as a product of states for the one-particle systems, $|\psi\rangle_{12} \neq |\psi\rangle_1 \otimes |\psi\rangle_2$ (where "$\otimes$" denotes the tensor product). As combining states by the tensor product is the rule of composition for quantum systems, an entangled state cannot be regarded as being composed of states of the one-

[17] As there is evidence that according to quantum theory the universe is an entangled system, this model of entanglement might lead to the view that the universe is the only existing object (mereological monism).

[18] Trivially, a and b themselves are sums over the condition of being a and of being b, respectively.

[19] Given that a is a part of c and b is a part of c, and a and b are distinct, c could only fail to be the mereological sum of a and b if c extended beyond a and b, i.e. if there were a third object d, which is part of c and distinct from a and b. Then, however, we would be confronted with, at least, a three-particle system – in contradiction with our assumptions.

particle systems. It is one of the very surprising and unique features of the quantum realm that such entangled states exist.

There is disagreement among quantum theorists whether the states of the one-particle systems are well-defined and one cannot derive the entangled state from them, or the states of the one-particle systems are not well-defined at all, such that there is an additional, trivial reason for supervenience to fail. The disagreement concerns the question what exactly counts as quantum state, either it is rays in Hilbert space ("ray view") or statistical operators ("statistical operator view"). In both cases, however, if we take quantum theory at face value and interpret it realistically, especially if we assume that quantum states correctly describe the properties of quantum systems, we reach the result that quantum macro properties do not reduce to quantum micro properties (failure of supervenience / micro-reductionism, Maudlin 2006/1998). This is the central and undisputed feature of entangled systems that we shall exclusively make use of in the following. Our subsequent considerations are therefore compatible with both views of quantum states.

There is a second irreducibility claim associated with entanglement that is not so often stated. It concerns entanglement of three or more particles ("multipartite entanglement") and says that there are entangled states of several objects (n≥3) that cannot be reduced to binary entanglement relations. In the strongest case, it is not even possible to reduce an entanglement relation among n objects to entanglement relations among any l and m objects (with l+m=n). We shall call this the "relational irreducibility claim (of entanglement)". In this way, there are entanglement relations that are irreducibly n-ary relations. This contrasts with other n-ary relations, e.g. "standing in an equilateral triangle", that can be reduced to binary relations.

6. Modeling irreducible entangled properties

The failure of micro-reductionism constitutes an important boundary condition for the quantum mereologist. In addition to the basic mereological question, which objects there are (i) and which objects stand in the parthood relation (ii), any appropriate mereological model must answer two more questions.

In order to formulate these question, we assume that in the models we consider each predicate P corresponds to a property *P* (mind the italics!).[20] Analogously to the arity of a predicate (the number of its places; also called "adicity" or "valency") we speak of the arity of properties and relations, too.

Here are the additional questions:

(iii) Of which arity is the irreducible entangled property?
(iv) Which object(s) bear the irreducible entangled property?

There are four prima facie answers to questions (iii) and (iv) for the case of an entangled two-particle system:

Model A: The entangled property is a binary relation *R* holding between the one-particle objects a and b, such that "Rab" is true. (The two-particle system c might or might not exist.) As micro-reduction fails, *R* is not reducible to unary properties of the one-particle objects (unlike *being taller* which can be reduced to the size of each object); *R* is a fundamental

[20] It is of course not trivial to formulate models such that they fulfill the assumption.

relation (Teller 1986). *R* might, for instance, be the symmetric two-place relation *having opposite spin to*.[21]

Model B: The entangled property is a unary property *P* and is carried by the two-particle object c, such that "Pc" is true. The one-particle systems a and b might or might not exist. If a and b exist, due to the failure of micro-reduction, *P* cannot be reduced to unary properties of a and b. In order to be distinct from model A, the position furthermore requires that *P* is not reducible to a relation *R* holding between a and b.

Model C: The entangled property is a unary property P^{col} and is carried by a and b collectively. The idea here is unusual in a twofold way: First, several micro objects can carry a macro property jointly such that the macro object need not exist as the bearer of the macro property (c might or might not exist.). Second, attributing a *unary* predicate to several objects requires *plural* attribution as in "Huey, Dewey and Louie stand in an equilateral triangle". Due to the failure of micro-reduction, property P^{col} is not reducible to unary properties carried by each a and b. (Note that we do not rule out that P^{col} is reducible to a *relation R*; more on the question how exactly to understand the collectively carried property P^{col} in Section 12.)

Model D: When talking about an entangled property, what we really mean is a pair of two unary properties P_A and P_B, such that each a and b bears one of them: P_Aa and P_Bb. (c might or might not exist.) P_A and P_B are pretty special: Since micro-reductionism fails, P_A and P_B must involve an essential reference to the respective other object, e.g. P_A might be *having opposite spin to particle b*.[22] We call such unary properties that essentially refer to other objects "*extrinsic* unary properties". Being extrinsic does not make them any less unary (as they are carried by *one* object). It is therefore crucial to distinguish between relations and extrinsic unary properties. In contrast, unary properties lacking essential reference to other objects are called "*intrinsic* unary properties".[23] The irreducibility claim then amounts to the fact that neither P_A nor P_B can be reduced to *intrinsic* unary properties of the one-particle systems.

Note that models A to D (about the irreducible entanglement property and its bearer(s)) are somewhat (but not completely) independent of mereological models I–III, since some of the models A to D leave open whether certain objects exist and whether existing objects relate as part and whole. We shall only integrate the two classes of models at the end of this paper. Before that we shall now make the models A to D precise and make explicit logical dependences between them; in order to do this, we first need to introduce a formal tool: the lambda operator.

7. The lambda operator

Roughly, the lambda operator is a formal device which allows to define *complex predicates* in the context of some formal language. As this paper *applies* logic, we permit ourselves to introduce the

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[21] The binary relation *having opposite spin to* would be reducible to unary spin properties if there were fixed unary spin values. The reason why reduction does not work in the entangled quantum case, however, is that in such states it is not true that the spin values are fixed.

[22] Maudlin (1998, 638) calls this view "joint state mode": "[E]ach particle has a spin state, but one that can only be specified by referring to the other particle. […] In this case the spin state of particle [a] simply cannot be specified without mentioning particle [b]".

[23] For a sound definition of intrinsicality see Lewis 1983, p. 197.

lambda operator here without going into technical details or varieties.[24] As an example, consider the complex predicate of being-both-F-and-G, which may be notated as "λx (Fx $\wedge$ Gx)". Its extension will contain just those objects which satisfy both the condition of being F and the condition of being G. Note that "λx (Fx $\wedge$ Gx)" is just a predicate, not a formula. In contrast, "λx (Fx $\wedge$ Gx) d" is a formula.[25] As a moment's reflection on the extension of "λx (Fx $\wedge$ Gx)" shows, the formula "λx (Fx $\wedge$ Gx) d" is equivalent to "Fd $\wedge$ Gd". In fact, this observation may *basically*[26] be generalized to the following rule, where *x* is a meta-variable, *d* is a meta-constant, $\ulcorner\Phi[x]\urcorner$ indicates that *x* occurs somewhere in Φ and $\ulcorner\Phi[d]\urcorner$ is the result of replacing every occurrence of *x* in $\Phi[x]$ by *a*):

$\ulcorner\lambda x\ (\Phi[x])\ d\urcorner$ is equivalent to $\ulcorner\Phi[d]\urcorner$.

A move, which exploits this rule, is called "lambda conversion". Since an equivalence holds in both directions, both "λx(Fx) d $\rightarrow$ Fd" and "Fd $\rightarrow$ λx(Fx) d" are true due to lambda conversion.

Another transformation involving the lambda operator is called "lambda abstraction": Take some formula with n occurrences of some constant *d*, replace at least one of them with some variable *x* and bind the result by $\ulcorner\lambda x\urcorner$, thus creating the new predicate $\ulcorner\lambda x\ (\Phi[x])\urcorner$. For example, "$\lambda$x (Fx)" is created by lambda abstraction from "Fd".[27]

The feature, which concerns us most, is relating predicates of different arity. For instance, the binary predicate "R" may be used to produce the formula "Rde" ("Romeo loves Juliet"). Now, by lambda abstraction, we may form the predicate "λx(Rxe)", which, in the example, expresses the property of loving Juliet, and the predicate "λy(Rdy)", which expresses the property of being loved by Romeo (or of being someone whom Romeo loves). Lambda conversion yields: Rde $\equiv$ λx(Rxe) d. It also yields: Rde $\equiv$ λy(Rdy) e. By propositional logic, we may deduce from this the following slightly redundant formula: Rde $\equiv$ λx(Rxe) d $\wedge$ λy(Rdy) e. Being told that Romeo loves Juliet provides just the same information as being told that Romeo has the property of loving Juliet, or as being told that Juliet has the property of being loved by Romeo; and, of course, the same information as – for reasons of symmetry – being told *both*. By using lambda abstraction, we can package information about two entities standing in a certain relation as information about each of them having one of two corresponding relational properties. Let us call this feature of the lambda operator "separate packaging".

We may also proceed the other way around, from the corresponding relational properties to the holding of the relation. If we know that "λx(Rxe) d" is true, or that "λy(Rdy) e" is true, we know that just the appropriate conditions for the truth of "Rde" obtain.

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[24] For our purpose, we won't have to go beyond the introduction of the lambda operator in Gamut 1991, vol. II, chapter 4. The context, in which it is introduced there, is an extensional theory of types. For our application, it is not necessary to explain how this theory works. Let it suffice to say that it is (literally) infinitely more complex than first order predicate logic, but that it incorporates first order predicate logic as its basic level, which, on that level, may be extended to CEM. So there is no problem of using the lambda operator together with notation for mereological sums.

[25] If the lambda operator is in play, formulae tend to have a lot of brackets in order to indicate argument places. We drop them for sake of readability (in analogy with "Fd" instead of "F(d)". Some readers might prefer "λx (Fx $\wedge$ Gx) (d)" to the formula above.

[26] There is a slight complication with the move from right to left in this equivalence if free variables assume the rôle of constants (Gamut 1991, vol. II, 109f), which need not worry us here.

[27] Note that lambda abstraction is not *quite* the right to left move of lambda conversion, since creating a predicate is not the same as exploiting a true implication (predicates are neither true nor false). The idea behind both is pretty similar, though.

A different step we can take is to (so to say) *relationalize properties*. Let us say that Romeo is happy ("H"), and Rosaline (formal name: "f") is hungry ("G").[28] So the following is true: $Hd \wedge Gf$. By lambda abstraction we may form the predicate "$\lambda x(Hx \wedge Gf)$" ("being happy while Rosaline is hungry"). And, indeed, lambda conversion tells us that "$\lambda x(Hx \wedge Gf)\ d$" is true, too: Romeo is happy while Rosaline is hungry. We may even, by applying lambda abstraction once more, form the predicate "$\lambda y\ \lambda x(Hx \wedge Gy)$". Sticking to the example, and brackets aside, the result is that the following is true: $\lambda y\ \lambda x(Hx \wedge Gy)\ d\ f$.[29] Romeo and Rosaline stand in just that (admittedly: artificial) relation to each other in which any x and any y stand if and only if x is happy, while y is hungry.[30]

8. Lurking inconsistency – specifying model A

We are now able to see that model A *in a literal reading* is inconsistent and inappropriate and needs some specification.

Recall that model A considers the crucial entanglement property as a relation *R* between a and b ("Rab" being true) and, due to the failure of micro-reduction, forbids that *R* is reducible to unary properties carried by a and b. In order to see the inconsistency, let us define the following unary predicates with the help of the lambda operator:

(D1) P_A ("having opposite spin to b") := $\lambda x(Rxb)$

(D2) P_B ("having opposite spin to a") := $\lambda x(Rax)$

Hence we have defined the unary predicates "P_A" and "P_B" as (relational) lambda abstracts. By lambda conversion, we have:[31]

(E1) $(Rab \equiv \lambda x(Rxb)\ a) \wedge (\lambda x(Rxb)\ a \equiv P_A a)$

(E2) $(Rab \equiv \lambda x(Rax)\ b) \wedge (\lambda x(Rax)\ b \equiv P_B b)$

(Hence, also "$P_A a \equiv P_B b$" is true.)

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[28] Readers of Shakespeare's *Romeo and Juliet* will recall that Romeo used to be in love with Rosaline (I.2) before he met Juliet (I.5), after which she is not mentioned again.

[29] Friends of brackets may prefer „$\lambda y\ \lambda x(Hx \wedge Gy)\ (d)\ (f)$" or even „$\lambda y\ (\lambda x(Hx \wedge Gy)\ (d))\ (f)$".

[30] Technically speaking, the language being used contains only unary predicates, many of which mimic many-place-predicates as a result of Currying. But when applying it, we shall simply treat it as if it contained many-place-predicates.

[31] Proof 1:

*	1	Rab	assumption
*	2	$\lambda x(Rxb)\ a$	1, lambda conversion
*	3	$\lambda y(Ray)\ b$	1, lambda conversion
*	4	$\lambda x(Rxb)\ a \wedge \lambda y(Ray)\ b$	2, I$\wedge$

Proof 2:

*	1	$\lambda x(Rxb)\ a \wedge \lambda y(Ray)\ b$	assumption
*	2	$\lambda x(Rxb)\ a$	1, I$\wedge$ (working with "$\lambda y(Ray)\ b$" instead would be fine)
*	3	Rab	2, lambda conversion

These reflections reveal that, for logical reasons, it is always possible to reformulate the holding of a binary relation *R* of a and b in terms of unary properties P_A or P_B, i.e. there are always unary properties P_A or P_B such that *R* is reducible to P_A or P_B. Therefore, the irreducibility requirement in model A, that relation *R* is not reducible to *any* unary properties, is too strong: For the one, it would make model A an inconsistent position. For the other, it would be an inappropriate description of entangled systems since the failure of micro-reduction in these systems is consistent with the entangled property being an extrinsic unary property such as P_A and P_B above (which is what model D claims).

What is required then in order to make model A consistent and appropriate, is to weaken the irreducibility claim by adding a qualification to the unary properties to which the relation *R* is assumed to be irreducible: One should only forbid reductions to such unary properties of a and b that do not involve an essential reference to another object (what we have called "intrinsic unary properties").

For these reasons, we abandon model A and introduce the consistent and more appropriate

Model A': The entangled property is a binary relation *R* holding between the one-particle objects a and b ("Rab" being true), and *R* is not reducible to *intrinsic* unary properties of a and b.

9. Model A' is equivalent to Model D

We shall now show that model A' is equivalent to model D. Recall that according to both models the entanglement property is had by a and b. While model A considers the crucial entanglement property as a relation *R* (i.e. *having opposite spin to*) between a and b, model D claims that the entanglement property consists in a pair of unary properties P_A and P_B that, however, essentially relate to the respective other object: e.g. P_A might be *having opposite spin to b* and P_B might be *having opposite spin to a*. Such properties seem strange: on the one hand, they are clearly unary; on the other hand, involving the other object makes them relational in a certain sense. Isn't that a contradiction? Not necessarily. A consistent, and in fact the best, way to understand such properties, it seems to us, is to construct them with the lambda operator as in the definitions (D1) and (D2) above.

By the immediate logical consequences (E1) and (E2) of these definitions we have already proven that the claims about the entangled property and its bearer of both models – "Rab" according to model A', and "$P_A a \wedge P_B b$" according to model D – are equivalent. Then, we still need to show that their irreducibility claims are equivalent. As the statements about properties are equivalent, this is straightforward: Both models assume that the crucial entangled properties are not reducible to intrinsic unary properties of a and b.

10. Models A' and B are distinct

The lambda operator might seem to level the difference between model A' (which assumes "Rab" to be true) and B (assuming "Pc" to be true). There are two strategies: one tries to reduce model B to model A' and the other proceeds vice versa.

First, a proponent of model A' might attempt to reduce model B, which says that the entangled property is a unary property *P'* carried by the two-particle object c, by defining the corresponding predicate as follows:

$$P' := \lambda x(a \ll x \wedge b \ll x \wedge Rab).$$

According to this definition, if “Rab” is true and a and b are (proper) parts of c, “P’ c” is true as well. The idea here is that a unary property *P’* of an object c can be reduced to a relation *R* holding among its parts a and b.

This reduction, however, does not work in the case of the above models for two reasons. First, it is clear that the presented reduction is only possible, when a, b and c exists and c is the sum of a and b. Model B, however, also applies in cases, in which only c but not a and b exist. Then, the envisaged reduction is not possible.

Second, and more importantly, even in cases where a, b and c exist, the reduction fails. For model B requires that the entangled (spin) property is a unary property. According to the reductionist model, however, while there is a unary property *P’ involving* the entangled spin property *R* (plus mereological relations), the entangled spin property *is not* unary but binary.

Third, vice versa, a proponent of model B might try to reduce model A’ in a similar way. He might refer to a complex binary relation *R’* by defining the corresponding binary predicate as follows:

$$R' := \lambda x\, \lambda y(x \ll c \wedge y \ll c \wedge Pc).$$

Thus, *R’* is that relation in which a stands to b exactly if both a and b are proper parts of c, while c has the property *P*. Since c is the sum of a and b, “R’ab” is true in model B, if a and b exist, c is the sum of a and b, and c is P. If that is how *R’* is defined, we will have:

$$R'ab \equiv \quad \lambda x(x \ll \sigma(a,b) \wedge P\,\sigma(a,b))\, a \quad \wedge \quad \lambda x(x \ll \sigma(a,b) \wedge P\,\sigma(a,b))\, b$$

(a stands in relation *R’* to b if and only if a has the property of being a proper part of the sum of a and b, while the sum of a and b is P, *and* b has the same property.) This alleged reduction of model A’ to model B does not work for the very same reasons as the parallel reduction of model B to A’ does not: *R’* only holds, when a and b exist, but model B does not require this generally; and the entangled spin property in this case is *P,* and *P* is not binary (as model A’ assumes).

In sum, it follows that, by the envisaged strategies, neither is model B reducible to model A’ nor vice versa. The two models are distinct.

11. Collective relational properties: irreducible relations

So far we have assumed that model A’ involves an irreducible, binary relation between a and b (or in the general case: an irreducible n-ary relation between the n objects). Such irreducible relations are one way of making clear the idea that the micro objects collectively carry a macro property. Then, no macro object is needed in order to carry the irreducible macro property.

Brenner (2018) furthermore proposes that we can think of the irreducible relation as being *multigrade*, i.e. as not having a fixed arity but rather being flexible as to how many objects carry it. According to Brenner, the macro property “having spin 0” can be carried by any number of objects. In this way, he defends relational properties against an objection by Schaffer (2010) who claims that the fact that intrinsic macro properties can be carried by a macro object consisting of *any* number of parts makes that model superior to collectively carried relational properties.

In sum, we note that model A’ comes in two variants: either as assuming n-ary relations or as assuming multigrade relations.

12. What are unary collective properties? Five readings of model C

There is a certain prima facie tension in model C in claiming that a *unary* property P^{col} is carried by *several* objects collectively. What might make sense of this claim? Prima facie, there are four possible readings:

(R1) P^{col} is in fact a unary *singular* property and there is a single composite entity c (distinct from a and b) that carries it.

(R2) P^{col} is a complex unary *plural* property essentially involving a binary relation carried by a and b (in the case of n particles: P^{col} essentially involves an n-ary relation carried by the n particles);

(R3) P^{col} can be analyzed into a binary relation carried by a and b (in the case of n particles: P^{col} can be analyzed into several binary relations holding between pairs of the n particles);

(R4) P^{col} is a *non-analyzable* unary plural property carried by a and b.

(R5) P^{col} is in fact a *complex* unary singular property and there is a single composite entity c (distinct from a and b) that carries it; P^{col} essentially involves a unary *plural* property, P^{p}, carried by a and b collectively.

We shall now argue that readings (R1), (R2), (R3) and (R5) are inappropriate in the present case. So the result will be: If model C is meant to be a further option besides models A' and B, it must be understood by reading (R4).

Let us start by saying why reading (R1) does not add an option to our taxonomy. Take an everyday example of (prima facie) plural predication: "the firefighters saved her from the burning house". Advocates of (R1) would analyze such cases as normal unary singular predication: "the ensemble of firefighters rescued her from the burning house", while the word "ensemble" is understood as referring to some single entity. There are different proposals for understanding such composite entities: they have been thought of as mereological sums, pluralities or the like.[32] According to the assumptions in this paper we restrict our considerations to sums.[33] Accordingly, to say that model C involves hidden reference to a single entity would be to assume that, in fact, it is the mereological sum c of a and b which carries the unary entangled property P^{col}: $\mathrm{P}^{\mathrm{col}}\mathrm{c}$. Then, however, model C reduces to model B. If model C is meant to be an option distinct from model B, we cannot interpret it in the sense of (R1).

Note that the other readings, in contrast, share the advantage of not being committed to the existence of a single entity as a bearer of the property in question. Plural predication according to (R2), (R3) and (R4) minimizes ontological commitments concerning objects and is even compatible with mereological nihilism. Analyzing these options requires plural logic, which is an emerging field recently explored in some depth (Oliver & Smiley 2016/2013). As we shall furthermore need to apply the lambda operator we are entering somewhat uncharted territory. To our knowledge, the lambda operator[34] has not been applied to plural terms yet, though it seems as if the application were straightforward (at least in the simple cases we shall consider). Therefore, in the following sections, we *pretend* having a resource at hand, whose availability the future must show. If the exploration in this paper is of any interest, it might motivate more basic research on this point.

[32] In other cases like "the even numbers are infinite", the single entity in question might be a set, "the set of even numbers is infinite".
[33] Our results might, mutatis mutandis, be transferred to models about pluralities etc. as well.
[34] To be distinguished from Boolos' lambda *notation* for plural logic, cf. Oliver & Smiley (2016), 31; 60-65.

13. A very short introduction to plural logic

In order to understand the remaining readings of model C we need to introduce some basic terminology of plural logic. We start with plural names: The characteristic feature of plural names (according to Oliver & Smiley 2016/2013) like “the Smiths” is that they refer to several objects without any reference to a single higher-level entity.[35] Note that a list of several names like “Huey, Dewey and Louie” counts as *one* (plural) name. Formally, we can define “d” as denoting Huey (and nobody else), “e” standing for Dewey, “f” for Louie and the plural name ”**c**” (boldface!) for the three of them. Let “$\prec$” denote the one-many relation *is (properly) among*, which is emphatically not the same as parthood. Finally, let us think of a *unary* plural predicate “F” (e.g. “jointly writing a paper”, “dancing tango”).[36] Characterizing a predicate as both unary and plural might sound surprising, but there is a clear sense. “Unary” means that adding *one* name to it will make a formula, which is a syntactical requirement. “Plural”, on the other hand, means that adding a *singular* name (referring to *one* object, in contrast to a plural name) to it will never yield a *true* formula, which is a semantical condition. How might the plural sentence “Huey, Dewey and Louie stand in an equilateral triangle” be expressed then? “F**c**” or, equivalent to it by lambda conversion, “$\lambda\mathbf{x}(F\mathbf{x})\ \mathbf{c}$” are good candidates.[37]

14. Reading (R2) of model C reduces to model A

We emphasize that reading (R2) does not say that P^{col} is in fact a binary (in the multipartite case: n-ary) relation as this would contradict the assumption of model C that P^{col} is unary, and, moreover, it would make model C straightforwardly coincide with model A. Rather, along the lines of our introduction to plural predicates above, the idea here is to understand P^{col} as a complex unary plural property essentially involving an n-ary relation. What exactly does this mean?

We start by considering the case of bipartite entanglement. Let us define “**c**” as a plural term that denotes a and b (once more: mind the boldface, also in what follows). Now “$P^{col}\mathbf{c}$” should be understood to be equivalent to

(plural) $\lambda\mathbf{x}\ (R'ab \wedge a \prec \mathbf{x} \wedge b \prec \mathbf{x} \wedge \forall y\ (y \prec \mathbf{x} \rightarrow y = a \vee y = b))\ \mathbf{c}$

In other words, the **c**s satisfy the condition of (properly) having a and b, and nothing else, among them which stand in relation R’ to each other.

What, according to this understanding of a collectively carried property, does the irreducibility claim of model C amount to? In the prima facie characterization of model C above we said that the collectively carried property P^{col} is not reducible to unary properties of each a and b. Since now we see that the essential part of a collectively carried property is a relational property *R’*, it is obvious that the same problems that arose for the relational model A emerges for model C (see Section 8). In order to avoid contradiction we must restrict irreducibility to *intrinsic* unary properties (let us call this model with improved irreducibility condition “model C’ ”).

[35] Plural constants in boldface are somewhat against the spirit of plural logic according to Oliver & Smiley, but we hope that this modified notation makes things easier to grasp here.

[36] Here we are only interested in what Oliver and Smiley call “collective plural predicates” (as opposed to distributive plural predicates).

[37] Note that without further information one cannot tell whether a unary plural predicate can be further analyzed. The predicate “standing in an equilateral triangle” obviously can be analyzed by pairwise relations, but there might be predicates for which this is not possible. The concept of a collective property is even more general than that of an n-ary relation (pace Brenner (2018), who assumes that collective properties are n-ary relations).

Given this precise formulation of model C' in reading (R3) for bipartite entanglement, we are now able to demonstrate that the model is equivalent to model A', that the entanglement property is a fundamental relation *R* carried by a and b. It is rather straightforward to show that (plural), the core claim of model C' in reading (R3), is equivalent to the core claim of model A', "Rab". First, it is obvious that in (plural), *R'* is the only candidate for representing the entangled property, so it is without sensible alternative to assume that *R'*=*R*. Then, the equivalence between (plural) and the truth of "Rab" is proven as follows:

(1) If anything satisfies any condition *while "*Rab" is true, then "Rab" is true. So (plural) implies "Rab".
(2) "$a \prec \mathbf{c} \wedge b \prec \mathbf{c} \wedge \forall y\, (y \prec \mathbf{c} \rightarrow y = a \vee y = b)$" is true by definition of "**c**". Therefore, if "Rab" is true,

$$a \prec \mathbf{c} \wedge b \prec \mathbf{c} \wedge \forall y\, (y \prec \mathbf{c} \rightarrow y = a \vee y = b) \wedge Rab$$

is true, and, by lambda conversion, (plural) is true. So "Rab" implies (plural). Finally, we still need to note that the irreducibility claims match: According to model A', *R* is not reducible to intrinsic unary properties of a and b. As *R'*=*R*, the irreducibility of *R* from model A' transfers to P^{col} from model C'. Vice versa, according to model C', P^{col} is not reducible to intrinsic unary properties of a and b; since the only candidate for reduction in P^{col} is *R'*, the irreducibility claim must hold of *R'*; and since *R'*=*R*, the irreducibility of property P^{col} from model C transfers to *R* from model A'. This completes our argument for the equivalence of model A' and model C in the simple case with two-particle entanglement. The idea easily generalises to the case of n (instead of just two) particles that are multipartitely entangled.[38] In sum, model C' in reading (R2) reduces to model A', and hence does not constitute an additional option in our taxonomy.

15. Excluding reading (R3) of model C

Model C in reading (R3) roughly amounts to the claim that the entanglement property P^{col} is analyzable. In this case it is crucial to distinguish the case of bipartite entanglement (n=2) from multipartite entanglement (n≥3). We shall argue that in the former case model C in reading (R3) reduces to model A'; and in the case of multipartite entanglement it is incompatible with the relational irreducibility claim.

In the case of bipartite entanglement, a proponent of model C in reading (R3) holds that P^{col} can be analyzed into a binary relation carried by a and b. Accordingly one would have to define "**c**" as a plural term that denotes a and b, and $P^{col}\mathbf{c}$ should be understood to be equivalent to (plural) (see Sect. 14). Again we need to adjust the irreducibility claim, so that, for the bipartite case, reading (R3) of model C' is equivalent to reading (R2) of the model. Hence, by the very same argument as for reading (R2) one can reduce model C' in reading (R3) to model A'.

That the two readings coincide is, however, an artifact of the bipartite case. The same is not true for any number of particles n≥3. In that case, (R3) amounts to the claim that P^{col} can be analyzed into several binary relations holding between pairs of the n particles. Let us, for the sake of simplicity (but

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[38] Let us agree that **c'** is a plural name for these n particles. For the proof to work in this more general case it is important that the arity of the relation R' matches the number of objects n (as reading (R2) assumes). Then, $P^{col}\mathbf{c'}$ should be understood to be equivalent with

(plural$_n$) $\lambda\mathbf{x}\, (R^n x_1 \ldots x_n \wedge x_1 \prec \mathbf{x} \wedge \ldots \wedge x_n \prec \mathbf{x}) \wedge \forall y\, (y \prec \mathbf{x} \rightarrow y = x_1 \vee \ldots \vee y = x_n))\mathbf{c'}$,

and the proof runs mutatis mutandis.

without loss of generality), consider the case of tripartite entanglement with three entangled objects a_1, a_2, a_3, denoted by **c'**. Reading (R3) then requires to claim that P^{col}**c'** is equivalent to

$$\lambda\mathbf{x}\,(R_1a_1a_2 \wedge R_2a_2a_3 \wedge R_3a_1a_3 \wedge a_1{\prec}\mathbf{x} \wedge a_2{\prec}\mathbf{x} \wedge a_3{\prec}\mathbf{x})\,\mathbf{c}',$$

where the R_i are binary entanglement relations.[39]

While there might be entangled states that are analyzable in this way, the difficulty with this reading is that tripartite entanglement in general is well known *not* to be analyzable into binary entanglement relations (see the relational irreducibility claim). So this suggestion must be discarded because it is in conflict with the irreducibility conditions for entanglement.

16. Reading (R4) of model C

In contrast to the readings discussed so far, reading (R4) of model C takes the model at face value: It assumes that the entangled property is both unary and plural and is not reducible to relational properties. While being unary and plural might prima facie sound contradictory, we have explained above that according to plural logic, it is perfectly sensible to have unary plural predicates, i.e. predicates that require one (as opposed to several) plural (as opposed to singular) name as in "the Smiths are dancing tango". This is also the way in which we understand Bohn's proposal, who calls such properties "*plural, collective* intrinsic properties" (Bohn 2012, 218; original emphasis).

It is obvious that there is a conceptual difference between a unary plural predicate (R4) and an n-place singular relational predicate (R2) or a unary singular predicate (R1). But is there an ontological difference? One might argue that all instances of correctly applied unary plural predicates are made true by, say, instantiations of n-place singular relational properties. One reason for this view might be that while one can consistently *speak* of unary plural predicates, it is ontologically mysterious what genuine unary plural properties are supposed to *be*. Unfortunately, we cannot discuss the issue satisfyingly here. We can only hint to the fact that the many clear examples that we have of unary plural predication would require the critics of the position to spell out what exactly they think to be mysterious about it. For the time being, we count it as a viable option in our overview.

17. Reading (R5) of model C reduces to (R4)

Reading (R5) is similar to reading (R1) in holding that, in fact, P^{col} is a unary *singular* property carried by the mereological sum c of a and b. Since we know that (R1) reduces to model B, it is crucial to emphasize that (R5) differs from (R1) in claiming that P^{col} is a complex property that essentially involves a unary plural property carried by a and b collectively (as (R4) claims). The suggestion here is to understand the entangled predicate as

$$P^{col} := \lambda x(a \ll x \wedge b \ll x \wedge a \prec \mathbf{c} \wedge b \prec \mathbf{c} \wedge \forall y\,(y \prec \mathbf{c} \rightarrow y = a \vee y = b) \wedge P^{p}\mathbf{c})$$

and to claim that the assertion

$$\lambda x(a \ll x \wedge b \ll x \wedge a \prec \mathbf{c} \wedge a \prec \mathbf{c} \wedge \forall y\,(y \prec \mathbf{c} \rightarrow y = a \vee y = b) \wedge P^{p}\mathbf{c})\ c$$

is true. This means that c has the property of having a and b as its proper parts, each of which, and nothing else, is among the **c**, and the **c** collectively carry P^p.

[39] To give a classical example, the plural property "standing in an equilateral triangle" can be analyzed in this way, if one chooses R1=R2=R3=:D as a binary relation of being some fixed nonzero distance apart.

This means that (R5) is based on (R4) similarly to how (R2) is based on model A. We have shown above that despite their prima facie distinctness, (R4) and (R2) are equivalent. By a very similar argument one can prove in the present case that (R5) is equivalent to (R4) – so (R5) does not really add a new proposal to our taxonomy.

To conclude, the only viable reading of model C is reading (R4).

III. Synthesis

18. Combining the models

The result of our investigation of property models is that there are three basic models to conceptualize the irreducible entanglement property and its bearer: either the entanglement property is a fundamental relation *R* carried by the one-particle objects a and b (model A'); or it is a unary singular property *P* carried by the two-particle object c (model B); or the entangled property is a unary plural property P^{col} carried collectively by a and b (model C). A fourth conceivable model, model D, that the entangled property is a unary singular property carried by a or b that essentially refers to the respective other object, is equivalent with model A'.

We shall now combine the basic property models A', B and C with the three basic mereological models (I)–(III). This will provide us with more specific, combined models each of which provides an answer to all of the questions (i)–(iv). The possible combinations are restricted by the fact that each property model requires the existence of certain objects, which, however, do not exist according to each mereological model. There are six possible combined models:

- **(RRH) Radical relational holism** (models II & A'): The one-particle-objects a and b exist, but there is no two-particle object c (i) and hence no part-whole relations (ii). The entangled property is a fundamental binary relation *R* (iii) holding between a and b (iv).
- **(MRH) Moderate relational holism** (models III & A'): a, b and c exist (i), and c is the mereological sum of a and b (ii). The entangled property is a fundamental binary relation *R* (iii) holding between a and b (iv).
- **(RMH) Radical monistic holism** (models I & B): The two-particle object c exists as a partless / undivided whole, i.e. the one-particle objects a and b do not exist (i and ii). c carries the entangled property (iv), which is a unary property (iii). The idea here is that when two separate quantum objects interact and get entangled, they fuse to yield a macro object and thereby stop existing (cf. Brenner 2018, §3.3)[40] Since c is spatially scattered, it is an extended simple.
- **(MMH) Moderate monistic holism** (models III & B): a, b and c exist (i), and c is the mereological sum of a and b (ii). The entangled property is a unary property *P* (iii) of c (iv). Calosi and Tarozzi (2014) seem to support this model of entangled systems. Schaffer (2010) holds an extreme variant thereof: the macro level is the whole universe and the parts, though existing, ontologically depend on the macro object.[41]
- **(RPH) Radical pluralistic holism** (models II & C in reading (R4)): The one-particle-objects a and b exist, but there is no two-particle object c (i) and hence no part-whole relations (ii). The entangled

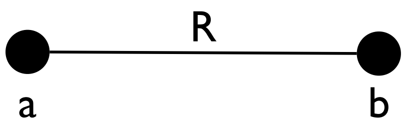
[40] Brenner mentions the position, but makes clear not to endorse it.

[41] Note that mereological models as depicted here do not involve relations of ontological dependence / priority. For a critique of Schaffer 2010 see Calosi 2014.

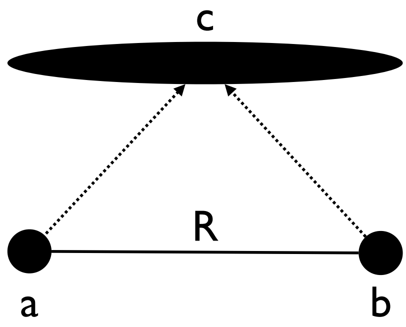
property is a fundamental unary plural property *P'* (iii) carried collectively by a and b (iv). (maybe Brenner 2018, §3.1)[42]

- **(MPH) Moderate pluralistic holism** (models III & C in reading (R4)): a, b and c exist (i), and c is the mereological sum of a and b (ii). The entangled property is a fundamental unary plural property *P'* (iii) carried collectively by a and b (iv).

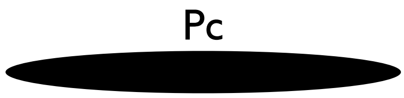
[42] Brenner (2018), who defends nihilism in the quantum realm, sketches a position in his §3.1 that seems to fall under model RPH, but the case is not clear since he speaks of an entangled state as a collectively instantiated "multigrade relation" (which points to RRH instead). His arguments, however, rather suggest that he means RPH.



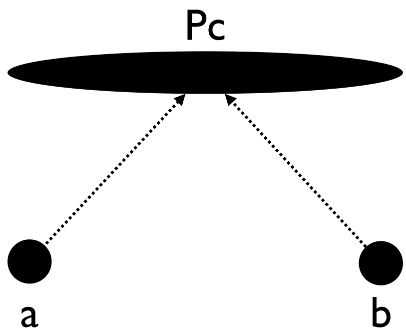
(d) Radical relational holism (RRH)



(e) Moderate relational holism (MRH)

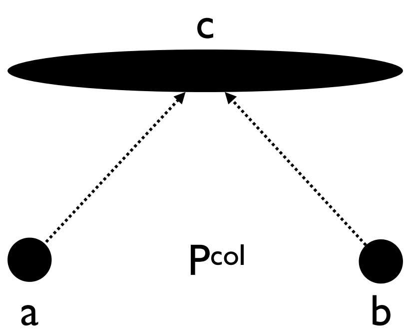


(f) Radical monistic holism (RMH)



(g) Moderate monistic holism (MMH)

(h) Radical pluralistic holism (RPH)



(i) Moderate pluralistic holism (MPH)

Figure 2: The possible part-property models for a two particle entangled system

If our argument in this paper is correct, these six models exhaust the possible answers to questions (i)–(iv). Recalling that a mereological graph does not indicate what is ontologically prior or fundamental (especially the arrows indicate only the asymmetry of the is-part-of relation), it is obvious that the models can be made more specific and differentiated by amending further features that are not mentioned in the four questions (e.g. ontological dependence/priority, emergence, …). We believe, however, that the four questions are basic, and possible answers to them provide a relevant taxonomy for mereological models of entangled systems.

Accounting for an irreducible entangled property, all models are in some sense holistic. The ontologically parsimonious positions are labelled “radical”. RRH and RPH assume that only the one-particle objects (and not the two-particle object) exist. All the property-bearing work is done by the one-particle objects: The irreducible entangled state is either accounted for by introducing a new kind of fundamental relation holding between the one-particle objects or by a unary plural property. In this way, both models circumvent the need for the level of two-particle objects.

RMH, in contrast, assumes that only the two-particle object (but not the one-particle objects) exists. Here, the property-bearing work is done on the macro level. Though the model does not imply this view, it is the position that is adequate for the mereological monist, who assumes that there is only one object, namely the whole universe.

19. Discussion

(1) In this paper we have developed four fundamental questions concerning the mereology of entangled quantum systems: (i) which objects do exist, (ii) in which mereological relations do they stand, (iii) of which arity is the entangled property and (iv) which object(s) bear(s) the entangled property. Questions (i) and (ii) define mereological models while (iii) and (iv) characterize property models of entanglement. Taken jointly, the four questions characterize part-property models of entanglement.

We have sharpened the conceivable models and have argued that certain models advanced in the debate are equivalent or not consistent with the quantum mechanical description. Our result is that there are six distinct part-property models each of which provides a characteristic pattern of answers to the four questions.

The overview of models makes clear that Bohn’s proposal of unary plural properties complements the usual options of monistic holism and relational holism by pluralistic holism. Brenner’s multigrade relations, in contrast, are a variant of relational holism. Relational holism and pluralistic holism make clear that irreducible (or emergent) macro properties do not per se require a macro object bearing the irreducible macro property, since one can conceive of irreducible macro properties that are carried collectively by the micro objects, and we have explained that one can either think of such collectively carried properties as n-place relations, as multigrade relations or as monadic plural properties.

Our result might prima facie not seem too surprising as our investigation does not yield a new model that has been completely unnoticed so far. While monistic holism and relational holism have been established positions, pluralistic holism is not very widespread, and largely unnoticed, but it has already been formulated by Bohn (2012). On the other hand, our overview is the result of a thorough investigation which has three main results that are not so obvious and especially cannot be read off from Figure 1: First, using the formal tools of the lambda operator and plural logic we have tried to make the possible positions clear and precise. Second, we have ruled out positions that are not distinct to others, so the positions presented here are distinct and provide fundamental options for the quantum mereologist. A third main result lies in what our overview does *not* mention: Having systematically investigated conceivable, prima facie sensible models we have not found any further model. Then, according to our examination, the six models presented exhaust the available models for conceiving of entangled quantum systems. One of these models must be the correct answer to the central mereological questions (i)–(iv) in entangled quantum systems. In this way the taxonomy developed here provides a suitable basis for systematically examining the question which mereological model is the most appropriate one for entangled quantum systems.

(2) The abstract mereological models are general in several respects and are hence compatible with a number of more specific models. First, they do not indicate ontological dependence but only parthood. They can be enriched to either let the whole depend ontologically on the parts or vice versa. Second, they are not committed to a certain theory concerning the individuation of the micro objects. Agreeing with the quantum mechanical description of systems with objects of the same kind, we have taken care to symmetrically distribute all properties among the micro objects, which allows for different interpretations concerning the identity of the micro objects (non-individuals vs. objects with weak identity). Third, the mereological models are also indifferent concerning the specific nature of the entangled quantum state. We have just presupposed a realistic reading of the quantum state but not a specific interpretation (e.g. textbook quantum theory, GRWm, propensity interpretation, etc.).

(3) In order to establish our results, we have used the formal tool lambda operator as well as the formalism of plural logic. Especially the latter has been developed more sharply only in recent years and seems to be indispensable when one tries to treat mereological questions in a precise way. Combining both tools has pushed formal considerations beyond what seems to be established at present and will require some future work.

(4) It is clear that the taxonomy developed here provides only a first step towards an appropriate mereological understanding of entangled quantum systems: It presents the possible, distinct models, which will need to be evaluated for their adequacy. We would like to stress that there does not seem to be a fast track to do so, and we shall shortly sketch in the following (5 and 6) why the two straightforward ways to tackle the question fail. One can either bring in more mereological assumptions, i.e. assume one of the main mereological positions (universalism, nihilism, moderate compositionalism), or one can bring in more features of the system under consideration, i.e., in the present case, further details of the quantum mechanical formalism.

(5) To assume one of the main mereological positions first of all requires independent reasons for the position assumed; and even when you think that you have good arguments for one of the positions, one finds that each of the positions restricts the models to some extent but ultimately leaves it underdetermined which of the models is the appropriate one.

A universalist cannot accept models according to which a and b exist, but not c (RRH and RPH). Positively, she can accept all the moderate positions as well as RMH (taking c as a simple). According to MRH and MPH, the entanglement property is a relation R carried by a and b, so c does not do any ontological job concerning the entangled property. If c does any ontological job at all, it is not specific to entanglement. Therefore, MRH and MPH are models that are only interesting for the universalist, who does not adhere to Ockhamistic principles like "If, if x existed, then x would not do any conceivable job, then x does not exist", especially not for mereological sums. Interestingly, a universalist might choose RMH as well, for instance if (s)he takes c to be an extended simple (see Simons 2004)..

A nihilist must refrain from assuming any model according to which composition occurs (the three moderate models): If she accepts the existence of c, she must deny the existence of a and b, and vice versa. So it is only the radical positions that she can accept. If the nihilist holds RRH or RPH, she will have to paraphrase away talk about c. The physics of entanglement might make this even harder than paraphrasing away talk about tables and chairs (van Inwagen 1990) or even biological organisms (Unger 1979).

A moderate compositionalist, who is neither a universalist nor a nihilist, claims that composition occurs under certain conditions. The positions compatible with moderate compositionalism depend on what the conditions for composition are. It might seem natural to think that entanglement is at least among the sufficient conditions for composition, and if this is the case moderate compositionalism is compatible with exactly those models that are available for the universalist (i.e. the moderate positions plus RMH). If, however, entanglement does not trigger composition, the position is compatible with those models that are available for the nihilist (i.e. the radical models).

In sum, each of the main mereological positions is compatible with more than one of the six models. Interestingly, RMH is the only model that all three positions are compatible with, which, however, does not speak for its truth. In order to determine one of the models one would need further assumptions, which might be grounded in the features of the specific system in question. Anyway, there is no shortcut to a solution from the main mereological positions.

(6) A second method to determine which of the models is the correct one is to bring in more features of the quantum mechanical formalism. In developing the six models we have only made use of the fact that the entangled property is not reducible to its parts (and we have assumed that there must be some object(s) that carry it). All other details of the quantum mechanical formalism have not been considered and could now possibly be adduced to argue for the one or the other model. We emphasize, however, that there is no innocent or straightforward way from such features of the formalism to ontological features that constrain mereological models, even when one assumes a realistic interpretation of quantum states and even when one has decided whether the ray view or the statistical operator view of quantum states is correct (see Section 5).

Let us shortly make one difficulty explicit. Say, one assumes the ray view which implies that the states of one-particle systems are not well-defined in themselves when the two-particle system is in an entangled state. One might intuitively think that this implies that the two-particle object exists and the one-particle objects do not. In order to make the inference one is committed to something like the principle "an object exists if and only if the corresponding quantum mechanical state exists". [43] However, such metaphysical principles that relate the well-definiteness of states with the well-definiteness of objects are far from established or obviously true. In fact, one can easily see that the principle in the given formulation is false: According to model RPH we only have the one-particle objects, although (according to the ray view) only the entangled quantum state is well-defined and the one particle states are not. As a consequence, a reliable inference from the formalism to mereological models would need a more thorough investigation.

What is certain, however, if the argument in this paper is correct, is that the appropriate mereological model is among the six models that we have derived for systems with irreducible entangled quantum states. In this sense, we have provided a framework for further investigations into the mereology of entangled quantum systems.

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[43] Cf. Jaeger's (2014, 154) quantum principle of individuation.

References

Bell, J. S. (1964). On the Einstein-Podolsky-Rosen Paradox. *Physics*, *1*(3), 195–200.

Bell, J. S. (2004). *Speakable and unspeakable in quantum mechanics* (2nd revised ed.). Cambridge: Cambridge University Press. (1st ed. 1987)

Bohm, D., & Hiley, B. J. (1993). *The Undivided Universe: An Ontological Interpretation of Quantum Theory*. London: Routledge.

Bohn, E. D. (2012). Monism, Emergence, and Plural Logic. *Erkenntnis*, *76*(2), 211-223. doi:10.1007/s10670-011-9280-4

Brenner, A. (2018). Science and the Special Composition Question. *Synthese*, 195(2), 657–678. doi:10.1007/s11229-016-1234-6

Calosi, C., Fano, V., & Tarozzi, G. (2011). Quantum Ontology and Extensional Mereology. *Foundations of Physics*, *41*(11), 1740-1755. doi:10.1007/s10701-011-9590-z

Calosi, C., & Tarozzi, G. (2014). Parthood and Composition in Quantum Mechanics. In C. Calosi & P. Graziani (Hgs.), *Mereology and the sciences: Parts and Wholes in the Contemporary Scientific Context* (pp. 53–84). Springer.

Calosi, C. (2014). Quantum mechanics and Priority Monism. *Synthese*, *191*(5), 915–928. doi:10.1007/s11229-013-0300-6

Einstein, A., Podolsky, B., & Rosen, N. (1935). Can Quantum Mechanical Description of Physical Reality Be Considered Complete? *Physical Review*, 47, 777–780.

Esfeld, M. (2001). *Holism in Philosophy of Mind and Philosophy of Physics*. Dordrecht: Kluwer.

French, S., & Krause, D. (2006). *Identity in Physics: A Historical, Philosophical, and Formal Analysis*. Oxford: Clarendon Press.

Friederich, S. (2011). How to Spell Out the Epistemic Conception of Quantum States. *Studies in History and Philosophy of Modern Physics*, 42(3), 149–157. doi:10.1016/j.shpsb.2011.01.002

Gamut, L. T. F. (1991). *Logic, Language, and Meaning*. Chicago: University of Chicago Press.

Ghirardi, G., Rimini, A., & Weber, T. (1986). Unified Dynamics for Microscopic and Macroscopic Systems. *Physical Review D*, *34*(2), 470–491.

Ghirardi, G., Grassi, R., & Benatti, F. (1995). Describing the Macroscopic World: Closing the Circle Within the Dynamical Reduction Program. *Foundations of Physics*, 25(1), 5–38. doi:10.1007/BF02054655

Hovda, P. (2009). What Is Classical Mereology? *Journal of Philosophical Logic, 38,* 55–82.

Howard, D. (1989). Holism, Separability, and the Metaphysical Implications of the Bell Experiments. In J. T. Cushing & E. McMullin (Eds.), *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem* (pp. 224–253). Notre Dame: University of Notre Dame Press.

Hüttemann, A. (2005). Explanation, Emergence, and Quantum Entanglement. *Philosophy of Science*, *72*, 114–127.

Jaeger, G. (2014). *Quantum Objects: Non-Local Correlation, Causality and Objective Indefiniteness in the Quantum World*. Berlin: Springer.

Krause, D. (2011). On a calculus of non-individuals: ideas for a quantum mereology. In L. H. de Araújo Dutra & A. Meyer Luz (Eds.), *Linguam, Ontologia e Ação* (pp. 92-106). Florianópolis: NEL/UFSC.

Leonard, H. S., & Goodman, N. (1940). The Calculus of Individuals and Its Uses. *The Journal of Symbolic Logic*, 5(2), 45–55.

Lewis, D. (1983). New Work for a Theory of Universals. *Australasian Journal of Philosophy*, *61*(4), 343–377.

Lyre, H. (2018). Quantum Identity and Indistinguishability. In C. Friebe, M. Kuhlmann, H. Lyre, P. M. Näger, O. Passon, and M. Stöckler (eds.), *The Philosophy of Quantum Physics* (1st. English ed., pp. 73–102, trans. W. D. Brewer). Cham: Springer International. doi:10.1007/978-3-319-78356-7 (original publication: *Philosophie der Quantenphysik*, 2nd ed., Springer Spektrum 2018)

Maudlin, T. (2006/1998). Part and Whole in Quantum Mechanics. In: Lange, M. (ed.), *Philosophy of Science: An Anthology*. Oxford: Blackwell. (Originally published in: E. Castellani (Hg.), *Interpreting Bodies: Classical and Quantum Objects in Modern Physics* (pp. 46-60). Princeton: Princeton University Press, 1998.)

Näger, P. M. (2016). The Causal Problem of Entanglement. *Synthese*, *193*(4), 1127–1155. doi:10.1007/s11229-015-0668-6

Oliver, A., & Smiley, T. J. (2016/2013). *Plural logic* (2nd ed.). Oxford: Oxford University Press. (1st ed. 2013)

Popper, K. (1957): The Propensity interpretation of calculus of probability, and the quantum theory, in S. Körner ed.: *Observation and Interpretation: A Symposium of Philosophers and Physicists* (pp. 65–70). London: Butterworths.

Redhead, M. L. G., & Teller, P. (1992). Particle Labels and the Theory of Indistinguishable Particles in Quantum Mechanics. *British Journal for the Philosophy of Science*, *43*, 201–218.

Schaffer, J. (2010). Monism: The Priority of the Whole. *Philosophical Review*, *119*(1), 31–76. doi:10.1215/00318108-2009-025

Saunders, S. (2006). Are Quantum Particles Objects? *Analysis*, *66*(289), 52–63. doi:10.1111/j.1467-8284.2006.00589.x

Schrödinger, E. (1935). Discussion of Probability Relations Between Separated Systems. *Mathematical Proceedings of the Cambridge Philosophical Society*, *31*(4), 555-563. doi:10.1017/S0305004100013554

Simons, P. M. (1987). *Parts: A Study in Ontology*. Oxford: Clarendon Press.

Simons, P. M. (2004). Extended Simples. *The Monist*, *87* (3), 371–385.

Tarski, A. (1929). Les fondements de la géométrie des corps. *Ksiega Pamiatkowa Pierwszego Polskiego Zjazdu Matematycznego* (*Annales de la Société Polonaise de Mathématiques*), 29–33.

Teller, P. (1986). Relational Holism and Quantum Mechanics. *British Journal for the Philosophy of Science*, *37*, 71–81.

Unger, P. (1979). I do not Exist. In: G. F. MacDonald (ed.), *Perception and Identity*, London: Macmillan, 235–251.