Fair Resource Allocation for OFDMA Femtocell Networks With Macrocell Protection

Vu Nguyen Ha, *Student Member, IEEE*, and Long Bao Le, *Senior Member, IEEE*

***Abstract*—We consider the joint subchannel allocation and power control problem for orthogonal frequency-division multiple-access (OFDMA) femtocell networks in this paper. Specifically, we are interested in the fair resource-sharing solution for users in each femtocell that maximizes the total minimum spectral efficiency of all femtocells subject to protection constraints for the prioritized macro users. Toward this end, we present the mathematical formulation for the uplink resource-allocation problem and propose an optimal exhaustive search algorithm. Given the exponential complexity of the optimal algorithm, we develop a distributed and low-complexity algorithm to find an efficient solution for the problem. We prove that the proposed algorithm converges and we analyze its complexity. Then, we extend the proposed algorithm in three different directions, namely, downlink context, resource allocation with rate adaption for femto users, and consideration of a hybrid access strategy where some macro users are allowed to connect with nearby femto base stations (FBSs) to improve the performance of the femto tier. Finally, numerical results are presented to demonstrate the desirable performance of the proposed algorithms.**

***Index Terms*—Femtocell networks, interference management, orthogonal frequency-division multiple access (OFDMA), power control, resource allocation, subchannel assignment (SA).**

I. Introduction

MASSIVE deployment of small cells such as femtocells provides an important solution to fundamentally improve the indoor throughput and coverage performance of wireless cellular networks [1]. Moreover, fourth-generation (4G) and beyond cellular networks are based on orthogonal frequency-division multiple access (OFDMA) that provides flexibility in radio resource management and robustness against adverse effects of multipath fading [2]. Hence, the design of an OFDMA femtocell tier so that it can coexist efficiently with the existing macrocell tier is an important research topic. One particular challenge in realizing this objective stems from the fact that femtocells are randomly deployed and they operate on the same frequency band with the macrocell tier, which can create strong cross-tier interference to the macrocells. Therefore, to successfully deploy the femtocell network, it is required to resolve many technical challenges, which range from resource allocation and synchronization [3] to protection of existing macrocells against cross-tier interference from femtocells [4].

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The authors are with the Institut National de la Recherche Scientifique-Énergie, Matériaux et Télécommunications, Université du Québec, Montréal, QC H3C 3P8, Canada (e-mail: hanguyen@emt.inrs.ca; long.le@emt.inrs.ca).

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There have been some existing works studying the resource allocation for OFDMA-based femtocell networks [5]–[12]. In [5], spectrum sharing and access control strategies are proposed for femtocell networks by using the water-filling algorithm and game theory technique. The authors in [6] propose two methods to mitigate the uplink interference for OFDMA femtocell networks. In the first method, femtocell user equipments (FUEs) that produce strong interference to macrocell user equipments (MUEs) are only allowed to use dedicated subchannels, whereas the remaining FUEs can utilize all subchannels assigned for the femto tier. In the second method, an auction-based algorithm is devised to optimize the channel assignment for both tiers to mitigate the co-tier interference. Interference avoidance and resource-allocation issues for OFDMA femtocell networks are also studied in [7] where a greedy algorithm is developed to enable self-organization of femtocells. In [8], Cheung *et al.* study the network performance where the macrocells and femtocells utilize separate sets of subchannels or share the whole spectrum under both open- and closed-access strategies. The quality-of-service (QoS)-aware admission control design for OFDMA femtocells is conducted in [9]. All these existing works, however, do not consider power control in their proposed resource-allocation algorithms.

In [10], an adaptive femtocell interference management algorithm comprising three control loops that run continuously and separately at macro and femto base stations (MBS and FBS) to determine initial femto maximum power, to decide target signal-to-interference-plus-noise ratios (SINRs) for FUEs, and to control the transmission power, respectively, is developed. However, subchannel assignment (SA) is not studied, and the maximum power constraint for each user is not considered in this paper. Spectrum sharing and power allocation (PA) are investigated in [11] and [12]. In particular, [11] aims to enhance the energy efficiency, whereas the main design objective of [12] is to achieve user-level fairness for cognitive femtocells. However, the algorithms developed in [11] and [12] do not provide QoS guarantees for users of both network tiers.

In this paper, we study the joint subchannel and PA problem for femtocell networks considering fairness for FUEs in each femtocell, protection of MUEs, and maximum power constraints. To the best of our knowledge, none of the existing works on the OFDMA-based femtocell network jointly consider all these design issues. In particular, we make the following contributions.

1) We formulate the uplink fair subchannel and PA problem, study its optimal structure, and present an algorithm to find its optimal solution. To overcome the exponential complexity of the centralized optimal exhaustive

search algorithm, we then develop a distributed and low-complexity resource-allocation algorithm. The proposed algorithm iteratively updates the SAs at each femtocell based on carefully designed assignment weights and transmission power values for the subchannels accordingly. We prove the convergence of the proposed algorithm and analyze its complexity.

2) We extend the proposed solution in three important directions. Specifically, we show how the proposed resource-allocation algorithm can be adapted for the downlink context. Moreover, we develop a distributed algorithm for the scenario where FUEs in different femtocells are allowed to choose different target rates per subchannel to better adapt to the network interference. Finally, we extend the proposed algorithm to implement the hybrid spectrum access where some MUEs are permitted to connect to nearby FBSs.
3) Numerical results are presented to demonstrate the efficacy of the proposed algorithms and their relative performance compared with the optimal algorithm. In particular, we demonstrate the impacts of target rates per subchannel (i.e., modulation schemes), maximum power values, rate adaptation, and hybrid access on the performance of the femtocell network.

The remainder of this paper is organized as follows. We describe the system model and the uplink problem formulation in Section II. In Section III, we present both optimal exhaustive search and suboptimal resource-allocation algorithms. Extensions for the downlink scenario, adaptive-rate transmission, and hybrid access control are described in Section IV. Numerical results are presented in Section V followed by conclusion in Section VI. Key notations used in this paper are summarized in Table III.

II. System Model and Problem Formulation

A. System Model

We consider the OFDMA-based two-tier macrocell–femtocell network where users of both tiers share the spectrum comprising N subchannels. We assume that there are M_{f} FUEs served by $(K-1)$ FBSs, which are underlaid by one macrocell serving M_{m} MUEs. Let $\mathcal{U}_k$ be the set of user equipments (UEs) in the kth cell, i.e., they are served by the base station (BS) k of the corresponding tier. For convenience, let $\mathcal{U}_{\mathsf{m}} \triangleq \mathcal{U}_1$ represent the set of MUEs and $\mathcal{U}_{\mathsf{f}} \triangleq \cup_{k=2}^{K}\mathcal{U}_k = \{M_{\mathsf{m}}+1,\ldots,M_{\mathsf{m}}+M_{\mathsf{f}}\}$ denote the set of all FUEs. In addition, let $\mathcal{U}$ and $\mathcal{B}$ be the sets of all UEs and BSs, respectively. Then, we have $\mathcal{U}=\mathcal{U}_{\mathsf{m}}\cup\mathcal{U}_{\mathsf{f}}=\{1,2,\ldots,M\}$ and $\mathcal{B}=\{1,2,\ldots,K\}$, where BS 1 is assumed to be the MBS and $\mathcal{B}_{\mathsf{f}}=\{2,\ldots,K\}$ denotes the set of all FBSs.

We assume the fixed BS association for all UEs of both tiers in the network (i.e., each UE is served by a fixed BS in the corresponding tier) to present the problem formulation here. Relaxation of this assumption under the hybrid access design will be considered in Section IV. Now, let $b_i \in \mathcal{B}$ denote the BS serving UE i, and let $\mathcal{N}=\{1,2,\ldots,N\}$ be the set of all orthogonal subchannels. We assume that there is no interference among transmissions on different subchannels. We consider a system with full frequency reuse where all N subchannels are allocated for UEs in all cells of either tier. To describe the SAs, let $\mathbf{A}\in\Re^{M\times N}$ be the SA matrix for all M UEs over N subchannels where

$$\mathbf{A}(i,n)=a_i^n=\begin{cases}1, & \text{if subchannel } n \text{ is assigned for UE } i\\ 0, & \text{otherwise.}\end{cases} \tag{1}$$

We assume that a subchannel can be allocated to, at most, one UE in any cell. Then, we have

$$\sum_{i\in\mathcal{U}_k} a_i^n \le 1, \forall k\in\mathcal{B} \text{ and } \forall n\in\mathcal{N}. \tag{2}$$

These constraints will be applied to all resource-allocation problems studied in this paper.

B. Physical-Layer Model

We assume that M-ary quadrature amplitude modulation (M-QAM) is adopted for communications on each subchannel where the constellation size s for s-QAM is chosen from a predetermined set $\mathfrak{M}$. To guarantee acceptable performance when a constellation size s is adopted on a particular subchannel, we need to ensure that the SINR on that subchannel is not smaller than a corresponding target SINR value $\bar{\gamma}(s)$. To determine $\bar{\gamma}(s)$, let $f(\gamma(s))$ be the expression of bit error rate (BER) for SINR $\gamma(s)$ when the constellation size s is adopted. Suppose we want to maintain the BER to be not greater than a target value $\overline{P}_e$. Then, the SINR must satisfy the following for the constellation size s:

$$\gamma(s) \le \bar{\gamma}(s) = f^{-1}(\overline{P}_e) \tag{3}$$

where $f^{-1}(.)$ denotes the inverse function of the BER function $f(.)$. To demonstrate this calculation, suppose that M-QAM modulation with square signal constellations (i.e., s-QAM where $s\in\mathfrak{M}\triangleq\{4,16,64,256,\ldots,s_{\mathsf{max}}\}=\{2^{2\sigma}|\sigma$ is a positive integer and $\sigma\le 1/2\log_2 s_{\mathsf{max}}\}$) is employed. According to [13], the BER of the s-QAM scheme with Gray encoding can be approximated as

$$f(\gamma(s)) \approx x_s\mathbb{Q}\left(\sqrt{y_s\gamma(s)}\right), \qquad s\in\mathfrak{M} \tag{4}$$

where $\mathbb{Q}(.)$ stands for the Q-function, $x_s=2(1-1/\sqrt{s})/\log_2 s$, $y_s=3/2(s-1)$, and $\gamma(s)$ is the SINR. Using the result in (3), the target SINR for the s-QAM modulation scheme can be calculated as

$$\bar{\gamma}(s)=\frac{\left[\mathbb{Q}^{-1}(\overline{P}_e/x_s)\right]^2}{y_s}, \qquad s\in\mathfrak{M} \tag{5}$$

where $\overline{P}_e$ is the target BER value. We assume that ideal Nyquist data pulses are used where the symbol rate on each subchannel is equal to BW/N, where BW is the total bandwidth of N subchannels. Then, if s-QAM modulation scheme is employed, the spectral efficiency per 1 Hz of system bandwidth is $\log_2 s/N$ (in bits per second per hertz).

C. Uplink Resource-Allocation Problem

Let p_i^n represent the transmission power of UE i over the subchannel n in the uplink where $p_i^n \geq 0$. We impose the following constraints on the total transmission power values:

$$\sum_{n=1}^{N} p_i^n \leq P_i^{\max}, \qquad i \in \mathcal{U} \tag{6}$$

where $P_i^{\max}$ is the maximum transmission power of UE i. Similar to the SAs, we define $\mathbf{P}$ as an $M \times N$ PA matrix where $\mathbf{P}(i,n) = p_i^n$. For convenience, we also define partitions of SA and PA matrices $\mathbf{A}$ and $\mathbf{P}$ as follows. In particular, let $\mathbf{A}_k, \mathbf{P}_k \in \Re^{|\mathcal{U}_k| \times N}$ represent the SA and PA matrices for UEs in cell k over N subchannels, respectively.

Let h_{ij}^n and η_i^n be the channel power gain from UE j to BS i and the noise power at BS i over subchannel n, respectively. Then, for a given SA and PA solution, i.e., given $\mathbf{A}$ and $\mathbf{P}$, the SINR achieved at BS b_i due to the transmission of UE i over subchannel n can be written as

$$\Gamma_i^n(\mathbf{A},\mathbf{P}) \frac{a_i^n h_{b_i i}^n p_i^n}{\sum_{j \notin \mathcal{U}_{b_i}} a_j^n h_{b_i j}^n p_j^n + \eta_{b_i}^n} = \frac{a_i^n p_i^n}{I_i^n(\mathbf{A},\mathbf{P})} \tag{7}$$

where $I_i^n(\mathbf{A},\mathbf{P})$ is the effective interference corresponding to UE i on subchannel n, which is defined as

$$I_i^n(\mathbf{A},\mathbf{P}) \triangleq \frac{\sum_{j \notin \mathcal{U}_{b_i}} a_j^n h_{b_i j}^n p_j^n + \eta_{b_i}^n}{h_{b_i i}^n}. \tag{8}$$

We assume that SAs for MUEs have been determined by a certain mechanism and fixed, whereas PAs over the corresponding subchannels for MUEs are updated to cope with the cross-tier interference due to transmissions of FUEs. This means that $\mathbf{A}_1$ is fixed while we need to determine $\mathbf{A}_k, 2 \leq k \leq K$ and the corresponding PAs. This assumption is quite natural since the macro tier has been deployed before the introduction of the femto tier; therefore, it would be more reasonable to keep the existing subchannel allocation mechanism and solution in the macro tier unchanged.

To protect the QoS of the licensed MUEs, we wish to maintain a predetermined target SINR $\bar{\gamma}_i^n$ for each of its assigned subchannel n. Specifically, we have the following constraints for the MUEs:

$$\Gamma_i^n(\mathbf{A},\mathbf{P}) \geq \bar{\gamma}_i^n, \qquad \text{if } a_i^n = 1, \ \forall i \in \mathcal{U}_{\mathsf{m}} \tag{9}$$

where $\bar{\gamma}_i^n = \bar{\gamma}(s^{\mathsf{m}})$, which is calculated as in (3) if MUE i employs s^{m}-QAM modulation scheme.

For FUEs, we assume that they all employ s^{f}-QAM for a predetermined s^{f}. Then, the spectral efficiency (in bits per second per hertz) achieved by FUE i on one subchannel can be written as

$$r_i^n(\mathbf{A},\mathbf{P}) = \begin{cases} 0, & \text{if } \Gamma_i^n(\mathbf{A},\mathbf{P}) < \bar{\gamma}_i^n \\ r_{\mathsf{f}}, & \text{if } \Gamma_i^n(\mathbf{A},\mathbf{P}) \geq \bar{\gamma}_i^n \end{cases} \tag{10}$$

where $r_{\mathsf{f}} = (1/N)\log_2 s^{\mathsf{f}}$, and $\bar{\gamma}_i^n = \bar{\gamma}(s^{\mathsf{f}})$, which is calculated as in (3). Note that we have assumed that FUEs in all femtocells employ the same constellation size s^{f} for ease of exposition. We will discuss the more general case in Section IV. In practice, we typically choose $s^{\mathsf{f}} \geq s^{\mathsf{m}}$ since FUEs can achieve higher transmission rates on each subchannel than MUEs due to their short distance to the associated FBS. This is indeed equivalent to $\bar{\gamma}(s^{\mathsf{f}}) \geq \bar{\gamma}(s^{\mathsf{m}})$. In the analysis, we mostly use the same notation $\bar{\gamma}_i^n$ to refer to the required target SINR for FUEs or MUEs where $\bar{\gamma}_i^n = \bar{\gamma}(s^{\mathsf{f}})$ for $\forall i \in \mathcal{U}_{\mathsf{f}}$ and $\bar{\gamma}_i^n = \bar{\gamma}(s^{\mathsf{m}})$ for $\forall i \in \mathcal{U}_{\mathsf{m}}$. Now, we can express the total spectral efficiency achieved by UE i for given SA and PA matrices $\mathbf{A}$ and $\mathbf{P}$ as

$$R_i(\mathbf{A},\mathbf{P}) = \sum_{n=1}^{N} r_i^n(\mathbf{A},\mathbf{P}). \tag{11}$$

To impose the max–min fairness for all FUEs associated with the same FBS, we define the minimum spectral efficiency of femtocell k as the smallest value of spectral efficiency values of all FUEs in that femtocell, e.g.,

$$R^{(k)}(\mathbf{A},\mathbf{P}) = \min_{i \in \mathcal{U}_k} R_i(\mathbf{A},\mathbf{P}). \tag{12}$$

The uplink resource-allocation problem for FUEs can be formulated as follows:

$$\max_{(\mathbf{A},\mathbf{P})} \sum_{2 \leq k \leq K} R^{(k)}(\mathbf{A},\mathbf{P}) = \sum_{2 \leq k \leq K} \min_{i \in \mathcal{U}_k} R_i(\mathbf{A},\mathbf{P}) \tag{13}$$

$$\text{s.t. constraints} \quad (2), (6), (9). \tag{14}$$

Therefore, the objective of this uplink resource-allocation problem aims to balance between attaining an equal rate for FUEs in each femtocell (max–min fairness [25]) and high total femtocell spectral efficiency subject to SA constraints, FUE power constraints, and protection constraints for MUEs. Note that the SINR constraints for FUEs are embedded in the calculation of FUE spectral efficiency in (10). The resource-allocation problem (13) and (14) can be transformed into a standard mixed integer program, which is, therefore, NP-hard.

III. Optimal and Suboptimal Algorithms

Here, we study the feasibility of a particular SA solution, which reveals the optimal structure of the optimization problem (13) and (14) based on which we develop optimal and suboptimal algorithms.

A. Feasibility of an SA Solution

The resource-allocation problem (13) and (14) involves finding a joint SA and PA solution. For a certain SA solution represented by matrix $\mathbf{A}$ satisfying the SA constraints (2), we can indeed find its "best" PA and verify its feasibility with respect to the constraints in (14). In particular, we wish to maintain the SINR constraints for FUEs and MUEs in (10) and (9), respectively. Specifically, for each subchannel n, we need to maintain SINR constraints $\Gamma_i^n(\mathbf{A},\mathbf{P}) \geq \bar{\gamma}_i^n$ for both MUEs and FUEs that are allocated with this subchannel.

Now, let $\mathcal{U}^n = \{i \in \mathcal{U} | a_i^n \neq 0\} = \{n_1, \ldots, n_{c^n}\}$ denote the set of UEs of both tiers that are assigned subchannel n and

$c^n = |\mathcal{U}^n|$ be the number of elements in set $\mathcal{U}^n$. Then, the SINR constraints for UEs in set $\mathcal{U}^n$, i.e., $\Gamma_i^n(\mathbf{A},\mathbf{P}) \geq \bar{\gamma}_i^n, i \in \mathcal{U}^n$, can be rewritten in a matrix form as

$$(\mathbf{I}^n - \mathbf{G}^n\mathbf{H}^n)\mathbf{p} \leq \mathbf{g}^n \tag{15}$$

where $\mathbf{g}^n = [g_{n_1}^n, \ldots, g_{n_{c^n}}^n]^T$ with $g_i^n = \eta_{b_i}^n \bar{\gamma}_i^n / h_{b_i i}^n$, $\mathbf{I}^n$ is $c^n \times c^n$ identity matrix, $\mathbf{G}^n = \mathrm{diag}\{\bar{\gamma}_{n_1}^n, \ldots, \bar{\gamma}_{n_{c^n}}^n\}$, $\mathbf{p}^n = [p_{n_1}^n, \ldots, p_{n_{c^n}}^n]^T$, and $\mathbf{H}^n$ is a $c^n \times c^n$ matrix defined as

$$\left[\mathbf{H}_{i,j}^n\right] = \begin{cases} 0, & \text{if } j = i \\ \frac{h_{b_{n_i} n_j}^n}{h_{b_{n_i} n_i}^n}, & \text{if } j \neq i. \end{cases} \tag{16}$$

To study feasible solutions for inequality (15), we recall some standard results due to the Perron–Frobenius theorem [14], [22], [23] and their application to the power control problem [17]. Let λ_i be the ith eigenvalue of matrix $\mathbf{G}^n\mathbf{H}^n$ and $\rho(\mathbf{G}^n\mathbf{H}^n) = \max_i |\lambda_i|$ be the maximum value of the modulus of all eigenvalues (i.e., the spectral radius of $\mathbf{G}^n\mathbf{H}^n$). The results in the Perron–Frobenius theorem can be applied for nonnegative matrix $\mathbf{G}^n\mathbf{H}^n$, which is the case for our model since all power channel gains, noise power values, and target SINRs are real and nonnegative numbers. Specifically, the Perron–Frobenius theorem implies the following fact, which has been clearly stated in [17].

Fact: If $\mathbf{G}^n\mathbf{H}^n$ has nonnegative elements, the following statements are equivalent.

1) There exists a nonnegative power vector $\mathbf{p}^n$ such that $(\mathbf{I}^n - \mathbf{G}^n\mathbf{H}^n)\mathbf{p}^n \geq \mathbf{g}^n$.
2) $\rho(\mathbf{G}^n\mathbf{H}^n) \leq 1$.
3) $(\mathbf{I} - \mathbf{G}^n\mathbf{H}^n)^{-1} = \sum_{k=0}^{\infty}(\mathbf{G}^n\mathbf{H}^n)^k$ exists and is positive elementwise.

From these results, if $\rho(\mathbf{G}^n\mathbf{H}^n) \leq 1$, then we can find a solution of $(\mathbf{I}^n - \mathbf{G}^n\mathbf{H}^n)\mathbf{p}^n = \mathbf{g}^n$ (i.e., the equality case of the considered inequality) as

$$\mathbf{p}^{n\star} = (\mathbf{I} - \mathbf{G}^n\mathbf{H}^n)^{-1}\mathbf{g}^n \tag{17}$$

which is also the Pareto-optimal solution of $(\mathbf{I}^n - \mathbf{G}^n\mathbf{H}^n)$ $\mathbf{p}^n \geq \mathbf{g}^n$ where Pareto optimality means that any feasible solution $\mathbf{p}^n$ for the inequality $(\mathbf{I}^n - \mathbf{G}^n\mathbf{H}^n)\mathbf{p}^n \geq \mathbf{g}^n$ is not smaller than $\mathbf{p}^{n\star}$ elementwise (i.e., there exists no feasible solution $\mathbf{p}^n$ for $(\mathbf{I}^n - \mathbf{G}^n\mathbf{H}^n)\mathbf{p}^n \geq \mathbf{g}^n$ so that $p_i^n \leq p_i^{n\star}, \forall i$ and $p_j^n < p_j^{n\star}$ for some j). These results have been used in [17] and several references therein.

In addition, this Pareto-optimal power vector can be achieved at the equilibrium by employing the following well-known distributed Foschini–Miljanic power updates [16], [17]:

$$p_i^n(l+1) := p_i^n(l)\frac{\bar{\gamma}_i^n}{\Gamma_i^n(l)} = I_i^n(l)\bar{\gamma}_i^n \tag{18}$$

where $\Gamma_i^n(l)$ and $I_i^n(l)$ are the SINR and effective interference achieved by UE i in iteration l, respectively. If there is no feasible solution for (15) (when $\rho(\mathbf{G}^n\mathbf{H}^n) > 1$), the transmission power values due to (18) will increase to infinity [16], [17]. Now, we are ready to state one important result for a given SA solution $\mathbf{A}$ that satisfies the SA constraints (2) in the following proposition.

Proposition 1: Suppose that we can find a finite Pareto-optimal PA solution $\mathbf{p}^n$ on the subchannel n for the considered SA $\mathbf{A}$, which is given in (17) (i.e., the spectrum radius $\rho(\mathbf{G}^n\mathbf{H}^n) < 1, \forall n$). Then, the underlying SA $\mathbf{A}$ is feasible if these Pareto-optimal PA vectors $\mathbf{p}^n$ satisfy the power constraints in (6).

Proof: The Pareto-optimal PA solution $\mathbf{p}^n$ on each subchannel n requires the minimum power values in the elementwise sense for all UEs, which are assigned subchannel n, to meet their SINR requirements. Therefore, the considered SA solution $\mathbf{A}$ is feasible if and only if the corresponding Pareto-optimal PA solutions $\mathbf{p}^n$ on all N subchannels satisfy the power constraints in (6). Therefore, we have completed the proof of the proposition. □

Since the number of possible SAs is finite, the result in this proposition pays the way to develop the optimal exhaustive search algorithm, which is presented in the following.

B. Optimal Algorithm

1) Exhaustive Search Algorithm: Based on the results in Section III-A, we can find the optimal solution for the resource-allocation problem (13) and (14) by performing exhaustive search as follows. For a fixed and feasible $\mathbf{A}_1$,[1] let $\Omega\{\mathbf{A}\}$ be the list of all potential SA solutions that satisfy the SA constraints (2) and the fairness condition $\sum_{n\in\mathcal{N}} a_i^n = \sum_{n\in\mathcal{N}} a_j^n = \tau_k$ for all FUEs $i, j \in \mathcal{U}_k$. Then, we sort the list $\Omega\{\mathbf{A}\}$ in the decreasing order of $\sum_{k=2}^{K}\tau_k$ to obtain the sorted list $\Omega^*\{\mathbf{A}\}$. Then, the feasibility of each SA solution in the list $\Omega^*\{\mathbf{A}\}$ can be verified, as presented in Section III-A. Among all feasible SA solutions, the feasible one achieving the highest value of the objective function (13) and its corresponding PA solution given in (17) is the optimal solution of the optimization problem (13) and (14).

2) Complexity Analysis: The complexity of the exhaustive search algorithm can be determined by calculating the number of the elements in the list $\Omega^*\{\mathbf{A}\}$ and the complexity involved in the feasibility verification for each of them. The number of the elements in $\Omega^*\{\mathbf{A}\}$, i.e., the number of potential SAs, is the product of the number of potential SAs for all femtocells, which satisfy the "fairness condition." Therefore, the number of potential SAs can be calculated as

$$T = \prod_{k=2}^{K}\sum_{\tau_k=0}^{\eta_k}\prod_{i=0}^{M_k-1} C_{N-i\tau_k}^{\tau_k} = \prod_{k=2}^{K}\sum_{\tau_k=0}^{\eta_k}\frac{N!}{(\tau_k!)^{M_k}(N-M_k\tau_k)!} \approx O\left((N!)^{(K-1)}\right) \tag{19}$$

where $\eta_k = \lfloor N/M_k \rfloor$ represents for the largest integer less than or equal to N/M_k, and $C_m^n = m!/n!(m-n)!$ denotes the *"m-choose-n"* operation. According to Section III-A, the complexity of the feasibility verification for each potential SA

[1]The SA solution for MUEs corresponding to $\mathbf{A}_1$ is feasible if there exists a PA solution that satisfies the constraints in (9) when there is no femto tier.

mainly depends on the eigenvalue calculation of the corresponding matrix and solving the linear system to find the PA solution for each subchannel. It requires $O(K^3)$ to calculate the eigenvalues of $\mathbf{G}^n\mathbf{H}^n$ [18] and $O(K^3)$ to obtain $\mathbf{p}^n$ by solving a system of linear equations [19]. Therefore, the complexity of the optimal exhaustive search algorithm is $O(K^3 \times N \times (N!)^{(K-1)})$, which is exponential in the numbers of subchannels and FBSs. This optimal exhaustive search algorithm will serve as a benchmark to evaluate the low-complexity algorithm developed in the following.

C. Suboptimal and Distributed Algorithm

To resolve the exponential complexity of the centralized optimal algorithm presented in Section III-B, we develop a low-complexity and distributed resource-allocation algorithm. To achieve max–min fairness as required by the objective function (13), our algorithm aims to assign the maximum equal number of subchannels to FUEs in each femtocell and to perform Pareto-optimal PA for FUEs and MUEs on each subchannel so that they meet the SINR constraints in (10) and (9). To achieve this design goal, we propose a novel subchannel allocation weight metric so that high weights are assigned for "bad allocations" requiring large transmission power values and *vice versa*.

The proposed resource-allocation algorithm is described in detail in Algorithm 1. The key operation in this algorithm is the iterative weight-based SA that is performed in parallel at all femtocells. The SA weight for each subchannel and FUE pair is defined as the multiplication of the estimated transmission power and a scaling factor capturing the quality of the corresponding allocation. Specifically, each UE i in cell k estimates the transmission power on subchannel n in each iteration l of the algorithm by using the Foschini–Miljanic power update given in (18) as follows:

$$p_i^{n,\min} = I_i^n(l)\bar{\gamma}_i^n. \tag{20}$$

The effective interference $I_i^n(l)$ given in (8) can be estimated by UE i as follows. The serving BS of UE i estimates the power channel gain $h_{b_i i}^n$ on subchannel n from this user to itself and transmits this channel state information to UE i. In addition, the BS of UE i measures the total received power on subchannel n due to both desired and interfering signals, as well as the Gaussian noise, which is then sent back to UE i. UE i can calculate $I_i^n(l)$ easily by using the gathered information (i.e., its transmission power, power channel gain to the BS, and the total received power at the BS) according to (8). We propose the following assignment weight for FUE i on subchannel n in cell k as

$$w_i^n = \chi_i^n p_i^{n,\min} \tag{21}$$

where the scaling factor χ_i^n is defined as follows:

$$\chi_i^n = \begin{cases} \alpha_i^n, & \text{if } p_i^{n,\min} \le \frac{P_i^{\max}}{\tau_k} \\ \alpha_i^n\theta_i^n, & \text{if } \frac{P_i^{\max}}{\tau_k} < p_i^{n,\min} \le P_i^{\max} \\ \alpha_i^n\delta_i^n, & \text{if } P_i^{\max} < p_i^{n,\min} \end{cases} \tag{22}$$

where τ_k denotes the number of subchannels assigned for each FUE in femtocell k; $\alpha_i^n \ge 1$ is a factor, which helps maintain SINR constraints of MUEs (i.e., it is increased if assigning subchannel n to FUE i tends to result in violation of the SINR constraint of the corresponding MUE); and $\theta_i^n, \delta_i^n \ge 1$ are another factors that are set higher if the assignment of subchannel n for FUE i tends to require transmission power larger than the average power per subchannel (i.e., $P_i^{\max}/\tau_k$) and the maximum power budget (i.e., $P_i^{\max}$), respectively. We will set δ_i^n as $\delta_i^n = \mu_i\theta_i^n$ where $\mu_i = N$ in Algorithm 1. Given the weights defined for each FUE i, femtocell k finds the SA for its FUEs by solving the following problem:

$$\begin{aligned} &\min_{\mathbf{A}_k} \sum_{i\in\mathcal{U}_k}\sum_{n\in\mathcal{N}} a_i^n w_i^n \\ &\text{s.t.} \quad \sum_{n\in\mathcal{N}} a_i^n = \tau_k, \ \forall i \in \mathcal{U}_k. \end{aligned} \tag{23}$$

This problem aims to find an assignment matrix $\mathbf{A}_k$ for which each FUE in femtocell k is assigned τ_k subchannels with minimum total weight (i.e., to attain low-power SAs). The optimization problem (23) can be transformed into the standard matching problem (for example, between "jobs" and "employees") as follows. Suppose we create τ_k virtual "employees" for each FUE in femtocell k, then we can consider the matching problem between $\tau_k M_k$ virtual "employees" (virtual FUEs) and N "jobs" (subchannels). In particular, FUE i is equivalent to τ_k virtual FUEs $\{i_1, \ldots, i_{\tau_k}\}$. Let the edge $v_{i_u}^n$ $(u \in \{1, \ldots, \tau_k\})$ between subchannel n and virtual FUE i_u represent the assignment of that subchannel to the corresponding FUE. Then, the weight $w_{i_u}^n$ of the edge $v_{i_u}^n$ is equal to w_i^n. After performing this transformation, the SA solution of the problem (23) can be found by using the standard Hungarian algorithm [20]. After running the Hungarian algorithm, if there exists a virtual FUE i_u, $u \in \{1, \ldots, \tau_k\}$, being matched with subchannel n, then we have $a_i^n = 1$; otherwise, we set $a_i^n = 0$. For further interpretation of Algorithm 1, let $W_k(l)$ denote the total minimum weight due to the optimal solution of (23). The main operations of Algorithm 1 can be summarized as follows.

1) In steps 2–10, the MBS needs to estimate the effective interference on all subchannels and calculates the transmission power values for the corresponding MUEs by using (20). Then, the MBS checks the power constraints (6) for its associated MUEs. For MUEs that can maintain the power constraints (6), the MBS will send the newly calculated transmission power values to them so that MUEs can update their power values accordingly (step 5). Otherwise, if the power constraint of any MUE is violated, the MBS will scale down the transmission power values to maintain the power constraints (step 7) and send the corresponding transmission power values to its MUEs. In addition, the MBS will inform all FBSs, which will find the FUE creating the largest interference to the victim MUE on the subchannel with the largest transmission power; then, we increase the parameter $\alpha_{m_i^*}^{n_i^*}$ corresponding to this FUE and subchannel pair by a factor of 2 (steps 8 and 9). This can be realized by allowing the

MBS to request nearby FBSs to report the transmission power on the victim subchannel based on which the MBS can identify and request the most interfering FUE m_i^* to update its parameter $\alpha_{m_i^*}^{n_i^*}$. The signaling required by these operations can be accommodated by the wired backhaul links [e.g., digital subscriber line (DSL) links].

2) In steps 15–20, each femtocell that has any FUEs' power constraints being violated in the previous iteration solves the SA problem (23) with the current weight values to update its SA solution. In addition, FBS k decreases the target number of assigned subchannels τ_k by one if the optimal total weight $W_k(l)$ of the problem (23) satisfies the condition in step 17. Each FBS can complete all required tasks in these steps since it knows current transmission power values and scaling factors χ_i^n of its FUEs based on which it can calculate all SA weights.

3) In step 12, each FBS estimates the effective interference on all subchannels and calculates the transmission power values for its FUEs by using (20). Then, each FBS will check the power constraints of its FUEs using the newly calculated transmission power values. For FUEs that have their power constraints satisfied, the corresponding FBS will send the newly calculated transmission power values to them so that they can update power values accordingly (steps 22 and 23). For any FUE that has its power constraint violated, its FBS scales down the transmission power values to meet the power constraints (step 25) and increase the $\theta_i^{n_i^*}$ parameter for the most interfering subchannel by a factor of 2 (steps 26 and 27). In both cases, each FBS must send the transmission power levels on all subchannels to the corresponding FUEs by using available control channels.

It can be observed that Algorithm 1 can be implemented distributively. In fact, the signaling information required in steps 8 and 9 must be sent over the wired backhaul link. In addition, each BS of either tier only needs to collaborate with its associated UEs to conduct all required tasks in other steps where the required signaling is sent over the air by using available control channels.

Algorithm 1 Distributed Uplink Resource Allocation

1: Initialization
- Set $p_i(0) = 0$ for all UE i, $i \in \mathcal{U}_\mathsf{f}$, feasible $\mathbf{A}_1$.
- Set $\tau_k = \lfloor N/|\mathcal{U}_k| \rfloor$ and $\varrho_k = 0$ for all $k \in \mathcal{B}_\mathsf{f}$.
- Set $\alpha_i^n = \theta_i^n = 1$, $\mu_i = N, \forall i \in \mathcal{U}_\mathsf{f}, n \in \mathcal{N}$.

2: **For the macrocell:**
3: MBS estimates $I_i^n(l)$ and calculates $p_i^{n,\min}$ for each MUE i as in (20). Let $\beta_i = \sum_{n\in\mathcal{N}} a_i^n(l) p_i^{n,\min}/P_i^{\max}$.
4: **if** $\beta_i \le 1$ **then**
5: Set $p_i^n(l+1) = a_i^n(l) p_i^{n,\min}, \forall n \in \mathcal{N}$.
6: **else if** $\beta_i > 1$ **then**
7: Set $p_i^n(l+1) = a_i^n(l) p_i^{n,\min}/\beta_i, \forall n \in \mathcal{N}$
8: Find $n_i^* = \arg\max_{n\in\mathcal{N}, c_n>1} a_i^n(l) p_i^{n,\min}$; $m_i^* = \arg\max_{m\in\mathcal{U}_\mathsf{f}} a_m^{n_i^*}(l) p_m^{n_i^*}(l) h_{1m}^*$
9: Set $\alpha_{m_i^*}^{n_i^*} = 2\alpha_{m_i^*}^{n_i^*}$ and $\varrho_{b_{m_i^*}} = 0$.
10: **end if**
11: **For each femtocell** $k \in \mathcal{B}_\mathsf{f}$:
12: Each FBS k estimates $I_i^n(l)$ and calculates $p_i^{n,\min}$ for each FUE i as in (20).
13: **if** $\varrho_k = 1$ **then**
14: Keep $\mathbf{A}_k(l) = \mathbf{A}_k(l-1)$
15: **else if** $\varrho_k = 0$ **then**
16: Calculate SA weights $w_{i_u}^n$ between $\mathcal{N}$ and $\cup_{i\in\mathcal{U}_k}\{i_1, \ldots, i_{\tau_k}\}$ as in (22), and run Hungarian algorithm with $\{w_{i_u}^n\}$ to obtain $W_k(l)$ and $\mathbf{A}_k(l)$.
17: **if** $W_k(l) > V \sum_{i\in\mathcal{U}_k} P_i^{\max}$ **then**
18: Set $\tau_k := \tau_k - 1$.
19: **end if**
20: **end if**
21: Let $\beta_i = \sum_{n\in\mathcal{N}} a_i^n(l) p_i^{n,\min}/P_i^{\max}$.
22: **if** $\beta_i \le 1$ **then**
23: Set $p_i^n(l+1) = a_i^n(l) p_i^{n,\min}, \forall n \in \mathcal{N}$ and $\varrho_{k,i} = 1$.
24: **else if** $\beta_i > 1$ **then**
25: Set $p_i^n(l+1) = a_i^n(l) p_i^{n,\min}/\beta_i, \forall n \in \mathcal{N}$
26: Find $n_i^* = \arg\max_{n\in\mathcal{N}} a_i^n(l) p_i^{n,\min}$
27: Set $\theta_i^{n_i^*} = 2\theta_i^{n_i^*}$ and $\varrho_{k,i} = 0$.
28: **end if**
29: Set $\varrho_k = \prod_{i\in\mathcal{U}_k} \varrho_{k,i}$.
30: Let $l := l + 1$, return to step 2 until convergence.

D. Convergence and Complexity Analysis of Algorithm 1

1) Convergence Analysis: The convergence of Algorithm 1 is stated in the following.

Proposition 2: Algorithm 1 converges to a feasible solution $(\mathbf{A}, \mathbf{P})$ of the optimization problem (13) and (14).

Proof: To prove this proposition, we will consider two possible scenarios. For the first scenario, there exists an iteration after which the scaling factors χ_i^n for SA weights given in (22) do not change. According to Algorithm 1, after this iteration, the SA solution will remain unchanged. In addition, Algorithm 1 simply updates transmission power values $p_i^{n,\min}$ for all UEs on their assigned subchannels by using the Foschini–Miljanic power updates. The authors in [16] have shown that these power updates indeed converge, which implies the convergence of Algorithm 1.

Now, suppose that the scaling factors χ_i^n are still changed over iterations, we will prove that the system will ultimately evolve into the first scenario previously discussed. First, it can be verified that the power $p_i^{n,\min}$ given in (20) is lower bounded by $\bar{\gamma}_i^n \eta_{b_i}^n / h_{b_i i}^n$. Therefore, if the scaling factors χ_i^n keep increasing over iterations, then the total weight $W_k(l)$ returned by the assignment problem (23) will increase over iterations as well. Therefore, there exist some femtocells k that decrease their number of assigned subchannels τ_k over iterations (see steps 17–19 of Algorithm 1). Since initial values of all τ_k are finite, this process will terminate after a finite number of iterations. Then, the system will be in the first scenario discussed above; therefore, Algorithm 1 converges. Therefore, we have completed the proof of the proposition. □

TABLE I
SUMMARY OF EXISTING ALGORITHMS

Papers	Objective function	UL/DL	SA/PA	Prt. MUE	Pw. Const.	CoTI/CrTI	Conv.	Complexity
Ref. [11]	Max-min number of assigned subchannels	Both	SA	Yes	No	No	NA	NA
Ref. [12]	Max sum rate	UL	SA	Yes	No	Both	NA	NA
Ref. [13]	Max sum PRB	Both	SA	No	No	NA	NA	$O(KN \log N)$
Ref. [16]	NA (Analysis)	DL	SA	No	No	Both	NA	NA
Ref. [17]	Min error between interference and predetermined thresholds at MUEs	UL	Both	Yes	No	Both	NA	NA
Ref. [18]	Max energy efficiency	Both	Both	No	No	No	Yes	NA
Ref. [19]	Max-min effective rate	DL	PA	No	No	CoTI	NA	NA
Zhang's paper	Max sum rate	UL	Both	Yes	Yes	CrTI	NA	NA
Our paper	Max-min rate	UL	Both	Yes	Yes	Both	Yes	$O(KN^3)$

UL: Uplink	DL: Downlink
SA: Subchannel assignment	PA: Power allocation
Prt. MUE: Protect the QoS of MUEs	Pw. Const.: Power constraint
CoTI: Co-tier interference among femtocells	CrTI: Cross-tier interference between macrocell and femtocells
Conv.: Convergence guarantee	PRB: Physical resource block (frequency and time)

2) Complexity Analysis and Comparison With Existing Algorithms: It is observed that the major complexity of Algorithm 1 is involved in solving the SA by using the Hungarian method in step 16. According to [20], the complexity of the Hungarian algorithm is $O(N^3)$. Therefore, the complexity of our proposed algorithm is $O(K \times N^3)$ for each iteration. However, the local SAs can be performed in parallel at all $(K-1)$ femtocells. Therefore, the runtime complexity of our algorithm is $O(N^3)$ multiplied by the number of required iterations, which is quite moderate according to our simulation results (i.e., tens of iterations).

For comparison purposes, we summarize how existing resource-allocation formulations and algorithms for two-tier macro–femto networks cover different design aspects in Table I where we write NA for the convergence and complexity aspects if they are not analyzed in these existing works. As we can see, the existing works consider different optimization objectives and cover some design issues although none of them accounts for all aspects. Our proposed algorithm captures all design aspects except the downlink scenario. Moreover, only Zhang *et al.* [27] and we consider the power constraints in our resource allocation. However, Zhang *et al.* aim to maximize the total throughput of the femtocell network while constraining the cross-tier interference from FUEs to the MBS. In their work, the co-tier interference among femtocells is assumed to be part of the noise power, which is not explicitly managed. Given their comparable design and our proposed algorithm, we will conduct performance comparison for the two algorithms in terms of throughput and fairness in Section V.

IV. FURTHER EXTENSIONS

A. Downlink Resource Allocation

For the downlink context, we can use the same notations as in the uplink system. However, h_{ij}^n and η_i^n are the power channel gain from BS j to UE i and the noise power at UE i, respectively. In the downlink system, we need to impose the power constraints for the BSs instead of UEs as follows:

$$\sum_{i \in \mathcal{U}_k} \sum_{n \in \mathcal{N}} p_i^n \leq P_{\mathsf{BS},k}^{\max}, \quad k \in \mathcal{B} \tag{24}$$

where $P_{\mathsf{BS},k}^{\max}$ is the maximum power of BS k. Then, the downlink resource-allocation problem can be formulated as

$$\max_{(\mathbf{A},\mathbf{P})} \sum_{2 \leq k \leq K} R^{(k)}(\mathbf{A},\mathbf{P}) \tag{25}$$

$$\text{s.t. constraints}(2), (9), (24). \tag{26}$$

We can employ Algorithm 1 to find a feasible solution of this problem with only some changes in the scaling factor χ_i^n of the assignment weight $w_i^n = \chi_i^n p_i^{n,\min}$ as follows:

$$\chi_i^n = \begin{cases} \alpha_i^n, & \text{if } p_i^{n,\min} \leq P_{\mathsf{BS},b_i}^{\max} \\ \infty, & \text{if } p_i^{n,\min} > P_{\mathsf{BS},b_i}^{\max}. \end{cases} \tag{27}$$

In fact, the structure of power constraints in the downlink context is less complicated than those in the uplink setting since we only need to maintain the total power constraint for each BS instead of all UEs connected with each BS. This explains the simpler structure of (27) compared with that of (22).

The signaling requirements for the downlink case are a bit different from those in the uplink one, which is described in the following. To complete the tasks in steps 2–10, each MUE needs to estimate the current effective interference on their allocated subchannels based on which the MUE calculates the transmission power values on the corresponding subchannels. All MUEs then send the updated transmission power values to the MBS. The MBS updates the transmission power values on all subchannels accordingly if the power constraint (24) is satisfied (see step 5). Otherwise, if the power constraint of the MBS is violated, the MBS scales down transmission power values on all subchannels by a factor $\beta^1 = \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{U}_1} a_i^n(l) p_i^{n,\min} / P_{\mathsf{BS},1}^{\max}$ to maintain the power constraint. In addition, the MBS informs all FBSs, which will find the FUE creating the largest interference to the victim MUE and increase parameter $\alpha_{m_i^*}^{n_i^*}$ by a factor of 2 (see steps 8 and 9). Here, the victim MUE i is the one that is allocated subchannels n_i^* satisfying $n_i^* = \arg\max_{n \in \mathcal{N}, c_n > 1} a_i^n(l) p_i^{n,\min}$.

The signaling in these tasks can be realized as follows. MUEs report the new transmission power values of their subchannels to the MBS via a control channel. In addition, the MBS requests nearby FBSs to report the transmission power values on subchannel n_i^*, and based on this, the MBS can identify and request the most interfering FUE m_i^* on subchannel n_i^* to update its parameter $\alpha_{m_i^*}^{n_i^*}$ by using the wired backhaul links (e.g., DSL links). In steps 15–20, each FBS can complete all required tasks in these steps by requiring its FUEs to report newly calculated transmission power values based on which it can calculate all SA weights. Note that each FBS knows the current values of scaling factor χ_i^n of the SA weights. The required signaling between FUEs and their corresponding FBSs can be accommodated by using available wireless control channels. Finally, each FBS that has its power constraint satisfied updates its transmission power (see steps 22 and 23). In addition, any FBS k that has its power constraint violated scales down transmission power values by a factor $\beta^k = \sum_{n\in\mathcal{N}}\sum_{i\in\mathcal{U}_k} a_i^n(l)p_i^{n,\min}/P_{\mathsf{BS},k}^{\max}$ and increases the $\theta_i^{n_i^*}$ parameter for the subchannel with largest transmission power by a factor of 2 (see steps 26 and 27).

B. Adaptive-Rate Resource Allocation

We can relax the fixed-rate assumption and enable FUEs in each femtocell k to choose any constellation size (modulation scheme) $s_k \in \mathfrak{M}$ for transmissions on their assigned subchannels. The target SINRs for different modulation schemes s_k-QAM $\bar{\gamma}(s_k)$ can be calculated as in (3). Hence, for a given (s_k, τ_k), the minimum spectral efficiency achieved by each FUE in femtocell k is

$$\mathrm{R}^{(k)}(s_k, \tau_k) = r_k\tau_k \qquad (28)$$

where τ_k is the number of subchannels assigned for each FUE in femtocell k, and $r_k = \log_2 s_k/N$. The resource-allocation problem for this adaptive-rate context is the same with (13) and (14) except that different femtocells k can choose different constellation sizes $s_k \in \mathfrak{M}$. Hence, we have the following SINR constraints for each FUE i connected with FBS b_i:

$$\Gamma_i^n(\mathbf{A}, \mathbf{P}) \geq \bar{\gamma}(s_{b_i}),\ \forall i \in \mathcal{U}_{\mathsf{f}},\ s_{b_i} \in \mathfrak{M}. \qquad (29)$$

The equal number of subchannels assigned to each FUE in femtocell k is at most $\lfloor N/M_k \rfloor$, and the maximum spectral efficiency of each subchannel is $\bar{r} = \log_2 s_{\mathsf{max}}/N$, where s_{max} is the maximum allowable constellation size in $\mathfrak{M}$. Hence, the maximum spectral efficiency achieved by any FUE is $\bar{r}\lfloor N/M_k \rfloor$.

Resource-allocation algorithm for this adaptive-rate setting can be performed by slightly adapting Algorithm 1, which is described in Algorithm 2. We list and sort all possible couples (s_k, τ_k) in the decreasing order of their $R^{(k)}(s_k, \tau_k) = r_k\tau_k$ to obtain a sorted list Θ_k. Then, the weight-based SAs can be performed by iteratively updating (s_k, τ_k) for all femtocells in the same order of the sorted list. In particular, if the current pair (s_k, τ_k) of any femtocell k results in violation of the condition, as specified in step 10 of Algorithm 2, then we update the pair (s_k, τ_k) by the next one in the list Θ_k (i.e., the element $(t_k + 1)$ in the sorted list Θ_k is chosen), which has lower value of $R^{(k)}(s_k, \tau_k)$. These updates are intuitive since the search space for this adaptive-rate setting comprises both the number of subchannels assigned for each FUE τ_k and the modulation scheme represented by s_k. These operations are repeated until convergence.

Algorithm 2 Adaptive-Rate Resource Allocation

1: Initialization
- Set $p_i^{(0)} = 0$ for all UE i, $i \in \mathcal{U}_{\mathsf{f}}$, feasible $\mathbf{A}_1$.
- Set $t_k = 1$ and $\varrho_k = 0$ for all $k \in \mathcal{B}_{\mathsf{f}}$.
- Set $\mu_i = 2$, $\alpha_i^n = 1, \forall i \in \mathcal{U}_{\mathsf{f}}, n \in \mathcal{N}$.

2: **For the macrocell:**
3: Update power values for MUEs and α_i^n factors as in steps 3–9 of Algorithm 1.
4: **For each femtocell** $k \in \mathcal{B}_{\mathsf{f}}$:
5: FBS k estimates $I_i^n(l)$ and calculates $p_i^{n,\min}$ for each FUE i using (20) based on the target SINR $\bar{\gamma}(s_k^{(t_k)})$.
6: **if** $\varrho_k = 1$ **then**
7: Keep $\mathbf{A}_k(l) = \mathbf{A}_k(l-1)$
8: **else if** $\varrho_k = 0$ **then**
9: Calculate SA weights $w_{i_u}^n$ between $\mathcal{N}$ and $\cup_{i\in\mathcal{U}_k}\{i_1, \ldots, i_{\tau_k^{(t_k)}}\}$, as in (22), and run Hungarian algorithm with $\{w_{i_u}^n\}$ to obtain $W_k(l)$ and $\mathbf{A}_k(l)$.
10: **if** $W_k(l) > V\sum_{i\in\mathcal{U}_k} P_i^{\max}$ **then**
11: Set $t_k := t_k + 1$, $\theta_i^n = 1, \forall i \in \mathcal{U}_k, n \in \mathcal{N}$.
12: **end if**
13: **end if**
14: Update power values for FUEs and factors as steps 21–29 of Algorithm 1.
15: Set $l := l + 1$, return to step 2 until convergence.

C. Resource Allocation With Hybrid Access

We can extend the considered problem to implement a hybrid access strategy, which can maintain the required QoS of MUEs while enhancing the performance of the femtocell tier. It can be observed that in the proposed algorithms (Algorithms 1 and 2), there may exist femtocells that do not utilize all available subchannels since doing so may prevent them from maintaining the target SINRs for all FUEs and MUEs. Therefore, it may be beneficial if we allow MUEs that potentially create and/or suffer from strong cross-tier interference from nearby femtocells to change their connections from the MBS to nearby FBS. However, this can only be performed if the subchannels assigned to the underlying MUE are not utilized by the target femtocell.

We present the resource-allocation algorithm with rate adaptation under such hybrid access in Algorithm 3. To interpret the operations of this algorithm, we first give definitions of involved quantities and parameters in the following. Let $b_i(l)$ and $\mathcal{N}_i(l)$ be the BS of UE i and the set of its assigned subchannels in iteration l, respectively. Note that we have assumed that each MUE has a fixed set of assigned subchannels and FUEs have fixed associated FBSs. Therefore, we have $b_i(l) = b_i$ for each FUE i and $\mathcal{N}_i(l) = \mathcal{N}_i$ for each MUE i as specified by $\mathbf{A}_1$. In addition, we can estimate the total required transmission power over the assigned subchannels of MUE i that is connected with BS k as

$$P_{i,k} = \bar{\gamma}(s^{\mathsf{m}}) \sum_{n \in \mathcal{N}_i} I_{i,k}^n(l) \tag{30}$$

where $I_{i,k}^n(l)$ is the effective interference corresponding to MUE i for its connection with BS k on subchannel n, which can be calculated as in (8). MUE i can determine $P_{i,k}$ for each potential FBS k by estimating $I_{i,k}^n(l)$ on each assigned subchannel n. This can be realized as we have discussed in the paragraph below (20). For the SA in any iteration l, we define the set of potential FBSs for MUE i as follows:

$$\mathcal{B}_i(l) = \{k | k \in \mathcal{B}_{\mathsf{f}}, \mathcal{N}_i \subseteq \mathcal{N} / \cup_{j \in \mathcal{U}_k} \mathcal{N}_j(l-1) \\ P_{i,k} < \min(P_{i,1}, P_i^{\max}), |\mathcal{N}_i| \leq Q_k\} \tag{31}$$

where Q_k is the number of unused subchannels at FBS k in the underlying iteration. Here, $\mathcal{B}_i(l)$ is the set of FBSs whose set of unused subchannels contains the set $\mathcal{N}_i$ of MUE i; connection between MUE i and the underlying FBS requires less power than connection between MUE i and the MBS. We also define the set $\mathcal{U}_{m,k}(l) = \{i \in \mathcal{U}_{\mathsf{m}} | b_i(l) = k\}$, which is the set of MUEs connecting with BS k in iteration l.

Algorithm 3 Adaptive-Rate Resource Allocation With Hybrid Access

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1: Initialization: Set parameters as step 1 of Algorithm 2 and $Q_k = 0, \forall k \in \mathcal{B}_{\mathsf{f}}$.
2: **For the macrocell:**
3: Estimate $P_{i,k}$, update $\mathcal{B}_i(l)$, set $\mathcal{B} = \cup_{i \in \mathcal{U}_{m,1}(l)} \mathcal{B}_i(l)$.
4: **if** $\mathcal{B} = \varnothing$ **then**
5: Go to step 10.
6: **else if** $\mathcal{B} \neq \varnothing$ **then**
7: Find $(k^\star, i^\star) = \arg\min_{i \in \mathcal{U}_{m,1}(l)} \min_{k \in \mathcal{B}_i} P_{i,k}$.
8: Set $b_{i^\star}(l) = k^\star$, $Q_{k^\star} = Q_{k^\star} - |\mathcal{N}_{i^\star}|$, $\varrho_{k^\star} = 0$, and go back to step 3.
9: **end if**
10: Update power values and all parameters as in steps 3–10 of Algorithm 1.
11: **For each femtocell** $k \in \mathcal{B}_{\mathsf{f}}$:
12: Update the subchannel, PA, and other parameters for all FUEs in cell k as steps 5–14 of Algorithm 2 where the Hungarian algorithm is run for the set of subchannels $\mathcal{N} / \cup_{i \in \mathcal{U}_{m,k}(l)} \mathcal{N}_i$ and the set of virtual UEs $\cup_{i \in \mathcal{U}_k} \{i_1, \ldots, i_{\tau_k^{(t_k)}}\}$.
13: **if** t_k changes **then**
14: Set $Q_k = N - \tau_k^{(t_k)} M_k$, $b_i(l+1) = 1, \forall i \in \mathcal{U}_{m,k}(l)$.
15: **else if** t_k does not change **then**
16: Set $Q_k = N - \sum_{i, b_i(l)=k} |\mathcal{N}_i(l)|$, $b_i(l+1) = k, \forall i \in \mathcal{U}_{m,k}(l)$.
17: **end if**
18: Let $l := l + 1$, return to step 2 until convergence.

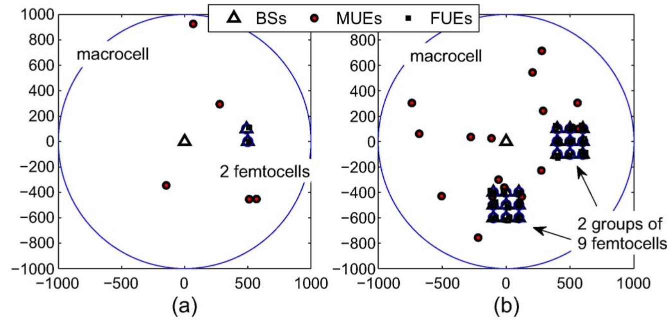

Fig. 1. Macrocell–femtocell networks used in the simulation.

In Algorithm 3, we do not change existing associations between any FBS k and its corresponding MUEs until the chosen pair (s_k, τ_k) in this femtocell is updated (i.e., the next pair (s_k, τ_k) in the sorted list Θ_k is chosen). Specifically, if t_k is updated (i.e., in step 13), then all MUEs currently connected with FBS k are forced to change their connections back to the MBS (see step 14); otherwise, we maintain the BS associations of all MUEs as they are (see steps 15 and 16). In addition, only MUEs currently connecting with the MBS (i.e., any MUE i with $b_i(l)$ equal to 1) are allowed to change their BS association in iteration l. In particular, the couple of FBS and MUE requiring the smallest transmission power will be chosen for association in each iteration (see step 7). Other operations corresponding to SA, PA, and updates of various parameters are performed as in Algorithm 2.

Algorithm 3 can be implemented as follows. To complete the tasks in steps 3–9, each MUE i upon estimating $P_{i,k}$ for all potential FBSs $k \in \mathcal{B}_i$ will send a connection request to the FBS that requires the minimum power (i.e., $\min_{k \in \mathcal{B}_i} P_{i,k}$) together with the value of $P_{i,k}$. If a particular FBS k receives several connection requests, then the FBS will grant the connection to only one MUE with the smallest value of $P_{i,k}$. Signaling information needed to exchange the connection request and granting messages can be accommodated by available control channels. In steps 13 and 14, if the next pair (s_k, τ_k) with index t_k in the sorted list Θ_k in femtocell k is chosen (i.e., t_k is increased by one), then FBS k simply requests all associated MUEs to change their connections back to the MBS. Otherwise, if t_k remains unchanged, then FBS k simply updates parameter Q_k based on the new SA assignment decisions in step 12.

V. Numerical Results

We obtain numerical results for two different networks (with a small and large number of femtocells and UEs) to evaluate the efficacy of our proposed algorithms. The network setting and UE placement for our simulations are shown in Fig. 1, where MUEs and FUEs are randomly located inside circles of radii of $r_1 = 1000$ m and $r_2 = 30$ m, respectively. The power channel gains h_{ij}^n are generated by considering both Rayleigh fading, which is represented by an exponentially distributed random variable with the mean value of one, and the path loss $L_{ij} = A_i \log_{10}(d_{ij}) + B_i + C \log_{10}(f_c/5) + WL \times n_{ij}$, where d_{ij} is the distance from UE j to BS i; (A_i, B_i) are set as (36, 40) and (25, 45) for MBS and FBSs, respectively;

TABLE II
TARGET SINRs FOR DIFFERENT CONSTELLATION SIZES WITH TARGET BER $\overline{P}_e = 10^{-3}$

s-QAM	4	16	64	256	1024
bits/symbol	2	4	6	8	10
$\bar{\gamma}(s)$	9.55	45.11	179.85	694.17	2667.32

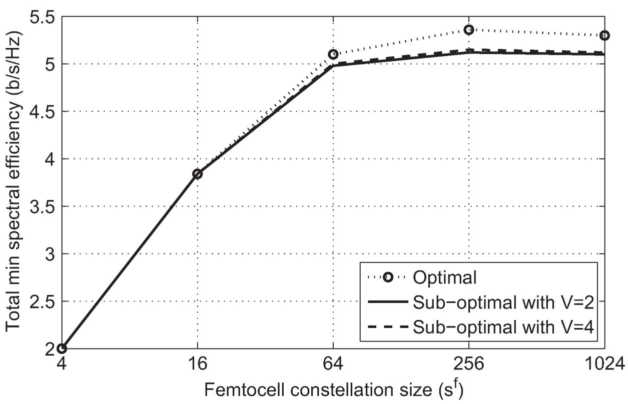

Fig. 2. Total minimum spectral efficiency for small network under optimal and suboptimal algorithms for $s^{\mathrm{m}} = 4$, $WL = 5$ dB, and $P_{\mathrm{m}}^{\max} = P_{\mathrm{f}}^{\max} = 0.01$ W.

$C = 20$; $f_c = 2.5$ GHz; WL is the wall-loss parameter; and n_{ij} is the number of walls between BS i and UE j. This path-loss model is chosen according to the general form of the path-loss formula in [24], which is suggested by the WINNER II channel modeling project. The noise power is set as $\eta_i = 10^{-13}$ W, $\forall i \in \mathcal{B}$.

To obtain simulation results, we use two modulation schemes (4- and 16-QAM) for MUEs and five modulation schemes (4-, 16-, 64-, 256-, and 1024-QAM) for FUEs. Moreover, we choose the target BER $\overline{P}_e = 10^{-3}$ whose target SINRs corresponding to different modulation schemes, which can be calculated by using (5), are given in Table II. Each simulation result is obtained by taking the average over 20 different runs where, for each run, UEs are randomly located and a feasible SA matrix for MUEs $\mathbf{A}_1$ is chosen so that each MUE is assigned N/M_1 subchannels.

A. Performance of Proposed Algorithms

In Fig. 2, we show the total minimum spectral efficiency of all femtocells [i.e., the optimal objective value of (13)] versus the constellation size of FUEs (s^{f}) for the small network due to both optimal and suboptimal algorithms (see Algorithm 1), which are presented in Section III. As shown, for the low constellation sizes (i.e., low target SINRs), Algorithm 1 can achieve almost the same spectral efficiency as the optimal algorithm, whereas for higher values of s^{f}, Algorithm 1 results in just slightly lower spectral efficiency than that due to the optimal algorithm. Moreover, when we increase the value of parameter V, a slightly better performance can be achieved. We then plot the minimum number of subchannels assigned for FUEs and the average SINRs over assigned subchannels versus iteration index for the small network in Fig. 3. This figure shows the convergence of Algorithm 1 in terms of both user SINR and number of assigned subchannels per FUE.

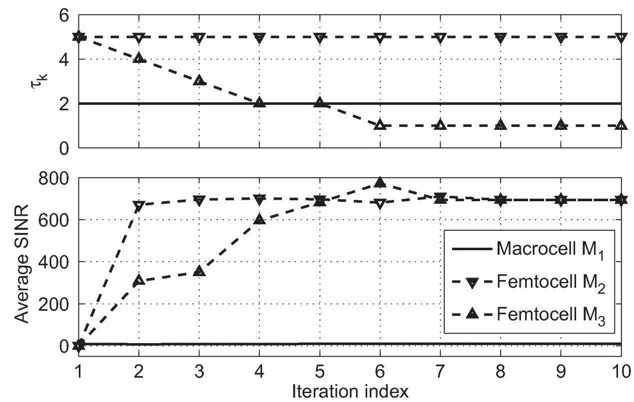

Fig. 3. Minimum number of subchannels assigned for FUEs and average SINRs versus iteration index where $s^{\mathrm{m}} = 4$, $s^{\mathrm{f}} = 256$, $WL = 5$ dB, and $P_{\mathrm{m}}^{\max} = P_{\mathrm{f}}^{\max} = 0.01$ W.

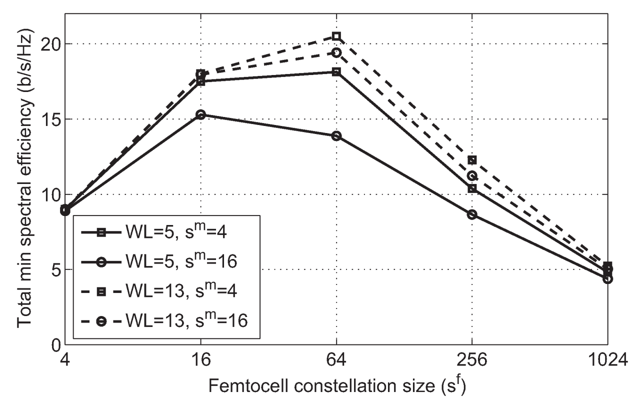

Fig. 4. Total minimum spectral efficiency versus QAM constellation size of femtocells with $P_{\mathrm{m}}^{\max} = P_{\mathrm{f}}^{\max} = 0.01$ W.

Fig. 4 shows the total femtocell minimum spectral efficiency versus the QAM constellation size, which is obtained by running Algorithm 1 for the large network. This figure shows that the total minimum spectral efficiency increases and then decreases as femtocell constellation size increases; it decreases as the constellation size of MUEs increases. These results can be interpreted as follows. Higher constellation sizes have higher target SINRs, which require higher transmission power and produce more interference to other users in the network. Therefore, the power constraints of FUEs and MUEs are more likely to be violated for higher constellation sizes (e.g., 254 or 1024), which limits the number of subchannels allocated for each FUE. Moreover, for low modulation levels (therefore, low target SINRs of FUEs), the spectral efficiency on each assigned subchannel increases with the increasing modulation level, whereas the number of subchannels assigned for each FUE does not decrease too much. Moreover, the total minimum spectral efficiency of femtocells increases with the increasing wall-loss value WL. This is because the higher wall loss reduces both co-tier and cross-tier interference to users of both tiers.

In Figs. 5 and 6, we plot the total femtocell minimum spectral efficiency versus the maximum power of FUEs $(P_{\mathrm{f}}^{\max})$ and MUEs $(P_{\mathrm{m}}^{\max})$, respectively, for different modulation levels of MUEs (the modulation scheme of FUEs is 256-QAM).

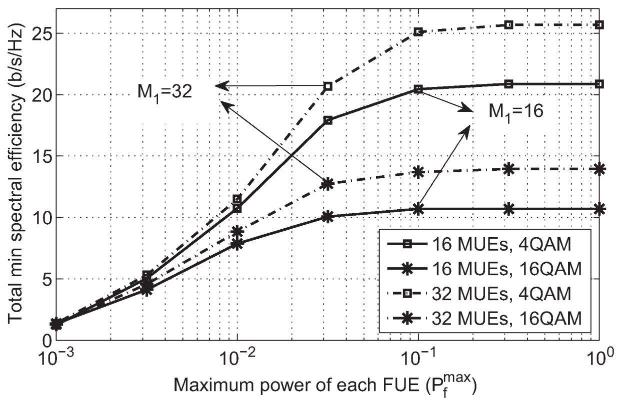

Fig. 5. Total minimum spectral efficiency versus $P_{\mathrm{f}}^{\max}$ for the large network with $s^{\mathrm{f}} = 256$, $WL = 5$ dB, and $P_{\mathrm{m}}^{\max} = 0.01$ W.

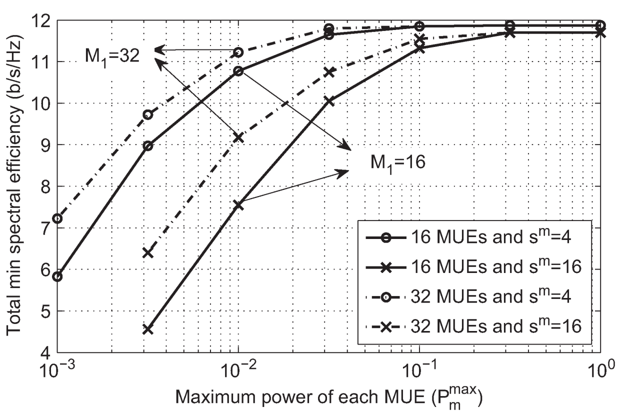

Fig. 6. Total minimum spectral efficiency versus $P_{\mathrm{m}}^{\max}$ for the large network with $s^{\mathrm{f}} = 256$, $WL = 5$ dB, and $P_{\mathrm{f}}^{\max} = 0.01$ W.

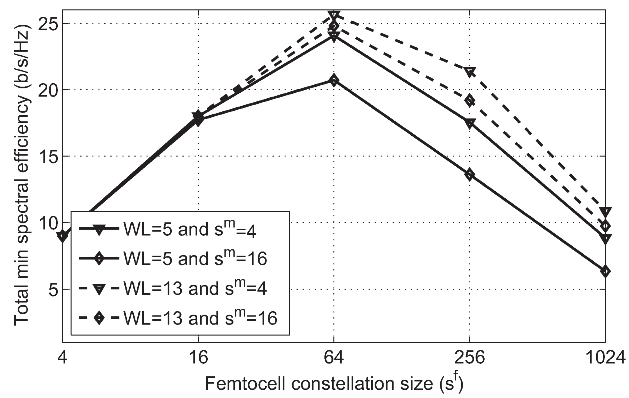

Fig. 7. Total minimum spectral efficiency of femtocells in the downlink with $WL = 5$ dB, $P_{\mathrm{BS},m}^{\max} = 0.2$ W, and $P_{\mathrm{BS},f}^{\max} = 0.05$ W.

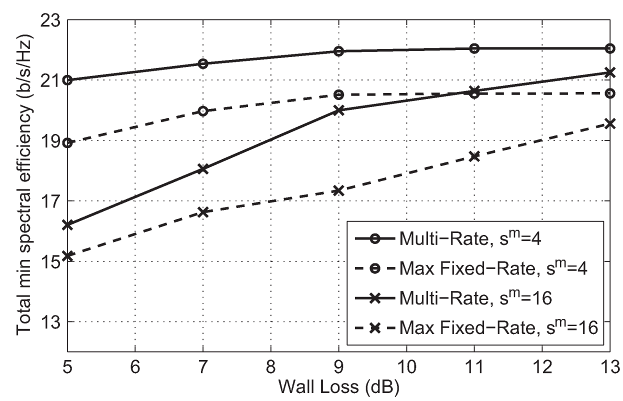

Fig. 8. Total minimum spectral efficiency versus wall loss for Algorithm 2 with $P_{\mathrm{m}}^{\max} = P_{\mathrm{f}}^{\max} = 0.01$ W.

These figures show that the total minimum spectral efficiency increases with the increase in maximum power budgets $P_{\mathrm{f}}^{\max}$ or $P_{\mathrm{m}}^{\max}$. However, this value is saturated as the maximum power budgets $P_{\mathrm{f}}^{\max}$ or $P_{\mathrm{m}}^{\max}$ become sufficiently large. In addition, as the number of MUEs increases, the total femtocell minimum spectral efficiency increases due to the better diversity gain offered by the macro tier.

In Fig. 7, we show the total minimum spectral efficiency of femtocells versus the QAM constellation sizes for the downlink by running the resource-allocation algorithm with the power constraints (24) and SA weights (27). This figure shows that the proposed algorithm works well for the downlink system where these results are similar to those for the uplink case presented in Fig. 4.

Fig. 8 presents the total minimum spectral efficiency of femtocells versus the wall-loss parameter WL achieved by the fixed-rate algorithm (i.e., Algorithm 1) and the adaptive-rate algorithm (i.e., Algorithm 2) whose results are indicated by "Max-Fixed Rate" and "Multi-Rate" in this figure, respectively. In Algorithm 2, each femtocell can choose one best modulation scheme among five schemes $(s^{\mathrm{f}} = 4, 16, 64, 256, \mathrm{and} 1024)$, whereas Algorithm 1 employs the maximum modulation level for all femtocells. This figure demonstrates that the great performance gain can be achieved by exploiting the adaptive-rate feature in performing resource allocation for FUEs.

In Fig. 9, we show the total minimum spectral efficiency of all femtocells versus $P_{\mathrm{f}}^{\max}$ under the hybrid and closed-access strategies, which are obtained by Algorithms 3 and 2, respectively. These results correspond to the large network with 32 MUEs and $WL = 5$ dB. As shown, the hybrid access strategy achieves higher performance than that under closed-access scheme. Moreover, the performance gap between two strategies becomes larger with increasing maximum power budget of FUEs. Under the closed access, the total spectral efficiency of femtocells is indeed limited by MUEs that are located far away from the MBS and close to some FBSs. The hybrid access strategy enables these victim MUEs to connect with nearby FBSs, which mitigates this problem and increases the spectral efficiency of the femto tier.

B. Performance Comparison With Zhang's Algorithm [27]

Performance comparison between our proposed algorithm and that in [27] is presented in the following. To obtain the simulation results in Fig. 10, the spectral efficiency of each FUE is obtained by running Zhang's algorithm and calculating the spectral efficiency by using the fixed-rate formula (10). In Fig. 10, we show the total spectral efficiency of all FUEs in all femtocells due to Zhang's algorithm [27] and Algorithm 1 as we vary the QAM constellation size of each FUE s^{f}. This figure shows that, for a fixed constellation size employed

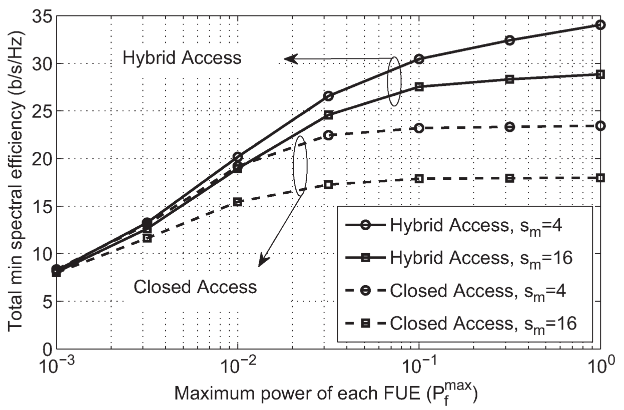

Fig. 9. Total minimum spectral efficiency versus $P_{\mathrm{f}}^{\max}$ under the adaptive-rate and hybrid access strategies for $P_{\mathrm{m}}^{\max} = 0.01$ W, $M_1 = 32$, and $WL = 5$ dB.

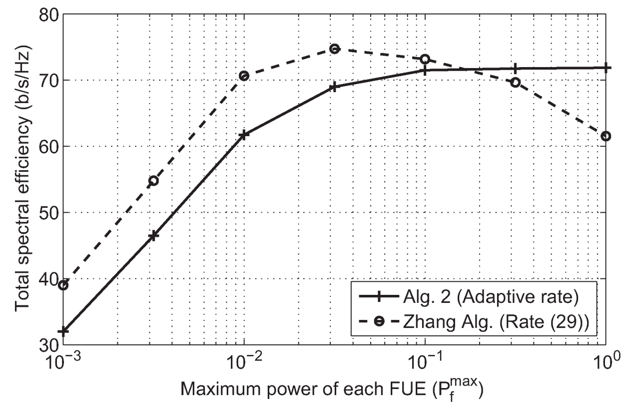

Fig. 11. Total spectral efficiency versus maximum power of each FUE with $P_{\mathrm{m}}^{\max} = 0.01$ W.

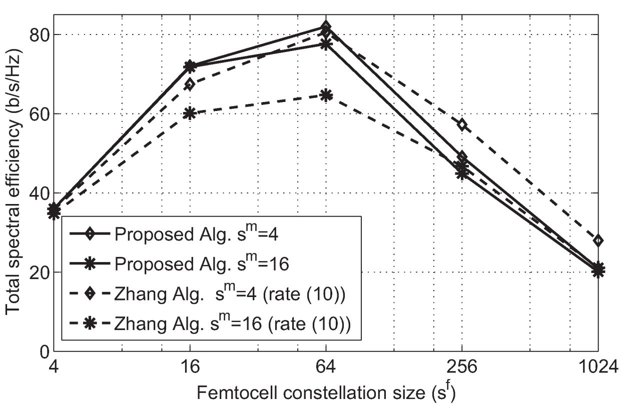

Fig. 10. Total spectral efficiency versus QAM constellation size of FUEs with $P_{\mathrm{m}}^{\max} = P_{\mathrm{f}}^{\max} = 0.01$ W.

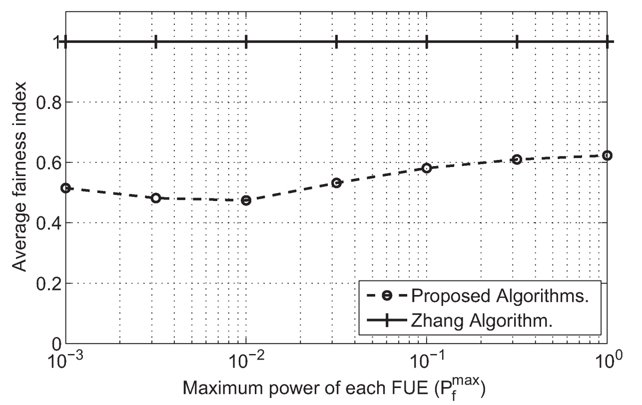

Fig. 12. Fairness index versus maximum power of each FUE with $P_{\mathrm{m}}^{\max} = 0.01$ W.

by MUEs, the total spectral efficiency achieved by Zhang's algorithm is lower than ours for low values of s^{f}, whereas Zhang's algorithm results in slightly higher spectral efficiency than our algorithm for larger values of s^{f}. The slightly better performance of Zhang's algorithm comes at the cost of severely unfair resource sharing among FUEs, as we will show in Fig. 12. It is worth recalling that Zhang *et al.* aim to maximize the total throughput without considering user fairness, while our formulation focuses on achieving an equal rate for FUEs in each femtocell (i.e., max–min fairness).

Fig. 11 shows the total spectral efficiency of Zhang's algorithm and our proposed algorithms with rate adaptation (i.e., Algorithm 2) as we vary the maximum power of each FUE $P_{\mathrm{f}}^{\max}$. Here, the spectral efficiency of FUEs under Zhang's algorithm is calculated by using the adaptive-rate formula (29). As evident, the total spectral efficiency due to Zhang's algorithm is higher than ours for low values of $P_{\mathrm{f}}^{\max}$, but Zhang's algorithm performs worse compared with our algorithm if $P_{\mathrm{f}}^{\max}$ becomes sufficiently large. The superior performance of Zhang's algorithm in the low-power regime is again due to its unfair resource-sharing nature. In fact, Zhang's algorithm does not account for the co-tier interference among femtocells, which explains why it performs worse compared with our algorithm in the high-power regime. This is because the transmission power values of FUEs increase with the larger power budget, which results in higher femtocell co-tier interference. The high co-tier interference degrades the performance of Zhang's algorithm since this type of interference is not managed in their work.

To compare the fairness of Zhang's and our proposed algorithms, we present the average fairness index achieved by FUEs in each femtocell, which is calculated as [26]

$$FI_k = \left(\sum_{i=1}^{M_k} R_i \right)^2 \Bigg/ M_k \left(\sum_{i=1}^{M_k} R_i^2 \right). \qquad (32)$$

This fairness index is widely employed in the literature to evaluate the level of fairness achieved by resource-allocation algorithms where an algorithm is fairer if its fairness index is higher and close to the maximum value of one and *vice versa*. Fig. 12 shows the average fairness index achieved by Zhang's and our proposed algorithms. As is evident, our algorithm provides maximum fairness for FUEs in each femtocell (since the rates of FUEs in each femtocell are equal), whereas the average fairness index of Zhang's algorithm is around 0.5–0.6, which is quite low. This implies that Zhang's algorithm may result in big differences in the rates achieved FUEs in each femtocell, which is not very desirable. Considering that the total spectral efficiency due to Zhang's and our proposed algorithms

TABLE III
SUMMARY OF KEY NOTATIONS

Notations	Description
a_i^n	Subchannel assignment variable for subchannel n and UE i
$\mathbf{A}$	Subchannel assignment matrix for all M UEs over N subchannels
$\mathbf{A}_k$	Subchannel assignment matrix for UEs in cell k over N subchannels
A_i, B_i, C	Coefficients in path loss formula
b_i	Base station serving UE i
BW	Total bandwidth of all N subchannels
$\mathcal{B}$	Set of all base stations
$\mathcal{B}_\mathsf{f}$	Set of all FBSs
$\mathcal{B}_i$	Set of potential FBSs for MUE i
$c^n = \lvert\mathcal{U}^n\rvert$	Number of elements in set $\mathcal{U}^n$
d_{ij}	Distance from UE j to BS i
f_c	Frequency
$f(\gamma(s))$	Bit error rate (BER) function for SINR $\gamma(s)$ as the constellation size s is adopted
h_{ij}^n	power channel gain from user j to BS i over subchannel n
$I_{i,k}^n$	Effective interference corresponding to MUE i for its connection with BS k on subchannel n
$I_i^n(\mathbf{A},\mathbf{P})$	Effective interference corresponding to UE i on subchannel n
K	Number of cells
L_{ij}	Path loss
M_k	Number of UEs served by base station k
$M_\mathsf{m}, M_\mathsf{f}$	Numbers of MUEs and FUEs, respectively
$\mathfrak{M}$	Set of constellation sizes
$\mathcal{N}$	Set of all subchannels
$\mathcal{N}_i$	Set of subchannels assigned to UE i
N	Number of subchannels
n_{ij}	Number of walls between BS i and UE j
$p_i^{n,\mathtt{min}}$	Required transmission power of UE i over its assigned subchannel n
p_i^n	Transmission power of UE i over subchannel n
$P_{i,k}$	Estimated required power for MUE i connecting with BS k
P_i^{max}	Maximum transmission power of UE i
$P_{\mathsf{BS},k}^{\mathsf{max}}$	Maximum power of BS k in the downlink
$\mathbf{P}$	Power allocation matrix whose elements are p_i^n
$\mathbf{P}_k$	Power allocation matrix for UEs over N subchannels in cell k
$\overline{P}_e$	Target BER
Q_k	Number of unused subchannels at FBS k
$\mathbb{Q}(.)$	Q function
$r_i^n(\mathbf{A},\mathbf{P})$	Spectral efficiency (bits/s/Hz) achieved by FUE i on subchannel n
$R_i(\mathbf{A},\mathbf{P})$	Spectral efficiency (bits/s/Hz) achieved by FUE i
s	Constellation size corresponding to the s-QAM modulation scheme
s^k	Constellation size of the MQAM modulation utilized in cell k
s_{max}	Maximum allowable constellation size in the set $\mathfrak{M}$
$s^\mathsf{m}, s^\mathsf{f}$	Constellation size of the MQAM modulation employed in macrocell and femtocells, respectively
$\mathcal{U}_k$	Set of all UEs served by base station k
$\mathcal{U}_{m,k}$	Set of MUEs connecting with BS k
$\mathcal{U}, \mathcal{U}_\mathsf{m}, \mathcal{U}_\mathsf{f}$	Set of all UEs, MUEs and FUEs, respectively
$\mathcal{U}^n$	Set of UEs in both tiers that are assigned subchannel n
w_i^n	Assignment weight of UE i over subchannel n
W_k	Total minimum weight of subchannel assignments in cell k
WL	Wall-loss parameter
$\alpha_i^n, \theta_i^n, \delta_i^n, \delta_i^n$	Other factors for calculating scaling factor χ_i^n
$\bar{\gamma}(s)$	Target SINR corresponding to constellation size s
$\bar{\gamma}_i^n$	Target SINR of UE i over subchannel n
$\Gamma_i^n(\mathbf{A},\mathbf{P})$	SINR of UE i over subchannel n
η_i^n	Noise power at BS i on subchannel n
τ_k	Number of subchannels assigned for each UE in cell k
χ_i^n	Scaling factor in SA weight
$\Omega\{\mathbf{A}\}$	List of all potential SA solutions
$\Omega^*\{\mathbf{A}\}$	Sorted list of $\Omega\{\mathbf{A}\}$ in decreasing order of $\sum_{k=2}^K \tau_k$

is not much different, whereas Zhang's algorithm is highly unfair, we would conclude that our proposed algorithms better balance between the throughput and fairness compared with Zhang's algorithm.

VI. Conclusion

We have proposed centralized optimal and distributed resource-allocation algorithms that maximize the total minimum spectral efficiency of the femtocell network while ensuring fairness among FUEs and QoS protection for all MUEs. Moreover, the proposed algorithm has been extended for three scenarios, namely, downlink context, adaptive-rate systems where FUEs in each femtocell can adaptively choose one in a predetermined set of modulation schemes, and hybrid access design where MUEs can associate with nearby FBSs to improve the performance of the femto tier. Extensive numerical results have been presented to demonstrate the impacts of different system parameters on the network performance and the efficacy of our proposed resource-allocation algorithms.

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Vu Nguyen Ha (S'11) received the B.Eng. degree from the French Training Program for Excellent Engineers in Vietnam, Ho Chi Minh City University of Technology, Ho Chi Minh City, Vietnam, and an addendum degree from the École Nationale Supérieure des Télécommunications de Bretagne, Bretagne, France, in 2007. He is currently working toward the Ph.D. degree with the Institut National de la Recherche Scientifique-Énergie, Matériaux et Télécommunications, Université du Québec, Montréal, QC, Canada.

From 2008 to 2011, he was a Research Assistant with the School of Electrical Engineering, University of Ulsan, Ulsan, Korea. His research interest includes radio resource management for wireless networks with special emphasis on heterogeneous networks.

Long Bao Le (S'04–M'07–SM'12) received the B.Eng. (with highest distinction) degree from Ho Chi Minh City University of Technology, Ho Chi Minh City, Vietnam, in 1999; the M.Eng. degree from the Asian Institute of Technology, Pathumthani, Thailand, in 2002; and the Ph.D. degree from the University of Manitoba, Winnipeg, MB, Canada, in 2007.

From 2008 to 2010, he was a Postdoctoral Research Associate with Massachusetts Institute of Technology, Cambridge, MA, USA. Since 2010, he has been an Assistant Professor with the Institut National de la Recherche Scientifique-Énergie, Matériaux et Télécommunications, Université du Québec, Montréal, QC, Canada, where he leads a research group working on cognitive radio and dynamic spectrum sharing, radio resource management, network control, and optimization.