Allan Pierce and adiabatic normal modes

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JASA REFLECTIONS

The *Reflections* series takes a look back on historical articles from *The Journal of the Acoustical Society of America* that have had a significant impact on the science and practice of acoustics.

Allan Pierce and adiabatic normal modes

Article: Extension of the method of normal modes to sound propagation in an almost-stratified medium
Author: Allan D. Pierce
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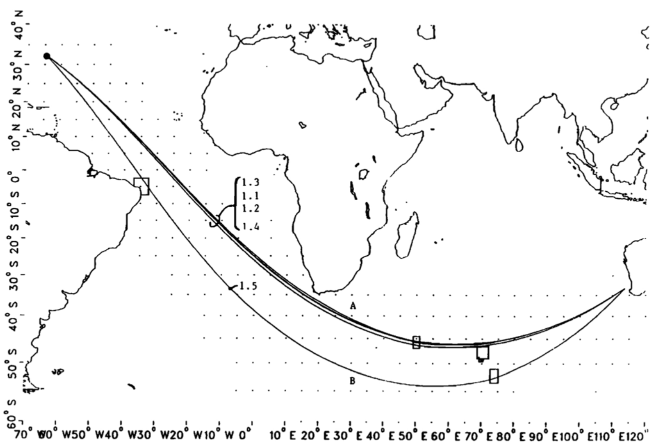
ARTICLE OVERVIEW

One of the beautiful things about science is that many of its main equations and concepts appear in a multitude of different fields. Change the medium and the boundary conditions, make a change in variables, and you have gone from solid state physics to aeroacoustics or from nuclear physics to underwater acoustics, but within the same formal, theoretical framework. Rethink the Born–Oppenheimer approximation of molecular physics and you have the adiabatic approximation to coupled normal mode theory. This interplay is one of the hallmarks of Pierce's 1965 paper.[1]

Interestingly, while the paper was primarily addressing the shallow-water acoustics problem, this was not the original inspiration, but rather a whole different problem—monitoring atmospheric nuclear tests. Widespread nuclear testing by the United States and the USSR followed World War II and led to so many above-ground nuclear tests in Nevada that Las Vegas was nicknamed "Atomic City."[2] The times were hyper-charged with the possibility of a nuclear exchange, especially following the Cuban missile crisis.



Mode one paths from Perth to Bermuda calculated with adiabatic mode theory. Reprinted with permission from Heaney et al., "Perth–Bermuda sound propagation (1960): Adiabatic mode interpretation," J. Acoust. Soc. Am. 90, 2586–2594 (1991).

Estimating the yield of nuclear tests was part of the defense research being done then (and now). In the early days, physicists like Enrico Fermi used remarkably simple methods of yield estimation,[3] but in 1965, more sophisticated means were needed to deal with larger and more remote tests. At this point, Allan Pierce enters the story. Both infrasound from nuclear tests and low frequency, shallow-water acoustic propagation, important at the time to anti-submarine warfare, are amenable to treatment by the method of coupled normal modes, being low frequency propagation in inhomogeneous, anisotropic waveguides. Allan's new work covered both of them.

In both the ocean and atmosphere, the first order description of the low frequency acoustic field is via propagation in a uniform, horizontally stratified waveguide. In underwater acoustics, this formalism was originally described using normal modes by Chaim Pekeris.[4] With the clear realization that neither the ocean nor the atmosphere is homogeneous and isotropic, the modal method was soon extended to a coupled normal mode formalism, shown in Eqs. (1) and (2):

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_n^2\right)\varphi_n=-\sum_m\left(A_{mn}+B_{mn}\frac{\partial}{\partial x}+C_{mn}\frac{\partial}{\partial y}\right)\varphi_m \tag{1}$$

$$\left(\frac{d^2}{dz^2}+\frac{\omega^2}{c^2}\right)Z_n=k_n^2Z_n, \tag{2}$$

where the φ_n are the coupled mode coefficients; the A, B, and C are mode coupling matrices that depend on the range and azimuthal variability of the medium; and the Z_n are the local vertical mode functions.

The coupled mode equations are significantly harder to solve as compared to the uncoupled case. The simplest thing one might think of doing to approximate the coupled mode equation solutions is to just turn off the mode coupling, i.e., set the right-hand side to zero. As long as the medium properties vary slowly with horizontal distance, this should work. Interestingly, this approach does not eliminate range dependent wave physics, because in formulating the coupled mode equations, we assumed that the sound speed depends on x and y, as well as z. The boundary conditions can also depend on x and y, such as variable topography.

The solution of the resulting "adiabatic" equation (each mode conserves its own energy), including horizontal refraction, is done in simple stages. First, one solves Eq. (2) on an *x*,*y* grid, usually near the straight line path (or great circle) connecting the source and receiver. These solutions give the vertical modes $Z_n(x, y, z)$ and the horizontal wavenumbers $k_n(x, y)$. An effective modal horizontal index of refraction is defined using $c_n(x, y) = \omega k_n(x, y)$, and one uses standard ray tracing techniques to get the modal paths via Eq. (1). The phase is simply the integral of $k_n(x, y)$ along the paths, and the amplitude of each mode is given by the product of mode function values at the source and receiver, which avoids solving the full coupled equations.

IMPACT OF THE ARTICLE

Pierce's paper was a landmark contribution and in ocean acoustics, the adiabatic approximation, sans horizontal refraction, became widely used, even as Weinberg and Burridge showed a more complete and detailed way to deal with refraction.[5] Only in recent years have 3D effects have been included more routinely, with one striking example being global modal propagation paths from Perth, Australia to Bermuda (Fig. 1).[6] Another recent example is the horizontal ducting of energy by nonlinear internal waves on continental shelves.[7,8]

In aeroacoustics, the adiabatic approximation allowed the calculation of modal pulse travel times, including horizontal refraction, which is important in that problem. This, in turn, allowed an estimation of the explosive yields of nuclear tests.[9]

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