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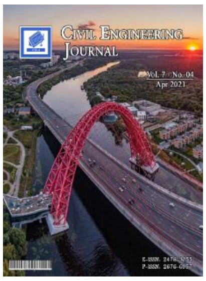

Determination of Reinforced Concrete Rectangular Sections Having Plastic Moments Equal to all IPE Profiles

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Abstract

The comparison between steel structures and reinforced concrete structures has always been governed by economy and response to earthquake. Steel structures being lighter and are thus more efficient to resist earthquake. On the other hand, they are more expensive (4 to 5 times). Theoretically, two structural elements having the same plastic moment have an equal failure or collapse load. Different profiles of IPE are realized in industry and all their characteristics are determined with a great precision (weight, geometrical characteristics and thus their plastic moment). Determining equivalent rectangular singly reinforced concrete cross-sections is not easy and seems impossible to be solved analytically. To a given profile it may be found a multitude of equivalent rectangular reinforced concrete cross-section (singly and doubly reinforced with different yield strengths and compositions of concrete). To take into consideration all these factors, it is absolutely necessary to construct three axis design charts with an appropriate choice of system of coordinates in order to cover all possible ranges of different parameters. The choice of all these possible rectangular reinforced concrete sections is governed by the plastic performance of these later. They must be under reinforced, allowing plastification of steel before failure in order to permit the redistribution phenomenon in plastic analysis. The exploitation of these different charts has revealed that the absolute majority of these rectangular reinforced concrete cross-section are reasonably well designed and are in conformity with the dimensions used in practice. The results of the present characterization using Eurocode 2 characteristics are compared to those of CP110. The impact does not seem to be very relevant.

Keywords: Reinforced Concrete; IPE Profile; Equivalent Sections; Plastic Moments; Reduced Moments; Reinforcement Ratio.

1. Introduction

The comparison between steel and reinforced concrete structures has been the subject of several studies which have generally focused on the economic aspect and seismic behaviour as well as the durability and strength of each material [1, 2]. Steel structures being lighter and thus more efficient to resist to earthquakes, but on the other hand they are more expensive (4 to 5 times).

All structural elements made of steel or reinforced concrete follow the same load transfer law (material strength), but they differ in several aspects, such as the material itself and its behaviour, plastic bending resistance capacity (loads and failure modes), stability and durability, etc. However, each type of structure has advantages and disadvantages. The dimensioning of the cross-sections of these structural elements is directly affected by the safety margin adopted by the different codes, and therefore the change of code has been the subject of several comparative

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studies [3, 4].

Knowledge of the real plastic capacity in bending of structural elements and identification of their collapse loads and failure modes play an important role in rational dimensioning [5, 6]. Ashrafzadeh and Kheyrolahi [7], studied the behaviour of folding steel structures by a durability method, where structural performance under seismic excitation was among the main objectives of their research. Al-Ansari and Afzal [8], presented a simple method for estimating the flexural design strength of reinforced concrete beam sections with irregular shapes. The method was based on structural safety and reliability.

Experiments for relevant loading rates and pressures reveal that steel and concrete exhibit complex non-linear behaviour that is difficult to capture in a single constitutive model [9]. The flexural capacity and ductility of reinforced concrete beams with several usual and non-usual geometrical shapes have been estimated theoretically and experimentally by several authors such as Yang et al. [10] and Nogueira and Rodrigues [11].

Previous characterisation has been carried out by Boussafel [12], using the design charts published in both codes: CP110-2:1972 [13] and BS: 8110-3:1985 [14]. The essential parameters taken into account are: f_{yk} = 410 MPa for the characteristic yield strength of reinforcement and three values for the characteristic cube strength of the concrete (f_{cu}) namely 25, 30 and 40 MPa and a neutral axis depth fixed at x = 0.50 d, in order to exploit to the maximum the concrete in compression. Characterisations of the equivalent sections in reinforced concrete and reinforced sand concrete have also been established using the material characteristics adopted by Eurocodes 2 and 3 [15, 16].

A characterisation of equivalent reinforced concrete rectangular sections using the characteristics adopted by two codes: CP110-2:1972 and BAEL Rules [17] was established by Boutlitht [18], this characterisation was associated with tests on the pure bending of three IPE beams (IPE 200, IPE 220 and IPE 240). The phenomenon of elastic instability (buckling) of the tested beams was strongly observed. In the majority of the test results, the experimental collapse load was higher than the theoretical collapse load. The rise between experimental and theoretical load varied between 17 % and 39 % for the series of profiles tested [19].

In general, analysis methods often consider reinforced concrete or steel structural elements as linear elastic elements. This assumption, acceptable for the serviceability limit state, is not valid for the ultimate limit state, which is generally characterised by significant cracking and plastification of certain parts of the structure. Consequently, considerable redistribution of forces in the structure and stresses in some elements are probable, which may have large influences on the overall behaviour of the structure in the ultimate limit state. Taking into account the plastic behaviour of the structure seems, therefore, essential to adequately describe and characterize the ultimate limit state of this structure [20, 21].

Two structural elements with the same plastic moment (M_p) subjected to the same loading and with the same support conditions have the same theoretical failure load, in other words they have the same bending plastic capacity [22]. The present paper presents an approach to determine simply and doubly reinforced rectangular concrete sections equivalent to the range of IPE sections using the material characteristics adopted by Eurocode 2 [23, 24].

Due to the fact that IPE profiles are produced in the factory and their geometric dimensions and the mechanical characteristics of the metal used are known, they are classified as plastic sections. The plastic modulus of these profiles and thus the plastic moments are directly given in the literature and tabulated. Whereas, reinforced concrete sections having the same plastic moment as a given profile may be multiple. The number of parameters is important (width, height, covering, characteristic strength of the concrete, characteristic strength of the steel, etc.). To overcome these difficulties it is, therefore, absolutely to solve the problem graphically by realizing a catalogue of three-axis design charts linking the reduced moment (M_u/bd^2), the ratio of tensioned reinforcements (ρ_{st}) and the ratio of compressed reinforcements (ρ_{sc}), using the material characteristics adopted by Eurocode 2, is essential.

In this parametric study and in order to cover all the parameters that can influence the dimensions and percentages of reinforcements of the rectangular section equivalent to a given profile, three practical classes of concrete were selected, namely C25, C30 and C40 having respectively values for the characteristic strength of concrete (f_{ck}) 25, 30 and 40 MPa. Also, two practical values for the characteristic yield strength of the reinforcement (f_{yk}) have also been taken, namely 400 and 500 MPa. The results of the present characterisation using Eurocode 2 characteristics are compared to those using CP110.

The main objective is to propose the best possible equivalent rectangular section, singly or doubly reinforced with:

- (ε_{cu} = 3.5 ‰, ε_{st} and $\varepsilon_{sc} > \varepsilon_y$) thus exploiting to the maximum the concrete and reinforcement steel used [25, 26];
- They should also be produced economically (minimisation of the concrete cross-section and reinforcement) [27, 28];

The present analysis was carried out in order to achieve the following objectives:

- Develop a catalogue of design charts similar to the one previously developed by British codes (CP110-2:1972 and BS: 8110-3:1985) [13, 14];
- Carry out a comparative study between the use of the two codes (CP110 and Eurocode 2) in order to formulate an opinion on the change of code;
- Set up an expert mini-system to find the best possible equivalent section in singly or doubly reinforced concrete from the point of view of economic and plastic performance.

The present paper is divided into seven sections. The introduction presents the problem that motivated this research, an overview of previous work as well as a summary of the main objectives are given. The second section presents the basic equations which are absolutely necessary for the graphical realisation of the design charts catalogue. The plastic moments for all IPE profiles are tabulated in section 3. The fourth section describes the procedures used to determine singly and doubly reinforced concrete sections equivalent to the various IPE profiles. The fifth section is devoted to the tabular and graphical presentation of the results. The effect of the different influencing parameters on the dimensioning of the equivalent sections is highlighted in section six. A comparative study between the use of two codes is established in order to highlight the impact of the change of codes. The last section is devoted to the general conclusions of the study. The flowchart of the research and characterisation methodology is presented in Figure 1.

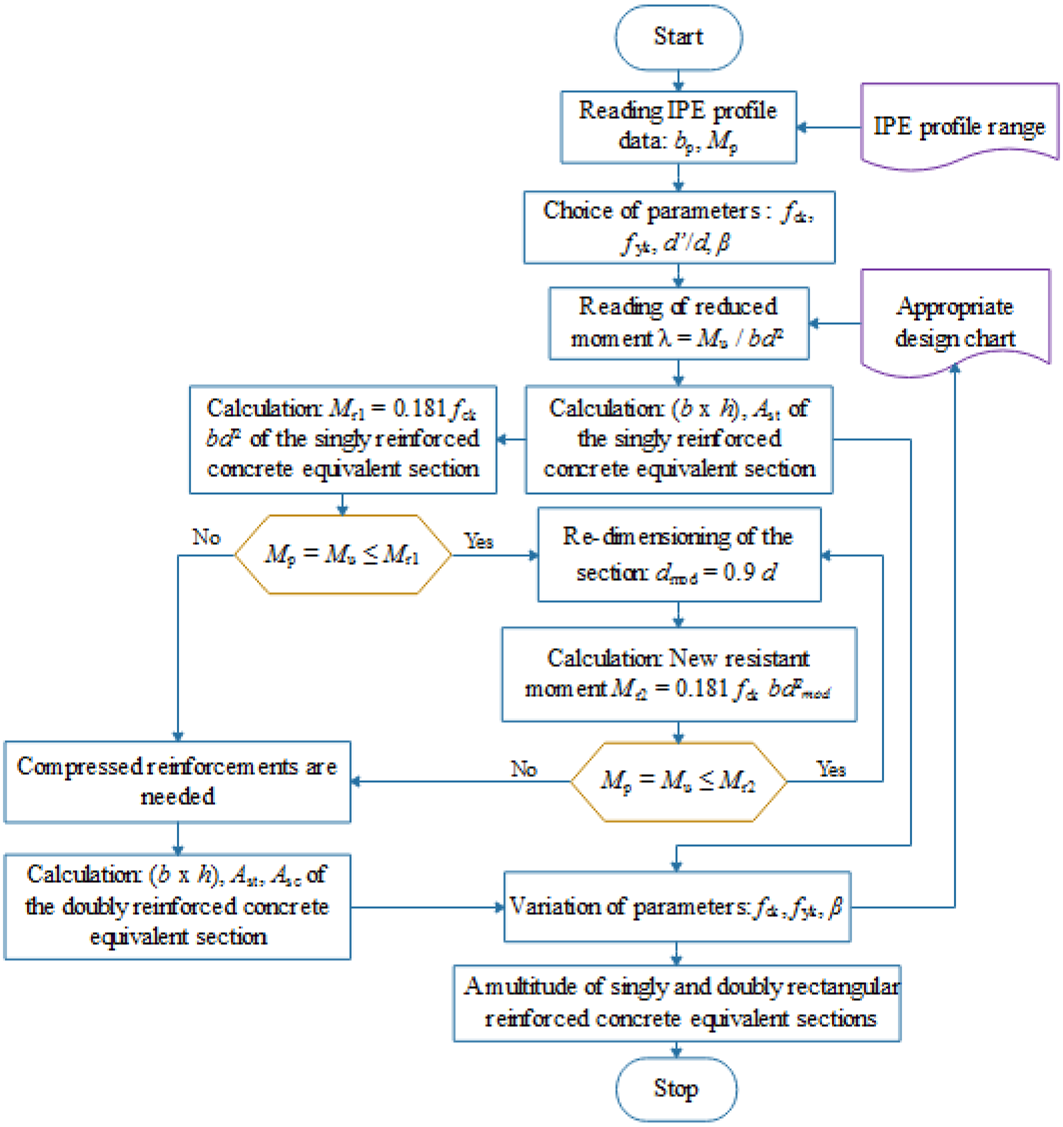

Figure 1. Flowchart of the research and characterization methodology

2. Design and Graphic Development of a Catalogue of Design Charts

2.1. Designing Equations

The design charts, by using the characteristics of the materials adopted by Eurocode 2, are obtained by plotting the reduced moment (M_u/bd^2), the tensioned reinforcement ratio ($\rho_{st} = 100A_{st}/bd$) and the compressed reinforcement ratio ($\rho_{sc} = 100A_{sc}/bd$) for singly and doubly reinforced concrete rectangular beams and for different values of characteristic steel strength (f_{yk}), concrete cylinder strength (f_{ck}) and parameter d'/d. This catalogue of design charts is similar to the one published in the British codes (CP110-2:1972 and BS: 8110-3:1985) and these design charts cannot be used to obtain the complete detailed design of any member but they may be used as an aid when analysing the cross section of a member at the ultimate limit state. The charts have been based on the assumptions laid down in BS EN 1992-1-1:2004+A1:2014 [20], use being made of the simplified rectangular stress block based on the Whitney principle which was previously been adopted by BAEL rules [17].

The plotting of this design charts is based on the digitization of the equations obtained by taking the balance of moments about the neutral axis of the section:

$$M_u = 0.567\, f_{ck}\, 0.8x\,(x - 0.8x/2) + f_{sc}\, A_{sc}(x - d') + f_{st}\, A_{st}(d - x) \tag{1}$$

$$f_{st}\, A_{st} = 0.454\, f_{ck}\, b\, x + f_{sc}\, A_{sc} \tag{2}$$

The Equations 1 and 2 can be written as:

$$\frac{M_u}{bd^2} = 0.454 f_{ck}\frac{x^2}{d^2}(1 - 0.40) + f_{sc}\frac{A_{sc}}{bd}\left(\frac{x}{d} - \frac{d\prime}{d}\right) + f_{st}\frac{A_{st}}{bd}\left(1 - \frac{x}{d}\right) \tag{1a}$$

$$f_{st}\frac{A_{st}}{bd} = 0.454 f_{ck}\frac{x}{d} + f_{sc}\frac{A_{sc}}{bd} \tag{2a}$$

For specified ratios of A_{sc}/bd, x/d and d'/d, the two non-dimensional Equations 1a and 2a can be solved to give values for A_{st}/bd and M_u/bd^2 so that a set of design charts such as the one shown in Figure 3, can be plotted. Before the equations can be solved, the steel stresses (f_{st}) and (f_{sc}) must be calculated for each value of x/d. This is achieved by first determining the appropriate strains from the strain diagram (or by applying Equations 3 and 4) and then by evaluating the stresses from the stress – strain curve of Figure 2.

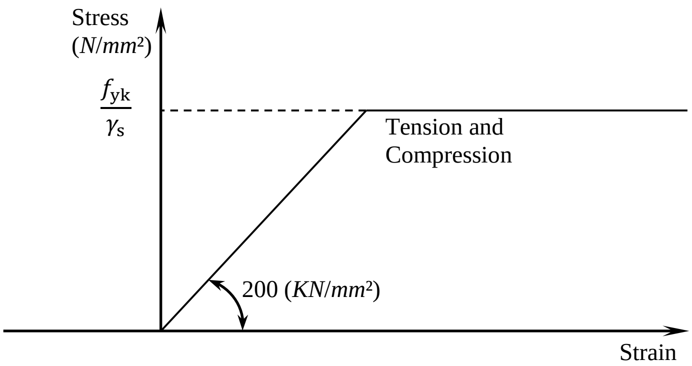

Figure 2. Design stress – strain curve for steel reinforcement [29]

$$\varepsilon_{st} = \varepsilon_{cc}\left(\frac{d-x}{x}\right) \tag{3}$$

$$\varepsilon_{sc} = \varepsilon_{cc}\left(\frac{x-d\prime}{x}\right) \tag{4}$$

2.2. Presentation of a Model from the Catalogue of Developed Design Charts

Only one design chart model of the developed catalogue is presented in this article. This catalogue developed for this study contains a series of charts for any value of the parameters (f_{ck}) (25, 30, 40 *MPa*) and (f_{yk}) (400, 500 *MPa*) and four values for the parameter d'/d (0.05, 0.10, 0.15, 0.20). The Figure 3 is an example of a design chart for (f_{yk} = 500 *MPa*, f_{ck} = 30 *MPa* and d'/d = 0.10). It is imperative to develop a catalogue of design charts, because the characterisation in an analytical way is almost impossible.

In addition, this catalogue will have two possible uses. The first is when the section is completely defined (geometrically and mechanically), the determination of the ultimate or plastic moment is very easy. The second is when the ultimate moment is known and it will be necessary to reinforce the section optimally.

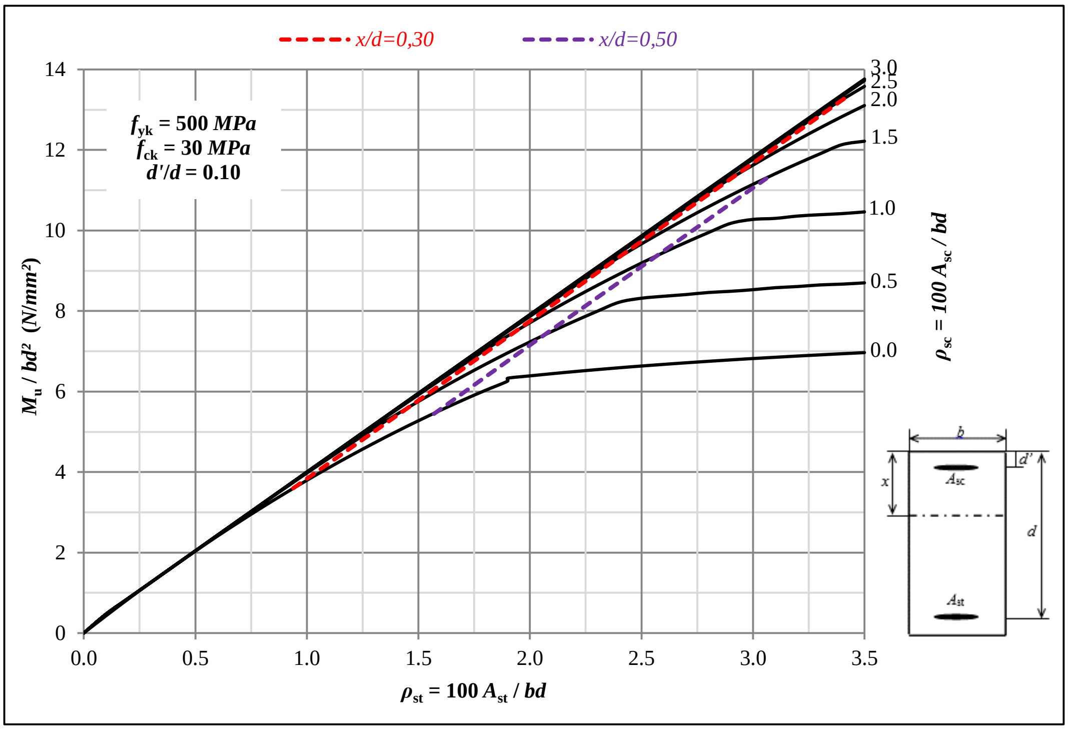

Figure 3. Typical design chart of the developed catalogue for (f_{yk} = 500 *MPa*, f_{ck} = 30 *MPa* and *d'/d* = 0.10)

3. Plastic Moments of all IPE Profile

For the calculation of the internal capacity namely the plastic moment the method of plastic analysis can be used, taking into account the total plastification of the section. Because the geometrical characteristics of the profiles are given, the calculation of the plastic moment is easy, it is just necessary to know the yield strength of the steel used (f_y). The design plastic moments of the IPE range are given in Table 1. The plastic bending moment of the cross-section is specified in Eurocode 3 [30]. All IPE profiles are classified as bending class 1, therefore their plastic moments are defined by multiplying the plastic bending modulus ($W_{pl,y}$) by the steel yield stress (f_y):

$$M_{pl,y} = W_{pl,y} \cdot f_y / \gamma_{M0} \tag{5}$$

Table 1. Design plastic moment of all IPE profiles

Profile	$W_{pl,y}$ ($\times 10^3$ mm^3)	$M_{pl,y}$ (*KN.m*)	Profile	$W_{pl,y}$ ($\times 10^3$ mm^3)	$M_{pl,y}$ (*KN.m*)
IPE 80	23.22	5.46	*IPE 270*	484.00	113.74
IPE 100	39.41	9.26	*IPE 300*	628.40	147.66
IPE 120	60.73	14.27	*IPE 330*	804.00	189.02
IPE 140	88.34	20.76	*IPE 360*	1019.00	239.50
IPE 160	123.90	29.11	*IPE 400*	1307.00	307.18
IPE 180	166.40	39.11	*IPE 450*	1702.00	399.92
IPE 200	220.60	51.85	*IPE 500*	2194.00	515.62
IPE 220	285.40	67.07	*IPE 550*	2787.00	654.95
IPE 240	366.60	86.16	*IPE 600*	3512.00	825.41

Where: Steel grade S235: (f_y = 235 *N/mm²*) is the yield strength of the profiles and (γ_{M0} = 1.00) is the partial safety coefficient.

4. Determination of Equivalent Reinforced Concrete Sections

4.1. Singly Reinforced Sections

4.1.1. Introduction

The determination of singly reinforced concrete sections equivalent to different IPE must be established by an optimal design. Singly reinforced concrete sections must be under reinforced (Concrete and reinforcement exploited to the maximum (ε_{cc} = ε_{cu}, ε_{st} and ε_{sc} exceeding ε_y) with *x*/*d* = 0.50 which corresponds to a plastic strain of the reinforcement). It is a question of determining the dimensions of the equivalent rectangular section (b x h) and the tensile reinforcement (A_{st}). This is an arduous and complex problem because it consists in determining several unknowns at the same time.

This operation can be carried out by exploiting the catalogue of design charts developed. These design charts must be designed according to the influential parameters, namely the reinforcement covering d'/d, the characteristic compressive strength of concrete (f_{ck}), the characteristic yield strength of steels (f_{yk}) and the position of the neutral axis x/d, etc. (See Figure 3).

4.1.2. Determination Process

Principles

The process requires knowing and setting the following parameters (f_{ck}, f_{yk} and d'/d) beforehand. It should be noted that each of the design charts in the catalogue developed has been plotted for a combination of these parameters. Thereafter, it is imperative to fix one of the geometric unknowns. For the present study, it was decided to fix the width of the equivalent concrete section (b) in proportion to the width of a given profile (b_p). This proportionality ratio is noted ($\beta = b/b_p$) and varies from 1.50 to 2.00 with a step of 0.25. This range corresponds to usual and practical rectangular cross-sections. In addition, the useful depth of the equivalent rectangular section (d) must also be fixed. In this study ($d = 0.9\ h$).

Thus, the process of determining equivalent sections can be started and with an additional requirement for x/d. In fact, for the steel strains to be plastic (x/d must be ≤ 0.50). As for the plastic equivalence, the ultimate moment of the rectangular cross-section (M_u) is taken equal to the plastic moment (M_p) of the profile concerned.

Procedure

Taking the appropriate design chart for a given combination of (f_{ck}, f_{yk} and d'/d). In this design chart, the intersection of the straight line ($x/d = 0.50$) with the curve ($\rho_{sc} = 0$) gives a point where its horizontal projection gives the value of the reduced moment (λ) and its vertical projection gives the value of (ρ_{st}).

$$M_u / b.d^2 = \lambda \tag{6}$$

From Equation 6 there is:

$$d = \sqrt{M_u / \lambda.b} = \sqrt{M_u / \lambda.\beta b_p} \tag{7}$$

So: $h = d / 0.9$;

$$\rho_{st} = 100\ A_{st}/bd \tag{8}$$

From Equation 8 there is:

$$A_{st} = \rho_{st} \frac{b.d}{100} \tag{9}$$

All unknowns (b, h and A_{st}) are consequently determined. By varying the values of the geometric ratio (β), the characteristic strength of concrete (f_{ck}) and the steel yield strength (f_{yk}). The result is a series of singly reinforced rectangular concrete sections for the IPE profile range.

Processing an Example

Taking an example, the case of the IPE 270 with ($\beta = 1.50$), the singly reinforced equivalent section is determined as follows:

- The data for IPE 270: $M_u = M_p = 113.74\ KN.m$, $b_p = 13.5\ cm$
- From the appropriate design chart, for example, ($f_{ck} = 30\ MPa$, $f_{yk} = 500\ MPa$, $d'/d = 0.10$) and with ($x/d = 0.50$). The values of the reduced moment (λ) and the tensile reinforcement ratio (ρ_{st}) can be deduced as follows:

  $\lambda = M_u / bd^2 = 5.416\ N/mm^2$ and $\rho_{st} = 100\ A_{st}/bd = 1.60$
- The results for the equivalent singly reinforced concrete section for this example are:

  $b = \beta\ b_p = 20.25\ cm$ $d = 32.2\ cm$
  $h = 35.8\ cm$ $A_{st} = 10.43\ cm^2$

4.2. Doubly Reinforced Sections

4.2.1. Introduction

Reinforced concrete beams used in practice are usually doubly reinforced sections (i.e. with mounting reinforcements). These doubly reinforced sections must be designed by exploitation to the maximum the concrete and tension and compression steel, the determination of the dimensions ($b\times h$) of these equivalent sections as well as their reinforcement areas (A_{st} and A_{sc}), must follow the steps below:

1) The existing data are:
   - The mechanical characteristics of the materials used, such as (f_{ck}) and (f_{yk});
   - The ultimate moment (M_u) which is taken equal to plastic moment (M_p) of a given profile;
   - The dimensions of the singly reinforced sections obtained (b and d).
2) The determination of the reinforcement areas (A_{st} and A_{sc}) as well as the dimensions of the doubly reinforced section (b x h) can be obtained analytically in this part. Using a simplified rectangular stress block based on the Whitney principle which was adopted by Eurocode 2 [20] and previously by BAEL rules [17], with a bilinear diagram for steel reinforcement (see Figure 2).

4.2.2. Determination Process

Principles and Procedure

The design of the doubly reinforced sections is established with a depth of the neutral axis (x/d=0.50), (this limit of 0.50 was taken in order to be able to make a comparison with the characterization established in previous contribution) [12]. This depth of the neutral axis ensures that the strains of the tensioned and compressed reinforcements are in the plastic range, ($\varepsilon_{sc} > \varepsilon_y$, $\varepsilon_{st} > \varepsilon_y$ and $\varepsilon_{cc} = 0.0035$), it is the optimal exploitation of the materials used.

For a singly reinforced concrete rectangular section, the resistant moment is calculated:

$$M_{r1} = 0.181\, f_{ck} b d^2 \tag{10}$$

$$\text{With : } z = d - 0.4\,x \quad \text{and } x = \frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_y}.d \tag{11}$$

First of all, the resistant moment (M_{r1}) of the singly reinforced section is calculated. Then comparing the ($M_u = M_p$) with (M_{r1}), when $M_u \leq M_{r1}$, the concrete section must be re-dimensioned and redesigned, modifying the value of (d) by (d_{mod}) where: ($d_{mod} = 0.9\, d$), until (M_u) is greater than (M_{r2}), (M_{r2} is the new resistant moment of the modified section), hence the need to use compressed reinforcements.

$$M_{r2} = 0.181\, f_{ck} b.\, d_{mod}^2 \tag{12}$$

Depending on the internal equilibrium of the moments, the compressed reinforcement area (A_{sc}) can be deduced as follows:

$$A_{sc} = \frac{M_u - 0.181.f_{ck}.b.d_{mod}^2}{0.87 f_{yk}(d_{mod} - d')} \tag{13}$$

Knowing that d'/d_{mod} = 0.10, hence:

$$A_{sc} = \frac{M_u - 0.181.f_{ck}.b.d_{mod}^2}{0.783.d_{mod}.f_{yk}} \tag{13a}$$

According to the equilibrium of forces, the tensioned reinforcement area (A_{st}) is given by:

$$A_{st} = \frac{0.227.f_{ck}.b.d_{mod} + 0.87.f_{yk}.A_{sc}}{0.87 f_{yk}} \tag{14}$$

Processing an Example

Taking the same example, the case of IPE 270 with (β = 1.50), the doubly reinforced equivalent section is determined as follows:

- The data for the singly reinforced section equivalent to IPE 270 are : $M_u = M_p$ = 113.74 *KN.m*, b_p = 13.5 *cm*, $b = \beta\, b_p$ = 20.25 *cm*, d = 32.2 *cm*, f_{ck} = 30 *MPa* and f_{yk} = 500 *MPa*

By applying the principles and procedure with the formulas of paragraph 4.2.2.1.

- The results for the equivalent doubly reinforced concrete section for this example are:

$b = \beta\, b_p = 20.25\ cm$ $d_{mod} = 29.0\ cm$ $h = 32.2\ cm$

$A_{st} = 11.07\ cm^2$ $A_{sc} = 1.89\ cm^2$

5. Results

5.1. Tabular Presentation of Results

All the different results of this theoretical analysis are presented in different tables, each table containing the dimensions of the singly and doubly reinforced concrete equivalent sections as well as their reinforcement ratios for the different influential parameters (f_{ck}, f_{yk} and β). In this article only two tables are presented, Table 2 represents a model for singly reinforced sections and Table 3 represents a model for doubly reinforced sections.

Table 2. Singly reinforced concrete rectangular equivalent cross-sections for f_{yk} = 500 N/mm^2, β =1.50, x/d = 0.50

f_{ck} (N/mm^2)				25			30			40	
Profile	**M_p (KN.m)**	**b (cm)**	**d (cm)**	**h (cm)**	**A_{st} (cm²)**	**d (cm)**	**h (cm)**	**A_{st} (cm²)**	**d (cm)**	**h (cm)**	**A_{st} (cm²)**
IPE 80	5.45	6.90	13.2	14.6	1.18	12.1	13.4	1.33	10.4	11.6	1.51
IPE 100	9.26	8.25	15.7	17.5	1.69	14.4	16.0	1.90	12.4	13.8	2.16
IPE 120	14.26	9.60	18.1	20.1	2.26	16.6	18.4	2.54	14.3	15.9	2.89
IPE 140	20.75	10.95	20.4	22.7	2.91	18.7	20.8	3.28	16.2	18.0	3.72
IPE 160	29.14	12.30	22.8	25.4	3.65	20.9	23.2	4.12	18.1	20.1	4.67
IPE 180	39.01	13.65	25.1	27.9	4.45	23.0	25.5	5.02	19.9	22.1	5.69
IPE 200	51.94	15.00	27.6	30.7	5.38	25.3	28.1	6.07	21.9	24.3	6.88
IPE 220	66.98	16.50	29.9	33.2	6.41	27.4	30.4	7.23	23.7	26.3	8.20
IPE 240	86.25	18.00	32.5	36.1	7.60	29.7	33.0	8.57	25.7	28.6	9.72
IPE 270	**113.74**	**20.25**	35.2	39.1	9.26	**32.2**	**35.8**	**10.43**	27.8	30.9	11.84
IPE 300	147.58	22.50	38.0	42.2	11.11	34.8	38.7	12.53	30.1	33.4	14.21
IPE 330	188.94	24.00	41.6	46.2	12.99	38.1	42.4	14.64	33.0	36.6	16.61
IPE 360	239.47	25.50	45.5	50.5	15.07	41.6	46.3	16.99	36.0	40.0	19.27
IPE 400	307.15	27.00	50.0	55.6	17.56	45.8	50.9	19.80	39.6	44.0	22.46
IPE 450	399.97	28.50	55.6	61.7	20.59	50.9	56.6	23.21	44.0	48.9	26.33
IPE 500	515.59	30.00	61.5	68.3	23.98	56.3	62.6	27.04	48.7	54.1	30.67
IPE 550	654.95	31.50	67.6	75.2	27.70	62.0	68.8	31.23	53.6	59.5	35.42
IPE 600	825.32	33.00	74.2	82.4	31.83	68.0	75.5	35.88	58.7	65.3	40.70

Table 3. Doubly reinforced concrete rectangular equivalent cross-sections for f_{yk} = 500 N/mm^2, β =1.50, x/d = 0.50

f_{ck} (N/mm^2)				25			30			40	
Profile	**M_p (KN.m)**	**b (cm)**	**h (cm)**	**A_{sc} (cm²)**	**A_{st} (cm²)**	**h (cm)**	**A_{sc} (cm²)**	**A_{st} (cm²)**	**h (cm)**	**A_{sc} (cm²)**	**A_{st} (cm²)**
IPE 80	5.45	6.90	13.2	0.22	1.29	12.1	0.24	1.41	10.4	0.29	1.64
IPE 100	9.26	8.25	15.7	0.33	1.85	14.4	0.34	2.02	12.4	0.42	2.34
IPE 120	14.26	9.60	18.1	0.43	2.47	16.6	0.45	2.69	14.3	0.55	3.12
IPE 140	20.75	10.95	20.4	0.56	3.19	18.7	0.59	3.48	16.2	0.68	4.02
IPE 160	29.14	12.30	22.8	0.71	4.00	20.9	0.75	4.37	18.1	0.86	5.05
IPE 180	39.01	13.65	25.1	0.85	4.87	23.0	0.89	5.32	19.9	1.04	6.15
IPE 200	51.94	15.00	27.6	1.03	5.90	25.3	1.09	6.44	21.9	1.26	7.43
IPE 220	66.98	16.50	29.9	1.23	7.02	27.4	1.29	7.66	23.7	1.51	8.86
IPE 240	86.25	18.00	32.5	1.45	8.32	29.7	1.57	9.10	25.7	1.83	10.52
IPE 270	**113.74**	**20.25**	35.2	1.76	10.13	**32.2**	**1.89**	**11.07**	27.8	2.24	12.82
IPE 300	147.58	22.50	38.0	2.13	12.17	34.8	2.26	13.29	30.1	2.64	15.37
IPE 330	188.94	24.00	41.6	2.50	14.23	38.1	2.66	15.54	33.0	3.07	17.95
IPE 360	239.47	25.50	45.5	2.87	16.49	41.6	3.10	18.04	36.0	3.60	20.85
IPE 400	307.15	27.00	50.0	3.39	19.24	45.8	3.60	21.02	39.6	4.22	24.30
IPE 450	399.97	28.50	55.6	3.93	22.54	50.9	4.19	24.63	44.0	4.93	28.49
IPE 500	515.59	30.00	61.5	4.60	26.26	56.3	4.91	28.70	48.7	5.73	33.18
IPE 550	654.95	31.50	67.6	5.35	30.35	62.0	5.60	33.12	53.6	6.58	38.30
IPE 600	825.32	33.00	74.2	6.10	34.85	68.0	6.43	38.05	58.7	7.66	44.05

5.2. Graphical Presentation of Results

5.2.1. Introduction

The cross-sectional dimensions equivalent to the individual IPE profiles obtained from this plastic analysis represent rectangular cross-sections that are usual in practice. The adequate exposure of the different results is the graphical presentation or more precisely the development of a series of curves by scanning the main influencing parameters. Their exploitation makes it easy to determine the most appropriate equivalent sections.

5.2.2. Choice of the Adopted Coordinate System

The choice of the coordinate system obeys to major constraint, that of having all the results on the same graph. The most judicious way is to opt for the logarithmic coordinate system. The best system that has proved successful is that of presenting the results by means of curves with three systems of coordinates.

The entire range of IPE profiles is shown on the x-axis. On the left side of the y-axis, the values ln (A_c) and ln (A_p) are shown, where: (A_c) is the area of the equivalent reinforced concrete section, (A_p) is the area of the IPE profile. On the right side of the y-axis, the ratios of reinforcement (ρ_{st} = 100 A_{st}/bd and ρ_{sc} = 100 A_{sc}/bd) are shown, hence the right scale takes an independent reading from the left one.

5.2.3. Presentation of the Developed Curves

In this analysis, curves are plotted to represent the obtained sections, either singly or doubly reinforced, equivalent to the different IPE profiles. Three classes of concrete (C25/30, C30/37 and C40/50) were considered having respectively the characteristic strengths of concrete, f_{ck} (25, 30, 40 N/mm^2). For each class of concrete there are two characteristic strengths (f_{yk}) namely 400 and 500 N/mm^2.

A typical selection of curves is presented in the present contribution. These curves representing the singly and doubly reinforced equivalent sections are shown in Figures 4 to 13, where the effect of (f_{ck}) and (f_{yk}) for a variation of the geometric ratio (β) are highlighted.

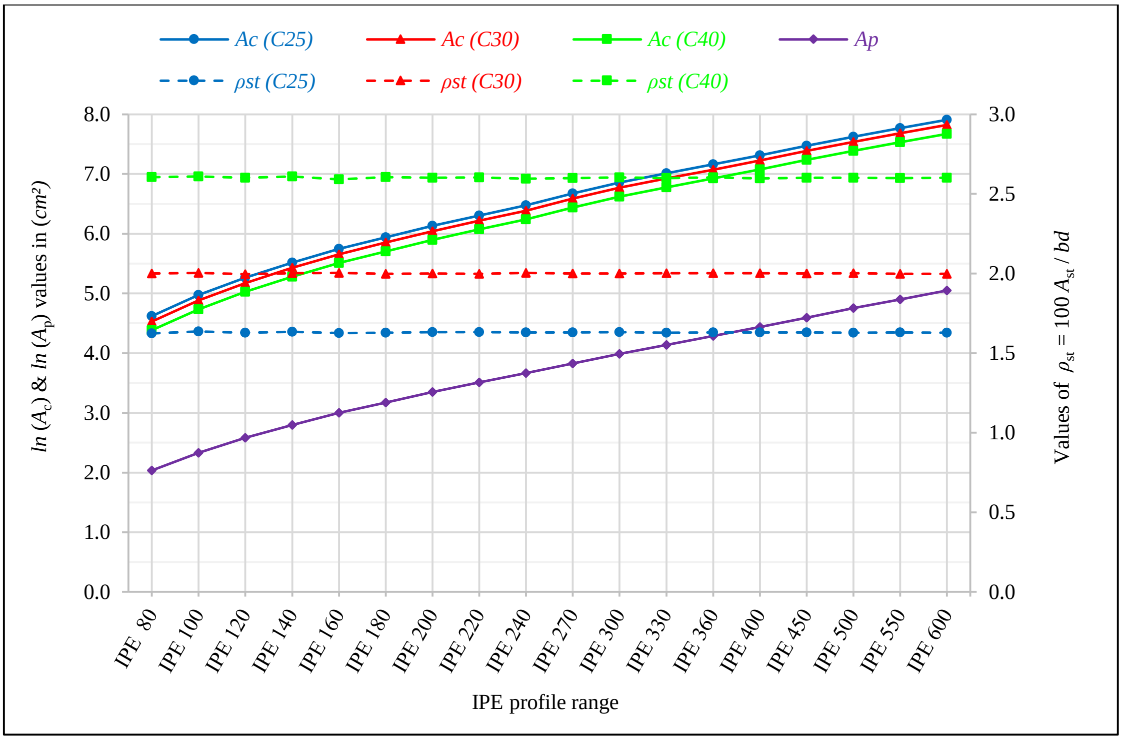

Figure 4. Singly reinforced concrete equivalent sections for: f_{yk} = 400 *MPa*, β = 1.50

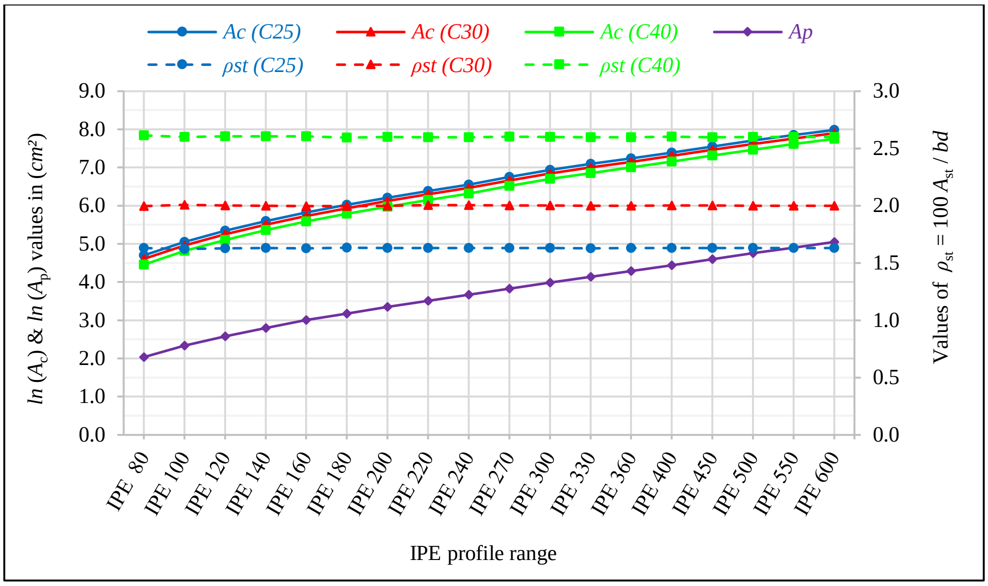

Figure 5. Singly reinforced concrete equivalent sections for: f_{yk} = 400 *MPa*, *β* = 1.75

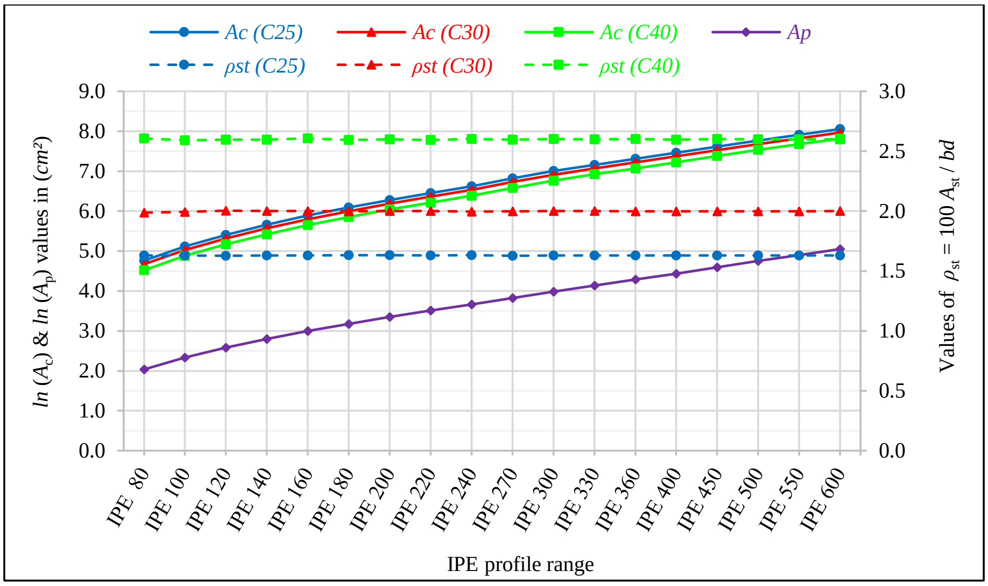

Figure 6. Singly reinforced concrete equivalent sections for: f_{yk} = 400 *MPa*, *β* = 2.00

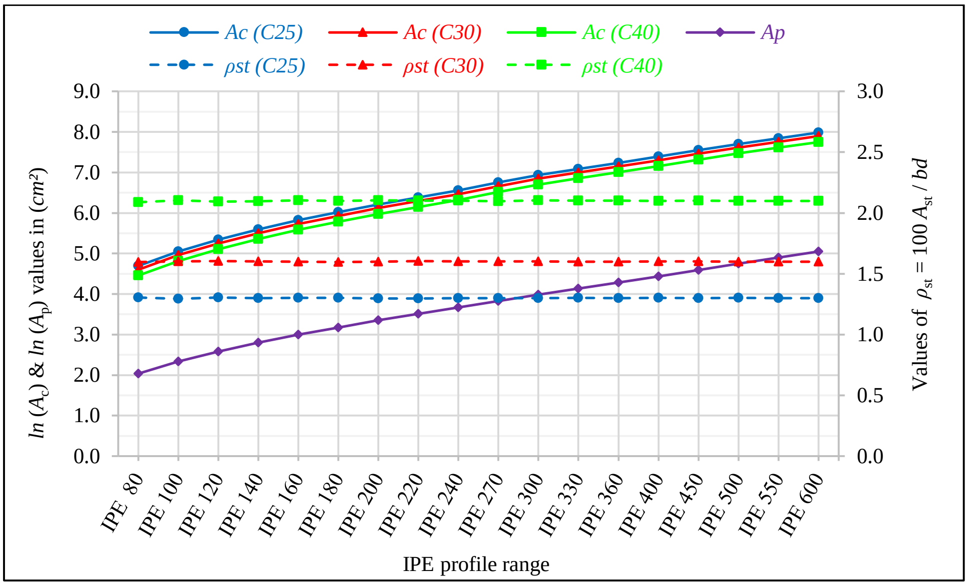

Figure 7. Singly reinforced concrete equivalent sections for: f_{yk} = 500 *MPa*, *β* = 1.75

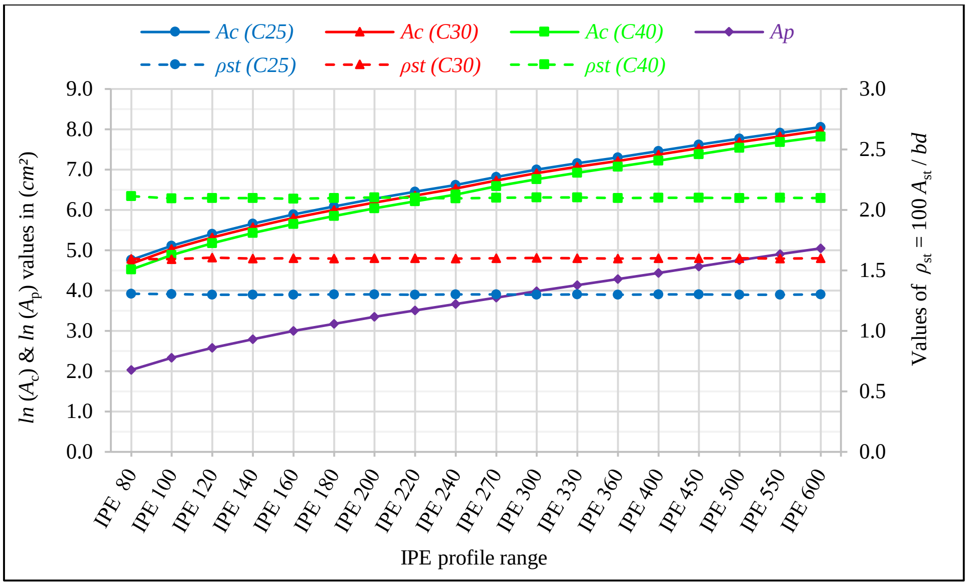

Figure 8. Singly reinforced concrete equivalent sections for: f_{yk} = 500 *MPa*, β = 2.00

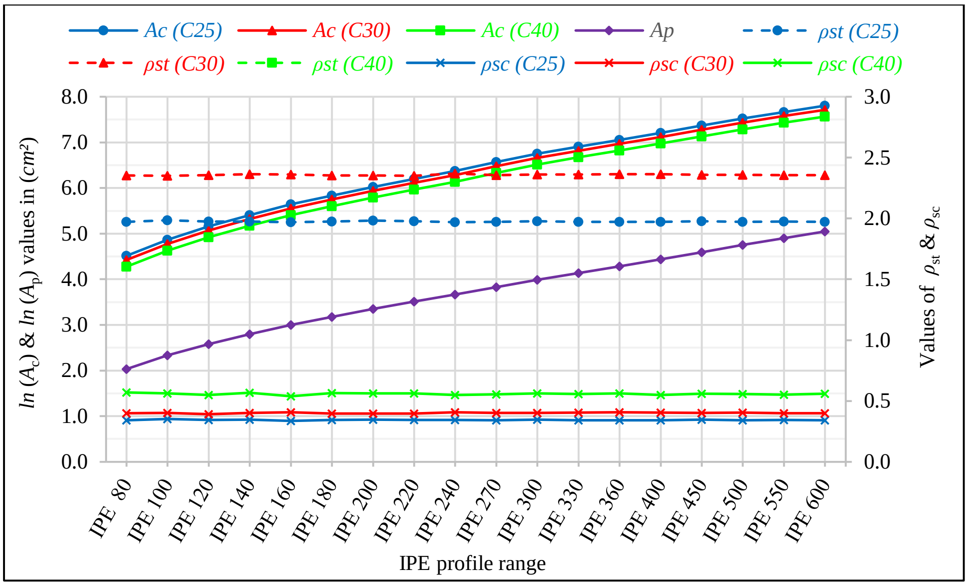

Figure 9. Doubly reinforced concrete equivalent sections for: f_{yk} = 400 *MPa*, β = 1.50

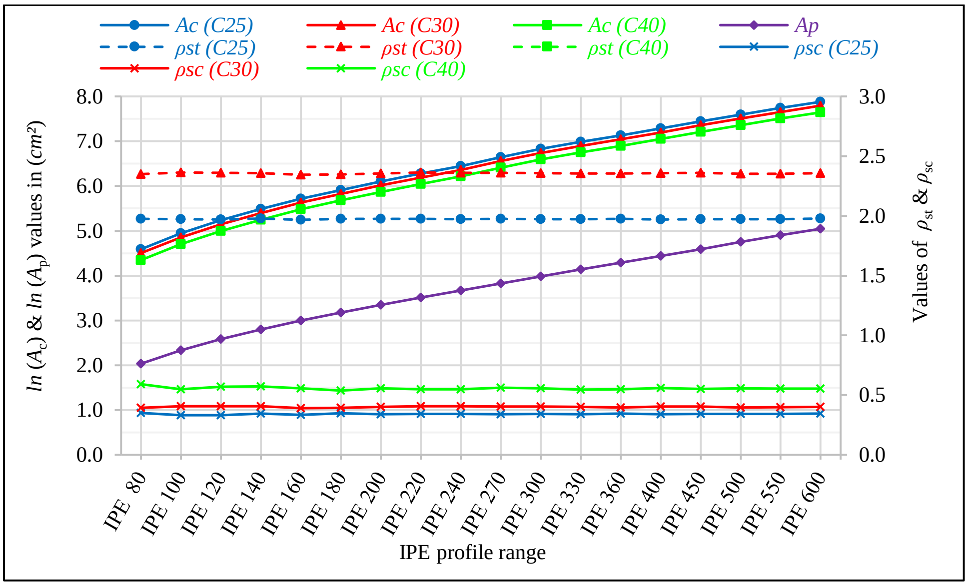

Figure 10. Doubly reinforced concrete equivalent sections for: f_{yk} = 400 *MPa*, β = 1.75

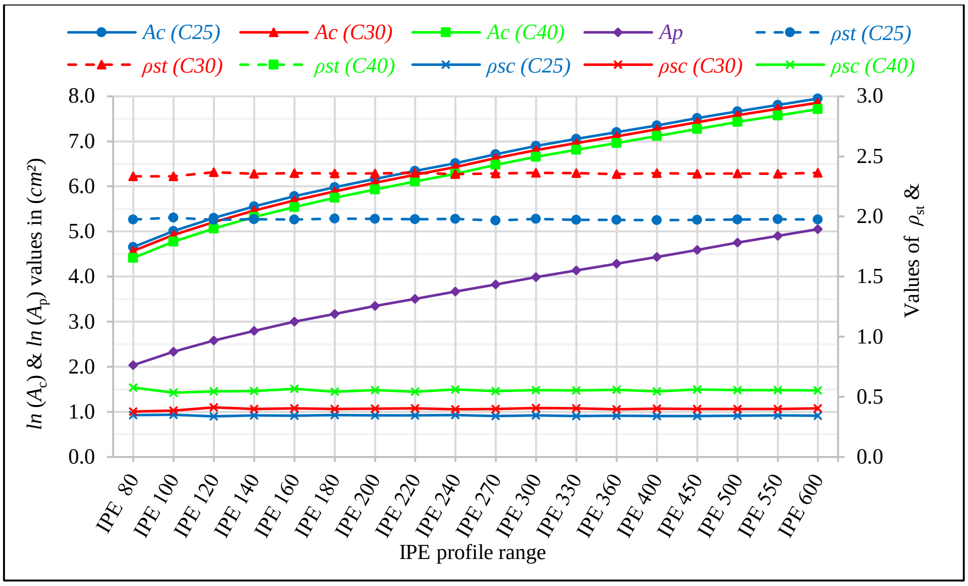

Figure 11. Doubly reinforced concrete equivalent sections for: f_{yk} = 400 *MPa*, β = 2.00

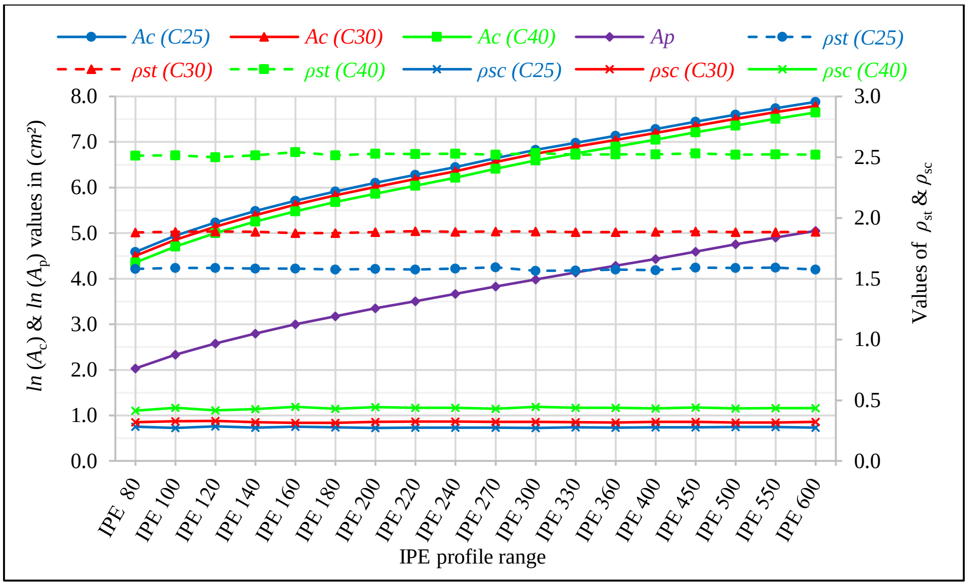

Figure 12. Doubly reinforced concrete equivalent sections for: f_{yk} = 500 *MPa*, β = 1.75

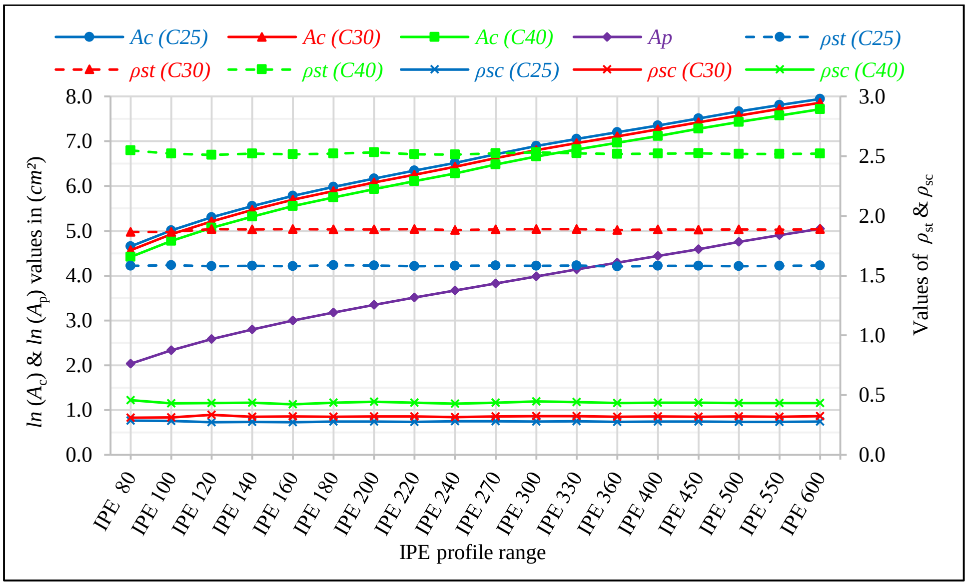

Figure 13. Doubly reinforced concrete equivalent sections for: f_{yk} = 500 *MPa*, β = 2.00

6. Effects of the Different Variables

6.1. Effects of the Characteristic Compressive Strength of Concrete (f_{ck}) and the Ratio (β)

6.1.1. Introduction

Figures 4 to 13 clearly illustrate the effect of increasing the characteristic compressive strength of concrete (f_{ck}) on the total height (h) of the equivalent rectangular section for both types of sections (singly and doubly reinforced). It can also be seen that the curves for the three values of (f_{ck}) have the same shape in the same graph.

6.1.2. Singly Reinforced Concrete sections

For the selected values of (β) and three ranges of (f_{ck}): i.e. (from 25 to 30 *MPa*, from 30 to 40 *MPa* and from 25 to 40 *MPa*), the effects of the variation of (f_{ck}) are given in Table 4. These values represent the average variation for the whole range of IPE.

- The height (h) decreases with the increase of (f_{ck}) for a given (f_{yk}). On the other hand, the tensioned reinforcement ratio (ρ_{st}) increases with the increase of (f_{ck}).
- The height (h) decreasing from 8.42 % to 21.03 %, whereas the ratio of tensioned reinforcement (ρ_{st}) increasing from 22.62 % to 61.53 %.

The maximum gain in concrete is around 20 % for the highest concrete strength (40 *MPa*), whereas the corresponding added reinforcement needed is around 60 % for the same strength. The results considerate coherent since the plastic moment is very slightly affected by the value of (f_{ck}) [29].

Table 4. Effects of the increase of (f_{ck}) on the height (h) and on the ratio of tensioned reinforcements (ρ_{st}) for singly reinforced sections

f_{yk} (*MPa*)	β	Decrease of height (h) in %			Increase of (ρ_{st}) in %		
		Variation of f_{ck} (*MPa*)			Variation of f_{ck} (*MPa*)		
		25 to 30	30 to 40	25 to 40	25 to 30	30 to 40	25 to 40
400	1.50	8.51 %	13.68 %	21.03 %	22.71 %	30.06 %	59.60 %
	1.75	8.53 %	13.65 %	21.02 %	22.78 %	30.03 %	59.65 %
	2.00	8.52 %	13.65 %	21.01 %	22.62 %	30.06 %	59.47 %
500	1.50	8.42 %	13.54 %	20.82 %	23.05 %	31.27 %	61.53 %
	1.75	8.50 %	13.58 %	20.92 %	23.10 %	31.20 %	61.51 %
	2.00	8.44 %	13.56 %	20.86 %	22.95 %	31.30 %	61.43 %

The effects of the increase of (β) (the β values varying from 1.50 to 2.00) on the total height (h) are given in Table 5:

- The total height (h) decreases with increasing (β) for a given set of (f_{ck}) and (f_{yk}), this decrease has been found constant and is around 13.40 %.

Table 5. Effects of the increase of (β) on the total height (h)

Section type	Decrease of height (h) in %		
	f_{ck} (*MPa*)	f_{yk} = 400 *MPa*	f_{yk} = 500 *MPa*
Singly reinforced sections	25	13.39 %	13.39 %
	30	13.40 %	13.40 %
	40	13.37 %	13.43 %

The values given in Table 5 represent the average of decrease in height for the three selected values of (β) (1.50, 1.75 and 2.00) and this for the whole range of IPE.

6.1.3. Doubly Reinforced Concrete Sections

As for doubly reinforced sections, also for the selected values of (β) and three ranges of (f_{ck}): i.e. (from 25 to 30 *MPa*, from 30 to 40 *MPa* and from 25 to 40 *MPa*), the effects of the variation of (f_{ck}) are given in Table 6. These values represent the average variation for the whole range of IPE.

- The height (h) also decreases with the increase of (f_{ck}) for a given (f_{yk}). On the other hand, the ratios of reinforcement (ρ_{st} and ρ_{sc}) increase;

- The height (h) decreasing from 8.37% to 21.07%, whereas the ratio of tensioned reinforcement (ρ_{st}) increasing from 19.10% to 60.37% and the ratio of compressed reinforcement (ρ_{sc}) increasing from 15.17% to 62.86;

Also for the doubly reinforced sections, the maximum gain in concrete is around 21% for the highest concrete strength (40 MPa), whereas the corresponding added reinforcement needed is around 61% for the same strength.

Table 6. Effects of the increase of (f_{ck}) on the height (h) and on the ratios of reinforcements (ρ_{st} and ρ_{sc}) for doubly reinforced sections

f_{yk} (*MPa*)	β	Decrease of height (h) in %			Increase of (ρ_{st}) in %			Increase of (ρ_{sc}) in %		
		Variation of f_{ck} (*MPa*)			Variation of f_{ck} (*MPa*)			Variation of f_{ck} (*MPa*)		
		25 to 30	30 to 40	25 to 40	25 to 30	30 to 40	25 to 40	25 to 30	30 to 40	25 to 40
400	1.50	8.46 %	13.75 %	21.05 %	19.43 %	34.25 %	60.33 %	16.79 %	38.99 %	62.29 %
	1.75	8.53 %	13.71 %	21.07 %	19.40 %	34.32 %	60.37 %	17.37 %	38.79 %	62.86 %
	2.00	8.43 %	13.70 %	20.97 %	19.26 %	34.17 %	60.02 %	15.95 %	38.70 %	60.73 %
500	1.50	8.40 %	13.57 %	20.83 %	19.24 %	33.81 %	59.55 %	15.79 %	36.09 %	57.56 %
	1.75	8.43 %	13.53 %	20.82 %	19.20 %	33.80 %	59.50 %	16.04 %	35.28 %	56.97 %
	2.00	8.37 %	13.59 %	20.82 %	19.10 %	33.87 %	59.44 %	15.17 %	36.38 %	56.99 %

For the doubly reinforced sections, the effects of varying (β) (the β values varying from 1.50 to 2.00) on the total height (h) are given in Table 7.

- Similarly to the singly reinforced sections, the total height (h) decreases with increasing (β) for a given set of (f_{ck}) and (f_{yk}), this decrease has been found constant for all tested sections and is around 13.39 %.

Table 7. Effects of the increase of (β) on the total height (h)

Section type	Decrease of height (h) in %		
	f_{ck} (*MPa*)	f_{yk} = 400 *MPa*	f_{yk} = 500 *MPa*
Doubly reinforced sections	25	13.42 %	13.42 %
	30	13.39 %	13.39 %
	40	13.34 %	13.40 %

6.2. Effects of the Variation of the Characteristic Yield Strength of Reinforcement (f_{yk})

6.2.1. Singly Reinforced Concrete Sections

For a given (f_{ck}) and for the same value of (β), the effects of the increase of the characteristic yield strength of reinforcement (f_{yk}) on the total height (h) of the equivalent sections seems to be minor and negligible (see Table 8).

Table 8. Effects of the increase of (f_{yk}) on the height (h) for singly reinforced sections

Section type	Variation of height (h) in %	
	f_{ck} (*MPa*)	f_{yk} 400/500 (*MPa*)
Singly Reinforced sections	25	- 0.08 %
	30	0.00 %
	40	- 0.14 %

- The effects of the variation of the characteristic yield strength of reinforcement (f_{yk}) on the tensioned reinforced ratio (ρ_{st}) are presented in Table 9. The tensioned reinforced ratio (ρ_{st}) decreases with the increase of the characteristic yield strength of the reinforcement (f_{yk}). For a given value of (β) and for different values of (f_{ck}) (25 *MPa*, 30 *MPa* and 40 *MPa*) the decrease in reinforcement needed has been found constant and is around 19.82 %.

Table 9. Effects of the variation of (f_{yk}) on the tensioned reinforcement ratio (ρ_{st}) for singly reinforced sections

f_{ck} (*MPa*)	Values of (ρ_{st}) for different (f_{yk})		Decrease of (ρ_{st}) in %
	f_{yk} = 400 *MPa*	f_{yk} = 500 *MPa*	f_{yk} = 400/500 (*MPa*)
25	1.63	1.30	20.25 %
30	2.00	1.60	20.00 %
40	2.60	2.10	19.23 %

6.2.2. Doubly Reinforced Concrete Sections

For the doubly reinforced sections, for a given (f_{ck}) and for the same value of (β), the effects of the increase of the characteristic yield strength of reinforcement (f_{yk}) on the total height (h) seems to be also minor and negligible (see Table 10):

Table 10. Effects of the increase of (f_{yk}) on the height (h) for doubly reinforced sections

Section type	Variation of height (h) in %	
	f_{ck} (*MPa*)	f_{yk} 400/500 (*MPa*)
Doubly reinforced sections	25	+ 0.08 %
	30	0.00 %
	40	- 0.16 %

- The effects of the variation of the characteristic yield strength of reinforcement (f_{yk}) on the ratios of reinforcement (ρ_{st} and ρ_{sc}) are presented in Table 11. The ratios of reinforcement (ρ_{st} and ρ_{sc}) decrease with the increase of the characteristic yield strength of the reinforcement (f_{yk}). For a given value of (f_{ck}), this decrease appears to be also constant and is around 20 % for (ρ_{st}) and around 19.63 % for (ρ_{sc}):

Table 11. Effects of the increase of (f_{yk}) on the ratios of reinforcement (ρ_{st}, and ρ_{sc}) for doubly reinforced sections

f_{ck} (*MPa*)	Values of (ρ_{st}) for different (f_{yk})		Decrease of (ρ_{st}) in %
	f_{yk} = 400 *MPa*	f_{yk} = 500 *MPa*	f_{yk} = 400/500 (*MPa*)
25	1.97	1.58	19.80 %
30	2.36	1.89	19.92 %
40	3.16	2.52	20.25 %
f_{ck} (*MPa*)	Values of (ρ_{sc}) for the different (f_{yk})		Decrease of (ρ_{sc}) in %
	f_{yk} = 400 *MPa*	f_{yk} = 500 *MPa*	f_{yk} = 400/500 (*MPa*)
25	0.34	0.28	17.65 %
30	0.40	0.32	20.00 %
40	0.56	0.44	21.43 %

6.3. Comparative Study between the use of Two Codes

6.3.1. Introduction

A similar characterisation was established in Boussafel (2003) studies [12] in which the design charts published in Part 2 of CP110 were used. This study was carried out for a characteristic yield strength of the reinforcement (f_{yk} = 410 *MPa*) and for three values of the characteristic cube strength of the concrete (f_{cu}) namely 25, 30 and 40 *MPa* with a neutral axis depth fixed at (x = 0.50 d). However, the present study is based on the characteristics of the materials adopted by Eurocode 2 and extended for a variation of (f_{yk}) between 400 and 500 *MPa* for the characteristic yield strength of the reinforcement and an (f_{ck}) taking the following values: 25, 30 and 40 *MPa* for the characteristic cylinder compressive strength of the concrete. The design charts used in this analysis must be designed and developed (a model of these design charts is shown in Figure 3).

6.3.2. Effect of Code Change on the Height of Equivalent Sections

Singly Reinforced Sections

In order to compare the sections obtained from the characterization carried out using two different codes. New design charts have been developed for a characteristic yield strength of reinforcement (f_{yk} = 410 *MPa*) and for characteristic cylinder strength (f_{ck}) taking three values of 20, 25 and 32 *MPa* (EC 2) corresponding respectively to characteristic cubic strengths (f_{cu}) 25, 30 and 40 *MPa* (CP110), because in the strength classes defined in the Eurocodes (C20/25 to C50/60) the ratios f_{ck}/f_{cu} range from 0.78 to 0.83. An example of this comparison is shown in Figure 14, for (f_{yk}=400 *MPa* and β =2.00), where the effect of the code change on the total height (h) of the equivalent sections is clearly shown.

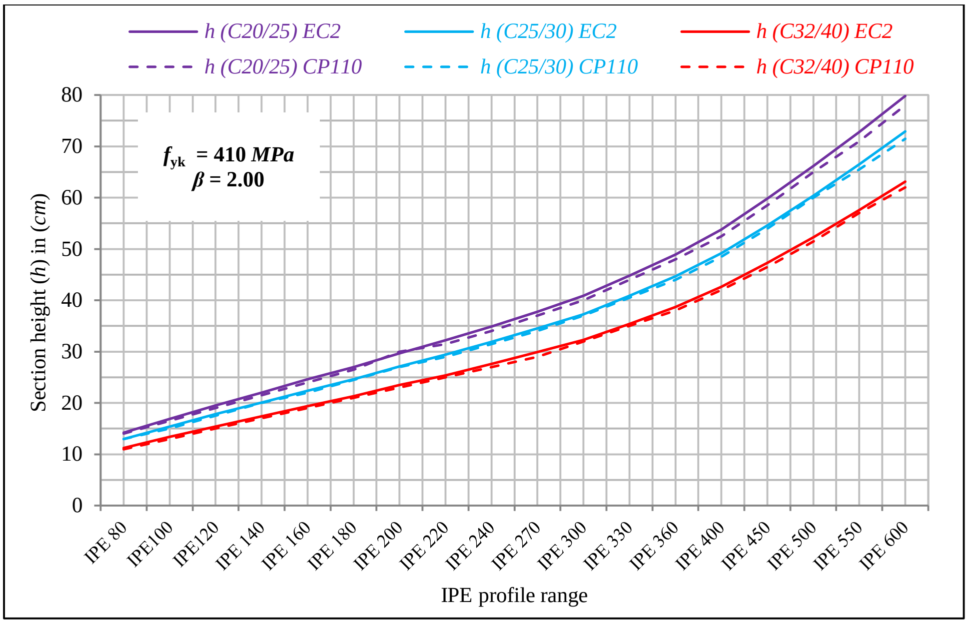

Figure 14. Effect of the code change on the height (*h*) of the singly reinforced sections for (f_{yk} = 410 *MPa* and β = 2.00)

According to the results, for a (f_{ck}) adopted by Eurocode 2 corresponding to a (f_{cu}) adopted by CP110 and for the same (f_{yk}), the heights of the equivalent sections obtained using CP110 are greater than those obtained using Eurocode 2. The effect on the height of the sections (*h*) and on the tension reinforcement ratio (ρ_{st}) is summarised in Table 12.

Table 12. Effect of code change on height (*h*) and on the tensioned reinforcement ratio (ρ_{st}) for singly reinforced sections

f_{yk} (*MPa*)	Variation of the height (*h*) in %			Variation of (ρ_{st}) in %		
	f_{ck} / f_{cu} 20/25 (*MPa*)	f_{ck} / f_{cu} 25/30 (*MPa*)	f_{ck} / f_{cu} 32/40 (*MPa*)	f_{ck} / f_{cu} 20/25 (*MPa*)	f_{ck} / f_{cu} 25/30 (*MPa*)	f_{ck} / f_{cu} 32/40 (*MPa*)
410	+ 2.00 %	+ 1.20 %	+ 1.90 %	- 5.93 %	- 3.77 %	+ 2.00 %

Doubly Reinforced Sections

As for the doubly reinforced sections, the total height (*h*) of the equivalent sections obtained using the CP110 code compared with those obtained using Eurocode 2 is increasing by an average ratio of 2%, (see an example in Figure 15). On the other hand, the ratio of tensioned reinforcement (ρ_{st}) decreases by about 5.30 %, while the decrease is about 23.80 % for the ratio of compressed reinforcement (ρ_{sc}).

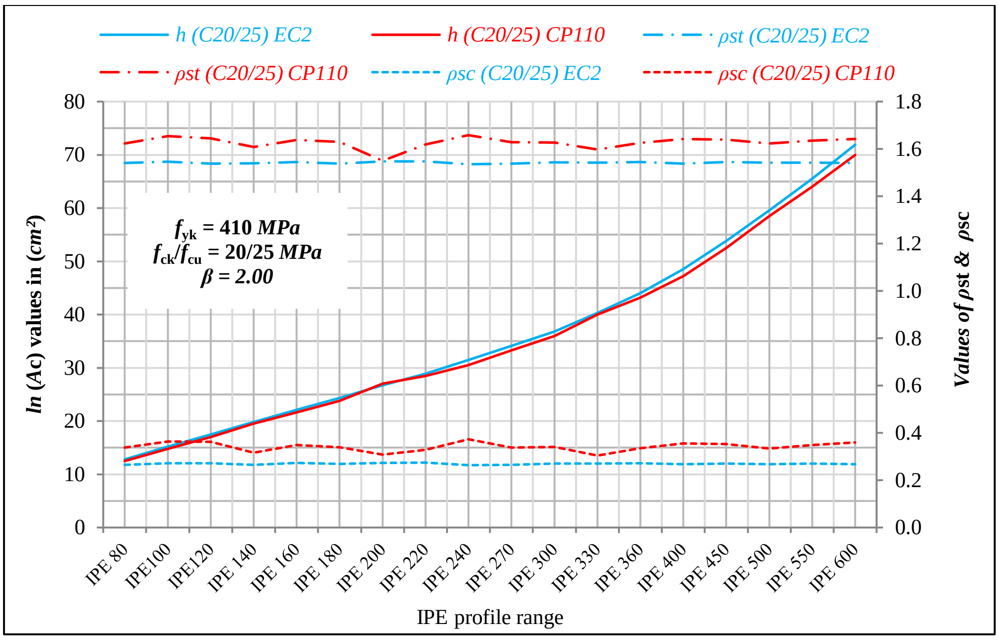

Figure 15. Effect of the code change on the height (*h*) of the doubly reinforced sections for the case: (f_{yk}=410 *MPa*, f_{ck}/f_{cu} = 20/25 *MPa* and β = 2.00) with presentation of reinforcement ratios

7. Conclusion

7.1. Importance of Developing the Catalogue of Design Charts

This study highlighted the importance of the design charts catalogue. The development of these design charts in three-axis graphs linking the reduced moment (M_u/bd^2) and the ratio of tensioned reinforcement (ρ_{st}) and the ratio of compressed reinforcement (ρ_{sc}) using the material characteristics adopted by Eurocode 2 was necessary and indispensable for the following reasons:

- Facilitation of the characterization operation (without this catalogue, the operation would have been impossible to be carried out analytically);
- Facilitation of the comparison between the characterisation obtained by two codes: CP110 and Eurocode 2;
- This catalogue can be used in the design of bent reinforced concrete beams that have rectangular cross-sections in the ultimate limit state (dimensioning and reinforcing);
- It can also be used to quickly check the quality of a bent singly reinforced concrete section (is it under- or over-reinforced);
- Finally, this catalogue allows to immediately determine the flexional capacity of a rectangular section if all the parameters (f_{ck}, f_{yk} and d'/d) of this section are known.

7.2. Characterization

From this characterisation, the study revealed the following points:

- All singly or doubly reinforced concrete sections equivalent to the various IPE profiles are in accordance with the sections used in practice;
- The three-axis graphs developed in this study set up a simple mini-system allowing the determination of singly and doubly reinforced concrete sections equivalent to the different IPE profiles which could facilitate decision making;
- Due to the large number of influential geometrical (d'/d, $\beta = b/b_p$) and mechanical (f_{ck}, f_{yk}) parameters, the characterisation allowed to obtain a multitude of reinforced concrete sections equivalent to a given profile and it is up to the user to opt for a practical choice;
- All that has been taken into account in the present study is the determination of practical rectangular reinforced concrete equivalent sections to different IPE profiles. The performance and the economy question have been left for future study.

7.3. Importance of Using a Given Code

The comparative study between the use of material characteristics adopted by two codes (CP110 and EC2) has shown that there are more or less important impacts on the equivalent sections obtained. This study revealed the following points:

- The variations are minor for the height (h), the percentage varies from (+1.2% to +2.0%) for singly reinforced sections and about (+2.0%) for doubly reinforced sections;
- The variations are more or less important for tensioned reinforcement ratio (ρ_{st}), the ratio varying from (-5.93% to +2.0%) for singly reinforced sections and about (-5.3%) for doubly reinforced sections. On the other hand, for the compressed reinforcement ratio (ρ_{sc}), the percentage is approximately (-23.8 %).

8. Nomenclature

A_c	Area of the equivalent reinforced concrete section
A_p	Area of the IPE profile
A_{sc}	Cross-sectional area of compression reinforcement
A_{st}	Cross-sectional area of tension reinforcement
b	Width of the reinforced concrete rectangular section
b_p	Width of a steel profile
d	Effective depth of tension reinforcement
d_{mod}	Modified effective depth of tension reinforcement ($d_{mod} = 0.9\ d$)
d'	Depth to compression reinforcement

f_{ck}	Characteristic cylinder strength of concrete [f_{ck} = (0.78 ÷ 0.83) f_{cu}]
f_{cu}	Characteristic cube strength of concrete
f_{sc}	Compressive steel stress
f_{st}	Tensile steel stress
f_y	Steel yield stress of a profile
f_{yk}	Characteristic yield strength of reinforcement
h	Overall depth of reinforced concrete rectangular section in plane of bending
$M_{pl,y}$	Plastic moment of a steel profile
M_{r1}	Resistant moment of a reinforced concrete rectangular section
M_{r2}	Resistant moment of the modified section
M_u	Ultimate moment of resistance or plastic moment of a reinforced concrete rectangular section
$W_{pl,y}$	Plastic modulus of a steel profile along the axis of strong inertia (y-y)
x	Neutral axis depth
β	Equivalent section width to IPE profile width ratio ($\beta = b / b_p$)
γ_{M0}	The partial safety coefficient (γ_{M0} = 1.00)
ε_{cc}	Compressive concrete strain
ε_{cu}	Boundary compressive concrete strain
ε_{sc}	Compressive steel strain
ε_{st}	Tensile steel strain
ε_y	Yield steel strain
λ	Reduced moment ($\lambda = M_u / bd^2$)
ρ_{sc}	Reinforcement ratio for compression reinforcement
ρ_{st}	Reinforcement ratio for tension reinforcement

9. Declarations

9.1. Author Contributions

Conceptualization, S.B. and M.L.S.; writing—original draft preparation, S.B. and M.L.S.; writing—review and editing, S.B. and M.L.S. All authors have read and agreed to the published version of the manuscript.

9.2. Data Availability Statement

The data presented in this study are available in article.

9.3. Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

9.4. Conflicts of Interest

The authors declare no conflict of interest.

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