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Modified Method for prefabricated vertical drain consolidation analysis

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ABSTRACT

Ground improvement with the prefabricated vertical drain (PVD) has become widely employed for soft ground treatment because of its economical and efficient method. While numerous numerical and analytical methods have been derived for PVD however, it is still an extensively high demand for a simpler and more accurate method for design steps. This paper proposes a method for solving the problem of one-dimensional (1D) consolidation with prefabricated vertical drains. The current approach introduces a 1D equivalent permeability, increasing linearly with depth to perform the consolidation of soft ground improved with PVD. The analytical solutions have been carried out and verified by analyses for two cases of one-way drainage and two-way drainage for uniform soil layer. The results show that the error of excess pore pressure determined by the proposed method is less than that obtained by the simpler method of Chai and smaller than 10% compared to the theoretical solution. The paper also compares the analytical solution with the FEM by ABAQUS software. It is found that the excess pore pressures and consolidation degrees obtained by these methods are similar and close to the theory. These confirm that the introduced 1D equivalent permeability can be employed to perform the consolidation of PVD improvement by analytical and FEM methods.

1 Introduction

The analytical solution of the consolidation problem derived by Terzaghi [1] has been widely applied for predicting consolidation settlement of constructions without treatment on soft soil. The Terzaghi's solution was considered as one-

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dimensional (1D) consolidation because it is treated with one vertical drain direction. To accelerate the consolidation rate, some ground improvement techniques could be applied such as sand drains or prefabricated vertical drains (PVDs). Excess pore water pressure in the soil treated by these techniques not only dissipates vertically to the layer sand drained blanket but also radially to the drain. There are many kinds of research on characteristics of vertical drains such as [2-14]. On the other hand, analytical solutions have been extensively studied by [15-23] where a review of the performance of PVDs, has been established and the objective was to improve the performance of the soft ground[24, 25]. However, most of them required sophisticated software or computation effort to analysis.

Moreover, Nguyen et al. [26] and Nghia-Nguyen et al. [23] developed a simple analytical solution and simple discrete element model for prefabricated vertical drain with and without vacuum consolidation. Nguyen and Kim [27] developed also a numerical solution based on the large theory in order to consider discharge capacity reduction where the obtained results were in good agreement with the measured data. furthermore, taking into account the effect of the variability of the soil's parameters on the design of prefabricated vertical drain become an essential aspect of the improvement of the soft ground [28]. Ngo et al. [29] Made an important study by proposing a plane strain models based on Indraratna and Redana's methods in order to predict the consolidation settlement. Nonetheless, the above solutions can easily be employed in a standard finite element (FEM) analysis software for designing of ground improvement projects by vertical drain.

Meanwhile, Chai [11] introduced a simplified method of model soft ground improved with PVD by substituting a system of PVD and soil with equivalent soil, which has an equivalent vertical permeability. The Chai's approach [11] is mainly used in modelling PVD with a finite element method for 1D, 2D or 3D problems. The Chai's approach is very simple and can be easily applied into FEM software for designing. One major issue which is related to the Chai's approach is the accuracyof the solution with the theoretical solution by Carrilo [30]. In the order words, Chai's approach showed the predicted results have certain differences from those of theoretical solution. Inspiration from the issue, this paper introduces a modified method which is not only a simple method in 1D but also more accurate than the previous approach by Chai's method. The verifications were carried out with ABAQUS and theoretical solution to demonstrate the effectiveness of the current approach. It is highly promising of wide applications by its simplicity and accuracy of the modified method.

2 Governing equation

To derive the governing equation for the 1D consolidation problem, Terzaghi [1] assumed that (a) soil is fully saturated, (b) water and soil particles are incompressible, (c) Darcy's law is valid, (d) strains are small, (e) all compressive strains within the soil mass occur in a vertical direction and (f) the coefficient of compressibility is constant. This study also applies those assumptions to solve the problem of 1D consolidation with PVD in which the coefficient of equivalent permeability, k_z, of permeability in the vertical and radial directions (see Fig. 1) is proposed to be linearly increased with depth and determined by the following equation:

$$k_z = k_{ini}(1 + a\,z) \tag{1}$$

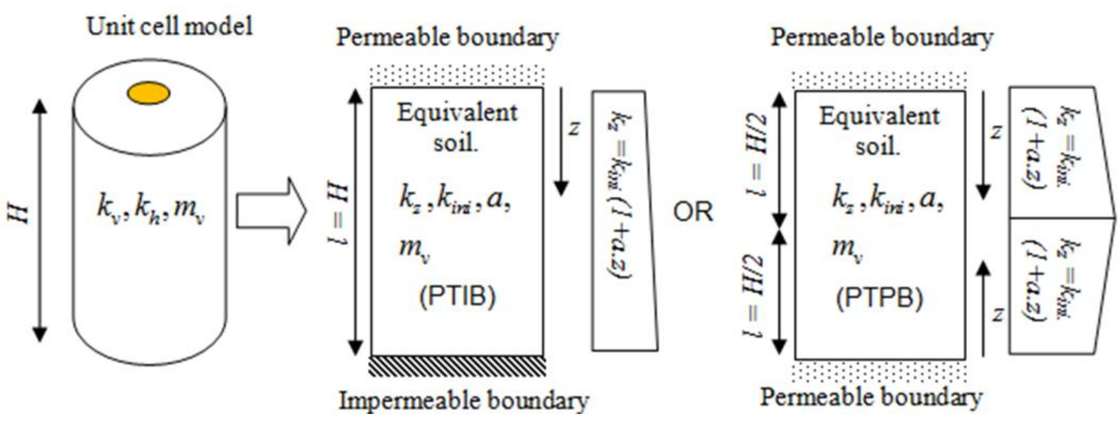

Fig. 1- Model for the modified method

l is the drainage length, $l = H$ for the case of one-way drainage with permeable top boundary and impermeable bottom boundary (PTIB), and in $l = H/2$ for the case of two-way drainage with permeable top boundary and permeable bottom boundary (PTPB).

The derivation of the differential equation for the pore water dissipation is detailed presented in Appendix A where the general equation is:

$$\frac{\partial u(z,t)}{\partial t} = \frac{\partial}{\partial z}\left(\frac{k_z}{m_v \gamma_w} \frac{\partial u(z,t)}{\partial z} \right) \tag{2}$$

If k_z is a constant ($k_z = k_v$), the Eq. (2) becomes the governing equation for Terzaghi's 1D consolidation theory

$$\frac{\partial u(z,t)}{\partial t} = \frac{k_v}{m_v \gamma_w} \frac{\partial^2 u(z,t)}{\partial z^2} \tag{3}$$

However, because k_z is assuming to be an increased function with depth, we can substitute Eq. (1) into Eq. (2) to obtain the governing equation as below:

$$\frac{\partial u(z,t)}{\partial t} = \frac{k_{ini}}{m_v \gamma_w} \left[a \frac{\partial u(z,t)}{\partial z} + (za+1) \frac{\partial^2 u(z,t)}{\partial z^2} \right] \tag{4}$$

Boundary and initial conditions are as follows:

$$\text{1)} \quad u(z=0,t) = 0 \tag{5}$$

$$\text{2)} \quad \frac{\partial u(z,t)}{\partial z} = 0 \qquad (z = l) \tag{6}$$

$$\text{3)} \quad u(z,t=0) = u_o \tag{7}$$

3 Solution

3.1 Excess pore pressure and average consolidation degree functions

Excess pore water pressure can be obtained by applying the separation method to the governing equation.

$$u(z,t) = Z(z)T(t) \tag{8}$$

where $Z(z)$ is Eigen function of depth and $T(t)$ is Eigen function of time.

From the above governing equation, the solution for pore pressure is obtained:

$$u(z,t) = \left\{ C_1 J_0\left(\sqrt{\alpha} R(z)\right) + C_2 Y_0\left(\sqrt{\alpha} R(z)\right) \right\} e^{-\alpha t} \tag{9}$$

where J_0 and Y_0 are Bessel functions of zero order of the first and second kind, respectively; C_1 and C_2 are integral constants; α is eigenvalue; and $R(z) = 2\sqrt{\frac{(az+1)m_v \gamma_w}{a^2 k_{ini}}}$ is the function of depth.

From the boundary condition (5), the integral constant C_1 can be expressed in relation with value C_2 as following

$$C_1 = -C_2 \frac{Y_0\left(\sqrt{\alpha}R(z)\right)}{J_0\left(\sqrt{\alpha}R(z)\right)} \tag{10}$$

From the boundary condition (6) , the root of eigenvalues m of α denoted as α_m is derived.

$$Y_1\left(\sqrt{\alpha_m}R(l)\right) - Y_0\left(\sqrt{\alpha_m}R(0)\right)\frac{J_1\left(\sqrt{\alpha_m}R(l)\right)}{J_0\left(\sqrt{\alpha_m}R(0)\right)} = 0 \tag{11}$$

The general solution of Eq. (8) is rewritten in form of a series with respect to each eigenvalue

$$u(z,t) = \sum_{m=1}^{\infty} C_m\left(M_0\left[\alpha_m, z\right]\right)e^{-\alpha_m t} \tag{12}$$

where

$$M_i\left[\alpha_m, z\right] = Y_i\left(\sqrt{\alpha_m}R(z)\right) - Y_0\left(\sqrt{\alpha_m}R(0)\right)\frac{J_i\left(\sqrt{\alpha_m}R(z)\right)}{J_0\left(\sqrt{\alpha_m}R(0)\right)} \quad \text{with } (i = 0,1) \tag{13}$$

The initial condition (7) is applied to Eq, (13) to determine the coefficient C_m

$$u(z,0) = \sum_{m=1}^{\infty} C_m\left(M_0\left[\alpha_m, z\right]\right) = u_o \tag{14}$$

Using Fourier's sine expansion for function (14), the coefficient C_m can be determined as

$$C_m = \frac{u_o A_1}{A_2} \tag{15}$$

where

$$A_1 = \int_0^l M_0\left[\alpha_m, z\right]dz = -\sqrt{\frac{k_{ini}}{m_v \gamma_w \alpha_m}} M_1\left[\alpha_m, l\right]$$

$$A_2 = \int_0^l \left(M_0\left[\alpha_m, z\right]\right)^2 dz == \frac{1}{a}\left\{(la+1)\left(M_0\left[\alpha_m, z\right]\right)^2 - \left(M_1\left[\alpha_m, z\right]\right)^2\right\}$$

Substituting Eq. (15) into Eq. (12) to obtain the function of excess pore pressure with depth

$$u(z,t) = \sum_{m=1}^{\infty} \frac{A_1}{A_2} u_o\left(M_0\left[\alpha_m, z\right]\right)e^{-\alpha_m t} \tag{16}$$

The average excess pore water pressure is calculated as an integral of excess pore water pressure in the whole length.

$$\bar{u}(t) = \frac{\int_0^l u(z,t)dz}{l} = \frac{\int_0^l M_0\left[\alpha_m, z\right]dz \sum_{m=1}^{\infty} \frac{A_1}{A_2} u_o e^{-\alpha_m t}}{l} \tag{17}$$

Equation (17) can be simplified by substituting above integration of $M_0\left[\alpha_m, z\right]$

$$\bar{u}(t)=\sum_{m=1}^{\infty}\frac{A_1^2 u_0}{A_2 l}e^{-\alpha_m t} \tag{18}$$

The average consolidation degree is obtained as below

$$\bar{U}(t)=1-\frac{\bar{u}(t)}{u_o}=1-\sum_{m=1}^{\infty}\frac{A_1^2}{A_2 l}e^{-\alpha_m t} \tag{19}$$

3.2 Equivalent consolidation coefficient

Carrillo's theoretical solution[30] combined the average degree of consolidation in the vertical direction and horizontal direction for the consolidation problem of a vertical drain (Appendix B). The average degree of consolidation is

$$\bar{U}_{vh}(t)=1-\sum_{m=1}^{\infty}\frac{8e^{-\left\{\left(\frac{\pi(2m-1)}{2}\right)^2\frac{c_v}{l^2}+\frac{8}{\mu}\frac{c_h}{D_e^2}\right\}t}}{\pi^2(2m-1)^2} \tag{20}$$

where $c_v=\frac{k_v}{m_v\gamma_w}$ and $c_h=\frac{k_h}{m_v\gamma_w}$.

To have approximation in the consolidation rate, both Eqs. (19) and (20) should have the same time ratio. As a result, α_m can be obtained as follows:

$$\alpha_m=\left\{\left(\frac{\pi(2m-1)}{2}\right)^2\frac{c_v}{l^2}+\frac{8}{\mu}\frac{c_h}{D_e^2}\right\} \tag{21}$$

The first eigenvalue in equation (21) is the most governing value, therefore $m=1$ is selected for Eq. (21)

$$\alpha_1=\left(\frac{\pi^2}{4}\frac{c_v}{l^2}+\frac{8}{\mu}\frac{c_h}{D_e^2}\right)\text{ or }\alpha_1=\frac{1}{m_v\gamma_w}\left(\frac{\pi^2}{4}\frac{k_v}{l^2}+\frac{8}{\mu}\frac{k_h}{D_e^2}\right) \tag{22}$$

where α_1 is the first eigenvalue. The value α_1 is then substituted to function (11) to determine the equivalent consolidation coefficient ($k_z=k_{ini}(az+1)$) which has two unknown variants: the initial permeability k_{ini} and increased coefficient a. To solve Eq. (11), one of two variants of k_{ini} or a is assumed first, and then the remaining value will easily be determined. It is proposed an experience function for determining the initial permeability k_{ini} which has a linear relationship with the drainage length both vertical and horizontal direction.

$$k_{ini}=\left(1+\frac{2.5\times l\times k_h}{\mu\times D_e\times k_v}\right)\times k_v \tag{23}$$

Finally, the increased coefficient a can be determined by the implicit function (11) and expressed in extended form as

$$Y_1\left(2\sqrt{\alpha_1}\sqrt{\frac{(al+1)m_v\gamma_w}{a^2k_{ini}}}\right)-Y_0\left(2\sqrt{\alpha_1}\sqrt{\frac{m_v\gamma_w}{a^2k_{ini}}}\right)\frac{J_1\left(2\sqrt{\alpha_1}\sqrt{\frac{(al+1)m_v\gamma_w}{a^2k_{ini}}}\right)}{J_0\left(2\sqrt{\alpha_1}\sqrt{\frac{m_v\gamma_w}{a^2k_{ini}}}\right)}=0 \tag{24}$$

The increased coefficient, a, is simply figured out by using the subroutine Goal Seek (a trial and error subroutine) in Microsoft Excel by following definition (equation 25) in an optional cell. The first trial value a should be 1 for faster to obtain a final solution.

$$= BesselY\left(2\sqrt{\alpha_1}\sqrt{\frac{(al+1)m_v\gamma_w}{a^2k_{ini}}},1\right) - BesselY\left(2\sqrt{\alpha_1}\sqrt{\frac{m_v\gamma_w}{a^2k_{ini}}},0\right)\frac{BesselJ\left(2\sqrt{\alpha_1}\sqrt{\frac{(al+1)m_v\gamma_w}{a^2k_{ini}}},1\right)}{BesselJ\left(2\sqrt{\alpha_1}\sqrt{\frac{m_v\gamma_w}{a^2k_{ini}}},0\right)} \quad (25)$$

4 Solution

4.1 Problem's description

Two cases are analyzed to verify the proposed method. They are permeable top and impermeable bottom case (PTIB) which is considered as one-way drainage and permeable top and permeable bottom case (PTPB) which is for two-way drainage. The PVD distributions and PVD parameters are given in Fig. 2 and Table 1 taken from the example in Chai [11]

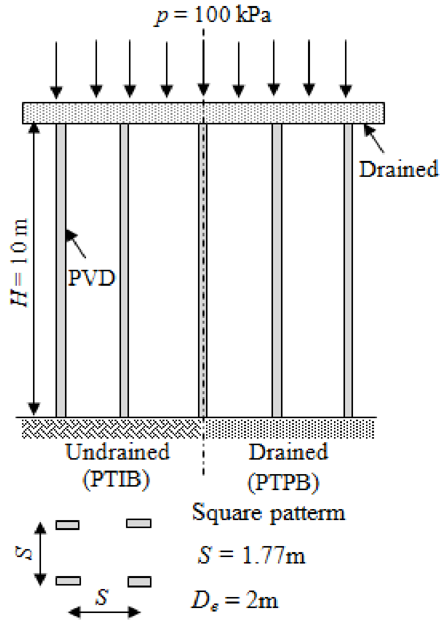

Fig. 2 – Theanalysis cases for verified proposed method

The coefficient of permeability in the vertical direction $k_v = c_v m_v \gamma_w = 0.0981$ $(m/year)$ and in the horizontal direction $k_h = c_h m_v \gamma_w = 0.1962$ $(m/year)$. These Results of the proposed method will be compared with the results that were obtained by Carrillo's theoretical solution[30]considering both vertical and horizontal consolidation and Chai's method[11] using the free strain assumption. Both theoretical solution and Chai's method[11] are defined in Appendix C.

1- Determining the increased coefficient for the one-way drainage (PTIB) with the drainage length $l = H = 10(m)$, and the PVD factor $\mu = 11.426$ from Eq.(3B), the first eigenvalue is determined as $\alpha_1 = 0.3747$ from Eq. (22). The initial permeability, k_{ini}, is given by Eq. (23) as 0.0312 ($m/year$). Substituting both α_1 and k_{ini} into Eq. (24) to determine the increased coefficient by Microsoft Excel software, a, to be 2.494. Finally, vertical permeability is $k_z = 0.0312(2.494z+1)$ ($m/year$).

2- Determining the increased coefficient for the two-way drainage (PTPB)Similar to the case of (PTIB), For the (PTPB) case, $l = H/2 = 5(m)$, $\mu = 10.436$ .Similar to the (PTIB) case, $\alpha_1 = 0.482$, $k_{ini} = 0.0216$ ($m/year$), and a = 1.202. Finally, vertical permeability is $k_z = 0.0216(1.202z+1)$ ($m/year$).

Table 1 – Assumed subsoil and drain parameters

Subsoil and Drain	
$c_v (m^2/year)$	1.0
$c_h (m^2/year)$	2.0
$d_w (m)$	0.1
D_e (m)	2.0
d_s (m)	0.3
k_h/k_s	5.0
q_w $(m^3/year)$	100
m_v (m^2/kN)	1/1000
γ_w (kN/m^3)	9.81

4.2 Validation using ABAQUS

The analytical results are also confirmed by the finite element method (FEM) to verify the proposed method. FEM simulations have been conducted by the ABAQUS software. The model includes 80 elements using eight-node biquadratic displacement and bilinear pore pressure (CPE8P).

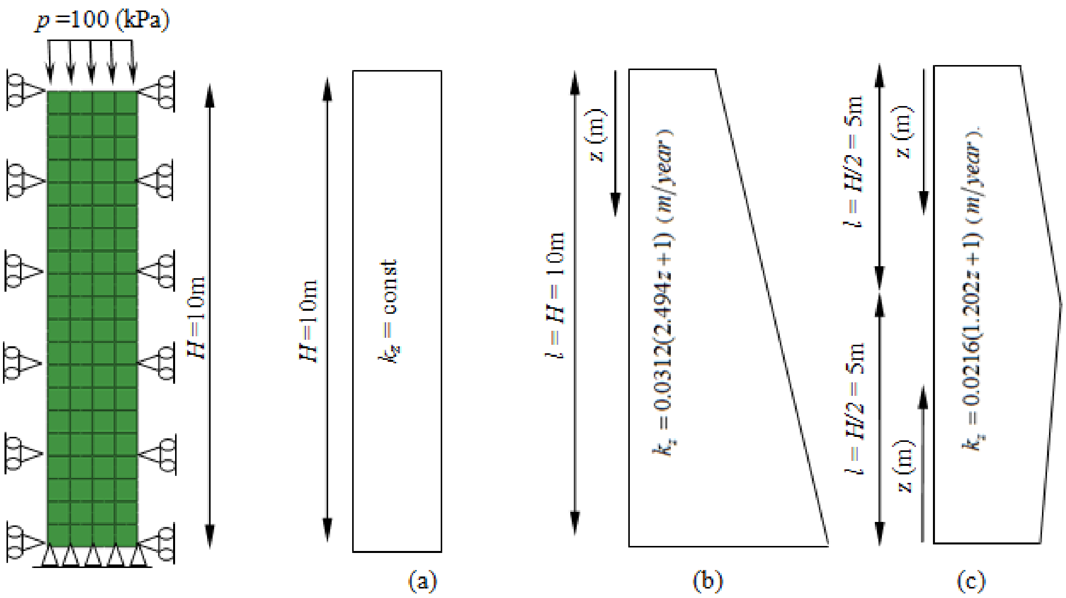

Fig. 3 – (a) ABAQUS model with a coefficient of permeability is constant with depth, (b) ABAQUS model for PTIB, (c) ABAQUS model for PTPB

The applied pressure of P = 100 kPa was applied on soil with thickness, H, of 10m, elastic modulus, E, of 1000 kPa. The Poisson's ratio ν is 0 to creates a nonlateral displacement condition to verify with an analytical solution with the assumption of no lateral displacement. Three simulations have been conducted to verify the analytical solution. Case one is the simulation for the assumption of constant permeability, $k_z = const$, to compare with the Terzaghi's solution [1] (Fig. 3(a)). Case two and three are the simulations with the proposed changing permeability for the (PTIB) and (PTTB) cases as shown in Fig. 3(a) and (b), respectively. k_z is calculated for every 0.5 m depth.

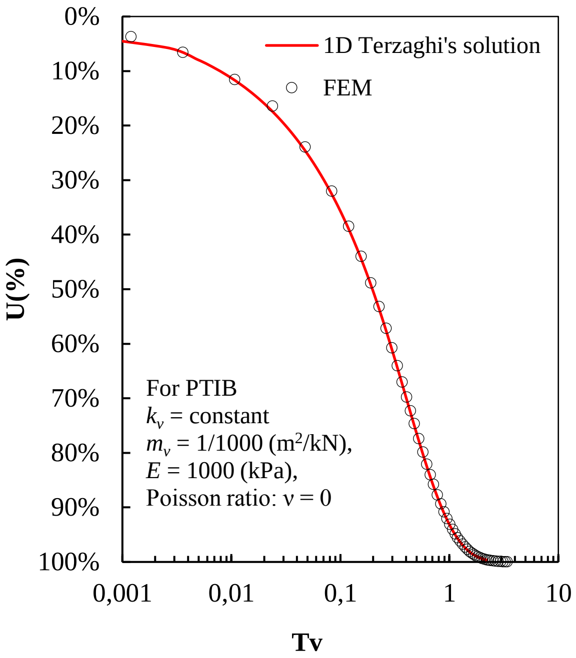

Fig. 4 – Comparison 1D Terzaghi's solution and FEM solution

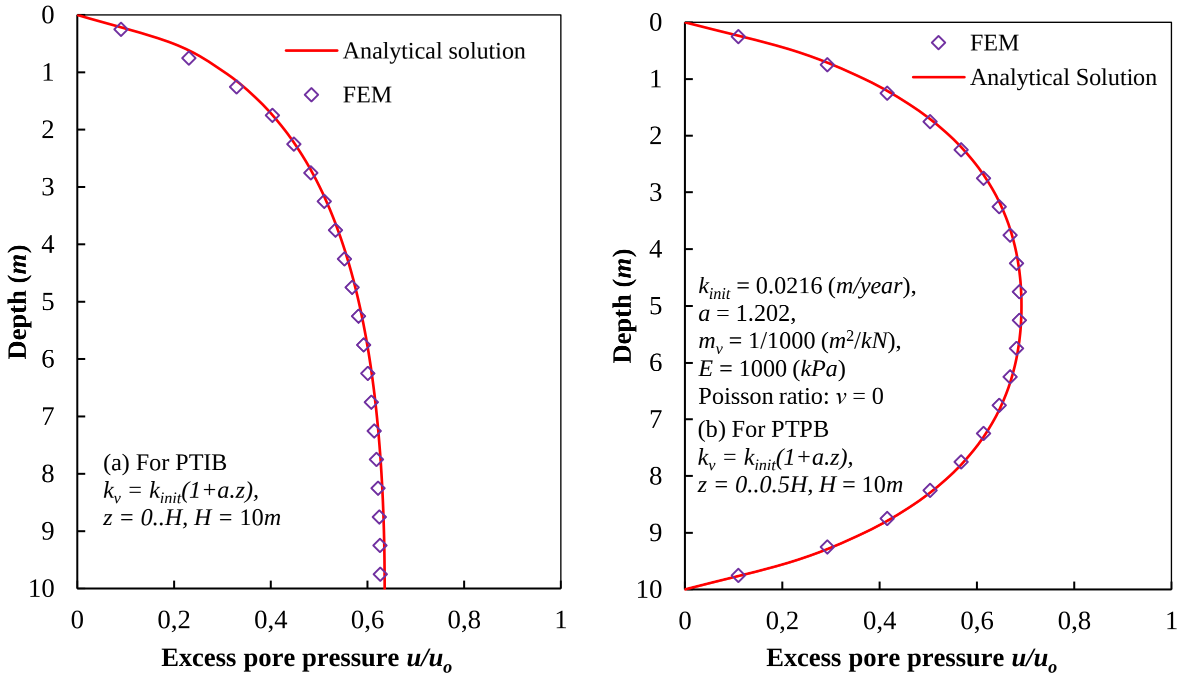

Fig. 5. (a) Comparison of the pore pressure distribution of analytical solution and FEM at U=50% for case PTIB, (b) Comparison of the pore pressure distribution of analytical solution and FEM at U=50% for case PTPB

The results of Case 1 are shown in Fig. 4. It is seen that the FEM employing the constant permeability provides the results similar to the Terzaghi's 1D solution [1]. It also means that the FEM model is suitable for solving the consolidation problem in 1D. Figure 5 shows a comparison of the excess pore pressures obtained by the analytical solution and FEM method for the cases of PTIB and PTPB at the average consolidation degree of 50%. Figure 6 shows a comparison of the average consolidation degree obtained by the analytical solution and FEM method for the cases of PTIB and PTPB.

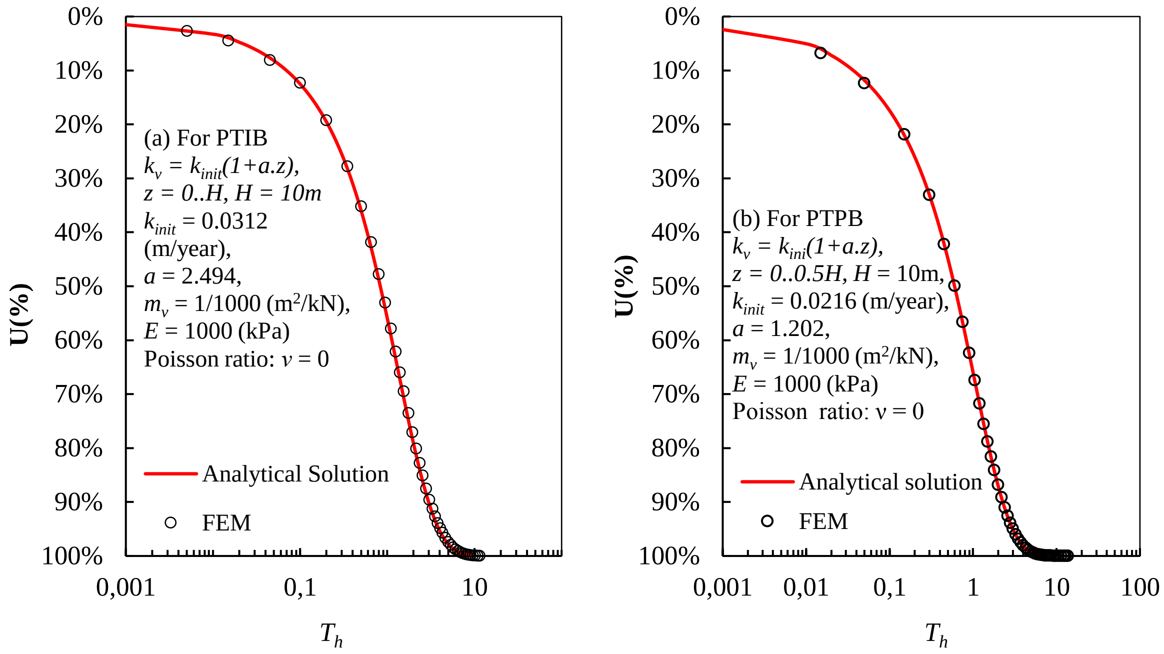

Fig. 6 – (a) Comparison consolidation degree of analytical solution and FEM for case PTIB, (b) Comparison consolidation degree of analytical solution and FEM for case PTPB

From the above comparison Figs. 5 and 6, which show that pore pressure and average consolidation degree of both methods are very matched. The user can apply FEM commercial software with the modified coefficient of permeability to achieve the same results as the present analytical solution.

4.3 Discussions

The present solution is compared with the theoretical and Chai's solution which are detailed in Appendix C.

(1) PTIB case with $l = H = 10m$ and $z = (0 \div H)$

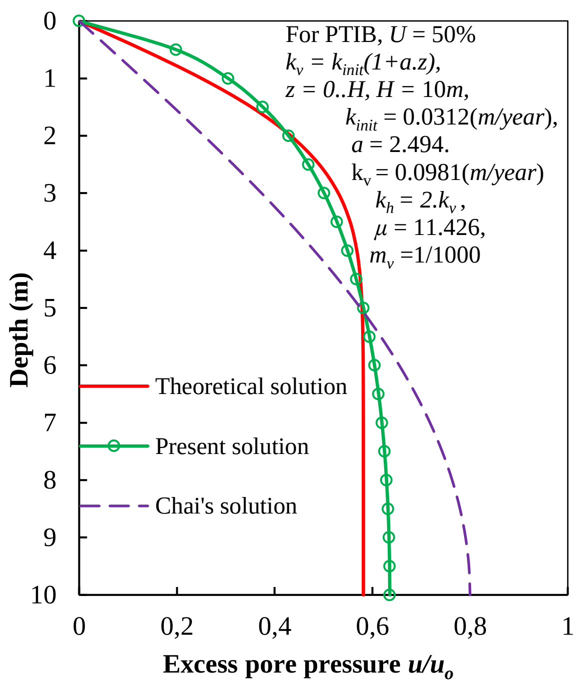

Fig. 7 – Comparison of Excess pore pressure distribution at consolidation degree U=50% for PTIB

For this case, the drainage length $l = H = 10m$, depth $z = (0 - H)$, factor of PVD geometry $\mu = 11.426$, and the equivalent permeability by Chai' method $k_{ev} = 0.117$. Chai's solution uses the 1D Terzghi's solution [1] to determine the excess pore pressure and average degree of consolidation. The results of excess pore pressure along the depth at the average consolidation at 50% are shown in Fig. 7.

It is obviously seen that the curve of excess pore pressure obtained by the proposed method is very much closer than the curve obtained by Chai's solution when it is compared to theory. Particularly, at the depth of 10m, the proposed solution provides a difference of excess pore pressure just 9% of the theory, while the error by the Chai's method is 37.5%. The comparison of the average degree of consolidation is shown in Fig. 8.

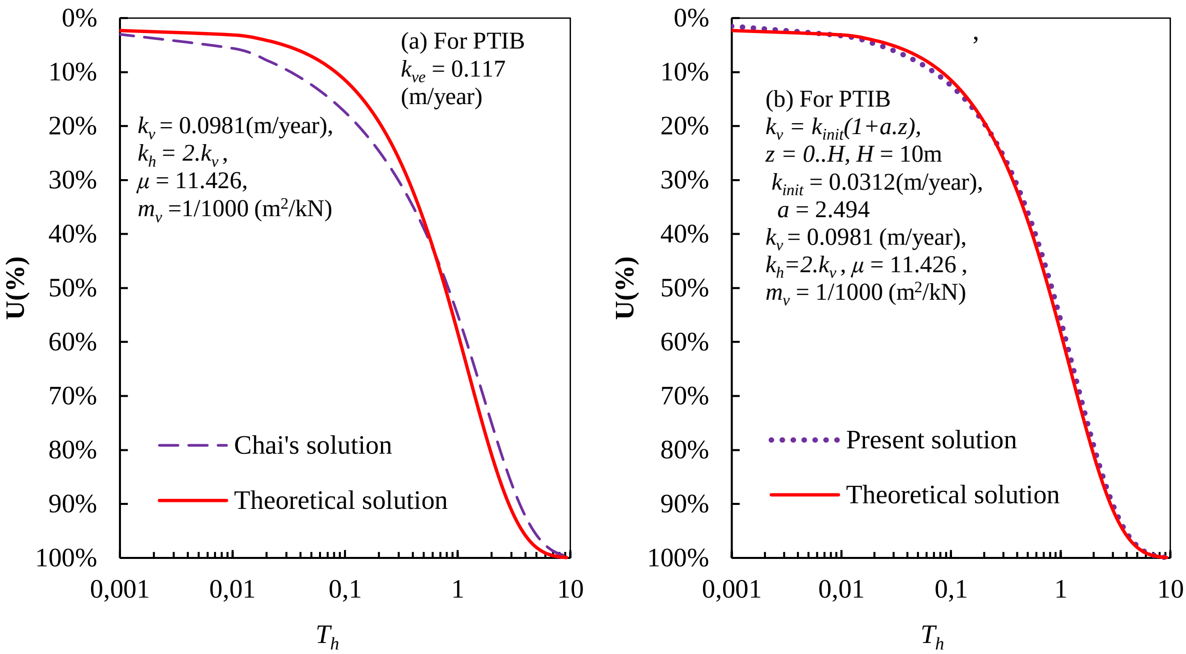

Fig. 8 – (a) Comparison of average degree of consolidation of Chai's solution and theoretical solution for PTIB, (b) Comparison of average degree of consolidation of the present solution and theoretical solution for PTIB

Fig. 8 shows that the obtained average degree of consolidationby the present solution is very closed and almost overlapped on the curved obtained by the theory, while the curve obtained by Chai's solution is fast at the beginning stage of the consolidation process but low at the end stage.

(2) PTPB case with $l = H/2 = 5m$ and $z = (0 \div H/2)$.

For this case, the geometry factor and the equivalent coefficient of permeability by Chai's solution will respectively be $\mu = 10.436$ and $k_{ev} = 0.392$ (m/year). The comparison of excess pore water pressure at U = 50% is shown in Fig. 9

It is seen that the excess pore pressure by the present solution is very close to the one by the theoretical solution. As compared to the theory, the difference of excess pore pressure at z = 5m obtained by the present is 5.7% while Chai's solution gives 24.7%. It can confirm that the present solution can predict the excess pore pressure with depth appropriately. Furthermore, the comparison of the average degree of consolidation is demonstrated in Fig. 10.

Fig. 10 shows that Chai' solution provides faster consolidation and smaller consolidation rate at the beginning and later consolidation process, respectively. The present solution performances the average degree of consolidation very close to the theory

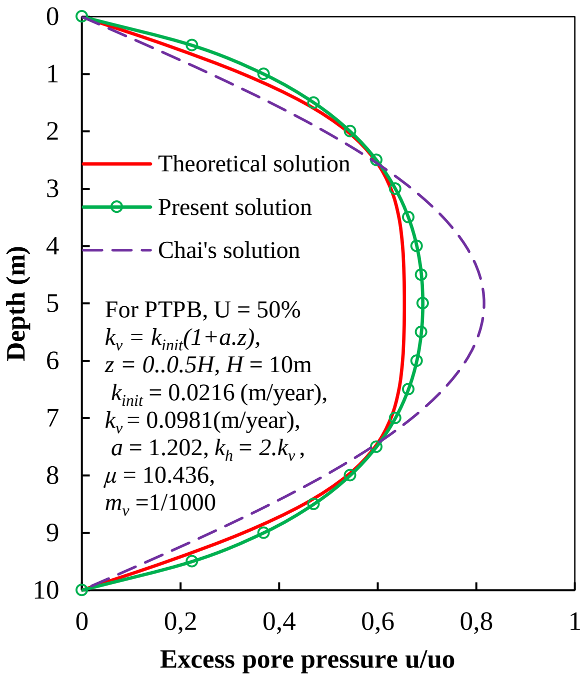

Fig. 9 – Comparison of excess pore pressure distribution at consolidation degree 50% for PTPB

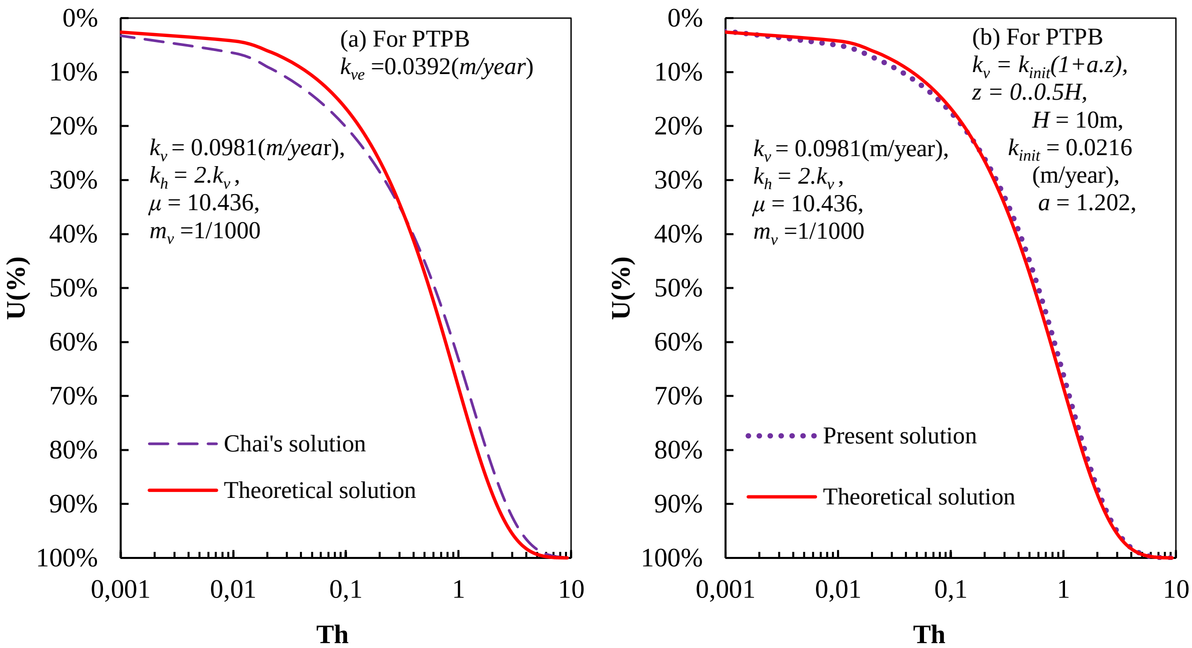

Fig. 10 – (a) Comparison of average degree of consolidation of Chai's solution [11] and theoretical solution for PTPB,(b) Comparison of average degree of consolidation of present solution and theoretical solution for PTPB

5 Conclusions

The problem of consolidation with PVD can be approximately modelled in 1D which is simple and more accurate than the Chai's approach [11]. The proposed method modified from the Terzaghi's theoretical consolidation in 1D [1] with the assumption of the linearly increased vertical permeability as $k_z = k_{ini}(az+1)$ which is a simple solution and easily implement. The analytical results solved from the proposed method are similar to the results by FEM using ABAQUS software. The coefficients of permeability with depth should be analytically calculated by using the Eqs. 22, 23 and 24 and then input to the FEM software for the analysis. The excess pore water pressure solved by the present method was fitted with the theoretical one very closely and had the maximum error of less than 10% and more accurate than Chai's approach [11]. The proposed method provides the average consolidation degrees almost the same as the theoretical one.

Acknowledgements

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Appendix A. Derivation of Eq. (2)

The continuity equation for one-dimensional flow in the vertical direction is:

$$\frac{\partial v_z}{\partial z}dx\cdot dy\cdot dz=-\frac{\partial}{\partial t}(n\cdot dx\cdot dy\cdot dz) \tag{A1}$$

The Darcy's Law is applied to the permeability of water

$$v=k_v i=-\frac{k_z}{\gamma_w}\frac{\partial u(z,t)}{\partial z} \tag{A2}$$

where i is the hydraulic gradient, k_z is the coefficient of permeability depending on the depth z, u is excess pore water pressure, and z is the depth from the top soil.

Substituting (A2) to (A1) and expressing the porosity n in terms of void ratio e, the following equation can be obtained

$$\frac{\partial}{\partial z}\left(\frac{k_z}{\gamma_w}\frac{\partial u(z,t)}{\partial z}\right)=\frac{1}{1+e}\frac{\partial e}{\partial t} \tag{A3}$$

In simplified analysis, assuming that the volume compression modulus m_v is the same for every state of consolidation and the rate of effective pressure change are equal to the rate of excess pore water pressure change, the equation (A3) can be rewritten

$$\frac{\partial}{\partial z}\left(\frac{k_z}{\gamma_w}\frac{\partial u(z,t)}{\partial z}\right)=m_v\frac{\partial u(z,t)}{\partial t} \tag{A4}$$

Rearrange equation (A4)

$$\frac{\partial u(z,t)}{\partial t}=\frac{\partial}{\partial z}\left(\frac{k_z}{\gamma_w m_v}\frac{\partial u(z,t)}{\partial z}\right) \tag{A5}$$

Appendix B. Derivation of Eq. (21)

The average consolidation degree in vertical direction by Terzaghi's consolidation solutions was

$$\bar{U}_v(t)=1-\sum_{1}^{m}\frac{8e^{\left\{-\left(\frac{\pi(2m-1)}{2}\right)^2\frac{c_v t}{l^2}\right\}}}{\pi^2(2m-1)^2} \tag{B1}$$

The average consolidation degree by horizontal drain of Hansbo's solution [17].

$$\bar{U}_h(t)=1-e^{-\frac{8}{\mu}\frac{c_h t}{D_e^2}} \tag{B2}$$

The value μ can be express as

$$\mu = \ln\left(\frac{n}{s}\right) + \frac{k_h}{k_s}\ln(s) - \frac{3}{4} + \pi\frac{2l^2 k_h}{3q_w} \quad \text{(B3)}$$

where, $n = D_e/d_w$, D_e is the diameter of a unit cell, d_w is the diameter of the drain, $s = d_s/d_w$, d_s is the diameter of the smeared zone, k_h and k_s are horizontal hydraulic conductivities of the natural soil and smeared zone, respectively; l is drainage length, and q_w is discharge capacity of PVD.

Carrillo's theoretical solution[30] for the average consolidation degree of both vertical and horizontal drain.

$$\bar{U}_{vh}(t) = 1 - \left\{1 - \bar{U}_v(t)\right\}\left\{1 - \bar{U}_h(t)\right\} \quad \text{(B4)}$$

Substitute equation (B1) and (B2) into equation (B4)

$$\bar{U}_{vh}(t) = 1 - \sum_{m=1}^{\infty}\frac{8e^{-\left\{\left(\frac{\pi(2m-1)}{2}\right)^2\frac{c_v}{l^2} + \frac{8}{\mu}\frac{c_h}{D_e^2}\right\}t}}{\pi^2(2m-1)^2} \quad \text{(B5)}$$

Appendix C. Theoretical solution and Chai's method [11]

Equation (21) from Carrillo's theoretical solution [30] can be used for determining the average consolidation degree of PVD by the combination of the vertical and horizontal drain. In order hand, the average excess pore water pressure with depth of PVD can be determined by the following equations.

$$\bar{u}(z,t) = u_0\bar{U}_{vh}(z,t) \quad \text{(C1)}$$

$$\bar{U}_{vh}(z,t) = 1 - \left\{1 - \bar{U}_v(z,t)\right\}\left\{1 - \bar{U}_h(t)\right\} \quad \text{(C2)}$$

$$\bar{U}_v(z,t) = 1 - \frac{u_v(z,t)}{u_0} \quad \text{(C3)}$$

$$u_v(z,t) = \sum_{1}^{m}\frac{4u_0}{\pi(2m-1)}\sin\left(\frac{\pi(2m-1)z}{2l}\right)e^{-\left(\frac{\pi(2m-1)}{2}\right)^2\frac{c_v t}{l^2}} \quad \text{(C4)}$$

Substitute (C4), (C3), (C2) and (2B) into (C1) to obtain the average excess pore water pressure with depth of theory method

$$\bar{u}(z,t) = u_o\left(1 - \sum_{1}^{m}\frac{4}{\pi(2m-1)}\sin\left(\frac{\pi(2m-1)z}{2l}\right)\times e^{-\left(\left(\frac{\pi(2m-1)}{2}\right)^2\frac{c_v}{l^2} + \frac{8}{\mu}\frac{c_h}{D_e^2}\right)t}\right) \quad \text{(C5)}$$

Chai's method[11] is utilized for this paper in simplified assumption that the change of the volume compressive coefficient (m_v) during consolidation state are the same in both horizontal and vertical direction. Then the equation can be rewritten under vertical consolidation coefficient c_v and horizontal consolidation coefficient c_h

$$c_{ve} = \left(1 + \frac{2.5\times l^2\times c_h}{\mu\times D_e^2\times c_v}\right)\times c_v \text{ or } k_{ve} = \left(1 + \frac{2.5\times l^2\times k_h}{\mu\times D_e^2\times k_v}\right)\times k_v \quad \text{(C6)}$$

$c_{ve}hg$ is the equivalent vertical consolidation coefficient by Chai's Method [11]. This equivalent coefficient c_{ve} is then substituted to Terzaghi's solutions [1] for the excess pore water pressure and average consolidation degree.

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