SUPPLY CHAIN CONTRACT DESIGN IN SUPPLIER- VERSUS BUYER-DRIVEN CHANNELS

A Dissertation

by

XINGCHU LIU

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2005

Major Subject: Industrial Engineering

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Approved as to style and content by:

Sıla Çetinkaya
(Chair of Committee)

Guy L. Curry
(Member)

Martin A. Wortman
(Member)

Catherine Yan
(Member)

Brett A. Peters
(Head of Department)

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ABSTRACT

Supply Chain Contract Design in Supplier- versus Buyer-Driven Channels. (May 2005)

Xingchu Liu, B.En.; M.S., Tsinghua University, P.R.China

Chair of Advisory Committee: Dr. Sıla Çetinkaya

In the context of supply contract design, the more powerful party has the liberty of withholding private information which also improves its bargaining power. Traditionally, the supplier (e.g., manufacturer) has been more powerful, and, hence, the existing literature in the area emphasizes supplier-driven contracts. However, in some current markets, such as the grocery channel, the bargaining power has shifted to the buyer (e.g., retailer). For example, in the United States, large retailers, such as Wal-Mart, exert tremendous market power over their suppliers. Also, with the advent of the Internet, buyers have gained access to much more information about multiple potential suppliers. Hence, this dissertation takes into account the recent trends in power shifting between suppliers and buyers, and it attempts to provide a comparison of optimal supply contract designs in supplier- versus buyer-driven channels. This research is unique in that we explore the impact of both power shifting and information asymmetry while designing optimal supply chain contracts under supply uncertainty and competition. Placing an emphasis on the cases of stochastic and/or price-sensitive demand, we work on several novel problems in stochastic modeling, nonlinear and dynamic optimization, and game theory. Hence, this research has roots in applied probability, optimization, inventory theory, game theory, and economics. The goal is to advance our practical knowledge of designing implementable contracts because such knowledge is crucial for optimizing supply chain performance

in the real world. This dissertation provides insights about

- the individual and joint impacts of the power structure and information asymmetry on supply chain performance,
- the value of information for contract design in supplier- versus buyer-driven channels,
- the impact of supply uncertainty and supplier competition on contract design in supplier- versus buyer-driven channels.

To my Parents and my Fiancé Xin Zhang

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CHAPTER I

INTRODUCTION

Interest in the field of supply chain management has grown tremendously during the past decade among both academics and practitioners. With that interest has come a growing body of work on supply chain contracts. Since few firms are large enough and few products are simple enough for one organization to manage the entire provision of goods, most supply chains require the coordination of independently managed entities who seek to maximize their own profits. For this reason, contractual arrangements have been used as an efficient means to coordinate entities within the supply chain to improve system-wide efficiency. This approach is called "channel coordination," a term adopted from the marketing literature. Channel coordination may be achieved by first identifying the intra-chain dynamics which cause inefficiency and then modifying the structure of these relationships contractually in order to more closely align individual incentives with global optimization. Supply chain contracts also help to divide the profits and to distribute the costs and risks arising from various sources of uncertainty, e.g. market demand, selling price, product quality, and delivery time, between the entities in the supply chain. Another important rationale for contracts is that they facilitate long-term partnerships by delineating mutual concessions that favor the persistence of the business relationship and make its terms more explicit.

The volume of work on contracts is enormous in the economics and operations management literature. Tirole (1988) and Tsay et al. (1999) provide excellent overviews. While the work in economics contributes a deeper understanding of the

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This dissertation follows the style and format of *Management Science.*

basic issues of motivation for a broad variety of contractual structures, the work in operations management focuses more explicitly on the modeling of material flows and their complicating factors, such as the impact of demand uncertainty, forecasting, and production capacity (Tsay et al. 1999).

A supply chain contract taxonomy provided by Tsay et al. (1999) classifies contracts into eight categories based on the types of clauses they contain. These are specification of decision rights, quality, pricing, minimum purchase commitments, quantity flexibility, buy-back or return policies, allocation rules, and lead time. We observe that the last six of these contract types approach the problem from the supplier's (e.g., manufacturer's or distributor's) perspective. These studies assume that buyers (e.g., retailers or customers) are passive decision makers in the sense that, for example, manufacturers can influence their retailers' decisions through various incentives, price schedules, and cooperation. This dissertation will concentrate on supply contract design issues where this assumption is violated so that the buyers act as the channel leaders. Such channels are called *buyer-driven* channels whereas the traditional work in supply contracts concentrates on *supplier-driven* channels.

The suppliers in monopolistic markets are able to charge above-competitive prices and, thereby, establish supplier-driven channels. However, as Messinger and Narasimhan (1995) point out, in some markets such as the grocery channel, the bargaining power has shifted to the buyer (e.g., retailer). For example, in the United States, large retailers such as Wal-mart exert tremendous market power over their suppliers. Also, with the advent of the Internet, buyers have access to much more information about multiple potential suppliers. Ertek and Griffin (2002) quote a recent survey by Forrester, which indicates that manufacturers of standard products expect approximately one-third of their sales to occur on-line within the next two years and that 40% of producers expect the Internet to squeeze prices. Thus far, however, only a limited number of research papers have analyzed this shift in the power structure, which is one of the important motivations of the dissertation.

Another deficiency in the current literature pertains to information asymmetry. Information asymmetry usually occurs in a supply chain when some entities in the chain are better informed than the others. Despite recent advances in information technology and the trend towards sharing information among supply chain partners, information asymmetry remains a key feature of real supply relationships. Since the entities in a supply chain may belong to different firms that have conflicting objectives, and/or they may not have access to private information, a system-wide optimal solution may not be implementable unless it can fully resolve any incentive alignment problems caused by asymmetric information in the system. This observation provides another important motivation for the dissertation in which we will develop optimal supply chain contracts via exploring the impact of both the power structure and information asymmetry. More specifically, our goal is to develop a supply contract design framework that considers decision power and information asymmetry issues in supplier- and buyer-driven channels. In order to achieve this goal, we attempt to

- generalize existing channel coordination mechanisms, such as price protection and return policies, which provide benchmarks for supply contracts,
- analyze optimal supply contract design mechanisms in buyer-driven channels,
- examine the impact of the power structure on supply contract design,
- evaluate the value of information in supply contract design for supplier- and buyer-driven channels, and
- provide practical solutions and insights to improve system-wide efficiency in supplier- and buyer-driven channels.

While focusing on these objectives, we also investigate the impact of supply uncertainty and supplier competition. Throughout this dissertation, we will refer to the supplier as *he* and the buyer as *she*.

I.1. Scope of the Dissertation

In order to develop a supply contract design framework that considers power structure and information asymmetry issues in supplier- and buyer-driven channels, this dissertation investigates the following four sets of problems.

I.1.1. Buyer's Impact

Recent work provides insight into several contract-based channel coordination mechanisms commonly used in practice, such as price protection and returns agreements (Lee et al. 2000, Taylor 2001), revenue sharing contracts (Cachon and Lariviere 2002), and supply contracts with options (Barnes-Schuster et al. 2002). By considering a two-period stochastic demand model and ignoring the possibility of returns or the disposal of unsold inventory, Lee et al. (2000) show that price protection policies provide a basis for channel coordination, i.e., a win-win contract. Taylor (2001) examines the use of three different channel policies that are used in declining price environments: price protection (P), midlife returns (M) and end-of-life returns (E). He shows that PEM guarantees channel coordination, i.e., again, a win-win solution.

However, these two papers, by Lee et al. (2000) and Taylor (2001), assume that the supplier (manufacturer) provides the contract and the buyer responds by passively choosing the order quantity. In practice, the buyer has impact on the retail market with pricing decisions (Emmons and Gilbert 1998) and sales efforts (Taylor 2002). Furthermore, the buyer has the liberty of withholding private information (Corbett and Tang 1999) which also improves her bargaining power. Therefore, there is a need to revisit and analyze price protection and returns agreements to consider the buyer's impact more explicitly. In this dissertation, we investigate the following problems in this area:

- The design of optimal price protection contracts with retail-pricing decisions.
- The design of optimal price protection contracts with buyer's sales effort considerations.
- The design of optimal price protection contracts with incomplete information about the buyer's cost structure.
- The design of optimal return policies with incomplete information about the buyer's cost structure.

I.1.2. Information Asymmetry

The effect of bargaining power on supply chain performance is an interesting issue that has arisen in recent years. Ertek and Griffin (2002) explore the impact of the power structure on a two-stage supply chain and develop pricing contracts in a case where the supplier has dominant bargaining power and a case where the buyer has dominant bargaining power. They consider a pricing scheme for the buyer that involves both a multiplier and a constant mark up and show that it is optimal for the buyer to set the mark-up to zero and use only a multiplier. The analysis in this paper is based on the assumption that each party in the supply chain has full information of the channel. However, private information usually exists in a supply chain, and it is often associated with the bargaining power. Therefore, the contract provider has to take information asymmetry into account.

Corbett and Groote (2000) derive an optimal quantity discount policy under asymmetric information about the buyer's holding costs. Corbett and Tang (1999) analyze three types of contracts from the supplier's point of view with information asymmetry considerations about the buyer's cost structure: the one-part linear contract, the two-part linear contract, and the two-part nonlinear contract. Under the one-part linear contract, the supplier charges a constant unit wholesale price. Under the two-part linear contract, the supplier charges a constant unit wholesale price but offers a fixed lump side payment to the buyer. Under a two-part nonlinear contract, the supplier offers a menu of contracts where each item on the menu consists of a pairing of a unit wholesale price and lump sum side payment. Corbett and Tang find that the value of information is higher under two-part contracts and that the value of offering two-part contracts is higher under full information. Also, they point out that more flexible contracts allow the supplier to trade with buyers with higher costs. Ha (2001) considers designing a contract to maximize the supplier's profit in the newsboy problem when demand is stochastic and price-sensitive and the supplier has incomplete information on the marginal costs of the buyer. He shows that the supplier's profit is lower than in the complete information case while the buyer's is improved.

As we have mentioned earlier, we seek to extend the current literature by exploring the impact of the power structure and information asymmetry simultaneously. In this dissertation, we explore the impact of the power structure and the value of information on designing contracts in a two-stage supply chain with a single-product where the product is shipped from a supplier to a buyer at a wholesale price and then sold to a price-sensitive market. Concentrating on a buyer-driven channel, we consider three types of contracts with and without information asymmetry considerations. These are i) one-part linear contracts, ii) two-part linear contracts, and iii) two-part nonlinear contracts. Under the one-part linear contract, the buyer declares a non-negative price multiplier, denoted by α, and states that she will set the retail price equal to the product of α and the wholesale price. Under the two-part linear contract, the buyer declares an α and charges a slotting fee to the supplier. Under the two-part nonlinear contract, the buyer offers a menu of contracts, where each item on the menu consists of a pair of α and a slotting fee, leaving it up to the supplier to select the pair of his choice. We compute the parameters of optimal supply contracts with information asymmetry considerations in buyer-driven channels for these three types of contracts. Utilizing our own and Corbett and Tang's (1999) results, we compare the performances of the supply chains (e.g., expected profits) under optimal buyer- and supplier-driven contracts. Hence, we explore the impact of the power structure on supply chain performance with, and without, information asymmetry considerations. Our investigations are highlighted below:

- The design of different types of optimal contracts for a buyer-driven channel with full information sharing.
- The design of different types of optimal contracts for a buyer-driven channel with incomplete information of the cost structure.
- The analysis of the impact of the power structure and information asymmetry on supply chain performance under different types of contracts.

I.1.3. Supply Uncertainty

The existing supply contract design literature ignores supply uncertainty issues. That is, this literature assumes that when an order is placed, it is either filled immediately (the case of zero lead time) or after a deterministic, or perhaps random, lead time. In reality, the supply of a product may sometimes be interrupted due to suppliers' random equipment breakdowns, maintenance durations, delays in raw material supply, etc. Hence, supply availability remains an important, but overlooked, issue. Another reason for unpredictability in the supply process is uncertainty in the yield quantity due to the random proportion of defective items received.

A properly designed supply contract provides an opportunity to improve system-wide profits under supply uncertainty by explicitly defining how to share the cost and risk caused by this uncertainty, i.e., how to coordinate the channel. With cost-sharing, the buyer can order more in each period so as to achieve optimal system-wide profits (therefore, the supplier and the buyer have a bigger pie to share) without increasing her cost. This is the main idea that we will implement in designing an optimal contract with supply uncertainty considerations.

When the supplier and the buyer are two independent entities with incentive conflicts, the buyer may not have access to complete information about the supplier's cost structure (such as setup cost) or supply uncertainty. To provide more implementable supply chain contracts, we further develop optimal contracts with information asymmetry considerations under conditions of supply uncertainty. Based on our results on contract design addressing the above issues, we explore further the value of supply uncertainty information.

Previous work on the value of flow uncertainty information focuses on demand uncertainty. For example, Cachon and Fisher (2000) find that the value of sharing demand information is very limited under stationary demand. Lee et al. (2000), by contrast, suggest that the value of demand information sharing can be quite high, especially when demands are significantly correlated over time. However, to the best of our knowledge, there is no work on the value of supply uncertainty information in the literature, which is an important motivation of the dissertation. The problems investigated in this area are summarized below:

- The design of optimal cost-sharing contracts under supply uncertainty with full information.
- The design of optimal cost-sharing contracts under supply uncertainty with incomplete information about supplier's costs.
- The design of optimal cost-sharing contracts with incomplete information about supply quality/yield/availability.
- Exploration of the value of supply uncertainty information and the impact of the power structure.

I.1.4. Supplier Competition

Most studies to date have focused on markets consisting of exclusive dealers that sell only one producer's brand; little attention has been given to the larger segment of most consumer goods markets in which retailers sell multiple (often highly substitutable) brands at the same location. This latter channel structure represents numerous markets including those consisting of specialty stores, such as consumer electronics, sporting goods, and automobile parts, etc. As Tsay et al. (1999) point out, "another deficiency in the current literature is the lack of attention to competition, either between multiple buyers or multiple suppliers. Buyers that share a common supplier and compete in the same consumer market might behave in a way that obstructs their competitors' access to suppliers ... Multiple suppliers to a common buyer might need to alter their price, service, lead time, or flexibility offerings in light of the competitive environment."

Although the consideration of competition is rare in the operations management literature, there is substantial coverage in the economics literature. For example, Choi (1991) analyzes a channel structure with two competing manufactures and a common retailer and studies three different power structures, i.e., supplier dominant, balanced power, and buyer dominant structures. He shows that all channel members are better off when no one dominates the structure. His work is followed by Trivedi (1998), who models a channel structure in which there are duopoly manufacturers and duopoly common retailers. Trivedi shows that the presence of competitive effects at both the retail and manufacturer levels of distribution has a significant impact on profits and prices.

Our research is based on the above work in the economics literature, but it focuses on designing effective contracts to improve supply chain performance. In particular, we investigate the following problems:

- The design of supply contracts under supply uncertainty and supplier competition.

- The design of supply contracts under power structure, supply uncertainty, and supplier competition considerations.

- Exploration of the impact of power structure.

This research has roots in applied probability, optimization, inventory theory, game theory, and economics. We intend to develop optimization models aimed at minimizing entity/system costs or maximizing entity/system profits for the purpose of optimal contract design. Our focus will be on probabilistic demand problems. From the methodology perspective, our optimization models are all stochastic modeling problems that require unconstrained/constrained dynamic or nonlinear optimization techniques, depending on the factors considered.

As previously indicated, current supply chain contract practices and the literature will be enriched by more research on power shifting, information asymmetry, supply uncertainty, and competition considerations. This dissertation is unique in that we explore the impact of both power shifting and information asymmetry while designing optimal supply chain contracts under supply uncertainty and competition. Hence, this research will advance our practical knowledge of designing implementable contracts, and such knowledge is crucial for optimizing supply chain performance in the real world. This dissertation will also shed light on the value of supply uncertainty information and the role of suppliers' and buyer's competition in the design of supply chain contracts. These are two important practical issues neglected in the existing literature.

In the next section, we present a classification of the supply chain contracts literature, which helps demonstrate the importance of our research and how our work will fit in with current academic interest and recent trends in supply chain practice.

I.2. Related Literature

Contracts are, of course, a major consideration in law, and the literature in this area is enormous. There is also a substantial literature on contracts in the economics literature. To distinguish our supply chain contract study, we will define a supply chain contract as follows: a supply chain contract is a mutual commitment between supplier(s) and buyer(s) on operational details including funds, goods, and information flow to improve the individual and/or joint entities' performances. By defining the purpose as "to improve the individual and/or joint entities' performances," we distinguish our concentration from the work in law which focuses on the legality of contracts. Also, our concentration is different from the work in economics since our emphasis is on operational details.

In supply contract design, the issues of who controls what decisions and how entities will be compensated are critical. An understanding of contractual forms and their economic implications is, therefore, an important part of evaluating supply chain performance. Due to the importance of supply contracts, the field has developed in many directions. A taxonomy of work in this area would be very helpful for understanding supply chain contracts. However, we still need to define what constitutes a supply contract paper even after we restrict our search by the above definition. As Tsay et al. (1999) point out , "the challenge arises because, broadly speaking, all literature on inventory theory could qualify." Therefore, we will further narrow our search to those works in which the analysis explicitly offers guidance for negotiating the terms of the relationship between the suppliers and buyers. Thus, we review in depth only those papers which treat the terms of the contractual relationship as decision variables.

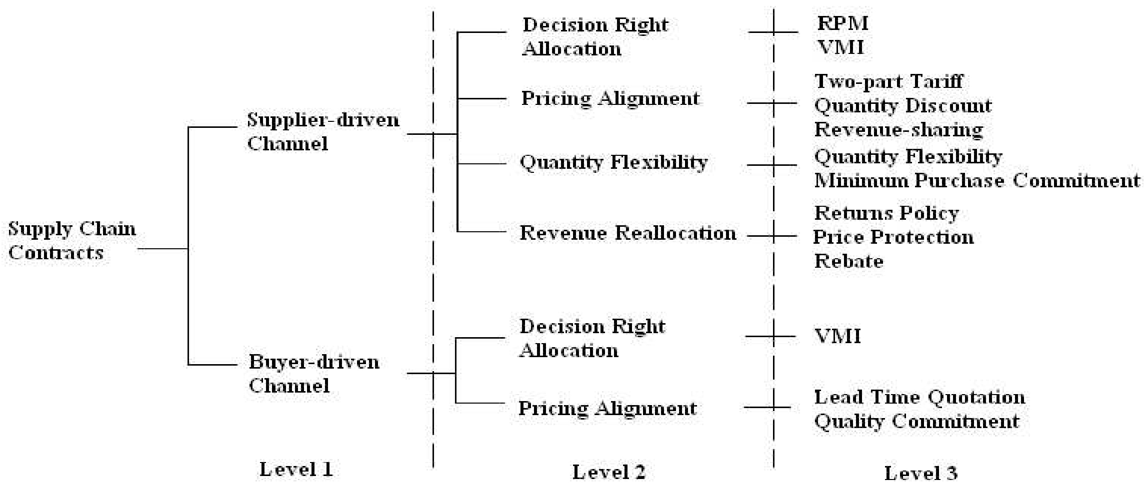
Tsay et al. (1999) provide a good taxonomy based on contract clauses. In their work, supply contracts are classified into eight categories: specification of decision rights, pricing, minimum purchase commitments, quantity flexibility, buyback or returns policies, allocation rules, lead time, and quality. Because several novel works on supply contracts, such as price protection, revenue-sharing, etc., have appeared in the last few years, the above taxonomy needs to be extended to include more categories. Expanding this taxonomy will be necessary as long as innovative work on new contract clauses continues to appear in the future. In other words, the major shortcoming of this taxonomy is that it does not provide a stable structural classification that allows for the incorporation of new specific clauses. To address this shortcoming, in the following section, we present a three-level classification, which is further narrowed down to the one-supplier-one-buyer supply chain structure. The reason for focusing on the one-supplier-one-buyer structure, rather than including multiple-buyer and multiple-supplier structures, is that the work on supply chain contract design based on the one-supplier-one-buyer structure is ample and more in depth, which provides an insightful understanding of intra-chain dynamics under exciting contractual arrangements. The work on multiple-buyer and multiple-supplier structures, on the other hand, is relatively scarce and rarely offers optimal policies.

I.2.1. A Classification of the Supply Chain Contract Literature

It is worth noting that although a supply contract is a mutual commitment, the final terms reached by both parties depend on their bargaining power. Usually one of the entities initiates the contract. For example, a returns policy is initiated by a supplier who takes responsibility for unsold inventory; a lead time quotation policy is provided by a buyer to reduce the risk of supply uncertainty. As the leader of the channel, one party initiates a contract and thus has the flexibility to adjust the contract and the opportunity to gain more profit than the follower. The bargaining power structure is a key issue in supply contract design, no matter what clauses are considered. Therefore, as the top level of the taxonomy, we classify the supply contract design literature into two categories: supplier-driven and buyer-driven channels.

The second level of the taxonomy has four categories under the supplier-driven channel: decision right allocation, pricing alignment, quantity flexibility, and revenue reallocation. Classification on the second level is based on the location of the main thrust in the contract through the time-line of the supply chain. That is, after the *decision right* is specified, the *pricing scheme* is determined, and then a certain *order quantity* is shipped, and, finally, the *revenue* is realized. Similarly, the second level has two categories under the buyer-driven channel: decision right allocation and pricing alignment. Although we expect more categories will be added under the buyer-driven

Figure 1 A Classification of the Supply Chain Contract Literature



channel in the future, currently only a limited number of papers analyze the buyer-driven channel. The third level of the taxonomy is based on the contract clauses. Figure 1 summarizes the classification.

Below, we briefly describe the concept represented by each category at the second level as well as which category the clauses of the third level fit into.

- Decision Right Allocation in supplier-driven and buyer-driven channels: Each party's decision rights are specified and a way is provided to adjust the cost structure and reallocate the risk so that the entities' information and incentives are more aligned with the system-wide optimum. *Resale Price Maintenance* (RPM) and *Vendor Managed Inventory* (VMI) fit in this category.

- Pricing Alignment in supplier-driven channel: The supplier specifies a pricing scheme that favors the buyer in exchange for her additional commitment or responsibility. The category includes *Two-Part Tariff*, *Quantity Discount*, and

Revenue-Sharing.

- Pricing Alignment in the buyer-driven channel: The buyer aligns her payment with the supplier's commitment or realized service level. *Lead Time Quotation* and *Quality Commitment* belong in this category.

- Quantity Flexibility in the supplier-driven channel: The supplier provides the buyer with flexibility against demand uncertainty. The quantity the buyer ultimately purchases may deviate from a previous planning estimate, subject to certain constraints and/or financial consequences. The category includes *Quantity Flexibility* and *Minimum Purchase Commitment*.

- Revenue Reallocation in the supplier-driven channel: The supplier is willing to share the cost and partially return his revenue to the buyer after the demand is realized so that the buyer's behavior is better aligned with the system-wide optimum. *Returns Policy*, *Price Protection*, and *Rebate* fit here.

We note that several papers are candidates for more than one of the above categories. For example, the work of Wang et al. (2004) combines RPM, VMI, and revenue sharing. This phenomenon is normal since a proper combination of different types of contracts provides an opportunity to capitalize on their merits simultaneously and improve the system efficiency. In fact, we expect many more such combinations to appear in the future. The purpose of our classification is to help explain the intra-chain dynamics under different contractual schemes. We do not intend to tear the literature into isolated pieces. The way that we fit a paper into a category is based on which category the paper fits first based on the time-line of the supply chain. Therefore, we place the work of Wang et al. (2004) into the decision right allocation category.

In the next section, we will review the supply chain contract literature based on our third level of classification.

I.2.2. A Review of the Supply Chain Contract Literature

Since Tsay et al. (1999) provide an excellent review of supply contracts, here we try to avoid overlap by focusing on more recent work. However, since we also attempt to provide a complete picture of the supply chain contract literature, we must review some of the classical work to demonstrate the roots of certain important ideas.

I.2.2.1. Resale Price Maintenance and Vendor Managed Inventory

We note that the economics literature has contributed a great deal to Resale Price Maintenance, in which the supplier is allowed to dictate the conditions of the retail price that the buyer may charge. Rather than summarize this vast literature, we refer the reader to Katz (1989) for a review. In this section, we concentrate on reviewing three recent papers on VMI, in which the supplier assumes the management of the buyer's inventory, making such decisions as when and how much inventory to ship to the buyer. The first paper focuses on the consignment stock which could be part of a VMI scheme to offer superior customer service under competitive pressure. The second paper considers a (z, Z) VMI contract, while the last paper designs a contract that includes both RPM and VMI.

Corbett (2001) considers consignment stock to help reduce cycle stock by providing the supplier with an additional incentive to decrease batch size. Under consignment stock, the supplier owns the inventory held at the buyer's site until it is consumed. Corbett analyzes both supplier-driven and buyer-driven channels. First, the optimal menu of contracts is derived in a buyer-driven channel when the supplier has private information about the setup cost. He shows how consignment stock can help reduce the impact of this information asymmetry. Then, he studies consignment under the assumption that the supplier cannot observe the buyer's backorder cost. The optimal menu of contracts on a consigned stock level is derived, and he shows that the supplier effectively has to overcompensate the buyer for the cost of each stockout.

Fry et al. (2001) model a type of vendor-managed inventory (VMI) agreement that occurs in practice called a (z, Z) contract. They investigate the savings due to better coordination of production and delivery that is facilitated by such an agreement in a buyer-driven channel. The optimal replenishment and production policies for a supplier are found to be up-to policies. Numerical analysis is conducted to compare the performance of a single supplier and a single retailer operating under a (z, Z) VMI contract with the performance of those operating under a traditional retailer-managed inventory (RMI) with information sharing. They show that the (z, Z) type of VMI agreement performs significantly better than the RMI in many settings but worse in others.

Wang et al. (2004) consider a consignment contract with revenue sharing where a supplier decides on the retail price and delivery quantity for his product and retains ownership of the goods. For each item sold, the retailer deducts a percentage from the selling price and remits the balance to the supplier. They assume that the retailer is the leader and the supplier is the follower. They show that, under such a contract, both the overall channel performance and the performance of individual firms depend critically on the demand price elasticity and on the retailer's share of the channel cost. In particular, the (expected) channel profit loss, compared with that of a centralized system, increases with demand price elasticity and decreases with retailer's cost share, while the profit share extracted by the retailer decreases with price elasticity and increases with retailer's cost share.

I.2.2.2. Two-Part Tariff, Quantity Discount, and Revenue-Sharing

Monahan (1984) first considers the economic implications, from the supplier's point of view, of offering quantity discounts to the buyer. Assuming that the supplier follows a lot-for-lot policy, he shows that a supplier may adjust his present pricing schedule to entice his buyer to increase the present order size by a factor of "k" and increase his profit as a result. Banerjee (1986a) incorporates supplier's inventory carrying costs and develops a generalized version of Monahan's model and demonstrates its equivalence with the joint economic lot size approach suggested by Banerjee (1986b). Lee and Rosenblatt (1986) extend Monahan's model by explicitly incorporating constraints imposed on the amount of discount that can be offered and relaxing the implicit assumption of the lot-for-lot policy adopted by the supplier.

Moorthy (1987) argues that a two-part tariff is superior to quantity discounting because it is simpler; it separates the coordination problem from the profit-sharing problem (coordination is achieved by setting the wholesale price equal to the supplier's production cost, and a lump-sum transfer allows the profits to be split arbitrarily), and leads to fewer legal problems (charging all retailers the same wholesale cost is more consistent with the Robinson-Patman Act). Many subsequent papers in the marketing literature have considered variants of this model.

Weng (1995) studies a policy coordinated through quantity discounts and franchise fees. He shows that the optimal all-unit quantity discount policy is equivalent to the optimal incremental quantity discount policy for achieving channel coordination. Hofmann (2000) analyzes the impact of all-units quantity discounts on channel coordination in a system consisting of one supplier and a group of heterogeneous buyers with a cash-flow-oriented lot-sizing model. His work provides insight into the efficiency of channel coordination through quantity discounts, as well as the influence of the number of price breaks and the heterogeneity of the system. Khouja (2000) extends to the case in which demand is price-dependent, and multiple discounts with prices under the control of the supplier are used to sell excess inventory. Two algorithms for determining the optimal number of discounts under fixed discounting costs for a given order quantity and realization of demand are developed.

Corbett and Groote (1999) derive the optimal quantity discount policy under asymmetric information of the buyer's holding cost. Corbett et al. (2004) analyze three types of contracts: the one-part linear contract, the two-part linear contract and the two-part nonlinear contract with information asymmetry consideration of the buyer's cost structure. They find that the value of information is higher under two-part contracts and the value of offering two-part contracts is higher under full information. Also, they point out that more flexible contracts allow the supplier to trade with buyers with higher costs. Ha (2001) designs a contract to maximize the supplier's profit in the newsboy problem when demand is stochastic and price-sensitive and the supplier has incomplete information about the marginal cost of the buyer. He shows that the supplier's profit is lower than in the complete information case while that of the buyer is improved.

Cachon and Lariviere (2002) study revenue-sharing contracts in a supplier-driven channel with revenues determined by each retailer's purchase quantity and/or price. They demonstrate that revenue-sharing does coordinate a supply chain with a single retailer. Also, they find that a number of other supply chain contracts (e.g., returns policy, quantity discount, quantity-flexibility, rebate) do not effectively coordinate all of the supply chains that they consider. However, they acknowledge several limitations of revenue-sharing contracts. Revenue sharing does not coordinate competing retailers when each retailer's revenue depends on its order quantity and the vector of the retail price. In addition, revenue sharing does not coordinate a supply chain with retail effort dependent demand.

I.2.2.3. Lead Time Quotation and Quality Commitment

Grout and Christy (1993) discuss the incentives faced by a supplier when quoting a delivery time to a buyer. During the contracting process associated with the one-time purchase of some item, the buyer offers a lump-sum bonus of B for on-time delivery, and the supplier in turn specifies a delivery time A. The supplier's risk is due to uncertainty in the production time. If the supplier completes production prior to time A, he collects B but incurs a cost of α per unit of time for holding the item until delivery. If instead, a production delay causes tardiness, the supplier incurs a cost of β per unit of time late. The authors show that a first-best outcome exists under certain pairings of (A, B).

Reyniers and Tapiero (1995) use a simple game-theoretic formulation of a supplier-producer channel to examine the impact of the contract structure on the supplier's quality and the producer's inspection practices as well as its implications for the quality of the end product. Since suppliers often have more information about the quality of the parts than the producer, Lim (2001) investigates contract design when there is incomplete information regarding the quality of the parts. He shows that when the supplier and the producer have to share damage costs, an optimal contract is one where the supplier compensates the producer by the same amount, regardless of his quality type. However, a supplier with low quality is more likely to be offered a contract with an inspection scheme, while a supplier with high quality is likely to be constrained with a warranty scheme.

I.2.2.4. Quantity Flexibility and Minimum Purchase Commitment

Traditional inventory theory generally assumes that the buyer can order any quantity from the supplier at any time. Certainly the buyer prefers to avoid any constraints on her ability to meet her own customers' demand. However, this may be undesirable from the supplier's point of view for a variety of reasons, such as the amplification of demand variance, referred to as "bullwhip effect" (see Lee et al 1997). When signing a contract, since the buyer prefers no quantity commitment while the supplier prefers a fixed quantity commitment for the whole horizon, a compromised contract is finally reached.

One type of these contracts is where the buyer agrees in advance to accept delivery of at least a certain quantity of stock, either in each individual order or cumulatively over some period of time. The supplier may offer the buyer a lower unit cost on items purchased under the contract. Anupindi and Akella (1993) study a class of contracts that require the buyer to commit to purchase a certain minimum quantity in every period of the horizon. Orders for each period are permitted to be adjusted upwards at a price premium. Bassok and Anupindi (1997) analyze a supply contract for a single product that specifies that the cumulative orders placed by the buyer over a finite horizon be at least as large as a given quantity. The demand for the product is uncertain, and the buyer places orders periodically.

Another type of these contracts is when the buyer agrees in advance to accept delivery of a fixed quantity of stock in each period and may adjust the quantity (after obtaining further information such as realized market demand), which is called quantity flexibility. When a buyer's estimate does not entail enforceable commitment, buyers commonly overstate their intended purchase, only to refuse undesired product later on. Quantity flexibility is a way to encourage the buyer to forecast and plan more deliberately and honestly. In exchange, the supplier might need to provide a price break to give the buyer an incentive to participate. Li and Kouvelis (1999) develop valuation methodologies for different types of supply contracts. A "time-inflexible contract" requires the buyer to specify not only how many units she will purchase, but also the timing of her purchases. A "time-flexible contract" allows the buyer to specify the purchase amount over a given period of time without specifying the exact time of purchase. Other than time flexibility, the supplier may offer "quantity flexibility" to the buyer as well, i.e., purchase quantities could be within a pre-specified quantity window. The objective is to minimize the net present value of the purchase cost plus inventory holding costs. Milner and Rosenblatt (2002) analyze a two-period supply contract which allows the buyer to adjust her order after observing the initial demand and demonstrates that flexible contracts can reduce the potentially negative effect of correlation of demand between two periods. Barnes-Schuster, Bassok and Anupindi (2002) investigate the role of options in providing flexibility to a buyer to respond to market changes. They show that, in general, channel coordination can be achieved only if the exercise price is allowed to be piecewise linear.

I.2.2.5. Returns Policy, Price Protection, and Rebate

The supplier often uses returns policies to encourage retailers to stock and price items more aggressively. Pasternack (1985) models a returns policy in a single period setting in which demand is uncertain. He assumes that the retail price is determined exogenously. Pasternack's results indicate that global optimization, i.e., channel coordination, can be achieved with either of the following forms of returns policy: (1) the buyer receives a partial credit for all unsold units, or (2) the buyer gets a full credit for the return of only up to a certain portion of her original order. Emmons and Gilbert (1998) demonstrate the need to incorporate the self-interested behavior of buyers into decisions relating to the establishment of a supplier's pricing policy. They show that uncertainty tends to increase the retail price and under certain conditions, a supplier can increase his own profit by offering to repurchase excess stock from the retailer at the conclusion of the period.

Price protection is a commonly used practice between manufacturers and retailers in the personal computer industry, motivated by the drastic declines in product value during the product life cycle. It is a form of rebate given by the supplier to the buyer for units unsold at the retailer when the price drops during the product life cycle. Lee et al. (2000) show that a properly chosen price protection credit coordinates the channel when the buyer has a single buying opportunity. They also show that when the buyer has two buying opportunities, channel coordination is achieved when the price protection is set endogenously together with the wholesale prices. Taylor (2001) further examines three channel policies that are used in declining price environments: price protection, midlife returns and end-of-life returns, and he shows that the three policies together guarantee both coordination and a win-win outcome.

A rebate is a payment from a supplier to a retailer based on buyer sales to the end consumers. Two common forms of channel rebates are linear rebates, in which a rebate is paid for each unit sold, and target rebates, in which the rebate is paid for each unit sold beyond a specified target level. Taylor (2002) shows that when demand is not influenced by the sales effort, a properly designed target rebate achieves channel coordination and a win-win outcome but that coordination cannot be achieved by a linear rebate in a way that is implementable. When demand is influenced by the sales effort, a properly designed target rebate and returns contract achieves coordination and a win-win outcome.

Through the above supply chain contract literature review, we can confirm that the buyer's impact (in a supplier-driven channel) is rarely examined and the number of papers investigating the buyer-driven channel is limited. Thus, our first and second sets of problems "Buyer's Impact" and "Information Asymmetry" enrich the literature on both supplier- and buyer-driven channels. In addition, the literature reveals that there is little contract design work on "Supply Uncertainty"; therefore, we must carry out our investigation from the ground level. Fortunately, substantial work has been done on supply uncertainty in inventory theory, and we will review that relevant literature in the next section.

I.2.3. A Review of the Supply Uncertainty Literature

Substantial work has been done on supply uncertainty in inventory theory. Silver (1976) initiates a main stream of literature on supply uncertainty which assumes that the quantity of (good) products received by the buyer is (stochastically) proportional to his/her order quantity. That is, Silver extends the classical economic order quantity model by considering the possibility that the quantity received may be a random variable whose expected value is proportional to the original order quantity. He shows that the optimal order quantity depends on the mean and standard deviation of the amount received. Kalro and Gohil (1982) extend Silver's model to include complete and partial backlogging of demands. Mak (1985) extends this model even further by assuming that the fraction of demand lost during a stockout period is itself a random variable. Gerchak and Parlar (1990) consider the problem of jointly determining yield variability and lot sizes when yield variability can be reduced through appropriate investments. Another stream of literature considers supply uncertainty issues as a consequence of random capacity. Parlar and Berkin (1991) consider a generalization of Silver's model in which supply is not always available. That is, the supplier has periods of supply availability and unavailability with random durations. Wang and Gerchak (1996) study the effect of random supply capacity on optimal lot sizing in a continuous review environment.

Some recent research concentrates on the impact of the production process by which defective products are delivered. Porteus (1986) and Rosenblatt and Lee (1986) incorporate the effects of imperfect production processes into the classical economic order quantity model with a deterministic and constant demand rate. Porteus assumes that there is a constant probability that the process goes out of control while producing each unit whereas Rosenblatt and Lee assume that the time-to-failure is exponentially distributed. Kim and Hong (1999) extend Rosenblatt and Lee's model by considering a general time-to-failure distribution. Chung and Hou (2003) provide a more generalized model by allowing shortages. In the above models with imperfect yields, the yield distribution itself is assumed to be known and given. However, investments to improve the production process can have an impact on this distribution. Cheng (1991) considers a constant yield rate and assumes that the unit production cost of the item increases with the yield rate. Cheng shows that for a specific form of the relationship between yield rate and unit production cost, a closed-form expression for the economic production quantity can be obtained. Tripathy et al. (2003) extend Cheng's model by assuming that demand exceeds supply and that the unit cost of production is directly related to reliability and inversely related to the demand rate.

While the above papers focus on continuous-time models, Henig and Gerchak (1990) analyze the structure of inventory policies in the presence of stochastically proportional yield in a periodic review setting with random demand. Erdem and Özekici (2002) consider a periodic-review single item inventory model in a randomly changing environment where all model parameters are specified by the state of the environment. They also consider random yield specified by the random capacity of the vendor, and they show that the optimal policy is a state-dependent-base-stock policy. Yano and Lee (1995) provide a comprehensive review of lot sizing models with random yields that summarizes both continuous-time and discrete-time models.

Since the above papers concentrate on minimizing the cost of a specific entity in the supply chain, they offer only a sub-optimal solution when used for decision making without coordination in a supplier-buyer system. Some recent work highlights the importance of supply uncertainty for effective supply chain coordination. For example, Affisco et al. (2002) propose a quality-adjusted joint economic lot size model, and they study the effect of quality improvement and setup reduction on system-wide cost. Khouja (2003) formulates two-stage models in which the proportion of defective products increases with increased production lot sizes, and he shows that quality considerations can lead to significant reductions in optimal production lot sizes. A basic assumption underlined in these papers is that the system has a single decision maker, and, therefore, the system-wide optimal solution can be easily implemented. However, effective supply chain coordination practices actually require resolving the entities incentive alignment conflicts.

A properly designed supply contract should provide an opportunity to improve system-wide profits under supply uncertainty by explicitly defining how to share the cost and risks caused by this uncertainty, i.e., by addressing the question of how to coordinate the channel. The difficulty in analysis is that placing emphasis on the operational-level details of incentive alignment obstructs the simultaneous analysis of supply uncertainty information. Starbird (2001) examines the effect of rewards, penalties, and inspection policies on the behavior of an expected cost minimizing supplier. However, to the best of our knowledge, there is no previous work on supply contracts under supply uncertainty considerations. Our work seeks to fill this gap.

I.3. Organization of the Dissertation

The dissertation is organized as follows. Chapter II examines the buyer's impact explicitly under price protection and returns policies. In Chapter III, we study three types of contracts in a buyer-driven channel with information asymmetry considerations, and we explore the impact of the power structure. In Chapter IV, we develop supply contracts under supply uncertainty. Supplier competition issues are addressed in Chapter V. The contributions of the dissertation are summarized in Chapter VI.

CHAPTER II

BUYER'S IMPACT

Recent work provides insight into several contract-based channel coordination mechanisms commonly used in practice, such as price protection and returns agreements (Lee et al. 2000, Taylor 2001, Lu et al. 2002), revenue sharing contracts (Cachon and Lariviere 2002), and supply contracts with options (Barnes-Schuster et al. 2002). Lee et al. (2000) explore the use of price protection with a two-period model, ignoring the possibility of returns or the disposal of unsold inventory. They show that price protection is an instrument for channel coordination. If products have long manufacturing lead times so that the retailer has a single buying opportunity, a properly chosen price protection credit coordinates the channel. If, contrarily, products have shorter manufacturing lead times so that the retailer has multiple buying opportunities, price protection alone cannot guarantee channel coordination when wholesale prices are exogenous. However, when the price protection credit is set endogenously together with the wholesale prices, channel coordination is restored. Taylor (2001) examines three channel policies that are used in declining price environments: price protection (P), midlife returns (M) and end-of-life returns (E). He shows that EM (i.e., midlife and end-of-life returns) achieves channel coordination if the wholesale prices and the return rebates are set properly. However, such a policy may not be implementable because it may require the manufacturer to be worse off as a result of coordination. If P is used in addition to EM and the terms are set properly, then PEM guarantees both coordination and a win-win outcome. Lu et al. (2002) further identify protection policies and/or conditions under which the supply chain can be coordinated and a win-win situation can be guaranteed. They also provide algorithms to determine win-win policy parameters.

However, these three papers, by Lee et al. (2000), Taylor (2001), and Lu et al. (2002), assume that the supplier (manufacturer) provides the contract and the buyer responds by passively choosing the order quantity. In practice, the buyer has impact on the retail market with pricing decisions (Emmons and Gilbert 1998) and sales efforts (Taylor 2002). Furthermore, the buyer has the liberty of withholding private information (Corbett and Tang 1999) which also improves the buyer's bargaining power. Therefore, there is a need to revisit and analyze price protection and returns agreements to consider the buyer's impact more explicitly.

Emmons and Gilbert (1998) study the role of returns policies and the impact of pricing on market demand. They model the relationship between a manufacturer and a retailer in a single period setting with price dependent demand uncertainty. Using a multiplicative model of demand uncertainty, they demonstrate that uncertainty tends to increase the retail price and that under certain conditions, a manufacturer can increase his own profit by offering to repurchase excess stock from the retailer at the conclusion of the period.

Retailers can also influence demand by merchandising, doing point-of-sale or other advertising, providing attractive shelf space, and guiding consumer purchases with sales personnel. Taylor (2002) shows that when demand is influenced by retailer sales effort, a properly designed target rebate and returns contract achieves coordination and a win-win outcome. Other contracts, such as linear rebate and returns or target rebate alone, cannot achieve coordination in a way that is implementable.

In this chapter, we generalize existing channel coordination mechanisms policies, such as price protection and return policies and explicitly examine the buyer's impact. The remainder of this chapter is organized as follows. In the next section, the impact of the pricing decision on price protection design is examined and four demand-curves are carefully explored. In section II.2, the impact of sales efforts is investigated. We further examine the impact of information asymmetry on price protection and returns policiy design in section II.3 and section II.4, respectively. Section II.5 provides concluding remarks.

II.1. Impact of Pricing Decisions

In this section, we examine the impact of pricing decisions on price protection designs under both deterministic and stochastic demand cases. We start with the price-sensitive deterministic demand case to investigate the dynamics of an optimal price protection design, and then we study the stochastic price-sensitive demand case which provides a better representation of real life applications.

II.1.1. Deterministic Demand Case

Consider a two-period model with deterministic price-sensitive demand. We assume that products have long lead times, so the buyer has a single buying opportunity at the beginning of the product life cycle. The buyer chooses an order quantity at the beginning of period 1, and the units are delivered ready for sale in period 1 at full price. At the beginning of period 2, another new product is introduced to the market. The launch of the new product reduces the attractiveness of the existing product. Consumers react to the introduction of the new product in one of two ways. Some consumers prefer to purchase the existing product at a discount; other consumers prefer to buy the new product. The buyer can adjust the retail price to influence the demand. Since the demand is deterministic, the buyer knows exactly how much the demand will be in the two periods at certain prices.

Assume the demand d_i in period i satisfies $d_i = a_i - b_i p_i$ where a_i, b_i are positive constants; p_i is the retail price in period i, $i = 1, 2$ satisfying

$$p_i \leq \frac{a_i}{b_i}$$

so the demand is always positive. The notation is summarized below:

p_i: selling price per unit at the market in period i,

d_i: market demand in period i,

w: supplier's wholesale price per unit to the buyer,

s: supplier's cost per unit,

c_i: buyer's handling cost per unit in period i,

h_i: holding cost per unit in period i,

g_i: loss of goodwill cost per unit in period i,

Q: order quantity from the buyer at the beginning of period 1,

$\bar{\beta}$: price protection factor (the supplier offers $\bar{\beta}(p_1 - p_2)$ for each unsold unit at the end of period 1 to the buyer).

Also, we make the following assumptions.

ASSUMPTION 1 $0 < s < w \leq p_1$, $0 \leq g_2 \leq g_1$.

ASSUMPTION 2 s, c_i, h_i, g_i, are exogenous; w, p_i, $\bar{\beta}$ and Q are endogenous.

We begin our analysis by considering the integrated channel. Let x be the leftover stock at the end of period 1. The channel's total profit in period 2, denoted by $\pi_{J2}(x)$, is given by

$$\pi_{J2}(x) = -c_2 x + p_2 x - \frac{1}{2} h_2 x.$$

Since the buyer knows exactly how much the demand is for a given retail price, it is always true that $x = Q - d_1 = d_2 = a_2 - b_2 p_2$. Therefore, the total profit of the channel when Q units are ordered at the beginning of period 1, denoted by $\pi_J(Q, p_1)$, is

$$\pi_J(Q, p_1) = -(s + c_1)Q + p_1 d_1 - \frac{1}{2}h_1(Q + d_2) + \pi_{J2}(Q - d_1).$$

Since $Q = d_1 + d_2$, the total profit can be expressed as a function of p_1 and p_2 as follows:

$$\begin{aligned} \pi_J(p_1, p_2) &= -b_1 p_1^2 + \left[b_1(s + c_1) + a_1 + \frac{1}{2}h_1 b_1\right] p_1 + [b_2(s + c_1) + h_1 b_2 + a_2 + b_2 c_2 \\ &\quad + \frac{1}{2}h_2 b_2]p_2 - b_2 p_2^2 - (s + c_1)(a_1 + a_2) - \frac{1}{2}a_1 h_1 - a_2 h_1 - c_2 a_2 - \frac{1}{2}h_2 a_2. \end{aligned}$$

The first-order conditions of the total profit function are given by

$$\begin{aligned} \frac{\partial \pi_J(p_1, p_2)}{\partial p_1} &= -2b_1 p_1 + b_1(s + c_1) + a_1 + \frac{1}{2}h_1 b_1 = 0, \\ \frac{\partial \pi_J(p_1, p_2)}{\partial p_2} &= -2b_2 p_2 + b_2(s + c_1) + h_1 b_2 + a_2 + b_2 c_2 + \frac{1}{2}h_2 b_2 = 0. \end{aligned}$$

Further, we have

$$\begin{aligned} \frac{\partial \pi_J^2(p_1, p_2)}{\partial p_1^2} &= -2b_1 < 0, \\ \frac{\partial \pi_J^2(p_1, p_2)}{\partial p_2^2} &= -2b_2 < 0, \\ \frac{\partial \pi_J^2(p_1, p_2)}{\partial p_1 \partial p_2} &= \frac{\partial \pi_J^2(p_1, p_2)}{\partial p_2 \partial p_1} = 0. \end{aligned}$$

Thus, the total profit function is strictly concave in p_1 and p_2. The optimal retail prices and order quantity are unique as follows

$$p_1^* = \frac{1}{2}\left[\frac{a_1}{b_1} + (s + c_1) + \frac{1}{2}h_1\right], \quad (2.1)$$

$$p_2^* = \frac{1}{2}\left[\frac{a_2}{b_2} + (s + c_1 + c_2) + h_1 + \frac{1}{2}h_2\right], \quad (2.2)$$

$$Q^* = a_1 - b_1 p_1^* + a_2 - b_2 p_2^*. \quad (2.3)$$

Next we consider the situation in which the supplier offers no price protection to the independent buyer. The buyer orders Q and receives the amount in time for sales in period 1. The buyer's optimal retail prices can be achieved by replacing the s with w in (2.1) and (2.2) as follows

$$\hat{p}_1 = \frac{1}{2}\left[\frac{a_1}{b_1} + (w + c_1) + \frac{1}{2}h_1\right], \tag{2.4}$$

$$\hat{p}_2 = \frac{1}{2}\left[\frac{a_2}{b_2} + (w + c_1 + c_2) + h_1 + \frac{1}{2}h_2\right], \tag{2.5}$$

$$\hat{Q} = a_1 - b_1\hat{p}_1 + a_2 - b_2\hat{p}_2. \tag{2.6}$$

Comparing (2.1–2.3) with (2.4–2.6), we have the following observation.

PROPOSITION 1 $\hat{p}_1 > p_1^*$, $\hat{p}_2 > p_2^*$, $\hat{Q} < Q^*$.

Proof: Since $w > s$, it is easy to verify by comparison. ∎

Proposition 1 shows that, without price protection, the independent buyer's optimal retail prices are strictly higher than the optimal prices of the integrated channel, while the optimal quantity is strictly less than the integrated channel. Note that this is a form of quantity distortion driven by double marginalization (Spengler 1950).

Now, we consider a price protection to align the buyer's incentive with the channel optimum. The price protection allows the supplier to offer to the buyer $\bar{\beta}(p_1 - p_2)$ $(0 \leq \bar{\beta} \leq 1)$ for each unit unsold at the end of period 1. Then the total profit to the buyer, denoted by $\pi_B(p_1, p_2)$, is given by

$$\begin{aligned}\pi_B(p_1, p_2) = {} & -b_1p_1^2 + \left[b_1(w + c_1) + a_1 + \frac{1}{2}h_1b_1\right]p_1 + [b_2(w + c_1) + h_1b_2 + a_2 \\ & + b_2c_2 + \frac{1}{2}h_2b_2]p_2 - b_2p_2^2 - (w + c_1)(a_1 + a_2) - \frac{1}{2}a_1h_1 - c_2a_2 \\ & + \frac{1}{2}h_2a_2 + \bar{\beta}(p_1 - p_2)(a_2 - b_2p_2).\end{aligned} \tag{2.7}$$

When the retail prices are exogenous, Lee et al. (2000) show that price protection is able to achieve the channel coordination. However, price protection fails to coordinate the channel when the buyer's pricing decision is considered explicitly, as shown in the following theorem.

THEOREM 1 Under price protection, the buyer's optimal retail prices are unique. Let $\tilde{p}_1$, $\tilde{p}_2$ represent the optimal retail prices in period 1 and 2, respectively. Then, $\tilde{p}_1 > \hat{p}_1 > p_1^*$. Price Protection alone can NOT achieve channel coordination.

Proof: The first-order conditions of the total profit function (2.7) are given by

$$\frac{\partial \pi_B(p_1,p_2)}{\partial p_1} = -2b_1p_1 + b_1(w+c_1) + a_1 + \frac{1}{2}h_1b_1 + \bar{\beta}(a_2 - b_2p_2) = 0, \tag{2.8}$$

$$\frac{\partial \pi_B(p_1,p_2)}{\partial p_2} = -2b_2p_2 + b_2(w+c_1) + h_1b_2 + a_2 + b_2c_2 + \frac{1}{2}h_2b_2 - \bar{\beta}(a_2 + b_2p_1) + 2b_2\bar{\beta}p_2 = 0. \tag{2.9}$$

Note that the determinant of the Hessian matrix $(4(1-\bar{\beta})b_1b_2 - \bar{\beta}^2b_2)$ is not always positive; thus the total profit function is not necessarily concave.

For a given p_2, the optimal value of $\tilde{p}_1$ is unique and can be achieved from (2.8) such that

$$\tilde{p}_1 = \frac{1}{2}\left(\frac{a_1}{b_1} + (w+c_1) + \frac{1}{2}h_1 + \frac{\bar{\beta}}{b_1}(a_2 - b_2p_2)\right). \tag{2.10}$$

Substituting (2.10) into (2.9), we can verify the uniqueness of the optimal value of p_2 with the second-order condition given by

$$\frac{\partial^2 \pi_B(p_2)}{\partial p_2^2} = -\frac{\bar{\beta}^2 b_2^2}{4b_1} - (1-\bar{\beta})2b_2. \tag{2.11}$$

Thus, the buyer's optimal retail prices are unique.

It can be easily verified that $\tilde{p}_1 > \hat{p}_1$ by comparing (2.10) with (2.4). Thus, $\tilde{p}_1 > p_1^*$ from Proposition 1. Since in the integrated channel, the optimal retail prices are unique, the retail price in period 1 under price protection is always higher than the optimal retail price in the integrated channel. Thus, price protection alone can not induce the buyer to set $\tilde{p}_1 = p_1^*$. Therefore, the channel can not be coordinated by price protection alone. ■

To provide a contract to coordinate the channel, we introduce the following term into the contract:

DEFINITION 1 The recommended retail price, represented by T, is a retail price upper-bound in period 1 recommended by the supplier. The supplier will provide price protection to the buyer only if the buyer sets her retail price p_1 satisfying $p_1 \leq T$.

To achieve channel coordination, the following theorem specifies the contract terms.

THEOREM 2 Let

$$\begin{aligned} T &= \frac{1}{2}\left[\frac{a_1}{b_1} + (s + c_1) + \frac{1}{2}h_1\right], \text{ and} \\ \bar{\beta} &= \frac{w - s}{\frac{a_1}{2b_1} - \frac{s}{2} - \frac{c_1}{2} - c_2 - \frac{3}{4}h_1 - \frac{1}{2}h_2}. \end{aligned} \quad (2.12)$$

Under the contract pair $(T, \bar{\beta})$, channel coordination is achieved.

Proof: From Theorem 1, we know that $\tilde{p}_1 > p_1^*$, so the supplier needs a way to force the buyer to reduce her retail price in period 1. By recommending a retail price upper bound T, now the supplier offers price protection only when $p_1 \leq T$.

To achieve channel coordination, let $T = p_1^*$. Since

$$\frac{\partial \pi_B(p_1, p_2)}{\partial p_1} > 0$$

when $p_1 \leq T$, the buyer will set $p_1 = T$ to receive the price protection and maximize her profit at the same time.

After p_1 is determined, the buyer will adjust p_2 to maximize her total profit as follows

$$p_2 = \frac{1}{2(1-\bar{\beta})}\left[w + c_1 + c_2 + h_1 + \frac{1}{2}h_2 + \frac{a_2}{b_2} - \frac{1}{2}\bar{\beta}(\frac{a_1}{b_1} + s + c_1 + \frac{1}{2}h_1 + \frac{a_2}{b_2})\right].$$

To induce the buyer to set $p_2 = p_2^*$, we obtain (2.12). ■

From the above analysis, we can see that price protection itself can not achieve channel coordination in the models when the buyer's pricing decision is considered explicitly. The supplier has to specify additional terms in the contract to align the buyer's incentive with the system-wide optimal solution. Thus, considering the buyer's pricing decision is important for the design of implementable contracts.

Lau and Lau (2002) point out that when a price-demand relationship is needed in inventory/pricing models, very often a convenient (typically linear) function is arbitrarily chosen. The common-wisdom implication is that any downward-sloping demand curve will lead to similar conclusions. They show that while the common-wisdom implication is valid for a single-echelon system, assuming different demand-curve functions can lead to very different results in a multi-echelon system. In some situations, a very small change in the demand-curve appearance leads to very large changes in the models optimal solutions. They put forward four representative demand-curves: linear, exponential, iso-elastic, and algebraic.

Note that for ease of analysis we assume a linear demand-curve throughout the above analysis. To thoroughly explore the impact of the buyer's pricing decision, we further design a pair of recommended retail prices and price protections under three other demand curves: iso-elastic, exponential, and algebraic. The optimal retail prices and contracts are summarized in Table I.

From Table I, we see that the price protection factor is always in the format $(w-s)/L$ where L depends on the demand curves. $\bar{\beta}$ always increases as w_1 increases. That is, to coordinate the channel, the higher the wholesale price charged by the supplier, the more protection he must provide to the buyer. Additionally, different demand-curves result in distinct retail prices. Obviously, the retial price p_1 under the iso-elastic demand-curve is always lower than the one under the algebraic demand-curve. Therefore, the supplier needs to consider the impact of market demand and the buyer's pricing decision explicitly to precisely decide the order quantity and the recommended retail price, which in turn determines his own profit.

Table I Price Protection Contracts with Four Demand Curves

Linear $d = a - bp$	*Iso-elastic* $d = Kp^{-\alpha}$
$p_1 = \frac{1}{2}(\frac{a_1}{b_1} + (s + c_1) + \frac{1}{2}h_1)$ $p_2 = \frac{1}{2}(\frac{a_2}{b_2} + (s + c_1 + c_2) + h_1 + \frac{1}{2}h_2)$ $T = \frac{1}{2}(\frac{a_1}{b_1} + (s + c_1) + \frac{1}{2}h_1)$ $\bar{\beta} = \frac{w-s}{\frac{a_1}{2b_1} - \frac{s}{2} - \frac{c_1}{2} - c_2 - \frac{3}{4}h_1 - \frac{1}{2}h_2}$	$p_1 = \frac{\alpha_1}{\alpha_1 - 1}(s + c_1 + \frac{1}{2}h_1)$ $p_2 = \frac{\alpha_2}{\alpha_2 - 2}(s + c_1 + c_2 + h_1 + \frac{1}{2}h_2)$ $T = \frac{\alpha_1}{\alpha_1 - 1}(s + c_1 + \frac{1}{2}h_1)$ $\bar{\beta} = \frac{w-s}{\frac{\alpha_1}{\alpha_1 - 1}(s+c_1+\frac{1}{2}h_1) - (s+c_1+h_1+c_2+\frac{1}{2}h_2)}$
Exponential $d = \gamma e^{-\alpha p}$	*Algebraic* $d = (Kp + b)^{-\alpha}$
$p_1 = \frac{1}{\alpha_1} + s + c_1 + \frac{1}{2}h_1$ $p_2 = \frac{1}{\alpha_2} + s + c_1 + h_1 + c_2 + \frac{1}{2}h_2$ $T = \frac{1}{\alpha_1} + s + c_1 + \frac{1}{2}h_1$ $\bar{\beta} = \frac{w-s}{\frac{1}{\alpha_1} - \frac{1}{2}h_1 - c_2 - \frac{1}{2}h_2}$	$p_1 = \frac{\alpha_1}{\alpha_1 - 1}(s + c_1 + \frac{1}{2}h_1 + \frac{b_1}{\alpha_1 K_1})$ $p_2 = \frac{\alpha_2}{\alpha_2 - 1}(s + c_1 + c_2 + h_1 + \frac{1}{2}h_2 + \frac{b_2}{\alpha_2 K_2})$ $T = \frac{\alpha_1}{\alpha_1 - 1}(s + c_1 + \frac{1}{2}h_1 + \frac{b_1}{\alpha_1 K_1})$ $\bar{\beta} = \frac{w-s}{\frac{\alpha_1}{\alpha_1 - 1}(s+c_1+\frac{1}{2}h_1+\frac{b_1}{\alpha_1 K_1}) - (s+c_1+h_1+c_2+\frac{1}{2}h_2)}$

Note that the implementation of price protection is based on an implicit assumption that the retail price will drop. That is, $p_1 > p_2$. To verify this condition under the different demand-curves, we compare the value of p_1 and p_2 in Table I which results in some interesting insights. If the demand-curve is linear, the condition $p_1 > p_2$ depends on the ratio of a_i and b_i, not on the individual value of a or b. That is, it depends on the maximal possible retail price, not the maximal possible demand or the price-sensitivity of the demand. When demand curves are iso-elastic or exponential, it depends on the shape of the curve (α), not the scale of the curve K or γ. When the demand curve is algebraic, all of the parameters affect condition satisfaction.

To demonstrate the above insights more clearly, we assume $h_1 = h_2 = c_1 = c_2 = 0$ and summarize the results in Table II.

Table II Price Protection with Neglectable Holding Cost and Handling Cost

	Linear $d = a - bp$	*Iso-elastic* $d = Kp^{-\alpha}$	*Exponential* $d = \gamma e^{-\alpha p}$	*Algebraic* $d = (Kp + b)^{-\alpha}$
Optimal Price	$p_1^* = \frac{1}{2}(\frac{a_1}{b_1} + s)$	$p_1^* = \frac{\alpha_1}{\alpha_1 - 1}s$	$p_1^* = \frac{1}{\alpha_1} + s$	$p_1^* = \frac{\alpha_1}{\alpha_1-1}(s + \frac{b_1}{\alpha_1 K_1})$
	$p_2^* = \frac{1}{2}(\frac{a_2}{b_2} + s)$	$p_2^* = \frac{\alpha_2}{\alpha_2 - 1}s$	$p_2^* = \frac{1}{\alpha_2} + s$	$p_1^* = \frac{\alpha_2}{\alpha_2-1}(s + \frac{b_2}{\alpha_2 K_2})$
$p_1^* > p_2^*$	$\frac{a_1}{b_1} > \frac{a_2}{b_2}$	$\alpha_1 < \alpha_2$	$\alpha_1 < \alpha 2$	$(\alpha_1 - \alpha_2)s + (\alpha_1 - 1)\frac{b_2}{K_2}$ $-(\alpha_2 - 1)\frac{b_1}{K_1} < 0$
Rec. Price	$T = \frac{1}{2}(\frac{a_1}{b_1} + s)$	$T = \frac{\alpha_1}{\alpha_1 - 1}s$	$T = \frac{1}{\alpha_1} + s$	$T = \frac{\alpha_1}{\alpha_1-1}(s + \frac{b_1}{\alpha_1 K_1})$
Protection	$\bar{\beta} = \frac{w-s}{\frac{1}{2}(\frac{a_1}{b_1} - s)}$	$\bar{\beta} = \frac{w-s}{\frac{1}{\alpha_1 - 1} - s}$	$\bar{\beta} = \frac{w-s}{\frac{1}{\alpha_1}}$	$\bar{\beta} = \frac{w-s}{\frac{1}{\alpha_1-1}s + \frac{b_1}{(\alpha_1-1)K_1}}$

Since we assume that demand is price-sensitive deterministic, in the next section we study the stochastic price-sensitive demand case in order to provide better representation of real life applications.

II.1.2. Stochastic Demand Case

Consider a two-period model with stochastic price-sensitive demand. Again, we assume that products have long lead times, so the buyer has a single buying opportunity at the beginning of the product life cycle. The buyer chooses an order quantity and a retail price at the beginning of period 1, and the units are delivered ready for sale in period 1 at full price. Unmet demand is lost. At the beginning of period 2, another new product is introduced to the market. Assume the expected demand D_i in period i is $D_i = a_i - b_i p_i$ where p_i is the retail price in period i, $i = 1, 2$. Along with the idea in Table II, we assume

$$a_2 < a_1 \text{ and } \frac{a_2}{b_2} < \frac{a_1}{b_1}.$$

That is, some customers may choose to buy the new products, and other customers may be willing to buy the old products at a reduced price in period 2. Thus, the maximal possible demand is lower and the demand approaches zero at a smaller price in period 2. We introduce additional notation as follows:

$D_i(p_i)$: expected demand in period i with a given retail price p_i,

ξ_i: random variable denoting the demand in period i,

$\phi_i(\cdot)$: density distribution function of demand in period i,

$\Phi_i(\cdot)$: cumulative distribution function of demand in period i,

ϵ_i: positive random variable with mean equal to 1,

$\psi_i(\cdot)$: density distribution function of ϵ_i,

$\Psi_i(\cdot)$: cumulative distribution function of ϵ_i,

θ: discount factor for costs in period 2.

The demand faced by the buyer is uncertain and price-sensitive. The actual demand is the product of $D_i(p_i)$ and the positive random variable ϵ_i. Thus, the density function of demand in period i, can be expressed as:

$$\phi_i(\xi_i, p_i) = \frac{1}{D_i(p_i)} \psi_i\left(\frac{\xi_i}{D_i(p_i)}\right), \quad \xi_i \geq 0, i = 1, 2.$$

Channel coordination can be achieved if the supplier replicates the performance of the fully integrated channel. To do so, the supplier has to provide an incentive to induce the buyer to choose the "right" order quantity and retail price that is optimal for an integrated channel. Thus, we begin our analysis by considering an integrated channel. To solve the decision problem for the integrated channel, we work backward starting with period 2. At the end of period 1, if the leftover stock is x, the channel's expected profit in the second period, denoted by $\pi_{J2}(x, p_2)$, is given by

$$\begin{aligned}
\pi_{J2}(x,p_2) &= -c_2 x + \int_0^x [p\xi_2 - h_2(x-\xi_2)]d\phi_2(\xi_2) + \int_x^\infty [p_2 x - g_2(\xi_2 - x)]d\phi_2(\xi_2) \\
&= -g_2 D_2(p_2) + (p_2 + g_2 - c_2)x - \int_0^x [(p_2+g_2+h_2)(x-\xi_2)]d\phi_2(\xi_2) \\
&= -g_2 D_2(p_2) + (p_2 + g_2 - c_2)x - \int_0^{\frac{x}{D_2(p_2)}} [(p_2+g_2+h_2)(x \\
&\quad -D_2(p_2)\xi_2)]\psi_2(\xi_2)d\xi_2. \qquad (2.13)
\end{aligned}$$

Let $\Gamma_2(x) = \int_0^x \xi_2\psi_2(\xi_2)d\xi_2$. Substituting $\Gamma_2(\cdot)$ into (2.13) leads to

$$\begin{aligned}
\pi_{J2}(x,p_2) &= -g_2 D_2(p_2) - (p_2+g_2+h_2)\left[x\Psi_2(\frac{x}{D_2(p_2)}) - D_2(p_2)\Gamma_2(\frac{x}{D_2(p_2)})\right] \\
&\quad + (p_2+g_2-c_2)x.
\end{aligned}$$

The first-derivative and second-derivative of $\pi_{J2}(x, p_2)$ with respect to p_2 are given by

$$\begin{aligned}
\frac{\partial \pi_{J2}(x,p_2)}{\partial p_2} &= g_2 b_2 + x - x\Psi_2(\frac{x}{a_2 - b_2 p_2}) + [a_2 - b_2 p_2 - b_2(p_2 + g_2 \\
&\quad + h_2)]\Gamma_2(\frac{x}{a_2 - b_2 p_2}), \\
\frac{\partial^2 \pi_{J2}(x,p_2)}{\partial p_2^2} &= -2b_2\Gamma_2(\frac{x}{a_2 - b_2 p_2}) - (p_2+g_2+h_2)b_2^2\frac{x^2}{(a_2-b_2p_2)^3}\psi_2(\frac{x}{a_2-b_2p_2}) < 0.
\end{aligned}$$

Thus, $\pi_{J2}(x, p_2)$ is a strictly concave function of p_2. The optimal value of p_2 is unique. The optimal retail price p_2^* satisfies

$$g_2 b_2 + x - x\Psi_2(\frac{x}{a_2 - b_2 p_2}) + [a_2 - b_2(2p_2 + g_2 + h_2)]\Gamma_2(\frac{x}{a_2 - b_2 p_2}) = 0.$$

To maintain analysis tractability and provide insightful understanding, we assume $\psi_2(x) = 0.5$, $\forall x \in [0, 2]$ for the rest of this section. Then, we have

$$\pi_{J2}(x, p_2) = \begin{cases} (p_2 - c_2)x + (x - a_2 + b_2 p_2)g_2 - (p_2 + g_2 + h_2)\frac{x^2}{4D_2(p_2)}, & a_2 - b_2 p_2 \geq \frac{x}{2}, \\ (p_2 + h_2(a_2 - b_2 p_2) - (h_2 + c_2)x, & a_2 - b_2 p_2 \leq \frac{x}{2}. \end{cases}$$

Assume a_2 is large enough that $x < a_2$ is always true. That is, no matter how many products are left at the end of the first period, there always exists a proper price so that all products can be expected to be sold (or the expected demand is greater than x). Then, the optimal price is

$$p_2^* = \frac{1}{b_2}\left[a_2 - \sqrt{\frac{b_2(g_2 + h_2 + \frac{a_2}{b_2})x^2}{4(x + b_2 g_2)}}\right]. \tag{2.14}$$

In the second period, a new product comes into the market. We further assume $g_2 = 0$, that is, the loss of goodwill is small and negligible in period 2 since the customers have the option to buy the new product. Substituting (2.14) into $\pi_{J2}(x, p_2)$, the maximal profit in period 2 for a given leftover stock x is given by

$$\pi_{J2}^*(x) = \left(\frac{a_2}{b_2} - c_2\right)x - \frac{x^2}{4b_2}. \tag{2.15}$$

From (2.15), we note that the maximal expected profit only depends on a_2, b_2, c_2, not on h_2. That is, the maximal expected profit in period 2 depends on the market demand and the handling cost, but not the inventory cost. We may check the concavity of $\pi_{J2}^*(x)$ with respect to x as follows

$$\frac{\partial \pi_{J2}^*(x)}{\partial x} = \left(\frac{a_2}{b_2} - c_2\right) - \frac{x}{2b_2}, \frac{\partial^2 \pi_{J2}^*(x)}{\partial x^2} = -\frac{1}{2b_2} < 0.$$

Thus, $\pi_{J2}^*(x)$ is a strictly concave function of x. Moving back to the first period, the expected profit of the integrated channel, denoted by $\pi_J(Q, p_1)$, is given by

$$\begin{aligned}
\pi_J(Q,p_1) &= -(s+c_1)Q + \int_0^Q [p_1\xi_1 - h_1(Q-\xi_1) + \theta\pi_{J2}(Q-\xi_1)]d\phi_1(\xi_1) \\
&\quad + \int_Q^\infty [p_1 - g_1(\xi_1 - Q) + \theta\pi_{J2}(0)]d\phi_1(\xi_1) \\
&= -(s+c_1)Q + \int_0^{\frac{Q}{D_1(p_1)}} [-\frac{D_1^2(p_1)}{4b_2}\xi_1^2 + (p_1 + h_1 - \theta\tilde{k} + \frac{Q}{2b_2})D_1(p_1)\xi_1 \\
&\quad + (\theta\tilde{k} - h_1)Q - \frac{Q^2}{4b_2}]\psi_1(\xi_1)d\xi_1 + \int_{\frac{Q}{D_1(p_1)}}^\infty [(p_1+g_1)Q \\
&\quad -g_1 D_1(p_1)\xi_1]\psi_1(\xi_1)d\xi_1 \\
\text{where } \tilde{k} &= \frac{a_2}{b_2} - c_2.
\end{aligned}$$

The first-derivative and second-derivative of $\pi_J(Q,p_1)$ with respect to Q and p_1 are given by

$$\begin{aligned}
\frac{\partial \pi_J(Q,p_1)}{\partial Q} &= -(s+\hat{c}_1) + \int_0^{\frac{Q}{D_1(p_1)}} \left[\frac{D_1(p_1)\xi_1}{2b_2} + (\theta\tilde{k} - h_1) - \frac{Q}{2b_2}\right]\psi_1(\xi_1)d\xi_1 \\
&\quad + \int_{\frac{Q}{D_1(p_1)}}^\infty (p_1+g_1)\psi_1(\xi_1)d\xi_1, \qquad (2.16) \\
\frac{\partial^2 \pi_J(Q,p_1)}{\partial Q^2} &= \int_0^{\frac{Q}{D_1(p_1)}} (-\frac{1}{2b_2})\psi_1(\xi_1)d\xi_1 - \frac{p_1+g_1+h_1-\theta\tilde{k}}{D_1(p_1)}\psi_1(\frac{Q}{D_1(p_1)}) < 0, \\
\frac{\partial \pi_J(Q,p_1)}{\partial p_1} &= \int_0^{\frac{Q}{D_1(p_1)}} [\frac{2b_1 D_1(p_1)}{4b_2}\xi_1^2 + D_1(p_1)\xi_1 - (p_1 + h_1 - \theta\tilde{k} \\
&\quad + \frac{Q}{2b_2})b_1\xi_1]\psi_1(\xi_1)d\xi_1 + \int_{\frac{Q}{D_1(p_1)}}^\infty [Q + g_1 b_1 \xi_1]\psi_1(\xi_1)d\xi_1, \\
\frac{\partial^2 \pi_J(Q,p_1)}{\partial p_1^2} &= \int_0^{\frac{Q}{D_1(p_1)}} \left[-\frac{b_1^2}{2b_2}\xi_1^2 - 2b_1\xi_1\right]\psi_1(\xi_1)d\xi_1 - \frac{b_1^2 Q^2(p_1+g_1+h_1-\theta\tilde{k})}{D_1^3(p_1)} \\
&\quad \psi_1(\frac{Q}{D_1(p_1)}) < 0.
\end{aligned}$$

For a given p_1, the optimal order quantity Q is unique. Also, for a given Q, the optimal retail price p_1 is unique.

When the buyer is independent, all of the above formulations are true except for replacing s with w. It is easy to see from (2.16) that for a given p_1, the independent buyer will order less than the optimal quantity in an integrated channel.

When the demand is stochastic, we can not use $\bar{\beta}(p_1 - p_2)$ as a protection mechanism for each unsold unit at the end of period 1 to induce the buyer to order more because the pricing decision in period 2 (p_2) depends on the realized demand in period 1 and is unknown at the beginning of period 1 when the contract is signed. To explicitly specify the protection level, the supplier can provide the protection as r dollars per unit for products unsold at the end of period 1. Next, we examine how the supplier should set the protection level to coordinate the channel.

PROPOSITION 2 Protection credit r will always induce the independent buyer to set a higher retail price for a given Q.

Proof: The independent buyer's expected profit in period 1, denoted by $\hat{\pi}_B(Q, p_1)$, and the first-order conditions are given by

$$\begin{aligned}
\hat{\pi}_B(Q, p_1) &= \pi_J(Q, p_1) + \int_0^{\frac{Q}{D_1(p_1)}} \theta r(Q - D_1(p_1)\xi_1)\psi_1(\xi_1)d\xi_1 - (w + s)Q, \\
\frac{\partial \hat{\pi}_B(Q, p_1)}{\partial Q} &= \frac{\partial \pi_J(Q, p_1)}{\partial Q} + \theta r \frac{Q}{2D_1(p_1)} - w + s, \\
\frac{\partial \hat{\pi}_B(Q, p_1)}{\partial p_1} &= \frac{\partial \pi_J(Q, p_1)}{\partial p_1} + \frac{1}{4}\theta r b_1 \frac{Q^2}{D_1^2(p_1)}.
\end{aligned}$$

Since

$$\frac{\partial^2 \pi_J(Q, p_1)}{\partial p_1^2} \leq 0, \ \frac{1}{4}\theta r b_1 \frac{Q^2}{D_1^2(p_1)} > 0,$$

we have $\bar{p}_1 \geq p_1^*$ where

$$\frac{\partial \hat{\pi}_B(Q, p_1)}{\partial p_1}|_{\bar{p}_1} = 0, \ \frac{\partial \pi_J(Q, p_1)}{\partial p_1}|_{p_1^*} = 0.$$

■

Proposition 2 shows that since the protection credit r reduces the buyer's cost and risk of uncertain demand, the buyer will likely take more risk by increasing her retail price such that the expected amount of unsold units at the end of period 1 is larger than in the integrated channel. Therefore, price protection alone can not align the independent buyer's pricing decision with the channel optimum. An additional term needs to be specified in the contract in order to coordinate the channel. A common way of providing an incentive to the buyer is to offer a quantity discount (Monahan 1988). The following theorem provides the pairing of a quantity discount and a protection credit to coordinate the channel.

THEOREM 3 Let

$$K(Q,p_1) = -\frac{\theta r Q^2}{4D_1(p_1)} + (w-s)Q + k \tag{2.17}$$

where k is a constant and K is the discount provided by the supplier. The pair of $(r, K(Q,p_1))$ coordinates the channel.

Proof: The buyer's expected profit in period 1, denoted by $\tilde{\pi}_B(Q,p_1)$, and the first-order conditions are given by

$$\begin{aligned}
\tilde{\pi}_B(Q,p_1) &= \hat{\pi}_B(Q,p_1) + K(Q,p_1), \\
\frac{\partial \tilde{\pi}_B(Q,p_1)}{\partial Q} &= \frac{\partial \pi_J(Q,p_1)}{\partial Q} + \theta r \frac{Q}{2D_1(p_1)} - w + s + \frac{\partial K(Q,p_1)}{\partial Q}, \\
\frac{\partial \tilde{\pi}_B(Q,p_1)}{\partial p_1} &= \frac{\partial \pi_J(Q,p_1)}{\partial p_1} + \frac{1}{4}\theta r b_1 \frac{Q^2}{D_1^2(p_1)} + \frac{\partial K(Q,p_1)}{\partial p_1}.
\end{aligned}$$

To align the buyer's incentive with the integrated channel, the supplier is aiming at

$$\frac{\partial \pi_J(Q,p_1)}{\partial Q} = \frac{\partial \pi_J(Q,p_1)}{\partial p_1} = 0.$$

Thus, we have

$$\begin{cases} \theta r \frac{Q}{2D_1(p_1)} - w + s + \frac{\partial K(Q,p_1)}{\partial Q} = 0, \\ \frac{1}{4}\theta r b_1 \frac{Q^2}{D_1^2(p_1)} + \frac{\partial K(Q,p_1)}{\partial p_1} = 0, \end{cases}$$

and (2.17) is obtained. A proper value of k needs to be determined to satisfy the buyer's reservation profit level. ■

Besides pairing a quantity discount and a protection credit to coordinate the channel, we can also use the concept in the deterministic price-sensitive demand case to achieve another coordination policy below.

THEOREM 4 Suppose the optimal solution in the integrated channel is (Q^*, p_1^*). Then, under the contract of (T, r) where

$$\begin{aligned} T &= p_1^*, \quad \text{and} \\ r &= \frac{w - s}{\Psi_1(\frac{Q^*}{D_1(p^*)})}, \end{aligned} \tag{2.18}$$

channel coordination is achieved.

Proof: First, we note that in either the centralized case or the decentralized case, the buyer's optimal pricing decision in period 2 remains the same for a given left over stock. To simplify our proof, we assume that the price in period 2 is fixed. For the unfixed price case, the proof is similar.

For the independent buyer with price protection and for the integrated channel, respectively, the first-order conditions are as follows

$$\begin{aligned} \frac{\partial \hat{\pi}_B(Q, p_1)}{\partial Q} &= p_1 + g_1 - w - \hat{c}_1 - [p_1 + g_1 + h_1 - \theta(p_2 + g_2 + \hat{c}_2] \int_0^{\frac{Q}{D_1(p_1)}} \psi_1(\xi_1) d\xi_1 \\ &\quad - \theta(p_2 + g_2 + h_2) \int_0^{\frac{Q}{D_1(p_1)}} \Psi_2(Q - D_1(p_1)\xi_1)\psi_1(\xi_1) d\xi_1 \\ &\quad + \theta r \Psi_1(\frac{Q}{D_1(p_1)}) = 0, \end{aligned} \tag{2.19}$$

$$\frac{\partial \pi_J(Q,p_1)}{\partial Q} = p_1 + g_1 - s - \hat{c}_1 - [p_1 + g_1 + h_1 - \theta(p_2 + g_2 + \hat{c}_2] \int_0^{\frac{Q}{D_1(p_1)}} \psi_1(\xi_1) d\xi_1 - \theta(p_2 + g_2 + h_2) \int_0^{\frac{Q}{D_1(p_1)}} \Psi_2(Q - D_1(p_1)\xi_1)\psi_1(\xi_1) d\xi_1 = 0. \quad (2.20)$$

Setting $p_1 = p^*$ and comparing (2.19) with (2.20), we obtain (2.18). ■

Theorem 4 shows that price protection and a recommended retail price together can also achieve channel coordination. From the analysis in this section, it is obvious that price protection alone fails to coordinate the channel due to the impact of the buyer's pricing decision. Therefore, it is important to take the buyer's pricing decision into account when developing implementable supply contracts.

II.2. Impact of Sales Efforts

Besides pricing decisions, buyers can also influence demand by merchandising, doing point-of-sale or other advertising, providing attractive shelf space, and guiding consumer purchases with sales personnel. In this section, we employ a two-period model in which a buyer makes order quantity and sales effort decisions and then observes demand. We will focus on a price protection contract in which the returns policy is a special case under this setting. Specifically, let demand in period 1 be given as $e\xi_1$, where e is the level of effort. Since a new product is introduced into the market in period 2, we assume that the buyer loses interest in expending sales effort on the old product in period 2. The cost to the buyer of exerting e units of effort is $V(e)$. We make the following assumption about cost function $V(e)$.

ASSUMPTION 3 $V(\cdot)$ is convex, increasing, and $V(0) = 0$.

Thus, the marginal cost of effort is increasing. This modeling approach is consistent with the implications of the sales response models having multiplicative error terms that are used in empirical studies of the relationship between advertising and sales (Taylor 2002). In this section, we also assume that the retail prices are fixed in order to concentrate on investigation of the impact of the sales effort.

In an integrated channel, the total expected profit of the channel is given by

$$\begin{aligned}\pi_J(Q,e) &= -(s+c_1)Q + \int_0^{\frac{Q}{e}} [p_1 e\xi_1 - h_1(Q - e\xi_1) + \theta\pi_{J2}(Q - e\xi_1)]d\Psi_1(\xi_1) \\ &\quad + \int_{\frac{Q}{e}}^{\infty} [p_1 Q - g_1(e\xi_1 - Q) + \theta\pi_{J2}(0)]d\Psi_1(\xi_1) - V(e).\end{aligned}$$

Let $Q_0 = Q/e$, then we have

$$\begin{aligned}\pi_J(Q_0,e) &= -(s+c_1)Q_0 e + \int_0^{Q_0} [p_1 e\xi_1 - h_1 e(Q_0 - \xi_1) + \theta\pi_{J2}[e(Q_0 \\ &\quad - \xi_1)]d\Psi_1(\xi_1) + \int_{Q_0}^{\infty} [p_1 e Q_0 + g_1 e(\xi_1 - Q_0)]d\Psi_1(\xi_1) - V(e), \\ \pi_{J2}[e(Q_0 - \xi_1)] &= -g_2\mu_2 + (p_2 + g_2 - c_2)e(Q_0 - \xi_1) - \int_0^{e(Q_0-\xi_1)} [(p_2 + g_2 \\ &\quad + h_2)[e(Q_0 - \xi_1) - \xi_2]]d\Psi_2(\xi_2).\end{aligned}$$

The first-order and second-order conditions of $\pi_{J2}(x)$ are given by

$$\begin{aligned}\frac{\partial \pi_{J2}(x)}{\partial x} &= p_2 + g_2 - c_2 - (p_2 + g_2 + h_2)\Psi_2(x), \\ \frac{\partial \pi_{J2}^2(x)}{\partial x^2} &= -(p_2 + g_2 + h_2)f_2(x) < 0.\end{aligned}$$

Thus, $\pi_{J2}(x)$ is a concave function of x. Next, we examine the impact of price protection on the buyer's sales effort decision.

PROPOSITION 3 In a decentralized channel, the optimal sales effort level with price protection is higher than without price protection for a given Q_0.

Proof: Let $\breve{\pi}_B(Q_0,e)$ and $\bar{\pi}_B(Q_0,e)$ denote the buyer's total expected profit with and without protection, respectively. Checking the first-derivative and second-derivative of $\bar{\pi}_B(Q_0, e)$ with respect to e, we have

$$\begin{aligned}\frac{\partial \bar{\pi}_B(Q_0,e)}{\partial e} &= -(w+c_1)Q_0 + \int_{Q_0}^{\infty}(p_1Q_0 + g_1(\xi_1 - Q_0))d\Psi_1(\xi_1) + \int_0^{Q_0}[p_1\xi_1 \\ &\quad - h_1(Q_0-\xi_1) + \theta[p_2 + g_2 - c_2 - (p_2+g_2+h_2)\Psi_2(e(Q_0 \\ &\quad -\xi_1)]]d\Psi_1(\xi_1) - V'(e), \qquad (2.21)\\ \frac{\partial \bar{\pi}_B^2(Q_0,e)}{\partial e^2} &= \int_0^{Q_0} -\theta(p_2+g_2+h_2)f_2[e(Q_0-\xi_1)](Q_0-\xi_1)d\Psi_1(\xi_1) \\ &\quad -V''(e) < 0, \qquad (2.22)\end{aligned}$$

where $V'(e)$ and $V''(e)$ are the first-derivative and second-derivative of $V(e)$ with respect to e.

With protection, the buyer's total expected profit is

$$\breve{\pi}_B(Q_0,e) = \bar{\pi}_B(Q_0,e) + \theta[\int_0^{Q_0} re(Q_0-\xi_1)d\Psi_1(\xi_1)]. \qquad (2.23)$$

The first-derivative of $\breve{\pi}_B(Q_0, e)$ with respect to e is given by

$$\frac{\partial \breve{\pi}_B(Q_0,e)}{\partial e} = \frac{\partial \bar{\pi}_B(Q_0,e)}{\partial e} + \theta\int_0^{Q_0} rQ_0 d\Psi_1(\xi_1). \qquad (2.24)$$

For a given Q_0, the optimal sales effort level is unique from (2.22). Also $\bar{\pi}_B(Q_0, e)$ and $\breve{\pi}_B(Q_0, e)$ are both concave functions over e. Let $e1^*$ and $e2^*$ denote the buyer's optimal sales effort level with and without protection. Comparing (2.21) with (2.24), it can be easily verified that $e1^* > e2^*$. ■

Proposition 3 shows a counter-intuitive result. With price protection, the buyer faces a lower risk of demand uncertainty; thus, she should be able to save some sales effort and some cost as a result. However, Proposition 3 shows that, in fact, the buyer expends more sales effort when price protection is available. The explanation is that price protection encourages the buyer to order a larger quantity by sharing the risk of demand uncertainty. Accordingly, the buyer will expend more sales effort to sell the larger quantity.

Next, we examine how the supplier may set the protection level to coordinate the channel.

PROPOSITION 4 Price protection itself can NOT coordinate the channel.

Proof: To align the independent buyer's order quantity with the integrated channel, the following equation has to be satisfied:

$$\frac{\partial \check{\pi}_B(Q_0, e)}{\partial Q_0} = \frac{\partial \pi_J(Q_0, e)}{\partial Q_0}.$$

The first-derivative and second-derivative of the buyer's total expected profit function over Q_0 in the integrated channel are given by

$$\begin{aligned}
\frac{\partial \pi_J(Q_0, e)}{\partial Q_0} &= [p_1 - g_1 - s - c_1]e - (p_1 - g_1 + h_1)e\Psi_1(Q_0) + e\int_0^{Q_0} \theta[p_2 + g_2 - c_2 \\
&\quad - (p_2 + g_2 + h_2)\Psi_2(e(Q_0 - \xi_1))]d\Psi_1(\xi_1), \\
\frac{\partial \pi_J^2(Q_0, e)}{\partial Q_0^2} &= -[p_1 + g_1 + h_1 - \theta(p_2 + g_2 - c_2)]ef_1(Q_0) - e^2\int_0^{Q_0} (p_2 + g_2 \\
&\quad + h_2)f_2(e(Q_0 - \xi_1)d\Psi_1(\xi_1) < 0.
\end{aligned}$$

With some algebra, we have $\theta\int_0^{Q_0} red\Psi_1(\xi_1) = (w - s)e$. Thus, the protection level has to satisfy

$$r = \frac{w - s}{\theta\Psi_1(Q_0)}.$$

To align the buyer's sales effort level with the integrated channel, the following equation has to be satisfied:

$$\frac{\partial \check{\pi}_B(Q_0, e)}{\partial e} = \frac{\partial \pi_J(Q_0, e)}{\partial e}.$$

With some algebra, the following condition needs to be satisfied:

$$\theta\int_0^{Q_0} r(Q_0 - \xi_1)d\Psi_1(\xi_1) = (w - s)Q_0. \tag{2.25}$$

However,

$$\theta \int_0^{Q_0} r(Q_0 - \xi_1) d\Psi_1(\xi_1) = \frac{(w-s)\int_0^{Q_0}(Q_0 - \xi_1) d\Psi_1(\xi_1)}{\Psi_1(Q_0)} < (w-s)Q_0,$$

which contradicts (2.25). Thus, r alone is unable to coordinate the channel. The sales effort level under r is lower than in the integrated channel. ■

To coordinate the channel, we present the following contract terms.

THEOREM 5 To coordinate the channel, the supplier can provide price protection together with cost sharing of the sales effort SE as follows:

$$r = \frac{w-s}{\theta \Psi_1(Q_0)}, \quad (2.26)$$

$$SE = \frac{(w-s)\Gamma_1(Q_0)}{\Psi_1(Q_0)}. \quad (2.27)$$

Proof: Replacing s with w in $\pi_J(Q_0, e)$, the total expected profit $\bar{\pi}_1(Q_0, e)$ for the independent buyer is achieved. With protection and sales effort sharing, the buyer's expected profit is $\bar{\pi}_1(Q_0, e) + \Upsilon$ where $\Upsilon = \theta[\int_0^{Q_0} re(Q_0 - \xi_1) d\Psi_1(\xi_1)] + SEe$.

Checking the first-order conditions, after simplification, we have

$$\frac{\partial \Upsilon(Q_0, e)}{\partial Q_0} = \theta \int_0^{Q_0} red\Psi_1(\xi_1) = (w-s)e, \quad (2.28)$$

$$\frac{\partial \Upsilon(Q_0, e)}{\partial e} = \theta \int_0^{Q_0} r(Q_0 - \xi_1) d\Psi_1(\xi_1) + SE = (w-s)Q_0. \quad (2.29)$$

Solving (2.28-2.29) for r and SE, we obtain (2.26) and (2.27). ■

In this section, we allow the buyer to make sales effort decisions to influence demand. The impact of the buyer's decision on the price protection contract design is examined explicitly. When the contract is signed between a supplier and buyer, they usually do not have complete information about each other. For example, the supplier may not be able to observe the buyer's handling cost c_1 at the beginning of period 1. In the next section, we design contracts for the supplier that take information asymmetry into account.

II.3. Price Protection with Information Asymmetry

To provide an optimal design of price protection that considers information asymmetry, we start with the case in which the demand is deterministic. The deterministic case provides us with an example of the difficulties of designing incentive-compatible contracts under conditions of information asymmetry. It also offers guidance on how to design a proper contract in the stochastic demand case.

II.3.1. Deterministic Demand Case

Consider a two-period model with deterministic price-sensitive demand which is similar to the model in Section II.1.1. The market demand in the final market is price-sensitive. In this section, we assume the demand-curve is linear. Further results can be easily achieved using other types of demand-curves based on the analysis in this section. To maximize her profits, it is sufficient for the buyer to select either Q or p as the other is then immediately determined.

In this section, for simplicity, we assume that the buyer's handling costs in period 1 and 2 are the same and denote them by c. In general, the supplier does not know the buyer's marginal cost c exactly; we assume that the supplier holds a prior distribution $F_S(c)$ with a continuous density function $f_S(c)$, defined on $[\underline{c}, \bar{c}]$, where $0 \leq \underline{c} \leq \bar{c} < \infty$.

Since the supplier has incomplete information, he may offer a menu of contracts to the buyer. The sequence of events is as follows. The supplier offers a menu of contracts $[w(Q), r(Q)]$ where $w(Q)$ is the wholesale price for a given Q and $r(Q)$ is the protection credit provided by the supplier to the buyer for each unit unsold by the end of period 1. The buyer chooses an order quantity Q based on her internal cost

c, pays the supplier $w(Q)Q$ and receives $r(Q)$ as protection. For any revealed choice of Q, the supplier can infer the buyer's true cost c, so, by the revelation principle (Fudenberg and Tirole 1991), we can reformulate this equivalently as a menu of contracts $[w(c), r(c)]$. A buyer announcing $\hat{c}$ then chooses a contract $[w(\hat{c}), r(\hat{c})]$.

The supplier's objective to is to maximize his expected profit as follows:

$$\max_{w(c),r(c)} \quad \int_{\underline{c}}^{\bar{c}} \pi_S(c, Q(c)) f_S(c) dc \tag{2.30}$$

$$s.t. \quad Q(c) = \arg\max_Q \pi_B(c, Q) \quad \forall \underline{c} \leq c \leq \bar{c} \tag{2.31}$$

The supplier's expected net profits in (2.30) depend on the quantity Q ordered by the buyer. Condition (2.31) is the buyer's incentive-compatibility constraint. Presented with any menu of contracts, a buyer with cost c will choose Q to maximize her profits.

Consider a menu of contracts $[w(c), r(c)]$. If a buyer with a marginal cost c chooses $\hat{c}$, then the buyer's profit is given by

$$\begin{aligned}\pi_B(\hat{c}, c, p_1, p_2) &= -(w(\hat{c}) + c)(a_1 - b_1 p_1 + a_2 - b_2 p_2) + p_1(a_1 - b_1 p_1) + p_2(a_2 - b_2 p_2) \\ &\quad + r(\hat{c})(a_2 - b_2 p_2) \\ &= -(w(\hat{c}) + c)(a_1 + a_2) + b_1(w(\hat{c}) + c)p_1 + b_2(w(\hat{c}) + c)p_2 + a_1 p_1 \\ &\quad - b_1 p_1^2 + a_2 p_2 - b_2 p_2^2 + a_2 r_2(\hat{c}) - b_2 r(\hat{c}) p_2. \end{aligned} \tag{2.32}$$

The buyer's optimal price decision satisfies the first-order conditions

$$\begin{aligned}\frac{\partial \pi_B(\hat{c}, c, p_1, p_2)}{\partial p_1} &= b_1(w(\hat{c}) + c) + a_1 - 2b_1 p_1 = 0, \\ \frac{\partial \pi_B(\hat{c}, c, p_1, p_2)}{\partial p_2} &= b_2(w(\hat{c}) + c) + a_2 - 2b_2 p_2 - b_2 r(\hat{c}) = 0.\end{aligned}$$

Thus, her unique optimal prices and order quantities, which can be easily verified, are

$$\begin{aligned} p_1^* &= \frac{1}{2}(\frac{a_1}{b_1} + w(\hat{c}) + c), \\ p_2^* &= \frac{1}{2}(\frac{a_2}{b_2} + w(\hat{c}) + c - r(\hat{c})), \\ q_1^* &= \frac{1}{2}[a_1 - b_1(w(\hat{c}) + c)], \\ q_2^* &= \frac{1}{2}[a_2 - b_2(w(\hat{c}) + c - r(\hat{c}))]. \end{aligned}$$

Therefore, if a buyer with cost c chooses $[w(\hat{c}), r(\hat{c})]$, then her optimal profit is given by

$$\pi_B^*(c, \hat{c}) = \frac{b_1}{4}\left[\frac{a_1}{b_1} - (w(\hat{c}) + c)\right]^2 + \frac{b_2}{4}\left[\frac{a_2}{b_2} - (w(\hat{c}) + c) + r(\hat{c})\right]^2.$$

The revelation principle states that there is an optimal contract under which the buyer will optimally announce $\hat{c} = c$. Requiring that the first-order condition solved at $\hat{c} = c$ gives the buyer's incentive-compatibility constraint:

$$\begin{aligned} \frac{\partial \pi_B^*(c, \hat{c})}{\partial \hat{c}} |_{\hat{c}=c} &= \frac{b_1}{2}\left[\frac{a_1}{b_1} - (w(c) + c)\right] w'(c) + \frac{b_2}{2}[\frac{a_2}{b_2} - (w(c) + c) \\ &\quad + r(c)][r'(c) - w'(c)] = 0. \end{aligned}$$

The above incentive-compatibility constraint derivation is based on the work of Laffont and Tirole (1993). Although it provides a useful method for describing the buyer's incentive-compatibility, the resulting constraints are often complicated and analytic results are difficult to achieve. To gain some further insight into this problem, we study a two-type case.

Assume that the supplier knows that c belongs to the two-point support $\underline{c}, \bar{c}$ with $\bar{c} > \underline{c}$. Incentive-compatibility says that the contract designed for type $\underline{c}$ (respectively type $\bar{c}$) is the one preferred by the type $\underline{c}$ buyer (resp. type $\bar{c}$) in the menu. The supplier has a prior distribution on the values of c characterized by $v = P(c = \underline{c})$ and designs the contract to maximize his expected profit.

Let $\underline{w}_1 = \underline{w}, \underline{w}_2 = \underline{w} - r$, $\bar{w}_1 = \bar{w}, \bar{w}_2 = \bar{w} - r$; the incentive-compatibility amounts to

$$\frac{b_1}{4}\left[\frac{a_1}{b_1} - \underline{w}_1 - \underline{c}\right]^2 + \frac{b_2}{4}\left[\frac{a_2}{b_2} - \underline{w}_2 - \underline{c}\right]^2 \geq \frac{b_1}{4}\left[\frac{a_1}{b_1} - \bar{w} - \underline{c}\right]^2 + \frac{b_2}{4}\left[\frac{a_2}{b_2} - \bar{w}_2 - \underline{c}\right]^2 \quad (2.33)$$

$$\frac{b_1}{4}\left[\frac{a_1}{b_1} - \bar{w} - \bar{c}\right]^2 + \frac{b_2}{4}\left[\frac{a_2}{b_2} - \bar{w}_2 - \bar{c}\right]^2 \geq \frac{b_1}{4}\left[\frac{a_1}{b_1} - \underline{w}_1 - \bar{c}\right]^2 + \frac{b_2}{4}\left[\frac{a_2}{b_2} - \underline{w}_2 - \bar{c}\right]^2 \quad (2.34)$$

Add up (2.33), (2.34) and after simplification, we have

$$b_1(\bar{w}_1 - \underline{w}_1) + b_2(\bar{w}_2 - \underline{w}_2) \geq 0.$$

The supplier's expected profit is given by

$$\begin{aligned} E[\pi_S] \quad = \quad & \frac{1}{2}v(\underline{w}_1 - s)(a_1 - b_1\underline{w}_1 - b_1\underline{c}) + \frac{1}{2}v(\underline{w}_2 - s)(a_2 - b_2\underline{w}_2 - b_2\underline{c}) + \frac{1}{2}(1-v)(\bar{w}_1 \\ & - s)(a_1 - b_1\bar{w}_1 - b_1\bar{c}) + \frac{1}{2}(1-v)(\bar{w}_2 - s)(a_2 - b_2\bar{w}_2 - b_2\bar{c}). \end{aligned}$$

The optimal solution without constraints (2.33),(2.34) can be easily obtained as follows

$$\begin{aligned} \underline{w}_1 &= \frac{1}{2b_1}(a_1 - b_1\underline{c} + b_1 s), \\ \underline{w}_2 &= \frac{1}{2b_2}(a_2 - b_2\underline{c} + b_1 s), \\ \bar{w}_1 &= \frac{1}{2b_1}(a_1 - b_1\bar{c} + b_1 s), \\ \bar{w}_2 &= \frac{1}{2b_2}(a_2 - b_2\bar{c} + b_2 s). \end{aligned}$$

It is easy to check that constraint (2.33) is binding. The following theorem provides the optimal menu of contracts for the supplier.

THEOREM 6 The supplier's optimal wholesale prices under a buyer's two-type cost structure are

$$
\begin{aligned}
\underline{w}_1^* &= \frac{(v-\lambda)\frac{a_1}{2} - (v-\lambda)\frac{b_1}{2}\underline{c} + \frac{1}{2}vb_1 s}{vb_1 - \lambda\frac{b_1}{2}}, \\
\underline{w}_2^* &= \frac{(v-\lambda)\frac{a_2}{2} - (v-\lambda)\frac{b_2}{2}\underline{c} + \frac{1}{2}vb_2 s}{vb_1 - \lambda\frac{b_2}{2}}, \\
\bar{w}_1^* &= \frac{\frac{1}{2}(1-v+\lambda)a_1 - \frac{1}{2}(1-v)b_1\bar{c} + \frac{1}{2}(1-v)b_1 s + \frac{\lambda}{2}a_1 - \frac{\lambda}{2}b_1\underline{c}}{(1-v)b_1 + \frac{b_1}{2}\lambda}, \\
\bar{w}_2^* &= \frac{\frac{1}{2}(1-v+\lambda)a_2 - \frac{1}{2}(1-v)b_2\bar{c} + \frac{1}{2}(1-v)b_2 s + \frac{\lambda}{2}a_2 - \frac{\lambda}{2}b_2\underline{c}}{(1-v)b_2 + \frac{b_2}{2}\lambda},
\end{aligned}
$$

where λ is the solution of

$$
\begin{aligned}
\frac{b_1}{4}A_1^2(vb_1 - \frac{\lambda b_1}{2})^{-2} + \frac{b_2}{4}B_1^2(vb_2 - \frac{\lambda b_2}{2})^{-2} &= \frac{b_1}{4}A_2^2((1-v)b_1 - \frac{\lambda b_1}{2})^{-2} \\
&\quad + \frac{b_2}{4}B_2^2((1-v)b_2 - \frac{\lambda b_2}{2})^{-2}
\end{aligned}
$$

and $A_1 = \frac{a_1}{2}v - \frac{1}{2}vb_1\underline{c} - \frac{1}{2}vb_1 s,\ A_2 = \frac{1}{2}(1-v)a_1 + (1-v)b_1\left[\frac{1}{2}\bar{c} - \underline{c}\right] - \frac{1}{2}(1-v)b_1 s,$
$B_1 = \frac{a_2}{2}v - \frac{1}{2}vb_2\underline{c} - \frac{1}{2}vb_2 s,\ B_2 = \frac{1}{2}(1-v)a_2 + (1-v)b_2\left[\frac{1}{2}\bar{c} - \underline{c}\right] - \frac{1}{2}(1-v)b_2 s.$

Based on Theorem 6, we obtain the following insights.

THEOREM 7 The supplier will provide less price protection (r^1) under incomplete information than under full information (r) for the buyer with cost $\underline{c}$ (resp. $\bar{c}$) and more protection for the other type buyer with cost $\bar{c}$ (resp. $\underline{c}$). Their relations are as follows

$$
\begin{aligned}
\underline{r}^1 &= \frac{v-\lambda}{v-\frac{\lambda}{2}}\underline{r}, \\
\bar{r}^1 &= \frac{1-v+\lambda}{1-v+\frac{\lambda}{2}}\bar{r}.
\end{aligned}
$$

Proof: Since $r = w_1 - w_2$, substituting $\underline{w}_1$, $\underline{w}_2$, $\bar{w}_1$, $\bar{w}_2$ leads to the value of r as follows:

$$
\underline{r} = \frac{a_1}{2b_1} - \frac{a_2}{2b_2} = \bar{r}.
$$

Similarly, we have

$$\underline{r}^1 = \frac{(v-\lambda)(\frac{a_1}{2b_1} - \frac{a_2}{2b_2})}{v - \frac{\lambda}{2}},$$
$$\bar{r}^1 = \frac{(1-v+\lambda)(\frac{a_1}{2b_1} - \frac{a_2}{2b_2})}{1 - v + \frac{\lambda}{2}}.$$

The relations of protection follow the comparison. ■

When the supplier has incomplete information about the buyer's cost structure, it is intuitively expected that the supplier needs to provide higher protection to the buyer to align her incentives with the channel optimum. Theorem 7 shows that this is not always true. In fact, the level of protection depends on the buyer's cost type.

From this deterministic demand case, we demonstrate that one of the difficulties in designing incentive compatible contracts under information asymmetry is the complicated incentive-compatibility constraints which result in difficulty in obtaining analytical results. The complexity of the constraints usually is due to the complex optimal response of the buyer facing a menu of contracts. One way to design a proper contract is to fix the buyer's decision parameter(s) in the contract, so that her response is easily trackable. In the next section, we will design an incentive-compatible contract under stochastic demand by incorporating this idea.

II.3.2. Stochastic Demand Case

In this section, we study the stochastic price-sensitive demand case. We assume that the prices in two periods are exogenous in order to concentrate on the impact of information asymmetry. The buyer has a reservation profit level π_B^{min} if she does not purchase. In general, the supplier does not know c; again we assume the supplier holds a prior distribution $F_S(c)$ with a continuous density function $f_S(c)$, defined on $[\underline{c}, \bar{c}]$, where $0 \leq \underline{c} \leq \bar{c} < \infty$. The supplier offers a menu of contracts $(Q(c), r(c))$

and charges a constant wholesale price w. Note that instead of designing a contract providing the wholesale price, we provide the order quantity in the contract so that the buyer's decision parameter is explicitly defined in the contract and her response is easier to track.

In the integrated channel, in which there is a single decision maker with full information of the whole system, the total expected profit in period 1 is given by

$$\begin{aligned}\pi_J(Q(c),c) &= -(s+c)Q+\int_0^Q [p_1\xi_1-h_1(Q-\xi_1+\theta\pi_{J2}(Q-\xi_1)]d\Phi_1(\xi_1)\\ &\quad +\int_Q^\infty [p_1-g_1(\xi_1-Q)+\theta\pi_{J2}(0)]d\Phi_1(\xi_1)\\ &= [p_1+g_1-s-c-[p_1+g_1+h_1-\theta(p_2+g_2-c_2)]\Phi_1(Q)\\ &\quad -\theta(p_2+g_2+h_2)\Phi_3(Q)]Q-g_1\mu_1-\theta g_2\mu_2+[p_1+g_1+h_1\\ &\quad -\theta(p_2+g_2-c_2)]\Gamma_1(Q)+\theta(p_2+g_2+h_2)\Gamma_3(Q),\end{aligned}$$

where $\Gamma_i(x)=\int_0^x \xi_i d\Phi_i(\xi_i)$, and $\Phi_3(\cdot)$ is the cumulative distribution function of $\xi_3 = \xi_1+\xi_2$, i.e., the convolution of $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$. Define $Q^*(c)$ as the optimal solution of the integrated channel, which can be easily achieved based on the work of Lee et al. (2000). In the decentralized channel, when there is no price protection, the buyer's expected profit is given by

$$\begin{aligned}\pi_B(Q(c),c) &= [p_1+g_1-w-c-[p_1+g_1+h_1-\theta(p_2+g_2-c_2)]\Phi_1(Q)\\ &\quad -\theta(p_2+g_2+h_2)\Phi_3(Q)]Q-g_1\mu_1-\theta g_2\mu_2+[p_1+g_1+h_1\\ &\quad -\theta(p_2+g_2-c_2)]\Gamma_1(Q)+\theta(p_2+g_2+h_2)\Gamma_3(Q).\end{aligned}$$

Now define $\pi_B(\hat{c}|c)$ as the expected profit of a buyer who has a marginal cost c and chooses to participate by signing the contract $(Q(\hat{c}),r(\hat{c}))$. Her expected profit is given by

$$\begin{aligned}\pi_B(\hat{c}|c) &= [p_1+g_1-w-c-[p_1+g_1+h_1-\theta(p_2+g_2-c_2)]\Phi_1(Q(\hat{c}))\\ &\quad -\theta(p_2+g_2+h_2)\Phi_3(Q(\hat{c}))]Q(\hat{c})-g_1\mu_1-\theta g_2\mu_2+[p_1+g_1+h_1\\ &\quad -\theta(p_2+g_2-c_2)]\Gamma_1(Q(\hat{c}))+\theta(p_2+g_2+h_2)\Gamma_3(Q(\hat{c}))\\ &\quad +\theta r(\hat{c})\Phi_1(Q(\hat{c}))Q(\hat{c})-\theta r(\hat{c})\Gamma_1(Q(\hat{c}))\\ &= \pi_J(Q(\hat{c}),c)-A(\hat{c}), \qquad (2.35)\\ \text{where } A(\hat{c}) &= (w-s)Q(\hat{c})+\theta r(\hat{c})\Gamma_1(Q(\hat{c}))-\theta r(\hat{c})\Phi_1(Q(\hat{c}))Q(\hat{c}).\end{aligned}$$

To simplify our exposition, it is more convenient to consider the equivalent representation of the contract menu as $(Q(c), A(c))$, from which $r(c)$ can be easily derived. The supplier's optimization problem can be formulated as

$$\max_{Q(c),A(c)} \int_{\underline{c}}^{\bar{c}} A(c)dF_S(c).$$

We now provide the optimal design of the menu of contracts in the following theorem.

THEOREM 8 When the supplier has incomplete information of the buyer's cost c, the optimal design of the menu of contracts is $(Q(c), A(c))$ where

$$\begin{aligned}Q(c) &= Q^*(c+z(c)), \qquad (2.36)\\ A(c) &= \pi_J(Q(c),c)-\int_{\underline{c}}^{\bar{c}} Q(u)du-\pi_B^{min}, \text{ and} \qquad (2.37)\\ z(c) &= \frac{F_S(c)}{f_S(c)}.\end{aligned}$$

Proof: With some algebra, it can be shown that

$$\pi_B(\hat{c}|c)=\pi_B(\hat{c}|\hat{c})+(\hat{c}-c)Q(\hat{c}).$$

To reveal the buyer's true cost, we have $\pi_B(\hat{c}|c) \leq \pi_B(c|c)$. Reverse the roles of c and $\hat{c}$ to get another inequality and combine the two inequalities to get

$$(\hat{c} - c)Q(\hat{c}) \leq \pi_B(c|c) - \pi_B(\hat{c}|\hat{c}) \leq (\hat{c} - c)Q(c). \tag{2.38}$$

Now apply the standard argument in incentive theory to divide (2.38) by $\hat{c} - c$ and then take the limit as $\hat{c} \to c$ to get $\pi'_B(c|c) = -Q(c)$ where $\pi'_B(c|c)$ is the first-derivative of $\pi_B(c|c)$ with respect to c. Integrate $\pi'_B(c|c)$ with the initial condition π_B^{min} to get

$$\pi_B(c|c) = \pi_B^{min} + \int_c^{\bar{c}} Q(u)du.$$

Now we can rewrite the supplier's objective function as follows:

$$\max_{Q(c),A(c)} \int_{\underline{c}}^{\bar{c}} [\pi_J(Q(c), c) - \pi_B(c|c)]dF_S(c) \quad = \quad \int_{\underline{c}}^{\bar{c}} [\pi_J(Q(c), c) - z(c)Q(c)]dF_S(c) - \pi_B^{min}.$$

The equality follows from integrating by part. Thus, according to the definition of $Q^*(c)$, the above problem can be easily solved and thus (2.36) and (2.37) are obtained. ■

Although the stochastic case is usually more complicated than the deterministic case, an analytical result is achieved in this section. The reason is that the order quantity is specified in the contract menu. Thus, the buyer needs only to choose one pair from the menu and does not need to make any other operational decisions. Therefore, her optimal response is easy for the supplier to track, and we can focus on the contract design itself. In the next section, we study a returns policy contract with a different menu design which offers a wholesale price instead of an order quantity. To our best knowledge, this is the first work to provide contracts specifying a wholesale price under information asymmetry due to the analysis complexity under the stochastic demand case.

II.4. Returns Policy with Information Asymmetry

In this section, we consider a one-period problem. The market demand is stochastic. The supplier offers a returns policy to the buyer to coordinate the channel. After the selling season is over, the buyer returns any left-overs to the supplier and receives a refund r for each unit returned. Pasternack (1985) shows that a returns policy can achieve channel coordination with full information of the buyer's cost structure.

In this section, we consider the information asymmetry case and make the same assumption of information as in Section II.3.2. When the supplier has incomplete information of the buyer's cost structure, one way of offering an optimal contract is by specifying the buyer's order quantity and payment in the contract as shown in Section II.3.2. A similar result can be easily achieved with a menu of contracts $(Q(c), r(c))$, providing pairings of order quantity and returns credit based on the approach in Section II.3.2.

In this section, we study another type of contract that pairs wholesale price and returns credit. As we pointed out in Section II.3.1, contracts involving wholesale prices under information asymmetry are analytically difficult. Here, we accept this challenge and seek to design an optimal returns policy with information asymmetry consideration.

We assume that the supplier initiates the process by offering a wholesale price w, at which he will sell items to the buyer, and a repurchase price r, at which he will buy items back from the buyer at the end of the season. In response to the offered wholesale and repurchase prices, the buyer determines a quantity Q to order from the supplier. As the supplier has incomplete information of the buyer's cost c, he offers a menu of contracts $(w(c), r(c))$ and leaves it to the buyer to select the pair of her choice.

Define $\pi_B(\hat{c}|c)$ as the expected profit of a buyer who has a marginal cost c and chooses to participate by signing the contract $(w(\hat{c}), r(\hat{c}))$. The buyer's expected profit is given by

$$\begin{aligned}\pi_B(\hat{c}|c) &= p\int_0^{Q(\hat{c})} x\phi(x)dx + pQ(\hat{c})\int_{Q(\hat{c})}^{\infty}\phi(x)dx - [w(\hat{c})+c]Q(\hat{c}) \\ &\quad + r(\hat{c})\int_0^{Q(\hat{c})}(Q(\hat{c})-x)\phi(x)dx.\end{aligned}$$

It is easy to achieve the buyer's optimal order quantity as

$$Q(\hat{c}) = \Phi^{-1}(\frac{p-c-w(\hat{c})}{p-r(\hat{c})}).$$

Since the optimal quantity depends on the demand distribution faced by the buyer, it is hard to track the buyer's behavior without specific knowledge of Φ. To gain some insight into the optimal design, we assume the demand is uniformly distributed on [0,1] for the rest of this section. Then, we have the optimal order quantity

$$Q(\hat{c}) = \frac{p-c-w(\hat{c})}{p-r(\hat{c})}.$$

The buyer's maximal expected profit is given by

$$\pi_B(\hat{c}|c) = \frac{1}{2}\frac{(p-c-w(\hat{c}))^2}{p-r(\hat{c})}.$$

By the revelation principle, solving the first-order condition at $\hat{c}=c$ gives the buyer's incentive-compatibility constraint:

$$\frac{\partial\pi_B(\hat{c}|c)}{\partial\hat{c}}\,|_{\hat{c}=c} = \frac{-2w'(c)(p-c-w(c))(p-r(c))+r'(c)(p-w(c)-c)^2}{2(p-r(c))^2} = 0,$$

where $w'(c)$ and $r'(c)$ are the first-derivatives of $w(c)$ and $r(c)$ with respect to c. After simplification, we have

$$r'(c)(p-w(c)-c) = 2w'(c)(p-r(c)) \quad or \quad w' = \frac{1}{2}Qr'.$$

To maximize his expected profit, the supplier faces the following optimization problem:

$$\begin{aligned}
\max_{w,r} \quad & \int_{\underline{c}}^{\bar{c}} \left[(w-s)Q - \frac{1}{2} r Q^2 \right] f_S dc \\
\text{s.t. } w' &= \frac{1}{2} Q r' \\
\text{where } Q &= \frac{p-c-w}{p-r}
\end{aligned}$$

The following theorem provides the property of the optimal solutions.

THEOREM 9 The supplier's optimal design of the menu of contracts is the solutions of the following two equations:

$$\begin{aligned}
w' &= \frac{1}{2}\frac{p-c-w}{p-r} r', \\
r' &= \frac{(2r-p)f_S + (w-s-r)(p-c-w)f_S'}{\frac{p}{2(p-r)} f_S}.
\end{aligned} \tag{2.39}$$

When the cost distribution is uniform, the optimal design can be solved through

$$w' = \frac{2r-p}{p}, \tag{2.40}$$

$$r' = 3pr - 2r^2 - p^2. \tag{2.41}$$

Proof: The proof relies on some fundamental results from optimal control theory as outlined below (see Kamien and Schwartz (1991), pp. 142–146 for details). Letting

$$\begin{aligned}
h &= (w-s)\frac{p-c-w}{p-r} f_S - \frac{1}{2} r \left(\frac{p-c-w}{p-r}\right)^2, \\
g_1 &= \frac{1}{2}\left(\frac{p-c-w}{p-r}\right) u, \text{ and} \\
g_2 &= u = r',
\end{aligned}$$

the above problem can be expressed as

$$\max \int_{\underline{c}}^{\bar{c}} h(c)dc$$
$$\text{s.t. } w'(c) = g_1(c), r'(c) = g_2(c).$$

The necessary and sufficient conditions for optimality can be stated in terms of the Hamiltonian

$$H = h + \lambda_1 g_1 + \lambda_2 g_2.$$

These conditions include the multiplier equations given by

$$\lambda_1' = -\left[\frac{p-c-2w+s}{p-r} f_S + \frac{r(p-c-w)}{(p-r)^2} f_S - \frac{1}{2(p-r)} u\lambda_1\right], \tag{2.42}$$

$$\begin{aligned} \lambda_2' &= -[\frac{(w-s)(p-c-w)}{(p-r)^2} f_S - \frac{1}{2}(\frac{p-c-w}{p-r})^2 f_S - r\frac{(p-c-w)^2}{(p-r)^3} f_S \\ &\quad + \frac{1}{2}\frac{p-c-w}{(p-r)^2} u\lambda_1], \end{aligned} \tag{2.43}$$

and the optimality condition given by

$$\frac{1}{2}\frac{p-c-w}{p-r}\lambda_1 + \lambda_2 = 0. \tag{2.44}$$

Differentiating (2.44), we have

$$\frac{\left[-1 - \frac{p-c-w}{2(p-r)}u\right](p-r) + u(p-c-w)}{(p-r)^2}\lambda_1 + \frac{p-c-w}{2(p-r)}\lambda_1' + \lambda_2' = 0.$$

Substituting (2.42),(2.43) into (2.44), we have

$$\begin{aligned} \lambda_1 &= r(\frac{p-c-w}{p-r})^2 f_S - (w-s)\frac{p-c-w}{p-r} f_S, \\ \lambda_2 &= \frac{1}{2}(w-s)(\frac{p-c-w}{p-r})^2 f_S - \frac{1}{2}r(\frac{p-c-w}{p-r})^3 f_S. \end{aligned} \tag{2.45}$$

Differentiating (2.45) and substituting it into (2.43), we obtain (2.39). When the cost distribution is uniform, we have $f_S' = 0$; thus (2.40) and (2.41) are obtained. ■

Based on Theorem 9, an optimal design of the menu of contracts can be solved numerically for any instance. The difficulty in our analysis is due to the wholesale price design in the menu. The buyer will make her own order decision, and it is hard to track her behavior with the information asymmetry consideration.

II.5. Summary

In this chapter, we explicitly examine the buyer's impact while designing price protection and returns policies contracts. We show that when the buyer is making pricing or sales effort decisions to influence the market demand, a price protection or returns policy alone is not able to coordinate the channel. Additional terms have to be introduced into the contracts to achieve channel coordination. We provide optimal contracts with pricing, sales effort, and information asymmetry considerations. Our investigations are highlighted below:

- The design of optimal price protection contracts with retail-pricing decisions under deterministic and stochastic price-sensitive demand.
- The design of optimal price protection contracts with buyer's sales effort considerations.
- The design of optimal price protection contracts and returns policies with information asymmetry consideration.

We have shown how to provide optimal supply contracts when the supplier has incomplete information of the buyer's cost, but there is also another type of information asymmetry existing in practice. Note that in Section II.2, since the buyer may improve the market demand with sales effort, the supplier needs to provide cost sharing of the sales effort together with price protection to coordinate the channel. However, the supplier may not be able to observe the buyer's level of sales effort. Therefore, the cost sharing mechanism is difficult to implement. This type of information asymmetry is called Moral Hazard (Laffont and Martimort 2002). This situation is also sometimes referred to as hidden action. It is shown that Moral Hazard is not an issue with a risk-neutral agent despite the non-observability of the effort. A system-wide optimal solution is still implementable (see Laffont and Martimort 2002, pp. 154 for details). One possible mechanism of coordination under a non-observable sales effort is to provide a rebate to the buyer and set up a target rebate level simultaneously (Taylor 2002). A similar target protection level can also be developed together with price protection to coordinate the channel.

CHAPTER III

INFORMATION ASYMMETRY

In the context of supply contract design, the common wisdom is that the party holding the bargaining power realizes the greater profit. Traditionally, the supplier (e.g., manufacturer) has been more powerful, and, hence, the existing literature in the area emphasizes supplier-driven contracts. In fact, suppliers in monopolistic markets are able to charge above-competitive prices and, thereby, easily establish supplier-driven channels. However, in some markets, such as the grocery channel, the bargaining power has shifted to the buyer (e.g., retailer) (Messinger and Narasimhan 1995). In the United States, large retailers, such as Wal-mart, exert tremendous market power over their suppliers. Furthermore, with the advent of the Internet, buyers have access to much better information about alternative potential suppliers. As quoted in Ertek and Griffin (2002), a recent survey by Forrester indicates that manufacturers of standard products expect approximately one-third of their sales to occur on-line within the next two years. Also, 40% of these manufacturers expect the Internet to squeeze prices. Thus far, however, only a limited number of research papers have analyzed this shift in the power structure.

This chapter considers the bilateral monopoly setting that has been analyzed previously from the supplier's perspective (Corbett and Tang 1999, Corbett et al. 2004, Corbett and Groote 2000, and Ha 2001). However, we focus on the case where the buyer has the dominant bargaining power and establishes a buyer-driven channel by offering the terms of the contract. In this context, the bilateral monopoly setting is, in fact, of practical importance. Using Wal-Mart as an example, it is obvious that any major cosmetics brand-name targeting budget-conscious, medium-income female customers, such as Revlon, Maybelline, Cover Girl, etc., would be greatly interested in one-on-one negotiations with the price-cutting retailer and vice versa. The list of examples can be easily extended by including the market shares of the few retail giants for brand-name household items from personal hygiene products to small electric appliances. This is due to increased consolidation at the retail level; larger retailers have greater bargaining influence over their suppliers (Carstensen 2000).

In fact, consolidation in grocery retailing has been a powerful trend in recent years. At the national level, the top five supermarket chains accounted for 20% of the U.S. grocery sales in 1993, but 42% of the sales seven years later (Swenson 2000). Some observers predict the near term emergence of four or five chains with more than 60% of all U.S. supermarket sales (Foer 2000). As a natural consequence of these trends, buyer-driven channels have become more prevalent, and the underlying supply contract problems addressed in this chapter are increasingly important.

The three general types of contracts, i.e., one-part linear schemes, two-part linear schemes, and two-part nonlinear schemes, that have been studied in the context of supplier-driven channels are still applicable in the context of buyer-driven channels. In the ACNielsen's 2000 Annual Survey of Trade Promotion Practices (ACNielsen 2000), 85% of the retailers reported charging slotting fees whereas 42% of the manufacturers reported that they were charged increased slotting allowances. As a general practice, powerful retailers are seeking to provide more general contracts, and slotting fees are identified as a mechanism through which their influence is exercised (Dimitri 2001). In order to investigate the impact of these practices, this chapter first examines the impact of the power structure under the one-part linear scheme and then under the more general schemes involving slotting fees. By deriving the buyer's optimal contracts and profits for different contract types and providing a comparative analysis of our results with the early work on supplier-driven channels in Corbett and Tang (1999) and Corbett et al. (2004), we aim to quantify the financial value of power in contract design.

Despite recent advances in information technology and the trend towards sharing information among supply chain partners, information asymmetry remains a key feature of real supply chain relationships. Since the entities in a supply chain may belong to different firms that have conflicting objectives and/or may not have access to private information, a system-wide optimal solution is not implementable unless it can fully resolve any incentive alignment problems caused by asymmetric information. For this reason, we investigate a total of six scenarios for the buyer-driven channel: three general contracts, each under full and incomplete information about the supplier's cost structure. We develop the counterpart buyer-driven models which complement the supplier-driven models studied in Corbett and Tang (1999) and Corbett et al. (2004) and derive the buyer's optimal contracts and profits for the three different contract types. Obviously, by applying our results to any practical problem instance, one can easily answer a set of questions for the buyer-driven channel similar to those addressed in Corbett and Tang (1999) and Corbett et al. (2004) for the supplier-driven channel, such as the value of information and the value of contract flexibility. However, our main focus is on the impact of power shifting in the channel. Hence, we use our work here, in combination with the analytical work presented in Corbett and Tang (1999) and Corbett et al. (2004), to seek an answer to the fundamental question "**Is bargaining power as beneficial as it is believed to be?**" By searching for an answer to this question, we obtain results addressing the following additional questions:

- From the system's perspective, which power structure is better under different contract types?
- What is the value of power for the individual parties under different contract

types and information structures? That is, what is the impact of power on the buyer's and supplier's profits under three types of contracts with full and asymmetric information?

Our analysis leads to several interesting observations. First, we prove that a shift of power from the supplier to the buyer improves the system efficiency under a one-part linear contract. Secondly, in the case of full information, we provide an analytical confirmation that the bargaining power is beneficial for both the buyer and the supplier and that the value of the bargaining power is higher under the more general contract type. Thirdly, by investigating the interaction between the power structure and information asymmetry, we find that the bargaining power is not necessarily beneficial for either party and that this tenet of common wisdom does not hold. In other words, it is not always wise for either the buyer or the supplier to pursue the bargaining power when accurate information is not available. Finally, we demonstrate that the more general types of contracts may not increase the value of holding the power when information asymmetry exists. That is, the value of holding the power may not be significantly higher under more general contract types, and further, the party without accurate information does not necessarily benefit from the power even when a two-part nonlinear contract is allowed. Our findings indicate that sometimes one party can forfeit the bargaining power and retain a higher profit!

The remainder of this chapter is organized as follows. Section III.1 introduces the modeling framework. Buyer's optimal contracts are analyzed in Section III.2. The impacts of power structure, information asymmetry, and contract type on channel performance are explored in Section III.3. Finally, Section III.4 provides concluding remarks.

III.1. Power Structure and Information Asymmetry

We consider the same setting, with a single supplier and a single buyer, analyzed in Corbett and Tang (1999) and Corbett et al. (2004) with the exception that our preliminary goal is to derive the *buyer's* optimal contracts and profits by focusing on the buyer-driven channel. The supplier provides the product to the buyer at a wholesale price w who in turn resells it to a final market with price-sensitive demand. Demand per period for the product is denoted by d, and the selling price is denoted by p where d decreases linearly in price, i.e., $d = a - bp$, where $a \geq 0$ and $b \geq 0$ are known parameters. Thus, the buyer's order quantity d is uniquely determined by the selling price p. In this context, the corresponding supplier- and buyer-driven contract design models are developed by considering the supplier and the buyer as, respectively, the first movers in a Stackelberg game to reflect channel power. Throughout the rest of the chapter, we refer to the supplier as *he* and the buyer as *she*.

The supplier's marginal cost is denoted by s whereas the buyer's marginal cost is denoted by c. Naturally, $p > s + c$, and since $d = a - bp$, to assure nonnegative demand, we assume $a - b(s + c) > 0$. In the case of full information, each party knows the actual value of the other party's marginal cost. However, in general, there is information asymmetry in the channel, and, hence, each party may keep his/her marginal cost private so that the other party does not know the corresponding actual value. If this is the case in the buyer-driven channel, then the buyer holds a prior distribution $F_B(s)$ of the supplier's marginal cost s (with continuous density function $f_B(s)$, and mean μ_s, and second moment γ_s,) where the distribution is defined on a finite interval $[\underline{s}, \bar{s}]$. Similarly, if information asymmetry exists in the supplier-driven channel, then the supplier holds a prior distribution $F_S(c)$ (with continuous density function $f_S(c)$, mean μ_c, and second moment γ_c,) where the distribution is defined on a finite interval $[\underline{c}, \bar{c}]$.

Under these assumptions, for a given contract, i.e., when the contractual parameter values are known, the realized profit functions of the supplier and the buyer are given, respectively, by

$$\pi_S = (w - s)(a - bp) - L \quad \text{and} \quad \pi_B = (p - w - c)(a - bp) + L,$$

where L represents a lump-sum side payment from the supplier to the buyer. When $L > 0$, the side payment can be interpreted as a slotting fee, which is common when dealing with large retailers. When $L < 0$, this lump sum can be seen as a franchise fee which is more common when the supplier's product is a strong brand.

In the supplier-driven channel, the supplier designs and provides the contract while the reverse occurs in the buyer-driven channel. As we have already mentioned, the three types of contracts that have been studied in Corbett and Tang (1999) for the supplier-driven channel include the following:

- A basic one-part linear scheme, in which the supplier can only specify a constant wholesale price w independent of the order quantity q selected by the buyer.

- A two-part linear scheme, denoted by (w, L), in which the supplier offers a unit wholesale price w and a per-period lump-sum side payment L independent of the order quantity q.

- A two-part nonlinear contract, denoted by $(w(q), L(q))$, in which both the actual wholesale price $w(q)$ and the side payment $L(q)$ offered by the supplier depend on the order quantity q selected by the buyer.

In order to develop the contract terms in the corresponding buyer-driven channel that complements the above described supplier-driven channel, let us first examine the buyer's response to the three types of contracts in the above list. In the supplier-driven channel under the corresponding one-part linear scheme, after the supplier specifies the constant wholesale price w, the buyer follows with the order quantity q, and, thus, the retail price p is uniquely determined. For a given w, the buyer's optimization problem is given by $\max_p \pi_B = (p-w-c)(a-bp)$. The optimal solution is

$$p = \alpha w + \beta, \tag{3.1}$$

where $\alpha = 1/2$ and $\beta = a/(2b) + c/2$. That is, after the supplier declares the wholesale price w, the buyer responds with a specific relationship between w and p. This relationship includes a multiplier α and a markup β. Under the two-part linear scheme, the side payment is independent of p so it does not affect the buyer's response. Under the two-part nonlinear scheme, the menu of contracts is designed to reveal the buyer's true marginal cost. That is, the buyer with cost c chooses $(w(c), L(c))$ and responds in the same way as in Expression (3.1) (see Section 4 in Corbett et al. 2004). Consequently, in order to examine the corresponding buyer-driven channel, we let the buyer move first and specify the relationship between w and p in the same way as in Expression (3.1). However, as we explain in Remark 1 below, without loss of generality, it suffices to analyze the case where $\beta = 0$. Hence, the three types of contracts we consider for the buyer-driven channel include the following:

- A one-part linear scheme in which the buyer declares a non-negative price multiplier α and states that the retail price will be equal to the product of α and the wholesale price w.

- A two-part linear scheme, denoted by (α, L), in which the buyer charges a slotting fee L along with declaring an α value.

- A two-part nonlinear contract, denoted by $(\alpha(w), L(w))$, in which the buyer offers a menu of contracts where each item on the menu consists of a pair including multiplier $\alpha(w)$ and slotting fee $L(w)$, both of which depend on the wholesale price w provided by the supplier. From this menu, the supplier selects the desirable pair.

REMARK 1 *Ertek and Griffin (2002) analyze the buyer-driven one-part linear contract by considering the pricing scheme given by Expression (3.1), and they prove that, under full information, it is optimal for the buyer to set* $\beta = 0$ *as outlined below. Observe that, for the one-part linear contract, considering given values of* α *and* β*, the supplier's optimization problem is given by* $\max_w \pi_S = (w-s)(a-bp)$ *where* p *satisfies Expression (3.1). The optimal solution is*

$$w = \frac{a - bp}{b\alpha} + s. \tag{3.2}$$

Observe that, when a*,* b*,* p*, and* s *are fixed, the optimal* w *decreases as* α*, and, hence, the buyer's profit, increases. Also, observe that the largest* α *can be obtained by setting* $\beta = 0$*. Therefore, under full information, we let*

$$p = \alpha w, \tag{3.3}$$

without loss of generality. Under the two-part linear scheme, the side payment is independent of w *so that it does not affect the supplier's response. Under the two-part nonlinear scheme, the menu of contracts is designed to reveal the supplier's's true marginal cost. That is, the supplier with cost* s *chooses* $(\alpha(s), L(s))$ *and responds in the same way as in Expression (3.2) (see Section III.2.1).*

Since three types of contracts are considered under full and asymmetric information for each channel structure, there are a total of twelve cases of interest, all of which are summarized in Table III. In Table III, Cases BF1, BF2, and BF3 (resp.

SF1, SF2, and SF3) represent the corresponding one-part linear, two-part linear, and two-part nonlinear schemes for the buyer-driven (resp. supplier-driven) channel under full information. Similarly, Cases BA1, BA2, and BA3 (resp. SA1, SA2, and SA3) represent the corresponding one-part linear, two-part linear, and two-part nonlinear schemes for the buyer-driven (resp. supplier-driven) channel under asymmetric information. As we have mentioned earlier, Cases SF1, SF2, SF3, SA1, SA2, and SA3 have been analyzed in Corbett and Tang (1999), and the corresponding results are summarized in the table on p.84 for the sake of completeness. Cases BF1, BF2, BF3, BA1, BA2, and BA3 are the focus of the current chapter, and they provide the foundation for a comparative analysis of buyer- and supplier-driven channels for the purpose of quantifying the financial value of power in contract design.

The specifics of the setting we consider are as follows. In Cases BF1, BF2, and BF3 (resp. SF1, SF2, and SF3), the buyer (resp. supplier) knows s (resp. c) and offers a one-part linear contract α (resp. w), a two-part linear contract (α, L) (resp. (w, L)), and a two-part nonlinear menu of contracts $(\alpha(w), L(w))$ (resp. $(w(q), L(q))$), respectively. Cases BA1, BA2, and BA3 (resp. SA1, SA2, and SA3) are analogous except that the buyer (resp. supplier) does not know s (resp. c) and only holds a prior $F_B(s)$ (resp. $F_S(c)$). In the buyer-driven (resp. supplier-driven) channel, the supplier (resp. buyer) chooses a wholesale price w (resp. an order quantity q) based on his (resp. her) marginal cost s (resp. c). Then, all sales and financial transactions take place simultaneously. Under information asymmetry, i.e., in Cases BA1, BA2, and BA3 (resp. SA1, SA2, and SA3), for any revealed choice of w (resp. q), the buyer (resp. supplier) can infer the supplier's (resp. buyer's) true cost s (resp. c). Therefore, by applying the revelation principle (Fudenberg and Tirole 1991), one can reformulate the buyer-driven (resp. supplier-driven) two-part nonlinear menu of contracts $(\alpha(w), L(w))$ (resp. $(w(q), L(q))$) as a menu of contracts $(\alpha(s), L(s))$ (resp.

Table III Problem Cases in Supplier- and Buyer-driven Channels

Contract Type	**Supplier-Driven**	**Buyer-Driven**
1. one-part linear contract under full information	*Case SF1: the supplier offers a one-part linear contract at wholesale price w; full information about c*	*Case BF1: the buyer offers a one-part linear contract with multiplier α; full information about s*
2. one-part linear contract under incomplete information	*Case SA1: the supplier offers a one-part linear contract at wholesale price w; incomplete information about c*	*Case BA1: the buyer offers a one-part linear contract with multiplier α; incomplete information about s*
3. two-part linear contract under full information	*Case SF2: the supplier offers a two-part linear contract (w, L); full information about c*	*Case BF2: the buyer offers a two-part linear contract (α, L); full information about s*
4. two-part linear contract under incomplete information	*Case SA2: the supplier offers a two-part linear contract (w, L); incomplete information about c*	*Case BA2: the buyer offers a two-part linear contract (α, L); incomplete information about s*
5. two-part nonlinear contract under full information	*Case SF3: the supplier offers a two-part menu of contracts $(w(q), L(q))$; full information about c*	*Case BF3: the buyer offers a two-part menu of contracts $(\alpha(w), L(w))$; full information about s*
6. two-part nonlinear contract under incomplete information	*Case SA3: the supplier offers a two-part menu of contracts $(w(q), L(q))$; incomplete information about c*	*Case BA3: the buyer offers a two-part menu of contracts $(\alpha(w), L(w))$; incomplete information about s*

$(w(c), L(c)))$. The supplier (resp. buyer) announces $\hat{s}$ (resp. $\hat{c}$) and then chooses the contract $(\alpha(\hat{s}), L(\hat{s}))$ (resp. $(w(\hat{c}), L(\hat{c})))$.

III.2. Optimal Contracts for the Buyer-Driven Channel

We begin our analysis by determining the supplier's optimal wholesale price under the contracts offered by the buyer. Next, we present the buyer's optimal contracts for the six cases of interest.

III.2.1. The Supplier's Optimal Wholesale Price

Given the contract parameters, the following proposition provides an expression of the supplier's optimal wholesale price denoted by w^*.

PROPOSITION 5

- *In Cases BF1, BF2, BF3, BA1, and BA2,*

$$w^* = \frac{a}{2\alpha b} + \frac{s}{2}. \tag{3.4}$$

- *In Case BA3, letting* $\alpha'(s)$ *and* $L'(s)$ *denote the first derivatives of* $\alpha(s)$ *and* $L(s)$ *with respect to* s*, respectively, and considering the incentive-compatibility constraint given by*

$$IC: \quad L'(s) = \frac{b^2 s^2 \alpha^2(s) - a^2}{4b\alpha^2(s)} \alpha'(s), \ \forall \, \underline{s} \leq s \leq \bar{s}, \tag{3.5}$$

for the supplier, we have

$$w^* = \frac{a}{2\alpha(s)b} + \frac{s}{2} \tag{3.6}$$

Proof: In Case BF1, the buyer declares an α value, and, the supplier solves

$$\mathbf{SP}_{BF1} \quad \max_{w} \pi_S = (w - s)(a - bp).$$

Since $p = \alpha w$, the optimal wholesale price is given by (3.4).

In Case BF2, the buyer also charges a slotting fee L so that the supplier solves

$$\mathbf{SP}_{BF2} \quad \max_{w} \pi_S = (w - s)(a - bp) - L.$$

However, since L is fixed and independent of w, this does not affect the supplier's wholesale price decision. In Section III.2.3, we show that Cases BF2 and BF3 are equivalent so that the optimal wholesale price is given by Expression (3.4) in all three cases.

In Case BA1, the supplier solves the same problem as $\mathbf{SP}_{BF1}$, so the optimal wholesale price is again given by Expression (3.4). We also note that the supplier's problem is the same as $\mathbf{SP}_{BF2}$ in Case BA2, and, as a result, the optimal wholesale price is given by Expression (3.4) as well.

In Case BA3, the buyer offers a menu of $(\alpha(s), L(s))$, and the supplier chooses which cost parameter $\hat{s}$ to announce. Once he announces $\hat{s}$, $\alpha(\hat{s})$ and $L(\hat{s})$ are fixed. In this case, it can be easily shown that the supplier's optimal wholesale price is given by

$$w = \frac{a}{2\alpha(\hat{s})b} + \frac{s}{2}$$

so that, according to the contract, the buyer sets the retail price at

$$p = \frac{a}{2b} + \frac{s\alpha(\hat{s})}{2}. \tag{3.7}$$

Consequently, in order to determine the marginal cost value to be announced, the supplier solves the following problem:

$$\mathbf{SP}_{BA3} \quad \max_{\hat{s}} \pi_S(s, \hat{s}) = \left(\frac{a}{2b\alpha(\hat{s})} - \frac{s}{2}\right)\left(\frac{a}{2} - \frac{bs\alpha(\hat{s})}{2}\right) - L(\hat{s}). \tag{3.8}$$

The revelation principle states that there is an optimal contract under which the supplier will optimally announce $\hat{s} = s$. Taking the first derivative of the above expression of $\pi_S(s, \hat{s})$ with respect to $\hat{s}$, setting it equal to zero, and requiring that $\hat{s} = s$, we have Expression (3.5) which represents the incentive-compatibility constraint for the supplier. This constraint, in turn, guarantees that the supplier announces his true marginal cost s by choosing $(\alpha(s), L(s))$ so the supplier's optimal wholesale price is given by Expression (3.6). ■

III.2.2. The Buyer's Optimal Contract in Case BF1

When the buyer has complete information about the supplier's marginal cost s, the buyer knows the supplier's optimal wholesale price w^* and needs to determine the value of α so that her profit is maximized. Recalling Expressions (3.3) and (3.4), and noting that $q = a - bp$, it is easy to show that the buyer's problem is

$$\mathbf{BP}_{BF1} \quad \max_{\alpha} \pi_B(\alpha) = (p - w - c)q = \left(\frac{a}{2b} + \frac{s\alpha}{2} - \frac{a}{2b\alpha} - \frac{s}{2} - c\right)\left(\frac{a}{2} - \frac{bs\alpha}{2}\right).$$

The following proposition provides a method for computing the buyer's optimal markup value in Case BF1 denoted by[1] α^*_{BF1}.

PROPOSITION 6 *In Case BF1, α^*_{BF1} is the unique positive solution of*

$$\alpha^3 - \left(\frac{1}{2} + \frac{c}{s}\right)\alpha^2 - \frac{a^2}{2b^2s^2} = 0. \tag{3.9}$$

Proof: Observe that the above expression of $\pi_B(\alpha)$ is concave, and, hence, the optimal value of α, denoted by α^*_{BF1}, satisfies

$$\frac{a^2}{4b\alpha^2} - \frac{bs^2\alpha}{2} + \frac{bs^2}{4} + \frac{bcs}{2} = 0. \tag{3.10}$$

After some algebra, Expression (3.10) leads to Expression (3.9) for which it is easy to verify that there is a unique and finite positive solution. ■

Since Expression (3.9) is a third degree polynomial, we do not have a closed form solution of α^*_{BF1}, but a numerical value can be easily obtained. It is worth noting that here we implicitly assume that the buyer's reservation profit level is zero, and, thus, the buyer will trade as long as her profit is non-negative. Otherwise, the buyer needs to determine her cut-off level (Corbett et al. 2004, Ha 2001). This assumption

[1] Similar notation is used in the remainder of the chapter, where the superscript (*) represents the optimal value of the contract parameter of interest, and the subscript (SF1, BF1, etc.) represents one of the twelve cases of interest.

is useful in that it allows us to concentrate on a detailed investigation of the impact of power structure, information asymmetry, and contract type.

III.2.3. The Buyer's Optimal Contract in Cases BF2 and BF3

Using the same argument presented for Case BF1, it is easy to show that the buyer can determine the contract of (w, L) that optimizes her profit by solving the following problem:

$$\begin{aligned} \mathbf{BP}_{BF2} \quad \max_{\alpha} \pi_B(\alpha) &= \left(\frac{a}{2b} + \frac{s\alpha}{2} - \frac{a}{2b\alpha} - \frac{s}{2} - c\right)\left(\frac{a}{2} - \frac{bs\alpha}{2}\right) + L \\ s.t. \;\; \pi_S &\geq \pi_S^- \end{aligned} \tag{3.11}$$

Here, Condition (3.11) represents the supplier's individual rationality constraint. That is, the supplier's net profits must exceed his reservation profit level denoted by π_S^-.

PROPOSITION 7 *In Case BF2, the buyer's optimal contract is as follows:*

$$\alpha^*_{BF2} = 1 + \frac{c}{s}, \quad \textit{and} \tag{3.12}$$

$$L^*_{BF2} = -\pi_S^- + \frac{a^2 s}{4b(s+c)} - \frac{as}{2} + \frac{bs(s+c)}{4}.$$

Proof: The proof relies on two observations. First, because the buyer has complete information, she sets Expression (3.11) to be binding. Second, the buyer's profit is thus given by $\pi_B = \pi_J - \pi_S^-$ where π_J denotes the joint profit so that problem $\mathbf{BP}_{BF2}$ is equivalent to maximizing the joint profit π_J. This completes the proof. ■

Observe that, in Case BF2, it follows from Expressions (3.4) and (3.12) that the supplier's optimal wholesale price is given by

$$w^*_{BF2} = \frac{as}{2b(s+c)} + \frac{s}{2}. \tag{3.13}$$

Since $a - b(s+c) > 0$, we have $w > s$. This is different from the two-part linear contract in the supplier-driven channel where the supplier sets $w = s$. That is, in the supplier-driven channel, the supplier extracts profit only through L. Note that using Expressions (3.3), (3.12), and (3.13), we have

$$p^*_{BF2} - w^*_{BF2} - c = \frac{ac}{2b(s+c)} - \frac{c}{2} > 0, \text{where } p^*_{BF2} = \alpha^*_{BF2} w^*_{BF2}$$

and, thus, in the corresponding buyer-driven channel, the buyer also extracts a per unit profit from the sale of individual items. Since the buyer is able to extract all of the profit from the supplier in Case BF2, the flexibility to offer a nonlinear contract will not increase the buyer's profit. As a result, we have the following corollary.

COROLLARY 1 *The optimal contract is the same in Cases BF2 and BF3.*

III.2.4. The Buyer's Optimal Contract in Case BA1

In this case, the buyer holds a prior distribution $F_B(s)$ of the supplier's marginal cost s (with continuous density function $f_B(s)$), where the distribution is defined on the finite interval $[\underline{s}, \bar{s}]$. The buyer solves

$$\mathbf{BP}_{BA1} \quad \max_{\alpha} E_s[\pi_B(\alpha)] = \int_{\underline{s}}^{\bar{s}} \left(\frac{a}{2b} + \frac{s\alpha}{2} - \frac{a}{2b\alpha} - \frac{s}{2} - c \right) \left(\frac{a}{2} - \frac{bs\alpha}{2} \right) dF_B(s).$$

PROPOSITION 8 *In Case BA1, α^*_{BA1} is the unique positive solution of*

$$\alpha^3 - \left(\frac{1}{2} + \frac{c\mu_s}{\gamma_s} \right) \alpha^2 - \frac{a^2}{2b^2\gamma_s} = 0. \tag{3.14}$$

Proof: It is easy to verify that $E_s[\pi_B(\alpha)]$ is concave in α, and the first order optimality condition leads to Expression (3.14) which has a unique positive solution. ■

III.2.5. The Buyer's Optimal Contract in Case BA2

The buyer's optimization problem in Case BA2 is given by

$$\mathbf{BP}_{BA2} \quad \max_{\alpha} E_s[\pi_B(\alpha)] = \int_{\underline{s}}^{\bar{s}} [\left(\frac{a}{2b} + \frac{s\alpha}{2} - \frac{a}{2b\alpha} - \frac{s}{2} - c\right)\left(\frac{a}{2} - \frac{bs\alpha}{2}\right) + L]dF_B(s) \tag{3.15}$$

$$s.t. \;\; \pi_S \geq \pi_S^- \tag{3.16}$$

PROPOSITION 9 *In Case BA2, the buyer's optimal contract is as follows:*

$$\alpha^*_{BA2} = \frac{1}{2} + \frac{1}{2}\frac{\bar{s}^2}{\gamma_s} + \frac{c\mu_s}{\gamma_s}, \quad and \tag{3.17}$$

$$L^*_{BA2} = \left(\frac{a}{2b\alpha} - \frac{\bar{s}}{2}\right)\left(\frac{a}{2} - \frac{b\bar{s}\alpha}{2}\right) - \pi_S^-. \tag{3.18}$$

Proof: For any given α, the buyer will always choose the highest L that still satisfies the supplier's individual rationality constraint given by Expression (3.16). Observe that $\pi_S(s)$ is decreasing in s so the constraint holds for all s if it holds at $s = \bar{s}$. This observation leads to Expression (3.18). Using Expression (3.18) in Expression (3.15) and solving for α, we obtain Expression (3.17). ■

III.2.6. The Buyer's Optimal Contract in Case BA3

In this case, the buyer has the flexibility to offer a two-part nonlinear menu of contracts $(\alpha(s), L(s))$. By selecting any specific pair, the supplier reveals a marginal cost value $\hat{s}$ which will be his true marginal cost s according to the revelation principle. To maximize her expected profit, the buyer solves the problem $\mathbf{BP}_{BA3}$. Observe that, in the formulation, Expression (3.19) is identical to Expression (3.5) which represents the supplier's incentive-compatibility constraint. Under this constraint, presented with a menu of contracts $(\alpha(s), L(s))$, the supplier with cost s will choose w^* to maximize his profit.

$$\mathbf{BP}_{BA3} \quad \max_{\alpha(s)} E_s[\pi_B(\alpha(s))] = \int_{\underline{s}}^{\bar{s}} [\left(\frac{a}{2b} + \frac{s\alpha(s)}{2} - \frac{a}{2b\alpha(s)} - \frac{s}{2} - c\right)\left(\frac{a}{2} - \frac{bs\alpha(s)}{2}\right) + L]dF_B(s)$$

$$s.t. \quad L'(s) = \frac{b^2s^2\alpha^2(s) - a^2}{4b\alpha^2(s)}\alpha'(s) \tag{3.19}$$

$$\pi_S \geq \pi_S^-. \tag{3.20}$$

PROPOSITION 10 *In Case BA3, the buyer's optimal contract is as follows:*

$$\alpha^*_{BA3} = 1 + \frac{c}{s} + \frac{F_B(s)}{sf_B(s)}, \quad \textit{and} \tag{3.21}$$

$$L^{*'}_{BA3} = \frac{b^2s^2{\alpha^*_{BA3}}^2 - a^2}{4b{\alpha^*_{BA3}}^2}\alpha^{*'}_{BA3}. \tag{3.22}$$

Proof: The proof relies on some fundamental results from optimal control theory as outlined below (see Kamien and Schwartz 1991, pp. 142–146 for details.) Letting

$$\begin{aligned} h(s) &= \left(\frac{a}{2b} + \frac{s\alpha(s)}{2} - \frac{a}{2b\alpha(s)} - \frac{s}{2} - c\right)\left(\frac{a}{2} - \frac{bs\alpha(s)}{2}\right) f_B(s) + L(s)f_B(s), \\ g_1 &= \frac{b^2s^2\alpha^2(s) - a^2}{4b\alpha^2(s)}u, \quad \text{and} \\ g_2 &= u = \alpha'(s), \end{aligned}$$

the above problem representing Case BA3 can be expressed as

$$\max \int_{\underline{s}}^{\bar{s}} h(s)ds \tag{3.23}$$

$$s.t. \quad L'(s) = g_1 \tag{3.24}$$

$$\alpha'(s) = g_2 \tag{3.25}$$

$$\pi_S \geq \pi_S^- \tag{3.26}$$

The necessary and sufficient conditions for optimality can be stated in terms of the Hamiltonian

$$H = h + \lambda_1 g_1 + \lambda_2 g_2.$$

These conditions include the state equations given by Constraints (3.24) and (3.25), the multiplier equations given by

$$\lambda_1' = -\frac{\partial H}{\partial L} = -\left(\frac{\partial h}{\partial L} + \lambda_1\frac{\partial g_1}{\partial L} + \lambda_2\frac{\partial g_2}{\partial L}\right), \lambda_2' = -\frac{\partial H}{\partial u} = -\left(\frac{\partial h}{\partial \alpha} + \lambda_1\frac{\partial g_1}{\partial \alpha} + \lambda_2\frac{\partial g_2}{\partial \alpha}\right),$$

and the optimality condition given by

$$\frac{\partial H}{\partial u} = \frac{b^2s^2\alpha^2(s) - a^2}{4b\alpha^2(s)}\lambda_1 + \lambda_2 = 0. \tag{3.27}$$

Then, it is easy to show that $\lambda_1' = -f_B(s)$ so that $\lambda_1 = -F_B(s)$, whereas

$$\lambda_2' = -\left(\frac{a^2}{4b\alpha^2(s)}f_B(s) - \frac{bs^2\alpha(s)}{2}f_B(s) + \frac{bs^2}{4}f_B(s) + \frac{bcs}{2}f_B(s) + \frac{2a^2}{4b\alpha^3(s)}u\lambda_1\right). \tag{3.28}$$

Differentiating Expression (3.27), we have

$$\lambda_2' = -\left[\left(\frac{2bs}{4} + \frac{2a^2}{4b\alpha^3(s)}u\right)\lambda_1 - \frac{b^2s^2\alpha^2(s) - a^2}{4b\alpha^2(s)}f_B(s)\right], \tag{3.29}$$

and substituting Expression (3.29) into Expression (3.28) leads to Expression (3.21).

Since we have ignored Expression (3.20) in the above discussion, we need to check whether or not it is satisfied for all $s \in [\underline{s}, \bar{s}]$. Recalling Expression (3.8), $\pi_S(s)$ is decreasing in s because

$$\begin{aligned}\frac{d\pi_S(s)}{ds} &= \left(-\frac{a}{2b\alpha^2(s)}\alpha'(s) - \frac{1}{2}\right)\left(\frac{a}{2} - \frac{bs\alpha(s)}{2}\right) - \left(\frac{b\alpha(s)}{2} + \frac{bs\alpha'(s)}{2}\right)(\frac{a}{2b\alpha(s)} \\ &\quad -\frac{s}{2}) - \frac{b^2s^2\alpha^2(s) - a^2}{4b\alpha^2(s)}\alpha'(s) \\ &= \frac{bs\alpha(s) - a}{2} < 0.\end{aligned}$$

Consequently, we can set $L(\bar{s})$ such that Expression (3.20) is binding, and this completes the proof. ■

Table IV Supplier-driven and Buyer-driven Optimal Supply Contracts

Contract Type	*Supplier Driven*	*Buyer Driven*
1. one part linear under full information: $w;\alpha$	$w^*_{SF1} = \frac{a}{2b} + \frac{1}{2}(s-c)$ $p^*_{SF1} = \frac{3a}{4b} + \frac{1}{4}(s+c)$ $q^*_{SF1} = \frac{1}{4}a - \frac{1}{4}b(s+c)$ $\pi^*_{S,SF1} = \frac{1}{8b}[a-b(s+c)]^2$ $\pi^*_{B,SF1} = \frac{1}{16b}[a-b(s+c)]^2$	$w^*_{BF1} = \frac{a}{2b\alpha^*_{BF1}} + \frac{s}{2}$ $p^*_{BF1} = \frac{a}{2b} + \frac{s}{2}\alpha^*_{BF1}$ $q^*_{BF1} = \frac{1}{2}a - \frac{1}{2}sb\alpha^*_{BF1}$ $\pi^*_{S,BF1} = \frac{a^2}{4b\alpha^*_{BF1}} - \frac{as}{2} + \frac{bs^2\alpha^*_{BF1}}{4}$ $\pi^*_{B,BF1} = \frac{a^2-b^2s^2{\alpha^*_{BF1}}^2}{4b} - \frac{a^2-b^2s^2{\alpha^*_{BF1}}^2}{4b\alpha^*_{BF1}} - \frac{ac-bcs\alpha^*_{BF1}}{2}$ *where* ${\alpha^*_{BF1}}^3 - (\frac{1}{2} + \frac{c}{s}){\alpha^*_{BF1}}^2 - \frac{a^2}{2b^2s^2} = 0$
2. one part linear under asymm. information: $w;\alpha$	$w^*_{SA1} = \frac{a}{2b} + \frac{1}{2}(s-\mu_c)$ $p^*_{SA1} = \frac{3a}{4b} + \frac{1}{4}(s+c) + \frac{1}{4}(c-\mu_c)$ $q^*_{SA1} = \frac{1}{4}a - \frac{1}{4}b(s+c) - \frac{1}{4}b(c-\mu_c)$ $E_c[\pi^*_{S,SA1}] = \frac{1}{8b}[a-(b(s+\mu_c)]^2$ $\pi^*_{B,SA1} = \frac{1}{16b}[a-b(s+2c-\mu_c)]^2$	$w^*_{BA1} = \frac{a}{2b\alpha^*_{BA1}} + \frac{s}{2}$ $p^*_{BA1} = \frac{a}{2b} + \frac{s}{2}\alpha^*_{BA1}$ $q^*_{BA1} = \frac{1}{2}a - \frac{1}{2}sb\alpha^*_{BA1}$ $\pi^*_{S,BA1} = \frac{a^2}{4b\alpha^*_{BA1}} - \frac{as}{2} + \frac{bs^2\alpha^*_{BA1}}{4}$ $E_s[\pi^*_{B,BA1}] = \frac{a^2-b^2\gamma_s{\alpha^*_{BA1}}^2}{4b} - \frac{a^2-b^2\gamma_s{\alpha^*_{BA1}}^2}{4b\alpha^*_{BA1}} - \frac{ac-bc\mu_s\alpha^*_{BA1}}{2}$ *where* ${\alpha^*_{BA1}}^3 - (\frac{1}{2} + \frac{c\mu_s}{\gamma_s}){\alpha^*_{BA1}}^2 - \frac{a^2}{2b^2\gamma_s} = 0$
3. two part linear under full information: $(w,L);(\alpha,L)$	$w^*_{SF2} = s$ $L^*_{SF2} = \pi^-_B - \frac{1}{4b}[a-b(s+c)]^2$ $p^*_{SF2} = \frac{1}{2b}[a+b(s+c)]$ $q^*_{SF2} = \frac{1}{2}[a-b(s+c)]$ $\pi^*_{S,SF2} = -\pi^-_B + \frac{1}{4b}[a-b(s+c)]^2$ $\pi^*_{B,SF2} = \pi^-_B$	$w^*_{BF2} = \frac{as}{2b(s+c)} + \frac{s}{2}$ $L^*_{BF2} = -\pi^-_S + \frac{a^2s}{4b(s+c)} - \frac{as}{2} + \frac{bs}{4}(s+c)$ $p^*_{BF2} = \frac{1}{2b}[a+b(s+c)]$ $q^*_{BF2} = \frac{1}{2}[a-b(s+c)]$ $\pi^*_{S,BF2} = \pi^-_S$ $\pi^*_{B,SF2} = -\pi^-_S + \frac{1}{4b}[a-b(s+c)]^2$ $\alpha^*_{BF2} = \frac{s+c}{s}$

Table IV Continued.

Contract Type	*Supplier Driven*	*Buyer Driven*
4. two part	$w^*_{SA2} = s + \bar{c} - \mu_c$	$w^*_{BA2} = \frac{a}{2b\alpha^*_{BA2}} + \frac{s}{2}$
linear under	$L^*_{SA2} = \pi^-_B - \frac{1}{4b}[a - b(s+\bar{c}) - b(\bar{c} - \mu_c)]^2$	$L = (\frac{a}{2b\alpha^*_{BA2}} - \frac{\bar{s}}{2})(\frac{a}{2} - \frac{b\bar{s}\alpha^*_{BA2}}{2}) - \pi^-_S$
asymm. information:	$p^*_{SA2} = \frac{1}{2b}[a + b(s + \bar{c} + c - \mu_c)]$	$p^*_{BA2} = \frac{a}{2b} + \frac{s}{2}\alpha^*_{BA2}$
$(w, L); (\alpha, L)$	$q^*_{SA2} = \frac{1}{2}[a - b(s + \bar{c} + c - \mu_c)]$	$q^*_{BA2} = \frac{1}{2}a - \frac{1}{2}sb\alpha^*_{BA2}$
	$E_c[\pi^*_{S,SA2}] = -\pi^-_B + \frac{1}{4b}[a - b(s + 2\bar{c} - \mu_c)]^2 + \frac{1}{2}(\bar{c} - \mu_c)[a - b(s + \bar{c})]$	$\pi^*_{S,BA2} = \frac{a^2}{4b\alpha^*_{BA2}} - \frac{as}{2} + \frac{bs^2\alpha^*_{BA2}}{4} - L$
	$\pi^*_{B,SA2} = \pi^-_B + \frac{1}{4b}[a - b(s + \bar{c} + c - \mu_c)]^2 - \frac{1}{4b}[a - b(s + 2\bar{c} - \mu_c)]^2$	$E_s[\pi^*_{B,BA2}] = \frac{a^2 - b^2\gamma_s{\alpha^*_{BA2}}^2}{4b} - \frac{a^2 - b^2\gamma_s{\alpha^*_{BA2}}^2}{4b\alpha^*_{BA2}} - \frac{ac - bc\mu_s\alpha^*_{BA2}}{2} + L$
		$\alpha^*_{BA2} = \frac{1}{2} + \frac{\bar{s}^2}{2\gamma_s} + \frac{c\mu_s}{\gamma_s}$
5. two part nonlinear under full information: $(w, L); (\alpha, L)$	*Case SF3: results are the same as Case SF2*	*Case BF3: results are the same as Case BF2*
6. two part	$w^*_{SA3} = s + \frac{F_S(c)}{f_S(c)}$	$w^*_{BA3} = \frac{a}{2b\alpha^*_{BA3}} + \frac{s}{2}$
nonlinear under	$L^{*'}_{SA3} = \frac{1}{2}[a - b(w + c)]w'$	$L^{*'}_{BA3} = \frac{b^2s^2{\alpha^*_{BA3}}^2 - a^2}{4b{\alpha^*_{BA3}}^2}{\alpha^*_{BA3}}'$
asymm. information:		$\alpha^*_{BA3} = 1 + \frac{c}{s} + \frac{F_B(s)}{sf_B(s)}$
$(w, L); (\alpha, L)$		

III.3. The Impact of Power Structure, Information Asymmetry, and Contract Type

Using the analysis above and the results summarized in Table IV, we are now able to explore the impacts of power structure, information asymmetry, and contract type. For this purpose, by assuming that both parties will trade as long as their profits are nonnegative, we can concentrate on our investigation. In Section III.3.1, we examine the impact of information asymmetry on the buyer's profit margin in the buyer-driven channel. In Section III.3.2, we evaluate the impact of the power structure on joint profits under full information in both supplier- and buyer-driven channels. The simultaneous impact of power structure, information asymmetry, and contract type is investigated in Section III.3.3.

III.3.1. The Impact of Information Asymmetry on the Buyer's Profit Margin

Considering the case where the buyer holds a prior distribution $F_B(s)$ of the supplier's marginal cost s, suppose that the supplier's real marginal cost $\hat{s}$ is equal to μ_s so the buyer has an unbiased estimator of the supplier's cost. Then, we have the following theorem regarding the impact of information asymmetry on the buyer's profit margin.

THEOREM 10 *Information asymmetry decreases the buyer's profit margin ($p-w-c$) under the one-part linear contract and increases the buyer's profit margin under the two-part nonlinear contract. Specifically, we have*

$$\alpha^*_{BA1} \leq \alpha^*_{BF1}, \quad w^*_{BA1} \geq w^*_{BF1}, \quad p^*_{BA1} \leq p^*_{BF1},$$

$$\alpha^*_{BA3} \geq \alpha^*_{BF3}, \quad w^*_{BA3} \leq w^*_{BF3}, \quad p^*_{BA3} \geq p^*_{BF3},$$

$$p^*_{BA1} - w^*_{BA1} - c \leq p^*_{BF1} - w^*_{BF1} - c, \tag{3.30}$$

$$p^*_{BA3} - w^*_{BA3} - c \geq p^*_{BF3} - w^*_{BF3} - c. \tag{3.31}$$

Under the two-part linear contract, information asymmetry decreases the buyer's profit margin when

$$\frac{\bar{s}^2 - \gamma_s}{\delta_s^2} < \frac{2c}{\mu_s}, \tag{3.32}$$

and it increases the buyer's profit margin when

$$\frac{\bar{s}^2 - \gamma_s}{\delta_s^2} > \frac{2c}{\mu_s} \tag{3.33}$$

where δ_s^2 is the buyer's estimation variance of s.

Proof: Considering the one-part linear contract and recalling Expression (3.9), first we show that if $s_1 \leq s_2$, then $\alpha_1 \geq \alpha_2$ where s_1, s_2, α_1, and α_2 satisfy the following two expressions:

$$\alpha_1^3 - \left(\frac{1}{2} + \frac{c}{s_1}\right)\alpha_1^2 - \frac{a^2}{2b^2 s_1^2} = 0, \tag{3.34}$$

$$\alpha_2^3 - \left(\frac{1}{2} + \frac{c}{s_2}\right)\alpha_2^2 - \frac{a^2}{2b^2 s_2^2} = 0. \tag{3.35}$$

Note that if $s_1 \leq s_2$, then

$$\frac{c}{s_1} \geq \frac{c}{s_2}, \quad \text{and} \quad \frac{a^2}{2b^2 s_1^2} \geq \frac{a^2}{2b^2 s_2^2}.$$

Also, recalling Expression (3.10), we have

$$\frac{a^2}{4b\alpha_1^2} - \frac{bs_1^2\alpha_1}{2} + \frac{bs_1^2}{4} + \frac{bcs_1}{2} = 0,$$

which is equivalent to Expression (3.34). Then, if $\alpha_2 > \alpha_1$,

$$\frac{a^2}{4b\alpha_2^2} - \frac{bs_1^2\alpha_2}{2} + \frac{bs_1^2}{4} + \frac{bcs_1}{2} < 0,$$

and, hence,

$$\alpha_2^3 - (\frac{1}{2} + \frac{c}{s_1})\alpha_2^2 - \frac{a^2}{2b^2 s_1^2} > 0.$$

This, in turn, implies that

$$\alpha_2^3 - \left(\frac{1}{2} + \frac{c}{s_2}\right)\alpha_2^2 - \frac{a^2}{2b^2 s_2^2} \geq \alpha_2^3 - \left(\frac{1}{2} + \frac{c}{s_1}\right)\alpha_2^2 - \frac{a^2}{2b^2 s_1^2} > 0,$$

which contradicts Expression (3.35), and, therefore, if $s_1 \leq s_2$, $\alpha_1 \geq \alpha_2$.

Since $\gamma_s \geq \mu_s^2$, comparing Expressions (3.9) and (3.14), we have

$$\alpha^*_{BA1} \leq \alpha^*_{BF1},$$

Then, it follows from Expressions (3.3) and (3.4) that (also see the results summarized in Table IV)

$$w^*_{BA1} \geq w^*_{BF1} \quad \text{and} \quad p^*_{BA1} \leq p^*_{BF1},$$

and this inequality leads to Expression (3.30). Under the two-part linear and non-linear contracts, Expressions (3.31), (3.32), and (3.33) can be easily verified by a comparison of the corresponding selling price and wholesale price values obtained in Section III.2 and summarized in Table IV. ■

Theorem 10 implies that the impact of information asymmetry on the buyer's profit margin varies depending on the contract type. Under the one-part linear contract, information asymmetry decreases the buyer's profit margin such that a buyer with incomplete information should accept a smaller profit margin in order to maintain demand volume. Under the two-part linear contract, the impact of information asymmetry depends on the information structure, i.e., the mean and variance of the prior distribution $F_B(s)$ held by the buyer. If the variance is large, information asymmetry decreases the buyer's profit margin and the buyer should accept a smaller profit margin to maintain demand volume. On the other hand, if the variance is small, information asymmetry increases the buyer's profit margin, and the buyer may achieve a higher profit margin under incomplete information. The latter case is counter-intuitive. Since information asymmetry decreases the buyer's total profit, how could her profit margin be increased? To examine this phenomenon more precisely, one must take the slotting fee into account. If the variance is small, the buyer prefers to maintain a higher profit margin and to charge a smaller slotting fee at the same time.

Under the two-part nonlinear contract, information asymmetry always increases the buyer's profit margin which suggests that the buyer can use the additional flexibility offered by nonlinear contracts to maintain a high unit profit margin in a way that she cannot always do when restricted to offering linear contracts.

III.3.2. The Impact of Power Structure on the Joint Profit

The following theorem provides a comparison of the resulting joint profits in both supplier- and buyer-driven channels.

THEOREM 11 *Assume that both parties have full information. Under the one-part linear contract,*

$$\pi^*_{S,SF1} + \pi^*_{B,SF1} < \pi^*_{S,BF1} + \pi^*_{B,BF1} \tag{3.36}$$

where $\pi^*_{S,SF1}$ represents the supplier's optimal profit in the supplier-driven channel under full information and $\pi^*_{B,SF1}$ represents the buyer's profit in the supplier-driven channel under full information[2]*. Under the two-part linear contract,*

$$\pi^*_{S,SF2} + \pi^*_{B,SF2} = \pi^*_{S,BF2} + \pi^*_{B,BF2} = \pi^*_J \tag{3.37}$$

where π^*_J represents the corresponding optimal joint profit of the channel.

Proof: The joint profit function of the channel is given by

$$\pi_J = (p - s - c)(a - bp),$$

where π_J is a concave function of p. Hence, the corresponding optimal retail price is given by

$$p^*_J = \frac{a}{2b} + \frac{1}{2}(s + c).$$

[2]Similar notation is used in the remainder of the chapter, where the first subscript (S, B, or J) represents the supplier, buyer, or supplier-buyer pair, and the second subscript (SF1, BF1, etc.) represents one of the twelve cases of interest.

From Table 2, under the one-part linear contract, the joint profit in the supplier-driven channel is

$$\pi_{J,SF1} = \pi^*_{S,SF1} + \pi^*_{B,SF1} = \frac{3}{16b}[a - b(s+c)]^2.$$

Let $\pi_{J1} = \pi_{J,SF1} - \pi_{J,BF1}$ where $\pi_{J,BF1}$ denotes the joint profit in the buyer-driven channel. Recalling Expression (3.4), we know that

$$p^*_{BF1} = \frac{a}{2b} + \frac{s}{2}\alpha^*_{BF1}.$$

Then, substituting

$$p = \frac{a}{2b} + \frac{s}{2}\alpha$$

in the above expression of π_{J1} and solving $\pi_{J1} = 0$ for α leads to the following two roots:

$$\alpha_1 = \frac{a + bs + bc}{2bs}, \quad \text{and} \quad \alpha_2 = -\frac{a - 3bs - 3bc}{2bs}.$$

Observe that $\alpha_1 > \alpha_2$ since $a - b(s+c) > 0$. As a result, $\pi_{J1} < 0$ when $\alpha_2 < \alpha < \alpha_1$.

Next, we show that $\alpha_2 < \alpha^*_{BF1} < \alpha_1$. In order to show that $\alpha^*_{BF1} < \alpha_1$, we rewrite a as $a = b(s+c)+k$ where k is a positive constant and recall that α^*_{BF1} solves Expression (3.9). Then, using the above expressions of a and α_1, the left-hand side of Expression (3.9) reduces to

$$\frac{k(4b^2c^2 + bsk + 4bck + 4b^2sc + k^2)}{8b^3s^3},$$

which is positive. It follows that $\alpha^*_{BF1} < \alpha_1$ and $p^*_{BF1} < p^*_{SF1}$. Finally, in order to show that $\alpha_2 < \alpha^*_{BF1}$, we use the same expressions of a and α_2 in the left-hand side of Expression (3.9). After some algebra, we have

$$-\frac{k(4b^2c^2 + 16b^2s^2 - bsk - 4bck + 20b^2sc + k^2)}{8b^3s^3}.$$

Observe that the above quantity is negative as long as $4b^2c^2 + 16b^2s^2 - bsk - 4bck + 20b^2sc + k^2 > 0$ which is minimized at

$$k = \frac{bs}{2} + 2bc,$$

with a corresponding minimum value of

$$18b^2sc + \frac{63b^2s^2}{4} > 0.$$

It follows that $\alpha_2 < \alpha^*_{BF1}$. Therefore, $\pi_{J1} < 0$, and this completes the proof of Expression (3.36).

Under the two-part linear contract, Expression (3.37) holds because the corresponding optimal joint solutions are always realizable, so the leader extracts all of the additional system profits using the optimal contracts summarized in Table IV. ■

Theorem 11 implies that, under the one-part linear contract, the buyer-driven channel always performs better than the counterpart supplier-driven channel. That is, the shift of power from the supplier to the buyer actually improves the system efficiency. This is counter-intuitive in the sense that the performance of the channel is expected to depend on the cost and demand parameters so that one power structure dominates the other in some cases whereas the opposite is true in other cases. However, according to Theorem 2, the power structure itself determines the channel performance, and the buyer-driven channel always achieves a higher joint profit under the one-part linear contract. The intuitive explanation is that, in the supplier-driven channel, the buyer is the follower and responds to the supplier's wholesale price. Since the supplier seeks to maximize his own profit, the buyer is pushed to select a higher than system optimal retail price which limits the market demand. On the other hand, when the buyer is the leader, she selects the α value before the supplier declares the wholesale price, and, thus, the buyer has more freedom to warrant a higher market demand by choosing a relatively smaller retail price which is closer to the system optimal retail price. Under the two-part linear contract, obviously, the value of power shifting is higher than under the one-part linear contract since the party with the dominant bargaining power extracts all of the additional profits.

III.3.3. The Impact of Power Structure versus the Impact of Information Asymmetry

In this section, we investigate the impact of power structure and information asymmetry on individual profits under three types of contracts. The common wisdom is that the buyer (resp. supplier), with dominant bargaining power, will gain more profit in the buyer-driven (resp. supplier-driven) channel than in the supplier-driven (resp. buyer-driven) channel. The following theorem provides a formal presentation of the common wisdom which is true when each party has full information of the system under the assumption that the buyer (resp. supplier) does not have any reservation profit in the buyer-driven (resp. supplier-driven) channel.

THEOREM 12 *For the case of full information, we have*

$$\pi^*_{B,BF1} \geq 2\pi^*_{B,SF1}, \tag{3.38}$$

$$\pi^*_{S,SF1} \geq \pi^*_{S,BF1}, \tag{3.39}$$

$$\pi^*_{B,BF2} = \pi^*_{S,SF2} = \pi^*_J, \quad \textit{and} \tag{3.40}$$

$$\pi^*_{B,SF2} = \pi^*_{S,BF2} = 0. \tag{3.41}$$

Proof: The theorem cannot be verified analytically by a direct comparison of the buyer's profits in the two channels simply because we do not have a closed form expression for α. Hence, instead of a direct comparison of the $\pi^*_{B,BF1}$ and $\pi^*_{B,SF1}$ values, we complete the proof by analyzing the more general one-part linear scheme

(α, β).

First, we examine the case where the buyer uses only a mark-up in the contract, that is, $\alpha = 1$. In this case, given the value of β, it is easy to show that the supplier's optimal wholesale price is

$$w^*_{BF1} = \frac{a}{2b} + \frac{1}{2}(s - \beta),$$

whereas the buyer's optimal mark-up is

$$\beta^*_{BF1} = \frac{a}{2b} + \frac{1}{2}(c - s).$$

Thus, the optimal retail price is

$$p^*_{BF1} = \frac{3a}{4b} + \frac{1}{4}(s + c),$$

which is the same as the one in the supplier-driven channel. As a result, the buyer's optimal profit under the contract $(\alpha = 1, \beta = \frac{a}{2b} + \frac{1}{2}(c - s))$ is given by

$$\pi^{**}_{B,BF1} = \frac{[a - b(s + c)]^2}{8b} = 2\pi^*_{B,SF1}.$$

Now, recall Expression (3.2) in Remark 1, and note that for any fixed value of p, the corresponding α and β values can be calculated once w is determined. Expression (3.2) shows that when all other parameters are fixed, the supplier will select a smaller w as α increases. The largest α can be obtained by setting $\beta = 0$. Thus, there exists a contract (α, β), such that $\alpha > 1$ and $\beta = 0$, under which the buyer's profit is larger than $\pi^{**}_{B,BF1}$. Hence, $\pi^*_{B,BF1} \geq \pi^{**}_{B,BF1}$ so Expression (3.38) is verified.

Noting that $\pi^*_{S,BF1} + \pi^*_{B,BF1} \leq \pi^*_J$, we have

$$\pi^*_{S,BF1} \leq \pi^*_J - \pi^*_{B,BF1} \leq \frac{1}{8b}[a - b(s + c)]^2,$$

where

$$\pi_J^* = \frac{1}{4b}[a - b(s + c)]^2$$

is the optimal joint profit, so Expression (3.39) is verified.

Expressions (3.40) and (3.41) can be easily verified using the results summarized in Table IV. ■

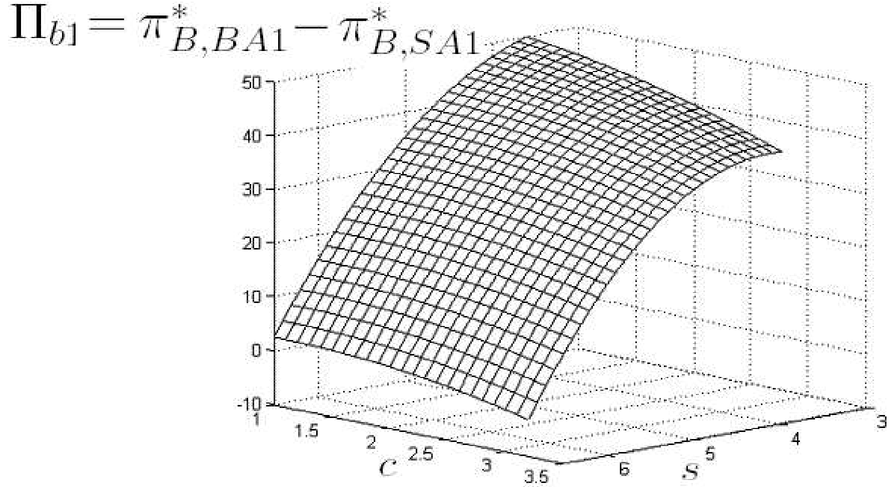
Naturally, the next question is whether or not the bargaining power remains a significant advantage under conditions of information asymmetry. For example, in the buyer-driven channel, when the buyer has incomplete information about the supplier's costs, is it still beneficial for the buyer to act as the channel leader? Below we discuss some numerical results that answer this question.

Suppose that $q = a - bp = 120 - 9p$ and the supplier's and buyer's prior distributions of c and s are on the bounded domains $[\underline{c}, \bar{c}] = [1, 3.6]$, $[\underline{s}, \bar{s}] = [3.6, 6.5]$, respectively. Let $\Pi_{b1} = \pi^*_{B,BA1} - \pi^*_{B,SA1}$, so that Π_b represents the buyer's relative benefit from being the leader in a buyer-driven channel rather than being the follower in a supplier-driven channel. Under these assumptions, Figure 2 illustrates the buyer's realized relative benefit under power shifting when c is uniformly distributed and s is exponentially distributed with

$$F_s(s) = \frac{1 - e^{\frac{3.6-s}{2.9}}}{1 - e^{-1}}.$$

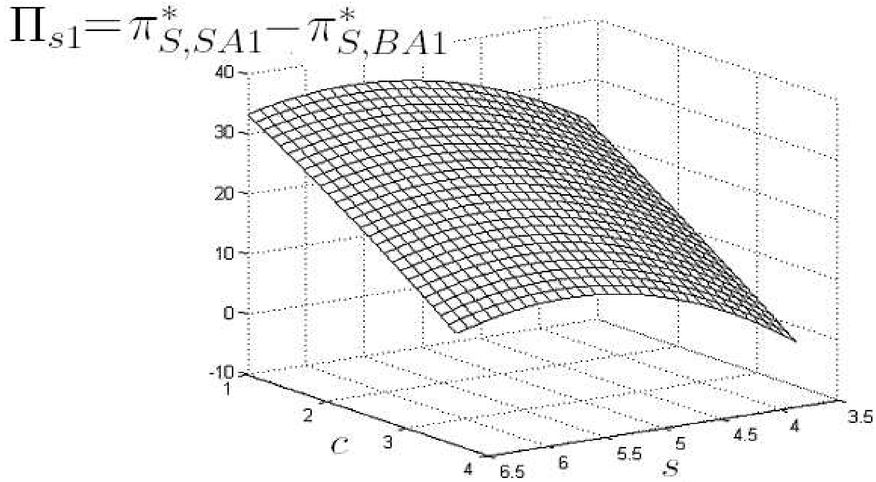
Observe that Π_{b1} is higher when the buyer's marginal cost c is smaller. This observation suggests that the buyer should keep the bargaining power when c is small. Also, Π_{b1} decreases as the supplier's marginal cost s increases, and, obviously, the supplier's (follower's) marginal cost has a more significant impact on Π_{b1} than the buyer's (leader's) marginal cost. In Figure 2, one interesting observation is that when c is close to $\bar{c}$ and s is close to $\bar{s}$, the corresponding Π_{b1} value is negative. That is,

Figure 2 The Impact of Power Structure and Information Asymmetry on the Buyer's Profit



considering those cases where the supplier's marginal cost is large (so the buyer is likely to underestimate its value), if the buyer's own marginal cost is large as well, then the buyer achieves a higher profit by giving up her bargaining power and acting as a follower! As a result, under information asymmetry, the buyer (leader) may potentially lose the relative benefit of the bargaining power to the supplier (follower.) According to Figure 3, this is also true for the supplier-driven channel where Π_{s1} represents the supplier's relative benefit of being the leader in a supplier-driven channel rather than being the follower in a buyer-driven channel, i.e, $\Pi_{s1} = \pi^*_{S,SA1} - \pi^*_{S,BA1}$. Observe that, in Figure 3, Π_{s1} is decreasing in c, and, again, the follower's (buyer's) marginal cost has a more significant impact than the leader's (supplier's) marginal cost on the supplier's relative benefit. Also, note that the supplier's relative benefit is negative when s is close to $\underline{s}$ and c is close to $\bar{c}$. That is, when the supplier's own marginal cost is small, and the buyer's marginal cost is large (so the supplier is likely to underestimate the buyer's cost), the supplier achieves a higher profit by acting as a follower. This phenomenon, in which the bargaining power under information

Figure 3 The Impact of Power Structure and Information Asymmetry on the Supplier's Profit



asymmetry reduces the leader's relative benefit, can be explained as follows. When information asymmetry exists in the channel, private information helps the follower without the dominant bargaining power improve his/her bargaining position, and, thus, the relative benefit of possessing the power is reduced and the follower's consequent profit might be improved. The above example shows that it is not always good to pursue the bargaining power, especially when accurate information is not available. Sometimes, in fact, one can forfeit the bargaining power and still have higher profit.

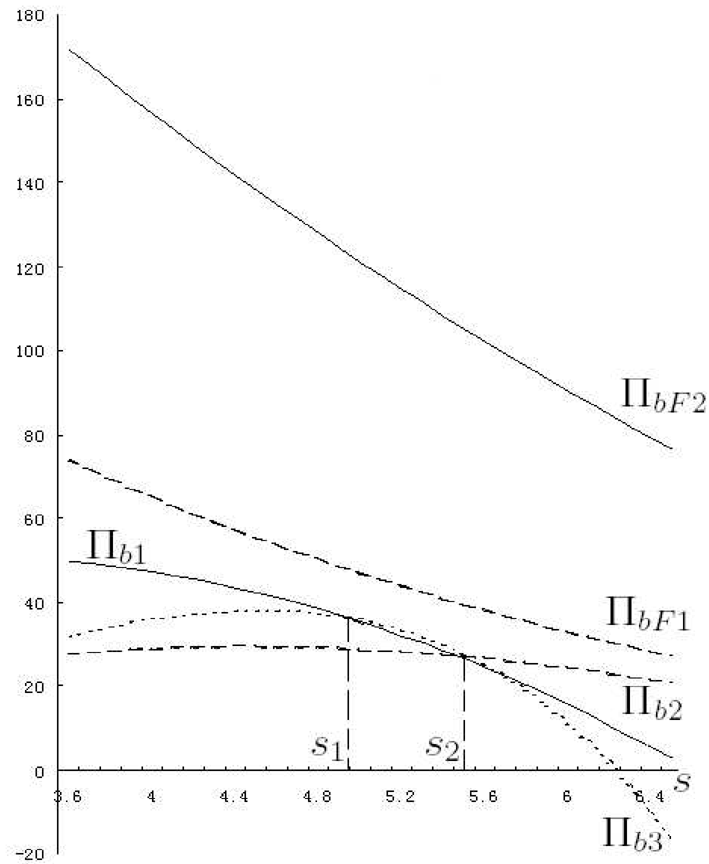
It is worth noting that Figures 2 and 3 illustrate the impact of power structure and information asymmetry under one-part linear contracts for buyer- and supplier-driven channels, respectively. Since two-part linear contracts help the leader of the channel exercise his/her influence and extract all profits under full information, the following questions arise:

- Should the leader offer more general contracts so that the advantage of leader-ship would be guaranteed?

- Is the relative benefit definitely positive under more general contracts?

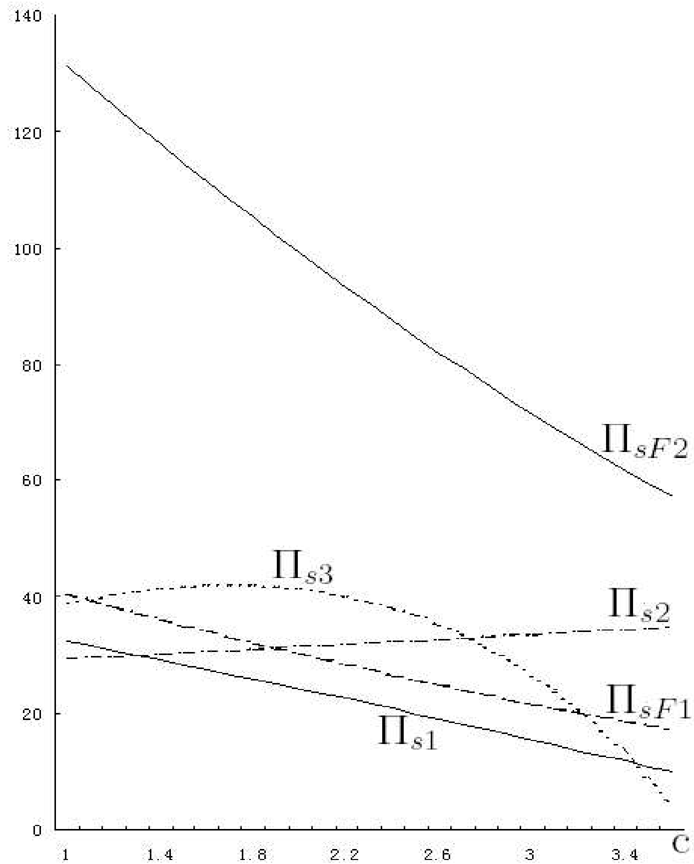
The answer to both of these questions is still NO! Under information asymmetry, more general contracts do not necessarily improve the relative benefits of holding the bargaining power. Considering the above numerical example and concentrating on the buyer-driven channel, suppose that the buyer offers both two-part linear and two-part nonlinear contracts. The buyer's relative benefits are negative when, for instance, $[c, s] = [3.2, 6.5]$ and $[c, s] = [1, 6.3]$ under the two-part linear and non-linear contracts, respectively. Therefore, even under two-part contracts, it is not necessarily more beneficial for the buyer to act as the leader rather than as the follower. A similar observation is true for the supplier-driven channel as well. Even under the two-part nonlinear contract, the supplier may achieve less profit as the leader than as the follower when, for instance, $[c, s] = [3.6, 5.7]$. Due to the existence of information asymmetry, being the leader is not necessarily as beneficial as being the follower, even under more general contracts. Figures 4 and 5 illustrate the relative value of power shifting under three types of contracts in full and incomplete information cases from the buyer's and supplier's perspectives, respectively. Assuming that $c = 1$, Figure 4 provides a comparison of the corresponding three types of contracts for the buyer-driven channel. We let $\Pi_{b2} = \pi^*_{B,BA2} - \pi^*_{B,SA2}$ and $\Pi_{b3} = \pi^*_{B,BA3} - \pi^*_{B,SA3}$, so that Π_{b2} and Π_{b3} represent the relative values of power shifting for the buyer under two-part linear and nonlinear contracts with an information asymmetry consideration. Also, we let $\Pi_{bF1} = \pi^*_{B,BF1} - \pi^*_{B,SF1}$ and $\Pi_{bF2} = \pi^*_{B,BF2} - \pi^*_{B,SF2}$, so that Π_{bF1} and Π_{bF2} represent the relative values of the power shifting to the buyer under the one-part and two-part linear contracts with full information. Similarly, $\Pi_{s1} = \pi^*_{S,SA1} - \pi^*_{S,BA1}$, $\Pi_{s2} = \pi^*_{S,SA2} - \pi^*_{S,BA2}$, $\Pi_{s3} = \pi^*_{S,SA3} - \pi^*_{S,BA3}$, $\Pi_{sF1} = \pi^*_{S,SF1} - \pi^*_{S,BF1}$, $\Pi_{sF2} = \pi^*_{S,SF2} - \pi^*_{S,BF2}$.

Figure 4 The Impact of Power Structure, Information Asymmetry, and Contract Type on the Buyer's Profit



The corresponding illustration for the supplier-driven channel is presented in Figure 5. Figure 4 illustrates that the two-part linear contract always dominates the one-part linear contract under full information, i.e., the relative value of the power shifting is always higher for the buyer under the two-part linear contract than under the one-part linear contract. This is consistent with Theorem 3, so the buyer extracts all of the profits of the channel. Also, in Figure 4, under full information, the relative value of the power shifting to the buyer depends purely on the contract type. Under incomplete information, the one-part linear contract dominates the two-part contracts

Figure 5 The Impact of Power Structure, Information Asymmetry, and Contract Type on the Supplier's Profit



when s is small ($s < s_1$), whereas the two-part linear contract dominates both the one-part linear and the two-part nonlinear contracts when s is large ($s > s_2$). The two-part nonlinear contract dominates the other two types of contracts when $s_1 < s < s_2$. Obviously, no contract always dominates the other two under asymmetric information where the relative value of the power shifting to the buyer does not purely depend on the contract type due to the existence of information asymmetry. Figure 5 also confirms that the relative value of power shifting does not depend purely on the contract type from the supplier's perspective.

These numerical results imply that information asymmetry dramatically reduces the benefits of possessing the bargaining power as well as the benefits of contract flexibility. Therefore, bargaining power and/or more general contracts alone are not necessarily helpful for extracting significant benefits in cases of incomplete information.

III.4. Summary

In this chapter, we explore the impact of power structure, information asymmetry, and contract type on the joint and individual profits in a two-stage supply chain with a single-product. Considering the buyer-driven channel, we derive optimal supply contracts with information asymmetry considerations. We show that the impact of information asymmetry on the buyer's profit margin is quite different under different types of contracts. Under the one part linear contract, information asymmetry decreases the buyer's profit margin. Under the two-part linear contract, the impact of information asymmetry depends on the buyer's estimation accuracy. Under the two-part nonlinear contract, information asymmetry increases the buyer's profit margin.

We also show that, from the system's perspective, the buyer-driven channel is more efficient than the supplier-driven channel under the corresponding one-part linear contract. We confirm the common wisdom that the bargaining power is beneficial for both the buyer and the supplier and that the value of possessing bargaining power is higher under more general contract types in full information cases. However, when information asymmetry exists in the supply chain, the bargaining power is not necessarily beneficial for either party and the common wisdom does not hold. Thus, it is not always wise for the buyer or the supplier to pursue the bargaining power when accurate information is not available. In addition, more general types of contracts may not increase the benefits of possessing power when information asymmetry exists. That is, the value of the bargaining power may be smaller under more general contract types. Hence, the party without accurate information may not benefit from having power even when a two-part nonlinear contract is allowed which means that sometimes one can forfeit the bargaining power and still gain higher profit. Our investigations are highlighted below:

- The design of different types of optimal contracts for a buyer-driven channel with full information sharing.
- The design of different types of optimal contracts for a buyer-driven channel with incomplete information of cost structure.
- The analysis of the impact of power structure and information asymmetry on supply chain performance under different types of contracts.

CHAPTER IV

SUPPLY UNCERTAINTY

The existing supply contract design literature ignores supply uncertainty issues. That is, this literature assumes that when an order is placed, it is either filled immediately (the case of zero lead time) or after a deterministic, or perhaps random, lead time. In reality, the supply of a product may sometimes be interrupted due to suppliers' random equipment breakdowns, maintenance durations, delays in raw material supply, etc. Hence, supply availability remains an important, but generally overlooked, issue. Another reason for unpredictability in the supply process is uncertainty in the yield quantity/quality due to the random proportion of defective items received.

A properly designed supply contract provides an opportunity to improve system-wide profits under supply uncertainty by explicitly defining how to share the cost and risk caused by this uncertainty, i.e., how to coordinate the channel. With cost-sharing, the buyer can order more in each period so as to achieve optimal system-wide profits (therefore, the supplier and the buyer have a bigger pie to share) without increasing her cost. This is the main idea that we implement in designing an optimal contract with supply uncertainty considerations.

As we have discussed in Chapter III, the power structure plays a key role in supply contract design. Therefore, it is important to first build a fundamental model that considers power structure and supply uncertainty and to present a framework on which we may develop more dedicated contractual mechanisms. Therefore, the reminder of this chapter is organized as follows. Section IV.1 presents a framework that incorporates power structure and supply uncertainty. Section IV.2 designs optimal contracts with continuous deterministic demand. Under a stochastic single-period demand, supply contracts are addressed in section IV.3. Concluding remarks are provided in Section IV.4.

IV.1. Optimal Contracts under Power Structure and Supply Uncertainty

In this section, we develop a one-supplier-one-buyer model in a single period that considers power structure and supply uncertainty. Note that supply uncertainty refers to several situations including supply availability, yield quantity, and yield quality. To develop the fundamental model and to provide insights into the dynamics of the channel, we assume that the supply availability and yield quantity is in statistical control. That is, the supplier's production process generates a known, constant proportion of defective items $\bar{p}$ (such an assumption has been made previously in the literature by Affisco et al. 2002 and Cheng 1991). Also, we assume that the demand is deterministic price-sensitive. Under this setting, the analysis to derive the supplier's optimal production lot size q_1, or the buyer's optimal order quantity Q_1, is the same as that without a supply uncertainty consideration (q_2 or Q_2) after adjustment with factor $\bar{p}$, that is, $(1-\bar{p})q_1 = q_2$ or $(1-\bar{p})Q_1 = Q_2$. Without loss of generality, we assume that $\bar{p} = 0$ in this section. However, even if each product is non-defective, the design, functions, and other aspects of the products may also affect the customer's satisfaction, and thus, the market demand. We assume that yield quality τ is a measurable attribute with values in the interval $[0, \infty)$. Also, we assume that the market demand function is linear in price and quality:

$$d = a - bp + c\tau.$$

The supplier has the option to improve the quality level with investment. The investment cost function is given by

$$C = (\nu + \zeta\tau)d + \kappa + \eta\tau^2.$$

Such an assumption has been made previously in the literature by Banker et al. (1998). Thus, the quality level selected by the supplier affects total cost in two ways:

1. Investment in a quality improvement program increases fixed production costs. The fixed cost $\kappa + \eta\tau^2$ is increasing and convex at the quality level τ.

2. The quality level also has an impact on the production cost per unit. Specifically, ν denotes the variable production costs per unit not including the quality related costs. Given a quality level τ selected by the supplier, the unit variable cost increases by $\zeta\tau$.

To develop the framework with power structure and supply uncertainty considerations, we study the simple contract design in which the supplier decides the quality level and the wholesale price and the buyer decides the retail price. Additional dedicated contractual mechanisms can be further developed based on the model that we present in this section. First we study the case where the supplier is the leader of the channel. The price and quality decisions take place in the following sequence in time:

i) Supplier (manufacturer) selects his quality level; buyer (retailer) observes the quality level;

ii) Supplier (manufacturer) selects the wholesale price;

iii) Buyer (retailer) selects the retail price.

Our model also reflects the assumption that price decisions are made after quality decisions, since the choice of a quality level reflects a long-term decision which cannot be changed as easily or as frequently as price. To solve this three-stage game, we first calculate the optimal retail price for the buyer assuming a given quality level and wholesale price, and then determine the optimal wholesale price assuming a given quality level. Lastly, the optimal quality level is decided for the supplier.

The profit functions of the supplier and buyer are given by:

$$\pi_S = (w - \nu - \zeta\tau)(a - bp + c\tau) - \kappa - \eta\tau^2, \tag{4.1}$$

$$\pi_B = (p - w)(a - bp + c\tau). \tag{4.2}$$

For a given quality level and wholesale price, the first-order condition characterizing the optimal retail price is given by

$$\frac{\partial \pi_B}{\partial p} = (a - bp + c\tau) - b(p - w) = 0.$$

We obtain the optimal retail price for a given quality level and wholesale price as follows:

$$p = \frac{a}{2b} + \frac{w}{2} + \frac{c\tau}{2b}. \tag{4.3}$$

Substituting (4.3) into (4.1), we have

$$\pi_S = (w - \nu - \zeta\tau)(\frac{a}{2} - \frac{bw}{2} + \frac{c\tau}{2}) - \kappa - \eta\tau^2.$$

The first-order condition characterizing the optimal wholesale price is given by

$$\frac{\partial \pi_S}{\partial w} = (\frac{a}{2} - \frac{bw}{2} + \frac{c\tau}{2}) - \frac{b}{2}(w - \nu - \zeta\tau) = 0.$$

Thus, we obtain the optimal wholesale price for a given quality level:

$$w = \frac{a}{2b} + \frac{c\tau}{2b} + \frac{\zeta\tau}{2} + \frac{\nu}{2}. \tag{4.4}$$

Substituting (4.4) into (4.3), we have

$$p = \frac{1}{4b}(3a + 3c\tau + b\nu + b\eta\tau). \tag{4.5}$$

Substituting (4.4) into (4.1), we have

$$\pi_S = (\frac{a}{2b} + \frac{c\tau}{2b} - \frac{\nu}{2} - \frac{\zeta\tau}{2})(\frac{a}{4} + \frac{c\tau}{4} - \frac{b\zeta\tau}{4} - \frac{b\nu}{4}) - \kappa - \eta\tau^2.$$

The first-order condition characterizing the optimal quality level is given by

$$\frac{\partial \pi_S}{\partial \tau} = (\frac{c}{2b} - \frac{\zeta}{2})(\frac{a}{4} + \frac{c\tau}{4} - \frac{b\zeta\tau}{4} - \frac{b\nu}{4}) + (\frac{c}{4} - \frac{b\zeta}{4})(\frac{a}{2b} + \frac{c\tau}{2b} - \frac{\nu}{2} - \frac{\zeta\tau}{2}) - 2\eta\tau = 0.$$

Finally, we obtain the optimal quality level

$$\tau = \frac{a\zeta - \frac{ac}{b} + c\nu - b\zeta\nu}{\frac{c^2}{b} - 2c\zeta + b\zeta^2 - 8\eta}. \tag{4.6}$$

Now we study the case when the supplier is the follower in the channel. The price and quality decisions take place in the following sequence in time:

i) Supplier (manufacturer) selects his quality level; buyer (retailer) observes the quality level;

ii) Buyer (retailer) announces her profit margin m;

iv) Supplier (manufacturer) selects the wholesale price.

REMARK 2 *Note that here the buyer announces only her profit margin while in Chapter III the buyer announces both the profit margin and the price multiplier. Thus, the buyer has less bargaining power in this section than in Chapter III. An analysis considering both the profit margin and a price multiplier can be easily conducted based on the framework in this section.*

To solve this three-stage game, we first calculate the optimal wholesale price assuming a given quality level and the buyer's profit margin, and then we determine the optimal profit margin assuming a given quality level. Lastly, the optimal quality level is determined.

The profit function of the supplier is given by:

$$\pi_S = (w - \nu - \zeta\tau)(a - b(w + m) + c\tau) - \kappa - \eta\tau^2. \tag{4.7}$$

The first-order condition characterizing the optimal retail price for a given quality level and the buyer's profit margin is as follows

$$\frac{\partial \pi_S}{\partial w} = (a - b(w + m) + c\tau) - b(w - \nu - \zeta\tau) = 0.$$

We obtain the optimal wholesale price for a given quality level and the buyer's profit margin:

$$w = \frac{1}{2b}(a - bm + c\tau + b\nu + b\zeta\tau). \tag{4.8}$$

Substituting (4.8) into (4.2), we have

$$\pi_B = m(a - bm - \frac{1}{2}(a - bm + c\tau + b\nu + b\zeta\tau) + c\tau).$$

The first-order condition characterizing the optimal profit margin for a given quality level is

$$\frac{\partial \pi_B}{\partial m} = (a - bm - \frac{1}{2}(a - bm + c\tau + b\nu + b\zeta\tau) + c\tau) - \frac{bm}{2} = 0.$$

Thus, we obtain the optimal profit margin for a given quality level:

$$m = \frac{1}{2b}(a + c\tau - b\nu - b\zeta\tau). \tag{4.9}$$

Since $p = w + m$, we have

$$p = \frac{1}{4b}(3a + 3c\tau + b\nu + b\eta\tau). \tag{4.10}$$

Recalling (4.5), note that the optimal retial prices are the same under both channels for a given quality level τ.

Substituting (4.9) into (4.7), we have

$$\pi_S = \frac{1}{2}(\frac{1}{4b}(a+c\tau+3b\nu+3b\zeta\tau)-\nu-\zeta\tau)(a+c\tau-\frac{1}{4}(3a+3c\tau + b\nu+b\zeta\tau))-\kappa-\eta\tau^2.$$

The first-order condition characterizing the optimal quality level is given by

$$\frac{\partial \pi_S}{\partial \tau} = \frac{1}{32}(\frac{c}{b}-\zeta)(a+c\tau-b\nu-b\zeta\tau)+\frac{1}{32}(c-b\zeta)(\frac{a}{b}+\frac{c\tau}{b}-\nu-b\zeta\tau)-2\eta\tau=0.$$

Finally, we obtain the optimal quality level:

$$\tau = \frac{a\zeta - \frac{ac}{b} + c\nu - b\zeta\nu}{\frac{c^2}{b} - 2c\zeta + b\zeta^2 - 32\eta}. \tag{4.11}$$

Comparing the quality levels under two power structures, we obtain the following theorem.

THEOREM 13 The optimal quality level is higher when the supplier is the follower of the channel than when he is the leader.

Proof: It is easy to check by comparing (4.6) with (4.11). ■

Theorem 13 shows that usually the supplier maintains a higher quality level when he does not have the dominant bargaining power in the channel. Since the buyer's optimal retail prices are the same under both power structures for a given quality level, the buyer will sell products at a higher price when she has the dominant bargaining power according to (4.10). The supplier will also increase the wholesale price accordingly. Therefore, it is important to take the power structure and supply uncertainty into account when designing supply contracts. Now we have presented the general framework with power structure and supply uncertainty considerations. Further investigation of more subtle considerations of supply uncertainty can be conducted under the general framework we have presented. In the next section, we will consider an infinite planning horizon problem with continuous deterministic demand, in which the proportion of defective items $\bar{p}$ has to be treated carefully.

IV.2. Optimal Contracts under Deterministic Demand

In this section, we seek to improve supply chain efficiency by designing supply chain contracts that include lot sizing and quality (defective items) considerations. Specifically, we consider a supply chain consisting of a single supplier and a single buyer where the supplier ships in response to orders from the buyer on a lot-for-lot basis. A proportion of products is defective in each lot. We investigate two cases. In the first case, the quality level is fixed and the buyer is seeking proper contracts that promotes cooperation with the supplier in order to reduce the system total cost and to properly share the savings. In the second case, the buyer initiates a supplier development program and helps the supplier improve his product quality level. Also, the buyer offers contracts as a formal mutual commitment to specify the targeted quality level, to share the investment expense to improve quality level and to distribute the resulting savings between both parties. In the first case, one critical piece of information that the buyer must have when offering contracts is the supplier's product quality level since the buyer needs to specify her lot size decision accordingly. However, the supplier may not be willing to share his full knowledge with the buyer because, for instance, there is lack of mutual trust or the supplier is an opportunist. In the second case, the knowledge of how much total effort and investment must be spent to improve the current quality to a certain level is also critical to the buyer offering contracts. It may depend on the supplier's current production conditions, management effectiveness, employee training level, etc. Again, the supplier may hesitate to release all of the information. Therefore, to design implementable supply chain contracts, we need to address the following questions:

- How can we design optimal contracts for the buyer when information asymmetry exists in the system?
- What is the impact of information asymmetry on the system and the individual parties?

To examine the situation where information asymmetry exists, we adapt the principal-agent contracting approach and derive optimal menus of contracts for the buyer. Our analysis leads to several interesting observations. Firstly, we show that the buyer will order a larger lot size than the channel optimum when she has incomplete information of the supplier's quality level. That is, the buyer tends to order more than the optimum to guarantee that she will have a large enough inventory of good units to reduce the average setup costs under asymmetric information. Secondly, a contract may be less effective than no form of contract when information asymmetry exists, which occurs when the hazard rate of the buyer's estimation of the supplier's quality level is small. Thirdly, when the buyer initiates a supplier development program to help the supplier improve his quality level, asymmetric information about the investment sensitivity evokes a lower quality requirement from the buyer than when the buyer has full information. Fourthly, the buyer may give up the supplier development program if her estimated hazard rate of the supplier's investment sensitivity is significantly small. Finally, through numerical examples, we show that variation in the buyer's estimation impairs her capability of extracting savings from the supplier while she can extract all of the savings when she has full information of the system. To our best knowledge, this work is the first attempt in the supply chain contract literature to investigate the effect of lot sizing, quality information, and supplier development programs.

IV.2.1. Supply Uncertainty in Statistical Control

We consider a stylistic setting in which a single supplier ships on a lot-for-lot basis in response to orders from a single buyer. Demand is deterministic. To concentrate explicitly on the quality issue in our analysis, we assume in this setting that the supplier's inventory carrying cost is negligible or that the supplier's production rate is infinite so that the supplier bears no inventory cost. This setting can be easily extended to a considerable carrying cost setting, however. Under our stated conditions, the joint economic lot size (JELS) of the channel may be obtained by minimizing the joint total relevant cost as

$$C_j(Q) = \frac{D}{Q}(S_s + S_b) + \frac{Q}{2}h_b,$$

where

D: annual demand or usage of the item,

S_b: buyer's ordering cost per order,

S_s: supplier's setup cost per setup,

h_b: buyer's annual inventory carrying cost,

Q: buyer's order lot size in units.

The result of classical optimization yields the following formula:

$$Q_j^* = \sqrt{\frac{2D(S_s + S_b)}{h_b}}. \tag{4.12}$$

The implicit assumption in this derivation is that all units produced by the supplier, in response to the buyer's order, are of acceptable quality. This is a strong assumption, and below we will analyze cases where defective units exist.

Let us look at a situation where the supplier operates a process that is in statistical control. That is, the process generates a known, constant proportion of defective items, p (such an assumption has been made previously in the literature by Affisco et al. 2002 and Cheng 1991). Under these circumstances, Deming (1981) proves that the buyer should consider only two inspection policies on receiving the parts from the supplier – zero inspection or 100% inspection. Full inspection is preferred when the cost of inspecting an incoming item is small compared to the cost of a defective item being released to the buyer's production process. We assume this to be the situation for the present research. To be consistent with Deming's ideas, we further assume that the buyer's inspection process is perfect, and that all rejected parts are disposed of by the supplier at his own expense.

Based on this scenario, we now adjust the model for the quality factor to provide the benchmark for the channel. The joint total relevant cost including quality may be written as

$$C_J(Q, \bar{p}) = \frac{D(S_s + S_b)}{\bar{p}Q} + \frac{\bar{p}Qh_b}{2} + \frac{D}{\bar{p}}[(1 - \bar{p})C_M + C_N], \tag{4.13}$$

where

- $\bar{p}$ proportion of good items produced by the supplier's process,
- C_M supplier's cost of disposing defective items per unit,
- C_N buyer's cost of inspection per unit.

The implicit assumption in the structure of the cost function (4.13) is that defective units are detected before they go into the buyer's inventory and are immediately returned to the supplier. The result of classical optimization yields the following formula for the quality-adjusted JELS:

$$Q^*_{J1} = \sqrt{\frac{2D(S_s + S_b)}{h_b\bar{p}^2}}. \tag{4.14}$$

Noting that in Expression (4.14), if $\bar{p} = 1$, then the quality is perfect and the quality-adjusted JELS simply reduces to the basic JELS expressed in Expression (4.12). Also note that Q^*_{J1} is inversely related to $\bar{p}$. This is intuitively evident because the required joint lot size is smaller if more items in the lot are acceptable.

Now, we formally introduce the notion that the system described above is being managed by two independent entities. We first examine the case in which the supplier and the buyer do not coordinate. Then, the supplier's and the buyer's total costs, denoted by C_{s1} and C_{b1}, respectively, may be written as

$$\begin{aligned} C_{s1} &= \frac{DS_s}{\bar{p}Q} + \frac{D}{\bar{p}}(1-\bar{p})C_M, \\ C_{b1} &= \frac{DS_b}{\bar{p}Q} + \frac{\bar{p}h_bQ}{2} + \frac{D}{\bar{p}}C_N. \end{aligned}$$

It is easy to derive the optimal lot size of the buyer:

$$Q^*_{b1} = \sqrt{\frac{2DS_b}{h_b\bar{p}^2}}. \tag{4.15}$$

In the absence of some form of coordination, the resulting lot size is smaller than the joint optimum, that is, $Q^*_{b1} < Q^*_{J1}$, and thus, the system's total cost increases. To improve the system's performance and thus to increase the competence of the channel, the buyer may seek to coordinate with the supplier. In the next section, we will show that the contracting approach may reduce the system's cost by modifying the structure of the relationships contractually in order to more closely align the individual incentives with the channel optimum.

IV.2.2. Contracting in Supply Uncertainty

With the facilitation of a contract, the buyer may build up a long-term partnership with the supplier by delineating mutual concessions that favor the persistence of the business relationship and make its terms more explicit. Obviously, the fundamental term in the contract is the proper lot size to improve the system's savings. Also, the way to distribute the total savings to each party is a critical term. Therefore, in the next section, we design supply contracts based on lot size and the buyer's sharing of savings when each party has full information of the system. When we closely examine both parties' objectives, we can see that, on one hand, both parties are glad to see more system savings so that they have a bigger pie to share; on the other hand, each party seeks to take more of the total savings. Due to these conflicting objectives, they may not be willing to share their private information. One critical piece of information is the product quality level $\bar{p}$. It determines the optimal lot size of the system and the total realizable savings. Therefore, in Section IV.2.2.2, we design supply contracts based on lot size and the buyer's sharing of the savings when the buyer has incomplete information of $\bar{p}$.

IV.2.2.1. Coordination on Lot Size Q under Full Information

It is clear that by coordinating on the jointly optimal lot size Q^*_{J1}, rather than choosing Q^*_{b1}, both supplier and buyer will be better off. Analogously to the basic principle of the joint economic lot sizing literature (see, e.g., Banerjee 1986, Lal and Staelin 1984, and Monahan 1984), we know that both parties will reduce their total costs by coordinating on Q^*_{J1}; i.e., $C_{s1}(Q^*_{b1}) - C_{s1}(Q^*_{J1}) \geq C_{b1}(Q^*_{J1}) - C_{b1}(Q^*_{b1})$. The buyer can implement this by placing her order lot size at Q^*_{J1} to the supplier and specifying that a sharing rate of $L \in [C_{b1}(Q^*_{J1}) - C_{b1}(Q^*_{b1}), C_{s1}(Q^*_{b1}) - C_{s1}(Q^*_{J1})]$ be charged to the supplier in return for the buyer's choosing $Q = Q^*_{J1}$. Under full information, the buyer can set the sharing rate at $L = C_{s1}(Q^*_{b1}) - C_{s1}(Q^*_{J1})$, extracting all of the savings from the supplier. Let C_s^{max} be the highest total net costs (after sharing) under which the supplier is still willing to contract with the buyer. With full information, the buyer can impose a sharing rate $L = C_s^{max} - C_{s1}(Q^*_{J1})$, pushing the supplier to incur the highest costs he will accept while keeping all remaining cost savings for herself. However, such a rate L, and the jointly optimal lot size Q^*_{J1}, depend on the supplier's product quality $\bar{p}$, so the buyer must know the supplier's product quality. In practice, this is often an unrealistic assumption, and below we show how the buyer can offer a menu of contracts when she does not know the exact product quality $\bar{p}$.

IV.2.2.2. Coordination under Incomplete Information of $\bar{p}$ with Contract $(Q(\bar{p}), L(\bar{p}))$

To address this problem, we follow the approach outlined in Laffont and Tirole (1993). The buyer is the entity taking the initiative and, therefore, is the principal, but the supplier holds private information about his product quality $\bar{p}$. Assume the buyer holds a prior distribution $F(\cdot)$, differentiable on its domain $[p_L, p_U]$ with $0 < p_L < p_U \leq 1$, density $f(\cdot)$, mean $\mu_{\bar{p}}$ and standard deviation $\delta_{\bar{p}}$. We make a common assumption as follows:

ASSUMPTION 4 *Increasing hazard rate:*

$$\frac{d}{d\bar{p}}\left(\frac{f(\bar{p})}{1-F(\bar{p})}\right) \geq 0.$$

Many common distributions satisfy Assumption 4, including uniform, normal, logistic, chi-squared, and exponential; see Bagnoli and Bergstrom (1989) for details on the hazard rate. Instead of proposing a single contract (Q, L) as in section IV.2.2.1, the buyer now offers a menu of contracts $(Q, L(Q))$, letting the supplier choose a specific pair from the menu. To model the contracting process, we parameterize each Q and L on $\bar{p}$. Note that offering a $(Q(\bar{p}), L(\bar{p}))$ menu is equivalent to a $(Q, L(Q))$ menu, although the equivalence relation need not exist in the closed form. The supplier's product quality $\bar{p}$ can be inferred from the $(Q(\bar{p}), L(\bar{p}))$. The contracting steps are:

Step 1. The supplier knows his product quality $\bar{p}$; the buyer does not.

Step 2. The buyer offers a menu $(Q(\cdot), L(\cdot))$, linking $Q(\hat{p})$ to compensation rate $L(\hat{p})$ for whatever product quality $\hat{p}$ the supplier announces.

Step 3. The supplier chooses a contract $(Q(\hat{p}), L(\hat{p}))$, effectively announcing $\hat{p}$.

Step 4. Production and delivery lot size are fixed at $Q(\hat{p})$, and the supplier pays sharing rate $L(\hat{p})$.

We make a common assumption here that contracting is a one-shot process, so that renegotiation is not allowed. From the revelation principle (Laffont and Tirole 1993), we can restrict our attention to menus of contracts in which the supplier announces his product quality truthfully. The intuitive explanation of the revelation principle is that the buyer can predict what product quality $\hat{p}$ any supplier with true quality $\bar{p}$ will announce, so she can take this knowledge of $\hat{p}(\bar{p})$ into account in designing the menu $(Q(\cdot), L(\cdot))$. This allows us to formulate an incentive-compatibility constraint, which ensures that a supplier with product quality $\bar{p}$ will indeed announce $\hat{p} = \bar{p}$. The optimization problem S_1 faced by a supplier with true product quality $\bar{p}$, who has to choose what product quality $\hat{p}$ to announce, is

$$S_1 \quad \min \quad C_{s1}(\bar{p}, \hat{p}) = \frac{DS_s}{\bar{p}Q(\hat{p})} + \frac{D}{\bar{p}}(1 - \bar{p})C_M + L(\hat{p}). \tag{4.16}$$

Letting $Q'(\hat{p})$ and $L'(\hat{p})$ denote the first derivatives of $Q(\hat{p})$ and $L(\hat{p})$ with respect to $\hat{p}$ respectively, the first-order condition of (4.16) for the supplier to minimize his own cost requires

$$-\frac{DS_s}{\bar{p}Q^2(\hat{p})}Q'(\hat{p}) + L'(\hat{p}) = 0. \tag{4.17}$$

In fact, (4.17) is also sufficient to provide the optimal product quality $\hat{p}$ for the supplier's optimization problem S_1 (see proof of proposition 11). Inducing truthful revelation is equivalent to ensuring that the optimum is reached at $\hat{p} = \bar{p}$; hence, we can formulate the incentive-compatibility constraint:

$$IC1: \quad L'(\bar{p}) = \frac{DS_s}{\bar{p}Q^2(\bar{p})}Q'(\bar{p}), \quad \forall \bar{p} \in [p_L, p_U]. \tag{4.18}$$

To make the contract acceptable to the supplier, regardless of $\bar{p}$, total costs under the contract may not exceed some predetermined and commonly known cost limit C_s^{max}. This is formulated as an individual rationality constraint:

$$IR1: \quad C_{s1}(\bar{p}, \bar{p}) \leq C_s^{max}, \quad \forall \bar{p} \in [p_L, p_U]. \tag{4.19}$$

To find the optimal menu of contracts, the buyer solves the following problem:

$$B_1 \quad \min_{Q(\cdot),L(\cdot)} \quad \int_{p_L}^{p_U} [\frac{DS_b}{\bar{p}Q(\bar{p})} + \frac{Q}{2}\bar{p}h_b + \frac{D}{\bar{p}}C_N - L(\bar{p})]f(\bar{p})d\bar{p}$$

$$s.t. \quad (IC1), (IR1), \quad \forall \bar{p} \in [p_L, p_U].$$

Solving the optimal control problem, B_1 yields the optimal menu of contracts below.

PROPOSITION 11 When the supplier holds private information about $\bar{p}$, the optimal menu of contracts $(Q^*_{JA}(\bar{p}), L^*_{JA}(\bar{p}))$, is such that:

$$Q^*_{JA} = \sqrt{\frac{2D(S_s + S_b + \frac{S_s(1-F(\bar{p}))}{\bar{p}f(\bar{p})})}{h_b\bar{p}^2}}, \quad \forall \bar{p} \in [p_L, p_U], \tag{4.20}$$

$${L^*_{JA}}'(\bar{p}) = \frac{DS_s}{\bar{p}{Q^*_{JA}}^2(\bar{p})}{Q^*_{JA}}'(\bar{p}), \quad \forall \bar{p} \in [p_L, p_U]. \tag{4.21}$$

Proof: The proof relies on some fundamental results from optimal control theory as outlined below (see Kamien and Schwartz (1981), pp. 142-146 for details.). Letting

$$h(\bar{p}) = [\frac{DS_b}{\bar{p}Q(\bar{p})} + \frac{Q}{2}\bar{p}h_b + \frac{D}{\bar{p}}C_N - L(\bar{p})]f(\bar{p}),$$

$$g_1 = \frac{DS_s}{\bar{p}Q^2(\bar{p})}u, \text{and}$$

$$g_2 = u = Q'(\bar{p}),$$

the above problem B_1 can be expressed as

$$\min \quad \int_{p_L}^{p_U} h(\bar{p})d\bar{p} \tag{4.22}$$

$$s.t. \quad L'(\bar{p}) = g_1, \tag{4.23}$$

$$Q'(\bar{p}) = g_2, \tag{4.24}$$

$$IR1: \quad C_{s1}(\bar{p},\bar{p}) \leq C_s^{max} \tag{4.25}$$

The necessary conditions for optimality can be stated in terms of the Hamiltonian

$$H = h + \lambda_1 g_1 + \lambda_2 g_2.$$

These conditions include the state equations given by Constraints (4.23) and (4.24), the multiplier equations given by

$$\lambda_1' = -\frac{\partial H}{\partial L} = -\left(\frac{\partial h}{\partial L} + \lambda_1 \frac{\partial g_1}{\partial L} + \lambda_2 \frac{\partial g_2}{\partial L}\right), \lambda_2' = -\frac{\partial H}{\partial \bar{p}} = -\left(\frac{\partial h}{\partial \bar{p}} + \lambda_1 \frac{\partial g_1}{\partial \bar{p}} + \lambda_2 \frac{\partial g_2}{\partial \bar{p}}\right),$$

and the optimality condition given by

$$\frac{\partial H}{\partial u} = \frac{DS_s}{\bar{p}Q^2}\lambda_1 + \lambda_2 = 0. \tag{4.26}$$

Then, it is easy to show that $\lambda_1'(\bar{p}) = f(\bar{p})$ and $\lambda_1(\bar{p}) = F(\bar{p}) - 1$, whereas

$$\lambda_2' = -[-\frac{DS_b}{\bar{p}Q^2} f(\bar{p}) + \frac{1}{2}\bar{p}h_b f(\bar{p}) - 2\lambda_1 \frac{DS_s}{\bar{p}Q^3} u], \tag{4.27}$$

Differentiating (4.26), we have

$$\lambda_2' = -[\frac{DS_s}{\bar{p}Q^2} f(\bar{p}) - \frac{DS_s}{\bar{p}^2 Q^2}\lambda_1 - 2\lambda_1 \frac{DS_s}{\bar{p}Q^3} u]. \tag{4.28}$$

Substituting (4.28) into (4.27) leads to (4.20). Since we have ignored Expression (4.25) in the above discussion, we need to determine whether or not it is satisfied for all $\bar{p} \geq p_L$. Recalling Expression (4.16), $C_{s1}(\bar{p}, \bar{p})$ is decreasing in $\bar{p}$ because

$$\frac{dC_{s1}(\bar{p})}{d\bar{p}} = -\frac{DS_s}{\bar{p}^2 Q(\bar{p})} - \frac{D}{\bar{p}^2} < 0.$$

Consequently, we can set $L(p_L)$ such that Expression (4.25) is binding.

To prove that IC1 (4.18) is sufficient for the supplier's optimization problem, we use a contradiction analogous to that in Laffont and Tirole (1993).

Assume there is a $\hat{p} \neq \bar{p}$ such that $C_{s1}(\bar{p}, \hat{p}) < C_{s1}(\bar{p}, \bar{p})$. This is equivalent to

$$\int_{\bar{p}}^{\hat{p}} \frac{\partial}{\partial x}[C_{s1}(\bar{p}, x)]dx < 0.$$

Since

$$\frac{\partial}{\partial x}[C_{s1}(x, x)] = 0$$

for all x, so that the previous expression is equivalent to

$$\int_{\bar{p}}^{\hat{p}} [\frac{\partial}{\partial x}[C_{s1}(\bar{p}, x)] - \frac{\partial}{\partial x}[C_{s1}(x, x)]]dx < 0,$$

which in turn is equivalent to

$$\int_{\bar{p}}^{\hat{p}} \int_{x}^{\bar{p}} \frac{\partial}{\partial x \partial u}[C_{s1}(u, x)]dudx < 0. \tag{4.29}$$

Since $Q(\bar{p})$ is decreasing in $\bar{p}$ according to Assumption 4, it is easy to verify that

$$\frac{\partial}{\partial x \partial u}[C_{s1}(u, x)] = \frac{DS_s}{u^2 Q^2(x)} \frac{\partial Q(x)}{\partial x} < 0.$$

So, if $\hat{p} > \bar{p}$, then $x > \bar{p}$ for all $x \in [\bar{p}, \hat{p}]$, so that the double integral in (4.29) is positive, which is a contradiction. A similar contradiction follows for the case $\hat{p} < \bar{p}$.

If the prior distribution $F(\bar{p})$ is uniform, we have

$$Q^*_{JA} = \sqrt{\frac{2D(S_s + S_b + S_s(\frac{p_U}{\bar{p}} - 1))}{h_b \bar{p}^2}},$$

and Q^*_{JA} is decreasing in $\bar{p}$, so the incentive-compatibility constraint is sufficient for the supplier's optimization problem. ■

Based on Proposition 11, we can explicitly examine the system's performance and individual decisions.

THEOREM 14 When the supplier holds private information about $\bar{p}$, the buyer's realized lot size is larger than the channel optimum. It holds that $Q^*_{JA} \geq Q^*_{J1} > Q^*_{b1}$. Further, the realized costs of the decentralized channel without contract ($C_J(Q^*_{b1})$), with contract and full information ($C_J(Q^*_{J1})$), and with contract and asymmetric information ($C_J(Q^*_{JA})$), satisfy the following relationships:

1. $C_J(Q^*_{J1}) \leq C_J(Q^*_{JA}) < C_J(Q^*_{b1})$ if

$$\frac{1 - F(\bar{p})}{\bar{p} f(\bar{p})} < 1 + \frac{S_s}{S_b}; \tag{4.30}$$

In this case, contracting is more effective than no form of contract;

2. $C_J(Q^*_{J1}) \leq C_J(Q^*_{b1}) \leq C_J(Q^*_{JA})$ if

$$\frac{1 - F(\bar{p})}{\bar{p} f(\bar{p})} \geq 1 + \frac{S_s}{S_b}; \tag{4.31}$$

In this case, contracting is less effective than no form of contract.

Proof: $Q^*_{JA} \geq Q^*_{J1} > Q^*_{b1}$ follows directly from Proposition 11.

To compare $C_J(Q^*_{JA})$ with $C_J(Q^*_{b1})$, recalling expression (4.13), we need only to compare

$$\frac{D(S_s + S_b)}{\bar{p} Q^*_{JA}} + \frac{\bar{p} Q^*_{JA} h_b}{2} \quad \text{with} \quad \frac{D(S_s + S_b)}{\bar{p} Q^*_{b1}} + \frac{\bar{p} Q^*_{b1} h_b}{2}.$$

Letting

$$C_{EOQ}(Q) = \frac{D(S_s + S_b)}{\bar{p} Q} + \frac{\bar{p} Q h_b}{2},$$

it is true that

$$\frac{C_{EOQ}(Q)}{C_{EOQ}(Q^*_{J1})} = \frac{1}{2}[\frac{Q}{Q^*_{J1}} + \frac{Q^*_{J1}}{Q}].$$

It follows that $C_{EOQ}(Q^*_{JA}) < C_{EOQ}(Q^*_{b1})$ if $Q^*_{JA}Q^*_{b1} < Q^{*2}_{J1}$. That is, $C_J(Q^*_{JA}) < C_J(Q^*_{b1})$ if

$$Q^*_{JA} \leq \frac{2D(S_s + S_b)}{h_b \bar{p}^2 Q^*_{b1}}. \tag{4.32}$$

Substituting (4.15) and (4.20) into (4.32), the inequality in (4.32) follows from (4.30).

Similarly, it can be shown that $C_J(Q^*_{b1}) \leq C_J(Q^*_{JA})$ if (4.31) holds. ■

Theorem 14 shows that the buyer will order a larger lot size than the channel optimum when she has incomplete information of the supplier's product quality. This is because the buyer does not know the exact quality level and thus she tends to order more than the optimum in order to guarantee that she will have enough good units per lot to satisfy demand as well as to reduce the average setup costs. Theorem 14 also shows that contracting may not always be more efficient than no form of coordination. If the hazard rate of the buyer's estimation of the supplier's true quality level is small, then (4.31) may hold and thus no contract is more effective. If the ratio S_s/S_b is large, that is, the potential for performance improvement under a contract is considerable, contracting tends to be more efficient than no form of coordination. For example, suppose that S_s = \$400/setup, S_b = \$250/order, the supplier's product quality level $\bar{p} = 0.785$, the buyer's prior distribution of the supplier's quality level is truncated normal over the interval $[0.75, 0.95]$ with a mean of 0.85 and a standard deviation of 0.023. Thus, $F(\bar{p}) = 0.0024$, $f(\bar{p}) = 0.3198$, we have

$$\frac{1 - F(\bar{p})}{\bar{p} f(\bar{p})} = 3.97 > 1 + \frac{S_s}{S_b} = 2.6, \frac{C_{EOQ}(Q^*_{JA})}{C_{EOQ}(Q^*_{b1})} = 1.25.$$

If $S_b = \$100$/order, then

$$\frac{1 - F(\bar{p})}{\bar{p} f(\bar{p})} = 3.97 < 1 + \frac{S_s}{S_b} = 5, \frac{C_{EOQ}(Q_{JA}^*)}{C_{EOQ}(Q_{b1}^*)} = 0.85.$$

Therefore, the buyer's information structure of the supplier's quality level $\bar{p}$ is critical to contracting efficiency. Sometimes the buyer is unable to improve the system's performance by offering a contract due to asymmetric information, especially when potential improvement is insignificant.

COROLLARY 2 When the buyer holds a prior uniform distribution, $C_J(Q_{b1}^*) \geq C_J(Q_{JA}^*)$, $\forall \bar{p} \in [p_L, p_U]$ only if

$$\frac{\delta_{\bar{p}}}{\mu_{\bar{p}}} \leq \frac{(1 + \frac{S_s}{S_b})}{\sqrt{3}(3 + \frac{S_s}{S_b})}. \tag{4.33}$$

If $\mu_{\bar{p}} \geq 0.75$, then $C_J(Q_{b1}^*) \geq C_J(Q_{JA}^*)$, $\forall \bar{p} \in [p_L, p_U]$. Also, if $\bar{p} = \mu_{\bar{p}}$, then $C_J(Q_{b1}^*) \geq C_J(Q_{JA}^*)$.

Proof: When the buyer holds a prior uniform distribution, we have the upper bound and lower bound of the domain of $\bar{p}$ as follows:

$$p_L = \mu_{\bar{p}} - \sqrt{3}\delta_{\bar{p}}, \tag{4.34}$$

$$p_U = \mu_{\bar{p}} + \sqrt{3}\delta_{\bar{p}} \tag{4.35}$$

Substituting (4.34) and (4.35) into (4.31), after simplification, we have (4.33). If $\mu_{\bar{p}} \geq 0.75$, noting that $p_U \leq 1$, we have

$$\frac{\delta_{\bar{p}}}{\mu_{\bar{p}}} \leq \frac{1}{3\sqrt{3}} \leq \frac{(1 + \frac{S_s}{S_b})}{\sqrt{3}(3 + \frac{S_s}{S_b})}.$$

If $\bar{p} = \mu_{\bar{p}}$, the left-hand side of inequality (4.30) can be simplified as

$$\frac{\sqrt{3}\delta_{\bar{p}}}{\mu_{\bar{p}}} \leq 1 \text{ since } pL \geq 0.$$

■

When the buyer holds a prior uniform distribution, contracting is more efficient than no form of contract as long as the buyer's expected quality level $\mu_{\bar{p}}$ is high enough (more than 0.75) or the buyer's estimation of the quality level is unbiased. Thus, if the supplier provides high quality products, it is always more efficient for the buyer to offer a contract to the supplier even when asymmetric information exists. In the following proposition, we provide the closed form of optimal contracts when the buyer's prior distribution of $\bar{p}$ is uniform.

PROPOSITION 12 When the prior $F(\bar{p})$ is uniform, and the buyer's ordering cost $S_b = 0$, the optimal menu of contracts are:

$$Q^*_{JA} = \sqrt{\frac{2DS_s p_U}{h_b \bar{p}^3}}, \quad \forall \bar{p} \in [p_L, p_U], \tag{4.36}$$

$$L^*_{JA} = -3\sqrt{\frac{h_b D S_s \bar{p}}{2p_U}} + C_s^{max} + \sqrt{\frac{2h_b D S_s p_L}{p_U}} - \frac{D}{p_L}(1-p_L)C_M, \quad \forall \bar{p} \in [p_L, p_U].$$

Proof: When $F(\bar{p})$ is uniform,

$$F(\bar{p}) = \frac{\bar{p} - p_L}{p_U - p_L},$$

and

$$\frac{(1 - F(\bar{p}))}{\bar{p} f(\bar{p})} = \frac{p_U}{\bar{p}} - 1. \tag{4.37}$$

Thus, substituting (4.37) into (4.20) we have (4.36), and

$$\frac{dQ^*_{JA}}{d\bar{p}} = -\frac{3Q^*_{JA}}{2\bar{p}}. \tag{4.38}$$

Substituting (4.38) into (4.21) leads to

$$\begin{aligned} {L_{JA}^{*}}'(\bar{p}) &= -\frac{3DS_s}{2\sqrt{\frac{2DS_s p_U \bar{p}}{h_b}}}, \\ L_{JA}^{*}(\bar{p}) &= -3\sqrt{\frac{h_b DS_s \bar{p}}{2p_U}} + X \quad \textit{for some constant } X. \end{aligned}$$

To solve for X, solving $C_{s1}(p_L, p_L) = C_s^{max}$, that is, when (4.19) is binding at $\bar{p} = p_L$, leads to

$$X = C_s^{max} + \sqrt{\frac{2h_b DS_s p_L}{p_U}} - \frac{D}{p_L}(1 - p_L)C_M.$$

■

Proposition 12 provides an explicit menu of contracts under the uniform case. Both the lot size and the sharing rate are decreasing in $\bar{p}$. A larger lot size Q will lead to a larger sharing rate L. From the supplier's perspective, a higher product quality level will incur a smaller order lot size from the buyer, which results in larger setup costs and less sharing. This is a tradeoff for the supplier to balance his setup costs and sharing rate, which is designed by the buyer under the incentive-compatibility constraint to make sure that the supplier will announce his true product quality.

IV.2.3. Contracting in Supplier Development Program

In this section, we study the case when the buyer initiates a supplier development program and both parties work closely to improve the supplier's product quality level in order to reduce the total costs. We consider $\bar{p}$ to be a decision variable. From the perspective of the system, the objective is to minimize the sum of the investment cost for increasing $\bar{p}$ and the quality-adjusted joint total relevant cost. Specifically, we seek to minimize

$$G(Q, \bar{p}) = ia_{\bar{p}}(\bar{p}) + C_J(Q, \bar{p}), \tag{4.39}$$

$$s.t \quad 0 < \bar{p}_0 \leq \bar{p} < 1,$$

where i is the cost of capital; $\bar{p}_0$ is the original proportion of good units produced by the supplier's process; $a_p(\bar{p})$ is a convex and strictly increasing investment function of $\bar{p}$. Thus, $ia_{\bar{p}}(\bar{p})$ represents the annual investment cost. We use the following logarithmic form, which is similar to the one employed in Porteus (1986):

$$a_{\bar{p}}(\bar{p}) = \beta - \alpha ln(1 - \bar{p}) \quad 0 < \bar{p}_0 \leq \bar{p} < 1,$$

where α denotes the investment sensitivity. The following proposition provides system-wide optimal quality improvement and determines the lot size decisions in the supplier development program.

PROPOSITION 13 The optimal lot size and the product quality level are unique and given by

$$\begin{aligned} \bar{p}^*_{J2} &= max\{\bar{p}_0, \bar{p}^*_{J3}\}, \\ Q^*_{J2} &= min\{Q^*_{J0}, Q^*_{J3}\}, \end{aligned}$$

where

$$\bar{p}^*_{J3} = \frac{\sqrt{(C_M + C_N)^2 D^2 + 4i\alpha(C_M + C_N)D} - (C_M + C_N)D}{2i\alpha}, \tag{4.40}$$

$$Q^*_{J0} = \sqrt{\frac{2D(S_s + S_b)}{h_b \bar{p}_0^2}},$$

$$Q^*_{J3} = \sqrt{\frac{2D(S_s + S_b)}{h_b \bar{p}^{*2}_{J3}}}. \tag{4.41}$$

Proof: For a given $\bar{p}$, there exists a unique optimal lot size Q^*_{J3} since

$$\frac{\partial^2 G}{\partial Q^2} = \frac{2D(S_s + S_b)}{pQ^3} > 0$$

and (4.41) can be easily obtained from

$$\frac{\partial G}{\partial Q} = 0.$$

Define $G_{\bar{p}}(\bar{p}) = G(Q^*_{J3}(\bar{p}), \bar{p})$ so that

$$G_{\bar{p}}(\bar{p}) = i(\beta - \alpha ln(1-\bar{p})) + \sqrt{2h_b D(S_s + S_b)} + \frac{D}{\bar{p}}[(1-\bar{p})C_M + C_N]. \tag{4.42}$$

Also, let $\bar{p}^*_{J3}$ denote the optimal minimizer of $G_{\bar{p}}(\bar{p})$. It follows that $Q^*_{J3}(\bar{p}^*_{J3})$ and $\bar{p}^*_{J3}$ minimizes (4.39). Therefore, in order to complete the proof, it is sufficient to compute $\bar{p}^*_{J3}$. Solving

$$\frac{\partial G_{\bar{p}}(\bar{p})}{\partial \bar{p}} = \frac{i\alpha}{1-\bar{p}} - \frac{(C_M + C_N)D}{\bar{p}^2} = 0 \tag{4.43}$$

for $\bar{p}$, we have

$$i\alpha \bar{p}^{*2}_{J3} + (C_M + C_N)D\bar{p}^*_{J3} - (C_M + C_N)D = 0,$$

and (4.40) is easily achieved. Noting that

$$\frac{\partial G_{\bar{p}}(\bar{p})}{\partial \bar{p}}|_{0^+} = -\infty, \frac{\partial G_{\bar{p}}(\bar{p})}{\partial \bar{p}}|_{1^-} = +\infty,$$

and

$$\frac{\partial^2 G_{\bar{p}}(\bar{p})}{\partial \bar{p}^2} > 0,$$

we conclude that $\bar{p}^*_{J3}$ is unique and $0 < \bar{p}^*_{J3} < 1$. ■

Proposition 13 provides the benchmark to the supplier development program. In the next section, we further investigate the case where both parties are managed independently, and examine the joint and individual performances.

IV.2.3.1. Coordination on Lot Size Q and Product Quality $\bar{p}$ under Full Information

Here, again, by properly targeting the improved quality level at $\bar{p}^*_{J3}$, and coordinating on the jointly optimal lot size Q^*_{J2} accordingly, both supplier and buyer will be better off, i.e., $G(Q^*_{b1}, \bar{p}_0) \geq G(Q^*_{J2}, \bar{p}^*_{J3})$. The buyer can implement this by placing her order lot size at Q^*_{J2}, urging and committing to help the supplier to improve his product quality level up to $\bar{p}^*_{J3}$, and specifying a cost/saving sharing rate L which includes the buyer's investment in a supplier development program and the saving-sharing rate from coordination. Let C_s^{max} be the highest total net costs (after sharing) under which the supplier is still willing to contract with the buyer. With full information, the buyer can impose a sharing $L = C_s^{max} - [C_{s1}(Q^*_{J2}, \bar{p}^*_{J3}) + ia_{\bar{p}}(\bar{p}^*_{J3})]$, pushing the supplier to incur the highest costs he will accept while keeping all remaining cost savings for herself. However, such a sharing L depends on how much total effort and investment have to be spent to improve the supplier's quality to a certain level. It may depend on the supplier's current production conditions, management effectiveness, employee training level, etc. Although the supplier is better informed about these conditions, he may hesitate to release all his information to the buyer. In the next section, we show how the buyer can offer a menu of contracts when she does not know the exact value of the supplier's investment sensitivity α.

IV.2.3.2. Coordination under Incomplete Information about α with Contract $(Q(\alpha), \bar{p}(\alpha), L(\alpha))$

Now assume that only the supplier observes α. We follow a contracting approach analogous to that in Section IV.2.2.2. The buyer offers a menu of contracts $(Q(\cdot), \bar{p}(\cdot), L(\cdot))$, letting the supplier choose according to the latter's investment sensitivity α. The contracting process is analogous to that in Section IV.2.2.2. We also assume that there is no renegotiation after the initial contracting.

The buyer holds a prior distribution $H(\alpha)$, differentiable on its domain $[\underline{\alpha}, \bar{\alpha}]$ with $0 < \underline{\alpha} < \bar{\alpha} < \infty$ with density $h(\alpha)$, mean μ_α and standard deviation δ_α. We make a common assumption as follows:

ASSUMPTION 5 *Decreasing reverse hazard rate:*

$$\frac{d}{d\alpha}\left(\frac{h(\alpha)}{H(\alpha)}\right) \leq 0.$$

Many common distributions satisfy Assumption 5, including uniform, normal, logistic, chi-squared, and exponential; see Bagnoli and Bergstrom (1989) and Corbett (2001) for details on the reverse hazard rate. The optimization problem S_2 faced by a supplier with a true investment sensitivity parameter α and having to choose what $\hat{\alpha}$ to announce is

$$\begin{aligned} S_2 \quad \min \quad C_{s2}(\alpha,\hat{\alpha}) &= i[\beta - \alpha \ln(1-\bar{p}(\hat{\alpha}))] + \frac{DS_s}{\bar{p}(\hat{\alpha})Q(\hat{\alpha})} + \frac{D}{\bar{p}(\hat{\alpha})}(1-\bar{p}(\hat{\alpha}))C_M \\ &\quad + L(\hat{\alpha}). \end{aligned} \tag{4.44}$$

Similarly to Section IV.2.2.2, we derive the supplier's incentive-compatibility constraint and individual-rationality constraint

$$\begin{aligned} IC2: \quad L'(\alpha) &= -\frac{i\alpha}{1-\bar{p}(\alpha)}\bar{p}'(\alpha) + DS_s[\frac{1}{\bar{p}^2(\alpha)Q(\alpha)}\bar{p}'(\alpha) \\ &\quad + \frac{1}{\bar{p}(\alpha)Q^2(\alpha)}Q'(\alpha)] + \frac{D}{\bar{p}^2(\alpha)}\bar{p}'(\alpha)C_M, \quad \forall \alpha \in [\underline{\alpha}, \bar{\alpha}] \qquad (4.45) \\ IR2: \quad C_{s2}(\alpha,\alpha) &\leq C_s^{max}, \quad \forall \alpha \in [\underline{\alpha}, \bar{\alpha}]. \qquad (4.46) \end{aligned}$$

To find the optimal menu of contracts, the buyer solves the following problem:

$$B_2 \quad \min_{Q(\cdot),\bar{p}(\cdot),L(\cdot)} \quad \int_{\underline{\alpha}}^{\bar{\alpha}} [\frac{DS_b}{\bar{p}(\alpha)Q(\alpha)} + \frac{Q(\alpha)}{2}\bar{p}(\alpha)h_b + \frac{D}{\bar{p}(\alpha)}C_N - L(\alpha)]h(\alpha)d\alpha$$

$$s.t. \quad IC2, IR2, \quad \forall \alpha \in [\underline{\alpha}, \bar{\alpha}].$$

Solving the optimal control problem, B_2 yields the optimal menu of contracts below.

PROPOSITION 14 When the supplier holds private information about α, the optimal menu of contracts, $(Q^*_{JAI}(\alpha), \bar{p}^*_{JAI}(\alpha), L^*_{JAI}(\alpha))$, is such that:

$$\begin{aligned}
\bar{p}^*_{JAI}(\alpha) &= max\{\bar{p}_0, \bar{p}^*_{JA\alpha}\}, \quad \forall \alpha \in [\underline{\alpha}, \bar{\alpha}], \\
Q^*_{JAI}(\alpha) &= min\{Q^*_{J0}, Q^*_{JA\alpha}\}, \quad \forall \alpha \in [\underline{\alpha}, \bar{\alpha}], \\
L'(\alpha) &= -\frac{i\alpha}{1 - \bar{p}^*_{JAI}(\alpha)} \bar{p}^{*'}_{JAI}(\alpha) + \frac{D}{\bar{p}^{*2}_{JAI}(\alpha)} \bar{p}^{*'}_{JAI}(\alpha) C_M, \quad \forall \alpha \in [\underline{\alpha}, \bar{\alpha}],
\end{aligned}$$

where

$$\bar{p}^*_{JA\alpha} = \frac{\sqrt{D^2(C_M + C_N)^2 + 4iD(C_M + C_N)(\alpha + \frac{H(\alpha)}{h(\alpha)})} - D(C_M + C_N)}{2i(\alpha + \frac{H(\alpha)}{h(\alpha)})}, \tag{4.47}$$

$$Q^*_{JA\alpha} = \sqrt{\frac{2D(S_s + S_b)}{h_b \bar{p}^{*2}_{JA\alpha}}}. \tag{4.48}$$

In addition, $\bar{p}^*_{JAI}(\alpha)$ is decreasing in α and $Q^*_{JAI}(\alpha)$ is increasing in α.

Proof: We use an approach similar to that in the proof of Proposition 11. The above problem B_2 can be expressed as

$$\min \quad \int_{\underline{\alpha}}^{\bar{\alpha}} h_2(\alpha) d\alpha \tag{4.49}$$

$$s.t. \quad L'(\alpha) = g_1, \tag{4.50}$$

$$\bar{p}'(\alpha) = g_2, \tag{4.51}$$

$$Q'(\alpha) = g_3, \tag{4.52}$$

$$IR2: \quad C_{s2}(\alpha, \alpha) \leq C_s^{max},$$

where

$$\begin{aligned}
h_2(\alpha) &= [\frac{DS_b}{\bar{p}(\alpha)Q(\alpha)} + \frac{Q(\alpha)}{2}\bar{p}(\alpha)h_b + \frac{D}{\bar{p}(\alpha)}C_N - L(\alpha)]h(\alpha), \\
g_1 &= -\frac{i\alpha}{1-\bar{p}(\alpha)}u + DS_s[\frac{1}{\bar{p}^2(\alpha)Q(\alpha)}u + \frac{1}{\bar{p}(\alpha)Q^2(\alpha)}v] + \frac{DC_M}{\bar{p}^2(\alpha)}u, \\
g_2 &= \bar{p}' = u, \\
g_3 &= Q' = v.
\end{aligned}$$

The necessary conditions for optimality can be stated in terms of Hamiltonian

$$H_2 = h_2 + \lambda_1 g_1 + \lambda_2 g_2 + \lambda_3 g_3.$$

These conditions include the state equations given by Constraints (4.50), (4.51), and (4.52), the multiplier equations given by

$$\lambda_1' = -\frac{\partial H_2}{\partial L} = -\left(\frac{\partial h_2}{\partial L} + \lambda_1\frac{\partial g_1}{\partial L} + \lambda_2\frac{\partial g_2}{\partial L} + \lambda_3\frac{\partial g_3}{\partial L}\right),$$

$$\lambda_2' = -\frac{\partial H_2}{\partial \bar{p}} = -\left(\frac{\partial h_2}{\partial \bar{p}} + \lambda_1\frac{\partial g_1}{\partial \bar{p}} + \lambda_2\frac{\partial g_2}{\partial \bar{p}} + \lambda_3\frac{\partial g_3}{\partial \bar{p}}\right), \text{and}$$

$$\lambda_3' = -\frac{\partial H_2}{\partial Q} = -\left(\frac{\partial h_2}{\partial Q} + \lambda_1\frac{\partial g_1}{\partial Q} + \lambda_2\frac{\partial g_2}{\partial Q} + \lambda_3\frac{\partial g_3}{\partial Q}\right),$$

and the optimality condition given by

$$\frac{\partial H_2}{\partial u} = -\frac{i\alpha}{1-\bar{p}}\lambda_1 + \frac{DS_s}{\bar{p}^2 Q}\lambda_1 + \frac{D}{\bar{p}^2}C_M\lambda_1 + \lambda_2 = 0, \tag{4.53}$$

$$\frac{\partial H_2}{\partial v} = \frac{DS_s}{\bar{p}Q^2}\lambda_1 + \lambda_3 = 0. \tag{4.54}$$

Then, it is easy to show that $\lambda_1 = H(\alpha)$ whereas

$$\begin{aligned}
\lambda_2' = & -[-\frac{DS_b}{\bar{p}^2 Q}h(\alpha) + \frac{1}{2}Qh_b h(\alpha) - \frac{D}{\bar{p}^2}C_N h(\alpha) - \frac{i\alpha}{(1-\bar{p})^2}u\lambda_1 - \frac{2DS_s}{\bar{p}^3 Q}u\lambda_1 - \frac{DS_s}{\bar{p}^2 Q^2}v\lambda_1 \\
& - \frac{2D}{\bar{p}^3}C_M u\lambda_1],
\end{aligned} \tag{4.55}$$

$$\lambda_3' = -[-\frac{DS_b}{\bar{p}Q^2}h(\alpha) + \frac{1}{2}\bar{p}h_b h(\alpha) - \frac{DS_s}{\bar{p}^2Q^2}u\lambda_1 - \frac{2DS_s}{\bar{p}Q^3}v\lambda_1]. \quad (4.56)$$

Differentiating (4.53), (4.54) results in

$$\lambda_2' = -[-\frac{i\alpha}{1-\bar{p}}h(\alpha) - \frac{i}{1-\bar{p}}\lambda_1 - \frac{i\alpha}{(1-\bar{p})^2}u\lambda_1 + \frac{DS_s}{\bar{p}^2Q}h(\alpha) - \frac{2DS_s}{\bar{p}^3Q}u\lambda_1 - \frac{DS_s}{\bar{p}^2Q^2}v\lambda_1$$
$$+ \frac{D}{\bar{p}^2}C_M h(\alpha) - \frac{2D}{\bar{p}^3}C_M u\lambda_1], \quad (4.57)$$
$$\lambda_3' = -[\frac{DS_s}{\bar{p}Q^2}h(\alpha) - \frac{DS_s}{\bar{p}^2Q^2}u\lambda_1 - \frac{2DS_s}{\bar{p}Q^3}v\lambda_1]. \quad (4.58)$$

Substituting (4.58) into (4.56), we have (4.48). Substituting (4.57) and (4.48) into (4.55) leads to

$$\frac{i(\alpha + \frac{H(\alpha)}{h(\alpha)})}{1-\bar{p}} = \frac{D(C_M + C_N)}{\bar{p}^2}. \quad (4.59)$$

and (4.47) is obtained.

Since we have ignored Expression (4.46) in the above discussion, we need to determine whether or not it is satisfied for all $\alpha \leq \bar{\alpha}$. Recalling Expression (4.44), $C_{s2}(\alpha)$ is increasing in α because

$$\frac{dC_{s2}(\alpha)}{d\alpha} = -iln(1 - \bar{p}(\alpha)) > 0,$$

Consequently, we can set $L(\alpha)$ such that Expression (4.46) is binding, and this completes the proof. ■

Proposition 14 shows that a larger value of investment sensitivity α would result in a lower quality level $\bar{p}$, which is intuitively evident since a larger α means a greater expense for a certain quality level. The buyer's order lot size increases according to the lower product quality in order to guarantee a certain amount of realized lot size $\bar{p}Q$.

THEOREM 15 The targeted quality level under asymmetric information is lower than under full information, that is,

$$\bar{p}^*_{JA\alpha} \leq \bar{p}^*_{J3}. \tag{4.60}$$

The optimal lot size under asymmetric information is larger, that is,

$$Q^*_{JA\alpha} \geq Q^*_{J3}. \tag{4.61}$$

Contracting under asymmetric information is worthwhile only if

$$\frac{D(C_M + C_N)(1 - \bar{p}_0)}{i\bar{p}_0^2} \geq \alpha + \frac{H(\alpha)}{h(\alpha)}. \tag{4.62}$$

Proof: (4.60) and (4.61) can be easily obtained by comparing (4.40) and (4.41) with (4.47) and (4.48). Contracting under asymmetric information is worthwhile only if $\bar{p}^*_{JA\alpha} \geq \bar{p}_0$. Note that $\bar{p}^*_{JA\alpha}$ is the root of equation (4.59). Thus, we need only to show that

$$\frac{i(\alpha + \frac{H(\alpha)}{h(\alpha)})}{1 - \bar{p}_0} \leq \frac{D(C_M + C_N)}{\bar{p}_0^2} \tag{4.63}$$

and (4.62) is achieved. ■

The asymmetric information results in a lower quality requirement from the buyer. Interestingly, the realized lot size $\bar{p}^*_{JA\alpha} Q^*_{JA\alpha}$ is the same as the joint optimal realized lot size $\bar{p}^*_{J2} Q^*_{J2}$. Recalling (4.13), the joint setup cost and inventory holding cost are

$$\frac{D(S_s + S_b)}{\bar{p}Q} \text{ and } \frac{\bar{p}Qh_b}{2},$$

respectively. Under an optimal menu of contracts, the resulting setup cost and inventory holding cost are system-wide optimal. However, the buyer specifies a lower quality level than the optimum to the supplier, which results in a lower investment expense for the supplier and higher inspection and disposal costs as well. The intuitive explanation is that since the buyer has incomplete information of the product quality level, she will conservatively request a lower quality level. After the buyer specifies the product quality level, she adjusts her order lot size accordingly to minimize the total setup and inventory holding cost so that she may ask for more compensation from the supplier without driving him away.

Contracting under asymmetric information may not be worthwhile. If the reverse hazard rate of the buyer's estimation of the supplier's true investment sensitivity is small, then (4.62) may not hold and thus no investment will be made. For example, suppose that $D = 1000$ units/year, $C_M = \$12$/unit, $C_N = \$5$/unit, $i = 0.1$, the supplier's original product quality level $\bar{p}_0 = 0.85$, investment sensitivity $\alpha = 14600$, the buyer's prior distribution of the supplier's investment sensitivity is truncated normal over the interval $[8750, 16250]$ with a mean of 12500 and a standard deviation of 1000. Thus, $H(\alpha) = 0.9823$, $f(\alpha) = 0.4399 \times 10^{-4}$, and we have

$$\alpha + \frac{H(\alpha)}{f(\alpha)} = 36929.59 > \frac{D(C_M + C_N)(1 - \bar{p}_0)}{i\bar{p}_0^2} = 35294.12, \bar{p}^*_{JA\alpha} = 0.845 < \bar{p}_0.$$

Thus, no investment decision will be made. The existence of information asymmetry may ruin the buyer's interest in initiating a supplier development program in this instance.

IV.2.4. Numerical Analysis

We first present an example to illustrate the form of the optimal contract and the performance of the system and each party. The parameters of the example are as follows: $D = 1000$ units/year, $C_p = \$25$/unit, $S_s = \$400$/setup, $S_b = \$20$/order, $r = 0.2$, $C_M = \$12$/unit, and $C_N = \$5$/unit. The initial product quality level is $\bar{p} = 0.85$. Investment may be made in improving the quality with $i = 0.1$, $\alpha = 12500$, and $\beta = -23714$. Suppose that the prior distribution of the supplier's quality level $\bar{p}$ is uniform over the interval $[0.75, 0.95]$ with a mean of 0.85 and a standard deviation of 0.0577. Suppose that the prior distribution of the supplier's investment sensitivity α is uniform over the interval $[8750, 16250]$ with a mean of 12500 and a standard deviation of 2165.13. Thus, the buyer's estimation of $\bar{p}$ and α is unbiased in this example. For simplicity, we assume that the supplier's highest cost for signing a contract with the buyer is $C_s^{max} = 6589.78$, which is the same as his total cost with no form of contract.

Table V presents five cases, including no form of contract, a contract with full information, a contract with asymmetric information of the quality level $\bar{p}$, a contract to improve the quality with full information, and a contract with asymmetric information of the investment sensitivity α. As Table V indicates, when each party has full information of the system, a contract based on the lot size can help reduce the total cost of the system by 22.22%, and the buyer extracts all of the savings from the supplier by initiating the contract. When the buyer has incomplete information of the supplier's quality level $\bar{p}$, the total savings are almost the same as in the full information case since the buyer's estimation is unbiased in this example. However, the buyer can only reduce her own cost by 13.78%, instead of 45.34% as in the full information case, due to the information asymmetry. The supplier's cost is reduced by 30.27% at the same time since he has full knowledge of his quality level. If a supplier development program is initiated by the buyer and both parties reach a contractual agreement, the total cost of the system can by further reduced by 28.20%, and the buyer extracts all of the savings from the supplier if she has full information of the system. If the buyer has incomplete information of the investment sensitivity to improve the quality level, her own cost can be reduced by 43% in contrast to 57.55% in the full information case. The total savings are almost the same as in the full information case since the buyer's estimation of α is unbiased. The supplier's cost is reduced by 13.36% at the same time. In this example, we observe that a contract

Table V Comprehensive Results under Supply Uncertainty

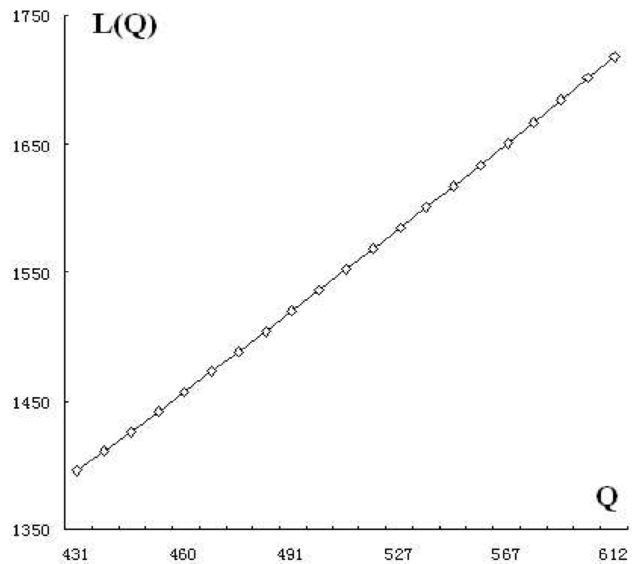
Variable	*No contract*	*Contract with full infor.*	*Contract with asym. $\bar{p}$*	*Contract to improve (full)*	*Contract to improve (asym. α)*
Q	105	482	612	438	446
L	-	3496.24	1718.29	3730.80	2905.25
$\bar{p}$	0.85	0.85	0.85	0.936	0.919
Supplier's cost	6589.78	6589.78	4595.01	6589.78	5709.34
% Savings	-	0	30.27%	0	13.36%
Buyer's cost	6329.57	3459.60	5457.27	2686.67	3607.58
% Savings	-	45.34%	13.78%	57.55%	43.00%
Total costs	12919.35	10049.38	10052.28	9276.45	9316.92
% Savings	-	22.22%	22.19%	28.20%	27.88%

would help reduce the system's total cost, and a supplier development program would help achieve further savings. If the buyer has full information of the system, she is able to extract all of the savings from the supplier. However, information asymmetry does impair the buyer's capability to extract savings. Next, we examine further how the buyer can design contracts under information asymmetry and the impact of information asymmetry on the system's and both individuals' costs.

Figure 6 illustrates the buyer's contract pairs $(Q, L(Q))$ when she has incomplete information of the supplier's quality level $\bar{p}$. We observe that a larger lot size Q will lead to a larger sharing rate L. From the supplier's perspective, a larger lot size means a smaller setup cost, and he would receive less savings as well. The tradeoff for the supplier, which is designed by the buyer under the incentive-compatibility constraint, is that he balances his setup costs and sharing rate which guarantees that the supplier will announce his true product quality.

Figure 7 illustrates the relationship of $\bar{p}$ and L in the buyer's contract menu

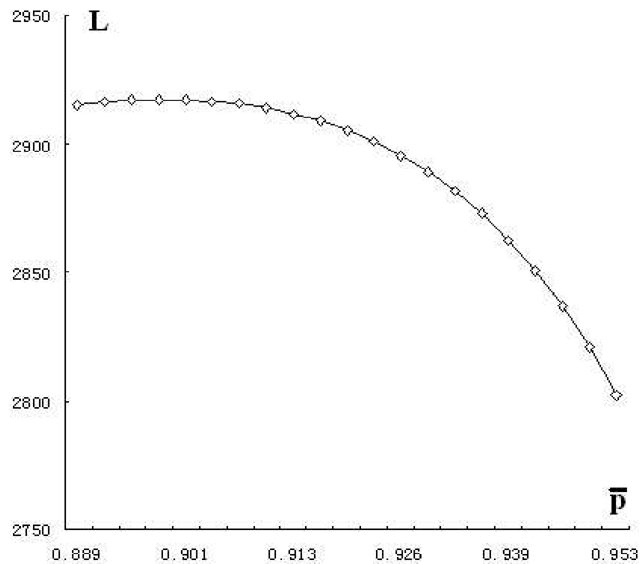
Figure 6 Buyer's Contract Menu $(Q, L(Q))$



$(Q, \bar{p}, L)$ when she has incomplete information of the supplier's investment sensitivity α. We observe that a higher quality expectation $\bar{p}$ is associated with a smaller sharing rate L. Recall Proposition 14 which states that the realized lot size $\bar{p}Q$ in the buyer's contract menu is the same as the joint optimal realized lot size, and thus the joint setup cost and inventory holding cost are system-wide optimal. Thus, from the supplier's perspective, a lower quality expectation means less investment expense, larger disposal costs, and less savings received. When the quality expectation is high, the buyer's sharing rate drops significantly. This is because the total investment to improve the quality level increases dramatically, and thus the buyer needs to expend much more effort and investment to help the supplier improve product quality as well as to make sure that the supplier is still willing to reveal his true investment sensitivity under the contract menu.

Obviously, when the buyer has incomplete information of the supplier's quality

Figure 7 $\bar{p}$ and L in Buyer's Contract Menu $(Q, \bar{p}, L)$

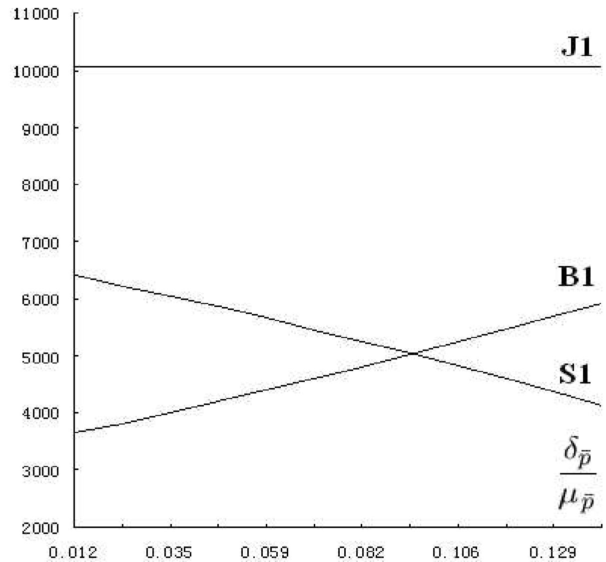


level $\bar{p}$ or investment sensitivity α, she is unable to extract all of the savings from the supplier. Next, we examine explicitly the impact of information asymmetry on the system's and both individuals' costs.

Figure 8 illustrates the impact of the coefficient of variation of the buyer's estimation of $\bar{p}$ on the joint total cost $T1$, the supplier's cost $S1$ and the buyer's cost $B1$. It indicates that the buyer's estimation variation has little impact on the system's cost. However, the buyer's cost increases significantly as the coefficient of variation increases. That is, the buyer's capability of extracting savings from the supplier is considerably impaired if she has information about $\bar{p}$ that is not accurate. Of course, the supplier's benefits from this and his cost are significantly reduced.

Figure 9 illustrates the impact of the coefficient of variation of the buyer's estimation of α on the joint total cost $T2$, the supplier's cost $S2$ and the buyer's cost $B2$. Again, we observe that the buyer's capability of extracting savings from the supplier

Figure 8 The Impact of the Coefficient of Variation $\frac{\delta_{\bar{p}}}{\mu_{\bar{p}}}$

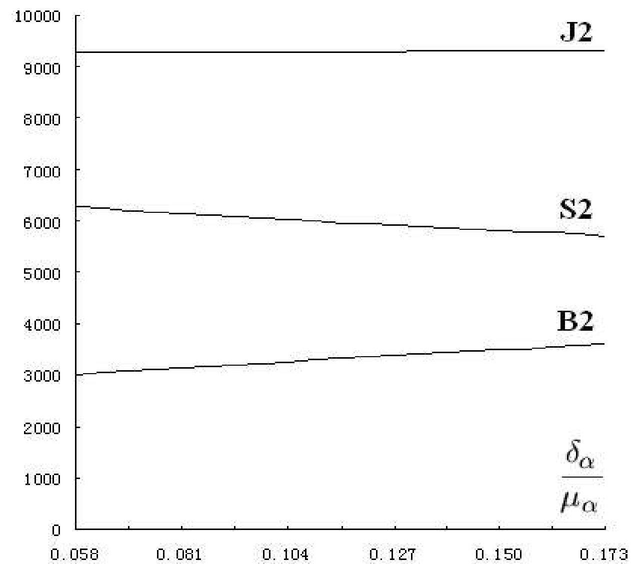


is considerably impaired if she has inaccurate information of α, while the supplier's benefits from this and his cost is significantly reduced.

The findings reported here clearly show that ignoring incentive conflicts and quality information issues can lead to undesirable behavior. The practical implications are significant. In designing a supply chain, it is tempting to take the perspective of a central planner and focus on improving overall efficiency. Few supply chains have any mechanism that can pass for a central planner, so this is rarely an option. This section proposes a framework for how lot size, quality level, and transactions should be structured to help reduce supply chain inefficiency due to individual incentives and private information.

In this section, to concentrate on the supply contract design, we assume that the supply is in statistical control. In the next section, we further develop the model when the supply is random.

Figure 9 The Impact of the Coefficient of Variation $\frac{\delta_\alpha}{\mu_\alpha}$



IV.2.5. Random Supply

In this section, we present a modification of the well-known single-vendor single-buyer integrated production-inventory model on the assumption that the percentage of defective items in the accepted lot is a random variable whose probability distribution is known to management from their experience. We also assume that the defective items are non-reworkable. We seek to demonstrate more closely how the possible presence of defective items in the accepted lot affects the structure as well as the cost of certain production-inventory systems. We present the total expected costs of the system and provide the optimal solutions. Further, we investigate the effect of defective items on the system performance. Our findings show that a high percentage of defective items, together with a large demand, has a significant impact on the total expected cost.

IV.2.5.1. Problem Formulation

The problem considered here concerns a single vendor supplying a single buyer with a product. The vendor manufactures, at a finite rate, in batches and incurs a batch setup cost. Each batch is despatched to the buyer in a number of shipments. Both vendor and buyer incur time-proportional holding costs. The buyer has to meet a fixed, level external demand. We summarize the additional notation below.

R: annual production rate,

h_s: annual inventory holding cost per unit for the vendor,

n: an integer multiplier, and thus the vendor's production batch size is nQ,

TC_S: the total expected annual cost for the vendor,

TC_B: the total expected annual cost for the buyer,

TC_J: $= TC_S + TC_B$, the total expected annual cost for both vendor and buyer.

Under the assumption of perfect quality, Hill (1997) shows that the total annual costs, denoted by C_J, are:

$$C_J = \frac{D}{nQ}(nS_b + S_s) + \frac{h_b Q}{2} + h_s \left[\frac{(n-1)Q}{2} - \frac{(n-2)DQ}{2R} \right],$$

and the joint optimal solution (Q^{**}, n^{**}) satisfies:

$$\begin{aligned} Q^{**}(n) = & \sqrt{\frac{2D(nS_b + S_s)}{n(h_b + h_s[(n-1) - \frac{(n-2)D}{R}])}}, \\ n^{**}(n^{**} - 1) \leq & \frac{S_s[2h_s D + R(h_b - h_s)]}{S_b h_s (R - D)} \leq n^{**}(n^{**} + 1). \end{aligned}$$

However, the quantity accepted may not exactly match the quantity requisitioned by the buyer due to the existence of defective items. Defective items may occur during production or in transit, i.e., during transportation or handling. We assume that the buyer inspects each incoming shipment and returns the defective items to the vendor. Further, we assume that the inspection time is negligible. We will discuss a considerable inspection time case in the concluding section. For simplicity, we assume that all of the defective items are non-reworkable and have to be disposed of and that there is no disposal cost or salvage value related to the defective items. The case in which the defective items are partially reworkable and there is a cost related to the reworking and disposal can be easily incorporated into our model.

Define $\bar{p}_i$ as the percentage of non-defective items in lot i received by the buyer where the $\bar{p}_i$'s are independent and identically distributed (i.i.d.) random variables with cumulative distribution $F(\cdot)$ and $0 < p_L \le \bar{p}_i \le p_U \le 1$. $E[\bar{p}_i] = \mu$ and $Var[\bar{p}_i] = \delta^2$. Further, we assume that the vendor's production rate is high enough so that he can always produce enough units to fulfill the buyer's order on time within consecutive shipments to the buyer.

ASSUMPTION 6

$$\frac{Qp_L}{D} > \frac{Q}{R}, \quad or \quad R > \frac{D}{p_L}.$$

Thus, shortages would not occur.

We use the renewal-reward theorem for deriving an expression of the total expected long-run average cost. Naturally, this quantity is a function of Q and n. Thus, the total expected long-run average cost is denoted by $TC_J(Q,n)$. The renewal-reward theorem simply states that

$$TC_J(Q,n) = \frac{\text{E[Production cycle cost]}}{\text{E[Production cycle length]}}. \tag{4.64}$$

The optimal joint policy parameters Q and n can be computed via minimizing $TC_J(Q,n)$. Hence, the mathematical problem can be stated as

$$\begin{aligned} \min \quad & TC_J(Q,n) \\ s.t. \quad & Q \ge 0,\ n \ge 0,\ \text{n is an integer.} \end{aligned}$$

Denote

$$X_i = \frac{\bar{p}_i Q}{D}, \quad i = 1, 2, ..., n.$$

Then, the X_i's are also i.i.d. random variables, which represent the buyer's replenishment cycle length. Letting $S_0 = 0$ and $S_i = \sum_{j=1}^{i} X_j$, we define

$$N(t) = \sup\{i : \ S_i \leq t\}.$$

It follows that $N(t)$ is a renewal process that registers the number of shipments by time t. Since the event $\{N(t) \geq i\}$ occurs if, and only if, $\{S_i \leq t\}$, we have

$$P\{N(t) \geq i\} = F^{(i)}(\frac{tD}{Q}), \tag{4.65}$$

where $F^{(i)}(\cdot)$ denotes the i-fold convolution of $F(\cdot)$.

IV.2.5.2. Expected Production Cycle Length

First, we need to clearly define the cycle since random shipments are involved in each cycle. We assume that the production of a batch is started as late as possible and that the despatch of the first shipment returns the vendor's stock to zero. Recall Assumption 6: the time to consume the last shipment of one batch will be greater than the time to produce the first shipment of the next batch, and, therefore, the despatch of the last shipment of one batch takes place before the production of the next batch starts. We define each production cycle as starting when the buyer receives the last shipment of a batch from the vendor and ending when the vendor despatches the last shipment of the next batch. Therefore, the buyer receives n shipments from the vendor in each cycle. It follows that

$$\text{E[Production Cycle Length]} = E[\sum_{i=1}^{n} X_i] = \frac{n\mu Q}{D}. \tag{4.66}$$

IV.2.5.3. Expected Production Cycle Cost

The total expected cost of a production cycle has two components: i) setup costs, ii) inventory costs. The setup costs can be easily obtained as $S_s + nS_b$. However, the computation of inventory costs is much more challenging due to the existence of defective items. We compute the buyer's and vendor's inventory costs, respectively, as follows.

Buyer's Expected Inventory Cost

$Q\bar{p}_i$ units are accepted in the shipment i by the buyer. Thus, the replenishment cycle time of the buyer is

$$\frac{Q\bar{p}_i}{D}.$$

Since the buyer receives n shipments in each production cycle, the buyer's expected inventory cost, denoted by $E[IC_b]$, is

$$E[IC_b] = E[\sum_{i=1}^{n} \frac{h_b}{2} \frac{Q^2\bar{p}_i^2}{D}] = \frac{nh_bQ^2(\mu^2 + \delta^2)}{2D} \tag{4.67}$$

Vendor's Expected Inventory Cost

Since defective items exist in each shipment, and thus the shipment frequency is random, the shape of the vendor's inventory profile is stochastic. Therefore, the computation of the vendor's expected inventory cost is challenging. We consider two cases as follows.

Case 1. $n = 1$

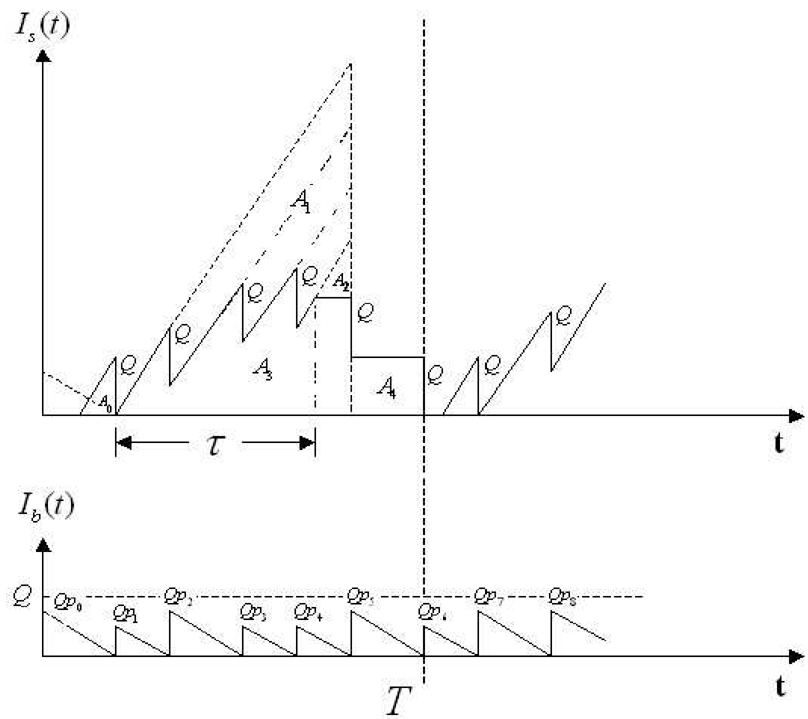
When $n = 1$, the vendor is implementing a lot-for-lot policy. Q units are produced in each production batch with the production rate R. Recall Assumption 6 which makes the lot-for-lot policy feasible. It follows that the vendor's inventory cost, which becomes deterministic, is

$$\frac{1}{2}h_s\frac{Q^2}{R}.$$

Case 2. $n > 1$

Let $I_s(t)$ and $I_b(t)$ denote the inventory level at time t of the vendor and the buyer, respectively. A realization of $I_s(t)$ and $I_b(t)$ is depicted in Figure 10.

Figure 10 Inventory Profile under Random Supply



Define

$$\tau = \frac{(n-1)Q}{R},$$

which is the production time length of a batch excluding the first shipment. Let T denote the production cycle length, that is, $T = \sum_{i=1}^{n} X_i$. Further, let A_0 denote the triangle area in the vendor's inventory profile between the beginning of a batch and the first shipment of the batch in the vendor's inventory profile. Let A_3 denote the total inventory between the first shipment of a batch and the first shipment right after the end of the batch production, if there is such a shipment; otherwise, let A_3 denote the total inventory between the first shipment of a batch and the end of the batch production. Let A_4 denote the total inventory between the first shipment right after the end of the batch production and the last shipment of the batch.

A_0 is deterministic and easily obtained as

$$A_0 = \frac{1}{2}\frac{Q^2}{R}.$$

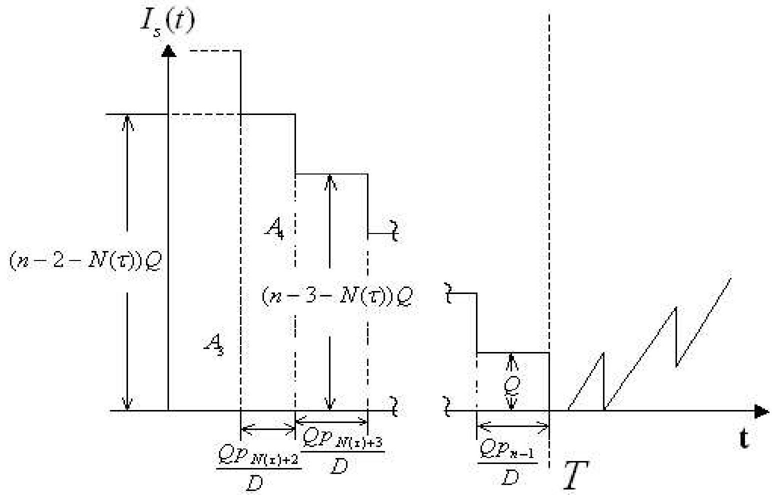
Now we calculate the expected value of A_4. Recall that $N(t)$ registers the number of shipments by time t. Thus, if $N(\tau) = n-1$, which means there are no shipments after the batch production according to the definition of τ, then $A_4 = 0$ since there are n shipments totally within the production cycle; if $N(\tau) = n-2$, which means there is only one shipment after the batch production, then $A_4 = 0$ as well since the shipment right after the end of the batch production is taken into account by A_3. If $N(\tau) \leq n-3$, there are $n-2-N(\tau)$ shipments within A_4 as shown in Figure 11. Thus, we have

$$A_4 = \begin{cases} 0, & if\ N(\tau) \geq n-2, \\ \sum_{i=2}^{n-1-N(\tau)} [n-i-N(\tau)] p_{N(\tau)+i} \frac{Q^2}{D}, & if\ N(\tau) \leq n-3\ and\ n \geq 3. \end{cases}$$

Thus, $E[A_4] = 0$ if $n = 2$. When $n \geq 3$, note that $N(\tau)$ and $p_{N(\tau)+i}$ are independent. We have

$$\begin{aligned} E[A_4] &= E[E[A_4|N(\tau)]] = \frac{Q^2}{D}\sum_{j=0}^{n-3} E[\sum_{i=2}^{n-1-j}(n-i-j)p_{i+j}]P(N(\tau)=j) \\ &= \frac{\mu Q^2}{D}\sum_{j=0}^{n-3}\sum_{i=2}^{n-1-j}(n-i-j)P(N(\tau)=j) \\ &= \frac{\mu Q^2}{2D}\sum_{j=0}^{n-3}(n-1-j)(n-2-j)P(N(\tau)=j) \end{aligned}$$

Figure 11 A Realization of A_4



We do not calculate the area of A_3 directly. Instead, we calculate the area of the large triangle first, which is $A_1 + A_2 + A_3$ in Figure 10. The extra inventory $A_1 + A_2$ occurs under the assumption that the vendor keeps producing after the first shipment of a batch until the first shipment after τ and that there is no shipment in-between. The total inventory of A_2 is the extra stock under the assumption that the vendor keeps producing after τ until the first shipment after τ. So $A_1 = 0$ if, in fact, there is no shipment between the first shipment of a batch and the end of production, that is, $N(\tau) = 0$. Obviously, $N(\tau) \leq n-2$ according to Assumption 6, and thus $N(\tau) = 0$ if $n = 2$. Let $A = A_1 + A_2 + A_3$. The length of the bottom of A is

$$\frac{Q}{D} \sum_{i=1}^{N(\tau)+1} \bar{p}_i$$

and the height of A is

$$\frac{QR}{D} \sum_{i=1}^{N(\tau)+1} \bar{p}_i.$$

It follows that

$$A = \frac{Q^2R}{2D^2}(\sum_{i=1}^{N(\tau)+1} \bar{p}_i)^2.$$

A_1 is a set of parallelograms as shown in Figure 10. Thus, we have

$$A_1 = \begin{cases} 0, & if\ n = 2, \\ \sum_{i=1}^{N(\tau)} \sum_{j=i+1}^{N(\tau)+1} \frac{Q^2 p_j}{D}, & if\ n > 2. \end{cases}$$

A_2 can be easily obtained as

$$A_2 = \frac{R}{2}(\sum_{i=1}^{N(\tau)+1} \frac{Q\bar{p}_i}{D} - \tau)^2.$$

Therefore, we have

$$A_3 = \begin{cases} \tau R \frac{Q\bar{p}_i}{D} - \frac{\tau^2 R}{2}, & if\ n = 2, \\ \tau R \sum_{i=1}^{N(\tau)+1} \frac{Q\bar{p}_i}{D} - \frac{\tau^2 R}{2} - \sum_{i=1}^{N(\tau)} \sum_{j=i+1}^{N(\tau)+1} \frac{Q^2 p_j}{D}, & if\ n > 2. \end{cases} \tag{4.68}$$

Now we calculate $E[A_3]$ when $n > 2$. Since $N(\tau) + 1$ is a stopping time for $\bar{p}_i, i = 1, 2, ...,$ by Wald's Equation, we have

$$E[\tau R \sum_{i=1}^{N(\tau)+1} \frac{Q\bar{p}_i}{D}] = \frac{\mu\tau RQ}{D} E[N(\tau) + 1]. \tag{4.69}$$

Define

$$I_i = \begin{cases} 1, & if\ N(\tau) + 1 \geq i, \\ 0, & \text{otherwise.} \end{cases}$$

Then I_i and $\bar{p}_i$ are independent since $N(\tau) + 1$ is a stopping time for p_i. Thus, we have

$$E[\sum_{i=1}^{N(\tau)}\sum_{j=i+1}^{N(\tau)+1}\frac{Q^2p_j}{D}] = E[\sum_{i=2}^{N(\tau)+1}(i-1)\frac{Q^2\bar{p}_i}{D}]$$
$$= \frac{Q^2}{D}E[\sum_{i=2}^{n-1}(i-1)\bar{p}_iI_i]$$
$$= \frac{Q^2}{D}\sum_{i=2}^{n-1}(i-1)E[\bar{p}_i]E[I_i]$$
$$= \frac{\mu Q^2}{D}\sum_{i=2}^{n-1}(i-1)P(N(\tau)+1\geq i)$$
$$= \frac{\mu Q^2}{D}\sum_{i=1}^{n-2}iP(N(\tau)\geq i) \quad (4.70)$$

From (4.68), (4.69), and (4.70), we have

$$E[A_3] = \begin{cases} \tau R\frac{Q\mu}{D} - \frac{\tau^2 R}{2} & if\ n=2 \\ \frac{\mu\tau RQ}{D}E[N(\tau)+1] - \frac{\tau^2R}{2} - \frac{\mu Q^2}{D}\sum_{i=1}^{n-2}iP(N(\tau)\geq i) & if\ n>2 \end{cases}$$

Hence, the vendor's expected holding costs, denoted by $E[IC_s]$, can be expressed as

$$E[IC_s] = h_s(A_0 + E[A_3] + E[A_4])$$
$$= \begin{cases} h_s[\frac{1}{2}\frac{Q^2}{R} + \tau R\frac{Q\mu}{D} - \frac{\tau^2R}{2}] & if\ n=2 \\ h_s[\frac{1}{2}\frac{Q^2}{R} + \frac{\mu\tau RQ}{D}E[N(\tau)+1] - \frac{\tau^2R}{2} - \frac{\mu Q^2}{D}\sum_{i=1}^{n-2}iP(N(\tau)\geq i) \\ +\frac{\mu Q^2}{2D}\sum_{j=0}^{n-3}(n-1-j)(n-2-j)P(N(\tau)=j)] & if\ n\geq 3 \end{cases} \quad (4.71)$$

When $n \geq 3$, recall (4.65), we have

$$E[N(\tau)+1] = 1+\sum_{i=1}^{n-2}F^{(i)}(\frac{(n-1)D}{R}), \quad (4.72)$$
$$P(N(\tau)\geq i) = F^{(i)}(\frac{(n-1)D}{R}), \quad (4.73)$$

$$P(N(\tau) = i) \quad = \quad F^{(i)}(\frac{(n-1)D}{R}) - F^{(i+1)}(\frac{(n-1)D}{R}). \qquad (4.74)$$

Let

$$Z = \frac{(n-1)D}{R}.$$

Substituting (4.72)-(4.74) into (4.71), we have

$$\begin{aligned} E[IC_s] &= h_s[\frac{1}{2}\frac{Q^2}{R} + \frac{\mu(n-1)Q^2}{D} - \frac{(n-1)^2Q^2}{2R} + \frac{\mu Q^2}{D}\sum_{i=1}^{n-2}(n-1-i)F^{(i)}(Z) \\ &\quad + \frac{\mu Q^2}{2D}\sum_{i=0}^{n-3}(n-1-i)(n-2-i)[F^{(i)}(Z) - F^{(i+1)}(Z)]] \\ &= h_s[\frac{1}{2}\frac{Q^2}{R} + \frac{\mu(n-1)Q^2}{D} - \frac{(n-1)^2Q^2}{2R} + \frac{\mu Q^2}{D}\sum_{i=1}^{n-2}(n-1-i)F^{(i)}(Z) \\ &\quad + \frac{\mu Q^2}{2D}\sum_{i=0}^{n-3}(n-1-i)(n-2-i)F^{(i)}(Z) - \frac{\mu Q^2}{2D}\sum_{i=1}^{n-2}(n-i)(n-1 \\ &\quad -i)F^{(i)}(Z)]] \qquad (4.75) \\ &= h_s[\frac{1}{2}\frac{Q^2}{R} + \frac{\mu(n-1)Q^2}{D} - \frac{(n-1)^2Q^2}{2R} + \frac{\mu Q^2}{2D}(n-1)(n-2)] \\ &= h_s[\frac{\mu n(n-1)Q^2}{2D} - \frac{n(n-2)Q^2}{2R}]. \qquad (4.76) \end{aligned}$$

When $n = 2$, we have

$$E[IC_s] \quad = \quad h_s[\frac{Q^2}{2R} + \frac{Q^2\mu}{D} - \frac{Q^2}{2R}] = \frac{h_s\mu Q^2}{D} \qquad (4.77)$$

It is easy to see that (4.77) is a special form of (4.76) when $n = 2$. In addition, it is easy to verify that the case $n = 1$ is a special case of $n > 1$ by comparing the expected holding costs.

IV.2.5.4. Long-Run Average Cost

Substituting (4.66), (4.67), and (4.76) into (4.64), we obtain the long-run average cost

$$TC_J(Q,n) = \frac{S_s + nS_b + \frac{nh_bQ^2(\mu^2+\delta^2)}{2D} + h_s[\frac{\mu n(n-1)Q^2}{2D} - \frac{n(n-2)Q^2}{2R}]}{\frac{n\mu Q}{D}}$$

$$= (S_s + nS_b)\frac{D}{n\mu Q} + \frac{h_bQ(\mu^2+\delta^2)}{2\mu} + h_s[\frac{(n-1)Q}{2} - \frac{(n-2)DQ}{2\mu R}]$$

The buyer's expected average cost is

$$TC_B = \frac{DS_b}{\mu Q} + \frac{h_bQ(\mu^2+\delta^2)}{2\mu},$$

which is the same as that derived by Silver (1976). The buyer's expected holding cost depends on both μ and δ. An interesting finding is that the vendor's expected average cost is

$$TC_S = \frac{DS_s}{n\mu Q} + h_s[\frac{(n-1)Q}{2} - \frac{(n-2)DQ}{2\mu R}]$$

and δ has no effect on it. The explanation is that since there are n shipments within a batch, the uncertainty of defective items neutralizes its own effect on the vendor's inventory and thus δ does not appear in the vendor's expected average cost. If we look only at the vendor's expected holding cost, it appears as if the vendor is producing at a constant rate μR.

Obviously, the model in Section IV.2.1 is a special case of $TC_J(Q,n)$ when $n = 1$. Thus, contracts under this general model can be developed based on the analysis in Section IV.2.2 and IV.2.3.

IV.2.5.5. Numerical Analysis

The joint optimal solution for the single-vendor single-buyer production-inventory model with defective items satisfies:

$$Q^*(n) = \sqrt{\frac{2D(nS_b + S_s)}{n(h_b(\mu^2+\delta^2)+h_s[(n-1)\mu - \frac{(n-2)D}{R}])}},$$
$$n^*(n^*-1) \leq \frac{S_s[2h_sD - \mu h_sR + Rh_b(\mu^2+\delta^2))]}{S_bh_s(\mu R - D)} \leq n^*(n^*+1).$$

Obviously, $Q^*(n) \geq Q^{**}(n)$ for a given n. That is, the joint optimal order quantity is larger with consideration of the defective items. This is intuitively evident since the existence of defective items would incur more frequent orders and thus increase setup costs while the larger order quantity would reduce the order frequency and thus relieve the effect of defective items. In addition, $Q^*(n)$ is decreasing in μ and δ. A large proportion of non-defective items and high uncertainty would both decrease the joint optimal order quantity.

Table VI Numerical Example 1 under Random Supply (D=1000)

	Solution 1: Perfect Quality	Solution 2: Defective items
Q	110	120
n	5	5
TC_B	500.13	501.46
TC_S	1458.44	1450.88
TC_J	1958.57	1952.34

Consider the example used by Banerjee (1986a), Goyal (1988), and Khouja (2003) in which $D = 1000$ units/year, $R = 3200$ units/year, $S_b = \$25$/order, $S_s = \$400$/set-up, $h_b = \$5$/unit/year, and $h_s = \$4$/unit/year. In addition, suppose that $\mu = 0.9$ and $\delta = 0.02$. Table VI shows two solutions to the problem. Solution 1 optimizes the whole supply chain without incorporating defective items. In solution 2, decectives are incorporated and the whole chain is optimized.

Under this setting, there is only a slight difference between the two solutions. Define

$$\alpha = \frac{C_j(Q^{**}, n^{**}) - TC_J(Q^*, n^*)}{TC_J(Q^*, n^*)}.$$

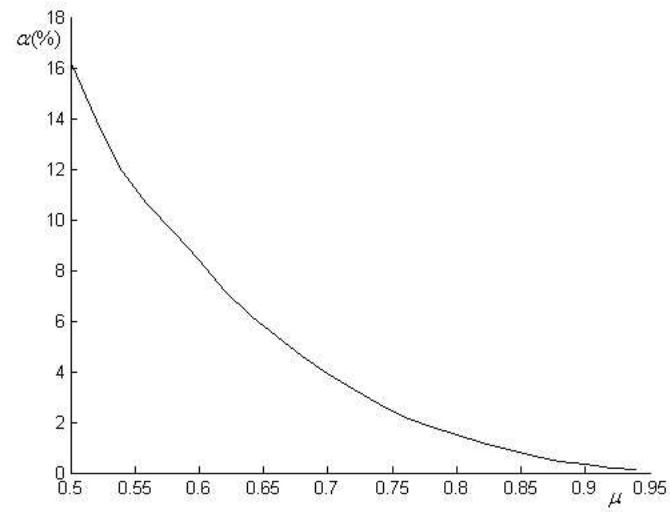
$\alpha_1 = 0.3\%$. The next example is under the same setting except that the demand is significantly larger ($D = 2750$). Table VII shows that when the demand is large,

Table VII Numerical Example 2 under Random Supply (D=2750)

	Solution 1: Perfect Quality	Solution 2: Defective items
Q	132	171
n	15	27
TC_B	875.88	831.88
TC_S	1036.22	991.56
TC_J	1912.1	1823.44

the difference between the two solutions is significant. $\alpha_1 = 4.9\%$. To examine the effects of incorporating defective items into the two-stage model as well as the impact of demand level, we plot the α versus the expected proportion of defective items μ and the demand D in Figures 12 and 13. In Figure 12, α is large when μ is small ($\alpha = 16\%$ when $\mu = 0.5$). That is, when the proportion of defective items is high, it has a significant impact on the joint optimal costs. In Figure 13, the increase in α becomes larger as demand increases. The defective items have little impact on the joint optimal costs when the demand is small, i.e., the production capacity is abundant to satisfy the demand. When the demand is large, i.e., the production capacity is barely adequate to satisfy the demand, the defective items have a significant effect on the joint optimal costs ($\alpha = 7.5\%$ when $D = 2800$).

Figure 12 α versus μ

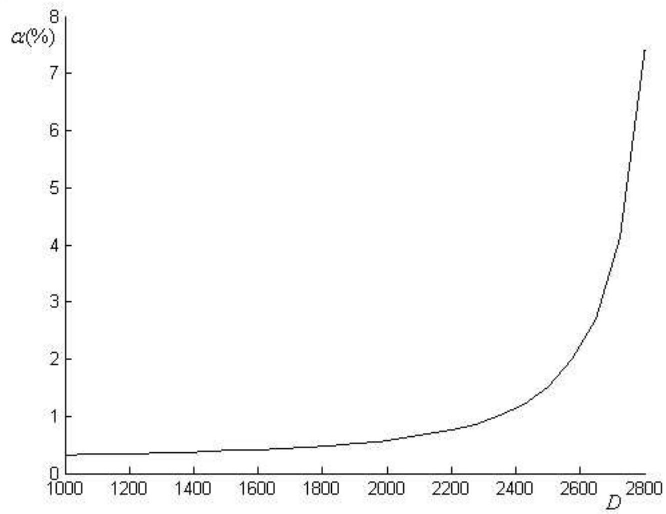


In our analysis, we assume that the inspection process time is negligible. For a considerable inspection process time as in Salameh and Jaber (2000) and Goyal and Cardenas-Barron (2002), the vendor's inventory profile remains the same because the buyer's order quantity and frequency does not change. Thus, our analysis can be easily extended by adopting the buyer's cost of Salameh and Jaber (2000).

IV.3. Optimal Contracts under Stochastic Demand

In this section, we study a general two-stage single-period model with uncertain supply and demand. When production activities are initiated or orders placed, the outputs or quantities received can be somewhat uncertain. The yield, or quantity received, might be itself uncertain, or possibly the usable portion of the yield varies.

Figure 13 α versus D



These uncertainties can affect inventory stocking decisions and production lot sizes.

A general analysis of production/inventory models with uncertain yield requires complete specification of the form of the dependence between the output level and the input level. A rather plausible assumption is that the output level is the product of the input level and a random variable which is independent of the input level. Let ξ be the random demand, with cumulative distribution Φ. Denote the supplier's lot size by q, and the resulting random yield by Y_q. Assume that Y_q is contingent on the input level q in the following manner

$$Y_q = \bar{p}q$$

where $\bar{p}$ is a non-negative random variable which is not contingent on q and is independent of ξ. When Y_q represents the proportion of nondefective discrete units in a lot size q, we clearly have $Y_q \leq q$ and $\bar{p} \leq 1$; in other applications, this might not be the case. For instance, if Y_q represents a random output measured in volume units (e.g., liquid), and q is input measured in weight units (e.g., powder), there is no obvious upper bound on Y_q. For the sake of generality then, we do not impose a prior upper bound on Y_q and $\bar{p}$.

The following notation is used throughout:

- p: unit retail price,
- c: unit production cost,
- w: unit wholesale price,
- g_b: unit shortage cost for the buyer,
- g_s: unit shortage penalty paid to the buyer by the supplier if there is such term in their agreement,
- h_b: unit overage value for the buyer; h_b can be positive (salvage value) or negative (holding cost). If $h_b > 0$, it is assumed that $h_b < c$,
- h_s: unit overage value for the supplier; h_s can be positive (salvage value) or negative (holding cost). If $h_s > 0$, it is assumed that $h_s < c$. Also, it is assumed that $h_s \geq h_b$. That is, the unit salvage value at the supplier's is at least as high as that at the buyer's since the supplier disposes of the item at the beginning of the period while the buyer disposes of the item at the end of the period; the unit holding cost for the buyer is at least as high as that for the supplier,
- Q: order quantity from the buyer,
- $F_{\bar{p}}$: cumulative distribution of $\bar{p}$,
- $\mu_{\bar{p}}$: mean value of $\bar{p}$.

Let π_S, π_B, and π_J denote the supplier's, buyer's and joint expected profits, respectively. To provide a benchmark, we first consider the centralized channel.

IV.3.1. The Centralized Model

There are two decision variables q and Q. In this centralized model, q is the input level of the chain while Q specifies the throughput level of the yield product. That is, if $Y_q > Q$, then $Y_q - Q$ units are discarded at the beginning of the period to save on holding costs and to achieve a higher salvage value.

The realized joint profit of the channel, denoted by Π_J, will be

$$\Pi_J(q,Q) = \begin{cases} -cpq + h_b(pq-\xi) + p\xi & if \ pq \leq Q \ and \ \xi \leq pq \\ -cpq - g_b(\xi - pq) + ppq & if \ pq \leq Q \ and \ \xi > pq \\ -cpq + h_s(pq-Q) + h_b(Q-\xi) + p\xi & if \ pq > Q \ and \ \xi \leq Q \\ -cpq + h_s(pq-Q) - g_b(\xi - Q) + pQ & if \ pq > Q \ and \ \xi > Q \end{cases} \tag{4.78}$$

where ξ represents the market demand. From (4.78), we have the expected joint profit of the channel as follows

$$\begin{aligned} \pi_J(q,Q) \ = \ & -c\mu_{\bar{p}}q + h_s\int_{\frac{Q}{q}}^{\infty}(pq-Q)dF_{\bar{p}}(\bar{p}) + \int_0^{\frac{Q}{q}}[h_b\int_0^{pq}(pq-\xi)d\Phi(\xi) \\ & -g_b\int_{pq}^{\infty}(\xi - pq)d\Phi(\xi) + p(\int_0^{pq}\xi d\Phi(\xi) + \int_{pq}^{\infty}pqd\Phi(\xi))]dF_{\bar{p}}(\bar{p}) \\ & + \int_{\frac{Q}{q}}^{\infty}[h_b\int_0^{Q}(Q-\xi)d\Phi(\xi) - g_b\int_Q^{\infty}(\xi - Q)d\Phi(\xi) \\ & +p(\int_0^{Q}\xi d\Phi(\xi) + \int_Q^{\infty}Qd\Phi(\xi))]dF_{\bar{p}}(\bar{p}). \end{aligned} \tag{4.79}$$

To find the optimal order quantity Q of the buyer, we derive the first derivative of $E[\pi_J(q,Q)]$ with respect to Q as follows

$$\frac{\partial \pi_J(q,Q)}{\partial Q} = [1 - F_{\bar{p}}(\frac{Q}{q})][-h_s + h_b\Phi(Q) + g_b(1-\Phi(Q)) + p(1-\Phi(Q))]. \tag{4.80}$$

Letting (4.80) equal 0 and solving for Q, we have

$$Q_J^* = \Phi^{-1}\left(\frac{g_b + p - h_s}{g_b + p - h_b}\right). \tag{4.81}$$

Since $h_s \geq h_b$, then Q_J^* in (4.81) exists. Furthermore,

$$\frac{\partial \pi_J(q,Q)}{\partial Q}|_0 = g_b + p - h_s > 0,$$

$$\frac{\partial \pi_J(q,Q)}{\partial Q} > 0 \ for \ all \ Q < Q_J^*,$$

and

$$\frac{\partial \pi_J(q,Q)}{\partial Q} \leq 0 \ for \ all \ Q > Q_J^*.$$

Thus, Q_J^* is the unique minimizer of (4.79). Note that Q_J^* is independent of q. That is, the buyer is able to make optimal order quantity decisions independently without considering the supplier's lot size decision.

To find the optimal lot size q for the supplier, we derive the first derivative of $\pi_J(q,Q)$ with respect to q as follows

$$\begin{aligned}\frac{\partial \pi_J(q,Q)}{\partial q} \quad = \quad & -c\mu_{\bar{p}} + h_s \int_{\frac{Q}{q}}^{\infty} p dF_{\bar{p}}(\bar{p}) + h_b \int_0^{\frac{Q}{q}} p\Phi(pq) dF_{\bar{p}}(\bar{p}) + g_b \int_0^{\frac{Q}{q}} p[1 \\ & -\Phi(pq)] dF_{\bar{p}}(\bar{p}) + p \int_0^{\frac{Q}{q}} p[1 - \Phi(pq)] dF_{\bar{p}}(\bar{p}).\end{aligned} \tag{4.82}$$

Substituting Q_J^* into (4.82) and solving

$$\frac{\partial \pi_J(q,Q)}{\partial q} = 0$$

for q, we have

$$\int_0^{\frac{Q_J^*}{q}} p[1 - \Phi(pq)] dF_{\bar{p}}(\bar{p}) = \frac{c\mu_{\bar{p}} - h_b \int_0^{\frac{Q_J^*}{q}} p dF_{\bar{p}}(\bar{p}) - h_s \int_{\frac{Q_J^*}{q}}^{\infty} p dF_{\bar{p}}(\bar{p})}{g_b + p - h_b}. \tag{4.83}$$

Deriving the second derivative of $\pi_J(q, Q)$ with respect to q leads to

$$\frac{\partial^2 \pi_J(q,Q)}{\partial q^2} = -\int_0^{\frac{Q}{q}} p^2 f_\xi(pq) dF_{\bar{p}}(\bar{p}) < 0.$$

Since

$$\frac{\partial \pi_J}{\partial q}|_{0^+} = \mu_{\bar{p}}(g_b + p - c) > 0,$$

$$\lim_{q \to \infty} \frac{\partial \pi_J}{\partial q} = \frac{\mu_{\bar{p}}(h_s - c)}{g_b + p - h_b} < 0,$$

the minimizer q_J^* exists and is unique.

When $Q_J^* = \infty$, (4.83) reduces to

$$\int_0^{\infty} p\Phi(pq) dF_{\bar{p}}(\bar{p}) = \frac{\mu_{\bar{p}}(g_b + p - c)}{g_b + p - h_b}.$$

We are not able to achieve the closed form of the optimal lot size q for the supplier analytically. However, since we know the optimal lot size q exists and is unique, for any instance we can solve (4.83) numerically and find the optimal lot size q for the supplier. Next, we examine the decentralized channel and the impact of supply uncertainty on the channel's performance.

IV.3.2. The Decentralized Model

To examine the impact of supply uncertainty on the channel's performance, we analyze two cases. In the first case, the buyer makes an order quantity decision without considering the supply uncertainty. In the second case, the buyer makes her decision taking the supply uncertainty into account.

In case 1, since the buyer does not consider supply uncertainty and assumes she will receive exactly as she orders, the buyer is facing the classical newsvendor's problem as follows:

$$\begin{aligned} min \quad \pi_B(Q) &= -wQ + h_b \int_0^Q (Q-\xi)d\Phi(\xi) - g_b \int_Q^\infty (\xi - Q)d\Phi(\xi) \\ &\quad +p[\int_0^Q \xi d\Phi(\xi) + \int_Q^\infty Q d\Phi(\xi)]. \end{aligned}$$

We can easily achieve the optimal order quantity Q^*_{D1} for the buyer

$$Q^*_{D1} = \Phi^{-1}\left(\frac{g_b + p - w}{g_b + p - h_b}\right). \tag{4.84}$$

Thus, the expected profit for the supplier is

$$\pi_S(q) = -c\mu_{\bar{p}}q + h_s \int_{\frac{Q^*_{D1}}{q}}^\infty (pq - Q^*_{D1})dF_{\bar{p}}(\bar{p}) + w[\int_0^{\frac{Q^*_{D1}}{q}} pqdF_{\bar{p}}(\bar{p}) + \int_{\frac{Q^*_{D1}}{q}}^\infty Q^*_{D1}dF_{\bar{p}}(\bar{p})].$$

To obtain the optimal lot size q for the supplier, we derive the first derivative of $\pi_S(q)$ with respect to q as follows

$$\frac{\partial \pi_S(q)}{\partial q} = w\int_0^{\frac{Q^*_{D1}}{q}} pdF_{\bar{p}}(\bar{p}) + h_s \int_{\frac{Q^*_{D1}}{q}}^\infty pdF_{\bar{p}}(\bar{p}) - c\mu_{\bar{p}}.$$

Define $\Lambda(\xi) = \int_0^\xi pdF_{\bar{p}}(\bar{p})$. Solving

$$\frac{\partial \pi_S(q)}{\partial q} = 0$$

for q leads to

$$q^*_{D1} = \frac{\Phi^{-1}\left(\frac{g_b+p-w}{g_b+p-h_b}\right)}{\Lambda^{-1}\left(\frac{\mu_{\bar{p}}(c-h_s)}{w-h_s}\right)}. \tag{4.85}$$

By comparing the optimal order quantity and lot size in the decentralized channel with the centralized channel, we achieve the following theorem.

THEOREM 16 $Q^*_{D1} < Q^*_J$ and $q^*_{D1} < q^*_J(Q^*_{D1})$.

Proof: $Q^*_{D1} < Q^*_J$ can be easily achieved by comparing (4.84) with (4.81). To show that $q^*_{D1} < q^*_J(Q^*_{D1})$, recall (4.83); it is equivalent to show that

$$\int_0^{\frac{Q^*_{D1}}{q^*_{D1}}} p[1-\Phi(pq^*_{D1})]dF_{\bar{p}}(\bar{p}) > \frac{c\mu_{\bar{p}} - h_b \int_0^{\frac{Q^*_{D1}}{q^*_{D1}}} pdF_{\bar{p}}(\bar{p}) - h_s \int_{\frac{Q^*_{D1}}{q^*_{D1}}}^{\infty} pdF_{\bar{p}}(\bar{p})}{g_b + p - h_b}. \quad (4.86)$$

Substituting (4.85) into the left-hand side of (4.86), we have

$$\begin{aligned} \int_0^{\frac{Q^*_{D1}}{q^*_{D1}}} p[1-\Phi(pq^*_{D1})]dF_{\bar{p}}(\bar{p}) &> \int_0^{\frac{Q^*_{D1}}{q^*_{D1}}} p[1-\Phi(Q^*_{D1})]dF_{\bar{p}}(\bar{p}) \\ &= \frac{w-h_b}{g_b+p-h_b} \int_0^{\frac{Q^*_{D1}}{q^*_{D1}}} pdF_{\bar{p}}(\bar{p}) \\ &= \left(\frac{w-h_b}{g_b+p-h_b}\right)\left(\frac{\mu_{\bar{p}}(c-h_s)}{w-h_s}\right) \end{aligned}$$

Substituting (4.85) into the right-hand side of (4.86), we have

$$\frac{\mu_{\bar{p}}(c-h_s) - (h_b - h_s)\frac{\mu_{\bar{p}}(c-h_s)}{w-h_s}}{g_b+p-h_b} = \left(\frac{w-h_b}{g_b+p-h_b}\right)\left(\frac{\mu_{\bar{p}}(c-h_s)}{w-h_s}\right).$$

■

Theorem 16 shows that the buyer's optimal order quantity in case 1 is always smaller than that in the centralized channel. The supplier's lot size in case 1 is also smaller than that in the centralized channel for a given order quantity Q^*_{D1}. Thus, the behaviors of both the buyer and the supplier deviate from the channel optimum due to the existence of supply uncertainty and the buyer's ignorance of it. An immediate question arises: if the buyer is aware of the supply uncertainty, how should she make the proper order quantity decision? Next, we study case 2 in which the buyer is aware of the supply uncertainty and selects the order quantity accordingly. Additionally, there is a penalty term g_s specified between the supplier and the buyer for each unit not delivered to the buyer.

In this case, the supplier's expected profit for a given order quantity Q is given by

$$\pi_S(q) = -c\mu_{\bar{p}}q + h_s\int_{\frac{Q}{q}}^{\infty}(pq-Q)dF_{\bar{p}}(\bar{p}) - g_s\int_0^{\frac{Q}{q}}(Q-pq)dF_{\bar{p}}(\bar{p})$$
$$+w[\int_0^{\frac{Q}{q}} pqdF_{\bar{p}}(\bar{p}) + \int_{\frac{Q}{q}}^{\infty} QdF_{\bar{p}}(\bar{p})].$$

The optimal lot size q for a given order quantity Q can be easily achieved as follows

$$q^*_{D2}(Q) = \frac{Q}{\Lambda^{-1}\left(\frac{\mu_{\bar{p}}(c-h_s)}{w+g_s-h_s}\right)}.$$

Let

$$K = \Lambda^{-1}\left(\frac{\mu_{\bar{p}}(c-h_s)}{w+g_s-h_s}\right),$$

then we have

$$q^*_{D2}(Q) = \frac{Q}{K}.$$

Based on the supplier's optimal response to the buyer's order quantity Q, the buyer's expected profit is given by

$$\pi_B(Q) = -w[\int_0^K \frac{pQ}{K}dF_{\bar{p}}(\bar{p}) + \int_K^{\infty} QdF_{\bar{p}}(\bar{p})] + g_s\int_0^K (Q-\frac{pQ}{K})dF_{\bar{p}}(\bar{p})$$
$$+\int_0^K [h_b\int_0^{\frac{pQ}{K}}(\frac{pQ}{K}-\xi)d\Phi(\xi) - g_b\int_{\frac{pQ}{K}}^{\infty}(\xi-\frac{pQ}{K})d\Phi(\xi)$$
$$+p[\int_0^{\frac{pQ}{K}}\xi d\Phi(\xi) + \int_{\frac{pQ}{K}}^{\infty}\frac{pQ}{K}d\Phi(\xi)]]dF_{\bar{p}}(\bar{p}) + \int_K^{\infty}[h_b\int_0^Q (Q-\xi)d\Phi(\xi)$$
$$-g_b\int_Q^{\infty}(\xi-Q)d\Phi(\xi) + p[\int_0^Q \xi d\Phi(\xi) + \int_Q^{\infty} Qd\Phi(\xi)]]dF_{\bar{p}}(\bar{p}).$$

To find the optimal order quantity Q for the buyer, we derive the first derivative of $\pi_B(Q)$ with respect to Q as follows

$$\begin{aligned}\frac{\partial \pi_B(Q)}{\partial Q} &= -w[\frac{\Lambda(K)}{K} + (1 - F_{\bar{p}}(K))] + g_s[F_{\bar{p}}(K) - \frac{\Lambda(K)}{K}] + \int_0^K [h_b \frac{p}{K}\Phi(\frac{pQ}{K}) \\ &\quad + \frac{p}{K} g_b(1 - \Phi(\frac{pQ}{K})) + \frac{p}{K} p[1 - \Phi(\frac{pQ}{K})]] dF_{\bar{p}}(\bar{p}) + \int_K^\infty [h_b \Phi(Q) \\ &\quad + g_b[1 - \Phi(Q)] + p[1 - \Phi(Q)]] dF_{\bar{p}}(\bar{p}) \\ &= -w[\frac{\Lambda(K)}{K} - F_{\bar{p}}(K))] - w + g_s[F_{\bar{p}}(K) - \frac{\Lambda(K)}{K}] + (g_b + p)\frac{\Lambda(K)}{K} \\ &\quad + (g_b + p)[1 - F_{\bar{p}}(K)] + \int_0^K [h_b \frac{p}{K}\Phi(\frac{pQ}{K}) - \frac{p}{K} g_b \Phi(\frac{pQ}{K}) \\ &\quad - \frac{p}{K} p \Phi(\frac{pQ}{K})] dF_{\bar{p}}(\bar{p}) + \int_K^\infty [h_b \Phi(Q) - g_b \Phi(Q) - p\Phi(Q)] dF_{\bar{p}}(\bar{p}).\end{aligned}$$

Solving

$$\frac{\partial \pi_B(Q)}{\partial Q} = 0$$

for Q, we have

$$\begin{aligned}\int_0^K \frac{p}{K}\Phi(\frac{pQ}{K}) dF_{\bar{p}}(\bar{p}) + \int_K^\infty \Phi(Q) dF_{\bar{p}}(\bar{p}) &= \frac{1}{(p + g_b - h_b)}[(p + g_b - w - g_s)[1 \\ &\quad + \frac{\Lambda(K)}{K} - F_{\bar{p}}(K))] + g_s]. \qquad (4.87)\end{aligned}$$

Denote the solution of (4.87) as Q^*_{D2}. Let

$$M = \frac{(p + g_b - w)[1 + \frac{\Lambda(K)}{K} - F_{\bar{p}}(K))] + g_s(F_{\bar{p}}(K) - \frac{\Lambda(K)}{K})}{(p + g_b - h_b)},$$

$$Q_2 = \Phi^{-1}(M).$$

Since

$$F_{\bar{p}}(K) \geq \frac{\Lambda(K)}{K},$$

we have $M \geq 0$ and $Q_2 \leq Q^*_{D2}$.

It is easy to verify that

$$\frac{\partial^2 \pi_B(Q)}{\partial Q^2} < 0,$$

and thus the minimizer Q^*_{D2} is unique. In addition, from (4.87) it is true that Q^*_{D2} is increasing in g_s. Obviously, the buyer would like to charge a higher penalty to the supplier when the supplier is unable to fulfill her order. However, a high penalty may result in poor channel performance as shown in the following theorem.

THEOREM 17 For a given Q, if $g_s > g_b + p - w + h_s - h_b$, then $q^*_{D2}(Q) < q^*_J(Q)$.

Proof: For a given Q define variables $q^*_J(Q)$, $q_1(Q)$, and $q_2(Q)$ as follows:

$$\begin{aligned}
\int_0^{\frac{Q}{q^*_J(Q)}} p[1-\Phi(pq^*_J(Q))]dF_{\bar{p}}(\bar{p}) &= \frac{c\mu_{\bar{p}} - h_b\int_0^{\frac{Q}{q^*_J(Q)}} pdF_{\bar{p}}(\bar{p}) - h_s\int_{\frac{Q}{q^*_J(Q)}}^{\infty} pdF_{\bar{p}}(\bar{p})}{g_b+p-h_b}, \\
\int_0^{\frac{Q}{q_1(Q)}} p[1-\Phi(pq_1(Q))]dF_{\bar{p}}(\bar{p}) &= \frac{\mu_{\bar{p}}(c-h_s)}{g_b+p-h_b}, \\
\int_0^{\frac{Q}{q_2(Q)}} pdF_{\bar{p}}(\bar{p}) &= \frac{\mu_{\bar{p}}(c-h_s)}{g_b+p-h_b}.
\end{aligned}$$

Obviously, $q_1(Q) < q_2(Q)$. If $g_s > g_b + p - w + h_s - h_b$, then $q_2(Q) < q^*_{D2}(Q)$. In addition, $q_1(Q) > q^*_J(Q)$ since

$$\frac{c\mu_{\bar{p}} - h_b\int_0^{\frac{Q}{q^*_J(Q)}} pdF_{\bar{p}}(\bar{p}) - h_s\int_{\frac{Q}{q^*_J(Q)}}^{\infty} pdF_{\bar{p}}(\bar{p})}{g_b+p-h_b} > \frac{\mu_{\bar{p}}(c-h_s)}{g_b+p-h_b}.$$

Thus, we have $q^*_{D2}(Q) > q^*_J(Q)$. ■

Theorem 17 shows that for a given order quantity Q from the buyer, if the buyer charges a penalty larger than $g_b + p - w + h_s - h_b$, the supplier will always choose a smaller lot size than the channel optimum. Although the buyer prefers a higher penalty, Theorem 17 shows that, from the system's perspective, it is not good to set the penalty level too high. To determine the optimal penalty level for the system, knowledge of the distributions of the supply uncertainty and the market demand is critical to the buyer. In the following section, we provide a consignment contract where the terms can be easily determined.

IV.3.3. Consignment Contract

Under a consignment contract, the supplier decides the delivery quantity for his product and retains ownership of the goods. For each item sold, the buyer deducts a percentage from the selling price; for each item unsold, the buyer shares a percentage of the cost. Also, the buyer specifies the maximal stock level. The contracting procedure is as follows:

1. The supplier decides the production input level and delivers all of the output to the retailer;

2. The buyer inspects the products and returns all of the defective items and overstock good items to the supplier;

3. The supplier disposes of all returned items;

4. Demand occurs. The buyer returns all unsold items to the supplier and remits his revenue share as well as his cost share.

Define r as the percentage of the revenue share where $0 \leq r \leq 1$ and β as the percentage of cost share where $0 \leq \beta \leq 1$.

To coordinate the channel, the buyer should set the maximal stock level at Q_J^*. Then, the supplier's expected profit is given by

$$\begin{aligned}\pi_S(q,Q) \quad = \quad & -c\mu_{\bar{p}}q + (\beta(c-h_s)+h_s)\int_{\frac{Q}{q}}^{\infty}(pq-Q)dF_{\bar{p}}(\bar{p}) + \int_0^{\frac{Q}{q}}[(\beta(c-h_b) \\ & + h_b)\int_0^{pq}(pq-\xi)d\Phi(\xi) + (1-r)p(\int_0^{pq}\xi d\Phi(\xi) \\ & + \int_{pq}^{\infty}pqd\Phi(\xi))]dF_{\bar{p}}(\bar{p}) + \int_{\frac{Q}{q}}^{\infty}[(\beta(c-h_b)+h_b)\int_0^{Q}(Q-\xi)d\Phi(\xi) \\ & +(1-r)p(\int_0^{Q}\xi d\Phi(\xi) + \int_Q^{\infty}Qd\Phi(\xi))]dF_{\bar{p}}(\bar{p}). \end{aligned} \tag{4.88}$$

To find the optimal lot size q for a given maximal stock level Q, we derive the first derivative of $\Pi_S(q,Q)$ with respect to q as follows

$$\begin{aligned}\frac{\partial \pi_S(q,Q)}{\partial q} &= -c\mu_{\bar{p}} + (\beta(c-h_s)+h_s)\int_{\frac{Q}{q}}^{\infty} p dF_{\bar{p}}(\bar{p}) + (\beta(c-h_b) \\ &\quad +h_b)\int_0^{\frac{Q}{q}} p\Phi(pq)dF_{\bar{p}}(\bar{p}) + (1-r)p\int_0^{\frac{Q}{q}} p[1-\Phi(pq)]dF_{\bar{p}}(\bar{p}). \qquad (4.89)\end{aligned}$$

Recall (4.81), substituting Q_J^* into (4.89) and solving

$$\frac{\partial \pi_S}{\partial q} = 0$$

for q, we have

$$\int_0^{\frac{Q_J^*}{q}} p[1-\Phi(pq)]dF_{\bar{p}}(\bar{p}) = \frac{(1-\beta)(c\mu_{\bar{p}} - h_b\int_0^{\frac{Q_J^*}{q}} pdF_{\bar{p}}(\bar{p}) - h_s\int_{\frac{Q_J^*}{q}}^{\infty} pdF_{\bar{p}}(\bar{p}))}{(1-r)p-(1-\beta)h_b-\beta c}.$$

Recall (4.83), to align the supplier's optimal lot size with the channel optimum, the buyer should set r and β satisfying

$$\frac{1-\beta}{(1-r)p-(1-\beta)h_b-\beta c} = \frac{1}{g_b+p-h_b}.$$

That is,

$$\begin{aligned}(r-\beta)p + (1-\beta)g_b + \beta c = 0, \quad \text{or} \\ r = \beta - \frac{(1-\beta)g_b+\beta c}{p}. \qquad (4.90)\end{aligned}$$

From (4.90), we can see that $r < \beta$ and r is increasing in β. That is, to coordinate the channel, the more revenue share that is taken by the buyer, the more cost she should share. The advantage of the consignment contract is that the buyer does not need to have specific knowledge of the distribution of the supply uncertainty to design the contract, and thus it makes the buyer's contract design easier.

IV.4. Summary

In this chapter, we seek to provide implementable contractual arrangements to coordinate a channel with supply uncertainty considerations. We first develop a general framework that includes power structure and supply uncertainty. We demonstrate that power structure and supply uncertainty are important considerations when designing supply contracts. Based on this general framework, we further develop supply contracts under supply uncertainty and deterministic demand with an infinite planning horizon. The findings reported clearly show that ignoring incentive conflicts and supply information issues can lead to undesirable behavior. We propose a model that shows how lot size, quality level, and transactions can be structured to reduce supply chain inefficiency due to individual incentives and private information. In addition, we develop supply contracts under supply uncertainty and stochastic demand in a single period. We examine the impact of both supply and demand uncertainty on channel performance and propose a consignment contract to help coordinate the channel. The problems investigated in this chapter are summarized below:

- A general framework of supply chain contract design under supply uncertainty in supplier- and buyer-driven channels.
- Exploration of the impact of the power structure.
- The design of optimal cost-sharing contracts under supply uncertainty and continuous deterministic demand.
- The design of optimal cost-sharing contracts under supply uncertainty and stochastic single-period demand.
- Exploration of the value of supply uncertainty information.

CHAPTER V

SUPPLIER COMPETITION

Most studies to date have focused on markets consisting of exclusive dealers that sell only one producer's brand; little attention has been given to the larger segment of most consumer goods markets in which retailers sell multiple (often highly substitutable) brands at the same location. This latter channel structure represents numerous markets including those consisting of specialty stores, such as consumer electronics, sporting goods, and automobile parts etc. As Tsay et al. (1999) point out, "another deficiency in the current literature is the lack of attention to competition, either between multiple buyers or multiple suppliers. Buyers that share a common supplier and compete in the same consumer market might behave in a way that obstructs their competitors' access to suppliers...Multiple suppliers to a common buyer might need to alter their price, service, lead time, or flexibility offerings in light of the competitive environment."

Although consideration of competition is rare in the operations management literature, there is substantial coverage of it in the economics literature. For example, Choi (1991) analyzes a channel structure with two competing manufactures and a common retailer and studies three different power structures, i.e., supplier dominant, balanced power, and buyer dominant structures. He shows that all channel members are better off when no one dominates the structure. His work is followed by Trivedi (1998), who models a channel structure in which there are duopoly manufacturers and duopoly common retailers. Trivedi shows that the presence of competitive effects at both the retailer and manufacturer levels of distribution has a significant impact on profits and prices.

As we discussed in Chapters III and IV, consideration of the power structure and supply uncertainty is important for implementable supply contract design. In this chapter, we follow the ideas in Chapters III and IV to develop a general framework with power structure, supply uncertainty, and supplier competition considerations. Under this framework, we further examine the case in which a single buyer is facing one stable supplier and one unstable supplier and provide optimal ordering policies for the buyer in each case. In addition, we study the case in which a single buyer is facing two unstable suppliers.

The reminder of this chapter is organized as follows. Section V.1 develops the general framework with power structure, supply uncertainty, and supplier competition considerations concentrating on the case of two stable suppliers. Section V.2 considers the one stable supplier and one unstable supplier case. The two unstable suppliers case is addressed in section V.3. Concluding remarks are provided in Section V.4.

V.1. Optimal Contracts under Power Structure, Supply Uncertainty, and Supplier Competition

In this section we develop a two-supplier-one-buyer model in a single period with power structure, supply uncertainty, and supplier competition considerations. This model is based on Choi (1991) and Banker et al. (1998). Similarly as in Section IV.1, in this section, we assume that the proportion of defectives is $\bar{p} = 0$. In addition, the model we construct is based on the following assumptions.

(1) There are two suppliers, labeled 1 and 2. Each supplier has one product, for which he must choose a wholesale price level w and a quality level τ. We assume that quality is a measurable attribute with values in the interval $[0, \infty)$. There is one buyer placing orders to the two suppliers. She sells the product to the retail market at price p_i for each unit of product i, $i = 1, 2$.

(2) We assume that the market demand function is linear in price and quality:

$$d_i = k_i\alpha - \beta p_i + \gamma p_j + \lambda\tau_i - \mu\tau_j,$$
$$i, j = 1, 2, i \neq j, \tag{5.1}$$

where $k_1 + k_2 = 1$. Here $k_i\alpha$ is the intrinsic demand potential parameter for firm i, $i = 1, 2$. Parameter β (λ) denotes the demand responsiveness to the firm's own price (quality), while γ (μ) denotes the demand responsiveness to the competitor's price (quality). We refer to β/γ (λ/μ) as the relative responsiveness to price (quality) and assume that it is greater than one, i.e.,

$$\beta > \gamma \text{ and } \lambda > \mu.$$

Thus, the demand d_i for each firm i's product is affected more by changes in its own price and quality than those of its competitors. This condition is necessary because if both firms were to raise their prices by \$1 or to decrease their quality by one unit, then both firms would lose sales (Tirole 1988).

(3) The supplier has the option to improve the quality level with investment. The investment cost function is given by

$$c_i = C_i(d_i, \tau_i) = (\nu + \zeta\tau_i)d_i + \kappa + \eta_i\tau_i^2, \; i = 1, 2.$$

To develop the framework while considering power structure and supply uncertainty, we study the simple contract design in which the suppliers decide the quality levels and wholesale prices and the buyer decides the retail prices. Additional dedicated contractual mechanisms can be further developed based on the model that we present in this section. First, we study the case when the suppliers are the leaders of the channel. The price and quality decisions take place in the following sequence in time:

i) Two suppliers simultaneously select their own quality levels; two suppliers and one buyer observe each supplier's quality level;

ii) Each supplier selects a wholesale price for his product;

iii) The buyer selects the retail prices for two products. Demand is realized based on the prices and quality levels.

Our model also reflects the assumption that price decisions are made after the quality decisions, since the choice of a quality level reflects a long-term decision which cannot be changed as easily or as frequently as price. To solve this three-stage game, we first calculate the optimal retail prices assuming given quality levels and wholesale prices, and then determine the optimal wholesale prices assuming given quality levels. At last, the optimal quality levels are determined.

The profit functions of the suppliers and the buyer are given by:

$$\pi_S^i = (w_i - \nu - \zeta\tau_i)(k_i\alpha - \beta p_i + \gamma p_j + \lambda\tau_i - \mu\tau_j) - \kappa - \eta_i\tau_i^2, \tag{5.2}$$

$$\pi_B = \sum_{i=1}^{2}(p_i - w_i)q_i, \tag{5.3}$$

$$i, j = 1, 2, i \neq j.$$

The buyer's reaction function given wholesale prices and quality levels can be derived from the first-order conditions for maximizing (5.3) as follows

$$\frac{\partial\pi_B}{\partial p_1} = k_1\alpha + \lambda\tau_1 - \mu\tau_2 - 2\beta p_1 + 2\gamma p_2 + \beta w_1 - \gamma w_2 = 0,$$
$$\frac{\partial\pi_B}{\partial p_2} = k_2\alpha + \lambda\tau_2 - \mu\tau_1 - 2\beta p_2 + 2\gamma p_1 + \beta w_2 - \gamma w_1 = 0.$$

Note that the Hessian matrix of (5.3) in terms of p_1 and p_2 is negative-definite since $\beta > \gamma > 0$. Thus, π_B is a joint concave function of p_1 and p_2. The optimal values of p_1 and p_2 are unique. We obtain the optimal retail prices for given quality levels and wholesale prices as follows

$$p_1 = \frac{w_1}{2} + \frac{\beta(k_1\alpha + \lambda\tau_1 - \mu\tau_2) + \gamma(k_2\alpha + \lambda\tau_2 - \mu\tau_1)}{2(\beta^2 - \gamma^2)}, \quad (5.4)$$

$$p_2 = \frac{w_2}{2} + \frac{\beta(k_2\alpha + \lambda\tau_2 - \mu\tau_1) + \gamma(k_1\alpha + \lambda\tau_1 - \mu\tau_2)}{2(\beta^2 - \gamma^2)}, \quad (5.5)$$

both of which are linear in wholesale prices. These expressions imply that only half the change in the wholesale price is reflected in the retail price, and the other half is absorbed by the buyer.

Substituting (5.4) and (5.5) into (5.2), the suppliers' Nash equilibrium of wholesale prices can be derived from the first-order conditions for maximizing (5.2) which are given by

$$\frac{\partial \pi_s^i}{\partial w_i} = \frac{1}{2}(k_i\alpha + \lambda\tau_i - \mu\tau_j - 2\beta w_i + \gamma w_j + \beta(\nu + \zeta\tau_i)) = 0. \quad (5.6)$$

Solving (5.6) results in the following wholesale prices

$$w_i^* = \nu + \zeta\tau_i + \frac{1}{W}(R_i + S\tau_i - T\tau_j),$$

$$where$$

$$W = 4\beta^2 - \gamma^2,$$

$$R = 2k_i\alpha\beta + k_j\alpha\gamma - \nu(\beta - \gamma)(2\beta + \gamma),$$

$$S = 2\beta\lambda - 2\beta^2\zeta - \gamma\mu + \zeta\gamma^2, \text{and}$$

$$T = 2\beta\mu - \beta\gamma\zeta - \gamma\lambda, \quad (5.7)$$

and the corresponding profits and demand quantities at the equilibrium prices are computed as

$$\pi_s^{i*} = \frac{\beta}{2W^2}(R_i + S\tau_i - T\tau_j)^2 - \kappa - \eta_i\tau_i^2, \quad (5.8)$$

$$q_i^* = \frac{\beta}{2W}(R_i + S\tau_i - T\tau_j). \quad (5.9)$$

Having characterized the price equilibrium, we now are able to analyze the quality equilibrium. Differentiating (5.8) with respect to τ_i and equating it to zero, we obtain the following reaction function for firm i that gives the best action for firm i given that firm j chooses τ_j:

$$\tau_i = \frac{\beta S}{2W^2\eta_i - \beta S^2}(R_i - T\tau_j). \tag{5.10}$$

Finally, we obtain the optimal quality levels given by

$$\tau_i^* = \frac{s\beta[R_i(2W^2\eta_j - \beta S^2) - R_j ST\beta]}{(2W^2\eta_i - \beta S^2)(2W^2\eta_j - \beta S^2) - S^2T^2\beta^2}. \tag{5.11}$$

Now we study the case when the suppliers are the followers of the channel. The price and quality decisions take place in the following sequence in time:

i) Two suppliers simultaneously select their own quality level; two suppliers and one buyer observe each supplier's quality level;

ii) The buyer announces her profit margin m;

iii) Each supplier selects a wholesale price for his product; demand is realized based on the prices and the quality levels.

To solve this three-stage game, we first calculate the optimal wholesale prices for given values of quality levels and buyer's profit margin, and then we determine the optimal profit margin for given quality levels. At last, the optimal quality levels are determined.

The profit functions of the suppliers are given by:

$$\pi_S^i = (w_i - \nu - \zeta\tau_i)(k_i\alpha - \beta(w_i + m_i) + \gamma(w_j + m_j) + \lambda\tau_i - \mu\tau_j) - \kappa - \eta_i\tau_i^2,$$
$$i, j = 1, 2, i \neq j. \tag{5.12}$$

The suppliers' reaction functions, given the buyer's profit margins and quality levels, can be derived from the first-order conditions as follows:

$$\begin{aligned}
\frac{\partial \pi_S^1}{\partial w_1} &= k_1\alpha - \beta(w_1+m_1) + \gamma(w_2+m_2) + \lambda\tau_1 - \mu\tau_2 - \beta(w_1 - \nu - \zeta\tau_1) = 0,\\
\frac{\partial \pi_S^2}{\partial w_2} &= k_2\alpha - \beta(w_2+m_2) + \gamma(w_1+m_1) + \lambda\tau_2 - \mu\tau_1 - \beta(w_2 - \nu - \zeta\tau_2) = 0.
\end{aligned}$$

It can be easily seen that the second-order Jacobian matrix is negative-definite, implying that a unique Nash equilibrium exists between the two suppliers. The resulting reaction functions are as follows

$$\begin{aligned}
w_1 &= \frac{1}{2\beta}(k_1\alpha - \beta m_1 + \gamma p_2 + \lambda\tau_1 - \mu\tau_2 + \beta\nu + \beta\zeta\tau_1),\\
w_2 &= \frac{1}{2\beta}(k_2\alpha - \beta m_2 + \gamma p_1 + \lambda\tau_2 - \mu\tau_1 + \beta\nu + \beta\zeta\tau_2).
\end{aligned}$$

or, equivalently,

$$\begin{aligned}
w_1 &= \frac{1}{\beta}(k_1\alpha - \beta p_1 + \gamma p_2 + \lambda\tau_1 - \mu\tau_2 + \beta\nu + \beta\zeta\tau_1),\\
w_2 &= \frac{1}{\beta}(k_2\alpha - \beta p_2 + \gamma p_1 + \lambda\tau_2 - \mu\tau_1 + \beta\nu + \beta\zeta\tau_2).
\end{aligned}$$

The buyer applies these suppliers' reaction functions to maximize her own profit by setting optimal retail prices. Her optimization problem is given by

$$\max_{p_1,p_2} \pi_B = (p_1 - w_1(p_1,p_2))q_1(p_1,p_2) + (p_2 - w_2(p_1,p_2))q_2(p_1,p_2). \tag{5.13}$$

Solving the optimization problem (5.13) with respect to retail prices p_1 and p_2 leads to:

$$\begin{aligned}
p_i &= \frac{2\beta a_i + \gamma a_j}{2(4\beta^2 - \gamma^2)} + \frac{\beta(k_i\alpha + \lambda\tau_i - \mu\tau_j) + \gamma(k_j\alpha + \lambda\tau_j - \mu\tau_i)}{2(\beta^2 - \gamma^2)}\\
& \quad where\\
a_i &= k_i\alpha + \lambda\tau_i - \mu\tau_j + \beta(\nu + \zeta\tau_i),
\end{aligned}$$

and from the suppliers' reaction functions, we obtain the following equilibrium wholesale prices:

$$w_i^{**} = \nu + \zeta\tau_i + \frac{1}{2W}(R_i + S\tau_i - T\tau_j).$$

Note that, after rearrangement, the retail prices are the same as those when the suppliers are the leaders, while the wholesale prices are lower. That means, when the suppliers do not have the dominant bargaining power, they have to charge less to the buyer though the market demand remains the same. The corresponding profit margins can be derived using $m_i = p_i - w_i$ as follows

$$m_i^{**} = -\frac{1}{2}(\nu + \zeta\tau_i) + \frac{\beta(k_i\alpha + \lambda\tau_i - \mu\tau_j) + \gamma(k_j\alpha + \lambda\tau_j - \mu\tau_i)}{2(\beta^2 - \gamma^2)},$$

and the corresponding profit and demand quantities at the equilibrium prices are computed as

$$\pi_s^{i**} = \frac{\beta}{4W^2}(R_i + S\tau_i - T\tau_j)^2 - \kappa - \eta_i\tau_i^2, \tag{5.14}$$

$$q_i^{**} = \frac{\beta}{2W}(R_i + S\tau_i - T\tau_j). \tag{5.15}$$

Having characterized the price equilibrium, we now analyze the quality equilibrium. Differentiating (5.14) with respect to τ_i and equating it to zero, we obtain the following reaction function for firm i that gives the best action for firm i given that firm j chooses τ_j:

$$\tau_i = \frac{\beta S}{4W^2\eta_i - \beta S^2}(R_i - T\tau_j). \tag{5.16}$$

Solving (5.16) for τ_i, we obtain the optimal quality levels as

$$\tau_i^* = \frac{s\beta[R_i(4W^2\eta_j - \beta S^2) - R_jST\beta]}{(4W^2\eta_i - \beta S^2)(4W^2\eta_j - \beta S^2) - S^2T^2\beta^2}. \tag{5.17}$$

Comparing the quality levels under two power structures, we obtain the following theorem.

THEOREM 18 The optimal quality levels are higher when the suppliers are the leaders of the channel than when they are the followers.

Proof: The theorem is easy to prove by comparing (5.11) with (5.17). ∎

The conclusion of Theorem 18 is opposite to the conclusion of Theorem 13 under the single-supplier-single-buyer case. That is, when suppliers compete, they choose to increase their quality levels when they have the bargaining power, rather than to decrease the quality level as when there is no supplier competition. The intuitive explanation is that when there is a single supplier, he is willing to improve his product quality level so that the market demand will increase. However, when suppliers compete, they have less interest in improving quality since their market share is less than in the no competition case, and they invest more than they can gain due to the competition. In this section, we assume that the proportion of defectives is $\bar{p} = 0$ and that the demand is deterministic. In the next two sections, we will consider supplier competition under stochastic supply and demand.

V.2. Optimal Contracts under One Stable Supplier and One Unstable Supplier

In this section, we consider a supply chain consisting of one buyer and two suppliers where one supplier is stable and the other is unstable. Ciarallo et al. (1994) consider a one-supplier one-buyer supply chain in which the supplier's production capacity is uncertain. Our model is based on their work and our interest lies in the optimal ordering policy for the buyer under supplier competition.

Supplier 1 has infinite capacity and delivers the order quantity, u, received from the buyer on time at cost $\$s_1$/unit; while supplier 2 has variable capacity, Y, and charges $\$s_2$ for each delivered unit where $s_2 \leq s_1$. We consider the case where Y is a random variable with cumulative distribution $F_y(y)$. The actual amount of units received from supplier 2 may be limited by his capacity. If the order quantity exceeds the actual capacity that is available, the actual amount is determined by the capacity. Otherwise, the complete order quantity is received successfully. In terms of the order quantity, v, and the uncertain capacity, y, the actual amount received is $\min(u, y)$. The orders are intended to satisfy a random demand, ξ, with distribution $\Phi(\xi)$.

It is assumed that the inventory holding and backlog penalty costs are linear, with per unit costs of h and g_b, respectively. The unit sale price is w. The single-period profit for the buyer, $\pi_B(x, u, v)$, is a function of the starting inventory, x, as well as orders u and v given by:

$$\begin{aligned}
\pi_B(x,u,v) \quad = \quad & -s_1 u - s_2 v[1 - F_y(v)] - s_2 \int_0^v y f_y(y)dy + [1 - F_y(v)][w \int_0^{x+u+v} \xi \\
& \phi(\xi)d\xi + w \int_{x+u+v}^{\infty} (x+u+v)\phi(\xi)d\xi - g_b \int_{x+u+v}^{\infty} (\xi - x - u \\
& -v)\phi(\xi)d\xi - h \int_0^{x+u+v} (x+u+v-\xi)\phi(\xi)d\xi] \\
& + \int_0^v [w \int_0^{x+u+y} \xi\phi(\xi)d\xi + w \int_{x+u+y}^{\infty} (x+u+y)\phi(\xi)d\xi \\
& -g_b \int_{x+u+y}^{\infty} (\xi - x - u - y)\phi(\xi)d\xi - h \int_0^{x+u+y} (x+u+y \\
& -\xi)\phi(\xi)d\xi] f_y(y)dy \qquad (5.18)
\end{aligned}$$

In (5.18), note that the number of units received is given by $x + u + v$, if the capacity is greater than v. Otherwise, it is $x + u + y$, where y is the realization of capacity. Also, note that the expectation is taken over both the demand and the capacity distributions.

The following proposition provides the optimal ordering policy for the buyer.

PROPOSITION 15 When $x < X$, the optimal ordering policy for the buyer is

$$(u^*, v^*) = \begin{cases} (X-x, 0) & if\ s_1 = s_2 \\ (u^{**}, X-x-u^{**}) & if\ s_2 < s_1 < s_3(x) \\ (0, X-x) & if\ s_1 \geq s_3(x) \end{cases}$$

where

$$\begin{aligned} X &= Q^{-1}(\frac{g_b + w - s_2}{g_b + w + h}), \\ s_3(x) &= [1 - F_y(X-x)]s_2 + \int_0^{X-x} [(w+g_b) - (w+g_b+h)Q(x+y)]f_y(y)dy, \end{aligned}$$

$$-s_1 + [1 - F_y(X-x-u^{**})]s_2 + \int_0^{X-x-u^{**}} [(w+g_b) - (w+g_b+h)Q(x+u^{**} \\ +y)]f_y(y)dy = 0.$$

When $x \geq X$, the optimal ordering policy is $(u^*, v^*) = (0, 0)$.

Proof: We compute the optimal values of u and v using the first partial derivatives of (5.18), with respect u and v. For this purpose, we have

$$\begin{aligned} \frac{\partial \pi_B(x,u,v)}{\partial u} &= -s_1 + [1 - F_y(v)][w \int_{x+u+v}^{\infty} \phi(\xi)d\xi + g_b \int_{x+u+v}^{\infty} \phi(\xi)d\xi \\ &\quad -h \int_0^{x+u+v} \phi(\xi)d\xi] + \int_0^v [w \int_{x+u+y}^{\infty} \phi(\xi)d\xi + g_b \int_{x+u+y}^{\infty} \phi(\xi)d\xi \\ &\quad -h \int_0^{x+u+y} \phi(\xi)d\xi]f_y(y)dy \\ &= -s_1 + [1 - F_y(v)][(w+g_b) - (w+g_b+h)Q(x+u+v)] \\ &\quad + \int_0^v [(w+g_b) - (w+g_b+h)Q(x+u+y)]f_y(y)dy, \end{aligned} \tag{5.19}$$

$$\frac{\partial \pi_B(x,u,v)}{\partial v} = [1 - F_y(v)][(-s_2 + w + g_b) - (w+g_b+h)Q(x+u+v)]. \tag{5.20}$$

First, we show that for a given pair of x and u, the optimal order quantity v^* minimizing (5.18) is given by

$$v^* = \begin{cases} Q^{-1}(\frac{g_b+w-s_2}{g_b+w+h}) - x - u \quad if\ x+u < Q^{-1}(\frac{g_b+w-s_2}{g_b+w+h}) \\ 0 \quad if\ x+u \geq Q^{-1}(\frac{g_b+w-s_2}{g_b+w+h}) \end{cases} \tag{5.21}$$

Setting (5.20) equal to zero and solving for v, we have

$$v^{**} = Q^{-1}(\frac{g_b+w-s_2}{g_b+w+h}) - x - u$$

as the unique v value that satisfies the first-order condition. Noting that,

$$\frac{\partial \pi_B(x,u,v)}{\partial v} \geq 0\ for\ all\ v \leq v^{**},$$

and

$$\frac{\partial \pi_B(x,u,v)}{\partial v} < 0\ for\ all\ v > v^{**},$$

we conclude that (5.21) is correct. That is, since v^* must be nonnegative, $v^* = v^{**}$ when $v^{**} \geq 0$ and $v^* = 0$ when $v^{**} < 0$.

Below we consider two cases with $x < X$ and $x \geq X$ respectively.

When $x < X$, we substitute v^{**} into (5.19). The resulting expression is denoted by $g(u)$ and given by

$$\begin{aligned} g(u) &= -s_1 + [1 - F_y(X-x-u)]s_2 + \int_0^{X-x-u} [(w+g_b) - (w+g_b+h)Q(x+u \\ &\quad +y)]f_y(y)dy. \end{aligned} \tag{5.22}$$

Now, we show that $g(u)$ is decreasing in u when $0 \leq u \leq X - x$. For any u_a and u_b satisfying $0 \leq u_a < u_b \leq X - x$, we have

$$
\begin{aligned}
g(u_a) - g(u_b) &= [F_y(X-x-u_b) - F_y(X-x-u_a)]s_2 + \int_0^{X-x-u_a} [(w+g_b) - (w \\
&\quad +g_b+h)Q(x)+u_a+y)]f_y(y)dy - \int_0^{X-x-u_b} [(w+g_b) - (w \\
&\quad +g_b+h)Q(x)+u_b+y)]f_y(y)dy \\
&\geq [F_y(X-x-u_b) - F_y(X-x-u_a)]s_2 + \int_{X-x-u_b}^{X-x-u_a} [(w+g_b) \\
&\quad -(w+g_b+h)Q(x)+u_a+y)]f_y(y)dy \\
&> [F_y(X-x-u_b) - F_y(X-x-u_a)]s_2 + \int_{X-x-u_b}^{X-x-u_a} [(w+g_b) \\
&\quad -(w+g_b+h)Q(X)]f_y(y)dy = 0.
\end{aligned}
$$

Next, we set (5.22) equal to 0 and solve for u. If $s_1 = s_2$, it is easy to see that $u^* = X - x$ is the solution. If $s_2 < s_1 \leq s_3$, there exists a unique solution u^{**} minimizing $\pi_B(x, u, v^*)$. If $s_1 > s_3(x)$, $u^* = 0$ minimizes $\pi_B(x, u, v^*)$.

When $x \geq X$, it is easy to check that $u^* = v^* = 0$ minimizes $\pi_B(x, u, v)$. ■

Proposition 15 shows that the total order-up-to level from the buyer to both suppliers is X, which does not depend on the cost of supplier 1 (s_1). Also, if supplier 1's product is too expensive ($s_1 \geq s_3(x)$) for the buyer, then the buyer will give up supplier 1 and place orders with supplier 2 only. Since supplier 2 is unstable, the buyer will give up supplier 2 if supplier 1 and supplier 2 provide the products at the same price. Therefore, the buyer is balancing price and supply uncertainty when she is placing orders in order to maximize her own expected profit.

V.3. Optimal Contracts under Two Unstable Suppliers

In this section, we consider the case where both suppliers are unstable. Our interest is in the optimal ordering policy for the buyer in this environment.

Suppose both suppliers have variable capacities, Y_i, $i = 1, 2$. The buyer requests u and v units from supplier 1 and 2, and pays \$$s_1$ and \$$s_2$ for each received unit, respectively. Without the loss of generality, we assume $s_1 \leq s_2$.

Y_i is a random variable with cumulative distribution $F_i(y_i)$. The actual amount of units received from both suppliers may be limited by their capacities. If the placed order exceeds the actual capacity that is available, the actual amount is determined by the capacity. Otherwise, the complete order quantity is received successfully. In terms of the order quantity, u and v, and the uncertain capacity, y_i, the actual amounts received are $\min(u, y_1)$ and $\min(v, y_2)$, respectively. The orders are intended to satisfy a deterministic demand, D.

Again, we assume that the inventory holding and backlog penalty costs are linear, with per unit costs of h and p, respectively. The unit sale price is w. The single-period profit, $\pi_B(x, u, v)$, is a function of the starting inventory, x, as well as orders u and v. The following proposition provides the optimal ordering policy for the buyer.

PROPOSITION 16 Let

$$U = F_1^{-1}\left(\frac{s_2 + h}{w + g_b + h}\right), V = F_2^{-1}\left(\frac{s_1 + h}{w + g_b + h}\right), W = F_1^{-1}\left(\frac{s_2 - s_1}{w + g_b - s_1}\right).$$

The optimal order quantity pair for the buyer is as follows:

$$(u^*, v^*) \quad = \quad \begin{cases} (D - x - V, D - x - U) \quad if\ x \leq D - U - V \\ (u^{**}, D - x - u^{**}) \quad if\ D - U - V < x \leq D - W \\ (D - x, 0) \quad if\ D - W < x < D \\ (0, 0) \quad if\ x \geq D \end{cases}$$

where

$$(w + g_b - s_2)F_2(D - x - u^{**}) - (w + g_b - s_1)F_1(u^{**}) - s_1 + s_2 = 0.$$

Proof: We consider five cases as follows

- *Case 1:* $x < D$, $x + u \leq D$, $x + v \leq D$, and $x + u + v \geq D$.
- *Case 2:* $x < D$, $x + u \geq D$, $x + v \leq D$.
- *Case 3:* $x < D$, $x + u \leq D$, $x + v \geq D$.
- *Case 4:* $x < D$, $x + u + v \leq D$.
- *Case 5:* $x \geq D$.

In Case 1, the buyer's expected profit function $\pi_B(x, u, v)$ is given by

$$\begin{aligned}
\pi_B(x,u,v) \;=\; & -s_1 u[1 - F_1(u)] - s_1 \int_0^u y_1 f_1(y_1) dy_1 - s_2 v[1 - F_2(v)] \\
& -s_2 \int_0^v y_2 f_2(y_2) dy_2 + [1 - F_1(u)][1 - F_2(v)][wD - h(x + u + v - D)] \\
& \{+[1 - F_2(v)] \int_0^{D-x-v} [w(x + y_1 + v) - g_b(D - x - y_1 - v)] f_1(y_1) dy_1 \\
& + \int_{D-x-v}^{u} [wD - h(x + y_1 + v - D)] f_1(y_1) dy_1\} + [1 - F_1(u)]\{\int_0^{D-x-u} \\
& [w(x + u + y_2) - g_b(D - x - u - y_2)] f_2(y_2) dy_2 + \int_{D-x-u}^{v} [wD - h(x \\
& +u + y_2 - D)] f_2(y_2) dy_2\} + \int_0^{D-x-v} [\int_0^{v} [w(x + y_1 + y_2) - g_b(D - x \\
& -y_1 - y_2)] f_2(y_2)] f_1(y_1) dy_1 + \int_{D-x-v}^{u} [\int_0^{D-x-y_1} [w(x + y_1 + y_2) \\
& -g_b(D - x - y_1 - y_2)] f_2(y_2) dy_2 + \int_{D-x-y_1}^{v} [wD - h(x + y_1 \\
& +y_2)] f_2(y_2) dy_2] f_1(y_1) dy_1
\end{aligned} \tag{5.23}$$

The first derivatives of (5.23), with respect to the orders, u and v, are given by:

$$\frac{\partial \pi_B(x,u,v)}{\partial u} = [1-F_1(u)][(w+g_b+h)F_2(D-x-u)-s_1-h], \quad (5.24)$$

$$\frac{\partial \pi_B(x,u,v)}{\partial v} = [1-F_2(v)][(w+g_b+h)F_1(D-x-v)-s_2-h]. \quad (5.25)$$

Solving (5.24) and (5.25) simultaneously for u and v, we have $(u^*, v^*) = (D - x - V, D - x - U)$. It is optimal only when $x + u^* + v^* \geq D$, that is, $x \leq D - U - V$.

When $x > D - U - V$, the constraint $x + u + v \geq D$ is binding. That is, $x + u + v = D$ or $v = D - x - u$. Thus, we have the buyer's expected profit function as follows:

$$\begin{aligned} \pi_B(x,u,D-x-u) = & -s_1 u[1-F_1(u)] - s_1\int_0^u y_1 f_1(y_1)dy_1 - s_2(D-x-u)[1 \\ & -F_2(D-x-u)] - s_2\int_0^{D-x-u} y_2 f_2(y_2)dy_2 + [1-F_1(u)] \\ & \int_0^{D-x-u}[w(x+u+y_2) - g_b(D-x-u-y_2)]f_2(y_2)dy_2 \\ & +[1-F_2(D-x-u)]\int_0^u [w(D-u+y_1) - g_b(u \\ & -y_1)]f_1(y_1)dy_1 + \int_0^u\int_0^{D-x-u}[w(x+y_1+y_2) - g_b(D-x \\ & -y_1-y_2)]f_2(y_2)dy_2 f_1(y_1)dy_1 + [1-F_1(u)][1-F_2(D-x \\ & -u)]wD. \end{aligned} \quad (5.26)$$

The first-order and second-order conditions of (5.26), with respect to the order u are given by:

$$\frac{\partial \pi_B(x,u,v)}{\partial u} = (w+g_b-s_2)F_2(D-x-u) - (w+g_b-s_1)F_1(u) - s_1 + s_2, \quad (5.27)$$

$$\frac{\partial \pi^2(x,u,v)}{\partial u^2} = -(w+g_b-s_2)f_2(D-x-u) - (w+g_b-s_1)f_1(u) \leq 0.$$

Set (5.27) equal to 0 and denote the root as u^{**}. Since $s_1 \leq s_2$, $u^{**} > D - x$ if

$$x > D - F_1^{-1}\left(\frac{s_2 - s_1}{w + g_b - s_1}\right).$$

In this case, the optimal order quantity is $D - x$. In addition, it is easy to check that

1. If $x < D$, $x + u \geq D$, $x + v \leq D$, then the optimal order pair is $(D - x, 0)$;

2. If $x < D$, $x + u \leq D$, $x + v \geq D$, then the optimal order pair is $(0, D - x)$;

3. If $x < D$, $x + u + v \leq D$, then the optimal order pair always satisfies that $u^* + v^* = D$;

4. If $x \geq D$, then the optimal order pair is $(0, 0)$. ■

Proposition 16 shows that the buyer should adjust her ordering policy according to the initial stock level. When the initial stock level is lower than $D - U - V$, the buyer should place orders with both suppliers, and her total order-up-to level will be larger than the market demand. However, since both suppliers are unstable, the buyer's orders may not be fulfilled completely. When the initial stock level is larger than $D-U-V$ while less than $D-W$, the buyer's total order-up-to level should equal the market demand. If the initial stock level is larger than $D - W$ but smaller than D, the buyer should order from supplier 1 only since he provides cheaper products. If the initial stock level is larger than D, no order should be placed.

V.4. Summary

In this chapter, we study a simple setting that includes considerations of power structure, supply uncertainty, and competition between two suppliers. We show that the optimal quality level is higher when the supplier is the follower of the channel than when he is the leader. In this setting, we further examine the case in which a single buyer is facing one stable supplier and one unstable supplier. We also study the case in which a single buyer is facing two unstable suppliers. We provide optimal ordering policies for the buyer in both cases. Our contributions are highlighted below:

- The design of supply contracts under supply uncertainty and supplier competition.
- The design of supply contracts under power structure, supply uncertainty, and supplier competition considerations.
- Exploration of the impact of power structure.

CHAPTER VI

SUMMARY AND CONCLUSION

This dissertation investigates supply chain contract design with power structure, information asymmetry, supply uncertainty, and supplier competition considerations. Our goals are to develop a supply contract design framework that considers power structure and information asymmetry issues in supplier- and buyer-driven channels and to advance practical knowledge of the design of implementable contracts. In order to achieve our objectives, we investigate four sets of problems: buyer's impact, information asymmetry, supply uncertainty, and supplier competition.

In Chapter II, we explicitly examine the buyer's impact on designing price protection and returns policies contracts. We show that when the buyer is making pricing or sales effort decisions to influence market demand, price protection or returns policies alone are not able to coordinate the channel. Additional terms must be introduced into the contracts to achieve channel coordination. Threre, we provide optimal contracts with pricing, sales effort, and information asymmetry considerations. Our investigations are highlighted below:

- The design of optimal price protection contracts with retail-pricing decisions under deterministic and stochastic price-sensitive demand.
- The design of optimal price protection contracts with the consideration of the buyer's sales effort.
- The design of optimal price protection contracts and returns policies with the consideration of information asymmetry.

In Chapter III, we explore the impact of power structure, information asymmetry, and contract type on joint and individual profits in a two-stage supply chain with a single-product. Considering a buyer-driven channel, we derive optimal supply contracts with the considerations of information asymmetry. We show that the impact of information asymmetry on the buyer's profit margin is quite different under different types of contracts. Under a one-part linear contract, information asymmetry decreases the buyer's profit margin. Under a two-part linear contract, the impact of information asymmetry depends on the buyer's estimation accuracy. Under a two-part nonlinear contract, information asymmetry increases the buyer's profit margin. We also show that, from the system's perspective, the buyer-driven channel is more efficient than the supplier-driven channel under the corresponding one-part linear contract. We confirm the common wisdom that the bargaining power is beneficial for both the buyer and the supplier and that the value of possessing bargaining power is higher under more general contract types in full information cases. However, when information asymmetry exists in the supply chain, the bargaining power is not necessarily beneficial for either party and the common wisdom does not hold. Thus, it is not always wise for the buyer or the supplier to pursue the bargaining power when accurate information is not available. In addition, more general types of contracts may not improve the benefit of possessing power when information asymmetry exists. That is, the value of the bargaining power may be smaller under more general contract types. Hence, the party without accurate information may not benefit from having power even when a two-part nonlinear contract is allowed, which means that sometimes one can forfeit the bargaining power and still gain higher profit. Our investigations are highlighted below:

- The design of different types of optimal contracts for a buyer-driven channel with full information sharing.

- The design of different types of optimal contracts for a buyer-driven channel with incomplete information of cost structure.

- The analysis of the impact of power structure and information asymmetry on supply chain performance under different types of contracts.

In Chapter IV we seek to provide implementable contractual arrangements to coordinate the channel with the consideration of supply uncertainty. We first develop a general framework that incorporates power structure and supply uncertainty and show that both are important when designing supply contracts. Based on the general framework, we further develop supply contracts under conditions of supply uncertainty and deterministic demand with an infinite planning horizon. The findings reported clearly show that ignoring incentive conflicts and supply information issues can lead to undesirable behavior. We propose a model for how lot size, quality level, and transactions should be structured to help reduce supply chain inefficiency due to individual incentives and private information. In addition, we develop supply contracts under supply uncertainty and stochastic demand in a single period. We examine the impact of both supply and demand uncertainty on channel performance and propose a consignment contract to help coordinate the channel. The problems investigated in this chapter are summarized below:

- A general framework of supply chain contract design under supply uncertainty in supplier- and buyer-driven channels.
- An exploration of the impact of the power structure.
- The design of optimal cost-sharing contracts under supply uncertainty and continuous deterministic demand.
- The design of optimal cost-sharing contracts under supply uncertainty and stochastic single-period demand.

- An exploration of the value of supply uncertainty information.

Finally, in Chapter V, we first develop a general framework with power structure, supply uncertainty, and supplier competition considerations. We show that the optimal quality level is higher when the supplier is the follower of the channel than when he is the leader. Under this framework, we further examine the case in which a single buyer is facing one stable supplier and one unstable supplier. In addition, we study the case in which a single buyer is facing two unstable suppliers. We provide optimal ordering policies for the buyer in both cases. Our investigations are highlighted below:

- The design of supply contracts under supply uncertainty and supplier competition.
- The design of supply contracts under power structure, supply uncertainty, and supplier competition considerations.
- Exploration of the impact of the power structure.

This research has roots in applied probability, optimization, inventory theory, game theory, and economics. We develop optimization models aimed at minimizing entity/system costs or maximizing entity/system profits for the purpose of optimal contract design. Our focus is on probabilistic demand problems. From the methodology perspective, our optimization models are all stochastic modeling problems that require unconstrained/constrained dynamic or nonlinear optimization techniques, depending on the factors considered.

As previously indicated, current supply chain contract practice, as well as the supply chain literature, will be enriched by this research on power shifting, information asymmetry, supply uncertainty, and competition considerations. This dissertation is unique in that we explore the impact of both power shifting and information asymmetry while designing optimal supply chain contracts under supply uncertainty and competition. Hence, this research will advance practical knowledge about designing implementable contracts, and such knowledge is crucial for optimizing supply chain performance in the real world. This dissertation also sheds light on the significance of supply uncertainty information and the role of suppliers' and buyer's competition in the design of supply chain contracts. These are two important practical issues previously neglected in the literature.

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VITA

Xingchu Liu received his B.En. in automation from Tsinghua University, China in 1998, and his M.S. in control theory and control engineering from Tsinghua University in 2000. In September 2000, he enrolled in the doctoral program at Texas A&M University, Department of Industrial Engineering. His research interests are in supply chain management, inventory theory, stochastic modeling, and optimization. His permanent address is 157 Shizi St., Harbin, Heilongjiang, China.